PS7_Solutions

February 27, 2019

0.0.1 Problem Set 7 Keertana V. Chidambaram

Problem 1

```
In [1]: import pandas as pd
        import numpy as np
        from sklearn.model_selection import train_test_split, LeaveOneOut, KFold, cross_val_sc
        from sklearn.linear_model import LogisticRegression
        from sklearn.metrics import classification_report, mean_squared_error
        from sklearn.metrics import confusion_matrix
        import matplotlib.pyplot as plt
        from matplotlib.ticker import MultipleLocator
        import statsmodels.api as sm
        import scipy.interpolate as si
```

```
In [2]: data = pd.read_csv('data/strongdrink.txt')
```

```
In [3]: data.head()
```

```
Out[3]:    cultivar   alco  malic   ash   alk  magn  tot_phen  flav  nonfl_phen  \
        0         1  14.23   1.71  2.43  15.6   127      2.80  3.06        0.28
        1         1  13.20   1.78  2.14  11.2   100      2.65  2.76        0.26
        2         1  13.16   2.36  2.67  18.6   101      2.80  3.24        0.30
        3         1  14.37   1.95  2.50  16.8   113      3.85  3.49        0.24
        4         1  13.24   2.59  2.87  21.0   118      2.80  2.69        0.39

           proanth  color_int   hue  OD280rat  proline
        0     2.29       5.64  1.04      3.92     1065
        1     1.28       4.38  1.05      3.40     1050
        2     2.81       5.68  1.03      3.17     1185
        3     2.18       7.80  0.86      3.45     1480
        4     1.82       4.32  1.04      2.93      735
```

```
In [4]: #Solution 1.a.
        y = data['cultivar']
        x_vars = ['alco', 'malic', 'tot_phen', 'color_int']
        X = data[x_vars]
        X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.25, random_sta
```

```
In [5]: LogReg = LogisticRegression(solver='newton-cg',multi_class='multinomial').fit(X_train,
        index = pd.DataFrame(['beta0', 'beta1', 'beta2', 'beta3', 'beta4'])
        j1 = pd.DataFrame([LogReg.intercept_[0]] + list(LogReg.coef_[0]))
        j2 = pd.DataFrame([LogReg.intercept_[1]] + list(LogReg.coef_[1]))
        result = pd.concat([index, j1, j2], axis=1)
        result.columns = ['coefficient','j = 1','j = 2']
        result
```

```
Out[5]:   coefficient      j = 1      j = 2
        0       beta0 -24.010989  22.802446
        1       beta1   1.700403  -1.468044
        2       beta2  -0.265605  -0.333053
        3       beta3   1.223894   0.664012
        4       beta4   0.022756  -0.922712
```

```
In [6]: y_pred=LogReg.predict(X_test)
        target_names = ['j=1', 'j=2', 'j=3']
        print(classification_report(y_test, y_pred, target_names=target_names))
```

```
             precision    recall  f1-score   support

         j=1       0.87      1.00      0.93        13
         j=2       1.00      0.90      0.95        21
         j=3       1.00      1.00      1.00        10

   micro avg       0.95      0.95      0.95        44
   macro avg       0.96      0.97      0.96        44
weighted avg       0.96      0.95      0.96        44
```

```
In [7]: error_rates = pd.DataFrame([1 - 0.87, 0, 0])
        error_rates.index = target_names
        error_rates.columns = ['error rate']
        error_rates
```

```
Out[7]:      error rate
        j=1        0.13
        j=2        0.00
        j=3        0.00
```

j=2,3 are the categories with the lowest error rates. But j=3 has the best f1 score. So j=3 has best prediction according to this model.

```
In [8]: data['cultivar'].value_counts()
```

```
Out[8]: 2    71
        1    59
        3    46
        Name: cultivar, dtype: int64
```

j=2 has the most number of observations but it is not the best predicted group in this model.

```
In [9]: print('Test set MSE = ', mean_squared_error(y_test, y_pred))
```

```
Test set MSE =  0.045454545454545456
```

```
In [10]: #Solution 1.b.
```

```
In [11]: Xvars = X.values
         yvars = y.values
         N_loo = Xvars.shape[0]
         loo = LeaveOneOut()
         loo.get_n_splits(Xvars)
         MSE_vec = np.zeros(N_loo)
         y_pred = np.zeros(N_loo)

         for train_index, test_index in loo.split(Xvars):
             X_train, X_test = Xvars[train_index], Xvars[test_index]
             y_train, y_test = yvars[train_index], yvars[test_index]
             LogReg = LogisticRegression(solver='newton-cg',multi_class='multinomial').fit(X_t
             y_pred[test_index] = LogReg.predict(X_test)

             MSE_vec[test_index] = y_test != y_pred[test_index]

         MSE_loo = MSE_vec.mean()
         print('test estimate MSE loocv=', MSE_loo)
```

```
test estimate MSE loocv= 0.07954545454545454
```

```
In [12]: print(classification_report(yvars, y_pred))
```

```
              precision    recall  f1-score   support

           1       0.90      0.93      0.92        59
           2       0.91      0.90      0.91        71
           3       0.96      0.93      0.95        46

   micro avg       0.92      0.92      0.92       176
   macro avg       0.92      0.92      0.92       176
weighted avg       0.92      0.92      0.92       176
```

```
In [13]: error_rates = pd.DataFrame([1 - 0.90, 1 - 0.91, 1 - 0.96])
         error_rates.index = target_names
         error_rates.columns = ['error rate']
         error_rates
```

```
Out[13]:      error rate
         j=1        0.10
         j=2        0.09
         j=3        0.04
```

Clearly, the error rates for all 3 categories have increased as compared to part a.

```
In [14]: #Solution 1.c.
```

```
In [15]: Xvars = X.values
         yvars = y.values
         k = 4
         kf = KFold(n_splits=4, shuffle=True, random_state=10)
         kf.get_n_splits(Xvars)

         MSE_vec_kf = np.zeros(k)
         y_pred = np.zeros(len(yvars))

         k_ind = int(0)
         for train_index, test_index in kf.split(Xvars):
             X_train, X_test = Xvars[train_index], Xvars[test_index]
             y_train, y_test = yvars[train_index], yvars[test_index]
             LogReg = LogisticRegression(solver='newton-cg',multi_class='multinomial').fit(X_t

             y_pred[test_index] = LogReg.predict(X_test)

             err = y_pred[test_index] != y_test
             MSE_vec_kf[k_ind] = err.mean()
             k_ind += 1

         MSE_kf = MSE_vec_kf.mean()
         print('test estimate MSE k-fold (k=4) =', MSE_kf)
```

```
test estimate MSE k-fold (k=4) = 0.09090909090909091
```

```
In [16]: print(classification_report(yvars, y_pred))
```

```
              precision    recall  f1-score   support

           1       0.87      0.93      0.90        59
           2       0.91      0.87      0.89        71
           3       0.96      0.93      0.95        46

   micro avg       0.91      0.91      0.91       176
   macro avg       0.91      0.91      0.91       176
weighted avg       0.91      0.91      0.91       176
```

In [17]:
```
error_rates = pd.DataFrame([1 - 0.87, 1 - 0.91, 1 - 0.96])
error_rates.index = target_names
error_rates.columns = ['error rate']
error_rates
```

Out[17]:

	error rate
j=1	0.13
j=2	0.09
j=3	0.04

The error rates are the same for j = 2,3 but it has increased for j=1.

Problem 2

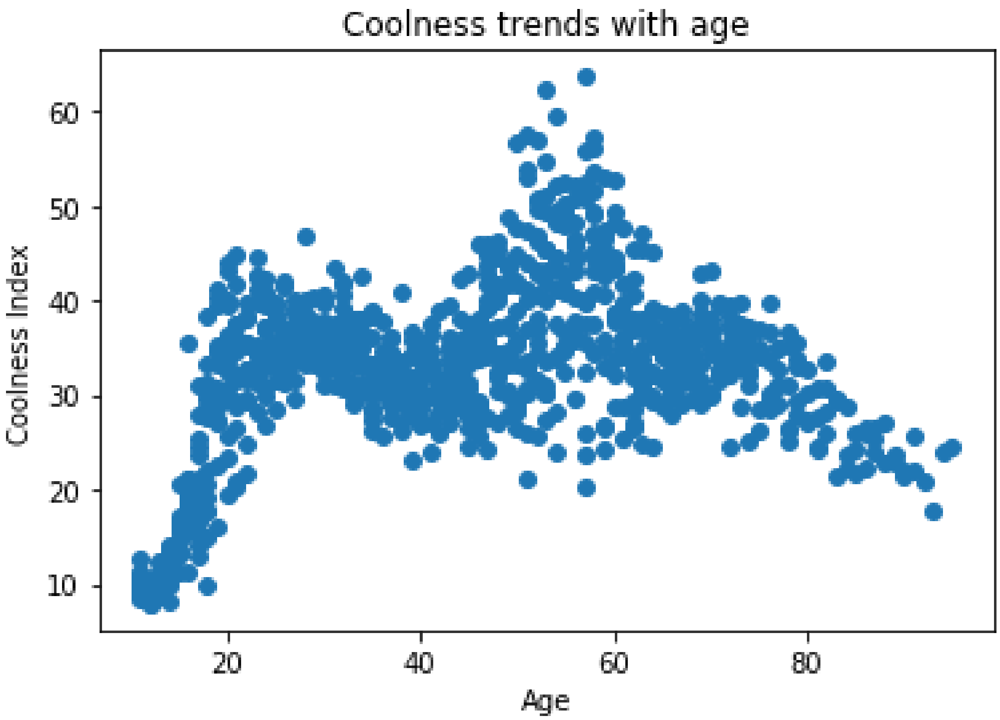
In [18]:
```
data = pd.read_csv('data/CoolIndex.txt', names = ['age', 'coolness'])
```

In [19]:
```
data.head()
```

Out[19]:

	age	coolness
0	11.0	10.981602
1	11.0	11.364925
2	11.0	10.190227
3	11.0	9.903725
4	11.0	8.997918

In [20]:
```
#Solution 2.a.
X = data['age']
y = data['coolness']
plt.scatter(X, y)
plt.xlabel('Age')
plt.ylabel('Coolness Index')
plt.title('Coolness trends with age')
plt.show()
```



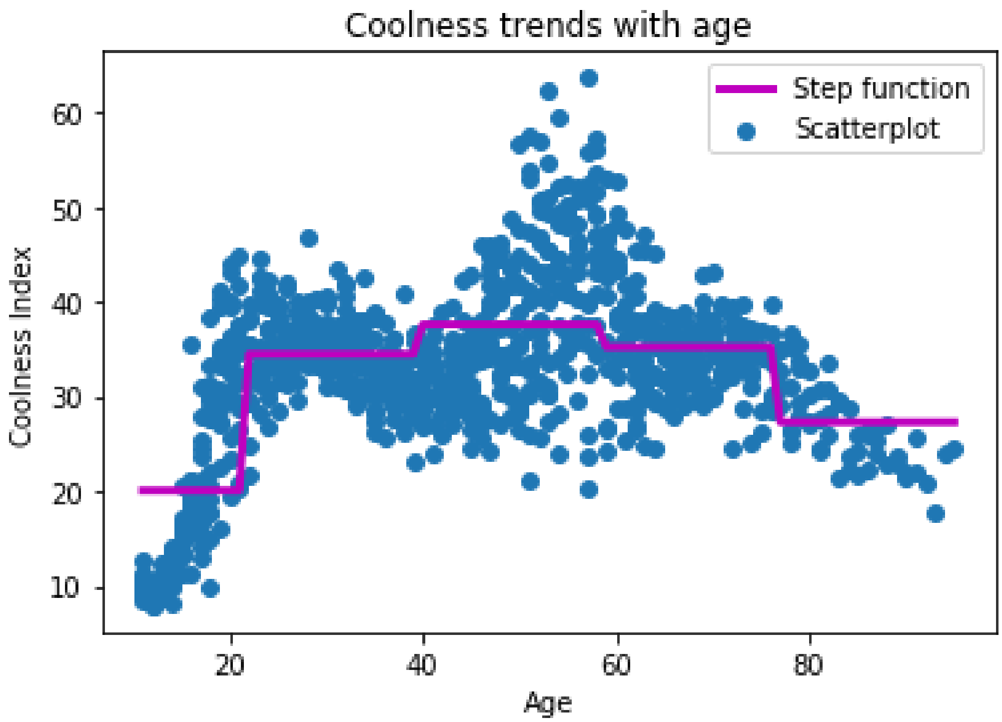
In [21]:
```
#Solution 2.b.
```

In [22]:
```
data['bin1'] = 1
data['bin1'] = data['bin1'].where((data.age >= 11) & (data.age < 22), 0)
data['bin2'] = 1
data['bin2'] = data['bin2'].where((data.age >= 22) & (data.age < 40), 0)
data['bin3'] = 1
data['bin3'] = data['bin3'].where((data.age >= 40) & (data.age < 59), 0)
data['bin4'] = 1
data['bin4'] = data['bin4'].where((data.age >= 59) & (data.age < 77), 0)
data['bin5'] = 1
data['bin5'] = data['bin5'].where((data.age >= 77) & (data.age <= 95), 0)
```

In [23]:
```
xvars = ['bin1', 'bin2','bin3','bin4','bin5']
X = data[xvars]
y = data['coolness']
result = sm.OLS(y, X).fit()
```

In [24]:
```
X = data['age']
y = data['coolness']
plt.scatter(X, y, label='Scatterplot')
plt.plot(X, result.predict(), color = 'm', label = "Step function", linewidth=3.0)
plt.xlabel('Age')
plt.ylabel('Coolness Index')
```

```
plt.title('Coolness trends with age')
plt.legend()
plt.show()
```



In [25]:
```
result.summary()
```

Out[25]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:              coolness   R-squared:                       0.429
Model:                           OLS   Adj. R-squared:                  0.427
Method:                Least Squares   F-statistic:                     178.7
Date:               Wed, 27 Feb 2019   Prob (F-statistic):          3.73e-114
Time:                       01:28:08   Log-Likelihood:                -3214.5
No. Observations:                956   AIC:                             6439.
Df Residuals:                    951   BIC:                             6463.
Df Model:                          4
Covariance Type:           nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
bin1          20.1025      0.562     35.746      0.000      18.999      21.206
bin2          34.4758      0.431     80.006      0.000      33.630      35.321
```

```
bin3          37.6351      0.424     88.814      0.000      36.804      38.467
bin4          35.2254      0.485     72.560      0.000      34.273      36.178
bin5          27.2964      0.936     29.175      0.000      25.460      29.132
==============================================================================
Omnibus:                       80.102   Durbin-Watson:                   1.236
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              101.718
Skew:                           0.714   Prob(JB):                     8.17e-23
Kurtosis:                       3.719   Cond. No.                         2.21
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spec
"""
```

Coefficients for bin 1 to 5 correspond to the estimated value for beta 1 to 5 respectively. A 73 year old belongs in bin 4, thus predicted value = 35.2254, alternatively we can also show:

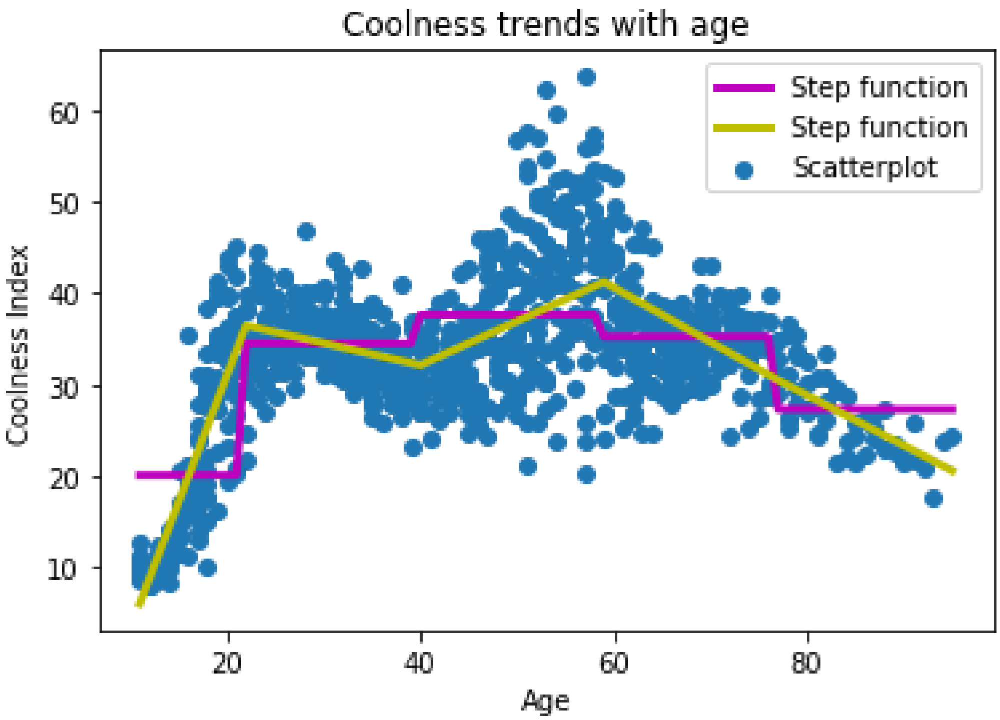
```
In [26]: result.predict([0, 0, 0, 1, 0])

Out[26]: array([35.22540004])

In [27]: #Solution 2.c.

In [28]: data2 = data.groupby('age').mean()
         data2['age'] = data2.index
         knots = [22, 40, 59, 77]
         spl_lin_cool = si.LSQUnivariateSpline(np.array(data2['age']), np.array(data2['coolness

In [29]: X = data['age']
         y = data['coolness']
         X2 = data2['age']
         plt.scatter(X, y, label='Scatterplot')
         plt.plot(X, result.predict(), color = 'm', label = "Step function", linewidth=3.0)
         plt.plot(X2, spl_lin_cool(X2), color = 'y', label = "Step function", linewidth=3.0)
         plt.xlabel('Age')
         plt.ylabel('Coolness Index')
         plt.title('Coolness trends with age')
         plt.legend()
         plt.show()
```



Predicted coolness of a 73 year-old:

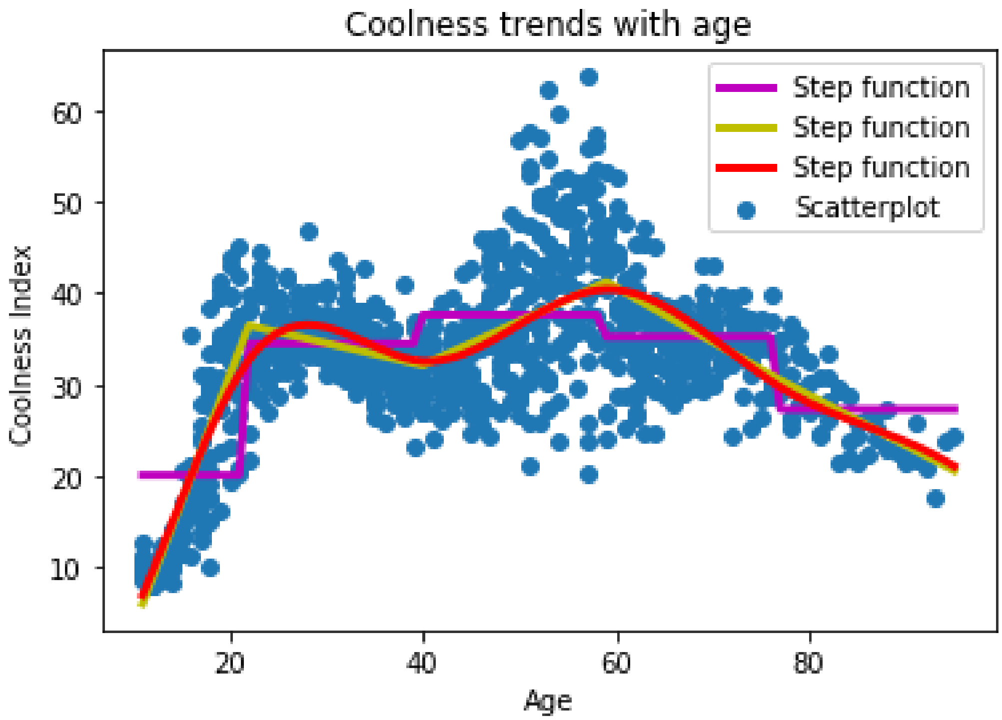
```
In [30]: spl_lin_cool(73)

Out[30]: array(32.86784862)

In [31]: #Solution 2.d.

In [32]: data2 = data.groupby('age').mean()
         data2['age'] = data2.index
         knots = [22, 40, 59, 77]
         spl_cub_cool = si.LSQUnivariateSpline(np.array(data2['age']), np.array(data2['coolness

In [33]: X = data['age']
         y = data['coolness']
         X2 = data2['age']
         plt.scatter(X, y, label='Scatterplot')
         plt.plot(X, result.predict(), color = 'm', label = "Step function", linewidth=3.0)
         plt.plot(X2, spl_lin_cool(X2), color = 'y', label = "Step function", linewidth=3.0)
         plt.plot(X2, spl_cub_cool(X2), color = 'r', label = "Step function", linewidth=3.0)
         plt.xlabel('Age')
         plt.ylabel('Coolness Index')
         plt.title('Coolness trends with age')
         plt.legend()
         plt.show()
```



Predicted coolness of a 73 year-old:

```
In [34]: spl_cub_cool(73)

Out[34]: array(32.64230107)

In [ ]:
```