Explosive Rational Bubbles in Stock Prices?

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A number of recent studies address the problem of assessing the contributions of market fundamentals and rational bubbles to stock-price fluctuations—see, for example, Olivier Blanchard and Mark Watson, 1982; Robert Flood, Robert Hodrick, and Paul Kaplan, 1986; and Kenneth West, 1986, 1987. A rational bubble reflects a self-confirming belief that an asset's price depends on a variable (or a combination of variables) that is intrinsically irrelevant—that is, not part of market fundamentals—or on truly relevant variables in a way that involves parameters that are not part of market fundamentals. A basic difficulty involved in testing for the existence of rational bubbles, pointed out by Flood and Peter Garber, 1980, and emphasized by James Hamilton and Charles Whiteman, 1985, is that the contribution of hypothetical rational bubbles to asset prices would not be directly distinguishable from the contribution to market fundamentals of variables that the researcher cannot observe. For example, as Hamilton, 1986, shows, a researcher who is unable to observe or to infer changes in the expectations of market participants, especially if they involve the probable future occurrence of relevant events that are infrequent and discrete, might falsely conclude that rational bubbles exist. In the present context, the probabilities that investors attach to possibilities for future tax treatment of dividend income could act like such an unobservable variable.

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Diba and Grossman, 1984, and Hamilton and Whiteman, 1985, propose an empirical strategy based on stationarity tests for obtaining evidence against the existence of explosive rational bubbles without precluding the possible effect of unobservable variables on market fundamentals. The present paper implements such tests for explosive rational bubbles in stock prices using a model that assumes a constant discount rate, but that allows unobservable variables to affect market fundamentals and also allows different valuations of expected capital gains and expected dividends. If the first differences of the unobservable variables and the first differences of dividends are stationary (in the mean) and if rational bubbles do not exist, then the model implies that first differences of stock prices are stationary. The model also implies, using an argument adapted from John Campbell and Robert Shiller, 1987, that, if the levels of the unobservable variables and the first differences of dividends are stationary, and if rational bubbles do not exist, then stock prices and dividends are cointegrated of order (1,1).

These theoretical results do not imply that the finding that first differences of stock prices are nonstationary, or that stock prices and dividends are not cointegrated, would establish the existence of rational bubbles. A finding that stock prices and dividends are not cointegrated could result from the nonstationarity of the unobservable variables in market fundamentals, and a finding that stock-price changes are nonstationary could result from the nonstationarity of changes in these unobservable variables. Such findings also could arise from the inappropriateness of the implicit assumption that dividends are generated by an ARIMA process.

The converse inference, however, is possible. That is, evidence that first differences of stock prices have a stationary mean and/or evidence that stock prices are cointegrated with dividends would be evidence against the existence of rational bubbles. Except by extremely unlikely coincidence, misspecification of market fundamentals could not offset the contribution of a nonstationary rational bubble to stock prices. In addition to analyzing the stationarity properties of the observed time-series of real stock prices and dividends, this paper also examines the stationarity properties of simulated time-series of hypothetical rational bubbles to determine whether the stationarity tests can detect the relevant nonstationarity when it is present.

Because it looks for evidence against the existence of rational bubbles, the analysis in the present paper, in contrast to the strategy for finding rational bubbles suggested by West, 1986, 1987, does not require the specification of a true difference equation relating stock prices only to other observable variables. West observes that, if we could find such a true difference equation, and if the data rejected the implied market-fundamentals solution for stock prices, then we could conclude that rational bubbles exist. The problem with this approach is that diagnostic tests—as reported, for example, by Flood et al., 1986—reject the difference equations linking stock prices to dividends implied by a constant discount rate as well as by extended models that relate the discount rate to the intertemporal marginal rate of substitution or that incorporate different valuations for capital gains and dividends. These results underscore the need for an empirical strategy that does not preclude the possibility that market fundamentals for stock prices depend on unobservable variables in addition to dividends.

I. The Model

The theoretical model consists of a single equation that relates the current stock price to the present value of next period's expected stock price and dividend payments and to an unobservable variable—that is,

$$(1)\quad P_t=(1+r)^{-1}E_t(P_{t+1}+\alpha d_{t+1}+u_{t+1}),$$

where

- P_t is the stock price at date t relative to a general index of prices of goods and services;
- r is a constant real interest rate that is appropriate for discounting expected capital gains;
- E_t is the conditional expectations operator;
- α is a positive constant that valuates expected dividends relative to expected capital gains;
- d_{t+1} is the real before-tax dividend paid to the owner of the stock between dates t and $t+1$; and
- u_{t+1} is a variable that market participants either observe or construct, but that the researcher does not observe.

(As suggested above, this unobservable variable could involve the probabilities that investors attach to possibilities for future tax treatment of dividend income.) If α were equal to unity and u_{t+1} were equal to zero for all t, equation (1) would state that the expected real rate of return from holding equity, including expected dividends and expected capital gains, equals the constant r. The information set of market participants at date t on which E_t is based contains at least the current and past realizations of P_t, d_t, and u_t.

Equation (1) is a first-order expectational difference equation. Because the eigenvalue, $1+r$, is greater than unity, the forward-looking solution for the stock price involves a convergent sum, as long as $E_t(\alpha d_{t+j}+u_{t+j})$ does not grow with j at a geometric rate equal to or greater than $1+r$. This forward-looking solution, denoted by F_t and referred to as the market-fundamentals component of the stock price, is

$$(2)\quad F_t=\sum_{j=1}^{\infty}(1+r)^{-j}E_t(\alpha d_{t+j}+u_{t+j}).$$

With α equal to unity and u_t equal to zero for all t, equation (2) would say that the market-fundamentals component of the stock price equals the present value of expected real dividends discounted at the constant rate r.

The general solution to equation (1) is the sum of the market-fundamentals component,

F_t, and the rational bubbles component, B_t —that is,

$$(3) \qquad P_t = B_t + F_t,$$

where B_t is the solution to the homogeneous expectational difference equation

$$(4) \qquad E_t B_{t+1} - (1+r) B_t = 0.$$

A nonzero value of B_t would reflect the existence of a rational bubble—that is, a self-confirming belief that the stock price does not conform to the market-fundamentals component, F_t.

Solutions to equation (4) satisfy the stochastic difference equation

$$(5) \qquad B_{t+1} - (1+r) B_t = z_{t+1},$$

where z_{t+1} is a random variable (or combination of variables) generated by a stochastic process that satisfies

$$(6) \qquad E_{t-j} z_{t+1} = 0 \qquad \text{for all } j \geq 0.$$

The key to the relevance of equation (5) for the general solution of P_t is that equation (4) relates B_t to $E_t B_{t+1}$, rather than to B_{t+1} itself as would be the case in a perfect-foresight model.

The random variable z_{t+1} is an innovation, comprising new information available at date $t+1$. This information can be intrinsically irrelevant—that is, unrelated to F_{t+1} —or it can be related to truly relevant variables, like d_{t+1}, through parameters that are not present in F_{t+1}. The only critical property of z_{t+1}, given by equation (5), is that its expected future values are always zero.

Diba and Grossman, 1988, review and extend theoretical arguments for ruling out rational stock-price bubbles on the basis of the nonnegativity of stock prices and the optimizing decisions of asset holders. George Evans, 1985, develops another theoretical argument for ruling out rational bubbles by requiring that equilibrium rational expectations solutions to the model should be stable in the sense that, given a small disequilibrium deviation from rational expectations, the system should return to rational expectations equilibrium under a natural revision rule. The empirical analysis developed in the present paper complements these theoretical analyses.

II. Stationarity of Stock Prices and Dividends

Consider the market-fundamentals component of the stock price given by equation (2). Assume that the process generating d_t is nonstationary in levels, but that first differences of d_t and u_t are stationary. Then, if rational bubbles do not exist, stock prices are nonstationary in levels but stationary in first differences.

If, however, stock prices contain a rational bubble, then for simple specifications of the process generating z_t, differencing stock prices a finite number of times would not yield a stationary process. Specifically, from equation (5), first differences of a rational bubble would have the generating process

$$(7) \qquad [1-(1+r)L](1-L)B_t = (1-L)z_t,$$

where L denotes the lag operator. For example, if z_t is white noise, then an ARMA process that is neither stationary nor invertible generates $(1-L)B_t$. (The only exceptions to nonstationarity discussed in the literature involve rational bubbles that almost surely would burst at a finite future date, as in the specifications of Blanchard, 1979, and Blanchard and Watson, 1982. Such a rational bubble would have innovations with infinite variance, but, as Danny Quah, 1985, demonstrates, it also would have a stationary unconditional mean of zero.)

Allan Kleidon, 1986, analyzes the stationarity properties of stock prices, dividends, and their first differences for Data Set 1 in Robert Shiller, 1981. The work of Blanchard and Watson, 1982; Flood et al., 1986; and West, 1986, 1987, also uses this data set. The price series is Standard & Poor's Composite Stock Price Index for January of each year from 1871 to 1986 divided by the wholesale price index for that month. The dividend series is total dividends accruing to this portfolio of stocks for the calendar year divided by the average whole-

TABLE 1—SAMPLE AUTOCORRELATIONS OF REAL STOCK PRICES, DIVIDENDS, AND THEIR FIRST DIFFERENCES

Number of Lags Series	1	2	3	4	5	6	7	8	9	10
P_t	0.94	0.87	0.84	0.79	0.74	0.68	0.63	0.57	0.51	0.45
d_t	0.95	0.88	0.82	0.78	0.74	0.70	0.65	0.62	0.59	0.56
ΔP_t	0.06	−0.24	0.12	0.17	−0.00	−0.12	0.15	0.00	−0.07	−0.05
Δd_t	0.23	−0.16	−0.07	−0.03	−0.01	−0.01	−0.17	−0.13	0.06	0.14

Note: The price (P_t) and dividend (d_t) series contain 116 observations. Their first differences (ΔP_t and Δd_t) contain 115 observations.

TABLE 2—DICKEY-FULLER TEST RESULTS: NO LAGS

x_t:	P_t	d_t	ΔP_t	Δd_t
$\hat{\mu}$	0.0058	0.0007	0.0002	0.0001
	(0.0166)	(0.0005)	(0.0168)	(0.0004)
$\hat{\gamma}$	0.0006	0.00003	0.0001	0.000001
	(0.0003)	(0.00001)	(0.0003)	(0.000006)
$\hat{\rho}$	0.90	0.87	0.06	0.23
	(0.04)	(0.05)	(0.10)	(0.10)
Standard Error of Estimate	0.071	0.002	0.072	0.002
Φ_3	2.55	3.43	42.38	30.89

Note: Regressions are of the form $x_t = \mu + \gamma t + \rho x_{t-1} +$ residual. "Standard errors" are in parentheses below coefficients. Sample size is 100 in all cases. The statistic Φ_3, calculated like the F-statistic, tests the null hypothesis $(\gamma, \rho) = (0,1)$ against the alternative $(\gamma, \rho) \neq (0,1)$. The rejection region is the set of values of Φ_3 above 5.47 (6.49) for a test of size 0.10 (0.05).

sale price index for the year. Tables 1, 2, and 3 report results similar to Kleidon's results.

Table 1 presents sample autocorrelations of these real stock prices and dividends, and their first differences, for one through ten lags. The autocorrelations of the undifferenced price and dividend series both drop off slowly as lag length increases, suggesting nonstationary means. Their patterns correspond closely to what would be expected for integrated moving average processes according to a formula presented by Dean Wichern, 1973. In contrast, autocorrelations of the differenced series, both for prices and dividends, are consistent with the assumption that these series have stationary means. Thus the autocorrelation patterns suggest that the nonstationarity of real stock prices is attributable to their market-fundamentals component and that explosive rational bubbles do not exist in stock prices.

Tables 2 and 3 report Dickey-Fuller, 1981, tests for unit roots in the autoregressive representations of real stock prices, dividends, and their first differences. For each time-series, the estimated OLS regression is

$$x_t = \mu + \gamma t + \rho x_{t-1} + \sum_{i=1}^{k} \beta_i \Delta x_{t-i} + \text{residual}, \tag{8}$$

where Δ is the difference operator. The tables report the statistic Φ_3 of David Dickey and Wayne Fuller, 1981, which is calculated as one would calculate the F-statistic for $(\gamma, \rho) = (0,1)$. The regressions in Table 2 set k equal to zero to test the null hypothesis that x_t follows a random walk with drift against the general alternative $(\gamma, \rho) \neq (0,1)$. The regressions in Table 3 set k equal to four and, thereby, allow Δx_t to follow an AR(4) process. Each regression discards the

TABLE 3—DICKEY-FULLER TEST RESULTS: FOUR LAGS

x_t:	P_t	d_t	ΔP_t	Δd_t
$\hat{\mu}$	0.0046	0.0008	−0.0009	0.0001
	(0.0159)	(0.0005)	(0.0164)	(0.0004)
$\hat{\gamma}$	0.0007	0.00003	0.0001	−0.000001
	(0.0003)	(0.00001)	(0.0002)	(0.000006)
$\hat{\rho}$	0.88	0.83	0.17	0.03
	(0.05)	(0.06)	(0.24)	(0.21)
$\hat{\beta}_1$	0.16	0.35	−0.07	0.25
	(0.10)	(0.10)	(0.22)	(0.19)
$\hat{\beta}_2$	−0.15	−0.14	−0.31	0.01
	(0.10)	(0.11)	(0.19)	(0.16)
$\hat{\beta}_3$	0.21	0.09	−0.14	0.05
	(0.10)	(0.10)	(0.14)	(0.13)
$\hat{\beta}_4$	0.17	0.04	−0.04	−0.01
	(0.10)	(0.10)	(0.11)	(0.10)
Standard Error of Estimate	0.068	0.002	0.070	0.002
Φ_3	3.12	4.42	6.41	10.45

Note: Regressions are of the form $x_t = \mu + \gamma t + \rho x_{t-1} + \Sigma_{i=1}^{4} \beta_i \Delta x_{t-i} + \text{residual}$. "Standard errors" are in parentheses below coefficients. Sample size is 100 in all cases. The statistic Φ_3, calculated like the F-statistic, tests the null hypothesis $(\gamma, \rho) = (0,1)$ against the alternative $(\gamma, \rho) \neq (0,1)$. The rejection region is the set of values of Φ_3 above 5.47 (6.49) for a test of size 0.10 (0.05).

first few observations to adjust sample size to 100. The rejection region, from Dickey and Fuller's Table VI, is the set of values of Φ_3 above 5.47 (6.49) for a test of size 0.10 (0.05).

For the undifferenced time-series of real stock prices and dividends, the statistic Φ_3 does not reject the null hypothesis $(\gamma, \rho) = (0,1)$. For both of the differenced series, the statistic rejects the null hypothesis. The rejections are stronger in Table 2 than in Table 3, probably because the regressions of Table 3 include the regressors Δx_{t-i}, which in most cases do not have significant coefficients and, consequently, reduce the power of the unit root test.

The results reported in Tables 2 and 3 support the impression, based on the sample autocorrelations in Table 1, that both real stock prices and dividends are nonstationary in levels but stationary in first differences. For sample sizes of 100, unit root tests have low power against alternatives slightly less than unity—see, for example, G. B. A. Evans and N. E. Savin, 1984. Accordingly, we cannot have much faith in the result that the undifferenced series are nonstationary and not borderline stationary. The critical finding for our purposes, however, is that, contrary to what the existence of explosive rational bubbles would imply, the data strongly reject the null hypothesis of a nonstationary mean for first differences of real stock prices. In fact, point estimates of ρ for the ΔP_t regressions of Tables 2 and 3 do not differ significantly from zero.

III. Cointegration of Stock Prices and Dividends

Rearranging terms in equation (2) and substituting the resulting expression for F_t into equation (3) yields

$$(9) \quad P_t - \alpha r^{-1} d_t = B_t + \alpha r^{-1}\left[\sum_{j=1}^{\infty} (1+r)^{1-j} E_t \Delta d_{t+j}\right] + \sum_{j=1}^{\infty} (1+r)^{-j} E_t u_{t+j}.$$

If the unobservable variable in market fundamentals, u_t, is stationary in levels, if dividends are first-difference stationary, and if rational bubbles do not exist, then the sum given by the right-hand side of equation (9) is stationary. Thus, although P_t and d_t are nonstationary, their linear combination $P_t - \alpha r^{-1} d_t$, given by the left-hand side of equation (9), is stationary.

Clive Granger and Robert Engle, 1987, define the components of a vector y_t of time-series to be cointegrated of order (d, b) if all components of y_t are integrated of order d—that is, have a stationary, invertible, nondeterministic ARMA representation after differencing d times—and if there exists a vector δ, other than the null vector, such that $\delta' y_t$ is integrated of order $d - b$ for some $b > 0$. They call δ the cointegrating vector. Using their terminology, equation (9) says that if the processes generating Δd_t and u_t are stationary and if B_t equals zero, then P_t and d_t are cointegrated of order (1,1) with cointegrating vector $(1, -\alpha r^{-1})$.

Drawing on the work of James Stock, 1987, Granger and Engle develop tests for cointegration that involve obtaining an estimate of the cointegrating vector from a cointegrating regression and then applying tests for stationarity to the residuals from this regression. For a test of stationarity of the left-hand side of equation (9), the cointegrating regression would be the OLS regression of P_t on d_t.

One test for stationarity of residuals suggested by Granger and Engle would reject the null hypothesis of no cointegration if the Durbin-Watson statistic of the cointegrating regression exceeds the critical values they tabulate. Another test suggested by Granger and Engle involves estimating Dickey-Fuller regressions of the form

$$(10) \quad \Delta v_t = -\rho v_{t-1} + \sum_{i=1}^{k} \beta_i \Delta v_{t-i} + \text{residual},$$

on the residuals v_t of the cointegrating regression. Granger and Engle tabulate the critical values for statistics denoted ξ_2 and ξ_3, calculated analogously to t-ratios for ρ in equation (10), with k set equal to zero for ξ_2 and to four for ξ_3.

Estimation of the cointegrating regression of P_t on d_t yields a point estimate for αr^{-1} of 30.50 and a Durbin-Watson statistic of 0.61, which is above the 1 percent critical value of 0.51. John Campbell and Shiller, 1987, also estimate such a cointegrating regression and calculate Granger and Engle's ξ_2 and ξ_3 statistics. They find that the statistic ξ_2 rejects the null hypothesis of no cointegration at the 5 percent level, but the statistic ξ_3 (narrowly) fails to reject even at the 10 percent level.

The results of cointegration tests for P_t and d_t, thus, are mixed. The Durbin-Watson statistic rejects the null hypothesis that P_t and d_t are not cointegrated at the 1 percent level, the statistic ξ_2 rejects the null at the 5 percent level, but the statistic ξ_3 fails to reject at the 10 percent level. Moreover, the point estimate for αr^{-1} is somewhat implausible. Specifically, with α set equal to unity, this point estimate would imply a value for r of about 0.033, well below its sample mean of about 0.08. If α is less than unity, then the implied value of r will be even lower than 0.033. (As Terry Marsh and Robert Merton, 1983, emphasize, if the logarithms of stock prices and dividends follow integrated stochastic processes, then a regression of stock prices on dividends, in levels, yields inefficient and possibly biased estimates. This bias could account for the implausibly low values of the required rate of return implied by the cointegrating regression of P_t on d_t.)

IV. Stationarity of the Unobservable Variable

Nonstationarity of the unobservable variable in market fundamentals would be a potential source of lack of cointegration of stock prices and dividends. To explore this possibility, note that equation (1) implies

$$(11) \quad P_{t+1} + \alpha d_{t+1} - (1+r) P_t = e_{t+1} - u_{t+1},$$

where $e_{t+1} = P_{t+1} + \alpha d_{t+1} + u_{t+1} - E_t(P_{t+1} + \alpha d_{t+1} + u_{t+1})$.

TABLE 4—BHARGAVA TESTS OF THE RANDOM-WALK HYPOTHESIS

Statistic	R_1	R_2	N_1	N_2
Null Hypothesis	Random Walk	Random Walk with Drift	Random Walk	Random Walk with Drift
Alternative Hypothesis	Stationary	Stationary	Unstable	Unstable
Rejection Region for test of size 0.05	Above 0.26	Above 0.35	Below 0.006	Below 0.022
$P_t - d_t/0.01$	0.15	0.19	0.05	0.19
$P_t - d_t/0.02$	0.40	0.45	0.12	0.64
$P_t - d_t/0.03$	0.62	0.60	0.31	1.11
$P_t - d_t/0.04$	0.48	0.49	0.44	0.97
$P_t - d_t/0.05$	0.35	0.38	0.35	0.76
$P_t - d_t/0.06$	0.28	0.32	0.26	0.62
$P_t - d_t/0.07$	0.24	0.27	0.21	0.53
$P_t - d_t/0.08$	0.21	0.25	0.18	0.47

Note: The statistics R_1, R_2, N_1, and N_2 are von Neumann-type ratios that yield most powerful invariant tests of the random-walk hypothesis against one-sided stationary and explosive alternatives.

Because the assumption of rational expectations implies that e_{t+1} is not serially correlated, stationarity of the left-hand side of equation (11) is equivalent to stationarity of u_{t+1}. (Of course, in a finite sample, even if u_{t+1} is nonstationary, the left-hand side of equation (11) can appear stationary if most of its variability results from movements in the forecast error e_{t+1}.) Stationarity of the left-hand side of equation (11) implies that the variables $P_{t+1} + \alpha d_{t+1}$ and P_t are cointegrated of order (1,1) with cointegrating vector $[1, -(1+r)]$.

For the present data, the tests suggested by Granger and Engle find cointegration between $P_{t+1} + \alpha d_{t+1}$ and P_t for values of α between 0.5 and 2, which correspond to varying the valuation for dividends from one-half to twice the valuation for capital gains. With α set equal to unity, for example, the Durbin-Watson statistic of the cointegrating regression is 1.82 (well above the 1 percent critical value of 0.51), Granger and Engle's ξ_2 statistic has a value of 8.74 (again above the 1 percent critical value of 4.07), and their statistic ξ_3 has a value of 3.32 (which is below the critical value of 3.77 at the 1 percent level but comfortably rejects the null hypothesis of no cointegration at the 5 percent level).

As Campbell and Shiller, 1987, point out, the difference $P_t - \alpha r^{-1} d_t$ is equivalent to a linear combination of the variables Δd_{t+1}, ΔP_{t+1}, and $P_{t+1} - \alpha d_{t+1} - (1+r)P_t$. Accordingly, the conclusion that Δd_{t+1}, ΔP_{t+1}, and $P_{t+1} + \alpha d_{t+1} - (1+r)P_t$ are all stationary would imply that $P_t - \alpha r^{-1} d_t$ is stationary, independently of the model of stock prices. Thus, the apparently mixed results of Section III on the hypothesis that stock prices and dividends are not cointegrated are puzzling. (Using the same test, Campbell and Shiller, 1986, find that the logarithm of the ratio of dividends to stock prices and the logarithm of dividends are stationary, but, contrary to what an algebraic identity would imply, they fail to reject the hypothesis that the logarithm of stock prices is nonstationary.)

V. Bhargava Tests

Given these problems, alternative tests of the hypothesis that $P_t - \alpha r^{-1} d_t$ is not stationary seem to be in order. To investigate the stationarity properties of $P_t - \alpha r^{-1} d_t$ further, this section reports von Neumann-type ratios, suggested by Alok Bhargava, 1986, that yield most powerful invariant tests of random-walk hypotheses against the one-

TABLE 5—AUTOCORRELATIONS OF FIRST DIFFERENCES OF SIMULATED RATIONAL BUBBLE SERIES

Simulation Number	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}
1	0.94	0.89	0.84	0.80	0.75	0.71	0.67	0.63	0.59	0.56
2	0.93	0.89	0.83	0.79	0.75	0.71	0.67	0.63	0.60	0.57
3	0.92	0.87	0.80	0.77	0.73	0.70	0.65	0.62	0.57	0.55
4	0.93	0.87	0.82	0.79	0.75	0.72	0.67	0.63	0.59	0.56
5	0.91	0.85	0.78	0.74	0.70	0.66	0.63	0.60	0.56	0.53
6	0.95	0.90	0.85	0.80	0.76	0.71	0.67	0.64	0.60	0.56
7	0.94	0.89	0.84	0.80	0.76	0.72	0.68	0.63	0.59	0.56
8	0.93	0.88	0.84	0.79	0.76	0.72	0.68	0.64	0.60	0.57
9	0.90	0.85	0.82	0.77	0.73	0.70	0.64	0.61	0.58	0.55
10	0.65	0.65	0.62	0.56	0.53	0.45	0.51	0.43	0.42	0.36
11	0.94	0.88	0.83	0.78	0.75	0.71	0.67	0.62	0.58	0.55
12	0.91	0.85	0.82	0.76	0.73	0.68	0.64	0.61	0.56	0.53
13	0.92	0.87	0.82	0.78	0.74	0.70	0.67	0.63	0.60	0.56
14	0.62	0.64	0.55	0.60	0.51	0.46	0.44	0.43	0.39	0.40
15	0.80	0.80	0.73	0.71	0.65	0.65	0.58	0.52	0.53	0.48
16	0.94	0.89	0.84	0.80	0.75	0.71	0.67	0.63	0.59	0.56
17	0.90	0.86	0.81	0.76	0.72	0.68	0.65	0.61	0.59	0.55
18	0.93	0.89	0.84	0.79	0.75	0.70	0.66	0.62	0.59	0.56
19	0.94	0.89	0.84	0.80	0.76	0.72	0.68	0.64	0.60	0.56
20	0.94	0.89	0.85	0.80	0.76	0.72	0.67	0.64	0.60	0.57
21	0.46	0.42	0.41	0.29	0.35	0.31	0.27	0.22	0.24	0.29
22	0.93	0.88	0.83	0.78	0.74	0.70	0.66	0.62	0.58	0.54
23	0.93	0.88	0.83	0.79	0.74	0.70	0.66	0.62	0.59	0.56
24	0.94	0.90	0.85	0.80	0.75	0.71	0.67	0.63	0.60	0.56
25	0.94	0.89	0.84	0.80	0.75	0.71	0.67	0.64	0.61	0.57
26	0.93	0.88	0.83	0.78	0.74	0.70	0.66	0.63	0.59	0.55
27	0.83	0.78	0.75	0.70	0.68	0.67	0.62	0.58	0.53	0.49
28	0.94	0.89	0.84	0.80	0.75	0.71	0.67	0.64	0.60	0.56
29	0.56	0.53	0.48	0.51	0.47	0.40	0.41	0.35	0.35	0.35
30	0.94	0.89	0.84	0.80	0.75	0.71	0.68	0.64	0.60	0.56
31	0.89	0.84	0.80	0.75	0.72	0.69	0.65	0.61	0.55	0.53
32	0.93	0.88	0.83	0.79	0.74	0.70	0.66	0.61	0.58	0.55
33	0.11	0.17	0.21	0.17	0.19	0.12	0.23	0.02	0.18	0.06
34	0.94	0.89	0.85	0.80	0.75	0.71	0.67	0.63	0.59	0.56
35	0.40	0.30	0.30	0.25	0.24	0.30	0.31	0.16	0.27	0.20
36	0.93	0.89	0.84	0.79	0.75	0.70	0.66	0.63	0.59	0.56
37	0.94	0.90	0.85	0.80	0.75	0.71	0.67	0.64	0.60	0.56
38	0.94	0.89	0.84	0.80	0.76	0.72	0.68	0.64	0.60	0.56
39	0.95	0.90	0.85	0.81	0.76	0.72	0.68	0.64	0.60	0.57
40	0.94	0.89	0.84	0.80	0.76	0.71	0.68	0.64	0.60	0.56
41	0.95	0.90	0.85	0.80	0.76	0.72	0.68	0.64	0.60	0.57
42	0.94	0.89	0.84	0.80	0.75	0.71	0.67	0.64	0.60	0.57
43	0.90	0.86	0.82	0.77	0.73	0.70	0.67	0.61	0.57	0.54
44	0.95	0.90	0.85	0.81	0.76	0.72	0.68	0.64	0.60	0.56
45	0.84	0.81	0.74	0.71	0.66	0.61	0.58	0.56	0.51	0.49
46	0.94	0.89	0.85	0.81	0.76	0.72	0.68	0.64	0.61	0.57
47	0.94	0.89	0.84	0.80	0.76	0.71	0.67	0.63	0.60	0.57
48	0.93	0.89	0.84	0.79	0.76	0.72	0.68	0.64	0.60	0.57
49	0.93	0.89	0.84	0.79	0.74	0.70	0.66	0.62	0.59	0.56
50	0.90	0.85	0.81	0.76	0.73	0.69	0.64	0.58	0.56	0.53

Note: Table reports the autocorrelations of first differences of simulated rational bubbles series: $B_t = 1.05B_{t-1} + z_t$, where z_t is normally distributed white noise, and B_0 is set equal to zero. For each simulation, r_k, $k = 1,\dots,10$, is the autocorrelation coefficient at lag k.

sided stationary and explosive alternatives. Tests against one-sided explosive alternatives are relevant because the existence of explosive rational bubbles would imply that $P_t - \alpha r^{-1} d_t$ has an explosive, rather than a unit, root.

Table 4 reports the Bhargava tests for $P_t - \alpha r^{-1} d_t$. The statistic R_1 rejects the null hypothesis of a simple random walk in favor of the stationary alternative for values of $\alpha^{-1} r$ between 0.02 and 0.06, and the statistic R_2 rejects the null hypothesis of a random walk with drift in favor of the stationary alternative for values of $\alpha^{-1} r$ between 0.02 and 0.05. The results of tests based on the statistics R_1 and R_2, concur with the results of two of the Granger and Engle tests reported above and suggest that $P_t - \alpha r^{-1} d_t$ is stationary. (The values of r implied by the tests based on R_1 and R_2, however, still seem somewhat implausibly low.)

The statistics N_1 and N_2 in Table 4 pertain to testing the null hypotheses that $P_t - \alpha r^{-1} d_t$ follows either a simple random walk or a random walk with drift against the one-sided explosive alternative. For all values of $\alpha^{-1} r$, these statistics fail to reject the null hypothesis that $P_t - \alpha r^{-1} d_t$ has a unit root. In sum, the Bhargava tests strongly suggest that stock prices and dividends are cointegrated, and, thus, are consistent with the finding that the first differences of stock prices and dividends and any unobservable variable in market fundamentals are all stationary.

VI. Stationarity Properties of Simulated Rational Bubbles

To verify that our tests would detect explosive rational bubbles if they were present, we applied the same tests to the time-series of simulated rational bubbles with standard normal innovations. The simulations set B_0 equal to zero and r equal to 0.05.

The statistic N_1 of Bhargava rejected at the 5 percent level the null hypothesis of a simple random walk in favor of the unstable alternative in 95 out of 100 simulations. For the same 100 simulations, the statistic N_2 rejected, at the 5 percent level, the null hypothesis of a random walk with drift in favor of the unstable alternative in 94 cases.

First differences of the simulated rational bubbles series also exhibited strong signs of nonstationarity. Table 5 reports the sample autocorrelations of the differenced time-series for the first 50 simulations. The patterns of autocorrelation coefficients in all but six cases (simulations numbered 10, 14, 21, 29, 33, and 35) strongly suggest nonstationarity. The autocorrelation function starts at a value of 0.8 or higher and drops off very slowly. For simulations numbered 10, 14, 21, 29, and 35, the starting values are lower, but the autocorrelations still drop off slowly. (Wichern's results indicate that the latter criterion is a more reliable sign of nonstationarity.) Only for simulation number 33 does the pattern of autocorrelations resemble those of differenced time-series of stock prices and dividends reported in Table 1 above.

The simulation results reported above, of course, do not mean that stationarity tests would detect a rational bubbles component even if its contribution to stock-price fluctuations is quantitatively small. If, however, the excess volatility in stock prices found by West, 1986, were attributable to rational bubbles, then innovations in these rational bubbles would account for 80 to 95 percent of the variance of stock-price innovations. It is likely then that the stationarity properties of stock prices and dividends would reflect the existence of explosive rational bubbles.

VII. Summary

This paper reports empirical tests for the existence of explosive rational bubbles in stock prices. The analysis focuses on a model that defines market fundamentals to be the sum of an unobservable variable and the expected present value of dividends, discounted at a constant rate, and defines a rational bubble to be a self-confirming divergence of stock prices from market fundamentals in response to extraneous variables. The pattern of autocorrelations in the data as well as Dickey-Fuller tests both indicate that stock prices and dividends are nonstationary before differencing, but are stationary in first differences. In contrast, first differences of simulated time-series of rational bubbles exhibit strong signs of nonstationarity.

If the nonstationarity of dividends accounts for the nonstationarity of stock prices, then stock prices and dividends are cointegrated. Although application of the cointegration tests suggested by Granger and Engle produced somewhat mixed results, these mixed results probably reflect low power of the tests rather than either the existence of rational bubbles or the presence of a nonstationary unobservable variable in market fundamentals. Most importantly, alternative tests suggested by Bhargava indicate that the relevant linear combination of stock prices and dividends is neither explosive nor has a unit root. In contrast, time-series of simulated rational bubbles failed the Bhargava tests. In sum, the analysis supports the conclusion that stock prices do not contain explosive rational bubbles.

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