Bachelor thesis

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Finding a long-run nonlinear stock priceŰdividend relationship could be seen as evidence that intrinsic bubbles are relevant in the long run and, hence, are important in explaining the long-run excessive volatility of stock prices. To test for a nonlinear long-run priceŰdividend relationship, we test for nonlinear cointegration between prices and dividends. Breitung (2001) shows that residual-based linear cointegrations, like Granger and HallmanŠs (1991) nonlinear cointegration test, are inconsistent for some classes of nonlinear functions. He suggests a nonparametric cointegration test that uses the rank transformation between two variables and claims it to be more powerfull than parametric cointegration tests if the cointegration relationship is nonlinear. The hypothesis that prices and dividends are linearly cointegrated is already rejected. Therefore we can savely assume that when Breitung's test gives us evidence for cointegration, it is for nonlinear cointegration.

We haven't rejected the hypothesises that P_t and D_t are integrated of order one. The null hypothesis that is being tested is that there exist monotonic functions f and g such that

$$f(P_t) - g(D_t) = u_t \tag{1}$$

where u_t is integrated of order one, which means prices and dividends are not cointegrated. With a rank-based test we don't have to know the specific functions f and g, only that f and g are monotonic functions. Monotonic transformations don't change ranks.

$$R_T(f(P_t)) = R_T(P_t) \tag{2}$$

$$R_T(g(D_t)) = R_T(D_t) \tag{3}$$

where $R_T(P_t)$ returns the rank of P_t among $P_1871, ..., P_2009$. Prices and dividends are assumed to be random walks, therefore their rank transformations follow ranked random walks. When there is a big rank transformation between the series, it seems unlikely that the two series are cointegrated. We assess the size of the transformation by measuring the difference in ranks at each time instant using δ_t.

$$\delta_t = R_T P_t - R_T(D_t)$$

With this measure we make the following statistics

$$\kappa_T = T^{-1} \sup_t |\delta_t| \xi_T = T^{-3} \sum_{t=1871}^{2009} \delta_t^2$$

These statistics have to be corrected if the considered time series are correlated, which is obviously the case with prices and dividends. To measure the correlation of the ranked series, the following statistic is defined

$$\rho_T^R = \frac{\sum_{t=1872}^{2009} \Delta R_T(P_t) \Delta R_T(D_t)}{\sqrt{(\sum_{t=1872}^{2009} \Delta R_T(P_t)^2)(\sum_{t=1872}^{2009} \Delta R_T(D_t)^2)}} \tag{4}$$

Breitung (2001) shows that for moderate values of correlation between time series, the ranked correlation has a downward bias in absolute value. With $\rho_T^R = 0.68$ we therefore use the following adjusted test statistics

$$\kappa_T^{**} = \frac{\kappa_T}{\widehat{\sigma}_{\Delta\delta}(1 - 0.174(\rho_T^R)^2)} \xi_T^{**} = \frac{\xi_T}{\widehat{\sigma}_{\Delta\delta}^2(1 - 0.462\rho_T^R)} \tag{5}$$

where $\widehat{\sigma}_{\Delta\delta}^2 = T^{-2} \sum_{t=1872}^{2009} (\delta_t - \delta_{t-1})^2$. we can use one sided testing since we do not want to investigate a relationship in which prices and dividends have an inverse relationship.