LPP: Simplex Methods:- Numerical Examples

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January 17, 2022

Note:-

1) The RHS of constraints have to be +ve, if not make them positive

Numerical Example (a1):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Simplex Method: Condensed Tableau

Numerical Example (a1):

$$\max : Z = x_1 + 3x_2 + 0s_1 + 0s_2$$

Subject to

$$x_1 + x_2 + s_1 = 10$$

$$x_1 + 4x_2 + s_2 = 16$$

$$x_1, x_2 \geq 0$$

Slack variables(Basic variables) :

$$s_1, s_2 \geq 0$$

Numerical Example (a1):

$$\max : Z = x_1 + 3x_2 + 0s_1 + 0s_2$$

Subject to

$$-x_1 - x_2 + 10 = s_1$$

$$-x_1 - 4x_2 + 16 = s_2$$

$$x_1, x_2 \geq 0$$

Slack variables(Basic variables) :

$$s_1, s_2 \geq 0$$

Numerical Example (a1):

Table 0:

$-x_1$	$-x_2$	1	BV
1	1	10	$= s_1$
1	4*	16	$= s_2$
-1	-3*	0	=Z

pivotal element as $\frac{16}{4}$ ratio is minimum positive ratio

most negative

Note:- Pivot is always >0

0 → swapped without sign!

Numerical Example (a1):

Table 1:

$-x_1$	$-s_2$	1	BV
3/4*	-1/4	6	$= s_1$
1/4	1/4	4	$= x_2$
-1/4*	3/4	12	=Z

divided by pivot

becomes $\frac{1}{\text{pivot}}$

divided by (-pivot)

for other elements

$$\text{new}(S) = \frac{ps - rq}{p}$$

$p \to$ pivot

$$\begin{array}{ccc} p & & r \\ & \square & \\ q & & s \end{array}$$

Repeating the same steps on the newly obtained table

Numerical Example (a1):

Table 2:

$-s_1$	$-s_2$	1	BV
4/3	-1/3	8	$= x_1$
- 1/3	1/3	2	$= x_2$
1/3	2/3	14	$=Z$

Optimal Solution : → as no negative elements in the last row (DO NOT consider the '1' column)

$$x_1^* = 8, x_2^* = 2, Z^* = 14,$$

$S_1, S_2 = 0$,

Note:- If in the last row, there is a '0', Alternate optimal solution is present

Numerical Example (b1):

$$\min : Z = x_1 + 3x_2$$

convert to a maximization problem & then use simplex method

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (b1):

$$\max : -Z = -x_1 - 3x_2 + 0s_1 + 0s_2$$

Subject to

$$x_1 + x_2 + s_1 = 10$$

$$x_1 + 4x_2 + s_2 = 16$$

$$x_1, x_2 \geq 0$$

Slack variables(Basic variables) :

$$s_1, s_2 \geq 0$$

Numerical Example (b1):

$$\max : -Z = -x_1 - 3x_2 + 0s_1 + 0s_2$$

Subject to

$$-x_1 - x_2 + 10 = s_1$$

$$-x_1 - 4x_2 + 16 = s_2$$

$$x_1, x_2 \geq 0$$

Slack variables(Basic variables) :

$$s_1, s_2 \geq 0$$

no negative element in the last row

Numerical Example (b1):

Table 0:

$-x_1$	$-x_2$	1	BV
1	1	10	$= s_1$
1	4*	16	$= s_2$
1	3	0	=-Z

Optimal Solution **:**

$$s_1^* = 10, s_2^* = 16, x_1^* = 0, x_2^* = 0, Z^* = 0$$

(Big M method)

When ARTIFICIAL variables get involved

Numerical Example (a2):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 = 10$$

$$x_1 + 4x_2 = 16$$

$$x_1, x_2 \geq 0$$

Artificial variables have to be $= 0$ in the final table.
Else, the solution is infeasible.

Numerical Example (a2):

imp →

$$\max : Z = x_1 + 3x_2 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 + a_1 = 10$$

$$x_1 + 4x_2 + a_2 = 16$$

$$x_1, x_2 \geq 0$$

$$M : is \quad a \quad large \quad positive \quad number$$

Artificial variables (Basic variables) :

$$a_1, a_2 \geq 0$$

Use M to drive out artificial variables

Simplex Method: Condensed Tableau

Numerical Example (a2):

$$\max : Z = x_1 + 3x_2 - Ma_1 - Ma_2$$
$$= x_1 + 3x_2 - M(a_1 + a_2)$$
$$= x_1(2M + 1) + x_2(5M + 3) - 26M$$

Subject to

$$-x_1 - x_2 + 10 = a_1$$
$$-x_1 - 4x_2 + 16 = a_2$$
$$x_1, x_2 \geq 0$$

Artificial variables (Basic variables) :

$$a_1, a_2 \geq 0$$

After redefining the objective function, the methodology is the same to reach the final table.

Numerical Example (a2):

Table 0:

$-x_1$	$-x_2$	1	BV
1	1	10	$= a_1$
1	* 4	16	$= a_2$
-2M-1	-5M-3	-26M	=Z

Numerical Example (a2):

Table 1:

$-x_1$	$-a_2$	1	BV
* 3/4	-1/4	6	$= a_1$
1/4	1/4	4	$= x_2$
-(3M+1)/4	(5M+3)/4	-6M+12	=Z

Numerical Example (a2):

Table 2:

$-a_1$	$-a_2$	1	BV
4/3	-1/3	8	$= x_1$
- 1/3	1/3	2	$= x_2$
(3M +1)/3	(3M +2)/3	14	=Z

Optimal Solution **:**

$a_1, a_2 = 0$, $x_1^* = 8, x_2^* = 2, Z^* = 14,$

Note:- Solution is feasible as all artificial variables are $= 0$.

Convert to maximization
problem & then follow the
steps used in (a2) problem

Numerical Example (b2):

$$\min : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 = 10$$

$$x_1 + 4x_2 = 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (b2):

$$\max : -Z = -x_1 - 3x_2 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 + a_1 = 10$$

$$x_1 + 4x_2 + a_2 = 16$$

$$x_1, x_2 \geq 0$$

$$M : is \quad a \quad large \quad positive \quad number$$

Artificial variables (Basic variables) :

$$a_1, a_2 \geq 0$$

Use M to drive out artificial variables

Numerical Example (b2):

$$\max : -Z = -x_1 - 3x_2 - Ma_1 - Ma_2$$

$$= -x_1 - 3x_2 - M(a_1 + a_2)$$

$$= x_1(2M - 1) + x_2(5M - 3) - 26M$$

Subject to

$$-x_1 - x_2 + 10 = a_1$$

$$-x_1 - 4x_2 + 16 = a_2$$

$$x_1, x_2 \geq 0$$

Artificial variables (Basic variables) :

$$a_1, a_2 \geq 0$$

Simplex Method: Condensed Tableau

Numerical Example (b2):

Table 0:

$-x_1$	$-x_2$	1	BV
1	1	10	$= a_1$
1	* 4	16	$= a_2$
-2M+1	-5M+3	-26M	=-Z

Numerical Example (b2):

Table 1:

$-x_1$	$-a_2$	1	BV
* 3/4	-1/4	6	$= a_1$
1/4	1/4	4	$= x_2$
-(3M-1)/4	(5M-3)/4	-6M-12	=-Z

Numerical Example (b2):

Table 2:

$-a_1$	$-a_2$	1	BV
4/3	-1/3	8	$= x_1$
- 1/3	1/3	2	$= x_2$
(3M -1)/3	(3M -2)/3	-14	=-Z

$$\textit{Optimal} \quad \textit{Solution}:$$

$$x_1^* = 8, x_2^* = 2, Z^* = 14,$$

Case of surplus + artificial variables

Numerical Example (a3):

$$\min : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \geq 10$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (a3):

$$\max : -Z = -x_1 - 3x_2 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 - x_3 + a_1 = 10$$

$$x_1 + 4x_2 - x_4 + a_2 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$M : is \quad a \quad large \quad positive \quad number$$

Surplus variables : $x_3, x_4 \geq 0$ → not Basic variables
Artificial variables (Basic variables) :

$$a_1, a_2 \geq 0$$

Use M to drive out artificial variables

Numerical Example (a3):

$$\max : -Z = -x_1 - 3x_2 - Ma_1 - Ma_2$$

$$= -x_1 - 3x_2 - M(a_1 + a_2)$$

$$= x_1(2M - 1) + x_2(5M - 3) - x_3M - x_4M - 26M$$

Subject to

$$-x_1 - x_2 + x_3 + 10 = a_1$$

$$-x_1 - 4x_2 + x_4 + 16 = a_2$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Artificial variables (Basic variables) :

$$a_1, a_2 \geq 0$$

Simplex Method: Condensed Tableau

Algorithm is the same after this point until final table is reached

Numerical Example (a3):

Table 0:

$-x_1$	$-x_2$	$-x_3$	$-x_4$	1	BV
1	1	-1	0	10	$= a_1$
1	* 4	0	-1	16	$= a_2$
-2M+1	-5M+3	M	M	-26M	=-Z

Numerical Example (a3):

Table 1:

$-x_1$	$-a_2$	-x3	-x4	1	BV
* 3/4	-1/4	-1	1/4	6	$= a_1$
1/4	1/4	0	-1/4	4	$= x_2$
-(3M-1)/4	(5M-3)/4	M	-(M-3)/4	-6M-12	=-Z

Numerical Example (a3):

Table 2:

$-a_1$	$-a_2$	-x3	-x4	1	BV
4/3	-1/3	-4/3	1/3	8	$= x_1$
- 1/3	1/3	1/3	-1/3	2	$= x_2$
(3M-1)/3	(3M-2)/3	1/3	2/3	-14	=-Z

Optimal Solution **:**

$a_1, a_2, x_3, x_4 = 0$,

$$x_1^* = 8, x_2^* = 2, Z^* = 14,$$

all artificial variables are zero

Numerical Example (a4):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \geq 10$$

$$x_1 + 4x_2 \geq 16$$

$$x_1, x_2 \geq 0$$

Simplex Method: Condensed Tableau

Numerical Example (a4):

$$\max : Z = x_1 + 3x_2 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 - x_3 + a_1 = 10$$

$$x_1 + 4x_2 - x_4 + a_2 = 16$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$M : is \quad a \quad large \quad positive \quad number$$

Surplus variables : $x_3, x_4 \geq 0$
Artificial variables (Basic variables) :

$$a_1, a_2 \geq 0$$

Use M to drive out artificial variables

Numerical Example (a4):

$$\max : Z = x_1 + 3x_2 - Ma_1 - Ma_2$$
$$= x_1 + 3x_2 - M(a_1 + a_2)$$
$$= x_1(2M + 1) + x_2(5M + 3) - x_3 M - x_4 M - 26M$$

Subject to

$$-x_1 - x_2 + x_3 + 10 = a_1$$
$$-x_1 - 4x_2 + x_4 + 16 = a_2$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Artificial variables (Basic variables) :

$$a_1, a_2 \geq 0$$

Numerical Example (a4):

Table 0:

$-x_1$	$-x_2$	$-x_3$	$-x_4$	1	BV
1	1	-1	0	10	$= a_1$
1	* 4	0	-1	16	$= a_2$
-2M-1	-5M-3	M	M	-26M	=Z

Numerical Example (a4):

Table 1:

$-x_1$	$-a_2$	-x3	-x4	1	BV
* 3/4	-1/4	-1	1/4	6	$= a_1$
1/4	1/4	0	-1/4	4	$= x_2$
-(3M+1)/4	(5M+3)/4	M	-(M+3)/4	-6M+12	=Z

Numerical Example (a4):

Table 2:

$-a_1$	$-a_2$	-x_3	-x_4	1	BV
4/3	-1/3	-4/3	1/3	8	$= x_1$
- 1/3	1/3	1/3	-1/3	2	$= x_2$
(3M+1)/3	(3M+2)/3	-1/3	-2/3*	14	=Z

Simplex Method: Condensed Tableau

Numerical Example (a4):

Table 3:

$-a_1$	$-a_2$	-x_3	-x_1	1	BV
4	-1	-4	3	24	$= x_4$
1	0	-1	1	10	$= x_2$
M+3	M	-3*	2	30	=Z

In 3rd column all the elements are negative.
So this LPP is unbounded.

If the pivotal column has all element ≤ 0, the problem is unbounded

Numerical Example (a5):

$$\min : Z = 2x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 20$$

$$x_1 + x_2 \geq 40$$

$$x_1, x_2 \geq 0$$

Numerical Example (a5):

$$\max : -Z = -2x_1 - 3x_2 - Ma_1$$

Subject to

$$x_1 + x_2 + s_1 = 20$$

$$x_1 + x_2 - x_3 + a_1 = 40$$

$$x_1, x_2, x_3 \geq 0$$

$$M : is \quad a \quad large \quad positive \quad number$$

Slack variable (Basic Variable) : $s_1 \geq 0$
Surplus variables $x_3 \geq 0$
Artificial variable (Basic variable) :

$$a_1 \geq 0$$

Use M to drive out artificial variable.

Numerical Example (a5):

$$\max : -Z = -2x_1 - 3x_2 - Ma_1$$

$$= -2x_1 - 3x_2 - M(40 - x_1 - x_2 + x_3)$$

$$= x_1(M - 2) + x_2(M - 3) - x_3M - 40M$$

Subject to

$$-x_1 - x_2 + 20 = s_1$$

$$-x_1 - x_2 + x_3 + 40 = a_1$$

$$x_1, x_2, x_3, s_1, a_1 \geq 0$$

Basic variables :

$$s_1, a_1 \geq 0$$

Numerical Example (a5):

Table 0:

$-x_1$	$-x_2$	$-x_3$	1	BV
1*	1	0	20	$= s_1$
1	1	-1	40	$= a_1$
-M+2	-M+3	M	-40M	=-Z

Numerical Example (a5):

Table 1:

$-s_1$	$-x_2$	$-x_3$	1	BV
1	1	0	20	$= x_1$
-1	0	-1	20	$= a_1$
M-2	1	M	-40-20M	=-Z

Since artificial variable ($a_1 = 20$) $\neq 0$
has a positive value, this LPP is infeasible.

Numerical Example (E1):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (E1):

$$\max : Z = x_1 + 3x_2 + 0s_1 + 0s_2$$

Subject to

$$x_1 + x_2 + s_1 = 10$$

$$x_1 + 4x_2 + s_2 = 16$$

$$x_1, x_2 \geq 0$$

Slack variables(Basic variables) :

$$s_1, s_2 \geq 0$$

objective function coefficients

Numerical Example (E1):

Table 0:

SIMP	CV	1	3	0	0	b
CB	BV/V	x_1	x_2	s_1	s_2	XB
0	s_1	1	1	1	0	10
0	s_2	1	4*	0	1	16
*	*	-1	- 3	0	0	0

→ variables

→ constraints

coefficients of BV in obj. function

Basic variables

$0\times 1+0\times 1-1$
$=-1$
(inner product with CB column)

$0\times 10+0\times 16$
$=0$ (value of objective function)

1) Select pivot same like condensed table.

Numerical Example (E1):

Table 1:

SIMP	CV	1	3	0	0	b
CB	**BV/V**	x_1	x_2	s_1	s_2	**XB**
0	s_1	**3/4***	**0**	**1**	**-1/4**	**6**
3	x_2	**1/4**	**1**	**0**	**1/4**	**4**
*	*	**-1/4**	**0**	**0**	**3/4**	**12**

divide row by pivot

coefficients of s_1 & x_2 in obj. function.

pivotal column header

column elements except pivot = 0

for other elements, apply the pq/rs method as used in condensed method

Repeat till no negative elements in the last row

Numerical Example (E1):

Table 2:

SIMP	CV	1	3	0	0	b
CB	BV/V	x_1	x_2	s_1	s_2	XB
1	x_1	1	0	4/3	-1/3	8
3	x_2	0	1	-1/3	1/3	2
*	*	0	0	1/3	2/3	14

can be calculated using pq/rs method (or) inner product method

Optimal Solution :

$$x_1^* = 8, x_2^* = 2, Z^* = 14,$$

Condensed Table :- Basic Variables vs non basic variables

non-condensed Table :- Basic Variables vs all variables

Numerical Example (E2):

$$\max : Z = 2x_1 + 8x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (E2):

$$\max : Z = 2x_1 + 8x_2 + 0s_1 + 0s_2$$

Subject to

$$x_1 + x_2 + s_1 = 10$$

$$x_1 + 4x_2 + s_2 = 16$$

$$x_1, x_2 \geq 0$$

Slack variables(Basic variables) :

$$s_1, s_2 \geq 0$$

Numerical Example (E2):

Table 0:

SIMP	CV	2	8	0	0	b
CB	BV/V	x_1	x_2	s_1	s_2	XB
0	s_1	1	1	1	0	10
0	s_2	1	* 4	0	1	16
*	*	-2	- 8	0	0	0

Numerical Example (E2):

Table 1:

SIMP	CV	2	8	0	0	b
CB	BV/V	x_1	x_2	s_1	s_2	XB
0	s_1	*3/4	0	1	-1/4	6
8	x_2	1/4	1	0	1/4	4
*	*	0	0	0	2	32

Optimal Solution :

$$x_1^* = 0, x_2^* = 4, Z^* = 32,$$

It has alternate optimal solutions. (as there is a zero in the last row for a non-basic variable)

To get another solution, find pivot in the column containing '0' in the last row

Numerical Example (E2):

Table 2:

SIMP	CV	2	8	0	0	b
CB	BV/V	x_1	x_2	s_1	s_2	XB
2	x_1	1	0	4/3	-1/3	8
8	x_2	0	1	-1/3	1/3	2
*	*	0	0	0	2	32

***Optimal Solution* :**

$$x_1^* = 0, x_2^* = 4, Z^* = 32,$$

$$x_1^* = 8, x_2^* = 2, Z^* = 32,$$

If there are > 1 solution, then there are ∞ number of solutions.

Simplex Method: Condensed Tableau

For ease, use only condensed Table of this form.

* In presence of constant in objective function, add at the end. Ignore it during computation

Numerical Example (C1):

$$\max : Z = x_1 + 3x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (C1):

$$\max : Z = x_1 + 3x_2 + 0s_1 + 0s_2$$

Subject to

$$x_1 + x_2 + s_1 = 10$$

$$x_1 + 4x_2 + s_2 = 16$$

$$x_1, x_2 \geq 0$$

Slack variables(Basic variables) :

$$s_1, s_2 \geq 0$$

Simplex Method: Condensed Tableau

Same condensed Table used initially, but in a slightly different form

Numerical Example (C1):

Table 0:

SIMP	CN	1	3	b
CB	BV/NV	x_1	x_2	XB
0	s_1	1	1	10
0	s_2	1	4*	16
*	*	-1	-3	0

Note:- Use this table for all problems, do not use the table used in very initial problems

Note:- Algorithm is exactly the same as for the first table

Numerical Example (C1):

Table 1:

SIMP	CN	1	0	b
CB	BV/NV	x_1	s_2	XB
0	s_1	3/4 *	-1/4	6
3	x_2	1/4	1/4	4
*	*	-1/4	3/4	12

Numerical Example (C1):

Table 2:

SIMP	CN	0	0	b
CB	BV/NV	s_1	s_2	XB
1	x_1	4/3	-1/3	8
3	x_2	-1/3	1/3	2
*	*	1/3	2/3	14

***Optimal Solution* :**

$$x_1^* = 8, x_2^* = 2, Z^* = 14,$$

Numerical Example (C2):

$$\max : Z = 2x_1 + 8x_2$$

Subject to

$$x_1 + x_2 \leq 10$$

$$x_1 + 4x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Numerical Example (C2):

$$\max : Z = 2x_1 + 8x_2 + 0s_1 + 0s_2$$

Subject to

$$x_1 + x_2 + s_1 = 10$$

$$x_1 + 4x_2 + s_2 = 16$$

$$x_1, x_2 \geq 0$$

Slack variables(Basic variables) :

$$s_1, s_2 \geq 0$$

Numerical Example (C2):

Table 0:

SIMP	CN	2	8	b
CB	BV/NV	x_1	x_2	XB
0	s_1	1	1	10
0	s_2	1	* 4	16
*	*	-2	-8	0

Numerical Example (C2):

Table 1:

SIMP	CN	2	0	b
CB	BV/NV	x_1	s_2	XB
0	s_1	*3/4	-1/4	6
8	x_2	1/4	1/4	4
*	*	0	2	32

Numerical Example (C2):

Table 2:

SIMP	CN	0	0	b
CB	BV/NV	s_1	s_2	XB
2	x_1	4/3	-1/3	8
8	x_2	-1/3	1/3	2
*	*	0	2	32

$$\textit{Optimal} \quad \textit{Solution}:$$

$$x_1^* = 8, x_2^* = 2, Z^* = 32,$$

$$x_1^* = 0, x_2^* = 4, Z^* = 32,$$

Numerical Example -7: Condensed Tableau

$$\max : Z = 4x_1 + x_2 + x_3$$

Subject to

$$x_1 + x_2 + x_3 = 10$$

$$x_1 + 4x_2 + x_3 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Numerical Example -7:

$$\max : Z = 4x_1 + x_2 + x_3 - Ma_1 - Ma_2$$

Subject to

$$x_1 + x_2 + x_3 + a_1 = 10$$

$$x_1 + 4x_2 + x_3 + a_2 = 16$$

$$x_1, x_2, x_3 \geq 0$$

Artificial variables:

$$a_1, a_2 \geq 0$$

Big M method is easier with this format of table as the last row is calculated directly using inner product

Table 0:

SIMP	CN	4	1	1	b
CB	BV/NV	x_1	x_2	x_3	XB
-M	a_1	1	1	1	10
-M	a_2	1	* 4	1	16
*	*	-2M-4	- 5M-1	-2M-1	-26M

coefficient of x_1, x_2, x_3 in optimization function

coefficient of a_1, a_2 in optimization function

$(-M \times 1) + (-M \times 4) - (1)$

$a_1 + x_1 + x_2 + x_3 = 10$

① → interchanged along with their coefficients in objective function

pivot replaced by $\frac{1}{\text{pivot}}$

Table 1:

SIMP	CN	4	-M	1	b
CB	BV/NV	x_1	a_2	x_3	XB
-M	a_1	*3/4	-1/4	- 3/4	6
1	x_2	1/4	1/ 4	1/4	4
*	*	(-3M-15)/4	(5M+1)/4	(-3M-3)/4	-6M+4

divide by (-pivot)

divide by pivot

for all other elements, use pq/rs rule (or) calculate using inner product method.

Table 2:

SIMP	CN	-M	-M	1	b
CB	BV/NV	a_1	a_2	x_3	XB
4	x_1	4/3	-1/3	1	8
1	x_2	-1/3	1/ 3	0	2
*	*	M+5	M-1	3	34

***Optimal Solution* :**

$$x_1^* = 8, x_2^* = 2, x_3^* = 0, Z^* = 34$$

Note :- To use Big-M method in minimization problems, first convert to maximization problem, then append the $-Ma_i$'s to the RHS.