Homework 3

ELEC 540: Advanced Wireless Communications

5 Problems, *** points

Problems are taken from *Fundamentals of Wireless Communications,* by David Tse and Pramod Viswanath.

1. (Book Problem 5.13, 30 points)
Consider a system with 1 transmit antenna and L receive antennas. Independent $\mathcal{CN}(0, N_0)$ noise corrupts the signal at each of the receive antennas. The transmit signal has a power constraint of P.

1. Suppose the gain between the transmit antenna and each of the receive antennas is constant, equal to 1. What is the capacity of the channel? What is the performance gain compared to a single receive antenna system? What is the nature of the performance gain?
2. Suppose now the signal to each of the receive antenna is subject to independent Rayleigh fading. Compute the capacity of the (fast) fading channel with channel information only at the receiver. What is the nature of the performance gain compared to a single receive antenna system? What happens when $L \to \infty$?
3. Give an expression for the capacity of the fading channel in part 2. with CSI at both the transmitter and the receiver. At low SNR, do you think the benefit of having CSI at the transmitter is more or less significant when there are multiple receive antennas? How about when the operating SNR is high?
4. Now consider the slow fading scenario when the channel is random but constant. Compute the outage probability and quantify the performance gain of having multiple receive antennas.

2. (Book Problem 5.14, 20 points)
Consider a MISO slow fading channel.

1. Verify that the Alamouti scheme radiates energy in an isotropic manner.
2. Show that a transmit diversity scheme radiates energy in an isotropic manner if and only if the signals transmitted from the antennas have the same power and are uncorrelated

3. (Book Problem 5.15, 30 points)
Consider the MISO channel with L transmit antennas and channel gain vector $\mathbf{h} = [h_1, \cdots, h_L]^t$. The noise variance is N_0 per symbol and the total power constraint across the transmit antennas is P.

1. First, think of the channel gains as fixed. Suppose someone uses a transmission strategy for which the input symbols at any time is zero mean and has a covariance matrix $\mathbf{K}_x$. Argue that the maximum achievable reliable rate of communication under this strategy is no larger than $\log\left(1 + \frac{\mathbf{h}^t \mathbf{K}_x \mathbf{h}}{N_0}\right)$ bits/symbol.

2. Now suppose we are in a slow fading scenario and $\mathbf{h}$ is random and i.i.d. Rayleigh. The outage probability of the scheme in Part 1 is given by $P_{out}(R) = \mathbb{P}\left\{\left(1 + \frac{\mathbf{h}^t\mathbf{K}_x\mathbf{h}}{N_0}\right) < R\right\}$.

   Show that correlation never improves the outage probability: i.e., given a total power constraint P , one can do no worse by choosing $\mathbf{K}_x$ to be diagonal.
   Hint: Observe that the covariance matrix $\mathbf{K}_x$ admits a decomposition of the form $\mathbf{U}diag\{P_1, \cdots, P_L\}\mathbf{U}$.

4. (Book Problem 5.21, 20 points)
In Chapter 3, we have seen that one way to communicate over the MISO channel is to convert it into a parallel channel by sending symbols over the different transmit antennas one at at time.

1. Consider first the case when the channel is fixed (known to both the transmitter and the receiver). Evaluate the capacity loss of using this strategy at high and low SNR. In which regime is this transmission scheme a good idea?

2. Now consider the slow fading MISO channel. Evaluate the loss in performance of using this scheme in terms of (i) the outage probability $p_{out}(R)$ at high SNR; (ii) the ϵ-outage capacity C_ϵ at low SNR.

5. (Book Problem 5.24, 20 points)
(The price of channel inversion)

1. Consider a narrowband Rayleigh flat fading SISO channel. Show that the average power (averaged over the channel fading) to implement the channel inversion scheme is infinite for any positive target rate.

2. Suppose now there are $L > 1$ receive antennas. Show that the average power for channel inversion is now finite.

3. Compute numerically and plot the average power as a function of the target rate for different L to get a sense of the amount of gain from having multiple receive antennas. Qualitatively describe the nature of the performance gain.