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FINTECH 545

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Project 2

Problem 1

A. Calculate the Arithmetic Returns. Remove the mean, such that each series has 0 mean. Present the last 5 rows and the total standard deviation.

Date	SPY	AAPL	EQIX
12/27/2024	-0.011492	-0.014678	-0.006966
12/30/2024	-0.012377	-0.014699	-0.008064
12/31/2024	-0.004603	-0.008493	0.006512
1/2/2025	-0.003422	-0.027671	0.000497
1/3/2025	0.011538	-0.003445	0.015745

- Standard Deviation:
  - SPY: 0.008077
  - AAPL: 0.013483
  - EQIX: 0.015361

Computed the next day's price divided by the current day's price to calculate returns. Subtracted out the mean from the returns after computing the arithmetic return. Then computed the standard deviations on each stock based on the returns.

B. Calculate the Log Returns. Remove the mean, such that each series has 0 mean. Present the last 5 rows and the total standard deviation.

Date	SPY	AAPL	EQIX
12/27/2024	-0.01152	-0.01468	-0.00687
12/30/2024	-0.01241	-0.0147	-0.00797
12/31/2024	-0.00458	-0.00843	0.006602
1/2/2025	-0.00339	-0.02793	0.000613
1/3/2025	0.011494	-0.00336	0.015725

- Standard Deviation:
  - SPY: 0.008078
  - AAPL: 0.013446
  - EQIX: 0.015270

Computed the natural logarithm of the ratio between the next day's price and the current day's price to calculate logarithmic returns. Subtracted out the mean from the returns after computing the logarithmic return. Then computed the standard deviations on each stock based on the returns.

Problem 2

A. Calculate the current value of the portfolio given today is 1/3/2025

PV = SPY (100 * ≈ $591) + AAPL (200 * ≈ $243) + EQIX (150 * ≈ $959)

PV = $251,862.50

B. Calculate the VaR and ES of each stock and the entire portfolio at the 5% alpha level assuming arithmetic returns and 0 mean return, for the following methods:

a. Normally distributed with exponentially weighted covariance with lambda=0.97

	VaR	**ES**
SPY	827.85	1038.16
AAPL	946.08	1186.42
EQIX	2933.51	3678.74
Portfolio	3856.32	4835.98

Used exponentially weighted covariance to create the covariance matrix, with the decay rate set at lambda=0.97. Fitted a normal distribution on the returns. Finally, the VaR and ES were calculated for individual stocks and the entire portfolio.

b. T distribution using a Gaussian Copula

	VaR	**ES**
SPY	775.20	1036.30
AAPL	1032.19	1469.24
EQIX	3400.29	4887.77
Portfolio	4407.74	6140.67

Implemented a t-distribution-based risk model using a Gaussian copula approach. First, it fits t-distributions to each stock's returns, then simulates correlated uniform variables using a multivariate normal distribution and correlation matrix before transforming them back to t-distributed returns. Finally, the VaR and ES were calculated for each stock and the overall portfolio.

c. Historic simulation using the full history.

	VaR	ES
SPY	873.32	1088.41
AAPL	1082.07	1452.52
EQIX	3672.82	4757.45
Portfolio	4580.03	6118.67

Performed a historical simulation by calculating profit and loss values for individual stocks based on historical returns. Then aggregated these values to compute the portfolio's historical profit and loss. Finally, the VaR and ES were calculated for individual stocks and the entire portfolio.

C. Discuss the differences between the methods.

The normally distributed with exponentially weighted covariance assumes a normal distribution with more recent events attributing to a higher weight. Normal distribution can underestimate tail risk and moreover assumes a linear dependence. The t-distribution improves upon the normal distribution for cases with fat tails. The t-distribution is better at capturing events with excess kurtosis. However, the t-distribution still assumes a linear dependence. The historical simulation assumes no distribution, therefore no linear dependence, but is still limited due to the dependence on historical data. If the historical data does not contain extreme market conditions the historical simulation can still underestimate risk.

Problem 3

A. Calculate the implied volatility

Implied Volatility: 0.3351

B. Calculate the Delta, Vega, and Theta. Using this information, by approximately how much would the price of the option change if the implied volatility increased by 1%. Prove it.

- Delta: 0.6659
- Vega: 5.6407
- Theta: -5.5446

Used the Vega approach to compute the expected price change as described below:

Call using Vega

$$\Delta P = Vega * \ \Delta\, Implied\ Volatility * 100$$

$$\Delta P = 0.056407 * \ 0.01 \ * 100$$

$$\Delta P = 0.0564$$

Then I used the generalized Black Scholes Merton with the new implied volatility = original implied volatility + 1% as described below:

Call using gbsm:

$$Call = Se^{(b-r)T}\Phi(d1) - Xe^{-rT}\Phi(d2)$$

$$Call = 31e^{(-0.1)0.25}\Phi(\frac{\ln\left(\frac{31}{30}\right)+\left(\frac{0.3451^2}{2}\right)}{0.3451*\sqrt{0.25}}) - 30e^{-0.1*.25}\Phi(\left(\frac{\ln\left(\frac{31}{30}\right)+\left(\frac{0.3451^2}{2}\right)}{0.3451*\sqrt{0.25}}\right) - 0.3451 * \sqrt{0.25})$$

$$Call = 3.056$$

C. Calculate the price of the put using Generalized Black Scholes Merton. Does Put-Call Parity Hold?

I calculated the Put price as described below:

Put using gbsm:

$$Put = Xe^{-rT}\Phi(-d_2) - Se^{(b-r)T}\Phi(-d_1)$$

$$Put = 30e^{-0.1*0.25}\Phi(-(\left(\frac{\ln\left(\frac{31}{30}\right)+\left(\frac{0.3351^2}{2}\right)}{0.3351*\sqrt{0.25}}\right) - 0.3451 * \sqrt{0.25})) -$$

$$31e^{(-0.1)0.25}\Phi(-\frac{\ln\left(\frac{31}{30}\right)+\left(\frac{0.3351^2}{2}\right)}{0.3351*\sqrt{0.25}})$$

$$Put = 1.2593$$

The put value should balance the put-call parity in the following equation so the put price from the gbsm should equal the missing P value:

Put-call parity:

$$C_0 + X * e^{-r*t} = P_0 + S_0$$

$$3.00 + 30 * e^{-0.10*0.25} = P_0 + 31$$

$$32.2593 = P_0 + 31$$

$$P_0 = 1.2593$$

Since the Put price is the same Put-Call parity holds.

D. Given a portfolio of
   a. 1 call
   b. 1 put
   c. 1 share of stock

   Assuming the stock's return is normally distributed with an annual volatility of 25%, the expected annual return of the stock is 0%, there are 255 trading days in a year, and the implied volatility is constant. Calculate VaR and ES for a 20 trading day holding period, at alpha=5% using:

   d. Delta Normal Approximation

VaR	ES
4.75	5.96

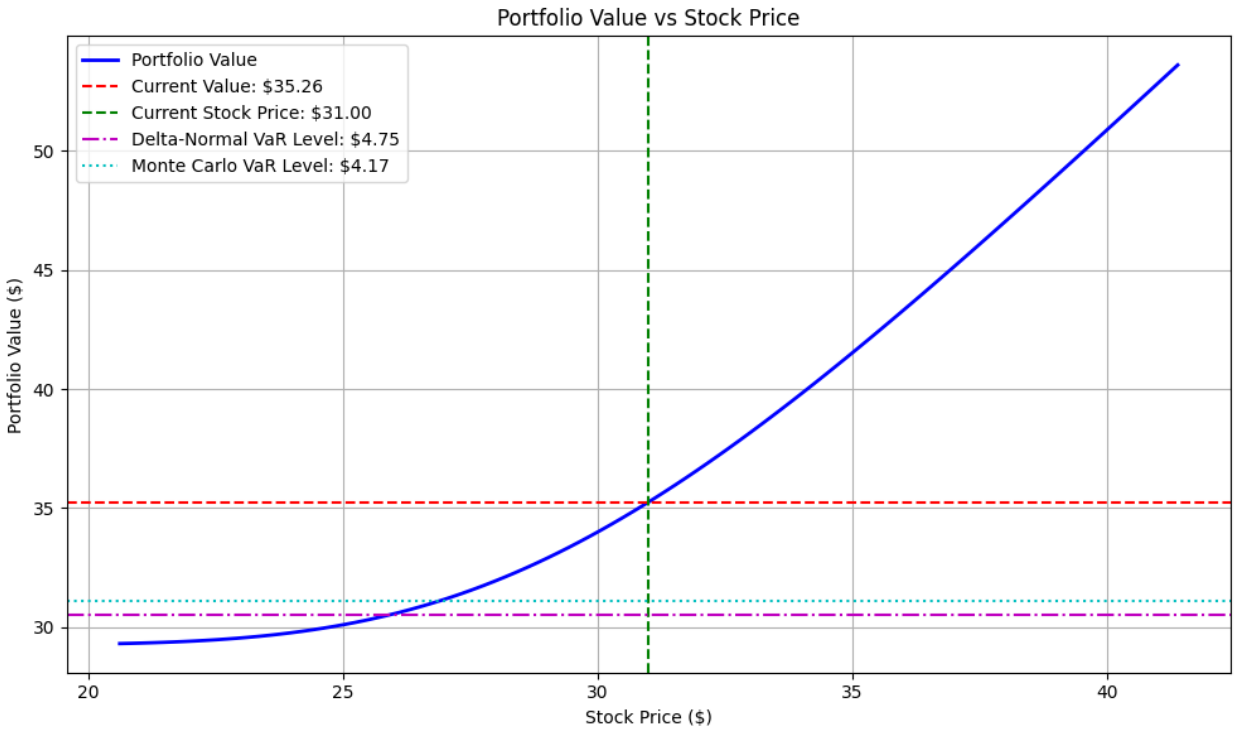
Used the delta normal approach as a linear approximation to estimate how portfolio value changes with stock price movements. Computationally this was very inexpensive.

   e. Monte Carlo Simulation

VaR	ES
4.17	4.60

Used the Monte Carlo approach using 100,000 possible stock price paths. For each simulated path, it recalculates portfolio value at the end of the holding period and determines the distribution of potential value changes. Computationally this was fairly expensive.

E. Discuss the differences between the 2 methods. Hint: graph the portfolio value vs the stock value and compare the assumptions between the 2 methods.



The underlying instrument has a non-linear payoff structure. Delta Normal will assume a linear distribution and will overestimate the risk. The payoff is positively convex surface. The positive convexity, gamma, means that as the price of the underlying falls, the change at which the loss decreases also decreases. The tangent line approximation in the delta normal approach is accurate near the current stock price but deviates significantly for larger price moves. Therefore, a delta normal on a long call will overestimate the risk. Monte Carlo takes convexity into account and leads to a more accurate assessment of risk compared to the delta normal approach. However, the delta normal approach is a significantly inexpensive compared to the Monte Carlo approach in terms of computation.