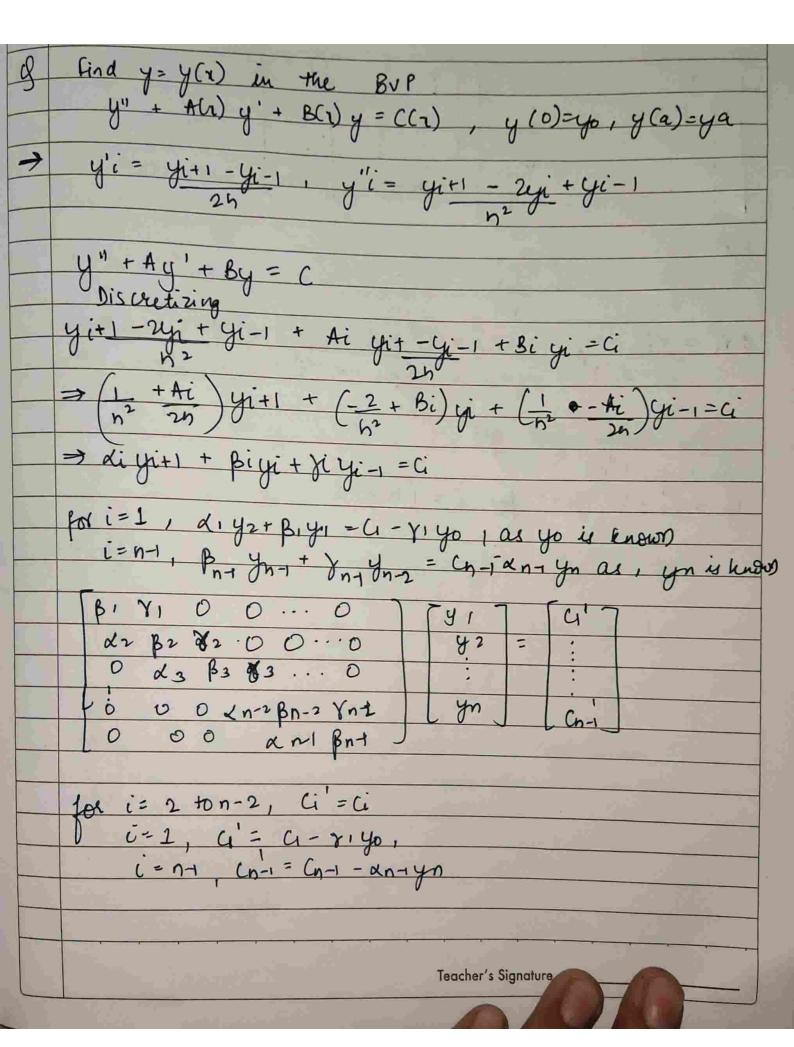
Keviti P. Charantinath 19MA 20059

ANT Assignment 1

Find Error in the differenciation of lagrange polynomial we know  $E(x) = f(x) - pn(x) = T(x-xi) f^{n+1}(f)$   $\Rightarrow E'(x) = f'(x) - pn'(x) = \frac{1}{(n+1)!} \frac{d}{dx} \int_{i=0}^{n} (x-xi)$ Error in differenciation  $= f'(x) - pn'(x) = \frac{1}{(n+1)!} \frac{d}{dx} \int_{i=0}^{n} (x-xi)$ 



Using tomo Thomas algo we get

$$\begin{bmatrix} 1 & Y_1' \\ 0 & 1 & Y_2' \\ \vdots & \vdots & 1 & Y_{n-2} \\ \vdots & \vdots & \ddots & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ \vdots \\ Y_{n-1} \end{bmatrix} = \begin{bmatrix} C_1'' \\ \vdots \\ C_{n-1} \end{bmatrix}$$

here
$$8i' = 8i \quad C'' = Ci'$$

$$4Yi' = Yi$$

$$8i - \lambda i Y'' = 1$$

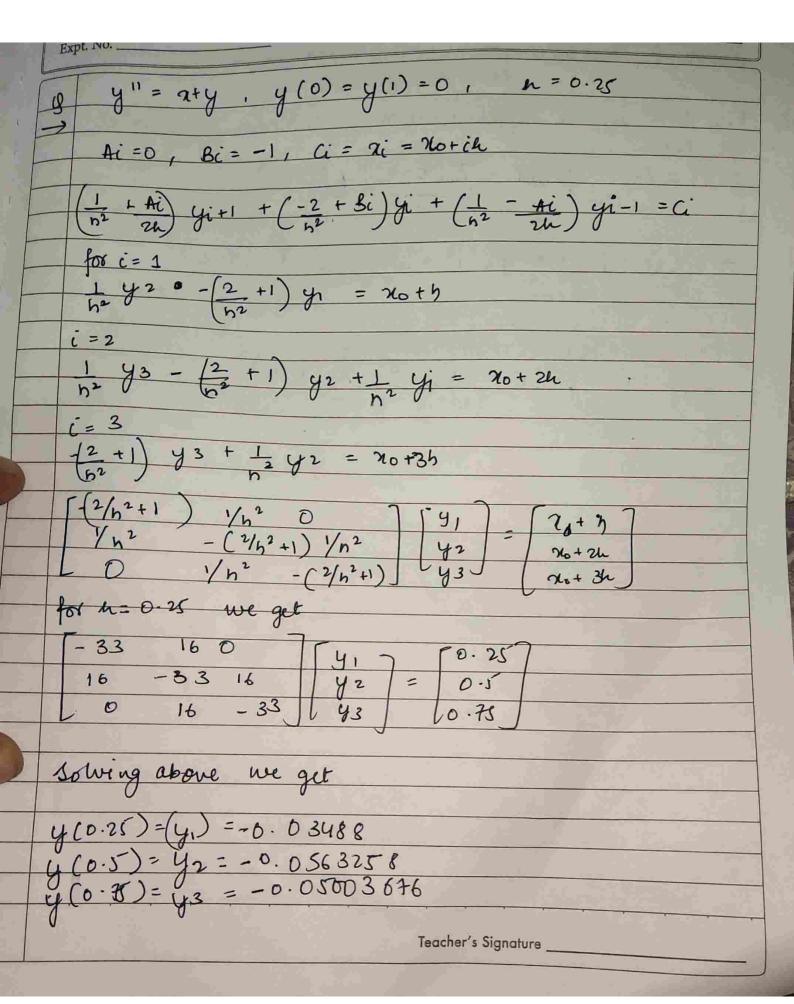
$$6i - \lambda i Y'' = 1$$

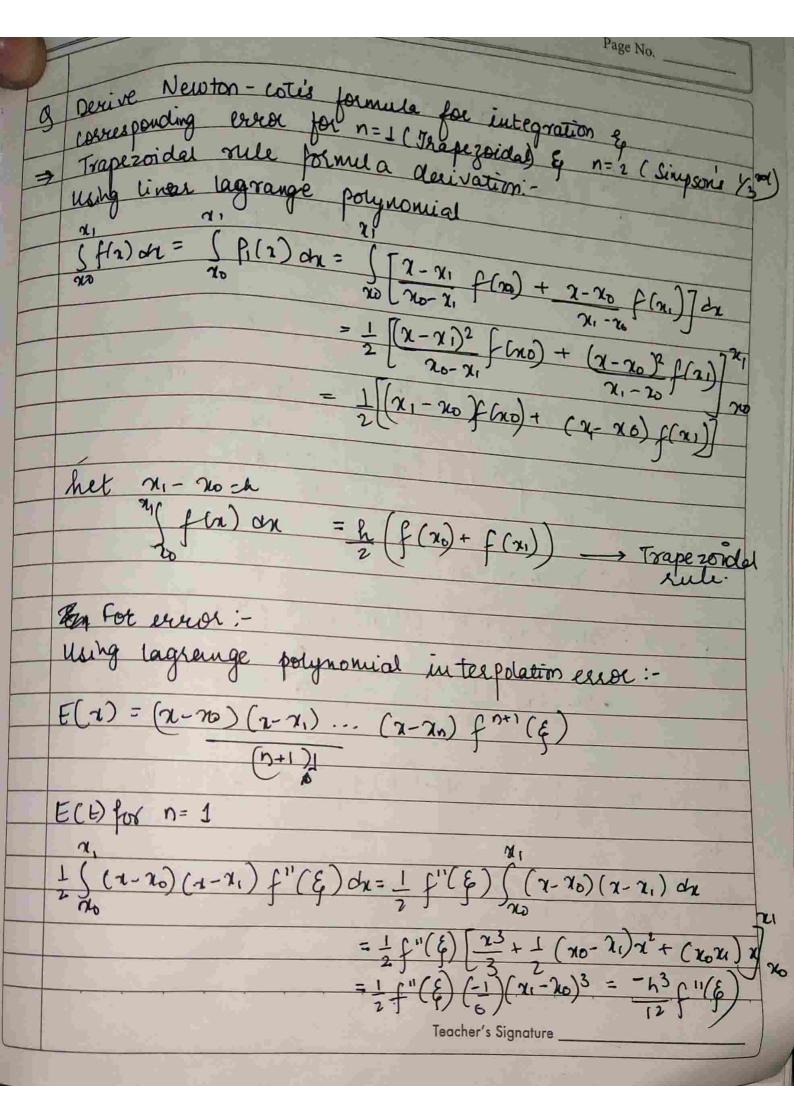
$$8i - \lambda i Y'' = 1$$

for i = 2 to n-1

Solving above we get  $y_{n-1} = C'_{n-1}$  $y_i = C'_i - \delta_i' y_{i+1}$  for i = 1 to n-2

	Page No.
8)	Discretize y"- 2xy'- 2y = -4x , n=0.1, y(0)-y'/0)=0
1	y = 2xy - 2y = -4x,  h = 0.1,  y(0) - y'(0) = 0 $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$ $y = -4x,  h = 0.1,  y(0) - y'(0) = 0$
	2 (1) - y'(1)=1  2nd Order forward difference  yo + 3 yo - 4 yr + y 2 = 0  2 (26 + 3) 1.00 (1)
	- (y + y = 0 - ()
	Backward diff. (2nd Order)  2410 - 410 = 1 => 2410 - 3410 - 449 + 48 = 1
	$\frac{3-1}{2n}\frac{y_{0}+4}{2n}\frac{y_{0}+(2-3)}{2n}\frac{y_{10}=1}{2n} \to 2$ $y_{i}^{"}-2\pi iy_{i}^{"}-2y_{i}^{"}=-4\pi i$
	⇒ yi+1-2yi+yi-1-2xi yi+1-yi-1-2yi = -4xi
	$\frac{1}{5^{2}} \left( \frac{1}{5^{2}} + \frac{1}{5} \right) y_{i-1} + \left( \frac{-2}{5^{2}} - \frac{2}{5} \right) y_{i} + \left( \frac{1}{5^{2}} + \frac{1}{5} \right) y_{i} + = -4x$
	for i-1 to n-1 -(3)
	Using (1), 8p (3), we obtain a tridiagonal  System of (n+1) x (n+1) dimensions, which can  be solved using Thomas algorithm
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	Teacher's Signature





Error of Trapezoidal for mula = -h3 f"(\$) Simpsons  $\frac{1}{3}$  rule derivation:

Taylor enpansion:

To  $f(x) + (x-x_1) f'(x_1) + (x-x_1)^2 f''(x_1) + \frac{(x-x_1)^3}{3!} f'''(x_1)$ No  $f(x) = \int_{10}^{10} \left[ f(x_1) + (x-x_1) f'(x_1) + \frac{(x-x_1)^3}{3!} f'''(x_1) + \frac{(x-x_1)^3}{3!} f''''(x_1) +$ + (2-2.) 4 f " ( = E) ) d1 here  $x_1 = (n_0 + x_2)/2 = (n_0 + x_0 + x_h)/2 = n_0 + 2$  $\int_{ab}^{a} f(x) dx = \left[ f(x_1) x + \left( \frac{x_1 - x_1}{x_1} \right)^2 + \frac{(x_1 - x_1)^3}{3!} f(x_1) + \frac$ No+2h= 712. & 20 + n = 211 Sf(x)dx = 2h f(xi) + h3 f"(xi) + h5 f4(%) =  $2h f(u) + \frac{h^2}{3} \left( \frac{f(u) - 2f(u) + f(u^2)}{h^2} \right) + \frac{h^2}{60} f''(\xi)$ \$ f(1) on = \frac{h}{3} [f(2) + 4f(21)] - \frac{h5}{90} f 4(6) here hs f4(E) is the cros

8. Funcation error for cos & its consistancy

Using Taylor expansion

yi+1 = yi + hyi' + h² y'' +

yi-1 = yi - hyi' + h²²¹ yi" - ...

① - ② , we get

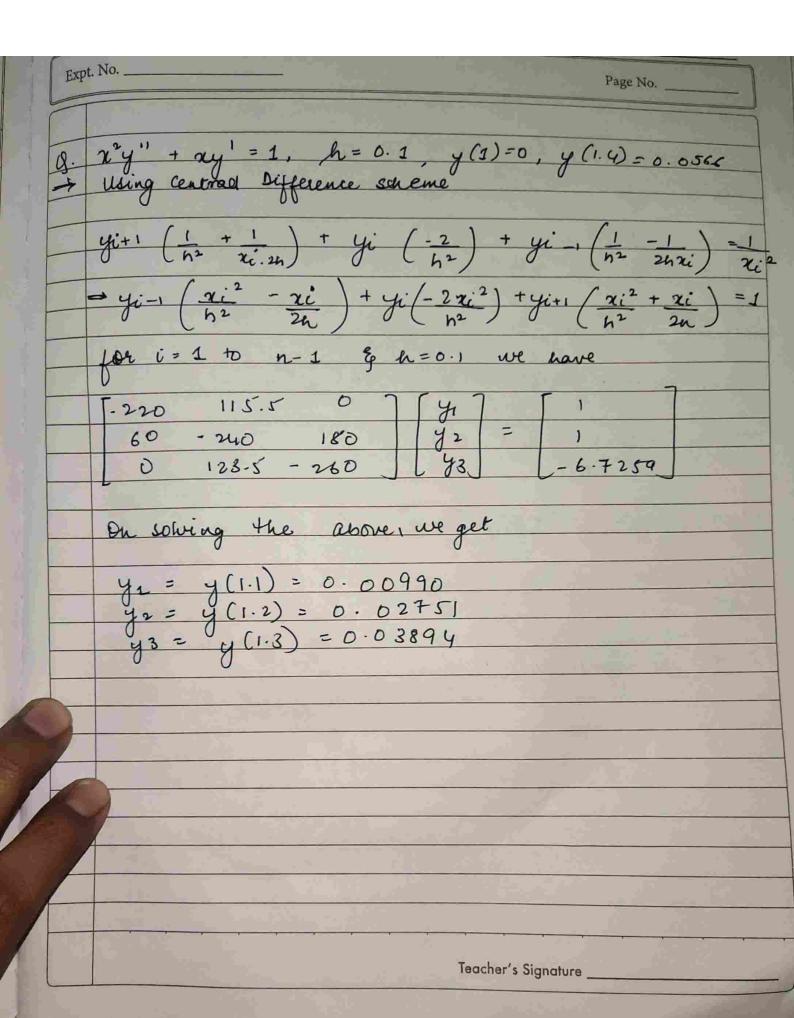
yi+1 - yi-1 = 2hyi' + 2h²yi'' + 2h 5 yi v +...

⇒ yi' = yi+1 - yi-1 + O(h²)

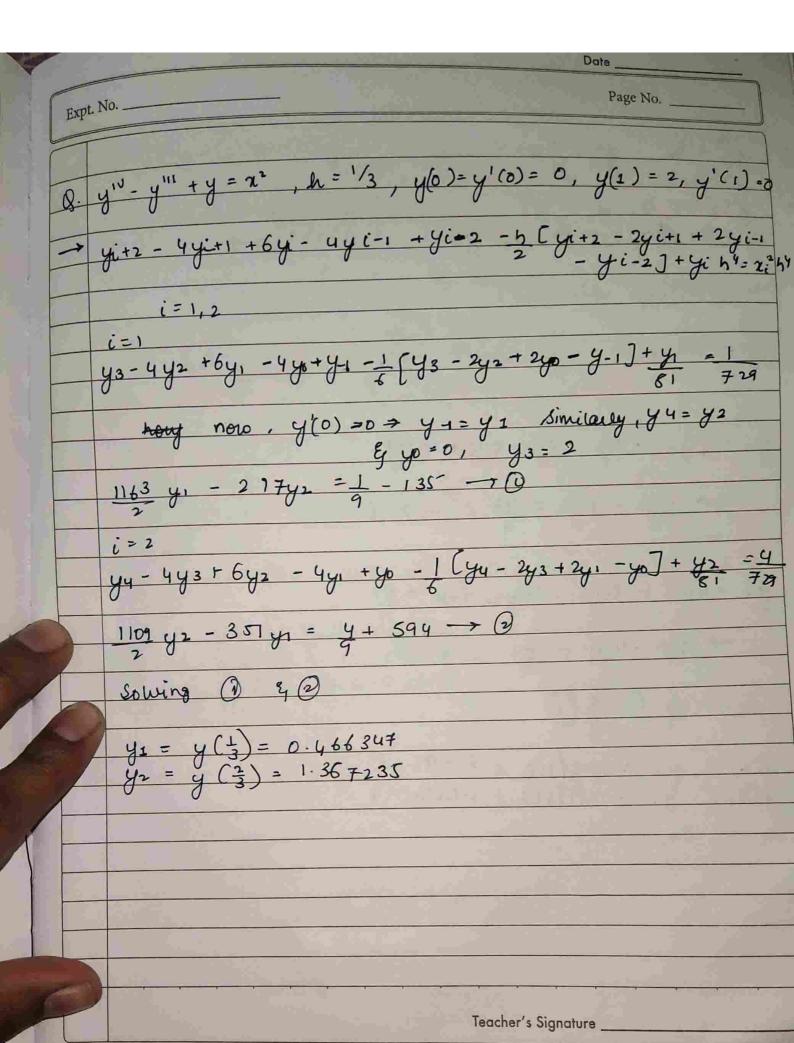
In TE = 0

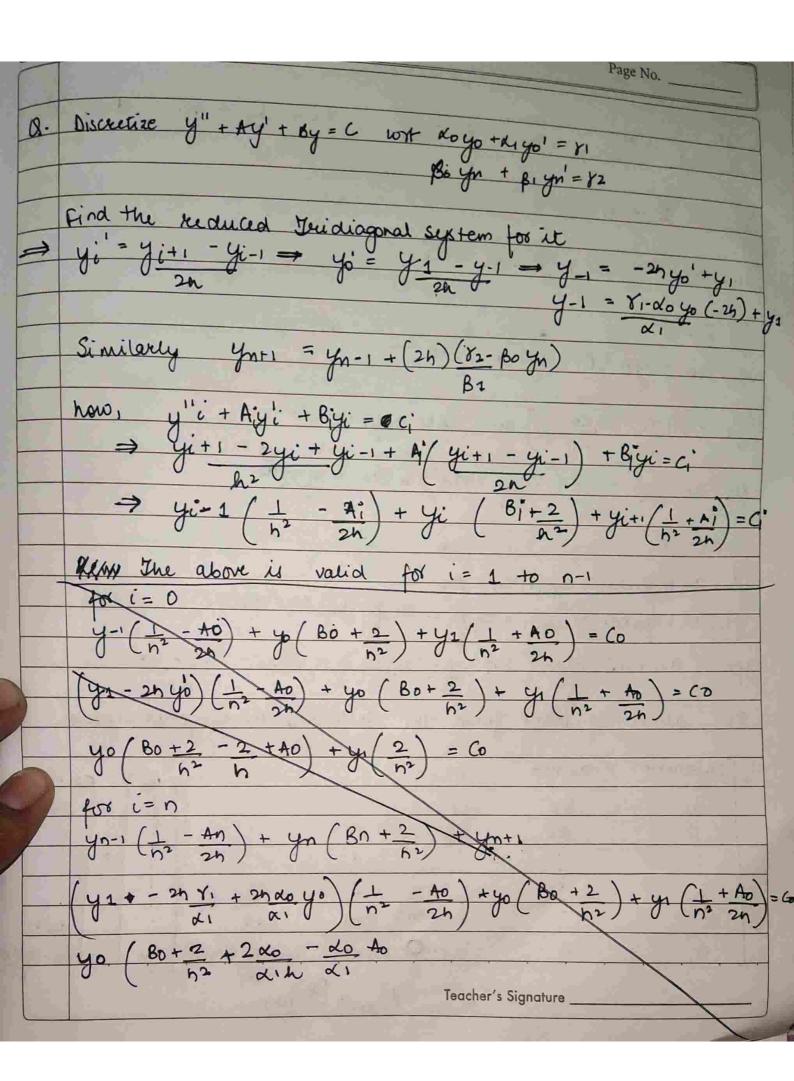
Lin TE = 0

.. TE is consistant



8. 
$$y'' + 2y = \frac{\pi^2}{3} + \frac{2}{3}x + u$$
,  $h = 1$ ,  $y(0) = y'(0) = y(3) = y'(3) = y'($ 





```
bi = Bi - \frac{2}{h^2}, Ci = \frac{1}{h^2} + \frac{Ai}{2h}, di = Ci
 ai = \frac{1}{n^2} - \frac{Ai}{2h},
 for i = 1: to n-1
yi-sai + yibi + yiti Ci = di
for €= 0
y-1 aio + yo bio + y2 co = do
/42 - 2h /1 +2h ×0 yo) ao + yobo + y, co = do
y2 (a0 + c0) + y0 (2hx0 a0+60) = d0 + 2h x1 a0
y'n-1 an + yn bn + yn+1 cn = olin
Yntan+ ynon + (yn++2h /2 - 2h Bo yn) cn = dn
yn-1 (an+cn) + yn (bn - Bo (2h) cn) = aln - 2h /2 cn
The above equations form the following tridiagonal system
                          [A]nxn[y]nxi = [B]nxi
```

Prove the following can be solved using Block Diagonal Method y''' + y'' + y' - 6y = 1, y(0) = y'(0) = 0, y(1) = 1, h=0.2r  $\Rightarrow \text{ Let } z = y' \implies \int_{x_{i-1}}^{z_{i-1}} dx \Rightarrow y_{i-y_{i-1}} = \frac{1}{2} (2i-2i-1)$   $zo = 0, y_0 = 0, z_n = 1 \qquad (n = 4)$ 

 $yi - yi - 1 = \frac{h}{2}(Zi - Zi - 1)$  for i = 1 to n gives n equations

Discretizing z'' + 4z' + 2 - 6y = 1  $\Rightarrow z_{i+1} - 2z_{i} + 2z_{i+1} + y^{2}(z_{i+1} - z_{i-1}) + z_{i} - 6y_{i} = 1$   $h^{2}$ In

 $\Rightarrow$  zi+1-2zi+2i+1+2zi+1-zi-1+2i-6yi-1for i=1+0 n-1 gives n-1 equations

In total we have n+n-1 = 2n-1 equations in variables z1, z2.... zn-1, y1, y2... ynm.

In total we have 2n+ variables of 2n 1 equations.

Hence, we can some this system to fing find the solution to the given problem

we get 
$$\frac{(pi-1+pi)}{2n^2}$$
  $yi-1 + (qi - pi + pi+1 - pi-1+pi)$   $yi$   $+ (pi+pi+1)$   $yi+1 = xi$ 

Dividing throughout by pi

$$\frac{1}{h^{2}} - \frac{Pi - Pi - 1}{2h^{2}Pi} \int yi - 1 + \left(\frac{qi}{pi} - \frac{1}{h^{2}} - \frac{Pi + 1 + Pi - 1}{2h^{2}pi}\right) yi + \left(\frac{1}{h^{2}} + \frac{Pi + 1 - pi}{2h^{2}pi}\right) yi$$

from forward & backward approximation Pi

we know:- Pi-Pi-I ~ pi & piti-pi ~ pi'

h

Mso, Pi+1+ Pi-1 = pi

Using above approximations we get

clearly 3 = 9.

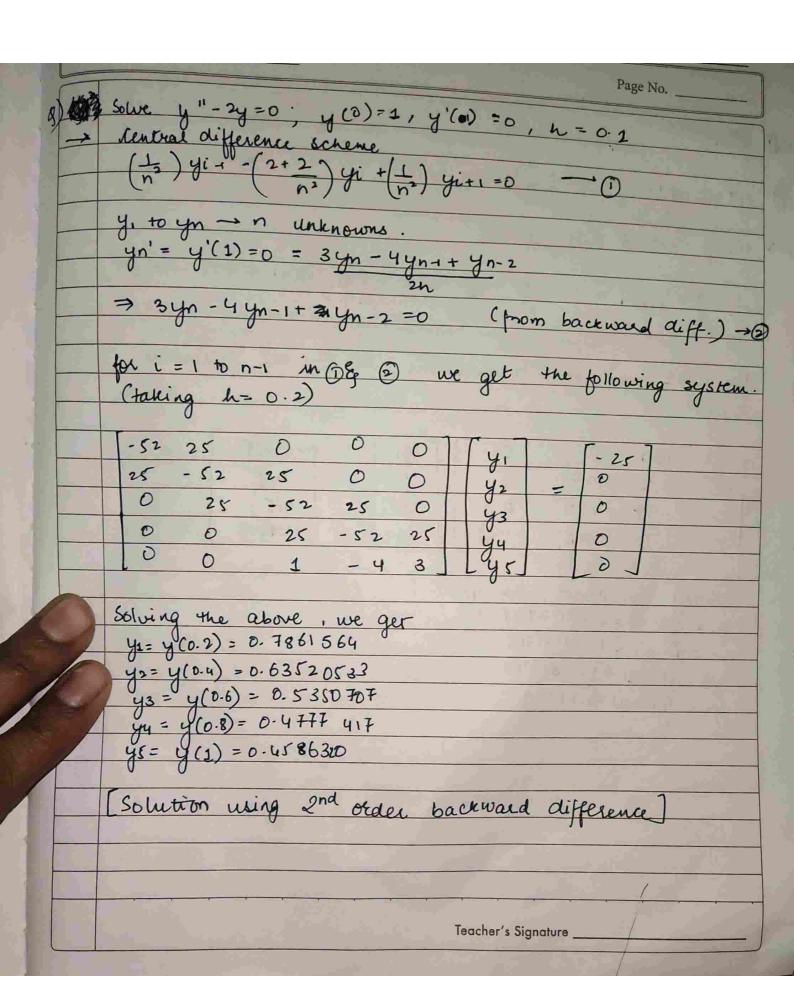
Hence proved

 $y'' + 2\pi y' + 2y = 42; \ y(0) = 1, \ y(0.5) = 1.279, \ h = 0.0$   $\rightarrow \text{ Using Central diff. Sheme}$   $\left(\frac{1}{n^2} - \frac{2\pi i}{2n}\right) y_i - i + \left(2 - \frac{n}{n^2}\right) y_i + \left(\frac{1}{n^2} + \frac{1}{2n}\right) y_i + i = 4\pi i \rightarrow 3$ 

For h=0.1, we have 4 unknowns - we get the following System

$$\begin{bmatrix}
-148 & 10 & 1 & 0 & 0 \\
98 & -198 & 10^2 & 0 \\
0 & 97 & -148 & 10^3 & y^3 \\
0 & 0 & 96 & -148
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4
\end{bmatrix} = \begin{bmatrix}
-493/5 \\
4/5 \\
6/5 \\
-131.416
\end{bmatrix}$$

Solving the above, we get y1 = y(0.1) = 1.0902945 y2 = y(0.2) = 1.16117143 y3 = y(0.3) = 1.25249 y4 = y(0.4) = 1.25249



Using central diffrence scheme yi+1 - 2yi+ yi-1 - 2yi =0 = (1) yi-1 - (2+2) yi+ (1) yi+1=0 ye to yn - n unknowns.  $\frac{y_{n+1}-y_{n-1}=0}{2n} \Rightarrow \frac{y_{n+1}=y_{n-1}}{1} \rightarrow 0$ ficticions pt. for i = 1 to n in 1 & using @ for i = n we get 28 0 0 -52 - 25 -52 25 0 0 0 25 -52 25 0 0 25 - 52 25 0 50 Solving above we get, y = y(0.2) = 0.7865088 y = y(0.4) = 0.63593833 = 4(0.6) = 0.536243 Ju = y(0-8) = 0.4794468 ys = y(1) = 0.46100658 Solution using ficticious pts Teacher's Signature \_