Equivalence Relations

Def1: A relation on a set A is called an equivalence relation if it is reflexive, and transitive.

Def2: Two elements a and b that are related by an equivalence relation are called equivalent. The notation $a \sim b$ is often used to denote that a and b are equivalent elements with respect to a particular equivalence relation.

To make sense the notion of equivalent elements,

- Every element should be equivalent to itsely, as the reflexive property guarantees for an equivalence relation.
- we can say a is related to b, by Symmetric property, b is related to a.
- B.coz an equivalence relation is transitive if a & b are equivalent & b & c are equivalent, so a & c are equivalent.

Ex:1 Let R be the relation on set of integers such that aRb if and only if $a=b$ or $a=-b$.

Ex:2 Let R be the relation on set of real numbers such that aRb if & only if $a-b$ is an integer. IS R an equivalence relation?

Sol: $\because$ $a-a=0$ is an integer for all real numbers, a, aRa for all real numbers a. So, R is reflexive.

→ Suppose aRb, then $a-b$ is an integer & $b-a$ is also an integer.

$\therefore$ bRa, R is Symmetric.

→ If aRb & bRc then $a-b$ & $b-c$ are integers.

$\therefore$ $a-c=(a-b)+(b-c)$ is also an integer..

$\therefore$ aRc. So, R is transitive.

$\therefore$ R is an equivalence Relation.

Ex:3 Congruence Modulo m Let m be an integer with $m>1$. show that relation $R=\{(a,b) \mid a \equiv b \pmod{m}\}$ is an equivalence relation on set of integers.

→ $a \equiv b \pmod{m}$ iff m divides $(a-b)$.

$a-a=0$ is divisible by m, $\therefore$ $a \equiv a \pmod{m}$, is reflexive.

→ $a \equiv b \pmod{m}$ $a-b$ is divisible by m, so

$a-b=Km$, where K is an integer.

$(b-a)=(-K)m$, So $b \equiv a \pmod{m}$. So it is Symmetric.

→ $a \equiv b \pmod{m}$ & $b \equiv c \pmod{m}$, Then m divides both $a-b$ & $b-c$. $a-b=Km$ & $b-c=lm$.

$a-c=(a-b)+(b-c)=Km+lm=(K+l)m$.

i.e., $a \equiv c \pmod{m}$. i.e. transitive.