Homework #2
COMP 582/ELEC 512
GRADUATE DESIGN AND ANALYSIS OF ALGORITHMS
Fall 2023

Due on: Thursday, September 7th, 6pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: Gradescope.

Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

August 30, 2023

Problem 1 (3 points)

A sequence $a_1, a_2, ..., a_n$ has a key element x if equal to more than half of the elements in the sequence are all the same and equal to x. Give a divide and conquer algorithm that finds and returns a dominant element in a sequence of n numbers or return "NULL" if no such element exists. You algorithm should run in $O(nlogn)$ time. Prove the correctness of your algorithm and analyze its runtime.

Problem 2 (2 points)

Let A and B be two sorted arrays of n elements each. We can easily find the median element in A – it is just the element in the middle – and similarly we can easily find the median element in B. (Let us define the median of $2k$ elements as the element that is greater than $k-1$ elements and less than k elements.) However, suppose we want to find the median element overall – i.e., the nth smallest in the union of A and B.

Give an $O(\log n)$ time algorithm to compute the median of $A \cup B$. You may assume there are no duplicate elements.

Prove the correctness of your algorithm and analyze its runtime.

Use divide and conquer approach to solve the problem.