Date

NSDE (MA20102) Asg-1

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→ 19MA20015

1. $\frac{dy}{dx} = \frac{1}{x^2+y}$ $\quad y(4) = 4 \quad y(4.2) = ? \quad h = 0.1$

$x_0 = 4 \quad y_0 = 4$

$y_{i+1} = y_i + h \cdot y_i' + \frac{h^2}{2} y_i'' \rightarrow$ 2nd order taylor's series

$y' = \frac{1}{x^2+y}$

$y'' = \frac{-1}{(x^2+y)^2}(2x + y') = \frac{-2x}{(x^2+y)^2} - \frac{1}{(x^2+y)^3} = -\frac{(2x^3+2xy+1)}{(x^2+y)^3}$

$\Rightarrow y(4.1) = y_1 = y_0 + (0.1) y_0' + \frac{(0.1)^2}{2} y_0''$

$= 4 + (0.1)\frac{1}{(4^2+4)} + \frac{(0.1)^2}{2} \cdot \frac{-(2\cdot 4^3 + 2\cdot 4\cdot 4 + 1)}{(4^2+4)^3}$

$= 4.004899375$

$y(4.2) = y_2 = y_1 + (0.1) y_1' + \frac{(0.1)^2}{2} y_1''$

$= y_1 + \frac{(0.1)}{(x_1^2+y_1)} + \frac{(0.1)^2}{2} \cdot \frac{-(2x_1^3 + 2x_1y_1 + 1)}{(x_1^2+y_1)^3}$

$= 4.009703626 - 0.000095188$

$= 4.009608441$

$y(4.2) \cong 4.0096084 \cong 4.01$

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2. $\frac{dy}{dx} = 3x + y^2$ $y(0) = 1$ $h = 0.2$ interval $[0, 0.4]$

$$y_{i+1} = y_i + h y_i' + \frac{h^2}{2} y_i'' + \frac{h^3}{6} y_i'''$$

$y' = 3x + y^2$

$y'' = 3 + 2y(3x + y^2) = 3 + 6xy + 2y^3$

$y''' = 6y + 6x(3x + y^2) + 6y^2(3x + y^2)$

$= 18y^2x + 6y^4 + 6y + 18x^2 + 6y^2x$

$= 6y + 18x^2 + 6y^4 + 24y^2x$

$y_1 = y_0 + (0.2) y_0' + \frac{(0.2)^2}{2} y_0'' + \frac{(0.2)^3}{6} y_0'''$

$= 1 + (0.2)(1) + \frac{(0.2)^2}{2}(5) + \frac{(0.2)^3}{6}(12) \approx 1.316$

$\Rightarrow y(0.2) \cong 1.316$

$y_2 = y_1 + (0.2) y_1' + \frac{(0.2)^2}{2} y_1'' + \frac{(0.2)^3}{6} y_1'''$

$= 1.316 + (0.2)(2.33185) + \frac{(0.2)^2}{2}(9.137444992) + \frac{(0.2)^3}{6}(34.89961203)$

$= 1.316 + 0.4663712 + 0.1827488998 + 0.04653281604$

$\cong 2.011652916$

$\therefore y(0.2) \cong 1.316$

$y(0.4) \cong 2.011652916$

3. $\frac{dy}{dx} = 2y + 3e^x$, $x_0 = 0$, $y_0 = 0$

$\Rightarrow y_{i+1} = y_i + h y_i' + \frac{h^2}{2} y_i''$

$y' = 2y + 3e^x$

$y'' = 2y' + 3e^x = 4y + 9e^x$

at $h = 0.1$

$y(0.1) = y_1 = y_0 + (0.1)(2y_0 + 3e^{x_0}) + \frac{(0.1)^2}{2}(4y_0 + 9e^{x_0})$

$= 0.3 + 0.045 = 0.345$

$y(0.1) \cong 0.345$

$y(0.2) = y_2 = y_1 + (0.1)(2y_1 + 3e^{x_1}) + \frac{(0.1)^2}{2}(4y_1 + 9e^{x_1})$

$= 0.345 + (0.1)(2(0.345) + 3e^{0.1}) + \frac{(0.1)^2}{2}(4(0.345) + 9e^{0.1})$

$= 0.802183967$

$\therefore y(0.1) \cong 0.345 \quad y(0.2) \cong 0.802183967$

4. $\frac{dy}{dx} = y - x \quad y(0) = 2 \quad$ let $h = 0.05$

$$y_n = y_{n-1} + h\left(f(x_{n-1}, y_{n-1})\right)$$

$\Rightarrow$

x	y' $y - x$	$y_n = y_{n-1} + h(y_{n-1} - x_{n-1})$
0	2	$= 2 + 0.05(2) = 2.1$
0.05	2.1	$= 2.1 + 0.05(2.05) = 2.2025$
0.1	2.2025	$= 2.2025 + 0.05(2.1025)$
		$= 2.307625$
0.15	2.307625	$= 2.307625 + 0.05(2.157625)$
		$= 2.41550625$

$\Rightarrow y(0.1) = 2.2025 \cong 2.20$

$y(0.2) = 2.41550625 \cong 2.42$

$\therefore y(0.1) = 2.20 \quad y(0.2) = 2.42$

5. $y' = x - y^2$ $y(0) = 1$ interval $[0, 0.6]$ $h = 0.2$

$$y_n = y_{n-1} + h(x_{n-1} - y_{n-1}^2)$$

x	y	$x - y^2$	$y_n = y_{n-1} + h(x_{n-1} - y_{n-1}^2)$
0	1	-1	$= 1 + 0.2(-1) = 0.8$
0.2	0.8	-0.44	$= 0.8 + 0.2(-0.44) = 0.712$
0.4	0.712	-0.106944	$= 0.712 + 0.2(-0.106944)$
			$= 0.6906112$
0.6	0.6906112	~~0.4230564704~~	~~$= 0.6906112 + 0.2(0.4230564704)$~~

$\therefore y(0) = 1$

$y(0.2) \cong 0.8$

$y(0.4) = 0.712$

$y(0.6) = 0.6906112$

6. $\frac{dy}{dx} = x + y^2$ $y(0)=1$ taking $h = 0.1$

$$y_n = y_{n-1} + h f(x_n, y_n)$$

(a) for $x = 0.1$

$x_0 = 0$ $y_0 = 1$

$y_1 = y_0 + (0.1) f(x_1, y_1)$

$= 1 + (0.1)(0.1 + y_1^2)$

$\Rightarrow 0.1 y_1^2 + 1.01 - y_1 = 0$

$\Rightarrow y_1^2 + 10.1 - 10 y_1 = 0$

on solving

$\Rightarrow y_1 \cong 1.14$

(b) for $x = 0.2$

$x_1 = 0.1$ $y_1 = 1.14$

$y_2 = y_1 + (0.1)(0.2 + y_2^2)$

$= 1.14 + (0.1)(0.2 + y_2^2)$

$= 1.14 + 0.02 + 0.1 y_2^2$

$y_2 = 1.16 + 0.1 y_2^2$

$\Rightarrow y_2^2 - 10 y_2 + 11.6 = 0$

on solving

$\Rightarrow y_2 \cong 1.339$

$\therefore y(0.2) \cong 1.339$

7. $\frac{dy}{dx} = \frac{x-y}{1+x}$ $\quad y(0)=1 \quad h=0.05$

$x_0=0 \quad y_0=1$

$$y_n = y_{n-1} + h\left(\frac{x_n - y_n}{1+x_n}\right)$$

$$\Rightarrow y(0.05) = y_1 = 1 + 0.05\left(\frac{0.05-y_1}{1+0.05}\right)$$

$$y_1 = 1 + \frac{0.0025}{1.05} - \frac{0.05y_1}{1.05}$$

$$\frac{22}{21}y_1 = \frac{421}{420} \Rightarrow y_1 = \frac{421}{440} = 0.956818181818$$

$$\Rightarrow y_1 = \frac{421.0}{440} = 0.956818181818$$

$$\Rightarrow y(0.1) = y_2 = \frac{421}{440} + 0.05\left(\frac{0.1-y_2}{1+0.1}\right)$$

$$\Rightarrow y_2 = \frac{421}{440} + \frac{0.005}{1.1} - \frac{0.05y_2}{1.1}$$

$$\frac{23}{22}y_2 = \frac{423}{440} \Rightarrow y_2 = \frac{423}{460} = 0.919565217391$$

$\therefore y(0.1) \cong 0.919565$

8. $\frac{dy}{dx} = x^2 + y \quad y_0 = 0.94 \quad x_0 = 0 \quad h = 0.1$

$\downarrow f(x,y)$

Using Modified Euler's method

$$y_1 = y_0 + h(f(x_0, y_0))$$
$$= 0.94 + 0.1[0.94]$$
$$= 1.034$$

$$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_0+h, y_1)]$$
$$= 0.94 + \frac{0.1}{2}[0.94 + (0.1)^2 + 1.034]$$
$$= 1.0392$$

$$y_1^{(2)} = 0.94 + \frac{0.1}{2}[0.94 + (0.1)^2 + 1.0392]$$
$$= 1.03946$$

$$y_1^{(3)} = 0.94 + \frac{0.1}{2}[0.94 + (0.1)^2 + 1.03946]$$
$$= 1.0394473$$

$$y_1^{(4)} = 0.94 + \frac{(0.1)}{2}[0.94 + (0.1)^2 + 1.0394473]$$
$$= 1.03947365$$

$$y_1^{(5)} = 1.03947368\overline{3}$$

$\therefore y_1 = y(0.1) \cong 1.0395$ [5 significant digits]

9. $\frac{dy}{dx} = x + |\sqrt{y}|$ $x_0 = 0$, $y_0 = 1$, $h = 0.2$ in interval $[0, 0.6]$

using modified euler's method.

step-1 $x_1 = 0.2$

$y_1 = 1 + 0.2(1) = 1.2$

$y_1^{(1)} = 1 + \frac{0.2}{2}(1 + 0.2 + |\sqrt{1.2}|) = 1.2295$

$y_1^{(2)} = 1 + \frac{0.2}{2}(1 + 0.2 + |\sqrt{1.2295}|) = 1.2309$

$y_1^{(3)} = 1 + \frac{0.2}{2}(1 + 0.2 + |\sqrt{1.2309}|) = 1.2309$

~~$y_1^{(4)} = x + \frac{0.2}{2}(1 + 0.2 + |$~~

step-2 $x_2 = 0.4$

$y_2 = 1.2309 + 0.2(0.2 + |\sqrt{1.2309}|) = 1.4927$

$y_2^{(1)} = 1.2309 + \frac{0.2}{2}(1.3094 + 0.4 + |\sqrt{1.4927}|) = 1.5240$

$y_2^{(2)} = 1.2309 + \frac{0.2}{2}(1.3094 + 0.4 + |\sqrt{1.5240}|) = 1.5253$

$y_2^{(3)} = 1.2309 + \frac{0.2}{2}(1.3094 + 0.4 + |\sqrt{1.5253}|) = 1.5253$

step-3 $x_3 = 0.6$

$y_3 = 1.5253 + 0.2(0.4 + |\sqrt{1.5253}|) = 1.8523$

$y_3^{(1)} = 1.5253 + \frac{0.2}{2}(1.6350 + 0.6 + |\sqrt{1.8523}|) = 1.8849$

$y_3^{(2)} = 1.5253 + \frac{0.2}{2}(1.6350 + 0.6 + |\sqrt{1.8849}|) = 1.8861$

$$y_3^{(4)} = 1.5253 + \frac{0.2}{2}\left(1.6350 + 0.6 + \left|\sqrt{1.8861}\right|\right) = 1.8861$$

$\therefore$ $y(0) = 1$

$y(0.2) \cong 1.2309$

$y(0.4) \cong 1.5253$

$y(0.6) \cong 1.8861$

10. $y' = xy$, $y(1) = 1$, $[1, 1.4]$, step size $h = 0.2$

$f(x, y) = xy$

(a) $K_1 = 0.2\ (1) = 0.2$

$K_2 = 0.2\ f(x_0 + 0.2, y_0 + 0.2) = 0.2 \times 1.2 \times 1.2$

$= 0.288$

$K = \frac{1}{2}(K_1 + K_2) = \frac{1}{2} \times 0.488 = 0.244$

$y_1 = y_0 + K \Rightarrow y_1 = 1 + 0.244$

$\Rightarrow y_1 = 1.244$ for $x_1 = 1.2$

(b) $K_1 = 0.2\ f(x_1, y_1)$

$= 0.2 \times 1.2 \times 1.244 = 0.29856$

$K_2 = 0.2\ f(1.2 + 0.2, 1.244 + 0.2)$

$= 0.2 \times 1.4 \times 1.444$

$= 0.4043$

$K = \frac{1}{2}(K_1 + K_2) = 0.35143$

$y_2 = y_1 + K = 1.244 + 0.35143 = 1.59543$

$\therefore$ $y(1.2) \cong 1.244$ $y(1.4) \cong 1.59543$

11. $\frac{dy}{dx} = \frac{1}{x+y}$ $\quad y(0)=1 \quad [0,2] \quad h=0.5$

$$f(x,y) = \frac{1}{x+y}$$

(a) for $x = 0.5$

$$k_1 = h f(x_0, y_0) = \frac{0.5}{0+1} = 0.5$$

$$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.5\left[\frac{1}{0+\frac{0.5}{2}+1+\frac{0.5}{2}}\right]$$

$$= 0.3333$$

$$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= 0.5\left(\frac{1}{0+\frac{0.5}{2}+1+\frac{0.3333}{2}}\right) = 0.3529$$

$$k_4 = h f(x_0 + h, y_0 + k_3)$$

$$= \frac{0.5}{0+0.5+1+0.3529} = 0.2698$$

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 0.3570$$

$$y_1 = y_0 + k = 1.3570 \Rightarrow y(0.5) \cong 1.3570$$

(b) for $x = 0.1$

$$k_1 = h f(x_1, y_1) = \frac{0.5}{0.5+1.3570} = 0.2693$$

$$k_2 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right) = \frac{0.5}{0.5+\frac{0.5}{2}+1.3570+\frac{0.2693}{2}}$$

$$= 0.2230$$

$$k_3 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= \frac{0.5}{0.5+\frac{0.5}{2}+1.3570+\frac{0.2230}{2}} = 0.2254$$

$$K_4 = h f(x_1 + h, y_1 + K_3)$$

$$= \frac{0.5}{1.3570 + 0.2254} = 0.1936$$

$$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) = 0.2266$$

$$y_2 = y_1 + K = 1.5836$$

$$\Rightarrow y(1) \cong 1.5836$$

(c) for $x = 1.5$

$$K_1 = h f(x_2, y_2) = \frac{0.5}{2.5836}$$

$$= 0.1935$$

$$K_2 = h f\left(x_2 + \frac{h}{2}, y_2 + \frac{K_1}{2}\right)$$

$$= \frac{0.5}{1 + \frac{0.5}{2} + 1.5836 + \frac{0.1935}{2}} = 0.1706$$

$$K_3 = h f\left(x_2 + \frac{h}{2}, y_2 + \frac{K_2}{2}\right)$$

$$= \frac{0.5}{1 + \frac{0.5}{2} + 1.5836 + \frac{0.1706}{2}}$$

$$= 0.1713$$

$$K_4 = h f(x_2 + h, y_2 + K_3)$$

$$= \frac{0.5}{1 + 0.5 + 1.5836 + 0.1713}$$

$$= 0.1536$$

$$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) = 0.1718$$

$$y_3 = y_2 + K \Rightarrow y(1.5) \cong 1.7554$$

(d) for $x = 2$

$$k_1 = h f(x_3, y_3)$$

$$= \frac{0.5}{3.2554} = 0.1536$$

$$k_2 = h f\left(x_3 + \frac{h}{2}, y_3 + \frac{k_1}{2}\right)$$

$$= \frac{0.5}{1.5 + \frac{0.5}{2} + 1.7554 + \frac{0.1536}{2}}$$

$$= 0.1396$$

$$k_3 = h f\left(x_3 + \frac{h}{2}, y_3 + \frac{k_2}{2}\right)$$

$$= \frac{0.5}{1.5 + \frac{0.5}{2} + 1.7554 + \frac{0.1396}{2}} = 0.13985$$

$$k_4 = h f(x_3 + h, y_3 + k_3)$$

$$= \frac{0.5}{1.5 + 0.5 + 1.7554 + 0.13985}$$

$$= 0.1284$$

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 0.14015$$

$$y_4 = y_3 + k = 1.89555$$

$\therefore y(0.5) \cong 1.3570$

$y(0.1) \cong 1.5836$

$y(1.5) \cong 1.7554$

$y(2) \cong 1.89555$

12. $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ $\quad y(0)=1 \quad h=0.1$

$$f(x,y) = \frac{y^2-x^2}{y^2+x^2}$$

(a) for $x = 0.1$

$$K_1 = h f(x_0, y_0) = 0.1\left[\frac{1^2-0^2}{1^2+0^2}\right] = 0.1$$

$$K_2 = h f\left(x_0+\frac{h}{2}, y_0+\frac{K_1}{2}\right) = 0.1\left[\frac{1.05^2-0.05^2}{1.05^2+0.05^2}\right]$$

$$= 0.0995$$

$$K_3 = h f\left(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}\right)$$

$$= 0.1\left[\frac{1.0498^2-0.05^2}{1.0498^2+0.05^2}\right] = 0.0995$$

$$K_4 = h f\left(x_0+h, y_0+K_3\right)$$

$$= 0.1\left\{\frac{1.0995^2-0.1^2}{1.0995^2+0.1^2}\right] = 0.09836$$

$$K = \frac{1}{6}\left[K_1+2K_2+2K_3+K_4\right] = 0.09940$$

$$y_1 = y_0 + K = 1.09940 \Rightarrow y(0.1) \cong 1.09940$$

(b) for $x = 0.2$

$$K_1 = h f(x_1, y_1)$$

$$= 0.1\left[\frac{1.0994^2-0.1^2}{1.0994^2+0.1^2}\right] = 0.09836$$

$$K_2 = h f\left(x_1+\frac{h}{2}, y_1+\frac{K_1}{2}\right)$$

$$= 0.1\left[\frac{1.1486^2-0.15^2}{1.1486^2+0.15^2}\right] = 0.0965$$

$$k_3 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 0.1\left[\frac{1.1477^2 - 0.15^2}{1.1477^2 + 0.15^2}\right] = 0.09664$$

$$k_4 = h f\left(x_1 + h, y_1 + k_3\right)$$

$$= 0.1\left[\frac{1.1960^2 - 0.2^2}{1.1960^2 + 0.2^2}\right] = 0.0946$$

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 0.09659$$

$$y_2 = y_1 + k = 1.1960 \Rightarrow y(0.2) \cong 1.1960$$

(c) For $x = 0.3$

$$k_1 = h f(x_2, y_2)$$

$$= 0.1\left[\frac{1.1960^2 - 0.2^2}{1.1960^2 + 0.2^2}\right] = 0.09456$$

$$k_2 = h f\left(x_2 + \frac{h}{2}, y_2 + \frac{k_1}{2}\right)$$

$$= 0.1\left[\frac{1.24328^2 - 0.25^2}{1.24328^2 + 0.25^2}\right] = 0.09223$$

$$k_3 = h f\left[x_2 + \frac{h}{2}, y_2 + \frac{k_2}{2}\right]$$

$$= 0.1\left[\frac{(1.2421)^2 - (0.25)^2}{(1.2421)^2 + (0.25)^2}\right] = 0.9221$$

$$k_4 = h f(x_2 + h, y_2 + k_3)$$

$$= 0.1\left[\frac{(1.2882)^2 - (0.25)^2}{(1.2882)^2 + (0.25)^2}\right] = 0.09274$$

$$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] = 0.09270$$

$$y_3 = y_2 + k = 1.2887 \Rightarrow y(0.3) \cong 1.2887$$

(d) for $x = 0.4$

$$k_1 = hf(x_3, y_3)$$

$$= 0.1\left[\frac{1.2887^2 - 0.3^2}{1.2887^2 + 0.3^2}\right] = 0.08972$$

$$k_2 = hf\left(x_3 + \frac{h}{2}, y_3 + \frac{k_1}{2}\right)$$

$$k_2 = 0.1\left[\frac{1.3336^2 - 0.35^2}{1.3336^2 + 0.35^2}\right] = 0.08711$$

$$k_3 = hf\left(x_3 + \frac{h}{2}, y_3 + \frac{k_2}{2}\right)$$

$$= 0.1\left[\frac{1.3772^2 - 0.35^2}{1.3772^2 + 0.35^2}\right] = 0.0879$$

$$k_4 = hf(x_3 + h, y_3 + k_3)$$

$$= 0.1\left[\frac{1.3766^2 - 0.35^2}{1.3766^2 + 0.35^2}\right] = 0.0879$$

$$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4] = 0.08764$$

$$y_4 = y_3 + k = 1.2887 + 0.08764$$

$$\Rightarrow y(0.4) = 1.3763$$

$\therefore y(0.2) \cong 1.1960$

$y(0.4) \cong 1.3763$

13. $\frac{dy}{dx} = x^2 + y^2 \quad y(0) = 1 \quad h = 0.2$

$x_0 = 0 \quad y_0 = 1$

$$y_{j+1} = y_j + \frac{h}{2}(K_1 + K_2)$$

$$K_1 = f\left(x_j + \frac{3-\sqrt{3}}{6}h,\ y_j + \frac{h}{4}K_1 + \frac{3-2\sqrt{3}}{12}hK_2\right)$$

$$K_2 = f\left(x_j + \frac{3+\sqrt{3}}{6}h,\ y_j + \frac{3+2\sqrt{3}}{12}hK_1 + \frac{h}{4}K_2\right)$$

(a)

$$y_1 = y_0 + \frac{h}{2}(K_1 + K_2)$$

$$K_1 = 0.000178632795 + (1 + 0.05K_1 - 0.00773502 69K_2)^2$$

$$K_2 = 0.02488033 + (1 + 0.0788675K_1 + 0.05K_2)^2$$

$$\Rightarrow K_1 = 1.092 \quad K_2 = 1.356$$

$$y(0.2) \cong y_1 \cong 1 + \frac{0.2}{2}(1.092 + 1.356) = 1.2448$$

$$\therefore y(0.2) = 1.2448$$

(b) $y_2 = y_1 + \frac{h}{2}(K_1 + K_2)$

$$K_1 = 0.058692317 18 + (1.2448 + 0.05K_1 - 0.00773502 69K_2)^2$$

$$K_2 = 0.127974 3495 + (1.2448 + 0.0788675K_1 + 0.05K_2)^2$$

$$\Rightarrow K_1 = 1.78994 \quad K_2 = 2.39519$$

$$y(0.4) = y_2 \cong 1.2448 + \frac{0.2}{2}(1.78994 + 2.39519)$$

$$\cong 1.66313$$

$$\therefore y(0.4) = 1.66313$$