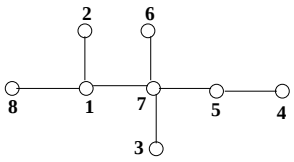
Indian Institute of Technology, Kharagpur

Date. April 12, 2021 Time: 55 mins Full Marks: 20
Fourth Class Test (Spring) Semester 2020-21 Subject Name: Discrete Mathematics

Instruction: Notations used are as explained in the class.

1. [2 **mark**] Write down the Prüfer code for the following tree.



2. [3 **mark**] Solve the recurrence relation

$$a_n = 6a_{n-1} - 8a_{n-2} + n4^n, n \geq 2$$

subject to the initial values $a_0 = 8, a_1 = 22$.

3. [3+2 = 5 **mark**] A certain disease is thought to be noncontagious. A research estimates the disease to be found in 1 in every 50 people. Let p_n be the probability that the n-th person studied has the disease and no one else before that person.

   i) Find a simple, closed form expression for the probability generating function of the sequence $\{p_n\}$.

   ii) Use the methods of generating functions to find the expected number of persons to be examined before finding a person with the disease, and the variance.

4. [3 **mark**] In a group G, for all $a, b \in G, (ab)^n = a^n b^n$ holds for three consecutive integers n. Prove that the group is abelian.

5. [2 + 1 = 3 **mark**] Consider the field $\mathsf{GF}(2^3)$ defined by $x^3 + x + 1$.

   i) Find the inverse and square root of 111.

   ii) Find all the quadratic residues in this field.

6. [2 + 1 + 1 = 4 **mark**] Let E be the modular elliptic curve defined by $y^2 = x^3 + 6x \pmod{13}$.

   i) Find all points of E (including the point at infinity).

   ii) Find $(4, 7) + (4, 7)$.

   iii) Find $(4, 7) + (5, 5)$.

———-The End———