9-08-21

Recap:

Single Qubit Operations

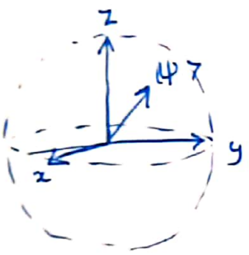
$\alpha|0\rangle + \beta|1\rangle \rightarrow \boxed{X} \rightarrow \beta|0\rangle + \alpha|1\rangle$

$\alpha|0\rangle + \beta|1\rangle \rightarrow \boxed{Z} \rightarrow \alpha|0\rangle - \beta|1\rangle$

$\alpha|0\rangle + \beta|1\rangle \rightarrow \boxed{H} \rightarrow \alpha|+\rangle + \beta|-\rangle$

Unitary - $UU^{\dagger} = I$
↳ transpose of complex conjugate

- All operations - single qubit
- Every operation is sequence of rotations & phase changes



$$\begin{bmatrix} \cos\gamma/2 & -\sin\gamma/2 \\ \sin\gamma/2 & \cos\gamma/2 \end{bmatrix}$$ - example of rotation gate (Rotation along y-axis)

Ex 2: $\begin{bmatrix} e^{-i\beta/2} & 0 \\ 0 & e^{i\beta/2} \end{bmatrix}$ - rotation along z-axis (in xy plane)

Multiple Qubit Operations

Classical multiple bit ops	Qubit

→ AND, OR, NAND, NOR, XOR etc

→ Reversibility - Not true in classical bit operations

→ Universality - NAND and NOR Gates

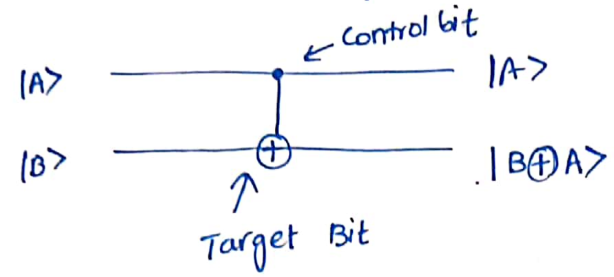
XOR cannot be Universal

Qubit System

→ are there any universal gates

→ Reversible

① Controlled NOT Gate



If control bit is classical 0, then it lets target bit as it is i.e unchanged, if control bit is classical 1, then target bit is inverted | flipped.

4 orthonormal states (2 qubits system)

$|00\rangle \longrightarrow |00\rangle$

$|01\rangle \longrightarrow |01\rangle$

$|10\rangle \longrightarrow |11\rangle$

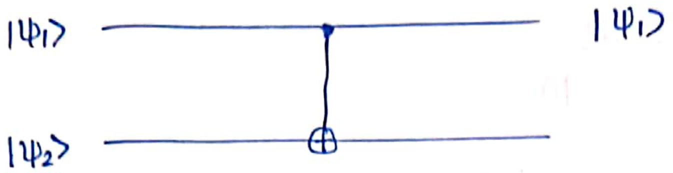
$|11\rangle \longrightarrow |10\rangle$

$$U_{CN} = \begin{array}{c c} \begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$U_{CN} U_{CN}^{\dagger} = I$$

$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$

$|\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$



$$U_{CN}|\psi_1\psi_2\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 . \alpha_2 \\ \alpha_1 . \beta_2 \\ \alpha_2 . \beta_1 \\ \beta_1 \beta_2 \end{bmatrix} = \begin{bmatrix} \alpha_1\alpha_2 \\ \alpha_1\beta_2 \\ \beta_1\beta_2 \\ \alpha_2\beta_1 \end{bmatrix}$$

- Any 2 qubit op $\rightarrow$ CNOT + single qubit operator
  $\downarrow$
  Universality

  i.e. CNOT is quantum parallel of NAND

<u>Measure in basis other than computational basis</u> ($|0\rangle$ & $|1\rangle$)

* Other basis can also be chosen for representing states

Eg: $|+\rangle$, $|-\rangle$

$|+\rangle \rightarrow \dfrac{|0\rangle + |1\rangle}{\sqrt{2}}$ $\qquad |-\rangle \rightarrow \dfrac{|0\rangle - |1\rangle}{\sqrt{2}}$

(measurements can only be done on orthonormal basis)

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$= \alpha\left(\frac{|+\rangle + |-\rangle}{\sqrt{2}}\right) + \beta\left(\frac{|+\rangle - |-\rangle}{\sqrt{2}}\right)$$

$$= \left(\frac{\alpha+\beta}{\sqrt{2}}\right)|+\rangle + \left(\frac{\alpha-\beta}{\sqrt{2}}\right)|-\rangle$$

Quantum Circuits

→ wire → connection

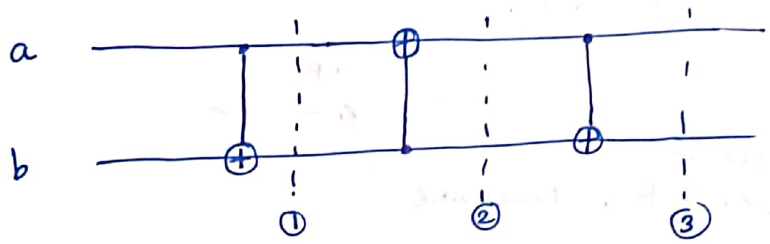
→ sequence → time change

→ Information reaching somewhere

classical

Circuits

① Swaps the state of two qubits



① - $|a, b\rangle \rightarrow |a, a \oplus b\rangle$

② - $|a, a\oplus b\rangle \longrightarrow |a \oplus (a \oplus b), a \oplus b\rangle = |b, a\oplus b\rangle$

$[\because a \oplus (a+b) = (a \oplus a) \oplus b = b]$

③ $|b, a\oplus b\rangle \longrightarrow \cancel{b\oplus} \; |b, a \oplus b \oplus b\rangle = |b, a\rangle$

↑

states are swapped

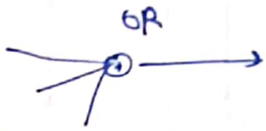
11-08-21

Recap - Qubit representation
- Single qubit ops - Unitarity
- Multiple qubit gates - CNOT gate
- Quantum Circuit
  Ex: swapping 2 qubits

(*) How are circuits (quantum) different than classical circuits?

① No feedback is possible
   - acyclic

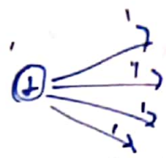
② No fan in



→ No fan in
Reason → Not reversible

③ No fan out

Reason → No cloning theorem

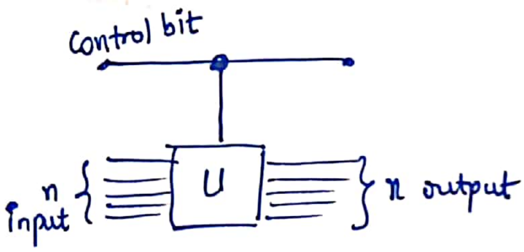


Classical

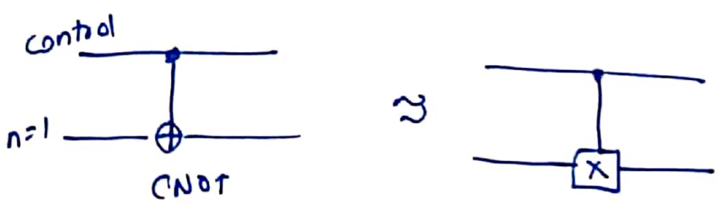
Quantum Gates

U is a unitary matrix acting on n-bits → regard it as a ~~gate on~~ quantum gate on n qubits
($UU^{\dagger} = I$)

Controlled-U



→ If control bit is 0, input is same as output
→ If control bit is 1, U is applied on input



Measurement

→ gate for measurement

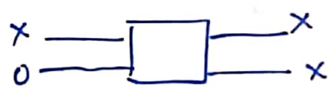
⇓

with prob according to $|\psi\rangle$

bit collapses → information is lost

* Lets see an example circuit which looks like qubit copying but its really not a copy

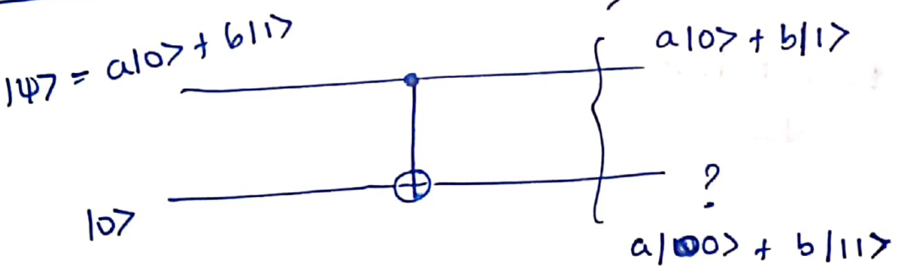
Classical



$x \oplus y = x \oplus 0 = x$ (bit is copied)

Why cant we do this with qubit ?

Quantum



We want $|\psi\rangle|\psi\rangle = a^2|00\rangle + ab|01\rangle + ab|10\rangle + b^2|11\rangle$

If it was $|\psi\rangle$ then this would have been new state

Doesnt match $a|00\rangle + b|11\rangle$

It only matches if $ab = 0$

i.e. if $|\psi\rangle = 0$ (or) $|\psi\rangle = 1$

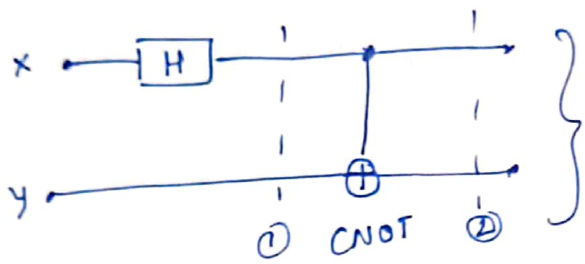
So it produces a copy only when $|\psi\rangle = |0\rangle$ (or) $|1\rangle$ not otherwise

* Information of a, b is not carried on to the copied state (or)

We don't get $|\psi\psi\rangle$ in general

Bell States (also called EPR states)

Circuit



$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

↓ one of Bell States

The bits are said to be entangled

Suppose $|xy\rangle$ is $|00\rangle$

$$|00\rangle \longrightarrow \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)|0\rangle \quad - ①$$

$$\downarrow$$

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}} \quad - ②$$

Bell Gate States

$$|\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\beta_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\beta_{10}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$|\beta_{11}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

The above bellstates can be obtained by following inputs to previous circuit.

$|xy\rangle = |00\rangle \longrightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}} = \beta_{00}$

$|xy\rangle = |01\rangle \longrightarrow \frac{|01\rangle + |10\rangle}{\sqrt{2}} = \beta_{01}$

$|xy\rangle = |10\rangle \longrightarrow \frac{|00\rangle - |11\rangle}{\sqrt{2}} = \beta_{10}$

$|xy\rangle = |11\rangle \longrightarrow \frac{|01\rangle - |10\rangle}{\sqrt{2}} = \beta_{11}$

$$|\beta_{xy}\rangle = \frac{|0,y\rangle + (-1)^x |1,y\rangle}{\sqrt{2}}$$

"Story"

Alice & Bob

↓

$|\psi\rangle$ has to communicate or send $|\psi\rangle$ to B

* Alice need not retain $|\psi\rangle$

→ Alice & Bob → entangle qubit pair (assume it was $|\beta_{00}\rangle$)

Alice has one of entagled pair

Bob has another

* Alice has means of classical communication (any no of 0's, 1's can be sent)