ELEC 533: Homework 5

Due on : Please check online

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Problem 1

With fixed λ for each integer $n \geq \lambda$, let $X_{1,n}, X_{2,n}, ..., X_{n,n}$ be independent random variables such that $P[X_{i,n} = 1] = \frac{\lambda}{n}$ and $P[X_{i,n} = 0] = 1 - \frac{\lambda}{n}$. Let $Y_n = X_{1,n} + X_{2,n} + ... + X_{n,n}$.

a) Find Φ_{Y_n} , the characteristic function of Y_n.

b) Find the limit of Φ_{Y_n} as n tends to ∞. What distribution does it correspond to?

Problem 2

Suppose g is a function from $\mathbb{R}$ to $\mathbb{R}$ with the following properties:

i) g is continuous and increasing

ii) $g(x) \leq 1 \;\; \forall\, x \geq 0$

iii) $g(0) = 0$

a) Suppose X is a r.v. (random variable). Show that $\mathcal{P}(|x| > b) \geq \mathrm{E}[g(x)] - g(b) \;\; \forall\, b \geq 0$

b) Suppose $\{X_i\}_{i=1}^{\infty}$ is a sequence of r.v. Show that $\{X_i\}_{i=1}^{\infty}$ converges i.P. to r.v. X if and only if $\lim\limits_{i\to\infty} \mathrm{E}[g(|X_i - X|)] = 0$

Problem 3

Suppose $\underline{X} \sim \mathcal{N}(\underline{\mu}, \not{\Sigma})$,

a) Show that $\mathrm{E}[\underline{X}] = \underline{\mu}$ and $\mathrm{cov}(\underline{X}, \underline{X}) = \not{\Sigma}$

b) Show that $A\underline{X} + \underline{b} \sim \mathcal{N}(A\underline{\mu} + \underline{b}, A \cdot \not{\Sigma} \cdot A^T)$.

c) Suppose $\not{\Sigma} > 0$ and $\not{\Sigma} = CC^T$ where C is invertible. Show that $C^{-1}(\underline{X} - \underline{\mu}) \sim \mathcal{N}(\underline{0}, I)$.

Problem 4

a) The random variables X_i are independent, with identical Cauchy densities $f_{X_i}(x) = \frac{\alpha/\pi}{\alpha^2 + x^2}, x \in \mathbb{R}$. Show that the density of their sample mean $\underline{X} = \frac{(X_1 + X_2 + ... + X_n)}{n}$ is also Cauchy.

b) Explain why CLT (Central Limit Theorem) does not hold for this sequence.