# Good proofs are:

1. correct
2. complete
3. clear
4. brief
5. "elegant"
6. well-organized
7. in order

## Fermat's Last Thm:

$$\forall n > 2, \neg\exists x, y, z \in \mathbb{N}^+$$
$$x^n + y^n = z^n$$

## Problem:

Find a sequence of moves to go from

| | | |
|---|---|---|
| A | B | C |
| D | E | F |
| H | G | |

to

| | | |
|---|---|---|
| A | B | C |
| D | E | F |
| G | H | |

**Legal Move**: Slide a letter into a adjacent blank square.

**Thm**: There is no sequence of legal moves to invert G&H and return all other letters to their original position.

**Natural Order**

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

**Row moves**

**Ex:**

| | | |
|---|---|---|
| A | B | C |
| D | G | |
| E | F | H |

$\Longrightarrow$

| | | |
|---|---|---|
| A | B | C |
| D | | G |
| E | F | H |

## Lemma 1:

A row move does not change the order of the items.

**Proof**: ~~Obvious~~. In a row move, we move an item from cell i into an adjacent cell i-1 or i+1. Nothing else moves. Hence the order of items is preserved.□

**Column moves**

**Ex:**

| A | B | C |
|---|---|---|
| D | F | |
| H | E | G |

$\Longrightarrow$

| A | B | C |
|---|---|---|
| D | F | G |
| H | E | |

**Ex:**

| A | B | C |
|---|---|---|
| D | | G |
| H | E | F |

$\Longrightarrow$

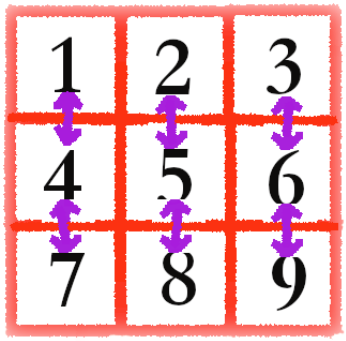
| A | | C |
|---|---|---|
| D | B | G |
| H | E | F |

## Lemma2:

A column move changes the relative order of precisely 2 paris of items.

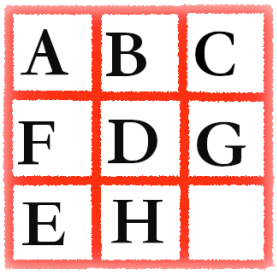
**Proof:** In a column move, we move an item in cell i to a blank spot in cell i-3 or i+3 . When an item moves 3 positions, it changes order with 2 items(i-1,i-2 or i+1,i+2).□

**Order Changes in Column moves:**




## Def:

A pair of letter L1&L2 form an inversion, also known as an inverted pair, if L1 precedes L2 in alphabet, but L1 appears after L2 in the puzzle.




**(D,F),(E,F),(E,G)** ——**3 inverions** in the left puzzle.

## Lemma 3:

During a move, the number of inversions can only increases by 2, decrease by 2 or stay same.

**Pf:** Row move :No changes (by lemma 1)