

NSDE Programming Assignment - 1

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Sec : 2

Question 1:

CODE:

```
h = 0.025; % change to 0.05 for second part
t = 0 : h : 0.5;
n = length(t);
Y = zeros(1, n);
Y(1) = 1;

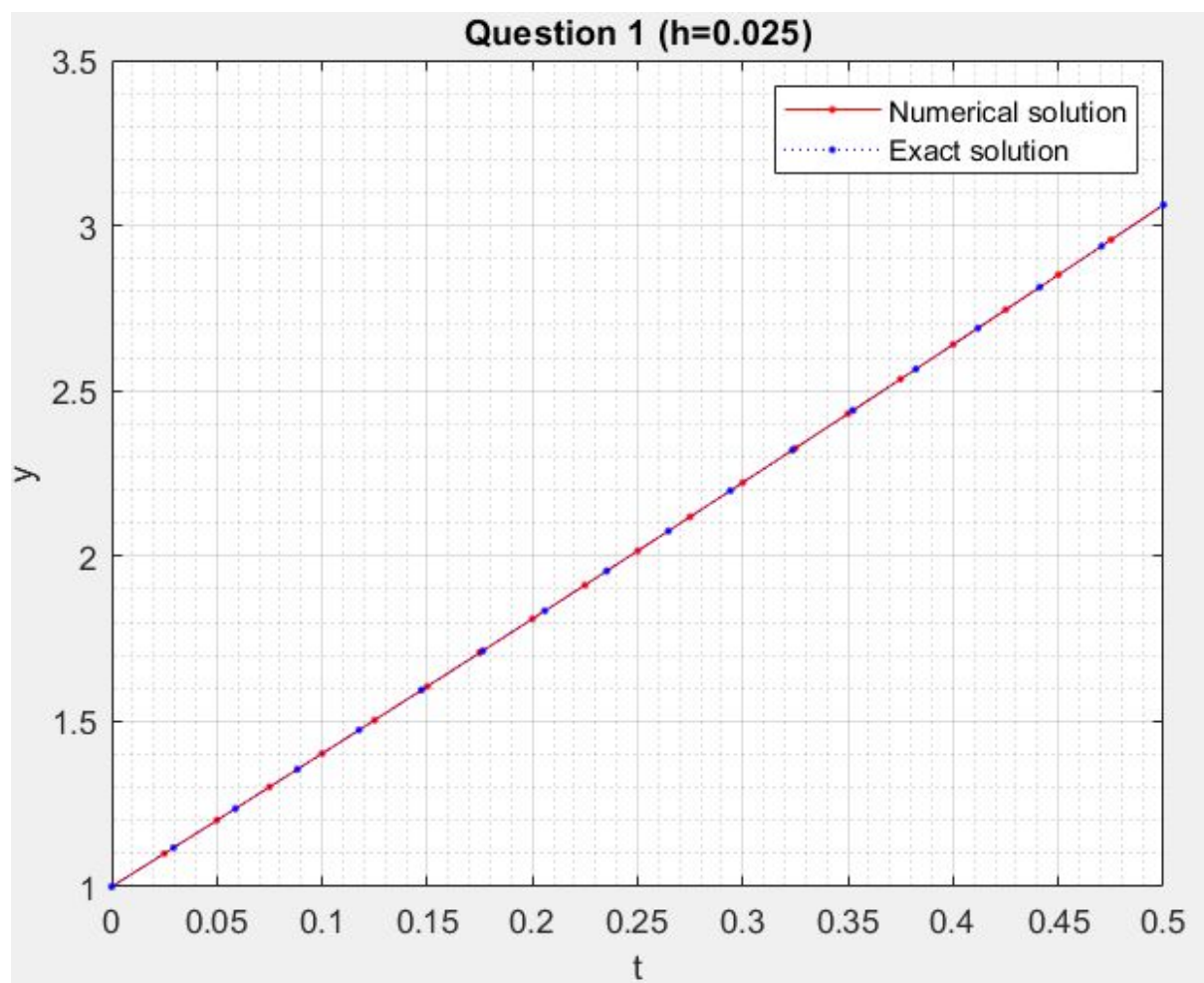
f = @(x,y) (2 + sqrt(y - 2 * x + 3));

for i = 1 : n-1
    k1 = h * f(t(i), Y(i));
    k2 = h * f(t(i) + h / 2, Y(i) + k1 / 2);
    Y(i + 1) = Y(i) + k2;
end
p = @(a) 1 + 4 .* a + (a.^2) ./ 4;

fprintf('Values for h = 0.025\n'); % change to 0.05 for second part
fprintf(' t      y(xn)      yn      e');
for i = 1 : n
    fprintf('\n%4.3f | %4.5f | %4.5f | %4.6f',t(i), Y(i), p(t(i)), abs(Y(i)-p(t(i))));
end
fprintf('\n');
plot(t, Y, 'r.-');
hold on;
fplot(p, [0 0.5], 'b.:');
grid on;
grid minor;
title({'Question 1 (h=0.025)'}); % change to 0.05 for second part
xlabel('t');
ylabel('y');
legend({'Numerical solution', 'Exact solution'});
```

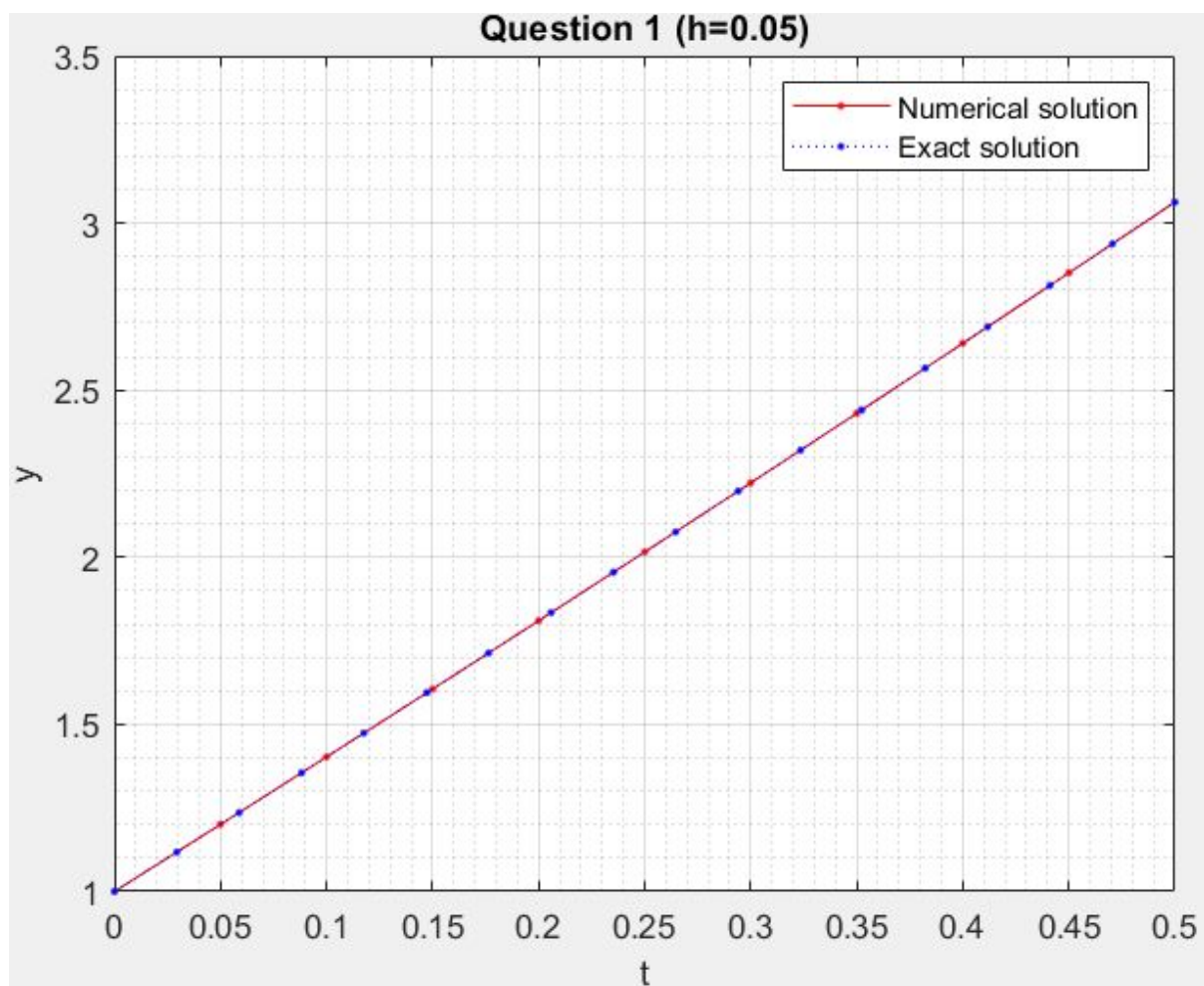
Values for $h = 0.025$

t	yn	y(tn)	e
0.000	1.00000	1.00000	0.000000
0.025	1.10016	1.10016	0.000000
0.050	1.20062	1.20063	0.000000
0.075	1.30141	1.30141	0.000001
0.100	1.40250	1.40250	0.000001
0.125	1.50391	1.50391	0.000001
0.150	1.60562	1.60563	0.000001
0.175	1.70765	1.70766	0.000002
0.200	1.81000	1.81000	0.000002
0.225	1.91265	1.91266	0.000002
0.250	2.01562	2.01563	0.000002
0.275	2.11890	2.11891	0.000003
0.300	2.22250	2.22250	0.000003
0.325	2.32640	2.32641	0.000003
0.350	2.43062	2.43063	0.000003
0.375	2.53515	2.53516	0.000004
0.400	2.64000	2.64000	0.000004
0.425	2.74515	2.74516	0.000004
0.450	2.85062	2.85062	0.000004
0.475	2.95640	2.95641	0.000005
0.500	3.06250	3.06250	0.000005



Values for $h = 0.05$

t	y_n	$y(t_n)$	e
0.000	1.00000	1.00000	0.000000
0.050	1.20062	1.20063	0.000002
0.100	1.40250	1.40250	0.000004
0.150	1.60562	1.60563	0.000006
0.200	1.80999	1.81000	0.000008
0.250	2.01562	2.01563	0.000010
0.300	2.22249	2.22250	0.000012
0.350	2.43061	2.43063	0.000014
0.400	2.63998	2.64000	0.000016
0.450	2.85061	2.85062	0.000017
0.500	3.06248	3.06250	0.000019



Question 2:

CODE:

```
h = 0.05; % change to 0.1 for second part
t = 1 : h : 2;
n = length(t);
Y = zeros(1, n);
Y(1) = 1;

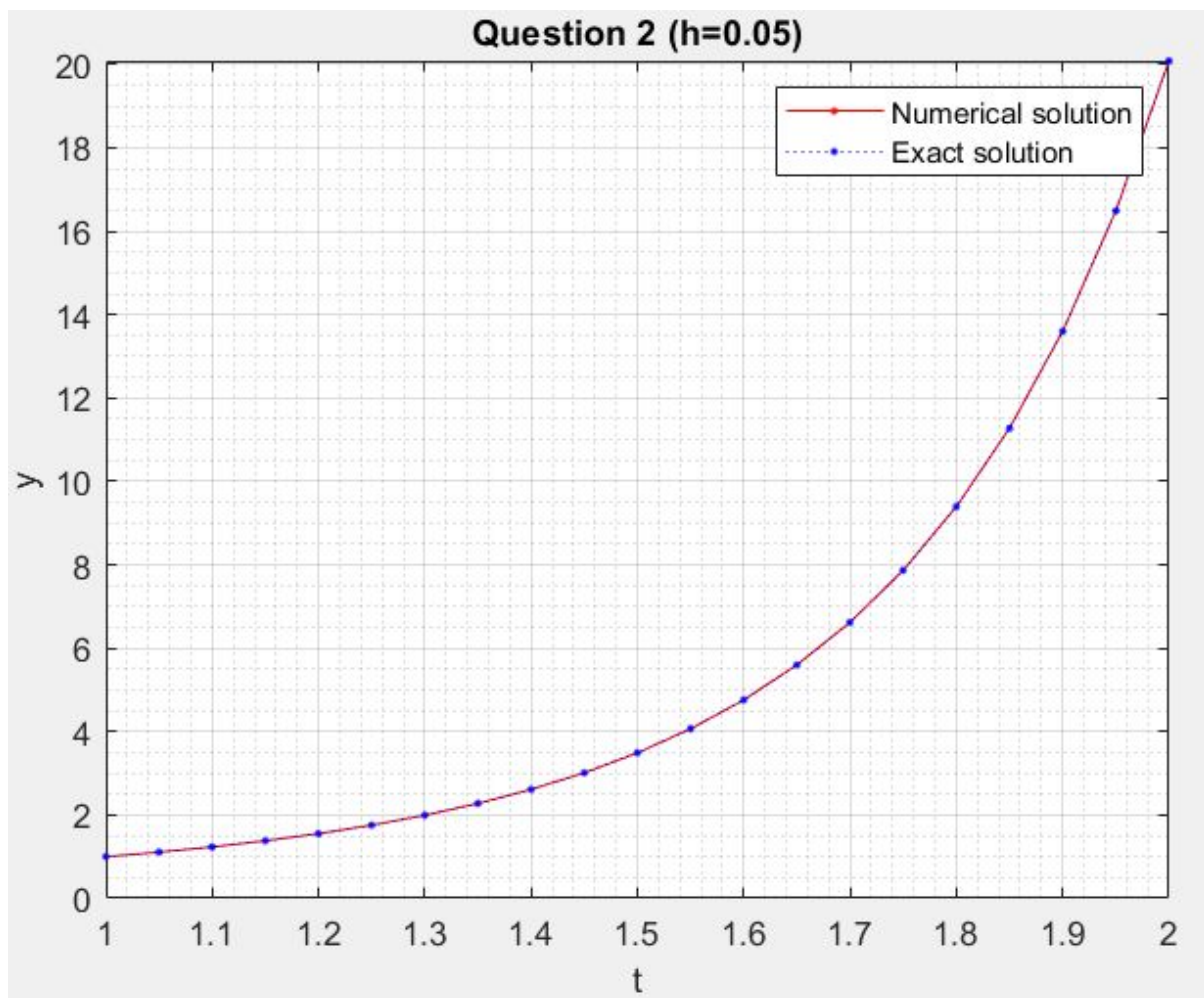
f = @(x, y) (2 * x * y);

for i = 1 : n-1
    k1 = h * f(t(i), Y(i));
    k2 = h * f(t(i) + h / 2, Y(i) + k1 / 2);
    k3 = h * f(t(i) + h / 2, Y(i) + k2 / 2);
    k4 = h * f(t(i) + h, Y(i) + k3);
    k = (1 / 6) * (k1 + 2 * k2 + 2 * k3 + k4);
    Y(i+1) = Y(i) + k;
end

g = @(a) (exp((a.^2) - 1));
fprintf('Values for h = 0.05\n'); % change to 0.1 for second part
fprintf(' t      yn      y(tn)      e');
for i = 1 : n
    fprintf('\n%4.3f | %4.5f | %4.5f | %4.6f', t(i), Y(i), g(t(i)), abs(Y(i)-g(t(i))));
end
fprintf('\n');
plot(t, Y, 'r.-');
hold on;
fplot(g, [1 2], 'b.:');
grid on;
grid minor;
xlabel('t');
ylabel('y');
title({'Question 2 (h=0.05)'}); % change to 0.1 for second part
legend({'Numerical solution', 'Exact solution'});
```

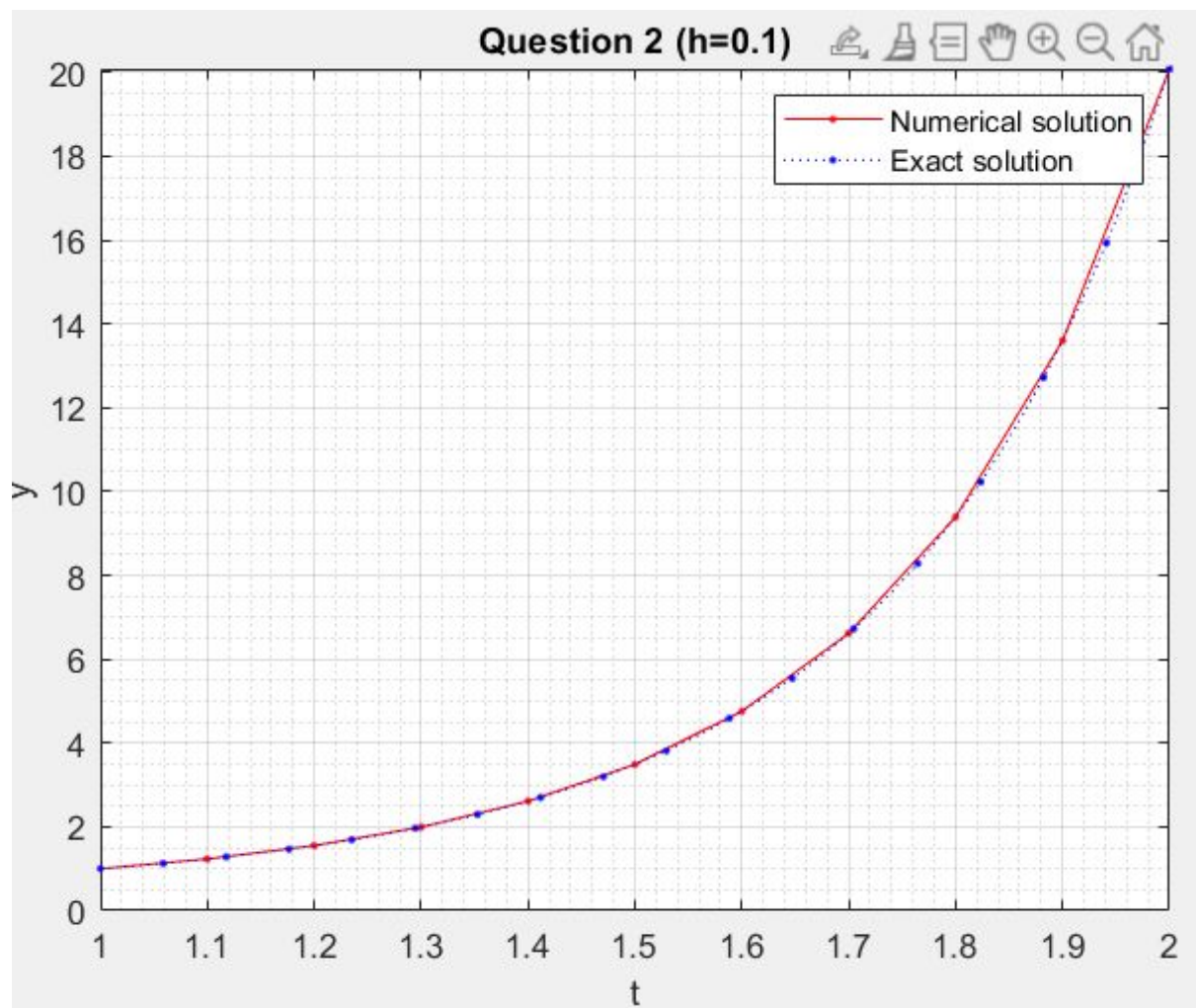

Values for $h = 0.05$

t	y_n	$y(t_n)$	e
1.000	1.00000	1.00000	0.000000
1.050	1.10794	1.10794	0.000000
1.100	1.23368	1.23368	0.000000
1.150	1.38057	1.38057	0.000000
1.200	1.55271	1.55271	0.000001
1.250	1.75505	1.75505	0.000001
1.300	1.99371	1.99372	0.000002
1.350	2.27618	2.27618	0.000003
1.400	2.61169	2.61170	0.000004
1.450	3.01168	3.01169	0.000006
1.500	3.49033	3.49034	0.000009
1.550	4.06534	4.06535	0.000013
1.600	4.75880	4.75882	0.000019
1.650	5.59848	5.59851	0.000027
1.700	6.61933	6.61937	0.000038
1.750	7.86556	7.86561	0.000054
1.800	9.39326	9.39333	0.000076
1.850	11.27390	11.27401	0.000108
1.900	13.59890	13.59905	0.000153
1.950	16.48559	16.48581	0.000216
2.000	20.08523	20.08554	0.000306



Values for $h = 0.1$

t	y_n	$y(t_n)$	e
1.000	1.00000	1.00000	0.000000
1.100	1.23367	1.23368	0.000004
1.200	1.55270	1.55271	0.000012
1.300	1.99369	1.99372	0.000029
1.400	2.61163	2.61170	0.000063
1.500	3.49021	3.49034	0.000132
1.600	4.75855	4.75882	0.000270
1.700	6.61883	6.61937	0.000541
1.800	9.39225	9.39333	0.001079
1.900	13.59691	13.59905	0.002145
2.000	20.08127	20.08554	0.004270



Question 3:

CODE:

```
h = 0.025; % change h to 0.05 for second part
t = 1:h:1.5;
n = length(t);
Y = zeros(1,n);
E = zeros(1,n);
Y(1) = 2;
f = @(x, y) (((y.^2) + x .* y - (x.^2)) ./ (x.^2));

for i = 1 : n - 1
    k = [0,0];
    s = fsolve(@(x) [x(1) - f(t(i) + (3 - sqrt(3)) * h / 6, Y(i) + h * x(1) / 4 + (3 - 2 * sqrt(3)) * h * x(2) / 12); x(2) - f(t(i) + (3 + sqrt(3)) * h / 6, Y(i) + h * x(2) / 4 + (3 + 2 * sqrt(3)) * h * x(1) / 12)], k);
    Y(i + 1) = Y(i) + h * (s(1) + s(2)) / 2;
end

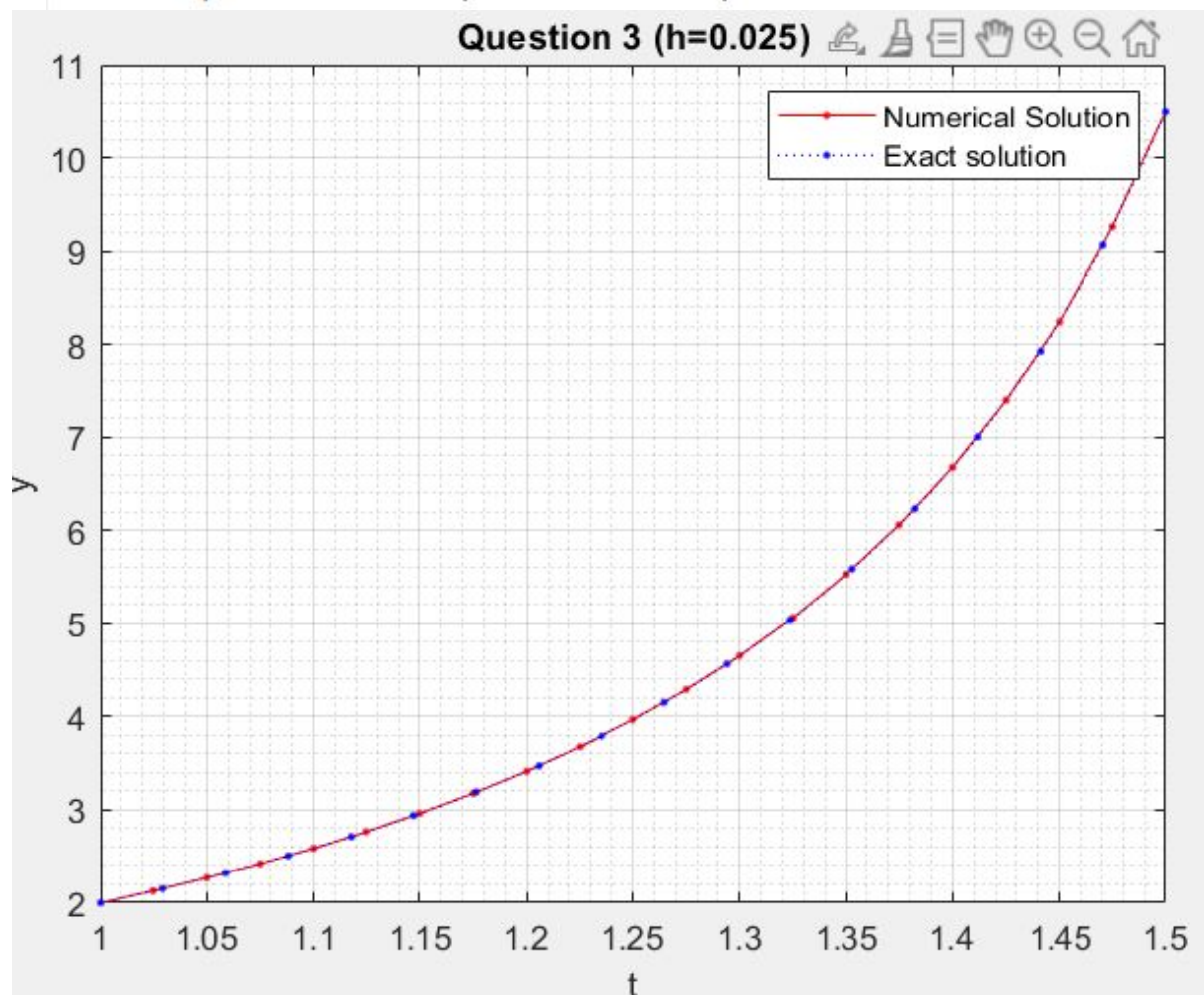
for i = 1 : n
    x = 1 + (i - 1) * h;
    E(i) = (x * (1 + (x.^2) / 3) / (1 - (x.^2) / 3)) - Y(i);
end

g = @(x) (x .* (1 + (x.^2) / 3) ./ (1 - (x.^2) ./ 3));
fprintf('Values for h = 0.025\n'); % change to 0.05 for second part
fprintf(' t      yn      y(tn)      e');
for i = 1 : n
    fprintf('\n%4.3f | %4.5f | %4.5f | %4.6f',t(i), Y(i), g(t(i)), abs(Y(i)-g(t(i))));
end
fprintf('\n');

plot(t, Y, 'r.-');
hold on;
fplot(g, [1 1.5], 'b.:');
hold on;
grid on;
grid minor;
xlabel('t');
ylabel('y');
title({'Question 3 (h=0.025)'}); % change h to 0.05 for second part
legend({'Numerical Solution', 'Exact solution'});
```

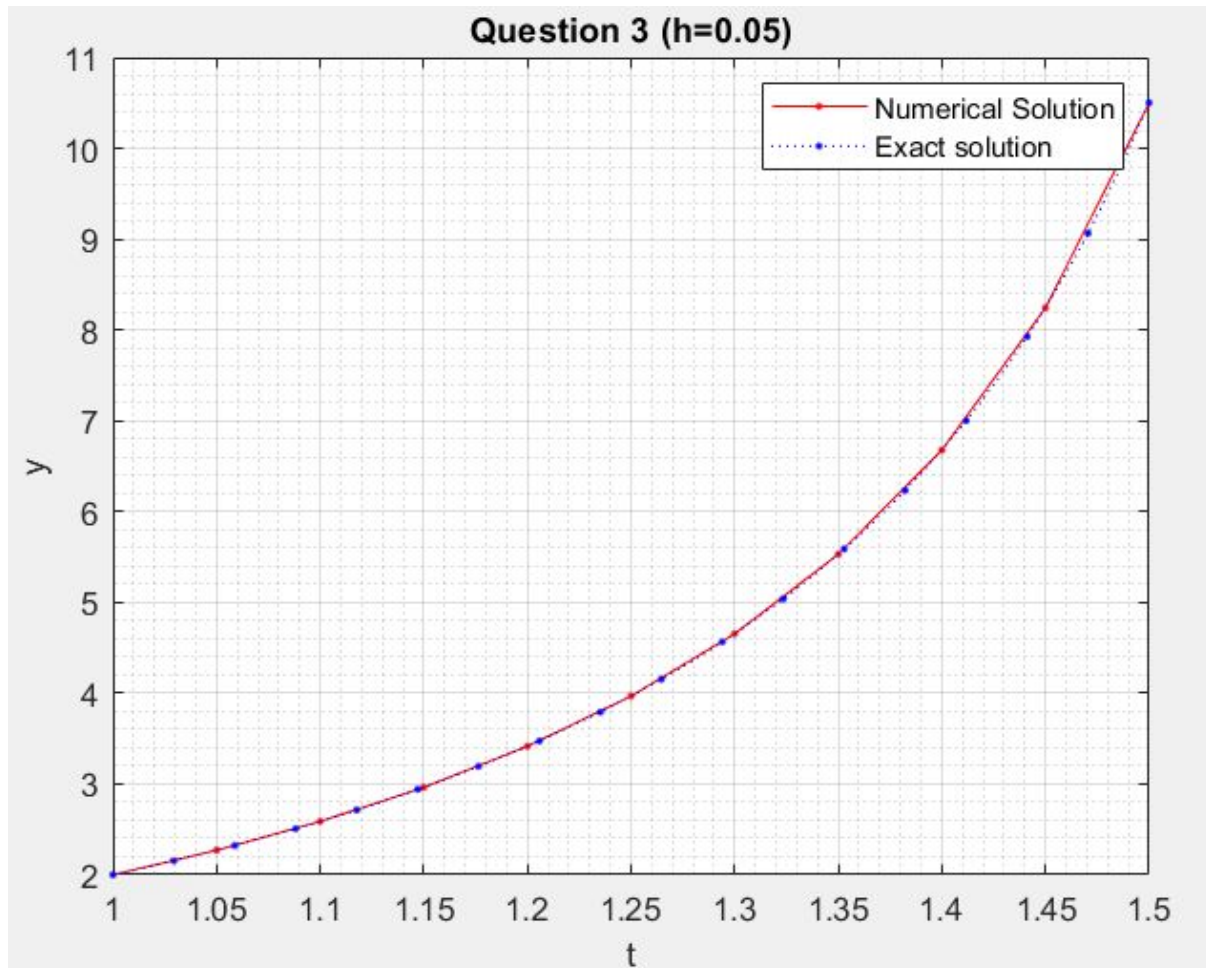

Values for $h = 0.025$

t	y_n	$y(t_n)$	e
1.000	2.00000	2.00000	0.000000
1.025	2.12986	2.12986	0.000000
1.050	2.27016	2.27016	0.000000
1.075	2.42212	2.42212	0.000000
1.100	2.58715	2.58715	0.000000
1.125	2.76689	2.76689	0.000000
1.150	2.96326	2.96326	0.000000
1.175	3.17853	3.17853	0.000000
1.200	3.41538	3.41538	0.000000
1.225	3.67704	3.67704	0.000001
1.250	3.96739	3.96739	0.000001
1.275	4.29117	4.29117	0.000001
1.300	4.65420	4.65420	0.000001
1.325	5.06375	5.06375	0.000002
1.350	5.52898	5.52898	0.000002
1.375	6.06162	6.06162	0.000003
1.400	6.67692	6.67692	0.000004
1.425	7.39511	7.39512	0.000005
1.450	8.24359	8.24359	0.000007
1.475	9.26040	9.26041	0.000010
1.500	10.49998	10.50000	0.000015



Values for $h = 0.05$

t	y_n	$y(t_n)$	e
1.000	2.00000	2.00000	0.000000
1.050	2.27016	2.27016	0.000001
1.100	2.58715	2.58715	0.000002
1.150	2.96326	2.96326	0.000004
1.200	3.41538	3.41538	0.000007
1.250	3.96738	3.96739	0.000012
1.300	4.65418	4.65420	0.000021
1.350	5.52894	5.52898	0.000036
1.400	6.67686	6.67692	0.000063
1.450	8.24348	8.24359	0.000118
1.500	10.49976	10.50000	0.000241



Question 4:

$h = 0.05$; **% change to 0.1 for second part**

$t = 0 : h : 1$;

$n = \text{length}(t)$;

$X = \text{zeros}(1, n)$;

$Y = \text{zeros}(1, n)$;

$X(1) = 1$;

$Y(1) = 2$;

$f = @(x, y) ((-3) * x + 4 * y)$;

$g = @(x, y) ((-2) * x + 3 * y)$;

for $i = 1 : n - 1$

$k1 = h * f(X(i), Y(i))$;

$l1 = h * g(X(i), Y(i))$;

$k2 = h * f(X(i) + k1 / 2, Y(i) + l1 / 2)$;

$l2 = h * g(X(i) + k1 / 2, Y(i) + l1 / 2)$;

$k3 = h * f(X(i) + k2 / 2, Y(i) + l2 / 2)$;

$l3 = h * g(X(i) + k2 / 2, Y(i) + l2 / 2)$;

$k4 = h * f(X(i) + k3, Y(i) + l3)$;

$l4 = h * g(X(i) + k3, Y(i) + l3)$;

$X(i+1) = X(i) + ((1 / 6) * (k1 + 2 * k2 + 2 * k3 + k4))$;

$Y(i+1) = Y(i) + ((1 / 6) * (l1 + 2 * l2 + 2 * l3 + l4))$;

end

$J = \text{zeros}(1, n)$;

$M = \text{zeros}(1, n)$;

$b = @(a) (3 * \exp(a) - 2 * \exp(-a))$;

$c = @(a) (3 * \exp(a) - \exp(-a))$;

for $i = 1 : n$

$J(i) = b(t(i))$;

$M(i) = c(t(i))$;

end

$\text{fprintf}('Values for h = 0.05 \backslash n')$; **% change to 0.1 for second part**

$\text{fprintf}(' \quad t \quad \quad x_n \quad \quad x(t_n) \quad \quad e \quad \quad y_n \quad \quad y(t_n) \quad \quad e')$;

for $i = 1 : n$

$\text{fprintf}('\backslash n \%4.3f \mid \%4.5f \mid \%4.5f \mid \%4.6f \mid \%4.5f \mid \%4.5f \mid \%4.6f', t(i), X(i), J(i), \text{abs}(X(i) - J(i)), Y(i), M(i), \text{abs}(Y(i) - M(i))))$;

end

$\text{fprintf}('\backslash n')$;

$\text{plot}(t, X, 'r.-', t, J, 'g.-', t, Y, 'b.-', t, M, 'm.-')$;

hold on;

grid on;

grid minor;

$\text{xlabel}('t')$;

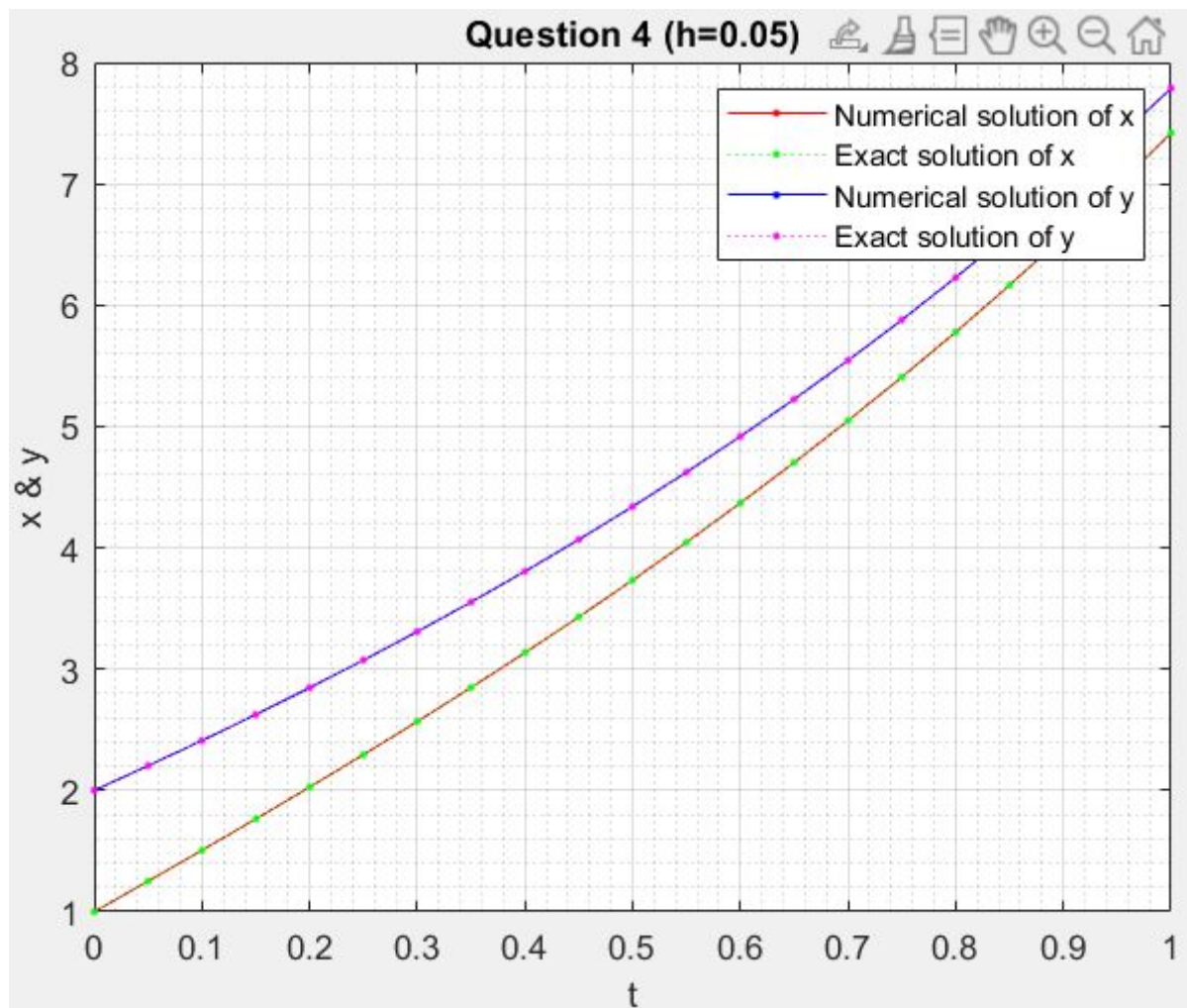
$\text{ylabel}('x \& y')$;

$\text{title}('Question 4 (h=0.05)')$; **% change to 0.1 for second part**

$\text{legend}('Numerical solution of x', 'Exact solution of x', 'Numerical solution of y', 'Exact solution of y')$;

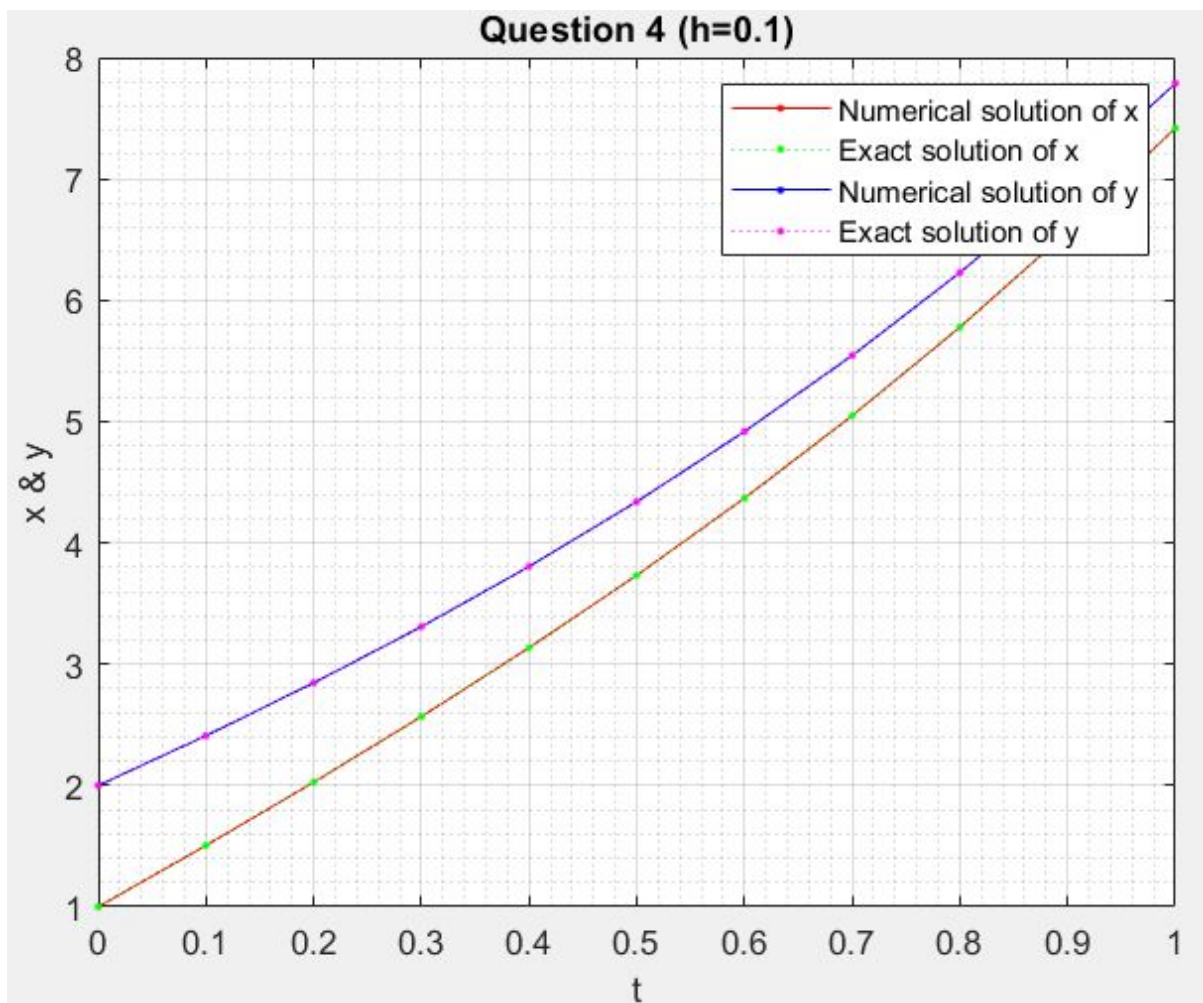
Values for $h = 0.05$

t	xn	x(tn)	e	yn	y(tn)	e
0.000	1.00000	1.00000	0.000000	2.00000	2.00000	0.000000
0.050	1.25135	1.25135	0.000000	2.20258	2.20258	0.000000
0.100	1.50584	1.50584	0.000000	2.41068	2.41068	0.000000
0.150	1.76409	1.76409	0.000000	2.62479	2.62479	0.000000
0.200	2.02675	2.02675	0.000000	2.84548	2.84548	0.000000
0.250	2.29447	2.29447	0.000000	3.07328	3.07328	0.000000
0.300	2.56794	2.56794	0.000000	3.30876	3.30876	0.000000
0.350	2.84783	2.84783	0.000000	3.55251	3.55251	0.000000
0.400	3.13483	3.13483	0.000000	3.80515	3.80515	0.000000
0.450	3.42968	3.42968	0.000000	4.06731	4.06731	0.000000
0.500	3.73310	3.73310	0.000000	4.33963	4.33963	0.000000
0.550	4.04586	4.04586	0.000000	4.62281	4.62281	0.000000
0.600	4.36873	4.36873	0.000000	4.91754	4.91754	0.000000
0.650	4.70253	4.70253	0.000000	5.22458	5.22458	0.000000
0.700	5.04809	5.04809	0.000000	5.54467	5.54467	0.000000
0.750	5.40627	5.40627	0.000000	5.87863	5.87863	0.000000
0.800	5.77796	5.77796	0.000000	6.22729	6.22729	0.000000
0.850	6.16411	6.16411	0.000000	6.59153	6.59153	0.000000
0.900	6.56567	6.56567	0.000000	6.97224	6.97224	0.000000
0.950	6.98365	6.98365	0.000000	7.37039	7.37039	0.000000
1.000	7.41909	7.41909	0.000000	7.78697	7.78697	0.000000



Values for $h = 0.1$

t	xn	x(tn)	e	yn	y(tn)	e
0.000	1.00000	1.00000	0.000000	2.00000	2.00000	0.000000
0.100	1.50584	1.50584	0.000000	2.41067	2.41068	0.000000
0.200	2.02675	2.02675	0.000001	2.84548	2.84548	0.000001
0.300	2.56794	2.56794	0.000001	3.30876	3.30876	0.000001
0.400	3.13483	3.13483	0.000002	3.80515	3.80515	0.000002
0.500	3.73310	3.73310	0.000002	4.33963	4.33963	0.000002
0.600	4.36873	4.36873	0.000003	4.91754	4.91754	0.000003
0.700	5.04808	5.04809	0.000004	5.54467	5.54467	0.000004
0.800	5.77796	5.77796	0.000005	6.22729	6.22729	0.000004
0.900	6.56566	6.56567	0.000006	6.97223	6.97224	0.000005
1.000	7.41908	7.41909	0.000007	7.78696	7.78697	0.000007



Question 5:

CODE:

```
h = 0.05; % change h to 0.1 for second part
x = 0 : h : 1;
n = length(x);
u = zeros(1, n);
y = zeros(1, n);
u(1) = 1;
y(1) = f(x(1), u(1));

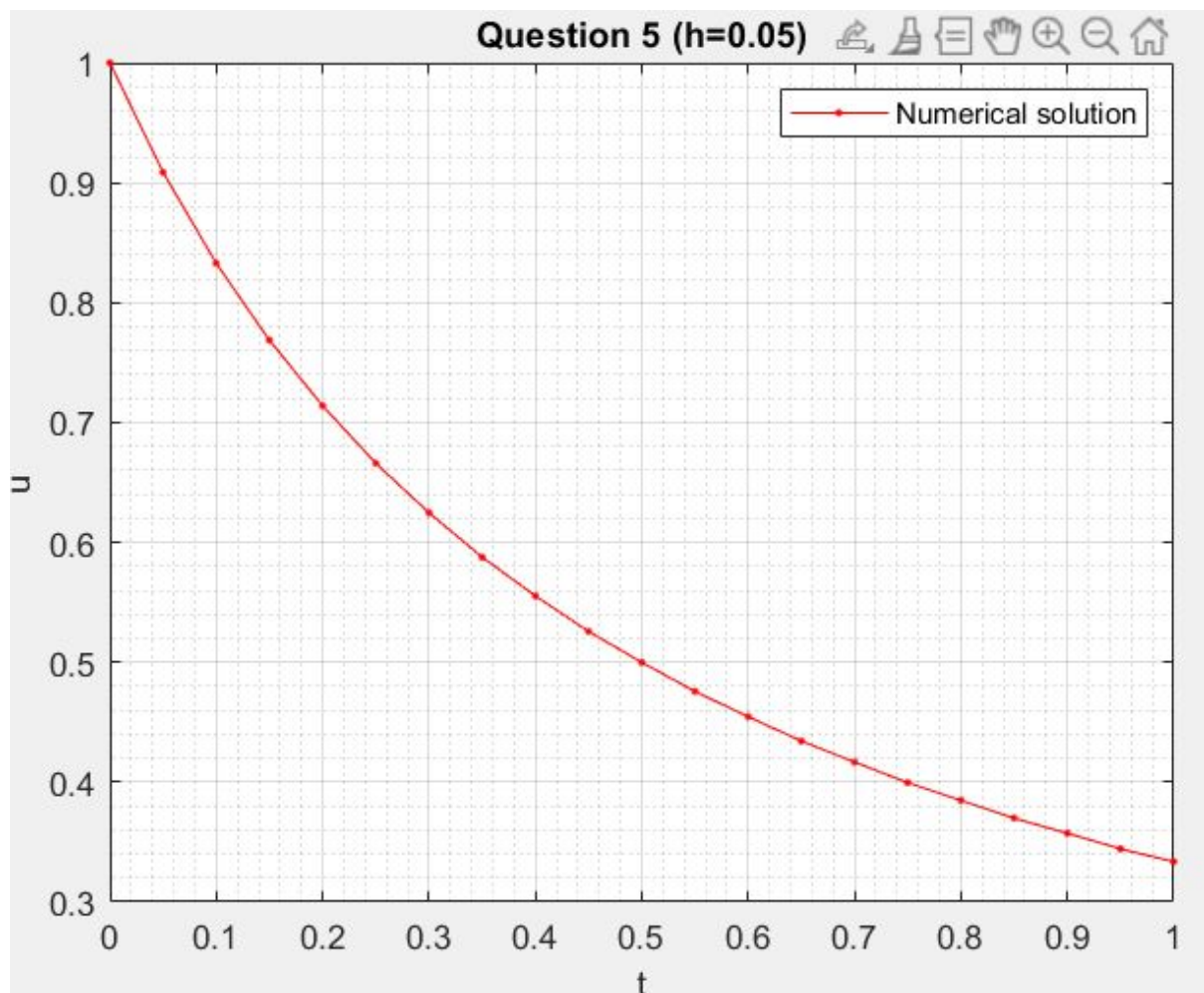
f = @(t, u) (-2 * u * u);
g1 = @(u1, u0, h) (h * u1 * u1 + u1 + h * u0 * u0 - u0);
g2 = @(u, h) (2 * h * u + 1);

for i = 1 : 4
    Up = u(i);
    while 1
        u(i + 1) = Up - g1(Up, u(i), h) / g2(Up, h);
        if (abs(u(i + 1) - Up) <= 0.0001)
            break;
        end
        Up = u(i + 1);
    end
    y(i + 1) = f(x(i + 1), u(i + 1));
end

for i = 4 : n - 1
    u(i + 1) = u(i - 3) + 4 * h * (2 * y(i) - y(i - 1) + 2 * y(i - 2)) / 3;
    y(i + 1) = f(x(i + 1), u(i + 1));
    u(i + 1) = u(i - 1) + h * (y(i + 1) + 4 * y(i) + y(i - 1)) / 3;
end
fprintf('Values for h = 0.05\n'); % change to 0.1 for second part
fprintf(' t      un');
for i = 1 : n
    fprintf('\n%4.3f | %4.5f', x(i), u(i));
end
fprintf('\n');
plot(x, u, 'r.-');
title('Question 5 (h=0.05)'); % change h to 0.1 for second part
grid on;
grid minor;
xlabel('t');
ylabel('u');
legend({'Numerical solution'});
```

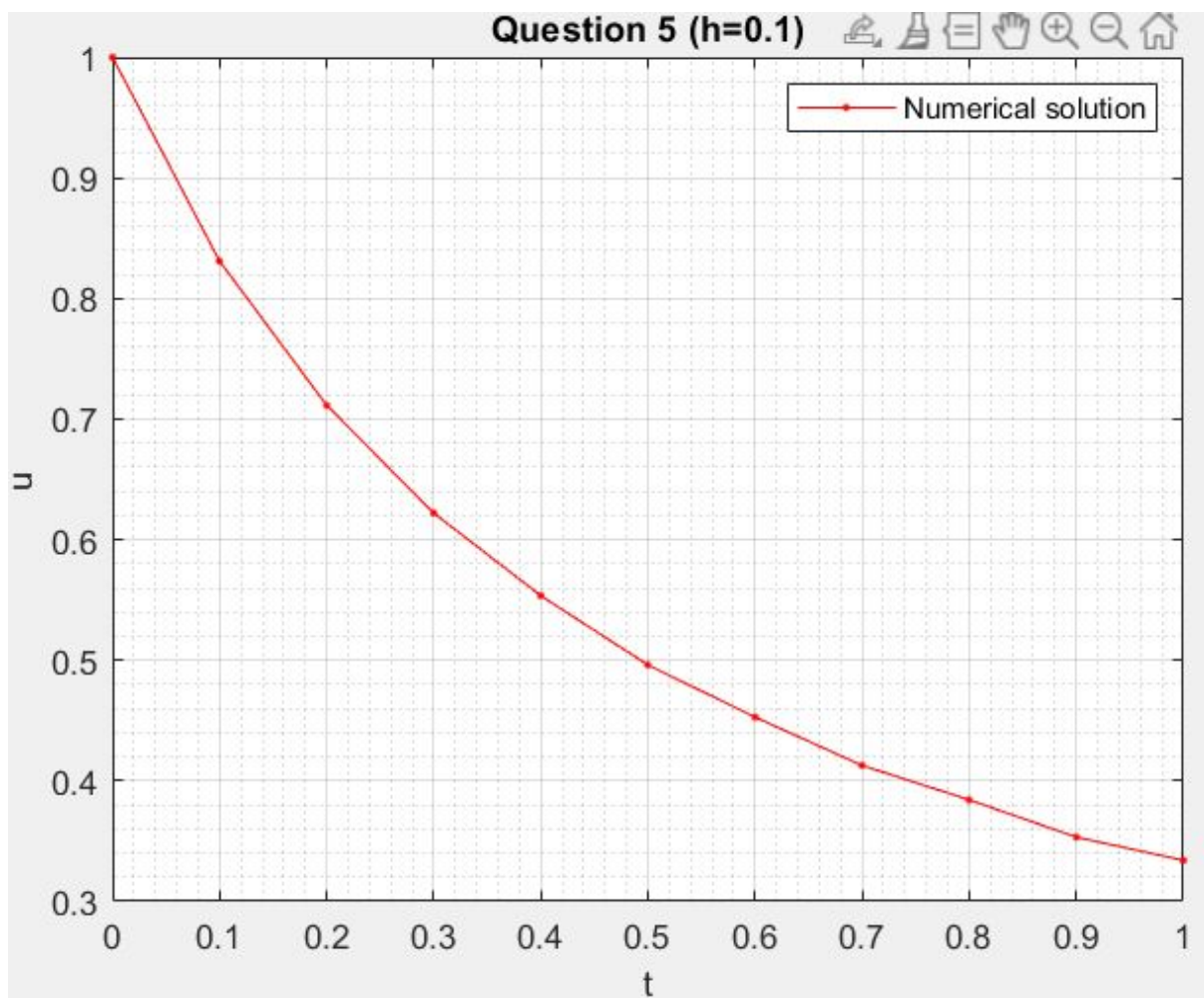
Values for $h = 0.05$

t	un
0.000	1.00000
0.050	0.90871
0.100	0.83275
0.150	0.76854
0.200	0.71385
0.250	0.66594
0.300	0.62461
0.350	0.58756
0.400	0.55530
0.450	0.52571
0.500	0.49986
0.550	0.47562
0.600	0.45450
0.650	0.43424
0.700	0.41670
0.750	0.39947
0.800	0.38471
0.850	0.36984
0.900	0.35729
0.950	0.34430
1.000	0.33354



Values for $h = 0.1$

t	un
0.000	1.00000
0.100	0.83095
0.200	0.71131
0.300	0.62202
0.400	0.55347
0.500	0.49605
0.600	0.45279
0.700	0.41274
0.800	0.38420
0.900	0.35327
1.000	0.33404



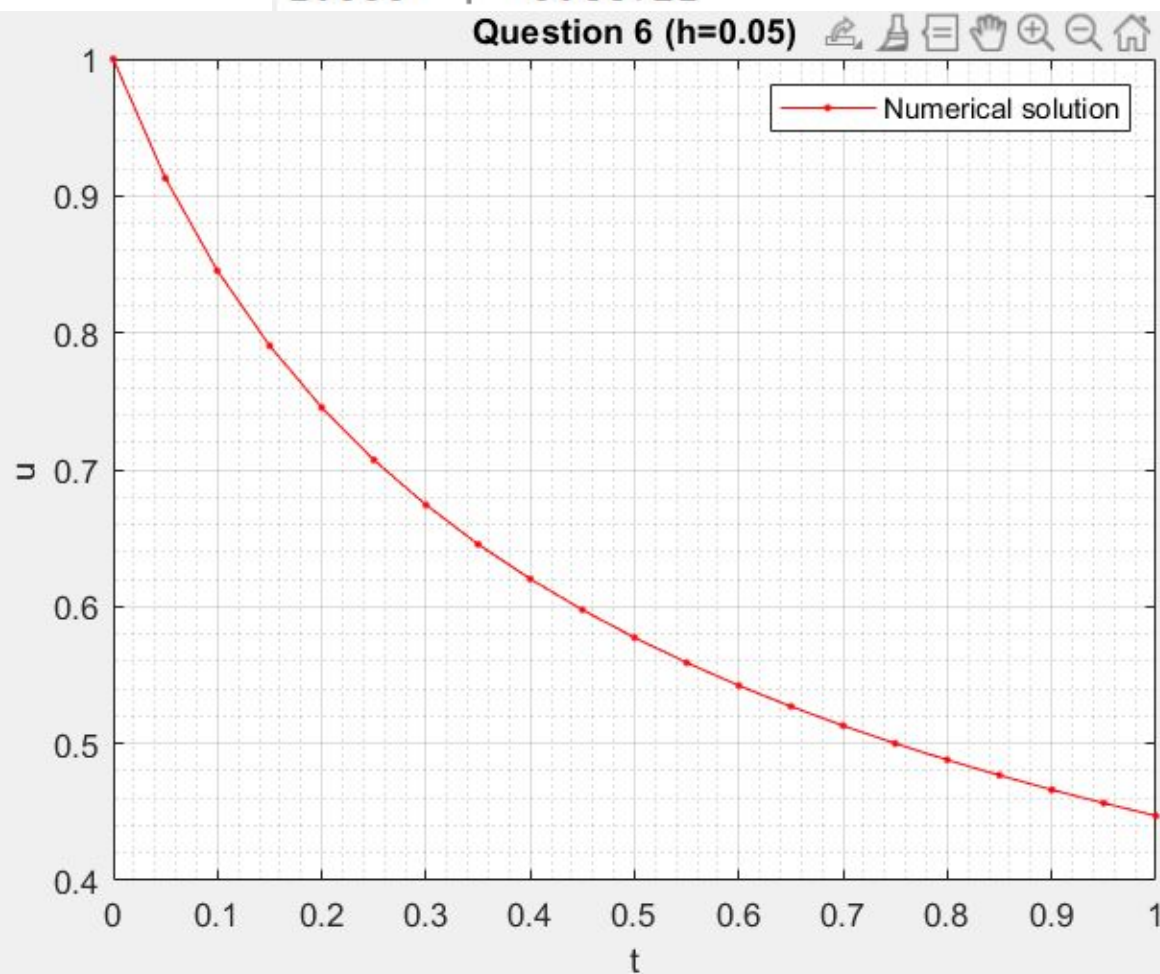
Question 6:

CODE:

```
h = 0.05; % change h to 0.1 for second part
x = 0 : h : 1;
n = length(x);
f = @(x, u) (-2 * u^3);
u = zeros(1, n);
y = zeros(1, n);
u(1) = 1;
y(1) = f(x(1), u(1));
for i = 1 : 4
    k1 = h * f(x(i), u(i));
    k2 = h * f(x(i) + h / 2, u(i) + k1 / 2);
    k3 = h * f(x(i) + h / 2, u(i) + k2 / 2);
    k4 = h * f(x(i) + h, u(i) + k3);
    k = (1 / 6) * (k1 + 2 * k2 + 2 * k3 + k4);
    u(i + 1) = u(i) + k;
    y(i + 1) = f(x(i + 1), u(i + 1));
end
for i = 4 : n-1
    u(i + 1) = u(i - 3) + 4 * h * (2 * y(i) - y(i - 1) + 2 * y(i - 2)) / 3;
    k = u(i + 1);
    y(i + 1) = f(x(i + 1), u(i + 1));
    u(i + 1) = u(i - 1) + h * (y(i + 1) + 4 * y(i) + y(i - 1)) / 3;
    y(i + 1) = f(x(i + 1), u(i + 1));
    while abs(u(i + 1) - k) > 0.0001
        k = u(i + 1);
        u(i + 1) = u(i - 1) + h * (y(i + 1) + 4 * y(i) + y(i - 1)) / 3;
        y(i + 1) = f(x(i + 1), u(i + 1));
    end
end
fprintf('Values for h = 0.05\n'); % change to 0.1 for second part
fprintf(' t      u\n');
for i = 1 : n
    fprintf('\n%4.3f | %4.5f', x(i), u(i));
end
fprintf('\n');
plot(x, u, 'r.-');
grid on;
grid minor;
xlabel('t');
ylabel('u');
title(['Question 6 (h=0.05)']); % change h to 0.1 for second part
legend(['Numerical solution']);
```


Values for $h = 0.05$

t	un
0.000	1.00000
0.050	0.91287
0.100	0.84515
0.150	0.79057
0.200	0.74535
0.250	0.70710
0.300	0.67419
0.350	0.64549
0.400	0.62017
0.450	0.59761
0.500	0.57734
0.550	0.55901
0.600	0.54232
0.650	0.52704
0.700	0.51298
0.750	0.50000
0.800	0.48794
0.850	0.47673
0.900	0.46625
0.950	0.45643
1.000	0.44721



Values for $h = 0.1$

t	un
0.000	1.00000
0.100	0.84514
0.200	0.74535
0.300	0.67419
0.400	0.62012
0.500	0.57734
0.600	0.54228
0.700	0.51299
0.800	0.48791
0.900	0.46626
1.000	0.44717

