MODERN ALGEBRA: ASSIGNMENT

TOTAL MARKS 10

DEADLINE OF SUBMISSION : 5TH APRIL, 2022 AT 23:59

(1) Consider the ring $\mathbb{Z}[i]$ and let $p = 11213 = 82^2 + 67^2$ be a prime integer.
(i) Find a maximal ideal I in $\mathbb{Z}[i]$ which contain 11213 with justification.
(ii) Find all of the irreducible elements α in $\mathbb{Z}[i]$ which divide 11213 in that ring.
(iii) Prove that $\mathbb{Z}[i]/I$ is isomorphic to $\mathbb{Z}/11213\mathbb{Z}$.

[3]

(2) Show that $R = \mathbb{Z}[\sqrt{-5}]$ is not and UFD. Give an example of an element in R which is irreducible but not prime. [2]

(3) Suppose that R is a PID. Suppose that a, b are nonzero elements of R and that they are relatively prime. Prove that $(a) \cap (b) = (ab)$. Furthermore, consider the map $\phi : R/(ab) \longrightarrow R/(a) \times R/(b)$ defined by $\phi(r + (ab)) = (r + (a), r + (b))$ for all $r \in R$. Prove that ϕ is a well-defined map and that it is a ring isomorphism. [3]

(4) Consider the polynomial $f(x) = 3x^5 + 15x^4 - 20x^3 + 10x + 20 \in \mathbb{Z}[x]$. Show that it irreducible over $\mathbb{Q}[x]$? Is it irreducible over $\mathbb{Z}[x]$? Justify your answer. [2]