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Intrinsic bubbles and regime-switching

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Abstract

Froot and Obstfeld (1991) allow for an intrinsic bubble in stock prices, using approximately a century of annual data for the US, in an attempt to model the widely documented deviations from the prices predicted by present values or fundamentals. However they assume that the log of real dividends follows a constant random walk with drift over the whole period. We show that this assumption is invalid, and that a Markov-switching model is a more appropriate representation of dividends. We then generalise the formulation of stock prices (including the intrinsic bubble) to allow for this, and show that regime-switching provides a better explanation for stock prices than the bubble. We show that when allowance is made both for the bubble and for regime-switching in the dividend process, the incremental explanatory contribution of the bubble is low.

JEL classification: C32; G12; E44

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1. Introduction

The persistent deviation of US stock prices from those predicted by present value models has been studied by many researchers in recent years. Several

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explanations for this divergence have been proposed, including time varying discount factors, noise traders, and fads (see, e.g. Shiller, 1989). Although some of these are appealing, they fail to account successfully for the bulk of the divergence between fundamentals and stock prices. Froot and Obstfeld (1991) are more successful in using an 'intrinsic bubble'. This bubble is a nonlinear function of current dividends, and thus models the apparent over-reaction of stock prices to them. Interestingly their analysis excludes consideration of regime changes, although they comment on the well-known similarities between the effects on asset prices of bubbles and those of future regime changes, and they speculate on the possibility that regime switching might provide an alternative explanation for stock prices. Moreover, several authors have argued that there have, in fact, been regime changes in US dividends.[1]

Since there appears to be regime-switching in dividend data, and it appears that an intrinsic bubble can go some way towards rationalising over-reaction of stock prices to current dividends, we extend the analysis of Froot and Obstfeld to allow for both features.[2] We can then address the question to what extent, once regime switching is allowed for, there remains a role for an intrinsic bubble. Allowing for stochastic regime switching not only changes the relationship between the dividend and the fundamental stock price, it also affects the relationship between the dividend and the bubble.

We find that a model accounting for regime changes accounts for most of the differences between stock prices and fundamentals, and also appears to provide a better characterization of the evolution of stock prices than the simple intrinsic bubble model without regime-switching. Although this model is outperformed by one which allows for both intrinsic bubbles and regime switching, the improvement obtained in the fit by including a bubble, once regime-switching has been allowed for, is relatively modest, and occurs in periods in which the bubble element of the estimated stock price is small. The bubble may simply be acting as proxy for any non-linearities in the data.

An unappealing feature of the intrinsic bubble is that it is permanent and its expected value is explosive. Therefore, it may be seen as encouraging that

[1] These include Cecchetti et al. (1990) and Bonomo and Garcia (1994). They do not, however, consider the possibility of a bubble, and therefore do not compare this alternative explanation of movements in stock prices.

[2] We use data on US stock prices and dividends reproduced by Robert Shiller listed in Shiller (1989), Chapter 26, where more details of the data can be found. The stock prices are January values for the Standard and Poor Composite Stock Price Index. Each observation in the dividend series is an average for the year in question. Nominal stock prices and dividends are deflated by the producer price index (Shiller's series 6 of prices for January each year) to get real stock prices and dividends. Froot and Obstfeld (1991) state that, in addition to the data on which their published results were based, they also used this data and found their results with respect to the existence of bubbles to be unaffected.

regime-switching provides a better explanation of the data than an intrinsic bubble.

2. Analysis of data on dividends

2.1. Random walk versus stochastic segmented trends and variances

Froot and Obstfeld (1991) do not reject the hypothesis that the logarithm of real dividends follows a random walk with drift. They also assume normality of the residuals. This assumption (and the constancy of unconditional moments) is central to the theory developed in their paper. We regress the change in the log of real dividends (d_t) on a constant μ, viz., $\Delta d_t = \mu + \epsilon_t$, for the period 1900–1987, where ϵ_t is a assumed to be white noise. We find that the assumption that ϵ_t is normally distributed is violated. A Jarque–Bera test for normality gives a test statistic of 21.5241 which has a $\chi^2(2)$ distribution under the null hypothesis. The errors also appear to contain ARCH effects ($\chi^2(4) = 13.6346$). Fig. 1 graphs the data for the change in the logarithm of real dividends. It suggests that during the period from 1952 dividend growth was more stable than in earlier periods. In contrast, the period between 1915 and 1945 appears to have been particularly volatile. Support for this possibility is provided by a rolling regression of the change in the logarithm of dividends against a constant, the results of which are shown in Figs. 2 and 3. Fig. 2 suggests some variation in the rate of growth of dividends, and Fig. 3 clearly shows a decline in the standard

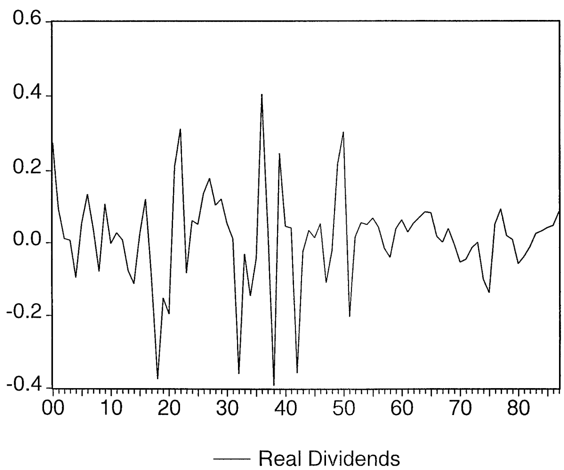

Fig. 1. Growth of real dividends.

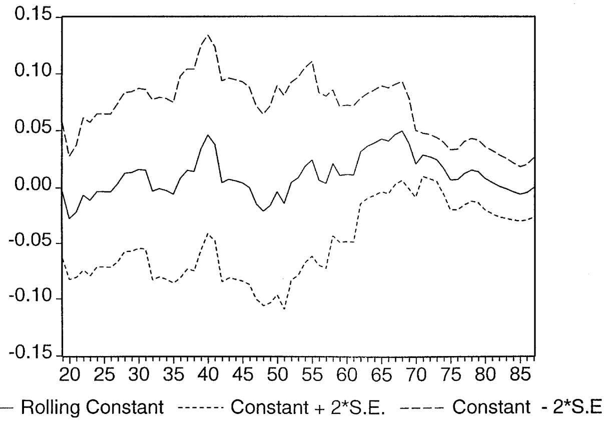

Fig. 2. Rolling regression of the first difference of real dividends against a constant. (The rolling regression is carried out using a window of 20 observations.)

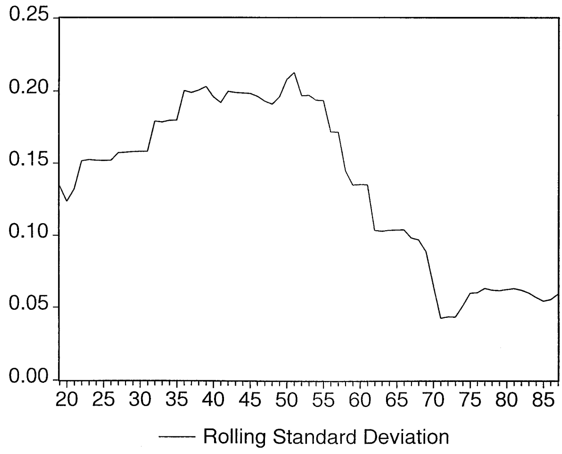

Fig. 3. Rolling regression of the first difference of real dividends against a constant. (The regression is carried out using a window of 20 observations.)

deviation of innovations in the second part of the sample. In the next section we analyze the mean-variance switching representation.

2.2. A switching mean–variance representation

Given the apparent non-constancy of the unconditional moments of the random walk model, we use the discrete regime-switching model to characterize

Table 1
Regime-switching models for dividends

μ_0	0.0007
	(0.0334)
μ_1	0.0223
	(0.0013)
σ_0	0.1997
	(0.0319)
σ_1	0.0546
	(0.0134)
p	0.8898
	(0.0984)
q	0.8410
	(0.1792)
Log-likelihood	151.2649
White test for autocorrelation $\chi^2(4)$	13.467**
White test for ARCH $\chi^2(4)$	7.526
White test of Markov specification $\chi^2(4)$	1.876
LM test for autocorrelation in regime 0 $\chi^2(1)$	0.248
LM test for autocorrelation in regime 1 $\chi^2(1)$	6.661**
LM test for autocorrelation across regimes $\chi^2(1)$	6.565
LM test for ARCH in regime 0 $\chi^2(1)$	3.024
LM test for ARCH in regime 1 $\chi^2(1)$	1.349
LM test for ARCH across regimes $\chi^2(1)$	0.004
Normality $\chi^2(2)$[a]	2.815

[a]This test is calculated using standardised residuals defined as $\Delta d_t - (\mu_0 P(s_t = 0 | I_t) + \mu_1 P(s_t = 1 | I_t))$ divided by their conditional standard deviation.
**Denotes significant at 1%.

the first difference of the logarithm of real dividends (see Hamilton, 1994). This model postulates the existence of an unobserved variable (denoted s_t) which takes the values 0 or 1. When $s_t = 0$ the first difference of the logarithm of real dividends, Δd_t, is distributed $N(\mu_0, \sigma_0^2)$; and when $s_t = 1$ it is distributed $N(\mu_1, \sigma_1^2)$. The states are assumed to follow a first order homogeneous Markov process with $p(s_t = 1 | s_{t-1} = 1) = p$ and $p(s_t = 0 | s_{t-1} = 0) = q$. The evolution of the logarithm of real dividends can therefore be written as

$$d_{t+1} = d_t + \mu_0(1 - s_{t+1}) + \mu_1 s_{t+1} + (\sigma_0(1 - s_{t+1}) + \sigma_1 s_{t+1})\epsilon_{t+1}, \qquad (1)$$

where ϵ_{t+1} is an i.i.d. standard normal variable. Table 1 shows the results obtained from estimating this model for the sample period 1900–1987. Both standard deviations are significantly different from zero at the 1% level, but the mean is not significantly different from zero in state 0.[3] If we compare the results presented in Table 2 with Froot and Obstfeld's results, for which $\mu = 0.011$ and $\sigma = 0.122$, we find their values to be approximately an average of the means and variances in states 0 and 1. We find that state 0 is a low-growth/high-variance state, and state 1 is a high-growth/low-variance state. Fig. 4 graphs the allocation of observations between the two states.

Finally, we perform specification tests on the model as in Engel and Hamilton (1990). The tests presented in Table 1 are White (1987) tests and Lagrange multiplier specification tests. White's is a score type test, which is based on the fact that if a maximum likelihood model is correctly specified, the score statistics should be serially uncorrelated. Hamilton (1996) extends this test to the Markov switching model.[4] The Jarque–Bera test now does not reject normality of the standardised residuals, in contrast to the results reported above (Section 2.1) for the random walk model for log dividends.

Table 1 appears to show the presence of some autocorrelation, though all other tests for misspecification are passed. Note that the hypothesis of the Markov specification has not been rejected.[5]

3. Models of stock prices

Froot and Obstfeld (1991) take as their point of departure the familiar condition that the real stock price should equal the present discounted value of the real dividend payment plus the real stock price next period:

$$P_t = e^{-r} E_t(D_t + P_{t+1}). \tag{2}$$

The real rate of discount r is assumed to be constant (the price P_t might be seen as the start-of-period price, with the dividend D_t paid at the end of the period).

[3] A statistical test confirms that the variances of dividends are significantly different in the two regimes. The LR test for the hypothesis $\sigma_0 = \sigma_1$ produces a test statistic 35.56 ($\chi^2(1)$), and the hypothesis is thus clearly rejected. The means are however not significantly different. The LR test for $\mu_0 = \mu_1$ gives a test statistic 0.13 ($\chi^2(1)$). Although we do not reject the equality of the mean growth rates, in the remainder of the paper we have reported results for the unrestricted regime-switching model. If anything, this introduces a small bias against our model, since in comparing it with alternatives this procedure involves the loss of one degree of freedom. It may be noted that a test on the Markov process, with the null hypothesis $p + q = 1$, gives a test statistic 7.18 ($\chi^2(1)$), and is therefore rejected. This confirms that the probability of being in a given state on date t is not independent of the state at date $t - 1$.

[4] Hamilton recommends, on the basis of Monte-Carlo experiments, that for small samples one might prefer to use the 1% critical value from the asymptotic distributions as a guide for a 5% small-sample test based on the Newey–Tauchen–White specification tests or Lagrange multiplier tests.

[5] Further evidence in favour of the two state model is found in Bonomo and Garcia (1994).

Any rational bubble B_t in the stock price satisfies

$$B_t = e^{-r}E_t(B_{t+1}). \tag{3}$$

The process that drives the log of dividends is assumed to be a random walk with drift μ:

$$d_{t+1} = \mu + d_t + \xi_{t+1}, \tag{4}$$

where $\xi_{t+1}N(0,\sigma^2)$.

The 'intrinsic bubble'[6] is postulated to be a non-linear function of the dividend D_t, which must satisfy Eq. (3). Froot and Obstfeld show that the following is such a function

$$B(D_t) = cD_t^{\lambda}. \tag{5}$$

The parameter λ is the positive root of the quadratic equation

$$\frac{\sigma^2}{2}\lambda^2 + \mu\lambda - r = 0, \tag{6}$$

and c is an arbitrary constant.

The present value (denoted by P_t^{pv}) is proportional to dividends:

$$P_t^{\text{pv}} = kD_t, \tag{7}$$

where $k = (e^r - e^{(\mu + \frac{1}{2}\sigma^2)})^{-1}$. In general the stock price may contain both the present value and a bubble component:

$$P_t = kD_t + cD_t^{\lambda}. \tag{8}$$

While the fundamental stock price implies that the price–dividend ratio (k) is constant, the bubble allows it to be a function of the current dividend. It allows the possibility of the stock-price over-reacting to the current dividend.

Because Eq. (8) contains both D_t and D_t^{λ} as explanatory variables, the data might be nearly colinear (at least for values of λ near 1). Froot and Obstfeld therefore estimate an equation for the price–dividend ratio:

$$\frac{P_t}{D_t} = k + cD_t^{\lambda - 1} + \zeta_t. \tag{9}$$

Their main conclusion is that the existence of this type of bubble cannot be rejected and it may account for most of the observed difference between the stock prices and the fundamentals.[7] Estimating this model using Shiller (1989) data, we also reject the null hypothesis of no intrinsic bubble. Table 2, columns

[6] Early work on rational bubbles is represented by Flood and Garber (1980) and Hamilton (1986).

[7] Notice that Eq. (9) augments Eq. (8) by a disturbance term which they interpret as a random measurement error ζ_t distributed $N(0,\theta^2)$.

Table 2
Models which allow for regime-switching and bubbles

	Regime-switching in dividends			No regime-switching	
	Bubble	No bubble		Two-step	Joint estimation
μ_0	0.0076	0.0128			
	(0.0077)	(0.0147)	μ	0.0141	0.0115
μ_1	0.0384	0.0651		(0.0163)	(0.0093)
	(0.0086)	(0.0081)			
σ_0	0.1526	0.1469	σ	0.1338	0.1328
	(0.0256)	(0.0231)		(0.0194)	(0.0163)
σ_1	0.0636	0.0694			
	(0.0312)	(0.0291)			
θ_0	2.9476	3.4156			
	(0.8712)	(0.4191)	θ	4.0266	4.0428
θ_1	2.2506	1.9240		(0.5030)	(0.5051)
	(0.5522)	(0.9685)			
k_0	15.0148	19.3796	k	15.3868	14.3958
	(2.7651)*	(3.0014)*		(2.1475)	(2.5599)*
k_1	17.9737	30.6301			
	(4.3213)*	(5.3667)*			
p	0.9636	0.9704			
	(0.0912)	(0.0634)			
q	0.9823	0.9782			
	(0.0520)	(0.0515)			
c_0	0.1529				
	(0.0733)		c	0.0339	0.0805
c_1	0.2874			(0.0131)	(0.0611)
	(0.0801)*				
λ	2.1488			2.78	2.5411
	(0.7293)*				(1.092)*
Log-likelihood	0.6475	− 16.1194		− 33.5911	− 33.9658
AIC	17.425	48.239		77.182	75.932
SIC	39.721	68.058		89.568	85.841
HQ	26.408	56.224		82.172	79.924
AR(1)	10.908	30.8599		49.2618	49.1687
AR(4)	13.628	33.0537		50.6311	50.5677
ARCH(1)	1.7428	19.2565		22.2035	21.9512
ARCH(4)	6.2686	19.9846		22.3975	22.2223

Notes: The figures in parentheses are autocorrelation and heteroskedasticity-consistent standard errors. The Akaike, Schwarz, and Hannan–Quinn model selection criteria are calculated as $\mathrm{AIC} = -2l_m + 2g$, $\mathrm{SIC} = -2l_m + 2g\ln(T)$, and $\mathrm{HQ} = -2l_m + 2g\ln(\ln(T))$, respectively, where l_m is the maximum value of the Gaussian log-likelihood function and g is the number of freely estimated parameters. The standard errors with an asterisk *, are asymptotic standard errors of restricted parameters calculated in the following way: given $\hat{x} = x(\hat{\mu}_0, \hat{\mu}_1, \hat{\sigma}_0, \hat{\sigma}_1, \hat{\theta}_0, \hat{\theta}_1, \hat{p}, \hat{q})$, asymptotic variance of $\hat{x}$ is

$$\mathrm{Var}(\hat{x}) = \hat{J}\,\mathrm{Var}(\hat{\mu}_0, \hat{\mu}_1, \hat{\sigma}_0, \hat{\sigma}_1, \hat{\theta}_0, \hat{\theta}_1, \hat{p}, \hat{q})\hat{J}',$$

where $J = [\partial x/\partial \mu_1, \ldots, \partial x/\partial q]$, and $x \in \{k_0, k_1, \lambda\}$. To calculate the standard error of c_1 use $\hat{c}_1 = c_1(\hat{c}_0, \hat{\mu}_0, \hat{\mu}_1, \hat{\sigma}_0, \hat{\sigma}_1, \hat{\theta}_0, \hat{\theta}_1, \hat{p}, \hat{q})$, and $J = [\partial c_1/\partial c_0, \partial c_1/\partial \mu_0, \ldots, \partial c_1/\partial q]$.

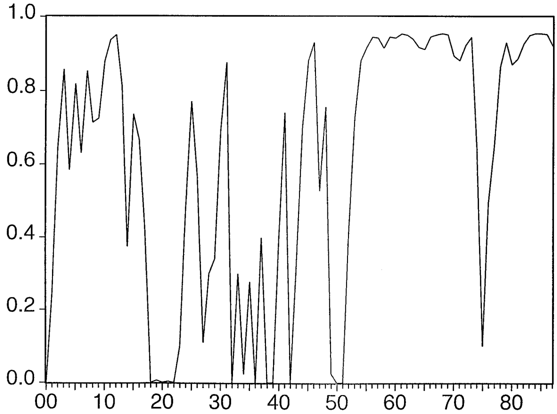

Fig. 4. Probabilities of state 1 – univariate filter. (These are filtered probabilities conditional on information available at time t.)

3 and 4 give our estimates. In column 3, a two stage procedure has been used. The parameters of the process for dividends (μ and σ of Eq. (4) above) are first estimated and used to compute a value for λ. This is then used in the estimation of the equation for stock prices, Eq. (9). In Table 2, column 4, Eqs. (4) and (9) are estimated jointly, with λ determined by Eq. (6) and k as defined above. Both sets of estimates display evidence of misspecification. Fig. 6 shows actual stock prices, the estimated fundamental, and the estimated fundamental-plus-bubble, obtained using the joint estimation procedure.

3.1. Allowing for regime shifts in the dividend process

We now assume that the dividend process is that set out in Eq. (1) in Section 2.2 above, with the state of the system following the two-state Markov process. We assume, as before in Eq. (2), that the price of shares equals the expected present discounted value of the end of period price plus the dividend.

The fundamental value of the stock price is once again proportional to the current dividend, but the factor of proportionality depends on the state.[8] Let $P_t^{\mathrm{pv}} = k_0 D_t$ when $s_t = 0$, and $P_t^{\mathrm{pv}} = k_1 D_t$ when $s_t = 1$. Then k_0 and k_1 satisfy

$$k_0 = e^{-r}(1 + qk_0a_0 + (1-q)k_1a_1) \tag{10}$$

[8] Cf. Cecchetti et al. (1990).

and

$$k_1 = e^{-r}(1 + pk_1a_1 + (1-p)k_0a_0),$$

where $a_0 = e^{(\mu_0 + 1/2\sigma_0^2)}$ and $a_1 = e^{(\mu_1 + 1/2\sigma_1^2)}$.

The intrinsic bubble can be shown to be $B_t = c_iD_t^\lambda$ when $s_t = i$, for $i = 0,1$, for some values of λ, c_0, and c_1. It satisfies $B_t = e^{-r}E(B_{t+1}|\Omega_t)$, where the information set Ω_t includes $(s_t,s_{t-1},\ldots,D_t,D_{t-1},\ldots,P_t,P_{t-1},\ldots)$. Therefore, when $s_t = 0$ the bubble satisfies

$$c_0D_t^\lambda = e^{-r}(c_0qD_t^\lambda e^{(\lambda\mu_0 + 1/2\lambda^2\sigma_0^2)} + (1-q)c_1D_t^\lambda e^{(\lambda\mu_1 + 1/2\lambda^2\sigma_1^2)})$$

and an analogous equation holds when $s_t = 1$. These two equations can be solved for λ and the ratio c_1/c_0. The first equation gives

$$\frac{c_1}{c_0} = \frac{e^r - qe^{(\lambda\mu_0 + 1/2\lambda^2\sigma_0^2)}}{(1-q)e^{(\lambda\mu_1 + 1/2\lambda^2\sigma_1^2)}}. \tag{11}$$

and the second gives

$$\frac{c_1}{c_0} = \frac{(1-p)e^{(\lambda\mu_0 + 1/2\lambda^2\sigma_0^2)}}{e^r - pe^{(\lambda\mu_1 + 1/2\lambda^2\sigma_1^2)}}. \tag{12}$$

Eqs. (11) and (12) have a unique positive solution for c_1/c_0 and λ. Eq. (12) has $c_1/c_0 < 1$ when $\lambda = 0$, and is increasing in λ so long as the denominator remains positive. Eq. (11) has $c_1/c_0 > 1$ for $\lambda = 0$, and is decreasing in λ, reaching zero when $qe^{(\lambda\mu_0 + 1/2\lambda^2\sigma_0^2 - r)} = 1$.

Consequently, there is just one combination of $\lambda(>0)$ and the ratio $c_1/c_0(>0)$ which solves Eqs. (11) and (12). Fig. 5 illustrates the solution.

Putting together these results, and expressing the stock price as the sum of the fundamental and bubble components we have a straightforward generalisation of Froot and Obstfeld's formulation

$$P_{s_t} = P_{s_t}^{\mathrm{pv}} + B_{s_t}(D_t), \tag{13}$$

where

$$P_{s_t}^{\mathrm{pv}} = (k_0(1-s_t) + k_1s_t))D_t \tag{14}$$

and

$$B_{s_t}(D_t) = (c_0(1-s_t) + c_1s_t))D_t^\lambda. \tag{15}$$

In the absence of the bubble, the Markov-switching model for dividends allows the price-dividend ratio to differ between states. The introduction of the bubble now adds a second component to the stock price which is both state-dependent

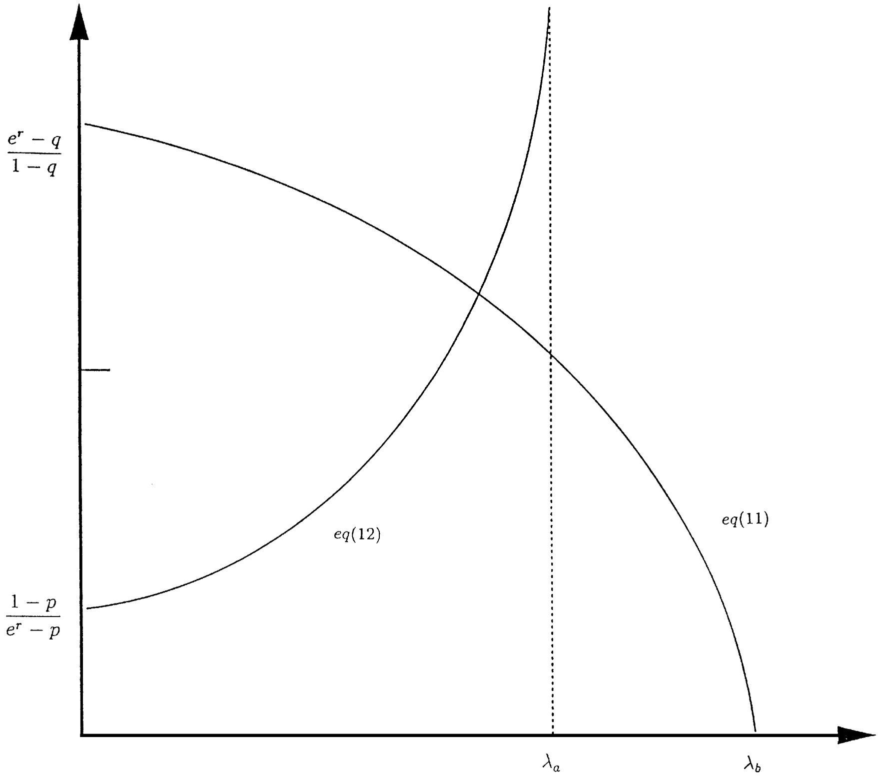

Fig. 5. The Determinants of c_1/c_0 from Eqs. (11) and (12)

$$\lambda_a = \frac{-\mu_1 \pm \sqrt{\mu_1^2 + 2\sigma_1^2(r - \log(p)}}{2\sigma_1^2}, \qquad \lambda_b = \frac{-\mu_0 \pm \sqrt{\mu_0^2 + 2\sigma_0^2(r - \log(q)}}{2\sigma_0^2}.$$

and elastic with respect to the current dividend. We divide through by dividends, and estimate a system for the price-dividend ratio and the log of dividends:

$$\frac{P_t}{D_t} = k_i + c_i D_t^{\lambda - 1} + \theta_i v_t \quad \text{in state } i, \text{ for } i = 0,1,$$

$$d_t = \mu_i + d_{t-1} + \sigma_i u_t, \tag{16}$$

where u_t and v_t are i.i.d. standard normal variables while θ_0 and θ_1 are positive constants.[9]

[9] Notice that, as in Froot and Obstfeld (1991), we have augmented the model with random disturbances which may be interpreted as measurement errors.

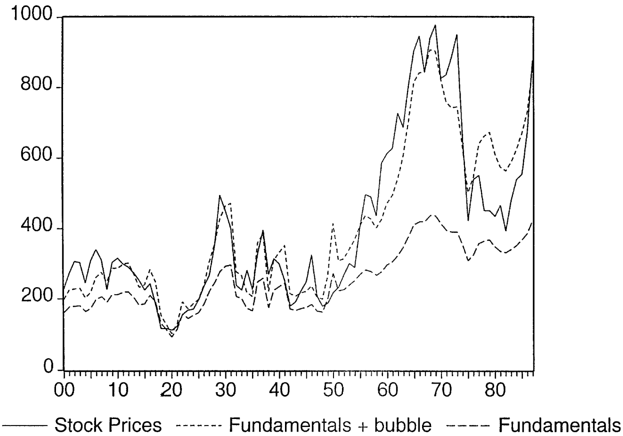

Fig. 6. Stock prices. (The fundamentals are constructed as in Eq. (7) while the bubble is constructed as in Eq. (5).)

The estimation procedure assumes that agents in the financial markets know the actual state of the system, s_t, at each point in time, whereas the econometrician does not, and has to make inferences of it based on the observable history of the system, i.e., the information contained in the history of stock prices and dividends.

We estimate the model subject to the theoretical restrictions on k_0, k_1, c_0, c_1, and λ implied by Eqs. (10)–(12). The estimation procedure is described briefly in Appendix A. The constant discounting factor is chosen as in Froot and Obstfeld to be the sample-average gross real return $r = 0.0816$.

4. Empirical results

Table 2, column 1, and Fig. 7 show the results of estimating the model described in Section 3.[10]

[10] To carry out specification tests for this model we compute the residuals as $pr_t - E(pr_t \,|\, \Omega_t : \hat{\psi})$, where $pr_t = (P_t/D_t)$ and $\hat{\psi}$ are estimates of the parameter vector, $\psi = (c_0, \mu_0, \mu_1, \sigma_0, \sigma_1, \theta_0, \theta_1, p, q, \lambda)$. The conditional expectation of pr_t is constructed by multiplying the probabilities of the states obtained from the filter by the functional form in those states. Based on the sample estimates $\hat{\psi}$, the predicted values of pr_t can be written as $E(pr_t|I_t{:}\hat{\psi}) = P(s_t = 0|I_t)(\hat{k}_0 + \hat{c}_0 D_t^{\lambda-1}) + P(s_t = 1|I_t)(\hat{k}_1 + \hat{c}_1 D_t^{\lambda-1})$. We then standardise the residuals by dividing them by the conditional standard deviation. Tests for AR (Godfrey–Breusch) and ARCH (Engle) errors are then performed. In Table 2 we show heteroskedasticity and autocorrelation-consistent standard errors. ML estimation was carried out using a variable-metric algorithm that approximates the Hessian according to the BFGS update. The pre-whitened quadratic spectral kernel with data-dependent bandwidth discussed in Andrews (1991) and Andrews and Monahan (1992) was used for the covariance matrix estimator.

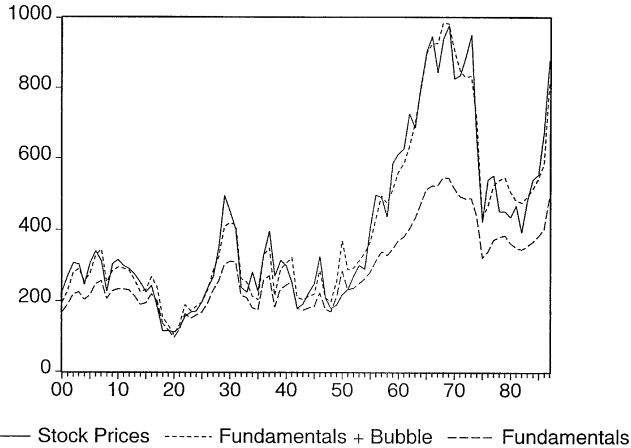

Fig. 7. Stock prices allowing for changes in regime for dividends and bubble. (The fundamentals are constructed as in Eq. (14) while the bubbles are constructed as in Eq. (15). The states are replaced by the filtered probabilities.)

The estimated means and variances of the logarithm of real dividends are mainly separated as in the univariate filter for dividends reported in Table 1. State 0 is a low-mean/high-variance state and state 1 a high-mean/low-variance state. The constants of proportionality between dividends and the fundamental share price are $k_0 = 15.01$ and $k_1 = 17.97$, respectively. The bubble coefficient is significant and is higher in state 1 ($c_1 = 0.29$) than in state 0 ($c_0 = 0.15$). The elasticity of the bubble with respect to the dividend is 2.14. The model shows no sign of ARCH errors of up to order 4. There is evidence of autoregressive errors, but the standard errors have been constructed so as to be robust with respect to this.

Fig. 7 shows the evolution of actual stock prices, predicted fundamental stock prices, and predicted fundamental-plus-bubble stock prices. The bubble term accounts for most of the large deviations between fundamentals and stock prices, especially those in the second part of the sample, where the bubble term appears to be particularly important.

The allocation of time periods to the two states is shown in Fig. 8. The period between 1900 and 1910 is attributed with high probability to state 1. The period between 1910 and 1955 is attributed mostly to state 0, with brief departures from it around 1930 and 1945. The period from 1955 to 1975 is attributed to the high growth/low-variance state (state 1). The remaining observations fall into state 0, with a final shift before the crash in 1987 to state 1. The periods for which observations are attributed to state 1 are ones in which the actual share price appears to be a long way above the fundamental price and the bubble component is particularly large. It is interesting to note that the allocation of observations to states when the joint model for both dividends and stock prices

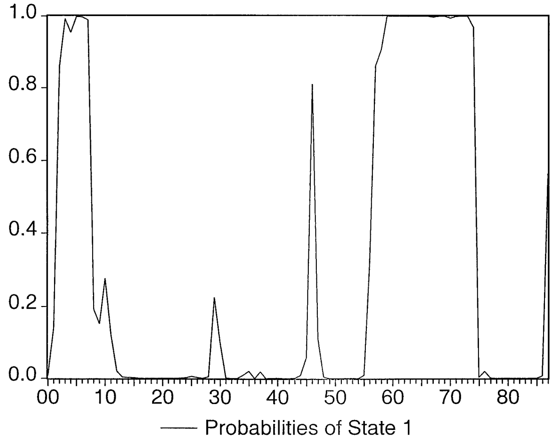

Fig. 8. Probabilities of state 1 allowing for changes in regime for dividends and bubbles. (Filtered probabilities obtained from the model as presented in Eq. (16).)

is estimated, displayed in Fig. 8, is much 'cleaner' than when the model for dividends alone is estimated, the results of which are displayed in Fig. 4. In Fig. 8, a long period, roughly from 1910 to 1955, is allocated substantially continuously to state 0, whereas in Fig. 4 this period shows much greater movement between the two states. This difference is presumably a result of the additional information on the state contained in the price–dividend ratio which reflects the perceptions of market participants, and it reveals the advantage of considering dividends and stock prices jointly. The difference between the figures is also reflected in the differences between the estimated values of p and q in Table 1 (the model for dividends alone) and Table 2 (the model for dividends and stock prices jointly). In Table 2 the estimated values of p and q are higher than in Table 1, indicating greater persistence of each state.

Table 2 (column 2) shows that the hypothesis that there is no bubble in stock prices is rejected (the likelihood ratio test statistic is 33.52, distributed $\chi^2(1)$). However, judging from Figs. 7 and 9, the improvement in fit obtained by including a bubble, once regime-switching has been allowed for, appears to occur in periods in which the bubble element of the estimated stock price is small. This suggests that the bubble may simply be acting as proxy for any non-linearities in the data.

More interesting is a comparison between the regime-switching model without the bubble (Fig. 9), and the bubble model without regime-switching (Fig. 6). Visual inspection of Figs. 6 and 9 suggests that both models account successfully for the boom in stock prices of the 1950s and 1960s and the collapse in the early 1970s. However, they offer two very different interpretations of these events. The intrinsic bubble diagnoses them as an over-reaction to the current dividend (the

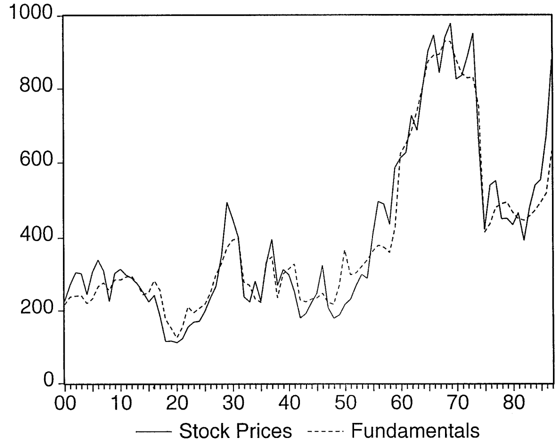

Fig. 9. Stock prices allowing for changes in dividend regimes. (The fundamentals are constructed as in Eq. (14). The states are replaced by the filtered probabilities obtained using Eq. (16) and setting the bubble parameters to zero.)

bubble component of the stock price has an elasticity of 2.54 with respect to the current dividend). The regime-switching model interprets the boom as a response of the present-value stock price to a change of regime into an era of rapidly growing dividends. The Akaike, Schwarz, and Hannan–Quinn specification criteria, reported in Table 2, all clearly favour regime-switching (column 2) over the bubble (column 4). For all three criteria there is a substantial fall in the value when one moves from the model with bubbles but no regime switching to the model with regime-switching but no bubble. The Akaike criterion falls from 76 to 48, the Schwartz from 86 to 68, and the Hannan–Quinn from 80 to 56 (all rounded to the nearest whole number). Since these two models are non-nested, there is no straightforward formal way of comparing them. Nevertheless, these criteria provide a useful heuristic comparison, and suggest that there is a clear difference between the models. The difference in values of the three criteria, and the differences in their change between one model and the other reflects the extent to which each penalises the number of degrees of freedom used up in estimating the models. Comparing Figs. 6 and 9 reinforces the view that allowing for regime-switching in the dividends process contributes at least as much to an explanation of the data as does allowing for the bubble.

5. Conclusions

We have developed a framework within which to compare intrinsic bubbles and regime-switching as explanations for the behaviour of stock prices. We find that dividends are appropriately modelled as a Markov process with two states, each with a separate mean and variance. Extending Froot and Obstfeld's analysis, stock prices are modelled as being in general the sum of fundamentals and an intrinsic bubble, consistently with the Markov process in dividends.

When stochastic regime-switching is introduced in place of the bubble, we find that fluctuations of stock prices that would otherwise have been interpreted as a bubble are now interpreted as shifts in the fundamental price resulting from a change of regime. This has two clear advantages. The first is that the predicted fundamental component of the stock price is based on a statistical model for dividends which is consistent with the data. The second is that the predicted fundamental component of the stock price has a clear interpretation whereas, as Froot and Obstfeld remarked, the interpretation of the bubble component is at best ambiguous.

Even though a model with both regime-switching and intrinsic bubbles outperforms a model with regime switching alone, the improvement appears to be relatively modest, and to occur in periods in which the bubble element of the stock price is relatively small.

When Froot and Obstfeld respond to their own observation that a rational bubble is not a credible explanation of the non-fundamental component of stock prices, by appealing to the impossibility of distinguishing between the bubble and explanations such as fads, noise trading, time varying discount factors, and time varying risk premia, they are effectively leaving a very large fraction of prices unexplained. It is doubtful that these alternatives are capable of explaining such a large fraction of prices. We conjecture that they may, however, be able to explain the very small unexplained (and non-fundamental) component from the regime-switching model.

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Appendix A.

The estimates of the regime-swtching models are obtained using procedures which are identical to those described by Hamilton (1994), except that in this case both the price and the dividend equation depend on the state. Also note that k_0 and k_1 satisfy the system described in Eq. (10). The restriction between

c_0 and c_1 imposed by the theory can only be solved numerically. The program calls a subroutine that solves Eqs. (12) and (11) numerically, so each line search is assured to satisfy the restrictions imposed by the model. The density of the data y_t conditional on the state s_t and the history of the system can be written as

$$P(y_t|s_t,y_{t-1},\dots,y_1) = \frac{1}{(2\pi)^{0.5}\sigma_{s_t}}\exp(-(\sigma_{s_t}^2)^{-1}(\Delta d_t - \mu_{s_t})^2)\frac{1}{(2\pi)^{0.5}\theta_{s_t}} \times \exp(-(\theta_{s_t}^2)^{-1}(pr_t - (k_{s_t} + c_{s_t}D_t^{\lambda-1}))^2),$$

where y_t is a 3×1 vector containing Δd_t, the dividend's rate of growth pr_t, the price dividend ratio and D_t real dividends.

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