Assignment 3

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1) $U_{n+2} + 28U_{n+1} - 28U_{n-1} - U_{n-2} = h(12f_{n+1} + 36f_n + 12f_{n-1})$

$\Rightarrow U_{n-2}[E^4 + 28E^3 - 28E - 1] - h[12E^3 + 36E^2 + 12E]f_{n-2} = 0$

$p(E) = E^4 + 28E^3 - 28E - 1$ $\quad p'(E) = 4E^3 + 84E^2 - 28$

$\sigma(E) = 12E^3 + 36E^2 + 12E$

$\sigma(-0.035) = -0.3764$

$\sigma(-27.96) = -234488.78$

$p'(-0.035) = -27.897$

$p'(-27.96) = -21792.24$

for convergence:

$E^4 + 28E^3 - 28E - 1 = 0$

$\Rightarrow E^3(E-1) + 29E^2(E-1) + 29E(E-1) + (E-1) = 0$

$\Rightarrow (E-1)(E^3 + 29E^2 + 29E + 1) = 0$

$\Rightarrow (E-1)\{(E+1)E^2 + 28E(E+1) + (E+1)\} = 0$

$\Rightarrow (E-1)\{(E+1)(E^2 + 28E + 1)\} = 0$

$\Rightarrow (E-1)(E+1)(E^2 + 28E + 1) = 0$

$E = 1, -1, \dfrac{-28 \pm \sqrt{780}}{2}$

$E = 1, -1, -0.085, -27.96$

So, Root Condition is not satisfied on $p(E)$

Hence, method is not convergent.

$\boxed{R_i = \left(\frac{1}{E_i}\right)\frac{\sigma(E_i)}{p'(E_i)}}$

$R_1 = 1$

$R_2 = -1 \times \dfrac{\sigma(-1)}{p'(-1)} = \dfrac{-1 \times (-12 + 36 - 12)}{(-4 + 84 - 28)}$

$R_2 = \dfrac{-1 \times 12}{52} = \dfrac{-12}{52}$

$k_8 = -0.3854$

$k_9 = -0.3848$

Ans

(2) $\rho(E) = E^2 - 1$

Find corresponding linear multistep method (Implicit)

$\rho(E) = (E-1+1)^2 - 1$

$= (E-1)^2 + 1 + 2(E-1) - 1$

$\rho(E) = (E-1)^2 + 2(E-1)$

order 2

$\sigma(E)$ must also be of order 2 for implicit method

$$\frac{\rho(E)}{\ln E} = \frac{(E-1)^2 + 2(E-1)}{\log(E-1+1)}$$

$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots\ldots$

$(1-x)^{-1} = 1 + x$

$$= \frac{(E-1)^2 + 2(E-1)}{(E-1) - \frac{(E-1)^2}{2} + \frac{(E-1)^3}{6} \ldots\ldots}$$

$$= \frac{[(E-1) + 2]}{\left[1 - \frac{(E-1)}{2} + \frac{(E-1)^2}{3} \ldots\ldots\right]}$$

$$= [(E-1)+2]\left[1 - \frac{(E-1)}{2} + \frac{(E-1)^2}{3} \ldots\ldots\right]^{-1}$$

$$= \{(E-1)+2\}\left[1 - \frac{1}{2}(E-1) \ldots\ldots\ldots\right]^{-1}$$

$$= \{(E-1)+2\}\left[1 + \frac{1}{2}(E-1) - \frac{1}{12}(E-1)^2 + \right] \frac{1}{24}(E-1)^3$$

$$\boxed{\sigma(E) = (E-1) + \tfrac{1}{2}(E-1)^2 + 2 + (E-1) - \frac{1}{6}(E-1)^2 + 0(E-1)^3 + O(E-1)^4}$$

$$= (E+1)\left[1 + \frac{(E-1)}{2} - \frac{1}{12}(E^2 + 1 - 2E)\right]$$

$$= (E+1)\left[\frac{E}{2} + \frac{1}{2} - \frac{E^2}{12} - \frac{1}{12} + \frac{E}{6}\right]$$

$$= (E+1)\left\{\frac{2E}{3} - \frac{E^2}{12} + \frac{5}{12}\right\} = \frac{2E^2}{3} + \frac{5}{12}E + \frac{2E}{3} - \frac{E^2}{12} + \frac{5}{12}$$

$$\boxed{\sigma(E) = \frac{7E^2}{12} + \frac{13E}{12} + \frac{5}{12}}$$

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$\sigma(E) = 2(E-1) + \frac{1}{3}(E-1)^2 + 2 = 2E - 2 + 2 + \frac{1}{3}(E^2 + 1 - 2E)$

$= 2E + \frac{E^3}{3} + \frac{1}{3} - \frac{2E}{3}$

$p(E) = E^2 - 1$

$$\boxed{\begin{aligned} \sigma(E) &= \frac{E^2}{3} + \frac{4E}{3} + \frac{1}{3} \\ p(E) &= E^2 - 1 \end{aligned}}$$

for stabality $p(E) = 0$

$E = \pm 1$

as the roots on $|E| = 1$ are simple roots

This method is stable.

$\boxed{\text{Ans}}$

$\Rightarrow (E^2 - 1) U_n - h \left[\frac{E^2}{3} + \frac{4E}{3} + \frac{1}{3}\right] f_n = 0$

$\Rightarrow U_{n+2} - U_n - h \left\{\frac{1}{3} f_{n+2} + \frac{4}{3} f_{n+1} + \frac{1}{3} f_n\right\} = 0$

$\Rightarrow U_{n+2} = U_n + \frac{h}{3}(f_{n+2} + 4f_{n+1} + f_n)$

Ans

Milne - Simpson method of Order 4

3) $U' = U + t$

$U(0) = 1$

Find $u(0.1)$

$\frac{dU}{dt} = U + t = f(U, t)$

$\frac{d^2U}{dt^2} = \frac{dU}{dt} + 1 = U + t + 1$

$\frac{d^3U}{dt^3} = \frac{dU}{dt} + 1 = U + t + 1$

By Taylor Series method: (Third Order)

$U(0.1) = U(0) + \frac{h}{1} \times U'(0) + \frac{h^2}{2} \times U''(0) + \frac{h^3}{6} \times U'''(0)$

$= 1 + h \times 1 + \frac{h^2}{2} \times 2 + \frac{h^3}{6} \times 2$

$= 1 + 0.1 + h^2 + \frac{h^3}{6}$

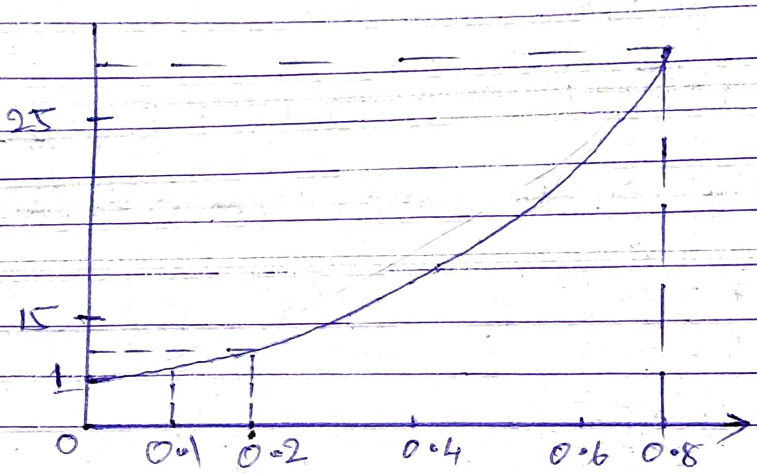
$$\boxed{U(0.1) = 1.1103} = \alpha$$

$u(0.2) = u(0) + \frac{h}{3}\{f_{0.2} + 4f_{0.1} + f_0\}$

We assumed $Y_{0.2}^{(0)} = Y(0.1) = 1.1103$
and
the completed 6 iterations to
correct $Y_{0.2}$

$U_{0.2} = Y_{0.2}^{(6)} = 1.24280460 = B$ Ans

Similarly
$U_{0.3} = 1.399970868 = 8$
Ans



Solution Plot obtained

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(4) ~~$U_{n+3} = U_n + \frac{3h}{8}[U_n' + 3U_{n+1}' + 3U_n$~~

$$U_{n+3} = U_n + \frac{3h}{8}\left[U_n' + 3U_{n+1}' + 3U_{n+2}' + U_{n+3}'\right]$$

$$U_n(E^3 - 1) - h\left[\frac{3}{8} + \frac{9}{8}E + \frac{9}{8}E^2 + \frac{3}{8}E^3\right]U_n' = 0$$

$p(\varepsilon) = \varepsilon^3 - 1$

$\varepsilon^3 - 1 = 0$

$\varepsilon = 1, \omega, \omega^2$

$k_1 = \frac{1}{\varepsilon_1}\frac{\sigma(\varepsilon_1)}{p'(\varepsilon_1)}$

$\boxed{k_1 = 1}$,

$\boxed{|\omega| = |\omega^2| = 1}$

But they are simple roots

$\sigma(\omega^2) = \frac{3}{8}(1 + 3\omega^2 + 3\omega + \omega^6)$

$= \frac{3}{8}(1 + 3\omega^2 + 3\omega + 1)$

$= \frac{3}{8}(2 + 3\omega^2 + 3\omega)$

$\sigma(\varepsilon) = \frac{3}{8}\{1 + 3\varepsilon + 3\varepsilon^2 + \varepsilon^3\}$

$p'(\omega^2) = 3\omega^4 = 3\omega$

$p'(\varepsilon) = 3\varepsilon^2$

$\sigma(\omega) = \frac{3}{8}(1 + 3\omega + 3\omega^2 + \omega^3)$

$= \frac{3}{8}(2 + 3\omega + 3\omega^2)$

$p'(\omega) = 3\omega^2$

$1 + \omega + \omega^2 = 0$

$k_2 = \frac{1}{\omega}\frac{(2 + 3\omega + 3\omega^2)}{8}$

$= \frac{2 + 3\omega + 3\omega^2}{8}$

$= \frac{2 + 3\omega + 3(-1-\omega)}{8} = \frac{2 + 3\omega - 3 - 3\omega}{8} = -\frac{1}{8}$

$k_3 = \frac{1}{\omega^2} \times \frac{\frac{3}{8}(2 + 3\omega^2 + 3\omega)}{3\omega} = \frac{2 + 3\omega^2 + 3\omega}{8}$

$\boxed{k_3 = -\frac{1}{8}}$ $\boxed{k_2 = -\frac{1}{8}}$

$\boxed{k_1 = 1}$

Ans

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Find interval of Absolute stability of :-

$$U_{n+1} = U_n + \frac{h}{12}\left[5U_{n+1}' + 8U_n' - U_{n-1}'\right]$$

for the IVP $y' = -y$
$y(x_0) = y_0$

$$[E^2 - E]U_{n-1} - h\left[\frac{5}{12}E^2 + \frac{8}{12}E - \frac{1}{12}\right]U'_{n-1} = 0$$

$$(E^2-E)U_{n-1} + h\left[\frac{5}{12}E^2 + \frac{8E}{12} - \frac{1}{12}\right]U_{n-1} = 0$$

$$\left(E^2 - E + \frac{5h}{12}E^2 + \frac{8h}{12}E - \frac{h}{12}\right)U_{n-1} = 0$$

$$\boxed{E^2\left(1+\frac{5h}{12}\right) + E\left(\frac{2h}{3} - 1\right) - \frac{h}{12} = 0}$$

Characteristic stabality EqD

Replace E by $\left(\frac{1+z}{1-z}\right)$
we get,

$$\frac{(1+z)^2}{(1-z)^2}\left(1+\frac{5h}{12}\right) + \frac{(1+z)}{(1-z)}\left(\frac{2h}{3}-1\right) - \frac{h}{12} = 0$$

$$\Rightarrow (1+z)^2\left(1+\frac{5h}{12}\right) + (1-z^2)\left(\frac{2h}{3}-1\right) - \frac{h}{12}(1-z)^2 = 0$$

$$\Rightarrow (1+z^2+2z)\left(1+\frac{5h}{12}\right) + \frac{2h}{3} - 1 - \left(\frac{2h}{3}\right)z^2 + z^2 - \frac{h}{12}(1+z^2-2z) = 0$$

$$\Rightarrow 1 + \frac{5h}{12} + z^2 + \frac{5h}{12}z^2 + 2z + \left(\frac{10h}{12}\right)z + \left(\frac{2h}{3}\right) - 1 - \left(\frac{2h}{3}\right)z^2 + z^2 - \frac{h}{12}$$
$$- \left(\frac{h}{12}\right)z^2 + \left(\frac{h}{6}\right)z = 0$$

$$\Rightarrow z^2\left\{1 + \frac{5h}{12} - \frac{2h}{3} + 1 - \frac{h}{12}\right\} + z\left\{2 + \frac{10h}{12} + \frac{h}{6}\right\}$$
$$+ \left(\cancel{1} + \frac{5h}{12} + \frac{2h}{3}\cancel{-1} - \frac{h}{12}\right) = 0$$

$$\Rightarrow z^2\left(2 - \frac{4h}{12}\right) + z\left(2 + \frac{12h}{12}\right) + \frac{12h}{12} = 0$$

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So, $a_0 = 2 - \frac{h}{3}$

$a_1 = 2 + h$

$a_2 = h$

Applying Routh-Hurwitz Criterion:

$a_0 > 0 \Rightarrow 2 - \frac{h}{3} > 0 \Rightarrow \frac{h}{3} < 2 : \boxed{h < 6}$

$a_1 > 0 \Rightarrow 2 + h > 0 \Rightarrow \boxed{h > -2}$

$a_2 > 0 \Rightarrow \boxed{h > 0}$

$\therefore \boxed{h \in (0, 6)}$

for absolute stabality

Ans

(6)

i) $\Delta^2 U_n - 3\Delta U_n + 2U_n = 0$

$U_{n+2} - 3U_{n+1} + 2U_n = 0$

$E^2 - 3E + 2 = 0$

$E^2 - 2E - E + 2 = 0$

$E(E-2) - (E-2) = 0$

$E = 1, 2$

$\boxed{U_n = C_1 \times (1)^n + C_2 \times (2)^n}$

Ans

ii) $\Delta^2 U_n + \Delta U_n + \frac{1}{4} U_n = 0$

$E^2 + E + \frac{1}{4} = 0$

$4E^2 + 4E + 1 = 0$

$(2E)^2 + 2.2E.1 + (1)^2 = 0$

$(2E+1)^2 = 0$

$E = -\frac{1}{2}, -\frac{1}{2}$

$\boxed{U_n = (C_1 + C_2 n)\left(-\frac{1}{2}\right)^n}$

Ans

6) iii) $\Delta^2 U_n - 2\Delta U_n + 2U_n = 0$

$E^2 - 2E + 2 = 0$

$E = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$

$U_n = (C_1 \cos n\theta + C_2 \sin n\theta)(\sqrt{2})^n$

where $\theta = \tan^{-1}(1) = \frac{\pi}{4}$

$\boxed{U_n = \left(C_1 \cos \frac{n\pi}{4} + C_2 \sin \frac{n\pi}{4}\right)(\sqrt{2})^n}$

Ans

6) iv) $\Delta^2 U_{n+1} - \frac{1}{3}\Delta^2 U_n = 0$

$$U_{n+3} - \frac{1}{3} U_{n+2} = 0$$

Let $U_n = A\varepsilon^n$

$$A\varepsilon^{n+3} - \frac{1}{3} A\varepsilon^{n+2} = 0$$

$$\varepsilon^{n+3} - \frac{1}{3}\varepsilon^{n+2} = 0$$

$$\varepsilon - \frac{1}{3} = 0 \qquad \boxed{\varepsilon = \frac{1}{3}}$$

$$U_n = C_1\left(\frac{1}{3}\right)^n$$

Ans

7) $(1-5\alpha)Y_{n+2} - (1+8\alpha)Y_{n+1} + \alpha Y_n = 0$

i) $(1-5\alpha)\varepsilon^2 - (1+8\alpha)\varepsilon + \alpha = 0$

Roots of this Eq^n should be less than 1 in magnitude

$D = (1+8\alpha)^2 - 4\alpha(1-5\alpha)$

$= (1+64\alpha^2) + 16\alpha - 4\alpha + 20\alpha^2$

$D = 84\alpha^2 + 12\alpha + 1 > 0$ [Always greater than 0]

So, roots are real and distinct

Putting $\varepsilon = \frac{1+z}{1-z}$

$$(1-5\alpha)\frac{(1+z)^2}{(1-z)^2} - (1+8\alpha)\frac{(1+z)}{(1-z)} + \alpha = 0$$

$$(1-5\alpha)(1+z)^2 - (1+8\alpha)(1-z^2) + \alpha(1-z)^2 = 0$$

$$(1-5\alpha)(1+z^2+2z) - (1-8\alpha)(1-z^2) + \alpha(1+z^2-2z) = 0$$

$$(1-5\alpha)(1+z^2+2z) - 1 + z^2 - 8\alpha + (8\alpha)z^2 + \alpha(1+z^2-2z) = 0$$

$$(1-5\alpha)(1+z^2+2z) - 1 + z^2 - 8\alpha + (8\alpha)z^2 + \alpha$$

$$1+z^2+2z-5\alpha-5\alpha z^2-(10\alpha)z - 1 + z^2 - 8\alpha + (8\alpha)z^2 + \alpha$$
$$+ \alpha z^2 - (2\alpha)z = 0$$

$$z^2(2+4\alpha) + z(2-12\alpha) + (1-5\alpha-1-8\alpha+\alpha) = 0$$

$$\boxed{z^2(2+4\alpha) + z(2-12\alpha) - 12\alpha = 0}$$

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Applying Routh - Hurwitz Criterion

$2+4\alpha>0 \Rightarrow 4\alpha>-2 \Rightarrow \boxed{\alpha>-\frac{1}{2}}$

$2-12\alpha>0 \Rightarrow 12\alpha<2 \Rightarrow \boxed{\alpha<\frac{1}{6}}$

$-12\alpha>0 \Rightarrow \boxed{\alpha<0}$

$\therefore \boxed{\alpha \in (-\frac{1}{2}, 0)}$

Ans

7) ii)

$(1-9\alpha) y_{n+3} - (1+19\alpha) y_{n+2} + 5\alpha y_{n-1} - \alpha y_n = 0$

$(1-9\alpha)\xi^4 - (1+19\alpha)\xi^3 + 5\alpha - \alpha\xi = 0$

$$\boxed{(1-9\alpha)\xi^4 - (1+19\alpha)\xi^3 - (\alpha)\xi + 5\alpha = 0}$$

Replace $\xi = \frac{1+z}{1-z}$

$$(1-9\alpha)\frac{(1+z)^4}{(1-z)^4} - (1+19\alpha)\frac{(1+z)^3}{(1-z)^3} - \alpha\frac{(1+z)}{(1-z)} + 5\alpha = 0$$

$$\Rightarrow (1-9\alpha)(1+z)^4 - (1+19\alpha)(1+z)^3(1-z) - \alpha(1+z)(1-z)^3 + 5\alpha(1-z)^4 = 0$$

$$\Rightarrow (1-9\alpha)\{1+2z+z^2\}^2 - (1+19\alpha)\{(1+z^2+2z)(1-z^2)\} - \alpha(1-2z+z^2)(1-z^2) + 5\alpha(1-2z+z^2)^2 = 0$$

$$\Rightarrow (1-9\alpha)\{1+4z^2+z^4+4z+4z^3+2z^2\} - (1+19\alpha)\{1-z^2+z^2-z^4+2z-2z^3\} - \alpha\{1-z^2-2z+2z^3+z^2-z^4\} + 5\alpha\{1+4z^2+z^4+2z^2-4z^3-4z\} = 0$$

$$\Rightarrow (1-9\alpha)(1+z^4+4z^3+6z^2+4z) - (1+19\alpha)(-z^4-2z^3+2z+1) - \alpha(-z^4+2z^3-2z+1) + 5\alpha(z^4-4z^3+6z^2-4z+1) = 0$$

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$$\Rightarrow 1+z^4+4z^3+6z^2+4z-9\alpha-(9\alpha)z^4-(36\alpha)z^3-(54\alpha)z^2$$
$$-(36\alpha)z \quad +z^4+2z^3-2z-1+(19\alpha)z^4+(38\alpha)z^3$$
$$-(38\alpha)z-19\alpha+\alpha z^4-(2\alpha)z^3+(2\alpha)z-\alpha$$
$$+(5\alpha)(z^4)-(20\alpha)z^3+(30\alpha)z^2-(20\alpha)z+5\alpha=0$$

$$\Rightarrow z^4\{2-9\alpha+19\alpha+\alpha+5\alpha\}+z^3\{4-36\alpha+2+38\alpha-2\alpha-20\alpha\}$$
$$+z^2\{6-54\alpha+30\alpha\}+z\{4-36\alpha-2-38\alpha+2\alpha-20\alpha\}$$
$$+(-9\alpha-19\alpha-\alpha+5\alpha)=0$$
$$\Rightarrow \underset{a_0}{z^4(2+16\alpha)}+\underset{a_1}{z^3(6-20\alpha)}+\underset{a_2}{z^2(6-24\alpha)}+\underset{a_3}{z(2-92\alpha)}+\underset{a_4}{(-24\alpha)}=0$$

Applying Routh - Hurwitz

$$\begin{vmatrix} a_1 & a_3 & 0 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} \qquad \begin{bmatrix} a_1 & a_3 & 0 & 0 \\ a_0 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & 0 \\ 0 & a_0 & a_2 & a_4 \end{bmatrix}$$

$2+16\alpha>0$ (i)

$6-20\alpha>0$ (ii)

$6-24\alpha>0$ (iii)

$2-92\alpha>0$ (iv) $\boxed{(6-20\alpha)(6-24\alpha)-(2-92\alpha)(2+16\alpha)>0}$

$-24\alpha>0$ (v)

$$a_1(a_2a_3-a_1a_4)-a_0(a_3^2)>0 \qquad a_1a_2-a_3a_0>0$$
$$a_1a_2a_3-a_1^2a_4-a_0a_3^2>0$$

$$a_1a_2a_3-a_1^2a_4-a_0a_3^2>0 \qquad \begin{matrix} a_0 & a_1 & a_2 & a_3 \\ a_1 & a_3 & 0 \\ a_0 & a_2 & 0 \\ 0 & a_1 & a_3 \end{matrix}$$
$$(6-20\alpha)(6-24\alpha)(2-92\alpha)+24(6-20\alpha)^2\alpha$$
$$-(2+16\alpha)(2-92\alpha)^2>0 \text{ (vi)}$$

$2+16\alpha>0 \Rightarrow \boxed{\alpha>-1/8}$

$6-20\alpha>0 \Rightarrow \boxed{\alpha<6/20=\frac{3}{10}}$

$24\alpha<6 \Rightarrow \boxed{\alpha<\frac{1}{4}}$

$92\alpha<2 \Rightarrow \boxed{\alpha<\frac{1}{46}}$

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$\alpha < 0 \Rightarrow \boxed{\alpha < 0}$

$\therefore \boxed{\alpha \in \left(-\frac{1}{8}, 0\right)}$

For $\alpha = 0$,

Condition (vi) > 0

for $\alpha = -\frac{1}{8}$,

$$(vi) = \left(6 + \frac{20}{8}\right)(6+3)\left(2 + \frac{92}{8}\right) - \cancel{24}^{3}\left(6 + \frac{20}{8}\right)^2$$

$$= \left(6 + \frac{5}{2}\right) 9 \times \left(2 + \frac{92}{8}\right) - 3\left(6 + \frac{5}{2}\right)^2$$

$$= 816 > 0$$

So, Condition (vi) is automatically satisfied in $\left(-\frac{1}{8}, 0\right)$

So, $\boxed{\alpha \in \left(-\frac{1}{8}, 0\right)}$

Ans

8) $Y_{\delta+1} - 2\lambda y_{\delta} + Y_{\delta-1} = 0$ ← Characteristic Stability Eq^n

dot $Y_{\delta} = A\, \varepsilon^{\delta}$

$\boxed{\varepsilon^2 - 2\lambda\varepsilon + 1 = 0}$

$\varepsilon^2 + 1 = 0$

$\boxed{\varepsilon = \pm i}$

$$\varepsilon = \frac{2\lambda \pm \sqrt{4\lambda^2 - 4}}{2} = \frac{2\lambda \pm 2\sqrt{\lambda^2 - 1}}{2}$$

$$\boxed{\varepsilon = \lambda \pm \sqrt{x^2 - 1}}$$

So, $\varepsilon = \lambda \pm i\sqrt{1 - \lambda^2}$

So, for $\boxed{-1 < \lambda < 1}$

$\boxed{1 - \lambda^2 > 0}$

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$$\boxed{U_n = (C_1 \cos n\theta + C_2 \sin n\theta)\,|\varepsilon_1|^n} \quad |\varepsilon| = \sqrt{\lambda^2 + (1-\lambda^2)} = 1$$

bounded

So, for $-1 < \lambda < 1$

$$\boxed{U_n = C_1 \cos n\theta + C_2 \sin n\theta}$$

Bounded

$|\varepsilon_1| = |\varepsilon_2| = 1$ and $\varepsilon_1, \varepsilon_2$ are complex conjugates

So, the solution remains bounded as $|\varepsilon_1| = |\varepsilon_2| = 1$ and both are simple roots

(9)

$$U_{j+1} = \frac{4}{3} U_j - \frac{1}{3} U_{j-1} + \frac{2}{3} h\, U'_{j+1}$$

$$\left(E^2 - \frac{4}{3}E + \frac{1}{3}\right) U_{j-1} - \frac{2}{3} h \times \lambda U_{j+1} = 0$$

$$\left(E^2 - \frac{4E}{3} + \frac{1}{3}\right) U_{j-1} - \frac{2h}{3} \lambda U_{j+1} = 0$$

$$\left(E^2 - \frac{4E}{3} + \frac{1}{3}\right) U_{j-1} - \frac{2h\lambda}{3} [E^2] U_{j-1}$$

$$U_{j-1} \left\{ E^2 - \frac{4E}{3} + \frac{1}{3} - \frac{2(h\lambda)}{3} E^2 \right\} = 0$$

$$E^2 - \left(\frac{2\bar{h}}{3}\right) E^2 - \frac{4E}{3} + \frac{1}{3} = 0$$

$$\boxed{\varepsilon^2 \left\{ 1 - \frac{2\bar{h}}{3} \right\} - \frac{4\varepsilon}{3} + \frac{1}{3} = 0}$$

Replace $\left(\varepsilon = \frac{1+z}{1-z}\right)$ $\quad \boxed{\varepsilon = \frac{1+z}{1-z}}$

$$\frac{(1+z)^2}{(1-z)^2}\left(1 - \frac{2\bar{h}}{3}\right) - \frac{4}{3}\frac{(1+z)}{(1-z)} + \frac{1}{3} = 0$$

$$(1+z)^2\left(1 - \frac{2\bar{h}}{3}\right) - \frac{4}{3}(1-z^2) + \frac{1}{3}(1+z^2-2z) = 0$$

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$$(1+z^2+2z)\left(1-\frac{2\bar{h}}{3}\right) - \frac{4}{3} + \frac{4}{3}z^2 + \frac{1}{3} + \frac{1}{3}z^2 - \frac{2z}{3} = 0$$

$$1-\frac{2\bar{h}}{3} + z^2 - \left(\frac{2\bar{h}}{3}\right)z^2 + 2z - \left(\frac{4\bar{h}}{3}\right)^2 - \frac{4}{3} + \frac{4z^2}{3} + \frac{1}{3} + \frac{z^2}{3}$$

$$-\frac{2z}{3} = 0$$

$$z^2\left\{1-\frac{2\bar{h}}{3}+\frac{4}{3}+\frac{1}{3}\right\} + z\left\{2-\frac{4\bar{h}}{3}-\frac{2}{3}\right\} + \left\{1-\frac{2\bar{h}}{3}-\frac{4}{3}+\frac{1}{3}\right\} = 0$$

$$z^2\left(\frac{8}{3}-\frac{2\bar{h}}{3}\right) + z\left(\frac{4}{3}-\frac{4\bar{h}}{3}\right) + \left(-\frac{2\bar{h}}{3}\right) = 0$$

for absolute stability

$$\frac{8}{3} - \frac{2\bar{h}}{3} > 0 \Rightarrow 2\bar{h} < 8 \Rightarrow \boxed{\bar{h} < 4}$$

$$\frac{4}{3} - \frac{4\bar{h}}{3} > 0 \Rightarrow \boxed{\bar{h} < 1}$$

$$-\frac{2\bar{h}}{3} > 0 \Rightarrow \boxed{\bar{h} < 0}$$

So, $\boxed{\bar{h} \in (-\infty, 0)}$

Absolutely Stable on $(-\infty, 0)$

So, method is A - stable (proved)

10)

$$U_{j+1} - (1+a)U_j + aU_{j-1} - h\left[\left\{\frac{1}{2}(1+a)+b\right\}U_{j+1}' + \left\{\frac{1}{2}(1-3a) - 2b\right\}U_j' + bU_{j-1}'\right] = 0$$

$$\boxed{U' = \lambda U}$$

~~$U_{j-1}[E^2 - (1+a)$~~

$$U_{j-1}\left[E^2 - (1+a)E + a\right] - \bar{h}\left[\left(\frac{1+a}{2}+b\right)E^2 + \left[\frac{1}{2}(1-3a) - 2b\right]E + b\right] U_{j-1} = 0$$

Characteristic Eqn

$$E^2 - (1+a)E + a - \bar{h}\left(\frac{1+a}{2} + b\right)E^2 + -\bar{h}\left(\frac{1-3a}{2} - 2b\right)E$$
$$- \bar{h}b = 0$$

$$\boxed{E^2\left\{1 - \bar{h}\left(\frac{1+a}{2} + b\right)\right\} + E\left\{-(1+a) + 2b - \left(\frac{1-3a}{2}\right)\right\} + (a - \bar{h}b) = 0}$$

Replace $\boxed{E = \frac{1+z}{1-z}}$

$$\Rightarrow \frac{(1+z)^2}{(1-z)^2}\left\{1 - \bar{h}\left(\frac{1+a}{2} + b\right)\right\} + \left(\frac{1+z}{1-z}\right)\left\{-(1+a) + 2b - \frac{(1-3a)}{2}\right\}$$
$$+ (a - \bar{h}b) = 0$$

$$\Rightarrow (1 + z^2 + 2z)\left\{1 - \bar{h}\left(\frac{1+a}{2} + b\right)\right\} + (1 - z^2)\left\{\frac{-2 - 2a + 4b - 1 + 3a}{2}\right\}$$
$$+ (a - \bar{h}b)(1-z)^2 = 0$$

$$1 \Rightarrow \left[1 - \bar{h}\left(\frac{1+a}{2} + b\right)\right] + z^2\left\{1 - \bar{h}\left(\frac{1+a}{2} + b\right)\right\}$$
$$+ z\left\{2 - 2\bar{h}\left(\frac{1+a}{2} + b\right)\right\} + (1 - z^2)\left\{\frac{a + 4b - 3}{2}\right\}$$
$$+ (a - \bar{h}b)(1 + z^2 - 2z) = 0$$

$$= \left[1 - \bar{h}\left(\frac{1+a}{2} + b\right)\right] + z^2\left\{1 - \bar{h}\left(\frac{1+a}{2} + b\right)\right\} \cancel{+ z\{2 - 2h}$$
$$+ z\left\{2 - 2\bar{h}\left(\frac{1+a}{2} + b\right)\right\} + \left\{\frac{a + 4b - 3}{2}\right\} - \left\{\frac{a + 4b - 3}{2}\right\}z^2$$
$$+ (a - \bar{h}b) + z^2(a - \bar{h}b) - 2(a - \bar{h}b)z = 0$$

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$$\boxed{\begin{aligned} a_0 &= 1-\bar{h}\left(\frac{1+a}{2}+b\right)-\left(\frac{a+4b-3}{2}\right)+a-\bar{h}b \\ a_1 &= 2-2\bar{h}\left(\frac{1+a}{2}+b\right)-2\left(a-\bar{h}b\right) \\ a_2 &= 1-\bar{h}\left(\frac{1+a}{2}+b\right)+\left(\frac{a+4b-3}{2}\right)+a-\bar{h}b \end{aligned}}$$

$a_0 > 0$

$$1-\bar{h}\left(\frac{1+a}{2}+b\right)-\frac{(a+4b-3)}{2}+a-\bar{h}b>0$$

$$1-\frac{(a+4b-3)}{2}+a>\bar{h}\left(b+\frac{1+a}{2}+b\right)$$

$$\frac{2-a-4b+3+2a}{\cancel{2}}>\bar{h}\times\frac{4b+a+1}{\cancel{2}}$$

$$\boxed{\bar{h}<\frac{a-4b+5}{a+4b+1}}$$

$a_1 > 0$

$$2-2\bar{h}\left(\frac{1+a}{2}+b\right)-2a+2b\bar{h}>0$$

$$2-2a>\bar{h}\left(2\left(\frac{1+a}{2}+b\right)-2b\right.$$

$$1-a>\bar{h}\left(\frac{1+a}{2}+b-b\right)$$

$$1-a>\bar{h}\left(\frac{1+a}{2}\right)$$

$$\boxed{\bar{h}<2\frac{(1-a)}{(1+a)}}$$

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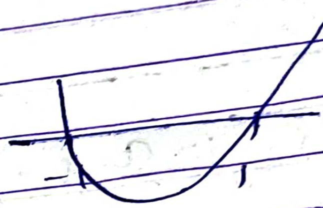
$a_2 > 0$

$$\Rightarrow \left(1 + \frac{a+4b-3}{2} + a\right) - \bar{h}\left(\frac{1+a}{2} + 2b\right) > 0$$

$$\Rightarrow \frac{2+a+4b-3+2a}{\cancel{2}} > \bar{h} \times \frac{1+a+4b}{\cancel{2}}$$

$$\Rightarrow (3a+4b-1) > \bar{h}(1+a+4b)$$

$$\boxed{\bar{h} < \frac{3a+4b-1}{1+a+4b}}$$



For A - stable

$$\boxed{\begin{array}{l} \dfrac{a-4b+5}{a-4b+1} \geq 0 \\ \dfrac{1-a}{1+a} \geq 0 \\ \dfrac{3a+4b-1}{1+4b+a} \geq 0 \end{array}}$$

$\Rightarrow \frac{a-1}{a+1} \leq 0$ $\boxed{a \in [-1, 1]}$

$\boxed{a \in (-1, 1)}$

Ans

Conditions on a and b for A - stable