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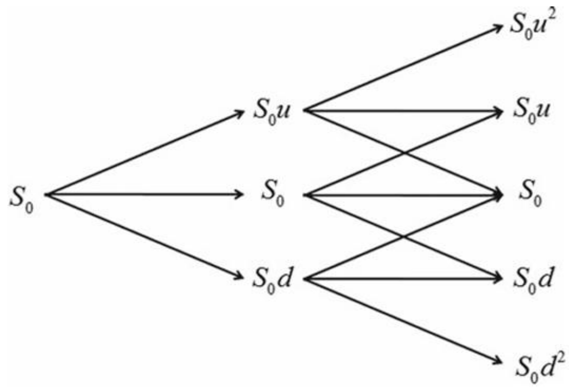
Investigation of trinomial tree models for option pricing



The trinomial tree model as proposed by Kamrad and Ritchken, extends the binomial tree model by adding another path. Whereas the binomial tree model only has an up and down path, the trinomial tree model adds the middle or stable path as well. The values at each node can be found by multiplying the value by its respective path:

$$u = e^{\lambda\sigma\sqrt{\delta t}}$$

$$d = m$$

$$d = \frac{1}{u}$$

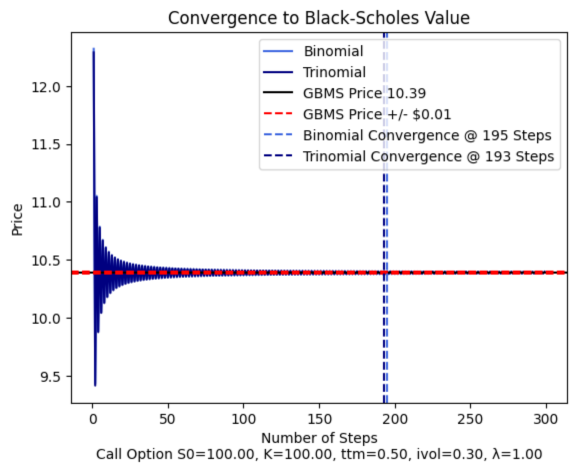
The respective probabilities for each path are the following:

$$p_u = \frac{1}{2\lambda^2} + \frac{(r - \frac{\sigma^2}{2})\sqrt{\delta t}}{2\lambda\sigma}$$

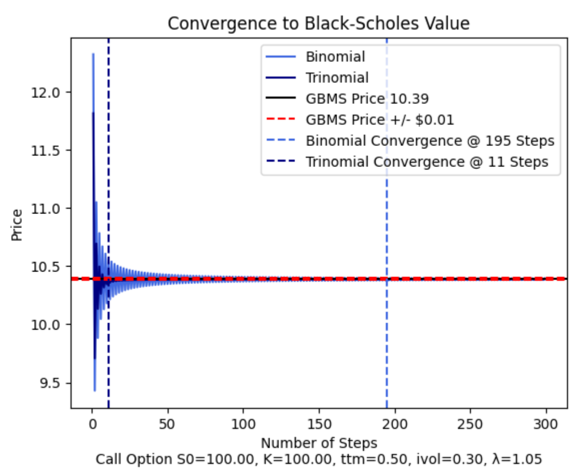
$$p_m = 1 - \frac{1}{\lambda^2}$$

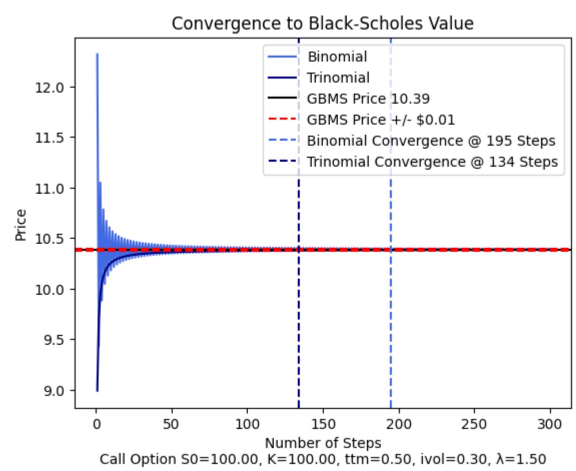
$$p_d = 1 - p_u - p_m$$

An important component for the Kamrad and Ritchken trinomial tree model is the parameter λ. If the parameter λ is equal to 1, then trinomial tree model reduces to a binomial tree model. See the following for a European call option with λ=1:

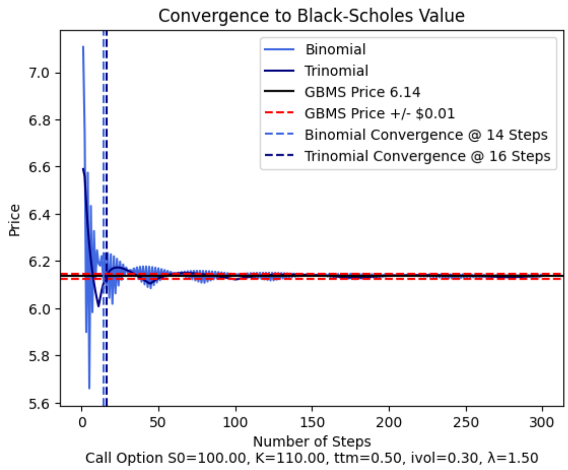


As λ increases, the probability of the middle node increases, and the probability of the up and down node decreases. Furthermore, as λ increases, u increases and d decreases, widening the value between the two nodes. This increases the range of possible asset prices. Therefore, the larger λ amplifies the discrete jumps within the tree, and replicates the geometric Brownian motion of the underlying asset, where the prices evolve multiplicatively rather than additively.





Kamrad and Ritchken state that "any value of λ, λ ≥ 1, yields a feasible set of probabilities" (Kamrad and Ritchken). When λ is too small, this could result in negative probabilities for u or d. The trinomial model is more computational expensive than the binomial model so the trinomial model must converge significantly faster than the binomial model for the advantage to appear.



In this example for an out-of-money option, the binomial model did converge faster than the trinomial model, proving the binomial model should be favored for this case.

Kamrad and Ritchken (1991) state the number of multiplications and additions are as follows:

Model	Number of multiplications	Number of additions
Trinomial	$3n^2$	$2n^2$
Binomial	$n^2 + n$	$\frac{1}{2}(n^2 + n)$

The trinomial model with n steps requires less computation than a binomial model with 2n steps. The authors also found that the trinomial model has less errors even with less computation than the binomial model for an at-the-money option.

References:

Kamrad, Bardia, and Peter Ritchken. "Multinomial approximating models for options with state variables." *Management science* 37.12 (1991): 1640-1652.

Lee, John, et al. "Binomial/Trinomial Tree Option Pricing Using Python." *Essentials of Excel VBA, Python, and R: Volume II: Financial Derivatives, Risk Management and Machine Learning*. Cham: Springer International Publishing, 2023. 313-334.