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FINTECH 545

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Final Project

Part 1:

Several steps were required prior to the calculation of holding period risk and returns attribution. The first was to calculate the daily arithmetic returns based on the price movement. The risk-free returns were subtracted from the daily stock returns to give the daily excess returns, i.e. the returns above the risk-free rate. Using the data prior to the holding period, the beta and alpha for each stock was computed, based on the market index, SPY, via a regression. Then the initial weights were calculated on a value-weighted basis at the start of the holding period, i.e., 1/2/2024. These initial weights were then stored and used throughout the project. The daily returns were then multiplied by the portfolio weights to give the daily weighted returns of the portfolio. These daily weighted returns were then summed across the dates to give the total daily portfolio excess returns, which were then multiplied by the portfolio beta to give the systematic returns. The idiosyncratic returns were simply the total portfolio excess returns, less the systematic returns. The total portfolio volatility was then computed as the annualized, based on 255 trading days, standard deviation of the portfolio's daily excess returns. The systematic and idiosyncratic risk attribution was computed using a regression of the systematic and idiosyncratic daily returns. The regression coefficients were then used to compute the component systematic and idiosyncratic risk by multiplying the coefficients by the total portfolio volatility.

Portfolio	Portfolio Beta	Total Period Excess Return %	Systematic Return %	Idiosyncratic Return %	Risk Free %	Total Portfolio Vol %	Component Systematic Vol %	Component Idiosyncratic Vol %
A Original	0.959041	7.874842	19.666727	-11.791884	5.227718	11.297868	10.655499	0.64237
B Original	0.914901	15.15034	18.761557	-3.611216	5.227718	10.932711	10.032225	0.900486
C Original	0.966868	23.041758	19.827221	3.214538	5.227718	12.243802	11.246533	0.997269
Total Original	0.946809	14.994702	19.415886	-4.421184	5.227718	10.948042	11.156675	-0.208632

Portfolio C has the highest return and alpha. This portfolio also takes the highest risk having the highest volatility. Portfolio B has the lowest comparative risk, having the lowest volatility among the portfolios.

The portfolio C also has a Beta closest to 1, indicating the portfolio returns are the most sensitive to that of the overall market. Portfolio A with the lowest idiosyncratic return suggests poor stock selection and underperformance relative to the CAPM expectation. The Total portfolio comprises the lowest contribution of idiosyncratic risk, suggesting much of the risk lies in the systematic or market risk. The Total portfolio containing all the holdings within the other portfolio has the most diversification among the portfolios so one would expect the portfolio to have a Beta close to 1 and lower idiosyncratic volatility due to the portfolio movements being largely systematic. Moreover, Portfolio C seems to have the best stock selection as the portfolio contains the highest overall idiosyncratic returns among the portfolios, meaning that the portfolio can produce an alpha. While this portfolio did have the largest idiosyncratic return, this was not without taking on the most risk, with the highest portfolio volatility.

Part 2:

To create the Sharpe Ratio optimization, the expected returns had to be determined. This was determined through the expected CAPM return. The underlying assumption is the expected market return would be the average return of SPY during the training period, r_m. Another large assumption is the risk-free rate would also be the average risk-free rate during the training period, r_f. These assumptions allow for the creation of expected returns for each of the individual stocks, $E(r_i) = r_f + \beta_i r_m$. To determine the selection of weights that maximize the Sharpe Ratio, certain conditions were required. The conditions set were no negative weights, meaning that short selling of securities was not allowed, and the weights must sum to 1, meaning allocation of securities could not surpass 100%, i.e., a margin account. Next the negative Sharpe Ratio algorithm was used to minimize the negative Sharpe Ratio, which in turn would maximize the Sharpe Ratio.

Portfolio	Portfolio Beta	Total Period Excess Return %	Systematic Return %	Idiosyncratic Return %	Risk Free %	Total Portfolio Vol %	Systematic Vol %	Realized Idiosyncratic Vol %	Predicted Idiosyncratic Vol %
A Original	0.959041	7.874842	19.666727	-11.791884	5.227718	11.297868	12.117795	5.867757	4.278479
A Optimal	1.013826	20.289414	20.790188	-0.500775	5.227718	11.699512	12.810024	4.596329	4.933395

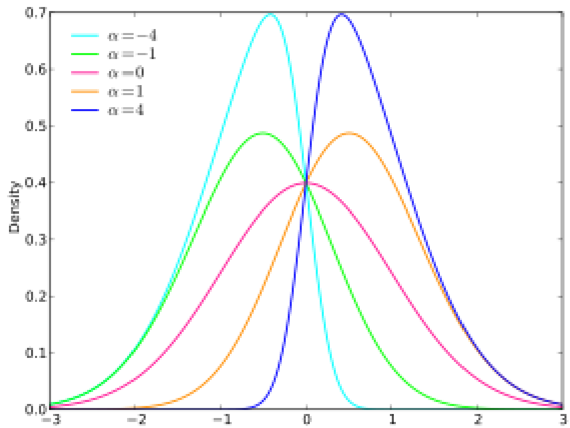
B Original	0.914901	15.15034	18.761557	-3.611216	5.227718	10.932711	11.560068	5.857195	3.757153
B Optimal	1.011493	21.09492	20.742345	0.352575	5.227718	11.699235	12.780545	4.708023	4.076027
C Original	0.966868	23.041758	19.827221	3.214538	5.227718	12.243802	12.216684	4.857763	3.850632
C Optimal	1.018251	25.36075	20.880917	4.479833	5.227718	12.749928	12.865927	4.752671	4.473697
Total Original	0.946809	14.994702	19.415886	-4.421184	5.227718	10.948042	11.963238	4.371185	2.302339
Total Optimal	1.000123	20.781178	20.509169	0.272009	5.227718	11.389648	12.636872	3.12271	3.149573

In all the optimized portfolios the Beta increased, indicating more market sensitivity. Interestingly, the optimized portfolios seemed to estimate the idiosyncratic risk better than the original portfolio, with the optimized portfolio having higher predicted idiosyncratic risk but lower deltas between the predicted and realized. The optimized portfolios produce substantially higher returns without taking on much more risk. For example, Portfolio A is a prime example of how an optimized Sharpe Ratio can substantially increase the return, ~7.9% in the original to ~20.3% while only taking on less than 0.5% volatility. In fact, through a proper allocation of weights to maximize the Sharpe Ratio, every portfolio was able to increase the total return when compared to the original portfolio. These increases in portfolio returns came at the cost of increase portfolio risk, emphasizing the tradeoff made by the Sharpe Ratio between risk and return. The portfolio will increase the return at the cost of overall risk to the portfolio.

Part 3:

Financial assets often produce characteristics that break normality assumptions. These characteristics can take the form of skewness and/or excess kurtosis. With normality assumptions broken, often traditional models can underestimate extreme risks. The Skew Normal and Normal Inverse Gaussian distributions address some of these limitations, offering enhanced flexibility for modeling financial assets. These models seek to better model these non-normal distributions especially in skewness and/or kurtosis, making them critical for accurate risk measurement.

The Skew Normal distribution can capture asymmetric behavior, making it useful for modeling financial returns that exhibit different tail behaviors. The Skew Normal distribution introduces a shape parameter α. This shape parameter, in addition to the scale and location parameters, allow for a more accurate representation of asymmetric distributions. Its density function skews left or right depending on α. Therefore, when α=0, the Skew Normal transforms into the Normal distribution. Furthermore, when α>0 the distribution is scaled to the right, and α<0 the distribution is scaled to the left. Lastly, the magnitude of |α|, controls the degree of skewness, as seen below.



Financial assets often display skewness in their returns. For example, a stock might exhibit a higher probability of large gains than large loses. Therefore, in the cases of a skewed distribution, the Skew Normal model improves estimates of tail risk metrics like VaR or ES. An important limitation of the Skew Normal model is the failure to address fat tails or excess kurtosis. This inability to model these fat tails limits its utility in cases with significant kurtosis.

The Normal Inverse Gaussian distribution is a model with four parameters μ, δ, α, β. It arises as a variance-mean mixture of normal and inverse Gaussian distributions. Importantly, the model seeks to address both skewness and kurtosis. The Normal Inverse Gaussian distribution can capture excess kurtosis and skewness. Therefore, the Normal Inverse Gaussian model improves upon the Skew Normal model in modeling those financial assets that exhibit fat tails and skewness in their distributions. For example, in the case of severe market crashes where the distribution might contain both negative skewness and excess kurtosis, the Normal Inverse Gaussian model would assign higher probabilities to such events.

The Skew Normal and Normal Inverse Gaussian distributions can be used to model asset returns with asymmetric behavior. For example, the models can more accurately attribute risk to the assets with higher probability of large gains than large losses. Moreover, while the Skew Normal addresses asymmetry, the Normal Inverse Gaussian's ability to model both skewness and kurtosis makes it important for identifying extreme risks, such as market crashes or sudden volatility spikes. Therefore, by accurately capturing the tail behavior, Normal Inverse Gaussian distributions can provide more realistic VaR and ES estimates, both of which are crucial metrics in risk management. Through the integration of these distributions into risk frameworks, these models enhance the identification of stress scenarios, and derivative pricing, ultimately fostering resilience in portfolio management.

Part 4:

Each stock was fit on all the various distributions and was scored using AIC. The AIC provides a goodness-of-fit for each distribution giving a measure of relative model quality when judged against peers. The distribution with the lowest AIC was then selected as the best fit model for each individual stock. These fitted distributions were then employed on the individual stocks to create simulations of returns used in the VaR and ES metrics.

Gaussian Copula

	VaR	**ES**
A	1.54%,	2.03%
B	1.39%,	1.86%
C	1.51%,	2.00%
Total	1.46%,	1.89%

Multivariate Normal

	VaR	**ES**
A	1.36%,	1.70%
B	1.49%,	1.87%
C	1.43%,	1.79%
Total	1.37%,	1.72%

The Gaussian Copula metrics better incorporates fatter tails and produces more conservative risk estimates than the Multivariate normal. These assumptions do appear in the portfolios with the Gaussian Copula seemingly have more conservative estimates on almost all portfolios in both VaR and ES. The discrepancies arise from the normality assumptions made in the multivariate normal approach. These assumptions cause the VaR and ES models to derive directly from the portfolio standard deviation, whereas the Gaussian Copula approach simulates asset returns from their individually fitted distributions. If some assets exhibit excess kurtosis the simulation will generate more extreme individual asset returns than a normal distribution would simulate. When aggregated into the portfolio, these extreme returns, even when combined using a Gaussian dependence structure, can lead to a portfolio return distribution with fatter tails than the multivariate normal model allows. While the Gaussian Copula method improves on the multivariate normal by allowing flexible individual distributions, the method still assumes a Gaussian dependence structure, meaning it does capture tail dependence. Moreover, financial markets often exhibit tail dependence, where assets tend to experience extreme negative returns simultaneously more often than predicted by simple correlation. Therefore, both the Gaussian Copula and multivariate normal approaches remain limited in fully modeling the risk metrics.

Part 5:

The Risk Parity portfolios were implemented by using the individual stock best fit distributions from the previous section. These distributions allowed for the creation of simulations which were used to achieve an equal contribution of the assets to the ES of the portfolio.

Portfolio	Portfolio Beta	Total Period Excess Return %	Systematic Return %	Idiosyncratic Return %	Risk Free %	Total Portfolio Vol %	Systematic Vol %	Component Systematic Vol %	Component Idiosyncratic Vol %
A Original	0.959041	7.874842	19.666727	-11.791884	5.227718	11.297868	12.117795	10.655499	0.64237
A Optimal	1.013826	20.289414	20.790188	-0.500775	5.227718	11.699512	12.810024	11.960745	-0.261233
A Risk Parity	0.907188	11.196286	18.603387	-7.4071	5.227718	10.663212	11.46261	10.014073	0.649139
B Original	0.914901	15.15034	18.761557	-3.611216	5.227718	10.932711	11.560068	10.032225	0.900486
B Optimal	1.011493	21.09492	20.742345	0.352575	5.227718	11.699235	12.780545	11.88011	-0.180875
B Risk Parity	0.870281	19.390555	17.846553	1.544002	5.227718	10.450466	10.996282	9.332187	1.118279
C Original	0.966868	23.041758	19.827221	3.214538	5.227718	12.243802	12.216684	11.246533	0.997269
C Optimal	1.018251	25.36075	20.880917	4.479833	5.227718	12.749928	12.865927	11.96472	0.785208
C Risk Parity	0.98125	26.761684	20.122155	6.639529	5.227718	12.576385	12.398411	11.518758	1.057627
Total Original	0.946809	14.994702	19.415886	-4.421184	5.227718	10.948042	11.963238	11.156675	-0.208632
Total Optimal	1.000123	20.781178	20.509169	0.272009	5.227718	11.389648	12.636872	12.266246	-0.876598
Total Risk Parity	0.931021	20.140962	19.092133	1.048829	5.227718	10.918321	11.763755	10.984829	-0.066509

The optimized Sharpe Ratio portfolios aimed for the highest risk-adjusted return, which can lead to concentrated positions in assets with high expected returns and low correlation. Risk parity, on the other hand, prioritizes balanced risk contributions, potentially sacrificing some expected return for better diversification, especially in the cases of tail events when using ES. An important consideration is the limitation of a CAPM based evaluation for performance attribution on the Risk Parity portfolio. The Risk Parity portfolio may be taking positions to balance risks related to skewness or excess kurtosis captured by the advanced distributions used in its construction. When analyzed through the narrow perspective of CAPM, these deliberate risk-balancing choices might display significant changes to the idiosyncratic return or risk components. This does not necessarily imply poor stock selection in the traditional CAPM sense but rather reflects portfolio characteristics related to the non-CAPM risk factors determined by ES. For example, in Portfolio B the Risk Parity portfolio increases the idiosyncratic volatility component but decreases the overall portfolio volatility. In fact, in almost all cases the Risk Parity portfolio decreased the overall portfolio volatility, showing that the portfolio did seek to produce a portfolio with a more balanced perspective on risk and not over allocate certain stocks with high volatility.

Conclusion:

The findings within the project often highlight the large assumptions made in the statistical distributions, risk estimates, and/or frameworks that are used when trying to model ex-post financial assets using ex-ante data. Regime changes happen, assets today may have fully priced in information that tomorrow may be worthless. Recently, large fluctuations in the market due to uncertainty in trade relations emphasize the inability of the market to predict the future. These models help in understanding the exposure a portfolio may have to certain fluctuations. Therefore, an understanding of the various assumptions and overall limitations of the models are as important today as ever to protect one’s portfolio against large fluctuations in the underlying value of the assets.

References:

Thorsten Dickhaus. *Lectures on Dependency: Selected Topics in Multivariate Statistics*. Springer, 2022.