

Tutorial Sheet-7

Graph Theory and Algorithms

- (1) Determine the minimum size of a maximal matching in the cycle C_n , $n \geq 3$.
- (2) Prove that every tree has at most one perfect matching.
- (3) Prove that every maximal matching in a graph G has at least $\frac{\alpha'(G)}{2}$ edges.
- (4) Prove that every component of the symmetric difference of two matchings is a path or an even cycle.
- (5) Let M be a matching in a graph G . A path P in G is called an **M -alternating path** if P contains edges alternately from M and edges not in M . An M -alternating path whose end vertices are unsaturated by M is called an **M -augmenting path**. Prove that a matching M in a graph G is maximum if and only if G contains no M -augmenting path.
- (6) Let G be a 3-regular simple graph with vertex connectivity 1. Prove that number of vertices in G is at least 10. Give example of such a graph on 10 vertices.
- (7) If G is a three regular graph then prove that $\kappa(G) = \kappa'(G)$.
- (8) Prove or disprove: If $\delta(G) = n - 2$ then $\kappa(G) = n - 2$.