Trading Rules from Forecasting the Collapse of Speculative Bubbles for the S&P 500 Composite Index
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Trading Rules from Forecasting the Collapse of Speculative Bubbles for the S&P 500 Composite Index*

Many recent studies documented the presence of speculative bubbles, defined as systematic and increasing deviations of actual prices from fundamentals, in asset prices. However, thus far, the usefulness of such models has been examined in the literature only from a statistical perspective. In this paper, we employ two-regime switching models of periodically partially collapsing speculative bubbles and examine the risk-adjusted profits of trading rules formed using inferences from them. Use of trading rules derived from an augmented model incorporating market volume leads to higher risk-adjusted returns than those obtained employing existing models or a buy-and-hold strategy.

I. Introduction

Stock prices, as well as the prices of many other traded assets, have historically shown tendencies to rise substantially over a protracted period and then to fall very quickly. At first blush, such movements in prices can be hard to reconcile with notions of investor rationality and market efficiency. However, an increasingly large literature developed in recent years, suggesting that investors acted rationally by continuing to pay ever-further-inflated prices, since they were being compensated for it. A persistent, systematic, and increasing deviation of prices from their "fundamental value," defined as the risk-adjusted present value of all expected future cash flows, is known as a *speculative bubble*.

Studies of bubble behavior in asset prices can be split into two classes: direct and indirect tests. Initially, studies on the relationship between actual prices and fundamental values focused on the indirect identification of speculative bubbles in financial data (see Shiller 1981; Blanchard and

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Watson 1982; West 1987; Diba and Grossman 1988). However, indirect tests of bubble presence suffered from potential problems of interpretation, since bubble effects in stock prices could not be distinguished from the effects of unobservable market fundamentals. For this reason, direct bubble tests, which test directly for the presence of a particular bubble specification in stock market returns, were developed (see Flood and Garber 1980; Flood, Garber, and Scott 1984; Summers 1986; Cutler, Poterbe, and Summers 1991; McQueen and Thorley 1994; Salge 1997; van Norden and Schaller 1999). Under these tests, researchers select the type of bubble they suspect might be present in the data, then examine whether this form of speculative bubble has any explanatory power for stock market returns.

To examine empirically the presence of periodically collapsing bubbles, researchers in recent years focused on the construction of direct bubble tests that can identify such stochastic bubble processes in financial and macroeconomic data. More specifically, Evans (1991) and van Norden and Schaller (1993) show how periodically collapsing speculative bubbles can induce regime switching behavior in asset returns. Regime switching in asset returns and speculative behavior have been linked in several studies. Van Norden and Schaller (1993) and van Norden (1996) show that a two-regime speculative behavior model has significant explanatory power for stock market and foreign exchange returns during several periods of observed market over- and undervaluations. Hall, Psaradakis, and Sola (1999) test for the presence of collapsing speculative bubbles in Argentinean monetary data using a univariate Markov-switching augmented Dickey-Fuller (ADF) test and find evidence of a speculative bubble in consumer prices in the period June 1986 to August 1988.

Research on speculative bubbles thus far focused only on the problem of bubble identification. To our knowledge, none of the papers in the literature examined speculative-bubbles as a base for the formulation of trading strategies that exploit knowledge about the implied probability of a stock market crash. One implication of this is that the speculative bubble literature has been argued to have obvious intellectual interest but little practical value for financial market participants. In this paper, we aim to address this issue by evaluating the financial usefulness of the van Norden and Schaller (1999) regime-switching model of speculative behavior and an augmented version of the model including market volume proposed by Brooks and Katsaris (2002). This is achieved by examining the profitability of trading rules formed on the basis of the probabilities of a crash and a rally in the next time period estimated from the models.

The remainder of this paper is organized as follows. We briefly describe the van Norden and Schaller switching regression model of periodically collapsing speculative bubbles and an augmented version of the model in Section II. The data are also described in Section II, with the results of model estimation presented in Section III. Section IV examines the forecasting ability of the augmented model and the van Norden-Schaller model by looking at the profitability of trading strategies formed on the basis of inferences from these models. Section V concludes.

II. Regime-Switching Models of Periodically Collapsing Speculative Bubbles and Data

Van Norden and Schaller formulate a periodically, partially collapsing, positive and negative speculative-bubble model that has a time-varying probability of collapse. They consider the following bubble process:

$$E_t(b_{t+1}) = \begin{cases} \dfrac{(1+r)b_t}{q(B_t)} - \dfrac{1-q(B_t)}{q(B_t)}u(B_t)p_t^a & \text{with probability } q(B_t) \\ u(B_t)p_t^a & \text{with probability } 1-q(B_t) \end{cases} \tag{1}$$

In (1), B_t is the size of the bubble relative to the actual price $p_t^a(B_t = b_t/p_t^a)$; $u(B_t)$ is a continuous and everywhere differentiable function such that $u(0) = 0$ and $0 \leq \partial u(B_t)/\partial B_t < 1$; $q(B_t)$ is the probability of the bubble continuing to exist, which is a negative function of the absolute relative size of the bubble; and b_t is the bubble component at period t, defined as the difference between the actual and fundamental price at that time. Two measures of fundamentals are employed in this paper, a measure based on a multiple of dividends and a measure based on Campbell and Schiller (1987); calculations using both methods are detailed in the appendix.

As noted by Kindleberger (1989), a crash becomes more likely as the relative size of the bubble becomes larger. To incorporate this and to ensure that the estimates of the probability of survival are bounded between zero and one, van Norden and Schaller employ a probit specification and allow the probability of survival to depend on the relative absolute size of the bubble:

$$q(B_t) = \Omega(\beta_{q,o} + \beta_{q,b} \mid B_t \mid), \tag{2}$$

where Ω is the standard normal cumulative density function, $\Omega(\beta_{q,o})$ is the constant probability of being in the surviving regime when the size of the bubble is equal to zero, and $\beta_{q,b}$ is the sensitivity of the probability of survival to the absolute relative size of the bubble. Van Norden and Schaller also allow for partial bubble collapses by letting the expected bubble size in the collapsing state be a function of the relative bubble size.

The van Norden-Schaller model can be used to specify asset returns as state dependent, where the state is unobservable. This implies that the security's gross returns are given, under certain assumptions about the dividend process, by the following nonlinear switching model:[1]

$$E_t(r_{t+1} \mid W_{t+1} = S) = \left[M(1 - B_t) + \frac{MB_t}{q(B_t)} - \frac{1 - q(B_t)}{q(B_t)} u(B_t) \right] \\ \text{with probability } q(B_t), \tag{3}$$

$$E_t(r_{t+1} \mid W_{t+1} = C) = [M(1 - B_t) + u(B_t)] \text{ with probability } 1 - q(B_t), \tag{4}$$

where $(r_{t+1} \mid S)$ denotes the gross return of period $t + 1$ conditioned on the fact that the state at time t is the survival state (S) and on all other available information at time t, W_t, is an unobserved indicator that determines the state in which the process is at time t; C denotes the collapsing state; and M is the gross fundamental return on the security. To estimate the model, van Norden and Schaller linearize equations (3) and (4) and derive a linear regime switching model for gross stock market returns where $(q(B_t))$ is the probability of being in regime S:

$$\begin{aligned} r_{t+1}^S &= \beta_{S,0} + \beta_{S,b} B_t + u_{S,t+1}, \\ r_{t+1}^C &= \beta_{C,0} + \beta_{C,b} B_t + u_{C,t+1}, \\ P(W_{t+1} = S) = q(B_t) &= \Omega(\beta_{q,0} + \beta_{q,b} \mid B_t \mid). \end{aligned} \tag{5}$$

In (5), $u_{S,t+1}$ and $u_{C,t+1}$ are the unexpected gross returns of period $t + 1$ in the surviving and the collapsing regimes, respectively, and are assumed zero mean and constant variance independently and identically distributed normal random variables.

Brooks and Katsaris (2002) proposed that the preceding model could be augmented by the inclusion of a measure of abnormal volume. Abnormal volume is argued to be employed by investors as a useful explanatory variable in determining the probability of bubble survival, and it can be considered a sign that other investors are selling the bubbly asset. Under this setting, higher volume is observed in the market simply because more and more orders are being filled, as investors who already hold the asset believe the market collapse to be imminent. This increase in volume is abnormal, since it is higher than in normal market conditions (where buy and sell orders are driven buy portfolio allocation

1. For a derivation of the equations, the reader is referred to the working paper version, van Norden and Schaller (1997), available at the Bank of Canada Web site: http://www.bankofcanada.ca/en/res/wp97-2.htm.

and investment-consumption decisions) and it is higher than during the early stages of the bubble expansion (since, although volume increases, it is not as high as possible because of the reluctance of holders of the asset to sell too early). As the bubble expands and more investors sell because they perceive the crash to be imminent, holders of the asset observe an ever-increasing abnormal volume and thus adjust their own estimate of when the bubble will collapse even more. Once a sufficient number of investors perceive that the market no longer believes the bubble will continue to exist, they liquidate their holdings in the bubbly asset simultaneously, causing the bubble to collapse.

The model for periodically collapsing speculative bubbles proposed by Brooks and Katsaris (2002) is:

$$E_t(b_{t+1}) = \begin{cases} \dfrac{(1+i)b_t}{q(B_t, V_t^x)} - \dfrac{1-q(B_t, V_t^x)}{q(B_t, V_t^x)} u(B_t) p_t^a & \text{with probability } q(B_t, V_t^x) \\ u(B_t) p_t^a & \text{with probability } 1-q(B_t, V_t^x) \end{cases} \tag{6}$$

In (6), $q(B_t, V_t^x)$ is the probability of the bubble continuing to exist, which is a function of the relative absolute size of the bubble and the measure of abnormal volume where $\partial q(B_t, V_t^x)/\partial \mid B_t \mid < 0$ and $\partial q(B_t, V_t^x)/\partial V_t^x < 0$, and V_t^x is a measure of unusual volume in period t. The probability of the bubble continuing to exist is a negative function of the absolute (since we allow negative bubbles to exist) size of the bubble and the measure of abnormal volume. Under the assumption that dividends follow a geometric random walk, we can show that the expected next period gross returns are given by[2]

$$E_t(r_{t+1} | W_{t+1} = S) = \left[M(1 - B_t) + \frac{M}{q(B_t, V_t^x)} B_t - \frac{1 - q(B_t, V_t^x)}{q(B_t, V_t^x)} u(B_t) \right], \tag{7}$$

$$E_t(r_{t+1} | W_{t+1} = C) = [M(1 - B_t) + u(B_t)], \tag{8}$$

$$P(r_{t+1} | W_{t+1} = S) = q(B_t, V_t^x) = \Omega(\beta_{q,0} + \beta_{q,b}|B_t| + \gamma_{q,V} V_t^x), \tag{9}$$

where $\gamma_{q,V}$ is the sensitivity of the probability of survival to the measure of abnormal volume. Note that the measure of abnormal volume affects the expected returns on the asset only indirectly, through the probability

2. Proof of these equations is not presented here in the interests of brevity but is available in an appendix upon request from the authors.

of the process being in state S or C. Abnormal volume is thus suggested to signal an increase in the size of the tails of the distribution of expected returns that would signify a higher probability of a sharp collapse in the bubble. Again, a probit model is employed for the probability of survival ($P(r_{t+1}|W_{t+1} = S)$), since it satisfies the conditions set out previously and it ensures that probability estimates are bounded between zero and one. To estimate these equations, we linearize them by taking the first-order Taylor series approximation of the model around an arbitrary B_0 and $V_0^x$and arrive at a linear switching regression model:

$$r_{t+1} = \beta_{S,0} + \beta_{S,b}B_t + \beta_{S,V}V_t^x + u_{t+1}^S, \tag{10}$$

$$r_{t+1} = \beta_{C,0} + \beta_{C,b}B_t + u_{t+1}^C, \tag{11}$$

$$P(r_{t+1}|W_{t+1} = S) = q(B_t, V_t^x) = \Omega(\beta_{q,0} + \beta_{q,b}|B_t| + \gamma_{q,V}V_t^x), \tag{12}$$

where u_{t+1}^S is the unexpected gross return in the surviving regime and u_{t+1}^C is the unexpected gross return in the collapsing regime. Equation (10) states that the returns in the surviving regime are a function of the relative size of the bubble and the measure of abnormal volume. In effect, equation (10) implies that, as the bubble grows, investors demand higher returns to compensate them for the probability of a bubble collapse, and since abnormal volume signals a possible change in the long-run trend in equity prices, investors want to be compensated for this risk as well. This linear switching regression model has a probability of being in the surviving regime $P(r_{t+1}|W_{t+1} = S)$ that is a function of the relative size of the bubble and the measure of abnormal volume. We estimate the augmented model under the assumption of disturbance normality, using maximum likelihood to directly maximize the following log-likelihood function:

$$\ell(r_{t+1}|\xi) = \sum_{t=1}^{T} \ln\left[P(r_{t+1}|W_{t+1} = S)\frac{\omega\left(\dfrac{r_{r+1} - \beta_{S,0} - \beta_{S,b}B_t - \beta_{S,V}V_t^x}{\sigma_S}\right)}{\sigma_S} + P(r_{t+1}|W_{t+1} = C)\frac{\omega\left(\dfrac{r_{t+1} - \beta_{C,0} - \beta_{C,b}B_t}{\sigma_C}\right)}{\sigma_C}\right], \tag{13}$$

where ξ is the set of parameters over which we maximize the likelihood function including $\beta_{S,0}, \beta_{S,b}, \beta_{S,V}, \beta_{C,0}, \beta_{C,0}, \beta_{q,0}, \beta_{q,B}, \gamma_{q,V}, \sigma_S, \sigma_C$, ω is the standard normal probability density function (pdf); $\sigma_S(\sigma_C)$ is the standard deviation of the disturbances in the surviving (collapsing) regime; and $P(r_{t+1}|W_{t+1} = C) = 1 - P(r_{t+1}|W_{t+1} = S)$.

Note that the maximization of this log-likelihood function produces consistent and efficient estimates of the parameters in ξ, as it requires no assumptions about which regime generated a given observation. The model is similar to the models described in Goldfeld and Quandt (1976).

From the first-order Taylor series expansion, we can derive certain conditions that must hold if the periodically collapsing speculative-bubble model has explanatory power for stock market returns. If the preceding model can explain the variation in future returns, then this would be evidence in favor of the presence of periodically collapsing speculative bubbles in the data. These restrictions are

$$\beta_{S,0} \neq \beta_{C,0}, \tag{a}$$

$$\beta_{C,b} < 0, \tag{b}$$

$$\beta_{S,b} > \beta_{C,b}, \tag{c}$$

$$\beta_{q,b} < 0, \tag{d}$$

$$\gamma_{q,V} < 0, \tag{e}$$

$$\beta_{S,V} > 0. \tag{f}$$

Restriction (a) implies that the mean return across the two regimes is different, so there exist two distinct regimes, although we cannot say anything about the relative size of these coefficients. Restriction (b) implies that the expected return should be negative if the collapsing regime is observed. This means that the bubble must be smaller in the following period if the bubble collapses. Note that the opposite holds for negative bubbles: the larger is the negative bubble, the more positive the returns in the collapsing regime. Restriction (c) ensures that the bubble yields higher (lower) returns if a positive (negative) bubble is observed in the surviving regime than in the collapsing regime. Restriction (d) must hold, since the probability of the bubble continuing to exist is expected to decrease as the size of the bubble increases. Restrictions (a) to (d) are equivalent to the restrictions derived by van Norden and Schaller. The additional restriction (e) must hold, so that an abnormally high volume signals an imminent collapse of the bubble. Finally, restriction (f) states that the coefficient on the abnormal volume measure in the state equation must be greater than zero, since as volume increases, investors perceive an increase in market risk.

We examined the power of the model to capture bubble effects in the returns of the S&P 500 by testing the model against three simpler specifications that capture stylized features of stock market returns. These models are nested within the speculative bubble model. We repeated these tests and also examined the augmented model against the simpler van Norden-Schaller model using a likelihood ratio test. First, we examine whether the effects captured by the switching model can be explained by a more parsimonious model of changing volatility. To test this alternative, we follow van Norden and Schaller and jointly impose the following restrictions:

$$\beta_{C,0} = \beta_{S,0}, \tag{14a}$$

$$\beta_{S,b} = \beta_{C,b} = \beta_{S,V} = \beta_{q,b} = \gamma_{q,V} = 0, \tag{14b}$$

$$\sigma_S \neq \sigma_C. \tag{14c}$$

Restriction (14a) implies that the mean return across the two regimes is the same, and restriction (14b) states the bubble deviation has no explanatory power for next period returns or the probability of being in the surviving regime. The latter point suggests that a constant probability of being in the surviving regime, as stated in restriction (14c).

To separate restrictions (14a) and (14b), we examine whether returns can be characterized by a simple mixture of normal distributions model, which allows only mean returns and variances to differ across the two regimes. This mixture of normal distributions model implies the following restrictions:

$$\beta_{S,b} = \beta_{C,b} = \beta_{S,V} = \beta_{q,b} = \gamma_{q,V} = 0. \tag{15}$$

Another possible alternative is that of mean reversion in prices (fads) as described by Cutler et al. (1991). Under the fads model, returns are linearly predictable, although mean returns do not differ across regimes. Furthermore, the deviation of actual prices from the fundamentals has no predictive ability for the probability of being in the surviving regime. The returns in the two regimes are characterized by different variances of residuals but are the same linear functions of bubble deviations. The fads model is

$$\begin{aligned} r^S_{t+1} &= \beta_0 + \beta_b B_t + u_{S,t+1}, \\ r^C_{t+1} &= \beta_0 + \beta_b B_t + u_{C,t+1}, \\ q_t &= \Omega(\beta_{q,0}). \end{aligned} \tag{16}$$

In these equations, $u_{t+1} \sim N(0, \sigma^2_S)$ with probability q_t, $u_{t+1} \sim N(0, \sigma^2_C)$ with probability $1 - q_t$.

As a final statistical test, we also examined the robustness of our model against the more parsimonious van Norden-Schaller model by testing whether abnormal volume should be included in the speculative-bubble model. The restrictions of this last test are

$$\beta_{S,V} = \gamma_{q,V} = 0. \tag{17}$$

The data employed comprise 1,369 monthly observations on the S&P 500 for the period January 1888–January 2003. The S&P 500 prices and dividends used to calculate fundamental prices and gross returns are taken from Shiller.[3] We calculated monthly share volume by taking the sum of daily share volume reported by the New York Stock Exchange (NYSE) for the period January 1888–January 2003.[4] Unusual trading volume is defined as the percentage deviation of last month's volume from the 6-month moving average.[5] This moving average is constructed using only lagged volume figures that would have been included in agents' information sets. The monthly dividend and price series are transformed into real variables using the monthly U.S. consumer price–all items seasonally adjusted index reported in Shiller.

III. The Results

Figure 1 presents the bubble deviations calculated using both the dividend multiple and Campbell and Shiller approaches for the entire sample. Note that both bubble deviations are increasingly large in 1929, 1987, and the late 1990s. The bubble deviations are significantly negative in 1917, 1932, 1938, 1942, and 1982. The Campbell and Shiller measure of bubble deviations (dotted line) displays significantly more short-term variability whereas the dividend multiple measure has larger and more persistent broad swings.

The results for the augmented model of speculative behavior using the dividend multiple measure of fundamental values are presented in

3. Data are available at http://www.econ.yale.edu/~shiller/data.htm. For a description of the data used, see also Shiller (2000) and the description online. When the data were being gathered, Shiller's sample ended in January 2000. For this reason, we updated his sample until January 2003 using data obtained from Datastream. To verify that the two data sets are consistent with each other, we compared Shiller's data from January 1965 to January 2000 with the values from Datastream and found no differences.

4. Data are available at http://www.nyse.com/marketinfo/marketinfo.html.

5. We also examined unusual trading volume measures using 3-, 12-, and 18-month moving averages but found that the deviation from the 6-month moving average has the highest explanatory power in predicting both the level and the generating state of returns. The results for the other measures of abnormal volume are not presented for brevity and are available upon request from the authors.

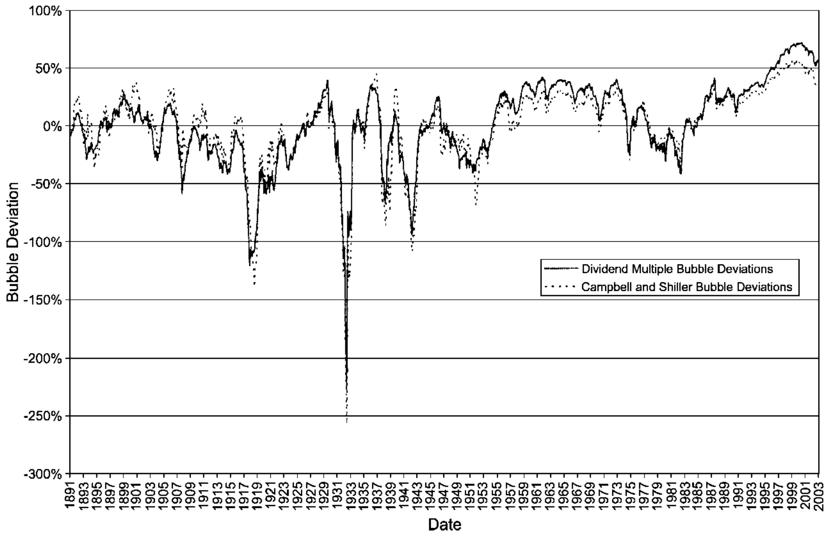

FIG. 1.—Bubble deviation of actual price from fundamental values, January 1888–January 2003.

the first panel of table 1, alongside the results of the van Norden-Schaller model for comparison. The second panel of the table contains the results of the likelihood ratio (LR) tests of the restrictions on the coefficients implied by the speculative-bubble model while the third panel of table 1 presents the results of the LR tests of the simpler volatility regimes, mixture of normals, fads, and van Norden-Schaller models.

From the first panel of table 1, we can see that all the coefficients of the augmented model, except the bubble coefficient of the surviving equation ($\beta_{S,\,b}$), have the correct sign and are of a financially meaningful magnitude. Specifically, the coefficient estimate of the intercept term in the surviving regime ($\beta_{S,0}$) is 1.0079, and it is highly significant, implying that when volume is normal and the bubble size is zero, the expected return in the surviving regime is 0.79% per month (9.90% on an annual basis).[6] The corresponding value of this coefficient in the van Norden-Schaller model is larger (1.0085), implying a return in the surviving regime when the bubble size and abnormal volume are zero of 10.69% on an annualized basis. On the other hand, the intercept coefficient in the collapsing regime equation ($\beta_{C,0}$) is −3.08% (−31.30% annualized) for the augmented model compared with −30.35% on an annual basis for the van Norden-Schaller model.[7]

6. Note that this coefficient is also statistically significantly different from 1 (p-value of t-test 0.0000).

7. These coefficients are also statistically significantly different from 1 (p-value of t-test 0.0255 for the augmented model and 0.0270 for the original van Norden-Schaller model).

TABLE 1 **Results of the Periodically Collapsing Speculative-Bubble Models for the Dividend Multiple Measure of Fundamental Values, January 1888–January 2003**

	Augmented Model			van Norden and Schaller Model		
Coefficient	Coefficient	SE	p-Value	Coefficient	SE	p-Value
$\beta_{S,0}$	1.0079	.0010	.0000	1.0085	.0010	.0000
$\beta_{S,b}$	−.0011	.0033	.3787	−.0002	.0034	.3981
$\beta_{S,V}$	.0113	.0032	.0008	—	—	—
$\beta_{C,0}$	.9692	.0138	.0000	.9703	.0134	.0000
$\beta_{C,b}$	−.0438	.0210	.0456	−.0424	.0201	.0436
$\beta_{q,0}$	1.9378	.1872	.0000	1.8318	.1671	.0000
$\beta_{q,b}$	−1.3572	.3083	.0000	−1.2770	.3092	.0001
$\gamma_{q,V}$	−.4745	.2044	.0270	—	—	—
σ_u^S	.0322	.0009	.0000	.0322	.0010	.0000
σ_u^C	.1047	.0106	.0000	.1025	.0108	.0000

Speculative Bubble Model Restrictions LR Tests

Restriction	LR Test Statistic	p-Value	LR Test Statistic	p-Value
$\beta_{S,0} \neq \beta_{C,0}$	8.9461	.0028	9.5164	.0020
$\beta_{C,b} < 0$	4.8699	.0273	4.9503	.0261
$\beta_{S,b} > \beta_{C,b}$	4.3570	.0369	4.5740	.0325
$\beta_{S,V} > 0$	15.1032	.0001	—	—
$\beta_{q,b} < 0$	21.0979	.0000	19.8993	.0000
$\gamma_{q,V} < 0$	5.4114	.0200	—	—

Robustness of Speculative-Bubble Models against Stylized Alternatives LR Tests

Volatility Regimes	46.6188	.0000	27.6269	.0000
Mixture of Normals	44.0115	.0000	25.0195	.0000
Fads	50.6741	.0000	31.6821	.0000
van Norden-Schaller	18.9919	.0001	—	—

NOTE.—

$$r_{t+1} = \beta_{S,0} + \beta_{S,b}\beta_t + \beta_{S,V}V_t^x + u_{t+1}^S \quad \text{with probability } q(B_t, V_t^x),$$
$$r_{t+1} = \beta_{C,0} + \beta_{C,b}\beta_t + u_{t+1}^C \quad \text{with probability } 1 - q(B_t, V_t^x),$$
$$P(r_{t+1} \mid W_{t+1} = S) = q(B_t, V_t^x) = \Omega(\beta_{q,0} + \beta_{q,b} \mid B_t \mid + \gamma_{q,V}V_t^x).$$

The van Norden and Schaller model coefficients are estimated using the model described in equation (13) in the text. The p-values are calculated using standard errors estimated from the inverse of the Hessian matrix at the optimum. The volatility regimes test imposes the restrictions $\beta_{S,0} = \beta_{C,0}$ and $\beta_{S,b} = \beta_{C,b} = \beta_{q,b} = \beta_{S,V} = \gamma_{q,b} = 0$. The mixture of normals test imposes the restrictions $\beta_{S,b} = \beta_{C,b} = \beta_{q,b} = \beta_{S,V} = \gamma_{q,b} = 0$. The fads test imposes the restrictions $\beta_{S,0} = \beta_{C,0}$, $\beta_{S,b} = \beta_{C,b}$, and $\beta_{q,b} = \beta_{S,V} = \gamma_{q,b} = 0$. The van Norden-Schaller test imposes the restrictions $\beta_{S,V} = \gamma_{q,b} = 0$.

According to restriction (a), the periodically collapsing speculative-bubble model implies that the intercepts across the two regimes must be statistically different from each other ($\beta_{S,0} \neq \beta_{C,0}$). From the second panel of table 1, there are two distinct return-generating regimes, since the null hypothesis that the intercepts across the two regimes are the same is rejected at the 1% level (p-value 0.0028).

Turning to the other coefficients, although the coefficient on the relative size of the bubble in the surviving regime ($\beta_{S,b}$) is negative[8] and statistically insignificant (coefficient estimate -0.0011 with p-value 0.3787), it is greater than the corresponding coefficient in the collapsing regime ($\beta_{C,b} = -0.0438$). The speculative bubble model requires the return in the collapsing regime to be a negative function of the size of the bubble ($\beta_{C,b} < 0$), while the coefficient of the bubble size in the surviving regime must be greater than the corresponding coefficient in the collapsing regime ($\beta_{S,b} > \beta_{C,b}$). From the second panel of table 1, the bubble coefficient in the collapsing regime is statistically smaller than zero at the 5% level (p-value of the LR test 0.0273), implying that, as the bubble size increases, the returns in the collapsing regime are more negative.[9] Furthermore, we can see that restriction (c) is satisfied, since $\beta_{S,b} > \beta_{C,b}$ at the 10% level (in the second panel of table 1 the p-value of the LR test 0.0369).

Examining the estimates of the bubble coefficients of the state equations of the van Norden-Schaller model, we see that, again, in the surviving state, the bubble coefficient is negative and statistically insignificant (-0.0002 with a p-value of 0.3981). In the collapsing regime, the bubble coefficient is smaller than in the surviving regime and approximately equal to the augmented model estimate. This implies that, if abnormal volume is zero, the two models yield approximately the same expected returns.

However, the augmented model incorporates abnormal volume in the surviving equation. The point estimate of the abnormal volume coefficient in the surviving state ($\beta_{S,V}$) is statistically significant (p-value 0.0008), and has the expected sign according to restriction (f). The likelihood ratio test shows that the coefficient is positive at the 1% level. This implies that, as volume increases, the expected returns for the next period increase, consistent with our conjecture that increased abnormal volume signals increased risk and thus investors demand a higher return. For example, in September 1929, the dividend multiple-bubble deviation measure was equal to 39.25% and volume for this month was 18.30% higher than the 6-month moving average. The expected return in the surviving regime for the next time period was 0.84% for the van Norden-Schaller model and 0.95% for the augmented model[10] (10.58%

8. It is not possible to derive an expected sign for this coefficient and the speculative-bubble model only implies that it should be greater in value than the bubble coefficient in the collapsing regime. Nevertheless, we should expect that, as the bubble increases in size, investors demand a higher return to compensate them for the increased risk of bubble collapse.

9. The opposite holds for negative bubbles, since they collapse by yielding positive abnormal returns.

10. Expected returns are calculated from the point estimates of the coefficients in table 1.

and 12.06% on an annual basis, respectively). The expected return for the collapsing regime for the van Norden-Schaller model was −4.63%, while it was −4.79% for the augmented model (−43.41% and −44.57% on an annual basis, respectively). This difference in expected returns is a direct result of the inclusion of abnormal volume in the surviving state equation. The real difference of our model, however, lies in the modeling of the classifying equation that gives the probability of being in the surviving regime in the next time period.

The coefficient estimates of the classifying equation for the augmented model and the van Norden-Schaller model are in favor of the presence of periodically collapsing speculative bubbles. As the bubble grows, the probability of being in the surviving regime in period $t + 1$ decreases, since the coefficient on the absolute bubble size is negative. For the augmented model, the intercept coefficient ($\beta_{q,0}$) implies that, when the bubble size and the measure of abnormal volume are jointly zero, there is a 2.63% of being in the collapsing regime in the next time period. This probability is calculated as $1 - \Omega(\beta_{q,0})$ using the point estimates shown in table 1. The corresponding probability for the van Norden-Schaller model is 3.35%.[11]

The point estimate of the bubble coefficient for the augmented model in the classifying equation ($\beta_{q,b}$) is −1.3572 and is highly significant. The size of ($\beta_{q,b}$), hence the estimated probability of survival, is not very different between the two models if we examine only the relative size of the bubble. The point estimate of the abnormal volume coefficient in the probability equation ($\gamma_{q,V}$) is negative, as expected, and statistically significant (−0.4745 with p-value 0.0270). Therefore, the probability of being in the collapsing regime should increase significantly prior to a bubble collapse if the augmented model is superior at forecasting regime changes. Indeed, in August 1987, the probability of collapse estimated from the augmented model increased by 16.90%, to a value of 7.75%,[12] which is 2.95 times greater than if the size of the bubble were zero and volume were normal (2.63 times greater than the average probability of being in the collapsing regime). The van Norden-Schaller model estimated probability of collapse, for the same month, increases by only 7.78% to a value of 8.84%, which is only 2.63

11. Note that the unconditional average bubble size over the entire sample is 1.86% (although statistically not different from zero) and the unconditional average of the measure for abnormal volume over the entire sample is 4.79% (although statistically not different from zero as well). This implies that the average probability of being in the collapsing regime in the next time period is 2.94% for the augmented model and 3.53% for the original van Norden-Schaller model.

12. This probability is calculated as $1 - \Omega(\beta_{q,0} + \beta_{q,b} \mid B_t \mid + \gamma_{q,V} V_t^x)$ for the augmented model and $1 - \Omega(\beta_{q,0} + \beta_{q,b} \mid B_t \mid)$ for the van Norden-Schaller model using the point estimates of table 1.

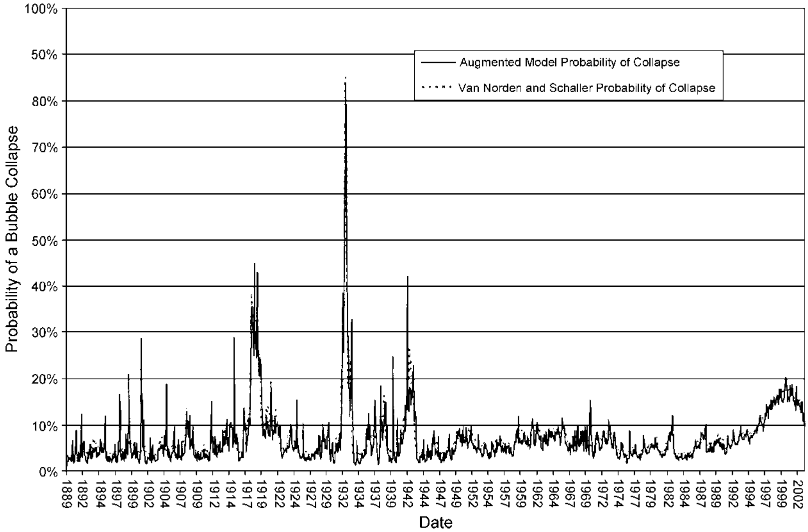

FIG. 2.—Probability of being in the surviving regime in $t+1$: Dividend multiple measure of bubble deviations, January 1888–January 2003. The probability of collapse is given by $1 - P(W_{t+1} = S) = 1 - \Omega(\beta_{q,0} + \beta_{q,b} \mid B_t \mid + \gamma_{q,V} V_t^x)$.

times greater than if the bubble size were equal to zero. The S&P 500 decreased by 3.54% in the next month and by 27.47% during the next 4 months.

The probabilities of collapse from both models, for the dividend multiple measure of fundamental values, are presented in figure 2. It is apparent that the augmented model yields a probability of collapse that is significantly more variable than the van Norden-Schaller model probability. Furthermore, the probability of being in the collapsing regime increases significantly before several bubble collapses; namely, August 1929, June 1932, June 1982, and October 2000.[13] This implies that the augmented model, incorporating an abnormal volume measure, is helpful in timing bubble collapses.

Finally, the standard deviations of the error terms are consistent with the theory of speculative bubbles, since they should have greater variance in the collapsing regime than in the surviving regime. This is because bubbles often collapse by yielding extreme negative returns (or positive returns in the case of price-decreasing bubbles). The error standard deviation in the surviving regime is 3.22% while in the collapsing regime it is 10.47% on a monthly basis.

13. Some of these periods were followed by market rallies, since we also are examining price-decreasing bubbles, which collapse yielding positive returns.

The third panel of table 1 presents the results of the LR tests of the augmented model against simpler models that capture well-documented properties of stock market returns. The LR test for the volatility regimes alternative rejects the volatility regimes model at the 1% level, implying either that the mean returns are different across the two regimes or the bubble deviation has predictive power for the returns of period $t + 1$ or for the probability of being in the surviving regime. Alternatively, both of the restrictions, may not be supported by the data. The test for the mixture of normal distributions separates the two restrictions, and the result of the LR test shows that the data reject the mixture of normals alternative in favor of the periodically collapsing speculative-bubble model. This shows that the measure of bubble deviations indeed has significant forecasting ability over the returns of the next period and the probability of residing in the surviving regime. The LR test statistic is 44.0115, signifying rejection of the null of the mixture of normal distributions at the 1% significance level.

We also examine the augmented model against a model of simple mean reversion in S&P 500 returns. Again, the fads model is strongly rejected in favor of the periodically collapsing speculative bubble. The implication of this rejection is that returns are a nonlinear function of the bubble deviations or bubble deviations can help classify returns into two regimes or both. Alternatively, the mean returns could be significantly different across the two regimes. Note that these results are consistent with the results of van Norden and Schaller even in this larger sample that contains the large bubble deviations of the 1990s.

As a final and more important statistical test, we examine whether abnormal volume has any explanatory power for the level of next period's returns and the generating state of returns. This is done by examining whether both abnormal volume coefficients are equal to zero. As seen from the third panel of table 1, the data reject the hypothesis that the measure of abnormal volume has no explanatory or classifying power in our switching regression model at the 1% level. This shows that abnormal volume is significant in explaining expected returns and it can be used to forecast the regime of the next time period.

To ensure that our model is robust to alternative specifications of fundamentals, we re-estimated the model using the bubble deviations calculated from the Campbell and Shiller measure of fundamental values. These fundamental values allow for predictable variation in the dividend-price ratio. The results for both models under this alternative fundamental specification are presented in table 2 and are roughly unchanged. In the first panel of table 2, we present the results of both speculative bubble models. The intercept coefficient estimates across the two regimes ($\beta_{S,0}$ and $\beta_{C,0}$) are 1.0079 and 0.9645, respectively, compared to 1.0079 and 0.9692 for the dividend multiple measure of

TABLE 2 **Results of the Periodically Collapsing Speculative Bubble Models for the Campbell and Shiller Measure of Fundamental Values, January 1888–January 2003**

	Augmented Model			van Norden and Schaller Model		
Coefficient	Coefficient	SE	p-Value	Coefficient	SE	p-Value
$\beta_{S,0}$	1.0079	.0010	.0000	1.0084	.0011	.0000
$\beta_{S,b}$	−.0005	.0036	.3951	.0004	.0036	.3963
$\beta_{S,V}$	.0113	.0032	.0010	—	—	—
$\beta_{C,0}$	.9645	.0152	.0000	.9656	.0154	.0000
$\beta_{C,b}$	−.0421	.0196	.0393	−.0411	.0198	.0467
$\beta_{q,0}$	1.9323	.1763	.0000	1.8538	.1729	.0000
$\beta_{q,b}$	−1.3723	.2888	.0000	−1.3430	.2959	.0000
$\gamma_{q,V}$	−.4427	.2031	.0372	—	—	—
σ_u^S	.0323	.0009	.0000	.0324	.0010	.0000
σ_u^C	.1064	.0108	.0000	.1046	.0112	.0000

Speculative Bubble Model Restrictions LR Tests

Restriction	LR Test Statistic	p-Value	LR Test Statistic	p-Value
$\beta_{S,0} \neq \beta_{C,0}$	9.3987	.0022	9.8948	.0017
$\beta_{C,b} < 0$	4.9973	.0254	5.1197	.0237
$\beta_{S,b} > \beta_{C,b}$	4.5657	.0326	4.8548	.0276
$\beta_{S,V} > 0$	14.6175	.0001	—	—
$\beta_{q,b} < 0$	27.6874	.0000	27.4445	.0000
$\gamma_{q,V} < 0$	4.8905	.0270	—	—

Robustness of Speculative-Bubble Models against Stylized Alternatives LR Tests

Volatility regimes	52.3807	.0000	33.9732	.0000
Mixture of normals	49.9112	.0000	31.5038	.0000
Fads	57.1273	.0000	38.7199	.0000
van Norden-Schaller	18.4074	.0001	—	—

NOTE.—

$$r_{t+1} = \beta_{S,0} + \beta_{S,b}\beta_t + \beta_{S,V}V_t^x + u_{t+1}^S \quad \text{with probability } q(B_t, V_t^x)$$
$$r_{t+1} = \beta_{C,0} + \beta_{C,b}\beta_t + u_{t+1}^C \quad \text{with probability } 1 - q(B_t, V_t^x)$$
$$P(r_{t+1} \mid W_{t+1} = S) = q(B_t, V_t^x) = \Omega(\beta_{q,0} + \beta_{q,b} \mid B_t \mid + \gamma_{q,V}V_t^x)$$

The van Norden and Schaller model coefficients are estimated using the model described in equation (13) in the text. The p-values are calculated using standard errors estimated from the inverse of the Hessian matrix at the optimum. The volatility regimes test imposes the restrictions $\beta_{S,0} = \beta_{C,0}$ and $\beta_{S,b} = \beta_{C,b} = \beta_{q,b} = \beta_{S,V} = \gamma_{q,b} = 0$. The mixture of normals test imposes the restrictions $\beta_{S,b} = \beta_{C,b} = \beta_{q,b} = \beta_{S,V} = \gamma_{q,b} = 0$. The fads test imposes the restrictions $\beta_{S,0} = \beta_{C,0}$, $\beta_{S,b} = \beta_{C,b}$, and $\beta_{q,b} = \beta_{S,V} = \gamma_{q,b} = 0$. The van Norden and Schaller test imposes the restrictions $\beta_{S,V} = \gamma_{q,b} = 0$.

fundamental values. The LR tests presented in the second panel of table 2 show that restriction (a) ($\beta_{S,0} \neq \beta_{C,0}$) is again supported (that is, the null hypothesis that $\beta_{S,0} = \beta_{C,0}$ is rejected) at the 1% level. The point estimate of the bubble coefficient in the collapsing equation ($\beta_{C,b}$) is smaller than zero (−0.0421), although now it is statistically significant at the 5% level (p-value 0.0393). The bubble coefficient in the surviving equation ($\beta_{S,b}$) is negative but statistically insignificant. However, the restriction that $\beta_{S,b}$ should be greater than ($\beta_{C,b}$) is supported by the data (the null hypothesis that $\beta_{S,b} = \beta_{C,b}$ is rejected at the 5% level).[14] The results for the van Norden-Schaller model also show that the statistical significance of the bubble coefficients in the state equations are qualitatively unaffected if we allow for time variation in the dividend growth rate.

Moreover, under this specification of fundamentals, bubble deviations still have significant power in predicting the generating state of returns. From the results of table 2, $\beta_{q,b}$ is smaller than zero and highly significant, as shown by the result of the *t*-test and the LR test. However, $\beta_{q,b}$ is now smaller in value (-1.3723). The size of this coefficient implies that, as the absolute size of the bubble increases, the probability that the bubble will continue to exist is significantly smaller than under the previous measure of bubble deviations. This could be caused by the fact that bubble deviations calculated from the Campbell and Shiller fundamental values are smaller on average than the dividend multiple measure of bubble deviations.

More important, the effect and the significance of abnormal volume is unchanged and $\beta_{S,V}$ is positive and statistically significant. Furthermore, $\gamma_{q,V}$ is negative (-0.4427) and statistically smaller than zero (LR test p-value 0.0270). The results show that the bubble deviations from fundamental values and the deviation of volume from the 6-month moving average have predictive ability for the generating state and the level of next period's returns.

Finally, the LR tests of the robustness of the speculative bubble models against stylized alternatives, presented in the third panel of table 2, again show that the augmented speculative-bubble model incorporates effects not captured by the other, more parsimonious models. More significant, the van Norden-Schaller LR test rejects the hypothesis that volume does not affect the levels or the generating state of future returns at the 1% level.[15]

14. Note that in the original van Norden and Schaller results, the bubble coefficient in the surviving equation was never significant, regardless of the specification of fundamentals and the significance of the bubble coefficient in the collapsing regime diminished under the Campbell and Shiller measure of fundamental values.

15. However, van Norden and Schaller estimate their model using part of our sample (January 1926–December 1989). To directly examine the validity and statistical significance of our model, we re-estimated our model using the original van Norden and Schaller sample. Again, in this subsample, the abnormal volume measure is highly significant both as a risk factor in the surviving state equation and as a classifying variable in the transition equation for both measures of fundamental values. We also examined our model for different subsamples (namely, 1888–1926, 1888–1948, 1926–54, 1954–74, 1974–89, 1974–2003, 1948–2003) to examine the robustness of the model, and the results were roughly unchanged. The results for these samples are not presented here for brevity and are available from the authors upon request.

IV. Predictive and Profitability Analysis

In the previous section, we showed that the augmented regime switching speculative-bubble model has explanatory power for S&P 500 returns. However, we said nothing about the ability of this model to forecast historical bubble collapses. In a previous study, van Norden and Vigfusson (1998) examined the size and the power of bubble tests based on regime-switching models and found that the tests are conservative but have significant power in detecting periodically collapsing speculative bubbles. However, their technique examines the econometric reliability of only the switching speculative behavior model developed by van Norden and Schaller.[16]

In this section, we examine the out-of-sample forecasting ability of the augmented model and the van Norden-Schaller model and investigate whether regime-switching speculative-bubble models can be used to create trading rules that could yield abnormal trading profits.[17] Although several bubble tests have been created, to our knowledge, all of them have been targeted at the identification of bubble presence and none of them has examined whether inferences from these tests can be used to make financially meaningful forecasts.[18] This approach also helps examine the predictive ability of our model and that of the van Norden-Schaller model in a financially intuitive way.

To ensure that the trading rules are formed using only information available to investors in real time, we cut the sample approximately in half and used the data from January 1888 to December 1945 to get initial estimates of the apparent bubble deviations and the coefficients of the augmented and van Norden-Schaller models. Using the point estimates of the two models, we then calculated the conditional probability of an unusually low and unusually high return for the next month (January

16. Billio and Pelizzon (2000) use a multivariate regime switching approach to calculate accurate value at risk (VaR) estimates. The speculative behavior models presented here could be used to construct VaR estimates for the next month.

17. Note that the fundamental values and the bubble model are constructed under the assumption of constant expected rates of return and investor risk neutrality. Indeed, the ex ante expected rate of return on the bubble is constant and equal to the expected rate of return on the bubble-free asset. For this reason, it is rational for a risk-neutral investor to hold the bubbly asset. However, the ex post realized return differs from the fundamental rate of return depending on which regime generates the bubble in a given period. This implies that, if an investor can time the bubble collapse and sell the bubbly asset, then he can earn excess returns. The investor is not able to time the bubble collapse with accuracy (if he could, then a bubble could never be formed). Nevertheless, it is logical that investors sell the bubbly asset because they do not believe that the bubble will continue to exist. If investors believed that the realized return on the bubble would be forever equal to the expected rate of return on the bubble-free asset, then they would never sell the bubbly asset and thus bubbles would grow for infinite time. Our trading rule is designed to show that there are “optimal” entry and exit times from the market.

18. Maheu and McCurdy (2000) use a Markov switching model to identify bull and bear runs in stock markets, which is able to pinpoint major market downturns, although they do not examine the model’s financial usefulness.

1946). We then proceeded to update our sample by one observation and re-estimate the models and the probabilities of a crash and a rally. We continue updating the sample period used by 1 month until the end of the sample (January 2003) was reached.

Note that we consider both probabilities, since we allow for positive and negative bubbles, and positive (negative) bubbles collapse by yielding negative (positive) returns. Using this rolling estimation, we can form forecasts from the two models using only information available to investors up to that point in time. We then proceeded to form trading rules using the inferences from both models and calculated the risk and return of the strategies each month. These trading rules are based on observation of the expected probability of a crash (rally) in the case of a positive (negative) bubble. We then evaluated the trading rules by calculating the profits (or losses) that an investor would have made if he were using the van Norden-Schaller model or the augmented model in an effort to time large market movements from January 1948 to January 2003. To calculate the conditional probability of a crash, we used the following equation:

$$P(r_{t+1} < K)_t = q(r_{t+1}|S)_t \omega\left(\frac{K - \beta_{S,0,t} - \beta_{S,b,t}B_t - \beta_{S,V,t}V_t^x}{\sigma_{u,t}^S}\right) + \left[q(r_{t+1}|C)_t\right]\omega\left(\frac{K - \beta_{C,0,t} - \beta_{C,b,t}B_t}{\sigma_{u,t}^C}\right), \tag{18}$$

where $q(r_{t+1}|S)_t = \Omega(\beta_{q,0,t} + \beta_{q,b,t}|B_t| + \gamma_{q,V,t}V_t^x)$, $q(r_{t+1}|C)_t = 1 - q(r_{t+1}|S)_t$, the time subscript attached to the coefficients denote the estimated values of the coefficients using data only up to and including time t, K is the threshold below which a return is classified as a crash given by $K = \mu_t - 2 \times (\sigma_{r,t})$, μ_t is the mean of past gross returns until time t, ω is the standard normal probability density function, and $\sigma_{r,t}$ is the standard deviation of past gross returns until time t. The conditional probability of observing an extreme positive return of at least two standard deviations above the mean of past returns in period $t + 1$ can be defined similarly.

The probabilities of a crash derived from the van Norden-Schaller model and the augmented model for the dividend multiple measure of fundamental values are presented in figure 3 together with markers to signify the 20 largest negative 1-month and 3-month returns observed for the S&P 500.[19] In the top part of the figure, we plotted the logarithm of the

19. At this point, it should be noted that we allowed for partial collapses in the specification of the bubble model; therefore, a bubble may partially collapse for several periods before starting to grow again. For this reason, we also examined the probability of a crash (rally) against the top 20 3-month negative returns as well as the top 20 draw-downs. A draw-down is defined as the cumulative return from the last local maximum to the next local minimum of the S&P 500 Index and thus refers to cumulative continuous losses.

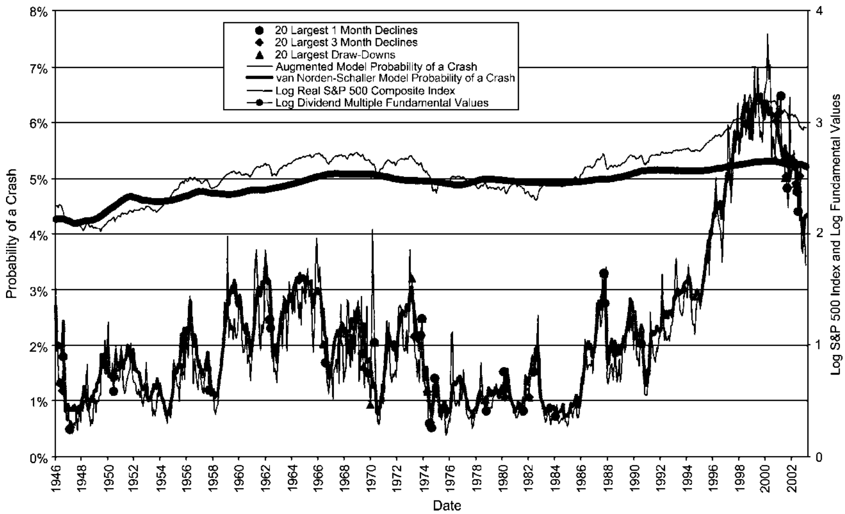

FIG. 3.—Probabilities of a crash from the van Norden-Schaller and augmented models, dividend multiple measure of fundamental values, January 1946–January 2003.

real S&P 500 index and the logarithm of the dividend multiple measure of fundamental values. From figure 3, we can see that the probability of a crash increases during several periods when a bubble is suspected, but more important, it is high before several of the 20 largest 1-month declines of the S&P 500. More specifically, we note that the probability of a crash increased by 19.40% in August 1987 to a value of 3.39%, suggesting that a bubble collapse was likely in September 1987. The corresponding probability from the van Norden-Schaller model increased by only 10.65% to a value of 3.45%. The average probability of a crash estimated using the augmented model for the previous year was 2.36%, while for the van Norden-Schaller model the average probability of a crash was 2.46%. Although the market did not crash until October 1987, the market declined for 4 consecutive months at the end of 1987, starting in September, and thus the behavior of the probability of a crash could be taken as evidence that the augmented model times bubble collapses more sharply than the van Norden-Schaller model. The probability of a crash is also high in several other periods, including 1962, 1970, and 2000. All these periods are followed by a strong correction in stock price levels.

In figure 4, we present the conditional probability of a rally calculated from the augmented model and the van Norden-Schaller model alongside with markers to signify the 20 largest 1-month, 3-month, and consecutive market advances.[20] The probability of a rally increases dramatically

20. The 20 largest consecutive market advances (draw-ups) are defined as the 20 largest consecutive positive returns, calculated as the return from the last local minimum to the next local maximum.

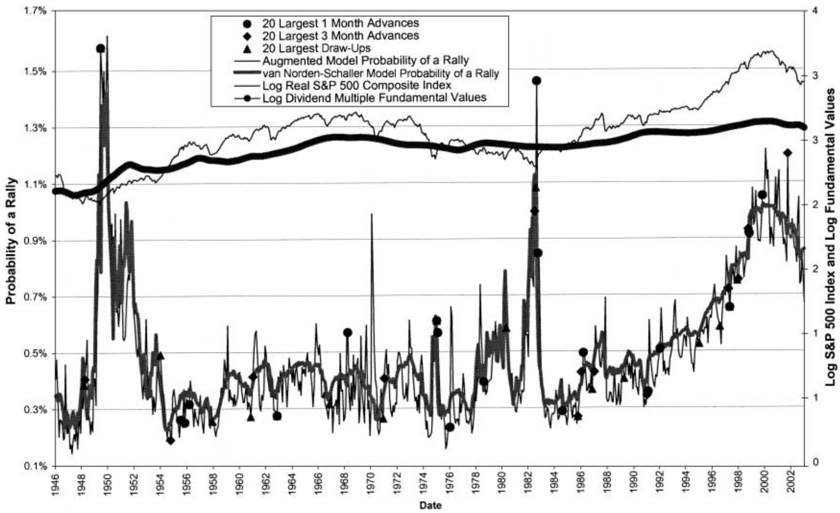

FIG. 4.—Probabilities of a rally from the van Norden-Schaller and augmented models, dividend multiple measure of fundamental values, January 1946–January 2003.

during several periods when a negative bubble appears to be present, especially in 1949–50 and 1982. Again, the probability of a rally estimated from the augmented model is significantly more variable than the corresponding probability derived from the van Norden-Schaller model. The same conclusions can be drawn by examining the probabilities of a crash and a rally produced by the Campbell and Shiller measure of fundamental values presented in figures 5 and 6. The probability of a

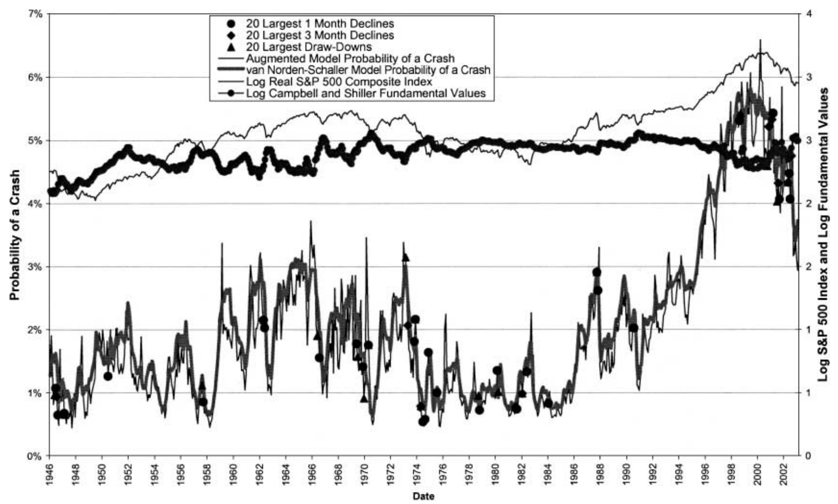

FIG. 5.—Probabilities of a crash from the van Norden-Schaller and augmented models, Campbell and Shiller measure of fundamental values, January 1946–January 2003.

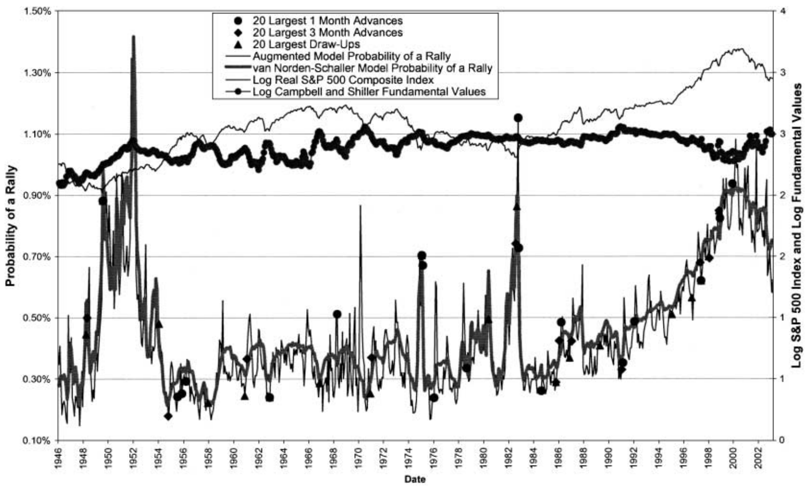

FIG. 6.—Probabilities of a rally from the van Norden-Schaller and augmented models, Campbell and Shiller measure of fundamental values, January 1946–January 2003.

crash spikes prior to market corrections and the probability of a rally increases significantly before negative bubble collapses.

However, we can see in several periods both probabilities increase simultaneously, thus damping the effects we seek to observe; namely, the conditional probability of a crash and a rally in the next period. This is because the conditional distribution of expected returns is a mixture of a low-variance (surviving state) and a high-variance (collapsing state) distribution. As the relative size of the bubble increases, the weight of the high-variance distribution increases, therefore both tails increase at the same time.

Based on the conditional probabilities of a crash and a rally from both the augmented model and the van Norden-Schaller model, we form a market-timing rule that can be used by the investor to determine when to be in or out of the market. The trading rule states that, when the probability of a crash (rally) crosses the upper 90% percentile[21] of its historical values, the investor should sell (buy) the index, investing his entire wealth in a risk-free asset (equities),[22] and maintain this position

21. Focusing on the 90th percentile is somewhat arbitrary but represents a trade-off between using too high a cutoff, which encourages the investor to remain in the market when the bubble has a historically high probability of collapse, while using too low a cutoff leads the investor out of the market too frequently, resulting in missed bull market opportunities. Our results are not qualitatively altered if an 85% or 95% cutoff is employed instead, although for the 95% cutoff, the results are inferior, since the investor stays in the market too long.

22. The risk-free rate for the period January 1946 to January 2003 is taken to be the monthly continuously compounded yield on 3-month Treasury bills. The data are taken from the Federal Reserve Bank of St. Louis Web site, http://www.stls.frb.org/fred/data/irates.html.

until the probability of a crash (rally) becomes lower than its historical median value, that is, until the bubble deflates.[23] When the appropriate probability becomes lower than its historical median value, the entire wealth should be invested in the S&P 500 Index. We include the probability of a rally in the strategy, since an investor should buy if there is a negative bubble and the probability of a rally is greater than the 90% percentile of its historical values. To ensure that we did not use any information not available to the investor in real time, we estimated the probabilities of a crash and a rally for every rolling sample based on the coefficient estimates of the rolling regression, then calculated the median value and the top 90% value recursively with a fixed starting point in January 1888.

We compared the two models by calculating the total holding period return for every month and examined the mean, standard deviation, skewness, and kurtosis of each trading rule's return distribution. We noted the number of trades that the trading rule generated over the trading period to adjust trading profits for transaction costs; we also noted the percentage of time that an investor following the rule would have invested in equities. To take into account transaction costs, we assume a 0.5% roundtrip cost paid upon exit from the market. We then compared the trading performance of the augmented model with the results of the van Norden-Schaller model and the results of a simple buy-and-hold strategy.

Furthermore, to examine the statistical significance of the profits generated from the trading rule, we formed 10,000 long random trading rules created by randomly generating series of zeros and ones, the length of which is equal to the number of months in our trading sample (January 1946–January 2003, or 685 months) using a binomial distribution. The probability of success (i.e., of a binomial draw of 1) is set equal to the percentage of time that trading rule would suggest the investor be in the market. We use this probability of success because it yields random trading rules with comparable average holding periods to our trading rules. To test for the statistical significance of the bubble rules, we compared the risk-adjusted returns and the other moments of the returns' distributions with those of the random rules. If our model yielded a risk-adjusted profit larger than 90%, 95%, or 99% of the random trading rules, we concluded that our abnormal profits are statistically significant at the 10%, 5%, or 1% level, respectively. Since this is done to compare the trading rule results to the results of other trading rules that would lead the investor in the market the same percentage of

23. We use the median to avoid any unwanted influence from extraordinarily large probabilities of a crash and a rally observed during the sample period (especially 1929–33). If we do not use a lower threshold, which forces the investor to wait before the bubble deflates, then the results are significantly affected, since the investor re-enters the market too early.

time, we compare the results only prior to adjusting for transaction costs. Finally, for every trading rule generated from the bubble models and the random rules, we calculated the wealth that an investor would have accumulated in January 2003 from an initial investment of $1 in January 1948.

The results of the augmented model and the van Norden-Schaller model trading rules are presented in table 3 alongside the results of the buy-and-hold strategy. The figures in parentheses show the percentage of randomly generated rules that would have led to higher average returns, lower standard deviation, higher skewness, and the like. Thus, in each case, the lower is the percentage value in parentheses, the better the relative performance of the bubble trading model would have been. The second column of table 3 contains the average of the real total monthly returns of the speculative-bubble models' trading rules, and the third column has the standard deviations of the total returns. Overall, we note that the augmented-model trading rule achieves a higher Sharpe ratio, since it yields higher average returns than the van Norden-Schaller model with lower standard deviations. For example, using the dividend multiple measure of fundamental values, an investor would have received an average return of 0.51% per month (6.26% continuously compounded annualized return) with a standard deviation of 2.09% if he were using the augmented model compared with 0.36% (4.39% annualized) if he was using the van Norden-Schaller model (standard deviation 1.78%). The difference in average returns decreases if the investor were using the Campbell and Shiller model to estimate fundamental values, although the augmented model still yields superior results.

Furthermore, the average return of the augmented model trading rule is statistically significant, since it is greater than the mean return of 99.99% of the random trading rules, while the van Norden-Schaller model manages to beat 99.90% of the randomly generated trading rules. The superiority of our model persists if we examine the reward to variability ratio, since the Sharpe ratio for our model is higher than the van Norden-Schaller model's ratio and higher than the Sharpe ratio of 99.99% of the random trading strategies with the same percentage of time in the market. Moreover, the Sharpe ratio of the augmented model trading rule is higher than the Sharpe ratio of the buy-and-hold strategy, and this superiority does not fade if we examine the Sharpe ratio after we take into account the transaction costs involved. For both measures of fundamental values, the augmented model trading rule yields significantly higher end-of-period wealth than the van Norden-Schaller model. This higher end-of-period wealth is achieved with higher skewness and lower kurtosis coefficients. Both these higher moments of the distribution of augmented-model returns would be more desirable to investors than those of the van Norden-Schaller model under some

TABLE 3 **Speculative Bubble Models' Trading Rules Results against Randomly Generated Trading Rules and the Buy-and-Hold Strategy, January 1946–January 2003**

Strategy	Mean Return	SD	Skewness	Excess Kurtosis	End-of-Period Wealth	Sharpe Ratio	Time in the Market (%)	Number of (Roundtrip) Trades	Adjusted Sharpe Ratio	Adjusted End-of-Period Wealth
Buy and hold	.60%	3.52%	−.60	1.76	$38.36	.1541	100.00	1	.1540	$38.17
Risk-free Investment	.05%	.44%	−1.64	6.67	$1.44	—	.00	—	—	—
Dividend Multiple Measure of Fundamentals										
Augmented model	.51%(.00%)	2.09%(12.23%)	.44(1.19%)	5.83(.20%)	$24.79(.00%)	.2172(.00%)	38.25	14	.2113	$23.11
van Norden Schaller model	.36%(.10%)	1.78%(42.74%)	1.37(.20%)	12.57(9.05%)	$9.70(.20%)	.1711(.10%)	24.53	11	.1663	$9.18
EC B V model	.38%(45.83%)	2.51%(.89%)	−.50(44.23%)	5.04(.40%)	$10.97(41.85%)	.1306(27.14%)	58.89	13	.1291	$10.28
EC model	.28%(73.26%)	2.58%(75.75%)	−.48(46.72%)	5.90(.60%)	$5.36(74.55%)	.0869(75.84%)	49.27	3	.0807	$4.80
Campbell and Shiller Measure of Fundamentals										
Augmented model	.43%(2.09%)	2.27%(9.05%)	−.23(22.17%)	7.06(1.09%)	$15.40(3.18%)	.1694(.99%)	45.54	15	.1640	$14.29
van Norden Schaller model	.37%(3.98%)	2.01%(4.47%)	.19(4.87%)	7.49(1.29%)	$1.09(3.28%)	.1564(1.39%)	36.79	12	.1517	$9.50
EC B V model	.34%(76.74%)	2.62%(4.57%)	−.71(84.10%)	5.29(.40%)	$8.07(74.25%)	.1089(67.50%)	61.37	9	.1084	$7.71
EC model	.32%(37.67%)	2.35%(24.25%)	−.32(27.14%)	6.55(.70%)	$7.36(36.88%)	.1129(33.60%)	45.92	6	.1007	$6.06

NOTE.—Trading rules are formed based on the conditional probability of a crash when a positive bubble is present and the conditional probability of a rally when a negative bubble is present. The investor either places his entire wealth in the S&P 500 Composite Index or in the 3-month U.S. Treasury bill. The end of period wealth is the real value of the investor's portfolio in January 2003 if the initial value of the portfolio in January 1946 was $1.00. All the numbers and the returns are in real terms. Figures in parentheses show the percentile ranking of the trading rule relative to 10,000 random trading rules with an equal percentage of time invested in the index. The mean return is the average monthly real total return and the standard deviation of returns is the standard deviation of total returns. The Sharpe ratio is the ratio of the mean excess return of a given trading rule over the corresponding standard deviation of returns. The percentage of time in the market is the percentage of months the trading rule produced a hold signal out of the 685 months in the sample. The number of trades is the total number of buy and sell orders produced by a given trading rule. The adjusted end of period wealth shows the end of period wealth net of transaction costs. Transaction costs are assumed to be 0.5% per roundtrip on the total value of the trade. EC denotes the error contamination model, see the text for details.

fairly weak assumptions concerning the shape of investor utility functions (see Scott and Horvath 1980 for higher moment preferences and Kraus and Litzenberger 1976 and references therein for skewness preference in asset pricing). For example, the augmented model with the dividend multiple measure of fundamental values yields 55% higher end-of-period wealth than the van Norden-Schaller model (\$24.79 against \$9.70), with a higher Sharpe ratio (0.22 compared with 0.17), positive skewness and lower kurtosis, with a similar number of trades at 28 (versus 22). Again, the augmented model's end-of-period wealth and Sharpe ratio are statistically significant, since they are higher than the end-of-period wealth of 99.90% and the Sharpe ratio of 99.99% the random trading rules.

The skewness of the distribution of total returns of the augmented model is significantly higher than the skewness of the returns to the buy-and-hold strategy, although the kurtosis coefficients of the speculative-bubble model trading rules' returns are also higher. However, the end-of-period wealth of the buy-and-hold strategy is considerably higher than the portfolio value of the augmented model trading rule (\$38.36 compared with \$24.79 and \$15.40). This implies that, if an investor used the augmented model to time entry to and exit from the market, he would have less wealth by January 2003 than if he had just held a portfolio of stocks that tracked the index. To test whether the augmented speculative behavior model leads to a higher Sharpe ratio because it causes the investor to be out of the market during volatile periods, we compared our trading results to the results of an error-contamination model suggested by van Norden and Vigfusson (1998). This model imposes the restriction that the level of returns across the two regimes is the same, although the variances are not, and that the probability of switching regimes is constant. If this model yields comparable results to the augmented and the van Norden-Schaller models, then this would be evidence that there is no nonlinear predictability of returns. The error-contamination model trading rule results are collected by imposing the following restrictions on the augmented model then following the rolling regression procedure described previously:

$$\beta_{C,0} = \beta_{S,0}, \tag{19}$$

$$\beta_{C,b} = \beta_{S,b}, \tag{20}$$

$$\beta_{S,V} = \beta_{q,b} = \gamma_{q,V} = 0, \tag{21}$$

$$\sigma_S \neq \sigma_C. \tag{22}$$

From the results of the error contamination model trading rule presented in table 3 (EC model in the table), we can see that the results of the augmented and the van Norden-Schaller trading rules demonstrate some nonlinear predictability of returns captured by the speculative behavior models. The error-contamination model leads to significantly lower end-of-period wealth and Sharpe ratios than both the augmented and van Norden-Schaller models. To further ensure that the results are not just attributable to the avoidance of high-volatility periods, we allowed for the probability of being in the surviving regime to be a function of bubble size and abnormal volume and we re-estimated the error-contamination model by imposing only the following restrictions and repeating the procedure described earlier:

$$\beta_{C,0} = \beta_{S,0}, \tag{23}$$

$$\beta_{C,b} = \beta_{S,b}, \tag{24}$$

$$\beta_{S,V} = 0, \tag{25}$$

$$\sigma_S \neq \sigma_C. \tag{26}$$

The results for this model again are inferior to the augmented model, implying that our model times the market more efficiently by capturing some nonlinear predictability of returns.

Examining the augmented model trading rule in more detail shows that the lower end-of-period wealth relative to the buy-and-hold strategy is caused by the large bubble deviation observed toward the end of the sample. Although this large and persistent bubble significantly decreases in value in the last 3 years of the sample, it does not fully collapse in our sample, thus it causes the speculative-bubble model to produce a large probability of a crash throughout the 1990s and at the end of the sample period. Figures 7 and 8 plot the real S&P 500 Composite Index together with the net excess wealth of the augmented model trading rule and of the van Norden-Schaller model trading rule, respectively, as a percentage of the wealth of the buy-and-hold strategy. On the plots of the S&P 500, we place markers that signify entry and exit times that the trading rules generated using the dividend multiple measure of fundamentals. In these figures, investor wealth has been adjusted for transaction costs assuming a 0.5% roundtrip cost paid upon exit from the market.

From figure 7, it is evident that the augmented model, estimated using the dividend multiple measure of fundamentals, forces the investor to be out of the market for long periods of time (especially in the 1960s and 1990s). Nevertheless, it produces higher wealth than the buy-and-hold strategy until December 1992. At the end of the sample, however, the buy-and-hold strategy yields significantly higher wealth, since the large observed bubble deviation does not fully reverse during the sample period employed. Furthermore, the augmented model underperforms the buy-and-hold strategy in the 1960s, since the fundamental values produce a large, persistent bubble that collapses in 1970. This causes the augmented model to produce large probabilities of extreme returns, thus

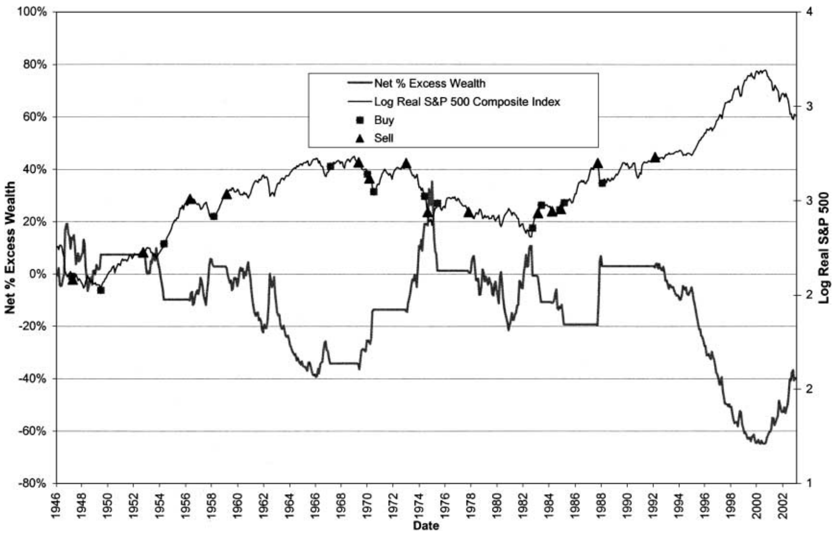

FIG. 7.—Net excess wealth generated using the augmented speculative-bubble model trading rule as a percentage of the buy-and-hold strategy wealth, dividend multiple measure of fundamentals, January 1946–January 2003.

the trading rule produces a sell signal for long periods of time, leading the investor to be out of the market 61.75% of the time.

Examining the trading rule results of the augmented model using the Campbell and Shiller fundamental values, we note that the augmented-model trading rule produces a persistent sell signal for shorter periods of time (see figure 9).[24] However, the augmented-model trading rule yields lower wealth than the buy-and-hold strategy for the entire sample. This is mainly due to the late entry of the investor in the market in the beginning of the 1950s, when the market experienced significant positive returns. The fact that the speculative behavior models cannot yield higher absolute wealth than the buy-and-hold strategy can be attributed to a number of possible reasons. First, it is possible that a bubble of the form assumed was not present in the data at the end of the sample period or the model used to estimate fundamental values is not adequate and does not capture fundamentals precisely. An alternative explanation is that this result is the consequence of a manifestation of the "peso problem,"[25] where the speculative-bubble models suggest that a market crash is imminent but do not suggest a precise date when this will occur. Such an event did not occur during the sample period, but this does not mean that

24. For comparison, we plotted the net excess wealth generated from the van Norden-Schaller model trading rule using the Campbell and Shiller (1987) measure of fundamentals in figure 10.

25. See Evans (1996) for a thorough description of the impact of peso problems for asset pricing models.

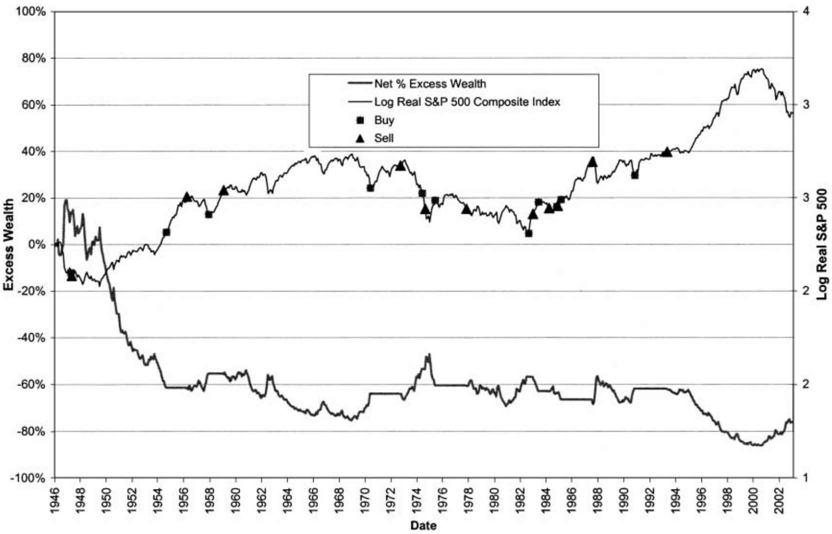

FIG. 8.—Net excess wealth generated using the van Norden-Schaller speculative-bubble model trading rule as a percentage of the buy-and-hold strategy wealth, dividend multiple measure of fundamentals, January 1946–January 2003.

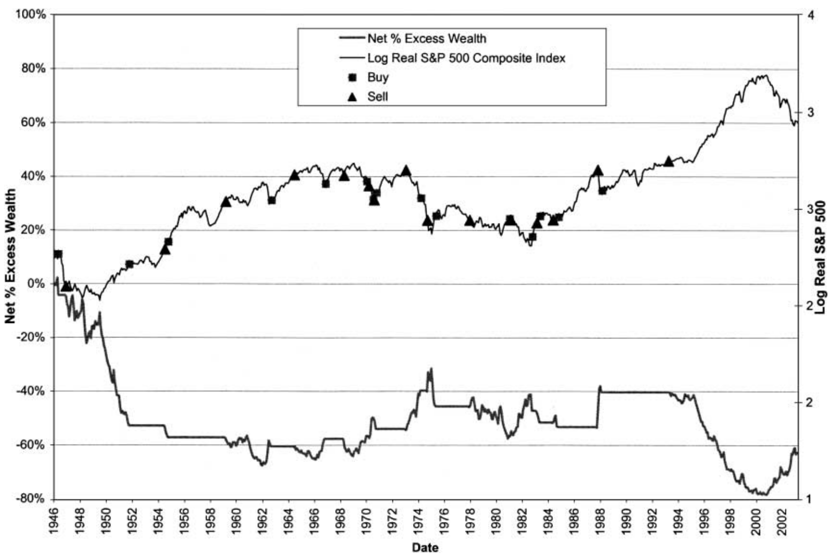

FIG. 9.—Net excess wealth generated using the augmented speculative-bubble model trading rule as a percentage of the buy-and-hold strategy wealth, Campbell and Shiller (1987) measure of fundamentals, January 1946–January 2003.

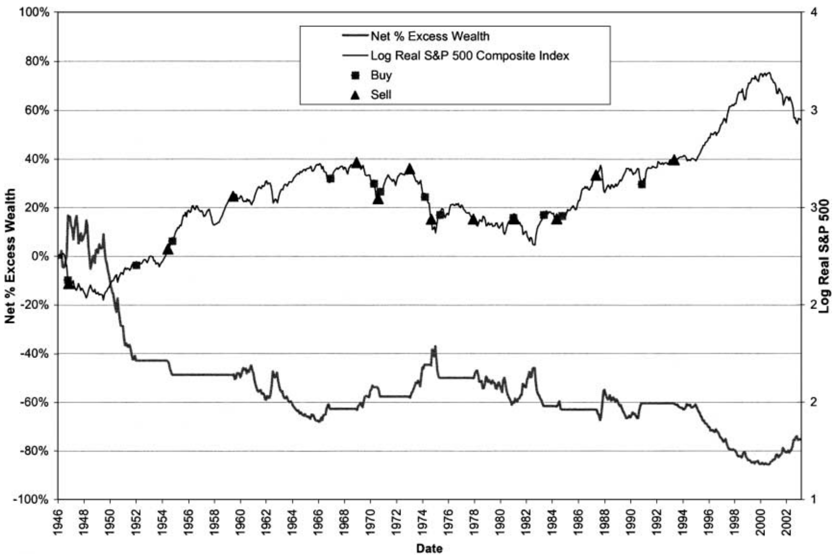

FIG. 10.—Net excess wealth generated using the van Norden-Schaller speculative-bubble model trading rule as a percentage of the buy-and-hold strategy wealth, Campbell and Shiller (1987) measure of fundamentals, January 1946–January 2003.

it was wrong to predict that it would; indeed, the market was subject to substantial falls in the end of the sample but both measures of fundamentals yield a significant positive bubble deviation in January 2003.

Finally, it should be noted that, while the augmented van Norden-Schaller model cannot improve upon a buy-and-hold rule in pure return terms, it does lead to a higher Sharpe ratio and is useful for investors as a way to reduce the risk of holding equity portfolios.

V. Conclusion

Although van Norden and Vigfusson (1998) examine the statistical power and reliability of regime-switching bubble models, to our knowledge no existing research has examined the financial usefulness of models of speculative bubbles. We test the out-of-sample forecasting ability of the regime switching model of van Norden and Schaller and an augmented version of it in a financially intuitive way. We construct trading rules based on inferences about the conditional probability of a crash and a rally and analyze the risk-adjusted returns obtained with the use of the van Norden-Schaller model and the augmented model. To ensure that we used no data not available to investors, we estimated the van Norden-Schaller and augmented models using rolling regressions with a fixed starting point. We examined the timing ability of the bubble models by comparing the returns of the speculative-bubble model trading rules with the returns on 10,000 randomly generated trading rules that have the same average proportion of the sample period invested in equities. We found that the augmented model consistently led to higher risk-adjusted returns than the van Norden-Schaller model and the randomly generated trading rules, although it did not beat the buy-and-hold strategy in risk-unadjusted (total end-of-period wealth) terms.

This result is mainly attributed to the fact that the estimated fundamental values yield a significant and persistent positive bubble deviation in the 1990s, causing the augmented model to produce large conditional probabilities of a crash. Therefore, the augmented-model trading rule forced the investor to remain out of the market for protracted periods of time, while a full correction did not occur by the end of our sample period. An alternative approach would be to allow the observed bubble deviations to follow a different generating process than the one described by Blanchard and Watson (1982) and used by van Norden and Schaller. More specifically, it could be that bubble deviations are systematic and persistent during long periods of time before entering a "critical" state, in which the probability of collapse is significantly higher. A bubble process that could produce such "distinct" periods of deterministic and explosive growth has been described in Evans (1991), and the investigation of the empirical usefulness of such speculative bubble models appears to be a fruitful avenue for further research.[26]

Appendix

Calculation of Fundamental Values

To construct fundamental values, we examine two different specifications that use only information on past prices and dividends. The models used are the dividend multiple model of van Norden and Schaller (1999) and a mathematical manipulation of Campbell and Shiller's (1987) VaR model of dividend components of prices. The dividend multiple model of fundamental values assumes a constant dividend-price ratio, whereas the Campbell and Shiller measure allows for predictable variation in the dividend growth rate. Both models assume constant discount rates.

Dividend Multiple Measure of Fundamentals

Van Norden and Schaller show that, if the discount rate is constant, then stock market prices follow the period-by-period arbitrage condition:

$$p_t = \frac{E_t(p_{t+1} + d_{t+1})}{(1+i)}. \tag{A1}$$

26. See Brooks and Katsaris (2004) for one possible such model.

Assuming that dividends follow a geometric random walk, that is, log dividends follow a random walk with a drift, it can be shown that the fundamental price of a stock will be equal to a multiple of current dividends:

$$p_t^f = \rho d_t, \tag{A2}$$

where

$$\rho = \frac{1+r}{e^{a+\sigma^2/2} - 1}.$$

We use the sample mean of the price-dividend ratio to approximate ρ. In this setting, the relative bubble size is given by

$$B_t = \frac{b_t}{p_t} = \frac{p_t - p_t^f}{p_t} = 1 - \frac{\rho d_t}{p_t}. \tag{A3}$$

Campbell and Shiller Fundamental Values

The dividend multiple measure of fundamental values assumes that the expected dividend growth rate is constant. To allow for predictable variation in the dividend growth rate, we estimated fundamental values based on the Campbell and Shiller (1987) dividend component of prices. It can be shown that the spread between stock prices and a constant multiple of current dividends is the optimal forecast of a multiple of the discounted value of all future dividend changes:

$$S_t \equiv p_t^f - \frac{1+i}{i} d_t = \frac{1+i}{i} \left[\sum_{g=1}^{\infty} \frac{1}{(1+i)^g} E_t(\Delta d_{t+g}) \right]. \tag{A4}$$

Using the VaR methodology developed by Campbell and Shiller, we examined whether changes in dividends could be forecasted by the spread between prices and the multiple of current dividends. If the changes in dividends could not be forecasted by the spread, this would imply that investors use only past dividends to form expectations about future dividends. If, on the other hand, investors include other variables in their information set, then this information is reflected in past prices and thus past realizations of the spread. This would imply that the spread has power to forecast future dividend changes. We examined this relationship using the following VaR:

$$\begin{bmatrix} \Delta d_t \\ S_t \end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \Delta d_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}, \tag{A5}$$

where both Δd_t and S_t are demeaned. This matrix equation can be rewritten more compactly as

$$z_t = A z_{t-1} + v_t. \tag{A6}$$

This VaR can be used to forecast future dividend changes conditional on the information set (H_t) described previously that contains data on past dividend growth rates and realizations of the spread. The present value of future dividend changes, equal to the fundamental spread, can be forecasted from equation (A6) using the following equation:

$$S_t^* = E_t(S_t^f | H_{t-1}) = \left(\frac{1+i}{i}\right)\omega'\left(\frac{1}{1+i}\right)A\left[I - \left(\frac{1}{1+i}\right)A\right]^{-1} z_t, \quad \text{(A7)}$$

where ω is a row vector that picks out Δd_{t-1}.[27] The fundamental price therefore can be calculated by solving equation (A7):

$$p_t^f = S_t^* + \left(\frac{1+i}{i}\right)d_{t-1}, \quad \text{(A8)}$$

where i is the average real total return over the entire period. We constructed fundamental values using equation (A8). The second measure of relative bubble size is thus given by

$$B_t = 1 - \frac{S_t^* + \left(\frac{1+i}{i}\right)d_{t-1}}{p_t}. \quad \text{(A9)}$$

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27. See Campbell and Shiller (1987) for more details concerning the methodology.

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