Patterns of Proof

证明的方式

The Axiomatic Method

公理化方法

The standard procedure for establishing truth in mathematics was invented by Eu- clid, a mathematician working in Alexandria, Egypt around 300 BC. His idea was to begin with five assumptions about geometry, which seemed undeniable based on direct experience.For example, one of the assumptions was "There is a straight line segment between every pair of points." Propositions like these that are simply accepted true are called axioms.

　　生活于公元前300年埃及亚历山大港的数学家欧几里得，发明了在数学中证实命题正确性的标准步骤。他的思想发轫于几何学上的五个假设，它们基于直接经验，似乎毋庸置疑。例如，其中一个假设是“两点之间只有一个直线段。”像这样不证自明的命题称为公理。

Starting from these axioms, Euclid established the truth of many additional propositions by providing "proofs".A proof is a sequence of logical deductions **from** axioms and previously-proved statements that concludes with the proposition in question.

从这些公理开始，通过提供“证明”，欧几里得证实了许多附加命题的正确性。证明是一系列逻辑推论，根据公理与之前证实的陈述推定讨论中的命题。

There are several common terms for a proposition that has been proved.The different terms hint at the role of the proposition within a larger body of work.

- Important propositions are called *theorems*.
- A *lemma* is a preliminary proposition useful for proving later propositions.
- A *corollary* is a proposition that follows in just a few logical steps from a lemma or a theorem.

可用多个常见术语来指代已证明的命题。不同的术语暗示着该命题在更大工作体系中的地位。

- 重要的命题称作**定理**。
- **引理**是个可用于证明后续命题的初级命题。
- 从某个引理或定理开始，仅推导几步，就可得到叫做**推论**的命题。

Euclid's axiom-and-proof approach, now called the axiomatic method, is the foundation for mathematics today. In fact, just a handful of axioms, collectively called Zermelo-Frankel Set Theory with Choice(ZFC), together with a few logical deduction rules, appear to be sufficient to derive essentially all of mathematics.

欧几里德的公理&证明方法，现在称为公理化方法，是现代数学的基石。事实上，只需几个公理——它们统称为包括选择公理的策梅洛-弗兰克尔集合论——加上一些逻辑推论规则，似乎就足以推导出大体上所有的数学理论。

2.1.1 Our Axioms

2.1.1 我们的公理

The ZFC axioms are important in studying and justifying the foundations of mathematics, but for practical purposes, they are much too primitive.Proving theorems in ZFC is a little like writing programs in byte code instead of a full-fledged programming language——by one reckoning, a formal proof in ZFC that 2 + 2 = 4 requires more than 20,000 steps! So instead of starting with ZFC, we're going to take a huge set of axioms as our foundation: we'll accept all familiar facts from high school math!

在研究及证明数学基础的合理性方面，ZFC公理价值很大，但对实际应用来讲，它们却太过简陋。用ZFC证明定理有点象用字节码而非完备的编程语言来编写程序——据估计，用规范的ZFC来证明2 + 2 = 4需要超过20,000 个步骤！所以与其从ZFC开始，倒不如把超大的公理集当作我们的基础：我们将把高中数学中的常见事实都默认为公理！

This will give us a quick launch, but you may find this imprecise specification of the axioms troubling at times.For example, in the midst of a proof, you may find yourself wondering, "Must I prove this little fact or can I take it as an axiom?"Feel free to ask for guidance, but really there is no absolute answer. Just be up front about what you're assuming, and don't try to evade homework and exam problems by declaring everything an axiom!

这让我们得以快速开始，但有时你也会认为公理的这种不精确的规范会让人苦恼。例如，你可能在证明时陷入疑惑，“我是否必须证明这一小处事实，还是可以把它当公理？”不要羞于寻求指导，但的确不存在绝对正确的答案。只需直面你的职责，不要把一切都声明为公理来逃避作业和考试中的问题！

2.1.2 Logical Deductions

2.1.2 逻辑推论

Logical deductions or **inference rules** are used to prove new propositions using previously proved ones.

使用之前证实的命题来证明一个新命题时，会用到**逻辑推论**或**推理规则**。

A fundamental inference rule is *modus ponens*. This rule says that a proof of P together with a proof that P *IMPLIES* Q is a proof of Q.

假言推理是一个基本的**推理规则**。该规则称，**P**为真且**P蕴涵Q**为真，即可证明**Q**为真。

Inference rules are sometimes written in a funny notation. For example, *modus ponens* is written:

推理规则有时用古怪的符号标记。例如，假言推理标记如下：

Rule 2.1.1

$$\frac{P, P \ \ IMPLIES \ \ Q}{Q}$$

规则2.1.1

$$\frac{P, P \text{ 蕴 涵 } Q}{Q}$$

When the statements above the line, called the antecedents, are proved, then we can consider the statement below the line, called the conclusion or **consequent**, to also be proved.

直线上面的陈述叫做**前件**，下面的陈述叫做**结论**或**后件**。证明了前件，就可以认为也证明了后件。

A key requirement of an **inference rule** is that it must be *sound*: any assignment of truth values that makes all the antecedents true must also make the consequent true. So if we start off with true axioms and apply sound inference rules, everything we prove will also be true.

一定得合理是推理规则的必要条件：只要指定真值，使所有前件为真，必然使后件也为真。所以如果我们从正确的公理着手，应用合理的推理规则，那么我们所证明的一切也都将为真。

You can see why modus ponens is a sound inference rule by checking the truth table of P *IMPLIES* Q. There is only one case where P and P IMPLIES Q are both true, and in that case Q is also true.

通过核查"P蕴涵Q"的真值表，你就会明白为什么要说假言推理是合理的推理规则。只在一种情形下**P**与**P蕴涵Q**都为真，同时**Q**也为真。

P	Q	$P \rightarrow Q$
F	F	T
F	T	T
T	F	F
T	T	T

There are many other natural, sound inference rules, for example:

也有很多自然合理的推理规则，如：

Rule 2.1.2

$$\frac{P \text{ } IMPLIES \text{ } Q, \text{ } Q \text{ } IMPLIES \text{ } R}{P \text{ } IMPLIES \text{ } R}$$

规则 2.1.2

$$\frac{P \text{ 蕴 涵 } Q, Q \text{ 蕴 涵 } R}{P \text{ 蕴 涵 } R}$$

Rule 2.1.3

$$\frac{P \text{ } IMPLIES \text{ } Q, \text{ } NOT(Q)}{NOT(P)}$$

规则 2.1.3

$$\frac{P\ \text{蕴涵}\ Q,\ \text{非}\ Q}{\text{非}\ P}$$

Rule 2.1.4

$$\frac{NOT(P)\ IMPLIES\ NOT(Q)}{Q\ \ IMPLIES\ \ P}$$

规则 2.1.4

$$\frac{\text{非}\ P\ \text{蕴涵}\ \text{非}\ Q}{Q\ \text{蕴涵}\ P}$$

On the other hand,

Non-Rule.

$$\frac{NOT(P)\ IMPLIES\ NOT(Q)}{P\ \ IMPLIES\ \ Q}$$

is not sound: if P is assigned **T** and Q is assigned **F**, then the antecedent is true and the consequent is not.

另一方面，

非规则

$$\frac{\text{非}\ P\ \text{蕴涵}\ \text{非}\ Q}{P\ \text{蕴涵}\ Q}$$

并不合理：如果指定P为**真**，Q为**假**，那么前件为真，但后件却不为真。

Note that a propositional inference rule is sound precisely when the conjunction(AND) of all its antecedents implies its consequent.

只有当某个**命题推理规则**所有前件的合取(AND)蕴涵着它的后件，才可以说**该规则**真正合理。