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1) Let the domain be animals

$P(x)$: x has scales
$Q(x)$: x is a dragon
$R(x)$: x has sharp claws

All animals with scales are dragons
$$\forall x \, (P(x) \rightarrow Q(x))$$

Some animals which are not dragons have sharp claws
$$\exists x \, (\neg Q(x) \wedge R(x))$$

Therefore, there are animals without scales which have sharp claws
$$\therefore \exists x \, (\neg P(x) \wedge R(x))$$

$$\begin{array}{l} \forall x \, (P(x) \rightarrow Q(x)) \\ \underline{\exists x \, (\neg Q(x) \wedge R(x))} \\ \therefore \exists x \, (\neg P(x) \wedge R(x)) \end{array}$$

Using resolution Principle

C1: $\forall x\ (P(x) \longrightarrow Q(x))$

C2: $\exists x\ (\neg Q(x) \land R(x))$

C3: $\neg(\exists x\ (\cancel{\forall Q(x)}\ \neg P(x) \land R(x))$ (negation of conclusion)

$\Leftrightarrow \forall x\ \neg(\neg P(x) \land R(x))$ De Morgan's law

$\Leftrightarrow \forall x\ (\neg(\neg P(x) \lor \neg R(x))$ De Morgan's law

$\Leftrightarrow \forall x\ (P(x) \lor \neg R(x))$ Double negation

(Here 'x' is some element of the domain)

C4: $\neg Q(c) \land R(c)$ Existential Instantiation from C2

C5: $\neg Q(c)$ From C4

C6: $R(c)$

C7: $P(\cancel{x}\,c) \rightarrow Q(c)$ Universal Instantiation from C1

$\Leftrightarrow$ ~~xPc~~ $\neg P(c) \lor Q(c)$

C8: $P(c) \lor \neg R(c)$ Universal Instantiation from C3

C9: $\neg P(c)$ Resolution of C5 & C7

C10: $P(c)$ Resolution of C6 & C8

C11: $\square$ Resolution of C9 & C10

We get an empty resolution.

Hence, it is valid

~~2) $P(x)$: x lives in small rate~~
~~$Q(x)$: x has a lot of savings ~~which makes~~ him rich~~
~~$R(x)$: x has a large expenditure which proves ~~he is rich~~~~
~~$S(x)$: x is rich~~

$$\cancel{P(x) \rightarrow Q(x)}$$
$$\cancel{\neg P(x) \rightarrow R(x)}$$
$$\cancel{P(x) \vee R(x)}$$

2) $P(x)$: x lives in a small rate
$Q(x)$: x has savings that make him rich
$R(x)$: x has a large expenditure, which proves he is rich

$$P(x) \rightarrow Q(x)$$
$$\neg P(x) \rightarrow R(x)$$
$$\underline{P(x) \vee R(x)}$$
$$Q(x) \vee R(x)$$

Using resolution principle

C_1: $P(x) \rightarrow Q(x)$
$\Leftrightarrow \neg P(x) \vee Q(x)$

C_2: $\neg P(x) \rightarrow R(x)$
$\Leftrightarrow P(x) \vee R(x)$

C_3: $P(x) \vee R(x)$

C_4: $\neg(Q(x) \vee R(x))$ (negation of conclusion)
$\Leftrightarrow \neg Q(x) \wedge \neg R(x)$ (De morgans law)

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C_5 : $\neg Q(x)$ } from C_4
C_6 : $\neg R(x)$
C_7 : $Q(x) \vee R(x)$ ~~from C_2~~ Resolution of C_1 & C_2

C_8 : $R(x)$ Resolution of C_5 & C_7
C_9 : $\square$ Resolution of C_6 & C_8

We get an empty resolution.
Hence the argument is valid.

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4) Given:- every integer $n \geq 4$ is sum of 2 primes (acc. to Gold bach conjecture)

To prove:- There are infinite even integers following the goldbach conjecture i.e they are a sum of two primes

Proof:- • A number is prime if it is only divisible by 1 & itself.
• Sum of 2 odd numbers is even
• All primes except '2' are odd.

Let us ~~prove~~ prove that there are infinite primes.

Let us assume that there are finite primes $p_1, p_2 \ldots p_n$ (n is the no. of primes).

Let $p = (p_1 \cdot p_2 \cdots p_n + 1)$

As $p > p_n$, p is composite.
Thus p/p_i should be '0' (p_i is one of the primes)
but

$$P/p_i = \left(\frac{p_1 p_2 \ldots p_n}{p_i} + \frac{1}{p_i}\right) \neq 0 \quad \therefore p \bmod p_i \neq 0$$

The same pattern continues for $i = 1, 2 \ldots n$

This contradicts our assumption as p turns out to be a prime and $p > p_n$.

Thus, there are infinite primes.
(Proved using contradiction)

As there are infinite primes, and all primes are odd except '2', we can say that there are infinite even numbers which are a sum of two primes.

Hence Proved

3) Given:- $\neg(\neg p \wedge q) \wedge (r \rightarrow \neg s)$

To find:- PCNF

Soln:-

$\neg(\neg p \wedge q) \wedge (r \rightarrow \neg s)$

$\wedge (\neg r \vee \neg s)$

$\Leftrightarrow (p \vee \neg q)$

$\Leftrightarrow ((p \vee \neg q) \vee (r \wedge \neg r) \vee (s \wedge \neg s)) \wedge$

$((\neg r \vee \neg s) \vee (p \wedge \neg p) \vee (q \wedge \neg q))$

$\Leftrightarrow [((p \vee \neg q \vee r) \wedge (p \vee \neg q \vee \neg r)) \vee (s \wedge \neg s)] \wedge$

$[((\neg r \vee \neg s \vee p) \wedge (\neg r \vee \neg s \vee \neg p)) \vee (q \wedge \neg q)]$

$\Leftrightarrow (p \vee \neg q \vee r \vee s) \wedge (p \vee \neg q \vee \neg r \vee s) \wedge$

$(p \vee \neg q \vee r \vee \neg s) \wedge (p \vee \neg q \vee \neg r \vee \neg s) \wedge$

$(\neg r \vee \neg s \vee p \vee q) \wedge (\neg r \vee \neg s \vee p \vee \neg q) \wedge$

$(\neg r \vee \neg s \vee \neg p \vee q) \wedge (\neg r \vee \neg s \vee \neg p \vee \neg q)$

$\Leftrightarrow (p \vee \neg q \vee r \vee s) \wedge (p \vee \neg q \vee \neg r \vee s) \wedge$

$(p \vee \neg q \vee r \vee \neg s) \wedge (p \vee \neg q \vee \neg r \vee \neg s) \wedge$

$(\neg p \vee q \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg r \vee \neg s) \wedge$

$(p \vee q \vee \neg r \vee \neg s)$

↳ PCNF

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5) ~~Given~~:- p is odd prime.

prove that:-

$$1^n + 2^n + 3^n + \dots + (p-1)^n \equiv \begin{cases} 0 \bmod p & \text{if } p-1 \nmid n \\ -1 \bmod p & \text{if } p-1 \mid n \end{cases}$$

proof:- we know that p is odd prime.

$\therefore$ $p-1$ is even.

If n is odd

$$1^n + 2^n + 3^n + \dots + (p-1)^n = 1^n + (-1)^n + 2^n + (-2)^n \dots$$
$$\equiv 0 \bmod p$$

If n is even and $(p-1) \mid n$, then every term in the sum is a $(p-1)$ power of an integer

So every term is $\equiv 1 \bmod p$.

and $(p-1)$ ~~and~~ terms sum to $(p-1) \equiv -1 \bmod p$

If n is even & $p \equiv 1 \bmod 4$ and $(p-1) \nmid n$, then quadratic residues come in pairs $a, (p-a)$, the sum is $\equiv 0 \bmod p$.

If n is even, & $p \equiv 3 \bmod 4$ & $(p-1) \nmid n$, the sum is $\equiv 0 \bmod p$

~~Hence~~ Hence if $(p-1) \mid n$, the sum $\equiv -1 \bmod p$

else sum $\equiv 0 \bmod p$