Keerti . P. Charantimath

NSOPDE

19MA20059

Assignment 2.

1) $y' = \lambda y \quad , \quad \lambda < 0;$

$$u_{n+1} = u_n + \frac{h}{4}(k_1 + 3k_2) \longrightarrow (1)$$

$$k_1 = f(t_n, u_n) \longrightarrow (2)$$

$$k_2 = f\left(t_n + \frac{h}{3}, u_n + \frac{h}{3}(k_1 + k_2)\right) \longrightarrow (3)$$

→ Determine the absolute stability of the implicit method.

Sol$^{\underline{n}}$:-

Test eq$^{\underline{n}}$:- $y' = \lambda y, \lambda < 0;$

$f(x,y) = \lambda y \Rightarrow f(x,u) = \lambda u$

from (2), $k_1 = \lambda u_n \longrightarrow (4)$

from (3), $k_2 = \lambda\left(u_n + \frac{h}{3}(k_1 + k_2)\right) \longrightarrow (5)$

$$k_2 = \lambda\left(u_n + \frac{h}{3}(\lambda u_n + k_2)\right)$$

$$k_2\left(1 - \frac{\lambda h}{3}\right) = \lambda u_n + \frac{\lambda^2 h}{3} u_n$$

$$k_2 = \frac{\left[\lambda u_n + \frac{\lambda^2 h}{3} u_n\right]}{\left(1 - \frac{\lambda h}{3}\right)} \longrightarrow (6)$$

put (4) & (6) in (1)

$$u_{n+1} = u_n + \frac{h}{4}\left(\lambda u_n + 3\frac{\left(\lambda u_n + \frac{\lambda^2 h}{3} u_n\right)}{\left(1 - \frac{\lambda h}{3}\right)}\right)$$

$$u_{n+1} = u_n E(\lambda h)$$

$$E(\lambda h) \Rightarrow \frac{\left(1 + \frac{\lambda h}{4}\right)\left(1 - \frac{\lambda h}{3}\right) + \frac{3\lambda h}{4} + \left(\frac{\lambda h}{4}\right)^2}{1 - \frac{\lambda h}{3}}$$

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$$E(\lambda h) \Rightarrow \frac{1 + \frac{\lambda h}{4} + \frac{3\lambda h}{4} - \frac{\lambda h}{3} - \frac{(\lambda h)^2}{12} + \frac{(\lambda h)^2}{4}}{1 - \frac{\lambda h}{3}}$$

$$E(\lambda h) \Rightarrow \frac{1 + \frac{2}{3}\lambda h + \frac{(\lambda h)^2}{6}}{1 - \frac{\lambda h}{3}}$$

For Absolute stability, $|E(\lambda h)| \leq 1$

$$\left| \frac{1 + \frac{2}{3}\lambda h + \frac{(\lambda h)^2}{6}}{1 - \frac{\lambda h}{3}} \right| \leq 1$$

OR

$$\left(\frac{1 + \frac{2}{3}\lambda h + \frac{1}{6}(\lambda h)^2}{1 + \frac{\lambda h}{3}} \right)^2 - 1^2 \leq 0$$

$$\left(\frac{1 + \frac{2\lambda h}{3} + \frac{\lambda^2 h^2}{6}}{1 + \frac{\lambda h}{3}} - \frac{1}{1} \right) \left(\frac{1 + \frac{2\lambda h}{3} + \frac{\lambda^2 h^2}{6}}{1 + \frac{\lambda h}{3}} + 1 \right) \leq 0$$

$$\left(\lambda h + \left(\frac{\lambda^2 h^2}{6} \right) \right) \left(2 + \frac{\lambda h}{3} + \frac{(\lambda h)^2}{6} \right) \leq 0$$

$$\lambda h \left(1 + \frac{\lambda h}{6} \right) \left(12 + 2\lambda h + \lambda^2 h^2 \right) \leq 0$$

$D = 4 - 48 < 0$, $\longrightarrow$ always +ve
$a = 1 > 0$

$$\lambda h \left(1 + \frac{\lambda h}{6} \right) \leq 0$$

$\boxed{\lambda h \in [-6, 0]}$ region of absolute stability

2) $u_{n+1} = u_n + \frac{1}{4}(3k_1 + k_2) \longrightarrow (1)$

$k_1 = hf(t_n + h/3,\ u_n + k_1/3) \rightarrow (2)$

$k_2 = hf(t_n + h,\ u_n + k_1) \longrightarrow (3)$

$\mapsto$ Determine the Absolute Stability

Test $eq^{\underline{n}}$:- $y' = \lambda y$

$f(x, y) = \lambda y \Rightarrow f(t, u) = \lambda u$

Now,

$k_1 = h\lambda(u_n + k_1/3)$

$k_1\left(1 - \frac{\lambda h}{3}\right) = \lambda h u_n$

$$k_1 = \left(\frac{\lambda h}{1 - \frac{\lambda h}{3}}\right) u_n \longrightarrow (4)$$

Also,

$k_2 = (u_n + k_1)h\lambda$

$k_2 = u_n\left(h\lambda + \frac{(h\lambda)^2}{1 - \frac{\lambda h}{3}}\right)$

$$k_2 = u_n\left(\frac{h\lambda + \frac{2}{3}h^2\lambda^2}{1 - \frac{h\lambda}{3}}\right) \longrightarrow (5)$$

Put (4) & (5) in (1)

$$u_{n+1} = u_n + \frac{1}{4}\left(\frac{3\lambda h}{1 - \frac{\lambda h}{3}}\right)u_n + \frac{1}{4}\left(\frac{\lambda h + 2/3(\lambda h)^2}{1 - \frac{\lambda h}{3}}\right)u_n$$

$$= u_n\left(1 + \frac{\frac{3}{4}\lambda h + \frac{\lambda h}{4} + \frac{\lambda^2 h^2}{6}}{1 - \frac{\lambda h}{3}}\right)$$

$$E(\lambda h) = \frac{1 - \frac{\lambda h}{3} + \lambda h + \frac{\lambda^2 h^2}{6}}{1 - \frac{\lambda h}{3}} = \left(\frac{1 + \frac{2\lambda h}{3} + \frac{\lambda^2 h^2}{6}}{1 - \frac{\lambda h}{3}}\right)$$

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for absolute stability $|E(\lambda h)| \leq 1$

$$\left(\frac{1+\frac{2}{3}\lambda h+\frac{\lambda^2h^2}{6}}{1-\frac{\lambda h}{3}}\right)^2 - 1^2 \leq 0$$

$$\left(\frac{1+\frac{2}{3}\lambda h+\frac{\lambda^2h^2}{6}}{1-\frac{\lambda h}{3}} - 1\right)\left(\frac{1+\frac{2}{3}\lambda h+\frac{\lambda^2h^2}{6}}{1-\frac{\lambda h}{3}} + 1\right) \leq 0$$

$$\lambda h\left(1+\frac{\lambda h}{6}\right)\left(\underbrace{12+2\lambda h+\lambda^2h^2}\right) \leq 0$$

$D = 4-48 < 0$ → always +ve
$a = 1 > 0$

$$\lambda h\left(1+\frac{\lambda h}{6}\right) \leq 0$$

$\boxed{\lambda h \in [-6, 0]}$ → region of absolute stability

Now

$y' = t^2 + y^2$, $y(1.0) = 2$, $h = 0.1$
$y(1.1) = ?$

$y_{n+1} = y_n + \frac{1}{4}(3k_1 + k_2)$
$k_1 = hf(t_n + h/3, y_n + k_1/3)$
$k_2 = hf(t_n + h, y_n + k_1)$

Iteration 1:- $n=0$, $t_0 = 1$, $y_0 = 2$

$y_1 = y_0 + \frac{1}{4}(3k_1 + k_2)$

$k_1 = 0.1 f(t_0 + h/3, y_0 + k_1/3)$

$k_1 = 0.1 f(1 + 0.1/3, 2 + k_1/3)$

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$$k_1 = (0.1)\left(\left(\frac{1+0.1}{3}\right)^2 + \left(\frac{2+k_1}{3}\right)^2\right)$$

$$10k_1 = 1.06\bar{7} + \frac{4 + k_1^2}{9} + \frac{4k_1}{3}$$

$$\frac{k_1^2}{9} - \frac{26k_1}{3} + 5.06\bar{7} = 0$$

Using N-R method.

$$g(k_1) = \frac{k_1^2}{9} - \frac{26}{3}k_1 + 5.06\bar{7}$$

$$g'(k_1) = \frac{2}{9}k_1 - \frac{26}{3}, \quad \text{Let } (k_1)_0 = 0$$

$$\cancel{(k_1)_{n+1} = (k_1)_n - \frac{g((k_1)_n)}{g'(k_1)}} \qquad (k_1)_{n+1} = (k_1)_n - \frac{g((k_1))_n}{g'((k_1)_n)}$$

$$(k_1)_1 = (k_1)_0 - \frac{g(k_1)_0}{g'(k_1)_0} = 0 - \frac{5.06\bar{7}}{-26/3} = 0.65512819$$

$$(k_1)_2 = (k_1)_1 - \frac{g(k_1)_1}{g'(k_1)_1} = 0.65512819 - \frac{(-0.5623118416)}{-8.521028}$$

$$(k_1)_2 = 0.58913763$$

$$(k_1)_3 = 0.58913763 - \frac{g(k_1)_2}{g'(k_1)_2} = 0.589194227$$

$$\left|(k_1)_3 - (k_1)_2\right| \le 10^{-5}$$

$$\boxed{k_1 = 0.589194227}$$

Now,

$$k_2 = 0.1 f(1+0.1,\ 2+0.589194227) = 0.1(1.1^2 + 2.589194227^2)$$

$$\boxed{k_2 = 0.79139267}$$

$$y_1 = y_0 + \frac{1}{4}(3k_1 + k_2) = 2 + \frac{1}{4}(3\times 0.589194227 + 0.79139267)$$

$$\boxed{y(1.1) \simeq y_1 = 2.639743839}$$

3) Given $y' = x+y$ initial condition:

$f(x,y) = x+y$ $\quad x(0) = 0$, $h = 0.1$

$y(0) = 1$

Classical R-K method

$y_{n+1} = y_n + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = hf(x_n, y_n)$

$k_2 = hf(x_n + h/2,\ y_n + k_1/2)$

$k_3 = hf(x_n + h/2,\ y_n + k_2/2)$

$k_4 = hf(x_n + h,\ y_n + k_3)$

Iteration 1, $n=0$

$y_1 = y_0 + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = 0.1\,(x_0 + y_0) = 0.1\,(0+1) = 0.1$

$k_2 = 0.1\,(0 + 0.1/2,\ 1 + 0.1/2) = 0.11$

$k_3 = 0.1\,(0 + 0.1/2,\ 1 + 0.11/2) = 0.1105$

$k_4 = 0.1\,(0 + 0.1,\ 1 + k_3) = 0.12105$

$y_1 = y_0 + 1/6\,(0.1 + 2\times 0.11 + 2\times 0.1105 + 0.12105)$

$y_1 = 1.110341667$, $x_1 = 0.1$

Iteration 2, $n=1$

$y_2 = y_1 + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = 0.1\,(x_1 + y_1) = 0.121034167$

$k_2 = 0.1\left(0.1 + 0.1/2 + 1.110341667 + \frac{0.121034167}{2}\right)$ ~~0.132085875~~

$k_2 = 0.132085875$

~~$k_3 = 0.132638460417$~~

$k_3 = 0.1\left(0.1 + 0.1/2 + 1.110341667 + \frac{0.132085875}{2}\right)$

$k_3 = 0.132638460417$

$k_4 = 0.1\,(0.1 + 0.1 + 1.110341667 + 0.1326384604\cancel{17})$

$k_4 = 0.14429801271$

$y_2 = y_1 + \frac{1}{6}\big(0.121034167 + 2(0.132085875) + 0.14429801272$
$+ 2(0.132638460417)\big)$

$y_2 = 1.24280514176$, $x_2 = 0.2$

Iteration 3, $n = 2$

$$y_3 = y_2 + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = 0.1\,(x_2 + y_2) = 0.144280514176$$

$$k_2 = 0.1\left(0.2 + \frac{0.1}{2} + 1.24280514176 + \frac{0.144280514176}{2}\right)$$

$$k_2 = 0.156494539885$$

$$k_3 = 0.1\left(0.2 + 0.1/2 + 1.24280514176 + \frac{0.156494539885}{2}\right)$$

$$k_3 = 0.1571052 4117$$

$$k_4 = 0.1\,(0.2 + 0.1 + 1.24280514176 + 0.157105 24117)$$

$$k_4 = 0.169991038293$$

$$y_3 = y_2 + 1/6\,(0.144280514176 + 2(0.156945398885) + 0.169991088293 + 2(0.157105524117))$$

$y_3 = 1.39971699419$, $x_3 = 0.3$

	x_i	y_i	$f_i = x_i + y_i$
$i=0$	0	1	1
$i=1$	0.1	1.110341667	1.210341667
$i=2$	0.2	1.24280574176	1.44280514176
$i=3$	0.3	1.39971699419	1.69971699419

Predictor $\quad y_4^{(P)} = y_0 + \frac{4h}{3}(2f_1 - f_2 + 2f_3)$

$$y_4^{(P)} = 1 + \frac{4(0.1)}{3}\left(2(1.210341667) - 1.44280514176 + 2(1.69971699419)\right)$$

$$y_4^{(P)} = 1.58364162408$$

Corrector $\quad y_4^{(c)} = y_2 + \frac{h}{3}(f_2 + 4f_3 + f_4)$

$$f_4 = x_4 + y_4^{(P)} = 0.4 + 1.58364162408$$
$$= 1.98364162408$$

$$y_4^{(c)} = 1.24280514176 + 0.1/3\,(1.44280514176 + 4(1.699716994) + 1.98364162408)$$

$$\boxed{y(0.4) \cong y_4^{(c)} = 1.58864896651} \longrightarrow \text{Ans}$$

4) $y' = x^3 - y^2 - 2$, $x = 0, y = 1$

Taylor series

$y(x+h) = y(x) + h\, y'(x) + \frac{h^2}{2!} y''(x) + \frac{h^3}{3!} y'''(x)$

$y'(x) = x^3 - y^2 - 2$
$y''(x) = 3x^2 - 2yy'$
$y'''(x) = 6x - 2(y'^2 + yy'')$

$y_0' = -3$, $y_0'' = 6$, $y_0''' = -30$

$y_1 = 1 + 0.1(-3) + \frac{(0.1)^2}{2}(6) + \frac{(0.1)^3(-30)}{6} = 0.725$ at $x_1 = 0.1$

$y_1' = -2.524625$
$y_1'' = 3.69070625$
$y_1''' = -17.4989868$

$y_2 = 0.725 + (0.1)\, y_1' + \frac{(0.1)^2}{2} y_1'' + \frac{(0.1)^3}{6} y_1''' = 0.48807453$ at $x_2 = 0.2$

$y_{-1} = y_0 + h y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0'''$ (where $h = -0.1$)

$y_{-1} = 1.335$ at $x_{-1} = -0.1$

i	x_i	y_i	f_i
-1	-0.1	1.335	-3.783225
0	0	1	-3
1	0.1	0.725	-2.524625
2	0.2	0.48807453	-2.23021675

Predictor:- $y_3^{(P)} = y_{-1} + \frac{4(0.1)}{3}(2f_0 - f_1 + 2f_2) = 0.2768922$ at $x = 0.3$
$f_3^{(P)} = -2.04871373$

Corrector:- $y_3^{(C)} = y_1 + h/3\,(f_1 + 4f_2 + f_3)$
$y_3^{(C)(1)} = 0.27686929$, $y_3^{(C)(2)} = 0.27516129$

$\boxed{y(0.3) \simeq y_3 = 0.27519314212}$

Predictor:- $y_4^{(P)} = y_0 + 4h/3\,(2f_1 - f_2 + 2f_3) = 0.07780056\,26$

at $x = 0.4$

Corrector:- $y_4^{(C)} = y_1 + h/3\,(f_1 + 4f_2 + f_3)$

$y_4^{(C)(1)} = 0.07583471$; $y_4^{(C)(2)} = 0.0758444776$

$$\boxed{y(0.4) \simeq y_4 = 0.075844776}$$

Predictor:- $y_5^{(P)} = y_1 + 4h/3\,(2f_2 - f_3 + 2f_4) = -0.114360946$

at $x = 0.5$

Corrector:- $y_5^{(C)} = y_2 + h/2\,(f_2 + 4f_3 + f_4)$

$y_5^{(C)(1)} = -0.114934172$; $y_5^{(C)(2)} = -0.114938553$

$$\boxed{y(0.5) \simeq y_5 = -0.114938553}$$

Predictor:- $y_6^{(P)} = y_2 + 4h/3\,(2f_3 - f_4 + 2f_5) = -0.3028763789$

at $x = 0.6$

Corrector:- $y_6^{(C)} = y_3 + h/2\,(f_3 + 4f_4 + f_5)$

$y_6^{(C)(1)} = -0.303166224$; $y_6^{(C)(2)} = -0.30317207951$

$$\boxed{y(0.6) \simeq y_6 = -0.30317207951}$$

5) $\frac{dy}{dx} = y+x, \quad y(0)=1, \quad h=0.1$ 19MA20059

Milne Simpson method of order 4

$y_{(i+1)} = y_{i-1} + \frac{h}{3}(f_{i+1} + 4f_i + f_{i-1})$

~~Iteration 1, i=0~~ Fourth order taylor series

~~$y_0 = y_0$~~ $y_1 = y_0 + hy_0' + \frac{h^2}{2}y_0'' + \frac{h^3}{6}y_0''' + \frac{h^4}{24}y_0^{iv}$

$= 1 + 0.1 + 0.01 + 0.0003333 + 0.0000083$

$y(0.1) \simeq y_1 = 1.1103416$

now

$y_{i+2} = y_i + \frac{h}{3}(f_{i+2} + 4f_{i+1} + f_i)$

For the given eqn

$\frac{29}{30} y_{i+2} = y_i + \frac{1}{30}(x_{i+2} + 4x_{i+1} + 4y_{i+1} + y_i)$

Iteration 1, $i=0$

$\frac{29}{30} y_2 = y_0 + \frac{1}{30}(x_2 + 4x_1 + x_0 + 4y_1 + y_0)$

$= 1 + \frac{1}{30}(0.6 + 4(1.1103416) + 1) = 1.2013789$

$\boxed{y(0.2) \simeq y_2 = 1.2428057}$

Iteration 2, $i=1$

$\frac{29}{30} y_3 = y_1 + \frac{1}{30}(x_3 + 4x_2 + x_1 + 4y_2 + y_1)$

$= 1.1103416 + \frac{1}{30}(1.2 + 4(1.2428057) + 1.1103416)$

$= 1.3530604$

$\boxed{y(0.3) \simeq y_3 = 1.399717655}$

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6) $y' = 2 - xy^2$, $y(0) = 10$, $h = 0.2$

Classical R-K Method.

$y_{n+1} = y_n + 1/6 \, (k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = h f(x_n, y_n)$

$k_2 = h f(x_n + h/2, \; y_n + k_1/2)$

$k_3 = h f(x_n + h/2, \; y_n + k_2/2)$

$k_4 = h f(x_n + h, \; y_n + k_3)$

Iteration 1 :- $n = 0$

$y_1 = y_0 + 1/6 \, (k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = (0.2)(2 - 0) = 0.4$

$k_2 = (0.2)(2 - (0.1)(10 + 0.2)^2) = -1.6808$

$k_3 = (0.2)\left(2 - (0.1)\left(10 - \frac{1.6808}{2}\right)^2\right) = -1.27796544432$

$k_4 = (0.2)(2 - (0.2)(10 - 1.27796544432)) = -2.64295454724$

$y_1 = y_0 + 1/6 \, (k_1 + 2k_2 + 2k_3 + k_4) = 8.6399189402$ at $x_1 = 0.2$

Iteration 2 :- $n = 1$

$k_1 = 0.2\,(2 - 0.2(y_1)^2) = -2.58592797178$

$k_2 = 0.2\,(2 - 0.3(y_1 + k_1/2)^2) = -2.8386648826$

$k_3 = 0.2\,(2 - 0.3(y_1 + k_2/2)^2) = -2.7282121873$

$k_4 = 0.2\,(2 - 0.4(y_1 + k_3)^2) = -2.3958621318$

$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 5.95399491732$ at $x_2 = 0.4$

Iteration 3 :- $n = 2$

$k_1 = 0.2\,(2 - 0.4(y_2)^2) = -2.43600448804$

$k_2 = 0.2\,(2 - 0.5(y_2 + k_1/2)^2) = -1.842862268.3$

$k_3 = 0.2\,(2 - 0.5(y_2 + k_2/2)^2) = -2.13261928$

$k_4 = 0.2\,(2 - 0.6(y_2 + k_3)^2) = -1.3523494$

$y_3 = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 3.8673636387$ at $x_3 = 0.6$

Iteration 4 , $n=3$

$k_1 = 0.2(2 - 0.6(y_3)^2) = -1.394780181 67$

$k_2 = 0.2(2 - 0.7(y_3 + k_1/2)^2) = -1.00682252$

$k_3 = 0.2(2 - 0.7(y_3 + k_2/2)^2) = -1842645$

$k_4 = 0.2(2 - 0.8(y_3 + k_3)^2) = -0.75184328$

$y_4 = y_3 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 2.77923069231$ at $x_4 = 0.8$

	x_i	~~y_i~~	$f_i = 2 - \frac{x_i y_i^2}{2}$	y_i
$i=0$	0	~~1.0~~		1.0
$i=1$	0.2	~~8.639918~~	-12.9296398586	8.6399189402
$i=2$	0.4		-12.1800221902	5.9539949 1732
$i=3$	0.6		-6.973900 90836	3.8673636387
$i=4$	0.8		-4.17929859286	2.7792306923

Predictor :- $y_5^{(P)} = y_1 + \frac{4h}{3}(2f_2 - f_3 + 2f_4)$

$= 8.6399189402 + 0.8/3\,(2\times(-12.1800221902) - (-6.973900 90836) + 2(-4.17929859286))$

$y_5^{(P)} = 1.7746547648$ $\quad f_5 = -1.14939953423$

Corrector :- $y_5^{(C)} = y_3 + h/3\,(f_3 + 4f_4 + f_5)$

$= 3.8673636387 + 0.2/3\,(-6.97390090836 + 4(-4.17929859286) - 1.149399 53423)$

$y_5^{(0)(C)} = 2.21133064608$, $f_5^{(0)} = -2.88998322629$

$y_5^{(1)(C)} = 2.09529 1733$, $f_5^{(1)} = -2.390247$

$y_5^{(2)(C)} = 2.128607451$, $f_5^{(2)} = -2.530969$

$y_5^{(3)(C)} = 2.1192259694 2$

$\boxed{y(1) \simeq y_5 = 2.11922596942}$

7) $y' = -y$, $y(0) = 1$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

Classical RK method

$y_{n+1} = y_n + 1/6 \, (k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = h \, f(x_n, y_n)$

$k_2 = h \, f(x_n + h/2, \; y_n + k_1/2)$

$k_3 = h \, f(x_n + h/2, \; y_n + k_2/2)$

$k_4 = h \, f(x_n + h, \; y_n + k_3)$

Iteration 1 , $n = 0$,

$k_1 = 0.1 \, (-1) = -0.1$

$k_2 = 0.1 \, (-(1 - \frac{0.1}{2})) = -0.095$

$k_3 = 0.1 \, (-(1 - 0.095/2)) = -0.09525$

$k_4 = 0.1 \, (-(1 - 0.09525)) = -0.090475$

$y_1 = y_0 + 1/6 \, (k_1 + 2k_2 + 2k_3 + k_4) = 0.9048375$ at $x_1 = 0.1$

Iteration 2 , $n = 1$

$k_1 = 0.1 \, (-y_1) = -0.09048375$

$k_2 = 0.1 \, (-(y_1 + k_1/2)) = -0.0859595$

$k_3 = 0.1 \, (-(y_1 + k_2/2)) = -0.08618577$

$k_4 = 0.1 \, (-(y_1 + k_3)) = -0.081865172$

$y_2 = y_1 + 1/6 \, (k_1 + 2k_2 + 2k_3 + k_4) = 0.818730923$, $x_2 = 0.2$

Iteration 3 , $n = 2$

$k_1 = 0.1 \, (-y_2) = -0.0818730923$

$k_2 = 0.1 \, (-(y_2 + k_1/2)) = -0.07777943$

$k_3 = 0.1 \, (-(y_2 + k_2/2)) = -0.07798412$

$k_4 = 0.1 \, (-(y_2 + k_3)) = -0.0740747468$

$y_3 = y_2 + 1/6 \, (k_1 + 2k_2 + 2k_3 + k_4) = 0.74081844154$, $x_3 = 0.3$

Iteration 4 , $n = 3$

$k_1 = 0.1 \, (-y_3) = -0.074081844 15$

$k_2 = 0.1 \, (-(y_3 + k_1/2)) = -0.0703777$

$k_3 = 0.1 \, (-(y_3 + k_2/2)) = -0.07056295$

$k_4 = 0.1 \, (-(y_3 + k_3)) = -0.0670255$

$y_4 = y_3 + 1/6 (k_1 + 2k_2 + 2k_3 + k_4) = 0.670320306597$ at $x_4 = 0.4$

i	x_i	y_i	f_i
0	0	1	-1
1	0.1	0.9048375	-0.9048375
2	0.2	0.818730923	-0.818730923
3	0.3	0.74081844	-0.74081844
4	0.4	0.6703203	-0.6703203

Predictor:- $y_5^{(P)} = y_1 + 4h/3 (2f_2 - f_3 + 2f_4)$

$y_5^{(P)} = 0.606532965867$, $f_5^{(P)} = -0.606532965867$

Corrector:- $y_5^{(C)} = y_3 + h/3 (f_3 + 4f_4 + f_5^{(P)})$

$$\boxed{y(0.5) \simeq y_5 = 0.606530686471}$$

Predictor:- $y_6^{(P)} = y_2 + 4h/3 (2f_3 - f_4 + 2f_5)$

$y_6^{(P)} = 0.548813862608$, $f_6^{(P)} = -0.548813862608$

Corrector:- $y_6^{(C)} = y_4 + h/3 (f_4 + 4f_5 + f_6^{(P)})$

$$\boxed{y(0.6) \simeq y_6 = 0.548811736384}$$

Predictor:- $y_7^{(P)} = y_3 + 4h/3 (2f_4 - f_5 + 2f_6)$

$y_7^{(P)} = 0.496587321827$, $f_7^{(P)} = -0.496587321827$

Corrector:- $y_7^{(C)} = y_5 + h/3 (f_5 + 4f_6 + f_7^{(P)})$

$$\boxed{y(0.7) \simeq y_7 = 0.496585188801}$$

Predictor:- $y_8^{(P)} = y_4 + 4h/3 (2f_5 - f_6 + 2f_7)$

$y_8^{(P)} = 0.449330996499$, $f_8^{(P)} = -0.449330996499$

Corrector:- $y_8^{(C)} = y_6 + h/3 (f_6 + 4f_7 + f_8^{(P)})$

$$\boxed{y(0.8) \simeq y_8 = 0.449328954604}$$

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8) $\frac{dy}{dx} = x - y^2$ $y(0) = 1$, $y(0.1) = 0.9117$, $y(0.2) = 0.8494$, $y(0.3) = 0.8061$

$f(x,y) = x - y^2$ $y(0.4) = ?$, $h = 0.1$

Adams-Moulton Method of order 5

$$y_{n+4} = y_{n+3} + h\left(\frac{251}{720} f_{n+4} + \frac{646}{720} f_{n+3} - \frac{264}{720} f_{n+2} + \frac{106}{720} f_{n+1} - \frac{19}{720} f_n\right)$$

Iteration 1, $n = 0$

$f_4 = f(0.4, y_4) = 0.4 - y_4^2$

$f_3 = f(0.3, 0.8061) = 0.3 - (0.8061)^2 = -0.3497972$

$f_2 = f(0.2, 0.8494) = -0.5214803$

$f_1 = f(0.1, 0.9117) = -0.7311962$

$f_0 = f(0,1) = -1$

$$y_4 = y_3 + 0.1\left(\frac{251}{720} f_4 + \frac{646}{720} f_3 - \frac{264}{720} f_2 + \frac{106}{720} f_1 - \frac{19}{720} f_0\right)$$

$$y_4 + (0.1)\frac{251}{720} y_4^2 = 0.8061 + \frac{0.1}{720}(-46.4050465) = 0.799654854$$

$$251 y_4^2 + 720 y_4 - 575.75149535 = 0$$

Using NR Method

$g(y_4) = 25.1 y_4^2 + 720 y_4 - 575.75149535$

$g'(y_4) = 50.2 y_4 + 720$

$(y_4)_{n+1} = (y_4)_n - \frac{g(y_4)_n}{g'(y_4)_n}$ Let $(y_4)_0 = 0$

$(y_4)_1 = (y_4)_0 - \frac{g(y_4)_0}{g'(y_4)_0} = 0.799654855$

$(y_4)_2 = (y_4)_1 - \frac{g(y_4)_1}{g'(y_4)_1} = 0.778540106$

$(y_4)_3 = (y_4)_2 - g(y_4)_2 / g'(y_4)_2 = 0.778525468$

since $|(y_4)_3 - (y_4)_2| \le 10^{-5}$

$$\boxed{y(0.4) = y_4 = 0.778525468}$$

9) ~~Given~~ $\frac{dy}{dx} = \frac{xy}{2}$ $y(0) = 1$, $y(0.1) = 1.0025$

$y(0.2) = 1.0101$

$f(x,y) = \frac{xy}{2}$ $y(0.3) = 1.0228$, $h = 0.1$

Adams - Bashforth of order 4

$$y_{n+1} = y_n + \frac{h}{24}(55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3})$$

Iteration 1 :- $n=3$

$y_4 = ?$

$f_0 = f(0,1) = 0$

$f_1 = f(0.1, 1.0025) = 0.05012$

$f_2 = f(0.2, 1.0101) = 0.10101$

$f_3 = f(0.3, 1.0228) = 0.15342$

$$y_4 = y_3 + \frac{h}{24}(55f_3 - 59f_2 + 37f_1 - 9f_0)$$

$$\boxed{y(0.4) \simeq y_4 = 1.040854729}$$

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10) $\alpha = ?$ for consistancy of multistep method

$$u_{n+1} = u_n + h/2\,(5 u'_n + \alpha u'_{n-1})$$

$$u_{n+1} - u_n = h\left(\frac{5}{2} u'_n + \frac{\alpha}{2} u'_{n-1}\right)$$

$\alpha_1 = 1, \quad \alpha_0 = -1, \quad \alpha_{-1} = 0$ | $\beta_0 = 5/2, \quad \beta_{-1} = \frac{\alpha}{2}$, $\beta_1 = 0$

$$C_0 = \Sigma \alpha_j = 1 + -1 + 0 = 0$$

$$C_1 = \Sigma (j\alpha_j - \beta_j) = (1.1 + 0 + 0) - \left(\frac{5}{2} + \frac{\alpha}{2} + 0\right) = 0$$

$$1 - \frac{5}{2} - \frac{\alpha}{2} = 0$$

$$\boxed{\alpha = -3}$$

We can say that numerical method is consistant with diff. eq^n if $\tau_j \to 0$ as $h \to 0$

$$\tau_j = \frac{1}{\beta h} \sum_{p=0}^{\infty} c_p h^p y^p(jh)$$

$$c_p = \Sigma \left(\frac{j^p \alpha_j}{p!} - \frac{j^{p-1}}{(p-1)!} \beta_j\right)$$

We have

$C_0 = 0$

hence $C_1 = 0$ has to be true for consistancy

Thus $\boxed{\alpha = -3}$