Definite Integration

Substitution :-

By the substitution $x = \phi(t)$ in the definite integral $\int_a^b f(x)\,dx$

The lower & upper limits change

$$\int_a^b f(x)\,dx = \int_{\phi^{-1}(a)}^{\phi^{-1}(b)} f(\phi(t))\,\phi'(t)\,dt$$

(OR)

$$LL \Rightarrow \quad x = \phi(t)$$
$$a = \phi(t) \Rightarrow \quad t_{LL} = \phi^{-1}(a)$$

$$UL \Rightarrow \quad b = \phi(t) \Rightarrow \quad t_{UL} = \phi^{-1}(b)$$

Properties.

$$\rightarrow \int_a^b f(x)\,dx = \int_a^b f(t)\,dt$$

$$\rightarrow \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\rightarrow \int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \quad ; \; a<c<b$$

(Note :- This property can be useful when $f(x)$ changes at 'c')

$$\rightarrow \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$\rightarrow \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\rightarrow \int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(x) \text{ is even} \\ 0 & \text{if } f(x) \text{ is odd} \end{cases}$$

$$\rightarrow \int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$$

<u>Note</u> :-

When UL is not defined $\longrightarrow$ take left limit
" LL " " " $\longrightarrow$ " right "

<u>Vallice formula</u>

$\rightarrow$ for $I = \int_0^{\pi/2} \sin^n x\,dx$ or $\int_0^{\pi/2} \cos^n x\,dx$

if n is even

$$I = \frac{(n-1)(n-3)}{n(n-2)} \cdots\cdots \frac{1}{2} \times \frac{\pi}{2}$$

if n is odd

$$I = \frac{(n-1)(n-3)}{n(n-2)} \cdots\cdots \frac{2}{3}$$

$\rightarrow$ for $I = \int_0^{\pi/2} \sin^n x \cos^m x \, dx$

$$I = \frac{(n-1)(n-3) \ldots . 1 \text{ or } 2 \; (m-1)(m-3) \ldots .. 1 \text{ or } 2}{(n+m)(n+m-2) \ldots . 1 \text{ or } 2} \times \frac{\pi}{2}$$

if both m & n are even

$$I = \frac{(n-1)(n-3) \ldots . 1 \text{ or } 2 \; (m-1)(m-3) \ldots .. 1 \text{ or } 2}{(n+m)(n+m-2) \ldots .. 1 \text{ or } 2}$$

if m or n is odd

Periodic function

$\rightarrow \int_a^{a+nT} f(x)\,dx = \int_0^{nT} f(x)\,dx = n \int_0^{T} f(x)\,dx$

$\rightarrow \int_{nT}^{mT} f(x)\,dx = (m-n) \int_0^{T} f(x)\,dx$

$\rightarrow \int_{a+nT}^{b+nT} f(x)\,dx = \int_a^{b} f(x)\,dx$

Note :-

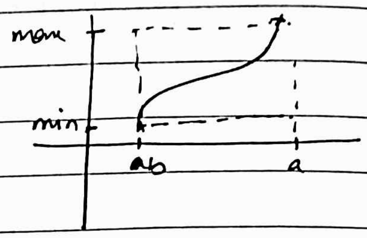
$\rightarrow \int_b^{a} f(x)\,dx = p \int_{b/p}^{a/p} f(xp)\,dx$

$\rightarrow \int_b^{a} f(x)\,dx = \int_{b-d}^{a-d} f(x+d)\,dx$

Property :-

$$\rightarrow \left| \int_a^b f(x)\,dx \right| \leq \int_a^b |f(x)|\,dx$$

$$\cancel{\rightarrow \min f(x)(b-a) \leq \int_b^a f(x)\,dx}$$



$$\rightarrow \left[\min f(x)\right](b-a) \leq \int_b^a f(x)\,dx \leq \left[\max f(x)\right](b-a)$$

equality only for const function

Differentiation of a fn definate integral

$$\rightarrow \frac{d}{dx} \int_{f_1(x)}^{f_2(x)} g(t)\,dt = f_2'(x)\, g\left(f_2(x)\right) - f_1'(x)\left(g(f_1(x))\right)$$

Definite integral as limit of sum

$$\lim_{n\to\infty} \sum_{r=a}^{b} \frac{1}{n} \xrightarrow[\text{with}]{\text{replace}} \int \quad dx$$

$$\frac{r}{n} \longrightarrow x$$

$$LL = \cancel{\frac{\alpha}{n}} \left(\lim_{n\to\infty} \frac{a}{n}\right) = \lim_{n\to\infty} \frac{\text{Lower value of } r}{n}$$

$$UL = \cancel{\frac{b}{n}} \left(\lim_{n\to\infty} \frac{b}{n}\right) = \lim_{n\to\infty} \frac{\text{Upper value of } r}{n}$$

Imp :-

$$\int_0^{\pi/2} \log(\sin x)\,dx = \frac{\pi}{2} \log \frac{1}{2}$$

↓ or $\cos x$

Indefinite integration.

Note:-

* use trigo ratios for substitution
* In integration, rationalize N^r
* convert

$$\cos, \sin \rightleftharpoons \cot, \tan$$

* take $\sqrt{ax+b}$ as t^2

* $\dfrac{1}{\cos^4\theta + \sin^4\theta}$ mutiply & Divide $\sec^4\theta$