STAT 641
Homework 4

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Question Group 1

1.1

1. The CDF of Weibull ($z \geq 0$):

$$F(z) = \int_0^z \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1} e^{-(\frac{x}{\lambda})^k} dx$$

using u substitution where:

$$u = (\frac{x}{\lambda})^k$$
$$du = \frac{k}{\lambda} \cdot (\frac{x}{\lambda})^{k-1} dx$$
$$dx = \frac{du}{\frac{k}{\lambda} \cdot (\frac{x}{\lambda})^{k-1}}$$

Thus we have:

$$\begin{aligned} F(z) &= \int_0^{(\frac{z}{\lambda})^k} \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1} e^{-u} \frac{du}{\frac{k}{\lambda} \cdot (\frac{x}{\lambda})^{k-1}} \\ &= \int_0^{(\frac{z}{\lambda})^k} e^{-u} du \\ &= (-e^{-u})_0^{(\frac{z}{\lambda})^k} \\ &= (-e^{-(\frac{z}{\lambda})^k} - (-1)) \\ &= (1 - e^{-(\frac{z}{\lambda})^k}) \end{aligned}$$

2. Quantile for p = .5:

$$
\begin{aligned}
p &= (1 - e^{-(\frac{z}{\lambda})^k}) \\
1 - p &= e^{-(\frac{z}{\lambda})^k} \\
-\ln(1-p) &= -(\frac{z}{\lambda})^k \\
(-\ln(1-p))^{\frac{1}{k}} &= (\frac{z}{\lambda}) \\
z &= (-\ln(1-p))^{\frac{1}{k}} \cdot \lambda \\
z &= (-\ln(1-.5))^{\frac{1}{3}} \cdot 2 \\
&\approx 1.77
\end{aligned}
$$

3. The survival function is:

$$
\begin{aligned}
S(t) &= 1 - F(t) \\
&= 1 - (1 - e^{-(\frac{z}{\lambda})^k}) \\
&= e^{-(\frac{z}{\lambda})^k} \\
&= e^{-(\frac{1}{2})^3} \\
&\approx 0.8825
\end{aligned}
$$

4. The hazard function is:

$$
\begin{aligned}
H(t) &= \frac{f(t)}{S(t)} \\
&= \frac{\frac{k}{\lambda}(\frac{t}{\lambda})^{k-1} e^{-(\frac{t}{\lambda})^k}}{e^{-(\frac{t}{\lambda})^k}} \\
&= \frac{\frac{3}{2}(\frac{1}{2})^{3-1} e^{-(\frac{1}{2})^3}}{e^{-(\frac{1}{2})^3}} \\
&\approx 0.375
\end{aligned}
$$

1.2

1. CDF of gompertz ($z \geq 0$):

$$
F(z) = \int_0^z \eta b e^{bx} e^{-\eta(e^{bx}-1)} dx
$$

Using u sub where:

$$
\begin{aligned}
u &= \eta(e^{bx} - 1) \\
du &= \eta b e^{bx} dx \\
dx &= \frac{du}{\eta b e^{bx}}
\end{aligned}
$$

We then have:

$$
\begin{aligned}
F(z) &= \int_0^{\eta(e^{bz}-1)} \eta b e^{bx} e^{-u} \frac{du}{\eta b e^{bx}} \\
&= \int_0^{\eta(e^{bz}-1)} e^{-u} du \\
&= (-e^{-u})_0^{\eta(e^{bz}-1)} \\
&= (-e^{-(\eta(e^{bz}-1))} - (-1)) \\
&= 1 - e^{-(\eta(e^{bz}-1))}
\end{aligned}
$$

2. The survival function is:

$$
\begin{aligned}
S(t) &= 1 - F(t) \\
&= 1 - (1 - e^{-(\eta(e^{bz}-1))}) \\
&= e^{-(\eta(e^{bz}-1))} \\
&= e^{-(2\cdot(e^{\frac{1}{2}}-1))} \\
&\approx 0.2732
\end{aligned}
$$

3. The hazard function is:

$$
\begin{aligned}
H(t) &= \frac{f(t)}{S(t)} \\
&= \frac{\eta b e^{bz} e^{-\eta(e^{bz}-1)}}{e^{-(\eta(e^{bz}-1))}} \\
&= \eta b e^{bz} \\
&= 2 \cdot \frac{1}{2} e^{\frac{1}{2}} \\
&\approx 1.6487
\end{aligned}
$$

Question Group 2

2.1

1. Iris Setosa: 4.5, 4.9, 5.3 for .1, .4, .8 quantiles respectively
   Iris Virginica: 5.8, 6.4, 7.2 for .1, .4, .8 quantiles respectively.
   This is the python code I wrote to solve this problem:

```
def iris_quantile(data, quantile):
    n = len(data)
    index = (n - 1) * quantile
    return data[int(index)]

if __name__ == "__main__":
    iris_setosa_data = [
        5.1, 4.9, 4.7, 4.6, 5.0, 5.4, 4.6, 5.0, 4.4, 4.9,
        5.4, 4.8, 4.8, 4.3, 5.8, 5.7, 5.4, 5.1, 5.7, 5.1,
        5.4, 5.1, 4.6, 5.1, 4.8, 5.0, 5.0, 5.2, 5.2, 4.7,
        4.8, 5.4, 5.2, 5.5, 4.9, 5.0, 5.5, 4.9, 4.4, 5.1,
        5.0, 4.5, 4.4, 5.0, 5.1, 4.8, 5.1, 4.6, 5.3, 5.0
    ]
    iris_setosa_data = sorted(iris_setosa_data)
    setosa_quantile = iris_quantile(iris_setosa_data, .1)
    print(setosa_quantile)
    setosa_quantile = iris_quantile(iris_setosa_data, .4)
    print(setosa_quantile)
    setosa_quantile = iris_quantile(iris_setosa_data, .8)
    print(setosa_quantile)

    virginica_data = [
        6.3, 5.8, 7.1, 6.3, 6.5, 7.6, 4.9, 7.3, 6.7, 7.2,
        6.5, 6.4, 6.8, 5.7, 5.8, 6.4, 6.5, 7.7, 7.7, 6.0,
        6.9, 5.6, 7.7, 6.3, 6.7, 7.2, 6.2, 6.1, 6.4, 7.2,
        7.4, 7.9, 6.4, 6.3, 6.1, 7.7, 6.3, 6.4, 6.0, 6.9,
        6.7, 6.9, 5.8, 6.8, 6.7, 6.7, 6.3, 6.5, 6.2, 5.9
    ]
    virginica_data = sorted(virginica_data)
    virginica_quantile = iris_quantile(virginica_data, .1)
    print(virginica_quantile)
    virginica_data = sorted(virginica_data)
    virginica_quantile = iris_quantile(virginica_data, .4)
    print(virginica_quantile)
    virginica_data = sorted(virginica_data)
    virginica_quantile = iris_quantile(virginica_data, .8)
    print(virginica_quantile)
```

3.1

1. f(5) = 0.0607, f(7) = 0.4006, wrote the following python code:

```
import math
def K(u):
    return 1 / math.sqrt(2 * math.pi) * math.e**(-u**2 / 2)

def kernel_estimator(n, h, kernel_function, data, y):
    result = 0
    coefficient = 1 / (n * h)
    for i in range(0, len(data)):
        input = (y - data[i]) / h
        result += kernel_function(input)
    return coefficient * result

if __name__ == "__main__":
    virginica_data = [
        6.3, 5.8, 7.1, 6.3, 6.5, 7.6, 4.9, 7.3, 6.7, 7.2,
        6.5, 6.4, 6.8, 5.7, 5.8, 6.4, 6.5, 7.7, 7.7, 6.0,
        6.9, 5.6, 7.7, 6.3, 6.7, 7.2, 6.2, 6.1, 6.4, 7.2,
        7.4, 7.9, 6.4, 6.3, 6.1, 7.7, 6.3, 6.4, 6.0, 6.9,
        6.7, 6.9, 5.8, 6.8, 6.7, 6.7, 6.3, 6.5, 6.2, 5.9
    ]
    f_5 = kernel_estimator(len(virginica_data), .5, K, virginica_data, 5)
    print("f_5: ", f_5)

    f_7 = kernel_estimator(len(virginica_data), .5, K, virginica_data, 7)
    print("f_7: ", f_7)
```

2. f(5) = 0.0416, f(7) = .25 I wrote the following code to do so:

```
def frequency_histogram(data, width, y):
    num_bins = int(data[-1] / width) + 1
    bin_freqs = [0] * num_bins
    for i in range(0, len(data)):
        bin_index = int(data[i] / width)
        bin_freqs[bin_index] += 1
    relative_freqs = [0] * num_bins
    for i in range(0, len(relative_freqs)):
        relative_freqs[i] = bin_freqs[i] / len(data)
    concentrations = [0] * num_bins
    for i in range(0, len(concentrations)):
        concentrations[i] = relative_freqs[i] / width
```

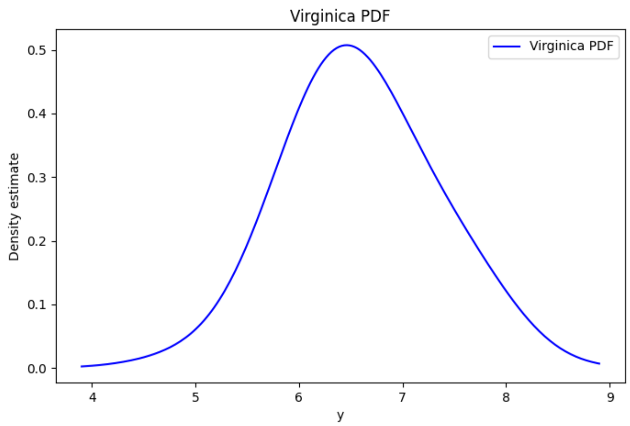
```
    bin_index = int(y / width)
    return concentrations[bin_index]

if __name__ == "__main__":
    virginica_data = [
        6.3, 5.8, 7.1, 6.3, 6.5, 7.6, 4.9, 7.3, 6.7, 7.2,
        6.5, 6.4, 6.8, 5.7, 5.8, 6.4, 6.5, 7.7, 7.7, 6.0,
        6.9, 5.6, 7.7, 6.3, 6.7, 7.2, 6.2, 6.1, 6.4, 7.2,
        7.4, 7.9, 6.4, 6.3, 6.1, 7.7, 6.3, 6.4, 6.0, 6.9,
        6.7, 6.9, 5.8, 6.8, 6.7, 6.7, 6.3, 6.5, 6.2, 5.9
    ]
    virginica_data = sorted(virginica_data)
    f_5 = frequency_histogram(virginica_data, .48, 5)
    print("f5: ", f_5)
    f_7 = frequency_histogram(virginica_data, .48, 7)
    print("f7: ", f_7)
```

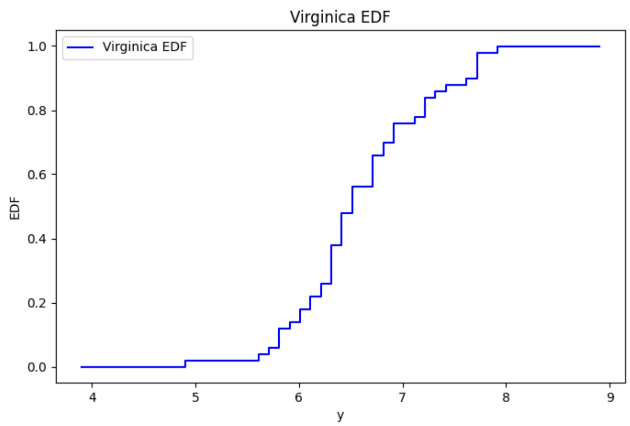
3. The point with the smallest contribution is 4.9 which had a weight of 5.894e-05

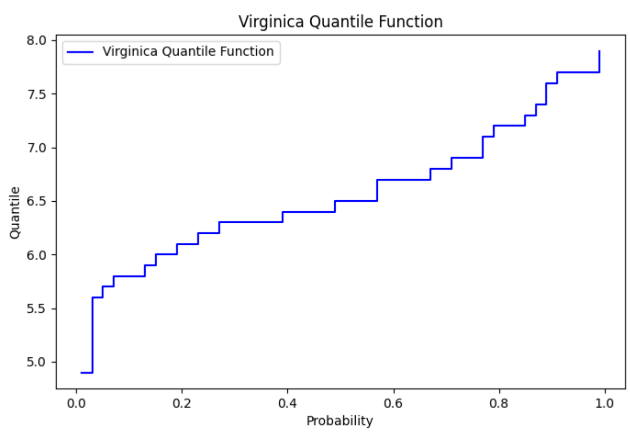
4. The point with the largest contribution is 7.1 which had a weight of 0.3910

4.1

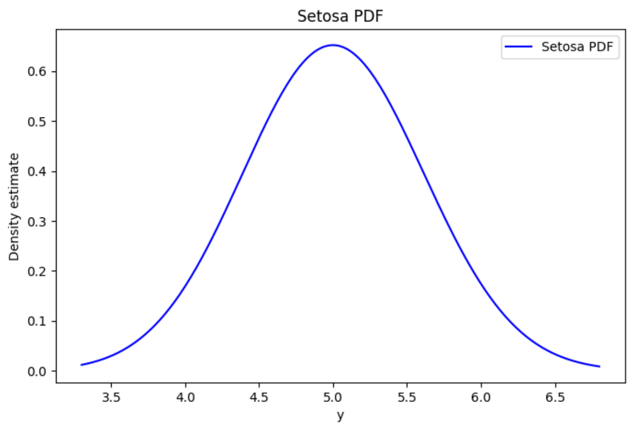
1. Virginica:

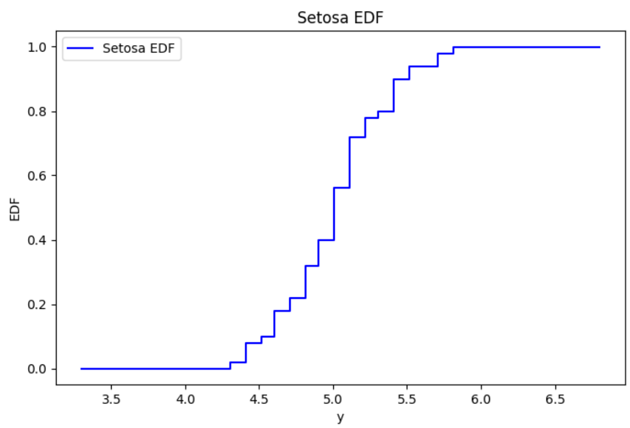


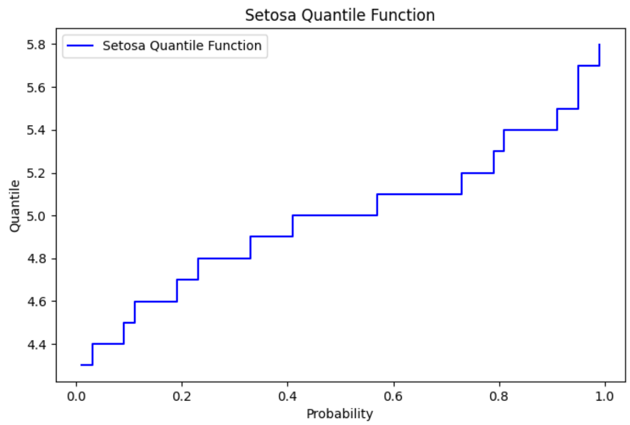

Virginica EDF
Virginica EDF
1.0
0.8
0.6
0.4
0.2
0.0
EDF
4
5
6
7
8
9
y


Virginica Quantile Function
Virginica Quantile Function
8.0
7.5
7.0
6.5
6.0
5.5
5.0
Quantile
0.0
0.2
0.4
0.6
0.8
1.0
Probability

Setosa:


Setosa PDF
Setosa PDF
Density estimate
y
0.0
0.1
0.2
0.3
0.4
0.5
0.6
3.5
4.0
4.5
5.0
5.5
6.0
6.5


Setosa EDF
Setosa EDF
EDF
y
0.0
0.2
0.4
0.6
0.8
1.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5



Generated with the following code:

```
import math
import numpy as np
import matplotlib.pyplot as plt
def K(u):
    return 1 / math.sqrt(2 * math.pi) * math.e**(-u**2 / 2)

def kernel_estimator(n, h, kernel_function, data, y):
    result = 0
    coefficient = 1 / (n * h)
    smallest_weight = 10000000
    smallest_weight_y = 0
    largest_weight = 0
    largest_weight_y = 0
    for i in range(0, len(data)):
        input = (y - data[i]) / h
        weight = kernel_function(input)
        if(weight < smallest_weight):
            smallest_weight = weight
            smallest_weight_y = data[i]
        if(weight > largest_weight):
            largest_weight = weight
            largest_weight_y = data[i]
        result += kernel_function(input)
    print("smallest weight: ", smallest_weight, " y: ", smallest_weight_y)
    print("largest weight: ", largest_weight, " y: ", largest_weight_y)
    return coefficient * result
```

```
def generate_graphs(data, prefix):
    data = sorted(data)
    n = len(data)
    h = 0.5

    y_values = np.linspace(min(data) - 1, max(data) + 1, 200)
    estimates = [kernel_estimator(n, h, K, data, y) for y in y_values]

    plt.figure(figsize=(8, 5))
    plt.plot(y_values, estimates, label=prefix + ' PDF', color='blue')
    plt.title(prefix + ' PDF')
    plt.xlabel('y')
    plt.ylabel('Density estimate')
    plt.legend()
    plt.show()

    edf_values = [np.sum(data <= y) / n for y in y_values]

    plt.figure(figsize=(8, 5))
    plt.step(y_values, edf_values, where='post', label=prefix + ' EDF', color='blue')
    plt.title(prefix + ' EDF')
    plt.xlabel('y')
    plt.ylabel('EDF')
    plt.legend()
    plt.show()

    n = len(data)

    probs = np.linspace(0, 1, n, endpoint=False) + 1/(2*n)

    plt.figure(figsize=(8, 5))
    plt.step(probs, data, where='post', color='blue', label=prefix + ' Quantile Functio
    plt.title(prefix + ' Quantile Function')
    plt.xlabel('Probability')
    plt.ylabel('Quantile')
    plt.legend()
    plt.show()

if __name__ == "__main__":
    virginica_data = [
        6.3, 5.8, 7.1, 6.3, 6.5, 7.6, 4.9, 7.3, 6.7, 7.2,
        6.5, 6.4, 6.8, 5.7, 5.8, 6.4, 6.5, 7.7, 7.7, 6.0,
        6.9, 5.6, 7.7, 6.3, 6.7, 7.2, 6.2, 6.1, 6.4, 7.2,
        7.4, 7.9, 6.4, 6.3, 6.1, 7.7, 6.3, 6.4, 6.0, 6.9,
        6.7, 6.9, 5.8, 6.8, 6.7, 6.7, 6.3, 6.5, 6.2, 5.9
    ]
```

```
iris_setosa_data = [
    5.1, 4.9, 4.7, 4.6, 5.0, 5.4, 4.6, 5.0, 4.4, 4.9,
    5.4, 4.8, 4.8, 4.3, 5.8, 5.7, 5.4, 5.1, 5.7, 5.1,
    5.4, 5.1, 4.6, 5.1, 4.8, 5.0, 5.0, 5.2, 5.2, 4.7,
    4.8, 5.4, 5.2, 5.5, 4.9, 5.0, 5.5, 4.9, 4.4, 5.1,
    5.0, 4.5, 4.4, 5.0, 5.1, 4.8, 5.1, 4.6, 5.3, 5.0
]
f_5 = kernel_estimator(len(virginica_data), .5, K, virginica_data, 5)
print("f_5: ", f_5)

f_7 = kernel_estimator(len(virginica_data), .5, K, virginica_data, 7)
print("f_7: ", f_7)
generate_graphs(virginica_data, prefix="Virginica")
generate_graphs(iris_setosa_data, "Setosa")
```

2. The distributions appear to be approximately normally distributed.

Question Group 3

1. B

2. B

3. B

4. C

5. C

6. C

7. B

8. B

9. C

10. B

11. C