

MATH 417 502

Homework 3

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Problem 1

Our system of equations can be re-written as:

$$\begin{aligned}x_1 + 2x_2 + 3x_3 - \lambda x_1 &= 0 \\4x_1 + 5x_2 + 6x_3 - \lambda x_2 &= 0 \\7x_1 + 8x_2 + 10x_3 - \lambda x_3 &= 0 \\x_1^2 + x_2^2 + x_3^2 - 1 &= 0\end{aligned}$$

The jacobian of this system is:

$$\begin{bmatrix}1 - \lambda & 2 & 3 & -\lambda x_1 \\4 & 5 - \lambda & 6 & -x_2 \\7 & 8 & 10 - \lambda & -x_3 \\2x_1 & 2x_2 & 2x_3 & 0\end{bmatrix}$$

Thus the Newton iteration looks like:

$$x^{n+1} = x^n - \begin{bmatrix}1 - \lambda & 2 & 3 & -\lambda x_1^n \\4 & 5 - \lambda & 6 & -x_2^n \\7 & 8 & 10 - \lambda & -x_3^n \\2x_1^n & 2x_2^n & 2x_3^n & 0\end{bmatrix}^{-1} \begin{bmatrix}x_1^n + 2x_2^n + 3x_3^n - \lambda x_1^n \\4x_1^n + 5x_2^n + 6x_3^n - \lambda x_2^n \\7x_1^n + 8x_2^n + 10x_3^n - \lambda x_3^n \\(x_1^n)^2 + (x_2^n)^2 + (x_3^n)^2 - 1\end{bmatrix}$$