ELEC 533: Homework 4

Due on : Please check the Course Timetable

prof. Behnaam Aazhang MWF 11:00 AM - 11:50 AM

Problem 1

Supose X and Y are jointly continuous

i) Show that $F_{Y|X}(b|x) = \int_{-\infty}^{b} \frac{f_{XY}(x,y)}{f_X(x)} dy$ and thus $f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)}$.

ii) Suppose $\int_{-\infty}^{\infty} |y| f_{Y|X}(y|x) dy < +\infty \;\; \forall x$. Show that $E[Y|X = x] = \int_{-\infty}^{\infty} y f_{Y|X}(y|x) dy \;\; \forall x$.

Problem 2

Suppose X and Y are jointly continuous random variables with joint pdf given by
$$f_{XY}(x,y) = \begin{cases} e^{-y} & \text{if } x > 0 \text{ and } y > x \\ 0 & otherwise \end{cases}$$

i) Show that f_{XY} is a legitimate joint pdf.

ii) Find the Marginal pdf's of X and Y.

iii) Find $E[Y|X = x]$ for $x > 0$.

Problem 3

Suppose $X_n \xrightarrow{i.p.} X$ and that there is a constant c such that $|X_n| \leq c$ for all n. Show that $X_n \xrightarrow{m.s.} X$.

Problem 4

Show that if $X_n \xrightarrow{D} X$ and $P(X = c) = 1$ for some constant c then $X_n \xrightarrow{i.p.} X$.

Problem 5

If $X_n \xrightarrow{i.p.} X$ and there is a random variable Y such that $P(|X_n| \leq y) = 1 \;\; \forall n$ and $E[Y^2] < +\infty$ then $X_n \xrightarrow{m.s.} X$.