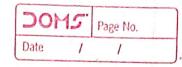
| -       |   |
|---------|---|
|         | Assignment 64 MM - 128 with restricted (1, 12) CA   |
|         | 19MA 20059 Reesti P. Charantinath (30) 2 = (30)   |
|         | D1 = 0+0 = 7 = 4e 1   |
| (1      | 12 d2y + (225+1) dy 0+ y=05 = (31) ) + (31))  |
|         | dn2 dn dn   |
|         | Adjoint is given by   |
|         | Adjoint is given by $My(x) = \frac{d^2}{dx^2} \left( \frac{x^2 \cdot y}{x^2 \cdot y} \right)^2 = \frac{d}{dx} \left( \left( \frac{2x^3 + 1}{x^3 + 1} \right) \cdot \frac{y}{x^3 + 1} \right)^2 + \frac{d}{dx} \left( \frac{2x^3 + 1}{x^3 + 1} \right) \cdot \frac{y}{x^3 + 1} $   |
|         | dz 12x2 dx. [(5.39)]  |
| 1       | = d (2xy + x2,y') - Au (6x2y) - (2x3+1)y' + y   |
|         | Lolubion of But an officer position   |
|         | = 2y + 2xy' + 2xy' + 22y' - 6x2y - (2x3+1) y'+y   |
|         |   |
|         | = n2y = (223 - 4 x +1) y = 3(2x2-1) y   |
| ro .    |   |
|         | Adjoint eq My(2)=0(=) 1) 1 -3(2x2-1)y = 0   |
|         | $\chi^{2}y'' - (2\chi^{3} - 4\chi + 1)y' - 3(2\chi^{2} - 1)y = 0$   |
| \$ 1101 | City of the Market College of the Market Col  |
| _2)_    | $\frac{2^{2} d^{2}y - 2n dy + 2y = 0}{dx^{2}} dx \qquad \frac{1}{2} dx + (r) drie = 0$ $\frac{1}{2} dx^{2} dx + \frac{1}{2} dx + $ |
|         | - U = Kink(n) + Book (b-n)e   |
|         | ANA PINE / 4. / HIVED NOW   |
|         | for the given eq = to be self adjoint   |
|         | d(22) # 1000 to should be equal to -22  |
| 7       | d(x2) # 1000 to should be equal to -2x  |
|         | but , d (22)= 2x + -22  |
|         | ohl 12  |
|         | Thus,   |
|         |   |
|         | or or or a self adjoint eg  |
|         | adjoint eg=   |
|         | Marille State of the state of t  |
|         |   |



|    |   | DOM5" Page No.      |
|----|---|---------------------|
|    | 19MA 20059  | Date / /            |
| 3) | Scos (n T2) ? -L < x < L 1=1                                      | Henre , { Sin (CE)} |
| P  | 1- (-) 1 n=0 (-1) h = 0 = (0) h                                   | (c) 4 (c) 4 - 1     |
|    | Let $\{fn\}_{n=0}^{\infty} = \{\cos(n\pi\alpha)\}_{n=0}^{\infty}$ | 1 = A = 1 and       |

918 185 = 1 Jal cosmix ) cosmix ) dx ard wind

= 1x0+1 x0=0 = (1) y

Thus Sios nkx? ~ ~ mutually orthogonal on -L<25L

for m=on, \fm(2). fn(2) dx = existence of the BUP.

 $\left[\cos\left((m-n)\pi x\right)-\cos(m+n)\pi x\right]dx$ 

= 0-0=0

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Hence, \Sin(n\overline{\tau})\frac{1}{n=1} is mutually extragonal on -LSxSL -y"(2) = >y(2), y(0) = 0 = y(L) 5) Case 1:- h=0 Gen. Sol =:- y(2) = A2 +1800 -- Sold Store of or Only trivial solution, y = 0 is possible in this case Case 2:- 20. Let 2= = pin p +0 1 = 100-9 Homogeneous fact: -y" = -μ²y ⇒ y" + -μ²y=0 Exercise Gen. Sol=: - Ae μα + Be-μα = y(x) Using BC's:- A=0, B=0 Only trivial solution, y=0 is possible in this case Case 3! 1- (270 ) Xet 2= 12 1 1 +0= Homogeneous part:  $-y''' = \mu^2 y \Rightarrow y'' + \mu^2 y = 0$ Gen so!  $\Rightarrow y(x) = A \cos(\mu x) + B \sin(\mu x)$ Using BC's:  $\Rightarrow A = 0$  (from  $\Rightarrow y(0) = 0$ )  $\Rightarrow y(L) = A \cos(\mu x) + B \sin(\mu L) = 0$  as  $\Rightarrow x = 0$ Bsin(µL) =0 Thuis Sies nig? is hustically orthogonal on -LS. For non-trivial solution to exist, B \$0, :. sin(µc)=0 µc = nx, n= a,1,2,.... as µ≠0 & L≠0 12= L (OK)2/L2 for  $m \neq \pi n$ ,  $(f_m(x) : f_n(x) dx = (disharq \ sinh f$ Then,  $x_n = \mu_n^2 = \frac{n^2 \kappa^2}{n^2}$  are the eigenvalues of the BVP, i.e. values where non-zero solutions to the BVP exist. The corresponding solutions are: yn(x) = B sin(ux) = B sin(n xx)

0-0-0

 $y''(r) + \lambda(y(r)) = 0$ , y'(0) = 0 = y'(1)Combining all three caled Gensol =:- y(x) = A x + B Using BC's or - & A = O , B is arbitrary and monager and i. y(x) = B, B = 0 has non-zero solutions to Thus,  $\lambda=0$  is an eigenvalue with eigen function,  $y_0(x)=B$ Case 2:-  $\lambda < 0$ , Let  $\lambda = -\mu^2$ ,  $\mu \neq 0 = 100$ Homogeneous part:-  $\mu'' - \mu^2 y = 0$ Gen  $Sol = \frac{1}{2} := y = A e^{\mu \pi} + B e^{\mu \pi} \Rightarrow y' = \mu A e^{\mu \pi} = \mu B e^{\mu \pi}$ Using  $Bc's! - \mu A - \mu B = 0 \Rightarrow A - B = 0 \Rightarrow A = B = 0$   $\mu A e^{\mu L} - \mu B e^{\mu L} = 0 \Rightarrow A e^{\mu L} = 0 \Rightarrow A = B = 0$ Only trivial sola y(x)=0 in this case, so x <0 in norto-an saeigenvalue for BUP given A sista mist Case 3:- x70, Let \ = \mu^2, \ \mu = 0 Yen sol?: - y(x) = A cos(ux) +B sin(ux) embrenges jan & Ob = y' = = - Ape Sidux) + Bu cos(per) soming while using BC's:- y'(0) = B = 0 => B=0 y'(L) = - Au sin(uL) = 01, now u = 0 & + +0 for 1) 19 18 + (24) 112 34 Mon = tmirial 1/801 = 8 + (24) 128 + (24) which is sinful = 0 1 = 1 + plu= n 70 n=1/2 11:00 prich  $(0 \pm 1/2 +$ BUP with  $\frac{1}{2}$  eigenfunctions  $\frac{1}{2}$   $\frac{1}{2}$  P200 19 M A 20059.

(1), h = 0 = (0), h '0 = (0), h) 4, (0), h Combining all three cases, eigenvect eigenvalues:  $\lambda_n = n^2 \kappa^2$ , n = 0, 1/2.  $\frac{1}{2} \cdot \frac{1}{2} \cdot$ corresponding eigenfunctions: - of yn(n) == A cos (n x 2), n = 0,11, n, ... y"(2) + xy(2)=0, y(0)=0, y(1)+y'(1)=0 Ŧ. Case 1:- 2=0 Gen Sol=: - y(2) = A2+B Su- = 1 to G> 1 -1826 Using BC's: - B=0, A=0 0= W=11-" BL TINDO WIGOWADWATE Only trivial solfx, by = 0 is possible A//g/00 200 WHEN - MEET - B = ACT-8=0 => 1=8=0 Case 2:- 1<0, Let 1=-µ2, µ≠0 Gensola: y= Aen+Be-Min, gl= uAend-uBe-Min Using Bc's:- A+B=0, Ae++Be-++ + MAE+- MBE-H=0+1 => A × 2 Msh(µ) × [tanh(µ)+µ]=0 AS MED, HARY / FOR MONTE XWELL GOLDS LOCAL (M) \$0,00 As \u + 0, cosh(\u) + 0; is tanh(\u) + - \uzan was ansomoth Thus A=0 (rubois 8+ (rubous A = (r)) -18102 not Only trivial sola, y=0 is possible. > XLO is not eigenvalue. 11/440 PCR: A CO) = Bre 0 => Ben . Case 3: 1 20 - neletin = per pe for in A = (1) Gen sol= :- y = A cospan) + B sin(plas), y'= - Aprosin(plas) + Bp cos(plas) Using BC's: A=0, Buccosu)+ Bsin u=0 => B(pcosu+sinu)=0 For non-trivial sol= 1 B ≠0, ... µ cosµ + sinµ=0 => tanp=-µ (as cos \upsilon \upsilon) ¿ Eigenvalue: = λ = μ² where μ satisfies tanμ=-μ, μ≠ο P.E.s Eigenfunction: = 4 = B sin (pea) who is