SUSY-Components

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Abstract

This is the documentation associated with the Mathematica notebooks SUSY-components for doing componentwise supersymmetry calculations.

Contents

1 Using the Notebooks

To run any of the notebooks, open and **Evaluate Initialization Cells**.

`Q:` Why is this in the form of notebooks and not a package (.wl)?

`A:` Global variables are very hard to deal with when importing through a Mathematica package. It's much simpler to initialize all variables in a single notebook rather than have the user set spacetime variables, field names, etc. every time.

1.1 Symbols in Mathematica

Symbols are typed using the escape key, denoted `esc`. The following are used in these notebooks:

Symbol	Mathematica Shortcut
Φ	`esc Phi esc`
θ	`esc theta esc`
θ_1	`esc theta esc ctrl+"dash" 1`

Other Greek letters can be typed similarly. For ease of use, we use $\theta_3 = \overline{\theta_1}$ and $\theta_4 = \overline{\theta_2}$.

1.2 Definitions

`Field` A scalar or spinor component of a superfield. All Berezinian derivatives on a field evaluate to zero.

`f` A placeholder argument used to hold differential operators (`DD`) in unevaluated form.

`NonCommutativeMultiply`
`**`
`Commutative`
`AntiCommutative`
Treated as `Times` for `Commutative` objects and anticommutative multiplication for `AntiCommutative` objects. Using with objects that are not specified is treated as standard `NonCommutativeMultiply`.

`Commutative Field` A bosonic field that has even parity. It commutes with all objects and `NonCommutitiveMultiply` will act equivalently to `Times`. Commuting-coordinate differentials (`DD`) of Commutative Fields are commutative.

`AntiCommutative Field` A fermionic field that has odd parity. It anticommutes with all anticommuting objects using `NonCommutativeMultiply`. Commuting-coordinate differentials (`DD`) of AntiCommutative Fields are anticommutative.

1.3 Differential Operators

`DD[x][f]` Derivative operator with respect to coordinate x and placeholder `f`. Coordinate `x` can be commuting or anticommuting.

`ActDD[a, B]` Acts with differential operator a on differential operator or superfield B, outputing another differential operator (using placeholder f) or superfield.

`Theta4Component[Φ]` Outputs the component of the superfield Φ proportional to $\theta_1\theta_2\theta_3\theta_4$.

`Theta2Component[Φ]` Outputs the component of the superfield Φ proportional to $\theta_1\theta_2$.

`Thetabar2Component[Φ]` Outputs the component of the superfield Φ proportional to $\theta_3\theta_4$.

`ComponentFields[Φ]` Outputs a list of all components fields for superfield Φ.

`TeXFormNice[a]` Outputs `a` in LaTeXform, fixing many small errors of TeXForm.

`SimplifyDD[a]`	Simplifies differential operators to a readable form (is called by `ActDD`, `AntiCommutatorDD`, and `CommutatorDD`).
`FactorThetas[a]`	Collects and sorts expression by powers of θ terms.
`IntegrateByParts[a]`	Takes a differential operator and integrates by parts if possible, throwing away boundary terms

1.4 Superfields

`AddCommutativeField[a,...]`	Adds one or more commutative fields `a` ... to the active list of commutative fields
`AddCommutativeField[]`	Displays the list of commutative fields
`AddAntiCommutativeField[a,...]`	Adds one or more anticommutative fields `a` ... to the active list of anticommutative fields
`AddAntiCommutativeField[]`	Displays the list of anticommutative fields
`FullSuperfield`	A generic superfield with all θ components (using 4 θ variables).
`FieldsQ[a]`	Checks if `a` is a field
`CommutativeQ[a]`	Checks if `a` is a commutative coordinate or field
`AntiCommutativeQ[a]`	Checks if `a` is an anticommutative coordinate or field
`CoordinatesQ[a]`	Checks if `a` coordinate
`CommutativeCoordinatesQ[a]`	Checks if `a` is a commutative coordinate
`AntiCommutativeCoordinatesQ[a]`	Checks if `a` is an anticommutative coordinate
`SUSYTransformations[Φ, component]`	Acts with all 4 SUSY generators on the `component` component of Φ. `component` can be `Theta4Component`, `Theta2Component`, `Thetabar2Component`, or a custom function.

1.5 Notebooks

There are three notebooks. They each have a different initialization setup depending on the dimension or structure of the spaces they work in.

2 Examples

2.1 Differential Operator Mechanics

2.1.1 Integrate By Parts

```
Input:  IntegrateByParts[z DD[z][A]]

Output:  -A
```

2.2 Verify SUSY Algebra

Verify that the SUSY generators $\{Q_i\}_i$ satisfy the anticommutation relations of the SUSY algebra.
`Input:`

```
AntiCommutatorDD[Q1, Q1dag]
AntiCommutatorDD[Q2, Q1dag]
AntiCommutatorDD[Q1, Q2dag]
AntiCommutatorDD[Q2, Q2dag]
```

(In the $\mathcal{N} = 2$, $d = 3$ case) `Output:`

```
- i DD[t][f]
- i DD[z][f]
- i DD[zbar][f]
i DD[t][f]
```

2.2.1 Verify Chiral Superfield

Verify that Φ is indeed a chiral superfield.

```
Input:
ActDD[D1dag, Φ]
ActDD[D2dag, Φ]
```

```
Output:
0
0
```

2.3 WZ Model

Compute the Wess-Zumino action (with superpotential) in components. In superspace we would write this in terms of a chiral superfield Φ and its complex conjugate $\overline{\Phi}$ as

$$\mathcal{L} = \int d^4\theta\, \overline{\Phi}\Phi + \int d^2\theta\, \Phi^2 + \int d^2\overline{\theta}\, \overline{\Phi}^2 \tag{1}$$

```
Input:
Theta4Component[Φbar ** Φ] +
Theta2Component[Φ ** Φ] + Thetabar2Component[Φbar ** Φbar]
Output:
```

2.4 SUSY Transformations

Compute the actions of the four SUSY generators on the top (θ^2) form of the chiral superfield Φ.

```
Input:   SUSYTransformations[Φ, Theta2Component]
```

3 v2 Features

3.1 Soon

- **Add more examples to the documentation**
- Add more examples of commonly used Lagrangians in components, e.g. **obtain scalar potential in superQED from superspace Lagrangian**
- Exponential form of vector superfield expression (in WZ gauge)

3.2 Someday

- Feature to solve for F-terms and D-terms
- Add a canonical ordering to `IntegrateByParts` for anticommutative fields, so they cancel properly
- Fin/paramterize moduli spaces, Higgs branches, etc.
- Easily implement gauge theory? - RG flow, symmetries, anomalies, etc.