Date

NSOPDE ASSIGNMENT - 4

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1) $x^2 y'' - 2y + x = 0 \quad y(2) = 0 \quad y(3) = 0$

$y'' = \frac{2y - x}{x^2} \qquad h = \frac{1}{4} = 0.25$

2nd order taylors series $\Rightarrow$

$y_{j+1} = y_j + h y_j' + \frac{h^2}{2} y_j''$ $\rightarrow$ differentiating

$\Rightarrow y'_{j+1} = y_j' + h y_j'' + \frac{h^2}{2} y_j'''$

$y''' = \frac{(2y'-1)x^2 - 2x(2y-x)}{x^4} = \frac{2y'x - 4y + x}{x^3}$

General soln $y(x) = x\, y_1(x) + (1-x)\, y_2(x)$

$x^2 y_1'' - 2y_1 + x = 0, \quad y_1'(2) = 0 \quad y_1(2) = 0$

$x^2 y_2'' - 2y_2 + x = 0, \quad y_2'(2) = 1 \quad y_2(2) = 0$

Using $y_{j+1} = y_j + h y_j' + \frac{h^2}{2}\left(\frac{2y_j - x_j}{x_j^2}\right)$

& $y'_{j+1} = y_j' + h\left(\frac{2y_j - x_j}{x_j^2}\right) + \frac{h^2}{2!}\left(\frac{2y_j' x_j - 4y_j + x_j}{x_j^3}\right)$

$j=0 \Rightarrow x_0 = 2 \quad y_1(2) = 0 \quad y_1'(2) = 0$

$y_1 = y_1(2.25) = -0.015625$

$y_1' = y_1'(2.25) = -0.1171875$

$j=1 \Rightarrow x_1 = 2.25 \quad y_1(2.25) = -0.015625$

$y_1'(2.25) = -0.1171875$

$y_1(2.5) = -0.05900366$

$y_1'(2.5) = -0.22494427$

$j=2 \Rightarrow x_2 = 2.5$

$y_1(2.75) = -0.12832977$

$y_1'(2.75) = -0.3264419796$

$j=3 \Rightarrow x_3 = 2.75$

$y_1(3) = -.222364477$

$y_1'(3) = -0.4236299912$

|||ly for y_2

$\Rightarrow y_2(2) = 0 \quad y_2'(2) = 1$

$y_2(2.25) = 0.234375 \quad y_2'(2.25) = 0.8984375$

$y_2(2.5) = 0.4479890046 \quad y_2'(2.5) = 0.825167181$

$y_2(2.75) = 0.6462606899 \quad y_2'(2.75) = 0.7706740612$

$y_2(3) = 0.8329065663 \quad y_2'(3) = 0.729100265$

$$\lambda = \frac{y(3) - y_2(3)}{y_1(3) - y_2(3)} = 0.7892821201$$

$\Rightarrow y(x) = 0.7892821201\, y_1(x) + (0.2107178799)\, y_2(x)$

$\therefore \; y(2) = 0 \quad y(2.25) \simeq 0.03705447$

$y(2.5) \simeq 0.0478275$

$y(2.75) \simeq 0.03489028$

$y(3) = 0$

2) $y'' = 2y - y'$ $\quad y(1) = 2e + \frac{1}{e^2}$ $\quad y(2) = 2e^2 + \frac{1}{e^4}$

$y''' = 2y' - y''$

$y^{iv} = 2y'' - y'''$

$h = \frac{1}{3}$ $\quad \left[\begin{array}{l} y(x) = \lambda y_1(x) + (1-\lambda) y_2(x) \\ y_1'(1) = 0 \quad y_2'(1) = 1 \end{array} \right]$

3rd order taylors series

$$y_{j+1} = y_j + h y_j' + \frac{h^2}{2} y_j'' + \frac{h^3}{6} y_j'''$$

$$y'_{j+1} = y_j' + h y_j'' + \frac{h^2}{2} y_j''' + \frac{h^3}{6} y_j^{iv}$$

$$y''' = 2y' - (2y - y') = 3y' - 2y$$

$$y^{iv} = 2(2y - y') - (3y' - 2y) = 6y - 5y'$$

$$\therefore y_{j+1} = y_j + h y_j' + \frac{h^2}{2}(2y_j - y_j') + \frac{h^3}{6}(3y_j' - 2y_j)$$

$$y'_{j+1} = y_j' + h(2y_j - y_j') + \frac{h^2}{2}(3y_j' - 2y_j) + \frac{h^3}{6}(6y_j - 5y_j')$$

$\Rightarrow$ taking $j = 0, 1, 2, 3$ for y_1

$y_1(1) = 5.571899894$ $\quad y_1'(1) = 0$

$y_1(4/3) = 6.1222099464$ $\quad y_1'(4/3) = 3.3018660385$

$y_1(5/3) = 7.7052033352$ $\quad y_1'(5/3) = 6.2776218510$

$y_1(2) = 10.3262471760$ $\quad y_1'(2) = 9.6036442025$

$\Rightarrow$ taking $j = 0, 1, 2, 3$ for y_2

$y_2(1) = 5.571899894$ $\quad y_2'(1) = 0$

$y_2(4/3) = 6.4185067481$ $\quad y_2'(4/3) = 4.1043354469$

$y_2(5/3) = 8.2685327322$ $\quad y_2'(5/3) = 7.0971620735$

$y_2(2) = 11.1880407769$ $\quad y_2'(2) = 10.5951247645$

$$\Rightarrow \quad \lambda = \frac{y(2) - y_2(2)}{y_1(2) - y_2(2)} \doteq -4.187066438.$$

$$\Rightarrow y(x) = (-4.187066438)\, y_1(x) + (5.187066438)\, y_2(x)$$

$\therefore$ $y(1) \cong 5.5718989679$

$y(4/3) \approx 7.659583 7393.$

$y(5/3) \cong 10.6272303439$

$y(2) \doteq 14.79642 7897.$

3) $y'' = y \qquad y'(0) = 3 \qquad y'(1) = e + \frac{2}{e} \qquad h = \frac{1}{4}$

$y''' = y' \quad y^{iv} = y'' = y$

$y(x) = \lambda y_1(x) + (1-\lambda) y_2(x)$ general soln

$\Rightarrow \; y_1'' = y_1 \qquad y_1'(0) = 3 \qquad y_1(0) = 0$

$y_2'' = y_1 \;, \quad y_2'(0) = 3. \qquad y_2(0) = 1$

3^{rd} order taylors series

$$\Rightarrow \quad y_{j+1} = y_j + h y_j' + \frac{h^2}{2} y_j'' + \frac{h^3}{6} y_j'''$$

$$y_{j+1}' = y_j' + h y_j'' + \frac{h^2}{2} y_j''' + \frac{h^3}{6} y_j^{iv}$$

$$\therefore \; y_{j+1} = y_j + h y_j' + \frac{h^2}{2} y_j + \frac{h^3}{6} y_j' \quad -(1)$$

$$= \left(1 + \frac{h^2}{2}\right) y_j + \left(h + \frac{h^3}{6}\right) y_j'$$

$$y_{j+1}' = y_j' + h y_j + \frac{h^2}{2} y_j' + \frac{h^3}{6} y_j \quad -(2)$$

$$= \left(h + \frac{h^3}{6}\right) y_j + \left(1 + \frac{h^2}{2}\right) y_j'$$

using (1) & (2)

$y_1(0) = 0 \quad , \quad y_1'(0) = 3$

$y_1(0.25) = 0.7578125 \quad y_1'(0.25) = 3.09375$

$y_1(0.5) = 1.562988 2813 \quad y_1'(0.5) = 3.381 8562826$

$y_1(0.75) = 2.4661026531 \quad y_1'(0.75) = 3.8823566437$

$y_1(1) = 3.5238678257 \quad y_1'(1) = 4.6266280944$

$|||^{ly}$

$y_2(0) = 1 \quad y_2'(0) = 3$

$y_2(0.25) = 1.7890625 \quad y_2'(0.25) = 3.3463541667$

$y_2(0.5) = 2.6902737088 \quad y_2'(0.5) = 3.9028523763$

$y_2(0.75) = 3.7602215343 \quad y_2'(0.75) = 4.7043908614$

$y_2(1) = 5.0660771905 \quad y_2'(1) = 5.8612507029$

$$\lambda = \frac{y'(1) - y_2'(1)}{y_1'(1) - y_2'(1)} = 1.998267337$$

$\therefore \; y(x) = (1.998267337)\, y_1(x) - (0.998267337)\, y_2(x)$

$y(0) = -0.9982673372$

$y(0.25) = -0.2716506916$

$y(0.5) = 0.4376560590$

$y(0.75) = 1.174226043)$

$y(1) = 1.9843305870$

4) $y'' = xy + 1$ $\quad y(0) + y'(0) = 1 \quad y(1) = 1$

$y''' = xy' + y \quad h = 0.25$

$y^{iv} = xy'' + 2y' = 2y' + x^2y + x$

general soln $\quad y(x) = \lambda y_1(x) + (1-\lambda) y_2(x)$

$y_1'' = xy_1 + 1 \quad , y_1(0) = 0 \quad y_1'(0) = 1$

$y_2'' = xy_2 + 1 \quad , y_2(0) = 1 \quad y_2'(0) = 0$

3rd order taylor's series

$y_{j+1} = y_j + h y_j' + \frac{h^2}{2} y_j'' + \frac{h^3}{6} y_j'''$

$y_{j+1}' = y_j' + h y_j'' + \frac{h^2}{2} y_j''' + \frac{h^3}{6} y_j^{iv}$

$\Rightarrow \quad y_{j+1} = y_j + h y_j' + \frac{h^2}{2}(x_j y_j + 1) + \frac{h^3}{6}(x_j y_j' + y_j)$

$y_{j+1}' = y_j' + h(x_j y_j + 1) + \frac{h^2}{2}(x_j y_j' + y_j) + \frac{h^3}{6}(2y_j' + x_j^2 y_j + x_j)$

by taking $j = 0, 1, 2, 3$

$y_1(0) = 0 \quad y_1'(0) = 1$

$y_1(0.25) = 0.28125 \quad y_1'(0.25) = 1.2552083333$

$y_1(0.5) = 0.6300489638 \quad y_1'(0.5) = 1.5486161974$

$y_1(0.75) = 1.0619547080 \quad y_1'(0.75) = 1.9310364569$

$y_1(1) = 1.6073904483 \quad y_1'(1) = 2.4721639205$

IIIly for y_2

$y_2(0) = 1 \quad y_2'(0) = 0$

$y_2(0.25) = 1.0338541667 \quad y_2'(0.25) = 0.28125$

$y_2(0.5) = 1.1463690864 \quad y_2'(0.5) = 0.6326552497$

$y_2(0.75) = 1.3575040218 \quad y_2'(0.75) = 1.0770041545$

$y_2(1) = 1.6954602514 \quad y_2'(1) = 1.6587514971$

$$\lambda = \frac{(b_0 y(b) + b_1 y'(b)) - (b_0 y_2(b) + b_1 y_2'(b))}{(b_0 y_1(b) + b_1 y_1'(b)) - (b_0 y_2(b) + b_1 y_2'(b))}$$

$\Rightarrow \lambda = 7.89669361$

$\Rightarrow y(x) = (7.89669361) y_1(x) - (6.89669361) y_2(x)$

$\therefore y(0) = -6.8966936$
$y(0.25) = -4.90923034$
$y(0.5) = -2.930852726$
$y(0.75) = -0.97635835$
$y(1) = 1$

5) $y'' = 6y^2 \quad y(0) = 1 \quad y\left(\frac{3}{10}\right) = \frac{100}{169} \quad h = 0.1$

Non linear BVP $\Rightarrow$

$y_1'' = 6y_1^2 \quad y_1(0) = 1 \quad y_1'(0) = s^{(0)} = -1.8 = \frac{-9}{5}$

$y_2'' = 6y_2^2 \quad y_2(0) = 1 \quad y_2'(0) = s^{(1)} = -1.9 = \frac{-19}{10}$

3^{rd} order taylor series

$\therefore y_{j+1} = y_j + h y_j' + \frac{h^2}{2} y_j'' + \frac{h^3}{6} y_j'''$

$y_{j+1}' = y_j' + h y_j'' + \frac{h^2}{2} y_j''' + \frac{h^3}{6} y_j^{iv}$

$$\left[\begin{aligned} y'' &= 6y^2 \\ y''' &= 12yy' \\ y^{iv} &= 12(y')^2 + 12yy'' \\ &= 12\left[(y')^2 + 6y^3\right] \end{aligned}\right.$$

using the above,
for $j = 0, 1, 2$

$y_1(0.1) \simeq 0.8464 \qquad y_1'(0.1) = -1.308$
$y_1(0.2) \simeq 0.734878 \qquad y_1'(0.2) = -0.94459$
$y_1(0.3) \simeq 0.655239 \qquad y_1'(0.3) = -0.578915$
~~$y(0.3) = 0.655239$~~

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$$\phi(s) = y_1(s, x) - y(x)$$

$$\Rightarrow \phi(s^{(0)}) = y_1(s, 0.3) - y(0.3)$$

$$\Rightarrow \phi(s^{(0)}) = 0.063516$$

$|||^{ly}$ for $y_2 \Rightarrow$

$y_2(0.1) = 0.8562 \qquad y_2'(0.1) = -1.414$
$y_2(0.2) = 0.713412 \qquad y_2'(0.2) = -1.065405$
$y_2(0.3) = 0.620620 \qquad y_2'(0.3) = -0.805635$

$$\Rightarrow \phi(s^{(1)}) = y_2(s, 0.3) - y(0.3)$$

$$\Rightarrow \phi(s^{(1)}) = 0.028904$$

Using secant method $\Rightarrow$ $s^{(k+1)} = s^{(k)} - \left[\dfrac{s^{(k)} - s^{(k-1)}}{\phi(s^{(k)}) - \phi(s^{(k-1)})}\right]\phi(s^{(k)})$

$$\Rightarrow s^2 = -1.983504$$

$|||^{ly}$ for y_3, $\quad y_3'' = 6y_3^2 \quad y_3(0) = 1 \quad y_3'(0) = s^{(2)}$
$= -1.983504$

$\Rightarrow$ $y_3(0.1) = 0.835616 \qquad y_3'(0.1) = -1.502520$
$y_3(0.2) = 0.703801 \qquad y_3'(0.2) = -1.158899$
$y_3(0.3) = 0.601140 \qquad y_3'(0.3) = -0.910636$

$$\phi(s^{(2)}) = y_3(s, 0.3) - y(0.3) = 0.009424$$

$$s^{(3)} = s^{(2)} - \left[\frac{s^{(2)} - s^{(1)}}{\phi(s^{(2)}) - \phi(s^{(1)})}\right]\phi(s^{(2)}) = -2.023909$$

exact value of $y'(0) = -2$

finally, $y'' = 6y^2$ $y(0) = 1$ $y'(0) = -2.023909$

by using 3rd order taylors series

$\therefore$ $y(0.1) \cong 0.823561$
$y(0.2) \cong 0.686829$
$y(0.3) \cong 0.577837$

6) $u'' = 2uu'$ $0 < x < 1$ $u(0) = \frac{1}{2}$ $u(1) = 0.5$
$h = 0.25$
$u'(0) = s^{(0)} = 0.09$

using taylor series method of 2nd order

$\Rightarrow u(0.25) = 0.525312 5$ $u'(0.25) = 0.1158 1875$
$u(0.5) = 0.55 80697523$ $u'(0.5) = 0.1510727121$
$u(0.75) = 0.6011072498$ $u'(0.75) = 0.2005349985$
$u(1) = 0.6587749395$ $u'(1) = 0.2723773241$

$\phi(s^{(0)}) = 0.6587749395 - 0.5 = 0.1587749395$
here $a_1 = 0$ $a_0 = 1$ $b_1 = 0$ $b_0 = 1$
1st variational eqn.
$v'' = (2u')v + 2uv'$ $v(0) = a_1/a_0 = 0$ $v'(0) = 1$
$v''' = 2u''v + 2u'v' + 2u'v' + 2u(2u'v + 2uv')$
$= 4uv(2u+1) + 4u^2v'$

$v(0.25) = 0.28125$ $v'(0.25) = 1.28125$
$v(0.5) = 0.5661179077$ $v'(0.5) = 1.321787053$
$v(0.75) = 0.9480130672$ $v'(0.75) = 1.807454544$
$v(1) = 1.479663193$ $v'(1) = 2.579715205$

$\phi'(s^{(0)}) = 2.579715205$
$s^{(1)} = s^{(0)} - \dfrac{\phi(s^{(0)})}{\phi'(s^{(0)})} = 0.0284525318 2$

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$s^{(1)} = 0.028452 \quad u(0) = 1/2 \quad u'(0) = s^{(1)}$

after 1 iteration

$u(0.25) = 0.5080022746 \qquad u'(0.25) = 0.03650540306$

$u(0.5) = 0.5180177767 \qquad u'(0.5) = 0.04487942813$

$u(0.75) = 0.5306906454 \qquad u'(0.75) = 0.0658134865 61$

$u(1) = 0.5471525886 \qquad u'(1) = 0.07519194 72$

7) $y'' = (3/2)\, y^2 \quad y(0) = 1 \quad y(1) = 4 \quad h = 0.25$

$s^{(0)} = 0.9$

IVPs $\Rightarrow$ $u'' = \frac{3u^2}{2} \quad u(0) = 1 \quad u'(0) = 0.09$

$v'' = 3uv \quad v(0) = 0 \quad v'(0) = 1$

2nd order RK method $\Rightarrow$ $\bar{u}_{j+1} = \bar{u}_j + \frac{h}{2}\left[\bar{K}_1 + \bar{K}_2\right]$

$K_{i1} = f_i(t_j, u_{1j}, u_{2j})$

$K_{i2} = f_i(t_j + h, u_{1j} + hK_{11}, u_{2j} + hK_{21})$

$f_1 = u_2 \quad f_2 = \frac{3u_1^2}{2} \quad u_1 = u \quad u_2 = u'$

by using the above eqns

$\Rightarrow u(0.25) = 1.069375 \qquad u'(0.25) = 0.4735324219$

$u(0.5) = 1.2413626 16 \qquad u'(0.5) = 0.9524697108$

$u(0.75) = 1.5517135 35 \qquad u'(0.75) = 1.651815 15$

$u(1) = 2.077533646 \qquad u'(1) = 2.827015009$

$\phi(s^{(0)}) = 2.077533646 \quad u_1 = v \quad f_1 = u_2$

$u_2 = v' \quad f_2 = 3uu_1$

b

$v(0.25) = 0.25 \qquad v'(0.25) = 1.09375$

$v(0.5) = 0.5485009766 \qquad v'(0.5) = 1.403910522$

$v(0.75) = 0.9633119141 \qquad v'(0.75) = 2.077960919$

$v(1) = 1.622938157 \qquad v'(1) = 3.497482172$

$$\phi'(s^{(0)}) = 3.497482172$$

$$s^{(1)} = s^{(0)} - \frac{\phi(s^{(0)})}{\phi'(s^{(0)})} = -0.504008359$$

$$s^{(1)} = u'(0) \qquad u(0) = 1$$

$\therefore$

$u(0.25) = 0.9208729102 \qquad u'(0.25) = -0.173282294$

$u(0.5) = 0.9761258906 \qquad u'(0.5) = 0.4570416432$

$u(1) = 1.120161981 \qquad u'(1) = 0.8586224008$

8) (i) $y'' = y + x \quad y(0) = 0 \quad y(1) = 0$

at $x = x_j$

$$\frac{y(x_{j+1}) - 2y(x_j) + y(x_{j-1})}{h^2} = y(x_j) + x_j$$

$$\Rightarrow y(x_{j-1}) - (2+h^2)\, y(x_j) + y(x_{j+1}) = h^2 x_j$$

(a) $h = \frac{1}{2}$

$$\cancel{y(0)} - \left(2 + \frac{1}{4}\right) y\left(\frac{1}{2}\right) + \cancel{y(1)} = \frac{1}{4} \times \frac{1}{2}$$

$$\Rightarrow y\left(\frac{1}{2}\right) = -\frac{1}{18}$$

(b) $h = 1/3$

$$y(0) - \frac{19}{9} y(1/3) + y(2/3) = \frac{1}{27}$$

$$y(1/3) - \frac{19}{9} y(2/3) + y(1) = \frac{2}{27}$$

$$y(1/3) = -\frac{37}{840} = -0.04404761905$$

(c) $h = 1/4$

$$y(0) - \frac{33}{16} y(1/4) + y(1/2) = \frac{1}{64}$$

$$y(1/4) - \frac{33}{16} y(1/2) + y(3/4) = \frac{1}{32}$$

$$y(1/2) - \frac{33}{16} y(3/4) + y(1) = 3/64$$

$y(1/4) = -2657/76164 = -0.03488524762$

$y(1/2) = -65/1154 = -0.05632582322$

$y(3/4) = -3811/76164 = -0.0500367627 8$

(ii) $x^2 y'' = 2y - x \quad y(2) = 0 \quad y(3) = 0$

$$\frac{y(x_{j+1}) - 2y(x_j) + y(x_{j-1})}{h^2} = \frac{2y(x_j) - x_j}{x_j^2}$$

$$y(x_{j+1}) - y(x_j)\left[2 + \frac{2h^2}{x_j^2}\right] = \frac{-h^2}{x_j} + y(x_{j-1})$$

(a) $h = 1/2$

$$y(3) - y(5/2)\left(2 + \frac{\cancel{4}^2}{\cancel{4}\times 25}\right) + y(2) = \frac{-\cancel{2}}{\cancel{4}_2 \times 5} = -\frac{1}{10}$$

$$y(5/2) \cdot 2 \times \frac{26}{\cancel{2}5} = \frac{1}{\cancel{10}_2}$$

$$y(5/2) = \frac{5}{104} = 0.04807692308$$

(b) $h = 1/3$

$$y(2) - y(7/3)\left(2 + \frac{2\times 9}{9 \times 49}\right) + y(8/3) = \frac{-3}{9\times 7}$$

$$y(7/3) - y(8/3)\left(2 + \frac{2\times 9}{9\times 64}\right) + y(3) = \frac{-3}{9\times 8}$$

$$y(7/3) = 217/4932 = 0.04399837794$$

$$y(8/3) = 52/1233 = 0.04217356042$$

(iii) $y'' - 3y' + 2y = 0$ $\quad 2y(0) - y'(0) = 1$ $\quad h = 0.5$

$y(1) + y'(1) = 2e + 3e^2$

$$\frac{y(x_{j+1}) - 2y(x_j) + y(x_{j-1})}{h^2} - 3\,\frac{y(x_{j+1}) - y(x_{j-1})}{2h}$$

$$+ 2y_j = 0$$

$x_0 = 0 \quad x_1 = \frac{1}{2} \quad x_2 = 1$

$\Rightarrow -y_0' = 1 - 2y_0 \Rightarrow \dfrac{-(y_1 - y_{-1})}{\cancel{2} \times \frac{1}{\cancel{2}}} = 1 - 2y_0$

$\Rightarrow y_{-1} = 1 - 2y_0 + y_1$

$y_2 + y_2' = 2e + 3e^2$

$y_3 = y_1 - y_2 + 2e + 3e^2$

$4y_{j+1} - 8y_j + 4y_{j-1} - 3y_{j+1} + 3y_{j-1} + 2y_j = 0$

$y_{j+1} - 6y_j + 7y_{j-1} = 0$

$j = 0, 1, 2 \Rightarrow$

$y_1 - 6y_0 + 7y_{-1} = y_1 - 6y_0 + 7 - 14y_0 + 7y_1 = 0$

$\Rightarrow 8y_1 - 20y_0 = -7$

$\Rightarrow y_2 - 6y_1 + 7y_0 = 0$

$y_1 - y_2 + 2e + 3e^2 - 6y_2 + 7y_1 = 0$

$\Rightarrow 8y_1 - 7y_2 = -2e - 3e^2$

$y(0) = 1.593159221$

$y(0.5) = 3.107898052$

$y(1) = 7.495273767$.

(iv) $y'' = 2yy'$ $y(0) = \frac{1}{2}$ $y(1) = 1$

at x_j $\frac{y(x_{j+1}) - 2y(x_j) + y(x_{j-1})}{h^2} - 2y(x_j)\left(\frac{y(x_{j+1}) - y(x_{j-1})}{2h}\right) = 0$

(a) $h = 1/2$ $y_1^{(0)} = 1/4$

$y_2 = 2y_1 + y_0 = hy_1(y_2 - y_0)$

$1 - 2y_1 + \frac{1}{2} = \frac{1}{2} \times y_1 \times \frac{1}{2}$

$\Rightarrow y_1 = 2/3$

(b) $h = 1/3$ $y_1^{(0)} = 4/5$ $y_2^{(0)} = 3/5$

$y_2 - 2y_1 + y_0 = \frac{1}{3} y_1 (y_2 - y_0)$

$\Rightarrow 2y_1y_2 + 11y_1 - 6y_2 - 3 = 0 = f_1$

$y_3 - 2y_2 + y_1 = \frac{1}{3} y_2 (y_3 - y_1)$

$\Rightarrow y_1 y_2 + 3y_1 - 7y_2 + 3 = 0 = f_2$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial y_1} & \frac{\partial f_1}{\partial y_2} \\ \frac{\partial f_2}{\partial y_1} & \frac{\partial f_2}{\partial y_2} \end{bmatrix} = \begin{bmatrix} 2y_2 + 11 & 2y_1 - 6 \\ y_2 + 3 & y_1 - 7 \end{bmatrix}$$

$$\begin{bmatrix} y_1^{(1)} \\ y_2^{(1)} \end{bmatrix} = \begin{bmatrix} y_1^{(0)} \\ y_2^{(0)} \end{bmatrix} - \left[J\, y_1^{(0)}, y_2^{(0)}\right]^{-1} \begin{bmatrix} f_1(y_1^{(0)}, y_2^{(0)}) \\ f_2(y_1^{(0)}, y_2^{(0)}) \end{bmatrix}$$

by solving

$\Rightarrow y_1^{(1)} = y(1/3) = \frac{231}{389} = 0.5938303342$

$y_2^{(1)} = y(2/3) = \frac{1452}{1945} = 0.7465295630$

#

(v) $y'' = \frac{3}{2}y^2 \quad y(0) = 4 \quad y(1) = 1$

at $x_j \rightarrow \frac{y(x_{j+1}) - 2y(x_j) + y(x_{j-1})}{h^2} = \frac{3}{2}y^2(x_j)$

(a) $h = 1/2 \qquad y_1^{(0)} = 7/2$

$y_2 - 2y_1 + y_0 = \frac{3}{8}y_1^2$

$f \Rightarrow \frac{3}{8}y_1^2 + 2y_1 - 5 = 0 \qquad f' \Rightarrow \frac{3}{4}y_1 + 2$

Newton Raphson $\Rightarrow$

$y_1^{(1)} = y_1^{(0)} - \frac{f(y_1^{(0)})}{f'(y_1^{(0)})} = 3.5 - \frac{6.59375}{4.625}$

$\Rightarrow y_1^{(1)} = 2.07432432$

(b) $h = 1/3. \quad y_1^{(0)} = 2 \quad y_2^{(0)} = 3$

$y_2 - 2y_1 + y_0 = \frac{y_1^2}{6} \Rightarrow y_1^2 + 12y_1 - 6y_2 - 24 = 0 \Rightarrow f_1$

$y_3 - 2y_2 + y_1 = \frac{y_2^2}{6} \Rightarrow y_2^2 - 6y_2 + 12y_1 - 6 = 0 \Rightarrow f_2$

$J = \begin{bmatrix} 2y_1 + 12 & -6 \\ -6 & 2y_2 - 6 \end{bmatrix} = \begin{bmatrix} 16 & -6 \\ -6 & 0 \end{bmatrix}$

$\begin{bmatrix} y_1^{(1)} \\ y_2^{(1)} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 16 & -6 \\ -6 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -14 \\ 9 \end{bmatrix}$

$= \begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 0 & 6/36 \\ 6/36 & 16/36 \end{bmatrix} \begin{bmatrix} -14 \\ 9 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 4/3 \end{bmatrix}$

$\therefore y_1^{(1)} = y(1/3) = 0.5$

$y_2^{(1)} = y(2/3) = 4/3 = 1.333$