Homework 5

ELEC 540: Advanced Wireless Communications
Due: 11/27/17

2 Problems, 50 points

Problems are taken from *Fundamentals of Wireless Communications,* by David Tse and Pramod Viswanath.

1. (Book Problem 6.22, 25 points)
(Selective feedback) The downlink of IS-856 has K users each experiencing i.i.d. Rayleigh fading with average SNR of 0 dB. Each user selectively feeds back the requested rate only if its channel is greater than a threshold γ. Suppose γ is chosen such that the probability that no one sends a requested rate is ϵ. Find the expected number of users that sends in a requested rate. Plot this number for K = 2, 4, 8, 16, 32, 64 and for $\epsilon = 0.1$ and $\epsilon = 0.01$. Is selective feedback effective?

Solutions: The probability that no one user sends a request rate is the probability that all users channel is less than γ, that is

$$\begin{aligned}
\mathbb{P}\{\text{no one sends a request rate}\} &= \prod_{k=1}^{K} \mathbb{P}\{\text{user k's channel is less than}\gamma\} \\
&= \prod_{k=1}^{K} \mathbb{P}\{|h_k|^2 \text{SNR} < \gamma\} \\
&= \left(1 - e^{-\gamma/\text{SNR}}\right)^K \\
&= (1 - e^{-\gamma})^K,
\end{aligned}$$

where we used SNR = 0dB = 1. We need this probability to be ϵ, thus

$$(1 - e^{-\gamma})^K = \epsilon$$

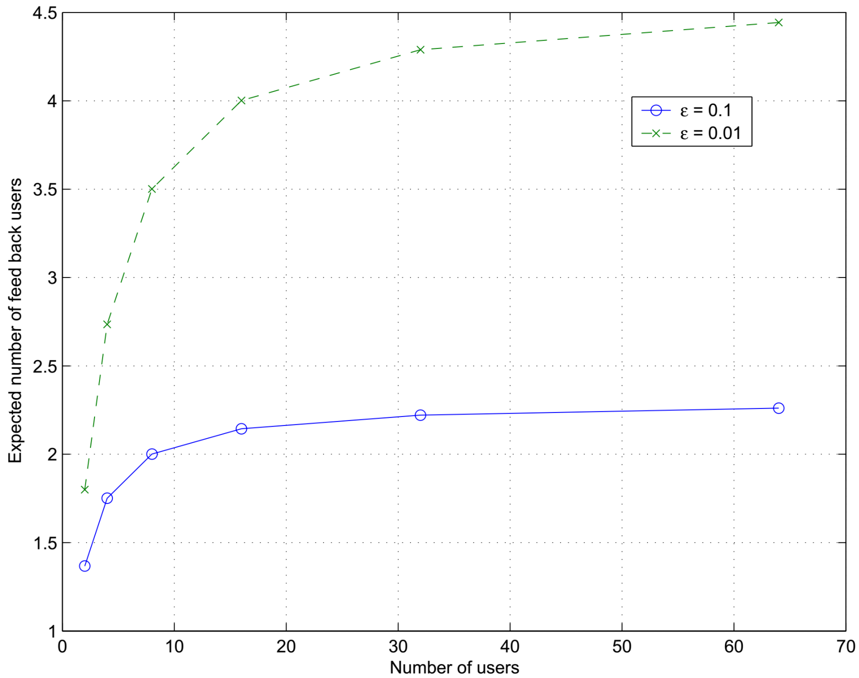
and the solution for Γ is

$$\gamma = -ln(1 - \epsilon^{1/K})$$

Now, the probability that any user sends a request is $p = e^{-\gamma}$, and the number of users that sends in a requested rate is a Binomial random variable with parameter p, hence

$$E[\text{number of users that sends in a requested rate}] = Kp = Ke^{-\gamma} = K(1 - \epsilon^{1/K})$$

The expected number of users that sends in a requested rate for different K and ϵ is shown in the following figure.



2. (Book Problem 8.4 (1), 25 points)
For i.i.d. Rayleigh fading, show that the distribution of H and that of HU are identical for every unitary matrix U. This is a generalization of the rotational invariance of an i.i.d. complex Gaussian vector.

Solutions: Let $\mathbf{A} = \mathbf{HU}$. Then, $\mathbf{A}$ is zero mean. Let $\mathbf{a}_i$ and $\mathbf{h}_i$ denote the i^{th} columns of $\mathbf{A}$ and $\mathbf{H}$ respectively. Then,

$$\begin{aligned} E[\mathbf{a}_j^* \mathbf{a}_i] &= \mathbf{U}^* E[\mathbf{h}_j^* \mathbf{h}_i] \mathbf{U}, \\ &= \mathbf{U}^* \delta_{ij} \mathbf{I} \mathbf{U}, \\ &= \delta_{ij} \mathbf{I}. \end{aligned}$$

Also,

$$\begin{aligned} E[\mathbf{a}_j^t \mathbf{a}_i] &= \mathbf{U}^t E[\mathbf{h}_j^t \mathbf{h}_i] \mathbf{U}, \\ &= \mathbf{0}. \end{aligned}$$

Thus, $\mathbf{A}$ has i.i.d. $\mathcal{CN}(0,1)$ entries.