Indefinate Integration

formulae

→ $\int dx = x + c$

→ $\int x^n\, dx = \dfrac{x^{n+1}}{n+1} + c \qquad n \neq 1$

→ $\int \dfrac{1}{x}\, dx = \log x + c$

→ $\int e^x\, dx = e^x + c$

$\int \log x\, dx = x(\log x - 1) + c$

→ $\int a^x\, dx = \dfrac{a^x}{\log a} + c$

→ $\int \sin x\, dx = -\cos x$

→ $\int \cos x\, dx = \sin x$

→ $\int \tan x\, dx = \log|\sec x| + c$

→ $\int \sec x\, dx = \log|\sec x + \tan x| + c$

$= \log\left|\tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)\right| + c$

→ $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + c$

$= \log\left|\tan \dfrac{x}{2}\right| + c$

→ $\int \cot x\, dx = -\log|\operatorname{cosec} x| + c$

$= \log|\sin x| + c$

$\rightarrow \int \sec^2 x \, dx = \tan x + c$

$\rightarrow \int \operatorname{cosec}^2 x \, dx = -\cot x + c$

$\rightarrow \int \sec x \tan x \, dx = \sec x + c$

$\rightarrow \int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

$\rightarrow \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$

$= -\cos^{-1} x + c$

$\rightarrow \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

$= -\cot^{-1} x + c$

$\rightarrow \int \frac{1}{|x|\sqrt{x^2-1}} = \sec^{-1} x + c$

$= -\operatorname{cosec}^{-1} x + c$

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☆ $\rightarrow$ If $N^r = \sin x + \cos x$

take $t = \sin x - \cos x$ or vice versa

$\rightarrow \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$\rightarrow \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$

$\rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = \log \left| x + \sqrt{x^2+a^2} \right| + c$

$\rightarrow \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \left| \left(\frac{x}{a} \right) \right| + c$

→ $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + c$

→ $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$

→ To Integrate $\frac{1}{a\cos^2 x + b\sin^2 x}$ Multiply & Divide by $\boxed{\sec^2 x}$

→ $\int \frac{1}{\text{Quad}}\,dx$ or $\int \frac{1}{\sqrt{\text{Quad}}}\,dx$ or $\int \text{Quad}\,dx$ or $\int \sqrt{\text{Quad}}\,dx$

can be solved in 3 steps :-

- x^2 should have coeff 1
- convert to form of $(x+a)^2 + b$
- Apply formulae

→ $\int \frac{\text{linear}}{\text{Quad}}\,dx$ or $\int \frac{\text{linear}}{\sqrt{\text{Quad}}}\,dx$ or $\int \text{linear}\,(\text{Quad})\,dx$ or

$\int \text{linear}\sqrt{\text{Quad}}\,dx$

* express linear as $A\frac{d}{dx}\text{Quad} + B$

→ For $\frac{\text{Quad}}{\text{BiQuad}}$ $\frac{x}{x^4}$ form take $t = x+\frac{1}{x}$ or $x-\frac{1}{x}$

→ If $\int \text{inverse}\,dx$ or $\int \log\,dx$ appears, use by parts method

* → $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x) + c$

→ $\int e^{f(x)}\left(1+xf'(x)\right)dx = xe^{f(x)} + c$

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$$\rightarrow \int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right| + c$$

$$\rightarrow \int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{\cancel{a^2+x^2}\; x^2-a^2} - \frac{a^2}{\cancel{x}2}\log\left|x+\sqrt{x^2-a^2}\right| + c$$

$$\rightarrow \int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$$

$\rightarrow$ for $\int \frac{dx}{a\cos x + b\sin x + c}$ use $t = \tan\frac{x}{2}$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$\rightarrow$ for $\int \frac{a\cos x + b\sin x + c}{d\cos x + e\sin x + f}\,dx$

* express $N^r = A\frac{d}{dx}D^r + BD^r + k$

$\rightarrow$ for $\int \frac{a\cos x + b\sin x}{\cancel{\cos}\ c\cos x + d\sin x}\,dx$

express $N^r = A\frac{d}{dx}D^r + BD^r$

$$\rightarrow \int e^{ax}\sin(bx+c)\,dx = \frac{e^{ax}}{b^2+a^2}\left(a\sin(bx+c) - b\cos(bx+c)\right) + k$$

$$\rightarrow \int e^{ax}\cos(bx+c)\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\,\cancel{\sin}\cos(bx+c) + b\sin(bx+c)\right) + k$$

$\rightarrow \dfrac{1}{\pm a\sin x \pm b\cos x}$ Multiply N^r & D^r by $\sqrt{a^2+b^2}$

$\rightarrow$ In case of $f(x) = \dfrac{2}{x^2 \pm \frac{1}{x^2}}$ take $f(x) = \dfrac{1+\frac{1}{x^2}+1-\frac{1}{x^2}}{x^2 \pm \frac{1}{x^2}}$

$\rightarrow \sec x + \tan x = t \Rightarrow \sec x - \tan x = \dfrac{1}{t}$

$\therefore \sec x = \dfrac{1}{2}\left(t + \dfrac{1}{t}\right)$

Partial fractions

Form 1 :- Lin $f(x) = \dfrac{N^r}{(a_1x+b_1)(a_2x+b_2)+\ldots\ldots}$

Express $\dfrac{N^r}{(a_1x+b_1)(a_2x+b_2)+\ldots\ldots} = \dfrac{A}{(a_1x+b_1)} + \dfrac{B}{(a_2x+b_2)} + \ldots\ldots$

Form 2 :- $f(x) = \dfrac{N^r}{(a_1x+b_1)^2(a_2x+b_2)\ldots\ldots}$

Express $f(x) = \dfrac{A_1}{(a_1x+b_1)} + \dfrac{A_2}{(a_1x+b_1)^2} + \dfrac{B}{(a_2x+b_2)} + \ldots\ldots$

?

Form 3 :- $f(x) = \dfrac{N^r}{(a_1x^2+b_1x+c_1)(a_2x+b_2)\ldots\ldots}$ (If $a_1x^2+b_1x+c$ cannot be expressed as factors)

Express $f(x) = \dfrac{A_1x+A_2}{(a_1x^2+b_1x+c)} + \dfrac{B}{a_2x+b_2} + \ldots\ldots$

Note :- Degree of N^r must be lesser than D^r, if not, then divide N^r by D^r

→ <u>Integration by parts</u>

I L A T E

$$\int \underset{\text{I}}{u}\,\underset{\text{II}}{v}\,dx = \overset{u\,v_1}{u\int v\,dx} - \overset{\int u' v_1\,dx}{\int \frac{du}{dx}\left(\int v\,dx\right)dx}$$

$$\to \int \frac{1}{(x^2+a^2)(x^2+b^2)}\,dx = \frac{1}{b^2-a^2}\int \frac{\overset{(x^2+b^2)-(a^2+x^2)}{b^2-a^2}}{(x^2+a^2)(x^2+b^2)}\,dx$$

$$\to \int \sin^m x \cos^n x\,dx = \int \sin^m x \cos^{n-1} x\,(\cos x\,dx)$$

take $\sin x = t$

(n is odd)

If m is odd vice versa

$$\to \int \frac{1}{\underset{\text{or } \cos}{\sin(x-a)}\,\underset{\text{or } \cos}{\sin(x-b)}} \longrightarrow \frac{1}{\sin(a-b)}\int \frac{\sin(a-b)}{\sin(x-a)\sin(x-b)}$$

$$= \frac{1}{\sin(a-b)}\int \frac{\sin\left((x-b)-(x-a)\right)}{\sin(x-a)\sin(x-b)}$$

Similar adjustments in other cases as well