STAT 641
Homework 4

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June 16, 2025

Question Group 1

1.1

1. The CDF of Weibull:

$$F(z) = \int_0^z \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}e^{-(\frac{x}{\lambda})^k}dx$$

using u substitution where:

$$\begin{aligned} u &= (\frac{x}{\lambda})^k \\ du &= \frac{k}{\lambda}\cdot(\frac{x}{\lambda})^{k-1}dx \\ dx &= \frac{du}{\frac{k}{\lambda}\cdot(\frac{x}{\lambda})^{k-1}} \end{aligned}$$

Thus we have:

$$\begin{aligned} F(z) &= \int_0^{(\frac{z}{\lambda})^k} \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1}e^{-u}\frac{du}{\frac{k}{\lambda}\cdot(\frac{x}{\lambda})^{k-1}} \\ &= \int_0^{(\frac{z}{\lambda})^k} e^{-u}du \\ &= (-e^{-u})_0^{(\frac{z}{\lambda})^k} \\ &= (-e^{-(\frac{z}{\lambda})^k} - (-1)) \\ &= (1 - e^{-(\frac{z}{\lambda})^k}) \end{aligned}$$

2. Quantile for p = .5:

$$
\begin{aligned}
p &= (1 - e^{-(\frac{z}{\lambda})^k}) \\
1 - p &= e^{-(\frac{z}{\lambda})^k} \\
-\ln(1-p) &= -(\frac{z}{\lambda})^k \\
(-\ln(1-p))^{\frac{1}{k}} &= (\frac{z}{\lambda}) \\
z &= (-\ln(1-p))^{\frac{1}{k}} \cdot \lambda \\
z &= (-\ln(1-.5))^{\frac{1}{3}} \cdot 2 \\
&\approx 1.77
\end{aligned}
$$

3. The survival function is:

$$
\begin{aligned}
S(t) &= 1 - F(t) \\
&= 1 - (1 - e^{-(\frac{z}{\lambda})^k}) \\
&= e^{-(\frac{z}{\lambda})^k} \\
&= e^{-(\frac{1}{2})^3} \\
&\approx 0.8825
\end{aligned}
$$

4. The hazard function is:

$$
\begin{aligned}
H(t) &= \frac{f(t)}{S(t)} \\
&= \frac{\frac{k}{\lambda}(\frac{t}{\lambda})^{k-1}e^{-(\frac{t}{\lambda})^k}}{e^{-(\frac{t}{\lambda})^k}} \\
&= \frac{\frac{3}{2}(\frac{1}{2})^{3-1}e^{-(\frac{1}{2})^3}}{e^{-(\frac{1}{2})^3}} \\
&\approx 0.375
\end{aligned}
$$