Homework 4

ELEC 540: Advanced Wireless Communications

3 Problems, 60 points

Problems are taken from *Fundamentals of Wireless Communications,* by David Tse and Pramod Viswanath.

1. (Book Problem 6.1, 20 points)
The sum constraint in (6.6) applies because the two users send independent information and cannot cooperate in the encoding. If they could cooperate, what is the maximum sum rate they can achieve, assuming still individual power constraints P_1 and P_2 on the two users? In the case $P_1 = P_2$, quantify the cooperation gain at low and at high SNR. In which regime is the gain more significant?

Solutions:

Channel model is $y[m] = x1[m] + x2[m] + w[m]$.

The signal power at receiver is

$$P = \mathbb{E}[(x_1[m] + x_2[m])^2] = P_1 + P_2 + 2\mathbb{E}[x_1[m]x_2[m]].$$

When the two users can cooperate, they can choose the correlation of $x_1[m]$ and $x_2[m]$ to be one and thus get the largest total power

$$P = P_1 + P_2 + 2\sqrt{P_1 P_2},$$

and hence the maximum sum rate they can achieve is

$$C_{coop} = \log\left(1 + \frac{P_1 + P_2 + 2\sqrt{P_1 P_2}}{N_0}\right).$$

In the case of $P_1 = P_2 = P$,

$$C_{coop} = \log\left(\frac{1 + 4P}{N_0}\right),$$

whereas the sum rate without cooperation is

$$C_{no_coop} = \log\left(\frac{1 + 2P}{N_0}\right).$$

At high SNR,

$$\frac{C_{coop}}{C_{no_coop}} \simeq \frac{\log(4P/N_0)}{\log(2P/N_0)} \to 1 \quad \text{as } P \to \infty.$$

At low SNR,

$$\frac{C_{coop}}{C_{no_coop}} \simeq \frac{4P/N_0}{2P/N_0} = 2 \quad \text{as } P \to 0.$$

Thus in lower SNR region the cooperative gain is more effective.

2. (Book Problem 6.2, 20 points)
Consider the basic uplink AWGN channel in (6.1) with power constraints P_k on user k (for $k = 1, 2$). In Section 6.1.3, we stated that orthogonal multiple access is optimal when the degrees of freedom

are split in direct proportion to the powers of the users. Verify this. Show also that any other split of degrees of freedom is strictly suboptimal, i.e., the corresponding rate pair lies strictly inside the capacity region given by the pentagon in Figure 6.2. Hint: Think of the sum rate as the performance of a point-to-point channel and apply the insight from Exercise 5.6.

Solutions:

For orthogonal multiple access channel the rates of the two users satisfy

$$R_1 < \alpha \log\left(1 + \frac{P_1}{\alpha N_0}\right)$$
$$R_2 < (1-\alpha)\log\left(1 + \frac{P_2}{(1-\alpha)N_0}\right)$$

When the degrees of freedom are split proportional to the powers of the users, we have

$$\alpha = \frac{P_1}{P_1 + P_2}.$$

Thus the sum rate satisfy

$$R_1 + R_2 < \frac{P_1}{P_1+P_2}\log\left(1 + \frac{P_1}{\frac{P_1}{P_1+P_2}N_0}\right) + \frac{P_2}{P_1+P_2}\log\left(1 + \frac{P_2}{\frac{P_2}{P_1+P_2}N_0}\right) = \log\left(1 + \frac{P_1+P_2}{N_0}\right),$$

which is the optimal sum rate.

For arbitrary split of degrees of freedom, from the strictly concavity property of $log(1+x)$, we have

$$\begin{aligned} R_1 + R_2 &< \alpha\log\left(1 + \frac{P_1}{\alpha N_0}\right) + (1-\alpha)\log\left(1 + \frac{P_2}{(1-\alpha)N_0}\right) \\ &\le \log\left(1 + \left(\alpha\frac{P_1}{\alpha N_0} + (1-\alpha)\frac{P_2}{(1-\alpha)N_0}\right)\right) \\ &= \log\left(1 + \frac{P_1+P_2}{N_0}\right), \end{aligned}$$

and equality holds only when

$$\frac{P_1}{\alpha} = \frac{P_2}{(1-\alpha)},$$

that is when the degrees of freedom are split proportional to the powers of the users. Any other split of degrees of freedom are strictly sub-optimal.

3. (Book Problem 6.3, 20 points)
Calculate the symmetric capacity, (6.2), for the two-user uplink channel. Identify scenarios where there are definitely superior operating points.

Solutions;

The symmetric capacity is

$$C_{sym} = \frac{1}{2}\log\left(1 + \frac{P_1+P_2}{N_0}\right)$$

There are three scenarios of capacity region shown in Figure 1. In scenario I and II, the point A is superior to the symmetric rate point, in scenario III we do not have a superior point.

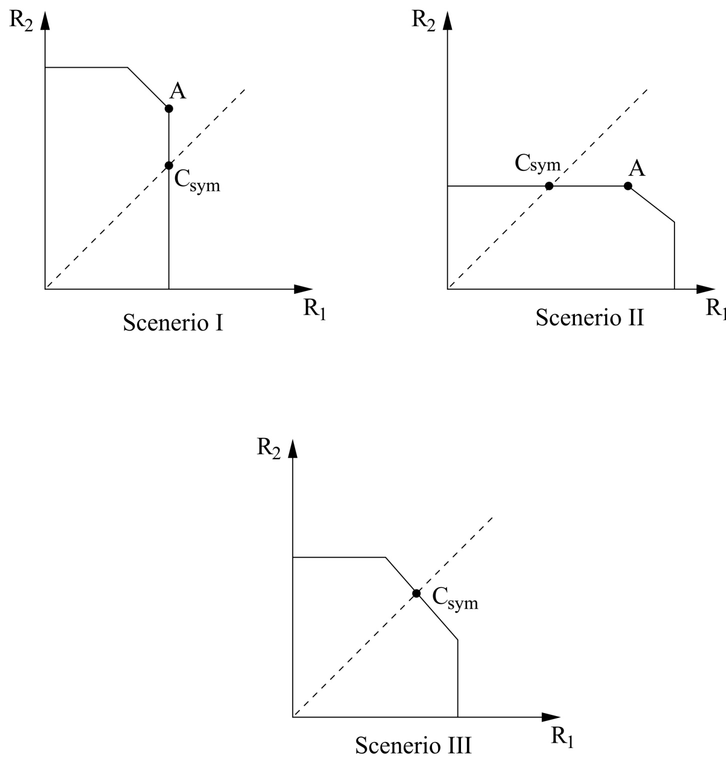

Figure 1: Figure for Problem 3