

SFA Test 1

Q1)

c) Regular Expression:- Patterns formed only using symbols $\epsilon, \phi, +, \cdot, *$ and elements of Σ are known as regular expressions

Eg:- Let $\Sigma = \{a, b\}$

$a^* + (bb)^*$ \rightarrow regular expression

a) DFA:- Deterministic Finite Automata is a structure $M = (Q, \Sigma, \delta, s, F)$ where Q is the set of states possible, Σ is the alphabet of possible inputs, s is the start state, F is the final states and δ is a function from $Q \times \Sigma$ to Q which moves from one state to other in response to an input.

Eg:-

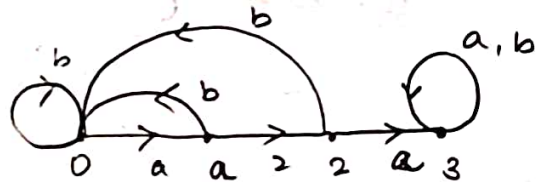
	a	b
$s \rightarrow 0$	1	0
1	2	0
2	3	0
$F \rightarrow 3$	3	3

$A =$ strings with 3 continuous a's

$\Sigma = \{a, b\}$

$F = 3$

$s = 0$



b) NFA:- Non Deterministic finite Automata is a structure $N = (Q, \Sigma, \Delta, s, F)$ where Q is the set of possible states, Σ is the alphabet of possible inputs, s is the set of start states, F is the set of final states and Δ is a function from $Q \times \Sigma$ to 2^Q which gives the possible states to move to from a given state in response to an input

Eg:- $A =$ strings with 1 on 2nd position from right

	0	1
$s \rightarrow \phi$	ϕ	ϕ
0	0	1, 0
Final $\rightarrow 1$	1	1

e.) CFL :- context free language is the language generated by context free grammar G and is denoted by $L(G)$.

Eg :- $\{a^n b^n \mid n \geq 0\}$

a) NPDA :- Non Deterministic Push Down Automata (NPDA) is a type of NFA which is associated with stack. NPDA is a structure of the form $N = (Q, \Sigma, \Gamma, \delta, s, \perp, F)$ where Q is the set of all possible states, Σ is the alphabet of inputs, s is the start state, F is the Final state, \perp is the starting stack, Γ is the alphabet of stack. δ and δ is the transition function



Q2) a) Pumping lemma

Let A be a regular language $\exists k \geq 0$ such that for string $xyz \in A$, $|y| \geq k$, there exists u, v, w such that $v \neq \epsilon$, $y = uvw$ and $\forall i \geq 0, xu v^i w z \in A$.

Pumping lemma is a necessary condition and not sufficient condition. Hence non-regular languages can also satisfy pumping lemma.

We can only prove that, if a language does not satisfy pumping lemma, it is non-regular.

Informal meaning of pumping lemma:-

If we pump v even ∞ times $xu v^i w z$ still stays in A .

Q2) b)

Pumping lemma for CFL

Let A be any CFL, $\exists k \geq 0$ such that for all $z \in A$, $|z| \geq k$

Then we can write $z = uvwxy$, st $vx \neq \epsilon$ and $|vwx| \leq k$. Also, $\forall i \geq 0$ $uv^iwx^iy \in A$

Proving $a^n b^n c^n$ is not CFL using pumping lemma

Let us assume A is a CFL i.e. A satisfies pumping lemma. Let $z = a^n b^n c^n$, As $|z| \geq n$ and $z \in L$,

there exist

$$z = uvwxy \Rightarrow |vwx| \text{ has to be } \leq n \text{ \& } |vx| \geq 1 \text{ as } vx \neq \epsilon$$

$$\forall i \geq 0, uv^iwx^iy \in A$$

Let us take the case where uvw consists of only "a"
when $i=0$, $uv^iwx^iy \notin A$

This is a contradiction.

Hence, A is not a CFL (proved using pumping lemma)

Q3) a) S is a regular language and ~~is~~ a CFL too. If L is a regular language & ~~non~~ non deterministic CFL, S is not a CFL.
 If L is deterministic CFL, S is not a CFL.
Proof:-

L is a regular language and let M be the automata that accepts the strings of L .

Consider automata N which accepts the start states of M and starts with accept states of M .

The transition b/w states of N is opp. to that of M .

$\therefore N$ accepts the reverse strings of L .

Thus S is regular.

Proof 2:-

We know that $L = L(G)$ where G is a CFG.

Let $G = (V, T, P, S)$.

Construct $H = (V, T, P', S)$

Then $S = S(\#)$

Thus S is a CFL

here P' is reverse of P for each production.

i.e. $A \rightarrow a \Rightarrow$ production of G ,

$A \rightarrow a' \Rightarrow$ production of H

a' is reverse of a