Mathematical Model of a Balanced Transportation Problem! min:  $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} \chi_{ij}$  $\sum_{j=1}^{n} \chi_{ij} = \alpha_{i,j} = \alpha_{i,j} = \alpha_{i,j} = \alpha_{i,j}$  $\sum_{i=1}^{m} \chi_{ij} = b_{j,j} = 1,2,..,n$ where dis >0 His For a Balanced T.P  $\sum_{i=1}^{m} a_i = \sum_{i=1}^{n} b_i$ Generally Transportation brokens are minization Type. All the constraints are equality Type. 9+ has m+n constraints and mn variables.

Dual of a Transportation Portun:  $max: Z' = \sum_{i=1}^{m} u_i a_i + \sum_{j=1}^{m} u_j b_j$ subject to Ui+U; < (i) where Ui and U; are free variables. 8+ has (m+n) variables and man constraints. All the variables one free. Example: m=3, n=3 min: Z = 3 3 Cij Vij subject to 3 dis = di, x=1,2,3  $\frac{3}{2} \chi_{ij} = b_j, j = 1/2/3$   $\stackrel{\cdot}{=} 1$ Vis 70, i=1,2,3 91+92+93 =

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## Dual of the T.P:

max:  $Z' = 9_1 4_1 + 9_2 4_2 + 9_3 4_3$ +  $b_1 2_1 + b_2 2_2 + b_3 2_3$ +  $b_1 2_1 + b_2 2_2 + b_3 2_3$ Subject to  $4_1 + 2_2 \le C_{ij}$  i = 1, 2, 3i = 1, 2, 3Ui and i = 1, 2, 3

0.15
CII CIO CI3 COI CO2 CO3 COI CO32 CO33 RHS
(1) (12 (13 (2) (2) (2) (3) (3)
1 21 1/2 X 1/22 1/23 X31 X32 X33 Var
711 712 V3 121 122 - 1 20 New
1 1 1 2 1 2 1 2 1 2 1 2 1
21/26/22/49/25
M 2 2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
41/41/1/1/0/0/0/0/0/0/0/0/0/0/0/0/0/0/0/
81(41)
12 11 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1
42 42 000 000 0000 00000000000000000000
4 4 00000000000000000000000000000000000
03 43 0 0 0 0
11 0 0 1 0 0 1 0 1 52
40 61 -10 -10 10
(4) 20 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0001
14 20 000 000
46 le3 00 1 10 10 10 10 10 10 10 10 10 10 10 1
100 (011 1) 42+61 = 29

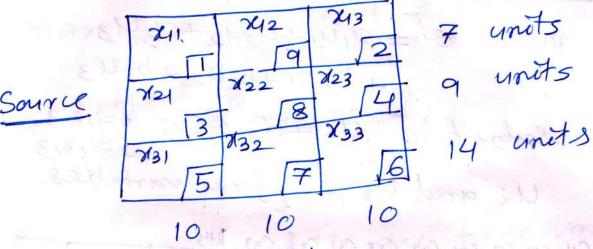
 $41+124 \leq C11$  \\  $41+122 \leq C12$  \\  $41+123 \leq C13$ 

42+14=621 42+12=622 42+12=622 42+12=623

 $U_3 + \mathcal{U}_1 \leq C_{31}$  \\  $U_3 + \mathcal{U}_2 \leq C_{32}$  \\  $C(_3 + \mathcal{U}_3 \leq C_{33})$ 

41,42,43, 14,72,123 are free

T. P. Example 1 Destination



Z = 211 + 9242 + 2243 +3 1/21 +8 1/22 +4 1/23

+5×31+7×32+6×33

subject to 211 + 212+213=7 7/21 + X22 + X23 = 9  $\chi_{31} + \chi_{32} + \chi_{33} = 14$  $\times 11 + \times 21 + \times 31 = 10$ X12+X22+X32=10 243 + 23 + 23 + 23 = 10Nis 7,0 Vijo

Dual of T.P: max: Z/= 7-41+942+1443 +10201+10202+10203 subject to 41+rey = 1 41+22=9 41+U3 =2 U2+24 53 U2+22 = 8 42+113 = 4 713+14 55 U3+U2 = 7 U3 +U3 = 6 where Ui and Il; are free In general

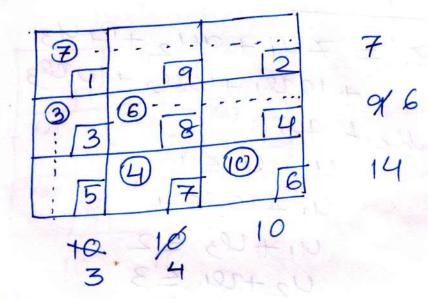
man: Z' \( \text{min: Z}\)
For obtained seln:

man: Z' = min: Z

man: Z' = min: Z

5

Solution:



NWCR:

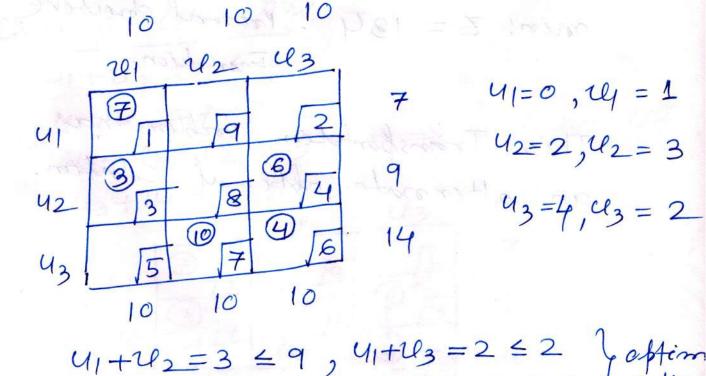
$$741 = 7$$
  
 $11 = 7$   
 $121 = 3$ ,  $122 = 6$   
 $132 = 4$   $133 = 10$   
Fearsible Solution

$$Z = 7 \times 1 + 3 \times 3 + 6 \times 8 + 4 \times 7 + 10 \times 6$$
  
= 152

Check for appirorality:

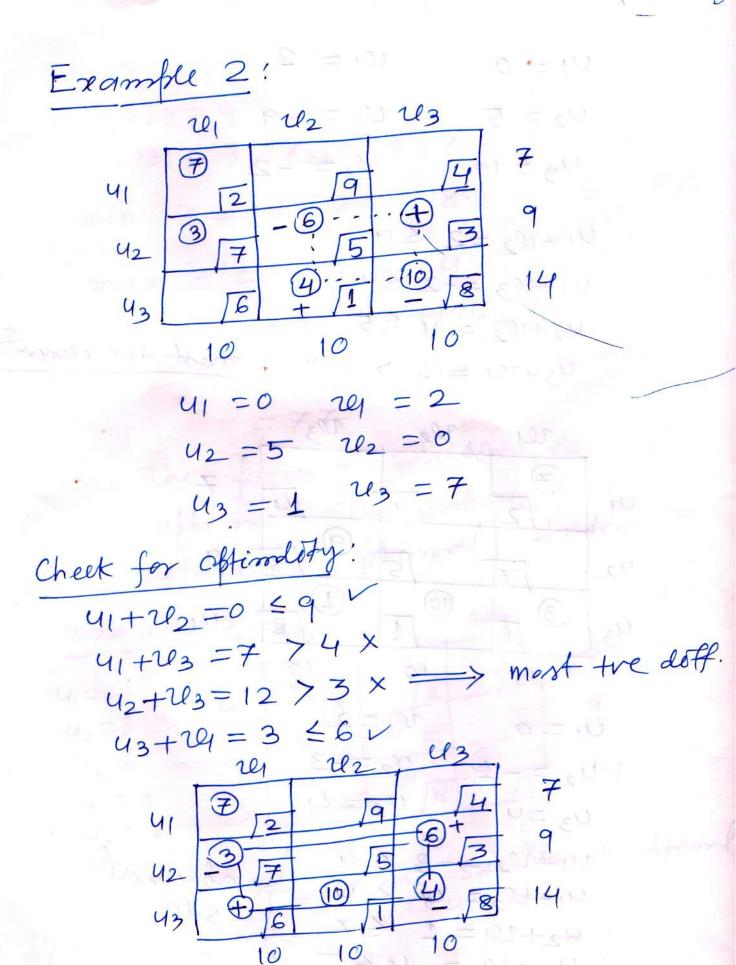
## Check the other constraints:

$$41 + 12 = 5 \le 9$$
 $41 + 12 = 5 \le 9$ 
 $41 + 12 = 3 \times 72 \times 9$ 
 $42 + 12 = 6 \times 4 \times 7$ 
 $42 + 12 = 3 \times 5 \times 9$ 
 $41 + 12 = 3 \times 5 \times 9$ 
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 $41 + 12 = 3 \times 9$ 
 $41 + 12$ 



 $u_2+u_2=5 \le 8$ ,  $u_3+ve_1=5 \le 1$ 

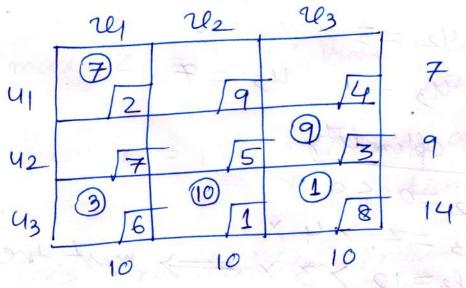
 $\gamma_{11}=7, \gamma_{21}=3, \gamma_{23}=6$  $\chi_{32} = 10$ ,  $\chi_{33} = 4$ min: Z = 134  $u_1 = 0$ ,  $u_2 = 2$ ,  $u_3 = 4$  $U_1 = 1$ ,  $U_2 = 3$ ,  $U_3 = 2$ max: z'= 0x7+9x2+4x14 +1x10 +3x10+2x10 = 134max 1 Z' = 134: Dual dijective function min: Z = 134: Primal objective Function. This Transportation Porblim has an alternate oftimal solution.



$$41 = 0$$
  $101 = 2$ 
 $41 = 0$   $101 = 2$ 
 $41 = 0$   $102 = -9$ 
 $41 = 0$   $101 = 0$ 
 $101 = 0$ 
 $101 = 0$ 
 $101 = 0$ 
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 $101 = 0$ 
 $101 = 0$ 
 $101 = 0$ 
 $101 = 0$ 

$$41+12=2 \le 9$$
  
 $41+12=2 \le 9$   
 $41+12=-2 \le 9$   
 $42+12=-9 \le 5$   
 $43+12=12 > 6 \times$ 

most tre eliment



$$41 = 0$$
 $12 = 2$ 
 $41 = 0$ 
 $41 = 2$ 
 $42 = -3$ 
 $43 = 4$ 
 $41 + 12 = -3 \le 9$ 
 $41 + 12 = -3 \le 9$ 
 $41 + 12 = 4 \le 9$ 
 $41 + 12 = 4 \le 9$ 
 $42 + 12 = 4 \le 7$ 
 $42 + 12 = -4 \le 5$ 
 $42 + 12 = -4 \le 5$ 

$$\chi_{11} = 7$$
 $\chi_{23} = 9$ 
 $\chi_{31} = 3$ ,  $\chi_{32} = 10$ ,  $\chi_{33} = 1$ 
 $\chi_{31} = 3$ ,  $\chi_{32} = 10$ ,  $\chi_{33} = 1$ 
 $\chi_{31} = 3$ ,  $\chi_{32} = 10$ ,  $\chi_{33} = 1$ 
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 $\chi_{31} = 3$ ,  $\chi_{32} = 10$ ,  $\chi_{33} = 1$ 
 $\chi_{31} = 3$ ,  $\chi_{32} = 10$ ,  $\chi_{33} = 1$ 
 $\chi_{10} = 10$ 
 $\chi_{10} =$ 

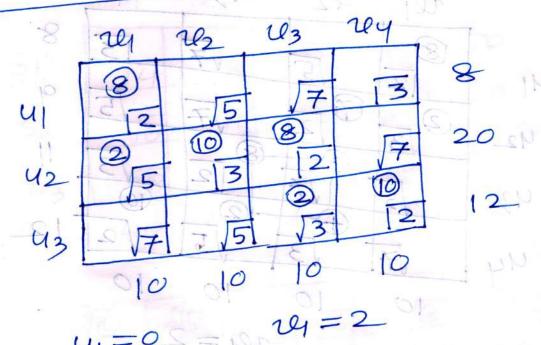


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Example 4



$$u_1 = 0$$
 =  $u_2 = 0$   $u_2 = 0$   $u_3 = -1$ 

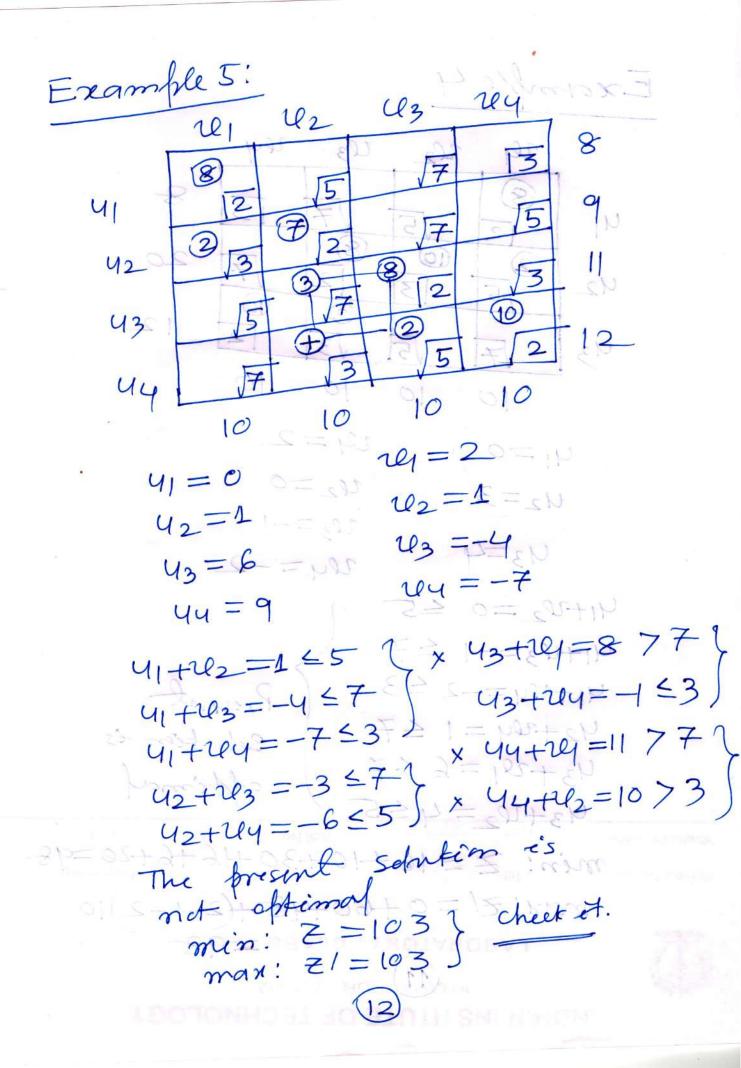
$$43=4$$
  $104=-2=0$ 

$$43+204=1 \le 7$$
  
 $43+20_1=6 \le 7$   
 $43+20_2=4 \le 5$ 

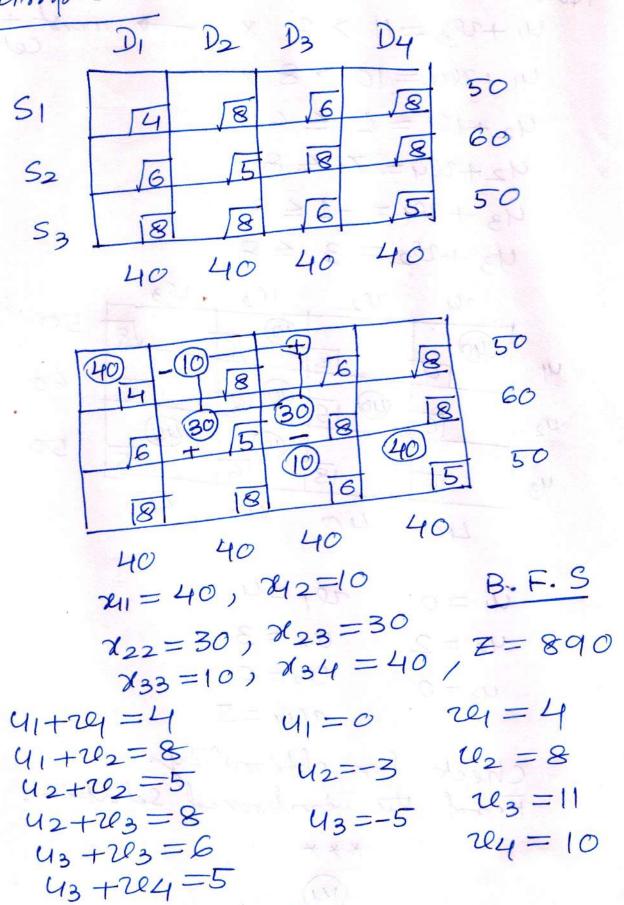
Present Schrkim is aptimal

min: Z = 16 + 10 + 30 + 16 + 6 + 20 = 98man: Z = 0 + 60 + 48 + (2 - 1 - 2)10= 98

11



Example: 6



Now 41+23=1176 x 41+24=10>8x 42+24=1 = 6 42+24=7 =8 43+29=-1 < 8 U3+U2=3 <8 1/2 10 8 60 20 8 40 15 16 42 50 10 8 43 40 40 40 2,=4 103=6 43 = 0 14 = 5 check for obtionality. of the improved solut \* \* \*