Assignment - 4 (submission deadline: 2 April, 2021)

Note: Unless otherwise stated, notation used is as defined in the class.

1. Which of the following groupoids are semigroups? Which are groups?
   (a) $(\mathbb{N}, \star)$ where $a \star b = ab$ for all $a, b \in \mathbb{N}$.
   (b) $(\mathbb{N}, \star)$ where $a \star b = b$ for all $a, b \in \mathbb{N}$.
   (c) $(\mathbb{Z}, \star)$ where $a \star b = a - b$ for all $a, b \in \mathbb{Z}$.
   (d) $(\mathbb{Z}, \star)$ where $a \star b = a + b + ab$ for all $a, b \in \mathbb{Z}$.
   (e) $(\mathbb{R}, \star)$ where $a \star b = a|b|$ for all $a, b \in \mathbb{R}$.
   (f) $(\mathbb{R}, \star)$ where $a \star b = 2^a b$ for all $a, b \in \mathbb{R}$.
2. Let $(G, \star)$ be a group and $a, b \in G$. Suppose that $a^2 = e$ and $a \star b \star a = b^7$. Show that $b^{48} = e$.
3. Let G be a group generated by the elements a and b such that $o(a) = 4, a^2 = b^2$, and $ba = a^3b$. Find $o(b)$ and $|G|$.
4. If $G = \langle g \rangle$ is a cyclic group of order 30, then find all distinct elements of (a) order 5 (b) order 6.
5. Let (G, $*$) be a group and $a, b \in G$ where $b \neq e$. If $o(a) = 3$ and $a * b * a^{-1} = b^2$ find $o(b)$.
6. Justify your answer: $(Q^+, \star)$ is not a abelian group where $a \star b = \frac{ab}{2} \ \ \forall a, b \in \mathbb{Q}$
7. Let S be the set of all roots of the equation $x^5 = 1$. Does S forms a commutative group w.r.t multiplication?
8. Let $G = (\mathbb{Z}, +)$ and $H = (3\mathbb{Z}, +)$. Find all the distinct right cosets of H.
9. Find the order of the permutation given below and check if it is even or odd permutation. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 3 & 5 & 1 & 2 \end{pmatrix}$
10. Find fg, gf, f^{-1}, g^{-1} where f $= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 5 & 6 & 1 \end{pmatrix}$, g $= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 4 & 5 & 3 & 2 \end{pmatrix}$.

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