Page No. MM - Assignment 5 Keerti P. Charantimath 19MA20059  $\frac{P_{0}(\tau)=1}{P_{3}(\tau)=\frac{1}{2}}, \quad \frac{P_{1}(\tau)=\pi}{5\tau^{3}-3\tau}, \quad \frac{P_{2}(\tau)=\frac{1}{2}(3\tau^{2}-1)}{(35\tau^{4}-30x^{2}+3)/8}$ From the generating fun =  $\int_{0}^{\infty} P_{n}(z)$ , we have:  $\sum_{n=0}^{\infty} P_{n}(z) t^{n} = (1-2\pi t + t^{2})^{-1/2} = (1-t(2\pi-t))^{-1/2}$ Expanding R+K:- $\frac{8}{5} P_{n}(x) t^{n} = 1 + \frac{1}{2} t(2x - t) + \frac{1}{2} \frac{\sqrt{3}}{2} \cdot t^{2} (2x - t)^{2}$ + \frac{1}{2} \times \frac{5}{2} \times \frac{5}{(21-t)}^3 + \frac{1}{2} \times \frac{3}{2} \times \frac{5}{2} \times \frac{t}{2} \frac{t}{2} \frac{4}{2} \times \frac{1}{2} \times \fra  $= 1 + \frac{1}{2} t(27-t) + \frac{3}{8} t^{2}(2x-t)^{2} + \frac{5}{16} t^{3}(2x-t)^{3} + \frac{35}{128} t^{4}(2x-t^{4})^{\frac{1}{2}}$ Comparing powers of t from l+s & R+s  $P_{2}(x) = \frac{1}{1}$ ,  $P_{1}(x) = \frac{1}{2} \times 2x = x$ ,  $P_{2}(x) = \frac{1}{2} \left(3x^{2} - 1\right)$ ,  $P_{3}(x) = \frac{1}{2} \left(5x^{3} - 3x\right)$ ,  $P_{4}(x) = \frac{1}{2} \left(35x^{4} - 30x^{2} + 3\right)$ where  $P_{n}(x)$  is the coefficient  $\int_{0}^{\infty} t^{n}$ . Hence Proved  $P = \chi^{11} + 2\chi^{3} + 2\chi^{2} - \chi - 3$ 

 $P_3(x) = 1 (5x^3 - 3x) \Rightarrow 2P_3(x) + 3P_1(x) = x^3$ 

 $P_{4}(1) = \frac{1}{8} \left( 357^{4} - 302^{2} + 3 \right) \Rightarrow \frac{1}{8} P_{4}(1) - 3P_{6}(1) + 30 \left( \frac{2}{3} P_{2}(1) + \frac{1}{3} P_{6}(1) \right) - \frac{1}{3} P_{6}(1) + \frac{1}{3} P_{6}(1)$ 

=> 24 = 8 Py(2) + 20 P2(2) + 23 Po(2)
35 25 7 355

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 $P = \alpha^4 + 2\alpha^3 + 2\alpha^2 - \alpha - 3$ MOW,  $\frac{8 P_4 + 4 P_2 + 3 P_0}{7 + 3 P_1} + 2 \left(\frac{2 P_3 + 3 P_1}{5}\right)$ 

+2(Po) - 3Po +2+P2(2) - P.

 $\frac{=8 P_{0} + (4 + 21) P_{2} + 4 P_{3} + (6 P_{-1}) P_{1} + \frac{4}{3} + 2 - 3) P_{0}}{5}$ 

P = 8 Py + 47 4 Ps + 40 P2 + 1 Pr - 224 Po 35 4/1 5 21 5 105

hence proved. (1900) all the shind a

Pln+1 + Pin = Por + 3P1 + 5P2 + 5P2 + ... + (2n+1) Pn

Po=Pi as Po=1 & Pi=2.

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2 1-1 Mso, from securrence relation (2n+) Pn = Pn+1 - Pn-1.

3 P1 = P2 - Po 1 5 P2 = P3' - P1' 7 B = P4' - P2'

 $\frac{3P_{1} = P_{2} - P_{0}}{(2n-3)P_{n-2} = P_{n-1} - P_{n-3}} \frac{5P_{2} + F_{0}}{(2n-1)P_{n-1}} \frac{7P_{n-2}}{(2n-1)P_{n-1}} \frac{(2n-1)P_{n-1}}{(2n-1)P_{n-1}} = \frac{P_{n-1} - P_{n-1}}{(2n-1)P_{n-1}}$ 

Adding all of the above, we get = 1-11+19 = Pn+1 + Pn' - Pn' = Pn+1 + Pn'

hence proved

Integrating born discler incast S-13 (1-22) Pm Pn dr =0

Now as  $P_m(x)$  is a solution of Legendre's equation, it satisfies:  $(1-x^2) \cdot P_m'' - 2xP_m' + m(m+1) \cdot P_m = 0$   $\Rightarrow (1-x^2) \cdot P_m'' - 2xP'(x) = -m(m+1) \cdot P_m = 0$ 

NE PAVENT (1-596+16) 12 = 2 60(4) P.

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	Putting (ii) in (i), we gett:
	C'an (m+1) Pm. Pa ch
	$= m(m+1) \int rm(D, ln(1) dl$
	9 (1912) 18 - (0) (4)
	=0 as m + n is given, so
\$ 1	Pm & Pn are orthagonal on -15x()
	hence proved = = 9 01 + 19 1 = 9
5)	5-1 22 Parti Pa - de = 2n (n+1) borond
į	(2n-1)(2n+1)(2n+3)
	From recuraire relation: (2n H) 2 fn = (2+1) Pn +1 +n Pn-1
	we get: 2 Pn-1 = 1 x (n Pn+ (n-1) Pn-2) -0
	- 109 - 10 = 20 Flore mittales more more of the
	a(Pn+1) - 1 x ((n+2) Pn+2 +(n+1) (An)) -1
= (5)	(2n+3)
1-01-	$(1) \times (2) \Rightarrow$
	$\chi^{2} P_{n+1} P_{n-1} = \frac{1}{6} \times (n(n+2) P_{n} P_{n+2}) + (n+1) P_{n}^{2} + \frac{1}{6} n(n+1) P_{n}^{2$
	13+ 1417 = 10 - 1 (2n+3) 14 (1+02) 7 - 142 = 193 + 3
	(n-1) (n+2) Pn-2 Pn+2+ (n2-1) Pn Pn-2)
	Themes for red
	Integrating both sides, we get
E	$\int_{-1}^{2^{2}} \frac{P_{n+1}P_{n-1}}{(2n-1)(2n+3)} = \frac{n(n+1)}{(2n-1)(2n+3)} \int_{-1}^{2^{2}} \frac{P_{n}}{(2n+1)(2n+3)} \int_{-1}^{2^{2}} \frac{P_{n}}{(2n+3)(2n+3)} \int_{-1}^{2^{2}} \frac{P_{n}}{(2n+3)(2n$
	= 2n(n+1)
	(2n-1)(2n+1)(2n+3)
	hence proved
1953	The state of the s
6)	$\frac{1-h^{2}}{(1-27h+h^{2})^{3/2}} = \frac{\sum (2n+1) \ln(x) h^{n}}{n=0}$
	$(1-27h+h^2)^{3/2}$ $n=0$
لع	We know, (1-2xt++2)-1/2 = & Pn(x)+0
	n=0

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	Partially dwit to -1/2 (1-2nt+t3)3/2 (2t-22) = 2 nPn(x)t
	ne, manual manual manual manual manual manua
	Multiplying by t on both
	t(2-t) (1-27++2) -3(2 = 5 n Pr (1) + 1)
	Multiplying by t on both sides. $t(x-t)(1-2xt+t^2)^{-3/2} = \sum_{n=1}^{\infty} n \cdot P_n(x) t^n$ $\int_{-1}^{2} (2t-t^2) \left(1-2xt+t^2\right)^{-3/2} = 0$
	$\Rightarrow (2t - \xi^2) (1 - 2n + + \xi^2)^{-3/2} = 0$
	$= \frac{(nt - t^2)(1 - 2nt + t^2)^{-3/2}}{n + 2nt} = \frac{2n}{2n} \frac{(n + 2n)}{n} = \frac{2n}{2n} \frac{(n + 2n)}{n}$
	t(1-22+++2)-1/2= 2 Pn(2)+n+1
	2=7) - 2 × Pn(2) + n + 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2
	bastrally dwst and
	partially dwelfart trando third out the season
	partually dwst = t = 1000 = 1000   10
	[ [J] - 2 [ JAP 3 0 ] - Tribert Andrews and [ ] - 1 [ ] - 1 Andrews All [ ] - 1 Andrew
	$\frac{2}{5}(2n+1)(Pn(x)t^n) = (1-2zt+t^2)^{-1/2} + 2x(tx-t^2)(1-2zt+t^2)^{-3/2}$
y' fn	$=\frac{1-2xt+t^2}{3/2}\cdot\left(1-2xt+t^2+2xt-2ta\right)$
	$\frac{\sum_{n=0}^{\infty} (2n+1) (Pn(n)t^n)}{(1-2nt+t^2)^{-3/2}} = \frac{(1-2nt+t^2)^{-3/2}}{(1-2nt+t^2)^{-3/2}} = \frac{(1-2nt+t^2)^{-3/2}}{(1-2nt+t^2)^{-3/2}} = \frac{(1-2nt+t^2)^{-3/2}}{(1-2nt+t^2)^{-3/2}} = \frac{(1-2nt+t^2)^{-3/2}}{(1-t^2)^{-3/2}} = (1-2nt+t^2)^{-3$
	hence proved.  SET OFT OFT OFT OFT OFT OFT OFT OFT OFT OF
1	1 = Epich av 2/145-1 & 21, = 2 (2019) (245-1)00 Fine
7.	$f_n(1)=1$
->	
	We know, (1-2x++2) = 5 Pn(2)+10 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
6	- (12-26++2) = 3 = E Pn(D tin) (++1) 3 = 38
	71-12)-1/2 = 6 0 ( ) +0
	\$ 50 (U+F) (U+F) - (U+F) (34 ) = 5 (U) 2 = 5
	$\Rightarrow$ $\leq P_n(1) t^n = 1 + t + t^2 + t^3 + \dots$
	n=0 3+0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
On I	Egnating the coefficient of the from both sides, we have
14. 11	x Pn(1) 1 = 1 ≤ Y" h=0,1,2.0. + " 1 - 1 0 0
	hence browled & B 2000 & 12 min 12 200 Condons
į	Tan in (k-1) (k-1) En se Lite un Maria de Maria de Maria
(Tar	2 ( 1 ( 1 ) (2 ) 30 by K= 1 of /2 K+0 & M = 1 +0 A