# STAT 641
# Homework 4

Keegan Smith

June 16, 2025

## Question Group 1

### 1.1

1. The CDF of Weibull:

$$F(z) = \int_0^z \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1} e^{-(\frac{x}{\lambda})^k} dx$$

using u substitution where:

$$\begin{aligned} u &= (\frac{x}{\lambda})^k \\ du &= \frac{k}{\lambda} \cdot (\frac{x}{\lambda})^{k-1} dx \\ dx &= \frac{du}{\frac{k}{\lambda} \cdot (\frac{x}{\lambda})^{k-1}} \end{aligned}$$

Thus we have:

$$\begin{aligned} F(z) &= \int_0^{(\frac{z}{\lambda})^k} \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1} e^{-u} \frac{du}{\frac{k}{\lambda} \cdot (\frac{x}{\lambda})^{k-1}} \\ &= \int_0^{(\frac{z}{\lambda})^k} e^{-u} du \\ &= (-e^{-u})_0^{(\frac{z}{\lambda})^k} \\ &= (-e^{-(\frac{z}{\lambda})^k} - (-1)) \\ &= (1 - e^{-(\frac{z}{\lambda})^k}) \end{aligned}$$

2. Quantile for p = .5:

$$
\begin{aligned}
p &= (1 - e^{-(\frac{z}{\lambda})^k}) \\
1 - p &= e^{-(\frac{z}{\lambda})^k} \\
-\ln(1-p) &= -(\frac{z}{\lambda})^k \\
(-\ln(1-p))^{\frac{1}{k}} &= (\frac{z}{\lambda}) \\
z &= (-\ln(1-p))^{\frac{1}{k}} \cdot \lambda \\
z &= (-\ln(1-.5))^{\frac{1}{3}} \cdot 2 \\
&\approx 1.77
\end{aligned}
$$

3. The survival function is:

$$
\begin{aligned}
S(t) &= 1 - F(t) \\
&= 1 - (1 - e^{-(\frac{z}{\lambda})^k}) \\
&= e^{-(\frac{z}{\lambda})^k} \\
&= e^{-(\frac{1}{2})^3} \\
&\approx 0.8825
\end{aligned}
$$

4. The hazard function is:

$$
\begin{aligned}
H(t) &= \frac{f(t)}{S(t)} \\
&= \frac{\frac{k}{\lambda}(\frac{t}{\lambda})^{k-1} e^{-(\frac{t}{\lambda})^k}}{e^{-(\frac{t}{\lambda})^k}} \\
&= \frac{\frac{3}{2}(\frac{1}{2})^{3-1} e^{-(\frac{1}{2})^3}}{e^{-(\frac{1}{2})^3}} \\
&\approx 0.375
\end{aligned}
$$

# 1.2

1. CDF of gompertz:

$$
F(z) = \int_0^z \eta b e^{bx} e^{-\eta(e^{bx}-1)} dx
$$

Using u sub where:

$$
\begin{aligned}
u &= \eta(e^{bx} - 1) \\
du &= \eta b e^{bx} dx \\
dx &= \frac{du}{\eta b e^{bx}}
\end{aligned}
$$

We then have:

$$
\begin{aligned}
F(z) &= \int_0^{\eta(e^{bz}-1)} \eta b e^{bx} e^{-u} \frac{du}{\eta b e^{bx}} \\
&= \int_0^{\eta(e^{bz}-1)} e^{-u} du \\
&= (-e^{-u})_0^{\eta(e^{bz}-1)} \\
&= (-e^{-(\eta(e^{bz}-1))} - (-1)) \\
&= 1 - e^{-(\eta(e^{bz}-1))}
\end{aligned}
$$

2. The survival function is:

$$
\begin{aligned}
S(t) &= 1 - F(t) \\
&= 1 - (1 - e^{-(\eta(e^{bz}-1))}) \\
&= e^{-(\eta(e^{bz}-1))} \\
&= e^{-(2\cdot(e^{\frac{1}{2}}-1))} \\
&\approx 0.2732
\end{aligned}
$$

3. The hazard function is:

$$
\begin{aligned}
H(t) &= \frac{f(t)}{S(t)} \\
&= \frac{\eta b e^{bz} e^{-\eta(e^{bz}-1)}}{e^{-(\eta(e^{bz}-1))}} \\
&= \eta b e^{bz} \\
&= 2 \cdot \frac{1}{2} e^{\frac{1}{2}} \\
&\approx 1.6487
\end{aligned}
$$

# Question Group 2

## 2.1

1. Iris Setosa: 4.5, 4.9, 5.3 for .1, .4, .8 quantiles respectively
   Iris Virginica: 5.8, 6.4, 7.2 for .1, .4, .8 quantiles respectively.
   This is the python code I wrote to solve this problem:

```
def iris_quantile(data, quantile):
    n = len(data)
    index = (n - 1) * quantile
    return data[int(index)]

if __name__ == "__main__":
    iris_setosa_data = [
        5.1, 4.9, 4.7, 4.6, 5.0, 5.4, 4.6, 5.0, 4.4, 4.9,
        5.4, 4.8, 4.8, 4.3, 5.8, 5.7, 5.4, 5.1, 5.7, 5.1,
        5.4, 5.1, 4.6, 5.1, 4.8, 5.0, 5.0, 5.2, 5.2, 4.7,
        4.8, 5.4, 5.2, 5.5, 4.9, 5.0, 5.5, 4.9, 4.4, 5.1,
        5.0, 4.5, 4.4, 5.0, 5.1, 4.8, 5.1, 4.6, 5.3, 5.0
    ]
    iris_setosa_data = sorted(iris_setosa_data)
    setosa_quantile = iris_quantile(iris_setosa_data, .1)
    print(setosa_quantile)
    setosa_quantile = iris_quantile(iris_setosa_data, .4)
    print(setosa_quantile)
    setosa_quantile = iris_quantile(iris_setosa_data, .8)
    print(setosa_quantile)

    virginica_data = [
        6.3, 5.8, 7.1, 6.3, 6.5, 7.6, 4.9, 7.3, 6.7, 7.2,
        6.5, 6.4, 6.8, 5.7, 5.8, 6.4, 6.5, 7.7, 7.7, 6.0,
        6.9, 5.6, 7.7, 6.3, 6.7, 7.2, 6.2, 6.1, 6.4, 7.2,
        7.4, 7.9, 6.4, 6.3, 6.1, 7.7, 6.3, 6.4, 6.0, 6.9,
        6.7, 6.9, 5.8, 6.8, 6.7, 6.7, 6.3, 6.5, 6.2, 5.9
    ]
    virginica_data = sorted(virginica_data)
    virginica_quantile = iris_quantile(virginica_data, .1)
    print(virginica_quantile)
    virginica_data = sorted(virginica_data)
    virginica_quantile = iris_quantile(virginica_data, .4)
    print(virginica_quantile)
    virginica_data = sorted(virginica_data)
    virginica_quantile = iris_quantile(virginica_data, .8)
    print(virginica_quantile)
```

# 2.2

1. f(5) = 0.0607, f(7) = 0.4006, wrote the following python code:

```
import math
def K(u):
    return 1 / math.sqrt(2 * math.pi) * math.e**(-u**2 / 2)

def kernel_estimator(n, h, kernel_function, data, y):
    result = 0
    coefficient = 1 / (n * h)
    for i in range(0, len(data)):
        input = (y - data[i]) / h
        result += kernel_function(input)
    return coefficient * result

if __name__ == "__main__":
    virginica_data = [
        6.3, 5.8, 7.1, 6.3, 6.5, 7.6, 4.9, 7.3, 6.7, 7.2,
        6.5, 6.4, 6.8, 5.7, 5.8, 6.4, 6.5, 7.7, 7.7, 6.0,
        6.9, 5.6, 7.7, 6.3, 6.7, 7.2, 6.2, 6.1, 6.4, 7.2,
        7.4, 7.9, 6.4, 6.3, 6.1, 7.7, 6.3, 6.4, 6.0, 6.9,
        6.7, 6.9, 5.8, 6.8, 6.7, 6.7, 6.3, 6.5, 6.2, 5.9
    ]
    f_5 = kernel_estimator(len(virginica_data), .5, K, virginica_data, 5)
    print("f_5: ", f_5)

    f_7 = kernel_estimator(len(virginica_data), .5, K, virginica_data, 7)
    print("f_7: ", f_7)
```