ELEC 533: Homework 1

Due on Please check the Course Timetable

Professor Behnaam Aazhang

Problem 1

Using axioms of probability prove the following properties of probability

i) $A_1, A_2, ..., \in \mathcal{F} \Rightarrow P\left(\bigcup\limits_{i=1}^{\infty} A_i\right) \leq \sum\limits_{i=1}^{\infty} P(A_i)$

ii) If $A_n \nearrow A$ then $P(A_n) \to P(A)$

iii) If $P(B) > 0$ and $A_1, A_2, ... \in \mathcal{F}$ and $A_i A_j = \emptyset$ for $i \neq j$ then $P\left(\bigcup\limits_{i=1}^{\infty} A_i | B\right) = \sum\limits_{i=1}^{\infty} P(A_i | B)$

Problem 2

Show that the following are valid probability mass functions

i) $p_X(a) = \begin{cases} \frac{n!}{(n-a)!a!} \Theta^a (1 - \Theta)^{n-a} & : a = 0, 1, 2, .., n \quad 0 \leq \Theta \leq 1 \\ 0 & : \qquad\qquad otherwise \end{cases}$

ii) $p_X(a) = \begin{cases} \frac{e^{-\lambda} \lambda^a}{a!} & : a = 0, 1, 2, ... \quad \lambda \geq 0 \\ 0 & : otherwise \end{cases}$

Problem 3

Suppose X is a random variable with cdf F_X. Prove the following

i) F_X is nondecreasing

ii) $\lim\limits_{a \to \infty} F_X(a) = 1$

iii) $\lim\limits_{a \to -\infty} F_X(a) = 0$

iv) F_X is right continuous

v) $P(a < X \leq b) = F_X(b) - F_X(a)$ if $b > a$

vi) $P(X = a) = F_X(a) - \lim\limits_{\substack{b \to a \\ b < a}} F_X(b)$

vii) Find expressions for $P(a \leq X \leq b)$, $P(a \leq X < b)$ and $P(a < X < b)$ in terms of F_X