Programming ASSIGNMENT
Numerical Solutions of ODEs & PDEs

1. Using the second order Runge – Kutta method, solve the I V P $y' = 2 + \sqrt{y - 2t + 3},\ \ y(0) = 1, h = 0.025, 0.05.$ in [0, 0.5]. Compare numerical solution with the exact solution $y(t) = 1 + 4t + \frac{1}{4}t^2$.
2. Consider the IVP $y' = 2ty,\ \ y(1) = 1, h = 0.05, 0.1$. Solve this using the classical 4th order Runge-Kutta method in the interval [1, 2] and plot these numerical results by comparing it with the exact solution $y = e^{t^2-1}$.

3. Using the implicit 4th order Runge - Kutta method, solve the I V P $y' = \dfrac{y^2 + ty - t^2}{t^2},\ \ y(1) = 2,\ h = 0.025, 0.05.$ in *[1, 1.5]*. Plot your numerical solution with the exact solution $y(t) = \dfrac{t(1 + t^2/3)}{1 - t^2/3}$.

4. Solve the following system of differential equations by taking h = 0.05, 0.1 in the interval [0, 1] using the explicit 4th order Runge - Kutta method.

$$x'(t) = -3x + 4y, \quad x(0) = 1$$
$$y'(t) = -2x + 3y, \quad y(0) = 2$$

Compare you results with the values of the exact solution

$$x(t) = 3e^t - 2e^{-t},\ y(t) = 3e^t - e^{-t}.$$

5. Solve the I V P: $\dfrac{du}{dx} = -2u^2, u(0) = 1,\ h = 0.05,\ 0.1,\ [0.0, 1.0]$ correct to 4 decimal places using the following Predictor – Corrector method:

$$P:\ u_{j+1} = u_{j-3} + \frac{4h}{3}(2f_j - f_{j-1} + 2f_{j-2}),\ C:\ u_{j+1} = u_{j-1} + \frac{h}{3}(f_{j+1} + 4f_j + f_{j-1}).$$

Calculate the starting values using the modified Euler method. Plot these numerical data.

6. Solve the I V P $\dfrac{du}{dx} = -2u^3, u(0) = 1,\ h = 0.05,\ 0.1,\ [0.0, 1.0]$ correct to 4 decimal places using the above given Predictor – Corrector method. Calculate the starting values using the 4thorder Classical Runge-Kutta method. Plot these numerical data.