第一次報告

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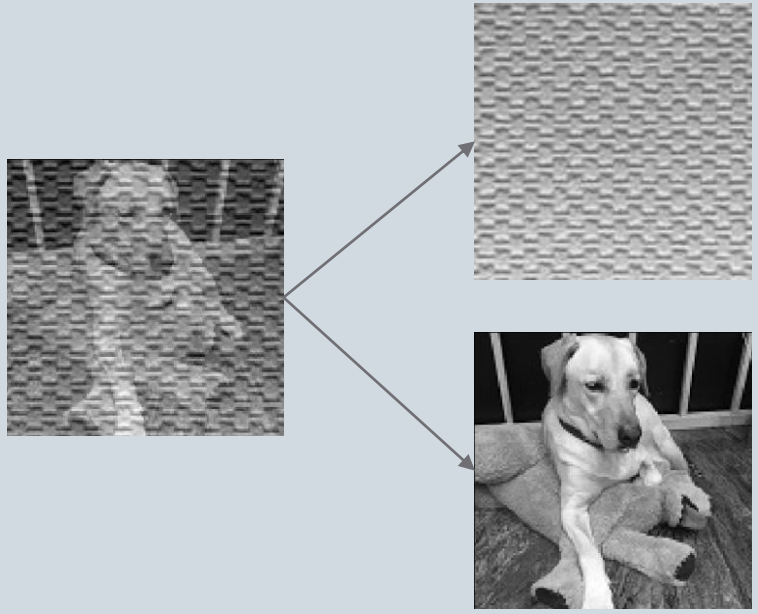
Problem

$$\|y_1 - D_1 x_1\|_F^2 + \|y_2 - D_2 x_2\|_F^2 + \lambda \left\| {D_1}^T D_2 \right\|_F^2 + \Gamma_x\,(x_1) + \Gamma_x\,(x_2) + \Gamma_D\,(D_1) + \Gamma_D\,(D_2)$$



- y_1 : picture 1
- y_2 : picture 2
- D_1 : dictionary for picture 1
- D_2 : dictionary for picture 2
- x_1 : sparse representation of picture 1 for D_1
- x_2 : sparse representation of picture 2 for D_2
- λ : regularization parameters
- Γ_x : constraint for x (make it sparse)
- Γ_D : constraint for D (normalize)

Proximal Algorithm

- Proximal operator : $prox_{\lambda f}(v) = \underset{x}{\operatorname{argmin}}(f(x) + \frac{1}{2\lambda}\|x - v\|^2)$
- Proximal gradient method :

  F(x) = f(x) + g(x), f(x) is continuously differentiable and g(x) is not continuous

  (1)gradient step

  $$v^t = x^t - \lambda \nabla f(x^t)$$

  (2)proximal operator step

  $$x^{t+1} = prox_x(v^t)$$

- When ∇f is Lipschitz continuous with constant L, $\lambda \in (0, \frac{1}{L}]$

Solution

$$\|y_1 - D_1 x_1\|_F^2 + \|y_2 - D_2 x_2\|_F^2 + \lambda \left\| {D_1}^T D_2 \right\|_F^2 + \Gamma_x(x_1) + \Gamma_x(x_2) + \Gamma_D(D_1) + \Gamma_D(D_2)$$

f : differentiable

g : nonconvex

- Let f be the red line part, and g be the blue line part
- Solving D_1, D_2, x_1, x_2 via proximal algorithm , respectively
- Re-compute Lipschitz constant each iteration

lambda = 0, iteration = 1000

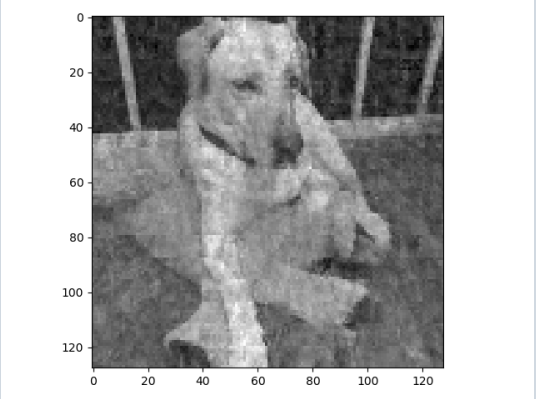
- before dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 220.18268975817733

$\|y_2 - D_2 x_2\|_F$ = 222.28837786521683

$\left\|{D_1}^T D_2\right\|_F$ = 15.93912746115688

$\|{y_1}^T y_2\|_F$ = 4652.819827829139



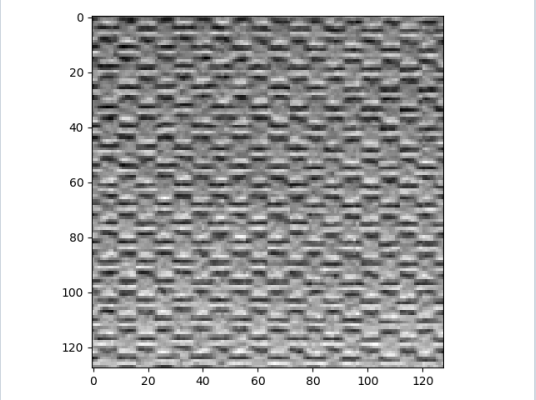
- after dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 34.72835875511873

$\|y_2 - D_2 x_2\|_F$ = 39.37764567597413

$\left\|{D_1}^T D_2\right\|_F$ = 33.47709294532105

$\|{y_1}^T y_2\|_F$ = 1377.193986001195



lambda = 1, iteration = 1000

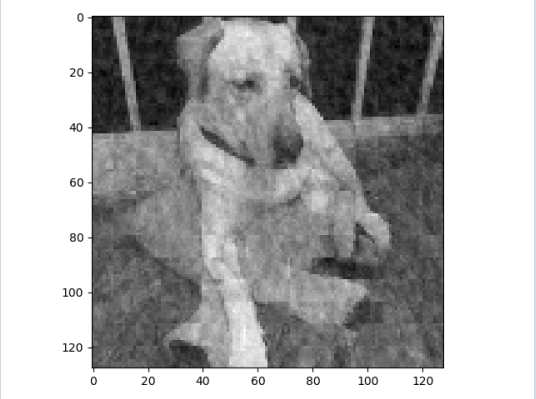
- before dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 220.18268975817733

$\|y_2 - D_2 x_2\|_F$ = 222.28837786521683

$\left\|{D_1}^T D_2\right\|_F$ = 15.93912746115688

$\|{y_1}^T y_2\|_F$ = 4652.819827829139



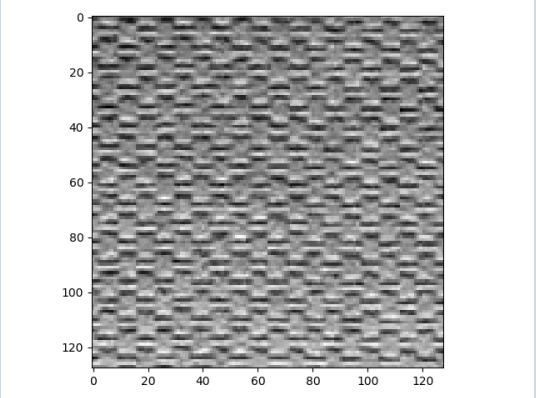
- after dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 32.52759512164004

$\|y_2 - D_2 x_2\|_F$ = 42.42109661711466

$\left\|{D_1}^T D_2\right\|_F$ = 21.11685321894725

$\|{y_1}^T y_2\|_F$ = 1090.59509862585



lambda = 5, iteration = 1000

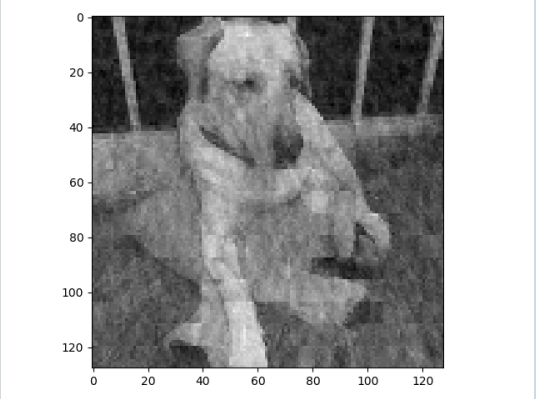
- before dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 220.18268975817733

$\|y_2 - D_2 x_2\|_F$ = 222.28837786521683

$\left\|{D_1}^T D_2\right\|_F$ = 15.93912746115688

$\|{y_1}^T y_2\|_F$ = 4652.819827829139



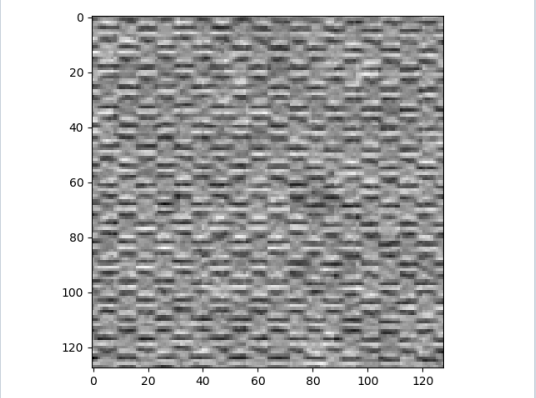
- after dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 36.97705376300321

$\|y_2 - D_2 x_2\|_F$ = 63.10484695717596

$\left\|{D_1}^T D_2\right\|_F$ = 14.558816422370223

$\|{y_1}^T y_2\|_F$ = 829.6839292239839



lambda = 10, iteration = 1000

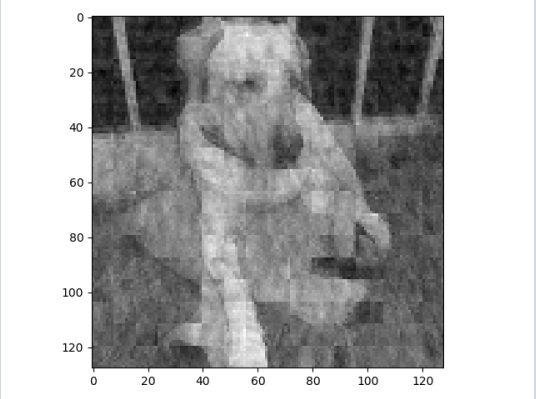
- before dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 220.18268975817733

$\|y_2 - D_2 x_2\|_F$ = 222.28837786521683

$\left\|{D_1}^T D_2\right\|_F$ = 15.93912746115688

$\|{y_1}^T y_2\|_F$ = 4652.819827829139



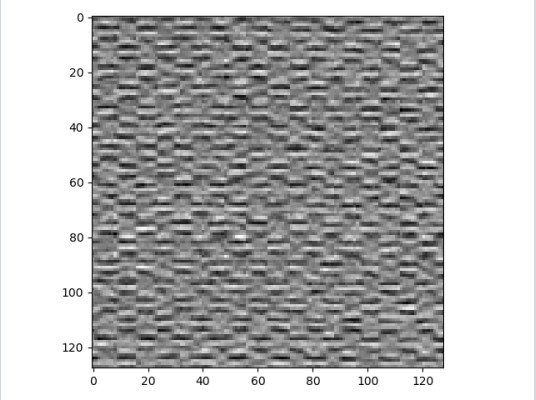
- after dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 42.907315307035326

$\|y_2 - D_2 x_2\|_F$ = 72.9588111422319

$\left\|{D_1}^T D_2\right\|_F$ = 7.658268715329088

$\|{y_1}^T y_2\|_F$ = 388.02419599750397



lambda = 20, iteration = 1000

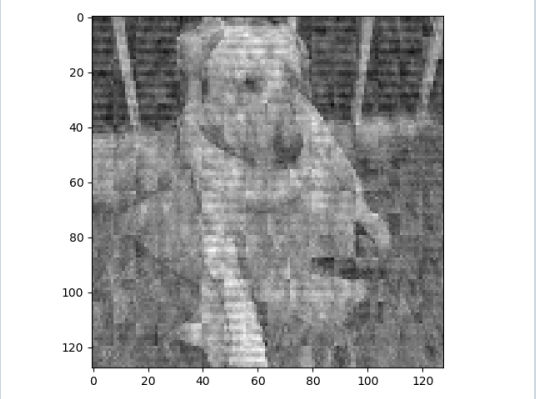
- before dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 220.18268975817733

$\|y_2 - D_2 x_2\|_F$ = 222.28837786521683

$\left\|{D_1}^T D_2\right\|_F$ = 15.93912746115688

$\|{y_1}^T y_2\|_F$ = 4652.819827829139



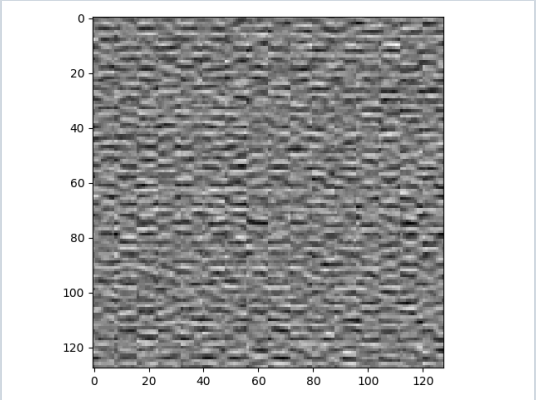
- after dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 65.82048708056374

$\|y_2 - D_2 x_2\|_F$ = 89.64607277704985

$\left\|{D_1}^T D_2\right\|_F$ = 5.808565750146154

$\|{y_1}^T y_2\|_F$ = 343.6802912825157



lambda = 50, iteration = 1000

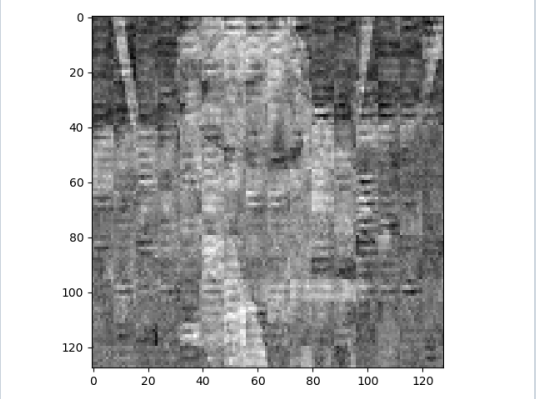
- before dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 220.18268975817733

$\|y_2 - D_2 x_2\|_F$ = 222.28837786521683

$\left\|{D_1}^T D_2\right\|_F$ = 15.93912746115688

$\|{y_1}^T y_2\|_F$ = 4652.819827829139



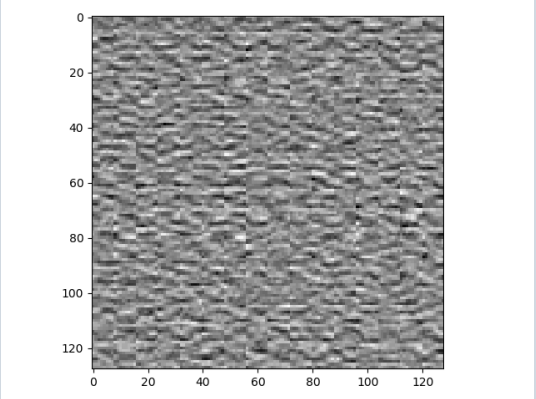
- after dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 84.74730986884552

$\|y_2 - D_2 x_2\|_F$ = 100.01402097663563

$\left\|{D_1}^T D_2\right\|_F$ = 2.509107459832805

$\|{y_1}^T y_2\|_F$ = 354.63651448064314



lambda = 100, iteration = 1000

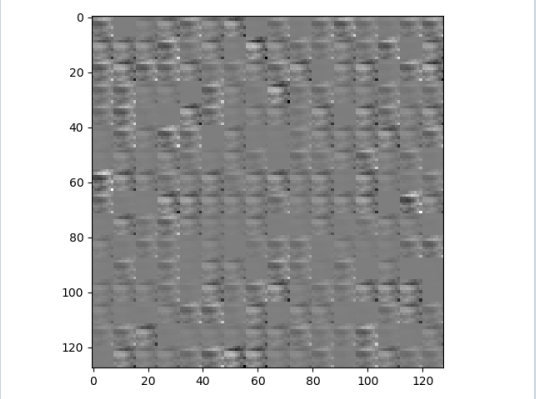
- before dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 220.18268975817733

$\|y_2 - D_2 x_2\|_F$ = 222.28837786521683

$\left\|{D_1}^T D_2\right\|_F$ = 15.93912746115688

$\|{y_1}^T y_2\|_F$ = 4652.819827829139



- after dictionary learning

$\|y_1 - D_1 x_1\|_F$ = 131.8448164823175

$\|y_2 - D_2 x_2\|_F$ = 131.53127587105865

$\left\|{D_1}^T D_2\right\|_F$ = 127.0613438885982

$\|{y_1}^T y_2\|_F$ = 203.13003871859212