

## Graph Theory and Algorithms

### Tutorial Sheet- 4

- (1) Prove or disprove:
  - (i) Every Eulerian simple graph with an even number of vertices has an even number of edges.
  - (ii) Every Eulerian bipartite graph has an even number of edges.
- (2) Which of the following are Hamiltonian graphs:  $K_{3,4}$ ,  $K_{3,3}$ , and the Petersen graph.
- (3) Let  $G$  be a bipartite graph with partite sets of different sizes. Prove that  $G$  is not Hamiltonian
- (4) Find a non-Hamiltonian graph  $G$  with the property that  $G - v$  is Hamiltonian for every vertex  $v$  in  $G$ .
- (5) Let  $G$  be a graph on  $n \geq 2$  vertices. Suppose  $\delta(G) \geq \frac{n-1}{2}$ . Prove that  $G$  contains a Hamiltonian path.
- (6) What is the maximum number of edges that an  $n$ -vertex simple non-Hamiltonian graph may have?
- (7) The line graph  $L(G)$  of a simple graph  $G$  is a graph whose vertex set is  $E(G)$  and two vertices in  $L(G)$  are adjacent if the corresponding edges have a common end vertex in  $G$ . Prove that line graph of an Eulerian graph is Hamiltonian.
- (8) Prove that cartesian product of two Hamiltonian graphs is a Hamiltonian graph.
- (9) Prove or disprove:  $Q_n$  is Hamiltonian for all  $n \geq 2$ .