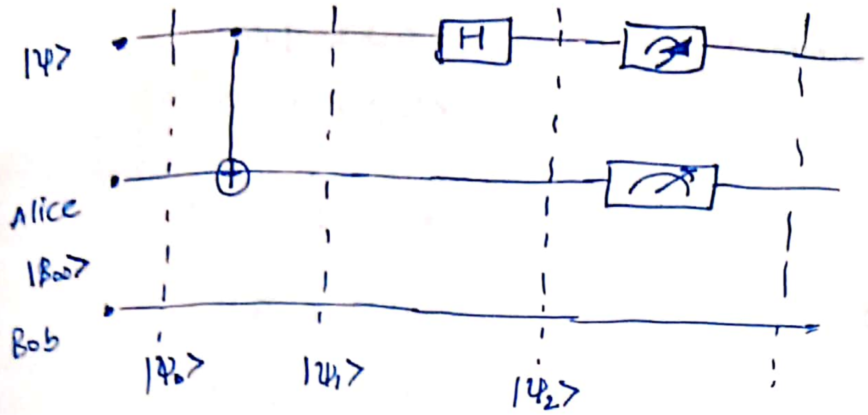
13-8-21

Quantum Teleportation



$$|\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\psi_0\rangle = |\psi\rangle|\beta_{00}\rangle \qquad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$= \left[(\alpha|0\rangle + \beta|1\rangle)\left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)\right]$$

$$= \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)\right]$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)\right]$$

As ψ passes through Hadamard gate,

$$\alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha|+\rangle + \beta|-\rangle$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left[\alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right)(|00\rangle + |11\rangle) + \beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)(|10\rangle + |01\rangle)\right]$$

$$= \frac{1}{2}\left[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\right]$$

$$= \frac{1}{2}\left[\begin{array}{l}\alpha|00\rangle|0\rangle + \alpha|01\rangle|1\rangle + \alpha|10\rangle|0\rangle + \alpha|11\rangle|1\rangle + \\ \beta|01\rangle|0\rangle + \beta|00\rangle|1\rangle - \beta|11\rangle|0\rangle - \beta|10\rangle|1\rangle\end{array}\right]$$

After regrouping of terms,

$$|\psi_2\rangle = \frac{1}{2}\left[\begin{array}{l}|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) + |10\rangle(\alpha|0\rangle - \beta|1\rangle) \\ + |11\rangle(\alpha|1\rangle - \beta|0\rangle)\end{array}\right]$$

$\sqrt{\alpha_r + \alpha_i}$ $\sqrt{\beta_r + \beta_i}$

$|\psi_2\rangle$ now has 4 terms where first term indicates Alice's qubits and next is state of Bob's qubit. If Alice performs any measurement, Bob's system will be as follows -

Alice's measurement	Bob's qubit
00	$\alpha\|0\rangle + \beta\|1\rangle$
01	$\alpha\|1\rangle + \beta\|0\rangle$
10	$\alpha\|0\rangle - \beta\|1\rangle$
11	$\alpha\|1\rangle - \beta\|0\rangle$

Based on Alice's measurement, Bob will end up with one of four states. But to know which state, Alice must tell the result to Bob.

Now Alice sends the measured classical bits i.e. 00/01/10/11

Now Bob can reconstruct $|\psi\rangle$ using appropriate gates

00 $\rightarrow$ doesn't need to do anything

01 $\rightarrow$ sends his qubit through X gate to get $|\psi\rangle$

10 $\rightarrow$ sends his qubit through Z gate to get $|\psi\rangle$

11 $\rightarrow$ sends his qubit through Z and then X to get $|\psi\rangle$

$|\psi\rangle$ got transferred $\rightarrow$ after knowing Alice's measurement, Bob does corresponding corrections

(1) Does this mean faster than light communication?

Even the value of state must have been changed at some point, the correction can only happen after the exchange of classical information to Bob.

Correction $\longrightarrow |\psi\rangle \rightarrow$ only after classical communication

↓

can't be faster than light

Q.

Is there a "copy" done ?

Bob got Alice's qubit

Is it a copy of $|\psi\rangle$?

↓

$\boxed{\text{No Cloning Theorem is violated}}$

(*) No its not

we can see that Alice's qubit ceases to be $|\psi\rangle$

only (at the most) 1 copy of $|\psi\rangle$ in existence at every point of time

So at end, Alice's qubit collapses and only Bob has qubit.

How do entangled bits behave ?

Ex: $|\beta_{00}\rangle = \dfrac{|00\rangle + |11\rangle}{\sqrt{2}}$

when any change happens to first bit, its corresponding other bit changes so

Ex: if one measures $|0\rangle$ other also becomes $|0\rangle$

so probability information of one of entangled qubits is related to another and performing operations on one affects the other instataneously irrespective of distance.