Assignment - 2 (submission deadline: 19 February, 2021)

Note: Unless otherwise stated, notation used is as defined in the class.

1. Given the Boolean function
$$F = x\overline{y}z + \overline{x}\ \overline{y}z + xyz$$
   (a) List the truth table of the function.
   (b) Draw the logic diagram using the original Boolean expression.
   (c) Simplify the algebraic expression using Boolean algebra.
   (d) List the truth table of the function from the simplified expression and show that it is the same as the truth table in part (a).
   (e) Draw the logic diagram from the simplified expression and compare the total number of gates with the diagram of part (b).

2. Simplify the following expression in (a) sum-of-product form, and (b) product-of-sum form.
$$A\overline{C} + \overline{B}D + \overline{A}CD + +ABCD$$

3. Simplify the following Boolean function in sum-of-products form by means of a four-variable map. Draw the logic diagram with (a) AND-OR gates; (b) NAND gates.
$$F(A, B, C, D) = \sum(0, 2, 8, 9, 10, 11, 14, 15)$$

4. Simplify the following Boolean function in product-of-sum form by means of a four-variables map. Draw the logic diagram with (a) OR-AND gates; (b)NOR gates
$$F(w, x, y, z) = \sum(2, 3, 4, 5, 6, 7, 11, 14, 15)$$

5. Simplify the Boolean function F together with the don't care conditions d in (a) sum-of-products form, and (b) product-of-sums form
$$F(w, x, y, z) = \sum(0, 1, 2, 3, 7, 8, 10)$$
$$d(w, x, y, z) = \sum(5, 6, 11, 15)$$

6. Use the tabulation procedure to generate the set of prime implicants and to obtain all minimal expressions for the following functions:
   (a) $f(w, x, y, z) = \sum(0, 1, 5, 7, 8, 10, 14, 15)$
   (b) $f(w, x, y, z) = \sum(0, 1, 2, 5, 7, 8, 9, 10, 13, 15)$

7. Simplify the sum-of-products expression for the function
$$f(x, y, z) = xyz + x\overline{y}z + x\overline{y}\ \overline{z} + \overline{x}yz + \overline{x}y\overline{z} + \overline{x}\ \overline{y}\ \overline{z}$$
   using (a) a k-map, (b)the Quine-McCluskey method.

8. Determine the cannonical sum-of-products representation of the following functions:

   (a) $f(x, y, z) = z + (\overline{x} + y)(x + \overline{y})$
   (b) $f(x, y, z) = x + \overline{(\overline{x}\ \overline{y} + \overline{x}z)}$

9. Simplify the algebraic expression:

$$(\overline{x} + xy\overline{z}) + (\overline{x} + xy\overline{z})(x + \overline{x}\ \overline{y}z)$$

10. Find the complement of

$$\overline{w} + (\overline{x} + y + \overline{y}\ \overline{z})(x + \overline{y}z)$$

    and then simplify it.

11. Given $A\overline{B} + \overline{A}B = C$, show that $A\overline{C} + \overline{A}C = B$

12. Show that $F^d(x_1, \ldots, x_n) = \overline{F(\overline{x}_1, \ldots, \overline{x}_n)}$ for a Boolean function $F(x_1, \cdots, x_n)$, where F^d stands for the dual of F.

13. Prove that the Cartesian product of two enumerable sets is enumerable.

14. Let S be an enumerable subset and T be an infinite non-enumerable subset of $\mathbb{R}$. Prove that

    (i) $S \cup T$ is non-enumerable.
    (ii) $S \cap T$ is at most enumerable.
    (iii) $S - T$ is at most enumerable.
    (iv) $T - S$ is non-enumerable.

15. Prove that the sets A and B are equipotent.

    (i) $A = \{x \in \mathbb{R} : 0 \leqslant x \leqslant 1\}$, $B = \{x \in \mathbb{R} : 0 \leqslant x < 1\}$
    (ii)$A = \{x \in \mathbb{R} : 0 \leqslant x \leqslant 1\}$, $B = \{x \in \mathbb{R} : a \leqslant x \leqslant b\}$
    (iii)$A = \{x \in \mathbb{R} : 0 \leqslant x \leqslant 1\}$, $B = \{x \in \mathbb{R} : 0 < x < 1\}$
    (iv)$A = \{x \in \mathbb{R} : x \geqslant 1\}$, $B = \{x \in \mathbb{R} : x > 1\}$

16. Determine whether each of these sets is countable or uncountable. For those that are countable, exhibit a one-to-one correspondence between the set of natural numbers and that set.

    (a) the integers greater than 10
    (b) the odd negative integers
    (c) the real numbers between 0 and 2
    (d) integers that are multiples of 10

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