~~Ex5ck~~

1. $h[n] = \underbrace{\cancel{a^n u[n]}}_{h_R[n]} + \underbrace{\cancel{-b^n u[-n-1]}}_{h_L[n]} \Rightarrow H(z) = H_R(z) + H_L(z)$ linearity

$$H_R(z) = \sum_{n=-\infty}^{\infty} a^n u[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n = \frac{1}{1-az^{-1}}, \text{ for } |az^{-1}|<1$$

$$= \frac{z}{z-a} \quad \text{part } L$$

$-\infty \to -1$

a) $H(z) = \frac{z}{z-a} + \frac{z}{z-b}$

b) poles: $z = a$, $z = b$

zeros: $z = 0$

c) converges for $|a| < |z| < |b|$

d) Not causal since ~~not~~ ~~neither~~ ~~right~~ right sided, this system is neither right nor left-sided

$$H_L(z) = \sum_{n=-\infty}^{\infty} -b^n u[-n-1] z^{-n}$$

$$= \sum_{n=-\infty}^{-1} -b^n z^{-n} = \sum_{n=1}^{\infty} -b^{-n} z^{n}$$

$$= \sum_{n=0}^{\infty} -b^{n-1} z^{-n+1}$$

$$= \sum_{n=0}^{\infty} -b^{-n} z^{n} + 1$$

$$= 1 - \sum_{n=0}^{\infty} (b^{-1}z)^n$$

$$= 1 - \frac{1}{1-|b^{-1}z|}$$

$$= \frac{z}{z-b} \quad \text{part } C, \quad |b^{-1}z| < 1$$

2. $H(z) = \frac{(z-1)(z+1)}{(z-0.5)(z+0.5)}$

Freq response is how LTI systems responds to ~~frequencies~~ points on complex unit circle

$\Downarrow$

$T\{e^{i\omega n}\} = H(e^{i\omega}) e^{i\omega n}$

a) poles: $z = \{\pm 0.5\}$

zeros: $z = \{\pm 1\}$

b) $H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$

, where LCCDE is of form $\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$

$\Downarrow$

$H(z) = \frac{(z-1)(z+1)}{(z-0.5)(z+0.5)} = \frac{Y(z)}{X(z)} \Rightarrow$ ~~$Y(z)(z-1)(z+1) =$~~

$Y(z)(z-0.5)(z+0.5) = X(z)(z-1)(z+1)$

$y[n+2] - 0.25\,y[n] = x[n+2] - x[n] \Leftarrow Y(z)(z^2 - 0.25) = X(z)(z^2 - 1)$

~~$.25 Y[k] - y[k+2] =$~~