ELEC 533: Homework 6

Due on : Please check the Course Timetable

Professor Behnaam Aazhang MWF 11:00 AM - 11:50 AM

Problem 1

Suppose that U and V are independent random variables with $E[U] = E[V] = 0$ and $Var(U) = Var(V) = 1$. Define random processes by $X_t = U\cos(t) + V\sin(t)$ and $Y_t = U\sin(t) + V\cos(t), t \in \mathbb{R}$.

a) Find $E[X_{t_1}X_{t_2}]$, $E[Y_{t_1}Y_{t_2}]$, and $E[X_{t_1}Y_{t_2}]$.

b) Is $E[X_{t_1}X_{t_2}]$ a function only of $t_2 - t_1$?

c) Is $E[X_{t_1}Y_{t_2}]$ a function only of $t_2 - t_1$?

Problem 2

Suppose W is a continuous random variable with pdf $f_W(w)$. Consider the random process $X_t = A\cos(Wt)$, where A is a constant and $t \in \mathbb{R}$.

a) Find the autocorrelation function of X_t.

b) Is X_t wide-sense stationary?

c) Suppose A and W are defined as above, and let Θ be a random variable independent of W, distributed uniformly on $[-\pi, \pi]$. Find the autocorrelation function of $Y_t = A\cos(Wt + \Theta)$.

d) Is Y_t wide-sense stationary?

Problem 3

Suppose X_1 and X_2 are random variables such that $E[X_1] = E[X_2] = E[X_1X_2] = 0$ and $Var(X_1) = Var(X_2) = \sigma^2$. Define a random process $Y_t = X_1\sin(t) + X_2\cos(t), t \in \mathbb{R}$.

a) Is Y_t wide-sense stationary?

b) Give an example of random variables X_1, X_2 such that Y_t is *strictly* stationary, and an example such that Y_t not strictly stationary.

Problem 4

Let Y_t and Z_t be independent random processes with autocorrelation functions $R_Y(t, s)$ and $R_Z(t, s)$. If we define $X_t = Z_tY_t$, what is $R_X(t, s)$?