Department of Mathematics, I I T Kharagpur
MA 31007 Mathematical Methods
Class Test 1, Autumn 2021 **Max. Marks : 20**
Time : 1hr and 30 minutes **No. of Students : 93**

Instruction : Answer **ALL** the questions.

Question 1. Prove that

(a) Solve the following differential equation,

$$xy'' + y' + \frac{y}{4} = 0,$$

by using the substitution $z = \sqrt{x}$.

(b) Prove that

$$\frac{d}{dx}\left\{xJ_n(x)J_{n+1}(x)\right\} = x\left\{J_n^2(x) - J_{n+1}^2(x)\right\}.$$

[3+2 = 5]

Question 2.

(a) Show that $J_n(x) = 0$ has no repeated roots except at $x = 0$.

(b) Express $J_4(x)$ in terms of J_0 and J_1.

[2+3=5]

Question 3.

(a) Prove that

(i) $J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \ldots) = 1$
(ii) $|J_0(x)| \leq 1$
(iii) $|J_n(x)| \leq 2^{-1/2}$, when $n \geq 1$.

(b) Evaluate $\lim\limits_{a,b\to\infty} {}_2F_1(a, b; \frac{1}{2}; \frac{x^2}{4ab})$.

[3+2 = 5]

Question 4.

(a) Find the solution of the following equstion

$$4x(1-x)y'' + y' + 8y = 0, \text{ about } x = 0.$$

(b) Show that
$\cos(x\sin\phi) = J_0 + 2\cos(2\phi)J_2 + 2\cos(4\phi)J_4 + \ldots$.

[3+2 = 5]

********** THE END **********