Discrete Mathematics and Applications(18IT C05) – UNIT III(Relations)

Short Answer Questions

1. Let A be the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation

   $R = \{(a, b) \mid a \text{ divides } b\}$? [CO3]

2. Consider these relations on the set of integers: [CO3]

$R_1 = \{(a, b) \mid a \leq b\}$,

$R_2 = \{(a, b) \mid a > b\}$,

$R_3 = \{(a, b) \mid a = b \text{ or } a = -b\}$,

$R_4 = \{(a, b) \mid a = b\}$,

$R_5 = \{(a, b) \mid a = b + 1\}$,

$R_6 = \{(a, b) \mid a + b \leq 3\}$.these are relations on an infinite set.

   Which of these relations contain each of the pairs $(1, 1)$, $(1, 2)$, $(2, 1)$, $(1, -1)$, and $(2, 2)$?

3. How many relations are there on a set with n elements? [CO3]

4. Consider the following relations on $\{1, 2, 3, 4\}$:

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$,

$R_2 = \{(1, 1), (1, 2), (2, 1)\}$,

$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$,

$R_4 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$,

$R_5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$,

$R_6 = \{(3, 4)\}$. Which of these relations are reflexive, symmetric ,antisymmetric ,Transitive?[CO3]

5. Is the "divides" relation on the set of positive integers reflexive? [CO3]

6. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. The relations $R_1 = \{(1, 1), (2, 2), (3, 3)\}$ and

   $R_2 = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$ What are the relations $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \oplus R_2$, $R_1 - R_2$,

   and $R_2 - R_1$? [CO3]

7. Let R_1 be the "less than" relation on the set of real numbers and let R_2 be the "greater than" relation on the set of real numbers, that is, $R_1 = \{(x, y) \mid x < y\}$ and $R_2 = \{(x, y) \mid x > y\}$.What are $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 - R_2$, $R_2 - R_1$, and $R_1 \oplus R_2$? [CO3]

8. What is the composite of the relations$R$ and S, where$R$ is the relation from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$

   with $R = \{(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)\}$ and S is the relation from $\{1, 2, 3, 4\}$ to $\{0, 1, 2\}$

   with $S = \{(1, 0), (2, 0), (3, 1), (3, 2), (4, 1)\}$? [CO3]

9. What results when the projection $P_{1,3}$ is applied to the 4-tuples $(2, 3, 0, 4)$,
(Jane Doe, 234111001, Geography, 3.14), and (a_1, a_2, a_3, a_4)? [CO3]

10. What relation results when the projection $P_{1,4}$ is applied to the relation in

TABLE 1 Students.

Student_name	*ID_number*	*Major*	*GPA*
Ackermann	231455	Computer Science	3.88
Adams	888323	Physics	3.45
Chou	102147	Computer Science	3.49
Goodfriend	453876	Mathematics	3.45
Rao	678543	Mathematics	3.90
Stevens	786576	Psychology	2.99

11. Suppose that the relation R on a set is represented by the matrix [CO3]

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

Is R reflexive, symmetric, and/or antisymmetric?

12. Find the matrix representing the relations $S \circ R$, where the matrices representing R and S are [CO3]

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{M}_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

13. Represent each of these relations on $\{1, 2, 3\}$ with a matrix [CO3]
(with the elements of this set listed in increasing order).

a) $\{(1, 1), (1, 2), (1, 3)\}$

b) $\{(1, 2), (2, 1), (2, 2), (3, 3)\}$

c) $\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$

d) $\{(1, 3), (3, 1)\}$

14. What is the reflexive closure of the relation $R = \{(a, b) \mid a < b\}$ on the set of integers? [CO3]

15. What is the symmetric closure of the relation $R = \{(a, b) \mid a > b\}$ on the set of positive integers?[CO3]

16. Which of these relations on $\{0, 1, 2, 3\}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack. [CO3]

a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$

b) $\{(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$

c) $\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$

d) $\{(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

e) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

17. Which of these collections of subsets are partitions of $\{1, 2, 3, 4, 5, 6\}$? [CO3]

a) $\{1, 2\}, \{2, 3, 4\}, \{4, 5, 6\}$ **b)** $\{1\}, \{2, 3, 6\}, \{4\}, \{5\}$

c) $\{2, 4, 6\}, \{1, 3, 5\}$ **d)** $\{1, 4, 5\}, \{2, 6\}$

18. Is (S,R) a poset if S is the set of all people in the world and $(a, b) \in R$, where a and b are people, if [CO3]

a) a is taller than b?

b) a is not taller than b?

c) $a = b$ or a is an ancestor of b?

d) a and b have a common friend?

Long Answer Questions

19. Show that The relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3, \ldots$. [CO3]

20. For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive. [CO3]

a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$

b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$

c) $\{(2, 4), (4, 2)\}$

d) $\{(1, 2), (2, 3), (3, 4)\}$

e) {(1, 1), (2, 2), (3, 3), (4, 4)}

f) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}

21. Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if [CO3]

a) x _= y. **b)** $xy \geq 1$.

c) $x = y + 1$ or $x = y - 1$.

d) $x \equiv y \pmod 7$. **e)** x is a multiple of y.

f) x and y are both negative or both nonnegative.

g) $x = y^2$. **h)** $x \geq y^2$.

22. How many nonzero entries does the matrix representing the relation R on $A = \{1, 2, 3, \ldots, 100\}$ consisting of the first 100 positive integers have if R is [CO3]

a) $\{(a, b) \mid a > b\}$? **b)** $\{(a, b) \mid a$ _= $b\}$?

c) $\{(a, b) \mid a = b + 1\}$? **d)** $\{(a, b) \mid a = 1\}$?

e) $\{(a, b) \mid ab = 1\}$?

23. Find the zero–one matrix of the transitive closure of the relation R where [CO3]

$$\mathbf{M}_R \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$$

24. Use Algorithm 1 to find the transitive closures of these relations on {1, 2, 3, 4}. [CO3]

a) {(1, 2), (2,1), (2,3), (3,4), (4,1)}

b) {(2, 1), (2,3), (3,1), (3,4), (4,1), (4, 3)}

c) {(1, 2), (1,3), (1,4), (2,3), (2,4), (3, 4)}

d) {(1, 1), (1,4), (2,1), (2,3), (3,1), (3, 2), (3,4), (4, 2)}

25. Use Warshall's algorithm to find the transitive closures of these relations on {*a*, *b*, *c*, *d*, *e*}. [CO3]

a) {*(a, c), (b, d), (c, a), (d, b), (e, d)*}

b) {*(b, c), (b, e), (c, e), (d, a), (e, b), (e, c)*}

c) {*(a, b), (a, c), (a, e), (b, a), (b, c), (c,a), (c,b), (d,a),(e, d)*}

d) {*(a, e), (b, a), (b, d), (c,d), (d,a), (d, c), (e,a), (e,b),(e, c), (e, e)*}

26. Draw the Hasse diagram for divisibility on the set [CO3]

a) {1, 2, 3, 4, 5, 6, 7, 8}. **b)** {1, 2, 3, 5, 7, 11, 13}.

c) {1, 2, 3, 6, 12, 24, 36, 48}.

d) {1, 2, 4, 8, 16, 32, 64}.

27. Answer these questions for the poset $(\{\{1\}, \{2\}, \{4\}, \{1, 2\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}, \subseteq)$.[CO3]

a) Find the maximal elements.

b) Find the minimal elements.

c) Is there a greatest element?

d) Is there a least element?

e) Find all upper bounds of {{2}, {4}}.

f) Find the least upper bound of {{2}, {4}}, if it exists.

g) Find all lower bounds of {{1, 3, 4}, {2, 3, 4}}.

h) Find the greatest lower bound of {{1, 3, 4}, {2, 3, 4}},if it exists.