STAT 641
Homework 4

Keegan Smith

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Problem 1

1. the trimmed mean is 913.5534 and the untrimmed mean is 955.3713. This would suggest that the data has some extreme values to the right, and is right skewed.

2. the survival function is:

$$S(t) = 1 - F(t)$$

We can derive $F(t)$ from the pdf, (and since t is a time any probability for $t < 0$ is 0, so we are only considering $t \geq 0$):

$$\begin{aligned} F(t) &= \int_0^t \lambda e^{-\lambda x} \\ &= (-e^{-\lambda x})_0^t \\ &= (-e^{-\lambda t} - (-1)) \\ &= 1 - e^{-\lambda t} \end{aligned}$$

So we have:

$$\begin{aligned} S(t) &= e^{-\lambda t} \\ \ln(S(t)) &= -\lambda t \end{aligned}$$

So if $S(t)$ is a good estimate, then the plot t vs $\ln(S(t))$ should be a linear plot with slope:

$$\frac{t}{-\lambda t} = \frac{1}{-\lambda}$$

3. We have the likelihood function:

$$
\begin{aligned}
L(\lambda; y) &= \Pi_{k=0}^{n-1} \lambda e^{-\lambda y_k} \\
&= \lambda^n e^{\sum_{k=0}^{n-1} -\lambda y_k} \\
&= \lambda^n e^{-\lambda \cdot \sum_{k=0}^{n-1} y_k}
\end{aligned}
$$

The log likelihood function is then:

$$
\begin{aligned}
\ln(L(\lambda; y)) &= \ln(\lambda^n e^{-\lambda \cdot \sum_{k=0}^{n-1} y_k}) \\
&= \ln(\lambda^n) + (-\lambda \cdot \sum_{k=0}^{n-1} y_k) \\
&= n \cdot \ln(\lambda) - \lambda \cdot \sum_{k=0}^{n-1} y_k
\end{aligned}
$$

The derivative of the log likelihood function w.r.t λ is:

$$
\begin{aligned}
\frac{d}{d\lambda}(n \cdot \ln(\lambda) - \lambda \cdot \sum_{k=0}^{n-1} y_k) &= n \cdot \frac{1}{\lambda} - \sum_{k=0}^{n-1} y_k \\
&= \frac{n}{\lambda} - \sum_{k=0}^{n-1} y_k
\end{aligned}
$$

Solving for the maximum:

$$
\begin{aligned}
\frac{n}{\lambda} - \sum_{k=0}^{n-1} y_k = 0 \\
\lambda &= \frac{n}{\sum_{k=0}^{n-1} y_k} \\
&= \frac{1}{\mu}
\end{aligned}
$$

Thus the MLE estimate for λ is $\frac{1}{955.3713} \approx 0.001047$