Product Form of Inverse(PFI)of a Basis Matrix and Revised Simplex Method (RSM)

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We wish to compute the inverse of a basis matrix, B_c, that is differ by one column from the basis matrix, B, whose inverse is known. The product form of the inverse allows us to determine this new inverse in an efficient manner. We want to find B_c^{-1}.

First, let us consider the following definitions:

B is the original basis matrix of size $m \times m$. Its inverse i.e. B^{-1} is known.

B_c is the new basis matrix, which is identical to B *except* for the column r.

c is the rth column of matrix B_c, the only column different from those in B. Let

$$\begin{aligned} \boldsymbol{e} &= (e_1, e_2, \ldots, e_{r-1}, e_r, e_{r+1}, ..., e_m)^T = B^{-1}c \qquad (1) \\ \eta &= \left(-\frac{e_1}{e_r}, -\frac{e_2}{e_r}, \ldots, -\frac{e_{r-1}}{e_r}, \frac{1}{e_r}, -\frac{e_{r+1}}{e_r}, \ldots, -\frac{e_m}{e_r}\right)^T, e_r \neq 0 \qquad (2) \end{aligned}$$

where e_r is the r-th component of $\boldsymbol{e}$ as computed in (1) and m is the total number of elements of the column vector $\boldsymbol{e}$. Thus,

$$B_c^{-1} = E_r B^{-1} \tag{3}$$

where B_c^{-1}= inverse of B_c
B^{-1} = inverse of the previous matrix
E_r= an identity matrix with its r-th column replaced by η.

We now use (3) to illustrate the computation of the inverse of a basis matrix that differs by only a single column from another basis matrix, whose inverse is known.

Example 1:

Consider the two matrices shown below. Both are non-singular and differ by only one column, the first. The inverse of B is given and we wish to find the inverse of B_c.

c_1

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B_c = \begin{pmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 0 & 1 \end{pmatrix} \qquad B_c^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

We first compute $\boldsymbol{e}$ from (1), where

$$c_1 = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$$

(i.e., the first column in B_c)

$$\boldsymbol{e} = B^{-1}c_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$$

Next, from (2) we establish η:

$$\eta = \begin{pmatrix} \frac{1}{2} \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} (2)^{-1} \\ -2/2 \\ -4/2 \end{pmatrix}$$

first element pivot as c_1 is different in B_c from B

Thus,

first column of B replaced by η

$$E_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

$$B_c^{-1} = E_1 B^{-1}$$

$$B_c^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

Example 2:

Consider the two matrices shown below. Both are non singular and differ by only one column, the second. The inverse of B is given and we wish to find the inverse of B_c.

c_2

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad B_c = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 4 & 1 \end{pmatrix} \quad B_c^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

We first compute $\boldsymbol{e}$ from (1), where

$$c_2 = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$$

(i.e., the second column in B_c)

$$\boldsymbol{e} = B^{-1}c_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$$

Next, from (2) we establish η:

$$\eta = \begin{pmatrix} -1 \\ \frac{1}{2} \\ -2 \end{pmatrix} = \begin{pmatrix} -2/2 \\ (2)^{-1} \\ -4/2 \end{pmatrix}$$

pivot as

$r = 2$

Thus,

$$E_2 = \begin{pmatrix} 1 & -1 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

replacing 2nd column with η

$$B_c^{-1} = E_2 B^{-1}$$

$$B_c^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

Example 3:

Consider the two matrices shown below. Both are non singular and differ by only one column, the third. The inverse of B is given and we wish to find the inverse of B_c.

$C_3, r = 3$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B_c = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{pmatrix} \qquad B_c^{-1} = \begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{4} \end{pmatrix}$$

We first compute $\boldsymbol{e}$ from (1), where

$$c_3 = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$$

(i.e., the third column in B_c)

$$\boldsymbol{e} = B^{-1}c_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$$

Next, from (2) we establish η:

$$\eta = \begin{pmatrix} -1/2 \\ -1/2 \\ 1/4 \end{pmatrix} = \begin{pmatrix} -2/4 \\ -2/4 \\ (4)^{-1} \end{pmatrix} \rightarrow \text{pivot as } r = 3$$

Thus,

$$E_3 = \begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{4} \end{pmatrix}$$

$\rightarrow$ r^{th} column of B replaced with η

$$B_c^{-1} = E_3 B^{-1}$$

$$B_c^{-1} = \begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{4} \end{pmatrix}$$

Let B be a basis matrix of size $m \times m$.
Let $B = I_{m\times m}$ (an identity matrix of size $m \times m$)
Then $B = B^{-1} = I_{m\times m}$.
Let $B_1, B_2, \ldots, B_m$ are m non-singular matrices of size $m \times m$.
B and B_1 are differ by first column.
B_1 and B_2 are differ by second column.
B_2 and B_3 are differ by third column.
B_3 and B_4 are differ by fourth column.
$\vdots$
B_{m-1} and B_m are differ by m-th column.
Now $B_1^{-1} = E_1 B^{-1} = E_1 I_{m\times m} = E_1$
Then $B_2^{-1} = E_2 B_1^{-1} = E_2 E_1 = E_2 E_1 B^{-1} = E_2 E_1 I_{m\times m}$
$B_3^{-1} = E_3 B_2^{-1} = E_3 E_2 E_1$
$B_4^{-1} = E_4 B_3^{-1} = E_4 E_3 E_2 E_1$
$B_m^{-1} = E_m B_{m-1}^{-1} = E_m E_{m-1} \ldots E_1$
where $E_r, (r = 1, 2, \ldots, m)$ is defined in equation (3).
↳ calculated by replacing r^{th} column in B by η_r

Example 4:

Consider four different matrices shown below. All are non-singular matrices and differ by only one column. The inverse of the matrices are computed as follows:

$$B = B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B_1 = \begin{array}{c} \quad\ c_1 \\ \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array} \qquad B_2 = \begin{array}{c} c_1 \quad\quad\ \\ \begin{pmatrix} 2 & 4 & 0 \\ 0 & 2 & 0 \\ 0 & 6 & 1 \end{pmatrix} \\ \quad\ c_2 \end{array}$$

$$B_3 = \begin{array}{ccc} c_1 & c_2 & \\ \multicolumn{3}{c}{\begin{pmatrix} 2 & 4 & 0 \\ 0 & 2 & 0 \\ 0 & 6 & 5 \end{pmatrix}} \end{array} c_3 = B_{new}$$

We first compute $\boldsymbol{e}$ from (1), where

$$c_1 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

B is basis for B_1

B_1 " " " B_2

B_2 " " " B_3

From B and B_1 we find c_1.

(i.e., the first column in B_1)

$$\boldsymbol{e} = B^{-1}c_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

Next, from (2) we establish η:

$$\eta = \begin{pmatrix} 1/2 \\ 0 \\ 0 \end{pmatrix}$$

Thus,

$$E_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow \text{only using } c_1$$

$$B_1^{-1} = E_1 B^{-1}$$

$$B_1^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Then we compute c_2.

$$c_2 = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix}$$

From B_1 and B_2 we find c_2.

(i.e., the second column in B_2)

as B1 is Basis for B2

$$\boldsymbol{e} = B_1^{-1} c_2 = \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix}$$

Next, from (2) we establish η:

$$\eta = \begin{pmatrix} -1 \\ 1/2 \\ -3 \end{pmatrix}$$

Thus,

$$E_2 = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1/2 & 0 \\ 0 & -3 & 1 \end{pmatrix}$$

replace r^{th} column in B

inf

$$B_2^{-1} = E_2 B_1^{-1} = E_2 E_1$$

$$B_2^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1/2 & 0 \\ 0 & -3 & 1 \end{pmatrix} \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & -1 & 0 \\ 0 & 1/2 & 0 \\ 0 & -3 & 1 \end{pmatrix}$$

Then we compute c_3.

$$c_3 = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$$

(i.e., the third column in B_3)

$$\boldsymbol{e} = B_2^{-1} c_3 = \begin{pmatrix} 1/2 & -1 & 0 \\ 0 & 1/2 & 0 \\ 0 & -3 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$$

Next, from (2) we establish η:

$$\eta = \begin{pmatrix} 0 \\ 0 \\ 1/5 \end{pmatrix}$$

Thus,

$$E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/5 \end{pmatrix}$$

imp

$$B_3^{-1} = E_3 B_2^{-1} = E_3 E_2 E_1 = B_{new}^{-1}.$$

$$B_3^{-1} = \underset{E_3}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/5 \end{pmatrix}} \underset{B_2^{-1}}{\begin{pmatrix} 1/2 & -1 & 0 \\ 0 & 1/2 & 0 \\ 0 & -3 & 1 \end{pmatrix}} = \begin{pmatrix} 1/2 & -1 & 0 \\ 0 & 1/2 & 0 \\ 0 & -3/5 & 1/5 \end{pmatrix}$$

Hence $B_{new}^{-1} = B_3^{-1} = \begin{pmatrix} 1/2 & -1 & 0 \\ 0 & 1/2 & 0 \\ 0 & -3/5 & 1/5 \end{pmatrix} = E_3 E_2 E_1$

Revised Simplex Method

Original simplex method calculates the stores all numbers in the simplex Tableau. Many are not needed.
Revised Simplex Method (more efficient for computing):
It is used in all commercially packages (e.g. IBM MPSX, CDC APEX III).

$$LPP \quad \max: \quad Z = c^T x$$

Subject to

$$\begin{aligned} Ax &\leq b, \quad b \geq 0 \\ x &\geq 0. \end{aligned}$$

For easo of purpose we use only slack variable ie $\leq$ constraints

Initially constraints becomes (standard form):

$$\begin{bmatrix} A & I \end{bmatrix} \begin{bmatrix} x \\ x_s \end{bmatrix} = \begin{bmatrix} b \end{bmatrix}$$

x_s = slack variables
Basis matrix: Column relating to basic variables.

$$B = \begin{pmatrix} B_{11} & \dots & \dots & \dots & B_{1m} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ B_{m1} & \dots & \dots & \dots & B_{mm} \end{pmatrix}_{m \times m}$$

Initially $B = I_{m \times m}, B^{-1} = I_{m \times m}$.

Basic variable values: $X_B = \begin{pmatrix} X_{B1} \\ \dots \\ \dots \\ \dots \\ X_{Bm} \end{pmatrix}$

At any iteration all the non-basic variables are zero.

$$BX_B = b$$

Therefore $X_B = B^{-1}b$ where B^{-1}, inverse basis matrix.
At any iteration, given the original b vector and the inverse matrix B^{-1}, X_B can be calculated.
$Z = c_B^T x_B$, where c_B =objective coefficients of basic variables.

Steps in the Revised Simplex Method

Step 1. Determine the entering variable, x_j, with associated vector P_j.
Compute $Y = c_B^T B^{-1}$
Compute $z_j - c_j = YP_j - c_j$ for all non-basic variables.
Select the largest negative value (For Max type LPP) among all $z_j - c_j$.
Break the ties arbitrarily. If all the $z_j - c_j \geq 0$, optimal solution is reached.
$X_B = B^{-1}b$
$Z = c_B^T X_B$
Otherwise go to Step 2.
Step 2. Determine leaving variable, x_r, with associated vector P_r.
Compute the current basic variable $X_B = B^{-1}b$
Compute constraint coefficients of entering variables for P_j:

$$\alpha^j = B^{-1}P_j$$

Leaving variable x_r must be associated with

$$\theta = \min_k \left\{ \frac{(B^{-1}b)_k}{\alpha_k^j}, \alpha_k^j > 0 \right\}.$$

using minimum ratio rule.
If $\alpha_k^j \leq 0, \ \forall k$, then the problem is unbounded.
Step 3. Determination of the next basis matrix and B_{next}^{-1}
For the given B^{-1} the B_{next}^{-1} is computed by
$B_{next}^{-1} = E_r \ B^{-1}$, where r is the column number of the entering vector
Set $B^{-1} = B_{next}^{-1}$
Go to Step 1.
Note E_r is computed using equation (3).

(See the next slide for the numerical example)

Revised Simplex Method: Extended Tableau

Numerical Example (R1):

$$\max : Z = 4x_1 + 2x_2 + x_3$$

Subject to

$$2x_1 + x_2 + x_3 \leq 14$$

$$x_1 + 2x_2 + x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

Introduce Slack variables(Basic variables) :

$$s_1, s_2 \geq 0$$

Revised Simplex Method: Extended Tableau

Numerical Example (R1):

$$\max : Z = 4x_1 + 2x_2 + x_3 + 0s_1 + 0s_2$$

Subject to

$$2x_1 + x_2 + x_3 + s_1 = 14$$

$$x_1 + 2x_2 + x_3 + s_2 = 10$$

$$x_1, x_2, x_3 \geq 0$$

where slack variables(Basic variables) :

$$s_1, s_2 \geq 0$$

Revised Simplex Method: Extended Tableau

$\to c_1$ $\to c_2$ $\to c_3$

Numerical Example (R1):

Table 0:

SIMP	CV	4	2	1	0	0	b
CB	BV/V	x_1	x_2	x_3	s_1	s_2	XB
0	s_1	2	1	1	1	0	14
0	s_2	1	2	1	0	1	10
*	*	-4	- 2	-1	0	0	0

$\to$ Basis Matrix for step 1

P_1 P_2 P_3

P_i = vector for i^{th} non basic variable

c_i = coeff " " " " " in obj fun

Step 1:
In this Example we have the Basis Matrix B and its Inverse: → as we have only 2 basic variables s_1 & s_2

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Basic Variables are s_1 and s_2.

$$C_B^T = (0,0), Y = C_B^T B^{-1} = (0,0)$$

$C_B^T = (a, b)$

a = coeff of s_1 in obj. fun

b = coeff of s_2 in obj fun

as s_1, s_2 are basic variables

Basic Variables are s_1 and s_2.
Non- Basic Variables are x_1, x_2, x_3.
For all Non-Basic Variables calculate $z_j - c_j = YP_j - c_j$.
where P_1, P_2, P_3 are the Non-basic vectors.
Hence $z_1 - c_1 = -4$, $z_2 - c_2 = -2$, $z_3 - c_3 = -1$.
x_1 is selected as the entering variable.

most negative

Step 2:

right most column

$$X_B = B^{-1}b, \alpha^1 = B^{-1}P_1 \longrightarrow \text{as } x_1 \text{ is pivot column}$$

It gives

$$X_B = \begin{bmatrix} 14 \\ 10 \end{bmatrix}, \alpha^1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad X_B./\alpha^1 = \begin{bmatrix} 7 \\ 10 \end{bmatrix} \rightarrow \text{min}$$

Pivot

Minimum ratio is min (14/2, 10/1) = 7 i.e. Row no. = 1

s_1 is selected as the departing variable.

Hence the 1st column of the Basis Matrix is B is replaced by P_1

$x_1 \rightarrow$ entering $\rightarrow$ as most neg (-4)

$s_1 \rightarrow$ departing $\rightarrow$ min ratio

$$B_{next} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$\rightarrow$ s_1 column replaced with P_1

Revised Simplex Method:

Step 3:
Then B_{next}^{-1} is computed using Product Form of Inverse (PFI).

$$B_{next}^{-1} = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix}$$

Set $B^{-1} = B_{next}^{-1}$ and go to Step 1.

Revised Simplex Method:

Step 1:

Now we have the Basis Matrix B and its Inverse:

$$B_1 = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, B_1^{-1} = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix}$$

Basic Variables are x_1 and s_2.

$$C_B^T = (4, 0), Y = C_B^T B^{-1} = (2, 0)$$

Non- Basic Variables are s_1, x_2, x_3.

For all Non-Basic Variables calculate $z_j - c_j = YP_j - c_j$.

where P_4, P_2, P_3 are the Non-basic vectors.

Hence $z_4 - c_4 = 2, z_2 - c_2 = 0, z_3 - c_3 = 1$. → can be used to find alternate sol^{n}

All $\underline{z_j - c_j \geq 0}$. An optimal solution is reached.

$$X_B = \begin{bmatrix} x_1 \\ s_2 \end{bmatrix} = B_1^{-1} b$$

→ from first table only

x_2 can be considered as entering variable as $z_2 - c_2 = 0$ in the final step. The rest then follows the same as before

Step 1 :(Contd)

$$X_B = \begin{bmatrix} x_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$Z = C_B^T X_B = (4, 0) \begin{bmatrix} 7 \\ 3 \end{bmatrix} = 28$$

Optimal Solution:

$$x_1^* = 7, x_2^* = 0, x_3^* = 0, Z^* = 28$$

This problem has alternate optimal solution: $(x_1^*, x_2^*, x_3^*) = (6, 2, 0)$.

as $z_2 - c_2 = 0$ in previous step

Note:- If required for calculating B_2^{-1}, we need to use Product form of inverse method with B_1 as basis matrix

Numerical Example -R2

$$\max : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \leq 16$$

$$x_1 + x_2 + 2x_3 \leq 14$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example R3

$$\max : Z = 4x_1 + 4x_2 + x_3$$

Subject to

$$x_1 + 6x_2 + x_3 \leq 40$$

$$6x_1 + x_2 + x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$

Revised Simplex Method:

Numerical Example R4

$$\max : Z = 8x_1 + 2x_2 + 8x_3$$

Subject to

$$4x_1 + x_2 + x_3 \leq 40$$

$$x_1 + x_2 + 4x_3 \leq 25$$

$$x_1, x_2, x_3 \geq 0$$

Revised Simplex Method:

Numerical Example R5

$$\max : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \leq 16$$

$$x_1 + x_2 + 2x_3 \leq 14$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example R6

$$\max : Z = 6x_1 + 6x_2 + x_3$$

Subject to

$$4x_1 + 2x_2 + x_3 \leq 26$$

$$2x_1 + 4x_2 + x_3 \leq 22$$

$$x_1, x_2, x_3 \geq 0$$

Revised Simplex Method:

Numerical Example -R7

$$\max : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \leq 16$$

$$x_1 + x_2 + 2x_3 \leq 14$$

$$4x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Revised Simplex Method:

Numerical Example R8

$$\max : Z = 4x_1 + 4x_2 + x_3$$

Subject to

$$x_1 + 6x_2 + x_3 \leq 40$$

$$6x_1 + x_2 + x_3 \leq 30$$

$$x_1 + x_2 + 3x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example R9

$$\max : Z = 8x_1 + 2x_2 + 8x_3$$

Subject to

$$4x_1 + x_2 + x_3 \leq 40$$

$$x_1 + 5x_2 + x_3 \leq 15$$

$$x_1 + x_2 + 4x_3 \leq 25$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example R10

$$\max : Z = x_1 + 4x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \leq 16$$

$$x_1 + x_2 + 2x_3 \leq 14$$

$$4x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Revised Simplex Method:

Numerical Example R11

$$\max : Z = 6x_1 + 6x_2 + x_3$$

Subject to

$$4x_1 + 2x_2 + x_3 \leq 26$$

$$2x_1 + 4x_2 + x_3 \leq 22$$

$$x_1 + x_2 + x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$