Assignment 5 (NSOPDE) Keertip. Charantimath

19MA20059

1. $\mu_t = \mu_{xx}$ $\quad x \in (0,1),\ t \in (0,\infty)$

$u(x,0) = 2x,\quad u_x(0,t) = 0,\quad u_x(1,t) = 1$

$h, k = 0.5$

Using derivative at $(x_m, t_n + k/2)$ | $(m, n+1/2)$ as

$$u_t\Big|_{(m,n+\frac{1}{2})} \simeq \frac{u_m^{n+1} - u_m^n}{k}$$

$$u_{xx}\Big|_{(m,n+1/2)} \simeq \frac{1}{2}\left\{ u_{xx}\Big|_{(m,n)} + u_{xx}\Big|_{(m,n+1)} \right\}$$

$$\simeq \frac{1}{2}\frac{\left[ u_{m+1}^n - 2u_m^n + u_{m-1}^n + u_{m-1}^{n+1} - 2u_m^{n+1} + u_{m+1}^{n+1} \right]}{h^2}$$

from $u_t = u_{xx}$, $\lambda = k/h^2$

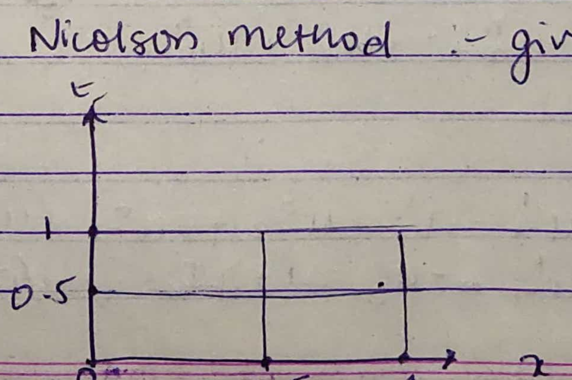
$$\Rightarrow \frac{u_m^{n+1} - u_m^n}{k} = \frac{1}{2h^2}\left[ u_{m+1}^n - 2u_m^n + 2_{m-1}^n + u_{m-1}^{n+1} - 2u_m^{n+1} + u_{m+1}^{n+1} \right]$$

$$2u_m^{n+1} - 2_m^n = \lambda \left[ u_{m+1}^n - 2_m^n + u_{m-1}^n + u_{m-1}^{n+1} - 2u_m^{n+1} + u_{m+1}^{n+1} \right]$$

$$-\lambda u_{m+1}^{n+1} + (2+2\lambda) u_m^{n+1} - \lambda u_{m-1}^{n+1} = \lambda u_{m-1}^n + (2-2\lambda) u_m^n + \lambda u_{m+1}^n$$

Crank Nicolson method :- given $h = k = 1/2 \Rightarrow \lambda = 2$

$u_0^n = 0$




$u_x(0,t) = 0$

$\Rightarrow \boxed{u_1^n = u_{-1}}$

$u_x(1,t) = 1$

$\Rightarrow \boxed{u_3^n = 1 + u_1^n}$

$u_m^0 = 2x_m$

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$\underline{n=0}$

a) $m=1 \Rightarrow -2u_{-1}' + 6u_0' - 2u_1' = 2+0+2$

$6u_0' - 4u_1' = 4$

$\Rightarrow 3u_0' - 2u_1' = 2 \longrightarrow (1)$

b) $m=2 \Rightarrow -2u_0' + 6u_1' - 2u_2' = 0 + 2(1) + 2(2)$

$\Rightarrow -2u_0' + 6u_1' - 2u_2' = 2$

$\Rightarrow -u_0' + 3u_1' - u_2' = 1 \longrightarrow (2)$

c) $m=3 \Rightarrow -2u_1' + 6u_2' - 2u_3' = 2(1) - 2(2) + 2(2)$

$\Rightarrow -4u_1' + 6u_2' = 4$

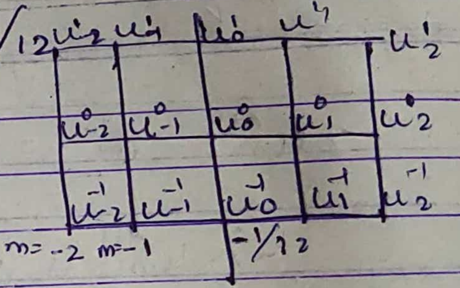
$\Rightarrow -2u_1' + 3u_2' = 2 \longrightarrow (3)$

from (1), (2) & (3)

$u_0' = 8/5$, $u_1'$ $7/5$, $u_2' = 8/5$

$\boxed{\therefore u(0.5, 0.5) = u_1' = 7/5 = 1.4}$

2) $u_t = u_{xx}$, $h = 0.5$, $\lambda = 1/3 \Rightarrow k = 1/12$




$u(x,0) = \cos(x\pi/2)$, $x \in [-1,1]$

$u(-1,t) = u(1,t) = 0$

$u_{-2}^n = u_2^n = 0$, $t > 0$

$u_m^0 = \cos\left(\frac{\pi x_m}{2}\right)$

$\lambda = 1/3$

$\Rightarrow -u_{m-1}^{n+1} + 8_m^{n+1} - u_{m+1}^{n+1} = u_{m-1}^n + 4u_m^n + u_{m+1}^n$

for $n=0$

a) $m=-1 \Rightarrow -u_{-2}^{1} + 8u_{-1}' - u_0' = u_{-2}^0 + 4u_{-1}^0 + u_0^0$

$\Rightarrow -u_{-2}' + 8u_{-1}' - u_0' = 1 + 2\sqrt{2} \longrightarrow (1)$

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b) $m=0 \Rightarrow -u_{-1}^1 + 8u_0^1 - u_1^1 = u_{-1}^0 + 4u_0^0 + u_1^0$

$\Rightarrow -u_{-1}^1 + 8u_0^1 - u_1^1 = 4+\sqrt{2} \longrightarrow (2)$

c) $m=1 \Rightarrow -u_0^1 + 8u_1^1 - u_2^1 = u_0^0 + 4u_1^0 + u_2^0$

$\Rightarrow -u_0^1 + 8u_1^1 - u_2^1 = 1+2\sqrt{2} \longrightarrow (3)$

Also $u_{-2}^n = u_2^n = 0 \longrightarrow (4)$

from (1), (2), (3) & (4).

$u_{-1}^1 = 0.5813 \approx u(-0.5,1)$
$u_{-2}^1 = 0 \approx u(-1,1)$
$u_0^1 = 0.8221 \approx u(0,1)$
$u_2^1 = 0 \approx u(1,1)$
$u_1^1 = 0.5813 \approx u(0.5\,1)$

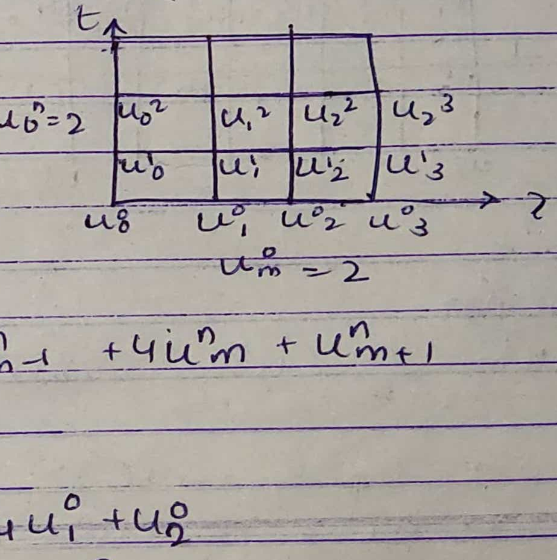
3) $u_t = u_{xx}$, $x \in (0,1)$, $h = 1/3$, $\lambda = 1/3$, $k = 1/27$

$u(x,0) = 2$

$u(0,t) = 2$, $t\,[0,\infty)$

$u_x(1,t) = -u(1,t)$

$\Rightarrow \boxed{u_4^n = u_2^n - 2/3\, u_3^n}$




Crank Nicolson for $\lambda = 1/3$

$\Rightarrow -u_{m-1}^{n+1} + 8u_m^{n+1} - u_{m+1}^{n+1} = u_{m-1}^n + 4u_m^n + u_{m+1}^n$

for $n=0$

$m=1 \Rightarrow -u_0^1 + 8u_1^1 - u_2^1 = u_0^0 + 4u_1^0 + u_2^0$

$\Rightarrow 8u_1^1 - u_2^1 = 14 \longrightarrow (1)$

$m=2 \Rightarrow -u_1^1 + 8u_2^1 - u_3^1 = u_1^0 + 4u_2^0 + u_3^0$

$\Rightarrow -u_1^1 + 8u_2^1 - u_3^1 = 12 \longrightarrow (2)$

$m=3 \Rightarrow -u_2^1 + 8u_3^1 - u_4^1 = u_2^0 + 4u_3^0 + u_4^0$

$\Rightarrow -3u_2^1 + 13u_3^1 = 16 \longrightarrow (3)$

Solving (1), (2) & (3) $\Rightarrow u_0^1 = 2$, $u_1^1 = 1.9950$,

$u_2^1 = 1.9597$, $u_3^1 = 1.6830$

for n=1

m=1 ⇒ $-u_0^2 + 8u_1^2 - u_2^2 = u_0^1 + 4u_1^1 + u_2^1$

⇒ $8u_1^2 - u_2^1 = 13.939622 \longrightarrow$ (4)

m=2 ⇒ $-u_1^2 + 8u_2^2 - u_3^2 = u_1^1 + 4u_2^1 + u_3^1$

⇒ $-u_1^2 + 8u_2^2 - u_3^2 = 11.5169811 \longrightarrow$ (5)

m=3 ⇒ $-u_2^2 + 8u_3^2 - u_4^2 = u_2^1 + 4u_3^1 + u_4^1$

⇒ $-3u_2^2 + 13u_3^2 = 14.294339 \longrightarrow$ (6)

solving (4), (5) & (6) ⇒ $u_0^2 = 2$, $u_1^2 = 1.977$

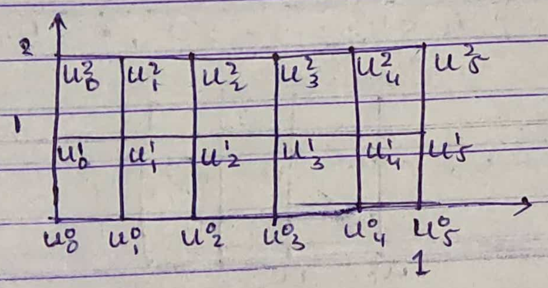
$u_2^2 = 1.8784$, $u_3^2 = 1.533$

4) $u_{tt} = u_{xx}$  $x \in (0,1)$, $t \in (0,\infty)$, $c = 1$, $k = 1$, $h = 1/5$

$u(0,t) = -\sin t = u_0^n$ | $u_t(x,0) = -\cos x$

$u(1,t) = \sin(1-t) = u_1^n$ | $\frac{u_m^1 - u_m^{-1}}{2} = -\cos x_m$  $r = 5$

$u(x,0) = \sin x = u_m^0$




explicit method

$u_m^{n+1} = r^2 u_{m-1}^n + 2(1-r^2)u_m^n + r^2 u_{m+1}^n - u_m^{n-1}$

$= 25u_{m-1}^n - 48u_m^n + 25u_{m+1}^n - u_m^{n-1}$

for n=0

$u_m^1 = \frac{25}{2}(u_{m-1}^0 + u_{m+1}^0) - 24u_m^0 - \cos(m/5)$

m=0 ⇒ $u_0^1 = -\sin(1)$

m=1 ⇒ $u_1^1 = 25/2(u_0^0 + u_2^0) - 24u_1^0 - \cos 1/5)$

$= -0.880401$

Similarly

| | |
|---|---|
| m=0 ⇒ $u_0^1 = -0.8415$ | m=3 ⇒ $u_3^1 = -0.5420$ |
| m=1 ⇒ $u_1^1 = -0.8804$ | m=4 ⇒ $u_4^1 = -0.3368$ |
| m=2 ⇒ $u_2^1 = -0.7257$ | m=5 ⇒ $u_5^1 = 0$ |

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for $n=1$,

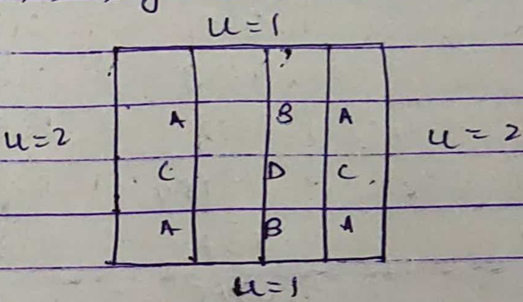
$u^2_m = 25\,(u'_{m-1} + u'_{m+1}) - 48u'_m - \sin(m/5)$

$\therefore$

| | |
|---|---|
| $m=0 \Rightarrow u^2_0 = -0.9093$ | $m=4 \Rightarrow u^2_4 = 1.8988$ |
| $m=1 \Rightarrow u^2_1 = 2.8812$ | $m=5 \Rightarrow u^2_5 = -0.8415$ |
| $m=2 \Rightarrow u^2_2 = -1.1175$ | |
| $m=3 \Rightarrow u^2_3 = -1.1085$ | |

5) $u_{xx} + u_{yy} = x^2 + y^2$, $x \in (-1,1)$, $y(-1,1)$, $h=k=1/2$

from symmetry




$u^1_1 = u^1_3 = u^3_1 = u^3_3 = A$

$u^3_1 = u^2_3 = C$

$u^1_2 = u^3_2 = B$

$u^2_2 = D$

$x_m = \frac{m-2}{2}$, $y_n = \frac{n-2}{2}$

$h^2 f(x,y) = \frac{1}{4}\,\frac{(m-2)^2 + (n-2)^2}{4}$

$u^n_m = \frac{1}{4}\left[u^n_{m-1} + u^n_{m+1} + u^{n-1}_m + u^{n+1}_m - \frac{(m-2)^2 + (n-2)^2}{16}\right]$

| | |
|---|---|
| at $m=1, n=1$ | at $m=2, n=1$ |
| $\Rightarrow 4A - B - C = 23/8$ | $\Rightarrow -2A + 4B - D = 15/16$ |
| at $m=1, n=2$ | at $m=2, n=2$ |
| $\Rightarrow -2A + 4C - D = 31/16$ | $\Rightarrow -B - C - 2D = 0$ |

Gauss siedal iterations for A, B, C, D with initial values 1, 1, 1, 1

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| Iterations | A | B | C | D |
|---|---|---|---|---|
| 1 | 1.2188 | 1.0938 | 1.3438 | 1.2188 |
| 2 | 1.3281 | 1.2031 | 1.4531 | 1.3281 |
| 3 | 1.3828 | 1.2575 | 1.5078 | 1.3828 |
| 4 | 1.4102 | 1.2852 | 1.5352 | 1.4102 |
| 5 | 1.4238 | 1.2988 | 1.5488 | 1.4238 |
| 6 | 1.4307 | 1.3057 | 1.5557 | 1.4307 |
| 7 | 1.4341 | 1.3091 | 1.5591 | 1.4341 |
| 8 | 1.4358 | 1.3108 | 1.5608 | 1.4358 |
| 9 | 1.4366 | 1.3116 | 1.5616 | 1.4366 |
| 10 | 1.4371 | 1.3123 | 1.5621 | 1.4371 |
| 11 | 1.4372 | 1.3124 | 1.5623 | 1.4374 |
| 12 | 1.4373 | 1.3124 | 1.5623 | 1.4374 |
| 13 | 1.4374 | 1.3124 | 1.5624 | 1.4375 |
| 14 | 1.4375 | 1.3125 | 1.5625 | 1.4375 |
| 15 | 1.4375 | 1.3125 | 1.5625 | 1.4375 |

$\Rightarrow u_1^1 = 1.4375, \quad u_1^2 = 1.5625, \quad u_1^3 = 1.4375$

$u_2^1 = 1.3125, \quad u_2^2 = 1.4375, \quad u_2^3 = 1.3125$

$u_3^1 = 1.4375, \quad u_3^2 = 1.5625, \quad u_3^3 = 1.4375$

6) $u_m^{n+1} = 2(1-p^2)u_m^n + p^2(u_{m-1}^n + u_{m+1}^n) - u_m^{n-1}$

$u_{xx} = u_{tt}, \quad x \in (0,1), \quad h = 0.5, k = 0.1, c = 1, p = 0.2$

$u(x,0) = x^2/16 \Rightarrow u_m^0 = m^2/40$

$u_t(x,0) = 0, \quad u_x(0,t) = t/5$

$u(1,t) = \frac{(1+t)^2}{10} \Rightarrow u_1^n = \frac{(n+10)^2}{1000}$

$u_t(x,0) = 0 \Rightarrow u_m^1 = u_m^{-1}$

$u_x(0,t) = t/5 \Rightarrow \frac{u_1^n - u_{-1}^n}{2h} = n/50 \Rightarrow u_{-1}^n = u_1^n - n/50$

for $n=0$

$u_m^1 = 1.92\, u_m^0 + 0.04\,(u_{m-1}^0 + u_{m+1}^0) - \cancel{u_m^{-1}}\ u_m^1$

$\Rightarrow u_0^1 = 0.001, \quad u_1^1 = 0.026, \quad u_2^1 = 0.121$

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for $n=1$

$$u^2_m = 1.92u^1_m + 0.04(u^1_{m-1} + u^1_{m+1}) - u^0_m - m^2/40$$

$$u^0_2 = 0.0032 \approx u(0, 0.2)$$
$$u^2_1 = 0.0298 \approx (0.5, 0.2)$$
$$u^2_2 = 0.1414 \approx (1, 0.2)$$

7) $\delta_t^2 u^n_m = r^2 \delta^2_x [\theta u^{n+1}_m + (1-2\theta) u^n_m + \theta u^{n-1}_m]$

$\theta = 1/2$, $u_{tt} = u_{xx}$, $c = 1$, $h = k = 0.25$, $r = 1$

$u(x,0) = \sin x$

$u(0,t) = -\sin t/5$

$u(1,t) = \sin(1 - t/5)$

$u_t(x,0) = -1/5 \cos x$ $\rightarrow$ $\dfrac{u^1_m - u^{-1}_m}{2k} = -\frac{1}{5}\cos m/4 \Rightarrow u^{-1}_m = u^1_m + \frac{1}{10}\cos(m/4)$

$$\delta^2_t u^n_m = u^{n+1}_m - 2u^n_m + u^{n-1}_m$$

RHS $\Rightarrow$ $r^2 \delta^2_x [\theta u^{n+1}_m + (1-2\theta)u^n_m + \theta u^{n-1}_m]$

$$= \frac{1}{2}[u^{n+1}_{m-1} - 2u^{n+1}_m + u^{n+1}_{m+1} + u^{n-1}_{m-1} - 2u^{n-1}_m + u^{n-1}_{m+1}]$$

now

$$2u^{n+1}_m - 4u^n_m + 2u^{n-1}_m = u^{n+1}_{m-1} - 2u^{n+1}_m + u^{n+1}_{m+1} + u^{n-1}_{m-1} - 2u^{n-1}_m + u^{n-1}_{m+1}$$

$$\Rightarrow \boxed{4u^{n+1}_m - u^{n+1}_{m+1} - u^{n+1}_{m-1} = 4u^n_m - 4u^{n-1}_m + u^{n-1}_{m-1} + u^{n-1}_{m+1}}$$

for $n=0$

$$4u^1_1 - u^1_0 - u^1_2 = 4u^0_1 - 4u^{-1}_1 + u^{-1}_0 + u^{-1}_2$$

a) $4u^1_1 - u^1_2 = 2\sin\frac{1}{4} + \frac{1}{20}(\cos 0 + \cos 2/4 - 4\cos 1/4) - \sin(1/20)$
$= 0.3449253 \rightarrow (1)$

b) $-u^1_1 + 4u^1_2 - u^1_3 = 2\sin 1/2 + 1/20(\cos 1/4 + \cos 3/4 - 4\cos 2/4)$
$= 0.8683646 \rightarrow (2)$

c) $-u^1_2 + 4u^1_3 = 2\sin 3/4 + 1/20(\cos 3/4 + \cos 1 - 4\cos 3/4) + \sin 19/20$
$= 2.1012494 \rightarrow (3)$

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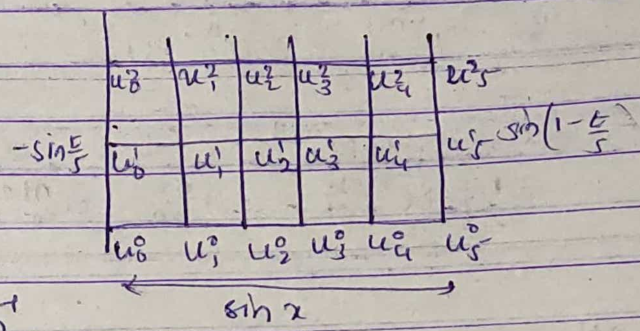
from ①, ② & ③

$u_0^1 = u(0, 0.25) = -0.04998$
$u_1^1 = u(0.25, 0.25) = 0.19194$
$u_2^1 = u(0.5, 0.25) = 0.42283$
$u_3^1 = u(0.75, 0.25) = 0.63102$
$u_4^1 = u(1, 0.25) = 0.813415$

8) $u_{tt} = \frac{1}{25} u_{xx}$ $\quad c = 1/5$

$u(0,t) = -\sin(t/5)$ , $u(x,0) = \sin x$
$u(1,t) = \sin(1 - t/5)$ , $u_t(x,0) = -\frac{1}{5}\cos x$
$u_m^1 - u_m^{-1} = -\frac{2}{5}\cos x_m$

$h = 1/5$, $k = 1$, $r = 1$




$u_m^{n+1} = r^2 u_{m-1}^n + 2(\cancel{1-r^2})u_m^n + r^2 u_{m+1}^n - u_m^{n-1}$

$= u_{m-1}^n + u_{m+1}^n - u_m^{n-1}$

$n = 0$

$u_m^1 = u_{m-1}^0 + u_{m+1}^0 - u_m^{-1}$
$\Rightarrow - u_m^1 - \frac{2}{5}\cos x_m$

for $m = 1, 2, 3, 4$
$u_0^1 = -\sin 1/5 = -0.19867$
$u_5^1 = \sin(1 - 1/5) = 0.717356$
$u_1^1 = (u_0^0 + u_2^0)0.5 - 1/5\cos 1/5 = -0.001304$
$u_2^1 = (u_1^0 + u_3^0)0.5 - 1/5\cos 2/5 = 0.197443$
$u_3^1 = (u_2^0 + u_4^0)0.5 - 1/5\cos 3/5 = 0.388320$
$u_4^1 = (u_3^0 + u_5^0)0.5 - 1/5\cos 4/5 = 0.563715$

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for $n=1$

$$u_m^2 = u_{m-1}^1 + u_{m+1}^1 - u_m^0 = u_{m-1}^1 + u_{m+1}^1 - \sin x_m$$

$$u_0^2 = -\sin(2/5) = -0.38941$$
$$u_5^2 = \sin(1-2/5) = 0.56464$$

for $m = 1, 2, 3, 4,$

$$u_1^2 = u_0^1 + u_2^1 - \sin x_1 = -0.1998949$$
$$u_2^2 = u_1^1 + u_3^1 - \sin x_2 = -0.002402$$
$$u_3^2 = u_2^1 + u_4^1 - \sin x_3 = 0.196516$$
$$u_4^2 = u_3^1 + u_5^1 - \sin x_4 = 0.38832$$

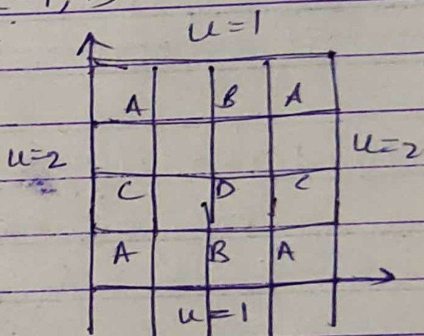
$\therefore$

| | |
|---|---|
| $u_0^2 = -0.3894$ | $u_3^2 = 0.1965$ |
| $u_1^2 = -0.1999$ | $u_4^2 = 0.3883$ |
| $u_2^2 = -0.0024$ | $u_5^2 = 0.5646$ |

9) $u_{xx} + u_{yy} = 8xy \qquad x \in (-1,1) \quad y \in (-1,1)$

at $x = -1, 1, \ u = 2 \qquad h, k = 0.5$

at $y = -1, 1, \ u = 1$




Symmetry:- $u_1^1 = u_3^1 = u_1^3 = u_3^3 = A$

$u_1^2 = u_3^2 = C$

$u_2^2 = D$

$u_2^1 = u_2^3 = B$

$x_m = \frac{m}{2} - 1 = (m-2)/2 \quad | \quad y_n = (n-2)/2$

$$u_m^n = \frac{1}{4}\left[u_{m-1}^n + u_{m+1}^n + u_m^{n+1} + u_m^{n-1} - \frac{(m-2)(n-2)}{2}\right]$$

| | |
|---|---|
| at $n=1, m=1$: $4A - B - C = 5/2$ | at $n=1, m=2$: $-2A + 4B - D = 1$ |
| at $n=2, m=2$: $\cancel{4DA}\ \cancel{D} - B + 2D - C = 0$ | at $n=2, m=\cancel{2}\ 1$: $-2A - D + 4C = 2$ |

Let $A = B = C = D = 1$

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| Iteration | A | B | C | D |
|---|---|---|---|---|
| 1 | 1.125 | 1.0625 | 1.3125 | 1.1875 |
| 2 | 1.2188 | 1.1563 | 1.4063 | 1.2812 |
| 3 | 1.2656 | 1.2031 | 1.4331 | 1.3281 |
| 4 | 1.2891 | 1.2266 | 1.4766 | 1.3576 |
| 5 | 1.3008 | 1.2383 | 1.4883 | 1.3633 |
| 6 | 1.3066 | 1.2441 | 1.4941 | 1.3691 |
| 7 | 1.3096 | 1.2471 | 1.4971 | 1.3721 |
| 8 | 1.3110 | 1.2485 | 1.4985 | 1.3743 |
| 9 | 1.3118 | 1.2496 | 1.4993 | 1.3744 |
| 10 | 1.3121 | 1.2498 | 1.4996 | 1.3746 |
| 11 | 1.3123 | 1.2499 | 1.4998 | 1.3748 |
| 12 | 1.3124 | 1.2450 | 1.4999 | 1.3749 |
| 13 | 1.3125 | 1.2450 | 1.500 | 1.375 |
| 14 | 1.3125 | 1.2480 | 1.500 | 1.375 |

$u_1^1 = 1.3125$ $\quad u_1^2 = 1.5$ $\quad u_1^3 = 1.3125$

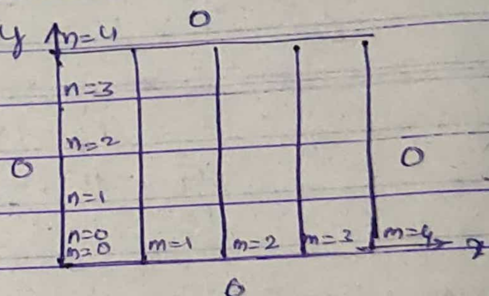
$u_2^1 = 1.25$ $\quad u_2^2 = 1.375$ $\quad u_2^3 = 1.25$

$u_3^1 = 1.3125$ $\quad u_3^2 = 1.5$ $\quad u_3^3 = 1.3125$

10) $u_{xx} + u_{yy} = -2$

$u = 0$ on the boundaries

$h = k = 0.5$




$x_m = -1 + mh = (m-2)/2$

$y_n = (n-2)/2$

$u_m^n = \frac{1}{4}\left[u_{m-1}^n + u_{m+1}^n + u_m^{n+1} + u_m^{n-1} - h^2(-2)\right] \longrightarrow (1)$

Symmetry :-

$u_1^1 = u_3^1 = u_1^2 = u_3^3 = A$

$u_2^2 = B$

$u_1^2 = u_2^1 = u_3^2 = u_2^3 = C$

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from (1)

| | | |
|---|---|---|
| $m=1, n=1$ | $m=2, n=1$ | $m=2, n=2$ |
| $8A-2C=1$ | $-4A-2B+4C=1$ | $8B-8C=1$ |
| $\hookrightarrow$ (2) | $\hookrightarrow$ (3) | $\hookrightarrow$ (4) |

from (2), (3), (4)

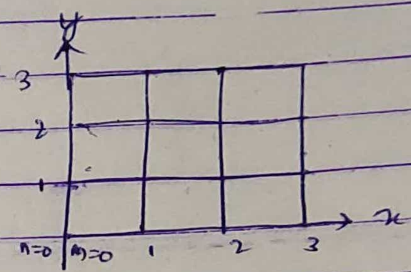
$A = 9/16$ , $C = 7/16$ , $B = 15/8 = 1.875$.

$\boxed{u(0,0) = 1.875}$

11) $u_{xx} + u_{yy} = 0$

$u(x,y) = e^{3x} \cos 3y$ on boundary

$h, k = \frac{1}{3}, \frac{1}{3}$ , $x, y \in (0,1)$



$x_m = m/3$ , $y_n = n/3$

$u_m^n = e^n \cos n$

using $u_1^1 = u_1^2 = u_2^1 = u_2^2 = 0$ as initial ~~gess~~ guess for ~~gst~~ Gauss Siedal iterations

$u_m^n = \frac{1}{4}[u_{m-1}^n + u_{m+1}^n + u_m^{n-1} + u_m^{n+1}]$

| Iterations | $u_1^1$ | $u_2^1$ | $u_1^2$ | $u_2^2$ |
|---|---|---|---|---|
| 1 | 0.8146 | 4.7640 | -0.5731 | -2.8707 |
| 2 | 1.8624 | 4.3082 | -1.0289 | -3.0986 |
| 3 | 1.6345 | 4.1943 | -1.1428 | -3.1555 |
| 4 | 1.5775 | 4.1658 | -1.1713 | -3.1698 |
| 5 | 1.5633 | 4.1587 | -1.1784 | -3.1733 |
| 6 | 1.5597 | 4.1569 | -1.1802 | -3.1742 |
| 7 | 1.5588 | 4.1565 | -1.1808 | -3.1745 |
| 8 | 1.5586 | 4.1564 | -1.1808 | -3.1745 |
| 9 | 1.5585 | 4.1563 | -1.1808 | -3.1745 |
| 10 | 1.5585 | 4.1563 | -1.1808 | -3.1745 |

$\boxed{u_1^1 = 1.5585}$ $\boxed{u_1^2 = -1.1808}$

$\boxed{u_2^1 = 4.1563}$ $\boxed{u_2^2 = -3.1745}$