1 Proof by Contradiction

To prove P is true, we assume P is False(i.e. ¬P is True) then you use that hypothesis to derive a falsehood or contradiction.

If P is true, then ¬P is false, and this means that $\neg P \implies F$ is true.

Ex: **Thm**:$\sqrt{2}$ is *irrational.* An irrational number is something that can't be expressed as the ratio of integers.

1.1 Proof(by Contradiction)

Assume for the purpose of contradiction, that $\sqrt{2}$ is **rational**.

$$\begin{aligned}
&\implies \sqrt{2} = a/b(a\ fraction\ in\ lowest\ terms, \text{i.e.}\ a\ and\ b\ have\ no\ common\ divisors.)\\
&\implies 2 = a^2/b^2\\
&\implies 2b^2 = a^2\\
&\implies a\ is\ even\ (2 \mid a)\\
&\implies 4 \mid a^2\\
&\implies 4 \mid 2b^2\\
&\implies 2 \mid b^2\\
&\implies b\ is\ even(2 \mid b)\\
&\implies a/b\ is\ not\ in\ lowest\ terms\\
&\implies contradiction(\Rightarrow\Leftarrow)
\end{aligned}$$

$$\implies \sqrt{2}\ is\ irrational.$$

2 Induction axiom