

CHE613A: The Structure & Rheology Of Complex Fluids (Assignment 2)

Keerthi Vasan M
Roll No.: 21102023
keerthi21@iitk.ac.in
August 12, 2022

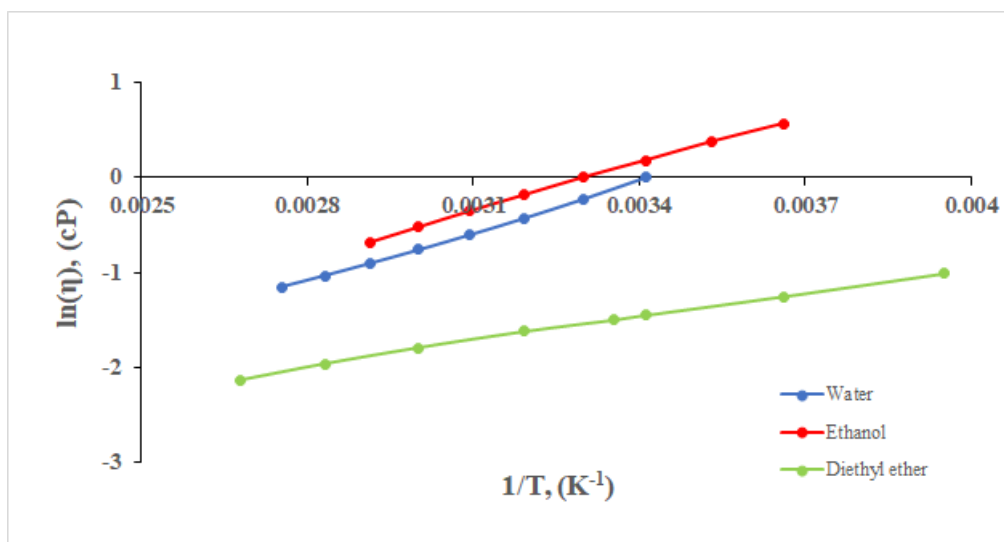
Question no.: 01 By using the data for variation of viscosity with respect to temperature obtain activation energy for flow in kJ/mol for three different liquids. Furthermore, mention latent heat of vaporization in kJ/mol (at the mean of above considered temperatures) and compare the same. Please mention the reference(s).

Answer: Reference: Viscosities of Simple Liquids - Temperature Variation. (2021, March 7). Retrieved August 19, 2021, from <https://chem.libretexts.org/@go/page/96239>

As stated on the literature, variation of viscosity with temperature for water, ethanol, and diethyl ether are as follows:

Water		Ethanol		Diethyl ether	
Temperature (°C)	η (cP)	Temperature (°C)	η (cP)	Temperature (°C)	η (cP)
20	1.002	0	1.773	-20	0.362
30	0.7975	10	1.466	0	0.2842
40	0.6529	20	1.200	20	0.2332
50	0.5468	30	1.003	25	0.222
60	0.4665	40	0.834	40	0.197
70	0.4042	50	0.702	60	0.166
80	0.3547	60	0.592	80	0.140
90	0.3147	70	0.504	100	0.118

One of the simplest model to explain the relationship between viscosity and temperature assumes that viscosity (η) depends on the temperature (T) as follows: $\eta = A \exp\left(\frac{E_a}{RT}\right)$ Here A is a constant (also known as pre-exponential factor), T is the absolute temperature (in K), R is the universal gas constant (J/mol K) and E_a is the activation energy (J/mol). Upon linearization, above equation reduces to $\ln \eta = \ln A + \left(\frac{E_a}{R}\right) \frac{1}{T}$. Using the above mentioned viscosity-temperature data table and the linearized model equation, we can obtain the fluid flow's activation energy from the slope of the plot.



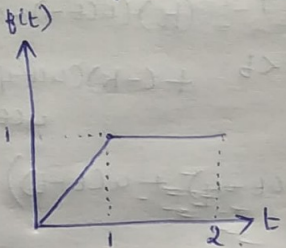
From the above plot following results were calculated, and shown below. It can be seen that the activation energy should be less than the heat of vaporization by a factor of 0.4. In other words, upper bound value for bond activation energy of a fluid is the latent heat of vaporization of that fluid.

Fluid	Slope $\left(\frac{E_a}{R}\right)$ (K)	Activation energy (E_a) (kJ/mol)	Latent heat of vaporization $\left(\approx \frac{E_a}{0.4}\right)$ (kJ/mol)
Water	1753.224	14.576	36.440
Ethanol	1690.136	14.051	35.129
Diethyl ether	866.1372	7.201	18.002

Question no.: 02 If ordinate (vertical axis) is $f(t)$ and abscissa (horizontal axis) is t , represent following functional forms in terms of unit step functions and obtain Laplace transform of the same. Please note that: $L(u(t-a)f(t-a)) = e^{-as}F(s)$ (You may have to use other Laplace transforms to including that of mentioned on the last page to obtain the results). You may find following website useful to understand the topic: <http://www.intmath.com/laplace-transformation/intro.php>

Answer:

①



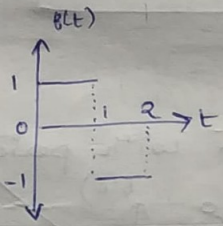
$$f(t) = \begin{cases} t & 0 < t < 1 \\ 1 & 1 < t < 2 \\ 0 & t > 2 \end{cases} = t[u(t) - u(t-1)] + [u(t-1) - u(t-2)]$$

$$\Rightarrow f(t) = (t-0)u(t-0) - (t-1)u(t-1) - u(t-2)$$

$$L(f(t)) = F(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}$$

$$F(s) = \frac{1 - e^{-s}}{s^2} - \frac{e^{-2s}}{s}$$

②

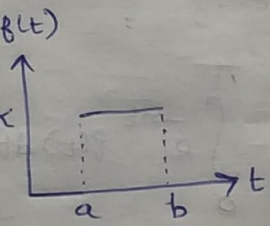


$$f(t) = \begin{cases} 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \end{cases} = [u(t) - u(t-1)] + (-1)[u(t-1) - u(t-2)]$$

$$f(t) = u(t) - 2u(t-1) + u(t-2)$$

$$L(f(t)) = F(s) = \frac{1 - 2e^{-s} + e^{-2s}}{s}$$

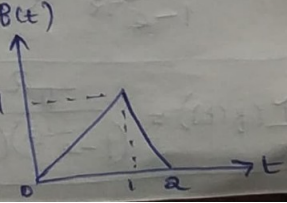
③



$$f(t) = \begin{cases} 0 & 0 < t < a \\ K & a < t < b \\ 0 & t > b \end{cases} = K[u(t-a) - u(t-b)]$$

$$L(f(t)) = \frac{K}{s} (e^{-as} - e^{-bs})$$

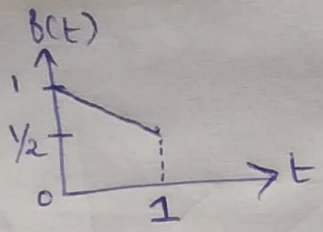
④



$$f(t) = \begin{cases} t & 0 < t < 1 \\ 2-t & 1 < t < 2 \end{cases} = t[u(t) - u(t-1)] + (t-1)[u(t-1) - u(t-2)]$$

$$L(f(t)) = \frac{1 - 2e^{-s} + e^{-2s}}{s^2} = \frac{(1 - e^{-s})^2}{s^2}$$

⑤



$$f(t) = \left(1 - \frac{t}{2}\right) [u(t-0) - u(t-1)]$$

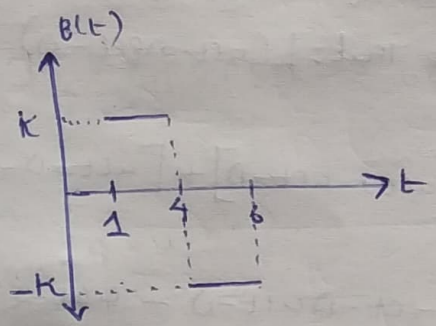
$$\Rightarrow f(t) = u(t) - \frac{t u(t)}{2} + \frac{(t-1) u(t-1)}{2} + \frac{u(t-1)}{2} - u(t-1)$$

$$\therefore L(f(t)) = \frac{e^{-0s}}{s} - \frac{e^{-0s}}{2s^2} + \frac{e^{-s}}{2s^2} + \frac{e^{-s}}{2s} - \frac{e^{-s}}{s}$$

$$= \frac{2s - 1 + e^{-s} + s e^{-s} - 2s e^{-s}}{2s^2}$$

$$L(f(t)) = \frac{e^{-s} [1 - s] + (2s - 1)}{2s^2}$$

⑥



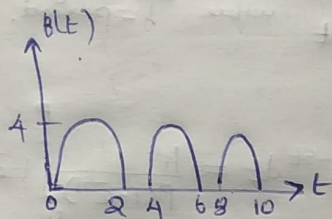
$$f(t) = \begin{cases} k & 1 < t < 4 \\ -k & 4 < t < 6 \end{cases} = (k)(u(t-1) - u(t-4)) + (-k)(u(t-4) - u(t-6))$$

$$f(t) = k(u(t-1) - 2u(t-4) + u(t-6))$$

$$L(f(t)) = k \left[\frac{e^{-s}}{s} - 2 \frac{e^{-4s}}{s} + \frac{e^{-6s}}{s} \right]$$

$$\Rightarrow L(f(t)) = F(s) = \frac{k e^{-s}}{s} (1 - 2e^{-3s} + e^{-5s})$$

⑦



Here $B(t)$ is a periodic with a period '4'

$$\therefore L(B(t)) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} B(t) dt \quad (1)$$

$$\text{Here } B(t) = \begin{cases} 4 \sin\left(\frac{\pi}{2}t\right) & , 0 < t < 2 \\ 0 & , 2 < t < 4 \end{cases} \quad (2)$$

$$\therefore L(B(t)) = \frac{4}{(1-e^{-4s})} \int_0^2 e^{-st} \sin\left(\frac{\pi}{2}t\right) dt = \frac{4}{1-(e^{-2s})^2} I \quad (3)$$

$$I = \int_0^2 e^{-st} \sin\left(\frac{\pi}{2}t\right) dt = \text{Imaginary part} \left\{ \int_0^2 e^{-st} e^{i\frac{\pi}{2}t} dt \right\}$$

$$I = \text{Imaginary part} \left\{ \left[\frac{\exp\left(\left[\frac{\pi}{2}i - s\right]t\right)}{\left[\frac{\pi}{2}i - s\right]} \right]_0^2 \right\}$$

$$I = \text{Imaginary part} \left\{ \frac{\exp(\pi i - 2s) - 1}{\frac{\pi}{2}i - s} \right\} = \text{I.P} \left\{ \frac{e^{\pi i} \cdot e^{-2s} - 1}{\frac{\pi}{2}i - s} \right\}$$

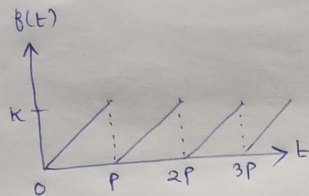
$$I = \text{I.P} \left\{ \frac{(-1)(e^{-2s}) - 1}{\frac{\pi}{2}i - s} \times \frac{\frac{\pi}{2}i + s}{\frac{\pi}{2}i + s} \right\} = \text{I.P} \left\{ \frac{\left(\frac{\pi}{2}i + s\right)(1 + e^{-2s})}{\frac{\pi^2}{4} + s^2} \right\}$$

$$\boxed{I = \frac{\frac{\pi}{2}(1 + e^{-2s})}{\frac{\pi^2}{4} + s^2}} \quad (4)$$

$$\therefore (4) \text{ in } (3), \quad L(B(t)) = \frac{4}{1-(e^{-2s})^2} \left[\frac{\frac{\pi}{2}(1 + e^{-2s})}{\frac{\pi^2}{4} + s^2} \right]$$

$$\boxed{L(B(t)) = \frac{2\pi}{(1-e^{-2s})(\frac{\pi^2}{4} + s^2)}}$$

⑧



Here $f(t) = \frac{kt}{P}$ is repeating with a period 'P'. We know that for periodic functions, Laplace transform is given

by $L(B(t)) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} B(t) dt$ where $T = \text{period}$.

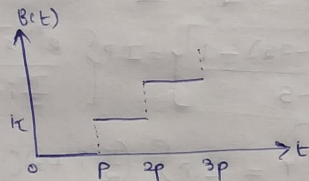
$$L(B(t)) = F(s) = \frac{1}{1-e^{-sP}} \int_0^P \frac{kt}{P} e^{-st} dt = \frac{k}{P(1-e^{-sP})} \int_0^P t e^{-st} dt$$

$$\left. \begin{aligned} u &= t, \quad V = e^{-st} \\ u' &\equiv 1, \quad \int V du = e^{-st} \left(-\frac{1}{s}\right) \\ u'' &= 0, \quad \int V du = e^{-st} \left(\frac{1}{s^2}\right) \end{aligned} \right\} \Rightarrow F(s) = \frac{k}{P(1-e^{-sP})} \left(-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2} \right)_0^P$$

$$F(s) = \frac{k}{P} \left(-\frac{Pe^{-sP}}{s} - \frac{e^{-sP}}{s^2} + \frac{1}{s^2} \right) \left(\frac{1}{1-e^{-sP}} \right)$$

$$\therefore F(s) = \frac{k}{P} \left(\frac{1}{s^2} - \frac{Pe^{-sP}}{s(1-e^{-sP})} \right)$$

⑨



$$B(t) = k(u(t-P) + u(t-2P) + u(t-3P) + \dots)$$

$$B(t) = k[u(t-P) + u(t-2P) + u(t-3P) + \dots]$$

$$L(B(t)) = k \left[\frac{e^{-sP}}{s} + \frac{e^{-2sP}}{s} + \frac{e^{-3sP}}{s} + \dots \right]$$

$$L(B(t)) = F(s) = \frac{ke^{-sP}}{s} (1 + e^{-sP} + e^{-2sP} + \dots \infty) \quad \text{--- (1)}$$

Terms in the bracket form a geometric series with a common ratio e^{-sP} . We know that, Sum of n terms of a geometric progression

is given by $S^n = \frac{a(r^n - 1)}{(r - 1)}$ a - First term

$$\therefore 1 + e^{-sP} + e^{-2sP} + \dots \infty = \frac{(1)(e^{-sPn} - 1)}{(e^{-sP} - 1)} \quad \rightarrow (2)$$

$$= \frac{e^{-sPn} - 1}{e^{-sP} - 1} \quad \rightarrow (3) \quad \left(\begin{array}{l} \text{In eqn (1),} \\ \text{Sum is for } \infty \\ \text{terms. So, } n = \infty \end{array} \right)$$

$$= \frac{-1}{e^{-sP} - 1} = \frac{1}{1 - e^{-sP}} \quad \rightarrow (4)$$

Substitute (4) in (1):

$$F(s) = \frac{ke^{-sp}}{s(1-e^{-sp})}$$