# Indian Institute of Technology, Kharagpur

Date. March 15, 2021 Time: 45 mins Full Marks: 10
Second Class Test (Spring) Semester 2020-21 Subject Name: Discrete Mathematics

**Instruction:** Notations used are as explained in the class.

1. [1 **mark**] Find the flaw with the following "proof" that $a^n = 1$ for all nonnegative integers $n$, whenever $a$ is a nonzero real number.
   *Basis Step:* $a^0 = 1$ is true by the definition of $a^0$.
   *Inductive Step:* Assume that $a^j = 1$ for all nonnegative integers $j$ with $j \leq k$. Then note that
   $$a^{k+1} = \frac{a^k \cdot a^k}{a^{k-1}} = \frac{1 \cdot 1}{1} = 1$$

2. [1 **mark**] Prove that $n! + 1$ and $(n + 1)! + 1$ are relatively prime.

3. [1 **mark**] Use the **Extended Euclidean Algorithm** to compute $17^{-1}$ mod 101.

4. [2 **mark**] Prove that if $a$ is any integer and $n$ is any non-negative integer, then $a$ and $a^{4n+1}$ have the same last digit.

5. [2 **mark**] Generalize the following patterns and show correctness by using induction.
   $$\begin{aligned} 1 \cdot 2 \cdot 3 \cdot 4 &= 5^2 - 1, \\ 2 \cdot 3 \cdot 4 \cdot 5 &= 11^2 - 1, \\ 3 \cdot 4 \cdot 5 \cdot 6 &= 19^2 - 1, \\ 4 \cdot 5 \cdot 6 \cdot 7 &= 29^2 - 1, \\ &\vdots \end{aligned}$$

6. [3 **marks**]
   ($a$) Determine whether there are any primitive roots mod 98; if so, how many will there be?
   ($b$) If there are primitive roots mod 98, find one.
   ($c$) If there are primitive roots, use the one you found in part (b) to construct another.

———-The End———