STAT 641
Homework 4

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Question Group 1

1.1

1. The CDF of Weibull:

$$F(z) = \int_0^z \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1} e^{-(\frac{x}{\lambda})^k} dx$$

using u substitution where:

$$\begin{aligned} u &= (\frac{x}{\lambda})^k \\ du &= \frac{k}{\lambda} \cdot (\frac{x}{\lambda})^{k-1} dx \\ dx &= \frac{du}{\frac{k}{\lambda} \cdot (\frac{x}{\lambda})^{k-1}} \end{aligned}$$

Thus we have:

$$\begin{aligned} F(z) &= \int_0^{(\frac{z}{\lambda})^k} \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1} e^{-u} \frac{du}{\frac{k}{\lambda} \cdot (\frac{x}{\lambda})^{k-1}} \\ &= \int_0^{(\frac{z}{\lambda})^k} e^{-u} du \\ &= (-e^{-u})_0^{(\frac{z}{\lambda})^k} \\ &= (-e^{-(\frac{z}{\lambda})^k} - (-1)) \\ &= (1 - e^{-(\frac{z}{\lambda})^k}) \end{aligned}$$

2. Quantile for p = .5:

$$\begin{aligned} p &= (1 - e^{-(\frac{z}{\lambda})^k}) \\ 1 - p &= e^{-(\frac{z}{\lambda})^k} \\ -\ln(1-p) &= -(\frac{z}{\lambda})^k \\ (-\ln(1-p))^{\frac{1}{k}} &= (\frac{z}{\lambda}) \\ z &= (-\ln(1-p))^{\frac{1}{k}} \cdot \lambda \\ z &= (-\ln(1-.5))^{\frac{1}{3}} \cdot 2 \\ &\approx 1.77 \end{aligned}$$

3. The survival function is:

$$\begin{aligned} S(t) &= 1 - F(t) \\ &= 1 - (1 - e^{-(\frac{z}{\lambda})^k}) \\ &= e^{-(\frac{z}{\lambda})^k} \\ &= e^{-(\frac{1}{2})^3} \\ &\approx 0.8825 \end{aligned}$$

4. The hazard function is:

$$\begin{aligned} H(t) &= \frac{f(t)}{S(t)} \\ &= \frac{\frac{k}{\lambda}(\frac{t}{\lambda})^{k-1} e^{-(\frac{t}{\lambda})^k}}{e^{-(\frac{t}{\lambda})^k}} \\ &= \frac{\frac{3}{2}(\frac{1}{2})^{3-1} e^{-(\frac{1}{2})^3}}{e^{-(\frac{1}{2})^3}} \\ &\approx 0.375 \end{aligned}$$

1.2

1. CDF of gompertz:

$$F(z) = \int_0^z \eta b e^{bx} e^{-\eta(e^{bx}-1)} dx$$

Using u sub where:

$$
\begin{aligned}
u &= \eta(e^{bx} - 1) \\
du &= \eta b e^{bx} dx \\
dx &= \frac{du}{\eta b e^{bx}}
\end{aligned}
$$

We then have:

$$
\begin{aligned}
F(z) &= \int_0^{\eta(e^{bz}-1)} \eta b e^{bx} e^{-u} \frac{du}{\eta b e^{bx}} \\
&= \int_0^{\eta(e^{bz}-1)} e^{-u} du \\
&= (-e^{-u})_0^{\eta(e^{bz}-1)} \\
&= (-e^{-(\eta(e^{bz}-1))} - (-1)) \\
&= 1 - e^{-(\eta(e^{bz}-1))}
\end{aligned}
$$