0.0.1 ***Determinant of a matrix of order one***

Let $A = [A]$ be the matrix of order 1, then determinant of A is defined to be equal to a.

0.0.2 ***Determinant of a matrix of order two***

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a matrix of order 2×2 ,

then the determinant of A is defined as:

det $(A) = |A| = \triangle = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{21} \end{vmatrix} = a_{11}a_{22} - a_{21} - a_{12}$

Example 1 Evaluate $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$.

Solution We have $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} = 2(2) - 4(-1) = 4 + 4 = 8.$

Example 2 Evaluate $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$

Solution We have

$$\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix} = x(x) - (x+1)(x-1) = x^2 - (x^2 - 1) = x^2 - x^2 + 1$$

0.0.3 ***Determinant of a matrix of order*** 3×3

Determinant of a matrix of order three can be determined by expressing it in terms of second order determinants. This is known as expansion of a determinant along a row(or a column). There are six ways of expanding a determinant of order 3 corresponding to each of three rows($R_1, R_2 and R_3$) and three columns($C_1, C_2 and C_3$) giving the same values as shown below.
Consider the determinant of square matrix $A = [a_{ij}]_{3\times3}$

i.e.,
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$