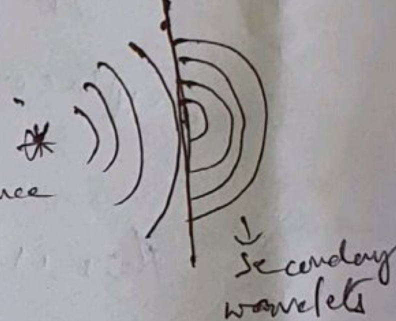
UNIT - I
Wave optics

Interference of Light

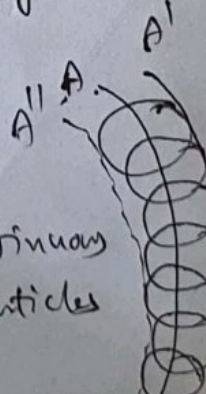
Huygen's Principle

1) Each point on the wavefront acts as a centre of new disturbance and emits its own set of spherical waves called secondary wavelets.

These secondary wavelets travel in all the directions with the velocity of light so long as they move in the same medium.



2) The envelope or the locus of these wavelets in the forward direction gives the position of the new wavefront at any subsequent time.



A. wavefront: "A wavefront at any time is defined as a continuous locus of all the neighbouring particles vibrating in the same phase."

* A spherical wavefront is obtained for a point source placed at a finite distance and the wavefront is plane when the point source is kept at infinite distance.
* An extended source of the light gives cylindrical wavefronts.

Superposition of waves

"The resultant displacement of a particle of the medium acted upon by two or more waves simultaneously is the algebraic sum of the displacements of the same particle due to individual waves, in the absence of the others".

If y_1 and y_2 are the individual displacements, then the resultant is given by

$$R = y_1 \pm y_2$$

This is known as principle of superposition.

If y_1 and y_2 are in the same directions,

$$R = y_1 + y_2$$

If y_1 and y_2 act opposite to each others,

$$R = y_1 - y_2$$

Interference of light:

* "The modification in the distribution of intensity of light in the region of superposition is called interference".

Interference can be due to
1) Division of wavefront
2) Division of amplitude

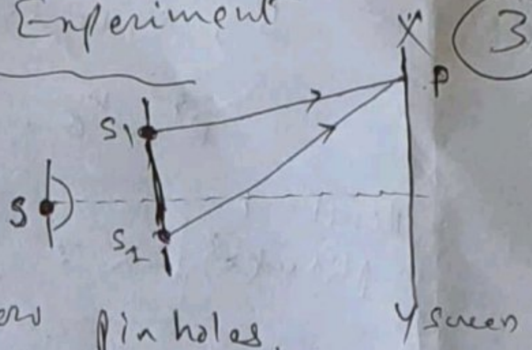
1) Division of wavefront:
The incident wavefront is divided into two parts by utilising the phenomena of reflection, refraction or diffraction. These two parts of the same wavefront travel unequal distances and reunite at some angle to produce interference bands.
Ex: Fresnel's biprism, Lloyd's mirror

2) Division of Amplitude:
The amplitude of incoming beam is divided into two parts either by parallel reflection or refraction. These divided parts reunite after travelling different paths and produce interference.
Ex: Newton's rings, Michelson's interferometer

---

coherent waves: The two light waves are said to be coherent, if they maintain constant phase difference with time.

Young's Double Slit Experiment



S is a source of monochromatic light and S_1 and S_2 are two narrow pin holes.

The waves arriving at S_1 and S_2 from S will be in phase at all the times. We shall investigate the resultant intensity at point P on the screen.

Let a_1 and a_2 be the amplitudes of the waves from S_1 and S_2 and δ be the phase difference between the two waves reaching at point P. If y_1 and y_2 are the displacement of the two waves, then,

$$y_1 = a_1 \sin\omega t \quad \text{— (1)}$$

and

$$y_2 = a_2 \sin(\omega t + \delta) \quad \text{— (2)}$$

The resultant displacement will be

$$y = y_1 + y_2$$

$$y = a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$$

$$y = a_1 \sin\omega t + a_2 \sin\omega t \cos\delta + a_2 \cos\omega t \sin\delta$$

$$y = \sin\omega t\,[a_1 + a_2 \cos\delta] + \cos\omega t\,[a_2 \sin\delta]$$

Let $a_1 + a_2 \cos\delta = R\cos\theta$ (3) and $a_2 \sin\delta = R\sin\theta$ (4)

$$y = \sin\omega t\, R\cos\theta + \cos\omega t\, R\sin\theta \quad \text{— (5)}$$

$$\Rightarrow y = R\sin(\omega t + \theta)$$

By squaring eqs. (4) and (5) and adding,

$$R^2\cos^2\theta + R^2\sin^2\theta = (a_1 + a_2\cos\delta)^2 + (a_2\sin\delta)^2$$

Resultant amplitude is

$$\boxed{R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\delta}} \quad - (6)$$

The resultant intensity $I = R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$ — (7)

Max Intensity:

From $I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

I is maximum when $\cos\delta = 1$, i.e.,

$$\delta = 2n\pi, \quad n = 0, 1, 2, 3 \ldots$$

or path diff $\Delta = \frac{\lambda}{2\pi} \times \delta$

$$\Delta = n\lambda$$

and $I_{max} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2$

Min Intensity:

From $I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

I is minimum when $\cos\delta = -1$, i.e.,

$$\delta = (2n+1)\pi, \quad n = 0, 1, 2, 3 \ldots$$

or path diff $\Delta = (2n+1)\frac{\lambda}{2}$

$I_{min} = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2$

4

Newton's Rings

Experimental Arrangement:

As shown in the Fig. 1, a plano-convex lens of large radius of curvature is kept on a plane glass plate.

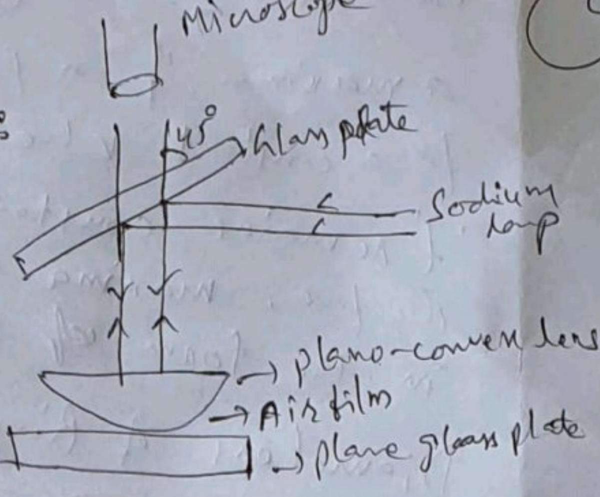


Fig. 1

Light from sodium lamp falls on a glass plate which is at an angle 45° with the vertical. The light from the lamp falls on the glass plate at is reflected towards the air film enclosed between the plano-convex lens and the plane glass plate.

The two rays which interfere are (Fig. 2)

1. A part of the light reflected from the curved surface of the lens
2. A part of the light reflected from the top of the plane glass plate

Formation of Newton's Rings:

The two rays (1) and (2) are responsible for the interference.

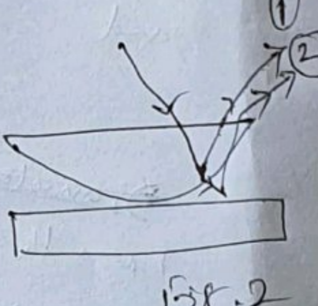


Fig. 2

The path diff $\Delta = 2\mu t \cos r + \frac{\lambda}{2}$

For air film $\mu = 1$ and for normal incidence $\cos r = 1$

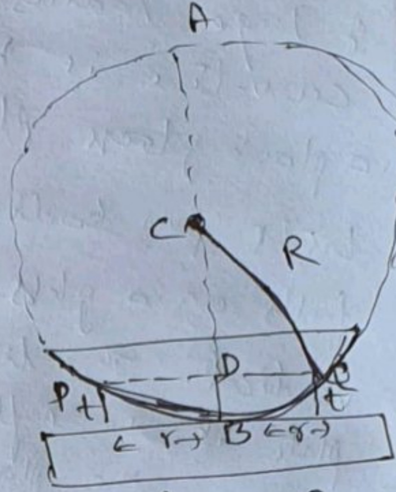
$\Rightarrow \Delta = 2t + \frac{\lambda}{2}$ (t is thickness of the air film)

At the point of contact, $t = 0$ and hence $\Delta = \frac{\lambda}{2}$ which is a condition for minima. Therefore, the central spot will be a dark spot.

From $2t+\frac{\lambda}{2}=n\lambda$ (maximum condition), a maximum of a particular order 'n' will occur for a constant value of 't'. In this case, 't' remains constant along a circle. Therefore, maxima and also minima will be in the form of circle.

Diameters of Rings

Let R be the radius of curvature of the plano-convex lens and 'r' be the radius of Newton's Ring corresponding to the constant air-film thickness 't'.



(Fig-3)

path diff $\Delta = 2t+\frac{\lambda}{2}$

$2t+\frac{\lambda}{2} = n\lambda$ (Maximum or bright ring)

$\Rightarrow 2t = (2n-1)\frac{\lambda}{2}$ (Bright Ring) —(1)

and $2t+\frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$ (minima or dark ring) —(2)

$\Rightarrow 2t = n\lambda$ (Dark ring)

~~Diameter of Dark Ring~~ ~~Diameter of Bright Ring~~

From the property of the circle, (Fig-3)

$PD \times DQ = BD \times DA$

$r \times r = t \times (2R-t)$

$\Rightarrow 2t = \frac{r^2}{R}$ —(3)

For Dark ring {from (2) and (3)},

$\frac{r^2}{R} = n\lambda$

$\Rightarrow r^2 = n\lambda R$

$\Rightarrow \boxed{D_n^2 = 4n\lambda R}$ —(4)

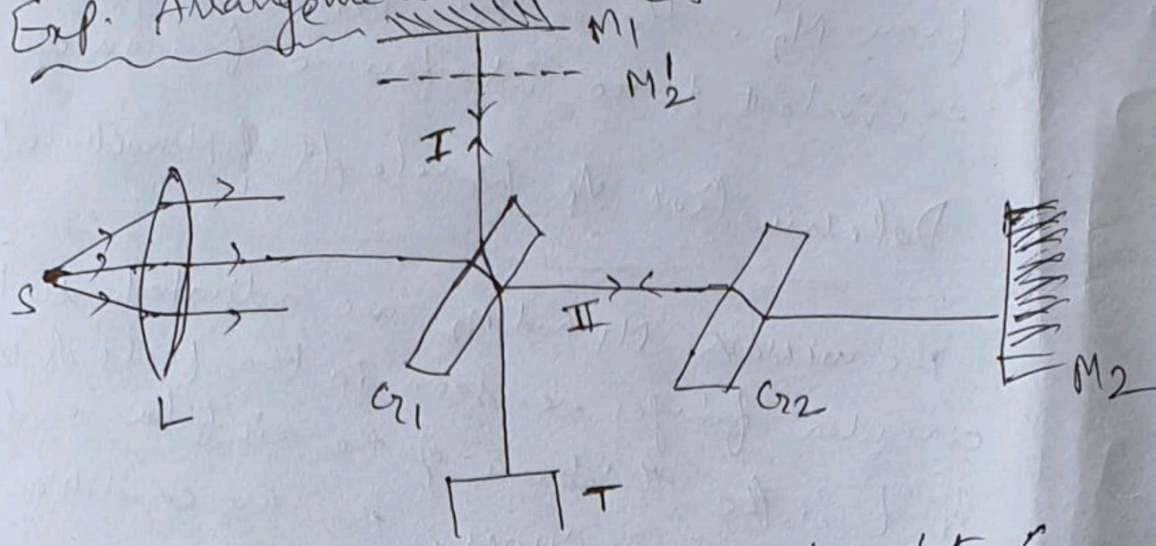
For Bright ring, from (1) and (3), we get

$\boxed{D_n^2 = (2n-1)2\lambda R}$ —(5)

Michelson Interferometer

Principle: Interference due to division of amplitude.

Exp. Arrangement and Working



Light from source S falls on a glass plate G_1 which has a reflective coating on the back. The coating reflects half of the beam to the plane mirror M_1. The transmitted beam travels to plane mirror M_2. The beams reflected by mirrors M_1 and M_2 are recombined again after passing through G_1. The reflected beam (I) goes through the glass plate G_1 three times (see fig.). The transmitted beam (II) goes through G_1 only once. Therefore, the beam I travels more distance that [illegible] beam II.

To make the two optical paths equal, a compensating glass plate G_2 with same thickness as G_1 is introduced in the path of beam II.

Let M_2' be the virtual image of M_2 formed by the reflected beam at the plate G_1. One of the interfering beams comes by reflection from M_1 and the other appears to come by reflection from M_2'. Hence, the system is optically equivalent to the interference from an air film.

Determination of wavelength of Monochromatic Light

The mirrors M_1 and M_2 are adjusted so that circular fringes are seen in the field of view. If 't' is the thickness of the air film enclosed between the two mirrors, then the condition for bright fringes is $2\mu t \cos r + \frac{\lambda}{2} = n\lambda$

Here $\mu = 1$, $\cos r = 1$ (for normal incidence)
(for air film)

$$\Rightarrow 2t + \frac{\lambda}{2} = n\lambda \quad \text{(central bright spot)}$$

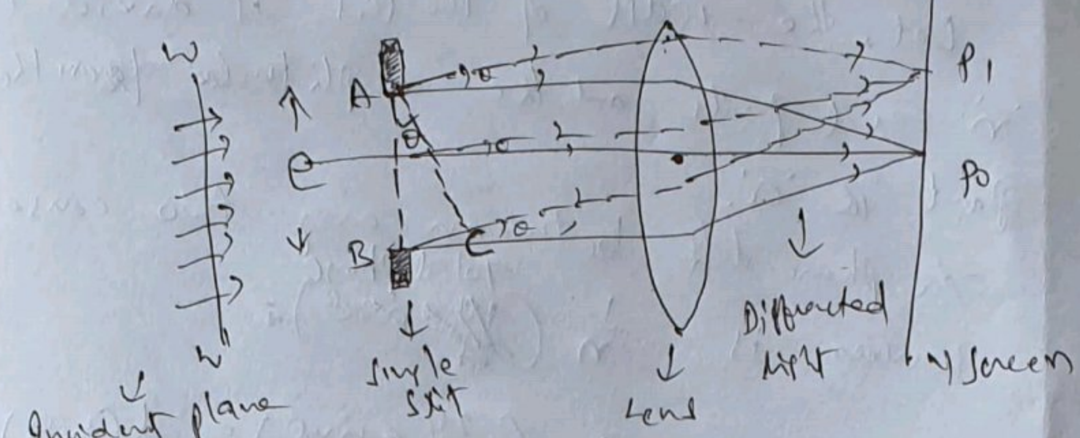
The position of the mirror M_1 is noted by taking the reading R_1 on the micrometer. When the mirror M_1 is moved through a distance $\frac{\lambda}{2}$ then the next bright spot appears. Further, the mirror M_1 is moved so that N bright spots appear and the reading R_2 is taken.

Let $x = R_1 \sim R_2$

and $x = N\frac{\lambda}{2}$

$$\therefore \boxed{\lambda = \frac{2x}{N}}$$

Fraunhofer Diffraction at Single slit



Let a plane wavefront WW' of monochromatic light of wavelength λ propagating normally to the slit be incident on it.

- The secondary wavelets without any deviation are focused at point P_0 and is a bright central image.
- The secondary wavelets with 'θ' deviation to the normal are focussed at point P_1. This may be a point of minimum or maximum intensity depending upon the path difference between the secondary waves coming from the corresponding points of the wavefront.

Expression for the Intensity:

In order to find the intensity at P_1, we draw a normal AC onto the path of wavelets with deviation. Then the path difference between secondary wavelets from A and B in the direction θ is given by

$$\sin\theta = \frac{BC}{AB}$$

$$\Rightarrow BC = AB\sin\theta$$

$$AB = e \Rightarrow BC = e\sin\theta \quad \text{--- (1)}$$

The corresponding phase difference $= \frac{2\pi}{\lambda} e \sin\theta$ — (2)

Let the width of the slit is divided into n equal parts and the amplitude from the each part is 'a'.

The phase diff between any two consecutive waves is $\frac{1}{n}$ (Total phase) $=$

$$= \frac{1}{n}\left(\frac{2\pi}{\lambda} e \sin\theta\right) = d \text{ (say)} \quad \text{— (3)}$$

The resultant amplitude of all the waves, i.e., each having an amplitude a and consecutive phase difference 'd' is given by

$$R = a \frac{\sin nd/2}{\sin d/2}$$

$$\Rightarrow R = a \frac{\sin(\pi e \sin\theta/\lambda)}{\sin(\pi e \sin\theta/n\lambda)}$$

$$\Rightarrow R = a \frac{\sin\alpha}{\sin\frac{\alpha}{n}}, \quad \alpha = \frac{\pi e \sin\theta}{\lambda}$$

$$\Rightarrow R = a \frac{\sin\alpha}{\alpha/n}, \quad (\alpha/n \text{ is very small})$$

$$\Rightarrow R = na \frac{\sin\alpha}{\alpha}$$

∴ The resultant amplitude $\boxed{R = A \frac{\sin\alpha}{\alpha}}$, $A = na$ (constant) — (4)

& the resultant Intensity

$$I = R^2 = A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{— (5)}$$

Conditions:

I. Principal Maximum

$$R = \frac{A \sin\alpha}{\alpha}$$

$$\Rightarrow R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \cdots\right]$$

$$\Rightarrow R = A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \cdots\right]$$

'R' will be maximum when $\alpha = 0$

$$\Rightarrow \alpha = \frac{\pi e \sin\theta}{\lambda} = 0$$

$$\Rightarrow \sin\theta = 0$$

$$\text{or } \boxed{\theta = 0}$$

When $\theta = 0$, i.e, the secondary wavelets normal to the slit are focussed at a point. There a maximum will be formed and this is called principal Maximum.

II. Minima:

from $R = \frac{A \sin\alpha}{\alpha}$

The intensity will be minimum when $\sin\alpha = 0$

$$\Rightarrow \alpha = \pm n\pi, \quad n = 1, 2, 3, \cdots$$

$$\Rightarrow \frac{\pi e \sin\theta}{\lambda} = \pm n\pi$$

$$\Rightarrow \boxed{e \sin\theta = \pm n\lambda}, \quad n = 1, 2, 3, \cdots$$

Secondary Maxima:

From $I = A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2$

$$\Rightarrow \frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

Solving, we get, $\alpha = \tan\alpha$

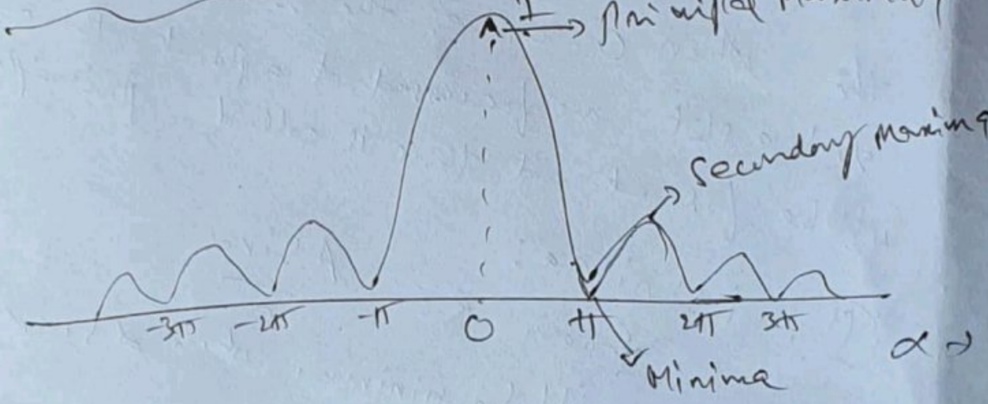
The solutions of the above equation are:

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots \text{ (approximately)}$$

$$\Rightarrow \alpha = \pm\frac{(2n+1)\pi}{2} \text{ (approx)}$$

or $\boxed{e\sin\theta = \pm\frac{(2n+1)\lambda}{2}}$, $n = 1, 2, 3, \ldots$

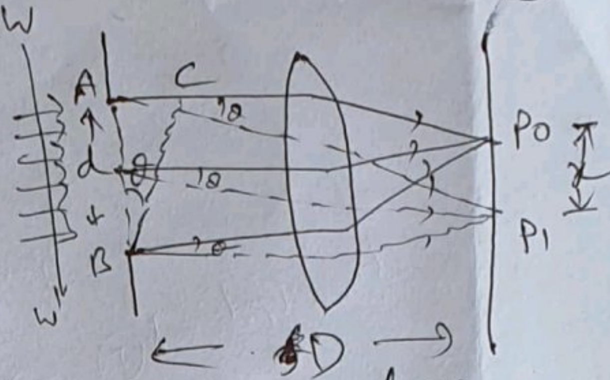
Intensity distribution Graph



Diffraction at circular Aperture

(8)

Let AB is a circular aperture of diameter 'd'. A plane wavefront WW' of a monochromatic light of wavelength λ propagating normally to the circular aperture is incident on it.



The wavelets travelling ~~along~~ along the normal are focused at P_0 which is the position of central maximum. The wavelet inclined at an angle 'θ' with the normal to the aperture meet at P_1 on the screen and $P_0P_1 = x$.

The path diff between extreme waves from points A and B is $AC = AB\sin\theta = d\sin\theta$

Proceeding in the similar way as in case of diffraction at single slit, we get.

$\theta = 0$ Principal Maximum

$d\sin\theta = m\lambda,\ m = 1, 2, 3 \ldots$ Minima

$d\sin\theta = (2m+1)\dfrac{\lambda}{2}$ (approx), $m = 1, 2, 3 \ldots$ Secondary Maxima

Here, the diffraction pattern consists of a central bright disc, called Airy's disc and is surrounded by alternate ~~bright band~~ dark and bright concentric rings called Airy's rings.

If the lens is kept very close to the aperture, then the screen distance $D \simeq f$, focal length of the lens.

Now, $\sin\theta = \theta = \dfrac{x}{f}$ — (1)

From Minima condition,

$d\sin\theta = m\lambda$

For $m = 1 \Rightarrow d\sin\theta = \lambda$ $\sin\theta = \dfrac{\lambda}{d}$ or $\theta = \dfrac{\lambda}{d}$ — (2)

From (1) & (2),

$x = \dfrac{f\lambda}{d}$

or the exact value,

$$\boxed{x = \frac{1.22 f\lambda}{d}}$$

x is the radius of Airy's disc.

Diffraction Grating

* A diffracting grating is a device made of transparent glass or plastic which contains a number of parallel and equidistant slits.

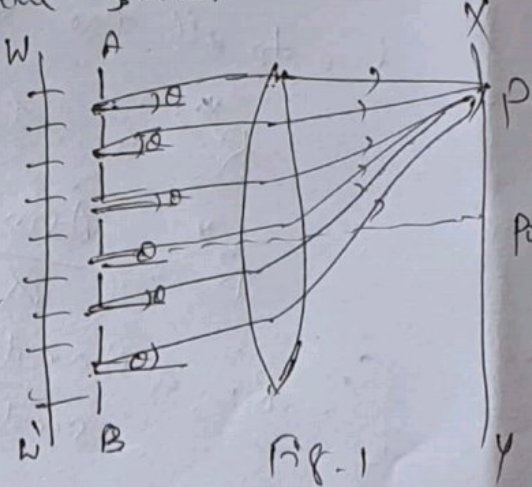


Fig. 1

As shown in Fig. 1, a plane transmission grating AB is placed perpendicular to the plane of the paper.

The width of the slit is 'e' and 'd' is the width of each opaque part. Then (e + d) is known as 'grating element'. Let there be 'N' number of slits.

Each slit sends secondary wavelets in all directions. The wavelets without any deviation are focused at point P_0 and forms central maximum. The inclined rays (with angle θ) are focused at point P_1.

To find the intensity at P_1, let us consider the case of single slit diffraction.

The resultant amplitude in case of a single slit diffraction $= \frac{A \sin\alpha}{\alpha}$, $\alpha = \frac{\pi e \sin\theta}{\lambda}$

In the present case, there are 'N' such waves and the corresponding path difference between two consecutive slits is $(e+d)\sin\theta$

[$\because$ $e\sin\theta$ is path diff in case of the single slit diffraction]

The corresponding phase diff $= \frac{2\pi}{\lambda}(e+d)\sin\theta$

$= 2\beta$ (say)

By vector addition, the resultant amplitude due to all ~~the~~ 'N' waves will be,

$$\cancel{R} = \cancel{a \sin}$$

$$R^* = \left[\frac{A\sin\alpha}{\alpha}\right]\left[\frac{\sin N\beta}{\sin\beta}\right] \quad \text{--- (1)}$$

$\left[\because R = a\frac{\sin n d/2}{\sin d/2}\right.$, here $a = A\sin\alpha$, $n = N$ and $\left. d = 2\beta\right]$

and $$I = R^{*2} = \left[\frac{A\sin\alpha}{\alpha}\right]\left[\frac{\sin N\beta}{\sin\beta}\right]^2 \quad \text{--- (2)}$$

Intensity Distribution:

principal maxima

~~The Inten~~ from $R^* = \left[\frac{A\sin\alpha}{\alpha}\right]\left[\frac{\sin N\beta}{\sin\beta}\right]$

The intensity will be maximum when $\sin\beta = 0$

$\Rightarrow \beta = \pm n\pi, \quad n = 0, 1, 2, 3 \ldots$

$\Rightarrow \frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$

or $\boxed{(e+d)\sin\theta = \pm n\lambda}$

$n = 0, 1, 2, 3 \ldots$

From Hospital's rule, for $\beta \to \pm n\pi$ $\left[\frac{\sin N\beta}{\sin\beta}\right]^2 = N$

10

Minima

$$R^* = \left(\frac{A \sin\alpha}{\alpha}\right)\left[\frac{\sin N\beta}{\sin\beta}\right]$$

R^* will be minimum for $\sin N\beta = 0$

$$\Rightarrow N\beta = \pm m\pi$$

$$\Rightarrow \boxed{N(e+d)\sin\theta = \pm m\lambda}$$

where 'm' has all integral values except $0, N, 2N, \ldots nN$.

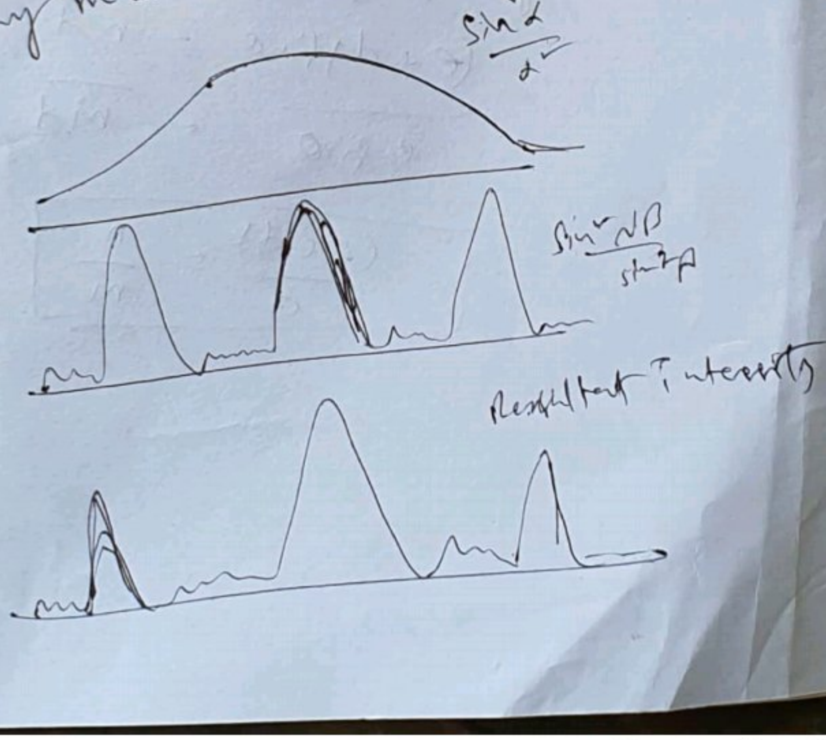
Secondary Maxima

From $I = \left(\frac{A \sin\alpha}{\alpha}\right)^2 \left[\frac{\sin N\beta}{\sin\beta}\right]^2$

$$\frac{dI}{d\beta} = 0$$

$$\Rightarrow N\tan\beta = \tan N\beta$$

The roots of the above equation represent secondary maxima.



I. Maximum Number of Orders:

For principal maxima,

$$(e+d)\sin\theta = n\lambda$$

$$\Rightarrow \quad n = \frac{(e+d)\sin\theta}{\lambda}$$

'n' will be maximum for $\sin\theta = 1$,

$$\boxed{\therefore n_{max} = \frac{(e+d)}{\lambda}}$$

II. Absent Spectra (or) Missing Orders:

Principal maxima in case of a grating:

$$(e+d)\sin\theta = n\lambda. \quad \text{--- (1)}$$

Minima in case of a single slit:

$$e\sin\theta = m\lambda, \quad \text{--- (2)}$$

$m = 1, 2, 3 \dots$

If these two conditions satisfy simultaneously, a particular maxima of order 'n' will be missing in the grating spectrum.

$$\frac{(e+d)\sin\theta}{e\sin\theta} = \frac{n\lambda}{m\lambda}$$

$$\boxed{\frac{(e+d)}{e} = \frac{n}{m}}$$

Rayleigh's Criterion of Resolution

"Two sources are resolvable by an optical instrument when the central maximum in the diffraction pattern of one falls over the first minimum in the diffraction of the other and vice-versa".

Resolving power of

Two objects that can be just resolved according to Rayleigh's criterion should have an angular seperation,

$$\boxed{\theta_R = 1.22 \frac{\lambda}{d}}$$

If $\theta > \theta_R$, two objects can be resolved.

If $\theta < \theta_R$, two objects can't be resolved.

Resolving Power of a Grating

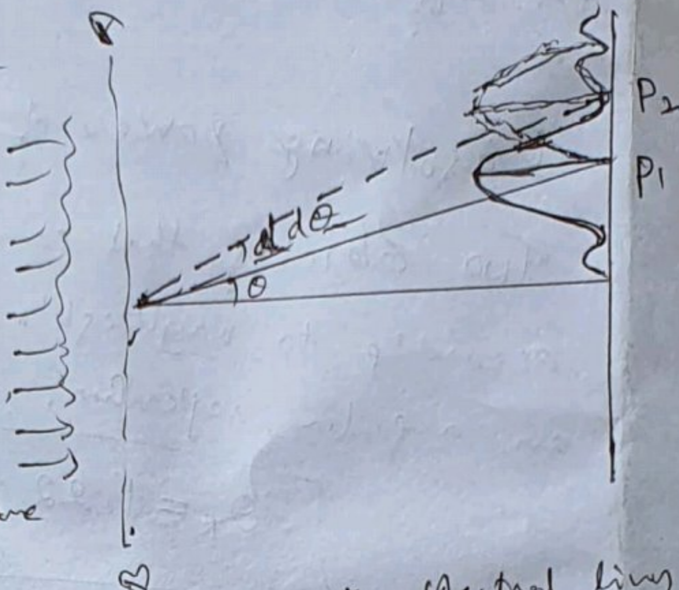
Def. The resolving power of a grating is defined as the capacity to form separate diffraction maxima of two wavelengths which are very close to each other".

This is measured by $\lambda/d\lambda$.

Expression:

Let a beam of light having two wavelengths λ and $\lambda+d\lambda$ is normally incident on the grating.

~~As shown in the figure, P_2 corresponds to the first minima due to~~



According to Rayleigh's criterion, the spectral lines can be resolved if one principal maximum falls on the first minima due to the second wavelength.

In the fig, P_2 corresponds to

(i) principal maximum of $(\lambda+d\lambda)$ in the direction $(\theta+d\theta)$

and (ii) first minimum of λ in the direction $(\theta+d\theta)$

Further first minimum of 'λ' is given by,

$$N(e+d)\sin(\theta+d\theta) = (nN+1)\lambda \quad \text{—(1)}$$

[$\because$ $N(e+d)\sin\theta = m\lambda$ is condition for minima and m does not take $0, N, 2N, \ldots$]

Principal maximum due to $(\lambda+d\lambda)$ is given by,

$$(e+d)\sin(\theta+d\theta) = n(\lambda+d\lambda) \quad \text{—(2)}$$

[$\because$ $(e+d)\sin\theta = n\lambda$ is principal maxm]

Multiplying eq.(2) by N and solving, we get

Resolving power $\boxed{\frac{\lambda}{d\lambda} = nN}$