STAT 641
Homework 7

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Problem 1

1. The rejection region is defined as:

$$R = X : T(X) > C$$

where $T(X)$ is the distribution of the test statistic X.
Since Y is normally distributed we can use the test statistic:

$$T = \frac{\sqrt{n}(\bar{Y} - \mu_o)}{\sigma}$$

Substituting our values we get:

$$T = \frac{\sqrt{36}(\hat{\mu} - 10)}{\hat{\sigma}}$$

And we have the critical value (obtained from the table):

$$t_{.025,35} = 2.0315$$

Thus our rejection region is:

$$R = (\hat{\mu}, \hat{\sigma}) : \frac{\sqrt{36}(\hat{\mu} - 10)}{\hat{\sigma}} > 2.0315$$

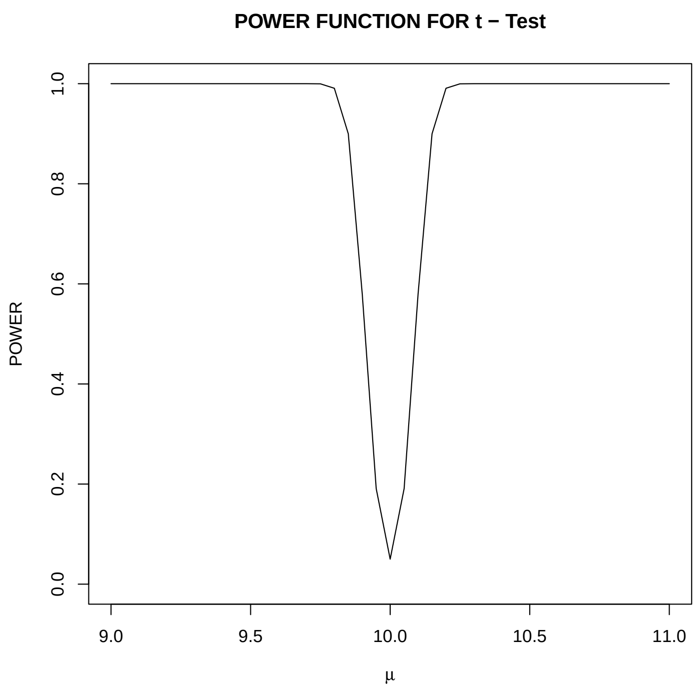
2. The power function is:

$$\gamma(\mu) = G(-t_{.025}) + 1 - G(t_{t_{.025}})$$

Where G is the cdf of the students T distribution centered at:

$$T = \frac{\sqrt{36}(\mu - 10)}{.27}$$

with $df = 35$.
Graphing this function in R:


POWER FUNCTION FOR t − Test
POWER
0.0
0.2
0.4
0.6
0.8
1.0
9.0
9.5
10.0
10.5
11.0
μ

The R code is:

```
n = 36
df = n-1
muo = 10
sigma = .27
mu = seq(9,11,.05)
delta = sqrt(n)*(mu-muo)/sigma
power = 1-pt(qt(.975,df),df,delta) + pt(qt(.025, df), df, delta)
#par(lab=c(15,20,4))
plot(mu,power,type="l",ylim=c(0,1),xlab=expression(mu),
ylab="POWER")
title("POWER FUNCTION FOR t - Test")
out = cbind(mu,delta,power)
```

The sample mean is 10, so our T statistic is 0, thus G is centered at 0, so:

$$\begin{aligned}\gamma(\mu) &= G(-t_{.025}) + 1 - G(t_{.025}) \\ &= .025 + 1 - .975 \\ &= .05\end{aligned}$$

3. To determine the sample size, we have $\alpha = .05$, $\beta = .2$, and $\phi = \frac{.2}{.27} = 0.7407$. From the table $A11$, this gives us a sample size of 13.

Problem 2

1. The null hypothesis is $\mu \geq 5$, with the alternative hypothesis being $\mu < 5$. σ is unkown, so we will be using t testing.
   We know that: $n = 20$, $\hat{\mu} = 4.2$, $S = 1.2$, and $\alpha = .05$, so we have the T statistic:

   $$T = \frac{\sqrt{20}(4.2 - 5)}{1.2} = -2.98142396999972$$

   the t critical value is:

   $$t_{.05} = -1.729$$

   (from the table).
   Since $T < t_{.05}$, we reject the null hypothesis. We also have the p-value for T which is approximately $.005 < .05$ from the table.

2. The probability that the test will be able to detect that the real mean is 4.1 or less can be given by the power function [ ]

$$\gamma(\mu) = G(-t_{.05})$$

where G is the cumulative t distribution centered at $\delta = \frac{\sqrt{20}(4.1-5)}{1.2}$ with $df = 19$. This can be solved with R:

```
result = pt(-qt(.05, 19), 19, 0.3727)
```

which yields a power of 0.9006489

3. For sample size estimation we know: $\alpha = .05$, $\beta = .1$, and $\phi = \frac{|4.4-5|}{1.2} = .5$. From the table this gives us 44

Problem 3

1. We will assume the population distribution is normal, since this is a manufacturing process.
The null hypothesis is that $\sigma \geq 1$, the alternative hypothesis is that $\sigma < 1$. We can calculate the test statistic:

$$\begin{aligned} TS &= \frac{(n-1) * S^2}{\sigma_o} \\ &= \frac{19 * .8458^2}{1} \\ &= 13.5922 \end{aligned}$$

we can determine the critical value with:

$$\begin{aligned} t_{.05} &= \chi^2_{19,.95} \\ &= 10.117 \end{aligned}$$

The test statistic is not less than the critical value, thus we fail to reject the null hypothesis.

2. We have the power function:

$$\begin{aligned} power &= G(\frac{\sigma_o^2}{\sigma_1^2}\chi^2_{n-1,1-\alpha}) \\ &= G(10.117) \\ &= .05 \end{aligned}$$

so the type II error rate for $\sigma_1 = 1$ is $1 - .05 = .95$. For $\sigma_1 = .9$:

$$\begin{aligned} power &= G(\frac{\sigma_o^2}{\sigma_1^2}\chi^2_{n-1,1-\alpha}) \\ &= G(\frac{1^2}{.9^2} \cdot 10.117) \\ &= G(12.4901) \\ &= 0.1363781 \end{aligned}$$

so the type II error is 0.8636219. For $\sigma_1 = .8$:

$$\begin{aligned} power &= G(\frac{\sigma_o^2}{\sigma_1^2}\chi^2_{n-1,1-\alpha}) \\ &= G(\frac{1^2}{.8^2} \cdot 10.117) \\ &= G(15.81) \\ &= 0.32994 \end{aligned}$$

so the type II error is 0.67006. For $\sigma_1 = .7$:

$$\begin{aligned} power &= G(\frac{\sigma_o^2}{\sigma_1^2}\chi^2_{n-1,1-\alpha}) \\ &= G(\frac{1^2}{.7^2} \cdot 10.117) \\ &= G(20.6469) \\ &= 0.6433 \end{aligned}$$

so the type II error is .3567. For $\sigma_1 = .6$:

$$\begin{aligned} power &= G(\frac{\sigma_o^2}{\sigma_1^2}\chi^2_{n-1,1-\alpha}) \\ &= G(\frac{1^2}{.6^2} \cdot 10.117) \\ &= G(28.1027) \\ &= 0.918528 \end{aligned}$$

so the type II error is .08147.

3. The bound for the upper 90% CI is given by:

$$
\begin{aligned}
CI &= \frac{\sqrt{n-1} \cdot S}{\sqrt{X^2_{1-\alpha,n-1}}} \\
&= \frac{\sqrt{19} \cdot .8458}{\sqrt{X^2_{.9,19}}} \\
&= \frac{\sqrt{19} \cdot .8458}{\sqrt{11.651}} \\
&= 1.0801
\end{aligned}
$$

This aligns with my results in part 1 since 1 is included the in the CI.

Problem 4

1. $S_+ = 5$, so we have:

   $$p = G(5, 20, .5) = .021$$

   where G is the cdf of the binomial distribution with $p = .5$, $n = 20$, and $x = 5$.
   .021 ¡ .05, so we reject the null hypothesis.

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