# FAST LOW-RANK SHARED DICTIONARY LEARNING FOR IMAGE CLASSIFICATION

*Final Project*


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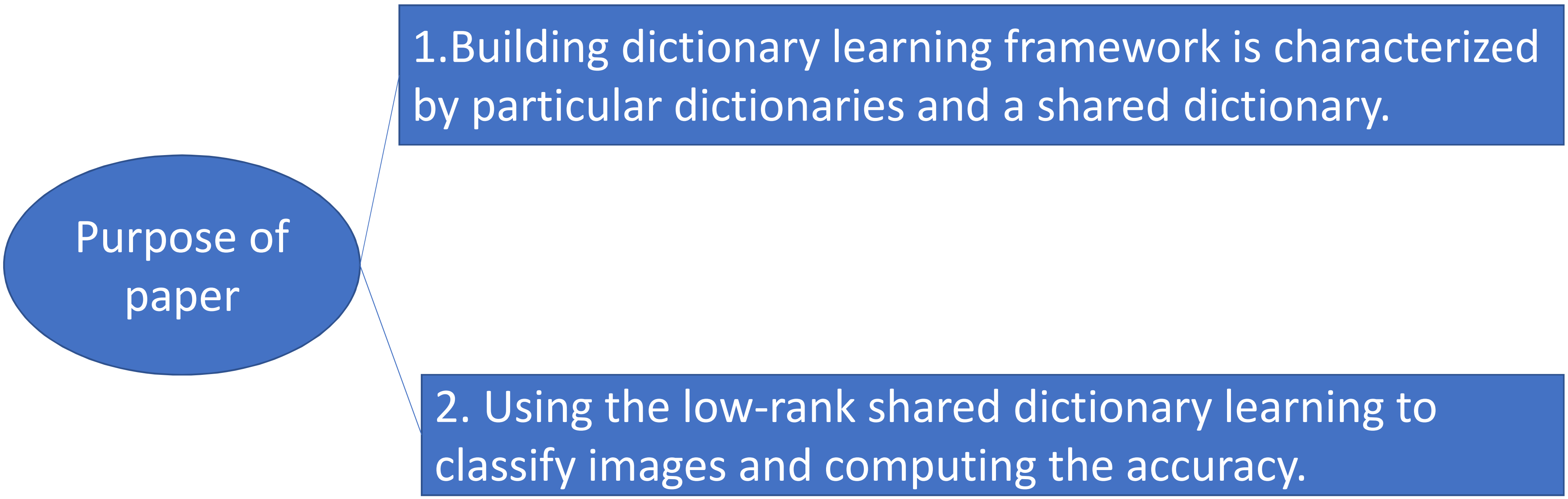
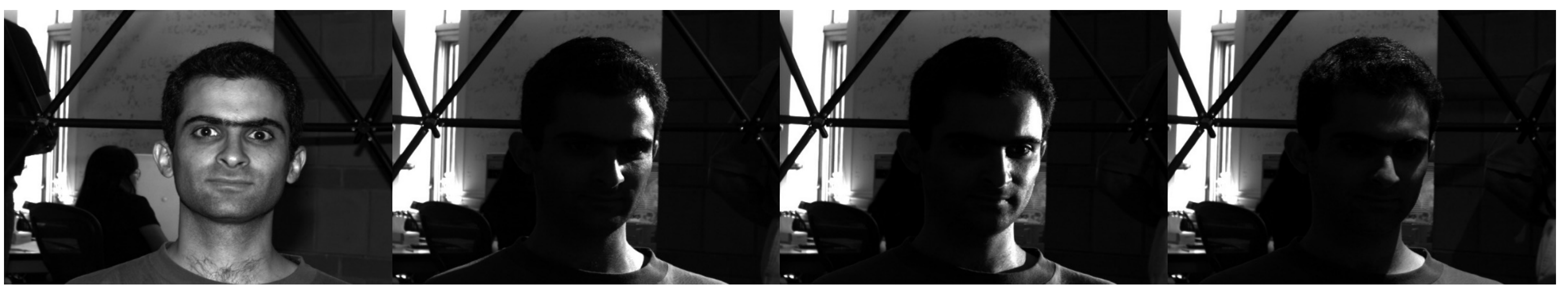

Purpose of paper
1.Building dictionary learning framework is characterized by particular dictionaries and a shared dictionary.
2. Using the low-rank shared dictionary learning to classify images and computing the accuracy.

# Dataset: The Extended Yale B

- This dataset contains about 2414 frontal face images of 38 individuals. We used the cropped and normalized face images of 192x168 pixels (see [2]).

- We randomly split the dataset into two halves. One half, which contains 32 images for each person, was used for training the dictionary. The other half was used for testing.

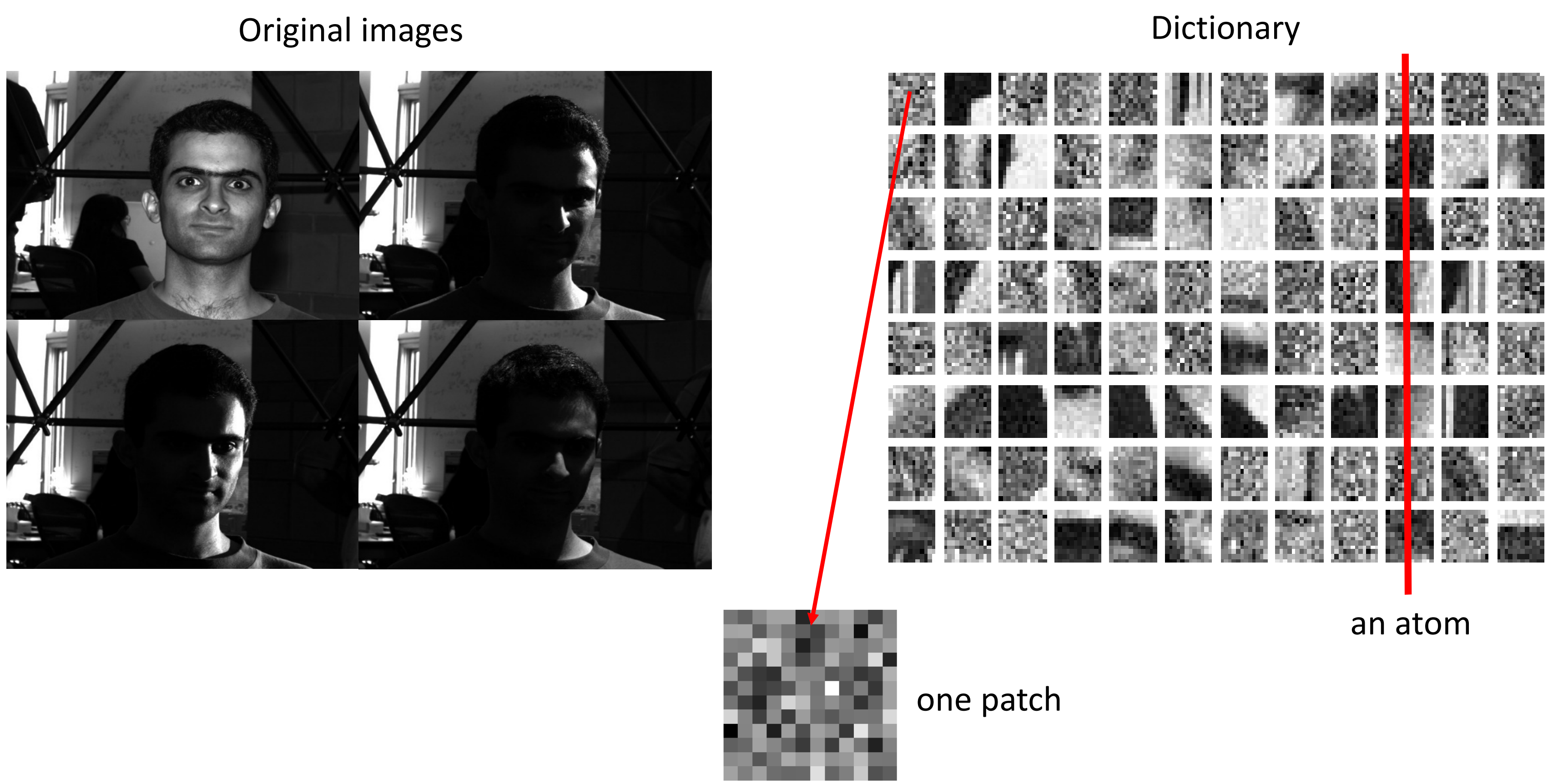


The extended Yale B

# Vectorization the dataset (see [2])

| | 1 | 2 | 3 | 4 | … | 2413 | 2414 |
|---|---|---|---|---|---|---|---|
| 1 | 31.75248 | 28.83727 | -29.2841 | 39.18376 | | -47.3624 | -19.6828 |
| 2 | -111.236 | -82.9543 | -39.9927 | 13.33082 | | 5.692033 | -3.7811 |
| 3 | 51.63764 | 55.73638 | 21.89475 | -51.5378 | | 23.22424 | 61.22354 |
| 4 | 14.09386 | 3.271673 | 1.735423 | 18.93872 | | 268.1512 | 162.8671 |
| 5 | -195.57 | -211.417 | -236.041 | -35.5466 | | -103.549 | -63.2441 |
| … | | | | | | | |
| 504 | 140.3989 | 127.1401 | 105.0745 | 29.25712 | | 131.8584 | 111.5676 |
| **Label** | **1** | **1** | **1** | **1** | | **38** | **38** |

- The learned dictionary contains 304 atoms, which corresponds to, on average, roughly 8 atoms for each person.



# Algorithm

**Algorithm 1** LRSDL Sparse Coefficients Update by FISTA [34]

**function** $(\hat{\mathbf{X}}, \hat{\mathbf{X}}^0) = \text{LRSDL\_X}(\mathbf{Y}, \mathbf{D}, \mathbf{D}_0, \mathbf{X}, \mathbf{X}^0, \lambda_1, \lambda_2)$.

1. Calculate:

$$\mathbf{A} = \mathcal{M}(\mathbf{D}^T\mathbf{D}) + 2\lambda_2\mathbf{I};$$
$$\mathbf{B} = 2\mathbf{D}_0^T\mathbf{D}_0 + \lambda_2\mathbf{I}$$
$$L = \lambda_{\max}(\mathbf{A}) + \lambda_{\max}(\mathbf{B}) + 4\lambda_2 + 1^4$$

2. Initialize $\mathbf{W}_1 = \mathbf{Z}_0 = \begin{bmatrix}\mathbf{X}\\ \mathbf{X}^0\end{bmatrix}, t_1 = 1, k = 1$

**while** not convergence and $k < k_{\max}$ **do**

3. Extract $\mathbf{X}, \mathbf{X}^0$ from $\mathbf{W}_k$.

4. Calculate gradient of two parts:

$$\begin{aligned}\mathbf{M} &= \mu(\mathbf{X}), \mathbf{M}_c = \mu(\mathbf{X}_c), \widehat{\mathbf{M}} = [\mathbf{M}_1, \dots, \mathbf{M}_C].\\ \mathbf{V} &= \mathbf{Y} - \frac{1}{2}\mathbf{D}\mathcal{M}(\mathbf{X})\\ \mathbf{G} &= \begin{bmatrix}\mathbf{A}\mathbf{X} - \mathcal{M}(\mathbf{D}^T(\mathbf{Y} - \mathbf{D}_0\mathbf{X}^0)) + \lambda_2(\mathbf{M} - \widehat{\mathbf{M}})\\ \mathbf{B}\mathbf{X}^0 - \mathbf{D}_0^T\mathbf{V} - \lambda_2\mu(\mathbf{X}^0)\end{bmatrix}\end{aligned}$$

5. $\mathbf{Z}_k = \mathcal{S}_{\lambda_1/L}(\mathbf{W}_k - \mathbf{G}/L)$ ($\mathcal{S}_\alpha()$ is the element-wise soft thresholding function. $\mathcal{S}_\alpha(x) = \text{sgn}(x)(|x| - \alpha)_+$).

6. $t_{k+1} = (1 + \sqrt{1 + 4t_k^2})/2$

7. $\mathbf{W}_{k+1} = \mathbf{Z}_k + \frac{t_k - 1}{t_{k+1}}(\mathbf{Z}_k - \mathbf{Z}_{k-1})$

8. $k = k + 1$

**end while**

9. OUTPUT: Extract $\mathbf{X}, \mathbf{X}^0$ from $\mathbf{Z}_k$.

**end function**

**Algorithm 2** LRSDL Algorithm

**function** $(\hat{\mathbf{X}}, \hat{\mathbf{X}}^0) = \text{LRSDL}(\mathbf{Y}, \lambda_1, \lambda_2, \eta)$.

1. Initialization $\mathbf{X} = \mathbf{0}$, and:

$$\begin{aligned}(\mathbf{D}_c, \mathbf{X}_c^c) &= \arg\min_{\mathbf{D},\mathbf{X}} \frac{1}{2}\|\mathbf{Y}_c - \mathbf{D}\mathbf{X}\|_F^2 + \lambda_1\|\mathbf{X}\|_1\\ (\mathbf{D}_0, \mathbf{X}^0) &= \arg\min_{\mathbf{D},\mathbf{X}} \frac{1}{2}\|\mathbf{Y} - \mathbf{D}\mathbf{X}\|_F^2 + \lambda_1\|\mathbf{X}\|_1\end{aligned}$$

**while** not converge **do**

2. Update $\mathbf{X}$ and $\mathbf{X}^0$ by Algorithm 1.

3. Update $\mathbf{D}$ by ODL [35]:

$$\begin{aligned}\mathbf{E} &= (\mathbf{Y} - \mathbf{D}_0\mathbf{X}^0)\mathcal{M}(\mathbf{X}^T)\\ \mathbf{F} &= \mathcal{M}(\mathbf{X}\mathbf{X}^T)\\ \mathbf{D} &= \arg\min_{\mathbf{D}}\{-2\text{trace}(\mathbf{E}\mathbf{D}^T) + \text{trace}(\mathbf{F}\mathbf{D}^T\mathbf{D})\}\end{aligned}$$

4. Update $\mathbf{D}_0$ by ODL [35] and ADMM [36] (see equations (17) - (20)).

**end while**

**end function**

# Implementation

- Input:
  - Dataset: *The Extended Yale B (38 classes* and *2414 images).*
  - Training samples: *Y_train* (*380 images*).
  - Testing samples: *Y_test* (2034 images).
  - Training labels: *label_train ([1 1 1 … 2 2 … 38 38]).*
  - Testing labels: *label_test ([1 1 1 … 2 2 … 38 38]).*
  - *k*: number of bases in class specific-dictionary c *(k=10)*.
  - *k0*: number of bases in shared dictionary *(k0=5)*.

# Implementation

- Input:
  - lambda1, lambda2 and eta in the cost function*

    Lambda1 = 0.001, lambda2 = 0.01 and eta = 0.02

  * Cost function

$$f\left(D,X,D_0,X^0\right)=\left\|Y-D_0X^0-DX\right\|_F^2+\sum_{i=1}^{C}\left(\left\|Y_i-D_0X_i^0-D_iX_i^i\right\|_F^2+\sum_{j\neq i}\left\|D_jX_i^j\right\|_F^2\right)+\lambda_1\left\|X\right\|_1$$

$$+\lambda_2\left(\sum_{i=1}^{C}\left(\left\|X_i-M_i\right\|_F^2-\left\|M_i-M\right\|_F^2\right)+\left\|X\right\|_F^2+\left\|X^0-M^0\right\|_F^2\right)+\lambda_1\left\|X^0\right\|_1+\eta\left\|D_0\right\|_*$$

# Implementation

- Output:
  - $D, D_0, X, X_0$: trained matrices as in the cost function.
  - *acc_lrsdl:* accuracy of testing samples.
  - *rt:* time (*seconds*)

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================= Summaray =================
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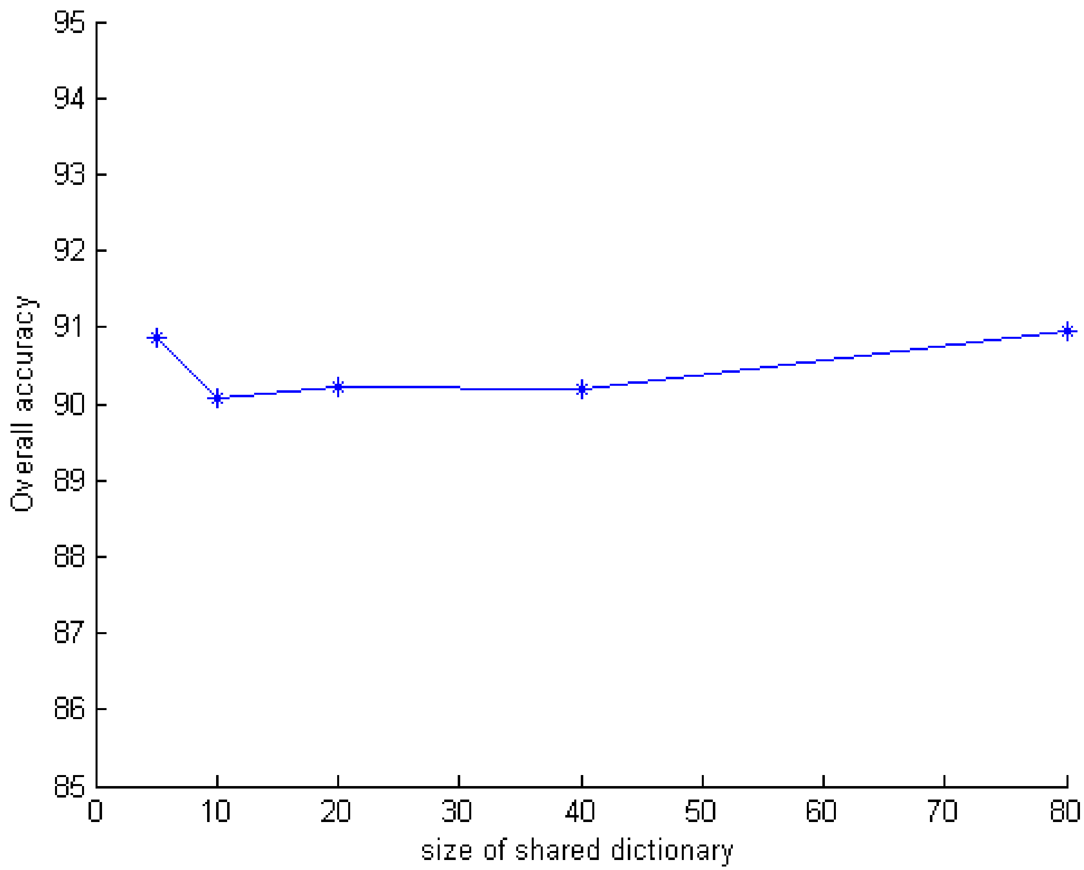
| Method | Accuracy |
| --- | --- |
| LRSDL | 90.86% |

- rt: 54 seconds

# Implementation

| k0 | Accuracy (%) | Time (s) |
|---|---|---|
| 5 | 90.86 | 54 |
| 10 | 90.07 | 127 |
| 20 | 90.22 | 196 |
| 40 | 91.19 | 264 |
| 80 | 90.95 | 337 |

# Implementation



# References

[1] T. H. Vu, V. Monga, *Fast Low-Rank Shared Dictionary Learning For Image Classification,* IEEE Trans. Image Process., vol. 26, no. 11, pp. 5160-5175, Nov. 2017.

[2] M. Yang, L. Zhang, X. Feng, and D. Zhang, *Fisher discrimination dictionary learning for sparse representation*, in Proc. IEEE International Conference on Computer Vision, pp. 543–550, Nov. 2011.

[3] A. Beck and M. Teboulle, *A fast iterative shrinkage-thresholding algorithm for linear invers problems*, SIAM J. Image. Sci, Vol. 2, No. 1, pp. 183–202, Nov. 2009.

[4] S. Boyd, N. Parith, E. Chu, B. Peleato, J. Eckstein, *Distributed optimization and statistical learning via the alternating direction method of multipliers*, Found. Trends Mach. Learn, Vol. 3, No. 1, pp. 1–122, Nov. 2009.