

STAT 641

Homework 4

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June 21, 2025

Problem 1

1. the likelihood function is:

$$L(\theta_1, \theta_2, \dots, \theta_k; y) = \prod_{i=1}^n f(y_i; \theta)$$

So plugging in the poisson distribution and our data we have:

$$\begin{aligned} L(\theta_1, \theta_2, \dots, \theta_k; y) &= \prod_{i=1}^n \frac{\lambda^{y_i} e^{-\lambda}}{y_i!} \\ &= \frac{\lambda^0 e^{-\lambda}}{0!} \cdot \frac{\lambda^1 e^{-\lambda}}{1!} \cdot \left(\frac{\lambda^2 e^{-\lambda}}{2!}\right)^2 \cdot \frac{\lambda^3 e^{-\lambda}}{3!} \cdot \frac{\lambda^4 e^{-\lambda}}{4!} \cdot \frac{\lambda^5 e^{-\lambda}}{5!} \\ &= e^{-\lambda} \cdot \lambda e^{-\lambda} \cdot \frac{\lambda^4 e^{-2\lambda}}{4} \cdot \frac{\lambda^3 e^{-\lambda}}{3!} \cdot \frac{\lambda^4 e^{-\lambda}}{4!} \cdot \frac{\lambda^5 e^{-\lambda}}{5!} \\ &= \frac{\lambda^{17} \cdot e^{-7\lambda}}{69120} \end{aligned}$$

2. The log likelihood is simply taking the natural log of the likelihood function, so we have:

$$\begin{aligned} \ln(L(\theta_1, \theta_2, \dots, \theta_k; y)) &= \ln\left(\frac{\lambda^{17} \cdot e^{-7\lambda}}{69120}\right) \\ &= \ln(\lambda^{17}) + \ln(e^{-7\lambda}) - \ln(69120) \\ &= \ln(\lambda^{17}) - 7\lambda - \ln(69120) \end{aligned}$$

3. The derivative of the log likelihood is as follows:

$$\begin{aligned} \frac{d}{d\lambda} (\ln(\lambda^{17}) - 7\lambda - \ln(69120)) &= 17 \cdot \lambda^{16} \cdot \frac{1}{\lambda^{17}} - 7 \\ &= 17 \cdot \frac{1}{\lambda} - 7 \end{aligned}$$

solving for the maximum we have:

$$\begin{aligned}17 \cdot \frac{1}{\lambda} - 7 &= 0 \\7 \cdot \lambda &= 17 \\ \lambda &= \frac{17}{7} \\ &\approx 2.4286\end{aligned}$$

Problem 2

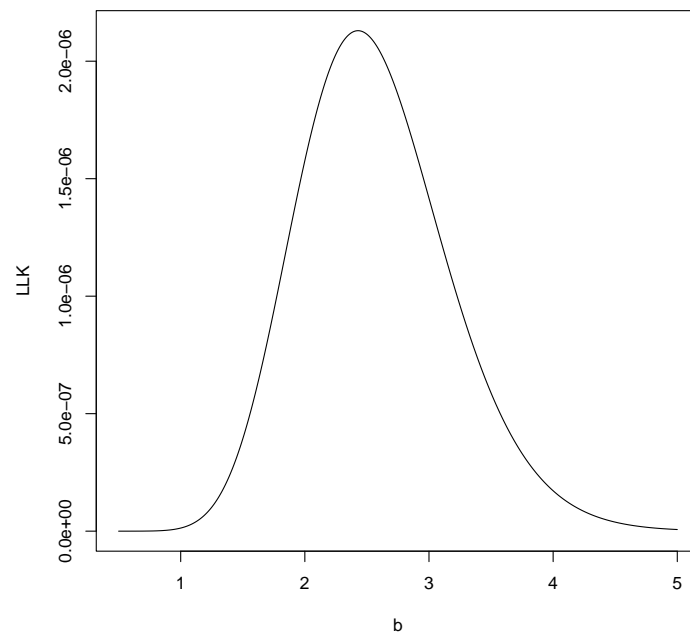
1. This is the R code for defining the log likelihood function:

```
b = seq(0, 10, .01)
LLK = b^(17) * exp(-7 * b) / 69120
```

2. This is the R code for plotting the log likelihood function:

```
b = seq(0.5, 5, .01)
LLK = b^(17) * exp(-7 * b) / 69120
plot(b, LLK, type="l")
```

This is the generated plot:



3. This is the R code for finding the max value:

```
b = seq(0.5, 5, .01)
LLK = b^(17) * exp(-7 * b) / 69120
plot(b, LLK, type="l")
LK_max = max(LLK)
bmax = which(LLK == LK_max)
print(LK_max)
print(bmax)
MLE = b[bmax]
print(MLE)
```

4. The numerical output was 2.43