ELEC 533: Homework 7

Due on : Please check the Course Timetable

Professor Behnaam Aazhang MWF 11:00 AM - 11:50 AM

Problem 1

Suppose $\{X_t; t \in \mathbb{R}\}$ is a wss, zero mean Gaussian random process with autocorrelation function $R_X(\tau); \tau \in \mathbb{R}$ and power spectral density $S_X(\omega); \omega \in \mathbb{R}$. Define the random process $\{Y_t; t \in \mathbb{R}\}$ by $Y_t = (X_t)^2$.

a) Find the mean function of Y_t

b) Find the autocorrelation function of Y_t as a function of τ

c) Does the power spectral density exist for Y_t ? What if we were to use the delta function?

Problem 2

Show that the autocorrelation function of a WSS process $\{X_t; t \in \mathbb{R}\}$ is continuous for all $\tau \in \mathbb{R}$ if it is contionuous at $\tau = 0$.

Problem 3

Let A and B be independent with $\Pr(A = 1)$=$\Pr(A = -1)$=$\Pr(B = 1)$=$\Pr(B = -1) = \frac{1}{2}$. Define the random process $\{X_t; t \in \mathbb{R}\}$ by $X_t = 2A + Bt$.

a) Sketch a possible sample path.

b) Find $\Pr(X_t \geq 0)$ for all $t \in \mathbb{R}$.

c) Find $\Pr(X_t \geq 0 \text{ for all } t \in \mathbb{R})$.

Problem 4

Suppose $\{X_n; n \in \mathbb{Z}\}$ and $\{Z_n; n \in \mathbb{Z}\}$ are mutually independent, i.i.d. zero mean Gaussian random process with autocorrelations

$R_x(k) = \sigma_x^2 \delta_k$ and $R_z(k) = \sigma_z^2 \delta_k$ where $\delta_k = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$

These process are used to construct new processes as follows

$Y_n = Z_n + rY_{n-1}$

$U_n = X_n + Y_n$

$W_n = U_n - rU_{n-1}$

Find the covariances and power spectral densities of $\{U_n\}$ and $\{W_n\}$. Find $E[(X_n - W_n)^2]$