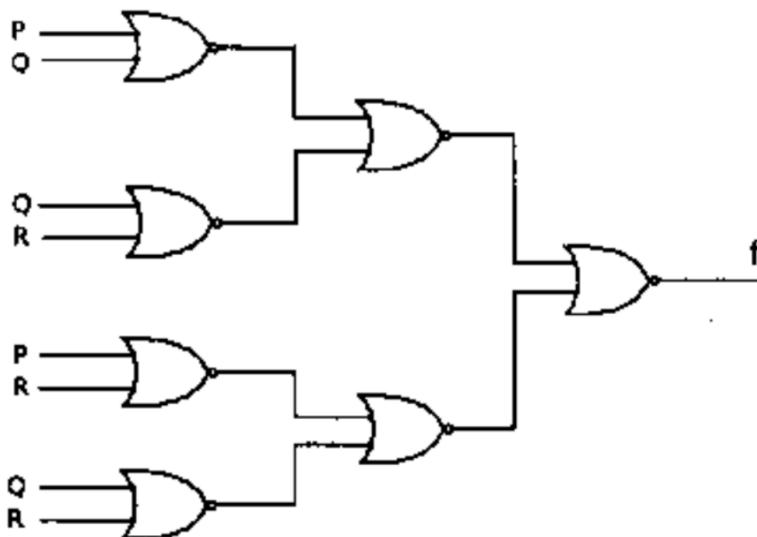


NOR Gate Circuit — Boolean Expression & Hardware Verification

Question

What is the Boolean expression for the output f of the combinational logic circuit of NOR gates given below?



Required: Find the Boolean expression for f and verify it using hardware.

Question Analysis

All gates are NOR. For a NOR gate:

$$\text{NOR}(X, Y) = \overline{X + Y}$$

Top branch

$$X_1 = \overline{P + Q}, \quad X_2 = \overline{Q + R}$$

These feed a NOR:

$$A = \overline{X_1 + X_2} = \overline{\overline{P + Q} + \overline{Q + R}} = (P + Q)(Q + R)$$

Bottom branch

$$Y_1 = \overline{P + R}, \quad Y_2 = \overline{Q + R}$$

These feed a NOR:

$$B = \overline{Y_1 + Y_2} = \overline{\overline{P + R} + \overline{Q + R}} = (P + R)(Q + R)$$

Final NOR

$$\begin{aligned} f &= \overline{A + B} \\ f &= \overline{(P + Q)(Q + R) + (P + R)(Q + R)} \end{aligned}$$

Factor $(Q + R)$:

$$\begin{aligned} f &= \overline{(Q + R)((P + Q) + (P + R))} \\ (P + Q) + (P + R) &= P + Q + R \end{aligned}$$

Using absorption:

$$\begin{aligned} (Q + R)(P + Q + R) &= (Q + R) \\ f &= \boxed{\overline{Q + R}} \end{aligned}$$

Truth Table

P	Q	R	$f = \overline{Q + R}$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Hardware Implementation

The above problem is implemented and tested in hardware using Arduino UNO board. We used 7447 and a common anode seven segment display to display output f as 0 or 1 for inputs P, Q, R as per the truth table and verified the expression.

Required Components & Pin Connections

Components

S.No	Component
1	Arduino UNO Board
2	Breadboard
3	7447 IC (BCD to 7-segment driver)
4	Seven Segment Display (Common Anode)
5	Resistors: 220Ω (for segments)
6	Jumper Wires
7	USB Cable

Arduino → 7447 (BCD Inputs)

7447 Input	7447 Pin	Arduino Pin
A (LSB)	7	D5
B	1	D6
C	2	D7
D (MSB)	6	D8

7447 Power & Control Pins

- 7447 pin 16 (VCC) → +5V
- 7447 pin 8 (GND) → GND
- pin 3 (LT) → +5V
- pin 4 (BI/RBO) → +5V
- pin 5 (RBI) → +5V

7447 → Seven Segment (Common Anode)

- Connect both COM pins of the 7-segment display → +5V
- Connect 7447 outputs to segments **through 220Ω resistors**:

	7447 Output	7447 Pin	Segment
a	13		A
b	12		B
c	11		C
d	10		D
e	9		E
f	15		F
g	14		G

Logic Description

- From Boolean simplification:

$$f = \overline{Q + R}$$

- To show f on 7-segment using 7447, we send BCD:

$$f = 0 \Rightarrow DCBA = 0000 \Rightarrow \text{Display 0}$$

$$f = 1 \Rightarrow DCBA = 0001 \Rightarrow \text{Display 1}$$

Arduino Source Code (Auto Cycling)

```
#include <Arduino.h>

const int A_pin = 5;
const int B_pin = 6;
const int C_pin = 7;
const int D_pin = 8;

void setup() {
    pinMode(A_pin, OUTPUT);
    pinMode(B_pin, OUTPUT);
    pinMode(C_pin, OUTPUT);
    pinMode(D_pin, OUTPUT);
}

void loop() {
    for (int n = 0; n < 8; n++) {

        int P = (n >> 2) & 1;
        int Q = (n >> 1) & 1;
        int R = (n >> 0) & 1;
    }
}
```

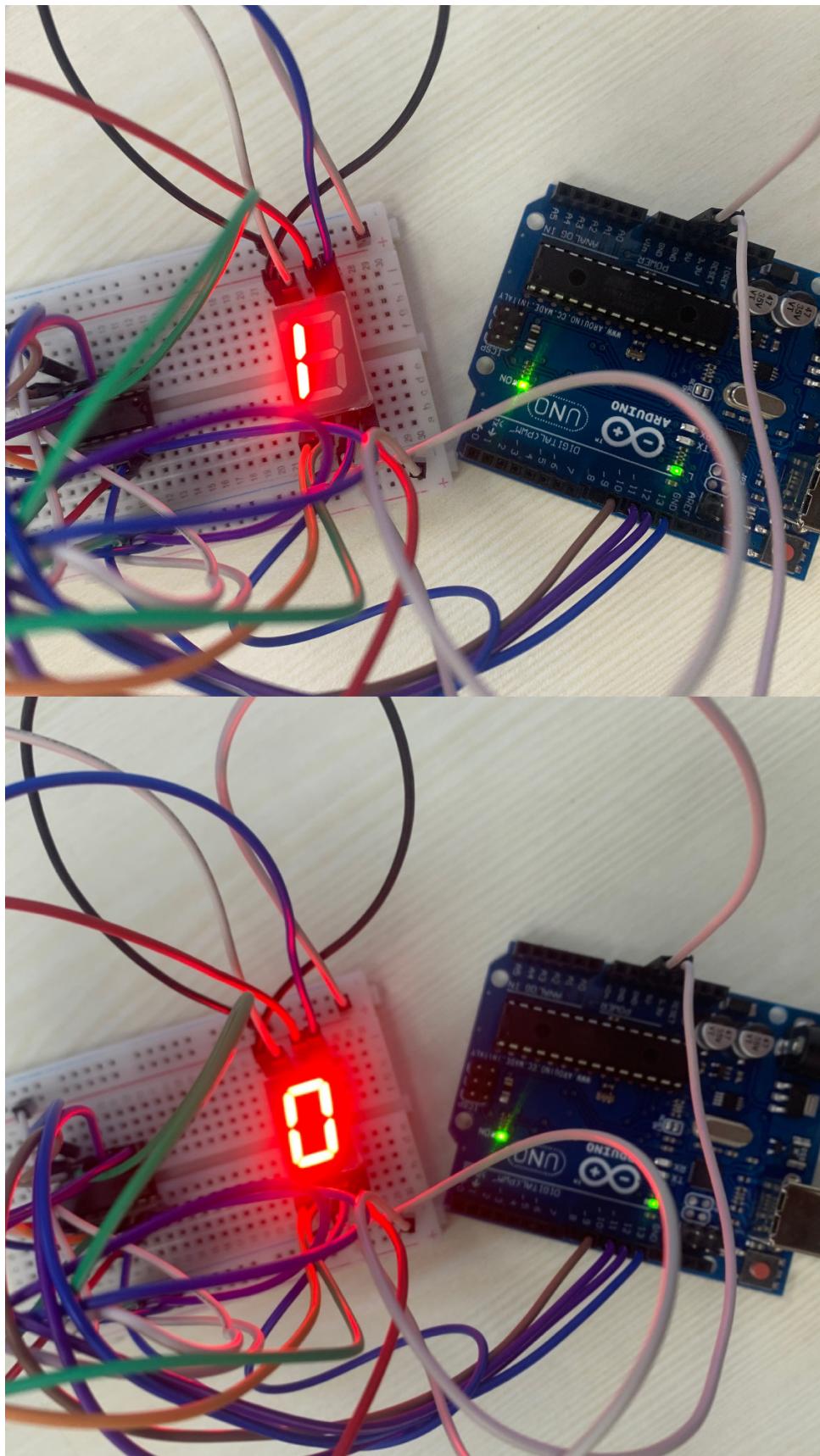
```
int f = !(Q || R);

digitalWrite(A_pin, f);
digitalWrite(B_pin, LOW);
digitalWrite(C_pin, LOW);
digitalWrite(D_pin, LOW);

delay(1000);
}

}
```

Setup / Output Image



Experimental Truth Table

P	Q	R	Observed f (7-seg)
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Conclusion

From the truth table and hardware verification, the NOR circuit output is confirmed as:

$$f = \overline{Q + R}$$

Hence the correct option is $\overline{Q + R}$.