NSDE Programming Assignment - 3

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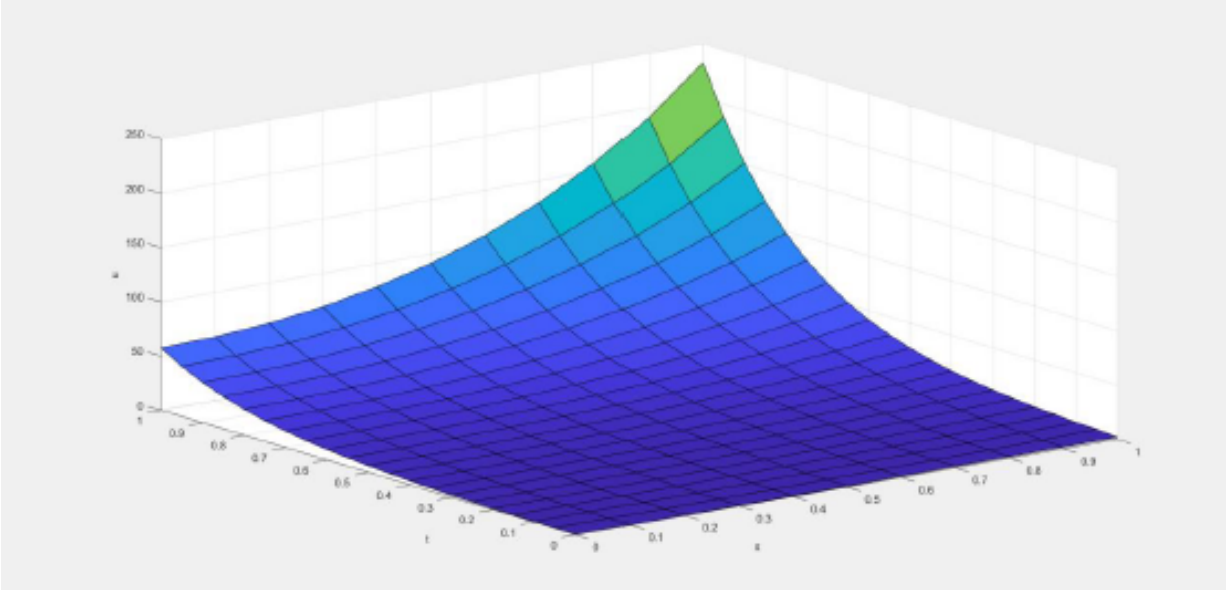
Question 1:

```
h=0.1;
k=0.05;
lambda = k / h^2;
x_init = 0;
x_final = 1;
t_init = 0;
t_final = 1;
u_t_0 = @(x) 2*x;
ux_x_0 = @(t) 0;
ux_x_n = @(t) 1;
x_itr = (x_final - x_init) / h;
t_itr = (t_final - t_init) / k;
Values = zeros(x_itr + 1, t_itr + 1);

for i=1:x_itr + 1
   Values(i, 1) = u_t_0(x_init + h * (i-1));
end
A = zeros(x_itr + 1, x_itr + 1);
B = zeros(x_itr + 1, 1);

syms u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1 eq
eq(u_m1_n2, u_m2_n2, u_m3_n2, u_m1_n1, u_m2_n1, u_m3_n1) = -1 * lambda * u_m1_n2
+ (2 + 2 * lambda) * u_m2_n2 - (lambda) * u_m3_n2 - 1 * lambda * u_m1_n1 - (2 - 2 *
lambda) * u_m2_n1 - lambda * u_m3_n1;

for j=1:t_itr
   for i=1:x_itr + 1
      if i==1
         temp_eqs = subs(eq, {u_m1_n2 u_m1_n1}, {u_m3_n2 u_m3_n1});
         temp_val = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1},
{0 0 0 0 0 0});

         A(1, 1) = subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {1 0 0 0}) -
temp_val;
         A(1, 2) = subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {0 1 0 0}) -
temp_val;
```

```
%            B(1, 1) = subs(temp_rhs, {u_m2_n1 u_m3_n1}, {Values(1, j) Values(2, j)});
         B(1, 1) = -1 * (subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {0 0
Values(1, j) Values(2, j)}) - temp_val);

      elseif i == x_itr + 1
         temp_eqs = subs(eq, {u_m3_n2 u_m3_n1}, {0.2 + u_m1_n2 0.2 + u_m1_n1});
         temp_val = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1},
{0 0 0 0 0 0});

         A(x_itr + 1, x_itr) = subs(temp_eqs, {u_m1_n2 u_m2_n2 u_m1_n1 u_m2_n1}, {1 0 0
0}) - temp_val;
         A(x_itr + 1, x_itr + 1) = subs(temp_eqs, {u_m1_n2 u_m2_n2 u_m1_n1 u_m2_n1}, {0
1 0 0}) - temp_val;
         B(x_itr + 1, 1) = -1 * (subs(temp_eqs, {u_m1_n1 u_m2_n1 u_m1_n2 u_m2_n2},
{Values(x_itr, j) Values(x_itr + 1, j) 0 0}) - temp_val);
      else
         temp_val = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1},
{0 0 0 0 0 0});
         A(i, i - 1) = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1}, {1
0 0 0 0 0}) - temp_val;
         A(i, i) = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1}, {0 1
0 0 0 0}) - temp_val;
         A(i, i + 1) = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1},
{0 0 1 0 0 0}) - temp_val;
%            B(i, 1) = subs(eq_rhs, {u_m1_n1 u_m2_n1 u_m3_n1}, {Values(i-1, j) Values(i, j)
Values(i + 1, j)});

         B(i, 1) = -1 * (subs(eq, {u_m1_n1 u_m2_n1 u_m3_n1 u_m1_n2 u_m2_n2 u_m3_n2},
{Values(i-1, j) Values(i, j) Values(i + 1, j) 0 0 0}) - temp_val);
      end
   end

   Values(:, j+1) = linsolve(A, B);
   A = zeros(x_itr + 1, x_itr + 1);
   B = zeros(x_itr + 1, 1);
end


X = 0:0.1:1
T = 0:0.05:1
surf(X.', T.', Values.')
xlabel('x');
ylabel('t');
zlabel('u');
```

t \ x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.05	0.609275	0.53113	0.585437	0.713919	0.887968	1.097205	1.345324	1.651573	2.058451	2.648709	3.578451
0.1	0.704092	0.801201	0.873885	0.987775	1.165639	1.419383	1.763117	2.215837	2.793635	3.482125	4.164499
0.15	1.07433	1.051269	1.152659	1.324354	1.566706	1.893605	2.320204	2.86065	3.533944	4.39267	5.62173
0.2	1.359187	1.43922	1.549981	1.749367	2.0481	2.454845	2.986386	3.670552	4.54583	5.640433	6.863805
0.25	1.848288	1.866076	2.023877	2.282612	2.655298	3.162851	3.828811	4.679115	5.741733	7.063387	8.825453
0.3	2.362102	2.447077	2.624439	2.941091	3.407185	4.039165	4.863265	5.916802	7.253101	8.93491	10.92492
0.35	3.073033	3.130244	3.368336	3.764695	4.341055	5.125076	6.151343	7.463403	9.111342	11.15707	13.78346
0.4	3.902421	4.011087	4.291211	4.782216	5.500229	6.47425	7.745695	9.369088	11.41888	13.99389	17.12297
0.45	4.972944	5.078382	5.439282	6.048528	6.93729	8.14487	9.723273	11.74076	14.28052	17.43971	21.42317
0.5	6.269561	6.423447	6.859897	7.616246	8.720165	10.21842	12.17527	14.67383	17.82335	21.77056	26.62585
0.55	7.902193	8.074834	8.625466	9.562425	10.93029	12.78786	15.21462	18.31541	22.22187	27.09004	33.17793
0.6	9.907757	10.13623	10.81127	11.9743	13.67118	15.9747	18.98378	22.82649	27.66796	33.72616	41.21909
0.65	12.40954	12.68142	13.52482	14.96564	17.06915	19.9252	23.6559	28.42182	34.42711	41.91932	51.25408
0.7	15.49978	15.84594	16.88641	18.67408	21.28244	24.82357	29.4494	35.35741	42.80029	52.10552	63.64268
0.75	19.34055	19.76254	21.05686	23.27216	26.50601	30.89651	36.63149	43.95713	53.18778	64.71108	79.04467
0.8	24.09519	24.62413	26.22537	28.97304	32.98252	38.42595	45.53667	54.61882	66.06061	80.3589	98.10602
0.85	29.99598	30.6472	32.63535	36.04105	41.01215	47.76115	56.57694	67.83741	82.02566	99.74345	121.7662
0.9	37.30735	38.1184	40.58101	44.80433	50.96756	59.33505	70.26532	84.22626	101.815	123.7898	151.0783
0.95	46.37583	47.37847	50.43359	55.66903	63.31043	73.68473	87.23611	104.5451	126.354	153.5922	187.4379
1	57.61623	58.86166	62.6479	69.13955	78.61343	91.47563	108.2771	129.7373	156.7746	190.5507	232.5027

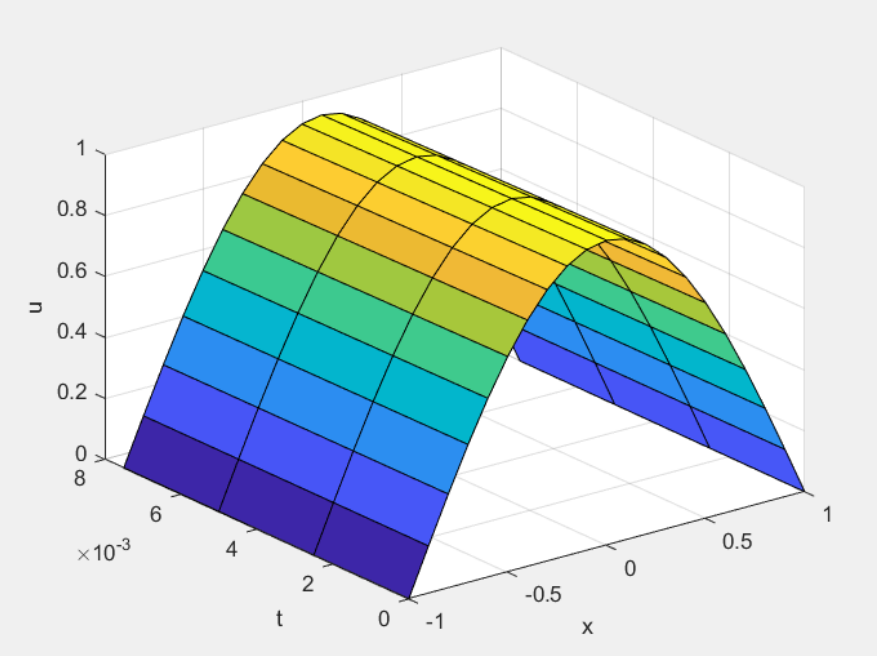


Question 2:

```
u_t_0 = @(x) cos(pi * x / 2);
ux_x_0 = @(t) 0;
ux_x_n = @(t) 0;
h=0.1;
lambda = 0.25;
k=lambda * h^2;
x_init = -1;
x_final = 1;
t_init = 0;
t_final = 0.0075;
x_itr = int16((x_final - x_init) / h);
t_itr = int16((t_final - t_init) / k);
Values = zeros(x_itr + 1, t_itr + 1);

for i=1:x_itr + 1
   Values(i, 1) = u_t_0(x_init + h * double(i-1));
end

for j=2:t_itr + 1
   Values(1, j) = 0;
   Values(x_itr + 1, j) = 0;
end

A = zeros(x_itr - 1, x_itr - 1);
B = zeros(x_itr - 1, 1);

syms u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1 eq
eq(u_m1_n2, u_m2_n2, u_m3_n2, u_m1_n1, u_m2_n1, u_m3_n1) = -1 * lambda * u_m1_n2
+ (2 + 2 * lambda) * u_m2_n2 - (lambda) * u_m3_n2 - 1 * lambda * u_m1_n1 - (2 - 2 *
lambda) * u_m2_n1 - lambda * u_m3_n1;

for j=1:t_itr
   for i=2:x_itr
      if i == 2
         temp_eqs = subs(eq, {u_m1_n2 u_m1_n1}, {Values(1, j+1) Values(1, j)});
         temp_val = subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {0 0 0 0});

         A(1, 1) = subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {1 0 0 0}) -
temp_val;
         A(1, 2) = subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {0 1 0 0}) -
temp_val;
%            B(1, 1) = subs(temp_rhs, {u_m2_n1 u_m3_n1}, {Values(1, j) Values(2, j)});
         B(1, 1) = -1 * (subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {0 0
Values(2, j) Values(3, j)}) - temp_val);
```

```
        elseif i == x_itr
            temp_eqs = subs(eq, {u_m3_n2 u_m3_n1}, {Values(x_itr + 1, j + 1) Values(x_itr + 1,
j)});
            temp_val = subs(temp_eqs, {u_m1_n2 u_m2_n2 u_m1_n1 u_m2_n1}, {0 0 0 0});

            A(x_itr - 1, x_itr - 2) = subs(temp_eqs, {u_m1_n2 u_m2_n2 u_m1_n1 u_m2_n1}, {1 0
0 0}) - temp_val;
            A(x_itr - 1, x_itr - 1) = subs(temp_eqs, {u_m1_n2 u_m2_n2 u_m1_n1 u_m2_n1}, {0 1
0 0}) - temp_val;
            B(x_itr - 1, 1) = -1 * (subs(temp_eqs, {u_m1_n1 u_m2_n1 u_m1_n2 u_m2_n2},
{Values(x_itr - 1, j) Values(x_itr, j) 0 0}) - temp_val);
        else
            temp_val = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1},
{0 0 0 0 0 0});

            A(i - 1, i - 2) = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1
u_m3_n1}, {1 0 0 0 0 0}) - temp_val;
            A(i - 1, i - 1) = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1
u_m3_n1}, {0 1 0 0 0 0}) - temp_val;
            A(i - 1, i) = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1}, {0
0 1 0 0 0}) - temp_val;
%             B(i, 1) = subs(eq_rhs, {u_m1_n1 u_m2_n1 u_m3_n1}, {Values(i-1, j) Values(i, j)
Values(i + 1, j)});

            B(i - 1, 1) = -1 * (subs(eq, {u_m1_n1 u_m2_n1 u_m3_n1 u_m1_n2 u_m2_n2
u_m3_n2}, {Values(i-1, j) Values(i, j) Values(i + 1, j) 0 0 0}) - temp_val);
        end
    end

    Values(2:x_itr, j+1) = linsolve(A, B);
    A = zeros(x_itr - 1, x_itr - 1);
    B = zeros(x_itr - 1, 1);
end



X = -1:0.1:1
T = 0:0.0025:0.0075
surf(X.', T.', Values.')
xlabel('x');
ylabel('t');
zlabel('u');
```

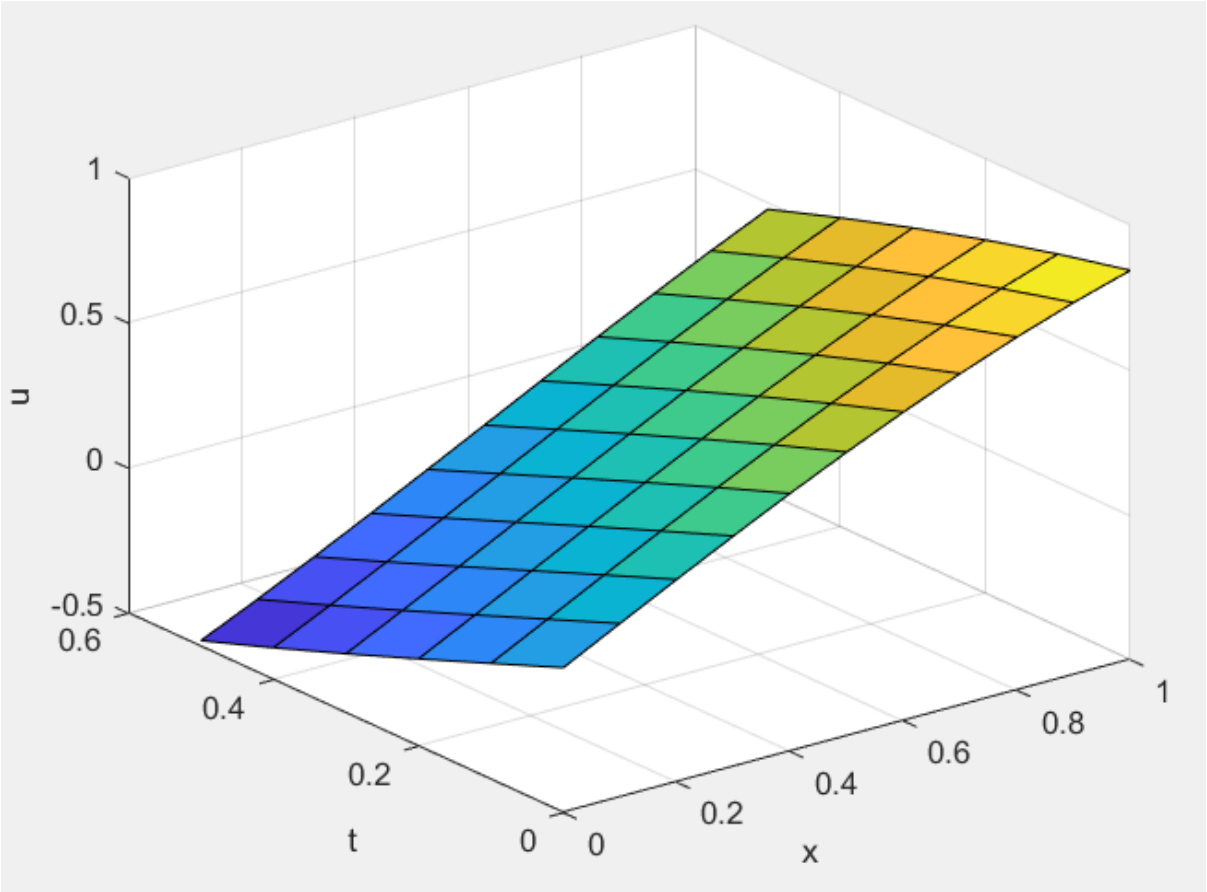
x \ t	0	0.0025	0.005	0.0075
-1	0	0	0	0
-0.9	0.156434	0.155474	0.15452	0.153572
-0.8	0.309017	0.307121	0.305236	0.303363
-0.7	0.45399	0.451204	0.448435	0.445683
-0.6	0.587785	0.584178	0.580593	0.57703
-0.5	0.707107	0.702767	0.698454	0.694168
-0.4	0.809017	0.804052	0.799118	0.794214
-0.3	0.891007	0.885538	0.880104	0.874703
-0.2	0.951057	0.94522	0.939419	0.933654
-0.1	0.987688	0.981627	0.975603	0.969616
0	1	0.993863	0.987764	0.981702
0.1	0.987688	0.981627	0.975603	0.969616
0.2	0.951057	0.94522	0.939419	0.933654
0.3	0.891007	0.885538	0.880104	0.874703
0.4	0.809017	0.804052	0.799118	0.794214
0.5	0.707107	0.702767	0.698454	0.694168
0.6	0.587785	0.584178	0.580593	0.57703
0.7	0.45399	0.451204	0.448435	0.445683
0.8	0.309017	0.307121	0.305236	0.303363
0.9	0.156434	0.155474	0.15452	0.153572
1	0	0	0	0



Question 3:

```
h=0.1;
k=0.1;
r = 1 * k/h;
x_init = 0;
x_final = 1;
t_init = 0;
t_final = 0.5;
f = @(x) sin(x);
g = @(x) -1 * cos(x);
u_x_0 = @(t) -1 * sin(t);
u_x_n = @(t) sin(1 - t);
x_itr = int16((x_final - x_init) / h);
t_itr = int16((t_final - t_init) / k);
Values = zeros(x_itr + 1, t_itr + 1);
for i=1:x_itr + 1
    Values(i, 1) = f((x_init + h * double(i-1)));
end
for j=1:t_itr + 1
    Values(1, j) = u_x_0((t_init + k * double(j-1)));
    Values(x_itr + 1, j) = u_x_n((t_init + k * double(j-1)));
end

for i=2:x_itr
    Values(i, 2) = 0.5 * (r^2 * f((x_init + double(i - 2) * h)) + 2 * (1 - r^2) * f((x_init + double(i - 1)
* h)) + r^2 * f((x_init + double(i) * h)) + 2 * k * g((x_init + double(i - 1) * h)));
end

for j=3:t_itr + 1
    for i=2:x_itr
       Values(i, j) = r^2 * Values(i-1, j-1) + 2 * (1 - r^2) * Values(i, j-1) + r^2 * Values(i + 1, j-1) -
Values(i, j-2);
    end
end

X = 0:0.1:1;
T = 0:0.1:0.5;
surf(X.', T.', Values.')
xlabel('x');
ylabel('t');
zlabel('u');
```

x \ t	0	0.1	0.2	0.3	0.4	0.5
0	0	-0.09983	-0.19867	-0.29552	-0.38942	-0.47943
0.1	0.099833	-0.00017	-0.1	-0.19883	-0.29567	-0.38956
0.2	0.198669	0.09967	-0.00032	-0.10015	-0.19897	-0.29581
0.3	0.29552	0.19851	0.099517	-0.00047	-0.10029	-0.1991
0.4	0.389418	0.295367	0.198364	0.099379	-0.0006	-0.1004
0.5	0.479426	0.389272	0.295229	0.198237	0.099263	-0.0007
0.6	0.564642	0.479288	0.389145	0.295113	0.198133	0.099263
0.7	0.644218	0.564515	0.479172	0.389041	0.295113	0.198237
0.8	0.717356	0.644102	0.564412	0.479172	0.389145	0.295229
0.9	0.783327	0.717253	0.644102	0.564515	0.479288	0.389272
1	0.841471	0.783327	0.717356	0.644218	0.564642	0.479426



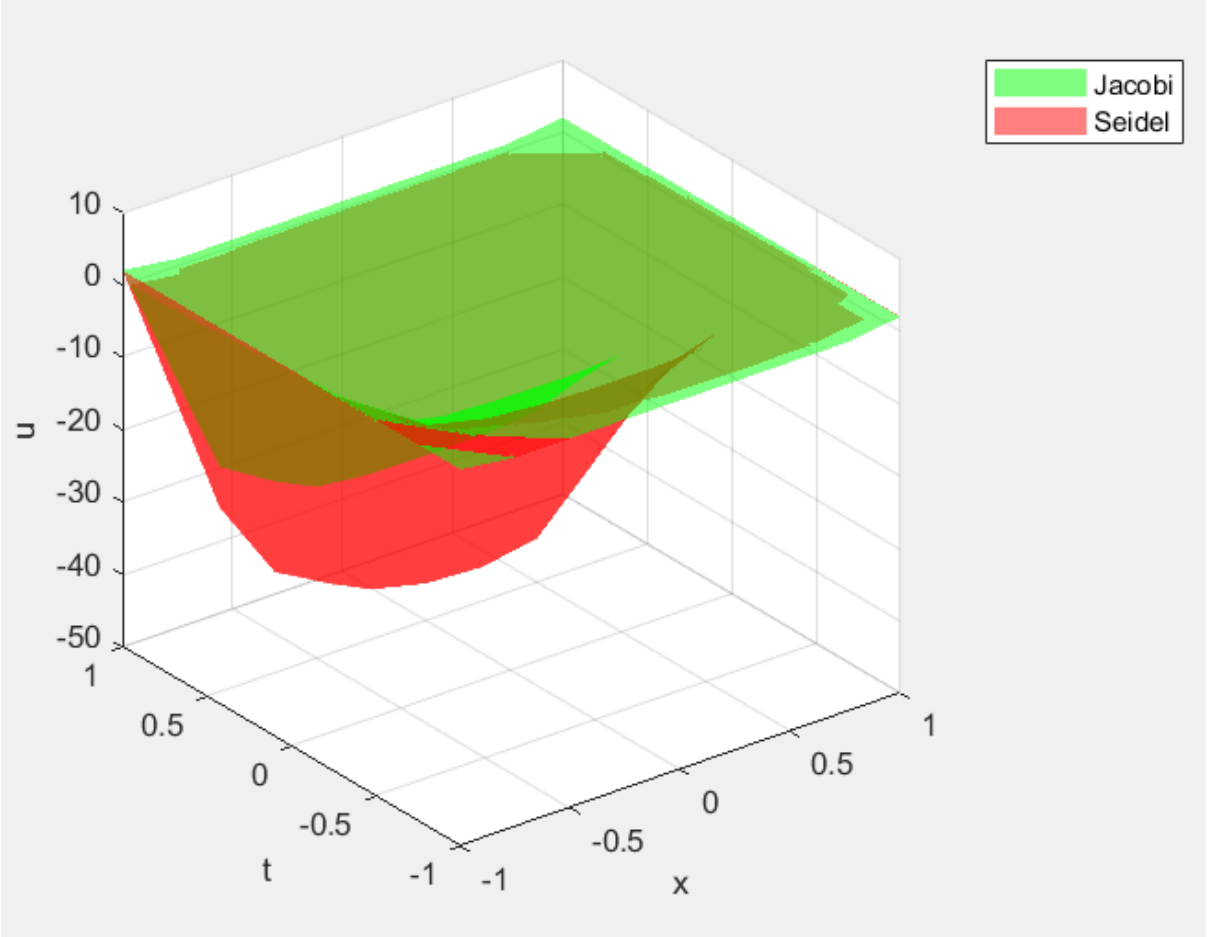
Question 4:

```
x_init = -1;
x_final = 1;
y_init = -1;
y_final = 1;
u_x_0 = 2;
u_x_n = 2;
u_y_0 = 1;
u_y_n = 1;
h=0.25;
k = 0.25;
iterations = 5;
f = @(x, y) x^2 + y^2;
x_itr = int16((x_final - x_init) / h);
y_itr = int16((y_final - y_init) / k);
Values = zeros(x_itr + 1, y_itr + 1);
for i=1:x_itr + 1
   Values(i, 1) = u_y_0;
   Values(i, y_itr + 1) = u_y_n;
end

for j=1:y_itr + 1
   Values(1, j) = u_x_0;
   Values(x_itr + 1, j) = u_x_n;
end

Values_jacobi = zeros(x_itr + 1, y_itr + 1);
Values_seidel = zeros(x_itr + 1, y_itr + 1);

Values_jacobi(:, :) = Values(:, :);
Values_seidel(:, :) = Values(:, :);

% Gauss jacobi
for k=1:iterations
   Values_temp = zeros(x_itr + 1, y_itr + 1);
   Values_temp(:, :) = Values_jacobi(:, :);
   for j=2:y_itr
      for i=2:x_itr
         Values_temp(i, j) = (0.25) * (Values_jacobi(i - 1, j) + Values_jacobi(i + 1, j) +
Values_jacobi(i, j-1) + Values_jacobi(i, j+1) - h^2 * f(x_init + h * double(i - 1), y_init + k *
double(j - 1)));
      end
   end
   Values_jacobi(:, :) = Values_temp(:, :);
end
```

```
% Gauss Seidel
for k=1:iterations
    for j=2:y_itr
        for i=2:x_itr
            Values_seidel(i, j) = (0.25) * (Values_seidel(i - 1, j) + Values_seidel(i + 1, j) +
Values_seidel(i, j-1) + Values_seidel(i, j+1) - h^2 * f(x_init + h * double(i - 1), y_init + k *
double(j - 1)));
        end
    end
end
X = -1:0.25:1;
Y = -1:0.25:1;
surf(X.', Y.', Values_jacobi.', 'FaceColor','g', 'FaceAlpha',0.5, 'EdgeColor','none');
hold on
surf(X.', Y.', Values_seidel.', 'FaceColor','r', 'FaceAlpha',0.5, 'EdgeColor','none');
xlabel('x');
ylabel('t');
zlabel('u');
legend('Jacobi', 'Seidel');
```

Gauss Jacobi Iterations									
x\y	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
-1	2	2	2	2	2	2	2	2	2
-0.75	1	-4.8133	-16.9886	-36.9648	-64.7002	-98.3356	-129.935	-130.328	1
-0.5	1	-7.39705	-23.6307	-49.9354	-86.2969	-129.886	-169.16	-164.801	1
-0.25	1	-8.22095	-25.6341	-53.7051	-92.4069	-138.594	-179.531	-173.217	1
0	1	-8.39733	-26.0552	-54.4657	-93.6224	-140.287	-181.502	-174.744	1
0.25	1	-8.22095	-25.6341	-53.7051	-92.4069	-138.594	-179.531	-173.217	1
0.5	1	-7.39705	-23.6307	-49.9354	-86.2969	-129.886	-169.16	-164.801	1
0.75	1	-4.8133	-16.9886	-36.9648	-64.7002	-98.3356	-129.935	-130.328	1
1	2	2	2	2	2	2	2	2	2

Gauss Seidel Iterations									
x\y	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
-1	2	2	2	2	2	2	2	2	2
-0.75	1	-6.08741	-21.1117	-45.9466	-80.6594	-122.373	-159.808	-156.375	1
-0.5	1	-10.7663	-33.6865	-70.9354	-122.523	-183.168	-233.798	-219.392	1
-0.25	1	-13.2386	-40.1385	-83.4095	-142.915	-211.83	-266.878	-245.294	1
0	1	-14.3891	-43.0919	-88.9997	-151.829	-223.945	-280.219	-255.14	1
0.25	1	-14.5702	-43.5088	-89.6667	-152.649	-224.639	-280.421	-254.842	1
0.5	1	-13.3633	-40.2563	-83.1574	-141.622	-208.533	-261.057	-239.056	1
0.75	1	-9.03937	-28.6362	-60.0631	-102.95	-152.509	-193.227	-181.72	1
1	2	2	2	2	2	2	2	2	2


Jacobi
Seidel
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