Homework #3
COMP 582
GRADUATE DESIGN AND ANALYSIS OF ALGORITHMS
Fall 2023

Due on: Thursday, September 14th, 6pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: Gradescope.

Please type your answers; handwritten assignments will not be accepted.
To get full credit, your answers must be explained clearly,
with enough details and rigorous proofs.

September 7, 2023

Problem 1

In class we considered the `Select` algorithm, which determines the ith smallest element in an array of size n in $O(n)$ time for the worst case input. The first step of this algorithm is division into $n/5$ groups of 5 elements each. Consider other two versions of this algorithm: the first one uses division into $n/3$ groups of 3 elements each and the second one uses division into $n/7$ groups of 7 elements each. Otherwise both algorithms implement the same routine as `Select`. Which algorithm is **asymptotically** faster, the one with division into groups of 3, groups of 5 (standard `Select`), or groups of 7? Prove your statement.

Problem 2

We will say that the pivot provides $x|n-x$ separation if x elements in the array are smaller than the pivot, and $n-x$ elements are larger than the pivot. Suppose Bob

knows a secret way to find a good pivot with $\frac{n}{3} | \frac{2n}{3}$ separation in constant time. But at the same time Alice knows her own secret technique, which provides separation $\frac{n}{4} | \frac{3n}{4}$, her technique also works in constant time. Alice and Bob applied their secret techniques as subroutines for the QuickSort algorithm. Whose algorithm works **asymptotically** faster? Prove your statement.