

MATHEMATICS

Maximum Marks: 100
Time allowed: 3 hours

Instructions to Candidates:

- Please check that this question paper contains 11 printed pages.
- Candidates must write the Code on the title page of the answer-book.
- Please check that this question paper contains 26 questions.
- Please write down the Serial Number of the question before attempting it.
- 15 minute time has been allotted to read this question paper.

General Instructions

- (i) All questions are compulsory.
- (ii) Please check that this question paper contains 26 questions.
- (iii) Questions 1–6 in Section A are very short-answer type questions carrying 1 mark each.
- (iv) Questions 7–19 in Section B are long-answer I type questions carrying 4 marks each.
- (v) Questions 20–26 in Section C are long-answer II type questions carrying 6 marks each.
- (vi) Please write down the serial number of the question before attempting it.

65/1/C

SECTION – A

Question numbers 1 to 6 carry 1 mark each.

1. If $x \in \mathbb{N}$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x .

2. Use elementary column operation $C_2 \rightarrow C_2 + 2C_1$ in the following matrix equation:

$$\begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

3. Write the number of all possible matrices of order 2×2 with each entry 1, 2 or 3.

4. Write the position vector of the point which divides the join of points with position vectors $3\vec{a} - 2\vec{b}$ and $2\vec{a} + 3\vec{b}$ in the ratio 2 : 1.

5. Write the number of vectors of unit length perpendicular to both the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$.

6. Find the vector equation of the plane with intercepts 3, -4 and 2 on x , y and z -axis respectively.

SECTION – B

Question numbers 7 to 19 carry 4 marks each.

7. Find the coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the XZ plane. Also find the angle which this line makes with the XZ plane.
8. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.
9. In a game, a man wins ₹ 5 for getting a number greater than 4 and loses ₹ 1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a number greater than 4. Find the expected value of the amount he wins/loses.

OR

A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white?

10. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x .

OR

If $y = 2 \cos(\log x) + 3 \sin(\log x)$, prove that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$.

11. If $x = a \sin 2t(1 + \cos 2t)$ and $y = b \cos 2t(1 - \cos 2t)$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.
12. The equation of tangent at $(2, 3)$ on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b .
13. Find:

$$\int \frac{x^2}{x^4 + x^2 - 2} dx$$

14. Evaluate:

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$$

OR

Evaluate:

$$\int_0^{\frac{3}{2}} |x \cos \pi x| dx$$

15. Find:

$$\int (3x + 1)\sqrt{4 - 3x - 2x^2} dx$$

16. Solve the differential equation:

$$y + x \frac{dy}{dx} = x - y \frac{dy}{dx}$$

17. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.
18. Solve the equation for x :

$$\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$$

OR

If $\cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \alpha$, prove that

$$\frac{x^2}{a^2} - 2\frac{xy}{ab} \cos \alpha + \frac{y^2}{b^2} = \sin^2 \alpha.$$

19. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2,800 as interest. However, if trust had interchanged money in bonds, they would have got ₹ 100 less as interest. Using matrix method, find the amount invested by the trust. Which value is reflected in this question?

SECTION – C

Question numbers 20 to 26 carry 6 marks each.

20. There are two types of fertilisers 'A' and 'B'. 'A' consists of 12% nitrogen and 5% phosphoric acid whereas 'B' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, a farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If 'A' costs ₹ 10 per kg and 'B' costs ₹ 8 per kg, then graphically determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost.
21. Five bad oranges are accidentally mixed with 20 good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of number of bad oranges drawn. Hence find the mean and variance of the distribution.
22. Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ to the plane

$$\vec{r} \cdot (2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0.$$

Also find image of P in the plane.

23. Show that the binary operation $*$ on $A = \mathbb{R} - \{-1\}$ defined as $a * b = a + b + ab$ for all $a, b \in A$ is commutative and associative on A . Also find the identity element of $*$ in A and prove that every element of A is invertible.
24. Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}r$.

OR

If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

25. If A, B, C are $(n \times n)$ matrices and

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \cos A & 1 + \cos B & 1 + \cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$

then show that $\triangle ABC$ is an isosceles triangle.

26. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours available for assembling. The profit is ₹ 5 each for type A and ₹ 6 each for type B. How many souvenirs of each type should the company manufacture in order to maximize profit? Solve the L.P.P. graphically.