MANAGERIAL CHALLENGE *Continued*

Discussion Questions

- Forecasting provides very useful projections for established products and services, but newly introduced offerings have wildly different success results. Name a few products that have exploded with exponentially increasing demand shortly after their introduction. How about products that have largely been ignored?
- Do you see any common features among established and new products that might prove useful to an economic forecaster?

[1]Based on "Adoption Rate of Internet by Consumers Is Slowing," *Wall Street Journal* (July 16, 2001), p. B1; "Has Growth of the Net Flattened?" *Wall Street Journal* (July 16, 2001), p. B8; "Behind the Fiber Glut," *Wall Street Journal* (July 26, 2001), p. B1; and "Innovation Outpaced the Marketplace," *Wall Street Journal* (September 26, 2002), p. B1.

THE SIGNIFICANCE OF FORECASTING

Accurately forecasting future business prospects is one of the most important functions of management. Sales forecasts are necessary for operations managers to plan the proper future levels of production. The financial managers require estimates of not only future sales revenue but also disbursements and capital expenditures. Forecasts of credit conditions must also be made so that the cash needs of the firm may be met at the lowest possible cost.

Public administrators and managers of not-for-profit institutions must also forecast. City government officials, for example, forecast the level of services that will be required of their various departments during a budget period. How many police officers will be needed to handle the public-safety problems of the community? How many streets will require repair next year, and how much will this cost? What will next year's school enrollment be at each grade level? The hospital administrator must forecast the health care needs of the community and the amount and cost of charity patient care.

SELECTING A FORECASTING TECHNIQUE

The forecasting technique used in any particular situation depends on a number of factors.

Hierarchy of Forecasts

The highest level of economic aggregation that is normally forecast is that of the national economy. The usual measure of overall economic activity is gross domestic product (GDP); however, a firm may be more interested in forecasting some of the specific components of GDP. For example, a machine tool firm may be concerned about plant and equipment expenditure requirements. Retail establishments are concerned about future levels and changes in disposable personal income rather than the overall GDP estimate.

The next levels in the hierarchy of economic forecasts are the industry sales forecast, followed by individual firm sales forecasts. A simple, single firm forecast might take the industry sales estimate and relate this to the expected market share of the individual firm. Future market share might be estimated on the basis of historical market shares as well as on changes that are anticipated in marketing strategies, new products and model changes, and relative prices.

Within the firm, a hierarchy of forecasts also exists. Managers often estimate company-wide or regional dollar sales and unit sales by product line. These forecasts are used by operations managers in planning orders for raw materials, employee-hiring needs, shipment schedules, and release-to-production decisions. In addition, marketing managers use sales forecasts to determine optimal sales force allocations, to set sales goals, and to plan promotions. The sales forecast also constitutes a crucial part of the financial manager's forecast of the cash needs of the firm. Long-term forecasts for the economy, the industry, and the firm are used in planning long-term capital expenditures for plant and equipment and for charting the general direction of the firm.

Criteria Used to Select a Forecasting Technique

Some forecasting techniques are quite simple, inexpensive to develop and use, and best suited for short-term projections, whereas others are extremely complex, require significant amounts of time to develop, and may be quite expensive. The technique used in any specific instance depends on a number of factors, including the following:

1. The cost associated with developing the forecasting model,
2. The complexity of the relationships that are being forecast,
3. The time period of the forecast (long-term or short-term),
4. The lead time needed to make decisions based on the forecast, and
5. The accuracy required of the forecasting model.

Evaluating the Accuracy of Forecasting Models

In determining the accuracy, or reliability, of a forecasting model, one is concerned with the magnitude of the differences between the observed (actual) (Y) and the forecasted values ($\hat{Y}$). Various measures of model accuracy are available. For example, in the discussion of regression analysis in Chapter 4, the coefficient of determination, or R^2, was used as a measure of the "goodness of fit" of the predicted values from the model to the patterns in the actual data. In addition, the mean prediction error, or root mean square error (RMSE),

$$\text{RMSE} = \sqrt{\frac{1}{n}\Sigma(Y_t - \hat{Y}_t)^2} \tag{5.1}$$

is often used to evaluate the accuracy of a forecasting model (where n is the number of observations). The smaller the value of the RMSE, the greater the accuracy.

WHAT WENT RIGHT • WHAT WENT WRONG

Crocs Shoes[2]

In 2002, a colorful foam clog that was lightweight and nearly indestructible appeared on the market. Crocs were an overnight sensation, and 100 million pairs were sold in seven years. The company forecasted double-digit sales growth for the next five years, and then did a very successful initial public offering that raised $200 million. The new capital was reinvested to ramp up Crocs' manufacturing capacity. Then the severe worldwide recession of 2008–2009 hit, and the bottom fell out of the market. No one needed replacements for a nearly indestructible fashion fad that couldn't help you look for a job. In one year (2007–2008), the company swung from a profit of $168 million to a loss of $185 million. After several years of retrenchment, Crocs are again profitable at less than one-tenth of the previous production capacity.

[2]Based on "Once-Trendy Crocs Could Be on Their Last Legs," *Washington Post* (July 16, 2009), p. C2.

ALTERNATIVE FORECASTING TECHNIQUES

The managerial economist may choose from a wide range of forecasting techniques. These can be classified in the following general categories:

1. Deterministic trend analysis
2. Smoothing techniques
3. Barometric indicators
4. Survey and opinion-polling techniques
5. Macroeconometric models
6. Stochastic time-series analysis
7. Forecasting with input-output tables

DETERMINISTIC TREND ANALYSIS

time-series data A series of observations taken on an economic variable at various points in time.

cross-sectional data Series of observations taken on different observation units (e.g., households, states, or countries) at the same point in time.

Data collected for use in forecasting the value of a particular variable may be classified into two major categories—time-series or cross-sectional data. **Time-series data** are defined as a sequence of the values of an economic variable at different points in time. **Cross-sectional data** are an array of the values of an economic variable observed at the same time, like the data collected in a census across many individuals in the population. No matter what type of forecasting model is being used, one must decide whether time-series or cross-sectional data are most appropriate.

Components of a Time Series

In the analysis of time-series data, time in years, months, or weeks is represented on the horizontal axis, and the values of the dependent variable are on the vertical axis. The variations that are evident in the time series in Figure 5.2 can be decomposed into four components:

secular trends Long-run changes (growth or decline) in an economic time-series variable.

cyclical variations Major expansions and contractions in an economic series that usually are longer than a year in duration.

seasonal effects Variations in a time series during a year that tend to appear regularly from year to year.

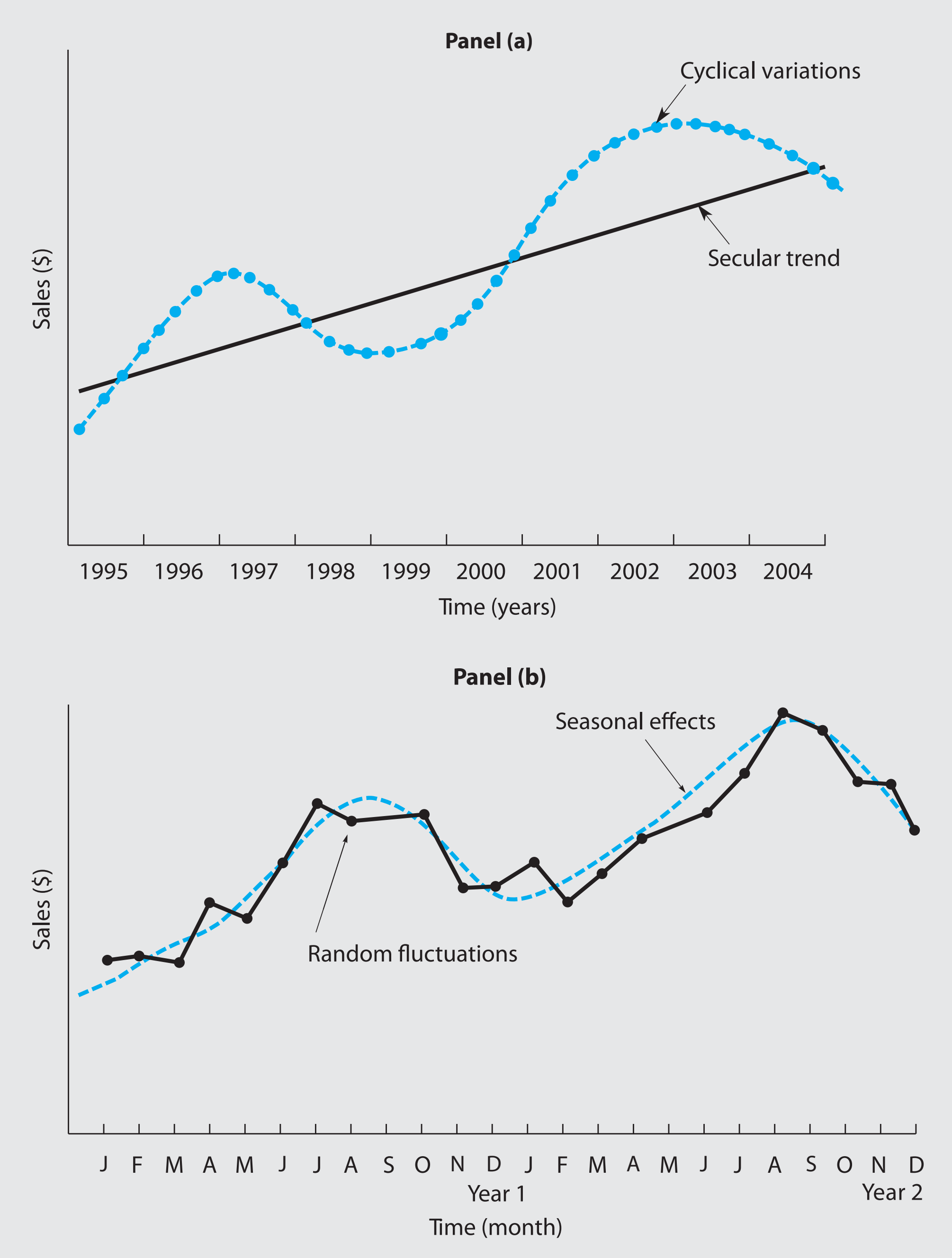
(a) ***Secular trends***. These are long-run trends that cause changes in an economic data series [*solid line* in Panel (a) of Figure 5.2]. For example, in empirical demand analyses, such factors as increasing population size or evolving consumer tastes may result in trend increases or decreases of a demand series over time.

(b) ***Cyclical variations***. These are major expansions and contractions in an economic series that are usually greater than a year in duration [*broken line* in Panel (a) of Figure 5.2]. For example, the housing industry appears to experience regular expansions following contractions in demand. When cyclical variations are present, regression estimates using the raw data will be distorted due to the presence of positive autocorrelation. Care must then be taken to specify an appropriate lag structure to remove the autocorrelation.

(c) ***Seasonal effects***. Seasonal variations during a year tend to be more or less consistent from year to year. The data in Panel (b) of Figure 5.2 (*broken line*) show significant seasonal variation. For example, two-thirds of Hickory Farms' (a retailer of holiday food gifts) annual sales occur between November and January.

(d) ***Random fluctuations***. Finally, an economic series may be influenced by random factors that are unpredictable [*solid line* in Panel (b) of Figure 5.2], such as hurricanes, floods, and tornados, as well as extraordinary government actions like a wage-price freeze or a declaration of war.

FIGURE 5.2 Secular, Cyclical, Seasonal, and Random Fluctuations in Time-Series Data



Some Elementary Time-Series Models

The simplest time-series model states that the forecast value of the variable for the next period will be the same as the value of that variable for the present period:

$$\hat{Y}_{t+1} = Y_t \tag{5.2}$$

For example, consider the sales data shown in Table 5.1 for the Buckeye Brewing Company. To forecast monthly sales, the model uses *actual* beer sales for March 2007 of 2,738 (000) barrels as the forecast value for April.

TABLE 5.1 BUCKEYE BREWING COMPANY'S MONTHLY BEER SALES (THOUSANDS OF BARRELS)

	YEAR		
MONTH	2012	2013	2014
January	2,370	2,446	2,585
February	2,100	2,520	2,693
March	2,412	2,598	2,738
April	2,376	2,533	
May	3,074	3,250	
June	3,695	3,446	
July	3,550	3,986	
August	4,172	4,222	
September	3,880	3,798	
October	2,931	2,941	
November	2,377	2,488	
December	2,983	2,878	

Where changes occur slowly and the forecast is being made for a relatively short period in the future, such a model may be quite useful. However, because Equation 5.2 requires knowledge of this month's sales, the forecaster may be faced with the task of speeding up the collection of actual data. Another problem with this model is that it makes no provision for incorporating special promotions by the firm (or its competitors) that could cause large sales deviations.

Further examination of the Buckeye beer sales data in Table 5.1 indicates a slight upward trend in sales—beer sales in most months are higher than in the same month of the previous year. Second, we note that sales are somewhat seasonal—beer sales are high during the summer months and low during the winter. The tendency for recent increases to trigger further increases in beer sales may be incorporated by slightly adjusting Equation 5.2 to yield this equation:

$$\hat{Y}_{t+1} = Y_t + (Y_t - Y_{t-1}) \quad [5.3]$$

For example, Buckeye's sales forecast for April 2014 using this model would be

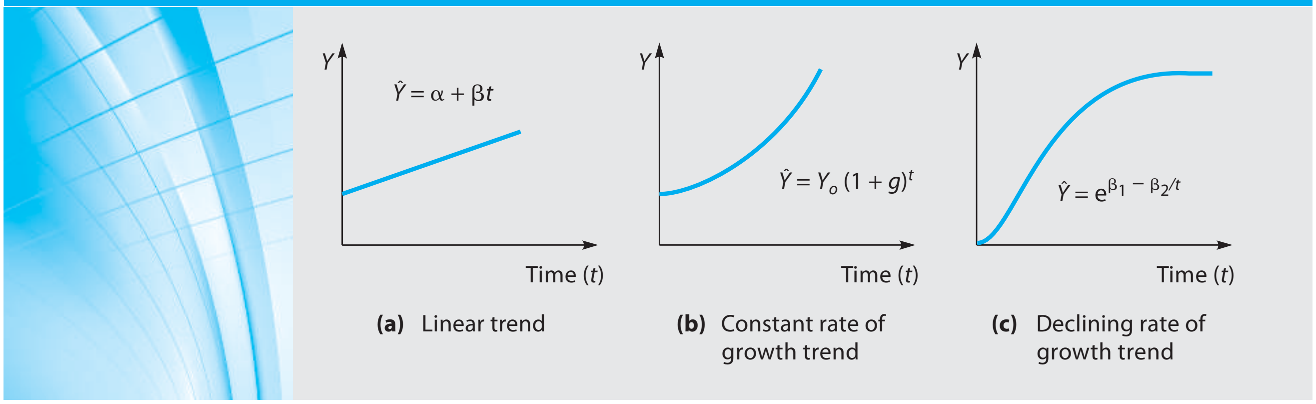
$$\hat{Y}_{t+1} = 2{,}738 + (2{,}738 - 2{,}693)$$
$$= 2{,}783(000) \text{ barrels}$$

Other forecasting models that incorporate trends and seasonal effects such as these are discussed in the next two sections.

Secular Trends

Long-run changes in an economic time series can follow a number of different types of trends. Three possible cases are shown in Figure 5.3. A *linear* trend is shown in Panel (a), Panel (b), and Panel (c) depict *nonlinear* trends. In Panel (b), the economic time series follows a *constant rate of growth* pattern. The earnings of many corporations follow this type of trend. Panel (c) shows an economic time series that exhibits a declining rate of growth. Sales of a new product may follow this pattern. As market saturation occurs, the rate of growth will decline over time.

FIGURE 5.3 Time-Series Growth Patterns



Linear Trends A linear time trend may be estimated by using *least-squares* regression analysis to provide an equation of a straight line of "best fit." (See Chapter 4 for a further discussion of the least-squares technique.) The equation of a linear time trend is given in the general form

$$\hat{Y} = \alpha + \beta t \qquad [5.4]$$

where $\hat{Y}_t$ is the forecast or predicted value for period t, α is the Y intercept or constant term, t is a unit of time, and β is an estimate of this trend factor.

Example

Linear Trend Forecasting: Prizer Creamery

Suppose one is interested in forecasting monthly ice cream sales of the Prizer Creamery for 2007. A least-squares trend line could be estimated from the ice cream sales data for the past four years (48 monthly observations), as shown in Figure 5.4. Assume that the equation of this line is calculated to be

$$\hat{Y}_t = 30{,}464 + 121.3t$$

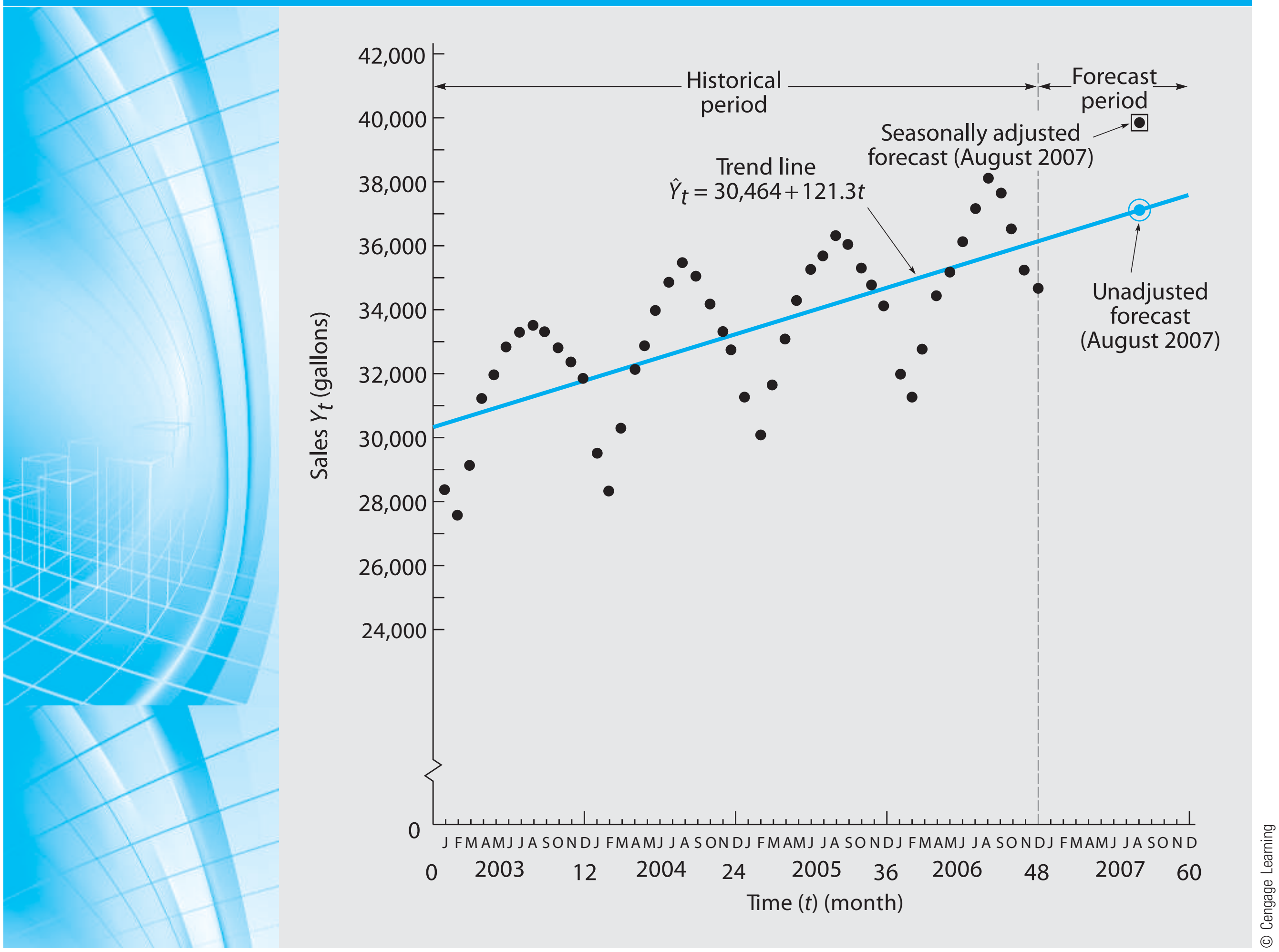
where $\hat{Y}_t$ = predicted monthly ice cream sales in gallons in month t
30,464 = number of gallons sold when $t = 0$
t = time period (months) (where December 2002 = 0, January 2003 = 1, February 2003 = 2, March 2003 = 3, …)

The coefficient (121.3) of t indicates that sales may be expected to increase by 121.3 gallons on the average each month. Based on this trend line and ignoring any seasonal effects, forecasted ice cream sales for August 2007 (t = 56) would be

$$\begin{aligned} Y_{56} &= 30{,}464 + 121.3(56) \\ &= 37{,}257 \text{ gallons} \end{aligned}$$

This *seasonally unadjusted* forecast is given by the August 2007 point (⊙) on the trend line in Figure 5.4. As can be seen in the graph, ice cream sales are subject to seasonal variations. Later in this section we will show how this seasonal effect can be incorporated into the forecast.

FIGURE 5.4 Prizer Creamery: Monthly Ice Cream Sales



Linear time trend forecasting is easy and inexpensive to do, but it is generally too simple and inflexible to be used in many forecasting circumstances. Constant rate of growth time trends is one alternative.

Constant Rate of Growth Trends The formula for the constant rate of growth forecasting model is

$$\hat{Y}_t = Y_0(1+g)^t \qquad [5.5]$$

where $\hat{Y}_t$ is the forecasted value for period t, Y_0 is the initial ($t = 0$) value of the time series, g is the constant growth rate per period, and t is a unit of time. The predicted value of the time series in period t, ($\hat{Y}_t$), is equal to the initial value of the series (Y_0) compounded at the growth rate (g) for t periods. Because Equation 5.5 is a nonlinear relationship, the parameters cannot be estimated directly with the ordinary least-squares method. However, taking logarithms of both sides of the equation gives

$$\log \hat{Y}_t = \log Y_0 + \log(1+g) \cdot t$$

or

$$\hat{Y}'_t = \alpha + \beta t \qquad [5.6]$$

where $\hat{Y}'_t = \log \hat{Y}_t$, $\alpha = \log Y_0$; and $\beta = \log (1 + g)$. Equation 5.6 is a linear relationship whose parameters can be estimated using standard linear regression techniques.

For example, suppose that annual earnings data for the Fitzgerald Company for the past 10 years have been collected and that Equation 5.6 was fitted to the data using least-squares techniques. The annual rate of growth of company earnings was estimated to be 6 percent. If the company's earnings this year ($t = 0$) are \$600,000, then next year's ($t = 1$) forecasted earnings would be

$$\begin{aligned}\hat{Y}_1 &= 600{,}000(1 + 0.06)^1 \\ &= \$636{,}000\end{aligned}$$

Similarly, forecasted earnings for the year after next ($t = 2$) would be

$$\begin{aligned}\hat{Y}_2 &= 600{,}000(1 + 0.06)^2 \\ &= \$674{,}160\end{aligned}$$

Declining Rate of Growth Trends The curve depicted in Figure 5.3, panel (c) is particularly useful for representing sales penetration curves in marketing applications. Using linear regression techniques, one can specify a semilog estimating equation,

$$\log \hat{Y}_t = \beta_1 - \beta_2(1/t)$$

and recover the β_1 and β_2 parameters of this nonlinear diffusion process as a new product spreads across a target population. β_1 and β_2 measure how quickly a new product or new technology or brand extension penetrates and then slowly (ever more slowly) saturates a market.

Seasonal Variations

When *seasonal variations* are introduced into a forecasting model, its short-run predictive power may be improved significantly. Seasonal variations may be estimated in a number of ways.

Ratio-to-Trend Method One approach is the *ratio to trend method.* This method assumes that the trend value is *multiplied by* the seasonal effect.

Example

Seasonally Adjusted Forecasts: Prizer Creamery (continued)

Recall in the Prizer Creamery example discussed earlier that a linear trend analysis (Equation 5.4) yielded a sales forecast for August 2007 of 37,257 gallons. This estimate can be adjusted for seasonal effects in the following manner. Assume that over a four-year period (2003–2006) the trend model predicted the August sales patterns shown in Table 5.2 and that actual sales are as indicated. These data indicate that, on the average, August sales have been 7.0 percent higher than the trend value. Hence, the August 2007 sales forecast should be seasonally adjusted *upward* by 7.0 percent to 39,865. The seasonally adjusted forecast is shown by the point ($\boxdot$) above the trend line in Figure 5.4. If, however, the model predicted February 2007 ($t = 50$) sales to be 36,529, but similar data indicated February sales to be 10.8 percent below trend on the average, the forecast would be adjusted *downward* to 36,529(1 − 0.108) = 32,584 gallons.

(continued)

TABLE 5.2 PRIZER CREAMERY'S AUGUST ICE CREAM SALES

(AUGUST)	FORECAST	ACTUAL	ACTUAL/FORECAST
2002	31,434	33,600	1.0689
2003	32,890	35,600	1.0824
2004	34,346	36,400	1.0598
2005	35,801	38,200	1.0670
2006	37,257	—	—
			Sum = 4.2781
	Adjustment factor = 4.2781/4 = 1.0695 (i.e., 1.07)		Average

Dummy Variables Another approach for incorporating seasonal effects into the linear trend analysis model is the use of *dummy variables.* A dummy variable is a variable that normally takes on one of two values—either 0 or 1. Dummy variables, in general, are used to capture the impact of certain qualitative factors in an econometric relationship, such as sex—male-0 and female-1. This method assumes that the seasonal effects are added to the trend value. If a time series consists of quarterly data, then the following model could be used to adjust for seasonal effects:

$$\hat{Y}_t = \alpha + \beta_1 t + \beta_2 D_{1t} + \beta_3 D_{2t} + \beta_4 D_{3t} \qquad [5.7]$$

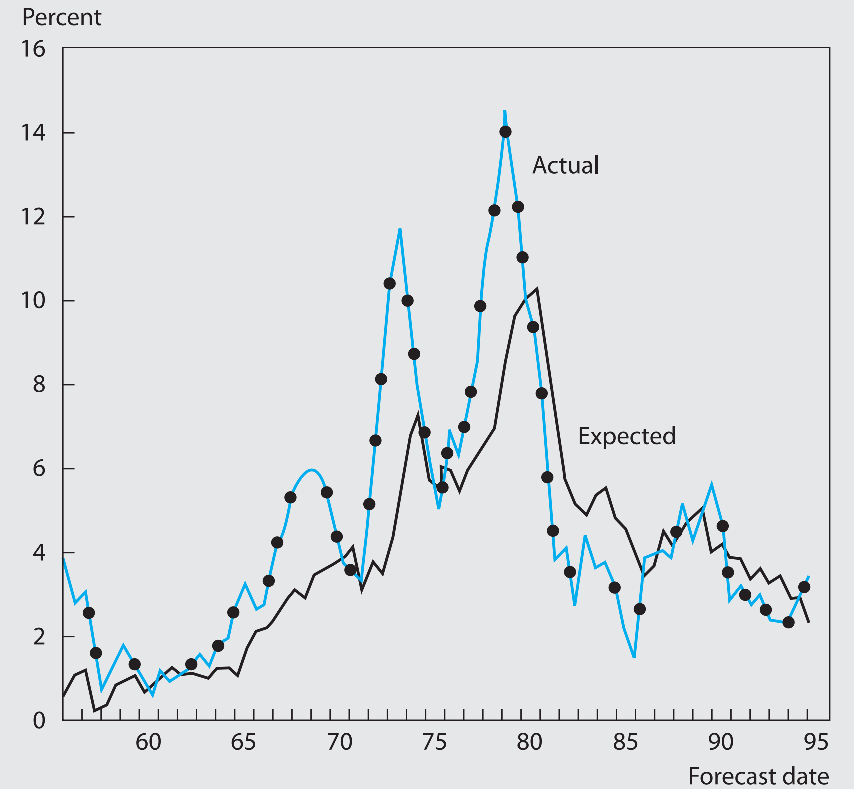
where $D_{1t} = 1$ for first-quarter observations and 0 otherwise, $D_{2t} = 1$ for second-quarter observations and 0 otherwise, $D_{3t} = 1$ for third-quarter observations and 0 otherwise, and α and β are parameters to be estimated using least-squares techniques. In this model the values of the dummy variables (D_{1t}, D_{2t}, D_{3t}) for observations in the fourth quarter of each year (base period) would be equal to zero. In the estimated model, the value $\beta_2\ D_{1t}$ represents the impact of a first-quarter observation (D_1) on values of the forecast, Y_t, relative to the forecast from the omitted class (fourth quarter), when D_{2t} and D_{3t} take values of 0.

The introduction of these trend and seasonality factors into a forecasting model should significantly improve the model's ability to predict short-run turning points in the data series, provided the historical causal factors have not changed significantly.[3]

The models of time-series trend forecasting discussed in this section may have substantial value in many areas of business. However, such models do not seek to relate changes in a data series to the causes underlying observed values in the series. For example, the nation's money supply series has at times proved very useful for forecasting inflationary pressure in the economy. But narrow definitions of the nation's money supply have gradually broadened to include bank-card lines of credit, which may have become a more important measure of household purchasing power. Inflation forecasts based on narrow money supply measures today would yield large errors between the actual and predicted inflation (see Figure 5.5).

[3]See more extensive discussion of these issues in F. Diebold, *Elements of Forecasting*, 4th ed. (Cincinnati: South-Western College Publishing, 2007).

FIGURE 5.5 Actual and Expected Inflation



Note: "Expected" is the inflation forecast for the year following the forecast date; "Actual" is the actual inflation rate over that period.

Source: Federal Reserve Bank of Philadelphia, Business Review, May/June 1996.

SMOOTHING TECHNIQUES

Smoothing techniques are another type of forecasting model, which assumes that a repetitive underlying pattern can be found in the historical values of a variable that is being forecast. By taking an average of past observations, smoothing techniques attempt to eliminate the distortions arising from random variation in the series. As such, smoothing techniques work best when a data series tends to change slowly from one period to the next with few turning points. Housing price forecasts would be a good application for smoothing techniques. Gasoline price forecasts would not. Smoothing techniques are cheap to develop and inexpensive to operate.

Example

Dummy Variables and Seasonal Adjustments: Value-Mart Company

The Value-Mart Company (a small chain of discount department stores) is interested in forecasting quarterly sales for next year (2008) based on Equation 5.7. Using

(continued)

quarterly sales data for the past eight years (2000–2007), the following model was estimated:

$$\hat{Y}_t = 22.5 + 0.25t - 4.5D_{1t} - 3.2D_{2t} - 2.1D_{3t} \quad [5.8]$$

where $\hat{Y}_t$ = predicted sales ($ million) in quarter t
22.5 = quarterly sales ($ million) when $t = 0$
t = time period (quarter) (where the fourth quarter of 1999 = 0; first quarter of 2000 = 1, second quarter of 2000 = 2, …)

The coefficient of t (0.25) indicates that sales may be expected to increase by $0.25 million on the average each quarter. The coefficients of the three dummy variables (−4.5, −3.2, and −2.1) indicate the change (i.e., reduction because the coefficients are negative) in sales in Quarters 1, 2, and 3, respectively, because of seasonal effects. Based on Equation 5.8, Value-Mart's quarterly sales forecasts for 2008 are shown in Table 5.3. On just this basis, Value-Mart would have ordered inventory for what proved to be a disastrous 2008–2009.

TABLE 5.3 VALUE-MART'S QUARTERLY SALES FORECAST (2008)

	TIME PERIOD	DUMMY VARIABLE			SALES FORECAST ($ MILLION) $\hat{Y} = 22.5 + 0.25t - 4.5D_{1t} - 3.2D_{2t} - 2.1D_{3t}$
QUARTER	t	D_{1t}	D_{2t}	D_{3t}	
1	33	1	0	0	26.25
2	34	0	1	0	27.80
3	35	0	0	1	29.15
4	36	0	0	0	31.50

Moving Averages

Moving averages are one of the simplest of the smoothing techniques. If a data series possesses a large random factor, a trend analysis forecast like those discussed in the previous section will tend to generate forecasts having large errors from period to period. In an effort to minimize the effects of this randomness, a series of recent observations can be averaged to arrive at a forecast. This is the moving average method. A number of observed values are chosen, their average is computed, and this average serves as a forecast for the next period. In general, a moving average may be defined as

$$\hat{Y}_{t+1} = \frac{Y_t + Y_{t-1} + \cdots + Y_{t-N+1}}{N} \quad [5.9]$$

where $\hat{Y}_{t+1}$ = forecast value of Y for one period in the future
Y_t, Y_{t-1}, Y_{t-N+1} = observed values of Y in periods $t, t-1, \ldots, t-N+1$, respectively
N = number of observations in the moving average

The greater the number of observations N used in the moving average, the greater the smoothing effect because each new observation receives less weight ($1/N$) as N increases. Hence, generally, the greater the randomness in the data series and the slower the turning point events in the data, the more preferable it is to use a relatively large number of past observations in developing the forecast. The most appropriate moving average period is the choice of N that minimizes the root mean square error (Equation 5.1).

Example

Moving Average Forecasts: Walker Corporation

The Walker Corporation is examining the use of various smoothing techniques to forecast monthly sales. The company collected sales data for 12 months (2006) as shown in Table 5.4 and Figure 5.6. One technique under consideration is a three-month moving average. Equation 5.9 can be used to generate the forecasts. The forecast for Period 4 is computed by averaging the observed values for Periods 1, 2, and 3.

$$\hat{Y}_4 = \frac{Y_3 + Y_2 + Y_1}{N} = \frac{1{,}925 + 1{,}400 + 1{,}950}{3} = 1{,}758 \qquad [5.10]$$

Similarly, the forecast for Period 5 is computed as

$$\hat{Y}_5 = \frac{Y_4 + Y_3 + Y_2}{N} = \frac{1{,}960 + 1{,}925 + 1{,}400}{3} = 1{,}762 \qquad [5.11]$$

TABLE 5.4 WALKER CORPORATION'S THREE-MONTH MOVING AVERAGE SALES FORECAST TABLE

		SALES ($1,000)		ERROR	
t	MONTH	ACTUAL Y_t	FORECAST $\hat{Y}_t$	$(Y_t - \hat{Y}_t)$	$(Y_t - \hat{Y}_t)^2$
1	January 2006	1,950	—	—	—
2	February	1,400	—	—	—
3	March	1,925	—	—	—
4	April	1,960	1,758	202	40,804
5	May	2,800	1,762	1,038	1,077,444
6	June	1,800	2,228	−428	183,184
7	July	1,600	2,187	−587	344,569
8	August	1,450	2,067	−617	380,689
9	September	2,000	1,617	383	146,689
10	October	2,250	1,683	567	321,489
11	November	1,950	1,900	50	2,500
12	December	2,650	2,067	583	339,889
13	January 2007	—	2,283	—	—
					Sum = 2,837,257

$$\text{RMSE} = \sqrt{2{,}837{,}257/9} = \$561(000)$$

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Note that if one subtracts $\hat{Y}_4$ from $\hat{Y}_5$, the result is the change in the forecast from $\hat{Y}_4$, or

$$\Delta\hat{Y}_4 = \hat{Y}_5 - \hat{Y}_4 = \frac{Y_4 + Y_3 + Y_2}{N} - \frac{Y_3 + Y_2 + Y_1}{N} = \frac{Y_4}{N} - \frac{Y_1}{N} \qquad [5.12]$$

(continued)

Adding this change to $\hat{Y}_4$, the following alternative expression for $\hat{Y}_5$ can be derived:

$$\hat{Y}_5 = \hat{Y}_4 + \frac{Y_4}{N} - \frac{Y_1}{N} \quad [5.13]$$

or, in general,

$$\hat{Y}_{t+1} = \hat{Y}_t + \frac{Y_t}{N} - \frac{Y_{t-N}}{N} \quad [5.14]$$

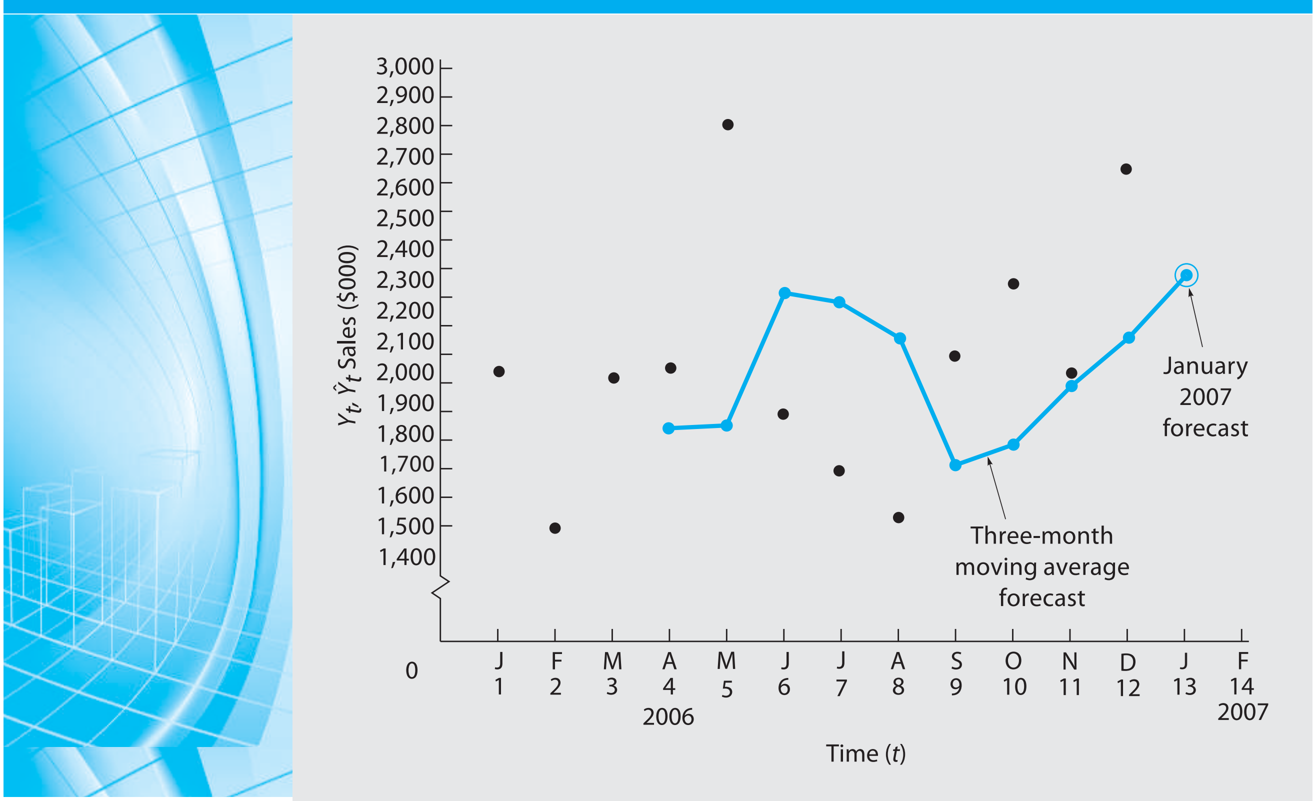
which indicates that each moving average forecast is equal to the past forecast, $\hat{Y}_t$, plus the weighted effect of the most recent observation, Y_t/N, minus the weighted effect of the oldest observation that has been dropped, Y_{t-N}/N. As N becomes larger, the smoothing effect increases because the new observation, Y_t, has a small impact on the moving average.

As shown in Table 5.4, Walker's forecast for January 2007 (t = 13) is \$2,283 (000). Note also that the root mean square error (RMSE) of the three-month (N) moving average period is \$561(000).

First-Order Exponential Smoothing

One criticism of moving averages as smoothing techniques is that they normally give equal weight (a weight of $1/N$) to all observations used in preparing the forecast, even though intuition often indicates that the most recent observation probably contains

FIGURE 5.6 Walker Corporation's Three-Month Moving Average Sales Forecast Chart



more immediately useful information than more distant observations. Exponential smoothing is designed to overcome this objection.[4]

Consider the following alternative forecasting model:

$$\hat{Y}_{t+1} = wY_t + (1 - w)\hat{Y}_t \quad [5.15]$$

This model weights the most recent observation by w (some value between 0 and 1 inclusive), and the past forecast by $(1 - w)$. A large w indicates that a heavy weight is being placed on the most recent observation.[5]

Using Equation 5.15, a forecast for $\hat{Y}_t$ may also be written as

$$\hat{Y}_t = wY_{t-1} + (1 - w)(\hat{Y}_{t-1}) \quad [5.16]$$

By substituting Equation 5.16 into 5.15, we get

$$\hat{Y}_{t+1} = wY_t + w(1 - w)Y_{t-1} + (1 - w)^2\hat{Y}_{t-1} \quad [5.17]$$

By continuing this process of substitution for past forecasts, we obtain the general equation

$$\hat{Y}_{t+1} = wY_t + w(1 - w)Y_{t-1} + w(1 - w)^2Y_{t-2} + w(1 - w)^3Y_{t-3} + \cdots \quad [5.18]$$

Equation 5.18 shows that the general formula (Equation 5.15) for an exponentially weighted moving average is a weighted average of all past observations, with the weights defined by the geometric progression:

$$w, (1 - w)w, (1 - w)^2w, (1 - w)^3w, (1 - w)^4w, (1 - w)^5w, \ldots \quad [5.19]$$

For example, a w of 2/3 would produce the following series of weights:

$$\begin{aligned} w &= 0.667 \\ (1 - w)w &= 0.222 \\ (1 - w)^2w &= 0.074 \\ (1 - w)^3w &= 0.024 \\ (1 - w)^4w &= 0.0082 \\ (1 - w)^5w &= 0.0027 \end{aligned}$$

With a high initial value of w, heavy weight is placed on the most recent observation, and rapidly declining weights are placed on older values.

Another way of writing Equation 5.15 is

$$\hat{Y}_{t+1} = \hat{Y}_t + w(Y_t - \hat{Y}_t) \quad [5.20]$$

This indicates that the new forecast is equal to the old forecast plus w times the error in the most recent forecast. A w that is close to 1 indicates a quick adjustment process for any error in the preceding forecast. Similarly, a w closer to 0 suggests a slow error correction process.

It should be apparent from Equations 5.15 and 5.20 that exponential forecasting techniques can be very easy to use. All that is required is last period's forecast, last period's observation, plus a value for the weighting factor, w. The optimal weighting factor is normally determined by making successive forecasts using past data with various values of w and choosing the w that minimizes the RMSE given in Equation 5.1.

[4]More complex double exponential smoothing models generally give more satisfactory results than first-order exponential smoothing models when the data possess a linear trend over time. See Diebold, *op. cit.*

[5]The greater the amount of serial correlation (correlation of values from period to period), the larger will be the optimal value of w.

Example

Exponential Smoothing: Walker Corporation (continued)

Consider again the Walker Corporation example discussed earlier. Suppose that the company is interested in generating sales forecasts using the first-order exponential smoothing technique. The results are shown in Table 5.5. To illustrate the approach, an exponential weight w of 0.5 will be used. To get the process started, one needs to make an initial forecast of the variable. This forecast might be a weighted average or some simple forecast, such as Equation 5.2:

$$\hat{Y}_{t+1} = Y_t$$

The latter approach will be used. Hence the forecast for Month 2 made in Month 1 would be \$1,950(000) ($\hat{Y}_{t+1} = 1{,}950$). The Month 3 forecast value is (using Equation 5.20)

$$\begin{aligned}\hat{Y}_3 &= 1{,}950 + 0.5(1{,}400 - 1{,}950)\\ &= 1{,}950 - 275 = \$1{,}675(000)\end{aligned}$$

Similarly, the Month 4 forecast equals

$$\begin{aligned}\hat{Y}_4 &= 1{,}675 + 0.5(1{,}925 - 1{,}675)\\ &= \$1{,}800(000)\end{aligned}$$

The remaining forecasts are calculated in a similar manner.

As can be seen in Table 5.5, Walker's sales forecast for January 2007 using the first-order exponential smoothing technique is \$2,322(000). Also, the root mean square error of this forecasting method (with $w = 0.50$) is \$491(000).

TABLE 5.5 WALKER CORPORATION: FIRST-ORDER EXPONENTIAL SMOOTHING SALES FORECAST

		SALES (\$1,000)		ERROR	
t	MONTH	ACTUAL Y_t	FORECAST $\hat{Y}_t$	$(Y_t - \hat{Y}_t)$	$(Y_t - \hat{Y}_t)^2$
1	January 2006	1,950	—	—	—
2	February	1,400	1,950	–550	302,500
3	March	1,925	1,675	250	62,500
4	April	1,960	1,800	160	25,600
5	May	2,800	1,880	920	846,400
6	June	1,800	2,340	–540	291,600
7	July	1,600	2,070	–470	220,900
8	August	1,450	1,835	–385	148,225
9	September	2,000	1,642	358	128,164
10	October	2,250	1,821	429	184,041
11	November	1,950	2,036	–86	7,396
12	December	2,650	1,993	657	431,649
13	January 2007	—	2,322	—	—
					Sum = 2,648,975

$$\text{RMSE} = \sqrt{2{,}648{,}975/11} = \$491(000)$$