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·	Keerti P. Charantimathusson I with MA exam 107
	19MA20059 - 12+32 forms - 19 - 57-571
	15 1 Th 1
7)	sim: to find 172.72 as a sum of two squares
	The state of the s
	we know that 17 = 1 (mod 4) & 7 = 3 (mod 4)
j.	Mso, I7= (1+4i)(1-4i) = (4+i)(4-i)
	The number of ways of that we can represent is $4x(2+1) = 12$
5	
	Consider
	AtiB = 7 (1+4i)(1+4i)
	(OR)
	7(1-41)(1-41)
	(OF)
1	7(1+4i)(1-4i)
	Multiplying A+iB with units in Z[i]
	(we know that ±1 and ±i are mrits in ZCiJ)
	(We know that
	(ase 1: taking unit as (+1) (ase 3: taking unis as(+c')
	(ase 1:- taking unit as (+1) (ase 3:- taking unis as(+i)) 7(1+4i)(1+4i), 7: (1-4i) (1-4i),
	7(1+4i)(1+4i), 7(1+4i)(1-4i) 7(1+4i)(1-4i)
	7(1+4i)(1-4i),
	7 (1-4i) (1-4i) 7i (144) (1-4i)
	1 1 1 2 2 C ()
_	(are 2: taking unit as (-1) (are 4: taking unit as (-i) -7(1+4i)(1+4i), -7(1+4i)(1+4i), -7(1+4i)(1+4i),
	-7i (1+4i)(1+4i)
	7: (1-41) (1-41)
	-7 (1+4i) (1-4i) [-41 (1+4))

For lace of the 12 casestiwengeting. I music $17^2 \cdot 7^2 = A^2 + B^2$ form. PROOF AMPI dim: to find. 172.72 as a sum of two squares We know that 17 = 1 (mod 4) & 7 = 3 (mod) Also : 17= (1+4i)(1-4i) = (4+i)(4-i) By Mimber of ways B that we can represent is 4X(2+1) = 19 Consider (ip41)(1441) = = 81+41) (20) 7 (1-4i) (1-4i) (90) (ib-1)(in+1)t Multiplying 44:8 with units in 20:7 (we know that ± 2 and ± i are north in ZC13) 元(トル)(ドル) 7 (1-4i) (1-4i), F(1-4i) (1-4i)

F(H4) (1-41)

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3)	$4 Z(6) \rightarrow center g g g$
	4 Z(b) → center & b Z Given. 5 → non-Abelian group J
alleger 2000	AS Z(G) & G, we know that Z(G) Bu is a
e de la companya de l	divisor of 191. We can prove this using the lagranges
	theorem
	q is order 125 (.e a = 125.
	This impries that Z(G) can be of orders 1, 5, 25, 125
	It is also given that $ z(G) \neq 1$.
	It is also obvious that \$(4) cannot be 125 as
	in that case Z(G)' would be same as Gr.
	But of is not-an abelian group. Thus, [ZCG) + 125.
1	
	We are left with orders or and s.
	Case 1:- 2 (Ca) = 25
	This case is not possible as
L.	16/2(6) = 6 / 2(6) = 5
	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
_	This leads to ETE(4) being regarded that G is abelian which is not true
	:. case 1 is not possible
लें .	
	Case 2: (2(G)) = 25 -> this is the only possible
	Case which does not had to G being abelian
2	or ZCa) being non-abelian.
ż	Hence, 12(9) = 25

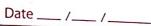


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16)	We know .
	f: F → R is non -zero ring homomorphism and F
	is a field (Criven)
	Ker(f) is a ideal & F.
ę	Fhas only two ideals (0) & (1)
-	
u"	ker(f) = (0) or (i)
	& Case 1: - ker(f)=(1)
-	
	1s & ker(f) => f(10) = 0e & f(1s) = 1a
	-> OR = IR
	Reads to a zero mapping Not possible
3.	Not possible
	f_{α} , g_{α} f_{α} f_{α} f_{α}
	(ase 2:- ker (f) = LO)
. ,	This is a possible case.
ı	∴ ker(f)=fo} ⇒ f is injective.
	weep, jed
	TRUE
1 C)	Suppose R is the subring of field of f.
	Suppose R is the subring of field of F. if a + 0 \in R then a - E R as a - E F & a.a = 1 C R we know that Ring is closed under multiplication
	we know that Ring is closed under multiplication
	: at ER
-	
	We know that field is commutative king which contains
	the mutiplicative inverse of all elements exist
	the metiplicative inverse of all elements exist. This is also true for R.
	i. Ru a field:
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	- July 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
1.e.	161 = 23
	INP know that any group I mine order is cyclic
-	We know that any group of prime order is cyclic and 93 is a prime number.
	We also know that every cyclic group is abelian-
	10 C C C C C C C C C C C C C C C C C C C
	Thus is abelian geoup.
	TRUE
1a.	For ZXZ, the additive identity snould be (0,0)
	But we see that
· -	$(a, o) \cdot (o, b) = (o, o)$ is true even when
	$a \neq 0$ and $b \neq 0$
91	(10.1.1.
	For ZXZ to be integral domain, (0,0) shout have
	no pero divisor.
	But the above statement is not frue in our case
	Thus
<u> </u>	Zxz is non an integral domain
	FALSE
	Commander of demain
1d.	Suppose R is an integral domain
1	Also Suppose R[z]/x \(\gamma\) not a field Thus, (z) is no maximal ideal
	Julia (1) 20 Marious income
	We also know that R has no zero divisors.
V	June (2) is a prime joka
	FALSE





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	among alter 18 1 manufactures and 18 1 manuf
6)	Z[J-2] - integral domain
	We need to make year 35 to 7 has division along the
	We need to prove that $2[J-2]$ has division algorith $N(x) = a^2 + 2b^2$ (here $x = a + bJ-2$ $\in \mathbb{Z}[J-2]$)
	(NEX 1- 4+ 00- = E = W 20)
	Suppose R= 255-27
	We wild to chack that & a bell and Br b non-zero
	7 g, y St. a=baty (whose &s is zeen on MC) SNG)
	Suppose $R = Z[J-2]$ We need to check that $\forall a, b \in R$, and the b non-zero, \overline{f}, q, v st. $a = bq + v$ (where $\& r$ is zero or $N(x) \in N(b)$)
F 2.	$x, y \in R, y \neq 0 \longrightarrow assume.$
	The state of the s
	AS e(i) is supplied of (-> or co(i) or to
	AS $R(i)$ is subfield of $C \rightarrow \chi \in R(i)$, $\chi \neq 0$ has multiplicative inverse
	nus much got case or 17 wasse
	let z= xy1 ER(i) , x,y & Z[J=2].
	let z= xy1 & R(i), x,y & Z[J-2], W= C+ d J-2 & Z[J2]
	·
	Suppose $Z = a+b\sqrt{-2}$, $a,b \in \mathbb{R}$ (here $ a-b \leq 1$ & $ b-d \leq \frac{1}{2}$)
	[b-d) \(\frac{2}{1}\)
	We see that, $Z = w + (z - w)$
	•
- 5	