SUSY-Components

v1
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Abstract

This is the documentation associated with the Mathematica notebooks SUSY-components for doing componentwise supersymmetry calculations.

Contents

1 Introduction

The superspace formalism has been used since the 1970s to aid in complicated supersymmetry calculations. Superfields (fields defined on superspace) package bosons and fermions related by supersymmetry into a single object, from which supersymmetric actions can be easily written. Not all supersymmetric actions can be expressed in terms of superfields, but the commonly studied examples such as $(3+1)D$ Minkowski superspace with $\mathcal{N}=1$ SUSY, and $(2+1)D$ with $\mathcal{N}=2$ do have superspace realizations.

This set of Mathematica notebooks was created to do superspace calculations in so-called Heisenberg superpsace, which does not allow for Minkowski indices. While there are many other supersymmetry packages for Mathematica availible, to get the flexibility needed for non-Minkowski superspace one needs to be able to implement index notation by hand, or see all components written out completely. In addition to a Heisenberg superspace notebook, there are also notebooks for doing component-wise computations in standard Minkowski $(3+1)D$, $\mathcal{N}=1$, and $(2+1)D$, $\mathcal{N}=2$ superspace.

An immediate difference between this package and similar ones is that this comes in the form of **notebooks**, as opposed to Wolfram Language (.wl) files. This is done for immediate ease-of-use, since there is a standard notation for anticommuting coordinates, a convention set by [1]. Since global variables can cause problems when importing through a Mathematica package, it's much simpler to predefine all objects with their standard conventions in a single notebook. Calculations can be done at the bottom of the notebooks, or using another notebook with the same kernel.

Section 2 gives the list of the defined functions and their definitions. Section 3 gives some examples and use-cases.

1.1 Anticommuting Objects

A distinguishing feature of superspace is the presence of anticommuting, or Grassmann-valued coordinates. These anticommuting dimensions are augmented on the standard spacetime coordinate system, now called the "bosonic" directions.

This set of notebooks constructs anticommuting objects using a modification of NonCommutativeMultiply (**). This type of multiplication reduces to standard (commuting) multiplication when one or both of the objects are commutative, and automatically recognizes anticommuting objects and their multiplication rules. An object that is not defined as commuting or anticommuting will not be simplified.

The rest of this paper describes how to use the notebooks and some examples of their usefulness. While they will solve most problems one might come across in an introductory SUSY textbook, the real value comes from using the anticommuting structure as a starting point for doing more complicated things.

1.2 Using the Notebooks

To run any of the notebooks, open and **Evaluate Initialization Cells**. Then calculations can be done at the bottom of the notebooks, or by opening a new notebook and running with the same kernel.

1.3 Symbols in Mathematica

Symbols are typed using the escape key, denoted `esc`. The following are used in these notebooks:

Symbol	Mathematica Shortcut
Φ	`esc Phi esc`
θ	`esc theta esc`

Other Greek letters can be typed similarly. For ease of use, we use $\theta_3 = \overline{\theta_1}$ and $\theta_4 = \overline{\theta_2}$.

2 Definitions

`Field` A scalar or spinor component of a superfield. All Berezinian derivatives on a field evaluate to zero.

`f` A placeholder argument used to hold differential operators (`DD`) in unevaluated form.

`NonCommutativeMultiply`
`**`
`Commutative`
`AntiCommutative`
Treated as `Times` for `Commutative` objects and anticommutative multiplication for `AntiCommutative` objects. Using with objects that are not specified is treated as standard `NonCommutativeMultiply`.

`Commutative Field` A bosonic field that has even parity. It commutes with all objects and `NonCommutitiveMultiply` will act equivalently to `Times`. Commuting-coordinate differentials (`DD`) of Commutative Fields are commutative.

`AntiCommutative Field` A fermionic field that has odd parity. It anticommutes with all anticommuting objects using `NonCommutativeMultiply`. Commuting-coordinate differentials (`DD`) of AntiCommutative Fields are anticommutative.

2.1 Functions

`DD[x][f]` $\partial_x f$ - Derivative operator with respect to coordinate x and placeholder `f`. Coordinate `x` can be commuting or anticommuting.

`ActDD[a, B]` Acts with differential operator a on differential operator or superfield B, outputing another differential operator (using placeholder f) or superfield.

`CommutatorDD[a, b]` Equivalent to `ActDD[a, b] - ActDD[b, a]`

`AntiCommutatorDD[a, b]` Equivalent to `ActDD[a, b] + ActDD[b, a]`

`Theta4Component[Φ]` Outputs the component of the superfield Φ proportional to $\theta_1\theta_2\theta_3\theta_4$.

`Theta2Component[Φ]` Outputs the component of the superfield Φ proportional to $\theta_1\theta_2$.

`Thetabar2Component[Φ]` Outputs the component of the superfield Φ proportional to $\theta_3\theta_4$.

`ComponentFields[Φ]` Outputs a list of all components fields for superfield Φ.

`TeXFormNice[a]` Outputs `a` in LaTeXform, fixing many small errors of TeXForm.

`SimplifyDD[a]` Simplifies differential operators to a readable form (is called by `ActDD`, `AntiCommutatorDD`, and `CommutatorDD`).

`FactorThetas[a]` Collects and sorts expression by powers of θ terms.

`IntegrateByParts[a]` Takes a differential operator and integrates by parts if possible, throwing away boundary terms

2.2 Superfield Operations

`AddCommutativeField[a,…]`
Adds one or more commutative fields `a` …to the active list of commutative fields

`AddCommutativeField[]` Displays the list of commutative fields

`AddAntiCommutativeField[a,…]`
Adds one or more anticommutative fields `a` …to the active list of anti-commutative fields

`AddAntiCommutativeField[]`
Displays the list of anticommutative fields

`FullSuperfield` A generic superfield with all θ components (using 4 θ variables).

`FieldsQ[a]` Checks if `a` is a field

`CommutativeQ[a]` Checks if `a` is a commutative coordinate or field

`AntiCommutativeQ[a]` Checks if `a` is an anticommutative coordinate or field

`CoordinatesQ[a]` Checks if `a` coordinate

`CommutativeCoordinatesQ[a]`
Checks if `a` is a commutative coordinate

`AntiCommutativeCoordinatesQ[a]`
Checks if `a` is an anticommutative coordinate

`SUSYTransformations[Φ, component]` Acts with all 4 SUSY generators on the `component` component of Φ. `component` can be `Theta4Component`, `Theta2Component`, `Thetabar2Component`, or a custom function.

3 Examples

`Example 1` Verify that the SUSY generators $\{Q_i\}_i$ satisfy the anticommutation relations of the SUSY algebra.

```
Input:
AntiCommutatorDD[Q1, Q1dag]
AntiCommutatorDD[Q2, Q1dag]
AntiCommutatorDD[Q1, Q2dag]
AntiCommutatorDD[Q2, Q2dag]
```

(In the $\mathcal{N} = 2$, $d = 3$ case)

```
Output:
- i DD[t][f]
- i DD[z][f]
- i DD[zbar][f]
i DD[t][f]
```

Note that the $\mathcal{N} = 2\, d = 3$ notebook uses complex spacial coordinates rather than real ones.

`Example 2` Verify that Φ is indeed a chiral superfield.

```
Input:
ActDD[D1dag, Φ]
ActDD[D2dag, Φ]

Output:
0
0
```

Wess and Bagger [1] is a canonical supersymmetry textbook, and SUSY-Components can solve some of the superspace-related exercises.

`Example 3` (WB 4.5) Evaluate $\{D, \overline{D}\}$ using the definitions of D and $\overline{D}$ as differential operators.

```
Input:
AntiCommutatorDD[D1, D1dag]
AntiCommutatorDD[D2, D1dag]
AntiCommutatorDD[D1, D2dag]
AntiCommutatorDD[D2, D2dag]


(In the N = 1, d = 4 case)

Output:
- 2 i DD[t][f] - 2 i DD[z][f]
- 2 i DD[x][f] - 2 DD[y][f]
- 2 i DD[x][f] + 2 DD[y][f]
- 2 i DD[t][f] + 2 i DD[z][f]
```

This is to be interpreted as the coordiante exapansion of

$$-2i\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}$$

`Example 4` (WB 4.6) Compute $\overline{D}F(x, \theta, \overline{\theta})$, where F is a general superfield.

```
Input:
AddCommutativeField[f, m, n, v1, v2, v3, v4, d]
AddAntiCommutativeField[ϕ1.ϕ2, ψ1, ψ2, λ1, λ2]
superfieldF = f + θ1**ϕ1 + θ2**ϕ2 + θ3**ψ1 + θ4**ψ2 +
θ1**θ2 m + θ3**θ4 n + (θ1**θ3 v1 + θ1**θ4 v2 + θ2**θ3 v3 +
θ2**θ4 v4) + θ1**θ2**θ3**λ1 + θ1**θ2**θ4**λ2 + θ1**θ3**θ4**χ1 +
θ2**θ3**θ4**χ2 + θ1**θ2**θ3**θ4 d

ActDD[D1dag, superfieldF]
ActDD[D2dag, superfieldF]
```

`Example 5` Compute the superfield $D_2 D_1 F$.

```
Input:
ActDD[D2,ActDD[D1, superfieldF]]
```

3.1 WZ Model

Compute the Wess-Zumino action (with superpotential) in components. In superspace we would write this in terms of a chiral superfield Φ and its complex conjugate $\overline{\Phi}$ as

$$\mathcal{L} = \int d^4\theta\,\overline{\Phi}\Phi + \int d^2\theta\,\Phi^2 + \int d^2\overline{\theta}\,\overline{\Phi}^2 \tag{1}$$

```
Input:
Theta4Component[Φbar ** Φ] +
Theta2Component[Φ ** Φ] +
Thetabar2Component[Φbar ** Φbar]
```

References

[1] J. Wess and J. Bagger. *Supersymmetry and supergravity*. Princeton University Press, Princeton, NJ, USA, 1992.