Tutorial Sheet-6

Graph Theory and Algorithms

(1) What are all simple graphs with the property that every connected subgraph is an induced subgraph?

(2) Prove that the center of a tree is a single vertex (K_1) or an edge (K_2).

(3) Prove that an edge e of a connected graph G is a cut edge (bridge) if and only if it belongs to every spanning tree of G.

(4) Find trees having prufer code in which
(a) all the entries are equal
(b) all the entries are distinct.

(5) Prove that a labelled $K_n - e$, where e is an edge in K_n, has $(n-2)n^{(n-3)}$ spanning trees.

(6) Prove Cayley's theorem applying the Matrix-Tree theorem.

(7) Show that the total number of vertices in a strictly binary tree is an odd integer.

(8) If T is a strictly binary tree on n vertices then prove that the total number of leaves in T is $\frac{n+1}{2}$.

(9) Find the total number of ternary trees (3-ary trees) of height 3 on 12 vertices so that the root vertex has 3 children and all the leaves are at the same level.