UNIT-2
AC CIRCUITS

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UNIT-II: AC Circuits

- Representation of sinusoidal waveforms
- Peak and RMS values
- Phasor Representation
- Real Power, Reactive Power, Apparent Power
- Power Factor
- Analysis of single-phase, ac circuits consisting of R
- Analysis of single-phase, ac circuits consisting of L
- Analysis of single-phase, ac circuits consisting of C
- Analysis of single-phase, ac circuits consisting of RL
- Analysis of single-phase, ac circuits consisting of RC
- Analysis of single-phase, ac circuits consisting of RLC

Topics to be Discussed

- Alternating Current
- Representation of AC
- Generation of AC
- Standard terminology
- Average value of pure sine wave, full rectified sine wave and half rectified sine wave
- A Sinusoid.
  - Some Definitions.
  - Phase Difference.
  - Physical Model for a Sinusoid.
- RMS or Effective Value.
- Concept of Phasors.
  - Operations on Phasors.
- Additions of Phasors Using Complex Numbers.
- Power and Power Factor.
- Purely Resistive Circuit.
- Purely Inductive Circuit.
  - Inductive Reactance.
- Purely Capacitive Circuit.
  - Capacitive Reactance

AC Circuits Alternating Current

- An alternating current (AC) is a current which changes periodically with respect to time both in magnitude and direction.
- The single greatest advantage of alternating current is that AC can be transformed and DC can not be transformed.
- The other advantage of AC is that it can be controlled by wide range of components. Ex: R, L and C
- It can be represented in three ways.

1. Graphical Form…..i) Wave form ii) Phasor form
2. Magnitude Form…i) Instantaneous ii) Peak iii) Average iv) RMS
3. Mathematical Form…i) Rectangular ii) Trigonometric iii) Polar

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Generation of AC voltage:

- The principle used for the generation of AC voltage is faradays law of electromagnetic induction.
- It states that whenever there is a relative motion between conductor and magnetic field in which it is kept, an emf gets induced in conductor. The relative motion may exist because of movement of conductors w.r.t to magnetic field (or) movement of magnetic field w.r.t to conductor.
- Hence generation of AC can be done in two ways.
  i. M.F is constant and conductor is movable
  ii. Conductor is constant and M.F is movable.

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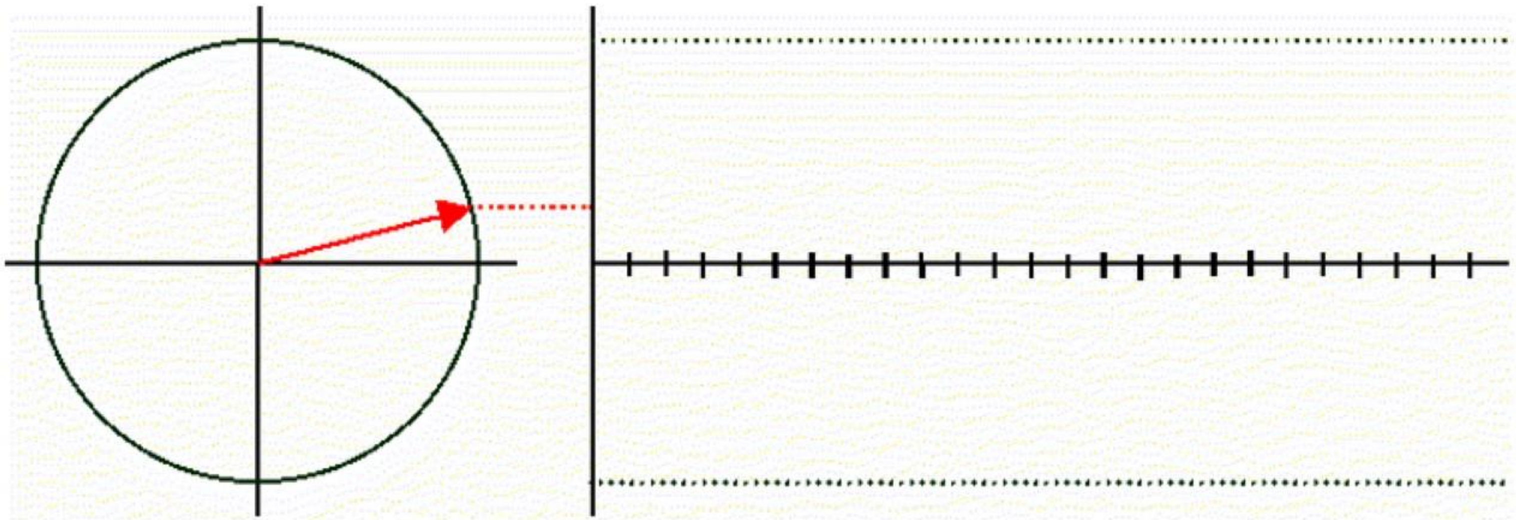
Generation of AC voltage:

A rotating bar generates a sinusoidal wave.



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Generation of AC voltage:

- According to faradays law of electromagnetic induction, the expression for the generated emf in each conductor is

$$e = B\,l\,V\,\mathrm{Sin}\theta \text{ Volts}$$

Where B= flux density of M.F (wb/m^2)

l = Active length of conductor

V = Linear velocity of each conductor

Let $E_m = B\,l\,V$

= Maximum value of induced EMF

Hence $e = E_m \,\mathrm{Sin}\theta$

$= E_m \,\mathrm{Sin}\,\omega t$

where $\theta = \omega t$

Similarly, $i = I_m \sin \omega t$

$v = V_m \,\mathrm{Sin}\,\omega t$

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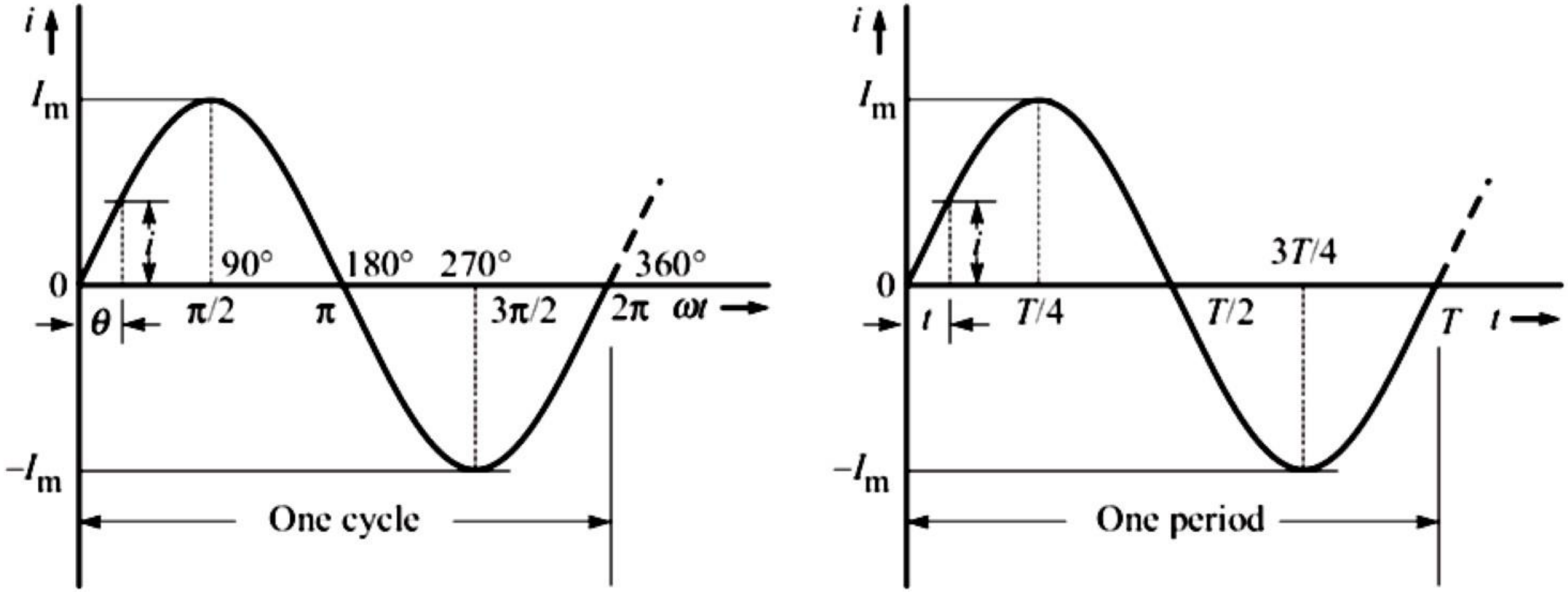
A Sinusoid:



(*a*) Current *i* versus angle ωt. (*b*) Current *i* versus time *t*.

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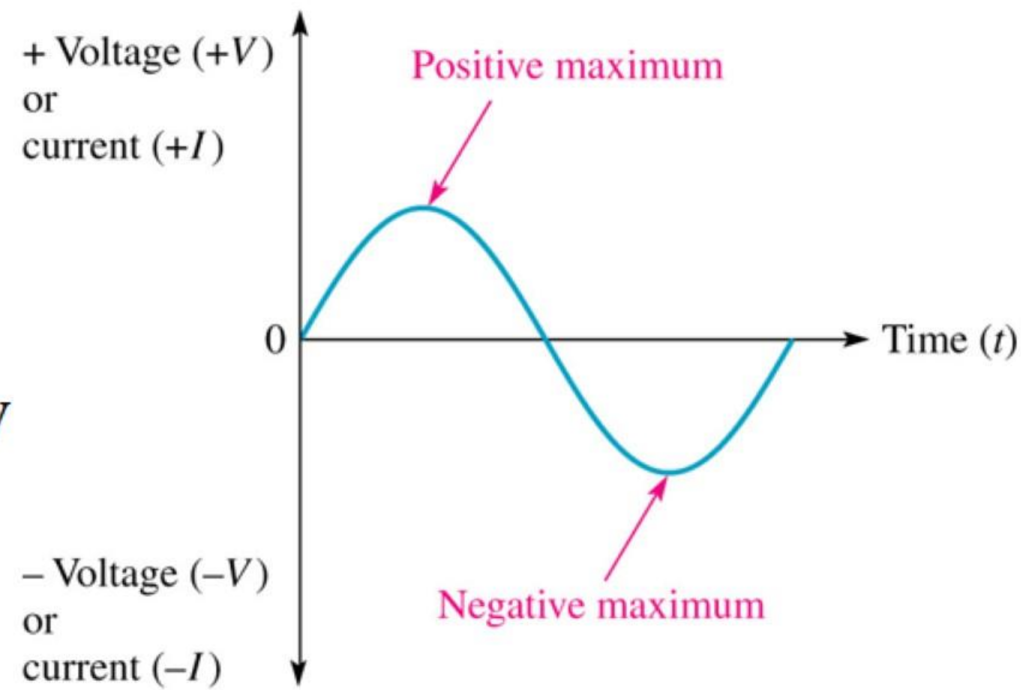
Standard terminology:

Sine waves

The sinusoidal waveform (sine wave) is the fundamental alternating current (ac) and alternating voltage waveform.

Cycle: Each repetition of a set of positive and negative Instantaneous values of alternating quantity is called cycle.



Instantaneous Value :

It is the value of the quantity at any instant of time.

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Standard terminology:

Sine waves

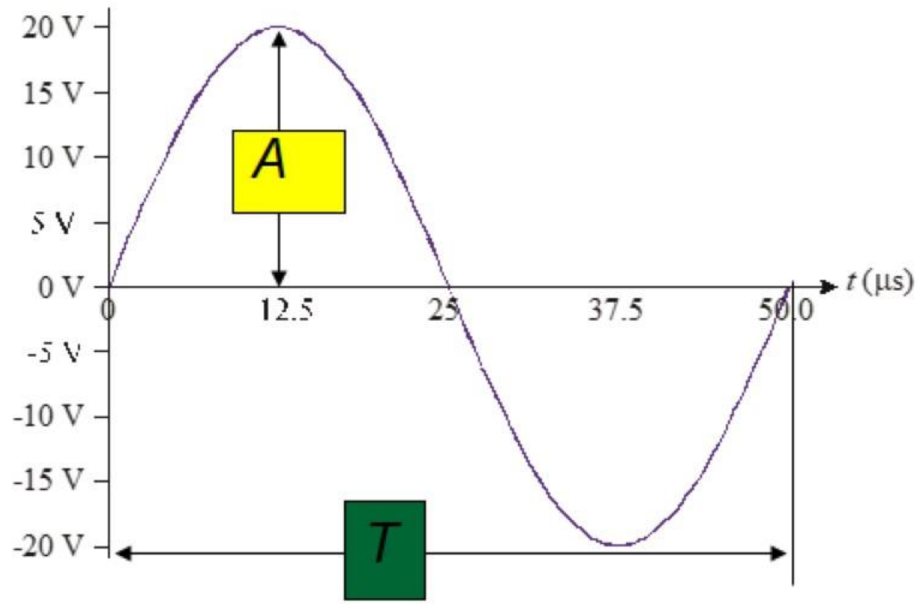
Sine waves are characterized by the amplitude and period. The **amplitude** is the maximum value of a voltage or current; the **period** is the time interval for one complete cycle.

Example

The amplitude (A) of this sine wave is

20 V

The period is 50.0 μs



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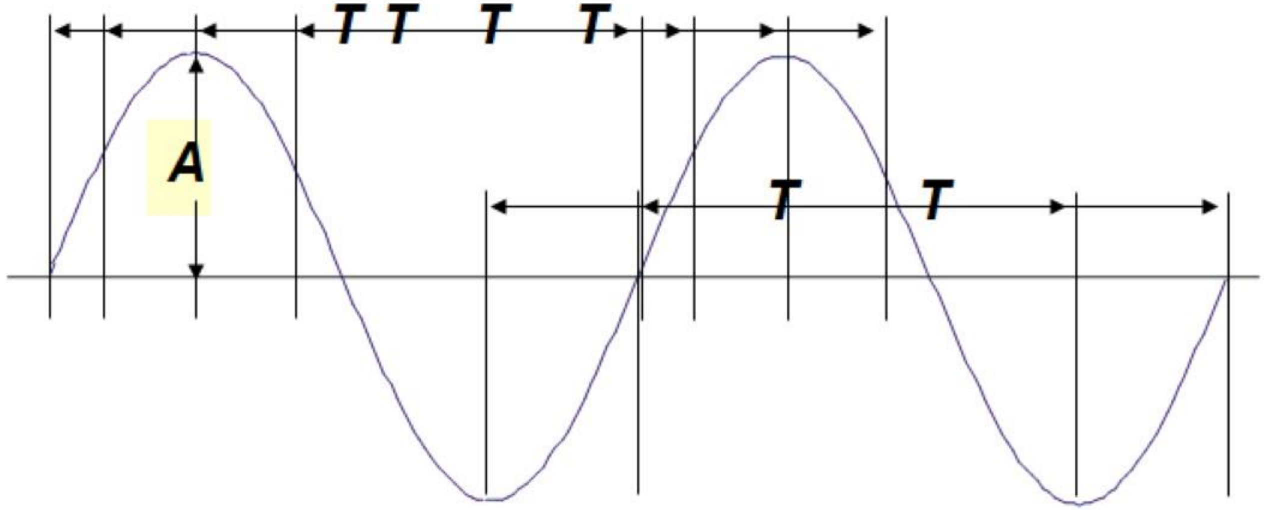
Standard terminology:

Sine waves

The period of a sine wave can be measured between any two corresponding points on the waveform.



By contrast, the amplitude of a sine wave is only measured from the center to the maximum point.

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Standard terminology:

Frequency

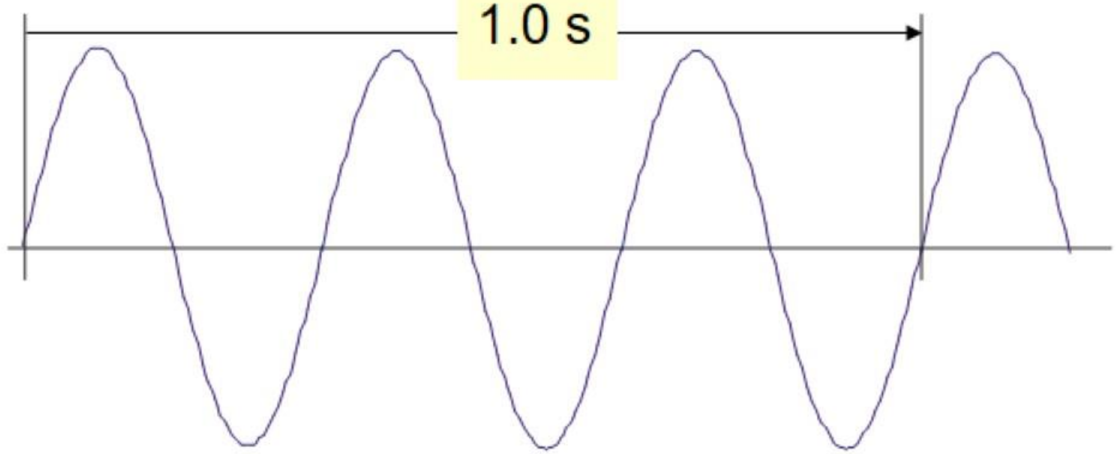
Frequency (f) is the number of cycles that a sine wave completes in one second.

Frequency is measured in **hertz** (Hz).

Example

If 3 cycles of a wave occur in one second, the frequency is

3.0 Hz



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Standard terminology:

Period and frequency

The period and frequency are reciprocals of each other.

$$\boxed{f = \frac{1}{T}} \quad \text{and} \quad \boxed{T = \frac{1}{f}}$$

Thus, if you know one, you can easily find the other.

(The 1/*x* key on your calculator is handy for converting between *f* and *T*.)

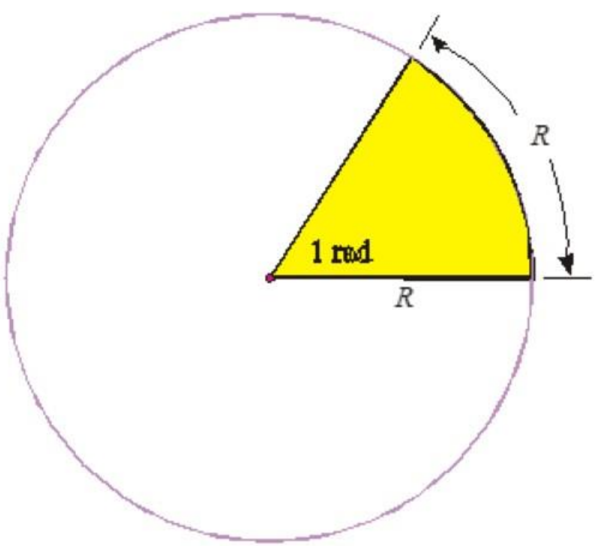
Example

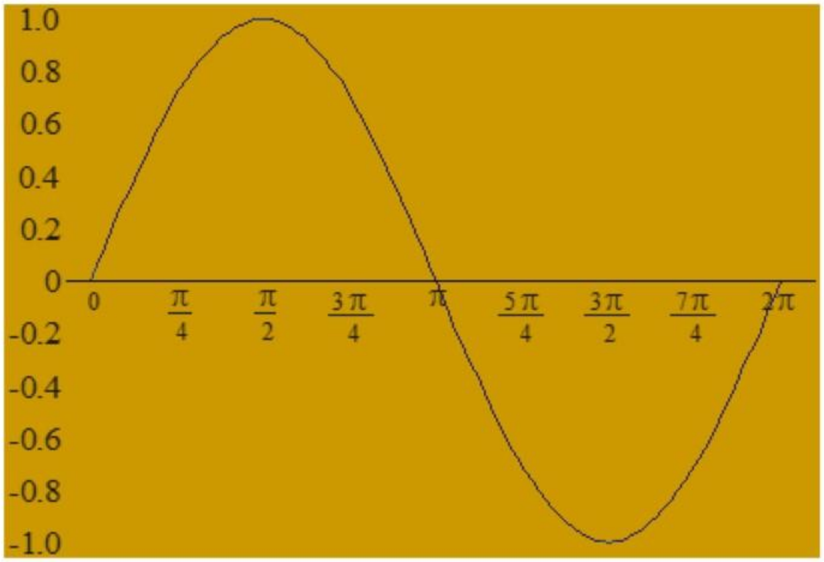
If the period is 50 μs, the frequency is 0.02 MHz = 20 kHz.

AC Circuits: Standard terminology

Angular measurement

Angular measurements can be made in degrees (°) or radians. The radian (rad) is the angle that is formed when the arc is equal to the radius of a circle. There are 360° or 2π radians in one complete revolution.





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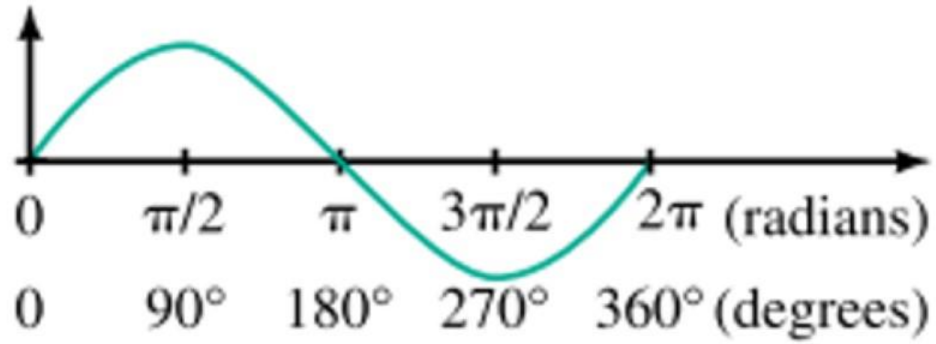
Standard terminology:

Radian Measure

Radians and degrees are related by :

$$2\pi \text{ radians} = 360^\circ$$



(b) Cycle length scaled in degrees and radians

For Conversion:

$$\text{rad} = \frac{2\pi \text{ rad}}{360^\circ} \times \text{degrees}$$

$$\alpha_{\text{radians}} = \frac{\pi}{180^\circ} \times \alpha_{\text{degrees}}$$

$$\text{deg} = \frac{360^\circ}{2\pi \text{ rad}} \times \text{rad}$$

$$\alpha_{\text{degrees}} = \frac{180^\circ}{\pi} \times \alpha_{\text{radians}}$$

AC Circuits

Standard terminology:

- **Angular Frequency :** Angular frequency, denoted as ω, is equal to the number of radians covered in one second. Its unit is rad/s. It is the ratio of angular displacement in one cycle to time period to complete one cycle
- Since one cycle covers 2π radians and there are f cycles in one second, the angular frequency is given as

$$\omega = 2\pi f \qquad \text{or} \qquad \omega = \frac{2\pi}{T}$$

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Modifications of basic EMF equation:

We know that emf equation can be written as

$$e = E_m \,\mathrm{Sin}\, \omega t$$
$$= E_m \,\mathrm{Sin}\, (2\pi f t)$$
$$= E_m \,\mathrm{Sin}\, (2\pi t / T)$$

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Note:

- In case of symmetrical alternating wave there exists two exactly similar half cycles whether sinusoidal or non-sinusoidal. In this case the average value over a complete cycle is zero. Hence for symmetrical waves the average value is taken for only one half cycle.
- In case of unsymmetrical alternating wave the average value must always be taken over whole cycle.

AC Circuits

Different Values of AC

- **Average (or Mean) Value :** It is the arithmetic sum of all the instantaneous values divided by the total number of instantaneous values over a period of half cycle.
- Here we are considering only half cycle because if we take average of +ve half cycle values and -ve half cycle values, it becomes zero. Hence there is no meaning.
- In other words, In case of symmetrical alternating wave there exists two exactly similar half cycles whether sinusoidal or non- sinusoidal. In this case the average value over a complete cycle is zero. Hence for symmetrical waves the average value is taken for only one half cycle.
- Since Pure sine wave is a symmetrical alternating wave, The average of a instantaneous values over a full cycle is zero.
- For this reason, whenever we talk of the average value of an ac wave, we mean average value of half-cycle only.

AC Circuits

Different Values of AC

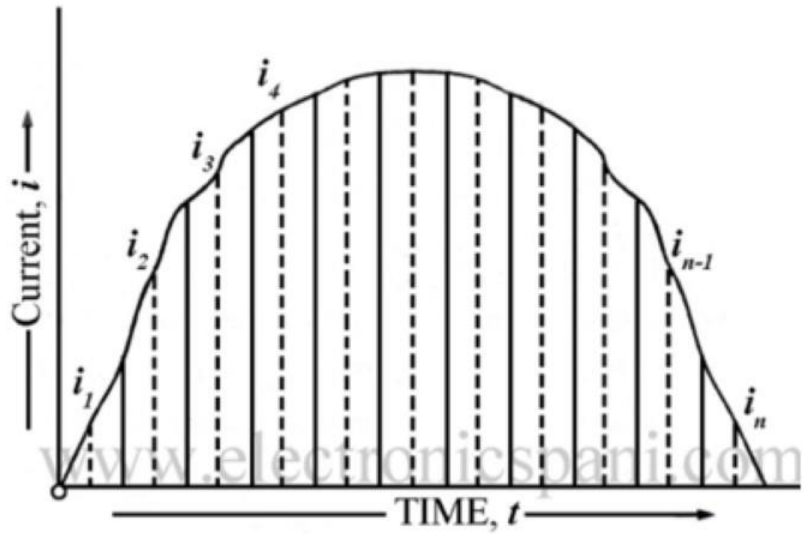
Average (or Mean) Value : It can be calculated using Graphical Method or Analytical Method.

i) Graphical method:

It is used when instantaneous values of AC quantity are known.

$$I_{avg} = \frac{\text{sum of all instantanious values of AC quantity over half cycle}}{\text{Total number of instantanious values of AC quantity over half cycle}}$$

$$V_{avg} = \frac{\text{sum of all instantanious values of AC quantity over half cycle}}{\text{Total number of instantanious values of AC quantity over half cycle}}$$



$$I_{avg} = \frac{i_1 + i_2 + i_3 + i_4 + ... + i_n}{n}$$

AC Circuits

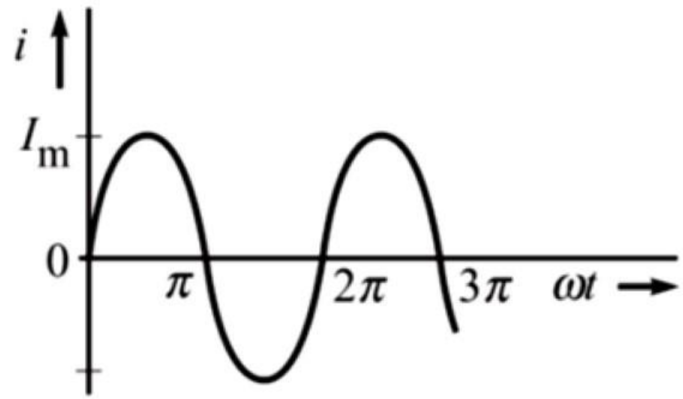
Different Values of AC

- **Average (or Mean) Value** **pure sine wave :**

ii) Analytical method:

- The average value can be calculated by taking ratio of area under the curve over half cycle to length of base for half cycle.

$$I_{avg} = \frac{\text{Area under the curve for half cycle}}{\text{Length of base over half cycle}}$$



$$I_{av} = I_{av\text{-half-cycle}} = \frac{1}{\pi}\int_0^{\pi} i\, d(\omega t)$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t\, d(\omega t)$$

$$= \frac{I_m}{\pi}\Big[-\cos \omega t\Big]_0^{\pi} = \frac{I_m}{\pi}[-\cos \pi + \cos 0] = \frac{2I_m}{\pi}$$

$$= 0.637\, I_m$$

Similarly, $V_{avg} = 0.637\ V_m$

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Different Values of AC

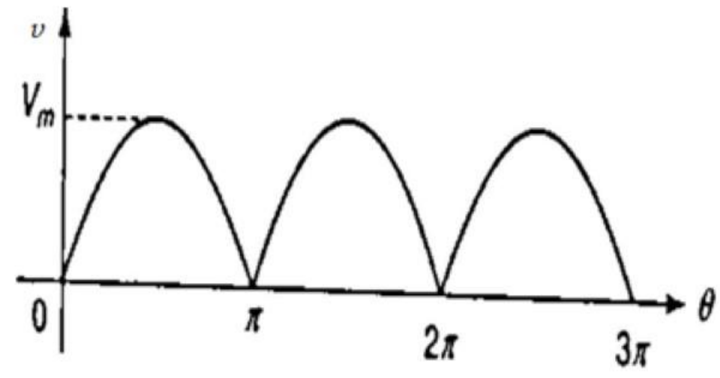
Average (or Mean) Value of Full rectified sine wave

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\, d\theta$$

$$= \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{\pi}[1+1] = 0.637\ V_m$$



Similarly, $I_{avg} = 0.637\ I_m$

- So the average value of pure sine wave and Full rectified sine wave are same

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Different Values of AC

Average (or Mean) Value of half rectified sine wave :

- The average value is given as

$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\, d\theta$$

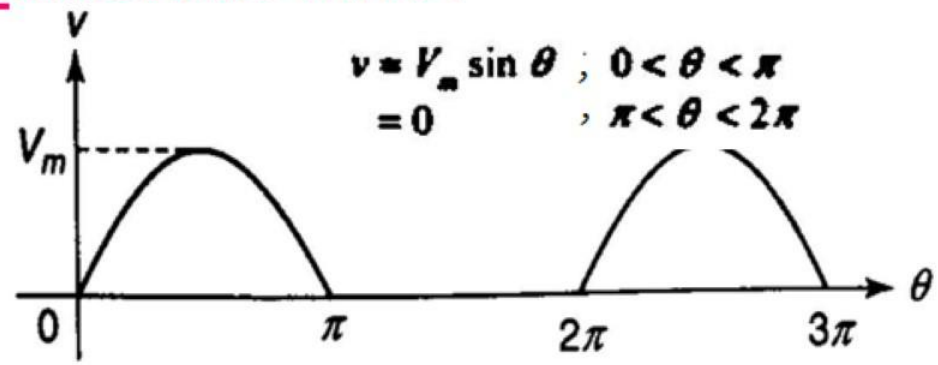
$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\theta\, d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m \sin\theta\, d\theta$$

$$= \frac{V_m}{2\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{2\pi}[1+1]$$

$$= 0.318\, V_m$$



- It is obvious that the average value of half-wave rectified current is just half of the average of full-wave rectified current.

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Different Values of AC

RMS or Effective Value

- It is equivalent direct current, which transfers the same charge as transferred by that ac current in the same time.
- **Note** that the rms value is always greater than the average value, except for a rectangular wave.
- For a rectangular wave, both the rms and average values are same.

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Different Values of AC

RMS or Effective Value pure sine wave:

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta}$$

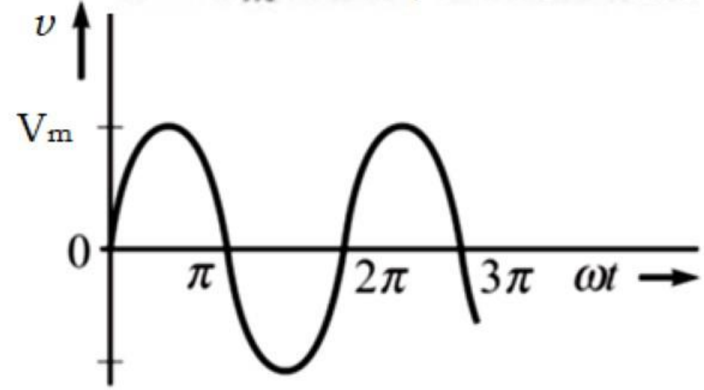
$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}}$$

$$v = V_m \sin\theta,\ 0 < \theta < 2\pi$$



$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{2\pi}{2} - 0 - 0 + 0\right]}$$

$$= \frac{V_m}{\sqrt{2}}$$

$$= 0.707\, V_m$$

Similarly , $I_{rms} = 0.707\ I_m$

AC Circuits

Different Values of AC

RMS or Effective Value full rectified sine wave

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta}$$

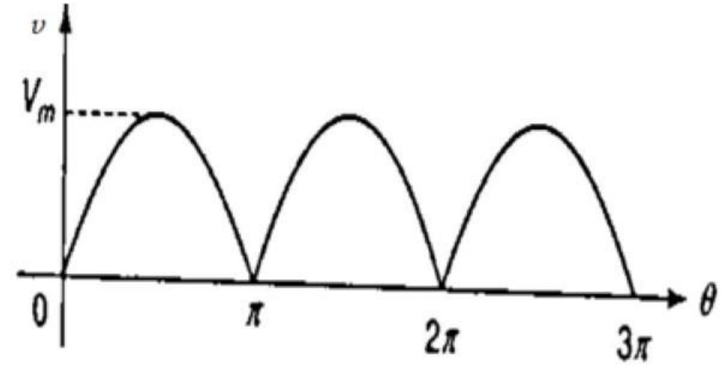
$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi} \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left[\frac{\pi}{2}-\frac{\sin 2\pi}{4}-0+\frac{\sin 0}{4}\right]}$$

$$= 0.707\, V_m$$



Similarly , $I_{rms} = 0.707\, I_m$

AC Circuits

Different Values of AC

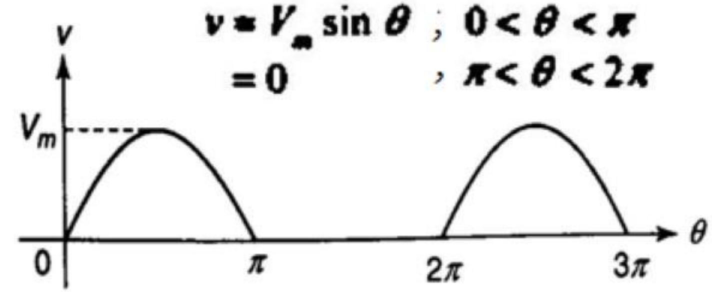
RMS or Effective Value Half rectified sine wave:

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi} \sin^2\theta\, d\theta}$$



$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]}$$

$$= 0.5\ V_m$$

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Different Values of AC

Form Factor

It is the ratio of rms value to the average value of an alternating quantity.

$$\text{Form Factor} = \frac{RMS}{AVERAGE}$$

For pure sinusoidal $= \dfrac{0.707Vm}{0.637Vm} = 1.11$

For full rectified sinusoidal $= \dfrac{0.707Vm}{0.637Vm} = 1.11$

For half rectified sinusoidal $= \dfrac{0.5Vm}{0.318Vm} = 1.571$

- Form factor has less value for less peaky wave. For square wave it is 1.0, for triangular wave it is 1.15.

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Different Values of AC

Peak (or Amplitude) Factor :

It is the ratio of maximum value to the rms value.

$$\text{Peak Factor} = \frac{MAXIMUM\ VALUE}{RMS}$$

For pure sinusoidal $= \dfrac{Vm}{0.707Vm} = 1.414$

For full rectified sinusoidal $= \dfrac{Vm}{0.707Vm} = 1.414$

For half rectified sinusoidal $= \dfrac{Vm}{0.2Vm} = 2$