# NSDE Programming ASSIGNMENT 3

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## Q1.

Use the Crank-Nikolson method to solve the parabolic partial differential equation $u_t = u_{xx}$, $x \in (0,1), t \in (0,\infty)$ with initial condition $u(x,0) = 2x$, boundary conditions $u_x(0,t) = 0$ and $u_x(1,t) = 1$. Use the central difference approximation for the boundary conditions. Take $h = 0.1,\ k = 0.05$. Plot the data for various values of (x_m, t_n, u_m,n).

## Matlab Code:

```
u_t_0 = @(x) 2*x;
ux_x_0 = @(t) 0;
ux_x_n = @(t) 1;

h=0.1;
k=0.05;

lambda = k / h^2;

x_init = 0;
x_final = 1;
t_init = 0;
t_final = 1;

x_itr = (x_final - x_init) / h;
t_itr = (t_final - t_init) / k;

Values = zeros(x_itr + 1, t_itr + 1);

for i=1:x_itr + 1
    Values(i, 1) = u_t_0(x_init + h * (i-1));
end

A = zeros(x_itr + 1, x_itr + 1);
B = zeros(x_itr + 1, 1);

syms u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1 eq
eq(u_m1_n2, u_m2_n2, u_m3_n2, u_m1_n1, u_m2_n1, u_m3_n1) = -1 * lambda *
u_m1_n2 + (2 + 2 * lambda) * u_m2_n2 - (lambda) * u_m3_n2 - 1 * lambda * u_m1_n1 -
(2 - 2 * lambda) * u_m2_n1 - lambda * u_m3_n1;
```

```
for j=1:t_itr
   for i=1:x_itr + 1
      if i==1
         temp_eqs = subs(eq, {u_m1_n2 u_m1_n1}, {u_m3_n2 u_m3_n1});
         temp_val = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1},
{0 0 0 0 0 0});

         A(1, 1) = subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {1 0 0 0}) -
temp_val;
         A(1, 2) = subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {0 1 0 0}) -
temp_val;
%          B(1, 1) = subs(temp_rhs, {u_m2_n1 u_m3_n1}, {Values(1, j) Values(2, j)});
         B(1, 1) = -1 * (subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {0 0
Values(1, j) Values(2, j)}) - temp_val);

      elseif i == x_itr + 1
         temp_eqs = subs(eq, {u_m3_n2 u_m3_n1}, {0.2 + u_m1_n2 0.2 + u_m1_n1});
         temp_val = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1},
{0 0 0 0 0 0});

         A(x_itr + 1, x_itr) = subs(temp_eqs, {u_m1_n2 u_m2_n2 u_m1_n1 u_m2_n1}, {1 0 0
0}) - temp_val;
         A(x_itr + 1, x_itr + 1) = subs(temp_eqs, {u_m1_n2 u_m2_n2 u_m1_n1 u_m2_n1}, {0
1 0 0}) - temp_val;
         B(x_itr + 1, 1) = -1 * (subs(temp_eqs, {u_m1_n1 u_m2_n1 u_m1_n2 u_m2_n2},
{Values(x_itr, j) Values(x_itr + 1, j) 0 0}) - temp_val);
      else
         temp_val = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1},
{0 0 0 0 0 0});
         A(i, i - 1) = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1}, {1
0 0 0 0 0}) - temp_val;
         A(i, i) = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1}, {0 1
0 0 0 0}) - temp_val;
         A(i, i + 1) = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1},
{0 0 1 0 0 0}) - temp_val;
%          B(i, 1) = subs(eq_rhs, {u_m1_n1 u_m2_n1 u_m3_n1}, {Values(i-1, j) Values(i, j)
Values(i + 1, j)});

         B(i, 1) = -1 * (subs(eq, {u_m1_n1 u_m2_n1 u_m3_n1 u_m1_n2 u_m2_n2 u_m3_n2},
{Values(i-1, j) Values(i, j) Values(i + 1, j) 0 0 0}) - temp_val);
      end
   end

   Values(:, j+1) = linsolve(A, B);
   A = zeros(x_itr + 1, x_itr + 1);
   B = zeros(x_itr + 1, 1);
end



X = 0:0.1:1;
T = 0:0.05:1;
```

```
surf(X.', T.', Values.')
xlabel('x');
ylabel('t');
zlabel('u');
```

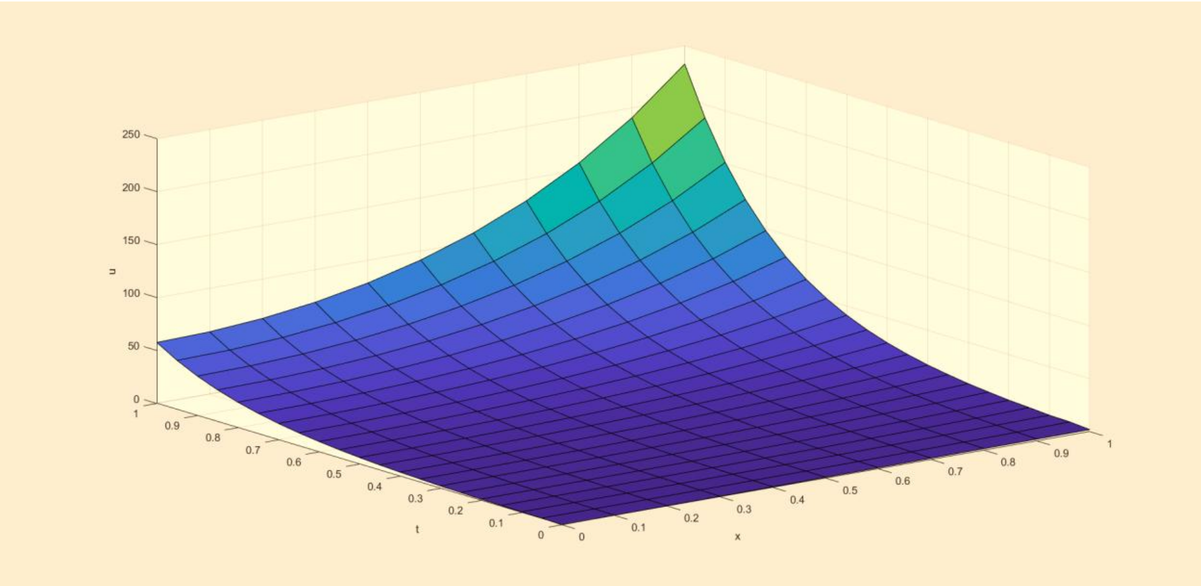
## Table:

| x \ t | 0 | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.609275 | 0.704092 | 1.07433 | 1.359187 | 1.848288 | 2.362102 |
| 0.1 | 0.2 | 0.53113 | 0.801201 | 1.051269 | 1.43922 | 1.866076 | 2.447077 |
| 0.2 | 0.4 | 0.585437 | 0.873885 | 1.152659 | 1.549981 | 2.023877 | 2.624439 |
| 0.3 | 0.6 | 0.713919 | 0.987775 | 1.324354 | 1.749367 | 2.282612 | 2.941091 |
| 0.4 | 0.8 | 0.887968 | 1.165639 | 1.566706 | 2.0481 | 2.655298 | 3.407185 |
| 0.5 | 1 | 1.097205 | 1.419383 | 1.893605 | 2.454845 | 3.162851 | 4.039165 |
| 0.6 | 1.2 | 1.345324 | 1.763117 | 2.320204 | 2.986386 | 3.828811 | 4.863265 |
| 0.7 | 1.4 | 1.651573 | 2.215837 | 2.86065 | 3.670552 | 4.679115 | 5.916802 |
| 0.8 | 1.6 | 2.058451 | 2.793635 | 3.533944 | 4.54583 | 5.741733 | 7.253101 |
| 0.9 | 1.8 | 2.648709 | 3.482125 | 4.39267 | 5.640433 | 7.063387 | 8.93491 |
| 1 | 2 | 3.578451 | 4.164499 | 5.62173 | 6.863805 | 8.825453 | 10.92492 |

| x \ t | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 |
|---|---|---|---|---|---|---|---|
| 0 | 3.073033 | 3.902421 | 4.972944 | 6.269561 | 7.902193 | 9.907757 | 12.40954 |
| 0.1 | 3.130244 | 4.011087 | 5.078382 | 6.423447 | 8.074834 | 10.13623 | 12.68142 |
| 0.2 | 3.368336 | 4.291211 | 5.439282 | 6.859897 | 8.625466 | 10.81127 | 13.52482 |
| 0.3 | 3.764695 | 4.782216 | 6.048528 | 7.616246 | 9.562425 | 11.9743 | 14.96564 |
| 0.4 | 4.341055 | 5.500229 | 6.93729 | 8.720165 | 10.93029 | 13.67118 | 17.06915 |
| 0.5 | 5.125076 | 6.47425 | 8.14487 | 10.21842 | 12.78786 | 15.9747 | 19.9252 |
| 0.6 | 6.151343 | 7.745695 | 9.723273 | 12.17527 | 15.21462 | 18.98378 | 23.6559 |
| 0.7 | 7.463403 | 9.369088 | 11.74076 | 14.67383 | 18.31541 | 22.82649 | 28.42182 |
| 0.8 | 9.111342 | 11.41888 | 14.28052 | 17.82335 | 22.22187 | 27.66796 | 34.42711 |
| 0.9 | 11.15707 | 13.99389 | 17.43971 | 21.77056 | 27.09004 | 33.72616 | 41.91932 |
| 1 | 13.78346 | 17.12297 | 21.42317 | 26.62585 | 33.17793 | 41.21909 | 51.25408 |

| x \ t | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 | 1 |
|---|---|---|---|---|---|---|---|
| 0 | 15.49978 | 19.34055 | 24.09519 | 29.99598 | 37.30735 | 46.37583 | 57.61623 |
| 0.1 | 15.84594 | 19.76254 | 24.62413 | 30.6472 | 38.1184 | 47.37847 | 58.86166 |
| 0.2 | 16.88641 | 21.05686 | 26.22537 | 32.63535 | 40.58101 | 50.43359 | 62.6479 |
| 0.3 | 18.67408 | 23.27216 | 28.97304 | 36.04105 | 44.80433 | 55.66903 | 69.13955 |
| 0.4 | 21.28244 | 26.50601 | 32.98252 | 41.01215 | 50.96756 | 63.31043 | 78.61343 |
| 0.5 | 24.82357 | 30.89651 | 38.42595 | 47.76115 | 59.33505 | 73.68473 | 91.47563 |
| 0.6 | 29.4494 | 36.63149 | 45.53667 | 56.57694 | 70.26532 | 87.23611 | 108.2771 |
| 0.7 | 35.35741 | 43.95713 | 54.61882 | 67.83741 | 84.22626 | 104.5451 | 129.7373 |
| 0.8 | 42.80029 | 53.18778 | 66.06061 | 82.02566 | 101.815 | 126.354 | 156.7746 |
| 0.9 | 52.10552 | 64.71108 | 80.3589 | 99.74345 | 123.7898 | 153.5922 | 190.5507 |
| 1 | 63.64268 | 79.04467 | 98.10602 | 121.7662 | 151.0783 | 187.4379 | 232.5027 |

Graph:



## Q2.

Using the Crank-Nicolson method with $h = 0.1$ and the mesh ratio parameter $r = 0.25$ find the solution of $u_t = u_{xx}$ with Initial condition $u(x,0) = \cos\frac{\pi x}{2}$, $-1 \le x \le 1, t = 0$; boundary conditions $u(-1,t) = u(1,t) = 0, \quad t > 0$ at the first 3 time steps. Plot the results for each time step in different frame.

## Matlab Code:

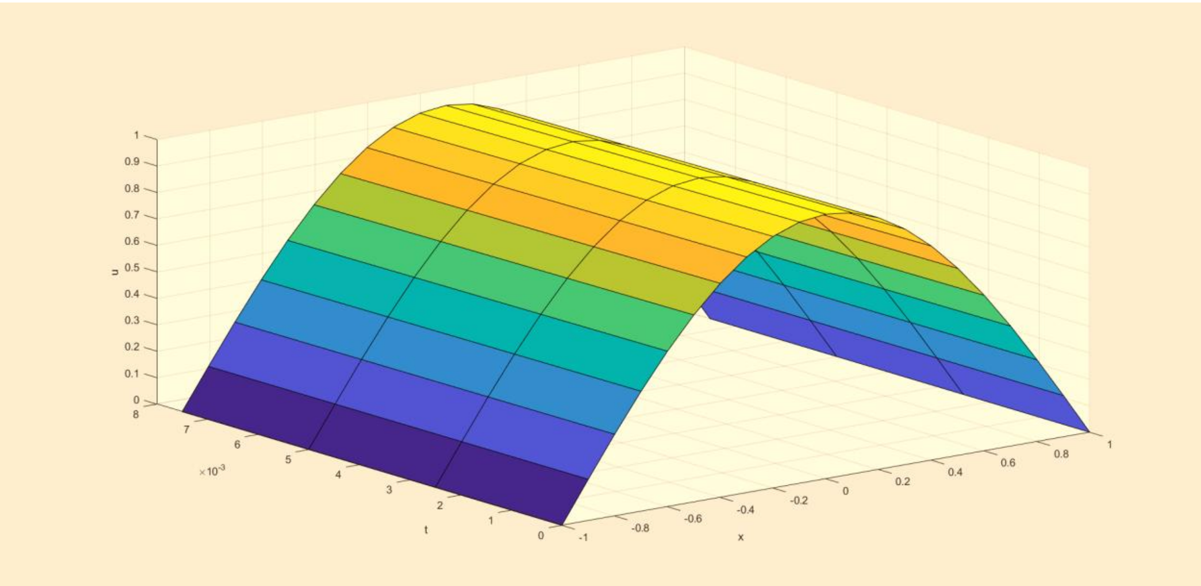
```
u_t_0 = @(x) cos(pi * x / 2);
ux_x_0 = @(t) 0;
ux_x_n = @(t) 0;

h=0.1;
lambda = 0.25;
k=lambda * h^2;

x_init = -1;
x_final = 1;
t_init = 0;
t_final = 0.0075;

x_itr = int16((x_final - x_init) / h);
t_itr = int16((t_final - t_init) / k);
Values = zeros(x_itr + 1, t_itr + 1);

for i=1:x_itr + 1
  Values(i, 1) = u_t_0(x_init + h * double(i-1));
end

for j=2:t_itr + 1
  Values(1, j) = 0;
  Values(x_itr + 1, j) = 0;
end

A = zeros(x_itr - 1, x_itr - 1);
B = zeros(x_itr - 1, 1);

syms u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1 eq
eq(u_m1_n2, u_m2_n2, u_m3_n2, u_m1_n1, u_m2_n1, u_m3_n1) = -1 * lambda *
u_m1_n2 + (2 + 2 * lambda) * u_m2_n2 - (lambda) * u_m3_n2 - 1 * lambda * u_m1_n1 -
(2 - 2 * lambda) * u_m2_n1 - lambda * u_m3_n1;

for j=1:t_itr
  for i=2:x_itr
    if i == 2
      temp_eqs = subs(eq, {u_m1_n2 u_m1_n1}, {Values(1, j+1) Values(1, j)});
      temp_val = subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {0 0 0 0});
```

```
            A(1, 1) = subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {1 0 0 0}) -
temp_val;
            A(1, 2) = subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {0 1 0 0}) -
temp_val;
%           B(1, 1) = subs(temp_rhs, {u_m2_n1 u_m3_n1}, {Values(1, j) Values(2, j)});
            B(1, 1) = -1 * (subs(temp_eqs, {u_m2_n2 u_m3_n2 u_m2_n1 u_m3_n1}, {0 0
Values(2, j) Values(3, j)}) - temp_val);

        elseif i == x_itr
            temp_eqs = subs(eq, {u_m3_n2 u_m3_n1}, {Values(x_itr + 1, j + 1) Values(x_itr + 1,
j)});
            temp_val = subs(temp_eqs, {u_m1_n2 u_m2_n2 u_m1_n1 u_m2_n1}, {0 0 0 0});

            A(x_itr - 1, x_itr - 2) = subs(temp_eqs, {u_m1_n2 u_m2_n2 u_m1_n1 u_m2_n1}, {1
0 0 0}) - temp_val;
            A(x_itr - 1, x_itr - 1) = subs(temp_eqs, {u_m1_n2 u_m2_n2 u_m1_n1 u_m2_n1}, {0
1 0 0}) - temp_val;
            B(x_itr - 1, 1) = -1 * (subs(temp_eqs, {u_m1_n1 u_m2_n1 u_m1_n2 u_m2_n2},
{Values(x_itr - 1, j) Values(x_itr, j) 0 0}) - temp_val);
        else
            temp_val = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1},
{0 0 0 0 0 0});

            A(i - 1, i - 2) = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1
u_m3_n1}, {1 0 0 0 0 0}) - temp_val;
            A(i - 1, i - 1) = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1
u_m3_n1}, {0 1 0 0 0 0}) - temp_val;
            A(i - 1, i) = subs(eq, {u_m1_n2 u_m2_n2 u_m3_n2 u_m1_n1 u_m2_n1 u_m3_n1}, {0
0 1 0 0 0}) - temp_val;
%           B(i, 1) = subs(eq_rhs, {u_m1_n1 u_m2_n1 u_m3_n1}, {Values(i-1, j) Values(i, j)
Values(i + 1, j)});

            B(i - 1, 1) = -1 * (subs(eq, {u_m1_n1 u_m2_n1 u_m3_n1 u_m1_n2 u_m2_n2
u_m3_n2}, {Values(i-1, j) Values(i, j) Values(i + 1, j) 0 0 0}) - temp_val);
        end
    end

    Values(2:x_itr, j+1) = linsolve(A, B);
    A = zeros(x_itr - 1, x_itr - 1);
    B = zeros(x_itr - 1, 1);
end


X = -1:0.1:1
T = 0:0.0025:0.0075
surf(X.', T.', Values.')
xlabel('x');
ylabel('t');
zlabel('u');
```

Table:

| x \ t | 0 | 0.0025 | 0.005 | 0.0075 |
|---|---|---|---|---|
| -1 | 6.12E-17 | 0 | 0 | 0 |
| -0.9 | 0.156434 | 0.155474 | 0.15452 | 0.153572 |
| -0.8 | 0.309017 | 0.307121 | 0.305236 | 0.303363 |
| -0.7 | 0.45399 | 0.451204 | 0.448435 | 0.445683 |
| -0.6 | 0.587785 | 0.584178 | 0.580593 | 0.57703 |
| -0.5 | 0.707107 | 0.702767 | 0.698454 | 0.694168 |
| -0.4 | 0.809017 | 0.804052 | 0.799118 | 0.794214 |
| -0.3 | 0.891007 | 0.885538 | 0.880104 | 0.874703 |
| -0.2 | 0.951057 | 0.94522 | 0.939419 | 0.933654 |
| -0.1 | 0.987688 | 0.981627 | 0.975603 | 0.969616 |
| 0 | 1 | 0.993863 | 0.987764 | 0.981702 |
| 0.1 | 0.987688 | 0.981627 | 0.975603 | 0.969616 |
| 0.2 | 0.951057 | 0.94522 | 0.939419 | 0.933654 |
| 0.3 | 0.891007 | 0.885538 | 0.880104 | 0.874703 |
| 0.4 | 0.809017 | 0.804052 | 0.799118 | 0.794214 |
| 0.5 | 0.707107 | 0.702767 | 0.698454 | 0.694168 |
| 0.6 | 0.587785 | 0.584178 | 0.580593 | 0.57703 |
| 0.7 | 0.45399 | 0.451204 | 0.448435 | 0.445683 |
| 0.8 | 0.309017 | 0.307121 | 0.305236 | 0.303363 |
| 0.9 | 0.156434 | 0.155474 | 0.15452 | 0.153572 |
| 1 | 6.12E-17 | 0 | 0 | 0 |

Graph:



## Q3.

Use the explicit method to solve the wave equation

$u_{tt} = u_{xx}$, $0 < x < 1, t > 0$ with boundary and initial conditions

$$u(0,t) = -\sin t,\ u(1,t) = \sin(1-t),\ u(x,0) = \sin x, \quad u_t(x,0) = -\cos(x).$$

Take step length along $x$-axis and $t$-axis as 0.1 and 0.1 respectively. Find solution up to $t = 0.5$.

## Matlab Code:

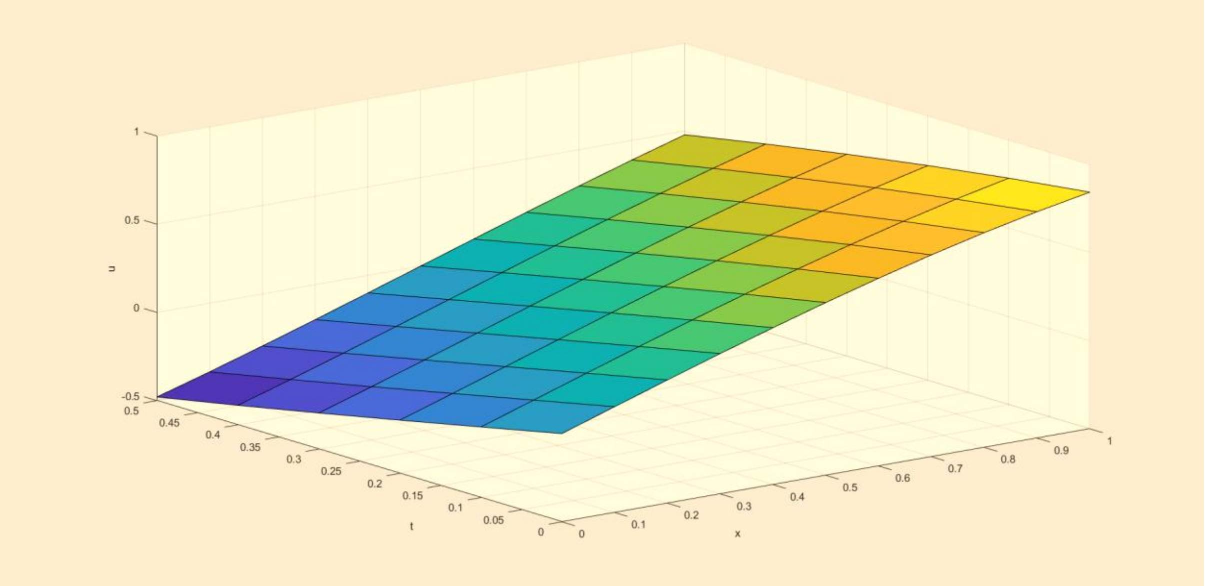
```
f = @(x) sin(x);
g = @(x) -1 * cos(x);

u_x_0 = @(t) -1 * sin(t);
u_x_n = @(t) sin(1 - t);

h=0.1;
k=0.1;
r = 1 * k/h;

x_init = 0;
x_final = 1;
t_init = 0;
t_final = 0.5;

x_itr = int16((x_final - x_init) / h);
t_itr = int16((t_final - t_init) / k);
Values = zeros(x_itr + 1, t_itr + 1);

for i=1:x_itr + 1
   Values(i, 1) = f((x_init + h * double(i-1)));
end

for j=1:t_itr + 1
   Values(1, j) = u_x_0((t_init + k * double(j-1)));
   Values(x_itr + 1, j) = u_x_n((t_init + k * double(j-1)));
end

for i=2:x_itr
   Values(i, 2) = 0.5 * (r^2 * f((x_init + double(i - 2) * h)) + 2 * (1 - r^2) * f((x_init + double(i -
1) * h)) + r^2 * f((x_init + double(i) * h)) + 2 * k * g((x_init + double(i - 1) * h)));
end

for j=3:t_itr + 1
   for i=2:x_itr
      Values(i, j) = r^2 * Values(i-1, j-1) + 2 * (1 - r^2) * Values(i, j-1) + r^2 * Values(i + 1, j-1) -
Values(i, j-2);
   end
end
```

```
X = 0:0.1:1;
T = 0:0.1:0.5;
surf(X.', T.', Values.')
xlabel('x');
ylabel('t');
zlabel('u');
```

## Table:

| x \ t | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|---|
| 0 | 0 | -0.09983 | -0.19867 | -0.29552 | -0.38942 | -0.47943 |
| 0.1 | 0.099833 | -0.00017 | -0.1 | -0.19883 | -0.29567 | -0.38956 |
| 0.2 | 0.198669 | 0.09967 | -0.00032 | -0.10015 | -0.19897 | -0.29581 |
| 0.3 | 0.29552 | 0.19851 | 0.099517 | -0.00047 | -0.10029 | -0.1991 |
| 0.4 | 0.389418 | 0.295367 | 0.198364 | 0.099379 | -0.0006 | -0.1004 |
| 0.5 | 0.479426 | 0.389272 | 0.295229 | 0.198237 | 0.099263 | -0.0007 |
| 0.6 | 0.564642 | 0.479288 | 0.389145 | 0.295113 | 0.198133 | 0.099263 |
| 0.7 | 0.644218 | 0.564515 | 0.479172 | 0.389041 | 0.295113 | 0.198237 |
| 0.8 | 0.717356 | 0.644102 | 0.564412 | 0.479172 | 0.389145 | 0.295229 |
| 0.9 | 0.783327 | 0.717253 | 0.644102 | 0.564515 | 0.479288 | 0.389272 |
| 1 | 0.841471 | 0.783327 | 0.717356 | 0.644218 | 0.564642 | 0.479426 |

## Graph:



Q4.

Using standard 5-point formula, derive the system of algebraic equations at the nodal points for the elliptic equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=x^2+y^2, \quad -1 < x < 1, -1 < y < 1,$ $u=2$ at $x=-1$ & $x=1$; $u=1$ at $y=-1$ & $y=1$. Take $h=k=0.25$.

Setup the (i) Gauss-Seidel and (ii) Gauss-Jacobi iterations for the system of equations. Take all the starting values for the iteration as ZEROS. Compare your Gauss Seidel and Gauss Jacobi solution on the plotter frame.

Matlab Code:

```
h=0.25;
k = 0.25;
iterations = 10;

f = @(x, y) x^2 + y^2;

x_init = -1;
x_final = 1;
y_init = -1;
y_final = 1;

u_x_0 = 2;
u_x_n = 2;
u_y_0 = 1;
u_y_n = 1;

x_itr = int16((x_final - x_init) / h);
y_itr = int16((y_final - y_init) / k);
Values = zeros(x_itr + 1, y_itr + 1);

for i=1:x_itr + 1
   Values(i, 1) = u_y_0;
   Values(i, y_itr + 1) = u_y_n;
end

for j=1:y_itr + 1
   Values(1, j) = u_x_0;
   Values(x_itr + 1, j) = u_x_n;
end

Values_jacobi = zeros(x_itr + 1, y_itr + 1);
Values_seidel = zeros(x_itr + 1, y_itr + 1);

Values_jacobi(:, :) = Values(:, :);
Values_seidel(:, :) = Values(:, :);

% Gauss jacobi
for k=1:iterations
   Values_temp = zeros(x_itr + 1, y_itr + 1);
```

```
    Values_temp(:, :) = Values_jacobi(:, :);
    for j=2:y_itr
        for i=2:x_itr
            Values_temp(i, j) = (0.25) * (Values_jacobi(i - 1, j) + Values_jacobi(i + 1, j) +
Values_jacobi(i, j-1) + Values_jacobi(i, j+1) - h^2 * f(x_init + h * double(i - 1), y_init + k *
double(j - 1)));
        end
    end

    Values_jacobi(:, :) = Values_temp(:, :);
end

% Gauss Seidel
for k=1:iterations
    for j=2:y_itr
        for i=2:x_itr
            Values_seidel(i, j) = (0.25) * (Values_seidel(i - 1, j) + Values_seidel(i + 1, j) +
Values_seidel(i, j-1) + Values_seidel(i, j+1) - h^2 * f(x_init + h * double(i - 1), y_init + k *
double(j - 1)));
        end
    end
end

X = -1:0.25:1;
Y = -1:0.25:1;
surf(X.', Y.', Values_jacobi.', 'FaceColor','b', 'FaceAlpha',0.5, 'EdgeColor','none');
hold on
surf(X.', Y.', Values_seidel.', 'FaceColor','g', 'FaceAlpha',0.4, 'EdgeColor','none');
xlabel('x');
ylabel('t');
zlabel('u');
legend('Jacobi', 'Seidel');
```

## Table:

1. Performing Gauss Jacobi Iterations

| x \ y | -1 | -0.75 | -0.5 | -0.25 | 0 | 0.25 | 0.5 | 0.75 | 1 |
|---|---|---|---|---|---|---|---|---|---|
| -1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| -0.75 | 1 | -4.8133 | -16.9886 | -36.9648 | -64.7002 | -98.3356 | -129.935 | -130.328 | 1 |
| -0.5 | 1 | -7.39705 | -23.6307 | -49.9354 | -86.2969 | -129.886 | -169.16 | -164.801 | 1 |
| -0.25 | 1 | -8.22095 | -25.6341 | -53.7051 | -92.4069 | -138.594 | -179.531 | -173.217 | 1 |
| 0 | 1 | -8.39733 | -26.0552 | -54.4657 | -93.6224 | -140.287 | -181.502 | -174.744 | 1 |
| 0.25 | 1 | -8.22095 | -25.6341 | -53.7051 | -92.4069 | -138.594 | -179.531 | -173.217 | 1 |
| 0.5 | 1 | -7.39705 | -23.6307 | -49.9354 | -86.2969 | -129.886 | -169.16 | -164.801 | 1 |
| 0.75 | 1 | -4.8133 | -16.9886 | -36.9648 | -64.7002 | -98.3356 | -129.935 | -130.328 | 1 |
| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

2. Performing Gauss Seidel iterations

| x \ y | -1 | -0.75 | -0.5 | -0.25 | 0 | 0.25 | 0.5 | 0.75 | 1 |
|---|---|---|---|---|---|---|---|---|---|
| -1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| -0.75 | 1 | -6.08741 | -21.1117 | -45.9466 | -80.6594 | -122.373 | -159.808 | -156.375 | 1 |
| -0.5 | 1 | -10.7663 | -33.6865 | -70.9354 | -122.523 | -183.168 | -233.798 | -219.392 | 1 |
| -0.25 | 1 | -13.2386 | -40.1385 | -83.4095 | -142.915 | -211.83 | -266.878 | -245.294 | 1 |
| 0 | 1 | -14.3891 | -43.0919 | -88.9997 | -151.829 | -223.945 | -280.219 | -255.14 | 1 |
| 0.25 | 1 | -14.5702 | -43.5088 | -89.6667 | -152.649 | -224.639 | -280.421 | -254.842 | 1 |
| 0.5 | 1 | -13.3633 | -40.2563 | -83.1574 | -141.622 | -208.533 | -261.057 | -239.056 | 1 |
| 0.75 | 1 | -9.03937 | -28.6362 | -60.0631 | -102.95 | -152.509 | -193.227 | -181.72 | 1 |
| 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

## Graph: