Mathematical Methods (MA31007)

Test-2

Time: 1hr 05 min, Date:21.9.21

12 - 1:05 P.M. Full Marks 25

All six questions are compulsory. No negative marking or part marking is there.

Q1. (a) The BVP $y''+y=x,\ y(0)=0$ and $y'(1)=0$ is reduced to the Fredholm integral equation $y(x)=\int_0^1 G(x,t)y(t)dt-\frac{1}{6}\left(k_1x-\frac{5}{k_2}x^3\right)$. Then $k_1=$_____ and $k_2=$______

Q1. (b) In Q1. (a), $G(x,t)$ is the Green's function of the associated homogeneous BVP and $G(x,t)=\frac{2}{k_3}x,\ 0\le x<t$ and $G(x,t)=k_4t,\ t<x\le 1$. Then $k_3=$_____ and $k_4=$______

Q1. (c) In Q1. (a), if the first boundary condition is changed to $y(0)=1$, then the Fredholm integral equation takes the form

$y(x)=\int_0^1 G(x,t)y(t)dt-\frac{1}{k_5}\left(x^3+k_6x+k_7\right)$. Then $k_5=$____ $k_6=$_____and $k_7=$_____

1+1+1=3M

Q2. (a) To solve $y''-2y'+y=xe^x\log x,\ x>0$, method of variation of parameter is adopted. The particular integral is of the form $\frac{1}{k_1}x^3e^x\log x-\frac{k_2}{k_3}x^3e^x$, k_2 and k_3 are prime to each other.

Then $k_1=$______ $k_2=$_____and $k_3=$ _______.

Q2. (b) Using the method of variation of parameter, the solution of $y''+9y=\varphi(x)$ satisfying the initial conditions $y(0)=0$ and $y'(0)=0$ is obtained as

$$y=c_1\cos 3x+c_2\sin 3x+\frac{1}{c_3}\int_0^x \phi(t)\sin c_4(x-t)\,dt.$$

Then $c_1=$______ $c_2=$_____ $c_3=$ _______and $c_4=$________

2+2=4M

Q3. (a) With the help of $1, x, x^2$ three functions φ_0, φ_1 and φ_2 are constructed which are orthogonal with respect to e^{-x} over $0 \le x < \infty$. If $\varphi_2(x)$ has the form $k_1 x^2 - k_2 x + k_3$, then $k_1 =$______ $k_2 =$_____and $k_3 =$ _______.

Q3. (b) Consider three functions $f_1(x) = a_0$, $f_2(x) = b_0 + b_1 x$ and $f_3(x) = c_0 + c_1 x + c_2 x^2$ where $a_0, b_0, b_1, c_0, c_1, c_2$ are real constants. If the given functions form an orthonormal set on the interval $-1 \le x \le 1$, then the values of $b_0 =$_____ and $c_1 =$______. If c_0 and c_2 are connected by $c_2 = -kc_0$, then $k =$________.

2+3=5M

Q4. (a) Consider a second order eigenvalue problem

$-P(x)y'' - Q(x)y' + R(x)y = \lambda y$ -----------------------------(1)

which is converted into Strum-Liouville eigenvalue problem

$Ly = \lambda r y$ where $Ly = -(py')' + qy$ ----------------------------(2)

with boundary conditions $\alpha_1 y(0) + \alpha_2 y'(0) = 0, \beta_1 y(l) + \beta_2 y'(l) = 0$, $p(x) > 0, r(x) > 0$.

Multiplying by a suitable factor $\mu(x)$, Eq. (1) is converted into Eq.(2) Then the form of $\mu(x)$ is (no constant is considered) (i) $\dfrac{e^{\int \frac{P}{Q} dx}}{Q}$ (ii) $\dfrac{e^{-\int \frac{P}{Q} dx}}{Q}$ (iii) $\dfrac{e^{\int \frac{Q}{P} dx}}{P}$ (iv) $\dfrac{e^{-\int \frac{Q}{P} dx}}{P}$

Q4. (b) Hence if $y'' + xy' + \lambda y = 0$, $y(0) = 0 = y(1)$, is reduced in the form of Eq. (2) of Q4.(a), then $p(x) = e^{\frac{x^2}{k_1}}$ and $r(x) = e^{\frac{2x^2}{k_2}}$. Then $k_1 =$______and $k_2 =$_____

Q4. (c) Define the Strum-Liouville problem as given in Q4(a). If u and v satisfy the S-L differential equation, then $\int_0^l (vLu - uLv)\, dx =$

(i) $(pvu')_0^l - (puv')_0^l$ (ii) $(-qvu')_0^l + (quv')_0^l$

(iii) $(-pvu')_0^l + (puv')_0^l$ (iv) $(qvu')_0^l - (quv')_0^l$

Q4. (d) In continuation to Q4.(c), if u and v both satisfy S-L boundary condition also, given in Q4(a), then the value of $\int_0^l (vLu - uLv)\, dx$ is _________ .

2+1+2+1=6M

Q5. (a) Consider the ODE $(5-x^2)y'' + \frac{1+x}{x}y' - \left(\frac{3}{x^2}+x\right)y = 0$. The number of singular points are _____

Q5. (b) If a power series solution $y(x) = \sum_{n=0}^{\infty} c_n x^n$ for $(x^2+1)y'' - 4xy' + 6y = 0$ about $x = 0$ is obtained, then the number of terms in the power series will be ______

Q5. (c) For the ODE in Q5.(b), if a recurrence relation between c_{n+2} and c_n is obtained as

$c_{n+2} = -\frac{(n-k_1)(n-k_2)}{(n+k_3)(n+k_4)}c_n, \quad n \geq 2$, then $k_1 =$ ______ $k_2 =$ ____ $k_3 =$ _____ and $k_4 =$ ___.

$k_1 < k_2, k_4 < k_3$. **1+1+1=3M**

Q6. (a) If a power series solution $y(x) = \sum_{n=0}^{\infty} c_n x^{r+n}, \quad c_0 \neq 0$ is found for $2x^2y'' - xy' + (1+x)y = 0$, then the roots of the indicial equation are k_1 and $\frac{1}{k_2}$, $k_1 < k_2$. Then $k_1 =$ ______ and $k_2 =$ ____

Q6. (b)If an ascending power series solution $y(x) = \sum_{m=0}^{\infty} c_m x^{k+m}, c_0 \neq 0$ is obtained for Legendre equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ and the roots of the indicial equation are k_1 and k_2 then $k_1 =$ ______ and $k_2 =$ ____ $k_1 < k_2$.

Q6. (c) If a recurrence relation between c_m and c_{m-2} for the problem in Q.6(b)

Is obtained as $c_m = \frac{(k+m-a-n)(k+m-b+n)}{(k+m)(k+m-1)}c_{m-2}$, then $a =$ ______ and $b =$ ____.

2+1+1=4M

******************************The End**************************************