

# Home Work: Simplex Method / Big-M Method / Two Phase Simplex Method

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# Two- Phase Simplex Method:

**Numerical Example -TPS1: Two different LPP are considered**

$$LPP - 1 : \max : Z = x_1 + 3x_2 + x_3$$

$$LPP - 2 : \min : Z = 3x_1 + x_2 + x_3$$

**Subject to**

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_3 = 16$$

$$x_1, x_2, x_3 \geq 0$$

# Two- Phase Simplex Method: LPP-1:

## Numerical Example -TPS1: Condensed Tableau

$$\max : Z = x_1 + 3x_2 + x_3$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_3 = 16$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS1:

### First Phase Objective Function:

$$\min : f1 = a_1 + a_2 = \sum \text{artificial variables}$$

$$\max : -f1 = -a_1 - a_2$$

### Second Phase Objective Function :

$$\max : Z = x_1 + 3x_2 + x_3$$

### Subject to

$$x_1 + x_2 + x_3 + a_1 = 10$$

$$x_1 + 4x_2 + x_3 + a_2 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Conditions remain same for both the phases

### Artificial variables:

$$a_1, a_2 \geq 0$$

**First Phase Objective Function:**

$$\min : f1 = a_1 + a_2$$

$$\max : -f1 = -a_1 - a_2$$

**Table 0:**

SIMP	CN	0	0	0	b
CB	BV/NV	$x_1$	$x_2$	$x_3$	XB
-1	$a_1$	1	1	1	10
-1	$a_2$	1	* 4	1	16
*	*	-2	- 5	-2	-26

# LPP: Numerical Example-TPS1

**First Phase Objective Function:**

**Table 1:**

<b>SIMP</b>	<b>CN</b>	<b>0</b>	<b>-1</b>	<b>0</b>	<b>b</b>
<b>CB</b>	<b>BV/NV</b>	$x_1$	$a_2$	$x_3$	<b>XB</b>
<b>-1</b>	$a_1$	$*3/4$	$-1/4$	$-3/4$	<b>6</b>
<b>0</b>	$x_2$	$1/4$	$1/4$	$1/4$	<b>4</b>
<b>*</b>	<b>*</b>	$-3/4$	$5/4$	$-3/4$	<b>-6</b>

# LPP: Numerical Example-TPS1

First Phase Objective Function:

Table 2:

SIMP	CN	-1	-1	0	b
CB	BV/NV	$a_1$	$a_2$	$x_3$	XB
0	$x_1$	4/3	-1/3	1	8
0	$x_2$	-1/3	1/3	0	2
*	*	1	1	0	0

*Optimal Solution :*

$$a_1 = 0, a_2 = 0, x_1^* = 8, x_2^* = 2, x_3^* = 0, -f_1^* = 0, f_1^* = 0,$$

Final Table of Phase-I

Hence, artificial  
variables successfully  
removed

# LPP: Numerical Example-TPS1

## First Table of Phase-II

### Second Phase Objective Function:

$$\max : Z = x_1 + 3x_2 + x_3$$

Table 3:

SIMP	CN	0	0	0	b
CB	BV/NV	$a_1$	$a_2$	$x_3$	XB
0	$x_1$	0	0	1	8
0	$x_2$	0	0	0	2
*	*	0	0	0	0

other columns same as the previous table

\* coefficients updated according to the actual objective function  
\* last row calculated using the inner product method

↓ ↓  
Zeros for artificial variable

lumn



# LPP: Numerical Example-TPS1

## Final Table of Phase-II

Second Phase Objective Function:

$$\max : Z = x_1 + 3x_2 + x_3$$

Table 4:

SIMP	CN	0	0	1	b
CB	BV/NV	$a_1$	$a_2$	$x_3$	XB
1	$x_1$	0	0	1	8
3	$x_2$	0	0	0	2
*	*	0	0	0	14

**Optimal Solution :**

$$x_1^* = 8, x_2^* = 2, x_3^* = 0, Z^* = 14,$$

indi - the presence of alternate optimal solution

# LPP: Numerical Example-TPS1

## Final Table of Phase-II

Second Phase Objective Function:

$$\max : Z = x_1 + 3x_2 + x_3$$

Table 5:

SIMP	CN	0	0	1	b
CB	BV/NV	$a_1$	$a_2$	$x_1$	XB
1	$x_3$	0	0	1	8
3	$x_2$	0	0	0	2
*	*	0	0	0	14

**Alternate Optimal Solution :**

$$x_1^* = 0, x_2^* = 2, x_3^* = 8, Z^* = 14,$$

>1 number  
of solutions  
indicate the  
exist of  
 $\infty$  solutions.

# Two Phase Simplex Method: LPP-2

## Numerical Example -TPS1:

### First Phase Objective Function:

$$\min : f_1 = a_1 + a_2$$

$$\max : -f_1 = -a_1 - a_2$$

### Second Phase Objective Function :

$$\min : Z = 3x_1 + x_2 + x_3$$

$$\max : -Z = -3x_1 - x_2 - x_3$$

### Subject to

$$x_1 + x_2 + x_3 + a_1 = 10$$

$$x_1 + 4x_2 + x_3 + a_2 = 16$$

$$x_1, x_2, x_3 \geq 0$$

### Artificial variables:

$$a_1, a_2 \geq 0$$

**First Phase Objective Function:**

$$\min : f1 = a_1 + a_2$$

$$\max : -f1 = -a_1 - a_2$$

**Table 0:**

SIMP	CN	0	0	0	b
CB	BV/NV	$x_1$	$x_2$	$x_3$	XB
-1	$a_1$	1	1	1	10
-1	$a_2$	1	* 4	1	16
*	*	-2	- 5	-2	-26

# LPP: Numerical Example-TPS1

**First Phase Objective Function:**

**Table 1:**

<b>SIMP</b>	<b>CN</b>	<b>0</b>	<b>-1</b>	<b>0</b>	<b>b</b>
<b>CB</b>	<b>BV/NV</b>	$x_1$	$a_2$	$x_3$	<b>XB</b>
<b>-1</b>	$a_1$	$*3/4$	$-1/4$	$-3/4$	<b>6</b>
<b>0</b>	$x_2$	$1/4$	$1/4$	$1/4$	<b>4</b>
<b>*</b>	<b>*</b>	$-3/4$	$5/4$	$-3/4$	<b>-6</b>

# LPP: Numerical Example-TPS1

**First Phase Objective Function:**

**Table 2:**

<b>SIMP</b>	<b>CN</b>	<b>-1</b>	<b>-1</b>	<b>0</b>	<b>b</b>
<b>CB</b>	<b>BV/NV</b>	<b><math>a_1</math></b>	<b><math>a_2</math></b>	<b><math>x_3</math></b>	<b>XB</b>
<b>0</b>	<b><math>x_1</math></b>	<b><math>4/3</math></b>	<b><math>-1/3</math></b>	<b>1</b>	<b>8</b>
<b>0</b>	<b><math>x_2</math></b>	<b><math>-1/3</math></b>	<b><math>1/3</math></b>	<b>0</b>	<b>2</b>
<b>*</b>	<b>*</b>	<b>1</b>	<b>1</b>	<b>0</b>	<b>0</b>

***Optimal Solution :***

$$a_1 = 0, a_2 = 0, x_1^* = 8, x_2^* = 2, x_3^* = 0, -f_1^* = 0, f_1^* = 0,$$

**Final Table of Phase-I**

# LPP: Numerical Example-TPS1

**First Table of Phase-II**

**Second Phase Objective Function:**

$$\max : -Z = -3x_1 - x_2 - x_3$$

**Table 3:**

<b>SIMP</b>	<b>CN</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>b</b>
<b>CB</b>	<b>BV/NV</b>	<b><math>a_1</math></b>	<b><math>a_2</math></b>	<b><math>x_3</math></b>	<b>XB</b>
<b>0</b>	<b><math>x_1</math></b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>8</b>
<b>0</b>	<b><math>x_2</math></b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>2</b>
<b>*</b>	<b>*</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>

## Final Table of Phase-II

Second Phase Objective Function:

$$\max : -Z = -3x_1 - x_2 - x_3$$

Table 4:

<b>SIMP</b>	<b>CN</b>	<b>0</b>	<b>0</b>	<b>-1</b>	<b>b</b>
<b>CB</b>	<b>BV/NV</b>	<b><math>a_1</math></b>	<b><math>a_2</math></b>	<b><math>x_3</math></b>	<b>XB</b>
<b>-3</b>	<b><math>x_1</math></b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>8</b>
<b>-1</b>	<b><math>x_2</math></b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>2</b>
<b>*</b>	<b>*</b>	<b>0</b>	<b>0</b>	<b>-2</b>	<b>-26</b>



# LPP: Numerical Example-TPS1

## Final Table of Phase-II

### Second Phase Objective Function:

$$\max : -Z = -3x_1 - x_2 - x_3$$

\* zeros in the last row of artificial variables do not imply presence of alternate sol<sup>n</sup>.

Table 4:

SIMP	CN	0	0	-3	b
CB	BV/NV	$a_1$	$a_2$	$x_1$	XB
-1	$x_3$	0	0	1	8
-1	$x_2$	0	0	0	2
*	*	0	0	2	-10

Optimal Solution :

$$x_1^* = 0, x_2^* = 2, x_3^* = 8, -Z^* = -10, Z^* = 10,$$

even though zeros are present in the last row, they correspond to artificial variables. Hence, they have no significance.

## Numerical Example -1

$$\max : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \leq 16$$

$$x_1 + x_2 + 2x_3 \leq 14$$

$$4x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

## Numerical Example 2

$$\max : Z = 4x_1 + 4x_2 + x_3$$

Subject to

$$x_1 + 6x_2 + x_3 \leq 40$$

$$6x_1 + x_2 + x_3 \leq 30$$

$$x_1 + x_2 + 3x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

## Numerical Example 3

$$\max : Z = 8x_1 + 2x_2 + 8x_3$$

Subject to

$$4x_1 + x_2 + x_3 \leq 40$$

$$x_1 + 5x_2 + x_3 \leq 15$$

$$x_1 + x_2 + 4x_3 \leq 25$$

$$x_1, x_2, x_3 \geq 0$$

## Numerical Example 4

$$\max : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \leq 16$$

$$x_1 + x_2 + 2x_3 \leq 14$$

$$4x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-5

$$\max : Z = 6x_1 + 6x_2 + x_3$$

Subject to

$$4x_1 + 2x_2 + x_3 = 26$$

$$2x_1 + 4x_2 + x_3 = 22$$

$$x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-6

$$\max : Z = 10x_1 + 10x_2 + 2x_3$$

Subject to

$$6x_1 + 2x_2 + x_3 = 58$$

$$2x_1 + 6x_2 + x_3 = 46$$

$$x_1 + x_2 + x_3 \leq 19$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-7

$$\max : Z = 6x_1 + 6x_2 + x_3$$

Subject to

$$8x_1 + 2x_2 + x_3 = 58$$

$$2x_1 + 8x_2 + x_3 = 52$$

$$2x_1 + 2x_2 + x_3 \leq 28$$

$$x_1, x_2, x_3 \geq 0$$



# Two Phase Simplex Method:

## Numerical Example -TPS-8

$$\max : Z = 8x_1 + 8x_2 + x_3$$

Subject to

$$6x_1 + 4x_2 + x_3 = 76$$

$$4x_1 + 6x_2 + x_3 = 74$$

$$4x_1 + 4x_2 + x_3 \leq 62$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-9

$$\max : Z = 7x_1 + 7x_2 + 2x_3$$

Subject to

$$4x_1 + 2x_2 + x_3 = 26$$

$$2x_1 + 4x_2 + x_3 = 22$$

$$x_1 + x_2 + x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-10

$$\max : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 = 16$$

$$x_1 + x_2 + 2x_3 = 14$$

$$4x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-11

$$\max : Z = 4x_1 + 4x_2 + x_3$$

Subject to

$$x_1 + 6x_2 + x_3 = 40$$

$$6x_1 + x_2 + x_3 = 30$$

$$x_1 + x_2 + 3x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-12

$$\max : Z = 8x_1 + 2x_2 + 8x_3$$

Subject to

$$4x_1 + x_2 + x_3 = 40$$

$$x_1 + 5x_2 + x_3 \leq 15$$

$$x_1 + x_2 + 4x_3 = 25$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-13

$$\max : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 = 16$$

$$x_1 + x_2 + 2x_3 = 14$$

$$4x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-14

$$\max : Z = 6x_1 + 6x_2 + x_3$$

Subject to

$$4x_1 + 2x_2 + x_3 = 26$$

$$2x_1 + 4x_2 + x_3 = 22$$

$$x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-15

$$\max : Z = 10x_1 + 10x_2 + 2x_3$$

Subject to

$$6x_1 + 2x_2 + x_3 = 58$$

$$2x_1 + 6x_2 + x_3 = 46$$

$$x_1 + x_2 + x_3 \leq 19$$

$$x_1, x_2, x_3 \geq 0$$



# Two Phase Simplex Method:

## Numerical Example -TPS-16

$$\min : Z = 6x_1 + 6x_2 + x_3$$

Subject to

$$8x_1 + 2x_2 + x_3 = 58$$

$$2x_1 + 8x_2 + x_3 = 52$$

$$2x_1 + 2x_2 + x_3 \leq 28$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-17

$$\min : Z = 8x_1 + 8x_2 + x_3$$

Subject to

$$6x_1 + 4x_2 + x_3 = 76$$

$$4x_1 + 6x_2 + x_3 = 74$$

$$4x_1 + 4x_2 + x_3 \leq 62$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-18

$$\min : Z = 7x_1 + 7x_2 + 2x_3$$

Subject to

$$4x_1 + 2x_2 + x_3 = 26$$

$$2x_1 + 4x_2 + x_3 = 22$$

$$x_1 + x_2 + x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-19

$$\min : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 = 16$$

$$x_1 + x_2 + 2x_3 = 14$$

$$4x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-20

$$\min : Z = 4x_1 + 4x_2 + x_3$$

Subject to

$$x_1 + 6x_2 + x_3 = 40$$

$$6x_1 + x_2 + x_3 = 30$$

$$x_1 + x_2 + 3x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

# Two Phase Simplex Method:

## Numerical Example -TPS-21

$$\min : Z = 8x_1 + 2x_2 + 8x_3$$

Subject to

$$4x_1 + x_2 + x_3 = 40$$

$$x_1 + 5x_2 + x_3 \leq 15$$

$$x_1 + x_2 + 4x_3 = 25$$

$$x_1, x_2, x_3 \geq 0$$