Department of Mathematics, I I T Kharagpur
MA 31007 Mathematical Methods
Class Test 2, Autumn 2021 **Max. Marks : 25**
Time : 1hr and 40 minutes **No. of Students : 93**

Instruction : Answer **ALL** the questions.

Question 1.

(a) Give definition of symmetric and skew-symmetric tensors with examples. Prove that a skew-symmetric tensor of second order has atmost $\frac{N(N-1)}{2}$ independent components.

(b) Let A^{ij} be a contravariant tensor and B_i a covariant vector. Are the quantities $A^{ij}B_k$ and $A^{ij}B_j$ tensors? If so write the tensors in a suitable notation and give the contravariant and covariant order and rank.

[3+2 = 5]

Question 2.

(a) Define Christoffel Symbols or Brackets of first and second kind. Determine the number of independent components of Christoffel symbols.

(b) Show that

$$[i, j, m] = g_{lm} \left\{ \begin{array}{cc} l & \\ i & j \end{array} \right\}.$$

[3+2=5]

Question 3.

(a) State and prove the Quotient law of tensors.

(b) Evaluate $\lim_{a\to\infty} {}_2F_1(1, a; 1; \frac{x}{a})$, where ${}_2F_1(1, a; 1; \frac{x}{a})$ is the hypergeometric function.

[4+1 = 5]

Question 4.

(a) Prove that the outer product of two vectors is a tensor of order two. Is the converse true? Justify.

(b) Show that if a tensor is skew-symmetric with respect to a pair of indices in one system of coordinates, then it is so in every system.

[3+2 = 5]

(P. T. O)

Question 5.

(a) Find the third derivative of the hypergeometric function ${}_2F_1(2,3;1;x)$ with respect to x.

(b) Show that

$$\int_0^1 \frac{uJ_0(xu)}{\sqrt{1-u^2}}du = \frac{\sin x}{x}.$$

[3+2 = 5]

********** THE END **********