# Homework #1
# COMP 582
# GRADUATE DESIGN AND ANALYSIS OF ALGORITHMS
# Fall 2023

**Due on:** August 31st, 6pm
**Late submissions:** will NOT be accepted
**Format:** Please start each problem on a new page.
**Where to submit:** On Gradescope.

August 24, 2023

## Problem 1 (2 points)

For each statement below explain if it is true or false and prove your answer. Be as precise as you can. The base of $\log$ is 2 unless stated otherwise.

1. $\sqrt{n} = \Theta(2 \log n^2)$
2. $3n \log n + n = O(\frac{n^2-n}{2})$
3. Let $f$ and $g$ be positive functions. If $f(n) + g(n) = \Omega(f(n))$ then $g(n) = O((f(n))^2)$.

## Problem 2 (2 points)

Prove by induction that $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ for all $n \geq 1$.

## Problem 3 (2 points)

Resolve the following recurrences. Use Master theorem, if applicable. In all examples, assume that $T(1) = 1$. To simplify your analysis, you can assume that $n = a^k$ for some $a, k$.

1. $T(n) = 8T(n/4) + n$
2. $T(n) = 8T(n/2) + n^3$
3. $T(n) = 2T(n/3) + \log n$
4. $T(n) = 30T(n/30) + n$