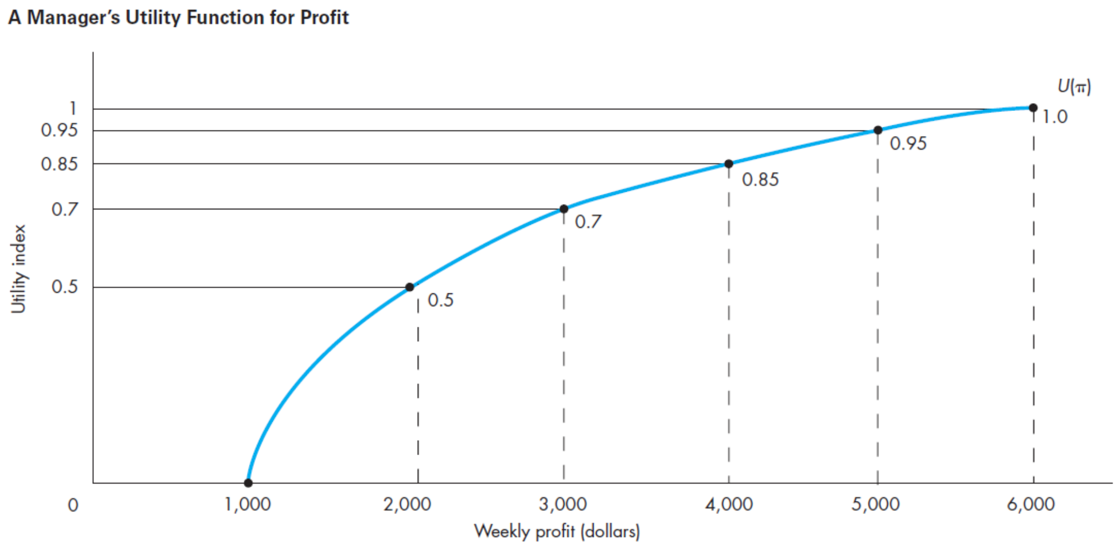
Working Examples

1. Let's look into the location decision problem facing Chicago Rotisserie Chicken. The manager calculates the expected utilities of the three risky location decisions using her own utility function for profit.



The expected utilities for the three cities are calculated as follows:

Atlanta $E(U_A) = 0U(1000) + 0.2U(2000) + 0.3U(3000) + 0.3U(4000) + 0.2U(5000) + 0U(6000)$
$= 0(0) + 0.2(0.5) + 0.3(0.7) + 0.3(0.85) + 0.2(0.95) + 0(1) = \mathbf{0.755}$

Boston $E(U_B) = 0.1U(1000) + 0.15U(2000) + 0.15U(3000) + 0.25U(4000) + 0.2U(5000) + 0.15U(6000)$
$= 0.1(0) + 0.15(0.5) + 0.15(0.7) + 0.25(0.85) + 0.2(0.95) + 0.15(1) = \mathbf{0.733}$

Cleveland $E(U_C) = 0.3U(1000) + 0.1U(2000) + 0.1U(3000) + 0.1U(4000) + 0.1U(5000) + 0.3U(6000)$
$= 0.3(0) + 0.1(0.5) + 0.1(0.7) + 0.1(0.85) + 0.1(0.95) + 0.3(1) = \mathbf{0.600}$

To maximize the expected utility of profits, the manager of Chicago Rotisserie Chicken chooses to open its new restaurant in Atlanta. Even though Boston has the highest expected profit [$E(\pi)$ = \$3,750], Boston also has the highest level of risk (σ = 1,545), and the risk-averse manager prefers to avoid the relatively high risk of locating the new restaurant in Boston. In this case of a risk-averse decision maker, the manager chooses the less risky Atlanta location over the more risky Cleveland location even though both locations have identical expected profit levels.

Profit	Utility	Marginal utility	Probabilities			Probability weighted utility		
(π)	[U(π)]	[ΔU(π)/ Δπ)]	Atlanta (P_A)	Boston (P_B)	Cleveland (P_C)	P_A x U	P_B x U	P_C x U
\$1000	0	--	0	0.1	0.3	0	0	0
2000	0.2	0.0002	0.2	0.15	0.1	0.04	0.03	0.02
3000	0.4	0.0002	0.3	0.15	0.1	0.12	0.06	0.04
4000	0.6	0.0002	0.3	0.25	0.1	0.18	0.15	0.06
5000	0.8	0.0002	0.2	0.2	0.1	0.16	0.16	0.08
6000	1	0.0002	0	0.15	0.3	0	0.15	0.3
					Expected utility =	0.50	0.55	0.50

To show what a risk-neutral decision maker would do, we constructed a utility function for profit that exhibits constant marginal utility of profit, which, as we have explained, is the condition required for risk neutrality. This risk-neutral utility function is presented in column 2 of above Table. Marginal utility of profit, in column 3, is constant, as it must be for risk-neutral managers.

From the table you can see that the expected utilities of profit for Atlanta, Boston, and Cleveland are 0.50, 0.55, and 0.50, respectively. For a risk-neutral decision maker, locating in Boston is the decision that maximizes expected utility. Recall that Boston also is the city with the maximum expected profit [E(π) = $3,750]. This is not a coincidence. As we explained earlier, a risk-neutral decision maker ignores risk when making decisions and relies instead on expected profit to make decisions in risky situations. Under conditions of risk neutrality, a manager makes the same decision by maximizing either the expected value of profit, E(π), or the expected utility of profit, E[U(π)].

2. Suppose the management at Dura Plastic is considering changing the size (capacity) of its manufacturing plant. Management has narrowed the decision to three choices. The plant's capacity will be:
   a. expanded by 20 percent,
   b. maintained at the current capacity, or
   c. reduced by 20 percent.

   The outcome of this decision depends crucially on how the economy performs during the upcoming year. Thus the performance of the economy is the "state of nature" in this decision problem. Management envisions three possible states of nature occurring:

   i. The economy enters a period of recovery,
   ii. economic stagnation sets in, or
   iii. the economy falls into a recession.

For each possible decision and state of nature, the managers determine the profit outcome, or payoff, shown in the payoff matrix.
A payoff matrix is a table with rows corresponding to the various decisions and columns corresponding to the various states of nature. Each cell in the payoff matrix gives the outcome (payoff) for each decision when a particular state of nature occurs.

The Maximax Criterion

	States of nature		
Decisions	Recovery	Stagnation	Recession
Expand plant capacity by 20%	$5 million	−$1 million	−$3.0 million
Maintain same plant capacity	3 million	2 million	0.5 million
Reduce plant capacity by 20%	2 million	1 million	0.75 million

For example, if management chooses to expand the manufacturing plant by 20 percent and the economy enters a period of recovery, Dura Plastic is projected to earn profits of $5 million. Alternatively, if Dura Plastic expands plant capacity but the economy falls into a recession, it is projected that the company will lose $3 million. The managers do not know which state of nature will actually occur, or the probabilities of occurrence, so the decision to alter plant capacity is made under conditions of uncertainty. To apply the **Maximax rule** to this decision, management first identifies the best possible outcome for each of the three decisions.
The best payoffs are:
$5 million for expanding plant size by 20 percent
$3 million for maintaining plant size
$2 million for reducing plant size by 20 percent

Each best payoff occurs if the economy recovers. Under the Maximax rule, management would decide to expand its plant. While the Maximax rule is simple to apply, it fails to consider "bad" outcomes in the decision-making process. The fact that two out of three states of nature result in losses when management decides to expand plant capacity, and neither of the other decisions would result in a loss, is overlooked when using the Maximax criteria. Only managers with optimistic natures are likely to find the Maximax rule to be a useful decision-making tool.

The Maximin Criterion

For managers with a pessimistic outlook on business decisions, the Maximin rule may be more suitable than the Maximax rule. Under the Maximin rule, the manager identifies the worst outcome for each decision and makes the decision associated with the maximum worst payoff. For Dura Plastic, the worst outcomes for each decision from above matrix are:

-$3 million for expanding plant size by 20 percent.
$0.5 million for maintaining plant size.
$0.75 million for reducing plant size by 20 percent

Using the Maximin criterion, Dura Plastic would choose to reduce plant capacity by 20 percent. The Maximin rule is also simple to follow, but it fails to consider any of the "good" outcomes.

The Minimax Regret Criterion

Managers concerned about their decisions not turning out to be the best once the state of nature is known (i.e., after the uncertainty is resolved) may make their decisions by minimizing the potential regret that may occur. The potential regret associated with a particular decision and state of nature is the improvement in payoff the manager could have experienced had the decision been the best one when that state of nature actually occurred. To illustrate, we calculate from matrix the potential regret associated with Dura Plastic's decision to maintain the same level of plant capacity if an economic recovery occurs. The best possible payoff when recovery occurs is $5 million, the payoff for expanding plant capacity. If a recovery does indeed happen and management chooses to maintain the same level of plant capacity, the payoff is only $3 million, and the manager experiences a regret of $2 million (= $5 - $3 million).

	States of nature		
Decisions	Recovery	Stagnation	Recession
Expand plant capacity by 20%	$0 million	$3 million	$3.75 million
Maintain same plant capacity	2 million	0 million	0.25 million
Reduce plant capacity by 20%	3 million	1 million	0 million

Above Table shows the potential regret for each combination of decision and state of nature. Note that every state of nature has a decision for which there is no potential regret. This occurs when the correct decision is made for that particular state of nature. To apply the **minimax regret rule**, which requires that managers make a decision with the minimum worst potential regret, management identifies the maximum possible potential regret for each decision from the matrix:

$3.75 million for expanding plant size by 20 percent
$2 million for maintaining plant size
$3 million for reducing plant size by 20 percent

Management chooses the decision with the lowest worst potential regret: maintain current plant capacity. For Dura Plastic, the minimax regret rule results in management's choosing to maintain the current plant capacity.

The Equal Probability Criterion

In situations of uncertainty, managers have no information about the probable state of nature that will occur and sometimes simply assume that each state of nature is equally likely to occur. In terms of the Dura Plastic decision, management assumes each state of nature has a one-third probability of occurring. When managers assume each state of nature has an equal likelihood of occurring, the decision can be made by considering the average payoff for each equally possible state of nature. This approach to decision making is often referred to as the equal probability rule.

To illustrate, the manager of Dura Plastic calculates the average payoff for each decision as follows:

$0.33 million [= (5 + (-1) + (-3)) /3] for expanding plant size
$1.83 million [= (3 + 2 + 0.5) /3] for maintaining plant size
$1.25 million [= (2 + 1 + 0.75) /3] for reducing plant size

Under the equal probability rule, the manager's decision is to maintain the current plant capacity since this decision has the maximum average return.