Keerti P. Charantimath
19MA20059

Date

ANT Test 2

Q2. Lax Wandroff

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{2\Delta x}\left[f(u_{i+1}^n) - f(u_{i-1}^n)\right] + \frac{\Delta t^2}{2\Delta x^2}\left[A_{i+\frac{1}{2}}\left(f(u_{i+1}^n) - f(u_i^n)\right) - A_{i-\frac{1}{2}}\left(f(u_i^n) - f(u_{i-1}^n)\right)\right]$$

$$f = \frac{u^2}{2}$$

$$A_{j+\frac{1}{2}} = \frac{A}{2}\left(u_j^n + u_{j+1}^n\right)$$

$$A_{j-\frac{1}{2}} = \frac{A}{2}\left(u_j^n + u_{j-1}^n\right)$$

$$A(u) = \frac{\partial f}{\partial u} = u$$

$$u(0,t) = 0, \quad u(x,0) = \sqrt{x}, \quad x \in [0,1]$$

$$\Delta x = \frac{1}{5} \qquad \frac{\Delta t}{\Delta x} = \frac{1}{2}$$

$$\boxed{n=0}$$

for $i=1$

$$u_1^1 = u_1^0 - \frac{1}{4}\left(\frac{(u_2^0)^2}{2} - \cancel{\frac{(u_0^0)^2}{2}}\right) + \frac{1}{8}\left(\left(\frac{u_1^0 + u_2^0}{2}\right)\left(\frac{(u_2^0)^2}{2} - \frac{(u_1^0)^2}{2}\right) - \left(\frac{u_1^0 + u_0^0}{2}\right)\left(\frac{(u_1^0)^2}{2} - \frac{(u_0^0)^2}{2}\right)\right)$$

$$u_1^1 = \sqrt{\frac{1}{5}} - \frac{1}{20} + \frac{1}{32}\left(\frac{\sqrt{2} - \sqrt{0}}{5\sqrt{5}}\right) = 0.40116 \cong \boxed{0.3908}$$

Ans(i)

for $i=2$

similarly

$$u_2^1 = \sqrt{\frac{2}{5}} - \frac{1}{20} + \frac{1}{32}\left(\frac{\sqrt{3} - \sqrt{1}}{5\sqrt{5}}\right) = 0.5845 \cong \boxed{0.5836}$$

Ans (ii)

$$u_3^1 = \sqrt{\frac{3}{5}} - \frac{1}{20} + \frac{1}{32}\left(\frac{\sqrt{4} - \sqrt{2}}{5\sqrt{5}}\right) = 0.7262 \cong \boxed{0.7259}$$

Ans(iii)

$$u_4^1 = \sqrt{4/5} - \frac{1}{20} + \frac{1}{32}\left(\frac{\sqrt{5} - \sqrt{3}}{5\sqrt{5}}\right) = 0.8458 \cong \boxed{0.8456}$$ Ans(iv)

Q1. ADI

$u_{ij}^0 = \sin \pi x_i \sin \pi y_j \qquad \delta x = \delta y = \frac{1}{3}, \; \delta t = \frac{1}{27}$

Step 1:-

$$\frac{u_{ij}^{n+1/2} - u_{ij}^{n}}{\frac{\delta t}{2}} = \left[\frac{u_{i+1,j}^{n+1/2} - 2u_{i,j}^{n+1/2} + u_{i+1,j}^{n+1/2}}{(\delta x)^2} + \frac{u_{i,j+1}^{n} - 2u_{i,j}^{n} + u_{i,j-1}^{n}}{(\delta y)^2} \right]$$

Step 2:-

$$\frac{u_{i,j}^{n+1} - u_{i,j}^{n+1/2}}{\frac{\delta t}{2}} = \left[\frac{u_{i+1,j}^{n+1/2} - 2u_{i,j}^{n+1/2} + u_{i-1,j}^{n+1/2}}{(\delta x)^2} + \frac{u_{i,j+1}^{n+1} - 2u_{i,j}^{n+1} + u_{i,j-1}^{n+1}}{(\delta y)^2} \right]$$

Tridiagonal system

$$\begin{bmatrix} (-27-9) & \left(\frac{9}{2}\right) \\ \left(\frac{9}{2}\right) & (-27-9) \end{bmatrix} \begin{bmatrix} u_{1,j} \\ u_{2,j} \end{bmatrix} = \begin{bmatrix} -16.87 \\ -16.87 \end{bmatrix}$$

Matrix after step 1:-

Ans(i) Ans(ii)

$$\begin{array}{c|cccc} i/j & 0 & & & 0 \\ & & 0.5357 & 0.5357 & \\ & & 0.5357 & 0.5357 & 0 \\ & 0 & & & 0 \end{array}$$

Matrix aftre step 2

Ans(iii) Ans(v) Ans(iv)

$$\begin{array}{c|cccc} i/j & 0 & & & 0 \\ & & 0.3827 & 0.3827 & \\ & & 0.3827 & 0.3827 & \\ & 0 & & & 0 \end{array}$$

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Q4) Impeicit scheme

$$\frac{\partial t}{\partial x} = \frac{\partial^2 t}{\partial x^2} + \left(\frac{\partial t}{\partial x}\right)^2 \qquad x \in [0,1]$$

at $x=0$, $\frac{\partial T}{\partial x} = 1$, $T=0$ when $x=1$

$T=0$ " $t=0$

$\Delta x = 0.025$, $\Delta t = 0.01$

$$F = \frac{T_j^{n+1} - T_j^n}{\Delta t} = \frac{T_{j+1}^{n+1} - 2T_j^{n+1} + T_{j-1}^{n+1}}{(\Delta x)^2} + \left(\frac{T_{j+1}^{n+1} - T_{j+1}^{n+1}}{2\Delta x}\right)^2$$

$$\frac{\partial F}{\partial T_{j+1}^{n+1}} = -\frac{1}{(\Delta x)^2} - \frac{T_{j+1}^{n+1}}{2(\Delta x)^2} + \frac{T_{j-1}^{n+1}}{2(\Delta x)^2}$$

$$\frac{\partial F}{\partial T_{j-1}^{n+1}} = -\frac{1}{(\Delta x)^2} - \frac{T_{j-1}^{n+1}}{2(\Delta x)^2} + \frac{T_{j+1}^{n+1}}{2(\Delta x)^2}$$

$$\frac{\partial F}{\partial T_j^{n+1}} = \frac{1}{\Delta t} + \frac{2}{(\Delta x)^2}$$

$$\frac{\partial F}{\partial T_{j-1}^{n+1}} \Delta T_{j-1}^{n+1} + \Delta T_j^{n+1} \frac{\partial F}{\partial T_j^{n+1}} + \Delta T_{j+1}^{n+1} \frac{\partial F}{\partial T_{j+1}^{n+1}} = -f$$

$$\frac{\partial T}{\partial x} = 1$$

$$\Delta T_0^n = \Delta x = 0.25$$

$$T_1^n - T_0^n = 0.25 \Rightarrow T_1^n - 0.25 = T_0^n$$

$T_1' = 0$; $T_2' = 0$, $T_3' = 0$, $T_0' = -0.25$

Tridiagonal system

$$\begin{bmatrix} 132 & -16 & 0 \\ -16 & 132 & -16 \\ 0 & -16 & 132 \end{bmatrix} \begin{bmatrix} \cdot \\ \vdots \end{bmatrix} = \begin{bmatrix} -7.5 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -0.058 \\ -0.07 \\ -0.0 \end{bmatrix}$$

$T_0' = -0.25 - 0.0582 = -0.308 \simeq \boxed{-0.2981}$ Ans(i)

$T_1' = -0.058$

$T_2' = -0.07$

$T_3' = -0.01$

first iteration

Next iteration

$T_0' = -0.2981$

$\boxed{T_1' = -0.0629}$ → Ans (ii)

Q3. ADI Sheme

$$\frac{\partial c}{\partial t} + u(x,y)\frac{\partial c}{\partial x} + v(x,y)\frac{\partial c}{\partial y} = v\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right)$$

$t>0, \; 0<x,y<1$

$c(x,y,0)=0, \quad x,y \in [0,1]$

$c(x,y,t)=0$

$\Delta x = \Delta y = \frac{1}{4} \qquad \Delta t = 0.01$ using formulae from Q1

$$\text{Coefficient of } C_{i+1,j}^{n+1/2} = \frac{\Delta t}{2} u_{i,j} \frac{1}{2\Delta x} - \frac{v}{(\Delta x)^2}\frac{\Delta t}{2}$$

$$= 0.01 u_{i,j} - 0.08 v$$

Ans (i)

$$\text{Coefficient of } C_{i,j}^{n+1/2} = 1 + \frac{2v}{(\Delta x)^2}\frac{\Delta t}{2}$$

Ans (ii)

$$= 1 + 0.16 v$$

Coefficient of $C_{i,j-1}^{n+1} \Rightarrow$

$$\frac{C_{i,j}^{n+1} - C_{i,j}^{n+1/2}}{\Delta t/2} + u_{i,j}\left[\frac{C_{i+1,j}^{n+1/2} - C_{i-1,j}^{n+1/2}}{2\Delta x}\right] + v_{i,j}\left[\frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1}}{2\Delta x}\right)$$

$$= v\left(\left(C_{i+1,j}^{n+1/2} - 2C_{i,j}^{n+1/2} + C_{i-1,j}^{n+1/2}\right)/(\Delta x)^2 + \left(C_{i,j+1}^{n+1} - 2C_{i,j}^{n+1} + C_{i,j-1}^{n+1}\right)/(\Delta y)^2\right)$$

$$= 0.01 v_{i,j} - 0.08 v$$

Ans (iii)

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Q5 $u_t = u_{xx}$, $u(x,0)=0$, $u(0,t)=0$
$u(1,t)=t$

$h=0.25$, $r=1$, $r = \frac{\Delta t}{(\Delta x)^2}$ $\Delta t = \frac{1}{16}$

$$2u_j^{n+1} - 2u_j^n = -r\, u_{j+1}^{n+1} - 2(1+r)\, u_j^{n+1} - r u_{j-1}^{n+1}$$
$$= 2u_j^n + r\left[u_{j+1}^n - u_j^n + u_{j-1}^n\right)$$

$r=1$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix}$$

$u\left(1, \frac{1}{16}\right) = \frac{1}{16}$ $d = \begin{bmatrix} 0 \\ 0 \\ 1/16 \end{bmatrix}$

$$A \begin{bmatrix} u_1' \\ u_2' \\ u_3' \end{bmatrix} = d$$

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} u_1' \\ u_2' \\ u_3' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1/16 \end{bmatrix}$$ → Tridiagonal system

Ans (i)

Ans (ii)

Sol

$u_1' = 0.0011$ → Ans (iii)

~~$u_2' = 0.0044$ → Ans (v)~~

~~$u_3' = 0.01674$ → Ans (iv)~~

$u_2' = 0.0044$ → Ans (v)

$u_3' = 0.01674$ → Ans (iv)

Q6) $u_{xx} + u_{yy} = 100$ SOR problem

$u(0,y) = 0$
$u(3,y) = 200$
$u_y(x,0) = 100$
$u_y(x,3) = -100$

$$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,0}}{2\Delta x} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{2\Delta y} = -100$$

$u_{1,4} = u_{1,2} = -200$

$i,j = 1,1$
$u_{2,1} - 4u_{1,1} + u_{0,1} + u_{1,2} + u_{1,0} = -200$

$i,j = 2,1$
$u_{3,1} - 4u_{2,1} + u_{1,1} + u_{2,2} + u_{2,0} = -200$

i,j $1,2$
$u_{2,2} - 4u_{1,2} + u_{0,2} + u_{1,3} + u_{1,1} = -200$

$$u_{1,1}^{(1)} = \frac{u_{2,1}^{(0)} + u_{0,1}^{(1)} + u_{1,2}^{(0)} + u_{1,0}^{(1)}}{4} + 25$$

$$u_{1,0}^{(1)} = \frac{u_{2,0}^{(0)} + u_{0,0}^{(0)} + u_{1,1}^{(0)} + u_{1,-1}^{(1)} + 25}{4}$$

$$u_{2,0}^{(0)} = \frac{u_{2,1}^{(0)} - 200 + 100 + 0 + 0 + u_{1,1}^{(1)} - 200 + 25}{4}$$