Programming ASSIGNMENT – 3

Numerical Solutions of Ordinary and Partial Differential Equations

1. Use the Crank-Nikolson method to solve the parabolic partial differential equation $u_t = u_{xx}$, $x \in (0,1), t \in (0,\infty)$ with initial condition $u(x,0) = 2x$, boundary conditions $u_x(0,t) = 0$ and $u_x(1,t) = 1$. Use the central difference approximation for the boundary conditions. Take $h = 0.1,\ k = 0.05$. Plot the data for various values of (x_m, t_n, u_m,n).

2. Using the Crank-Nicolson method with $h = 0.1$ and the mesh ratio parameter $r = 0.25$ find the solution of $u_t = u_{xx}$ with Initial condition $u(x,0) = \cos\dfrac{\pi x}{2}$, $-1 \le x \le 1, t = 0;$ boundary conditions $u(-1,t) = u(1,t) = 0, \quad t > 0$ at the first 3 time steps. Plot the results for each time step in different frame.

3. Use the explicit method to solve the wave equation $u_{tt} = u_{xx}$, $0 < x < 1, t > 0$ with boundary and initial conditions
$$u(0,t) = -\sin t,\ u(1,t) = \sin(1-t),\ u(x,0) = \sin x, \quad u_t(x,0) = -\cos(x).$$
Take step length along *x*-axis and *t*-axis as 0.1 and 0.1 respectively. Find solution up to $t = 0.5$.

4. Using standard 5-point formula, derive the system of algebraic equations at the nodal points for the elliptic equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = x^2 + y^2$, $\quad -1 < x < 1, -1 < y < 1,$ $u = 2$ at $x = -1 \,\&\, x = 1$; $u = 1$ at $y = -1 \,\&\, y = 1$. Take $h = k = 0.25$. Setup the (i) Gauss-Seidel and (ii) Gauss-Jacobi iterations for the system of equations. Take all the starting values for the iteration as ZEROS. Compare your Gauss Seidel and Gauss Jacobi solution on the plotter frame.

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