Indian Institute of Technology, Kharagpur

Date. January 25, 2021 Time: 45 mins Full Marks: 10
First Class Test (Spring) Semester 2020-21 Subject Name: Discrete Mathematics

Instruction: Notations used are as explained in the class.

1. [2 **mark**] Determine whether the following is valid. If valid, what rule of inferences being used? If not, what logical error occurs?

   All animals with scales are dragons.
   Some animals which are not dragons have sharp claws.

   Therefore, there are animals without scales which have sharp claws.

2. [2 **mark**] Sir Richard Empson (c. 1450-1510) minister to king Henry VII of England, reportedly was able to demonstate that any person was capable of paying a heavy tax.

   If the accused lives at a small rate, his savings must make him rich.
   If the accused maintains a large household, his expenditures proves he is rich.
   But, either he lives at a small rate, or he maintains a large expenditure.

   Therefore, he is rich, and consequently can play heavily to the king.

   This argument has been given the title of the Emperor's fork: whoever was found in the cross hairs of this argument was impaled on the prongs of this dilemma.
   Using resolution, determine whether Sir Empson's argument is valid or not.

3. [2 **mark**] Obtain pcnf of $\neg(\neg p \wedge q) \wedge (r \rightarrow \neg s)$.

4. [2 **mark**] The famous Goldbach conjecture says that every even integer $n \geq 4$ is a sum of two primes. This conjecture is not yet proved. Prove the weaker statement that there are infinitely many even integers that are sums of two primes.

5. [2 **mark**] For an odd prime p, establish the following fact:

$$1^n + 2^n + 3^n + \cdots + (p-1)^n \equiv \begin{cases} 0 \pmod{p}, & \text{if } (p-1) \not| n \\ -1 \pmod{p}, & \text{if } (p-1) \mid n \end{cases}$$

——-The End———