Remarks

If A, B are measurable sets, then $A+B$ need not measurable in $\mathbb{R}^d$.

If $A+B$ is measurable, then

$$m(A+B)^{1/d} \geq m(A)^{1/d} + m(B)^{1/d}.$$

Example: Let $A = \{0\} \times [0,1]$, $B = V \times \{0\}$, where V is non-measurable set in $[0,1]$. } in $\mathbb{R}^2$.

$$\left.\begin{aligned} m^*(A) &= \text{Area}(A) = 0 \\ m^*(B) &= \text{Area}(B) = 0 \end{aligned}\right\} A, B \text{ are measurable.}$$

$$A+B = (\{0\} \times [0,1]) + (V \times \{0\})$$

$$= \underline{V \times [0,1]} \quad \underline{\text{not measurable}}.$$