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Mathematical Methods
Assignment I

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1) If f is a linear combination of $f_1, f_2, \dots f_n$ then,

$$f = \sum_{i=1}^{n} \alpha_i f_i \quad , \alpha_i \text{ are constants}$$

now,

$$f(x) = 2x, \; f_1(x) = x \;\&\; f_2(x) = x^2$$

$\therefore$ $f(x) = \alpha_1 f_1(x) + \alpha_2 f_2(x)$ if $f(x)$ is a linear combination of $f_1(x)$ & $f_2(x)$

now, $2x = \alpha_1 x + \alpha_2 x^2$

for, $\alpha_1 = 2$, $\alpha_2 = 0$, the above eq^n is satisfied. i.e

$$f(x) = 2f_1(x) + 0f_2(x)$$

Thus, $f(x)$ is a linear combination of $f_1(x)$ & $f_2(x)$

2) $y_0 = \sin(x+1) = \cos 1 \sin x + \sin 1 \cos x$

$= 0.99 \sin x + 0.017 \cos x$

$= c_1 \sin x + c_2 \cos x$

now, $y_1 = \sin x$ & $y_2 = \cos x$

$y_0 = 0.99\, y_1 + 0.017\, y_2 \Rightarrow y_0$ is a linear combination of y_1 & y_2

3) a) $f(x) = 9\cos 2x$, $g(x) = 2\cos^2 x - 2\sin^2 x$

We know that

$$\cos 2x = \cos^2 x - \sin^2 x$$

Now,

$$g(x) = 2\cos^2 x - 2\sin^2 x = 2(\cos^2 x - \sin^2 x)$$

$$= 2\cos 2x = 9 . \frac{2}{9} . \cos 2x$$

$$= \frac{2}{9}(9\cos 2x)$$

$$g(x) = \frac{2}{9} f(x)$$

Verifying using Wronskian

$$W(f,g) = \begin{vmatrix} 9\cos 2x & 2\cos 2x \\ -18\cancel{\cos 2x}\sin 2x & -4\cancel{\cos 2x}\sin 2x \end{vmatrix} = 0$$

$\therefore$ $f(x)$ & $g(x)$ are linearly dependent

b) $f(t) = 2t^2$, $g(t) = t^4$

There exists no value 'α' for which $f(t)$ is equal to $(\alpha g(t))$

Verifying using Wronskian

$$W(f,g) = \begin{vmatrix} 2t^2 & t^4 \\ 4t & 4t^3 \end{vmatrix} = 8t^5 - 4t^5 = 4t^5 \neq 0$$

Thus, $f(t)$ & $g(t)$ are not linearly dependent.

4) $f(x) = 6^x$, $g(x) = 6^{x+2}$

Wronskian $W[f,g] = \begin{vmatrix} 6^x & 6^{x+2} \\ 6^x \log 6 & 36.6^x \log 6 \end{vmatrix} = 0$

Wronskian is '0' and $f(x)$ can be expressed as

$$f(x) = \frac{36}{36} 6^x = \frac{1}{36} 6^{x+2} = \frac{1}{36} g(x)$$

Thus, $f(x)$ & $g(x)$ are linearly dependent.

5) $dy/dx = \dfrac{4x^2 - 7x}{3y^2+2}$, $y(1) = 1$

$dy(3y^2+2) = (4x^2 - 7x)dx$

Integrate on both sides

$$\int (3y^2+2)dy = \int (4x^2 - 7x)dx$$

$$y^3 + 2y + c = \frac{4}{3}x^3 - \frac{7}{2}x^2$$

Now, $y(1) = 1$

$\therefore 1 + 2 + c = \frac{4}{3} - \frac{7}{2} \Rightarrow c = -31/6$

Final solution:-

$$\boxed{y^3 + 2y - (31/6) = (4/3)x^3 - (7/2)x^2}$$

6) $y''=2, \quad y'(0)=6, \quad y(0)=0$

$\cancel{\int y'' dy = \int 2}$

$\frac{d}{dx} y' = 2$

$\int dy' = \int 2\,dx \Rightarrow y' = 2x+c$

as $y'(0)=6$, $c=6$

$y' = 2x+6$

$\frac{dy}{dx} = 2x+6$

$\int dy = \int (2x+6)\,dx \Rightarrow y = x^2+6x+c'$

~~as ly y(0) = 0~~ as $y(0)=0 \Rightarrow c'=0$

Final solution $\Rightarrow$

$$\boxed{y = x^2+6x}$$

7) $y''+4y=0$

Characteric eq$\underline{n}$ $\Rightarrow r^2+4=0$

sol$\underline{n}$ of " " $\Rightarrow r = \pm 2i$

Actual solution form $\Rightarrow y = C_1 \sin 2x + C_2 \cos 2x$

a) $y(0) = -2$, $y(\pi/4) = 10$

$y = C_1 \sin 2x + C_2 \cos 2x$

$-2 = 0 + C_2 \Rightarrow C_2 = -2$ (using $y(0) = -2$) | $10 = C_1 + 0 \Rightarrow C_1 = 0$ (using $y(\pi/4) = 10$)

solution $\Rightarrow$ $y = 10 \sin 2x + (-2) \cos 2x$

b) $y(0) = -2$, $y(2\pi) = -2$

$y = C_1 \sin 2x + C_2 \cos 2x$

$-2 = 0 + C_2 \Rightarrow C_2 = -2$ (using $y(0) = -2$) | $-2 = 0 + C_2 \Rightarrow C_2 = -2$ (using $y(2\pi) = -2$)

As, there is no bound for C_1 , $C_1 \in \mathbb{R}$

solution:- $y = C_1 \sin 2x + (-2) \cos 2x$, where $C_1 \in \mathbb{R}$

c) $y(0) = -2$, $y(2\pi) = 3$

$y = C_1 \sin 2x + C_2 \cos 2x$

$-2 = 0 + C_2 \Rightarrow C_2 = -2$ (using $y(0) = -2$) | $3 = 0 + C_2 \Rightarrow C_2 = 3$ (using $y(2\pi) = 3$)

C_2 cannot be -2 & 3 simultaneously so no solution exists for this problem.