# STAT 641
# Homework 6

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## Problem 1

1. To create a reference plot, we must first make it such that the Weibull is a location-scale distribution. We can accomplish this (as outlined in Handout 8) by applying the transformation:

   $$X = \log(Y)$$

   so if $X$ follows a log-Weibull distribution, then $Y$ follows a Weibull distribution.
   The cdf of the log weibull is:

   $$\begin{aligned} P(\ln(Y) \leq x) &= P(Y \leq e^x) \\ &= 1 - e^{-(\frac{e^x}{\alpha})^\gamma} \\ &= 1 - e^{-e^{\frac{x-\theta_1}{\theta_2}}} \end{aligned}$$

   We can then solve for the quantile function:

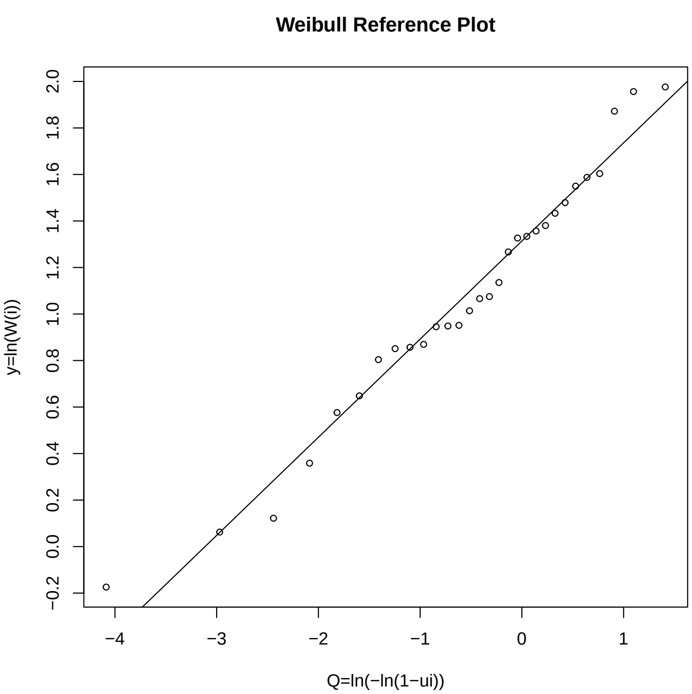
   $$\begin{aligned} s &= 1 - e^{-e^{\frac{x-\theta_1}{\theta_2}}} \\ \ln(-s+1) &= -e^{\frac{x-\theta_1}{\theta_2}} \\ \ln(-\ln(-s+1)) &= \frac{x-\theta_1}{\theta_2} \\ \theta_2 \cdot \ln(-\ln(-s+1)) + \theta_1 &= x \end{aligned}$$

   Thus we have:

   $$Q_z(u) = \ln(-\ln(-u+1))$$

   We can then make the reference plot with the following R code (continued from the previous R code):

```
\
x <- c(0.8402, 1.0644, 1.1298, 1.4314, 1.7795, 1.9121, 2.2343, 2.3424, 2.3559, 2.3855,
2.5734, 2.5815, 2.5893, 2.7562, 2.9040, 2.9295, 3.1124, 3.5490, 3.7684, 3.7953,
3.8846, 3.9766, 4.1918, 4.3887, 4.7106, 4.8918, 4.9716, 6.5018, 7.0740, 7.2158)
y = -log(x)
y = sort(y)
n = length(y)
weib= -y
weib= sort(weib)
i= 1:n
ui= (i-.5)/n
QW= log(-log(1-ui))
plot(QW,weib,abline(lm(weib~QW)),
main="Weibull Reference Plot",cex=.75,lab=c(7,11,7),
xlab="Q=ln(-ln(1-ui))",
ylab="y=ln(W(i))")
```



We will also compute the Anderson-Darling GOF test in R:

```
library(MASS)
mle <- fitdistr(x,"weibull")
shape = mle$estimate[1]
scale = mle$estimate[2]
a = -log(scale)
b = 1/shape
z = exp(-exp(-(y-a)/b))
A1i = (2*i-1)*log(z)
A2i = (2*n+1-2*i)*log(1-z)
s1 = sum(A1i)
s2 = sum(A2i)
AD = -n-(1/n)*(s1+s2)
ADM = AD*(1+.2/sqrt(n))
AD
ADM
```

This results in $A^2 = 0.3059062$. From the table in Handout 9, this corresponds with a p-value ¿ .25. Combining this with the fact that the reference plot is fairly straight, I am inclined to say the Weibull distribution is a good fit for the data.