INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

End Semester Exam-Spring, 2022

Department of Mathematics

Time : 1 hrs. Total Marks : 30,

Subject: MA 41002/MA 30002, Modern Algebra

Instruction: "No queries will be entertained during the examination".

(1) State weather the following statements are true or false with justification. No marks will be awarded without justification.
   (a) $\mathbb{Z} \times \mathbb{Z}$ is an integral domain.
   (b) Let $f : F \longrightarrow R$ be a non zero ring homomorphism where F is a field and R is a ring. Then f is injective.
   (c) Every subring of a field is also a field.
   (d) Every non zero prime ideal in $\mathbb{Z}[x]$ is also a maximal ideal.
   (e) Let G be group of order 23. Then G is abelian.

$[5 \times 2 = 10]$

(2) Let G be a finite abelian group of order 24. Is the mapping $f : G \longrightarrow G$ defined by $f(a) = a^7$ is an automorphism? Justify your answer.

(3) Let G be a non abelian group of order 125 and $Z(G) \neq 1$. Then find $|Z(G)|$ with justification.

(4) Let $R = \mathbb{Z}_5[x]$ and $f(x) = x^2 + x + 1$. Is $R/(f(x))$ a field? Justify your answer. Find the inverse of $2x + 1 + f(x)$ in $R/(f(x))$.

(5) Let $R = \mathbb{Z}[2i]$. Is R an UFD? Is 2 an irreducible element in R ? Is 2 a prime element in R? Justify all your answers.

(6) Let $R = \mathbb{Z}[\sqrt{-2}]$. Is R an Euclidean domain? Justify your answer. Find all the units in R.

(7) Determine all representations of the integer $17^2.7^2$ as a sum of two squares.

$[6 \times 3 = 18]$

(8) Let $R = \mathbb{C}[x, y]$ and $I = (x^2 + y^2 - 1, x + y - 1)$. Find all the maximal ideals of R/I with justification. [2]