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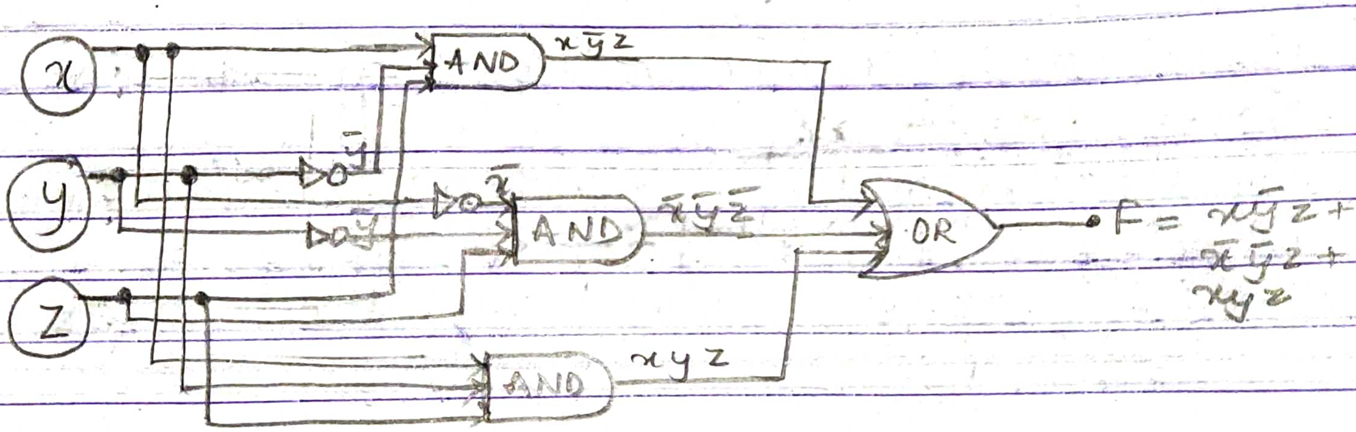
Assignment 2 (Discrete Maths)  19MA20059

1) a) $F = x\bar{y}z + \bar{x}\bar{y}z + xyz$

| $x$ | $y$ | $z$ | $\bar{x}$ | $\bar{y}$ | $\bar{z}$ | $x\bar{y}z$ | $\bar{x}\bar{y}z$ | $xyz$ | F |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |

b)




c) $F = x\bar{y}z + \bar{x}\bar{y}z + xyz + x\bar{y}z$ (Identity Law)

$= \bar{y}z(x+\bar{x}) + xyz + x\bar{y}z$ (Distributive Law)

$= \bar{y}z(1) + xyz + x\bar{y}z$ (Unit property)

$= \bar{y}z + xyz + x\bar{y}z$

$= \bar{y}z + xz(y+\bar{y})$ (Distributive Law)

$= \bar{y}z + xz$ (Unit property)

$\boxed{\therefore F = \bar{y}z + xz}$

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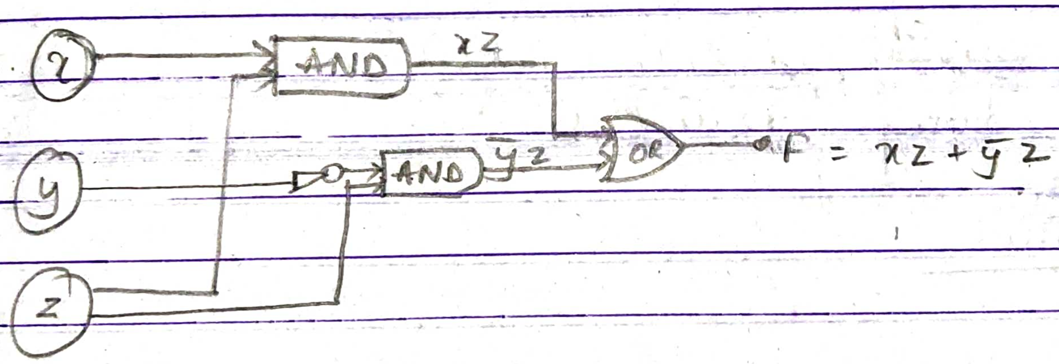
d)

| $x$ | $y$ | $z$ | $\bar{y}$ | $xz$ | $\bar{y}z$ | F |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |

The F column resembles the F column in this tables in (a)
Hence, $F = xz + \bar{y}z$ is a valid simplified eq$^{\underline{n}}$.

e) 


For part (b) : • 3 AND Gates • 3 NOT gates • 1 OR Gate → total 7 gates

For part (e) : • 2 AND gates • 1 NOT gate • 1 OR gate → total 4 gates

2)

$F = A\bar{C} + \bar{B}D + \bar{A}CD + ABCD$

$F = A\bar{C} + \bar{B}D + \bar{A}CD(B+\bar{B}) + ABCD$

$\quad + \bar{A}CDB + \bar{A}\bar{B}CD + ABCD$

$F = A\bar{C}(B+\bar{B}) + \bar{B}D(A+\bar{A}) + \bar{A}CDB + \bar{A}\bar{B}CD + ABCD$

$F = AB\bar{C} + A\bar{B}\bar{C} + A\bar{B}D + \bar{A}\bar{B}D + \bar{A}CDB + \bar{A}\bar{B}CD + ABCD$

$\quad + A\bar{B}D(C+\bar{C}) + \bar{A}\bar{B}D(C+\bar{C})$

$F = AB\bar{C}(D+\bar{D}) + A\bar{B}\bar{C}(D+\bar{D})$

$\quad + \bar{A}CDB + \bar{A}\bar{B}CD + ABCD$

$F = AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}\bar{C}D +$

$\bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}D + \bar{A}BCD + \bar{A}\bar{B}CD + ABCD$

$F = ABCD + \bar{A}BCD + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}D + A\bar{B}CD + A\bar{B}\bar{C}D +$

$A\bar{B}\bar{C}\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D}$

$F(A,B,C,D) = \Sigma(1,3,7,8,9,11,12,13,15)$

(sum of product form)

We know that

$\Sigma(1,3,7,8,9,11,12,13,15) = F = \Pi(0,2,4,5,6,10,14)$

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2) a) simplification sum of product ~~of~~ ~~sum~~ form

| AB \ CD | 00 ($\bar{C}\bar{D}$) | 01 ($\bar{C}D$) | 11 ($CD$) | 10 ($C\bar{D}$) |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ 00 | 0 | 1 1 | 1 3 | 2 |
| $\bar{A}B$ 01 | 4 | 5 | 1 7 | 6 |
| $AB$ 11 | 1 12 | 1 13 | 1 15 | 14 |
| $A\bar{B}$ 10 | 1 8 | 1 9 | 1 11 | 10 |

$F(A,B,C,D) = \Sigma(1,3,7,8,9,11,12,13,15)$

$F(A,B,C,\cancel{d}D) = A\bar{C} + \cancel{A}\bar{B}D + CD$

2) b) simplification of ~~sum~~ ~~of~~ product of sum form

| AB \ CD | 00 ($C+D$) | 01 ($C+\bar{D}$) | 11 ($\bar{C}+\bar{D}$) | 10 ($\bar{C}+D$) |
|---|---|---|---|---|
| $A+B$ 00 | 0 0 | 1 | 3 | 0 2 |
| $A+\bar{B}$ 01 | 0 4 | 0 5 | 7 | 0 6 |
| $\bar{A}+\bar{B}$ 11 | 12 | 13 | 15 | 0 14 |
| $\bar{A}+B$ 10 | 8 | 9 | 11 | 0 10 |

$F(A,B,C,D) = \pi(0,2,4,5,6,10,14)$

$F(A,B,C,D) = (A+\cancel{CD}D) \cdot (A+\bar{B}+C) \cdot (\bar{C}+D)$

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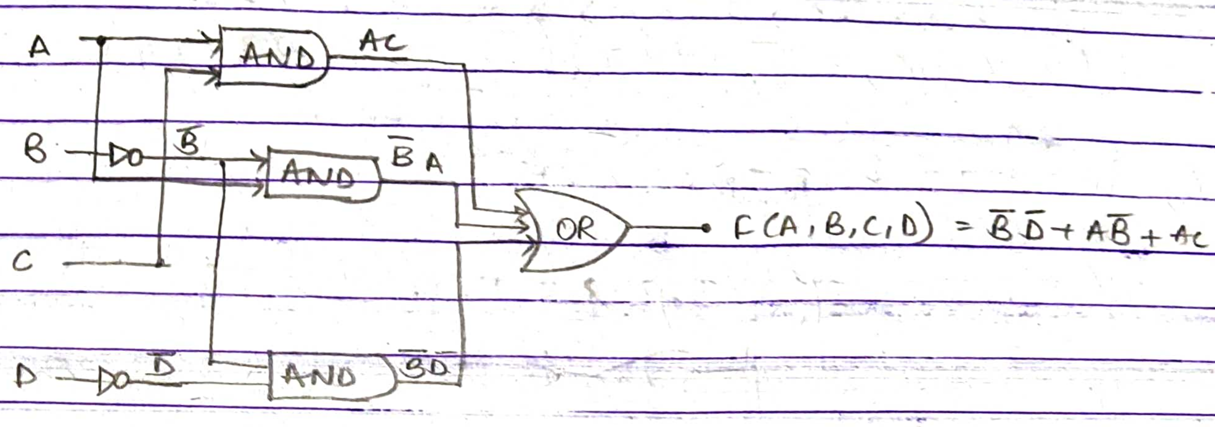
3) $F(A,B,C,D) = \Sigma(0,2,8,9,10,11,14,15)$

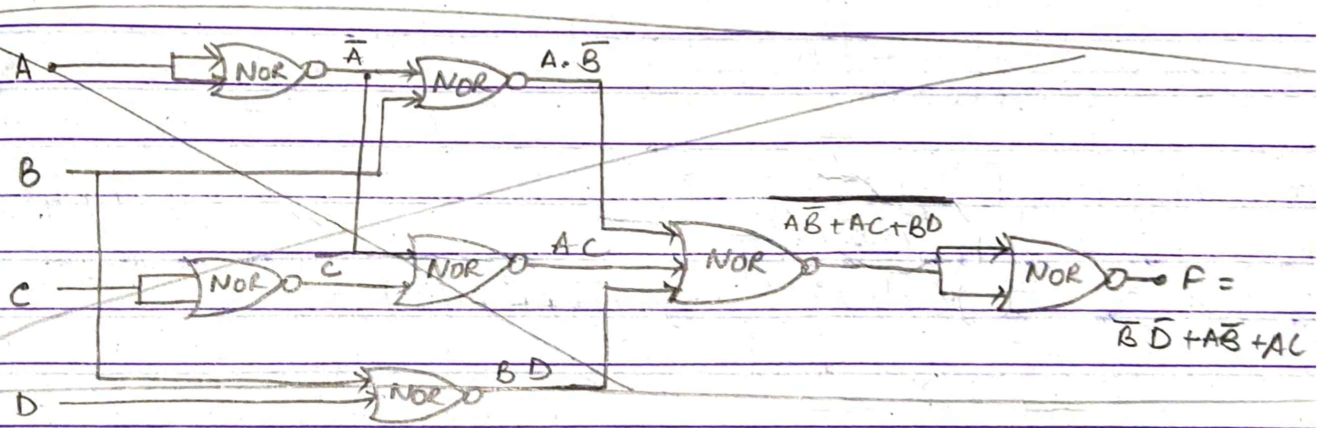
| AB \ CD | $\bar{C}\bar{D}$ 00 | $\bar{C}D$ 01 | $CD$ 11 | $C\bar{D}$ 10 |
|---|---|---|---|---|
| $\bar{A}\bar{B}$ 00 | $1_{0}$ | $_{1}$ | $_{3}$ | $1_{2}$ |
| $\bar{A}B$ 01 | $_{4}$ | $_{5}$ | $_{7}$ | $_{6}$ |
| $AB$ 11 | $_{12}$ | $_{13}$ | $1_{15}$ | $1_{14}$ |
| $A\bar{B}$ 10 | $1_{8}$ | $1_{9}$ | $1_{11}$ | $1_{10}$ |

$F(A,B,C,D) = \bar{B}\bar{D} + A\bar{B} + AC$

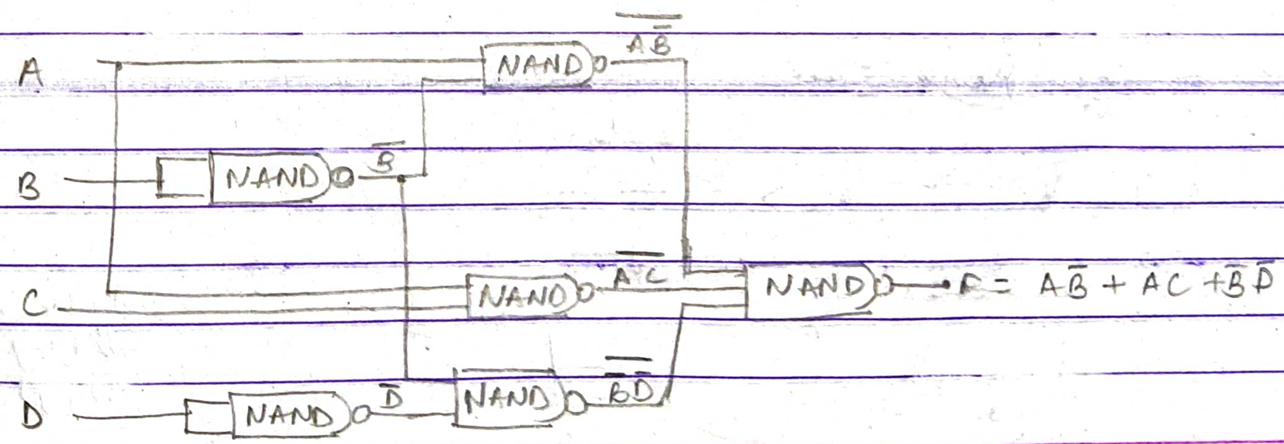
a)




b)




b)



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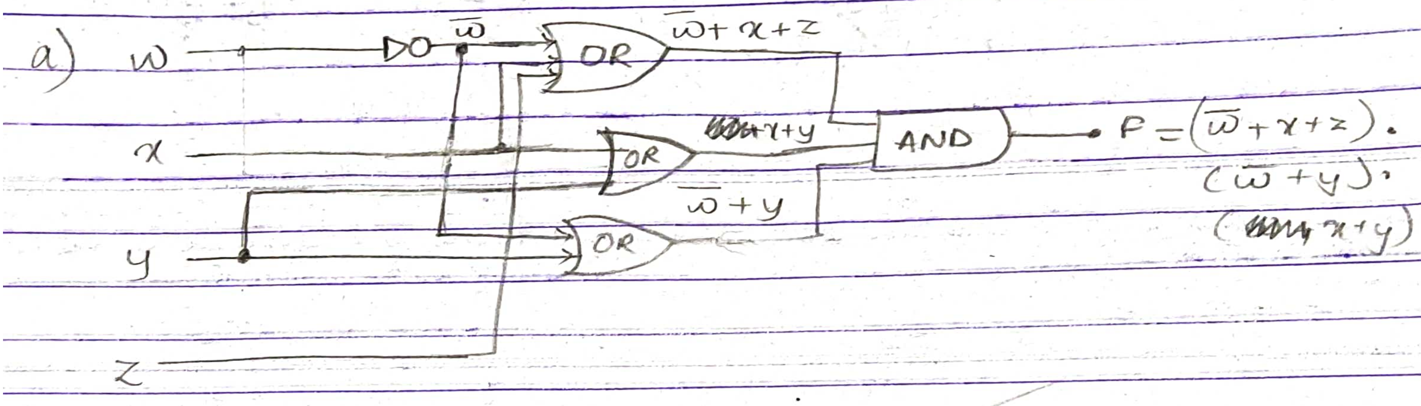
4) $F(w,x,y,z) = \Sigma(2,3,4,5,6,7,11,14,15)$
$= \pi(0,1,8,9,10,12,13)$

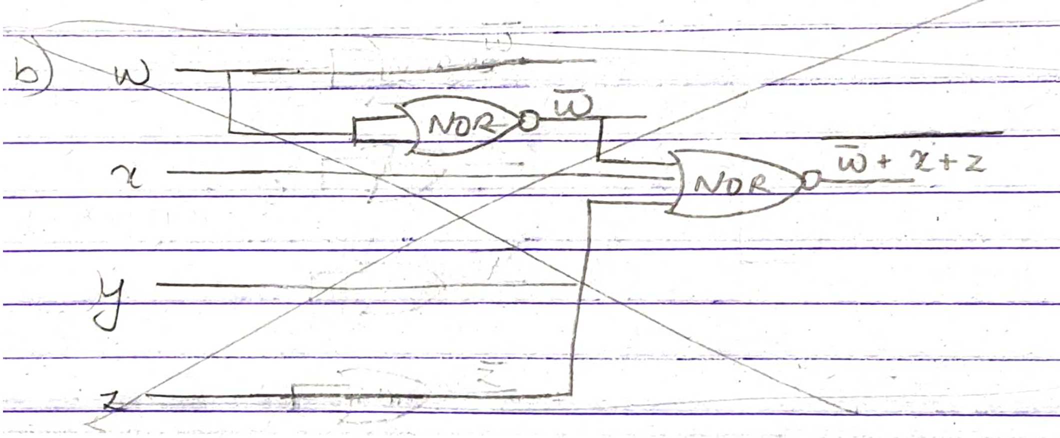
| wx \ yz | $y+z$ 00 | $y+\bar{z}$ 01 | $\bar{y}+\bar{z}$ 11 | $\bar{y}+z$ 10 |
|---|---|---|---|---|
| $w+x$ 00 | $0_0$ | $0_1$ | 3 | 2 |
| $w+\bar{x}$ 01 | 4 | 5 | 7 | 6 |
| $\bar{w}+\bar{x}$ 11 | $0_{12}$ | $0_{13}$ | 15 | 14 |
| $\bar{w}+x$ 10 | $0_8$ | $0_9$ | 11 | $0_{10}$ |

$F(w,x,y,z) = (\bar{w}+x+z).(\bar{w}+y).(x+y)$

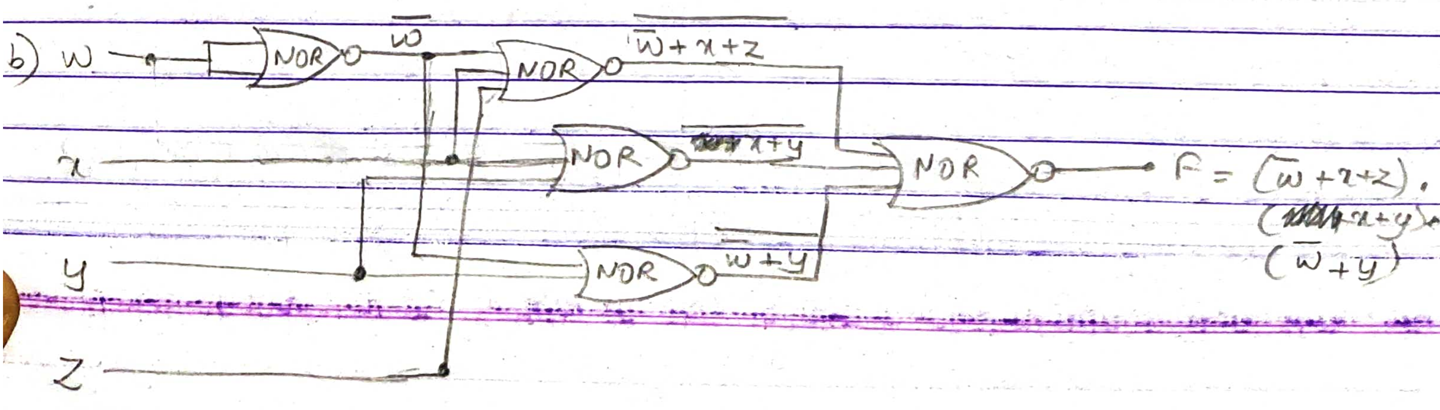
a)




b)




b)



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5) $F(w,x,y,z) = \Sigma(0,1,2,3,7,8,10)$

$d(w,x,y,z) = \Sigma(5,6,11,15)$

a)

| $wx \backslash yz$ | $\bar y \bar z$ 00 | $\bar y z$ 01 | $yz$ 11 | $y \bar z$ 10 |
|---|---|---|---|---|
| $\bar w \bar x$ 00 | $1_{0}$ | $1_{1}$ | $1_{3}$ | $1_{2}$ |
| $\bar w x$ 01 | $_{4}$ | $d_{5}$ | $1_{7}$ | $d_{6}$ |
| $wx$ 11 | $_{12}$ | $_{13}$ | $d_{15}$ | $_{14}$ |
| $w \bar x$ 10 | $1_{8}$ | $_{9}$ | $d_{11}$ | $1_{10}$ |

$F(w,x,y,z) = \bar x \bar z + \bar w \bar x + yz$

5) B) From the above map,

$$\overline{F} = x\bar{y} + x\bar{z} + wz$$

$$F = (\bar{x}+y).(\bar{x}+z).(\bar{w}+\bar{z})$$

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6) a) $f(w,x,y,z) = \Sigma(0,1,5,7,8,10,14,15)$

Step 0

| | minterms | bitstring | # of 1's |
|---|---|---|---|
| 1. | $\bar{w}\bar{x}\bar{y}\bar{z}$ | 0000 | 0 |
| 2. | $\bar{w}\bar{x}\bar{y}z$ | 0001 | 1 |
| 3. | $\bar{w}x\bar{y}z$ | 0101 | 2 |
| 4. | $\bar{w}xyz$ | 0111 | 3 |
| 5. | $w\bar{x}\bar{y}\bar{z}$ | 1000 | 1 |
| 6. | $w\bar{x}y\bar{z}$ | 1010 | 2 |
| 7. | $wxy\bar{z}$ | 1110 | 3 |
| 8. | $wxyz$ | 1111 | 4 |

Step 1

| term | bitstring |
|---|---|
| (1,5) $\bar{x}\bar{y}\bar{z}$ | _000 |
| (1,2) $\bar{w}\bar{x}\bar{y}$ | 000_ |
| (2,3) $\bar{w}\bar{y}z$ | 0_01 |
| (5,6) $w\bar{x}z$ | 10_0 |
| (3,4) $\bar{w}xz$ | 01_1 |
| (6,7) $wy\bar{z}$ | 1_10 |
| (4,8) $xyz$ | _111 |
| (7,8) $wxy$ | 111_ |

No more simplification possible

prime implicants :- $\bar{x}\bar{y}\bar{z}$, $\bar{w}\bar{x}\bar{y}$, $\bar{w}\bar{y}z$, $w\bar{x}z$, $\bar{w}xz$, $wy\bar{z}$, $xyz$, $wxy$

| | 1 $\bar{w}\bar{x}\bar{y}\bar{z}$ | 2 $\bar{w}\bar{x}\bar{y}z$ | 3 $\bar{w}x\bar{y}z$ | 4 $\bar{w}xyz$ | 5 $w\bar{x}\bar{y}\bar{z}$ | 6 $w\bar{x}y\bar{z}$ | 7 $wxy\bar{z}$ | 8 $wxyz$ |
|---|---|---|---|---|---|---|---|---|
| $\bar{x}\bar{y}\bar{z}$ | x | | | | x | | | |
| $\bar{w}\bar{x}\bar{y}$ ✓ | x | x | | | | | | |
| $\bar{w}\bar{y}z$ | | x | x | | | | | |
| $w\bar{x}z$ ✓ | | | | | x | x | | |
| $\bar{w}xz$ ✓ | | | x | x | | | | |
| $wy\bar{z}$ | | | | | | x | x | |
| $xyz$ | | | | x | | | | x |
| $wxy$ ✓ | | | | | | | x | x |

We dont find any single column with a single cross.
Also Row / column reduction is not possible

Hence

$f(w,x,y,z) = \bar{w}\bar{x}\bar{y} + w\bar{x}z + \bar{w}xz + wxy$

OR

$f(w,x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{w}\bar{y}z + wy\bar{z} + xyz$

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b) $f(w,x,y,z) = \Sigma(0,1,2,5,7,8,9,10,13,15)$

Step 0:-

| | Minterms | bitstring | # of 1's |
|---|---|---|---|
| 1. | $\bar{w}\bar{x}\bar{y}\bar{z}$ | 0000 | 0 |
| 2. | $\bar{w}\bar{x}\bar{y}z$ | 0001 | 1 |
| 3. | $\bar{w}\bar{x}y\bar{z}$ | 0010 | 1 |
| 4. | $\bar{w}x\bar{y}z$ | 0101 | 2 |
| 5. | $\bar{w}xyz$ | 0111 | 3 |
| 6. | $w\bar{x}\bar{y}\bar{z}$ | 1000 | 1 |
| 7. | $w\bar{x}\bar{y}z$ | 1001 | 2 |
| 8. | $w\bar{x}y\bar{z}$ | 1010 | 2 |
| 9. | $wx\bar{y}z$ | 1101 | 3 |
| 10. | $wxyz$ | 1111 | 4 |

Step 2

| Minterms | bitstring |
|---|---|
| (1,2) $\bar{w}\bar{x}\bar{y}$ | 000– ✓ |
| (1,3) $\bar{w}\bar{x}\bar{z}$ | 00–0 ✓ |
| (1,6) $\bar{x}\bar{y}\bar{z}$ | –000 ✓ |
| (2,4) $\bar{w}\bar{y}z$ | 0–01 ✓ |
| (2,7) $\bar{x}\bar{y}z$ | –001 ✓ |
| (3,8) $\bar{x}y\bar{z}$ | –010 ✓ |
| (6,7) $w\bar{x}\bar{y}$ | 100– ✓ |
| (6,8) $w\bar{x}\bar{z}$ | 10–0 ✓ |
| (4,5) $\bar{w}xz$ | 01–1 ✓ |
| (4,9) $x\bar{y}z$ | –101 ✓ |
| (7,9) $w\bar{y}z$ | 1–01 ✓ |
| (5,10) $xyz$ | –111 ✓ |
| (9,10) $wxz$ | 11–1 ✓ |

Step 3

| Minterms | | bitstring |
|---|---|---|
| (1,2,6,7) | $\bar{x}\bar{y}$ | –00– |
| (1,3,6,8) | $\bar{x}\bar{z}$ | –0–0 |
| (4,5,9,10) | $xz$ | –1–1 |
| (2,4,7,9) | $\bar{y}z$ | ––01 |

No more reduction possible

prime implicants = $\bar{x}\bar{y}$, $\bar{x}\bar{z}$, $xz$, $\bar{y}z$

| Sl No / Minterm value | 1<br>0 | 2<br>1 | 3<br>2 | 4<br>5 | 5<br>7 | 6<br>8 | 7<br>9 | 8<br>10 | 9<br>13 | 10<br>15 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{x}\bar{y}$ | × | × | | | | × | × | | | |
| $\bar{x}\bar{z}$ | × | | × | | | × | | × | | |
| $xz$ | | | | × | × | | | | × | × |
| $\bar{y}z$ | | × | | × | | | × | | × | |

We get

$f(w,x,y,z) = \bar{x}\bar{z} + xz + \bar{y}z$ or

$f(w,x,y,z) = \bar{x}\bar{z} + xz + \bar{x}\bar{y}$

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7)

a) $f(x,y,z) = xyz + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$

| $x \backslash yz$ | $\bar{y}\bar{z}$ 00 | $\bar{y}z$ 01 | $yz$ 11 | $y\bar{z}$ 10 |
|---|---|---|---|---|
| $\bar{x}$ 0 | 1 (0) | (1) | 1 (3) | 1 (2) |
| $x$ 1 | 1 (4) | 1 (5) | 1 (7) | (6) |

$f(x,y,z) = \bar{y}\bar{z} + xz + \bar{x}y$

b) $f(x,y,z) = xyz + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$

Step 0 | Step 1

| | Minterm | bitvalue | # of 1's | Minterm | bitvalue |
|---|---|---|---|---|---|
| 1. | $xyz$ | 1 1 1 | 3 | (1,2) $xz$ | 1 _ 1 |
| 2. | $x\bar{y}z$ | 1 0 1 | 2 | (1,4) $yz$ | _ 1 1 |
| 3. | $x\bar{y}\bar{z}$ | 1 0 0 | 1 | (2,3) $x\bar{y}$ | 1 0 _ |
| 4. | $\bar{x}yz$ | 0 1 1 | 2 | (4,5) $\bar{x}y$ | 0 1 _ |
| 5. | $\bar{x}y\bar{z}$ | 0 1 0 | 1 | (3,6) $\bar{y}\bar{z}$ | _ 0 0 |
| 6. | $\bar{x}\bar{y}\bar{z}$ | 0 0 0 | 0 | (5,6) $\bar{x}\bar{z}$ | 0 _ 0 |

| | $xyz$ | $x\bar{y}z$ | $x\bar{y}\bar{z}$ | $\bar{x}yz$ | $\bar{x}y\bar{z}$ | $\bar{x}\bar{y}\bar{z}$ |
|---|---|---|---|---|---|---|
| ✓ $xz$ | × | × | | | | |
| $yz$ | × | | | × | | |
| $x\bar{y}$ | | × | × | | | |
| ✓ $\bar{x}y$ | | | | × | × | |
| ✓ $\bar{y}\bar{z}$ | | | × | | | × |
| $\bar{x}\bar{z}$ | | | | | × | × |

$f(x,y,z) = xz + \bar{x}y + \bar{y}\bar{z}$

OR

$f(x,y,z) = yz + x\bar{y} + \bar{x}\bar{z}$

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8)

a) $f(x,y,z) = z + (\bar{x}+y)(x+\bar{y})$

$= z + \cancel{\bar{x}.x} + \bar{x}\bar{y} + xy + \cancel{y.\bar{y}}$

$= z + \bar{x}\bar{y} + xy$

$= z(x+\bar{x}) + \bar{x}\bar{y}(z+\bar{z}) + xy(z+\bar{z})$

$= xz + \bar{x}z + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} + xyz + xy\bar{z}$

$= xz(y+\bar{y}) + \bar{x}z(y+\bar{y}) + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} +$
$xyz + xy\bar{z}$

$= xyz + x\bar{y}z + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} +$
$xyz + xy\bar{z}$

$f(x,y,z) = xyz + x\bar{y}z + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} + xy\bar{z}$

$f(x,y,z) = \Sigma(0,1,3,5,6,7)$

↳ cannonical sum of product form

b) $f(x,y,z) = x + \overline{(\bar{x}\bar{y} + \bar{x}z)}$

$= x + (\overline{\bar{x}\bar{y}}).(\overline{\bar{x}z})$ (De-morgan's laws)

$= x + (x+y).(x+\bar{z})$

$= x + x.x + x\bar{z} + y\bar{z}$

$= x + x\bar{z}(y+\bar{y}) + y\bar{z}(x+\bar{x})$

$= x(y+\bar{y}) + xy\bar{z} + x\bar{y}\bar{z} + xy\bar{z} + \bar{x}y\bar{z}$

$= xy(z+\bar{z}) + x\bar{y}(z+\bar{z}) + xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z}$

$= xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z}$

$f(x,y,z) = xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz$

$f(x,y,z) = \Sigma(3,4,5,6,7)$

↳ cannonical sum of product form

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9)

$$f(x,y,z) = (\bar{x} + xy\bar{z}) + (\bar{x} + xy\bar{z})(x + \bar{x}\bar{y}z)$$

$$= \bar{x}(y+\bar{y}) + xy\bar{z} + \cancel{\bar{x}\cdot x} + \bar{x}\cdot\bar{x}\bar{y}z + x\cdot xy\bar{z} + \cancel{xy\bar{x}}\bar{y}z$$

$$= \bar{x}y(z+\bar{z}) + \bar{x}\bar{y}(z+\bar{z}) + xy\bar{z} + \bar{x}\bar{y}z$$

$$= \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} + xy\bar{z} + \bar{x}\bar{y}z$$

$$f(x,y,z) = \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} + xy\bar{z}$$

$$f(x,y,z) = \Sigma(0,2,3,1,6)$$

↳ Cannonical Sum of product form

| $x \backslash yz$ | $\bar{y}\bar{z}$ 00 | $\bar{y}z$ 01 | $yz$ 11 | $y\bar{z}$ 10 |
|---|---|---|---|---|
| $\bar{x}$ 0 | 1 $_0$ | 1 $_1$ | 1 $_3$ | 1 $_2$ |
| $x$ 1 | $_4$ | $_5$ | $_7$ | 1 $_6$ |

$$f(x,y,z) = \bar{x} + y\bar{z}$$

10)

$$F = \bar{w} + (\bar{x}+y+\bar{y}\bar{z})(x+\bar{y}z)$$

$$= \bar{w} + \cancel{\bar{x}\cdot x} + \bar{x}\bar{y}z + yx + \cancel{y\cdot\bar{y}}z + x\bar{y}\bar{z} + \bar{y}\cancel{\bar{z}\cdot}\bar{y}z$$

$$= \bar{w} + xy + \bar{x}\bar{y}z + x\bar{y}\bar{z}$$

$$\bar{F} = \overline{(\bar{w} + xy + \bar{x}\bar{y}z + x\bar{y}\bar{z})}$$

$$= w\cdot(\bar{x}+\bar{y})\cdot(\bar{x}+\bar{y}+z)\cdot(x+\bar{y}+\bar{z})$$

$$= (w\bar{x} + w\bar{y})\cdot(\cancel{\bar{x}\cdot x} + \bar{x}\bar{y} + \bar{x}\bar{z} + x\bar{y} + \bar{y}\cdot\bar{y} + \bar{y}\bar{z} + xz + \bar{y}z + \cancel{z\bar{z}})$$

$$= (w\bar{x} + w\bar{y})\cdot(\bar{x}\bar{y} + \bar{x}\bar{z} + x\bar{y} + \bar{y} + \bar{y}\bar{z} + xz + \bar{y}z)$$

$$= w\bar{x}\bar{y} + w\bar{x}\bar{z} + 0 + w\bar{x}\bar{y} + w\bar{x}\bar{y}\bar{z} + 0 + w\bar{x}\bar{y}z +$$
$$w\bar{x}\bar{y} + w\bar{x}\bar{y}\bar{z} + wx\bar{y} + w\bar{y} + w\bar{y}\bar{z} + wx\bar{y}z + w\bar{y}z$$

$$= w\bar{x}\bar{y}z + w\bar{x}\bar{y}\bar{z} + w\bar{x}y\bar{z} + w\bar{x}\bar{y}\bar{z} + w\bar{x}\bar{y}z + w\bar{x}\bar{y}\bar{z} +$$
$$wx\bar{y}\bar{z} + w\bar{x}\bar{y}z + w\bar{x}\bar{y}z + w\bar{x}y\bar{z} + w\bar{x}\bar{y}\bar{z} + wx\bar{y}z +$$
$$wx\bar{y}\bar{z} + wx\bar{y}z + wx\bar{y}z + w\bar{x}\bar{y}z + w\bar{x}\bar{y}\bar{z} + wx\bar{y}\bar{z} +$$
$$wx\bar{y}\bar{z} + wx\bar{y}z + wx\bar{y}z + w\bar{x}\bar{y}z$$

$$\bar{F} = w\bar{x}\bar{y}z + w\bar{x}\bar{y}\bar{z} + w\bar{x}y\bar{z} + wx\bar{y}z + wx\bar{y}\bar{z}$$

$$\bar{F} = \Sigma(8,9,10,12,13)$$

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11) Given :- $A\bar{B} + \bar{A}B = C$ $\longrightarrow$ (1)

To prove :- $A\bar{C} + \bar{A}C = B$

now

$$A\bar{B} + \bar{A}B = C$$

$$\overline{(A\bar{B} + \bar{A}B)} = \bar{C}$$

$$(\bar{A} + B) \cdot (A + \bar{B}) = \bar{C}$$

$$\bar{A} \cdot A + \bar{A}\bar{B} + AB + B \cdot \bar{B} = \bar{C}$$

$$\bar{A}\bar{B} + AB = \bar{C} \quad \longrightarrow \quad (2)$$

$$LHS = A\bar{C} + \bar{A}C$$

$$= A(\bar{A}\bar{B} + AB) + \bar{A}(A\bar{B} + \bar{A}B) \quad [\text{from } (1) \text{ \& } (2)]$$

$$= A \cdot \bar{A}\bar{B} + A \cdot AB + \bar{A} \cdot A\bar{B} + \bar{A} \cdot \bar{A}B$$

$$= AB + \bar{A}B$$

$$= B(A + \bar{A})$$

$$= B \quad = RHS$$

hence proved.

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16)

a) integers greater than 10

$f(x) = x+10$ $\quad f: \mathbb{N} \longrightarrow \mathbb{Z}$

| $x$ | 1 | 2 | 3 | 4 | 5 .... | $x \in \mathbb{N}$ |
|---|---|---|---|---|---|---|
| $f(x)$ | 11 | 12 | 13 | 14 | 15 ..... | |

↳ list of all integers greater than 10

This exibits a one-on-one correspondance b/w the set of natural numbers acc. to the function
$f(x) = x+10$ , $x \in \mathbb{N}$

Thus, integers greater than 10 forms a countable set

b) odd negative integers
$f(x) = -(2x-1)$ $\quad f: \mathbb{N} \longrightarrow \mathbb{Z}$

| $x$ | 1 | 2 | 3 | 4 | 5 .... | , $x \in \mathbb{N}$ |
|---|---|---|---|---|---|---|
| $f(x)$ | -1 | -3 | -5 | -7 | -9 .... | |

↳ list of all ~~the~~ odd negative integers

This enhibits a one-on-one correspondana b/w the set of natural numbers acc. to the function
$f(x) = -(2x-1)$ , $x \in \mathbb{N}$

Thus odd negative integers form a countable set

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d) integers that are multiples of 10

$f: \mathbb{N} \longrightarrow \mathbb{Z}$

$f(x) = 10x \quad , \quad x \in \mathbb{N}$

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | 10 | 20 | 30 | 40 | 50 ---- |

↳ list of integers that are multiples of 10

This exhibits a one-or one correspondance with natural numbers acc. to the function

$f(x) = 10x, \quad x \in \mathbb{N}.$

Thus, integers that are multiples of 10 form a countable set.

c) Real numbers between 0 & 2

→ let us assume that real numbers between 0 and 2 ~~are~~ ~~can~~ form a countable set.

→ The subset of all real no.s in [0, 1] is also countable

→ So, real numbers can be listed in an order $r_1, r_2, r_3$ ---

Let the decimal representation of these numbers be as follows,

$r_1 = .d_{11} \; d_{12} \; d_{13} \; d_{14}$ ---

$r_2 = .d_{21} \; d_{22} \; d_{23} \; d_{24}$ ---

$r_3 = .d_{31} \; d_{32} \; d_{33} \; d_{34}$ ---

$r_4 = .d_{41} \; d_{42} \; d_{43} \; d_{44}$

$\vdots$

where $d_{ij} \in \{0, 1, 2, \ldots 9\}$

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Let us ~~daw~~ draw a diagonal and construct a decimal number

$r = .\ \square\ \square\ \square\ \square\ ---$

| diff from $d_{11}$ | diff from $d_{22}$ | diff from $d_{33}$ | diff from $d_{44}$ |
|---|---|---|---|

r is ~~also~~ different from the numbers listed in the one-to-one correspondance at least at one decimal place.

But r is not in the list.

So, we found a number that is not in the list but is a decimal number in [0, 1]

This contradicts our assumption

Thus, real numbers between 0 and 2 form an uncountable set

Q12) $F^d(x_1, x_2, \ldots x_n) = \overline{F(\bar{x}_1, \ldots \bar{x}_n)}$

→ In case of dual all 1's are ~~converted~~ ~~as~~ converted to 0's & 0's ~~to~~ 1's. Also all '+' are converted to '.' & all '.' to '+'

Acc. to De Morgan's laws

$(\overline{x+y}) = \bar{x}\cdot\bar{y}$ & $(\overline{x.y}) = \bar{x}+\bar{y}$

Also

$\overline{\bar{x}+\bar{y}} = x\cdot y$, $(\overline{\bar{x}\bar{y}}) = x+y$, $\bar{1}=0$, $\bar{0}=1$

∴ RHS, $\overline{F(\bar{x}_1, \ldots \bar{x}_n)}$ would convert all the products to sums and all the sums to products.

Also all '0's would convert to '1's & '1's to '0's

∴ $F^d(x_1, x_2, \ldots x_n) = \overline{F(\bar{x}_1, \bar{x}_2, \ldots \bar{x}_n)}$

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13) Let S & T be 2 enumerable sets i.e
S & T are also countable

Let S be $\{s_1, s_2 \ldots s_k, \ldots\}$ &
T be $\{T_1, T_2 \ldots T_k \ldots\}$

Consider two cases:-
i) S & T are finite, then S×T will also be finite & finite set is both enumerable & countable
ii) S & T are countably infinite

We can define the function

$$f : S \times T \rightarrow \mathbb{N}$$

$(s_1, t_1)$ $(s_1, t_2) \rightarrow$ $(s_1, t_3)$ $(s_1, t_4) \rightarrow$
$\downarrow$
$(s_2, t_1)$ $(s_2, t_2)$ $(s_2, t_3)$

$(s_3, t_1)$ $(s_3, t_2)$
$\downarrow$
$(s_4, t_1)$

$(s_1, t_1) \rightarrow 1$, $k+l=2$
$(s_2, t_1) \rightarrow 2$, $(s_1, t_2) \rightarrow 3$ } $k+l=3$
$(s_1, t_3) \rightarrow 4$, $(s_2, t_2) \rightarrow 5$, $(s_3, t_1) \rightarrow 6$ } $k+l=4$

They are arranged in the order of $k+l$ & further in the order of S & t alternatively

mapping one-to-one. Hence S×T is countable
∴ S×T is enumerable

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14) S $\longrightarrow$ enumerable
T $\rightarrow$ non-enumerable subset of $\mathbb{R}$

(i) $S \cup T$ is non-enumerable
$\rightarrow$ As T is non-enumerable, $|T| > \aleph_0$
$\therefore |S \cup T| \geq |T|$
$\Rightarrow |S \cup T| \geq \aleph_0$
$\therefore$ An onto func$^n$ cannot be defined from $\mathbb{N}$ to $S \cup T$ $\therefore S \cup T$ is non-enumerable

ii) $S \cap T$ is at most enumerable
$\rightarrow$ $(S \cap T) \subseteq S$
There exists an onto ~~f~~ mapping from $\mathbb{N}$ to S
There are 2 cases.
a) S is finite, $\therefore S \cap T$ is finite. Hence its enumerable
b) S is countably infinite; $S \cap T$ can be atmost S $\therefore$ it can atmost be countably infinite
$\therefore S \cap T$ can either be finite or countably infinite
And we can find an onto mapping from $\mathbb{N}$ to $S \cap T$ with the help of the onto function from $\mathbb{N}$ to S.
Let S be $\{S_1, S_2, \dots S_k \dots\}$
$\therefore S \cap T$ will contain elements $\{S_1, S_2, \dots S_m, \dots\}$
$\therefore$ some elements will be absent from $S \cap T$ that were present in S
All the elements in $\mathbb{N}$ that were mapped to those elements in S that are not present in $S \cap T$ are mapped to any one element in $S \cap T$. This given an onto mapping from $\mathbb{N}$ to $S \cap T$. $\therefore S \cap T$ is enumerable.

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iii) $S-T$ is at most enumerable
→ $(S-T) \subseteq S$
There exists an onto mapping from $\mathbb{N}$ to $S$
There are 2 cases:
a) $S$ is finite $\therefore$ $S-T$ is finite, hence its enumerable
b) $S$ is countably infinite. $S-T$ can be finite or countably infinite as $|S-T| \leq \aleph_0$
Let $S = \{s_1, s_2 \ldots s_k \ldots\}$
& there exists an onto mapping from $\mathbb{N}$ to $S$. $S-T$ will contain elements of $S$ which are not in $T$.
$\therefore$ The numbers in $\mathbb{N}$ which were mapped to elements in $S$ which are not present in $S-T$ are mapped to any one element in $S-T$ $\therefore$ This is an onto mapping from $\mathbb{N}$ to $S-T$. $\therefore$ $S-T$ is enumerable

iv) $T-S$ is not enumeble
→ Consider that $T-S$ is enumerable
Therefore, there exists an onto mapping from $\mathbb{N}$ to $T-S$
$\therefore$ $|T-S| \leq \aleph_0$
$T-S = T \cap (S \cap T)^c$
As $T-S$ is enumerable, then it would be finite or countably infinite & as we proved earlier that $S \cap T$ is also enumerable ie either finite or countabley infite And, we know that the union of 2 countable sets is ~~a~~ also countable i.e.
$(T-S) \cup (S \cap T)$ will be countable
& $(T-S) \cup (S \cap T)$ is nothing ~~[illegible]~~ but $T$ which is given as non-enumerable.
Hence this a contradiction. Thus our assumption is wrong. $\therefore$ $T-S$ is not enumerable

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15)

i) $A = \{x \in R : 0 \leq x \leq 1\}$
$B = \{x \in R : 0 \leq x \leq 1\}$

$f(x) = \frac{x}{3}$ is a one-one mapping from A to B

$g(x) = x$ is a one-one mapping from B to A
So by Schroder - Bernstein theorem,
$|A| = |B|$
i.e A & B are equipotent

ii) $A = \{x \in R : 0 \leq x \leq 1\}$
$B = \{x \in R : a \leq x \leq b\}$

Let $f(x) = \frac{x-a}{b-a}$ be a funⁿ from B to A

Let $f(x) = f(y)$
$\frac{x-a}{b-a} = \frac{y-a}{b-a}$

$x = y \Rightarrow f$ is injective
as $a \leq x \leq b$
so $\frac{a-a}{b-a} \leq f(x) \leq \frac{b-a}{b-a}$ i.e $0 \leq f(x) \leq 1$

Hence $f$ is surjective fⁿ from B to A
$\therefore |B| = |A|$

iii) $A = \{x \in R, 0 \leq x \leq 1\}$
$B = \{x \in R, 0 < x < 1\}$

$f(x) = \frac{x}{3} + \frac{1}{2}$ is a one to one mapping from A to B

$g(x) = x$ is a one to one mapping from B to A
By Schroder Bersen theorem, $|A| = |B|$

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(v) $A = \{x \in R;\ x \geq 1\}$
$B = \{x \in R;\ x > 1\}$

$f(x) = 2x$ is a one-one mapping from A to B
$g(x) = x$ is a one-one mapping from B to A

Hence by schroder Berstein theorem
$|A| = |B|$