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19MA20059

Discrete Math test 2

1) Given:- T is a set of real numbers of the form

$$\frac{a_1}{3} + \frac{a_2}{3^2} + \frac{a_3}{3^3} + \cdots + \frac{a_n}{3^n} + \cdots$$

where a_n is 0 or 2

To prove:- T is an uncountable set

Proof:-

We can use Cantor's diagonal argument.

Let us assume that T is countable. Hence, an enumeration of T exists. Let the enumeration be

$T = \{t_1, t_2, t_3, \ldots\}$ where each

t_m is of the form $\frac{a_{1m}}{3} + \frac{a_{2m}}{3^2} + \cdots \frac{a_{nm}}{3^n}$,

and a_{ij} is either '0' or '2' $\forall\, i,j \in \mathbb{N}$

As T is countable, we can list its elements as follows

$$t_1 = \frac{a_{11}}{3} + \frac{a_{21}}{3^2} + \frac{a_{31}}{3^3} + \cdots + \frac{a_{n1}}{3^n} + \cdots$$

$$t_2 = \frac{a_{12}}{3} + \frac{a_{22}}{3^2} + \frac{a_{32}}{3^3} + \cdots + \frac{a_{n2}}{3^n} + \cdots$$

$\vdots$

$$t_m = \frac{a_{1m}}{3} + \frac{a_{2m}}{3^2} + \frac{a_{3m}}{3^3} + \cdots + \frac{a_{nm}}{3^n} + \cdots$$

$\vdots$

19MA20059

where $a_{ij} \in \{0, 2\}$ $\forall\, i, j \in \mathbb{N}$

Let's draw a diagonal and construct a number

$$t = \frac{a_{1t}}{3} + \frac{a_{2t}}{3^2} + \frac{a_{3t}}{3^3} + \cdots + \frac{a_{nt}}{3^n} + \cdots$$

where $a_{it} = \begin{cases} 0 & \text{if } a_{ii} \text{ is } 2 \\ 2 & \text{if } a_{ii} \text{ is } 0 \end{cases}$
$\forall\, i \in \mathbb{N}$

Thus t is a part of T as it is of the given form $\frac{a_1}{3} + \frac{a_2}{3^2} \cdots + \frac{a_n}{3^n} \cdots$,

$a_i \in \{0, 2\}$ $\forall \mathbb{N}$.

However, we will not see t in the list $\{t_1, t_2, \ldots t_n, \ldots\}$.

Suppose that $t = t_k$ for some $k \in \mathbb{N}$, then the a_{kt} from t will be equal to a_{kk} from t_k.
But, we have defined t such that $a_{kt} \neq a_{kk}$
Hence this is not possible.
This contradicts our assumption.
Hence, Cantor's Ternary set T is uncountable.
Hence proved.

19MA20059

Page No.:

3) $f(x,y,z) = xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z$

Step 0

	Minterms	Bitstring	# of 1
1	$xy\bar{z}$	110	2
2	$x\bar{y}z$	101	2
3	$x\bar{y}\bar{z}$	100	1
4	$\bar{x}yz$	011	2
5	$\bar{x}\bar{y}z$	001	1

Step 1

Term	Bitstring
(1,3) $x\bar{z}$	1 _ 0
(2,3) $x\bar{y}$	1 0 _
(4,5) $\bar{x}z$	0 _ 1
(2,5) $\bar{y}z$	_ 0 1

No more steps possible

	$xy\bar{z}$	$x\bar{y}z$	$x\bar{y}\bar{z}$	$\bar{x}yz$	$\bar{x}\bar{y}z$
$x\bar{z}$	×		×		
$x\bar{y}$		×	×		
$\bar{y}z$		×			×
$\bar{x}z$				×	×

After simplification, we get final answer as

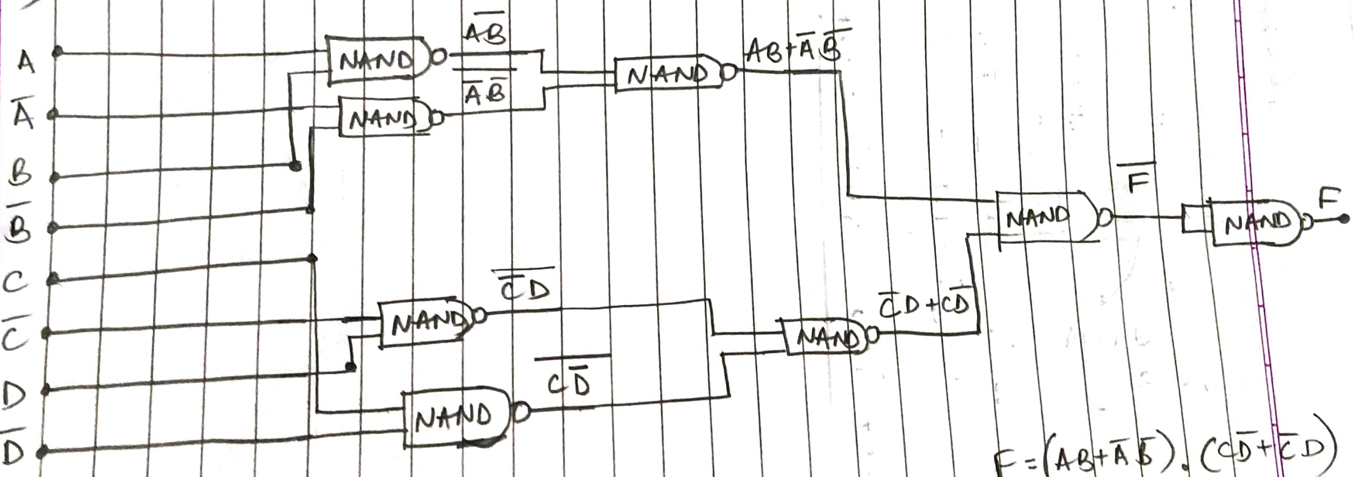
$f(x,y,z) = x\bar{z} + \bar{x}z + x\bar{y}$

(OR)

$f(x,y,z) = x\bar{z} + \bar{x}z + \bar{y}z$

19MA20059

2)



$F=(AB+\bar{A}\bar{B}).(C\bar{D}+\bar{C}D)$

19MA20059

4) $f = wx\bar{y} + \bar{y}z + \bar{w}y\bar{z} + \bar{x}y\bar{z}$

$f = wx\bar{y}(z+\bar{z}) + \bar{y}z(w+\bar{w})(x+\bar{x}) + \bar{w}y\bar{z}(x+\bar{x}) + \bar{x}y\bar{z}(w+\bar{w})$

$f = wx\bar{y}z + wx\bar{y}\bar{z} + wx\bar{y}z + w\bar{x}\bar{y}z + \bar{w}\bar{x}\bar{y}z + \bar{w}x\bar{y}z + \bar{w}xy\bar{z} + \bar{w}\bar{x}y\bar{z} + w\bar{x}y\bar{z} + \bar{w}\bar{x}y\bar{z}$

$f = wx\bar{y}z + wx\bar{y}\bar{z} + w\bar{x}\bar{y}z + \bar{w}\bar{x}\bar{y}z + \bar{w}x\bar{y}z + \bar{w}xy\bar{z} + \bar{w}\bar{x}y\bar{z} + w\bar{x}y\bar{z}$

$g = (w+x+\bar{y}+\bar{z})(\bar{x}+\bar{y}+z)(\bar{w}+y+\bar{z})$

$\bar{g} = \overline{(w+x+\bar{y}+\bar{z})(\bar{x}+\bar{y}+z)(\bar{w}+y+\bar{z})}$

$= \bar{w}\bar{x}yz + xy\bar{z} + w\bar{y}z$

$\bar{g} = \bar{w}\bar{x}yz + wxy\bar{z} + \bar{w}xy\bar{z} + w\bar{x}\bar{y}z + wx\bar{y}z$

$F = f \cdot g$

$f =$

$yz \backslash wx$	$\bar{w}\bar{x}$ 00	$\bar{w}x$ 01	wx 11	$w\bar{x}$ 10
$\bar{y}\bar{z}$ 00			1	
$\bar{y}z$ 01	1	1	1	1
yz 11				
$y\bar{z}$ 10	1	1		1

g

$yz \backslash wx$	$\bar{w}\bar{x}$ 00	$\bar{w}x$ 01	wx 11	$w\bar{x}$ 10
$\bar{y}\bar{z}$ 00	1	1	1	1
$\bar{y}z$ 01	1	1		
yz 11		1	1	1
$y\bar{z}$ 10	1			1

19MA 20059

$F = f \cdot g$

$yz \backslash wx$	$\bar{w}\bar{x}$ 00	$\bar{w}x$ 01	wx 11	$w\bar{x}$ 10
$\bar{y}\bar{z}$ 00			1	
$\bar{y}z$ 01	1	1		
yz 11				
$y\bar{z}$ 10	1			1

$$\boxed{F = f \cdot g = \bar{w}\bar{y}z + \bar{x}y\bar{z} + wx\bar{y}\bar{z}}$$

19MA20059

Page No.:

5) $B E + \bar{B} D \bar{E}$ is simplified version of $\bar{A} B E + B C D E + A C \bar{D} E + \bar{B} \bar{C} D \bar{E}$

AB \ CD	00	01	11	10
00		1	1	
01				
11				
10		1	X	

when $E = 0$

AB \ CD	00	01	11	10
00				
01	1	1	1	1
11	1	X	1	X
10				

when $E = 1$

The dont care conditions are

$d(A, B, C, D, E) = \Sigma m(22, 27, 29)$

for which

$f(A, B, C, D, E) = B E + \bar{B} D \bar{E}$ becomes the simplified version of

$g(A, B, C, D, E) = \bar{A} B E + B C D E + A \bar{C} \bar{D} E + \bar{B} \bar{C} D \bar{E}$