ELEC 533: Homework 3

Professor Behnaam Aazhang MWF 11:00 AM - 11:50 AM

Problem 1

a) Suppose that X and Y are jointly continuous. Show that

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x, y)dy, \forall x \in \mathbb{R}.$$

b) Suppose that X and Y are each continuous. Are they jointly continuous? Give a proof or counterexample.

Problem 2

Prove the following:

a) $F_{XY}(a, b)$ is nondecreasing (in a and b).

b) $\lim_{a \to \infty} F_{XY}(a, b) = F_Y(b)$.

c) Jointly continuous random variables X and Y are independent if and only if

$$f_{XY}(x, y) = f_X(x)f_Y(y).$$

Problem 3

a) Show that $-1 \leq \rho(X, Y) \leq 1$.

b) Show that jointly Gaussian random variables are independent if and only if $\rho(X, Y) = 0$.

Problem 4

Suppose X and Y are uniformly and independently distributed random variables on $(0, 1)$. Is $Z = XY$ also uniformly distributed? If not, what is its density?