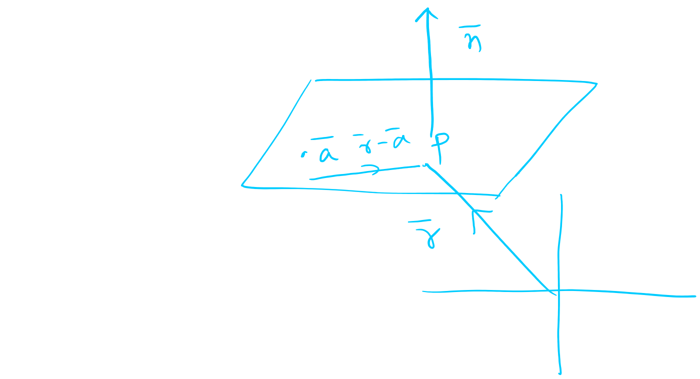


$\bar r = x\hat i + y\hat j + z\hat k$

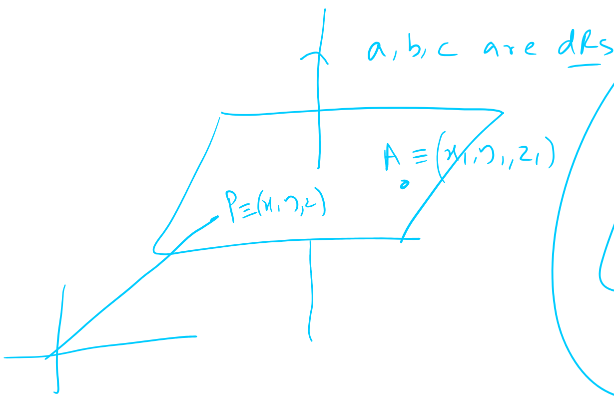
$P \equiv (x, y, z)$

$$\boxed{(\bar r - \bar a)\cdot \bar n = 0}$$

$$\boxed{\bar r \cdot \bar n = q}$$

a, b, c are dRs

$\bar n = a\hat i + b\hat j + c\hat k$



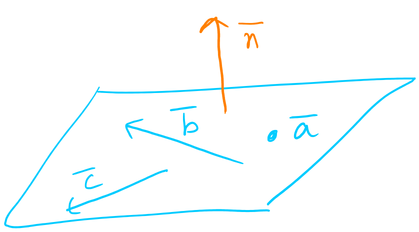
$$\boxed{a(x - x_1) + b(y - y_1) + c(z - z_1) = 0}$$

$$\boxed{ax + by + cz = q}$$

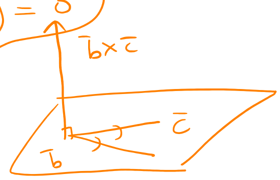
dRs of normal

$$\boxed{(\bar r - \bar a)\cdot \bar n = 0}$$

Q.1



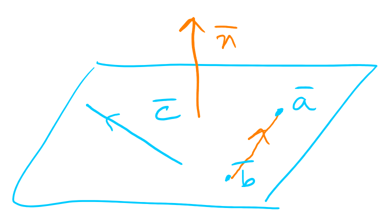
$(\bar r - \bar a)\cdot(\bar b \times \bar c) = 0$



$\bar b \times \bar c \cdot \bar b = 0$

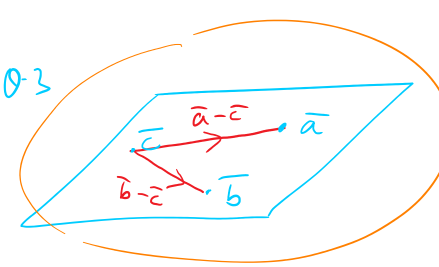
$\bar b \times \bar c \cdot \bar c = 0$

Q.2



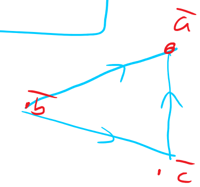
$(\bar r - \bar a)\cdot[(\bar a - \bar b)\times \bar c] = 0$

Q.3

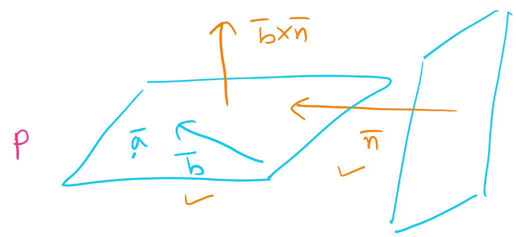


$$\boxed{(\bar r - \bar a)\cdot[(\bar a - \bar c)\times(\bar b - \bar c)] = 0}$$

$$\boxed{(\bar r - \bar a)\cdot\left[(\bar a - \bar c)\times(\bar b - \bar c)\right] = 0}$$



Find eqn of a plane passing through $\bar a$,
// to $\bar b$ & plane is $\perp$ to $\bar r \cdot \bar n = q$.



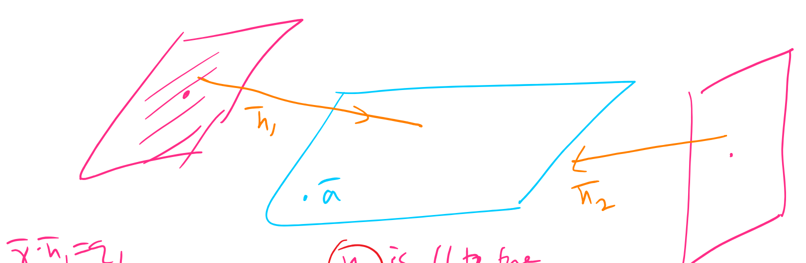
$\bar n$ is // to the required plane.

$$\boxed{(\bar r - \bar a)\cdot\left[\bar b \times \bar n\right] = 0}$$

Find eqn of a plane passing through $\bar a$

& $\perp$ to $\bar r \cdot n_1 = q_1$ & $\bar r \cdot \bar n_2 = q_2$.

(required plane $\perp$ to both $\bar r\cdot\bar n_1 = q_1$ & $\bar r \cdot \bar n_2 = q_2$)



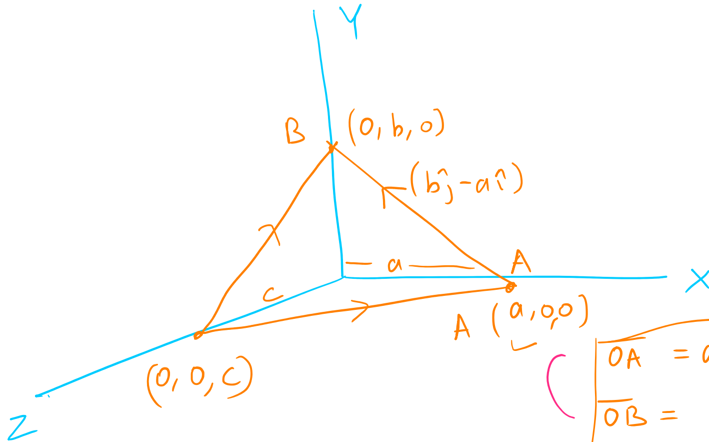
n_1 is // to the required plane.

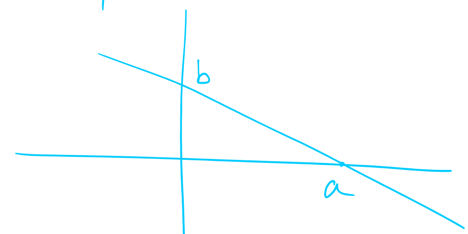
$\bar r \cdot \bar n_2 = q_2$.

n_2 is // to the required plane.

$$(\bar{r}-\bar{a})\cdot(\bar{n}_1\times\bar{n}_2)=0.$$

Find eqn of a plane who is making a, b, c intercepts with X-axis, Y-axis, Z-axis resp.





$$\boxed{\frac{x}{a}+\frac{y}{b}=1}$$

$$\overline{OA}=a\hat{i}$$
$$\overline{OB}=b\hat{j}$$
$$\overline{OC}=c\hat{k}$$

$$(\bar{r}-a\hat{i})\cdot\left[(b\hat{j}-a\hat{i})\times(a\hat{i}-c\hat{k})\right]=0$$

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\a_1&a_2&a_3\\b_1&b_2&b_3\end{vmatrix}$$

$$\cdot\left[ab(-\hat{k})-bc\,\hat{i}+ac(-\hat{j})\right]=0$$

$\hat{i}\times\hat{j}=\hat{k}$

$\hat{j}\times\hat{i}=-\hat{k}$ $\quad\hat{j}\times\hat{k}=\hat{i}$

$\hat{i}\times\hat{i}=\bar{0}$

$$-(\bar{r}-a\hat{i})\cdot\left[bc\hat{i}+ac\hat{j}+ab\hat{k}\right]=0$$

$$\boxed{(\bar{r}-a\hat{i})\cdot\left[bc\hat{i}+ac\hat{j}+ab\hat{k}\right]=0}$$

Cartesian form.

$$\left[(x-a)\hat{i}+y\hat{j}+z\hat{k}\right]\cdot\left[bc\hat{i}+ac\hat{j}+ab\hat{k}\right]=0.$$

$$(x-a)bc+acy+abz=0$$

$$bcx+acy+abz=abc.$$

Divide with abc.

$$\frac{\cancel{bc}\,x}{a\cancel{bc}}+\frac{\cancel{a}c\,y}{\cancel{a}b\cancel{c}}+\frac{\cancel{ab}z}{\cancel{ab}c}=1$$

$$\frac{\cancel{bc}\,x}{a\cancel{bc}} + \frac{\cancel{ac}\,y}{\cancel{a}b\cancel{c}} + \frac{\cancel{ab}\,z}{\cancel{ab}c} = 1$$

learn this $\boxed{\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1}$ ✓✓

$\times$ $(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \left[\frac{1}{a}\hat{i} + \frac{1}{b}\hat{j} + \frac{1}{c}\hat{k}\right] = 1$

Don't learn.

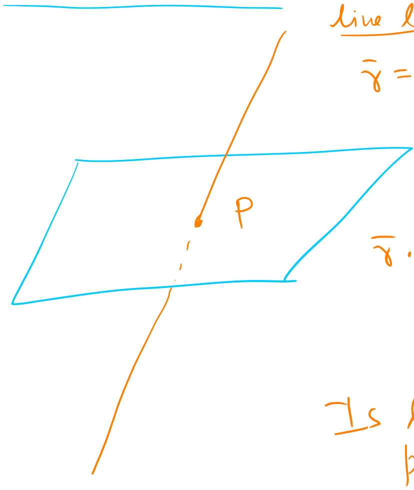
Intercept form of eqn of plane is

$$\boxed{\frac{x}{x_{int}} + \frac{y}{y_{int}} + \frac{z}{z_{int}} = 1}$$

Intersection of a plane & a line.

We will have to learn the process/steps to solve plane & line as it is needed in so many question.

Method-1 (vector form). [Though we prefer Cartesian form]



line ℓ

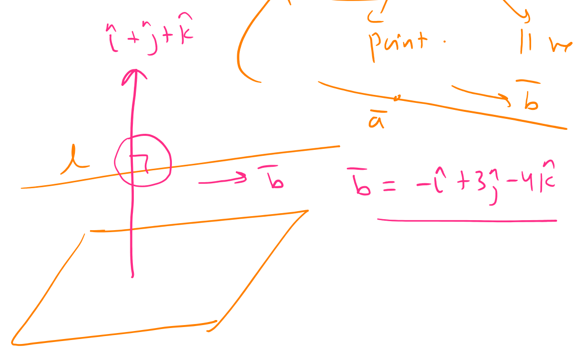
$$\bar{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(-\hat{i} + 3\hat{j} - 4\hat{k})$$

plane P

$$\bar{r}\cdot(\hat{i} + \hat{j} + \hat{k}) = 2.$$

$$\boxed{\bar{r} = \bar{a} + k\bar{b}}$$

point. $\parallel$ vector.



$\bar{b} = -\hat{i} + 3\hat{j} - 4\hat{k}$

Is line $\parallel$ to the plane?

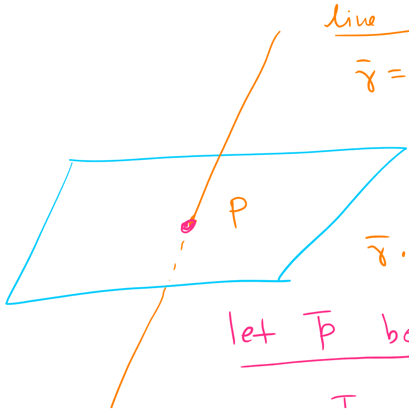
A) yes

B) No.

If $\bar{b}\cdot\bar{n} = 0$, then line $(\bar{r} = \bar{a} + k\bar{b})$ is $\parallel$ to plane $\bar{r}\cdot\hat{n} = q$

else line is not $\parallel$ to plane.

$$(-\hat{i} + 3\hat{j} - 4\hat{k})\cdot(\hat{i} + \hat{j} + \hat{k}) = -1 + 3 - 4 = -2 \neq 0$$



line

$$\bar{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(-\hat{i} + 3\hat{j} - 4\hat{k})$$

plane P

$$\bar{r}\cdot(\hat{i} + \hat{j} + \hat{k}) = 2$$

let P be

let $\bar{p}$ be

Step-1

$$\bar{p} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda_1(-\hat{i} + 3\hat{j} - 4\hat{k})$$

$$\bar{p} = (2-\lambda_1)\hat{i} + (3\lambda_1 - 1)\hat{j} + (1-4\lambda_1)\hat{k}$$

Replace $\bar{p}$ in eqn of plane.

Step-2 $\longrightarrow$ $\bar{p} \cdot [\hat{i} + \hat{j} + \hat{k}] = 2$

$$\left[(2-\lambda_1)\hat{i} + (3\lambda_1 - 1)\hat{j} + (1-4\lambda_1)\hat{k}\right] \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$$

Step-3

$$(2-\lambda_1) \times 1 + (3\lambda_1 - 1) \cdot 1 + (1-4\lambda_1) \cdot 1 = 2.$$

$$-2\lambda_1 + 2 = 2 \Rightarrow$$

$$\boxed{\lambda_1 = 0}$$

$$\bar{p} = 2\hat{i} - \hat{j} + \hat{k} \qquad P \equiv (2, -1, 1).$$

Cartesian form

$$\bar{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$$

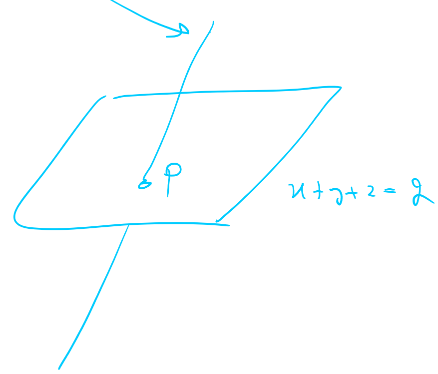
$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$$

Find point of intersection of plane $\boxed{x + y + z = 2}$

& line $\boxed{\dfrac{x-2}{-1} = \dfrac{y-(-1)}{3} = \dfrac{z-1}{-4}}$

$\bar{a} \equiv (2, -1, 1)$

$\bar{b} \equiv (-1, 3, -4)$



$\boxed{\text{Step-1}}$ let us assume P using line. where P is point of Int.

$$\frac{x-2}{-1} = \frac{y+1}{3} = \frac{z-1}{-4} = K.$$

$$x = 2 - K$$
$$y = 3K - 1$$
$$z = 1 - 4K.$$

$$\boxed{P \equiv (2-K, 3K-1, 1-4K).}$$

$$\left.\begin{aligned} y &= 3k-1 \\ z &= 1-4k \end{aligned}\right\}$$

Step-2 Replace P in the eqn of plane.

$$x + y + z = 2$$

$$\rightarrow \quad (2-k) + (3k-1) + (1-4k) = 2$$

Step-3 Solve for k. (Same step as in vector form)

$$-2k + 2 = 2$$

$$-2k = 0$$

$$\underline{\underline{k = 0}}$$

$$\boxed{P = (2,-1,1)}$$

Solve $2x + y - 3z = 1$

& $\rightarrow \dfrac{x}{1} = \dfrac{y-2}{2} = \dfrac{z+1}{3} = k$

$\bar{n} = 2\hat{i} + \hat{j} - 3\hat{k}$

$\bar{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\bar{n} \cdot \bar{b} = 2 + 2 - 9 \neq 0$

$\left(\frac{4}{5}, \frac{14}{5}, \frac{7}{5}\right)$

$$\rightarrow \begin{aligned} x &= x_1 + Ak \\ y &= y_1 + Bk \\ z &= z_1 + Ck \end{aligned}$$

$$\frac{x-x_1}{A} = \frac{y-y_1}{B} = \frac{z-z_1}{C} = k$$

$\dfrac{x_p}{1} = \dfrac{y_p - 2}{2} = \dfrac{z_p + 1}{3} = k \rightarrow$ address of point P

$$P = (x_p, y_p, z_p) = (k, 2k+2, 3k-1)$$

$$2x + y - 3z = 1$$

$$\boxed{2x_p + y_p - 3z_p = 1}$$

$$2(k) + (2k+2) - 3(3k-1) = 1$$

$2(k) + (2k+2)$

$4k - 9k + 2 + 3 = 1$

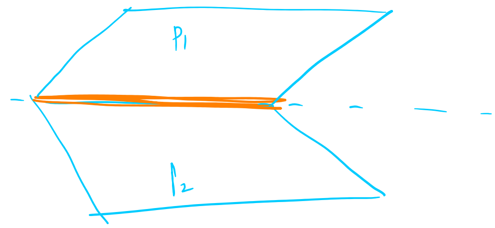
$-5k = -4$

$k = 4/5$

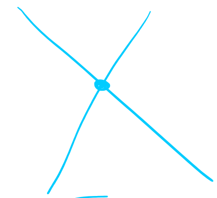
$P = \left(\frac{4}{5}, \frac{4}{5} + 2, \frac{12}{5} - 1\right)$

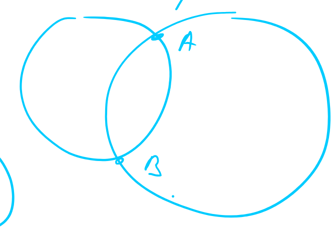
$= \left(\frac{4}{5}, \frac{14}{5}, \frac{7}{5}\right)$

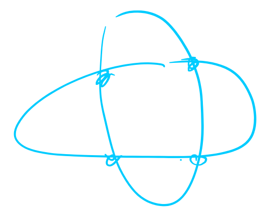
Family of Planes



Line of Intersection of two plane.







Family of plane passing through intersection of two given plane. → Line of Int.

is

$$\boxed{P_1 + \lambda P_2 = 0}$$

$$l_1 + k\,l_2 = 0$$
$$S_1 + k\,S_2 = 0$$

In vector form.

$P_1 \equiv \bar{r}\cdot\bar{n}_1 = q_1 \Rightarrow P_1 \equiv \boxed{\bar{r}\cdot\bar{n}_1 - q_1 = 0}$

$P_2 \equiv \bar{r}\cdot\bar{n}_2 = q_2 \Rightarrow P_2 \equiv \boxed{\bar{r}\cdot\bar{n}_2 - q_2 = 0}$

$$\boxed{(\bar{r}\cdot\bar{n}_1 - q_1) + \lambda(\bar{r}\cdot\bar{n}_2 - q_2) = 0}$$

Any plane passing through intersection of P_1 & P_2 can be assumed as

$P_1 + \lambda_1 P_2 = 0$

i.e. $(\bar{r}\cdot\bar{n}_1 - q_1) + \lambda_1(\bar{r}\cdot\bar{n}_2 - q_2) = 0.$

where λ_1 is address of the plane.

In Cartesian form (Mostly we use this)

$P_1 \equiv a_1x + b_1y + c_1z - q_1 = 0$

$P_2 \equiv a_2x + b_2y + c_2z - q_2 = 0$.

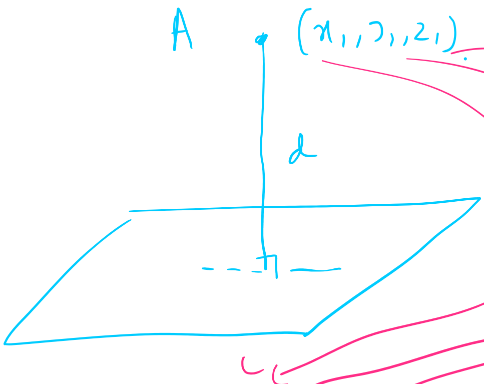
Any plane passing through line of intersection of P_1 & P_2 can be assumed as

$P_1 + \lambda_1 P_2 = 0$

$$\boxed{(a_1x + b_1y + c_1z - q_1) + \lambda_1 (a_2x + b_2y + c_2z - q_2) = 0}.$$

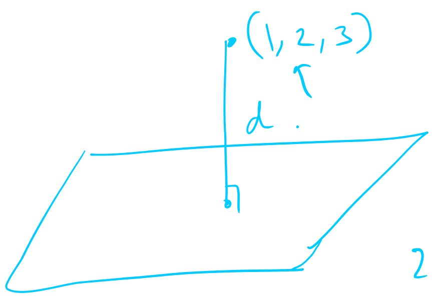
Where λ_1 is the address of the required plane in the family.

$\perp$ Distance of a point from a plane.



$$ax + by + cz - q = 0$$

$$d = \frac{|ax_1 + by_1 + cz_1 - q|}{\sqrt{a^2 + b^2 + c^2}}$$



$$2x + 3y - z = 3$$

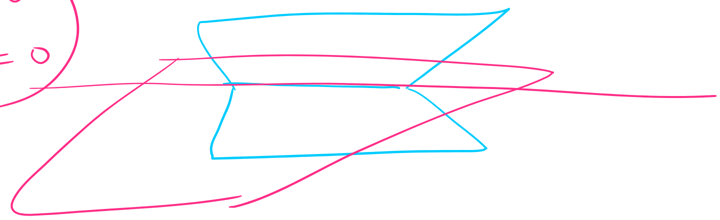
$$d = \frac{|2\times 1 + 3(2) - 3 - 3|}{\sqrt{2^2 + 3^2 + (-1)^2}}$$

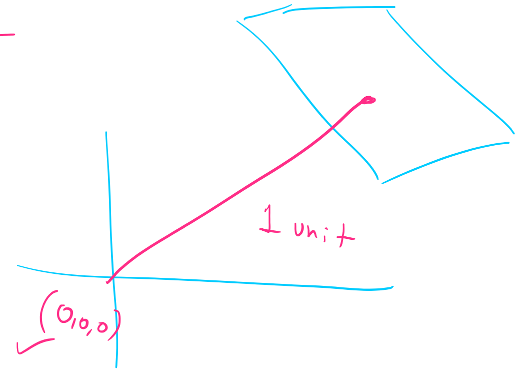
$$d = \frac{2}{\sqrt{14}} = \boxed{\frac{\sqrt{2}}{\sqrt{7}}}$$

Illustration - 43 *Find the cartesian as well as vector equation of the planes through the intersection of the planes* $\vec{r}\cdot(2\hat{i}+6\hat{j})+12=0$ *and* $\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})=0$ *which are at a unit distance from the origin.*

$2x+6y+12=0$

$3x-y+4z=0$





Let the required plane be :

$(2x+6y+12)+\lambda(3x-y+4z)=0$

$$\boxed{(2+3\lambda)x+(6-\lambda)y+4\lambda z+12=0}$$

$$\frac{|(\quad)0+(\quad)0+(\quad)0+12|}{\sqrt{(2+3\lambda)^2+(6-\lambda)^2+(4\lambda)^2}}=1$$

$144=4+9\lambda^2+\cancel{12\lambda}+36+\lambda^2-\cancel{12\lambda}+16\lambda^2$

$144=26\lambda^2$

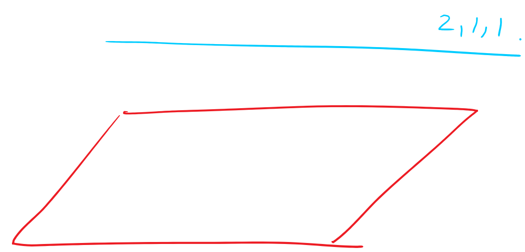
$104=\cancel{26}\lambda^2$

4

$\lambda^2=4$

$\boxed{\lambda=\pm 2}$

Illustration - 44 *Find the equation of the plane passing through the intersection of the planes* $4x - y + z = 10$ *and* $x + y - z = 4$ *and parallel to the line with direction ratios* 2, 1, 1. *Find also the* ~~*perpendicular*~~ *distance of* (1, 1, 1) *from this plane.*



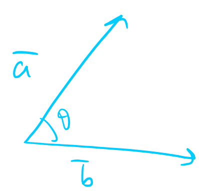
required plane.

Angle between two lines

$\bar{r} = a_1 + k\bar{b_1}$

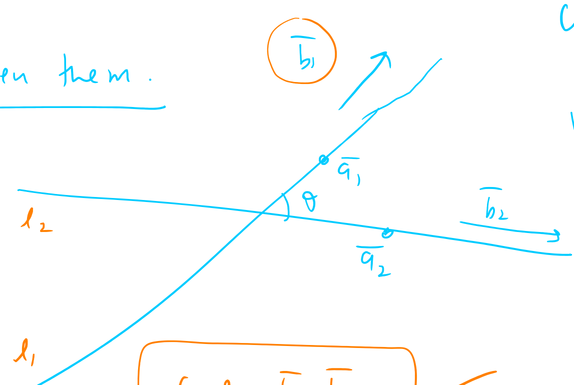
$\bar{r} = a_2 + k\bar{b_2}$

Find angle between them.



$\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|a||\bar{b}|}$

$\theta = \cos^{-1}\left(\dfrac{\bar{a}\cdot\bar{b}}{|a||\bar{b}|}\right)$



$$\boxed{\cos\theta = \frac{\bar{b_1}\cdot\bar{b_2}}{|\bar{b_1}||\bar{b_2}|}}$$

$l_1 \equiv \dfrac{x - x_1}{A_1} = \dfrac{y - y_1}{A_2} = \dfrac{z - z_1}{A_3}$

$l_2 \equiv \dfrac{x - x_2}{B_1} = \dfrac{y - y_2}{B_2} = \dfrac{z - z_1}{B_3}$

We will use vector form

vector $\parallel$ to first line l_1

$= \bar{b_1} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

vector $\parallel$ to second line l_2

$= \bar{b_2} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

$\cos\theta = \bar{b_1}\cdot\bar{b_2}$ $\quad A_1B_1 + A_2B_2 + A_3B_3$

$$\cos\theta = \frac{\bar b_1 \cdot \bar b_2}{|\bar b_1||\bar b_2|} = \frac{A_1B_1 + A_2B_2 + A_3B_3}{\sqrt{A_1^2+A_2^2+A_3^2}\,\sqrt{B_1^2+B_2^2+B_3^2}}$$

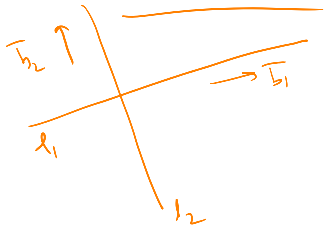
No need to learn.

If $l_1 \perp l_2$

Find Condition.

$l_1 \equiv \bar r_1 = \bar a_1 + k\bar b_1$

$l_2 \equiv \bar r_2 = \bar a_2 + k\bar b_2$



$$\cos\theta = \frac{\bar b_1 \cdot \bar b_2}{|\bar b_1||\bar b_2|}$$

$\theta = \pi/2$

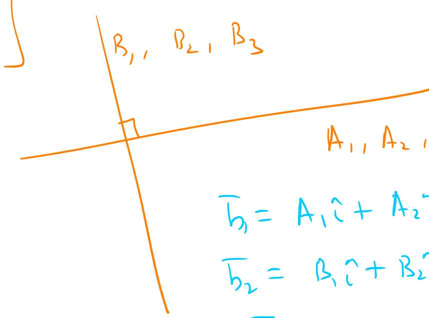
$$\cos\frac{\pi}{2} = 0 = \frac{\bar b_1 \cdot \bar b_2}{|\bar b_1||\bar b_2|}$$

$$\boxed{\bar b_1 \cdot \bar b_2 = 0}$$

$$l_1 \equiv \frac{x-x_1}{A_1} = \frac{y-y_1}{A_2} = \frac{z-z_1}{A_3}$$

$$l_2 \equiv \frac{x-x_2}{B_1} = \frac{y-y_2}{B_2} = \frac{z-z_3}{B_3}$$

If $l_1 \perp l_2$, what is the Condition.



$\bar b_1 = A_1\hat i + A_2\hat j + A_3\hat k$

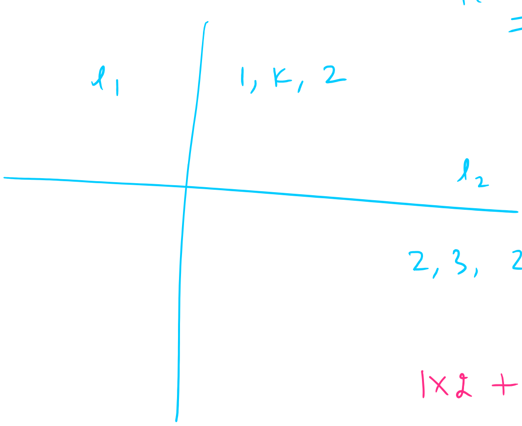
$\bar b_2 = B_1\hat i + B_2\hat j + B_3\hat k$

$$\cos\theta = \frac{\bar b_1 \cdot \bar b_2}{|\bar b_1||\bar b_2|}$$

$\bar b_1 \cdot \bar b_2 = 0$.

learn this

$$\boxed{A_1B_1 + A_2B_2 + A_3B_3 = 0}$$



If $l_1 \perp l_2$ find k.

$1 \times 2 + k(3) + 2(2k) = 0$

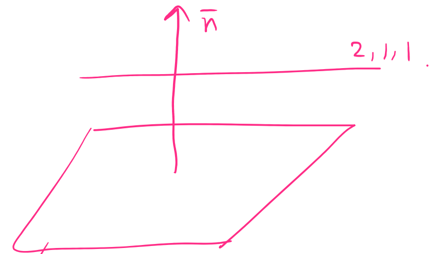
$7k = -2$

$$\boxed{k = -2/7}$$

Illustration - 44 *Find the equation of the plane passing through the intersection of the planes* $4x - y + z = 10$ *and* $x + y - z = 4$ *and parallel to the line with direction ratios* 2, 1, 1. *Find also the perpendicular distance of* (1, 1, 1) *from this plane.*

$(4x - y + z - 10) + \lambda(x + y - z - 4) = 0$

is the eqn of required plane.



$(4 + \lambda)x + (\lambda - 1)y + (1 - \lambda)z - 10 - 4\lambda = 0$

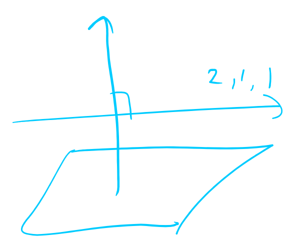
is the required plane.

Write eqn using $P_1 + \lambda P_2 = 0$

$\boxed{\text{drs of Normal} \equiv 4+\lambda,\ \lambda-1,\ 1-\lambda \quad \text{Normal is} \perp \text{to a line whose drs are } 2, 1, 1}$

$\Downarrow$

Apply result just discussed.



$(4 + \lambda)2 + (\lambda - 1)1 + (1 - \lambda)1 = 0$

$2\lambda + 8 - 1 + 1 = 0$

$2\lambda = -8 \qquad \boxed{\lambda = -4}$

$0x - 5y + 5z + 6 = 0 \qquad \boxed{5y - 5z - 6 = 0}$

Illustration - 47 *Find the equation of the plane through the intersection of the planes*

$ax + by + cz + d = 0$, $Ax + By + Cz + D = 0$ *and* $\perp$ *to XY-plane.*

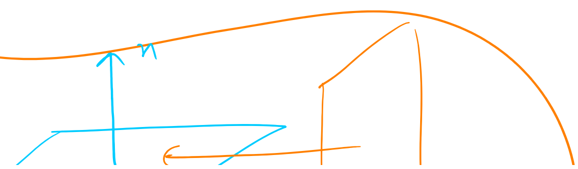
Required plane is

$(ax + by + cz + d) + \lambda(Ax + By + Cz + D) = 0$

$(a + \lambda A)x + (b + \lambda B)y + (c + \lambda C)z + d + \lambda D = 0$

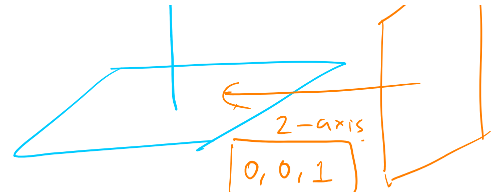
drs of Normal

$a + \lambda A,\ b + \lambda B,\ c + \lambda C$



dRs of Normal

$a+\lambda A, \quad b+\lambda B, \quad c+\lambda C$



dRs of z-axis

$\cos 90^\circ, \quad \cos 90^\circ, \quad \cos 0^\circ$

$\alpha = 90^\circ$
$\beta = 90^\circ$
$\gamma = 0^\circ$.

$0, \quad 0, \quad 1$.

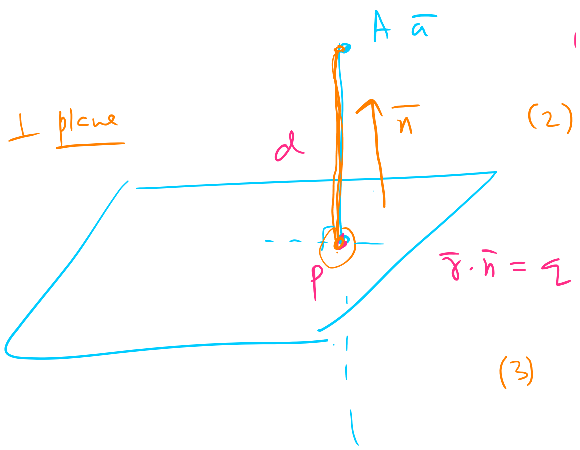
Perpendicular.

$0(a+\lambda A) + 0(b+\lambda B) + 1(c+\lambda C) = 0$

$\boxed{\lambda = \frac{-c}{C}}$

$\perp$ Distance of a point $A(\bar{a})$ from a plane $\bar{r}\cdot\bar{n} = q$.

AP $\perp$ plane



1) let foot of $\perp$ be P.

(2) Eqn of line AP.

$$\boxed{\bar{r} = \bar{a} + k\,\bar{n}}$$

(3) Assume P using eqn of line.

$$\boxed{\bar{p} = \bar{a} + k_1\bar{n}}$$

(4) Replace P in eqn of plane.

$$\bar{p}\cdot\bar{n} = q$$

$$(\bar{a} + k_1\bar{n})\cdot\bar{n} = q$$

$$\bar{a}\cdot\bar{n} + k_1|n_1|^2 = q$$

$$\boxed{k_1 = \frac{q - \bar{a}\cdot\bar{n}}{|n_1|^2}}$$

$$d = |\overline{AP}|$$

$$= |\bar{p} - \bar{a}|$$

$$= |k_1\bar{n}|$$

$$= \left| \frac{q - (\bar{a}\cdot\bar{n})}{|\bar{n}|^2}\,\bar{n} \right|$$

$$= \frac{|q - \bar{a}\cdot\bar{n}|\,\cancel{|\bar{n}|}}{|\bar{n}|^{\cancel{2}}}$$

$$\boxed{d = \frac{|\bar{a}\cdot\bar{n} - q|}{|n|}}$$

$$\boxed{ax + by + cz = q}$$

$$\bar{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\bar{n} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$d = \frac{|ax_1 + by_1 + cz_1 - q|}{\sqrt{a^2 + b^2 + c^2}}$$

$\bar{a}$

$\bar{r}\cdot\bar{n} = q$

$\bar{r}\cdot\bar{n} - q = 0$

$|\bar{a}\cdot\bar{n} - q|$

$|\bar{n}|$

1) How to Solve two planes to find eqn of line of Intersectin.

2) Pair of lines.

3) Shortest distance between pair of lines when they are non-Intersecting & non-parallel.