Decisions Under Risk and Uncertainty

Risk and Uncertainty

- **Risk:** A decision making condition under which a manager/entrepreneur _*can list*_ all outcomes and assign probabilities to each outcome
  - *Objective probabilities: based on scientific data*
  - *Subjective probabilities: based on hunches, gut feelings or personal experiences*

- **Uncertainty:** A decision making condition under which a manager/entrepreneur _*cannot list*_ all outcomes and assign probabilities to the various outcomes
  - *States of nature: future events or conditions that can influence the final outcome or payoff of a decision but cannot be controlled or affected by the manager*

Types of Risk

1. **Business Risk:** it is risk of doing business in a particular industry (e.g. Steel, Pharma) or economic system (e.g. India, China), it is then controlled by certain economy-wide or industry-wide factors which lead to variability of returns.
2. **Financial Risk:** arises when companies resort to financial leverage using debt financing. The more the co. resorts to debt financing, the greater is the financing risk.
3. **Liquidity Risk:** arises when asset cannot be liquidated easily in the secondary market

4. **Interest Rate Risk:** is the variability in a security's return resulting from changes in the level of interest rates. Other things being same, security price move inversely to interest rates. This risk affect bondholders more directly than equity investors.
5. **Market Risk:** refers to the variability of return due to fluctuations in the securities market. All securities are exposed to market risk but equity shares get the most affected. This risk includes a wide range of factors exogenous to securities themselves as depressions, wars, and politics.
6. **Inflation Risk:** with rise in inflation there is reduction of purchasing power, hence, this is also referred to as purchasing power risk and affects all securities. This risk is also directly related to interest rate risk as interest rate generally go up with inflation.

Total risk of scrip is the variance of its return. It consists of:

1. **Systematic risk:** arises due to macroeconomic variable and cannot be diversified and called as ***non-diversifiable risk*** also referred to as market risk. Some of the factors that may give rise to non-diversifiable risk are as follows:
    - Change in Tax rates
    - War and other calamities
    - Change in Inflation rates
    - Change in Economic policy
    - Industrial recession
    - Change in International oil prices and so on
2. **Unsystematic risk:** it is the risk specific to company or industry and hence can be eliminated by diversification and is called ***diversifiable risk***. Some factors that may cause it are:
    - Strikes in the company
    - Bankruptcy of a major Supplier
    - Exit/ Death of a key company officer
    - Unexpected entry of the new competitor into the market and so on

Concept of Return

Return is made up of two components:

1. **Periodic cash flows from the asset:** These are the cash flows that you receive in the intervening time periods between when the investment is made and when the investment matures. For e.g.
   – when we purchase the bonds issued by a company and we receive interest on these bonds periodically say after every six months;
   – when we purchase some equity shares of a company and we receive dividends every year; or
   – when an investment s made by a company in a capital project which generates a cash flow every year for the life of the project.
2. **Capital appreciation:** This represents an increase in the market price of the asset during the life of the investment. For e.g.
   – Market price of bonds purchased by you at Rs. 100 per bond increases to Rs 120 per bond after one year (Market price is more than Investment)
   – Market value of the project having an initial investment of Rs 50 lakhs increases to Rs 75 lakhs after 3 years

Measuring the Rate of Return

The measurement of return depends on investment horizon.

1. Single period return
2. Multiple period return

Return also depends on time.

1. Ex-post return: Historical return
2. Ex-ante return: Expected return

It can be a combination of single period ex post return, multiple period ex post return, single period ex ante return or multiple period ex ante return

Single-period Ex Post Return: $r_{it} = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}$

Where r_{it} is return on i*th* asset at time t; P_t and P_{t-1} is price of i*th* asset at time t and t-1; D_t is the cash flow generated by asset during the period. Return is the dividend yield and capital gain.

Illustration 1:

An investor Mr. X has purchased the shares of ITC Ltd. at Rs. 250 per share on 1 April, 2014. The company paid a dividend of Rs. 5 per share during the financial year 2014-15 and the price of ITC Ltd. as on 31st March, 2015 is Rs 375. If Mr. X decides to sell the shares of ITC, the rate of return earned by him is:

$$r = \frac{(375 - 250) + 5}{250} = \frac{125 + 5}{250}$$

= 0.52 or 52%

Illustration 2:

Suppose another investor Mr. Y who has purchased the bonds issued by Premium Steel Ltd. at Rs. 1,000 per bond. The bond has a coupon rate of 10% payable annually. At the end of one year, the bonds are trading at Rs. 1,200. If Mr. Y decides to sell the bonds, the rate of return earned by him is:

$$r = \frac{(1200 - 1000) + 100}{1000}$$

$$= 0.30 \text{ or } 30\%$$

Multiple-period Ex post Return: Suppose we want to calculate that over a period of two years, what has been the annual return on a security. How do we calculate this return? To understand this, let us consider a security whose market prices for the last two years are:

Time period	Market prices
2015	195
2014	250
2013	160

Single period return over 2 year period is:

Return = $(P_2 - P_0)/P_0$=(195 – 160)/160 = 0.21875 or 21.875%

Annual return: $FV_n = PV(1+r)^n$ 195 = $160(1+r)^2$ or r = 10.40%

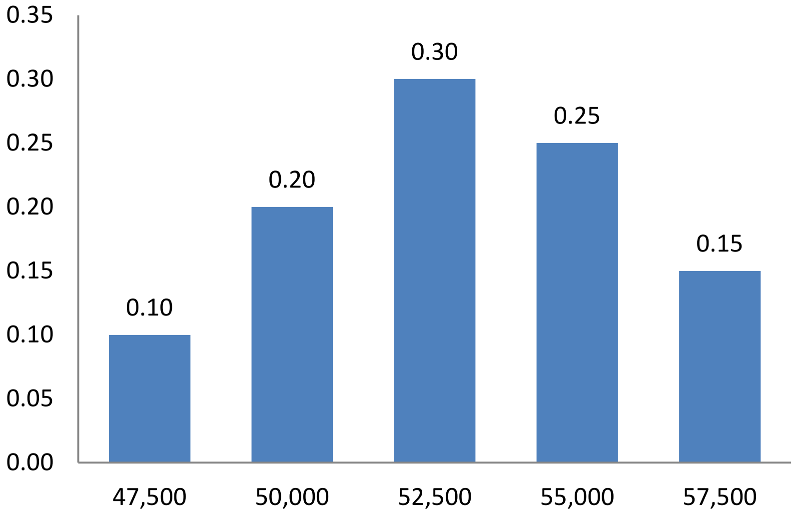
OR $(1+r)^n = 1+r_{0,n}$ where r = annual return over the period and $r_{0,n}$ is single-period return over the period

$1.21875 = (1+r)^2$ or r = $\sqrt{1.21875} - 1$ = 10.40%

Measuring Risk

- **Probability Distribution:** A table or graph showing all possible outcomes or payoffs of a decision and the probabilities that each outcome will occur

Outcome (Sales)	Probability (%)
47,500 units	10
50,000 units	20
52,500 units	30
55,000 units	25
57,500 units	15



From a probability distribution, the riskiness of a decision is reflected by the variability of outcomes indicated by the different probabilities of occurrence.

The nature of risk can be summarized by examining the central tendency of the probability distribution, through expected value, and by examining dispersion as measured by standard deviation and coefficient of variation.

Expected Value of a Probability Distribution

- Expected value: The weighted average of the outcomes, with the probabilities of each outcomes serving as the respective weights

$$E(X) = Expected \quad value \quad of \quad X = \sum_{i=1}^{n} p_i X_i$$

- Mean of the distribution: The expected value of the distribution
  - E(sales) = (0.10)47500 + (0.20)50000 + (0.30)52500 + (0.25)55000 + (0.15)57500 = 52,875
  - If only one of the five levels of sales can occur, the level that actually occurs will not equal the expected value of 52,875 but the expected value does indicate what the ***average*** value of the outcomes would be if the risky decision were to be repeated a large number of times.

Dispersion of a Probability Distribution

- **Variance:** The dispersion of a distribution about its mean. It is the probability-weighted sum of the squared deviations about the expected value of X

$$Variance(X) = \sigma_x^2 = \sum_{i=1}^{n} p_i \left[X_i - E(X)\right]^2$$

- If the expected values of two distributions are the same, the distribution with the higher variance is associated with the riskier decision
  – Even though if the expected values differ then also variance is often used to compare the riskiness of two decisions

Decision A			
Profit (X_i)	Probability (p_i)	p_iX_i	$[X - E(X)]^2p_i$
30	0.05	1.5	20
40	0.20	8	20
50	0.50	25	0
60	0.20	12	20
70	0.05	3.5	20
		E(X) = 50	$\sigma_A^2 = 80$

Decision B		
Probability (p_i)	p_iX_i	$[X - E(X)]^2p_i$
0.10	3	40
0.25	10	25
0.30	15	0
0.25	15	25
0.10	7	40
	E(X) = 50	$\sigma_B^2 = 130$

- Because variance is a squared term it is usually much larger than the mean, to avoid this scaling problem, the standard deviation is commonly used.
- **Standard deviation**: square root of the variance, higher the std. dev. the more riskier the decision
- Managers can compare the riskiness of various decisions by comparing std. dev., as long as the expected values are of similar magnitudes.
  - Suppose decision C has a mean outcome of $400 and decision D has a mean outcome of $5000 but the std. dev. remain 52.5 The dispersion of outcomes for decision D is much smaller *relative to its mean value of $5000* than is the dispersion of outcomes for decision C *relative to its mean value of $400*
- When the expected value of outcomes differ substantially, managers should measure the riskiness of a decision *relative* to its expected value.

- **Coefficient of Variation**: The std. dev. divided by the expected value of the probability distribution

  v = std. dev. / expected value = σ / E(X)

  – It measures the level of risk *relative* to the mean of the probability distribution.

  – In previous example, the two coefficients of variation are 52.5/400 = 0.131 and 52.5/5000 = 0.0105 i.e., the decision to be chosen is the one with the smallest coefficient of variation.

Decisions under Risk

- **Maximization of Expected value:** choosing the decision with the highest expected value (expected value rule)
- This rule *cannot* be applied when decisions have identical expected values and *should not* be applied when decisions have different levels of risk, except when the decision maker is risk neutral i.e. indifferent towards the level of risk associated

Mean – Variance Analysis

- Method of decision making that employs both the mean and the variance to make decisions
- Given two risky decisions, A and B, the mean-variance rules for decisions under risk are:
  - If decision A has a higher expected outcome *and* a lower variation than decision B, decision should be made.
  - If both decisions A and B have identical *variances* or *std. dev.*, the decisions with the higher expected value should be made.
  - If both decisions A and B have identical *expected values*, the decision with lower variance (std. dev.) should be made.

A decision maker prefers a higher expected return to a lower, other things equal, and a lower risk to a higher, other things equal.

Expected Utility: A Theory of Decision making under Risk

- **Which Rule is Best…???**
- **Expected Utility Theory:** of decision making under risk that accounts for a manager's attitude towards risk. (How managers actually make decisions under risk)
- Suppose a manager is faced with a decision to undertake a risky project, that may generate a range of possible profit outcomes π_1, π_2 ,…, π_n, will occur with probabilities p1, p2, …, pn respectively.
- **Expected Utility:** is the sum of the probability – weighted utilities of each possible profit outcomes.

$$E[U(\pi)] = p_1U(\pi_1) + p_2U(\pi_2) + \ldots + p_nU(\pi_n)$$

Manager's Utility Function for Profit

- The amount by which total utility increases with an additional dollar of profit earned by a firm is the **marginal utility of profit** i.e., $MU_{profit} = \Delta U(\pi) / \Delta \pi$
- The shape of the Utility curve determines the manager's attitude toward risk, which determines which choices a manager makes.
- Attitudes toward risk may be categorized as:
  - **Risk averse:** a decision maker who makes the less risky of two decisions that have the same expected value
  - **Risk loving:** a decision maker who makes the riskier of two decisions that have the same expected value
  - **Risk neutral:** a decision maker who ignores risk in decision making and considers only expected values of decisions

A manager's attitude toward risky decisions can be related to his or her marginal utility of profit. One who experiences diminishing (increasing) MU for profit will be a risk-averse (risk-loving) decision maker. One whose MU for profit is constant is risk-neutral.

- The process of deriving a utility function for profit is conceptually straight forward. It does, involve a substantial amount of subjective evaluation
- Let us consider a decision problem: Manager must decide where to locate the next restaurant. The profit outcomes for the three locations range from \$1,000 to \$6,000 per week
- Before the expected utilities of each location can be calculated, the manager must derive his/her utility function for profits covering the range of \$1,000 to \$6,000
- The manager begins the process of deriving $U(\pi)$ by assigning minimum and maximum values that the index will be allowed to take
- For the lower bound on the index i.e., \$1,000 suppose manager assigns a utility index value of 0 and for the upper bound suppose a utility index value of 1 is assigned (greater than lower bound) to the highest profit outcome of \$6,000

- Hence, U($1,000) = 0 and U($6,000) = 1
- Next a value of utility index for each of the remaining possible profit outcomes between $1,000 and $6,000 must be determined
- In this case, examining profit in increments of $1,000 is easy. To find, the value of the utility index for $5,000, the manager employs the following subjective analysis:
  - The manager begins by considering two decision choices A and B, where decision A involves receiving a profit of $5,000 with certainty and risky decision B involves receiving either a $6,000 profit with probability p or a $1,000 profit with probability 1 – p.
  - Now the probability p that will make the manager indifferent between the two decisions A and B must be determined. This is a subjective determination, any two managers likely will find different values of p depending on their individual preferences for risk.
  - Suppose, in this case, manager decides p = 0.95 makes decision A and B equally desirable. In effect the manager is saying that the expected utility of decision A equals the expected utility of decision B, i.e., $E(U_A) = E(U_B)$

- 1 x U($5,000) = 0.95 x U($6,000) + 0.05 x U($1,000)
- Only U($5,000) is unknown, so the manager can solve for the utility index for $5,000 profit as:

  U($5,000) = (0.95 x 1) + (0.05 x 0) = 0.95
- The utility index value of 0.95 is an indirect measure of the utility of $5,000 of profit.
- This procedure establishes another point on the utility function for profit. The sum of $5,000 is called the **certainty equivalent** of risky decision B because it is the dollar amount that the manager would be just willing to trade for the opportunity to engage in risky decision B.
- In other words, manager is indifferent between having a profit of $5,000 for sure or making a risky decision of having a 95% chance of earning $6,000 and a 5% chance of earning $1,000
- The utility index for $4,000, $3,000 and $2,000 can be established in exactly the same way

This is called **certainty equivalent method**

$$p_0 U(\pi_H) + (1 - p_0)\, U(\pi_L) = U(\pi_0)$$

Certainty Equivalent: The dollar amount that a manager would be just willing to trade for the opportunity to engage in a risky decision.

Principle: To implement the certainty method of deriving a utility of profit function, the following steps can be employed:

1. Set the utility index equal to 1 for the highest possible profit π_H and 0 for the lowest possible profit π_L.
2. Define a risky decision to have probability p_0 of profit outcome π_H and probability $(1 - p_0)$ of profit outcome π_L
3. For each possible profit outcome π_0 the manager determines subjectively the probability p_0 that gives risky decision the same expected utility as receiving π_0 with certainty. The certain sum π_0 is called the certainty equivalent of the risky decision.

Valuation Model for Risk

- Risk-adjusted discount rates are used to deal with any investment subject to risk
- These reflects the manager's or investor's trade-off between risk and return
- The difference between the expected or required rate of return on a risky investment and the rate of return on a riskless asset is called the **risk premium**
- The valuation model of the firm using risk-adjusted discount rate:

$$NPV = \sum_{t=1}^{n} \frac{R_t}{(1+k)^t} - C_0$$

where R_t reflects the net cash flow or return in each of the n time periods, k is the risk-adjusted discount rate, Σ refers to the sum of the present discounted value of all the future net cash flow from the investment, C_0 is the initial cost of the investment.

- An investment project is undertaken if NPV is greater than or equal to zero, or larger than that for an alternative project
- The method however, has the serious shortcomings that risk-adjusted rates are subjectively assigned by managers and investors, and variations in net cash flows or return are not explicitly considered.
- This approach is most useful for the evaluation of relatively small and repetitive investment projects.
- A better method for adjusting the valuation model for risk is the certainty equivalent approach.

Decision under Uncertainty

- **The Maximax Criterion:** Decision-making guide that calls for identifying the best outcome for each possible decision and choosing the decision with the maximum payoff of all the best outcomes.

- **The Maximin Criterion:** postulates that the decision maker should determine the worst possible outcome of each strategy and then pick the strategy that provides the best of the worst possible outcomes i.e. maximum of the minimum payoffs.

- **The Minimax Regret Criterion:** postulates that the decision maker should select the strategy that minimizes the maximum regret or opportunity cost of the wrong decision, whatever the state of nature that actually occurs.
  - **Regret** is measured by the difference between the payoff of a given strategy and the payoff of the best strategy under the same state of nature

- **The Equivalent Probability Criterion:** Decision making guide that calls for assuming each state of nature is equally likely to occur, computing the average payoff for each equally likely possible state of nature, and choosing the decision with the highest average payoff.

Concept of Diversification

- Generally, investing in a single security is riskier than investing in a “diversified group” of assets which might contain equity capital, bonds, real estate, saving accounts, and so on
- This “diversified group” of assets is called as **portfolio**
- We may create a portfolio within the same/different asset class
- It is possible to construct a portfolio in such a way that the total risk of the portfolio is less than the sum of the risk of the individual assets taken together

How does diversification helps in risk reduction?

- Let us understand the concept of diversification through a very simple illustration…

Temperature	Return on Sunflame Stock	Return on Duracool Stock	Return on Portfolio (50% Sunflame + 50% Duracool)
High	0%	20%	10%
Moderate	10%	10%	10%
Low	20%	0%	10%

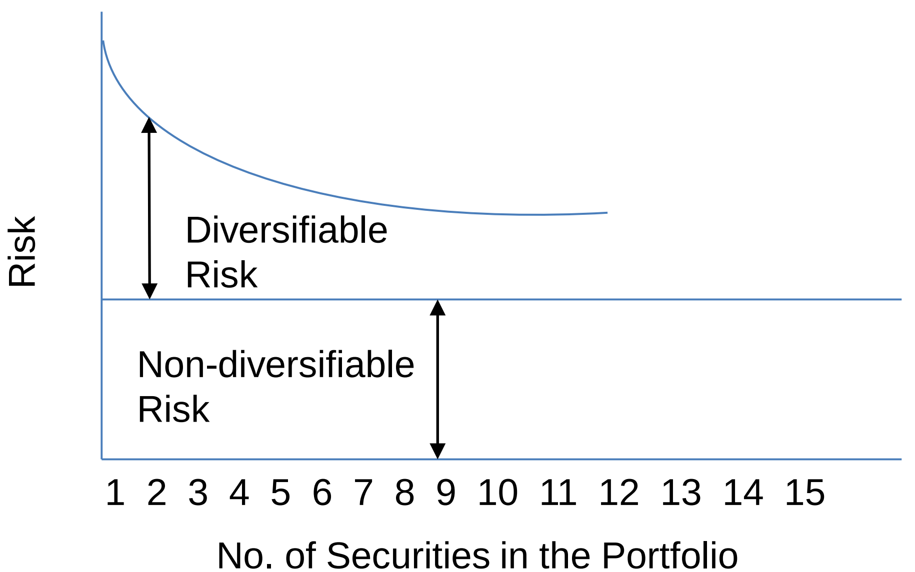
Possible Outcomes	Probabilities	Return on Sunflame Stock	Return on Duracool Stock	R_p
High	1/3	0%	20%	10%
Moderate	1/3	10%	10%	10%
Low	1/3	20%	0%	10%
Expected rate of return (k)		10%	10%	10%
Std. dev. (σ)		√66.67 = 8.16%	√66.67 = 8.16%	0

- From above, it can be noted that the portfolio earn 10%, no matter what the weather is. Through diversification two risky stocks have been combined to make a riskless portfolio
- Returns on two cos. are **perfectly negatively correlated** as they move in opposite direction

- Let us consider that "A" put his money equally into the stocks of 2 companies Sunflame Ltd., a manufacturer of room heating systems and Duracool Ltd., a manufacturer of air conditioners.
- If the temperatures soar high during summers in a particular year, the earnings of Duracool would go up leading to an increase in its share price and return to shareholders. The earnings of Sunflame would be on decline leading to decline in the share prices and investor's return. But, if there is a prolonged & severe winter, the situation would be just opposite.
- Whereas the return on each individual stock might vary, the return on A's portfolio could be quite stable because the decline in one will be offset by the increase in the other. In fact, at least in theory, the offsetting could eliminate this risk entirely.

- Fact that, returns do not move in perfect tandem means that risk can be reduced by diversification but since there is some positive correlation means that in practice risk can never be reduced to zero. So, there is a limit on the amount of risk that can be reduced through diversification due to two major reasons:

1. **Degree of correlation:** lower the positive correlation, the greater is the amount of risk reduction that is possible
2. **Number of stocks in the portfolio:** as the number of stock increases (up to a certain extent), the diversifying effect of each additional stock diminishes



- Figure indicates, the major benefits of diversification are obtained with the first 10 to 12 stocks, provided they belong to industries that are not closely related.
- It is also apparent that only one component of the total risk is being reduced and the other component remains constant whatever the no. of stocks in your portfolio is.