30-7-21

Classical Computing (bit)

- can be 0 (or) 1
- can be read, stored and perform~~ation~~ operations

Quantum Computing (qubit)

Qubit - fundamental mathematical object that quantum systems are made of and which possesses a state.

- Two states are $|0\rangle$ and $|1\rangle$ (dirac notation)
- But can be also in superposition of the above two states

Quantum Bits

Superposition is a linear combination of states

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

"psi"

α, β - complex numbers

- State of qubit is a vector in 2D complex vector space

$|0\rangle$ and $|1\rangle$ are computational basis states, form orthonormal basis for complex 2D vector space

Difference

Qubit cannot be examined without changing it to $|0\rangle$ or $|1\rangle$

Prob. of getting 0 - $|\alpha|^2$

Prob. of getting 1 - $|\beta|^2$

$$|\alpha|^2 + |\beta|^2 = 1$$

$|\psi\rangle$ is a unit vector in 2D complex vector space

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Quantum bit

Classical bit - 0, 1

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ $|0\rangle, |1\rangle$

Dirac Notation

$\alpha, \beta \in \mathbb{C}$ — complex no.

- Special states
- basic states and form orthonormal basis

Measured - 0, 1 $\rightarrow$ some state of the physical system

Eg: Electron $\rightarrow$ state (0)

$\downarrow$

Given proper energy $\rightarrow$ next state (1)

But it is possible for it to exist between these two states due to energy provided, but when measured it shows only 0 (or) 1 states

Prob of getting 0 - $|\alpha|^2$

prob. of getting 1 - $|\beta|^2$

$$|\alpha|^2 + |\beta|^2 = 1$$

Eg: If $|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$\downarrow$

~~So~~ If $|\psi\rangle$ is measured then there is 50% chance of getting 0 and 50% for 1.

* Hence there is no chance we can determine original state of qubit by measuring it (because once measured it collapses to 0 (or) 1)

* If we could have made infinite no. of compies of same state, then we can measure them which gives idea of $|\alpha|^2$, $|\beta|^2$

* $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$ (special state)

Visualising $|\psi\rangle$

- Discovered by Bloch

Bloch sphere

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$= (\alpha_r, \alpha_i)|0\rangle + (\beta_r, \beta_i)|1\rangle$$

4 variables (cant be visualized on 3D graph)

Using polar co-ordinates to express the state -

$$|\psi\rangle = |\alpha| e^{i\delta_1} |0\rangle + |\beta| e^{i\delta_2} |1\rangle$$

- four real parameters - $|\alpha|, |\beta|, \delta_1, \delta_2$

$$|\psi\rangle = e^{i\delta_1} \left(|\alpha| |0\rangle + |\beta| e^{i(\delta_2 - \delta_1)} |1\rangle \right)$$

↑
global phase

As only measurable quantities are $|\alpha|^2, |\beta|^2$
so $e^{i\delta_1}$ (global Phase) has no observable consequences and can be neglected

Eg: $|e^{i\delta}\alpha|^2 = (e^{i\delta}\alpha)^* (e^{i\delta}\alpha) = (e^{-i\delta}\alpha^*)(e^{i\delta}\alpha) \Rightarrow \alpha^*\alpha = |\alpha|^2$

so, $|\psi\rangle = |\alpha| |0\rangle + |\beta| e^{i(\delta_2 - \delta_1)} |1\rangle$

$$\delta_2 - \delta_1 = \phi$$

$$|\psi\rangle = |\alpha| |0\rangle + |\beta| e^{i\phi} |1\rangle$$

- Three real parameters - $|\alpha|, |\beta|, \phi$
where α cant have imaginary part
(i.e. δ_1 made to 0)

Essentially here δ_1 - reference phase (global phase) is absolute phase of qubit which isn't required but relative phase $\phi = \delta_2 - \delta_1$ plays an important role so for $|\psi'\rangle$ in sphere, $(\theta' = \frac{\theta}{2})$

$$|\psi'\rangle = \cos\theta' |0\rangle + e^{i\phi} \sin\theta' |1\rangle$$

$$\boxed{|\psi'\rangle = \cos\frac{\theta}{2} |0\rangle + e^{i\phi} \sin\frac{\theta}{2} |1\rangle}$$

where $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$

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$|\psi\rangle = \underbrace{\alpha|0\rangle + \beta|1\rangle}_{\text{Complex}}$

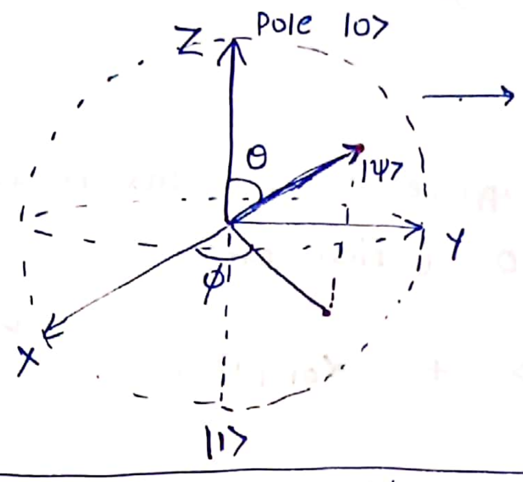
If try to measure the state, it collapses to 0 (or) 1

No reason provided

"Fundamental Postulates of Quantum Mechanics"

Bloch Sphere Visualization

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$



$0 \leq \theta \leq \pi$

$0 \leq \phi < 2\pi$

$$\boxed{|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle}$$

On surface $\longrightarrow$ all states $\rightarrow$ pure state (Prob of $|0\rangle + |1\rangle = 1$)

Inside sphere $\longrightarrow$ states $\longrightarrow$ impure state

(*) How much information does qubit store (or carry in it) ?

i.e. Points on sphere - infinite (its true but information cant be obtained)

α, β $\longrightarrow$ cannot be retrieved (because it collapses to 0 or 1)

So inorder to store more information, we need multiple qubits

2 - qubit system

4 states

$|00\rangle \quad |01\rangle \quad |10\rangle \quad |11\rangle$

4 basis states

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

α_{ij} — complex numbers

$|\alpha_{ij}|^2 =$ Prob of system returning i, j at measurement

⊛ In previous $|\psi\rangle$, suppose one bit was measured and it returned 0 (first one)

[Ratio of probabilities still remains same] ?

$$|\psi'\rangle = \alpha_{00}'|00\rangle + \alpha_{01}'|01\rangle$$

$$= \frac{\alpha_{00}}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}}|00\rangle + \frac{\alpha_{01}}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}}|01\rangle$$

suppose second qubit was measured now (assume 1)

$$|\psi''\rangle = 1 \cdot |01\rangle \qquad \left(\begin{array}{l}\alpha_{01}' = 1 \\ \alpha_{00}' = 0\end{array}\right)$$

α's can be manipulated or changes

$|\psi\rangle \rightarrow$ operated upon

(operators)

↓

circuit gates

Bell Gates (EPR) $\longrightarrow$ Einstein, Podolsky & Rosem

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

suppose first one is measured

if its 0 $\longrightarrow$ second must be 0

if its 1 $\longrightarrow$ second must be 1

so measurement of one qubit affects the other irrespective of their distance.

n qubits

Basis states ? $2^n \longrightarrow b_1 \ldots\ldots b_{2^n}$

$$|\psi\rangle = \sum_{i=1}^{2^n} \alpha_i |b_i\rangle$$

2^n α's stores 2^n α's

Eg: $n = 500 \longrightarrow 2^{500}$ values stored $\longrightarrow$ much greater than # of atoms in universe

Quantum computation

↓ How qubits can be operated upon

Circuits, gates, operators

Single qubit gate

1 Classical gates - NOT gate

like NOT gate in qubits

$$|0\rangle \longrightarrow |1\rangle$$

$$|1\rangle \longrightarrow |0\rangle$$

For $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$\downarrow$ NOT

$|\psi\rangle = \beta|0\rangle + \cancel{\alpha\beta}\ \alpha|1\rangle$

① X gate (operation)

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X|\psi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

② Z gate

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \longrightarrow \boxed{Z} \longrightarrow \alpha|0\rangle - \beta|1\rangle$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ -\beta \end{bmatrix}$$

③ H gate (Hadamard Gate)

$$Z = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \longrightarrow \boxed{H} \longrightarrow \frac{1}{\sqrt{2}}\cancel{[\alpha(|0\rangle + |1\rangle)} + \cdot$$

$$\frac{1}{\sqrt{2}}[(\alpha+\beta)|0\rangle + (\alpha-\beta)|1\rangle]$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \alpha+\beta \\ \alpha-\beta \end{bmatrix}$$

Scanned with CamScanner

6-08-21

Recap

Measuring qbits — 0 or 1 (collapses)

2 qubit system

first bit - measured - 0

$$|\psi\rangle = \frac{\alpha_{00}|00\rangle}{\sqrt{\alpha_{00}^2 + \alpha_{01}^2}} + \frac{\alpha_{01}|01\rangle}{\sqrt{\alpha_{00}^2 + \alpha_{01}^2}}$$

Algorithms
- Discrete Log
- Search Algo

Feymenn proposed : Quantum system simulations

n qubits — 2^n orthogonal states.

→ ~~Gate~~ Gates → operations → qubits

Single qubit operations

NOT

$|0\rangle \longrightarrow |1\rangle$

$|1\rangle \longrightarrow |0\rangle$

$\alpha|0\rangle + \beta|1\rangle \longrightarrow \beta|0\rangle + \alpha|1\rangle$

All operations are assumed to be linear in quantum systems

If it were non-linear → leads to anomalies like
- paradox
- Time travel
- faster than light travel
- time value negative

Matrix

NOT $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\psi : \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ $\alpha|0\rangle + \beta|1\rangle$

$X|\psi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$

Single qubit operations in Classical (VS) Quantum

Classical → only 1 non-trivial operation

Quantum → just one of example ... X and many other non-trivial gates

Rotations on bloch sphere ←

Adjoint of an operator / gate
↓
matrix (2×2)

say U is an operator for a qubit (after operation also the prob. have to sum to 1)

(1) $U^{\dagger}U = I$ ($U^{\dagger}$ - Transpose of complex conjugate of matrix)

verify for X :

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $X^{\dagger} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $X'X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Also called Unitary Constraint $\longrightarrow UU^{\dagger} = I$

verify for Z

$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $Z^{\dagger} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $ZZ^{\dagger} = I$

Hadamard Gate H

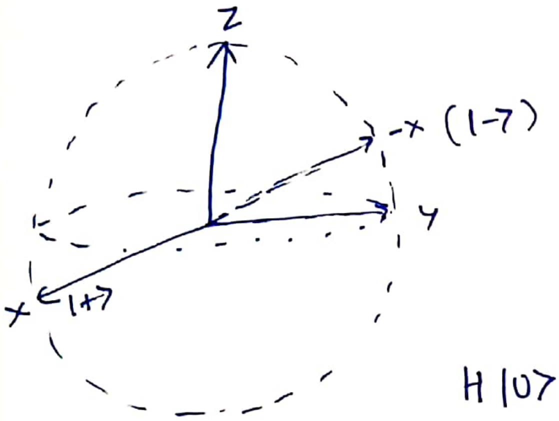
$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H|\psi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \alpha+\beta \\ \alpha-\beta \end{bmatrix}$$

$$H|\psi\rangle = \frac{\alpha+\beta}{\sqrt{2}}|0\rangle + \frac{\alpha-\beta}{\sqrt{2}}|1\rangle$$

$$= \alpha\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) + \beta\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)$$

$$= \alpha|+\rangle + \beta|-\rangle$$



$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$H|0\rangle = H\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = |+\rangle$$

$$H|1\rangle = H\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = |-\rangle$$

Hadamard Gate

1. Rotation about y-axis by 90° anticlockwise
2. Rotation about x-axis by 180° anticlockwise