Mathematical Methods (MA31007)

Test-1

Time: 55Min, Date:7.9.21

12 - 12:55 P.M. Full Marks 20

All seven questions are compulsory. No negative marking or part marking is there.

Q1. For the IVPs given below, find the largest interval in which a unique solution is guaranteed to exist

(a) $(x-3)y'+\ln(x)y=2x,\quad y(1)=2$

(b) $(x^2-81)y'+5e^{3x}y=\sin\ x,\quad y(1)=10\pi$

(c) $\sqrt{(16-x^2)}y''+\ln(x+1)y'+\cos(x)y=0,\quad y(0)=2, y'(0)=0$

Write the answer in the form of interval i.e. (a,b) or [a,b] in the blank space. No other form will be evaluated. **3M**

Q2. Consider the set of functions

(1) $f(x)=9\cos(2x)\quad g(x)=2\cos^2 x-2\sin^2 x$ for all x

(2) $f(t)=2t^2\quad g(t)=t^4$ for all t

Which one of the following options is correct?

(i) Both sets of functions are linearly independent

(ii) Both sets of functions are linearly dependent

(iii) The first set is linearly independent and the second set is linearly dependent

(iv) The first set is linearly dependent and the second set is linearly independent

1M

Q3. Consider the non-homogeneous BVP posed on $[0,\pi]$ as $y''+y=0$ with $y(0)=0$ and $y(\pi)=1$. Then which one of the following is true?

(i) Both the non-homogeneous BVP and the corresponding homogeneous BVP have no solution.

(ii) The non-homogeneous BVP has unique solution and corresponding homogeneous BVP has only trivial solution.

(iii) Both the non-homogeneous BVP and the corresponding homogeneous BVP have infinite number of solutions.

(iv) The non-homogeneous BVP has no solution and corresponding homogeneous BVP has infinite number of solutions. **1M**

Q4. (a) Let y_1 and y_2 be solutions of the differential equation $y''+p(t)y'+q(t)y=0$ where p and q are continuous on $[a,b]$. Then the Wronskian $W(y_1,y_2)(t)$ is given by (C is a constant depending on y_1, y_2)

(i) $Ce^{\int q(t)\,dt}$

(ii) $Ce^{-\int \frac{p(t)}{q(t)}\,dt}$

(iii) $Ce^{\int \frac{q(t)}{p(t)}\,dt}$

(iv) $Ce^{-\int p(t)\,dt}$

Q4.(b) If the Wronskian of two solutions of $t^4 y''-2t^3 y'-t^8 y=0$ on $[1,5]$ is $Ct^{\frac{m}{n}}$, C being a constant, then $m+n$ is

3+3=6M

Q5. The adjoint equation of $x^2 y''+(2x^3+1)y'+y=0$ is

(i) $x^2 y''+(2x+4x^3-2)y'-2y(1-3x^2)=0$

(ii) $x^2 y''-(3x+2x^3-1)y'+3y(1+2x^2)=0$

(iii) $x^2 y''+(4x-2x^3-1)y'+3y(1-2x^2)=0$

(iv) $x^2 y''-(4x+2x^3+1)y'-2y(1+3x^2)=0$

3M

Continued…….

Q6. Use method of variation of parameter to find a particular integral $y_p(t)$ of the ODE

$y''-2y'+y=\dfrac{e^t}{1+t^2}+3e^t$. The answer will be

(i) $y_p(t)=\dfrac{1}{2}e^t\ln(1+t)-t^2e^t\tan^{-1}t-\dfrac{2}{3}te^t$

(ii) $y_p(t)=-\dfrac{1}{2}e^t\ln(1+t^2)+te^t\tan^{-1}t+\dfrac{3}{2}t^2e^t$

(iii) $y_p(t)=\dfrac{1}{2}e^t\ln(1+t)-t^2e^t\cot^{-1}t+\dfrac{2}{3}t^2e^t$

(iv) $y_p(t)=-\dfrac{1}{2}e^t\ln(1+t^2)+te^t\cot^{-1}t+\dfrac{3}{2}te^t$

3M

Q7. Consider the BVP $y''-y=0$ in $[0,l]$ with $y(0)=y'(0), y(l)+\lambda y'(l)=0$, λ is a constant. Then which one of the following is true for the Green's function $G(x,t)$ of the BVP?

(i) $G(x,t)=\dfrac{1}{2}\left(\dfrac{1+\lambda}{1-\lambda}\right)e^{x-t+2l}+\dfrac{1}{2}e^{t-x}, 0\le x<t$

(ii) $G(x,t)=\dfrac{1}{2}\left(\dfrac{1-\lambda}{1+\lambda}\right)e^{x+t-2l}-\dfrac{1}{2}e^{x-t}, 0\le x<t$

(iii) $G(x,t)=-\dfrac{1}{2}\left(\dfrac{1+\lambda}{1-\lambda}\right)e^{x-t+2l}+\dfrac{1}{2}e^{t-x}, 0\le x<t$

(iv) $G(x,t)=-\dfrac{1}{2}\left(\dfrac{1-\lambda}{1+\lambda}\right)e^{x+t-2l}+\dfrac{1}{2}e^{x-t}, 0\le x<t$

3M

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