## 1. Integral domain, prime ideal and maximal ideal

(1) Let $R$ be an integral domain. Prove that units in the polynomial ring $R[x]$ are units of $R$.

(2) Is there an integral domain containing exactly 10 elements?

(3) Find the quotient field of the power series ring $R[|x|]$.

(4) Find integral domains among the rings $R = \mathbb{F}_5[x]/(x^2+x+1)$ and $S = \mathbb{F}_3[x]/(x^2 + x + 1)$.

(5) Determine maximal ideals of $\mathbb{Z}[x]$ and $F[|x|]$ where $F$ is a field.

(6) Prove that $m = (x + y^2, y + x^2 + 2xy^2 + y^4) \subset \mathbb{C}[x, y]$ is a maximal ideal.

(7) Consider the ideal $I = (y^2 + x^3 - 17)$ of $R = \mathbb{C}[x, y]$. Find generators of all maximal ideals in the quotient ring $R/I$.

(8) Show that $\mathbb{Z}_3[x]/(x^2 + x + 1)$ is not a field.

(9) How many elements are in $Z[i]/(3 + i)$? Give reason.

(10) Let $R = \{f : \mathbb{R} \longrightarrow \mathbb{R} | f$ is a continuous function$\}$. Show that $I = \{f \in R | f(0) = 0\}$ is a maximal ideal of $R$.

(11) Show that $\mathbb{Z}[i]/(1 - i)$ is a field. How many elements does this field have?

(12) In $\mathbb{Z}_5[x]$, let $I = (x^2 + x + 2)$. Find multiplicative inverse of $2x + 3 + I$ in $\mathbb{Z}_5[x]/I$.

(13) In $\mathbb{Z}[x]$ let $I = \{f(x) \in \mathbb{Z}[x] | f(0)$ is an even integer $\}$. Prove that $I = (2, x)$. Is $I$ a prime ideal of $\mathbb{Z}[x]$? Is $I$ a maximal ideal? How many elements does $\mathbb{Z}[x]/I$ have ?

(14) Prove that $(2 + 2i)$ is not a prime ideal in $\mathbb{Z}[i]$.

(15) Prove that $(3)$ is a maximal ideal in $\mathbb{Z}[i]$.