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SFA Exam 2

Ans1. NP class is a class of languages that has ~~[illegible]~~ an NDTM program which solves its in polynomial time.

NP class can also be defined as the class of language that is checked using a DTM given an input in polynomial time. Along with the input we also need to provide the certificate to DTM for verification.

Now, we need to prove that the above definitions are equivalent.

In NDTMS, if we non-deterministicaly find a certificate of the given NP class problem, we can run the turing machine using the certificate. This would verify the string and check if it is accepted.

In DTMS, we already have the input and certificate. We here again run the machine and check if the string is accepted.

Page No.

We see that ~~t~~ NDTMs & DTMs have similar computation. ~~&~~ Hence we can say that, due to similarity in the method of computation on NDTMs and DTMs, NP class definition with respect to both of them is same.

Date ___/___/_____

Ans2. Reduce SAT to 3SAT

Taking a claus of SAT, let us call it clause C_1

$(x_1 \vee x_2 \vee \dots x_k) \rightarrow$ clause C_1

Aim is to reduce clause C_1 to 3SAT

This is the 4 cases.

Case 1:- C_1 has 1 literal

We convert to 3-SAT ~~usin~~ in the following way

$(x_1 \vee x_1 \vee x_1)$

Case 2:- C_1 has 2 literals

3-SAT conversion $\longrightarrow (x_1 \vee x_2 \vee x_1)$

Case 3:- C_1 has 3 literals

3-SAT Conversion $\longrightarrow (x_1 \vee x_2 \vee x_3)$

Case 4:- C_1 has ≥ 4 literals

$\underset{\text{SAT}}{\phi} = (x_1 \vee x_2 \vee \dots x_k) \Longrightarrow \underset{\text{3SAT}}{\phi'} = (x_1 \vee x_2 \vee z_1) \wedge (x_3 \vee \bar{z}_1 \vee z_2) \wedge \dots \wedge (x_{k-1} \vee x_k \vee \bar{z}_{k-3})$

Hence this is how we reduce SAT to 3SAT when the number of literals are 1, 2, 3 or ≥ 4

Ans 3 Reduce vertex cover to set cover

let us suppose that we are given a set cover $\{S_1, S_2, \dots S_n\}$. Here, $S_1, S_2 \dots S_n$ are subsets of V. V is the set of elements. Our aim is to find collection of $\leq k$ (given) subsets of V whose union is V.

Now, Vertex Cover is an undirected graph $G = (V, E)$ and k is an integer. We need a subset $S \subseteq V$ of vertices which satisfies the condition
- $|S| \leq k$
- if $(a,b) \in E \Rightarrow$ either $a \in S$ or $b \in S$ or $(a,b) \in S$

We need to assume that we have a black box that solves set cover instances.
let $G = (V, E)$, k be an vertex cover instance
Also let $V = E$, $S_v = \{ k \in E : k$ is incident to $v \}$
is the set-cover instance

Date ___ / ___ / ______

We see that the set cover of size ≤ k is possible if and only if vortex cover size is k at most