Pitfalls in Testing for Explosive Bubbles in Asset Prices

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A number of studies (e.g., Robert J. Shiller, 1981; Olivier J. Blanchard and Mark Watson, 1982; Kenneth D. West, 1988) have argued that dividend and stock price data are not consistent with the "market fundamentals" hypothesis, in which prices are given by the present discounted values of expected dividends. These results have often been construed as evidence for the existence of bubbles or fads. (Related arguments have been made with respect to gold, bonds, and foreign exchange). A major problem with such arguments (e.g., James Hamilton and Charles Whiteman, 1985) is that apparent evidence for bubbles can be reinterpreted in terms of market fundamentals that are unobserved by the researcher.

Behzad T. Diba and Herschel I. Grossman (1984, 1988b) and Hamilton and Whiteman (1985) have recommended the alternative strategy of testing for rational bubbles by investigating the stationarity properties of asset prices and observable fundamentals.[1] In essence, the argument for equities is that if stock prices are not more explosive than dividends then it can be concluded that rational bubbles are not present, since they would generate an explosive component to stock prices.[2] Using unit-root tests, autocorrelation patterns, and cointegration tests to implement this procedure, Diba and Grossman (1988b p. 529) state that "the analysis supports the conclusion that stock prices do not contain explosive rational bubbles."

This paper shows that the above battery of tests is in fact unable to detect an important class of rational bubbles. The point is demonstrated by constructing rational bubbles that appear to be stationary when unit-root tests are applied, even though they are explosive in the relevant sense. Simulations show that, when such bubbles are present, stock prices will not appear to be more explosive than dividends on the basis of these tests, even though the bubbles are substantial in magnitude and volatility. The presence of rational bubbles in actual stock prices thus remains an open question.

I. The Model and Tests

I employ the standard model for stock prices,

$$(1) \qquad P_t = (1+r)^{-1} E_t(P_{t+1} + d_{t+1})$$

where P_t is the real stock price at t, d_{t+1} is the real dividend paid to the owner of the stock between t and $t+1$, and $0 < (1+r)^{-1} < 1$ is the discount rate, which is assumed to be constant.[3] I ignore tax issues and the possibility of unobserved fundamentals since these are not required for the points to be made in this paper. E_t denotes expectations conditional on information at time t.

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[1] John Y. Campbell and Shiller (1987) have also discussed this strategy.

[2] More precisely, if k-differences of dividends are sufficient to make the dividend series stationary and if bubbles are present, then k-differences of stock prices will not be sufficient to make the stock price series stationary. (For any time series x_t, the first difference of x_t is defined as $\Delta x_t = x_t - x_{t-1}$, and for $k \geq 2$, the k-difference of x_t is defined as the first difference of the $(k-1)$-difference of x_t.)

[3] There is considerable evidence against the hypothesis of constant expected real returns (e.g., Campbell and Shiller, 1989). However, bubble solutions can also be constructed for modifications of (1) which allow for time-varying discount rates, and issues analogous to those discussed in this paper would arise in such models.

The market-fundamentals solution to (1) is

$$F_t = \sum_{j=1}^{\infty} (1+r)^{-j} E_t d_{t+j} \tag{2}$$

provided the conditional expectations are defined and the sum converges. The entire class of solutions is given by

$$P_t = F_t + B_t$$

where B_t is any random variable that satisfies

$$B_t = (1+r)^{-1} E_t B_{t+1}. \tag{3}$$

The "rational bubble" B_t may depend either on d_t or on wholly extraneous variables.

Suppose that Δd_t is a stationary ARMA process and that there are no bubbles so that $P_t = F_t$. Then it can be shown that ΔP_t is a stationary ARMA process and that P_t and d_t (and the constant) are cointegrated (i.e., that there exists a stationary linear combination of P_t and d_t).[4] In particular, $P_t - r^{-1} d_t$ can be shown to be stationary.

Suppose instead that Δd_t is a stationary ARMA process but that $B_t \neq 0$. Then one knows (e.g., Stephen Beveridge and Charles R. Nelson, 1981) that, for some C_t,

$$E_t F_{t+j} \to C_t + \lambda j \quad \text{as} \quad j \to \infty$$

where $\lambda = E(\Delta F_t)$. In contrast, from (3) and the law of iterated expectations one finds that

$$E_t B_{t+j} = (1+r)^j B_t. \tag{4}$$

Thus, the conditional expectation of F_{t+j} grows linearly in the forecast horizon j, reflecting the unit root in the process, whereas the conditional expectation of B_{t+j} contains the root $1+r>1$ if $B_t \neq 0$. For large j, the conditional expectation of P_{t+j} will be dominated by the explosive root $1+r$ if a bubble is present. Furthermore,

$$\lim_{j \to \infty} E_t \Delta F_{t+j} = \lambda$$

and

$$E_t \Delta B_{t+j} = r(1+r)^{j-1} B_t$$

so that the conditional expectation of ΔP_{t+j} is stable if $B_t = 0$ but explosive if $B_t \neq 0$.

These types of consideration motivate the tests for bubbles implemented by Diba and Grossman (1988b). Specifically, they do the following:

(i) Test P_t, d_t, ΔP_t, and Δd_t for unit roots versus stable roots using the David A. Dickey and Wayne A. Fuller (1981) Φ_3 statistic. They are unable to reject the null hypothesis of a unit root in either d_t or P_t but are able to reject a unit root in favor of stable roots for both ΔP_t and Δd_t. They conclude that this is inconsistent with the existence of an explosive rational bubble in P_t.

(ii) Test for cointegration of P_t and d_t, using the Durbin-Watson statistic and the ξ_2 and ξ_3 statistics of Robert F. Engle and C. W. J. Granger (1987). Two of these three tests indicate cointegration, as do the tests of Alok Bhargava (1986). However, if Δd_t is stationary and P_t and d_t are cointegrated, then ΔP_t must be stationary, and this is inconsistent with the presence of a bubble. This evidence thus also appears to rule out explosive rational bubbles.

The central argument of this paper is that, when applied to periodically collapsing rational bubbles, the above procedure can, with high probability, incorrectly lead to the conclusion that these bubbles are not present. The explanation lies in the logic of the tests.

A maintained hypothesis of the Dickey-Fuller and Bhargava tests is that the process

[4]The stationary ARMA solution for ΔP_t can be calculated using the results of C. Gourieroux et al. (1982). It follows from Blanchard and Charles M. Kahn (1980) that this is the fundamentals solution (2). Campbell and Shiller (1987) show cointegration of F_t and d_t.

is linearly autoregressive, and under the null hypothesis H_0 it is assumed that there is a (largest) root of unity. If Δd_t is a stable linear autoregressive process, then under the nonbubble hypothesis the ΔP_t process will fall into the set of stable statistical alternatives to H_0. However, if a periodically collapsing bubble B_t is present then the ΔP_t process and the B_t process itself belong neither to H_0 nor to the explosive alternatives and indeed fall outside the maintained hypothesis of linear autoregressive processes. The key question then is whether B_t shows a greater resemblance, in terms of the distribution of test statistics, to stable, to unit root, or to explosive linear autoregressive processes. The answer from simulations, which can be most clearly exhibited in the case of the two-sided Bhargava tests, is that, unless the probability that the bubble does not collapse is very high, B_t will appear to be a stable linear autoregressive process. As a result, test procedures (i) and (ii) outlined above would erroneously come to the conclusion that bubbles are not present.

II. Periodically Collapsing Bubbles

Rational bubbles can take the form of deterministic time trends, explosive AR(1) processes or more complex stochastic processes. Bubbles do not appear to be empirically plausible unless there is a significant chance that they will collapse after reaching high levels. Blanchard (1979) and Blanchard and Watson (1982) describe bubbles that burst almost surely in finite time. However, Diba and Grossman (1988c) have shown that the impossibility of negative rational bubbles in stock prices implies theoretically that a bubble can never restart if it ever bursts (i.e., falls to 0).[5]

I therefore examine the following class of rational bubbles that are always positive but periodically collapse:[6]

$$(5a)\quad B_{t+1}=(1+r)B_t u_{t+1} \qquad \text{if } B_t \le \alpha$$

$$(5b)\quad B_{t+1}=\Big[\delta+\pi^{-1}(1+r)\theta_{t+1} \times\big(B_t-(1+r)^{-1}\delta\big)\Big]u_{t+1} \qquad \text{if } B_t > \alpha.$$

Here, δ and α are positive parameters with $0<\delta<(1+r)\alpha$, u_{t+1} is an exogenous independently and identically distributed positive random variable with $E_t u_{t+1}=1$, and θ_{t+1} is an exogenous independently and identically distributed Bernoulli process (independent of u) which takes the value 1 with probability π and 0 with probability $1-\pi$, where $0<\pi\le 1$.

It is straightforward to verify that process (5) satisfies (3) and that $B_t>0$ implies $B_s>0$ for all $s>t$. As long as $B_t \le \alpha$, the bubble grows at mean rate $1+r$. When eventually $B_t>\alpha$ the bubble "erupts" into a phase in which it grows at the faster mean rate $(1+r)\pi^{-1}$ as long as the eruption continues, but in which the bubble collapses with probability $1-\pi$ per period. When the bubble collapses, it falls to a mean value of δ, and the process begins again.

By varying the parameters δ, α, and π, one can alter the frequency with which bubbles erupt, the average length of time before collapse, and the scale of the bubble. For example, a high value of α generates bubbles with a long initial period of relatively steady slow growth. An example of a simulated bubble is given in Figure 2 of Section V.

III. Bhargava Tests of Simulated Bubbles

One set of tests employed by Diba and Grossman (1988b) are the Bhargava (1986)

[5]Other theoretical results, summarized in footnote 14, further restrict the possibility of rational asset price bubbles in existing rational-expectations models.

[6]Diba and Grossman (1988a) recognize the possibility of bubbles that "periodically shrink," and West (1987) provides examples of strictly positive bubbles that periodically collapse. An alternative to the use of two regimes in (5a) and (5b) to ensure positivity would be to restrict the range of u_{t+1}.

TABLE 1—BHARGAVA TEST: RESULTS FOR SIMULATED BUBBLES (PERCENTAGE OF TESTS REJECTING UNIT ROOT AT THE 5-PERCENT LEVEL)

		Probability per period that bubble does not collapse, π						
Test	Result	0.999	0.99	0.95	0.85	0.75	0.5	0.25
N_1	rejection in favor of explosive alternatives	78	32.5	0	0	0	0	0
	rejection in favor of stable alternatives	0	0	71.5	91.5	98.5	100	99.5
N_2	rejection in favor of explosive alternatives	94.5	61	11	6.5	2	2.5	0
	rejection in favor of stable alternatives	0	0	22	86.5	94.5	96	96

Notes: Results are for 200 simulated bubbles, generated by (5) with parameters as in text, each of sample size 100. N_1 and N_2 are the von Neuman type ratios described in Bhargava (1986). Five-percent critical points are taken from his table I with $T=100$: 0.006 and 0.17 are the critical points for N_1 for stable and explosive alternatives, respectively; 0.022 and 0.26 are the critical points for N_2 for stable and explosive alternatives, respectively.

tests. These have the advantage that they are the most powerful invariant tests of the null hypothesis of random walk (or random walk with drift) against one-sided stable and explosive alternatives. Using simulations, Diba and Grossman (1988b) show that, if bubbles are generated according to

$$B_{t+1}=(1+r)B_t+z_{t+1} \tag{6}$$

with $1+r=1.05$ and z_t standard normal white noise, then in 95 cases out of 100 the N_1 test and in 94 cases out of 100 the N_2 test, correctly reject the null hypothesis of a unit root, at a 5-percent level of significance, in favor of the explosive alternative.[7]

Table 1 reports the results of the Bhargava tests applied to periodically collapsing bubbles. Two hundred simulated bubbles, each of length 100 periods ("years"), are generated according to (5) with $1+r=1.05$, $\alpha=1$, $\delta=0.5$, and initial $B_t=\delta$. The variable u_{t+1} is chosen to be identically and independently distributed lognormal, scaled to have unit mean; that is, $u_t=\exp(y_t-\tau^2/2)$ where $y_t\sim\mathrm{IIN}(0,\tau^2)$, and for the simulations $\tau=0.05$.

[7]We focus on the N_1 and N_2 tests of Bhargava (1986) which allow for possible transients in the stable alternatives (the R_1 and R_2 tests can be employed when transients are known not to be present under the stable alternative). The null hypothesis for the N_1 test is a simple random walk, and the null hypothesis for the N_2 test is a random walk with drift.

The results of the test depend critically on π, the probability per period that the bubble does not collapse, and are thus reported for a range of values. As π approaches 1 the process (5) converges to

$$B_{t+1}=(1+r)B_t u_{t+1} \tag{7}$$

which is close to the case investigated by Diba and Grossman [eq. (6); (7) assumes a multiplicative disturbance which guarantees that B remains positive, instead of the additive disturbance in (6)]. It is thus not surprising that for π close to 1 the results of Table 1 are close to those obtained by Diba and Grossman.

However, for $\pi\le 0.95$, quite different results are obtained, with a larger number of rejections in favor of stable alternatives than in favor of explosive alternatives. Indeed, for $\pi\le 0.75$, more than 90 percent of the simulations reject the null hypothesis of a unit root in favor of stable alternatives for both the N_1 and N_2 statistics.

The qualitative nature of the results appears to be robust to moderate changes in the other parameters. Smaller values of α relative to δ generate more frequent eruptions and tend to yield a larger proportion of rejections in favor of stable alternatives. Very large values of α relative to δ generate infrequent eruptions and a larger proportion of rejections in favor of explosive alternatives.

The explanation for the Table 1 results was indicated above. The maintained hypothesis for the Bhargava tests is a first-order linear autoregressive process.[8] When $\pi < 1$, the bubble process in (5) is a complex nonlinear process which falls outside the maintained hypothesis. Unless π is close to 1, the pattern of periodic collapse generated by (5) looks more like a stable AR(1) process than an explosive one, despite the explosive root in the conditional expectation of the bubble sequence.

IV. Unit-Root and Cointegration Tests for Simulated Stock Prices and Dividends

Since bubbles, if they exist, are not directly observable, I now investigate the implications for tests based on observable stock prices and dividends. Two hundred replications were simulated, each of 100 "years" of stock price and dividend data, based on the following assumptions. Real dividends were assumed to be generated as a random walk with drift:

$$d_t = \mu + d_{t-1} + \varepsilon_t \tag{8}$$

where ε_t is $N(0, \sigma^2)$ white noise. The actual dividend process seems to be approximately consistent with (8), and I adopt the parameters $\mu = 0.0373$, $\sigma^2 = 0.1574$, and $d_0 = 1.3$ obtained by West (1988 p. 53) for the Standard and Poor 500 sample covering 1871–1980.[9]

With dividends generated by (8), equation (2) can be solved to yield

$$F_t = \mu(1+r)r^{-2} + r^{-1}d_t. \tag{9}$$

As before, $1+r$ is set at 1.05. Bubbles were simulated according to (5) with $\pi = 0.85$ and other parameters as specified as in Section III. No attempt was made to fit the actual price and dividend data as closely as possible, but the bubble series generated were scaled up by a multiplicative factor so that the simulated bubbles would provide most of the volatility of ΔP_t, where, of course, $P_t = F_t + B_t$. In the simulations reported, B_t was scaled up by a factor of 20, which ensures that the sample variance of ΔB_t is about three times the sample variance of ΔF_t.[10]

Table 2 reports the result of unit-root and cointegration tests using these simulated data. For dividends, the unit-root (Dickey-Fuller Φ_3) tests tend to indicate correctly that d has a unit root and that Δd is stationary.[11] For stock prices, the results are similar, except that in 25 percent of the cases, when no lags of ΔP_t are included, one would be led to the conclusion that P_t is stationary around a deterministic time trend. This test procedure thus clearly fails to find the bubble present, since the detection of rational bubbles is meant to be indicated by price series that are "*less* stationary" than dividend series.

All three cointegration tests reported in Table 2 also incorrectly indicate the absence of bubbles in the majority of simulations. Clearly, this is because the residuals from the cointegrating regression largely reflect the presence of periodically collapsing bubbles.[12] However, as in Section III, these

[8]In the case of the N_2 test, the maintained hypothesis allows for a deterministic time trend.

[9]An alternative, and perhaps more plausible, simple model for dividends is that log dividends follow a random walk with drift. Specification (8) is adopted in the text to maintain comparability with Diba and Grossman (1988b). Simulations based on the alternative specification are described in the Appendix.

[10]The median ratio of $\mathrm{Var}(\Delta B)/\mathrm{Var}(\Delta P)$, where $\mathrm{Var}(\Delta B)$ and $\mathrm{Var}(\Delta P)$ refer to sample variances, was 0.76. In 75 percent of the cases this ratio exceeded 0.54, and in 25 percent of the cases it exceeded 0.98.

[11]The proportion of rejections for d differs from 5 percent because the critical points used are based on an asymptotic approximation.

[12]Joe Mattey and Richard Meese (1986) present Monte Carlo evidence on the performance of a battery of test statistics under six different data-generation models, including one which incorporates a bursting bubble of the Blanchard and Watson (1982) type. One of their findings is that the cointegration-test critical values given in Engle and Granger (1987) do not appear to be appropriate under the null hypothesis of no bubbles for the I(1) dividend process they specify, and they also note low power for integration tests to detect the bubble alternative. Some of the other tests described by Mattey and Meese might detect the type of bubble investigated in this paper, as might the tests employed in Evans (1986). However, any such apparent detections would be subject to the argument noted in the Introduction that the results could be reinterpreted in terms of unobserved fundamentals.

TABLE 2—TEST RESULTS FOR SIMULATED STOCK PRICES WITH BUBBLES PRESENT AND DIVIDENDS GENERATED BY RANDOM WALK WITH DRIFT

A. *Percentage of Dickey-Fuller Φ_3 tests significant at the 5-percent level:*

	Series, x_t			
Number of lags	P	ΔP	d	Δd
None	25	98.5	2.5	100
Four	7	93.5	4	90

B. *Percentage of cointegration tests significant at the 5-percent level:*

Test		
$\xi_1 = \mathrm{DW}$	ξ_2	ξ_3
90	84.5	60

Notes: Dividends were generated by (8) with parameters set as in text. Bubbles were generated by (5) with parameters set as in text with $\pi = 0.85$. Prices were generated by (9) and $P_t = F_t + B_t$. There were 200 simulations of sample size 100 each. Φ_3 is the standard F statistic for the null hypothesis $(\gamma, \rho) = (0,1)$ in regressions of the form $x_t = \mu + \gamma t + \rho x_{t-1} + \varepsilon_t$ or $x_t = \mu + \gamma t + \rho x_{t-1} + \Sigma_{i=1}^{4} \beta_i \Delta x_{t-i} + \varepsilon_t$, with critical value from Dickey and Fuller (1981); ξ_1, ξ_2, and ξ_3 are the CRDW, DF, and ADF tests described in table I of Engle and Granger (1987) using the 5-percent critical values given in their table II. Cointegration tests are based on a regression of P_t on d_t and a constant.

bubbles (with $\pi = 0.85$) "appear" to be stationary when standard unit-root tests are applied.[13]

V. Example

To illustrate the findings of this paper, I report the results of a "typical" simulation obtained as follows. Of the 200 bubble series simulated, the one with median $\mathrm{Var}(\Delta B_t)$ was chosen. Similarly, of the 200 dividend series simulated, the one with median $\mathrm{Var}(\Delta d_t)$ was chosen. These were then used to generate the F_t and P_t series. The unit-root and cointegration tests described above are reported in Table 3. Also reported are the sample autocorrelations for P, ΔP, d, and Δd.

It is quite clear from Table 3 that the standard statistical procedures would indicate that a rational bubble is not present: both P and d appear to have a unit root, whereas both ΔP and Δd appear to be stationary. Furthermore, P and d appear to be clearly cointegrated. Yet, in fact, a bubble is present and 75.6 percent of the sample variance of ΔP_t is due to the presence of the bubble. Figures 1 and 2 plot P_t, F_t, and B_t for this example. Over the sample, there appear to be four bubble eruptions, each followed by a collapse. In the largest eruption, at its maximum, the bubble constitutes over half of the stock price.

VI. Conclusions

Periodically collapsing bubbles are not detectable by using standard tests to determine whether stock prices are "more explosive" or "less stationary" than dividends. Of course, this paper has not provided evidence that such bubbles are actually present in stock prices, and it should be noted that there are important theoretical arguments that restrict the possibility of rational bubbles.[14] However, unless a test that is based

[13]The Bhargava (1986) tests would give similar results for a range of assumed cointegrating vectors. In particular, for $P_t - r^{-1} d_t$, with $1 + r = 1.05$, the test would be applied to the actual bubble process, and the proportion of rejections can be read off of the column $\pi = 0.85$ in Table 1.

[14]In addition to the arguments ruling out negative bubbles and bubbles that restart, Jean Tirole (1982) has shown that rational bubbles cannot arise in a model with a fixed finite number of representative agents (due to the transversality condition), and Tirole (1985) has also shown that in nonstochastic

TABLE 3—RESULTS FOR "TYPICAL" SIMULATION

A. *Dickey-Fuller Φ_3 test statistics*:

Number of lags	Series: P	ΔP	d	Δd
None	5.13	58.56**	4.19	48.54**
Four	4.68	13.42**	4.15	13.37**

B. *Sample autocorrelations*:

Series	Number of lags: 1	2	3	4	5	6	7	8	9	10
P	0.85	0.72	0.63	0.54	0.44	0.36	0.30	0.26	0.21	0.15
ΔP	−0.10	−0.12	−0.04	0.03	−0.09	−0.10	−0.03	0.02	0.02	0.02
d	0.92	0.82	0.76	0.69	0.62	0.56	0.50	0.46	0.40	0.34
Δd	−0.01	−0.17	0.01	−0.04	−0.06	−0.09	−0.03	−0.03	−0.02	−0.08

C. *Cointegration tests*:

Test: $\xi_1 = \mathrm{DW}$	ξ_2	ξ_3
0.561**	−3.97*	−3.51*

Notes: The Φ_3 statistic and the cointegration tests are as described in the note to Table 2, with critical points taken from Dickey and Fuller (1981) and Engle and Granger (1987), respectively.

*Significant at the 5-percent level; **significant at the 1-percent level.

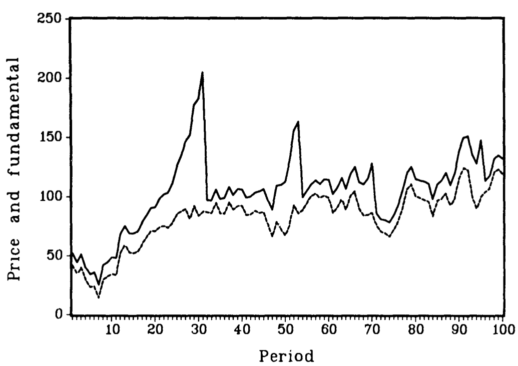

FIGURE 1. PRICE P (SOLID LINE) AND FUNDAMENTALS PRICE F (DASHED LINE) FOR EXAMPLE SIMULATION

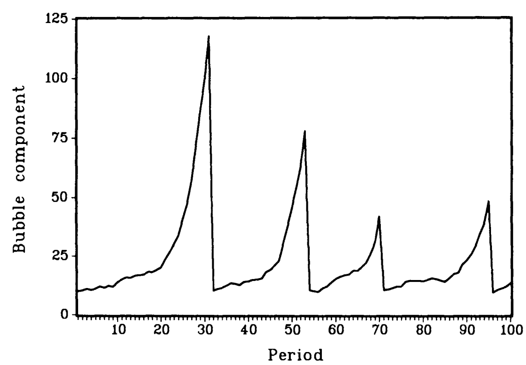

FIGURE 2. BUBBLE COMPONENT B FOR EXAMPLE SIMULATION

overlapping-generation models bubbles are ruled out if the interest rate is greater than the growth rate. (A related condition is cited in Stephen A. O'Connell and Stephen P. Zeldes [1988] for representative-agent models with a growing population.) Finally Evans (1989) provides theoretical reasons for doubting that bubbles, in rational-expectations models of form (1), can arise as the outcome of a learning process.

specifically on (3) can be devised, the way forward empirically would appear to be either to estimate a satisfactory parametric model of unobserved fundamentals or to estimate a satisfactory parametric model of rational bubbles.[15]

[15]Of course, it may also be necessary to take account of irrational elements in the market. James M.

TABLE A1—TEST RESULTS FOR SIMULATED STOCK PRICES WITH BUBBLES PRESENT AND LOG DIVIDENDS GENERATED BY RANDOM WALK WITH DRIFT.

A. *Percentage of Dickey-Fuller tests significant at the 5-percent level:*

	Series			
Number of lags	ln(*P*)	Δ ln(*P*)	ln(*d*)	Δ ln(*d*)
None	7.5	100	3.5	100
Four	6	90.5	6	89

B. *Percentage of Cointegration tests significant at the 5-percent level:*

Test		
$\xi_1 = \text{DW}$	ξ_2	ξ_3
90	85	61

Notes: Dividends and fundamentals prices were generated as in the Appendix. Bubbles were generated by (5) with parameters set as in the text and with $\pi = 0.85$. Prices were generated by $P_t = F_t + B_t$. There were 200 simulations of sample size 100 each. Test statistics are as in Table 2; cointegration tests are based on a regression of P_t on d_t and a constant.

APPENDIX

A second set of dividend simulations was constructed, based on the assumption that

$$\ln(d_t) = \mu + \ln(d_{t-1}) + \varepsilon_t$$

where ε_t is $\mathcal{N}(0, \sigma^2)$ white noise. The parameters $\mu = 0.013$ and $\sigma^2 = 0.016$ were again obtained by West (1988 p. 53) for the Standard and Poor 500 sample. For this case,

$$F_t = \frac{1+g}{r-g} d_t$$

where $1 + g = \exp(\mu + \sigma^2/2)$. The bubbles were scaled by a multiplicative factor of 250 so that the sample mean of $\text{Var}(\Delta B_t)$ would again be approximately three times the sample mean of $\text{Var}(\Delta F_t)$.

The results of the unit-root and cointegration tests are given in Table A1. For this table, unit-root tests were based on $\ln(P_t)$ and $\ln(d_t)$. (Cointegration continued to be tested by a regression of P_t on d_t and a constant.) The results are broadly similar to those in Table 2. The main difference is that P_t is now only slightly more likely to be found trend-stationary than d_t.

Poterba and Lawrence H. Summers (1988), who provide evidence of transitory components in stock prices, argue (inter alia) for the construction and testing of specific theories of "noise trading." On the other hand, periodically collapsing bubbles may be able to generate such apparent transitory components in stock prices.

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