NSDE Programming Assignment 2

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Q1.

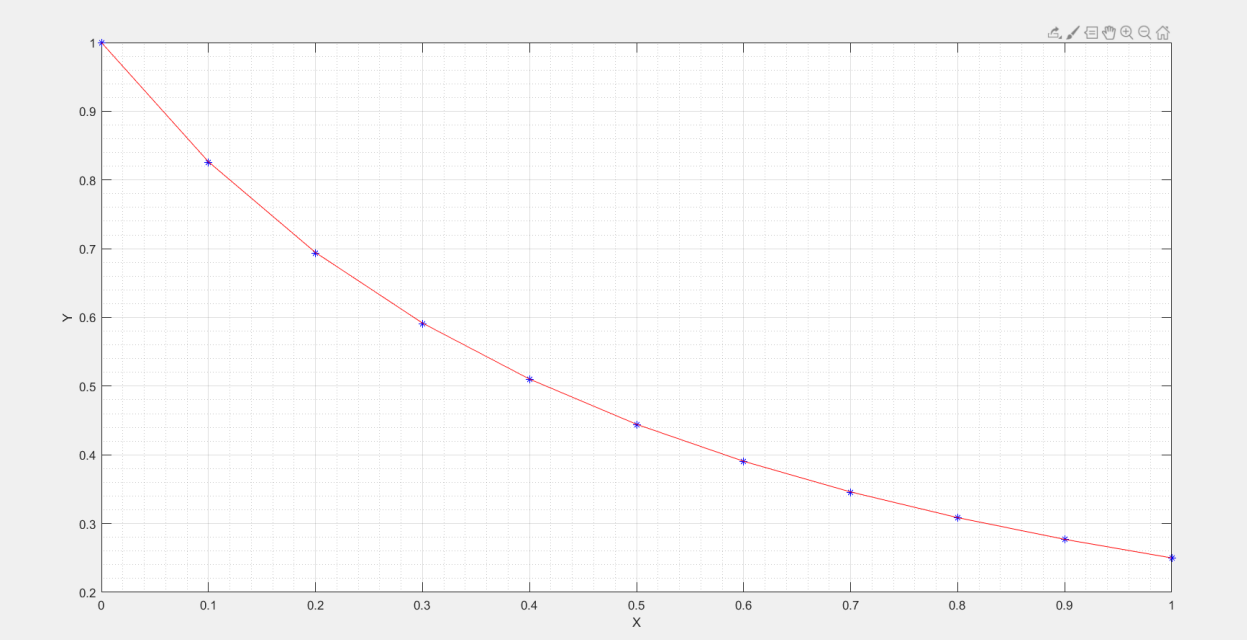
Matlab **Code:**

```
clear
x = 0:0.1:1;
n = length(x);
h = 0.1;
f2 = @(x,y) (6*y*y);
f3 = @(x,y) (12*x*y);
f4 = @(x,y) (72*x^3 + 12*y^2);
val = zeros(1,7);
g = zeros(1,7);
ans = zeros(1,n);
for j=1:7
   if j==1
      s=-9/5;
   elseif j==2
      s=-1.9;
   else
      s=val(j-1)-((g(j-1)/(g(j-1)-g(j-2)))*(val(j-1)-val(j-2)));
   end
   val(j)=s;
   y = zeros(1,n);
   y1 = zeros(1,n);
   y(1) = 1;
   y1(1) = s;
   for i=1:n-1
      y(i+1)=y(i)+h*y1(i)+(h*h/2)*f2(x(i),y(i))+(h*h*h/6)*f3(y(i),y1(i));
      y1(i+1)=y1(i)+h*f2(x(i),y(i))+(h*h/2)*f3(y(i),y1(i))+(h*h*h/6)*f4(y(i),y1(i));
   end
   g(j)=0.25 - y(n);
   ans = y;
end
act = zeros(1,n);
for i=1:n
   act(i)=1/(1+x(i))^2 ;
end
fprintf('%5s %20s %20s\n','X','Actual Y','Calculated Y');
for i=1:n
   fprintf('%5.4f %20.8f %20.8f\n',x(i),act(i),ans(i));
end
plot(x,ans,'*b');
hold on
plot(x,act,'r-');
hold on;
xlabel('X');
ylabel('Y');
grid on;
grid minor;
```

Table:

X	Actual Y	Calculated Y
0.0000	1.00000000	1.00000000
0.1000	0.82644628	0.82583208
0.2000	0.69444444	0.69363849
0.3000	0.59171598	0.59090184
0.4000	0.51020408	0.50946171
0.5000	0.44444444	0.44380781
0.6000	0.39062500	0.39010784
0.7000	0.34602076	0.34562849
0.8000	0.30864198	0.30837732
0.9000	0.27700831	0.27687399
1.0000	0.25000000	0.25000000

Graph:



Q2.

Matlab **Code:**

```
%boundary value problem shooting meathod
clear

u=zeros(1,11);
v=zeros(1,11);
Du=zeros(1,11);
Dz=zeros(1,11);
phi=zeros(1,5);
Dphi=zeros(1,5);
s=zeros(1,6);

h= 0.1;
x=(0:0.1:1); %values of x
uf=1.0;
s(1)= 0.09;

for k=1:5
   fprintf("\nFor s = %2.2f\n",s(k));
   Du(1)=s(k);
   u(1)=0; % u at x= 0 i.e. first element in column matrix u
   v(1)=0; Dv(1)= 1;

   for j=1:11
      % taylor series of order 3
      u(j+1)=u(j)+(0.1*Du(j))+(0.01*u(j)*Du(j))+((0.001/3)*(Du(j)*Du(j)+2*u(j)*u(j)*Du(j)));
      Du(j+1)=Du(j)+(0.2*Du(j)*u(j))+((0.01)*(Du(j)*Du(j)+2*u(j)*u(j)*Du(j)));
      % taylor series of order 3

v(j+1)=v(j)+(0.1*Dv(j))+(0.01*(Du(j)*v(j)+Dv(j)*u(j)))+((0.002/3)*(2*Du(j)*u(j)*v(j)+Du(j)*Dv(j)+D
v(j)*u(j)*u(j)));

Dv(j+1)=Dv(j)+0.2*(Du(j)*v(j)+Dv(j)*u(j))+0.02*(2*Du(j)*u(j)*v(j)+Du(j)*Dv(j)+Dv(j)*u(j)*u(j));
      fprintf("u(%2.2f)=%12.8f \n",x(j),u(j));
      plot(x(j),u(j),'+r')
      grid on
      hold on
   end
   phi(k)=u(11)-uf;
   Dphi(k)=v(11);
   s(k+1)=s(k)-(phi(k)/Dphi(k));

end
```

Output:

```
For s = 0.09
u(0.00)=  0.00000000
u(0.10)=  0.00900270
u(0.20)=  0.01802162
u(0.30)=  0.02707309
u(0.40)=  0.03617366
u(0.50)=  0.04534026
u(0.60)=  0.05459030
u(0.70)=  0.06394180
u(0.80)=  0.07341355
u(0.90)=  0.08302528
u(1.00)=  0.09279779
```

```
For s = 0.94
u(0.00)=  0.00000000
u(0.10)=  0.09461616
u(0.20)=  0.19103407
u(0.30)=  0.29118291
u(0.40)=  0.39729001
u(0.50)=  0.51209241
u(0.60)=  0.63914594
u(0.70)=  0.78331157
u(0.80)=  0.95156960
u(0.90)=  1.15446918
u(1.00)=  1.40888831
```

```
For s = 0.76
u(0.00)=  0.00000000
u(0.10)=  0.07646072
u(0.20)=  0.15409666
u(0.30)=  0.23414995
u(0.40)=  0.31801721
u(0.50)=  0.40735262
u(0.60)=  0.50420951
u(0.70)=  0.61124800
u(0.80)=  0.73205615
u(0.90)=  0.87167170
u(1.00)=  1.03747383
```

```
For s = 0.74
u(0.00)=  0.00000000
u(0.10)=  0.07444625
u(0.20)=  0.15000646
u(0.30)=  0.22785632
u(0.40)=  0.30931382
u(0.50)=  0.39593355
u(0.60)=  0.48963529
u(0.70)=  0.59289113
u(0.80)=  0.70901216
u(0.90)=  0.84260969
u(1.00)=  1.00037406
```

```
For s = 0.74
u(0.00)=  0.00000000
u(0.10)=  0.07442574
u(0.20)=  0.14996482
u(0.30)=  0.22779226
u(0.40)=  0.30922528
u(0.50)=  0.39581747
u(0.60)=  0.48948728
u(0.70)=  0.59270491
u(0.80)=  0.70877876
u(0.90)=  0.84231593
u(1.00)=  1.00000004
```

Q3.

Matlab **Code:**

```
clear
y=zeros(1,11);
v=zeros(1,11);
z=zeros(1,11);
w=zeros(1,11);
phi=zeros(1,5);
Dphi=zeros(1,5);
s=zeros(1,6);
b = [1 2 2 1];
h= 0.1;
x=(0:0.1:1);
yf=4.0;
s(1)=0.9;
dy = @(x,y,z,v,w) (z);
dz = @(x,y,z,v,w) ((3*y*y)/2);
dv = @(x,y,z,v,w) (w);
dw = @(x,y,z,v,w) (3*y*v); ay=zeros(1,4);az=zeros(1,4);av=zeros(1,4);aw=zeros(1,4);
for a=1:5
   fprintf("\nFor s = %2.2f\n",s(a));
   z(1)=s(a);
   y(1)=1; % u at x= 0 i.e. first element in column matrix u
   v(1)=0; w(1)= 1;

  for i = 1:11
   ay(1) = dy(x(i), y(i),z(i),v(i),w(i));
   az(1) = dz(x(i), y(i),z(i),v(i),w(i));
```

```
   av(1) = dv(x(i), y(i),z(i),v(i),w(i));
   aw(1) = dw(x(i), y(i),z(i),v(i),w(i));

   ay(2) = dy(x(i) + (h/2), y(i) + (h/2)*ay(1), z(i) + (h/2)*az(1), v(i) + (h/2)*av(1), w(i) +
(h/2)*aw(1));
   az(2) = dz(x(i) + (h/2), y(i) + (h/2)*ay(1), z(i) + (h/2)*az(1), v(i) + (h/2)*av(1), w(i) +
(h/2)*aw(1));
   av(2) = dv(x(i) + (h/2), y(i) + (h/2)*ay(1), z(i) + (h/2)*az(1), v(i) + (h/2)*av(1), w(i) +
(h/2)*aw(1));
   aw(2) = dw(x(i) + (h/2), y(i) + (h/2)*ay(1), z(i) + (h/2)*az(1), v(i) + (h/2)*av(1), w(i) +
(h/2)*aw(1));

   ay(3) = dy(x(i) + (h/2), y(i) + (h/2)*ay(2), z(i) + (h/2)*az(2), v(i) + (h/2)*av(2), w(i) +
(h/2)*aw(2));
   az(3) = dz(x(i) + (h/2), y(i) + (h/2)*ay(2), z(i) + (h/2)*az(2), v(i) + (h/2)*av(2), w(i) +
(h/2)*aw(2));
   av(3) = dv(x(i) + (h/2), y(i) + (h/2)*ay(2), z(i) + (h/2)*az(2), v(i) + (h/2)*av(2), w(i) +
(h/2)*aw(2));
   aw(3) = dw(x(i) + (h/2), y(i) + (h/2)*ay(2), z(i) + (h/2)*az(2), v(i) + (h/2)*av(2), w(i) +
(h/2)*aw(2));

   ay(4) = dy(x(i) + h, y(i) + h*ay(3), v(i) + h*av(3), z(i) + h*az(3), w(i) + h*aw(3));
   az(4) = dz(x(i) + h, y(i) + h*ay(3), v(i) + h*av(3), z(i) + h*az(3), w(i) + h*aw(3));
   av(4) = dv(x(i) + h, y(i) + h*ay(3), v(i) + h*av(3), z(i) + h*az(3), w(i) + h*aw(3));
   aw(4) = dw(x(i) + h, y(i) + h*ay(3), v(i) + h*av(3), z(i) + h*az(3), w(i) + h*aw(3));

   y(i+1) = y(i) + (h/6)*sum(b.*ay);
   z(i+1) = z(i) + (h/6)*sum(b.*az);
   v(i+1) = v(i) + (h/6)*sum(b.*av);
   w(i+1) = w(i) + (h/6)*sum(b.*aw);
   fprintf("u(%2.2f)=%12.6f \n",x(i),y(i));
   end
   phi(a)=y(11)-yf;
   Dphi(a)=v(11);
   s(a+1)=s(a)-(phi(a)/Dphi(a));

end
```

Output:

```
For s = 0.90          For s = 1.16          For s = 1.26
u(0.00)=    1.000000  u(0.00)=    1.000000  u(0.00)=    1.000000
u(0.10)=    1.081910  u(0.10)=    1.103973  u(0.10)=    1.111798
u(0.20)=    1.180287  u(0.20)=    1.225053  u(0.20)=    1.240934
u(0.30)=    1.298125  u(0.30)=    1.367019  u(0.30)=    1.391474
u(0.40)=    1.439397  u(0.40)=    1.534853  u(0.40)=    1.568778
u(0.50)=    1.609414  u(0.50)=    1.735210  u(0.50)=    1.780006
u(0.60)=    1.815348  u(0.60)=    1.977109  u(0.60)=    2.034884
u(0.70)=    2.067021  u(0.70)=    2.273001  u(0.70)=    2.346875
u(0.80)=    2.378126  u(0.80)=    2.640437  u(0.80)=    2.735039
u(0.90)=    2.768161  u(0.90)=    3.104780  u(0.90)=    3.227063
u(1.00)=    3.265596  u(1.00)=    3.703769  u(1.00)=    3.864416

          For s = 1.30          For s = 1.32
          u(0.00)=    1.000000  u(0.00)=    1.000000
          u(0.10)=    1.115220  u(0.10)=    1.116814
          u(0.20)=    1.247878  u(0.20)=    1.251114
          u(0.30)=    1.402170  u(0.30)=    1.407156
          u(0.40)=    1.583623  u(0.40)=    1.590543
          u(0.50)=    1.799623  u(0.50)=    1.808770
          u(0.60)=    2.060212  u(0.60)=    2.072029
          u(0.70)=    2.379312  u(0.70)=    2.394456
          u(0.80)=    2.776664  u(0.80)=    2.796115
          u(0.90)=    3.281013  u(0.90)=    3.306256
          u(1.00)=    3.935541  u(1.00)=    3.968871
```

Q4.

Part 4(i)

Matlab **Code**

```
function [X, Y, A, B, L, U] = finite_diff_method_dirichlet(D2y, x_0, y_0, x_n, y_n, h)
    syms y(x) y_i_plus_1 y_i y_i_minus_1
    Dy = diff(y);
    D2y_def = diff(Dy, x, 1);

    n = int16((x_n - x_0) / h);
    X = double([x_0, zeros(1, n-1), x_n]);
    Y = double([y_0, zeros(1, n-1), y_n]);

    A = zeros(n-1, n-1);
    B = zeros(n-1, 1);

    D2y_eq = D2y_def - D2y;
    D2y_eq_finite_approx = subs(D2y_eq, {D2y_def Dy y}, {((y_i_plus_1 - 2 * y_i +
    y_i_minus_1) / (h^2)) ((y_i_plus_1 - y_i_minus_1) / (2 * h)) y_i});

    for i=1:n-1

      X(i+1) = X(i) + h;

      if i==1

        B(i, 1) = -1 * subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i+1) Y(i)
    0 0});
        A(i, 1) = subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i+1) Y(i) 1
    0}) + B(i, 1);
        A(i, 2) = subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i+1) Y(i) 0
    1}) + B(i, 1);

      elseif i==n-1

        B(i, 1) = -1 * subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i+1) 0 0
    Y(i+1)});
        A(i, n-2) = subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i+1) 1 0
    Y(i+1)}) + B(i, 1);
        A(i, n-1) = subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i+1) 0 1
    Y(i+1)}) + B(i, 1);

      else

        B(i, 1) = -1 * subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i+1) 0 0
    0});
        A(i, i-1) = subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i+1) 1 0 0})
    + B(i, 1);
        A(i, i) = subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i+1) 0 1 0}) +
    B(i, 1);
        A(i, i+1) = subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i+1) 0 0 1})
    + B(i, 1);

      end
    end
```

```
        L = zeros(n-1, n-1);
        U = zeros(n-1, n-1);

        for i=1:n-1
            if i==1
                L(i, 1) = 1;
                U(i, 1) = A(i, 1);
            else
                L(i, i-1) = A(i, i-1) / U(i-1, i-1);
                L(i, i) = 1;
                U(i, i) = A(i, i) - L(i, i-1) * U(i-1, i);
            end

            if i<=n-2
                U(i, i+1) = A(i, i+1);
            end
        end

        T1 = zeros(n-1, 1);
        for i=1:n-1
            if i==1
                T1(i, 1) = B(i, 1);
            else
                T1(i, 1) = B(i, 1) - T1(i-1, 1) * L(i, i-1);
            end
        end

        for i=1:n-1
            j = n - i + 1;
            if i==1
                Y(j) = T1(j-1, 1) / U(j-1, j-1);
            else
                Y(j) = (T1(j-1, 1) - Y(j+1) * U(j-1, j)) / U(j-1, j-1);
            end
        end

end

syms y(x)
Dy = diff(y);
D2y = 2*y / (x^2) - 1/x;
x_0 = 2;
y_0 = 0;
x_n = 3;
y_n = 0;
h = 1/10;

[X, Y, A, B, L, U] = finite_diff_method_dirichlet(D2y, x_0, y_0, x_n, y_n, h);
```

Table:

X	Y
2.0	0
2.1	0.0186
2.2	0.0325
2.3	0.0420
2.4	0.0473
2.5	0.0487
2.6	0.0461
2.7	0.0399
2.8	0.0301
2.9	0.0167
3.0	0

Part 4(ii)

Matlab **Code:**

```
function [X, Y, A, B, L, U] = finite_diff_method_robinson(D2y, x_0, x_n, a0, a1, r1, b0, b1, r2, h)
    syms y(x) y_i_plus_1 y_i y_i_minus_1 y_minus_1 y_n_plus_1
    Dy = diff(y);
    D2y_def = diff(Dy, x, 1);

    n = int16((x_n - x_0) / h);
    X = double([x_0, zeros(1, n-1), x_n]);
    Y = zeros(1, n+1);

    A = zeros(n+1, n+1);
    B = zeros(n+1, 1);

    y_minus_1 = 2 * h * r1 / a1 + y_i_plus_1 -2 * h * a0 * y_i / a1;
    y_n_plus_1 = 2 * h * r2 / b1 + y_i_minus_1 - 2 * h * b0 * y_i / b1;

    D2y_eq = D2y_def - D2y;
    D2y_eq_finite_approx = subs(D2y_eq, {D2y_def Dy y}, {((y_i_plus_1 - 2 * y_i +
    y_i_minus_1) / (h^2)) ((y_i_plus_1 - y_i_minus_1) / (2 * h)) y_i});

    for i=1:n+1

      if i<=n
        X(i+1) = X(i) + h;
      end

      if i==1
        D2y_temp = subs(D2y_eq_finite_approx, y_i_minus_1, y_minus_1);

        B(i, 1) = -1 * subs(D2y_temp, {x y_i y_i_plus_1}, {X(i) 0 0});
        A(i, 1) = subs(D2y_temp, {x y_i y_i_plus_1}, {X(i) 1 0}) + B(i, 1);
        A(i, 2) = subs(D2y_temp, {x y_i y_i_plus_1}, {X(i) 0 1}) + B(i, 1);

      elseif i==n+1
        D2y_temp = subs(D2y_eq_finite_approx, y_i_plus_1, y_n_plus_1);

        B(i, 1) = -1 * subs(D2y_temp, {x y_i_minus_1 y_i}, {X(i) 0 0});
        A(i, n) = subs(D2y_temp, {x y_i_minus_1 y_i}, {X(i) 1 0}) + B(i, 1);
        A(i, n+1) = subs(D2y_temp, {x y_i_minus_1 y_i}, {X(i) 0 1}) + B(i, 1);

      else

        B(i, 1) = -1 * subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i) 0 0
    0});
        A(i, i-1) = subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i) 1 0 0}) +
    B(i, 1);
        A(i, i) = subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i) 0 1 0}) +
    B(i, 1);
        A(i, i+1) = subs(D2y_eq_finite_approx, {x y_i_minus_1 y_i y_i_plus_1}, {X(i) 0 0 1}) +
    B(i, 1);

      end
    end

    L = zeros(n+1, n+1);
```

```
        U = zeros(n+1, n+1);

        for i=1:n+1
            if i==1
                L(i, 1) = 1;
                U(i, 1) = A(i, 1);
            else
                L(i, i-1) = A(i, i-1) / U(i-1, i-1);
                L(i, i) = 1;
                U(i, i) = A(i, i) - L(i, i-1) * U(i-1, i);
            end

            if i<=n
                U(i, i+1) = A(i, i+1);
            end
        end

        T1 = zeros(n+1, 1);
        for i=1:n+1
            if i==1
                T1(i, 1) = B(i, 1);
            else
                T1(i, 1) = B(i, 1) - T1(i-1, 1) * L(i, i-1);
            end
        end

        for i=1:n+1
            j = n - i + 2;
            if i==1
                Y(j) = T1(j, 1) / U(j, j);
            else
                Y(j) = (T1(j, 1) - Y(j+1) * U(j, j+1)) / U(j, j);
            end
        end

end

syms y(x)
Dy = diff(y);
D2y = 3 * Dy - 2 * y;
x_0 = 0;
x_n = 1;
h = 1/10;
a0 = 2;
a1 = 1;
r1 = 1;
b0 = 1;
b1 = 1;
r2 = 2 * exp(1) + 3 * exp(2);

[X, Y, A, B, L, U] = finite_diff_method_robinson(D2y, x_0, x_n, a0, a1, r1, b0, b1, r2, h);
```

Table:

X	Y
0.0	1.9895
0.1	2.3122
0.2	2.6943
0.3	3.1480
0.4	3.6877
0.5	4.3311
0.6	5.0997
0.7	6.0196
0.8	7.1224
0.9	8.4470
1.0	10.0403

Part 4(iii)

Matlab **Code:**

```
% NR Method for non linear variables
y_sol = [1; 1; 1];% [y1; y2; y3]
y_sol_final = [0; 1; 1; 1; 1]; % [y0; y1; y2; y3; y4]
f1 = y_sol(2) - 2*y_sol(1) - (3/32)*(y_sol(1)^2);
f2 = y_sol(3) - 2*y_sol(2) + y_sol(1) - (3/32)*(y_sol(2)^2);
f3 = 1 - 2*y_sol(3) + y_sol(2) - (3/32)*(y_sol(3)^2);
y_pre=[f1 ; f2; f3];
df1_dy1 = -2 - (3/16)*y_sol(1);
df1_dy2 = 1;
df1_dy3 = 0;
df2_dy1 = 1;
df2_dy2 = -2 - (3/16)*y_sol(2);
df2_dy3 = 1;
df3_dy1 = 0;
df3_dy2 = 1;
df3_dy3 = -2 - (3/16)*y_sol(3);
J = ([df1_dy1,df1_dy2,df1_dy3 ; df2_dy1,df2_dy2,df2_dy3 ; df3_dy1,df3_dy2,df3_dy3]^(-1));
fprintf('Iteration y1 y2 y3\n');
fprintf('%i %12.10f %12.10f %12.10f\n',0,1,1,1);
for i = 1:10
   y_sol = y_sol - J*y_pre;
   fprintf('%i %12.10f %12.10f %12.10f\n',i,y_sol(1,1),y_sol(2,1),y_sol(3,1));
   f1 = y_sol(2) - 2*y_sol(1) - (3/32)*(y_sol(1)^2);
   f2 = y_sol(3) - 2*y_sol(2) + y_sol(1) - (3/32)*(y_sol(2)^2);
   f3 = 1 - 2*y_sol(3) + y_sol(2) - (3/32)*(y_sol(3)^2);
   y_pre=[f1 ; f2; f3];
   df1_dy1 = -2 - (3/16)*y_sol(1);
   df1_dy2 = 1;
   df1_dy3 = 0;
   df2_dy1 = 1;
   df2_dy2 = -2 - (3/16)*y_sol(2);
   df2_dy3 = 1;
   df3_dy1 = 0;
   df3_dy2 = 1;
   df3_dy3 = -2 - (3/16)*y_sol(3);
   J = ([df1_dy1,df1_dy2,df1_dy3 ; df2_dy1,df2_dy2,df2_dy3 ; df3_dy1,df3_dy2,df3_dy3]^(-1));
end
x_final=[0; 0.25; 0.5; 0.75; 1];
y_sol_final=[0; y_sol(1); y_sol(2); y_sol(3); 1];
fprintf('\n');
fprintf('i x_i y_i \n');
for i=1:5
   fprintf('%i %2.2f %12.10f \n',i,x_final(i),y_sol_final(i));
end
```

Table:

Iteration	y1	y2	y3
0	1.0000000000	1.0000000000	1.0000000000
1	0.2714285714	0.5000000000	0.7285714286
2	0.2253427019	0.4552468499	0.7043928902
3	0.2251100550	0.4549708427	0.7042377297
4	0.2251100479	0.4549708332	0.7042377242
5	0.2251100479	0.4549708332	0.7042377242
6	0.2251100479	0.4549708332	0.7042377242
7	0.2251100479	0.4549708332	0.7042377242
8	0.2251100479	0.4549708332	0.7042377242
9	0.2251100479	0.4549708332	0.7042377242
10	0.2251100479	0.4549708332	0.7042377242

i	x_i	y_i
1	0.00	0.0000000000
2	0.25	0.2251100479
3	0.50	0.4549708332
4	0.75	0.7042377242
5	1.00	1.0000000000