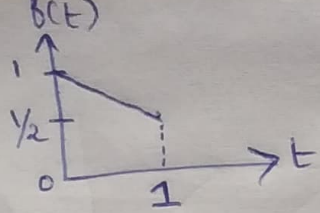
(5)



$$f(t) = \left(1 - \frac{t}{2}\right)\left[u(t-0) - u(t-1)\right]$$

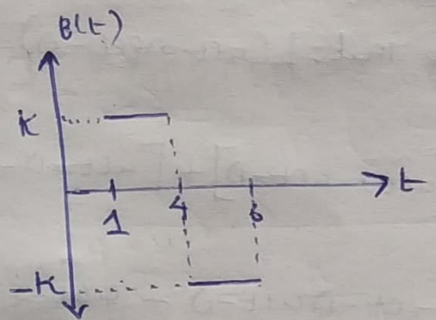
$$\Rightarrow f(t) = u(t) - \frac{t\,u(t)}{2} + \frac{(t-1)\,u(t-1)}{2} + \frac{u(t-1)}{2} - u(t-1)$$

$$\therefore L(f(t)) = \frac{e^{-0s}}{s} - \frac{e^{-0s}}{2s^2} + \frac{e^{-s}}{2s^2} + \frac{e^{-s}}{2s} - \frac{e^{-s}}{s}$$

$$= \frac{2s - 1 + e^{-s} + s e^{-s} - 2s e^{-s}}{2s^2}$$

$$\boxed{L(f(t)) = \frac{e^{-s}[1-s] + (2s-1)}{2s^2}}$$

(6)



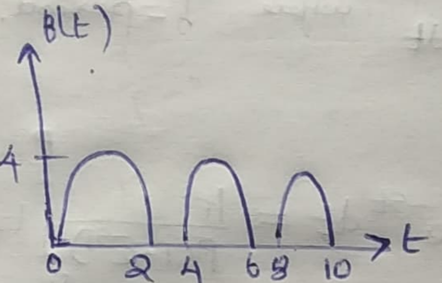
$$f(t) = \begin{cases} k & 1 < t < 4 \\ -k & 4 < t < 6 \end{cases} = (k)(u(t-1) - u(t-4)) + (-k)(u(t-4) - u(t-6))$$

$$f(t) = k\left(u(t-1) - 2u(t-4) + u(t-6)\right)$$

$$L(f(t)) = k\left[\frac{e^{-s}}{s} - \frac{2e^{-4s}}{s} + \frac{e^{-6s}}{s}\right]$$

$$\Rightarrow \boxed{L(f(t)) = F(s) = \frac{ke^{-s}}{s}\left(1 - 2e^{-3s} + e^{-5s}\right)}$$

7



Here $f(t)$ is a periodic with a period '4'

$$\therefore L(f(t)) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt \quad \text{— (1)}$$

Here $$f(t) = \begin{cases} 4\sin\left(\frac{\pi}{2}t\right), & 0<t<2 \\ 0, & 2<t<4 \end{cases} \quad \text{— (2)}$$

$$\therefore L(f(t)) = \frac{4}{(1-e^{-4s})} \int_0^2 e^{-st} \sin\left(\frac{\pi}{2}t\right) dt = \frac{4}{1-(e^{-2s})^2}\, I \quad \text{— (3)}$$

$$I = \int_0^2 e^{-st} \sin\left(\frac{\pi}{2}t\right) dt = \underset{\text{part}}{\text{Imaginary}} \left\{ \int_0^2 e^{-st} e^{\left(i\frac{\pi}{2}t\right)} dt \right\}$$

$$I = \text{Imaginary part} \left\{ \left[\frac{\exp\left(\left[\frac{\pi}{2}i - s\right]t\right)}{\left[\frac{\pi}{2}i - s\right]} \right]_0^2 \right\}$$

$$I = \text{Imaginary part} \left\{ \frac{\exp(\pi i - 2s) - 1}{\frac{\pi}{2}i - s} \right\} = \text{I.P} \left\{ \frac{e^{\pi i} . e^{-2s} - 1}{\frac{\pi}{2}i - s} \right\}$$

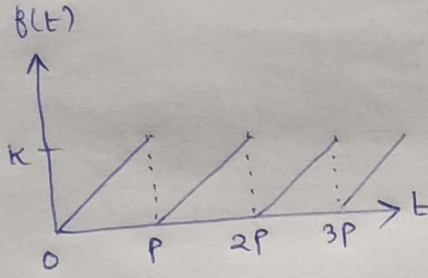
$$I = \text{I.P} \left\{ \frac{(-1)(e^{-2s}) - 1}{\frac{\pi}{2}i - s} \times \frac{\frac{\pi}{2}i + s}{\frac{\pi}{2}i + s} \right\} = \text{I.P} \left\{ \frac{\left(\frac{\pi}{2}i + s\right)(1+e^{-2s})}{\frac{\pi^2}{4} + s^2} \right\}$$

$$\boxed{I = \frac{\frac{\pi}{2}(1+e^{-2s})}{\frac{\pi^2}{4} + s^2}} \quad \text{— (4)}$$

$\therefore$ (4) in (3), $$L(f(t)) = \frac{4}{1-(e^{-2s})^2} \left[\frac{\frac{\pi}{2}(1+e^{-2s})}{\frac{\pi^2}{4} + s^2} \right]$$

$$\boxed{L(f(t)) = \frac{2\pi}{(1-e^{-2s})\left(\frac{\pi^2}{4} + s^2\right)}}$$

(8)



Here $f(t) = \frac{kt}{p}$ is repeating with a period 'p'. We know that for periodic functions, laplace transform is given by

$$L(f(t)) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt \quad \text{where } T = \text{period.}$$

$$L(f(t)) = F(s) = \frac{1}{1-e^{-sp}} \int_0^p \frac{kt}{p} e^{-st}\,dt = \frac{k}{p(1-e^{-sp})} \int_0^p t e^{-st}\,dt$$

$u = t, \quad V = e^{-st}$

$u' = 1, \quad \int v\,dt = e^{-st}\left(-\frac{1}{s}\right)$

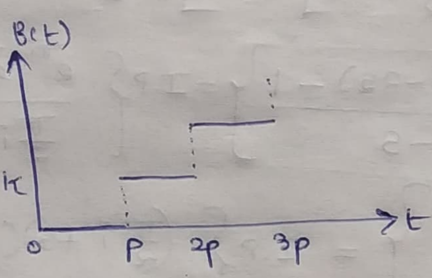
$u'' = 0, \quad \int v\,dt = e^{-st}\left(\frac{1}{s^2}\right)$

$$\Rightarrow F(s) = \frac{k}{p(1-e^{-sp})}\left(-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right)_0^p$$

$$F(s) = \frac{k}{p}\left(\frac{-pe^{-sp}}{s} - \frac{e^{-sp}}{s^2} + \frac{1}{s^2}\right)\left(\frac{1}{1-e^{-sp}}\right)$$

$$\therefore \boxed{F(s) = \frac{k}{p}\left(\frac{1}{s^2} - \frac{pe^{-sp}}{s(1-e^{-sp})}\right)}$$

(9)



$$f(t) = ku(t-p) + ku(t-2p) + k(u(t-3p)) + \dots$$

$$f(t) = k[u(t-p) + u(t-2p) + u(t-3p) + \dots]$$

$$L(f(t)) = k\left[\frac{e^{-sp}}{s} + \frac{e^{-2sp}}{s} + \frac{e^{-3sp}}{s} + \dots\right]$$

$$L(f(t)) = F(s) = \frac{ke^{-sp}}{s}\left(1 + e^{-sp} + e^{-2sp} + \dots \infty\right) \quad \text{—(1)}$$

Terms in the brackets form a geometric series with a common ratio e^{-sp}. We know that, Sum of 'n' terms of a geometric progression is given by $S^n = \frac{a(r^n - 1)}{(r-1)}$ a – First term

$$\therefore 1 + e^{-sp} + e^{-2sp} + \dots \infty = \frac{(1)(e^{-spn} - 1)}{(e^{-sp} - 1)} \rightarrow (2)$$

$$= \frac{e^{-\infty} - 1}{e^{-sp} - 1} \longrightarrow (3)$$

(In eqn (1), Sum is for '∞' terms. So, $n = \infty$)

$$= \frac{-1}{e^{-sp} - 1} = \frac{1}{1 - e^{-sp}} \longrightarrow (4)$$

Substitute (4) in (1); $\boxed{F(s) = \dfrac{k e^{-sP}}{s(1 - e^{-sP})}}$