

3<sup>rd</sup> Jan, 2024

## Computer Networks

Tentbook: Computer Networks by Tennenbaum.

Midterm, Final (40/60) → No attendance

### Class Hours

Monday	10-11	Thurs 10-11
Wednesday	8-10	

### Century of information processing

#### Communication

→ Broadcast

→ Point to point

- To contact particular device in network, we need IP address.
- For communication, we need channel.

#### DST Layers → Tomorrow.

Page rank algorithm?

#### Maths

→ Eigenvalues, characteristic matrix.

→ Probability.

4<sup>th</sup> Jan, 2023

Internet

Laptop

Desktop

Phones

Tablets

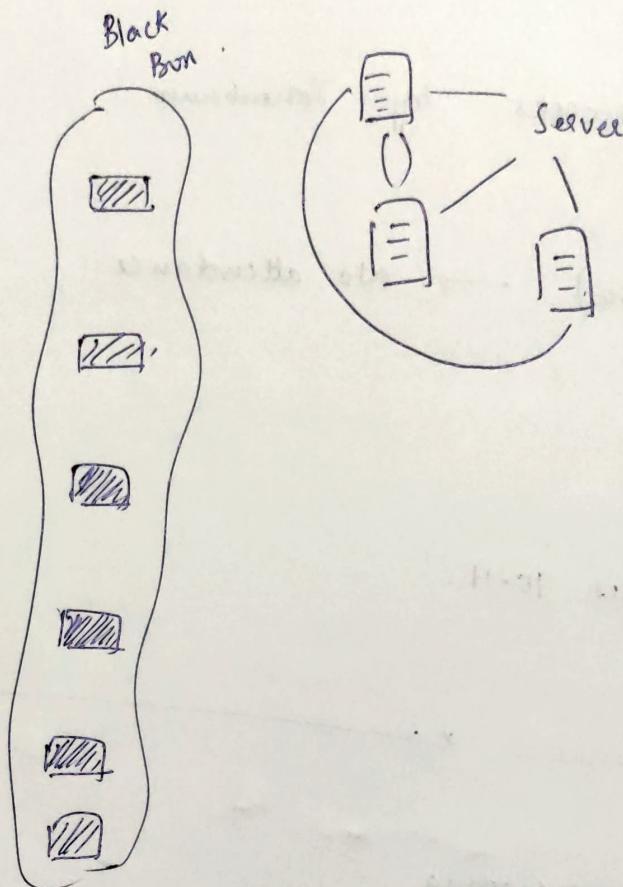
Smartwatches

Printers

Speaker

Mattress

Cars



(Internet) network of networks

Why?

We want network to be something that follows particular rules of communication (Protocols).

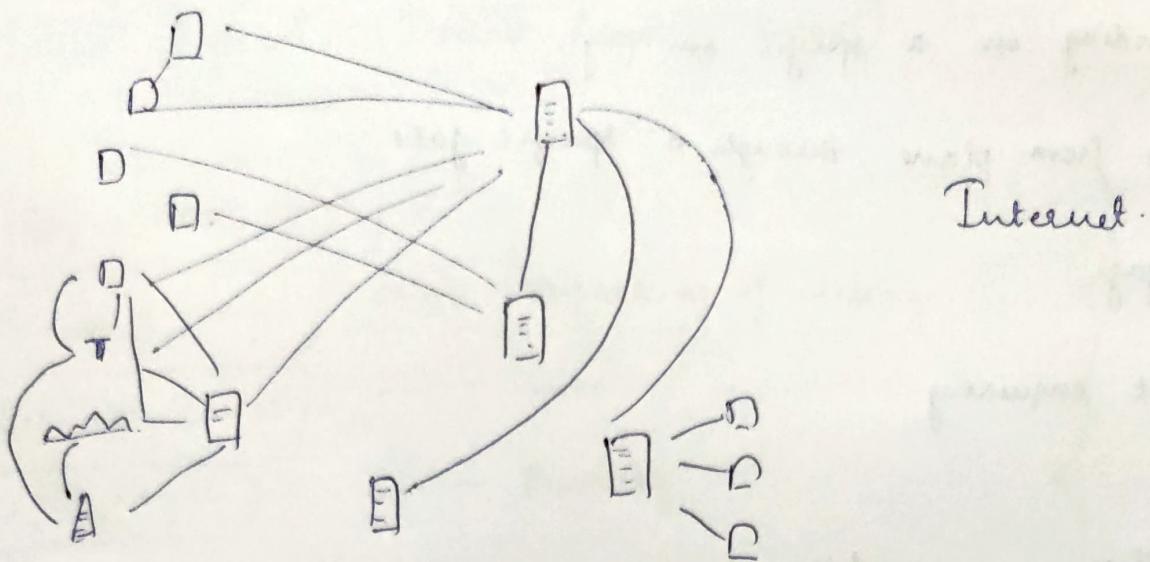
Rules governing a network.



Protocol.



Internet is governed.  
by several protocols.  
(http being one of them)



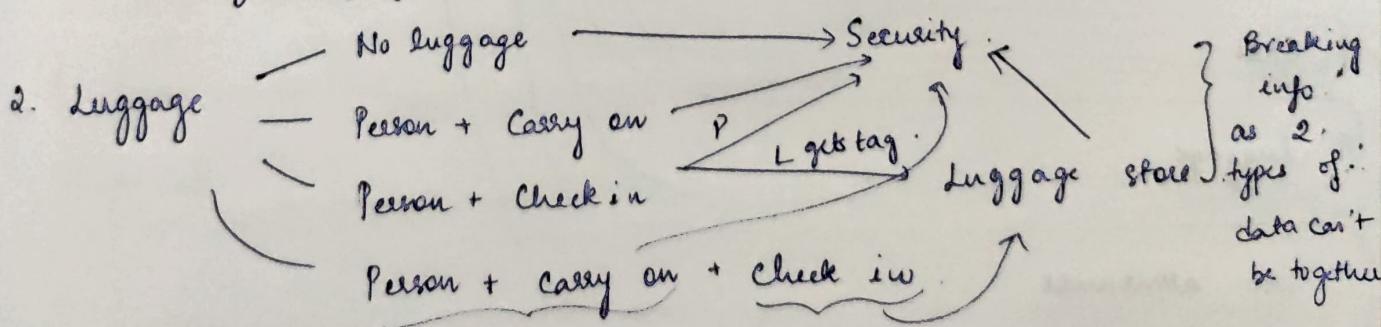
## Computer Network

(graph)

A network whose nodes are computing devices (computers, computerised cars, printers, phones, ...) & a link b/w two nodes indicates possibility of communication.

## Airline Communication

1. Ticket of a flight from A<sub>1</sub> to A<sub>2</sub>.



3. Go to the particular gate

4. Airplane on a fixed runway.

5. Airplane routing.

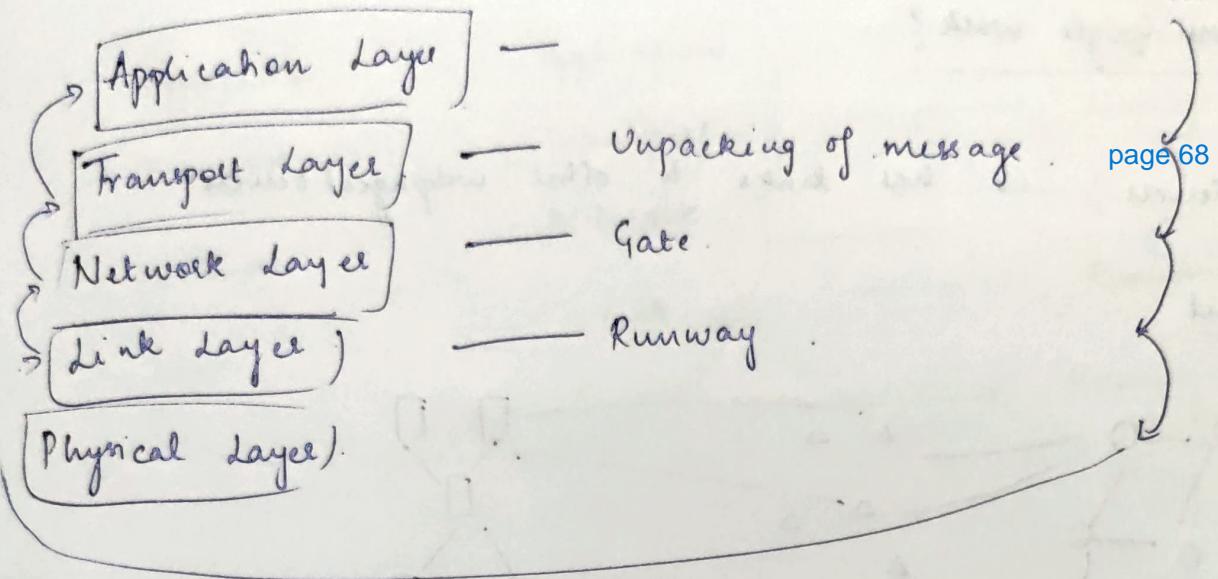
Runway 24

6. Landing on a specific runway.
7. Exit from plane through a specific gate.
8. Luggage
9. Ticket enquiring.

Same steps in a different way.

- 
1. Entry pass
  2. luggage
  3. Gate Management
  4. Runway.
  5. Routing - Routing - Routing.  
5½                    6.
  7. Runway.
  8. Gate
  9. luggage
  10. Exit assurance.

## 5 Layer Protocol (Internet Layering Protocol)



## OSI Layers

Open systems interconnection

Application.

Presentation

Session

Transport

Network.

Link

Physical.

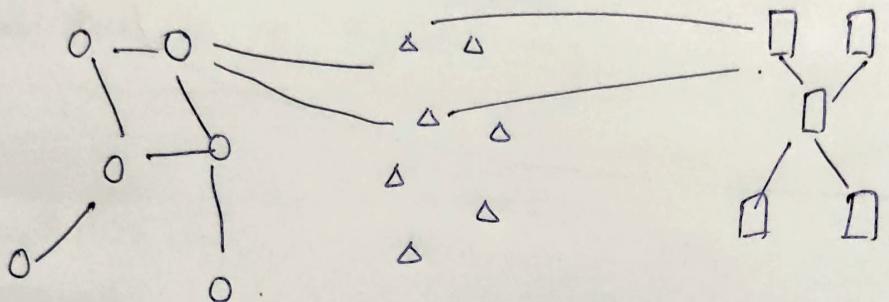
2 extra layers

8<sup>th</sup> Jan, 2023

## How does Google work?

Google Servers → has links to other webpages / servers.

## Internet



Users/  
Clients.

Electronic  
devices/routers.  
(decides which message  
goes which path)

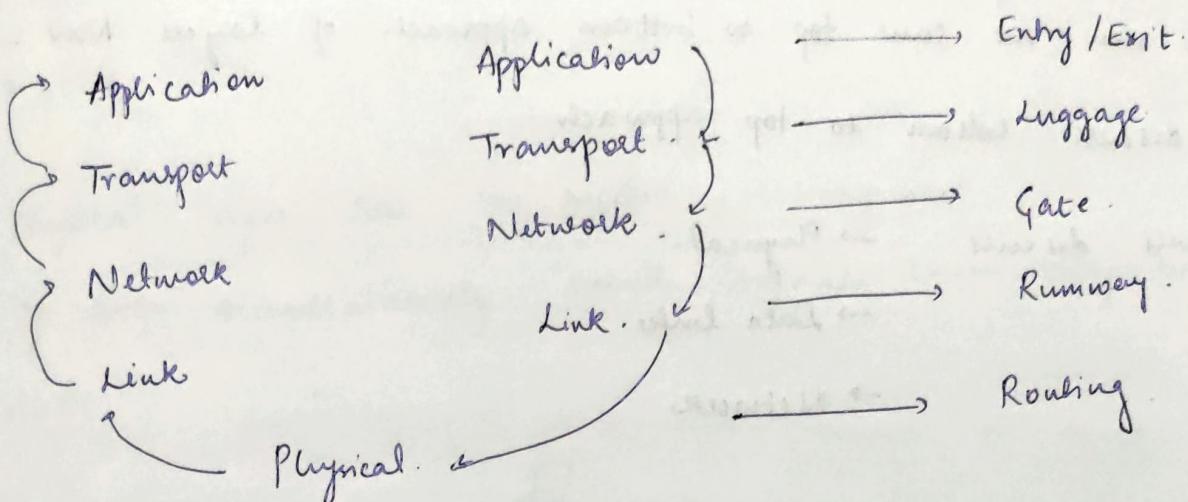
Servers.

Internet has several networks each having its own protocols

Basic of network → electronic transmissions. (electromagnetic  
transmissions in case of wired / wireless transmission)

Networks started from radio (transmitter sends at certain freq  
receiver tunes to that frequency to listen) → was also  
used for secure communication.

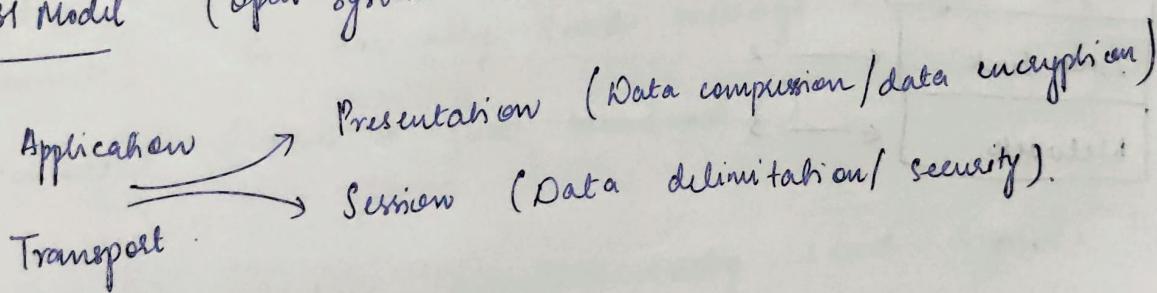
# Internet Layering Protocol



Datagram?

Traveling Salesman Problem?

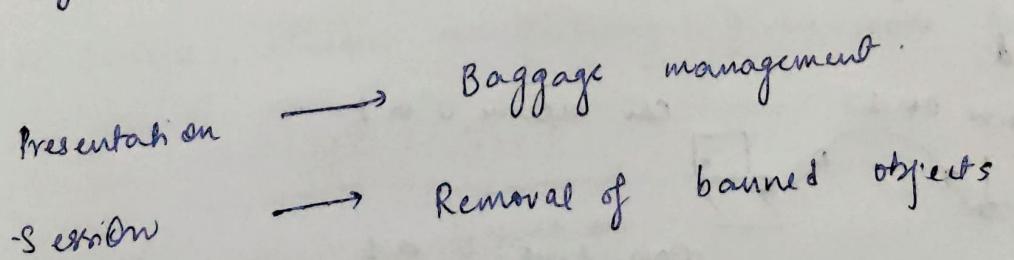
## OSI Model (Open Systems Interconnection)



Network.

Link.

Physical



Sort of combined with application or transport

\*  
Internet  
doesn't  
have  
these 2  
layers

10<sup>th</sup> Jan, 2024

So till now we saw top to bottom approach of layers. Now we will discuss bottom to top approach.

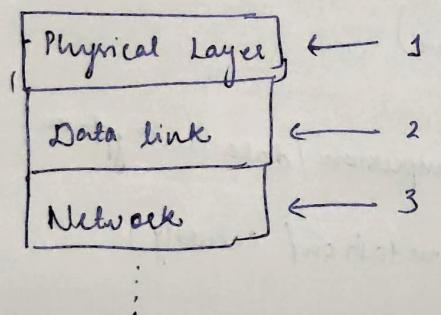
We will discuss → Physical.

→ Data link

→ Network

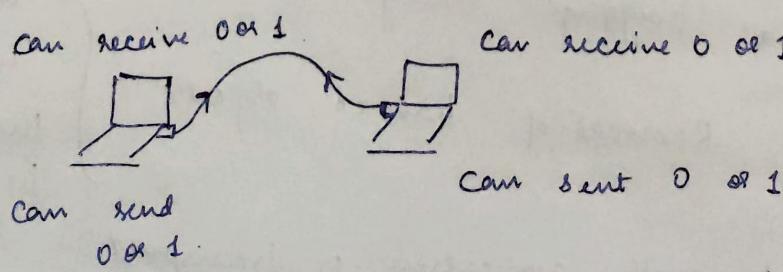
Each computer has an address unique to it → MAC Address

Bottom - Up View



Which layer is MAC address attached to? Data link layer

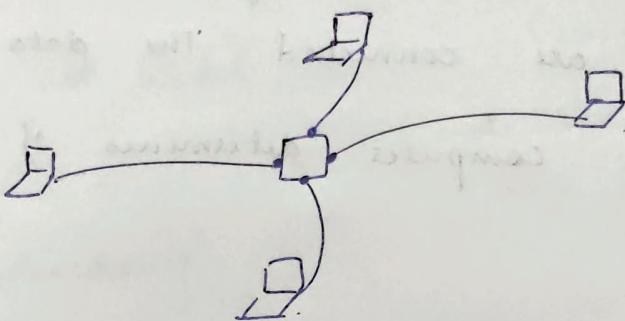
Physical layer: Through this, raw data. (seq. of 0's and 1's) gets transmitted.



You and your brother are in same room, want to play a game against each other in your own systems. How?

→ Wire.

- Physical Layer has no media management.
- If both simultaneously send signals then they will collide.



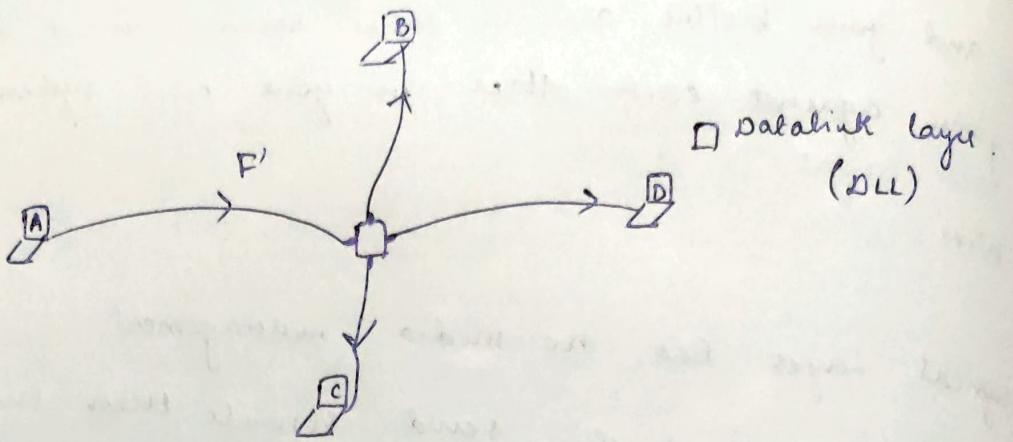
Physical layer only detects connectivity (graph theoretically connects). So basically sends message to any device connected. (will it just broadcast?)

- If 2 computers simultaneously send signals there will be collision.

### Data Link Layer

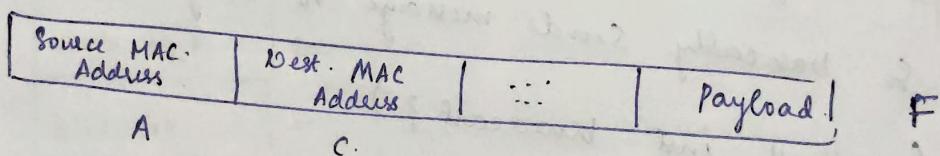
Mac Address: Unique manufacturing. Given code to every device.

32e : ff e : 7xx : f7 : ..... : 75  
Depend on manufacturer → One system



DLL check the sender, receiver & payload (data), sends to all systems that are connected. The data link layer of all the receiver computer determines if it's meant for it.

- Data link layer creates Frame.



$F \rightarrow F'$  (has 0s and 1s)

- A sends  $F'$ . B, C, D all get  $F'$ .
- All push it to their DLL.
- DLL decide whether the message is meant for them or not. If yes, it reads, otherwise it junks.
- So only C reads the message.
- collisions?

## Circuit Switching & Packet Switching

CS: A fixed communication channel is created between the two & that is used for continuous interaction.

### Advantages:

1. Consistency.
2. Routing not needed.
3. Faster.
4. Less redundancy.
5. More secure.

### Disadvantages:

1. Large no. of connections (complexity).
2. Adding a new user two-way communication is tough.

PS: Data is broken into small parts. potentially broadcasted.

If we pick up  $\alpha$  points between 0 & 1, the probability they will be same is 0. So if the packets are very small, collisions can be minimized.

\* Find MAC Address of your laptop and figure out what it means

What is IP address? Network Layer

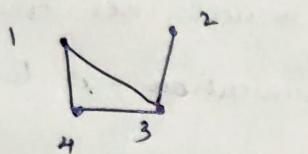
Will discuss in next class

## Basic Graph Theory

Graphs: Graphs are pairs  $G = (V, E)$  where  $V$  is a set &  
 $E$  is a set of size 2 subsets of  $V$ .

$$G = \left( \{1, 2, 3, 4\}, \{ \{1, 3\}, \{2, 3\}, \{1, 4\}, \{3, 4\} \} \right)$$

$V$                            $E$

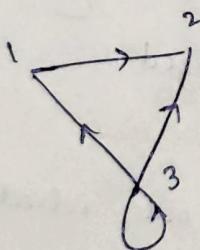


$V$ : Nodes / Vertices

$E$ : Links / Edges

In other words, graphs are collections of nodes and edges.

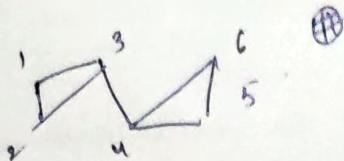
A graph is directed if every edge has a source & a target (every edge is an arrow).



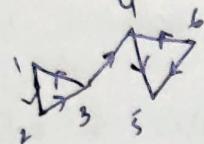
In a graph if a vertex is connected to itself by an edge then that edge is called a loop.

A path in a graph is a sequence of edges  $e_1 \dots e_k$ .

such that  $\forall i \quad e_i = xy \quad e_{i+1} = yz$  for some  $x, y, z \in V$



is a path from 1 to 6. If a graph  
is directed, then a **directed path** is a seq of edges.  
such that  $\forall i, e_i = \overrightarrow{xy} : e_{i+1} = \overrightarrow{yz} \quad x, y, z \in V$



$\overrightarrow{12}, \overrightarrow{23}, \overrightarrow{34}, \overrightarrow{45}$  is a directed path  
from 1 to 5.

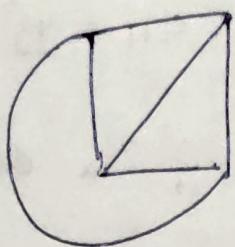
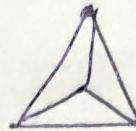
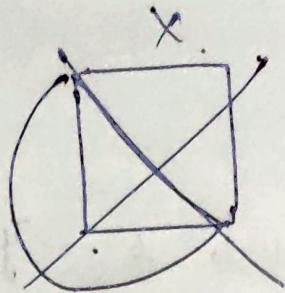
A **cycle** is a graph is a sequence of edges  $e_1 \dots e_n$   
such that  $e_i = xy$  &  $e_n = ux$  for some  $x, y, u \in V$ .

In graph  $\oplus$   $12, 23, 31, 21$  is a 3-cycle &  $1, 3, 34, 42, 21$  is a 5-cycle.

A graph is **connected** if for any  $x, y \in V \exists$  a path  
from  $x$  to  $y$ .

A graph is **planar** if it can be drawn on a plane  
without vertices crossing each other.  $\triangle$   $\square$

A graph is **complete** if any 2 vertices are connected  
by an edge.



Complete graph on 4 vertices is planar.

Complete on  $n$  vertices is  $K_n$ .

Q. What about  $\overline{K_5}$ ? Is it planar?

Putting 5 points on plane and connecting each pair.



Let's take triangle

(TRY!!).

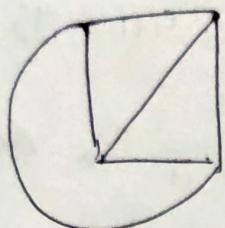
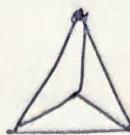
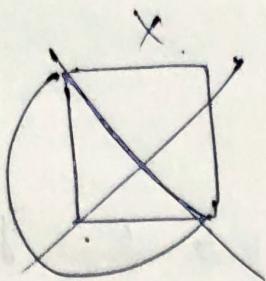
Check by keeping other

2. points in the diff

regions. All combinations should be tried.

We cannot get  $K_5$ .





=

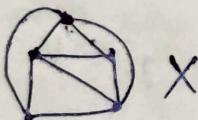
Complete graph on 4.

Vertices is planar.

Complete on  $n$  vertices is  $K_n$ .

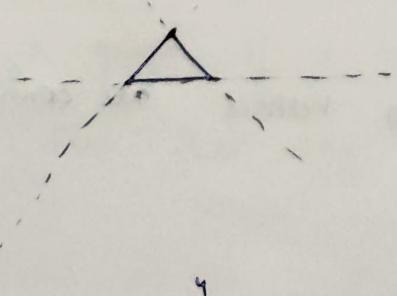
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Let's take triangle

(TRY!!).



Check by keeping other 2 points in the diff regions. All combinations should be tried.

We cannot get  $K_5$ .

15<sup>th</sup> Jan 2024

## Communication in Shared Medium

CSMA ?

Carrier Sense Multiple Access Protocols

Assume, transmissions are heard over limited distance.

- ① Let transmissions are heard over a distance of 50. A node at location 170 is transmitting for node at location 200. Node at location 240 starts transmission.

This is called Hidden Station Problem - I.

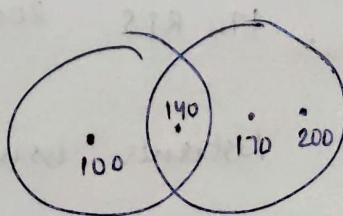
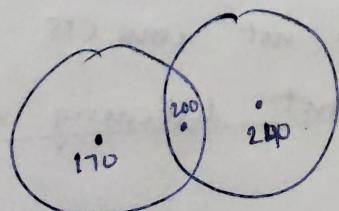
The idea "I will transmit when no one else will" fails here. 200 listens to both 170 & 240. But they cannot hear each other.

- ② A node at location 170 is transmitting for node at location 200. Node at location 100 starts transmission for node at location 140.

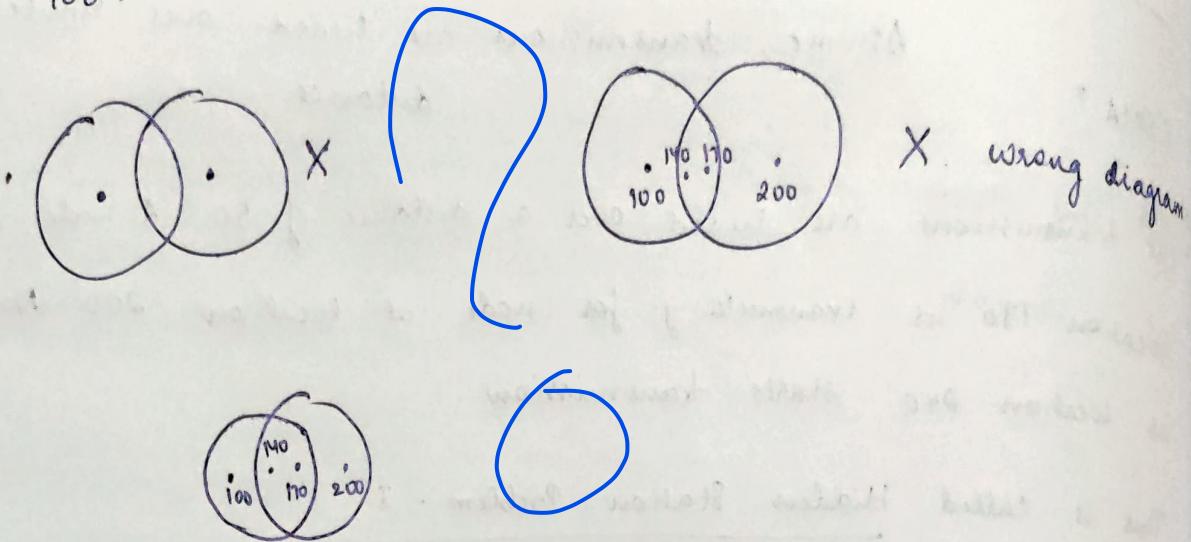
This is called Hidden Station Problem - II.

HSP 2

HSP 1



① A node at location 170 is transmitting for node at location 200. A node at location 120 wants to transmit (but does not) at 100.



This is called. Exposed Station Problem.

Solutions: When node wants to transmit, it issues RTS.

Listener issues CTS.

RTS: Request to Send.

CTS: Clear to Send.

170 issues RTS. 200 issues CTS. 240 gets CTS, will not issue RTS.

Solution ① : 170 RTS, 200 CTS 240 heard CTS hence no RTS

Solution ② : 170 RTS. 200 CTS 100 RTS. 140 listening 170 here does not issue CTS.

Solution ③ : Listener issues CTS if not listening any transmission.

170 RTS 200 CTS

140 RTS.

ers of 200 not  
listened.

100 CTS.

transmission of 170.  
doesn't reach 100

original protocol = guarantee of no collisions and

equal share of slots  $\rightarrow$  170, 120

new equal length

stamp present

(classical) token  
(new) stamp

(modified) token

length

push



and rounds present

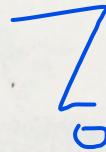
17<sup>th</sup> Jan, 2024

## Books

1. Computer Networks - Andrew Tanenbaum.
2. Data Communication & Networking - Behrouz Forouzan

HSP, ESP → belong to data link layer.

Encapsulating Security Payload



## Physical Layer Basics

Sending Signals.

Signals.  
— Digital - (Discrete)  
— Analog - (Continuous)

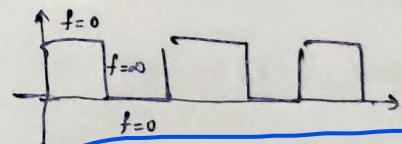
### Analog



for signals. we have.

- Frequency.
- Amplitude.
- Phase. (? check def)
- Wavelength.

### Digital



frequency changes between  
0 and  $\infty$

Freq.  $\propto \frac{1}{\text{Wavelength}}$

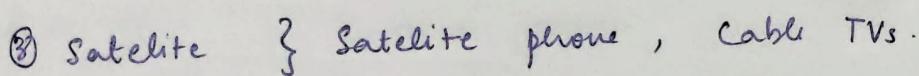
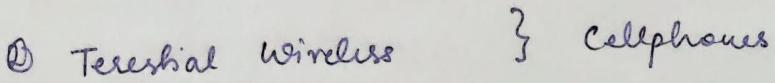
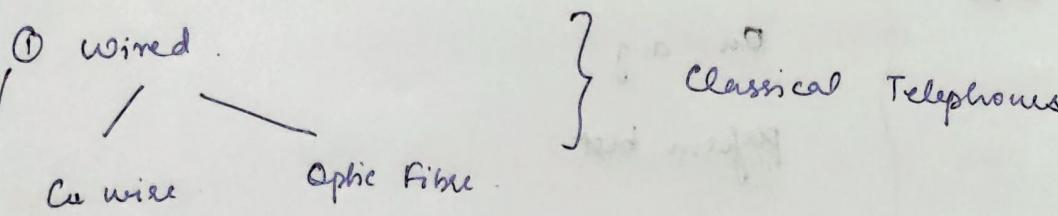
## Bandwidth (of communication channel.)

Range of frequencies that can go through that channel

\* Bandwidth can be theoretically  $\infty$  in optical fibre.

## Types of Communications

- ① wired
- ② Terrestrial wireless.
- ③ Satellite.



Point to point

Broadcasting

What is best form of communication?

Carrying information in hard disks (which has memory in terabytes) and taking a load of them to the location we need manually beats internet avg speed.

which is in Kbytes/sec

avg. speed of manual. > avg. internet  
hard disk transport speed.

Still we prefer internet as it allows instantaneous communication.

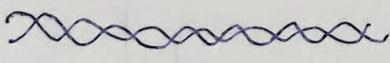
### Point to Point.

#### 1. Magnetic Media.

Hard disk      }  
USB .            }  
CD .             }  
FLOPPY           }

On avg.  
perform best.

#### 2. Twisted Pair Wires

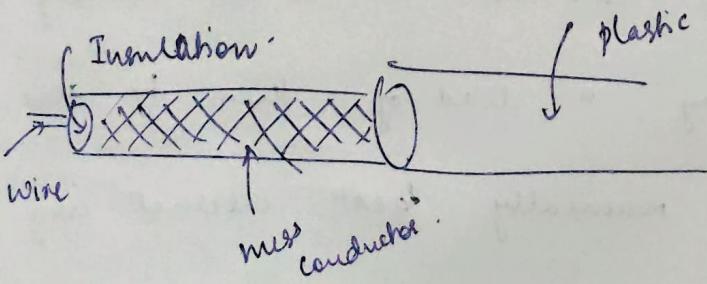


Category 3.

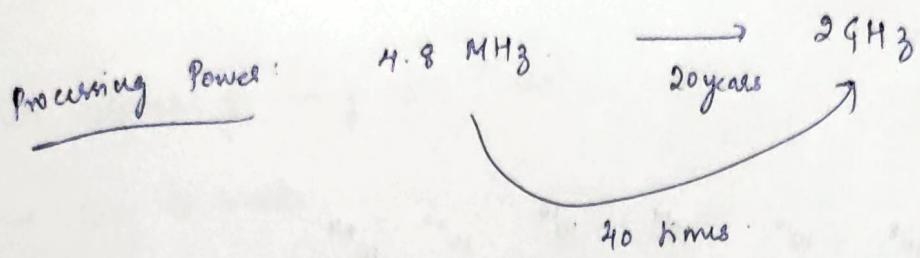


Category 5 . more twists, higher bandwidth and speed

#### 3. Coaxial Cable



## 2. Fibre Optics



So increased 20 times for a decade.

Communication Rate:  $56 \text{ Kbps} \xrightarrow[20 \text{ years}]{} 14 \text{ bps}$

125 times / decade.

communication tech was improving much more than processing power.

fibre optics:

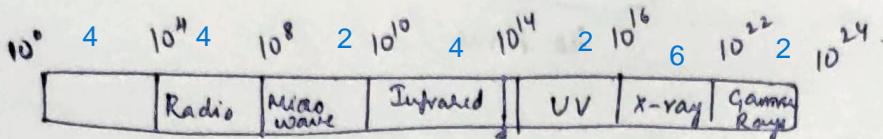
- ① More bandwidth.
- ② Much faster to build / install
- ③ Works for longer range.

disadvantage:

- ① Needs more care.

## Wireless Transmission

$f$  (Hz)



rich men in vegas use expensive girls

Visible light

## Electromagnetic Spectrum

It consists of everything except sound (which isn't electromagnetic waves).

Radio : Used by radio.

- ① Small freq / high wavelength.
- ② Penetrates solids.
- ③ Travel longer.

Radio < Microwave  
danger.

Microwave : Mobile phones, TV.

- ① All properties of radio hold.

Infrared : TV remotes

- ① Cannot penetrate solid.

Visible Light : Traffic lights

- Networking using LAN (rare)
- Easy to block.

## Satellite Communication

### 1. Geostationary

- Farthest from the Earth
- Appears to stay fixed

### 2. Medium Earth Orbiting Satellite

- Medium height
- Moves slowly

### 3. Low earth orbit

- Satellite (very close to earth).

## 2 Imp. Technologies to learn about Physical Layer

1. Multiplexing
2. Switching.

A wave looks like,

$$g(t) = \frac{c}{2} + \sum_{n=0}^{\infty} a_n \sin(2\pi n ft) + \sum_{n=0}^{\infty} b_n \cos(2\pi n ft)$$

Question: If  $g(t)$  is given then how do you find  $a_n, b_n$  &  $c$ ?

$$\int_0^T \sin(2\pi n ft) \cos(2\pi k ft) dt = 0 \quad \forall n, k$$

$$\int_0^T \sin(2\pi n ft) \sin(2\pi k ft) dt = \begin{cases} T/2 & n=k \\ 0 & \text{otherwise} \end{cases}$$

$$a_n = \frac{2}{T} \int_0^T g(t) \sin(2\pi n ft) dt$$

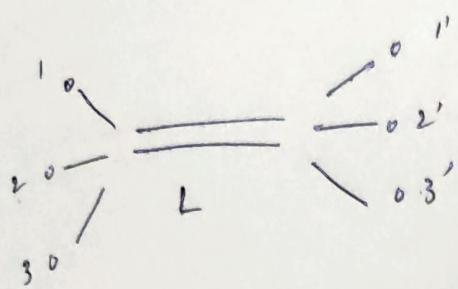
$$b_n = \frac{2}{T} \int_0^T g(t) \cos(2\pi n ft) dt$$

$$c = \frac{2}{T} \int_0^T g(t) dt$$

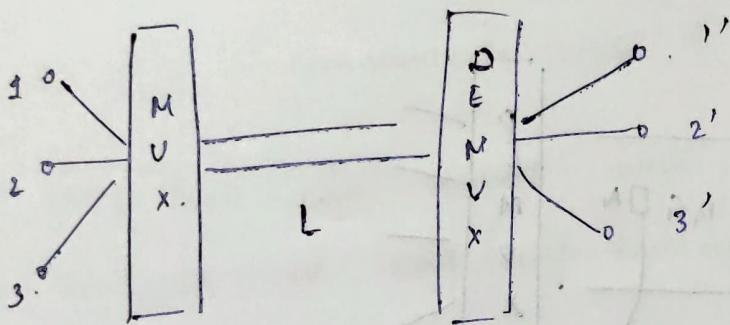
All wave extractions happens as following.

Communication through same link gives rise to multiplexing

## Multiplexing



1, 2, 3 want to send a message to 1' 2' 3' simultaneously through L.



One way to do it, especially useful in analog is

### Frequency Division Multiplexing

- Each user is assigned separate bandwidths
- Some space is kept between the bandwidths

Note: There is a variant of this called Wavelength Division Multiplexing.

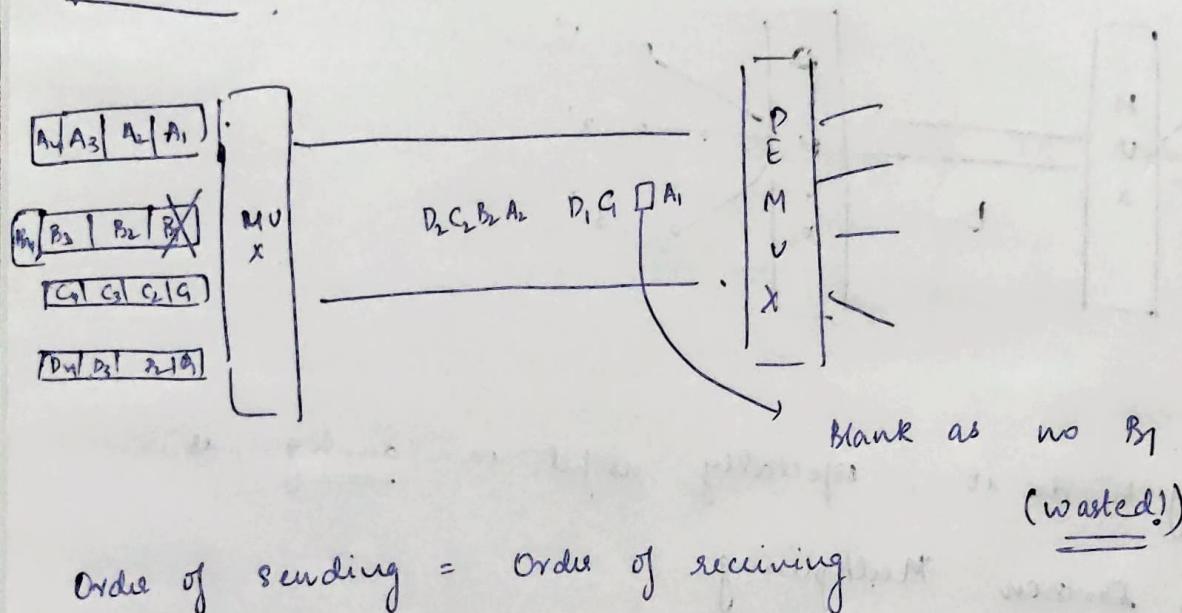
Division Multiplexing used for fibre optics mostly.

FDMA is bad for digital. so

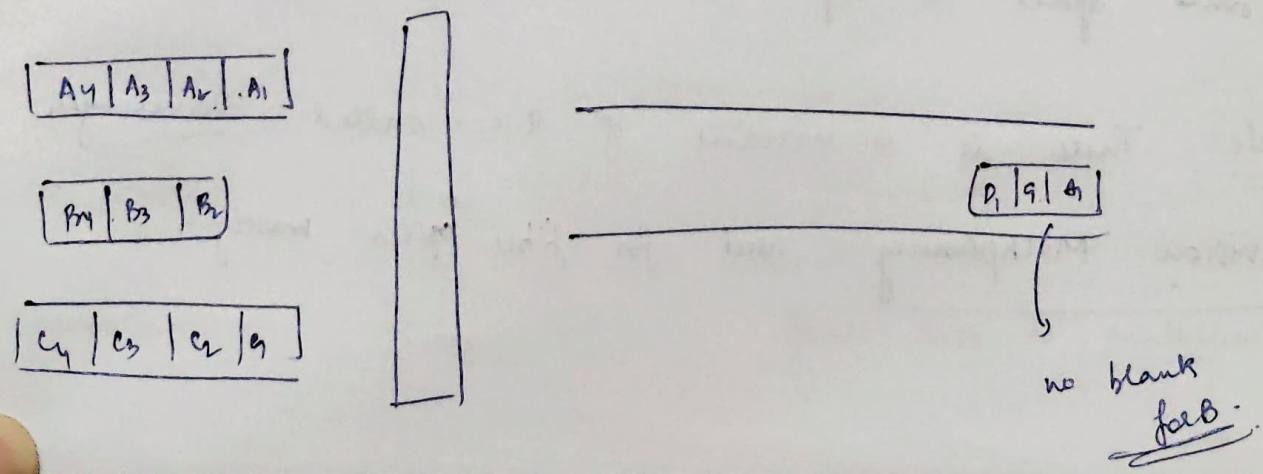
## Time Division Multiplexing

- Useful for digital.
- 2 types — Synchronous TDM.
  - Statistical TDM.

### Sync TDM



### Stat TDM



Saves slots and messages need addresses.

24th Jan 2024

## switching

Any communication channel has a limit (physically feasible)

N points.

If A wants to communicate with B.

→ Every point can connect with every other point. But this increases cost and infrastructure. Giving bandwidth to everyone isn't possible.

complete Graph also called.

Mesh topology

## Star Topology



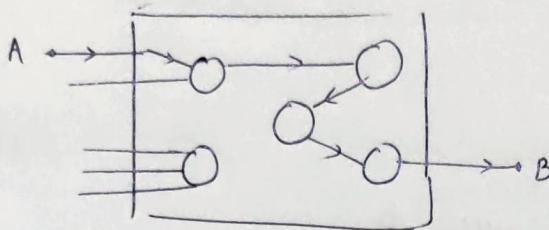
Every node communicates via central node.

## Circuit Switching

When fixed communication channel is created b/w. (two) users before communication.

## Stages of Circuit Switching

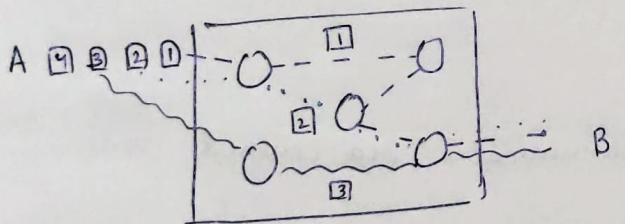
- ① Establishment of communication line
- ② Private Communication make communication
- ③ Ending the communication channel.



Communication only starts only after the channel is created.

→ Bandwidth wasted but better from user perspective. Traditional telephones used this

## Packet Switching:



Every packet chooses best path.

- Data is broken into packets & packets are sent one by one
- No fixed communication channel is created between users

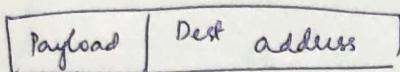
and packets are free to pick any available route.

→ This involves delay for queuing and reassembling

→ Needless to say, packets need destination address

\*

\* Routing Table helps choosing path for each packet.



PACKET.

## Physical Layer of Communication

① Point to Point Wired.

② Terrestrial - Wireless.

③ Satellite → Cable TV

→ wired Telephone

→ Mobile Phone

## Mobile Phone

### Characteristics

1G → Analog voice.

2G → Code Division Multiplexing, Digital Voice.

3G → Voice + Media Digital.

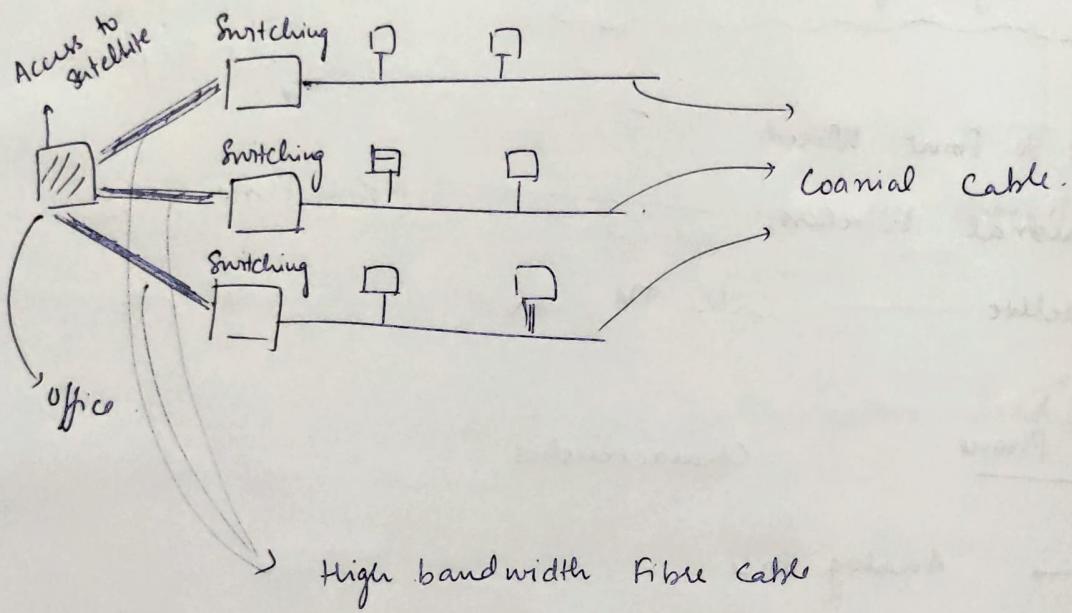
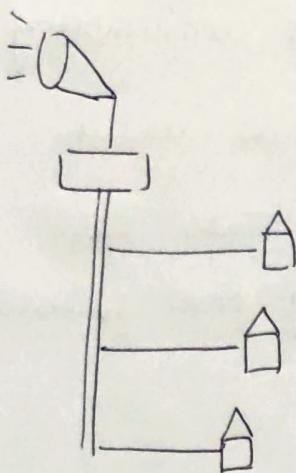
4G →



5G →



## Cable TV



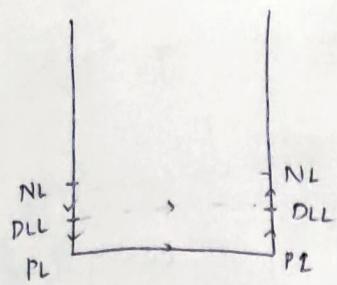
In circuit switching → cost depends on distance.

In packet switching → cost depends on amount of information

because in circuit switching, channel management is imp.

in packet switching, packet management is important.

## Data Link Layer



Network layer thinks DLL gets info from it and sends to DLL of receiver (it doesn't see the involvement of PL)

Physical Layer (PL)

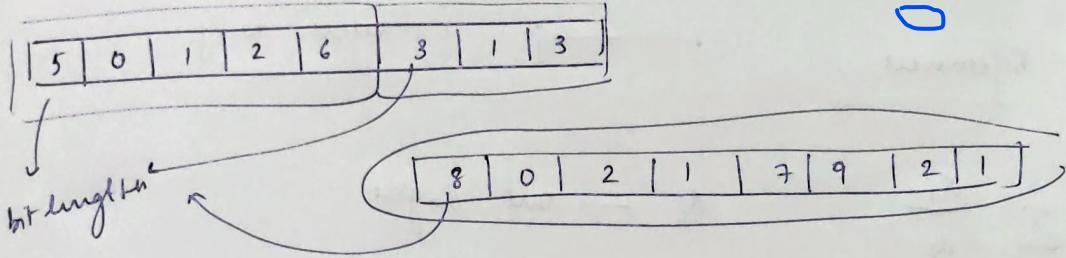
Packet

Data link layer (DLL)

[H | PL | T]

header payload trailer

frames



| 50 | 12 | 6 | | 31 | 3 | | 80 | 21 | 79 | 2 | 1 |

But errors occurs if 3 changes to 6

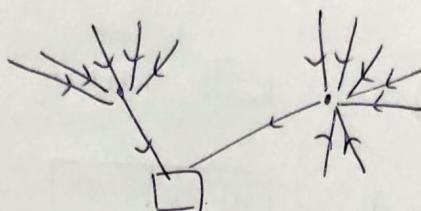
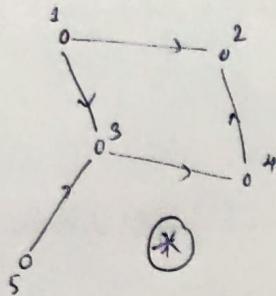
| 50 | 12 | 6 | | 61 | 3 | 80 | 3 | 0 | 1 | 7 | 9 | 2 | 1 | ... error identified.  
| 50 | 12 | 6 | | 61 | 3 | 80 | 3 | 0 | 1 | 7 | 9 | 2 | 1 |  
entire process ruined.

DLL will help us manage these things.

## Google Search

How does google rank various website ?? (How are Page ranks given?)

Page ranks  
given by  
Larry Page



No of references .

$\rightarrow$   $i \rightarrow j$   $i$  refers to  $j$

$$r_j = \sum_{i \rightarrow j} \frac{r_i}{d_i}$$

$d_i \rightarrow$  out degree  
(no. of stems going out)

for (\*)

$$\begin{aligned} r_1 &= 0 \\ r_2 &= \frac{r_1}{2} + r_4 \\ r_3 &= \frac{r_1}{2} + r_5 \\ r_4 &= r_3 \\ r_5 &= 0 \end{aligned}$$

1, 4 coming to 4

1 has 2 routes going out  
4, 5, 3 have only 1 route going out

Solve these equations to get rank of each node

### PROBLEM

→ These equations may not even have a solution

Many websites emit so number of variables very high  
Storing such large matrix → computations are high.

Search  $\pi$  such that  $P\pi = \pi$  eigen values.

$$r_{ij} = \frac{r_i}{d_j} + \frac{r_i'}{d_i'} + \frac{r_i''}{d_i''}$$

$$\left( \begin{array}{ccc} \frac{1}{d_1} & \frac{1}{d_1'} & \frac{1}{d_1''} \\ \vdots & \vdots & \vdots \\ \frac{1}{d_n} & \frac{1}{d_n'} & \frac{1}{d_n''} \end{array} \right)$$

P

?

thus can be done without storing P

29<sup>th</sup> Jan, 2024.



Sequence of 0s & 1s



Data Link Layer

Digital Signal? Both sender and receiver has a clock. Receiver periodically measures the signal and analyses the pattern of increase and decrease and analyses which is 0 and which is one

### Modulation

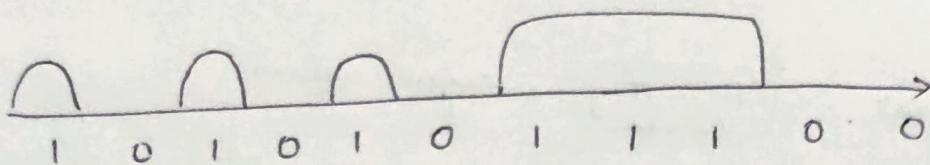
Data → Signal.

### Demodulation

Signal → Data.

Modulation is done by varying nature of signals over a ~~process~~ time intervals.

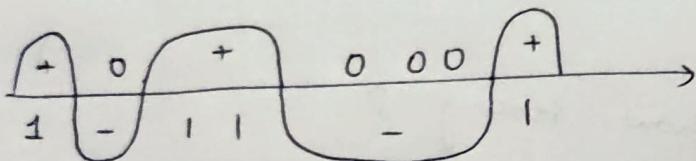
### Return to Zero (RZ)



Data

1 0 1 0 1 0 1 1 1 0 0

### Non return to Zero (NRZ)



Data

1 0 1 1 0 0 0 1

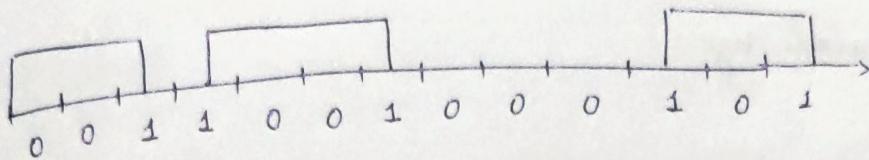
→ Clocks on both sides should be synchronized perfectly for these coding methodologies to work.

↓.

### Problem

- Sequence of 0s or sequence of 1's can be hard to detect (Variations are easy to detect and help in synchronization of clock)
- Exact 0 is hard to get

## Non-return to Zero Inverted (NRZI)



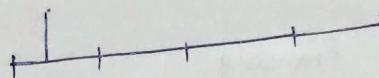
variation  
everytime  
1 comes,  
otherwise  
stays the  
same

solves the sequence of 1 problem

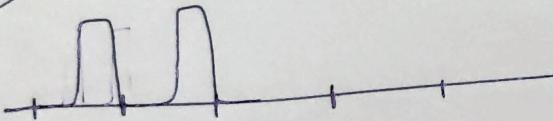
sequence of 0s problem remains.

## Manchester Coding

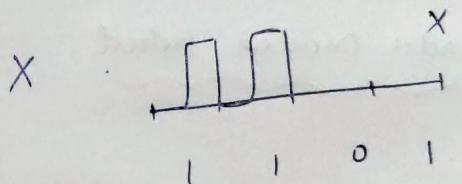
1 1 0 1



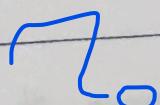
Final



1 1 0 1

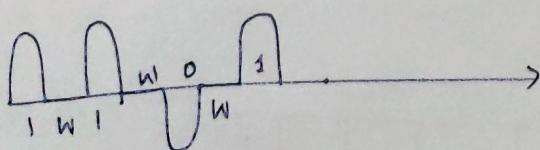


homework:



find a scheme that addresses both the weaknesses

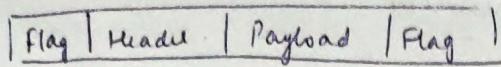
w → wait



half the time slots lost

## Framing

Fixed size  
Variable size

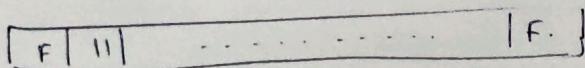


- Header → 1) Destination  
2) Protocol  
3) Error Detection Code

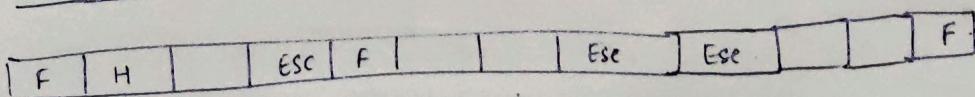
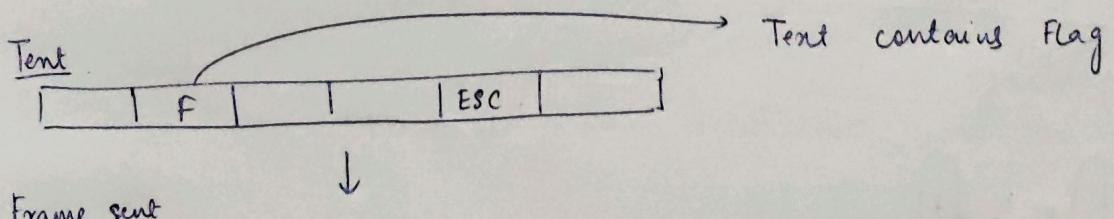
## Variable Sized Framing

Character Oriented Protocol → Byte Setup.

Character → ASCII Code



Use ESC character to denote beginning of a text used in the flag in the original body

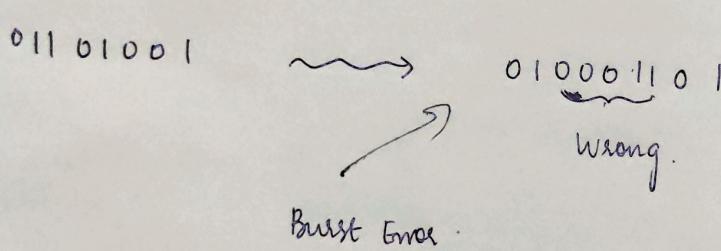
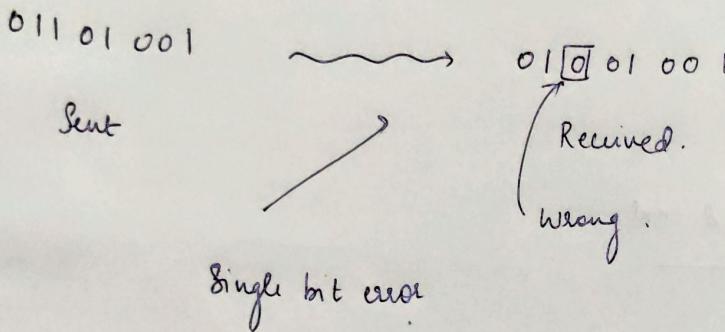


## ERROR

→ Single Bit Error  
 → Burst Error.

Single bit errors are rare. Why?

Errors mostly occur in higher layers and not physical layer



Data link layer has the following purposes → 1) Framing  
 2) Error Correction/  
 Detection

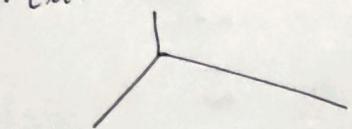
Suppose data is being sent at 1Mbps →  $10^6$  bits per second  
 (very slow).

So if any physical variation that causes error occurs, it's very unlikely that it happens to only 1 bit thus error can

occur for more than 14 sec.

- Error detection

- Error correction



Forward

Error

correction

Retransmission

(Send back)



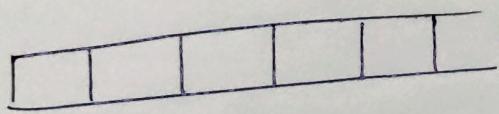
(Receiver tries to guess the correct message) More accurate.



Unpredictable but fast

Only if error is less, few bits only affected, only then is

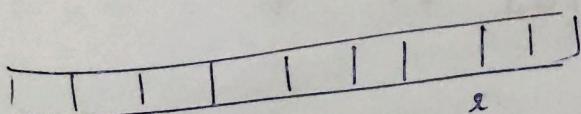
Forward Error Correction used.



← dataword

$k \leftarrow$  length

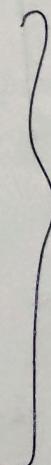
extra 2  
bits added



$k$

$$n = k + 2$$

codeword.



Encoding

Remainder classes when number divided by  $n$

$$\mathbb{Z}/n\mathbb{Z} = \{\bar{0}, \bar{1}, \dots, \bar{n-1}\}$$

$$\left. \begin{array}{l} \bar{a} + \bar{b} = \bar{a+b} \\ \bar{a} \cdot \bar{b} = \bar{ab} \end{array} \right\}$$

$$x = qn + a \quad 0 \leq a \leq n$$

$$y = q'n + b \quad 0 \leq b \leq n$$

Then

$$(x+y) = q''n + c \quad 0 \leq c \leq n$$

where  $a+b = qn+c$

$$\underline{\underline{n=2}}$$

$$\left. \begin{array}{l} \bar{0} + \bar{0} = \bar{0} \\ \bar{1} + \bar{1} = \bar{0} \\ \bar{0} + \bar{1} = \bar{1} \\ \bar{1} + \bar{0} = \bar{1} \end{array} \right\} \text{ XOR.}$$

$\left\{ n = p, p \text{ prime } \mathbb{Z}/p\mathbb{Z} \text{ is a field i.e. } \forall \bar{a} \neq \bar{0} \right\}$

$\left[ \begin{array}{l} \text{b such that } \bar{a} \cdot \bar{b} = \bar{1} \end{array} \right]$

prove this result



$$\left. \begin{array}{l} 6b = 4a + 1 \\ \frac{6b - 1}{4} = a \end{array} \right\}$$

Rough.

$$\left. \begin{array}{l} \bar{0} - \bar{0} = \bar{0} \\ \bar{0} - \bar{1} = \bar{1} \\ \bar{1} - \bar{0} = \bar{1} \\ \bar{1} - \bar{1} = \bar{0} \end{array} \right\}$$

even subtraction gives you XOR.

We mostly study block codes :

Generally, if data words are of size  $r$  and codewords of size  $n$  then encoding scheme is denoted by  $c(n, r)$

How many size  $r$  frames are there?  $2^r$

Encoding is one to one.

If  $n$  is sufficiently larger than  $r$ . It gives us lemmas to make the codewords further from each other. But there is a practical limitation on how much more  $n$  can be than  $r$ .

### Example

$$n = 3, r = 2$$

Dataword

00

01

10

11

Codeword

000

011

101

110

### Observation

1. If 1 bit is corrupted, can receiver detect?

Yes, as it will no longer be on the list.

2. But it cannot be correct.

3. Can receiver detect 2 bit error?

Nope.

### Example

$$r = 2 \quad n = 5$$

00

01

10

11

00000

01011

10101

11110

1. can detect 1 bit error
  2. can detect 2 bit error
  3. All code words have atleast 3 difference from each other so 1 bit errors can be corrected.
  4. 2 bit errors can not be corrected.
- } all errors can be detected.

01010 as transmitted, could be

00000
11110

~~01010~~

### Hamming Distance.

Hamming distance between 2 bit strings  $x, y$  of same size is defined by  $d(x, y) = \#$  of positions that they differ  
 $= \#$  of 1's in  $x \text{ XOR } y$

Minimum Hamming distance of an encoding scheme is the minimum Hamming distance between 2 code words

$C(r, n) \rightarrow$  possibility of  $n$  errors.

How to encode such that atleast  $s$  errors can be detected?

Key the hamming distance btw any 2 codewords as minimum  $s+1$ .

Theorem: If Hamming distance between any 2 codewords is atleast  $s+1$  then upto  $s$ -bit error gets detected.

How to encode such that s bit error gets corrected?

Hamming distance should be  $2s + 1$ .

Theorem: If Hamming distance between any 2 code words is at least  $2s + 1$  then receiver can detect & correct up to  $s$  bit errors.

### Linear Block Code

A block code is called a linear block code if XOR of any 2 codewords is a codeword.

One special type of Linear Block code is :-

### Simple Parity Check

$$n = 2s + 1$$

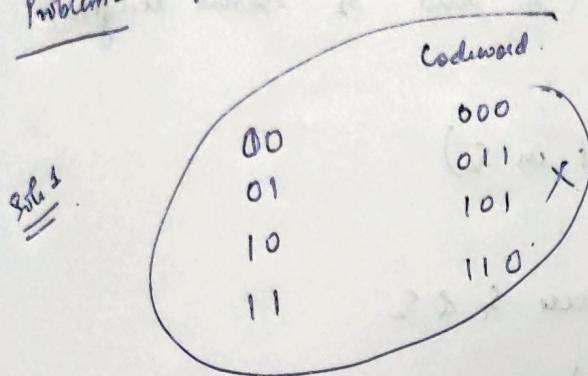
Add one extra bit at the end to make sum of 1's is even.

Exercise: Show that this is Linear Block Code.

1<sup>st</sup> Feb 2024

Problem 1: Why is a simple parity check code a LSC?

Problem 2: Give an example of a non-linear block code.



Use induction and develop from 1 bit.

Simple Parity Check code is set of all strings with even number of 1s. We need to prove that by XORing 2 such strings we get a string with even number of 1s.

1 bit Simple Parity Check code  $\rightarrow 0$ :

$$\underbrace{\text{O} \oplus \text{O} = \text{O}}$$

Base case done

$n-1$  bits induction

0s of 1<sup>st</sup> codeword cancel 0s of 2<sup>nd</sup> codeword.  $\rightarrow 2x$ .

1s of 1<sup>st</sup> codeword " 1s " " ".  $\rightarrow 2y$ .

$0 \oplus 1 \quad \} \quad 1. \text{ so only remaining 1s in } \oplus^3$   
 $1 \oplus 0 \quad \} \quad \text{result is as follows.}$

$$\frac{2(2n-1) - 2n - 2y}{2} = (2n-1) - n - y$$

Prove # of 1s in  $s_1 \text{ XOR } s_2$  ( $s_1$  and  $s_2$  same length)

$$\begin{array}{l} \#(\text{of 1s in } s_1) + \#(\text{of 1s in } s_2) \\ - 2 \cdot (\# \text{ of positions where } s_1 \text{ & } s_2 \\ \text{has 1's}) \end{array} \quad \begin{array}{r} 11011 \\ 10100 \\ \hline 21111 \end{array}$$

### Solution 2

Given an  $r$  and an  $n$ . for every data word of  $r$  bits give a codeword of  $n$  bits.

Dataword	Codeword
00	000
01	001
10	011
11	100
	111

010 not a codeword.

Example:

Can you produce a non-linear code of  $r =$  something,  $n =$  something such that it can always detect 2 errors.

[ b ]

Problem 3. Can you find a LBC with  $k=3$  &  $n=7$  that always detect 2 errors.

$k=3$	$n=7$	$3, 6, 9, \dots, 24$
000	0000000	
001	0010011	
010	0100101	
011	0110110	
100	1001001	
101	1011010	
110	1101100	
111	1111111	

is this linear?  
linear?

\* Simple Parity check can detect odd number of errors but it cannot tell how many places it happened.

→ detects 1 error is simple → make 4<sup>th</sup> bit parity and keep everything else 0.

→ Homework → turn it into a theorem.

$$n = 2k+1. \text{ theorem for this?}$$

$$\underbrace{7 = 2(3)+1.}_{\text{then if codewords in this codewords}}$$

data word

$s_1$

$s_1 \oplus s_1$  (XOR of all bits of  $s_1$ ).

$s_2$

$s_2 \oplus s_2$  (XOR of all bits of  $s_2$ ).

:

:

more distance of such codewords  $> 3$