(11)

a: Trunk 'A' contains a treasure.
b: Trunk 'B' contains a treasure.

$\neg a$: Trunk 'A' is a trap.
$\neg b$: Trunk 'B' is a trap.

Insp 1 :- At least one of the two trunks contains a treasure. $(a \vee b)$

Insp 2 :- A is a trap $(\neg a)$

Insp 1 $\leftrightarrow$ Insp 2

$(a \vee b) \leftrightarrow (\neg a)$

a	b	$a \vee b$	$\neg a$	$(a \vee b) \leftrightarrow (\neg a)$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F.

$\therefore$ Trunk B contains the Treasure.

(12)

a : Box 1 contains gold
b : Box 2 contains gold
c : Box 3 contains gold.

Message 1 : $\neg a$ $\qquad$ M_{iT} = Message i True
Message 2 : $\neg b$ $\qquad$ M_{iF} = Message i False.
Message 3 : b.

Only one message True, so

$$(M_{1T} \wedge M_{2F} \wedge M_{3F}) \vee (M_{1F} \wedge M_{2T} \wedge M_{3F}) \vee (M_{1F} \wedge M_{2F} \wedge M_{3T})$$

$$\equiv [(\neg a) \wedge \neg(\neg b) \wedge (\neg b)] \vee [\neg(\neg a) \wedge (\neg b) \wedge (\neg b)] \vee [\neg(\neg a) \wedge \neg(\neg b) \wedge b]$$

$$\equiv [\neg a \wedge (b \wedge \neg b)] \vee [a \wedge \neg b] \vee [a \wedge b]$$

$(\neg a \wedge F) = F$ $\qquad \equiv (a \wedge \neg b) \vee (a \wedge b)$ —— (1)

Scanned with CamScanner

Only one Box has gold

$$(a \wedge \neg b \wedge \neg c) \vee (\neg a \wedge b \wedge \neg c) \vee (\neg a \wedge \neg b \wedge c) \quad \text{—— (2)}$$

$\qquad\quad x \qquad\qquad\qquad y \qquad\qquad\qquad z$

a	b	c	¬a	¬b	¬c	x	y	z	x∨y∨z	a∧¬b	a∧b	
T	T	T	F	F	F	F	F	F	F	F	T	T
T	T	F	F	F	T	F	F	F	F	F	T	T
T	F	T	F	T	F	F	F	F	F	T	F	T
F	T	T	F	F	F	F	F	F	F	F	F	F
F	F	T	T	T	F	F	F	T	T	F	F	F
F	T	F	T	F	T	F	T	F	T	F	F	T
T	F	F	F	T	T	T	F	F	T	T	F	F
F	F	F	T	T	T	F	F	F	F	F	F	F

a:T, b:F c:F

Box1 contans gold.

(13).

$S(x)$: x shouts
$C(x)$: x cri'es.

Everyone shouts or cries : $\forall x\, [S(x) \vee C(x)]$
Not everyone cries : $\exists x\, (\neg C(x))$

Some people shout and does not cry : $\exists x\, [S(x) \wedge \neg C(x)]$

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for some person a in x

$C_1 \rightarrow S(a) \vee C(a)$

$C_2 \rightarrow \neg C(a)$

$C_3 \rightarrow \neg [S(a) \wedge \neg C(a)]$
$\quad \hookrightarrow \neg S(a) \vee C(a).$

$C_4 \rightarrow S(a)$ Using C_1 and C_2

$C_5 \rightarrow C(a)$ Using C_3 and C_4

$C_6 \rightarrow \square$ Using C_2 and C_5

Hence the conclusion holds.

14.) $P(x)$: x is a student
$Q(x)$: x goes to party
$R(x)$: x drinks too much

All student goes to party: $\forall x\ P(x) \rightarrow Q(x)$
Some students drink too much: $\exists x\ P(x) \wedge R(x)$.

Conclusion : $R(x) \rightarrow Q(x)$.
$\neg R(x) \vee Q(x)$.

Using Resolution

$$\forall x\ P(x) \rightarrow Q(x) \equiv \forall x \left[\neg P(x) \vee Q(x)\right]$$

Let there be a student 'a' from all values of x

C_1 : $\neg P(a) \vee Q(a)$
C_2 : $P(a)$
C_3 : $R(a)$.
C_4 : $\neg(\neg R(a) \vee Q(a))$
$\equiv R(a) \wedge \neg Q(a)$

C_5 : $Q(a)$ Using C_1 and C_2

C_6 : $R(a)$ Using C_4 and C_5

15.) "If n is a multiple of 3 then n is not a multiple of 7"

p: n is multiple of 3
q: n is multiple of 7

given $p \to \neg q$.
converse $\neg q \to p$.
"If n is not a multiple of 7 then n is a multiple of 3".

if we choose $n = 21$
$p \equiv T$
$q \equiv T$

So $p \to \neg q \equiv T \to F \equiv F$
So we found a contradicting case at $n = 21$

if we choose any number not a multiple of 3 and 7

$p \equiv F \quad q \equiv F$
$\neg q \to p \equiv T \to F \equiv F$
So converse is also false.

16.) u: n is a square of an even Integer. then n is a sum of two succesive odd Integers.

Let $n = 2a$
$(2a)^2 = (2k+1) + (2k-1)$.
$4a^2 = 4k$
$a^2 = k \quad \forall a$

if we choose $k = a^2 \quad a = 1, 2, 3 \ldots\ldots$

(a) $(2a)^2 = (2a^2+1) + (2a^2-1)$.

$a = 2 \quad 4^2 = 9 + 7$
$a = 3 \quad 6^2 = 19 + 17$
$\vdots$
so on.

(b) $p \equiv$ n is a square of even integer
$q \equiv$ n is a sum of 2 sucessive odd integer

converse of $p \to q$ is $q \to p$.

If n is a sum of 2 successive odd integers then n is a square of even Integer

for it to be false

$q = T$ $\quad (2k+1) + (2k-1) = n.$

$p = F$ $\quad (2a)^2 \neq n$

if we choose $k=3$.

$6^2 = 36 \neq 4k = 12.$

so converse is false.

(c) contrapositive of $p \to q$

$\neg q \to \neg p$.

If n is not a sum of 2 consecutive odd integers then n is not a square of even no.

$\neg q \equiv T$ $\quad$ since $p \to q$ and its contrapositive have same truth value so.

$\neg p \equiv F$ $\quad$ Contrapositive is True

17.) $(\neg P \vee \neg Q) \to (P \leftrightarrow \neg Q)$.

$[\neg p \vee \neg Q] \to [(P \to \neg Q) \wedge (\neg Q \to P)]$

$[\neg p \vee \neg Q \wedge \neg\neg Q \vee P]$

$[\neg p \vee (\neg Q \wedge Q) \vee P]$

$\neg p \vee F \vee p$.

$\neg p \vee p$.

$\neg(\neg p \vee \neg Q) \vee T$.

$p \wedge Q$.