Homework 2

ELEC 540: Advanced Wireless Communications

4 Problems, 100 points

Problems are taken from *Fundamentals of Wireless Communications,* by David Tse and Pramod Viswanath.

1. (Book Problem 3.13, 15 points)
The optimal coherent receiver for repetition coding is L branches of diversity is a maximal ratio combiner. For implementation reasons, a simple receiver one often builds is a *selection combiner.* It does detection based on received signal along the branch with the strongest gain only, and ignores the rest. For the i.i.d. Rayleigh fading model, analyze the high SNR performance of this scheme. How much inherent diversity gain can this scheme get? Quantify the performance loss from optimal combining.

2. (Book Problem 3.15, 25 points)
An $L \times 1$ MISO channel can be converted into a time-diversity channel with L diversity branches by simply transmitting over one antenna at a time.

1. In this way, any code designed for a time-diversity channel with L diversity branches can be used for a MISO (multi-input single-output) channel with L transmit antennas. If the code achieves k-fold diversity in the time-diversity channel, how much diversity can it obtain in the MISO channel? What is the relationship between the minimum product distance metric of the code when viewed as a time-diversity code and its minimum determinant metric when viewed as a transmit-diversity code?
2. Using this transformation, the rotation code can be used as a transmit diversity scheme. Compare the performance of this code and the Alamouti scheme in a 2×1 Rayleigh fading channel, using BPSK symbols. Which one is better? How about using QPSK symbols?
3. Use the permutation code (c.f. Figure 3.26) from Exercise 3.10 on the 2×1 Rayleigh fading channel and compare (via a numerical simulation) its performance with the Alamouti scheme using QPSK symbols (so the rate is the same in both the schemes).

3. (Book Problem 3.19, 35 points)
In this exercise we study the performance of space time codes (the subject of Section 3.3.2) in the presence of multiple receive antennas.

1. Derive, as an extension of (3.83), the pairwise error probability for space time codes with n_r receive antennas.
2. Assuming that the channel matrix has i.i.d. Rayleigh components derive, as an extension of (3.86), a simple upper bound for the pairwise error probability.
3. Conclude that the code design criterion remains unchanged with multiple receive antennas.

4. (Book Problem 3.20, 25 points)
We have studied the performance of the Alamouti scheme in a channel with two transmit and one receive antenna. Suppose now we have an additional receive antenna. Derive the ML detector for the symbols based on the received signals at both receive antennas. Show that the scheme effectively provides two independent scalar channels. What is the gain of each of the channels?