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REGIME SWITCHING AS A TEST FOR EXCHANGE RATE BUBBLES

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SUMMARY

This paper develops a new test for speculative bubbles, which is applied to data for the Japanese yen, the German mark and the Canadian dollar exchange rates from 1977 to 1991. The test assumes that bubbles display a particular kind of regime-switching behaviour, which is shown to imply coefficient restrictions on a simple switching-regression model of exchange rate innovations. Test results are sensitive to the specification of exchange rate fundamentals and other factors. Evidence most consistent with the bubble hypothesis is found using an overshooting model of the Canadian dollar and a PPP model of the Japanese yen.

1. INTRODUCTION

This paper develops a new test for speculative bubbles in exchange rates and then applies this test to data for three bilateral exchange rates over the 1977–91 period. Recent work in testing for bubbles has shifted from general tests that should detect any kind of bubble (Meese, 1986; West, 1987; Gros, 1989) to those that test for a particular form of bubble (Froot and Obstfeld, 1991; van Norden and Schaller, 1993a,b). An advantage of the latter is that such tests give more information about the kind of behaviour that produces significant evidence of bubbles. The test introduced below follows this newer approach. In particular, it focuses on a kind of stochastic bubble that each period is expected either to continue growing or to collapse (partially or completely). Given assumptions about the probability and size of these collapses, it is shown that such behaviour should lead to a particular kind of regime-switching behaviour in exchange rate innovations. Tests for such behaviour can be conducted using switching-regression techniques. These tests are applied to data for the Japanese yen, the German mark and the Canadian dollar exchange rates against the US dollar using different specifications of the underlying 'true' model of exchange rate fundamentals.

As discussed in Flood and Hodrick (1990), it is well understood that a bubble model will be observationally equivalent to a model without bubbles, but with a different specification of fundamentals. In the case studied below, the switching regression motivated by bubbles could also be motivated by the presence of regime switching in fundamentals. One example of such an alternative model would be the 'peso problem' considered by Krasker (1980). Therefore, as with all tests for bubbles, the results presented here should be interpreted as evidence of the kind of behaviour predicted by the bubble model, and not as definitive proof of the presence or absence of bubbles. Nonetheless, such a qualified conclusion should be interesting for a number of reasons.

First, any results consistent with bubbles will have implications for research on the efficiency

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of foreign exchange markets. If one interprets such results as evidence of bubbles, then this violates some definitions of market efficiency. If one instead interprets the results as evidence of switching in fundamentals, then this implies that empirical models of risk premiums need to take such regime-switching behaviour into account. In addition, the empirical link between regime-switching behaviour in exchange rates and other macroeconomic series then becomes another stylized fact that a satisfactory model of risk premiums needs to explain.

Second, the evidence presented below adds to the work on the univariate properties of exchange rate changes. In addition to recent research on autoregressive conditional heteroscedasticity models (such as Diebold, 1988) and semi-parametric estimators (such as Gallant, Hsieh and Tauchen, 1988), there has been particular interest in mixture of distribution models. Examples of these include Akgiray and Booth (1988), Bates (1988), Boothe and Glassman (1987), Engel and Hamilton (1990), Jorion (1988), and Tucker and Pond (1988). One of the key attractions of such models is their ability to capture the occasional occurrence of large, discrete exchange rate changes by appealing to a secondary data-generating processes that is observed only infrequently. By placing the mixture of distribution model in a multivariate context, the switching regression may offer new explanations of large abrupt exchange rate movements by linking them to other macroeconomic time series. Furthermore, the econometric techniques needed to estimate the simple switching-regression models used here are well established, and may be easier to compute than some univariate estimation methods, such as those proposed by Hamilton (1989).

Finally, tests of the switching-regression model also contribute to the growing literature on the predictability of returns in asset markets, revived in recent years by Fama and French (1988), Poterba and Summers (1988), Cecchetti, Lam and Mark (1990). In particular, these papers show that simple measures of the deviation of asset prices from their fundamental values help to predict future returns. For example, Cutler, Poterba and Summers (1991) show that such relationships exist across a wide range of asset markets, although the evidence for exchange rate markets is quite weak. Since the switching-regression model nests the linear regression they used, one can test for evidence of a more complex relationship and consider its sensitivity to a variety of assumptions about fundamental exchange rates.

The following section introduces a simple regime-switching model of bubbles that generalizes the model first suggested by Blanchard (1979). Section 3 discusses the econometric issues involved in the estimation and testing of such models, while Section 4 explains the data and various models of fundamental exchange rates used. Sections 5 and 6 review the empirical results, while the final section offers conclusions.

2. A REGIME-SWITCHING MODEL OF STOCHASTIC BUBBLES

We begin with a general model of exchange rate determination, that only requires that

$$s_t = f(X_t) + a \cdot E_t(s_{t+1}) \tag{1}$$

where s_t is the logarithm of the spot exchange rate, E_t is the operator for expectations conditional on information at time t, $0 < a < 1$, and X_t is a vector of other variables. Note that equation (1) is general enough to include examples of both fixed- and flexible-price monetary models, as well as models with imperfect international asset substitutability. Solving the equation forward gives the general result

$$s_t = \left(\sum_{j=0}^{T} a^j \cdot E_t(f(X_{t+j})) + a^{T+1} \cdot E_t(s_{T+1}) \right) \tag{2}$$

One solution to equation (1), which we will denote s_t^*, occurs when

$$\lim_{T \to \infty} a^{T+1} \cdot E_t(s_{T+1}) = 0 \tag{3}$$

therefore

$$s_t^* = \sum_{j=0}^{\infty} a^j \cdot E_t(f(X_{t+j})) \tag{4}$$

We refer to equation (4) as the fundamental solution, since it determines the exchange rate solely as a function of the current and expected behaviour of other macroeconomic variables.

However, equation (4) is not the only solution to equation (1). We define bubble solutions to be any other set of exchange rates and exchange rate expectations that satisfy equation (1) but where $s_t \neq s_t^*$. We define the size of the bubble b_t as

$$b_t \equiv s_t - s_t^* \tag{5}$$

Note that since s_t^* satisfies equation (1), it follows from equations (1) and (5) that

$$b_t = a \cdot E_t(b_{t+1}) \tag{6}$$

Since $a < 1$, this means the bubble must be expected to grow over time.

A considerable literature exists on the conditions under which such bubbles are feasible rational expectations solutions. Important contributions to this debate include Obstfeld and Rogoff (1983, 1986), Diba and Grossman (1987), Tirole (1982, 1985), Weil (1990), Buiter and Pesenti (1990), Allen and Gorton (1991), and Gilles and LeRoy (1992). In single-representative-agent models, a truly rational agent cannot expect to sell an overvalued asset (one with a positive bubble) before the bubble bursts. Therefore, bubbles should exist in such models only if they can be expected to grow without limit. Some researchers, such as Froot and Obstfeld (1991), have therefore suggested interpreting empirical tests for bubbles as tests of whether agents are fully rational, or whether they exhibit some form of myopia when considering events that are either very far in the future or occur with only very low probabilities. An alternative interpretation would be to consider evidence of bubbles as suggesting that non-representative-agent models (such as those of De Long *et al.*, 1990, Allen and Gorton, 1991, or Bulow and Klemperer, 1991) are required.

Blanchard (1979) and Blanchard and Watson (1982) examine a particular example of a process that satisfies equation (6) and captures some of the important features that have historically been attributed to bubbles. Blanchard considers a bubble process that moves randomly between two states, C and S. In state C, the bubble will collapse, so

$$E_t(b_{t+1} \mid C) = 0 \tag{7}$$

where the notation $E_t(X_j \mid C)$ (or $E_t(X_j \mid S)$) denotes the expectation of x_j conditional on the fact that the state j is C (or S) and on all other information available at time t. State S, where the bubble survives and continues to grow, occurs with a fixed probability q. Since

$$E_t(b_{t+1}) = (1-q) \cdot E_t(b_{t+1} \mid C) + q \cdot E_t(b_{t+1} \mid S) \tag{8}$$

equations (7) and (6) imply

$$E_t(b_{t+1} \mid S) = \frac{b_t}{a \cdot q} \tag{9}$$

Note that the lower the probability q of the bubble's survival, the faster the bubble must be expected to grow in the surviving state. The potentially large difference in the expected asset price between S and C implies that such bubble collapses could cause sudden and large price changes.

While it is a tractable and suggestive solution to equation (6), the Blanchard process seems unrealistically restrictive in at least two ways. First, it assumes that in state C the bubble is expected to collapse fully. However, there may be institutions in the real world that would tend to work against an instantaneous and complete collapse. For example, central banks may have a policy of smoothing sudden exchange rate changes as part of an effort to maintain orderly foreign exchange markets. Also, historical exchange rate movements that are sometimes attributed to bubbles, such as the rise and fall of the US dollar in 1984–5, tend to be reversed over a period of several months rather than in a single day.

It is therefore reasonable to allow for the possibility that the bubble is expected to collapse only partially in state C. In particular, equation (7) could be replaced with

$$E_t(b_{t+1} \mid C) = u(b_t) \tag{10}$$

where $u(\cdot)$ is a continuous and everywhere differentiable function such that $u(0)=0$ and $1 \geqslant u' \geqslant 0$. This means that the expected size of collapse will be a function of the relative size of the bubble, b_t, and that the bubble is not expected to grow (and may be expected to shrink) in state C.

Another restrictive feature of the Blanchard bubble process is the assumption of a constant probability of collapse. For example, Kindleberger (1989) describes the typical life-cycle of a 'bubble' or 'speculative mania'. He notes that as the bubble in the price of a particular asset grows, the prices of close substitutes become affected by the bubble, and that 'collapses' or 'panics' usually follow shortly thereafter.[1] One might therefore expect that the probability of the bubble's continued growth falls as the bubble grows, so that

$$q = q(b_t), \qquad \frac{d}{d|b_t|}\, q(b_t) < 0 \tag{11}$$

Note that the derivative of q is defined using the absolute value of b_t, since we wish to consider cases where b_t may be positive or negative. If we now use equations (10) and (11) with equation (6), we derive the revised counterpart to equation (9):

$$E_t(b_{t+1} \mid S) = \frac{b_t}{a \cdot q(b_t)} - \left(\frac{1-q(b_t)}{q(b_t)} \cdot u(b_t) \right) \tag{12}$$

We can see that in addition to replacing q with $q(b_t)$, the expected value of the bubble in state S is now lower by an additional factor that reflects its greater expected value in state C.

An interesting feature of the bubble model given in equations (10)–(12) is the structure it implies in exchange rate innovations. If we consider the unexpected change in the log exchange rate, $s_{t+1} - E_t(s_{t+1})$, this must be uncorrelated with all the information used to form $E_t(s_{t+1})$. However, if we could separate these innovations into those drawn from state C and those drawn from state S, this would no longer be the case.

To see this, note that equation (5) implies we can decompose the exchange rate innovation

[1]This is a very stylized description of Kindleberger's much richer narrative. The interested reader is referred to Kindleberger (1989) for more details.

into that arising from fundamentals and that arising from the bubble:

$$s_{t+1} - E_t(s_{t+1}) \equiv R_{t+1} = \varepsilon^*_{t+1} + [b_{t+1} - E_t(b_{t+1})] \tag{13}$$

where ε^*_{t+1} is the innovation in the fundamental exchange rate. If the bubble collapses at $t+1$, we observe state C and

$$R_{t+1} \mid C = \varepsilon^*_{t+1} + u(b_t) + \varepsilon^C_{t+1} - b_t/a \tag{14}$$

where ε^X_{t+1} is the expectational error term associated with equation (10) and $E_t(b_{t+1})$ is replaced using equation (6).[2] This expected value will generally be non-zero and a decreasing function of b_t since

$$\frac{d}{db_t} E(R_{t+1} \mid C) = u'(b_t) - \frac{1}{a} < 0 \tag{15}$$

Similarly, it can be shown that in the surviving state, S,

$$R_{t+1} \mid S = \frac{1 - q(b_t)}{a \cdot q(b_t)} \cdot [b_t - a \cdot u(b_t)] + \varepsilon^*_{t+1} + \varepsilon^S_{t+1} \tag{16}$$

where ε^S_{t+1} is the expectational error term associated with equation (12). The expectation will be an increasing function of b_t since

$$\frac{d}{db_t} E(R_{t+1} \mid S) = \frac{[1 - q(b_t)] \cdot [1 - a \cdot u'(b_t)]}{a \cdot q(b_t)} + \frac{-q'(b_t) \cdot [b_t - a \cdot u(b_t)]}{a \cdot q(b_t)^2} \tag{17}$$

which is unambiguously positive because both denominators and both numerators are always positive.

There are several points to note about the results in equations (14) and (16). They predict that the relationship between exchange rate innovations and deviations from fundamentals should be state-dependent if bubbles are present. Second, they provide a rationale for a mixture of distributions to be present in exchange rate innovations, since $\varepsilon^*_{t+1} + \varepsilon^S_{t+1}$ and $\varepsilon^*_{t+1} + \varepsilon^C_{t+1}$ will generally have different distributions. Third, testing for evidence of the relationships predicted by this bubble model will be difficult, since they depend on the regime generating the observation, which is not directly observed. They will also depend on the use of a measure of b_t, which requires an explicit model of fundamental exchange rates. The next section takes up the question of how one can reasonably test for these relationships and suggests various measures of b_t. The remainder of this section considers how one should interpret such test results.

If we test for the effects predicted by a model of bubbles, can we conclude whether or not bubbles are present? The answer is no, not without additional assumptions. There are two key problems. First, suppose we fail to find evidence to support the model. While this could be because the type of bubble described above does not exist, it could also be due to misspecification of b_t, which might then prevent us from finding the expected relationship between it and exchange rate innovations. Second, suppose we find evidence to support the

[2] Since $a<1$, $u(b_t)<b_t/a$ if $b_t<0$ and $u(b_t)>b_t/a$ if $b_t<0$. If we assume that $E(\varepsilon^*_{t+1} \mid C) = E(\varepsilon^*_{t+1} \mid S) = 0$, this means that, conditional on a bubble collapse, the expected innovation in the exchange rate will be non-zero. This assumption is equivalent to assuming that b_t is an extrinsic bubble. For a discussion of intrinsic bubbles and their relationship to nonlinearity, see Froot and Obstfeld (1991).

model. While this could be because the above type of bubble does exist, it might also be due to other phenomena. For example, Flood and Hodrick (1986) argue that bubbles will be observationally equivalent to process switching in fundamentals. The bubble process specified here is no exception. Consider the following example.

Suppose there are no bubbles but we misspecify the fundamental exchange rate s_t^* so that the actual exchange rate is given by

$$\tilde{s}_t = \sum_{j=0}^{\infty} a^j \cdot E_t(g(X_{t+j})) \tag{18}$$

If $E_t(g(X_{t+j})) > E_t(f(X_{t+j})), \forall j$, then $\tilde{s}_t > s_t^*$, so we would think there was a positive bubble present. Furthermore, because of the possibility of changes in fiscal, monetary or trade policies, X_{t+j} may be generated by distinct regimes. For example, it might be the case that fiscal policy can switch between a 'tight' and a 'loose' stance, and that the greater the government debt, the lower the probability that the loose stance will continue and the greater the expected change in stance. This would lead to a model of regime switching that is completely isomorphic to that described above, except that the size of the bubble, b_t, would now be replaced by some measure of the deviation of fiscal policy from its sustainable path. By misspecifying the fundamentals, however, any purported measure of bubbles could conceivably be correlated with such a deviation from the sustainable path. We could therefore find all the evidence suggested by the bubble model, even in the absence of bubbles.

As noted by Flood and Hodrick (1990), this kind of problem occurs in all bubble tests. They conclude that while this makes the interpretation of bubble test results difficult, it adds value to them as a diagnostic test of models of fundamentals. One interpretation of any evidence of bubbles found by the tests proposed below might be that bubbles are indeed present. An alternate interpretation would be that exchange rate fundamentals exhibit switching behaviour, which would itself be an important factor in modelling foreign exchange market risk premiums.

3. ESTIMATION AND HYPOTHESIS-TESTING ISSUES

As shown in the previous section, while the innovation in the exchange rate R_{t+1} should be uncorrelated with b_t, there may be a nonlinear relationship between these variables that takes the form of state-dependency; i.e., the relationship between R_{t+1} and b_t exists, but varies across states. If we knew with certainty which regime generated each observation of R_{t+1}, we could estimate these relationships using standard least-squares techniques on equations (14) and (16). Given uncertainty about the classification of R_{t+1} into these regimes, however, standard estimation techniques will give biased and inconsistent estimates, as shown by Lee and Porter (1984).

Nonetheless, as is discussed in the Appendix, consistent, efficient, asymptotically normal parameter estimates of such systems can still be obtained by maximum likelihood methods.[3] In particular, if we approximate equations (14) and (16) with first-order Taylor-series expansions,

[3] Inference requires that the series considered are stationary while rational bubbles of the kind described in Section 2 are explosive. However, the tests proposed below test the null hypothesis of no bubbles. Since the data may be assumed to be stationary under the null, tests of the null should have the correct size so standard inference procedures may be used. van Norden and Vigfusson (1996) find that the tests perform well for simulated (explosive) bubbles.

we are left with the following switching regression system:

$$E_t(R_{t+1} \mid S) = \beta_{S0} + \beta_{Sb} b_t \tag{19}$$

$$E_t(R_{t+1} \mid C) = \beta_{C0} + \beta_{Cb} b_t \tag{20}$$

$$Pr(\text{State}_{t+1} = S) = \Phi(\beta_{q0} + \beta_{qb1} b_t + \beta_{qb2} b_t^2) \tag{21}$$

$$Pr(\text{State}_{t+1} = C) = 1 - Pr(\text{State}_{t+1} = S) \tag{22}$$

where the model implies that $\beta_{Sb} > 0$, $\beta_{Cb} < 0$ and $\beta_{qb2} < 0$. A detailed derivation of these restrictions is given in the Appendix. This still requires us to choose a functional form for $\Phi(x)$ such that $\Phi(x)$ is everywhere increasing in x, but is bounded between 0 and 1 for all real values of x. The logit function $\Phi(x) \equiv (1 + e^x)^{-1}$ satisfies these requirements and is used below.

Having estimated the switching regression, we can then test the sign restrictions implied by the stochastic bubble model. However, one property of switching regressions is that such models are identified only up to a particular renaming of parameters that has the effect of swapping the names of the S and C regimes. In this case, this equivalence implies that

$$\begin{aligned} &\text{llf}(\beta_{S0}, \beta_{Sb}, \beta_{C0}, \beta_{Cb}, \beta_{q0}, \beta_{qb1}, \beta_{qb2}, \sigma_S, \sigma_S) \\ &\qquad = \text{llf}(\beta_{C0}, \beta_{Cb}, \beta_{S0}, \beta_{Sb}, -\beta_{q0}, -\beta_{qb1}, -\beta_{qb2}, \sigma_C, \sigma_S) \end{aligned} \tag{23}$$

where llf() is the log-likelihood function, so these alternative parameterizations cannot be distinguished without additional information. Therefore, the bubble model implies that one should find either $[\beta_{Sb} > 0, \beta_{Cb} < 0, \beta_{qb2} > 0]$, or $[\beta_{Sb} < 0, \beta_{Cb} > 0, \beta_{qb2} < 0]$

After estimating the model and testing its sign restriction, the results should be checked for evidence of misspecification that might in turn lead to inconsistent estimates or invalid inferences. Fortunately, White (1987) presents a general score-based test for misspecification in maximum-likelihood models that leads to several immediately useful tests in the switching-regression context considered here. Hamilton (1990) discusses these tests in the context of a Markov mixture of normal distributions model and presents evidence that White's test tends to over-reject the null hypothesis in small samples. Accordingly, all the tests statistics presented below are interpreted using 1% significance levels, as Hamilton suggests.

The results presented below test for three kinds of misspecification; omitted serial correlation, omitted heteroscedasticity, and Markov state-dependence. Specifically, first-order serial correlation in the derivative of the likelihood function with respect to β_{S0} and β_{C0} would indicate the presence of an AR(1) error process in regimes S and C respectively. Similar correlations in the derivatives with respect to σ_S and α_C would indicate the presence of ARCH(1) effects in their respective regimes. The presence of such first-order serial correlation in the derivative with respect to β_{q0} would be evidence of state-dependence in the classification probabilities and imply that a Markov-switching regression would be more appropriate. Testing for omitted ARCH or Markov-switching effects would seem to be particularly important given the popularity of ARCH and Markov-mixture models of financial time series.

In addition to testing the restrictions implied by the bubble model and its adequacy, this paper also aims to test a more general set of hypotheses. In particular, we wish to see whether the estimated switching-regression model gives additional information about the behaviour of mixtures of distributions in R_{t+1}, whether it gives evidence of a particular kind of nonlinearity in exchange rate behaviour, and whether it indicates that the distribution R_{t+1} is predictable. We will now consider these points in turn.

As noted in the previous section's discussion of equations (14) and (16), one implication of the stochastic bubble model is that the errors generating R_{t+1} will generally be from a mixture of distributions, which is assumed to be a mixture of normals. This means that the switching regression embodied by equations (19)–(21) will nest a general normal-mixture model as the special case where $\beta_{Sb} = \beta_{Cb} = \beta_{qb1} = \beta_{qb2} = 0$. This gives the model:

$$\begin{aligned} &R_{t+1} \sim N(\beta_{S0}, \sigma_S) \qquad \text{when State}_{t+1} = S \\ &R_{t+1} \sim N(\beta_{C0}, \sigma_C) \qquad \text{when State}_{t+1} = C \\ &Pr(\text{State}_{t+1} = S) = \Phi(\beta_{q0}) \end{aligned} \tag{24}$$

Note that equation (24) is more general than the restricted normal-mixture model estimated by Boothe and Glassman (1987), who also imposed the assumption of identical means, so $\beta_{S0} = \beta_{C0} = \beta_0$. It therefore seems logical to test both these null hypotheses against the general switching-regression alternative, which can be done using standard likelihood-ratio (LR) tests. A rejection of these null hypotheses would imply that there is a significant link between b_t and the behaviour of the mixing distributions, because it captures shifts either in their means, or in their mixing probabilities, or both.[4] A number of authors have noted that while Lagrange Multiplier and Wald tests should be asymptotically equivalent to the LR tests, they can give quite different results. The LR tests are thought to be the most reliable (see Engel and Hamilton (1990)) and are used below.

The switching-regression model also nests the linear regression model as the special case where $\beta_{S0} = \beta_{C0}$, $\beta_{Sb} = \beta_{Cb}$, $\beta_{qb1} = \beta_{qb2} = 0$, giving:[5]

$$\begin{aligned} &R_{t+1} = \beta_0 + \beta_b b_t + e_{t+1} \\ &e_{t+1} \sim N(0, \sigma_S) \text{ with prob } \Phi(\beta_q) \\ &e_{t+1} \sim N(0, \sigma_C) \text{ with prob } 1 - \Phi(\beta_q) \end{aligned} \tag{25}$$

One interpretation of equation (25), if R_{t+1} is measured as the return to holding foreign exchange, would be that it represents a linear model of exchange rate risk premiums. Alternatively, this regression has the same form as that used in Cutler, Poterba and Summers (1991) to describe non-rational speculative dynamics in a variety of asset markets. Any rejection of the restrictions implied by equation (25) could therefore be evidence of nonlinearities in exchange rate risk premiums, or of a more complex form of predictability than that considered by Cutler, Poterba and Summers (1991).

To summarize, maximization of the likelihood function allows estimation of the switching-regression system consistent with a model of stochastic bubbles. The bubble model implies testable coefficient restrictions on the switching-regression estimates. Furthermore, the switching regression can be tested against several simpler nested models that include both normal-mixture

[4] The Markov-switching model of Engel and Hamilton (1990) does not nest within this general switching model since it introduces state-dependent switching probabilities. Nonetheless, the switching regression can capture very similar effects. In the Markov-switching model, the probability of observing a given regime will vary over time depending on an unobserved state variable. In the switching regression, this probability varies as a function of the observed variable b_t. Given that b_t usually shows positive serial correlation, the dynamics of the two models can be quite similar. Formal tests of these two models could be done by estimating a Markov-switching regression. The bubble model would imply that the Markov behaviour should collapse to simple switching, while the Markov-mixture model would imply that all the coefficients on b_t should be insignificant. Instead, we simply test whether the switching-regression model incorrectly omits Markov-switching effects that are present in the data.

[5] In principle, one could also impose $\sigma_S = \sigma_C$, but this greatly complicates testing the null hypothesis against any switching alternative. Furthermore, given the possibility of heteroscedasticity in the data, it seems advisable to allow for time-variation in σ under the null hypothesis.

models and linear models of return predictability. All that remains is to specify appropriate measures of R_{t+1} and b_t before estimating the model. This will be done in the next section.

4. DATA AND MODELS OF FUNDAMENTAL EXCHANGE RATES

The empirical work presented below focusses on four of the most widely traded currencies in the world—the German mark (DM), the Japanese yen (¥), and the Canadian dollar (C$)—all measured relative to the US Dollar ($). Unless specified otherwise, all data are in natural logarithms and are measured monthly. The series cover most of the post-Bretton Woods floating exchange rate period, from September 1977 to October 1991. More details on all the data series used may be found in the Appendix.

The first series to be defined is the innovation in the exchange rate R_{t+1}. The most straightforward approach would be to assume that covered and uncovered interest parity hold, so that the log of the one-period forward exchange rate, f_t, is equal to the expected value of the log spot rate next period, s_{t+1}. This would suggest using

$$R_{t+1} = s_{t+1} - f_t \tag{26}$$

While tests of covered interest parity suggest it holds quite closely, there is considerable evidence rejecting uncovered interest parity. (See Hodrick, 1987, for an excellent summary, and for a discussion of the empirical points raised in the remainder of the paragraph.) If this were simply due to the presence of a constant risk premium, then only the mean of R_{t+1} would be affected, which would not affect any of the tests or restrictions proposed in Section 3. Certainly, most of the empirical evidence implies that the predictable component of $s_{t+1} - f_t$ accounts for only a small fraction of its total variance at the monthly frequency considered here. Furthermore, the fraction of this variance that represents risk premiums is disputed. The standard, rational, representative-agent paradigm implies that all of the predictability should be because of risk premiums. On the other hand, empirical models of the determinants of these premiums have had limited success, and studies using survey data on agents' forecasts of the spot rate imply that only a small fraction of the variance of the predictable component is due to risk premiums. See Frankel and Froot (1987, 1990) and Ito (1990). This suggests that equation (26) will give a reasonable measure of the exchange rate innovation.

The other series to be defined is the size of the bubble, b_t. Because of the lack of a widely agreed-upon empirical model of fundamental exchange rates, several different models were used to construct b_t. Note that because the tests and restrictions suggested in Section 2 are invariant to changes in the mean of b_t, the log of the fundamental exchange rate $\bar{s}_t$ need only be defined up to an additive constant. (Figure A1 in the Appendix compares the various estimates of the fundamental exchange rate with the actual exchange rate.)

The simplest exchange rate model tested uses the assumption of relative purchasing power parity (PPP), which implies that the real exchange rate should be constant. (In equation (4), this would imply $E_t(f(X_{t+j})) = (1-a) \cdot (p_t - p_t^f) \forall j$). Therefore, the measured real exchange rate will move one-for-one with the size of the bubble. The real exchange rate measure used here is the Morgan Guaranty real effective exchange rate index, which is a multilateral index based on general and wholesale price indices for 40 nations. To provide a benchmark for the bilateral exchange rate against the US dollar, indices for Canada, Germany and Japan are divided by that for the United States. (The use of a bilateral index based on normalized unit labour costs gave similar results.)

An alternative approach, common in the international trade literature, defines the fundamental real exchange rate as that rate which equilibrates the external sector of the economy. The deviation from this rate, b_t, should then be a function of the degree of external imbalance. The current account balance was therefore used as another measure of deviation from fundamentals, with an increase indicating a more undervalued (or less overvalued) exchange rate. Current accounts for all four nations are divided by GNP or GNE to provide a scale-free measure of imbalance, and the series for Canada, Germany and Japan were again measured relative to those for the United States. This measure is available on a quarterly basis only, and since it is already scale-free it is used in levels, not logarithms.

While these models of fundamental exchange rates may be quite simple, they have the advantage of being highly visible economic indicators. More sophisticated models of fundamentals are required to take account of realistic macroeconomic dynamics, however. One common approach is to use uncovered real interest parity, as in Shafer and Loopesko(1983), Campbell and Clarida(1987), Meese and Rogoff(1987), and Edison and Pauls (1991). As shown in the Appendix, the assumptions of UIP and long-run PPP imply that the long-term real interest rate differential gives us an index of the fundamental real exchange rate. This means that the difference between the actual real exchange rate and the real long-term interest rate differential is a measure of the bubble.

The final bubble measure we use is based on the sticky-price monetary (or 'overshooting') model of exchange rates. Earlier studies of bubbles in foreign exchange markets, such as those by Meese(1986), Woo (1985), West (1987), and Gros (1989), have all used variants of this model. As explained in the Appendix, the model used here is similar to those used by the above. As in these previous studies, the model's parameters must be calibrated indirectly, since parameter estimates based on exchange rate data would be inconsistent in the presence of bubbles.

At this point, it may be useful to review the effects of misspecification on tests for bubbles. As noted previously, significant evidence of bubbles can always be reinterpretted as evidence of misspecification of exchange rate fundamentals. However, should misspecification of the fundamentals lead to a false rejection of the null hypothesis of no bubbles?

For some kinds of bubble tests, the answer to this question would be 'yes'. For example, West (1987)'s misspecification test will tend to reject the null hypothesis of no bubbles if we misspecify the time series behaviour of fundamentals. The cointegration tests for bubbles suggested by Diba and Grossman (1987) will suggest that bubbles are present if the error between our measured and the true fundamentals follows an integrated process.

For the regime-switching test used in this paper, however, it is harder to argue that misspecification will lead us to find false evidence of bubbles. Specifically, the test is robust to misspecification in two ways.

First, both the likelihood ratio tests and the coefficient sign restrictions are invariant to linear transformations of b_t. Therefore misspecifying either the level of the bubble or its scale will have no effect on the regime-switching test for bubbles. The latter implies that, although the bubble measures shown above account for much of the observed movements in exchange rate, this assumption can be relaxed without affecting the validity of the test. It also implies that the adequacy of the models of fundamentals used in the test should not be judged by how well they fit the observed exchange rate. A more appropriate measure would be whether the resulting b_t can reasonably be expected to be highly *correlated* with the 'true' bubble (if any.)

Second, it should be remembered that the switching regression test will interpret as evidence of bubbles only a b_t which has explanatory power for the distribution of future innovations in the exchange rate, and only if it does so in a way that is consistent with the sign restrictions predicted by the model of bubbles. There is no reason to believe that arbitrary errors in model specification would have this effect. For example, if our measure of the bubble is simply i.i.d.

normally distributed 'noise' that is uncorrelated with the true fundamentals, we should be unable to reject the null hypothesis of no bubbles. Another example is the real interest rate parity measure of fundamentals for the Canada–US exchange rate. Figure A1 suggests that this measure may be suspect, as it is unusually volatile and bears little relationship to the actual exchange rate. However, Table I shows that it produces no evidence to support the bubble hypothesis.

As noted in Section 3, there is a special kind of specification error that could lead us to erroneously conclude that bubbles are present. The example presented there relies on a particular form of regime switching in fundamentals. Even then, however, the kind of misspecification required is quite specific. Our purported measure of bubbles must accurately measure the fundamental exchange rate up to but not including the effects of regime switching in the fundamentals. We would expect that adding more general kinds of misspecification would make it more difficult to find purported evidence of bubbles since this should simply reduce the explanatory power of b_t. Furthermore, it is not clear that regime switching in fundamentals can always be a plausible alternative to the conclusion that bubbles are present. For example, it is hard to rationalize the presence of regime switching in fundamentals when our model of fundamentals is purchasing power parity.

5. ESTIMATION AND TEST RESULTS

Tables I–III summarize the results of the estimated switching-regression model using the 1977–91 data on forward and spot exchange rates. As one would expect, the results are sensitive to the model of fundamentals used to construct b_t and to the particular exchange rate considered.

The results for Canada in Table I show that the overshooting model of fundamentals gives the highest values of the likelihood function and the most support for the bubble model. This bubble measure gives LR statistics that allow us to reject the three simpler models of regime switching in favour of the switching-regression alternative predicted by the bubble model. We also find that although two of the four bubble measures have a significant influence on R_{t+1} in state S, only the overshooting measure gives evidence that the size of the bubble also affects R_{t+1} in state C. Furthermore, the opposite signs on β_{Sb} and β_{Cb} that the overshooting measure gives are consistent with the predictions of the bubble model and allow us to reject the hypothesis that $\beta_{Sb} = \beta_{Cb}$. There is no significant evidence for any of the bubble measures that the classification probabilities are affected by the square of the bubble's size, nor is there any evidence of misspecification.

The results in Table II for Germany show that the external balance measure of the bubble has the best fit (as judged by the value of the likelihood function) and is the only measure that can reject either the unrestricted or the restricted normal-mixture models. However, it is unable to reject the null hypothesis of linear predictability. None of the bubble measures had more than one coefficient in the switching regression that was significant at the 5% level. The external balance measure comes close, with a significant β_{Sb} and a β_{Cb} with a marginal significance level of 5.2%. It should be noted, however, that both parameter estimates are negative (and significantly different), while the bubble model predicts that they should have opposite signs. Only the PPP measure produces evidence of misspecification, apparently due to the presence of both serial correlation and ARCH effects in regime S.

The Japanese results in Table III show that the PPP measure fits best, followed by the external balance measure. Both allow us to reject all three simpler switching models, while the other bubble measures fail to reject the null hypothesis of an unrestricted mixture of normal

Table I. Switching regression results for Canada: September 1977 to October 1991 (170obs.)[a]

Model of fundamentals	PPP		External balance		Real interest parity		Overshooting	
Log likelihood	498·27		497·42		495·53		502·69	
LR tests[b]	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)
Restricted normal mixture	10·05	7·3	8·22	14·4	2·47	78	**18·85**	0·2
Unrestricted normal mixture	8·47	7·5	6·63	15·7	0·88	92	**17·24**	0·2
Linear regression	9·02	6·0	6·29	17·8	2·34	67·7	**15·99**	0·3
Parameters[c]	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio
β_{Sb}	**0·0045**	3·68	0·003	1·95	−0·001	−0·5	**0·002**	4·47
β_{Cb}	−0·0008	−0·487	0·0007	0·36	−0·006	−0·439	**−0·003**	−1·98
β_{qb2}	−0·011	−0·028	0·338	−1·132	0·0753	0·214	−0·139	−0·45
Wald tests	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)
β_{Sb}	**5·65**	1·7	0·63	42·7	0·03	87·0	**9·79**	0·2
χ^2 *Diagnostics*[d]	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)
Joint test—$\chi^2(5)$	3·01	69·8	7·04	21·8	1·56	90·6	1·44	92·0
AR(1): regime S—$\chi^2(1)$	0·809	36·8	0·891	34·5	0·332	56·4	0·171	67·9
AR(1): regime C—$\chi^2(1)$	0·330	56·6	0·191	66·2	0·239	62·5	0·067	79·6
ARCH: regime S—$\chi^2(1)$	0·761	38·3	4·906	2·7	0·446	50·4	0·048	82·7
ARCH: regime C—$\chi^2(1)$	0·345	55·7	0·520	47·1	0·412	52·1	0·785	37·6
Markov effects—$\chi^2(1)$	0·454	50·0	0·237	62·6	0·204	65·2	0·140	70·8

Notes:
The likelihood-ratio statistics test various restrictions of the switching-regression model

$$(R^S_{t+1} - \beta_{S0} - \beta_{Sb}b_t) \sim N(0, \sigma_S)$$
$$(R^C_{t+1} - \beta_{C0} - \beta_{Cb}b_t) \sim N(0, \sigma_C)$$
$$Pr(R_{t+1} = R^S_{t+1}) = \Phi(\beta_{q0} + \beta_{qb1}b_t + \beta_{qb2}b_t^2)$$

where R^i_{t+1} is R_{t+1} when $\text{State}_{t+1} = i$, and $\Phi(.)$ is the logistic cumulative distribution function. The first row tests the *restricted normal-mixture model*, which implies that $\beta_{Sb} = \beta_{Cb} = \beta_{qb1} = \beta_{qb2} = 0$ and that $\beta_{S0} = \beta_{C0}$. The second row tests the *unrestricted normal-mixture model*, which drops the restriction that $\beta_{S0} = \beta_{C0}$, so the model becomes

$$R^S_{t+1} \sim N(\beta_{S0}, \sigma_S)$$
$$R^C_{t+1} \sim N(\beta_{C0}, \sigma_C)$$
$$Pr(R_{t+1} = R^S_{t+1}) = \Phi(\beta_{q0})$$

The third row tests the *linear regression model*, which imposes $\beta_{S0} = \beta_{C0}$, $\beta_{Sb} = \beta_{Cb}$, $\beta_{qb1} = \beta_{qb2} = 0$, giving

$$R_{t+1} = \beta_0 + \beta_b b_t + e_{t+1}$$
$$e_{t+1} \sim N(0, \sigma_S) \text{ with probability } \Phi(\beta_q)$$
$$e_{t+1} \sim N(0, \sigma_C) \text{ with probability } 1 - \Phi(\beta_q)$$

[a] Boldtype indicates significance at the 5% level except for diagnostic statistics, where it indicates significance at the 1% level.
[b] Tests restictions of the general switching regression. Statistics are distributed χ^2 with 5, 4 and 4 degrees of freedom, respectively, under the null.
[c] t-ratios and Wald tests are based on standard errors from the inverse of tbe Hessian. Robust standard errors gave similar results.
[d] Due to size distortion in finite samples noted by Hamilton (1990), tbe 1% critical values are used for inference instead of tbe usual 5%.

Table II. Switching regression results for Germany: September 1977 to October 1991 (170 obs.)[a]

Model of fundamentals	PPP		External balance		Real interest parity		Overshooting	
Log likelihood	326·06		327·25		325·72		324·02	
LR tests[b]	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)
Restricted normal mixture	8·86	11·5	**11·38**	4·4	8·47	13·2	4·89	42·9
Unrestricted normal mixture	8·85	6·5	**11·37**	2·2	8·46	7·5	4·88	29·9
Linear regression	8·83	6·5	4·86	30·1	5·15	27·3	3·89	41·9
Parameters[c]	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio
β_{Sb}	0·0008	0·02	**−0·044**	−3·29	**−0·010**	5·18	0·012	1·80
β_{Cb}	−0·0014	−0·46	−0·006	−1·94	−0·003	−0·75	−0·008	−0·62
β_{qb2}	−14·43	−0·19	3·97	1·24	−0·127	−0·32	0·069	0·12
Wald tests	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)
$\beta_{Sb} = \beta_{Cb}$	0·00	96·7	**7·55**	0·6	3·49	6·2	2·04	15·3
χ^2 *Diagnostics*[d]	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)
Joint test—$\chi^2(5)$	11·0	5·1	2·73	74·2	7·68	17·5	9·83	8·0
AR(1): regime S—$\chi^2(1)$	**8·272**	0·4	0·033	85·6	5·767	1·6	4·277	3·9
AR(1): regime C—$\chi^2(1)$	0·005	94·4	0·657	41·8	0·080	77·7	2·025	15·5
ARCH: regime S—$\chi^2(1)$	**8·272**	0·4	0·220	63·9	0·200	65·5	0·054	81·6
ARCH: regime C—$\chi^2(1)$	0·756	38·5	1·389	23·9	0·777	37·8	1·931	16·5
Markov effects—$\chi^2(1)$	0·880	34·8	0·876	34·9	0·000	100·0	1·723	18·9

See Notes to Table I.
[a–d] As Table I.

Table III. Switching regression results for Japan: September 1977 to October 1991 (170 obs.)[a]

Model of fundamentals	PPP		External balance		Real interest parity		Overshooting	
Log likelihood	327·25		325·55		321·47		321·98	
LR Tests[b]	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)
Restricted normal mixture	**28·73**	0·0	**25·28**	0·0	17·15	0·4	**18·26**	0·2
Unrestricted normal mixture	**18·66**	0·2	**15·18**	0·4	7·04	13·4	**8·15**	8·6
Linear regression	**26·15**	0·0	**13·83**	0·7	16·27	0·2	**16·65**	0·3
Parameters[c]	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio	Estimate	t-ratio
β_{Sb}	**−0·001**	−6·32	**−0·002**	−3·02	−0·002	−5·05	**−0·002**	−12·7
β_{Cb}	−0·006	−1·60	**−0·008**	−2·52	−0·003	−0·89	−0·004	−1·17
β_{qb2}	**−1·139**	−2·52	0·897	1·26	−0·240	−0·84	−0·562	−0·91
Wald Tests	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)
$\beta_{Sb} = \beta_{Cb}$	1·41	23·6	2·88	9·0	0·06	81·4	0·45	50·4
χ^2 *Diagnostics*[d]	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)	Statistic	p-value (%)
Joint test—$\chi^2(5)$	**15·8**	0·7	10·1	7·2	9·81	8·1	11·0	5·1
AR(1): regime S—$\chi^2(1)$	4·704	3·0	2·292	13·0	1·039	30·8	1·525	21·7
AR(1): regime C—$\chi^2(1)$	3·623	5·7	2·224	13·6	3·548	6·0	4·101	4·3
ARCH: regime S—$\chi^2(1)$	2·701	10·0	0·122	72·7	0·668	41·4	0·952	32·9
ARCH: regime C—$\chi^2(1)$	0·433	51·1	0·035	85·2	0·394	53·0	0·269	60·4
Markov effects—$\chi^2(1)$	0·022	88·2	0·619	43·1	0·008	92·9	2·803	9·4

See Notes to Table I.
[a–d] As Table I.

distributions. The estimate of β_{Sb} is negative, significant, and quite similar for all four measures of b_t Additional coefficients are significant for the PPP and external balance measures, which therefore give us binding restrictions on the bubble model's predictions. These are satisfied in the case of the PPP measure but not in the case of the external balance measure, where increases in the bubble are found to decrease expected returns in both regimes. We are never able to reject the hypothesis that $\beta_{Sb} = \beta_{Cb}$ at the 5% significance level for any of these measures. There is no significant evidence of misspecification, except in the case of the PPP measure. Even there the evidence is weak, with a significant joint test (with an LR statistic of 15·8 versus a critical value of 15·1) but no significant individual tests. The joint test presumably reflects some weak evidence of serial correlation, as the test statistics for first-order autocorrelation in each regime are closest to their critical values (4·7 and 3·6, compared to a critical value of 6·6).

To summarize these results, we can see that measures of apparent deviations of exchange rates from their fundamental values have some descriptive power for subsequent excess returns in a number of cases. Furthermore, for Canada and Japan there is evidence of significant nonlinearities in this relationship. Diagnostic tests also generally suggest that the simple switching regression adequately captures several aspects of the data. In particular, there is almost no evidence of omitted ARCH effects or Markov state-dependencies. None of the bubble measures produced evidence that supported all three of the parameter restrictions predicted by the bubble model. On the other hand, in two cases (the overshooting measure for Canada and the PPP measure for Japan) two of these three parameters were significant and had the predicted signs.

6. FURTHER IMPLICATIONS OF THE BUBBLE MODEL

While the above results give formal statistical tests of the bubble model, they provide little insight into the kind of behaviour that the model captures. In particular, to help assess the reasonableness of the bubble hypothesis, it would be useful to see whether such behaviour is consistent with popular accounts of speculative episodes. For that reason, we now examine more closely the behaviour of the switching regression in two of the cases above that most consistently support the bubble model; the overshooting model of fundamentals for the Canada–US exchange rate and the PPP model of fundamentals for the Japan–US exchange rate. Because none of results we considered above gave very strong support to the bubble model for the Germany–US exchange rate, we do not present further results for that exchange rate. In the graphs below, the data for Canada cover a slightly longer sample period, ending in January 1992. While the addition of these three extra observations has no noticeable effect on any of the test results reported above, it allows us to see the apparent collapse of a bubble in January 1992.

Figure 1 compares the spot Canada–US exchange rate with the calculated size of the bubble, b_t. Recall that the bubble size is identified only up to an arbitrary constant and is graphed with a mean of zero. Both series are dominated by the depreciation of the Canadian dollar over the 1983–6 period, which causes a rise in the bubble measure of 0·12–0·15. Therefore, if we believe the exchange rate was close to fundamentals at the beginning of this period, by early 1986 the Canadian dollar appeared to be 12–15% undervalued. In contrast, the currency is most overvalued from 1977–81 and in 1991, where the same benchmark suggests an overvaluation of about 2–5%.

The top of Figure 2 shows the *ex ante* probability of a collapse, calculated as $1 - q(b_t)$ from equation (A9) in the Appendix. This suggests two or three distinct bubble episodes. Not surprisingly, the most prominent one covers the undervaluation of the Canadian dollar from early 1985 to the end of 1986, and corresponds to the peaks in the size of the bubble. At its most extreme, the *ex ante* probability of a collapse exceeded 20% per month, suggesting the bubble had become quite fragile. A second episode is the apparent overvaluation of the Canadian dollar in 1991, which produces a short-lived but distinct surge in the probability of a collapse. Note that this bubble is apparently almost totally unwound by the sharp depreciation of the Canadian dollar in January 1992. The third episode is less distinct and corresponds to a

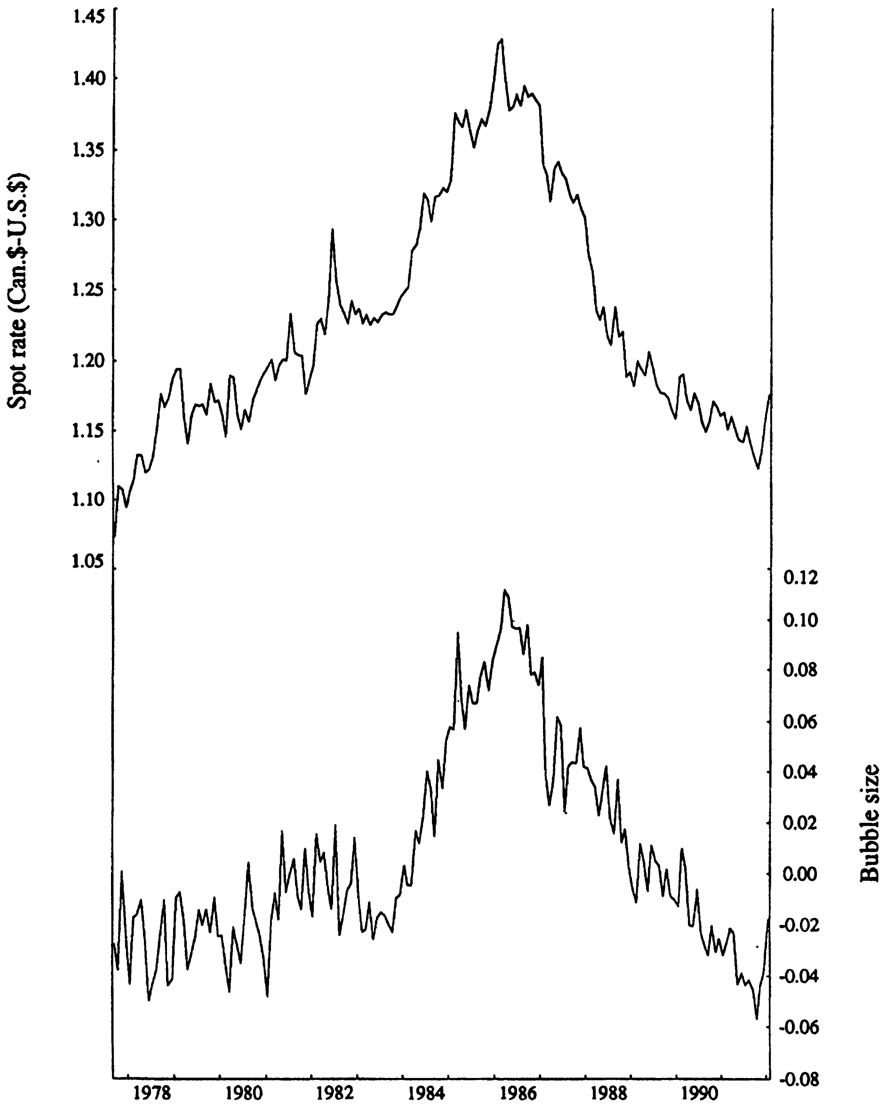

Figure 1. Spot exchange rates and bubble size: Canada—overshooting model. The bubble is constructed as the log of the spot exchange rate less the log of the fundamental exchange rate and is defined to have a mean of zero

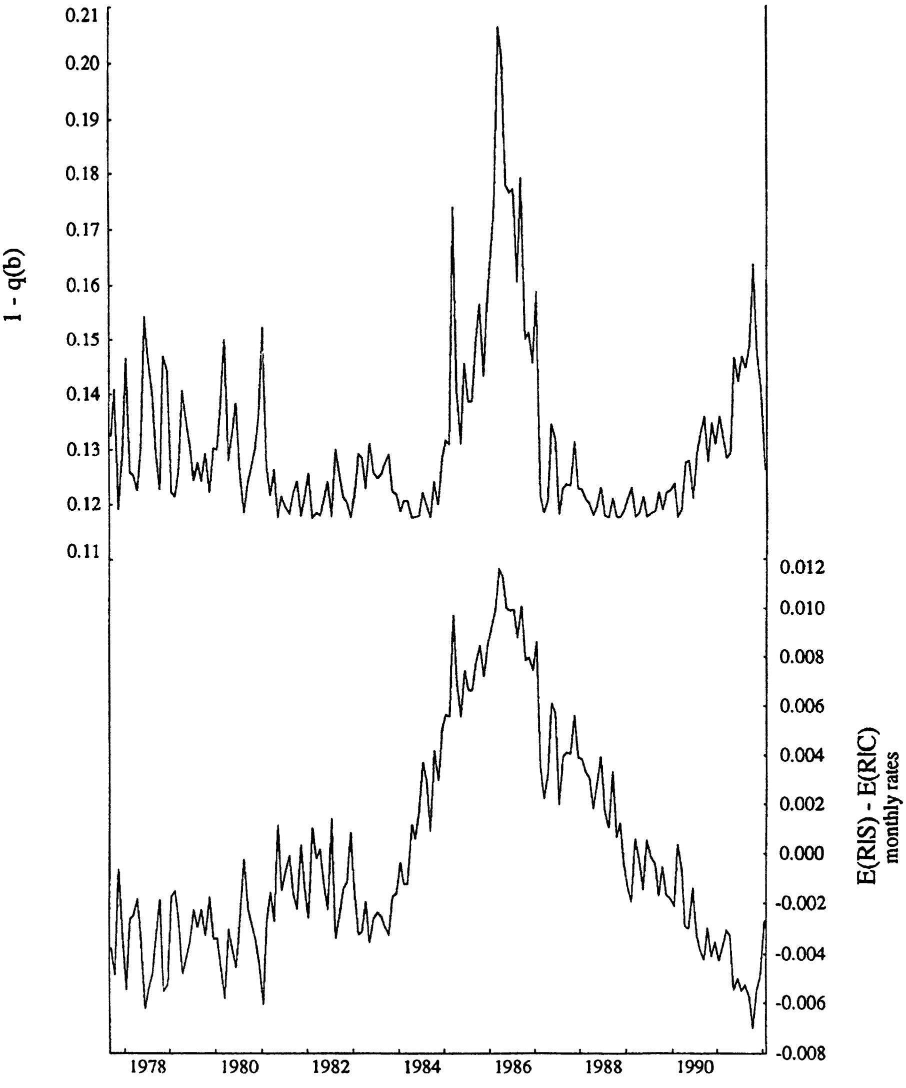

Figure 2. *Ex ante* probability of bubble collapse and expected difference across regimes: Canada—overshooting model

possible overvaluation of the Canadian dollar in the late 1970s. The peaks here are lower than in the other two episodes and there is greater month-to-month variability in the results, so the evidence is less definitive. Finally, we can also note that the probability of collapse is lowest in the 1982–3 and 1988–9 periods, suggesting that the exchange rate was close to its fundamental values at these times.

While the timing of the bubble episode in the late 1970s may be somewhat surprising, particularly in light of its apparent longevity, the first two episodes identified above seem to fit with the view of other market participants. The undervaluation of the Canadian dollar in early 1986 led the Canadian government and the Bank of Canada to take several steps to correct what they believed to be undue speculation against the Canadian dollar. This included intensive exchange market intervention in support of the Canadian currency, whose exchange rate they felt '... did not reflect the fundamentals of our economic and financial situation ...' (Bank of Canada, 1986, p. 15). The overvaluation in 1991 also corresponds to a period where several Canadian observers (including Courchene, 1991, and Harris, 1991) felt the currency was overvalued. However, it should be noted that the overvaluation covers a shorter period of time and was smaller than some of these observers suggested. The overshooting model finds a bubble that does not begin until 1990 at the earliest, and never exceeds C\$0·05 in size, whereas some observers suggested an overvaluation starting in 1987 and ranging from C\$0·10–0·15 in size.

To understand what this implies for the expected effects of bubble collapse, the bottom of Figure 2 shows the difference in expected returns to holding US dollars across states S and C. As in Figure 1, this series is graphed with a mean of zero. Since it is just a linear transformation of b_t, we find it has a similar shape, with peaks in the 1984–7 period and lows during 1977–81 and 1991. Relative returns in the two regimes change by less than 2% per month (27% per annum) from the peak of the undervaluation to the maximum overvaluation. This implies only a fraction of the bubble can be expected to be reversed in any given month, so bubbles may take some time to be eliminated, even conditional on a collapse.

The last two graphs present the same evidence for the Japanese data. Figure 3 shows that the bubble size is dominated by an apparent undervaluation of the yen starting in 1982 and declining rapidly after the Plaza Accord in September 1985. In addition, there appear to be two periods of yen overvaluation before and after the undervaluation, consisting of a brief period in the latter half of 1978 and a more prolonged episode centred in 1988. The variation in the bubble's size is much more pronounced than in the Canadian data, with a peak-to-trough change of roughly 70%, compared to a range of only 18% for Canada.

The probabilities of collapse in Figure 4 tend to confirm the timings of these bubble episodes, although they suggest that the undervaluation of the yen in the mid-1980s may be divided into two episodes, with a distinct peak and decline in 1982 followed by an even larger peak in 1985. This corresponds to a period where, probably more so than any other period since the breakdown of the Bretton Woods system, many observers (such as Krugman (1985)) felt the US dollar was overvalued. The probability of collapse shows even higher peaks for Japan than for Canada, reaching 44·6% per month in March 1985 as the U.S. dollar peaked against most overseas currencies. Variations in expected returns across regimes again reflect the movements of the bubble measure. Given the larger apparent size of the bubble in the yen-U.S. dollar exchange rate, however, it is perhaps surprising to see that differences in returns across regimes are expected to be roughly the same size as those in the Canadian data.

Caution must be exercised in drawing conclusions from Figures 2 and 4, since they present point estimates only and these estimates may be highly uncertain. However, it appears that the econometric evidence and the historical record can be consistent with the implications of the bubble model, given particular assumptions about fundamentals. As argued in the introduction, however, the switching regression can be useful in its own right as a descriptive device to characterize the behaviour of risk premiums and the distribution of excess returns. For that reason, we now turn briefly to consider two aspects of the switching-regression estimates that are not directly related to the bubble model.

The first of these is behaviour of expected excess returns, or risk premiums. The bubble model derived in Section 2 simply assumed that excess returns are unpredictable. The switching regression does not impose this restriction, however. It is straightforward to show that the expected value of R_{t+1} conditional on b_t is given by

$$E(R_{t+1} \mid b_t) = q(b_t) \cdot (\beta_{S0} + \beta_{Sb} b_t) + (1 - q(b_t)) \cdot (\beta_{C0} + \beta_{Cb} b_t) \tag{27}$$

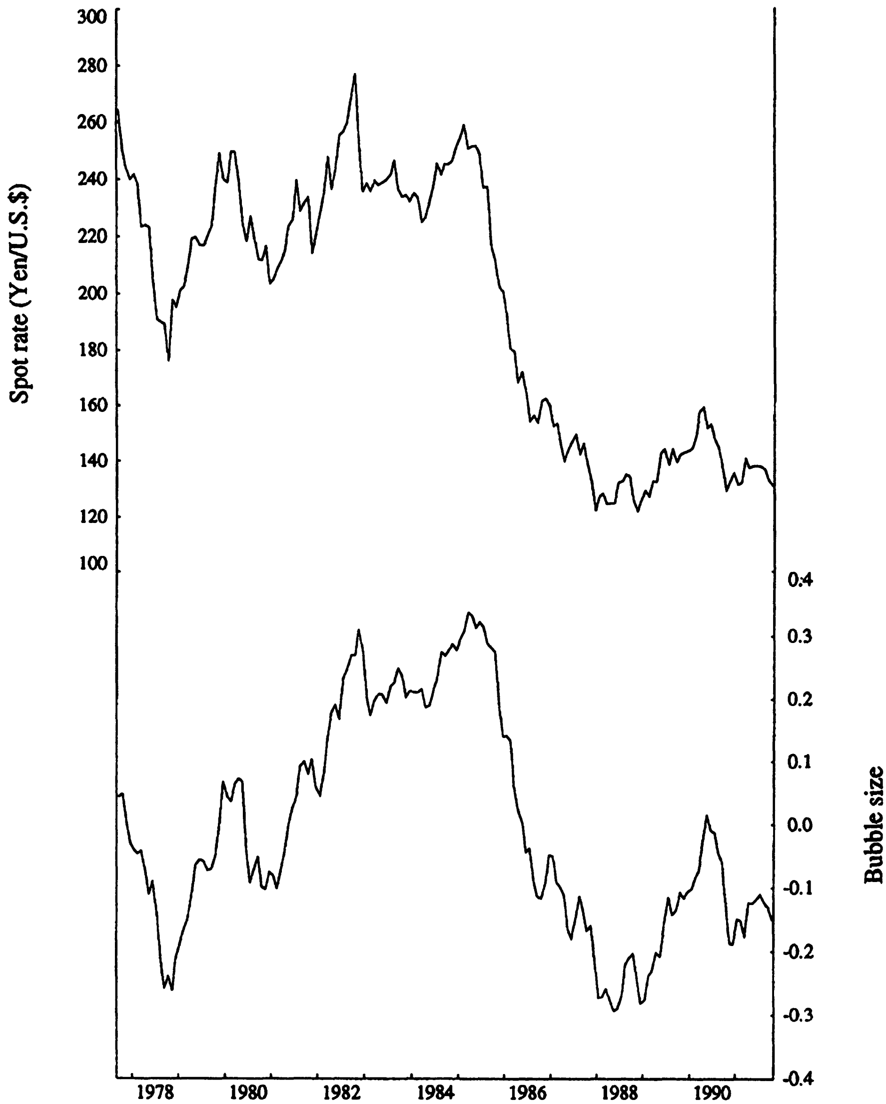

Figure 3. Spot exchange rates and bubble size: Japan—PPP model. The bubble is constructed as the log of the spot exchange rate less the log of the fundamental exchange rate and is defined to have a mean of zero

Figures 5 and 6 graph $E(R_{t+1} \mid b_t)$ alongside b_t for the Canadian and Japanese models discussed above. Again, it should be remembered that the figures do not display the confidence intervals associated with these point estimates, so caution should be exercised in their interpretation.

We see that for Canada, expected excess returns are almost perfectly correlated with b_t. This reflects the fact that β_{qb1}, β_{qb2} are quite small, so that $q(b_t)$ is roughly constant and $E(R_{t+1} \mid b_t)$ effectively collapses to a linear function of b_t. If we compare Figure 5 to Figure 2, we see that the range of expected excess returns is less than one third of the range of the difference in expected returns across regimes.

Figure 6 shows that results for Japan look quite different. The correlation between b_t and $E(R_{t+1} \mid b_t)$ is weaker and negative. This reflects both the non-linearity arising from a larger β_{qb2} and the negative estimates of β_{Sb} and β_{Cb}. Furthermore, the variation in expected excess returns over time is both larger than that for Canada, and is almost as great as the variation in expected returns across regimes. (See Figure 4.) The latter suggests that the bubble model may omit an important source of predictable variation in excess returns, that is itself correlated with deviations from the fundamental (PPP) exchange rate.

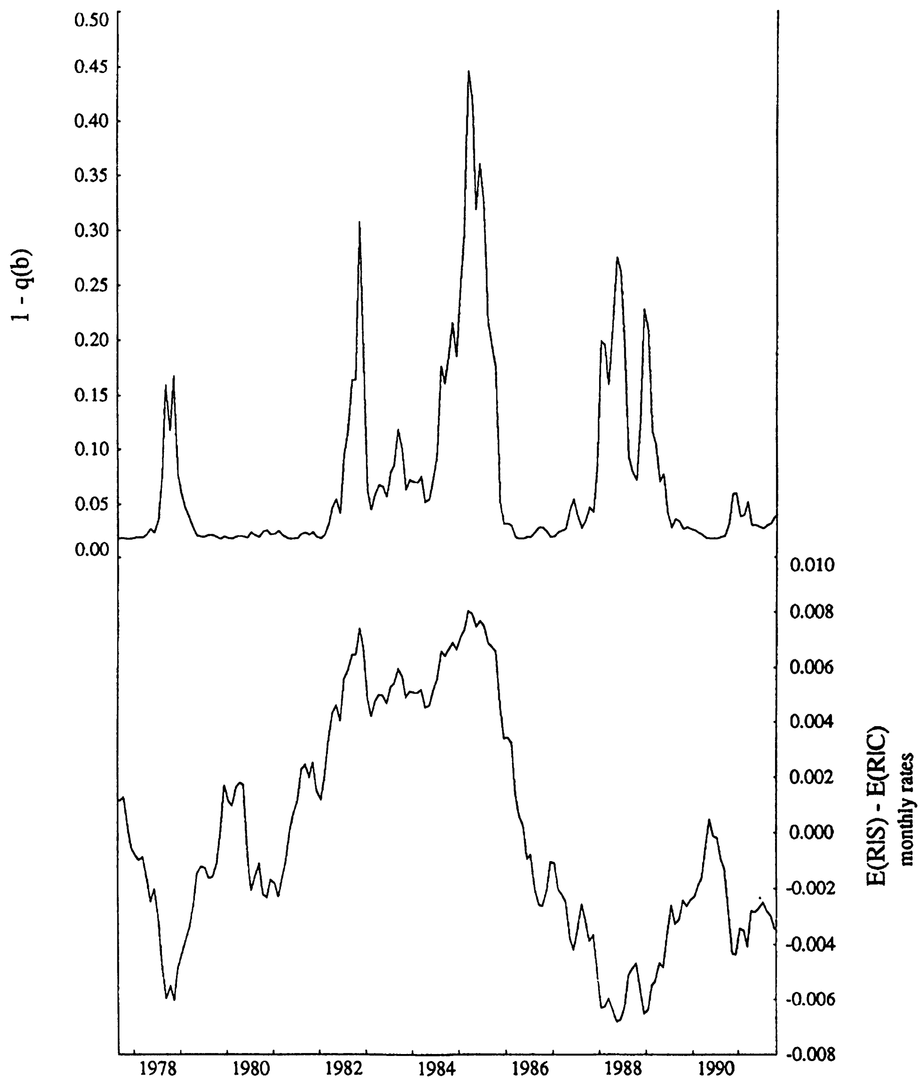

Figure 4. *Ex ante* probability of bubble collapse and expected difference in returns across regimes: Japan—PPP model

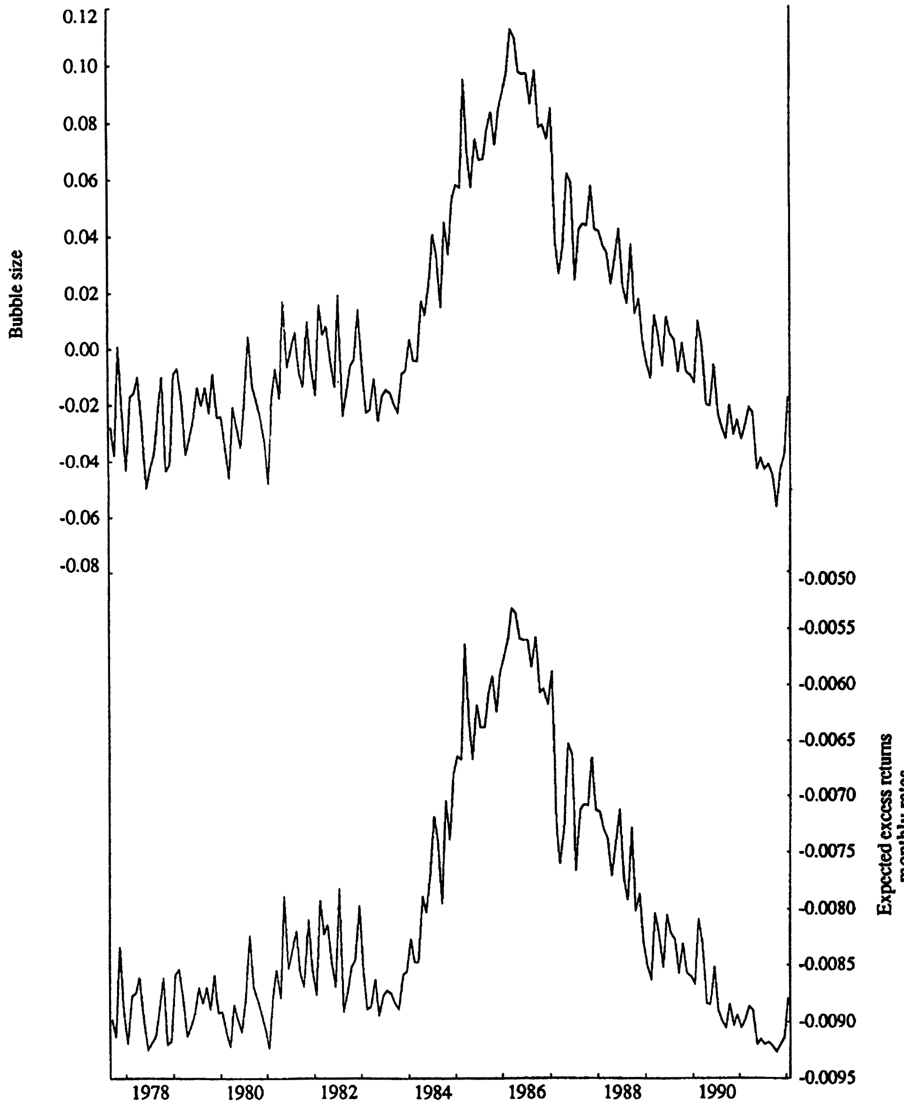

Figure 5. Expected excess returns: Canada—overshooting model. The bubble is constructed as the log of the spot exchange rate less the log of the fundamental exchange rate and is defined to have a mean of zero

The second aspect of the distribution of excess returns that we wish to consider is the predictable variation in their dispersion. One way to quantify this is to consider the conditional probability of observing an outlier of a given size, say two standard deviations from the sample mean. It can be shown that

$$Pr(R_{t+1} < x) = \varphi\left(\frac{x - \beta_{S0} - \beta_{Sb} b_t}{\sigma_S}\right) \cdot q(b_t) + \varphi\left(\frac{x - \beta_{C0} - \beta_{Cb} b_t}{\sigma_C}\right) \cdot (1 - q(b_t)) \tag{28}$$

and that

$$Pr(R_{t+1} > x) = \varphi\left(\frac{-x + \beta_{S0} + \beta_{Sb}b_t}{\sigma_S}\right) \cdot q(b_t) + \varphi\left(\frac{-x + \beta_{C0} + \beta_{Cb}b_t}{\sigma_C}\right) \cdot (1 - q(b_t)) \qquad (29)$$

where φ is the standard normal cumulative distribution function. Note that changes in x just give a monotonic transformation of the probabilities, so the conclusion below are robust to changes in the threshold value chosen.

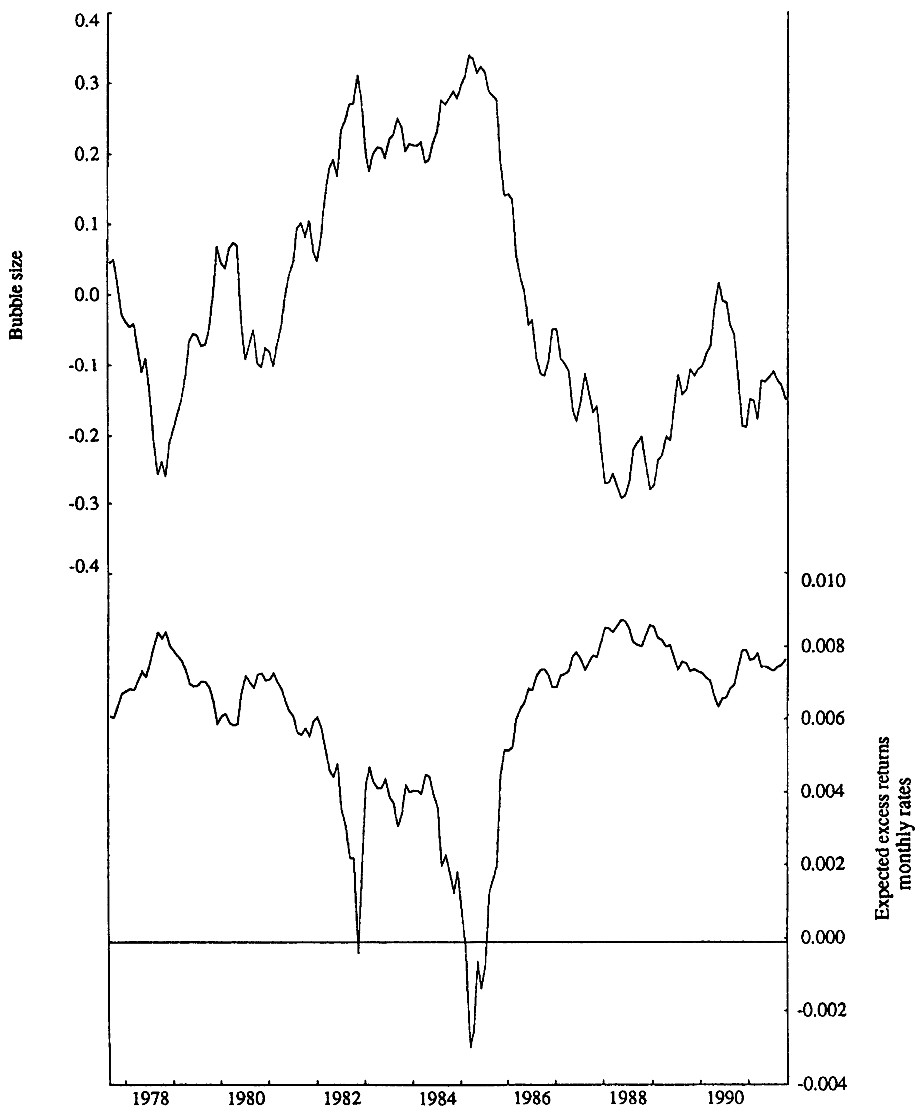

Figure 6. Expected excess returns: Japan—PPP model. The bubble is constructed as the log of the spot exchange rate less the log of the fundamental exchange rate and is defined to have a mean of zero

Figures 7 and 8 show the probabilities of observing an excess return two standard deviations either above or below the sample mean. If the expected value of returns $E(R_{t+1} \mid b_t)$ is fairly constant but its variance around this expectation is not, the probabilities of observing high and low outliers should be positively correlated. However, if $E(R_{t+1} \mid b_t)$ is quite variable and the variance around this expectation is stable, then the probabilities of observing high and low outliers should be negatively correlated. The figures show that for both Canada and Japan, the probabilities of observing high and low outliers are strongly negatively correlated, implying that variations in the conditional distribution of returns are dominated by shifts in its mean rather than its dispersion. This strengthens the conclusion drawn from the graphs of $E(R_{t+1} \mid b_t)$, that there is an important source of predictable variation in expected excess returns that is correlated with deviations from the fundamental exchange rate.

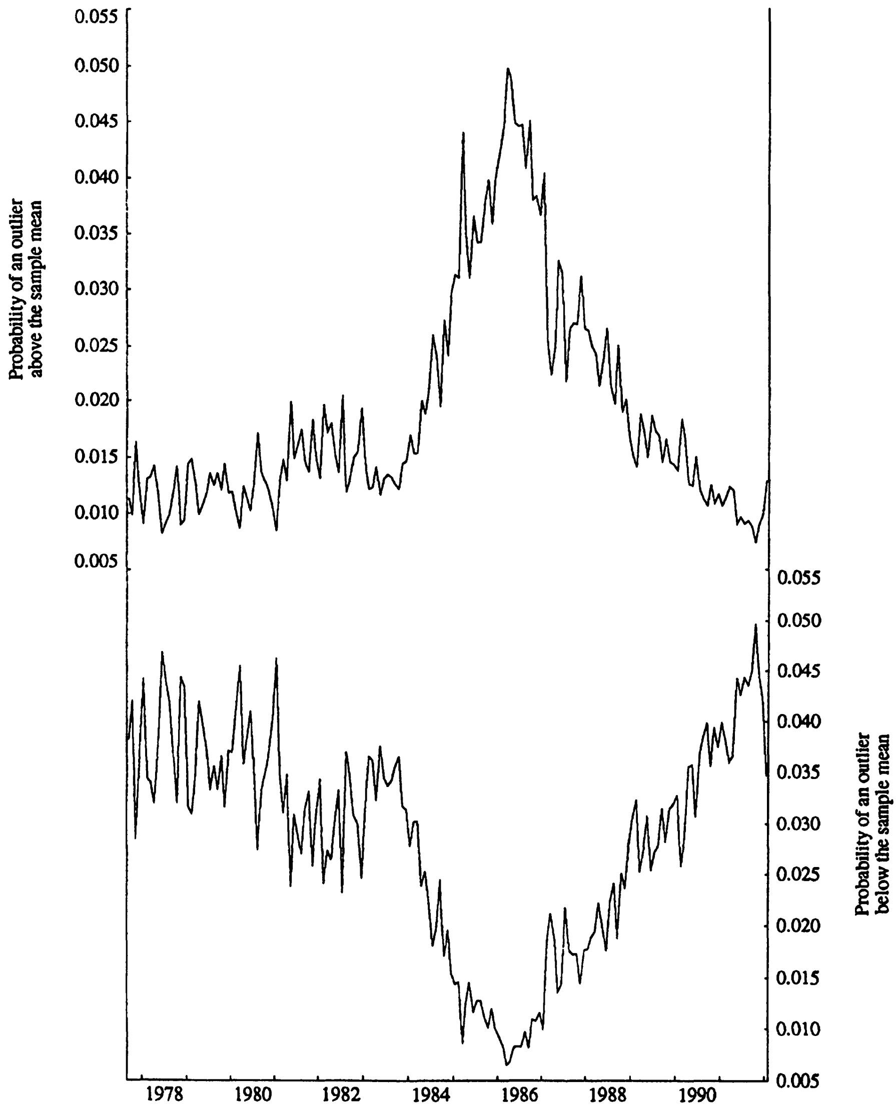

Figure 7. Probability of a two-standard-deviation outlier: Canada—overshooting model

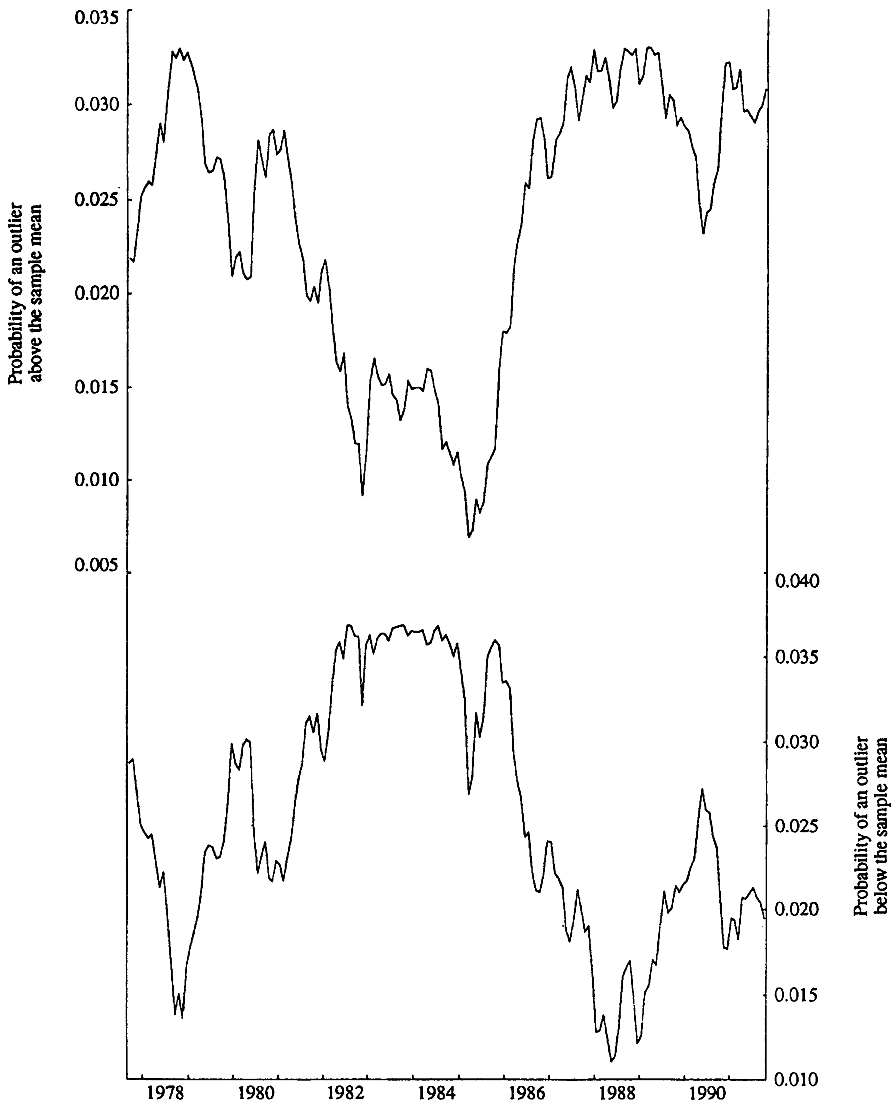

Figure 8. Probability of a two-standard-deviation outlier: Japan—PPP model

7. CONCLUSIONS

This paper has described a two-regime model of speculative bubbles. We have seen how simple restrictions on the behaviour of bubbles lead to a new set of testable predictions about the behaviour of exchange rate innovations. These predictions have implications for variety of current areas of research. They suggest generalizing univariate mixture-distribution models of innovations to a switching-regression framework and linking mixing behaviour to other macroeconomic variables. They also suggest a relationship between current macroeconomic variables and future exchange rate innovations that nests other empirical models of asset price dynamics.

The results presented above for the Canada/US, Germany/US and Japan/US exchange rates are sensitive to changes in the definition of the fundamental exchange rate and to the measurement of exchange rate innovations. In many cases, no significant evidence of bubbles is found. Whether this is due to their absence or a lack of power in the test is unclear. Evidence supporting the bubble model is strongest when using excess returns data and an overshooting model of fundamentals for the Canada/US exchange rate or a PPP model for the Japan/US exchange rate. Furthermore, the bubble episodes identified by the switching-regression model in these cases correspond well to periods that others have associated with deviations from fundamentals. Additional evidence also suggests that deviations from fundamental exchange rates in these cases have an important influence on expected excess returns, a fact which is not predicted by the bubble model.

Regardless of whether one accepts the bubble interpretation of the above results, they should be of interest to those studying the efficiency of foreign exchange markets. Obviously, the implication that bubbles may be present is of direct interest. However, even the interpretation that there are other, fundamental, causes of switching implies that the distribution of exchange rate innovations varies over time in a manner not previously considered. Since this time-variation may be predicted by other macroeconomic variables, such variation should affect exchange rate risk premiums. A satisfactory fundamental model of this switching behaviour and of the relationship between deviations from fundamentals and expected returns should therefore be an important ingredient in future research on risk premiums.

Distinguishing between switching in returns due to bubbles and that caused by process switching in fundamentals will be difficult and is beyond the scope of this paper. A useful way to proceed would be to specify a particular model of switching in fundamentals and examine whether actual switches in fundamentals correspond to apparent switches in exchange rate innovations. van Norden and Schaller (1993a) use such an approach to study historical crashes in US stock prices, and conclude that both bubbles and switches in fundamentals seem to play a role. Work by Lewis (1989) with monetary models of exchange rate determination has shown that while switches in monetary policy can explain the apparent predictability of forward-rate prediction errors in certain periods, they do not seem to fit during the significant US dollar appreciation in 1984–5. In contrast, the results here seem to fit well during this period, again suggesting that both factors may be at work.

APPENDIX: ESTIMATION OF THE SWITCHING REGRESSION MODEL

To understand the estimation procedure, suppose that in regime C:

$$R_{t+1} = R^C_{t+1} = h_C(b_t) + e^C_{t+1} \tag{A1}$$

and that in regime S:

$$R_{t+1} = R^S_{t+1} = h_S(b_t) + e^S_{t+1} \tag{A2}$$

where $e^C_{t+1} \equiv \varepsilon^*_{t+1} + \varepsilon^C_{t+1}$ and $e^S_{t+1} \equiv \varepsilon^*{}_{t+1} + \varepsilon^S_{t+1}$. This implies that we can write the probability density function of an observation conditional on it being generated by a given regime as

$$\phi_C(e^C_{t+1}) = \phi_C(R_{t+1} - h_C(b_t)) \tag{A3}$$

and

$$\phi_S(e^S_{t+1}) = \phi_S(R_{t+1} - h_S(b_t)) \tag{A4}$$

If we have no information on which regime generates each observation, we may denote the average probability that an observation comes from regime S as q. More generally, if we have a set of variables, M_t, that contain imperfect classifying information from variables dated t and earlier, we can write the probability that $R_{t+1} = R^S_{t+1}$ as $q(M_t)$. Therefore, the unconditional probability density function of each observation is

$$q(M_t) \cdot \phi_S(R_{t+1} - h_S(b_t)) + [1 - q(M_t)] \cdot \phi_C(R_{t+1} - h_C(b_t)) \tag{A5}$$

and the likelihood function for a set of T observations

$$\prod_{t=1}^{T} \{q(M_t) \cdot \phi_S(R_{t+1} - h_S(b_t)) + [1 - q(M_t)] \cdot \phi_C(R_{t+1} - h_C(b_t))\} \tag{A6}$$

Maximizing this likelihood function therefore estimates both equations (A1) and (A2) simultaneously with a set of parameters for $q(M_t)$, and can be shown to lead to consistent and efficient estimates without the need for *a priori* restrictions on which observations correspond to a given regime. (See Goldfeld and Quandt, 1973, and Kiefer 1978 for proofs.) A variety of other estimation approaches have been suggested, with various strengths and weaknesses. Hartley (1978) suggests using the EM algorithm, which is equivalent to maximizing the likelihood function but may be computationally easier. Mehta and Swamy(1975) propose a Bayesian approach, and Quandt and Ramsey (1978) suggest using a moment-generating-function method. It is well understood in this literature that the likelihood function is unbounded, but that a local maxima exists that has the desirable properties claimed above.

Transforming the bubble model from Section 2 into a form that can be estimated requires several additional pieces of information: functional forms for $h_S(b_t)$ and $h_C(b_t)$; a functional form and explanatory variables for $q(M_t)$; and distributional assumptions for e^S_{t+1}, e^C_{t+1} (which will imply functional forms for ϕ_S, ϕ_C). To keep the computational difficulty of estimation manageable, one can take a first-order Taylor series expansion of equations (14) and (16) around some arbitrary value b_0 to obtain:

$$h_S(b_t) = \beta_{S0} + \beta_{Sb} b_t \tag{A7}$$

$$h_C(b_t) = \beta_{C0} + \beta_{Cb} b_t \tag{A8}$$

where the model (from equations (15) and (17)) implies that $\beta_{Sb} > 0$, $\beta_{Cb} < 0$. Furthermore, equation (9) implies that b_t should give information on the probability of observing S or C, so one can use $M_t = b_t$. In choosing a form for $q(\cdot)$, it is important to ensure that $q(\cdot)$ will be bounded between 0 and 1, and that it is not monotonic in b_t (since it should be decreasing as either positive or negative bubbles grow in absolute value). One such candidate would be a logit function of the form $q(b_t) = \Phi(\beta_{q0} + \beta_{qb} \cdot b_t^2)$ where $\beta_{qb} > 0$ and $\Phi(x) \equiv (1 + e^x)^{-1}$ is the logistic cumulative distribution function. However, if the fundamental exchange rate s_t^*, is misspecified by some constant amount k so that the true size of the bubble is $\hat{b}_t = b_t + k$, this would force q to have its maximum at $\hat{b}_t = -k$ instead of $\hat{b}_t = 0$. This can be avoided by using the more general functional form

$$q(b_t) = \Phi(\beta_{q0} + \beta_{qb1} b_t + \beta_{qb2} b_t^2) \tag{A9}$$

which still has the testable implication from equation (11) that $\beta_{qb2} > 0$.[6] Finally, we will assume

[6]The reader may verify that $q(b_t)$ can still have a minimum at $\hat{b}_t = 0$ if $\beta_{qb1} = -2 \cdot k \cdot \beta_{qb2}$. However, so long as $\beta_{qb2} > 0$, $q(b_t)$ will eventually fall as $|b_t|$ becomes large relative to the size of the measurement error k.

that e^S_{t+1}, e^C_{t+1} follow i.i.d. normal distributions with mean 0 and standard deviations σ_S and σ_C. This means that the log-likelihood function for the bubble model, $\text{llf}(\cdot)$, can be written as

$$\text{llf}(\beta_{S0}, \beta_{Sb}, \beta_{C0}, \beta_{Cb}, \beta_{q0}, \beta_{qb1}, \beta_{qb2}, \sigma_S, \sigma_C)$$

$$= \sum_{t=1}^{T} \ln\left(\left(\Phi(\beta_{q0} + \beta_{qb1}\beta_t + \beta_{qb2}b_t^2)\cdot\phi\left(\frac{R_{t+1} - \beta_{S0} - \beta_{Sb}b_t}{\sigma_S}\right)\Big/\sigma_S\right)\right.$$
$$\left. + \left([1 - \Phi(\beta_{q0} + \beta_{qb1}b_t + \beta_{qb2}b_t^2)]\cdot\phi\left(\frac{R_{t+1} - \beta_{C0} - \beta_{Cb}b_t}{\sigma_C}\right)\Big/\sigma_C\right)\right) \quad \text{(A10)}$$

where $\phi(\cdot)$ is the standard normal probability density function. The model can then be estimated by maximizing the likelihood function directly, or by using an indirect method, such as the EM algorithm proposed by Hartley (1978). In practice, the EM algorithm tends to converge more slowly than direct estimation methods, but is more robust to the starting point. For that reason, the EM algorithm was used to produce good starting estimates for direct ML estimation.

Data Definitions

The spot exchange rates and the one-month forward premium/discount rates used to generate R_{t+1} were taken from the Bank of International Settlements database. The DM/US\$ spot rate is the official fixing at 13:00 Frankfurt time, while its forward rate is the middle market rate around noon Swiss time. The yen/US\$ spot and forward rates are the Tokyo market closing middle rate and the London middle market rate at around noon Swiss time, respectively. Both spot and forward rates for the C\$/US\$ are London middle market rates at around noon, Swiss time. Excess returns are calculated based upon the following conventions:

(1) The spot settlement date is two business days after the trading date except for the C\$/US\$, for which the settlement date is one business day after the trading date.
(2) The one-month maturity date is the same day as the spot settlement date moved forward to the next month, unless the maturity date is not a valid business date in either of the two home markets. In this case, the maturity date is delayed until the next business day valid in both countries. However, if the previous conventions take us out of the month, we move backwards to the first suitable one-month maturity date.
(3) However, if the spot settlement date is the last valid business day of the current month, then the one-month maturity date is the last business day of the next month.

PPP exchange rates are real effective exchange rates taken from Morgan Guaranty's *World Financial Markets*, which is based on wholesale price indices for 18 industrial countries and 22 less-developed countries.

The real long interest rates are constructed by using year-over-year CPI inflation as a proxy of expected inflation. The interest rate differential measures are then constructed by using the formula:

$$\text{diff} = \left(\frac{1 + r^{\text{US}}}{1 + r^{\text{other}}}\right)^{10} \quad \text{(A11)}$$

where r^{US} is the US real long-term interest rate and r^{other} the real rate for the differential country. The long-term interest rates are based on 7- to 10-year government bonds for each nation.

Finally, the overshooting-model exchange rates were calculated using data from a variety of sources. Money supplies were taken to be national measures of M1, prices were national CPIs, output measures were indices of industrial production, and interest rates were 30-day money market or commercial paper rates.

Models for Fundamental Exchange Rates

This section provides more detail on the real interest parity and the overshooting models of exchange rate fundamentals (Figure A1). Real interest parity requires that the real interest

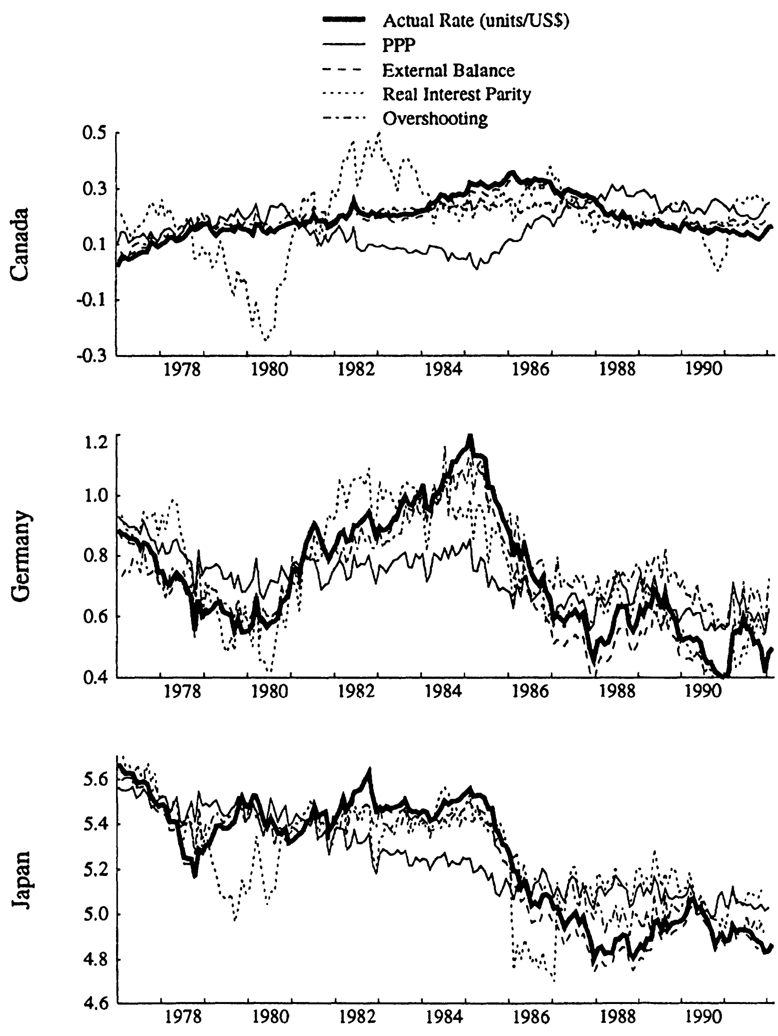

Figure A1. Actual and fundamental exchange rates (all series in logs) differentials correspond to expected changes in real exchange rates, or

$$E_t(Q_{t+k} - Q_t) = r_t - r_t^f \tag{A12}$$

$$Q_t \equiv s_t - p_t + p_t^f \tag{A13}$$

where Q_t is the logarithm of the real exchange rate at time t, p_t and p_t^f are the logarithms of the domestic and foreign price levels, r_t is the logarithm of the sum of 1 plus the k-period real interest rate at time t, and r_t^f is the corresponding transformation of the k-period foreign real interest rate. (Since nominal uncovered interest parity implies real uncovered interest parity, the discussion of the validity of the uncovered interest parity assumption in the context of defining R_{t+1} applies here also.) If one assumes that no changes are expected in the long-run real exchange rate, then

$$E_t(Q_{t+k}) \to \bar{Q} \qquad \text{as } k \to \infty \tag{A14}$$

where $\bar{Q}$ is this long-run value. This model can be used to determine fundamental exchange rates by using a suitably large k. This implies that the k-period real interest rate differential gives us an index of the fundamental real exchange rate, since equation (A12) then gives

$$\bar{Q} - (r_t - r_t^f) = Q_t \tag{A15}$$

To convert this into a measure of bubble size simply requires

$$b_t = s_t - (p_t - p_t^f) + (r_t - r_t^f) - \bar{Q} \tag{A16}$$

Note that the addition of a constant risk premium to equation (A13) would have the same effect as a shift in $\bar{Q}$, shifting the intercept term but maintaining a one-to-one relationship between $(r_t - r_t^f)$ and Q_t, and therefore maintaining the validity of equations (A15) and (A16).

Overshooting models of the exchange rate all begin with nominal uncovered interest parity

$$E_t(s_{t+1} - s_t) = i_t - i_t^f \tag{A17}$$

and substitute out the interest rates using money-demand equations of the general form

$$m_t - p_t = -a_0 \cdot i_t + a_1 \cdot y_t + a_2 \cdot (m_{t-1} - p_{t-1}) \tag{A18}$$

where m_t, p_t, y_t represent the log of relative money supplies, prices and output, respectively, and $0 < a_2 < 1$. (Woo, 1985, uses a slightly more general form, allowing for different coefficients on foreign and domestic output.) If we then assume that relative prices adjust slowly to their PPP values:

$$s_t - p_t = a_2 \cdot (s_{t-1} - p_{t-1}) \tag{A19}$$

West (1987) shows we can solve for the exchange rate, obtaining

$$\tilde{s}_t = \gamma s_{t-1} + E_t\left(\sum_{j=0}^{\infty} \lambda^j \cdot z_{t+j}\right) \tag{A20}$$

where

$$z_t = (\lambda / a_0) \cdot (m_t - a_2 m_{t-1} - a_1 y_t) \tag{A21}$$

$$\gamma = \{1 + a_0 - \sqrt{(1 + a_0)^2 - 4a_0 a_2}\}/(2a_0) \tag{A22}$$

and

$$\lambda = (2a_0)/\{1 + a_0 + \sqrt{(1 + a_0)^2 - 4a_0 a_2}\} \tag{A23}$$

Experimentation with alternative estimates of a_0, a_1 and a_2 showed that reasonable estimates of the fundamental exchange rate could be obtained using $a_0 = 0{\cdot}5$, $a_1 = 1{\cdot}0$ and $a_2 = 0{\cdot}9$ on monthly data. Results using a slightly different calibration were reported in van Norden (1993), but gave largely similar results. Instability of the money-demand functions in the 1980s could potentially cause this model to undergo structural shifts. Attempts to estimate parameters directly and allow for these shifts gave very poor results, with fundamental exchange rates that differed from actual rates by factors of 10 or more. In contrast, use of the constant parameter values given above gave fundamental rates that seemed reasonable over the full sample.

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