MATH 417 502
Homework 1

Keegan Smith

August 26, 2024

Problem 1

I implemented Heron's algorithm like so:

```python
import sys
sys.setrecursionlimit(20000)
MAX_DEPTH = 1000;
def heron_rec(num, approx, depth):
    if(depth >= MAX_DEPTH):
        return approx;
    next_approx = .5 * approx + .5 * (num / approx)
    return heron_rec(num, next_approx, depth  +1)

if __name__ == "__main__":
    num = int(input("Enter a number: "))
    result = heron_rec(num, num / 2, 1)
    print(f"sqrt of {num} is approx. {result}")
```

sqrt of 2 is approx. 1.414213562373095
sqrt of 10 is approx. 3.162277660168379
sqrt of 1000 is approx. 31.622776601683793
I implemented Heron's algorithm by recognizing the recursive nature of the formula. Because of this, I thought a recursive approach for the implementation would be most suitable, however an iterative solution is also very do-able (and also an iterative solution would have been much more efficient). All I needed were variables to keep track of the current approximation, the current depth (number of iterations), and the original number. The results of this implementation were quite good. Even with a depth of 1000, the algorithm is accurate up to around 17 significant figures. It is important to note that with larger numbers ($> 10^7$) this implementation starts to deteriorate; for instance, the output of 10^7 is 3162.277660168379, which is about $4.547473508864641e - 13$ off.

Problem 2

a.)

$$f(x) = x^3 - 2$$
$$f'(x) = 3x^2$$

Newton's method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$
$$x_{n+1} = x_n - \frac{x_n^3 - 2}{3x_n^2} \quad \text{Plugging in our values for } f(x), f'(x)$$

Thus we have:

$$g_3(x) = x - \frac{x^3 - 2}{3x^2}$$

b.) joe

Table 1: $g_1(x)$

Iteration	Approximation	Residual	Error
0	1.500000e+00	1.375000e+00	2.400790e-01
1	1.041667e+00	-8.697193e-01	2.182544e-01
2	1.331573e+00	3.609949e-01	7.165206e-02
3	1.211241e+00	-2.229805e-01	4.867957e-02
4	1.285568e+00	1.246405e-01	2.564725e-02
5	1.244021e+00	-7.476562e-02	1.589960e-02
6	1.268943e+00	4.327432e-02	9.022275e-03
7	1.254519e+00	-2.561763e-02	5.402498e-03
8	1.263058e+00	1.497488e-02	3.136712e-03
9	1.258066e+00	-8.820482e-03	1.854915e-03
10	1.261006e+00	5.172615e-03	1.085246e-03
11	1.259282e+00	-3.041309e-03	6.389589e-04
12	1.260296e+00	1.785456e-03	3.748109e-04
13	1.259701e+00	-1.049127e-03	2.203413e-04
14	1.260050e+00	6.161378e-04	1.293675e-04
15	1.259845e+00	-3.619614e-04	7.601172e-05

Table 2: $g_2(x)$

Iteration	Approximation	Residual	Error
0	1.500000e+00	1.375000e+00	2.400790e-01
1	8.888889e-01	-1.297668e+00	3.710322e-01
2	2.531250e+00	1.421829e+01	1.271329e+00
3	3.121475e-01	-1.969586e+00	9.477735e-01
4	2.052628e+01	8.646295e+03	1.926636e+01
5	4.746895e-03	-2.000000e+00	1.255174e+00
6	8.875865e+04	6.992493e+14	8.875739e+04
7	2.538684e-10	-2.000000e+00	1.259921e+00
8	3.103221e+19	2.988395e+58	3.103221e+19
9	2.076848e-39	-2.000000e+00	1.259921e+00
10	4.636825e+77	9.969245e+232	4.636825e+77
11	9.302260e-156	-2.000000e+00	1.259921e+00
12	inf	inf	inf
13	0.000000e+00	-2.000000e+00	1.259921e+00
14	inf	inf	inf
15	0.000000e+00	-2.000000e+00	1.259921e+00

Table 3: $g_3(x)$

Iteration	Approximation	Residual	Error
0	1.500000e+00	1.375000e+00	2.400790e-01
1	1.296296e+00	1.782757e-01	3.637525e-02
2	1.260932e+00	4.819286e-03	1.011175e-03
3	1.259922e+00	3.860583e-06	8.106711e-07
4	1.259921e+00	2.483791e-12	5.215828e-13
5	1.259921e+00	0.000000e+00	0.000000e+00
6	1.259921e+00	0.000000e+00	0.000000e+00
7	1.259921e+00	0.000000e+00	0.000000e+00
8	1.259921e+00	0.000000e+00	0.000000e+00
9	1.259921e+00	0.000000e+00	0.000000e+00
10	1.259921e+00	0.000000e+00	0.000000e+00
11	1.259921e+00	0.000000e+00	0.000000e+00
12	1.259921e+00	0.000000e+00	0.000000e+00
13	1.259921e+00	0.000000e+00	0.000000e+00
14	1.259921e+00	0.000000e+00	0.000000e+00
15	1.259921e+00	0.000000e+00	0.000000e+00