Department of Mathematics, I I T Kharagpur
End - Spring Semester 2017, Date of Examination: 20th April, 2017
Session: FN, Max. Marks : 50, Duration: 3hrs. Subject No: MA40002
Subject Name: Integral Equations and Variational Methods

Instructions :

(i) Answer ALL the questions.

(ii) No queries will be entertained during the examination.

(iii) Marks are indicated in the parenthesis besides each question.

Question 1.

a) Solve the following equation: [5]

$$u(x) = \frac{1}{1+x^2} + \frac{\pi^2}{32}x - x\int_0^1 [\tan^{-1} t]\ u(t)dt.$$

Will it have a unique solution in $[0,1]$?

b) Using method of successive approximations solve the nonlinear integral equation:

$$u(x) = \int_0^x \frac{1+u^2(t)}{1+t^2}dt.$$

[3]

c) Solve

$$\int_0^x (x-t)^{-\frac{1}{2}}u(t)dt = x^{\frac{1}{2}},\ x>0.$$

[4]

Question 2.

a) Solve the non homogeneous integral equation

$$u(x) - \int_0^\pi K(x,t)u(t)dt = \sin x$$

where

$$K(x,t) = \begin{cases} \sin(x+\frac{\pi}{4})\sin(t-\frac{\pi}{4}) & 0\le x\le t \\ \sin(t+\frac{\pi}{4})\sin(x-\frac{\pi}{4}) & t\le x\le \pi. \end{cases}$$

[5]

b) Investigate for solvability of the following integral equations for different values of the parameter λ [4]

$$u(x) - \lambda \int_{-1}^{1} (x^2 - 2xt)u(t)dt = x^3 - x.$$

c) Using Green's function reduce the following boundary-value problem to integral equation:

$$y''' + \lambda y = 2x; \ y(0) = y(1) = 0, \ y'(0) = y'(1).$$

[4]

Question 3.

a) Find the curve for which the functional

$$J(y) = \int_0^{x_1} \frac{\sqrt{1 + (y')^2}}{y} dx, \quad y(0) = 0$$

can have extrema if the point (x_1, y_1) can vary along the line $y = x - 5$. [5]

b) Justify true or false with proper reason:

(i) The functional

$$J(y) = \int_0^{\pi} (4y \cos x + y'^2 - y^2)dx, \ y(0) = 0, \ y(\pi) = 0$$

has infinite number of extremals. [2]

(ii) There exists a solution with corner points in the problem of the extremum for the functional

$$J(y) = \int_{x_0}^{x_1} (y'^2 + 2xy - y^2)dx, \ y(x_0) = y_0, \ y(x_1) = y_1.$$

[3]

iii) The family of curves $y = e^{(x+c)}$, $c \in \mathbb{R}$ form a proper field in the domain $D : \{(x, y) : x^2 + y^2 \leq 1\}$. [1]

Question 4.

a) Find the extremals with corner point for the functional

$$J(y) = \int_0^2 (y')^2(y'-1)^2 dx, \ y(0) = 0, \ y(2) = 1.$$

[4]

b) Test for an extremum(Weak or Strong) of the functional

$$J(y) = \int_0^a (1 - \exp(-y'^2))dx; \ y(0) = 0, \ y(a) = b, \ a > 0.$$

[5]

c) Test for an extremum(Weak or Strong) of the functional

$$J(y) = \int_{-1}^1 (y'^3 + y'^2)dx; \ y(-1) = -1, \ y(1) = 3.$$

[5]

THE END.