INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date: 23.02.2018, FN, Time: 2 Hrs, Full Marks: 30, Deptt: MA/ ME/PH
No. of Students: 85, Mid-Spring Semester Examination, 2018
Sub. No. MA40002/ MA51004/ MA61052,
Sub. Name Integral Equations and Variational methods

***Instruction**: Answer all the questions. Notations have their usual meanings. Each question carries **5** marks.*

1. Reduce the Volterra integral equation

$$y(x) = \frac{x^3}{6} - x + 1 + \frac{1}{2}\int_0^x \{2(x-t)\sin t - (x-t)^2(e^t + \cos t)\}\, y(t)dt$$

to an equivalent initial value problem.

2. If the boundary value problem given by

$$2y'' + y = 0;\ \ 0 < y < 1;\ \ y(0) = 0,\ \ y'(1) + 3y(1) = 1$$

is reduced to a Fredholm integral equation of the form

$$y(x) = f(x) + \lambda \int_0^1 K(x,t)\ y(t)dt,$$

find $f(x)$, λ and $K(x,t)$.

3. Solve $y(x) = \frac{9}{4} - \frac{x}{3} + \int_0^1 (x-t)\ y(t)dt$, using method of direct computation.

4. Solve $y(x) = \frac{3}{2}e^x - \frac{1}{2}xe^x - \frac{1}{2} + \frac{1}{2}\int_0^1 t\, y(t)dt$, using method of successive substitution.

5. (i) Find the iterative kernels $K_n(x,t); n = 1,2,3 \ldots$ corresponding to the kernel $K(x,t) = \sin(x+t)$.

   (ii) Hence find the resolvent kernel $R(x,t;\lambda)$ and solve the integral equation

$$y(x) = 1 + \frac{1}{2\pi}\int_0^\pi \sin(x+t)\, y(t)dt.$$

6. Find the eigenvalues and eigenfunctions of the homogeneous integral equation

$$y(x) = \lambda \int_{-1}^1 (5xt^3 + 4x^2t + 3xt)\ y(t)dt.$$

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