ELEC 533: Homework 8

Due on : Please check the Course Timetable

Professor Behnaam Aazhang MWF 11:00 AM - 11:50 AM

Problem 1

Suppose $\{X_t; t \in \mathbb{R}\}$ is a WSS random process with m.s. derivative $\{X_t'; t \in \mathbb{R}\}$.

a) Show that $\Pr(|X_{t+\tau} - X_t| \geq a) \leq \frac{2[R_X(0) - R_X(\tau)]}{a^2} \quad \forall\, a > 0$

b) Show that $cov(X_t, X_t') = 0 \quad \forall t \in \mathbb{R}$

Problem 2

Suppose $Y_t = S_t + N_t;\ t \in \mathbb{R}$ where $\{S_t;\ t \in \mathbb{R}\}$ and $\{N_t;\ t \in \mathbb{R}\}$ are zero mean WSS and orthogonal. Suppose that we wish to estimate the process $X_t = \int\limits_{-\infty}^{\infty} k(t-\tau) S_\tau d\tau,\ t \in \mathbb{R}$ with an estimate of the form $\widehat{X_t} = \int\limits_{-\infty}^{\infty} h(t-\tau) Y_\tau d\tau,\ t \in \mathbb{R}$, where k and h are impulse responses of linear-time invariant systems.

Show that:

$$\mathrm{E}[(X_t - \widehat{X_t})^2] = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \left[|K(\omega) - H(\omega)|^2 S_S(\omega) + |H(\omega)|^2 S_N(\omega) \right] d\omega$$

where K and H are the transfer function of k and h, respectively, and S_S and S_N are the power spectral densities of $\{S_t;\ t \in \mathbb{R}\}$ and $\{N_t;\ t \in \mathbb{R}\}$, respectively.

Problem 3

Show that:

$$\left| \int\limits_{\omega_1}^{\omega_2} S_{XY}(\omega) d\omega \right|^2 \leq \int\limits_{\omega_1}^{\omega_2} S_X(\omega) d\omega \int\limits_{\omega_1}^{\omega_2} S_Y(\omega) d\omega$$

From that if follows that if $S_X(\omega) = 0$ is an interval, then $S_{XY}(\omega) = 0$ in that interval.

Problem 4

Let $\{X_t;\ t \in \mathbb{R}\}$ be a continuous time, zero mean Gaussian random process with spectral density $S_X(\omega) = \frac{N_0}{2},\ \forall w \in \mathbb{R}$. Let $H(\omega)$ and $G(\omega)$ be the transfer functions of the two linear systems (time-invariant) with impulse responses $h(t)$ and $g(t)$, respectively. The process $\{X_t\}$ is passed through the filter $h(t)$ to obtain $\{Y_t;\ t \in \mathbb{R}\}$ and it is also passed through the filter $g(t)$ to obtain $\{V_t;\ t \in \mathbb{R}\}$.

a) Find $R_{YV}(t, s)$

b) Under what assumptions Y_t and V_t are independent ?

Problem 5

Consider the following mean-square differential equation

$$Y_t' + \alpha Y_t = X_t \quad \text{m.s.} \quad \text{with} \quad Y_0 = 0$$

where X_t is a Gaussian random process with $\mu_X(t) = 0$ and $R_X(\tau) = a^2\delta(\tau)$.

a) Find a differential equation to solve for the mean function of Y_t, with appropriate initial conditions.

b) Find a partial differential equation for crosscorrelation function between $\{Y_t;\ t \in \mathbb{R}\}$ and $\{X_t;\ t \in \mathbb{R}\}$ with appropriate boundary conditions.

c) Find a partial differential equation for autocorrelation function of $\{Y_t;\ t \in \mathbb{R}\}$ with appropriate boundary conditions.

d) Is Y_t wide sense stationary? Explain! Is Y_t a Gaussian random process?