Homework #5
COMP 582
GRADUATE DESIGN AND ANALYSIS OF ALGORITHMS
Fall 2023

Due on: Saturday, September 30th, 8pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: Gradescope.

Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

September 22, 2023

Problem 1

There are n coins given by a list $P = \{p_1, \ldots, p_n\}$ where $p_i \in \{1, \ldots, K\}$ is the price of the coin i. Design an algorithm that checks if it is possible to break the set of all items P into two parts P_A and P_B such that $P_A \cup P_B = P$, $P_A \cap P_B = \emptyset$ and $\sum_{i \in P_A} p_i = \sum_{i \in P_B} p_i$?

Problem 2

Assume you want to spend exactly A dollars. There are t items where each of them has unlimited supply and they are worth $C_1, C_2, ..., C_t$ dollars accordingly. Design a dynamic programming algorithm to compute the number of ways to spend exactly A dollars.

For example, when $A = 4$ and $C = \{1, 2\}$, you have three ways: $\{1, 1, 1, 1\}$, $\{1, 1, 2\}$, $\{2, 2\}$.

Problem 3

You are given an array $A[1..n]$, which consists of n randomly permutted distinct integers. An ith element of this array, $A[i]$, is said to be a *local spike*, if it is larger than all of its preceding elements (in other words, for all $j < i$, $A[i] > A[j]$). Show that the expected number of local spikes in A is $O(\log n)$. Hint #1: any array A has at least one local spike, since the first element of A trivially satisfies $A[1] > A[j]$ for all $j < 1$, because no such index j exists. Hint #2: a review of CLRS Appendix C and the related material will be helpful.