STAT 641
Homework 1

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Question Group 1

1.1

1. The CDF of Weibull:

$$F(z) = \int_0^z \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1} e^{-(\frac{x}{\lambda})^k} dx$$

using u substitution where:

$$\begin{aligned} u &= (\frac{x}{\lambda})^k \\ du &= k \cdot (\frac{x}{\lambda})^{k-1} dx \\ dx &= \frac{du}{k \cdot (\frac{x}{\lambda})^{k-1}} \end{aligned}$$

Thus we have:

$$\begin{aligned}
F(z) &= \int_0^{(\frac{z}{\lambda})^k} \frac{k}{\lambda}(\frac{x}{\lambda})^{k-1} e^{-u} \frac{du}{k \cdot (\frac{x}{\lambda})^{k-1}} \\
&= \frac{k}{\lambda} \cdot \int_0^{(\frac{z}{\lambda})^k} e^{-u} du \\
&= \frac{k}{\lambda} \cdot (-e^{-u})_0^{(\frac{z}{\lambda})^k} \\
&= \frac{k}{\lambda} \cdot (-e^{-(\frac{z}{\lambda})^k} - (-1)) \\
&= \frac{k}{\lambda} \cdot (1 - e^{-(\frac{z}{\lambda})^k})
\end{aligned}$$