Good proofs are:

1. correct
2. complete
3. clear
4. brief
5. "elegant"
6. well-organized
7. in order

Fermat's Last Thm:

$$\forall n > 2, \neg\exists x, y, z \in \mathbb{N}^+$$
$$x^n + y^n = z^n$$

Problem:

Find a sequence of moves to go from

A	B	C
D	E	F
H	G	

to

A	B	C
D	E	F
G	H	

Legal Move: Slide a letter into a adjacent blank square.

Thm: There is no sequence of legal moves to invert G&H and return all other letters to their original position.

Natural Order

1	2	3
4	5	6
7	8	9

Row moves

Ex:

A	B	C
D	G	
E	F	H

$\Longrightarrow$

A	B	C
D		G
E	F	H

Lemma 1:

A row move does not change the order of the items.

Proof: ~~Obvious~~. In a row move, we move an item from cell i into an adjacent cell i-1 or i+1. Nothing else moves. Hence the order of items is preserved.□

Column moves

Ex:

A	B	C
D	F	
H	E	G

$\Longrightarrow$

A	B	C
D	F	G
H	E	

Ex:

A	B	C
D		G
H	E	F

$\Longrightarrow$

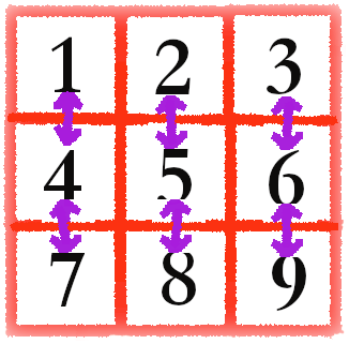
A		C
D	B	G
H	E	F

Lemma2:

A column move changes the relative order of precisely 2 paris of items.

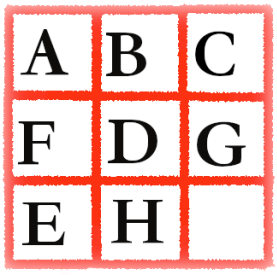
Proof: In a column move, we move an item in cell i to a blank spot in cell i-3 or i+3 . When an item moves 3 positions, it changes order with 2 items(i-1,i-2 or i+1,i+2).□

Order Changes in Column moves:



Def:

A pair of letter L1&L2 form an **inversion**, also known as an inverted pair, if L1 precedes L2 in alphabet, but L1 appears after L2 in the puzzle.



(D,F),(E,F),(E,G) ——3 inverions in the left puzzle.

Lemma 3:

During a move, the number of inversions can only increases by 2, decrease by 2 or stay same.

Pf: Row move :No changes (by Lemma 1)

Column move: 2 pairs change order (by Lemma 2)

A: both pairs were in order originally before the column move. $\Longrightarrow$ number of inversions $\uparrow 2$

B: both pairs were inverted originally before the column move. $\Longrightarrow$ number of inversions $\downarrow 2$

C: one of the pairs inverted while one of the pairs which is not inverted $\implies$ stays the same. $\square$

Corollary 1: During a move, the parity (evenness/oddness) of the number of inversions does not change.