Tutorial Sheet-8

Graph Theory and Algorithms

(1) If G is a 2-connected graph on n vertices then show that $diam(G) \leq \lfloor \frac{n}{2} \rfloor$.

(2) Let G be a graph on n vertices and m edges. If G is k-connected or k-edge connected then prove that $m \geq \frac{nk}{2}$.

(3) Applying Menger's theorem prove that Q_3 is 3-connected.

(4) Prove that if G is a planar graph of order $n \leq 11$ then G has a vertex of degree 4 or less.

(5) Prove or disprove: If G is a planar bipartite graph then G has a vertex of degree 3 or less.

(6) Let G be an n-vertex simple connected planar graph isomorphic to its dual graph. Then find the total number of edges in G in terms of n.

(7) Find the crossing number of K_6 and the Petersen graph.

(8) Prove or disprove: For every n-vertex graph G, $\chi(G) \leq n - \alpha(G) + 1$.

(9) Prove or disprove: If G is a connected graph and $a(G)$ is the average vertex degree of G then $\chi(G) \leq a(G) + 1$.

(10) A graph G is called **k-critical (or k-color critical)** if $\chi(G) = k$ and $\chi(H) < \chi(G)$ for every proper subgraph H of G. Then
(i) Give example of two color critical graphs.
(ii) Prove that for every k-critical graph G, $\delta(G) \geq k - 1$.

(11) Find the edge chromatic number of the Petersen graph.

(12) Prove that every bipartite r-regular graph G has $\chi_1(G) = \Delta(G) = r$.