Graph Theory and Algorithms
Tutorial Sheet- 2

(1) Draw all possible simple non-isomorphic graphs on four vertices.

(2) Prove that two simple graphs on 4 vertices are isomorphic if and only if they have the same degree sequence.

(3) Let $S = \{1, 2, 3, 4, 5\}$. Construct a graph G whose vertex set is the collection of all 2-subsets of S and two vertices are adjacent in G if and only if the corresponding 2-subsets are disjoint. Prove that G is isomorphic to the Petersen graph.

(4) Prove that in a group of six people, there must be three people who are mutually acquainted or three people who are mutually non-acquainted.

(5) Let G be a simple graph with $\delta(G) \geq k$. Show that: (i) G contains a path of length at least k.
(ii) If $k \geq 2$ then G contains a cycle of length at least $k + 1$.

(6) Let G be a self-complementary graph on n vertices. Prove that $n \equiv 0$ or $1 \pmod 4$.

(7) If G is a regular connected graph of degree k and girth 5 then prove that G has at least $k^2 + 1$ number of vertices and equality if $k = 2$ or 3.

(8) Prove that Petersen graph can not have a 7-cycle.

(9) Let G be a simple connected graph on $n \geq 2$ vertices. Suppose G does not have an induced subgraph isomorphic to P_4 or C_4. Prove that G contains a vertex of degree $n - 1$.

(10) Find the radius, diameter, center and periphery of the Petersen graph.