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CAN A WELL-FITTED EQUILIBRIUM ASSET-PRICING MODEL PRODUCE MEAN REVERSION?

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SUMMARY

In recent papers, Cecchetti *et al.* (1990) and Kandel and Stambaugh (1990) showed that negative serial correlation in long horizon returns was consistent with an equilibrium model of asset pricing. In this paper, we show that their results rely on misspecified Markov switching models for the endowment process. Once the proper Markov specification is chosen for the endowment process, the model does not produce mean reversion of the magnitude detected in the data. Furthermore, the small amount of mean reversion produced by the model is due only to small sample bias. We also show that this model is unable to predict negative excess returns, contrary to empirical evidence.

1. INTRODUCTION

The negative serial correlation detected in long horizon stock returns by Poterba and Summers (1988) and Fama and French (1988) could be interpreted as evidence in favour of the inefficiency of financial markets. According to this interpretation, stock prices take irrational long swings away from their fundamental value. However, Fama and French (1988) also propose an efficient market or rational pricing explanation based on equilibrium expected returns.

To support the efficient market explanation, one therefore needs an equilibrium model that can produce mean reverting expected returns. This is precisely the task undertaken with apparent success by Cecchetti *et al.* (1990) (hereafter CLM). In a Lucas exchange economy with an infinite amount of identical agents, they specify for the endowment process a Markov switching (MS) model (Hamilton, 1989) characterized by two states, one of low growth and one of normal growth, and estimate by maximum likelihood[1] the parameters of this model for three historical series on real consumption, GNP, and dividend growth rates.[2]

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[1] Markov switching models have often been used in asset pricing models because they allow closed-form solutions for asset returns. Using Tauchen's (1985) methodology, these discrete state space models have been frequently calibrated to match a predetermined number of moments of the endowment series. Although one can always use a large number of states to reproduce a few moments of the series, the decision about which moments to select is arbitrary. By fitting a Markov process to the series by maximum likelihood, we can test which model is the best, and we can possibly capture more moments in a parsimonious way.

[2] The justification for using these three series comes from the fact that the Lucas model does not provide a way to select any particular series among the three since it imposes an equality between consumption, dividends, and output.

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In an exchange economy asset-pricing model with levered equity, Kandel and Stambaugh (1990) specify for the endowment process a four-state MS model with two means and two variances. Once calibrated to the consumption growth rate, their model reproduces the unconditional moments of asset returns[3] as well as the U-shaped pattern of first-order autocorrelations.

In this paper, we show that if instead of imposing one particular MS model, one specifies a larger class of MS models and lets each historical series of endowment decide on the best model according to various testing procedures, the chosen specification is a two-state MS model with one mean and two variances. More specifically, we reject the CLM specification (two means and one variance) against this one-mean and two-variance specification and cannot reject the latter against the four-state specification of Kandel and Stambaugh (1990). The amount of mean reversion generated by the specification selected for the endowment process is substantially smaller than is found in the data, and the small sample bias stands as its only explanation.[4] The model is also unable to generate the negative serial correlation in excess returns which is found in the data. Moreover, it produces conditional expected returns that are always positive. This is not in accordance with the evidence on the statistical significance of predictions of negative excess returns (Pesaran and Potter, 1993).

The plan of the paper is as follows. In Section 2 we present the equilibrium asset pricing model and derive closed-form solutions for the returns. In Section 3 we estimate by maximum likelihood various MS models for the endowment process and select the best specification for each of the three series of consumption, GNP, and dividends. We then use the parameter estimates and the return formulae derived in Section 2 to generate the empirical distributions of the variance ratios at various lags for several specifications of the endowment process, and compare and interpret the respective implications of each specification for mean reversion. In Section 4, we examine the properties of the expected excess return function in this class of models in view of the empirical evidence regarding excess returns. Section 5 concludes.

2. THE MODEL

Our assumptions, apart from those related to the specification of the exogenous endowment process, are based on the standard Lucas (1978) model. In an exchange economy, an infinitely lived representative agent maximizes her intertemporal von Neumann–Morgenstern utility function over her lifetime, subject to her budget constraint, and receives each period an endowment of a non-storable good. There are a fixed number of assets that produce units of the same good. Assuming additive time separability, and constant discounting of a power utility function with constant relative risk aversion γ,[5] we obtain from the first-order condition

[3] The success of their exercise, especially for replicating the equity premium, relies on using a value of 55 for the coefficient of relative risk aversion.

[4] Our justification for using an exchange model where the consumption, dividends, and output are equal in equilibrium is that the contending papers for explaining negative autocorrelation in returns (CLM, 1990; Kandel and Stambaugh, 1990) are based on such a model. The main point of this paper is to show that the specifications chosen by both sets of authors are not the best in the class of MS models. It is not to say that a more 'realistic' model where equilibrium consumption and dividends are not equal as in Abel (1992), Bonomo and Garcia (1991b), CLM (1991) is not better to study this particular issue. Moreover, we carry the exercise for the three series of consumption, dividends, and GNP to show that the specification error is not limited to one particular series.

[5] We limit our investigation to an exchange model with a power utility function since it is the most commonly used model for the type of exercise we carry out. Our main interest is to estimate the best specification for the endowment process and to establish whether it generates the negative autocorrelation of returns observed in the data. Of course, one could investigate the implications of other forms of utility functions in the setting of an exchange economy, or an economy with production (see Rouwenhorst, 1988).

for an interior maximum the following formula for the equity price at time t:

$$P_t^e = \beta E_t\left[\left(\frac{D_{t+1}}{D_t}\right)^{-\gamma}(P_{t+1}^e + D_{t+1})\right] \tag{1}$$

where E_t denotes expectation conditional on information available at time t.

Iterating the above equation and rearranging gives:

$$P_t^e = D_t \sum_{j=1}^{\infty} \beta^j E_t\left(\frac{D_{t+j}}{D_t}\right)^{1-\gamma} \tag{2}$$

We postulate that the growth rate of the endowment process evolves according to a K-state MS model given by:

$$d_t - d_{t-1} = \alpha_0 + \alpha_1 S_{1,t-1} + \cdots + \alpha_{K-1} S_{K-1,t-1} + (\omega_0 + \omega_1 S_{1,t-1} + \cdots + \omega_{K-1} S_{K-1,t-1})\varepsilon_t \tag{3}$$

where $S_{i,t}$ is a function of the state of the economy, S_t, taking value 1 whenever $S_t = i$ and 0 otherwise; d_t is $\ln D_t$; ε_t is an $N(0, 1)$ error term, independent of S_t. The variable $\{S_t\}$ follows a Markov process with transition probability matrix π, whose representative element is $p_{ij} = \Pr[S_t = j \mid S_{t-1} = i)$ with $i, j = 0, \ldots, K-1$. Making $d_t - d_{t-1}$ a function of S_{t-1}, as in CLM, is equivalent to providing the agent with the foreknowledge of the state of the economy one period ahead, that is, S_{t+1} is known at time t.[6]

From equation (2) and the Markov structure of the endowment process growth rate, it can be seen that the price–dividend ratio is a function only of the present state of the economy, represented here as $\rho(S_t)$. The specific form of this function obviously depends on the specification and parameter values of the endowment process.[7] Then:

$$P_t^e = D_t \rho(S_t) \tag{4}$$

Defining the one-period equity return R_{t+1}^e as

$$R_{t+1}^e = \frac{P_{t+1}^e + D_{t+1}}{P_t^e} \tag{5}$$

and using equations (3) and (4), we arrive at the following return formula:

$$R_{t+1}^e = \frac{\rho(S_{t+1}) + 1}{\rho(S_t)} \exp(\alpha_0 + \alpha_1 S_{1,t} + \cdots + \alpha_{K-1} S_{K-1,t} + (\omega_0 + \omega_1 S_{1,t} + \cdots + \omega_{K-1} S_{K-1,t})\varepsilon_{t+1}) \tag{6}$$

3. CHOICE OF THE BEST ENDOWMENT PROCESS AND ITS IMPLICATIONS

3.1 Selection of the Best MS Model for the Endowment Process

Since we specify a general MS model with K states for the endowment process, the first step is to test whether the one-state model, i.e. the random walk model with drift, is rejected against

[6] A more natural informational assumption would be to restrict the agent's knowledge to the past states of the economy up to the state she is facing, that is, to assume that S_t (or equivalently $S_{1,t}, S_{2,t}, \ldots, S_{K-1,t}$) is known at time t but not S_{t+1}. The implications for mean reversion of this alternative formulation have been explored, but for space considerations, the results are not reported here (see Bonomo and Garcia, 1991a).

[7] We show in Bonomo and Garcia (1991a) that the price–dividend ratio is given by: $\rho(S_t) = \beta_{IK,t} M(I - \beta A)^{-1}\iota$, where $I_{K,t}$ is a k-dimension row vector with 1 in the column corresponding to the state at time t and zeros in the other columns, $A = \pi M$ where $M = \operatorname{diag}[\exp(\mu_0), \exp(\mu_0 + \mu_1), \ldots, \exp(\mu_0 + \mu_{k-1})]$, and ι is a k-dimension column vector of ones. The price parameters μ depend on the parameters of both the endowment process and the utility function:

$$\mu_0 = (1-\gamma)\alpha_0 + \frac{(1-\gamma)^2}{2}\omega_0^2; \qquad \mu_j = (1-\gamma)\alpha_j + \frac{(1-\gamma)^2}{2}(2\omega_0\omega_j + \omega_j^2) \text{ for } j = 1 \ldots K-1$$

the most general two-state model. If it is the case, we proceed further by taking the two-state model as the null and testing it against the most general three-state model. We continue in this fashion until the k-state model cannot be rejected against the $k+1$-state model. Once the optimal number of states, say k, is found, we determine whether it is possible to reject a more parsimonious k-state Markov model (with less than k means or variances) against the most general k-state model (with k means and k variances).

In the first part of the testing strategy aimed at determining the optimal number of states, the usual tests (likelihood ratio, Wald and Lagrange multiplier) do not have the standard asymptotic distribution.[8] Garcia (1992) derives analytically the asymptotic null distribution of the likelihood ratio test for two-state MS models, based on Hansen's (1991) distributional theory for models where nuisance parameters are not identified under the null. The 1 per cent and 5 per cent critical values of this distribution, used below to test the null of a random walk against the alternative of the most general two-state model, are 17·38 and 14·11, respectively.

The upper panel of Table I presents the maximum likelihood estimates of the three possible two-state models for each series of consumption, dividend, and GNP growth rates. Taking consumption as an example, the first model (C1) is the two-state MS model with two means ($\alpha_0, \alpha_0+\alpha_1$) and one variance ($\omega_0^2$). The second model (C2) refers to the one-mean (α_0) and two-variance ($\omega_0^2, (\omega_0+\omega_1)^2$) model. Finally, C3 is the two-state encompassing model with two means ($\alpha_0, \alpha_0+\alpha_1$) and two variances ($\omega_0^2, (\omega_0+\omega_1)^2$). The LR12 line gives the value of the likelihood ratio for each model with respect to the linear model. We look therefore first at the LR value for the C3, D3 and G3 models and we compare these values with the critical values given above. For the three models, the linear model is rejected at less than the 1 per cent level.

Next, we test the two-state model against the three-state model. The values of the likelihood ratio statistic for the two-state model 3 against the most general three-state model (three means and three variances) are shown on the LR23 line. They are, respectively, 3·08, 7·74, and 5·58 for the consumption, dividend, and GNP growth series. Although these values seem small relative to the 1 per cent and 5 per cent critical values given above for the heteroscedastic model, it is not assured that values of the same order will carry over to the likelihood ratio test of a null of a two-state MS model against the alternative of a three-state MS model. We generate, therefore, the empirical distribution of this statistic[9] and, based on this distribution, we are unable to reject the two-state model at p-values of 50 per cent and higher.

As mentioned in the introduction, Kandel and Stambaugh (1990) used another specification for the endowment process to replicate, in the same representative agent model framework, both the equity premium and the negative serial correlation of stock returns. Their model for the endowment is also a Markov process with four states but only two means and two variances. The difference with the previous two-state MS model is that all combinations of the two means and two variances can be accommodated. This specification is also less general than our general specification for the four-state model, because it imposes that for any state with

[8] The problem comes from two sources: under the null hypothesis, some parameters are not identified and the scores are identically zero. To clarify these two irregularities, let us take the case where one wants to test the null hypothesis of a linear model against the alternative hypothesis of a two-state homoscedastic Markov switching model. The null hypothesis can be expressed as either $\{\alpha_1=0\}$ or $\{p=0\}$ or $\{p=1\}$. To see the problem of unidentified parameters under the null, note that if $\{\alpha_1=0\}$, the transition probability parameter p is unidentified since any value between 0 and 1 will leave the likelihood function unchanged. As for the problem of identically zero scores, note that under $\{p=0\}$ or $\{p=1\}$, the scores with respect to p, q and α_1 will be identically zero under the null and the asymptotic formation matrix will be singular (see Garcia, 1992, for more details).

[9] To obtain the empirical distribution, we generated the series using the values of the parameters corresponding to the dividend model 3 and computed the likelihood ratio based on the most general two- and three-state models. Note that the distribution of the LR statistic is invariant to the values of the means and variances, as shown in Garcia (1992).

Table I. Maximum likelihood estimates of two-state modes and test results

	Consumption			Dividends			GNP		
	C1	C2	C3	D1	D2	D3	G1	G2	G3
α_0	0·0225	0·0200	0·0209	0·0171	−0·00008	0·0010	0·0240	0·0179	−0·0180
	(6·11)	(7·22)	(7·15)	(1·53)	−(0·01)	(0·14)	(5·78)	(5·58)	(5·54)
α_1	−0·0895	—	−0·0051	−0·370	—	−0·0093	−0·1756	—	−0·0013
	(−4·63)		(−0·75)	(−6·56)		(−0·43)	(−7·19)		(−0·05)
ω_0	0·0316	0·0179	0·0181	0·1050	0·0429	0·0428	0·0431	0·0310	0·0310
	(11·88)	(7·29)	(7·88)	(13·69)	(7·69)	(7·83)	(15·05)	(11·70)	(12·04)
ω_1	—	0·0290	0·0289		0·1400	0·1404		0·0781	0·0781
		(5·81)	(6·19)	—	(6·87)	(7·16)	—	(4·29)	(4·22)
p_{11}	0·5115	0·9891	0·9890	0·1753	0·8430	0·8367	0·5089	0·9305	0·9305
	(1·88)	(70·99)	(84·22)	(0·82)	(8·40)	(8·52)	(2·32)	(15·03)	(15·75)
p_{00}	0·9748	0·9875	0·9876	0·9508	0·8289	0·8238	0·9821	0·9858	0·9858
	(43·25)	(59·12)	(65·88)	(40·52)	(9·66)	(9·53)	(93·93)	(66·08)	(77·31)
L	273·33	282·48	282·72	179·67	191·74	191·81	294·41	311·366	311·367
LR12	30·68	48·98	49·46	17·38	41·52	41·76	39·86	73·78	73·78
χ^2 *p*-value	0·002	0·62	—	0·000	0·80	—	0·000	0·97	—
LR23		3·08			7·74			5·58	

Notes:
(1) Asymptotic *t*-ratios in parentheses.
(2) The values in C1, D1, G1 on the χ^2 *p*-value line give the probability values of the two-mean two-variance model under the null of the two-mean, one-variance model for consumption, dividend, and GNP respectively.
(3) The values in C2, D2, G2 on the χ^2 *p*-value line give the probability values of the two-mean two-variance model under the null of the one-mean, two-variance model for consumption, dividend, and GNP respectively.
(4) LR12 refers to the likelihood ratio statistic for the random walk model as the null and the corresponding two-state model as the alternative.
(5) LR23 refers to the likelihood ratio statistic for the one-mean, two-variance two-state model as the null and the most general three-state model as the alternative.

a specific mean and variance, there exists another state which has either the same mean or the same variance.[10] We estimate this four-state model by maximum likelihood for consumption, dividend, and GNP growth series[11] and report the results in Table II. The values of the likelihood ratio statistic for the null of the two-state model (1M2V) against this constrained four-state model (LR24) appear at the bottom of this table. They are 5·78, 9·86, and 1·92[12] for the consumption, dividend, and GNP series, respectively. Given that the highest value obtained for the dividend series (9·86) would translate into a *p*-value of 36 per cent with the empirical distribution for LR23, we can safely conclude that we are unable to reject the two-state model. The remaining task is to choose the most parsimonious specification among the three two-state models.

As indicated by the χ^2 *p*-values in the C1, D1, and G1 columns, the constraint $\omega_1 = 0$ is rejected for the three series. The results for testing the constraint $\alpha_1 = 0$ are presented in the C2, D2, and G2 columns. The χ^2 *p*-value line confirms that C2, D2, G2 cannot be rejected

[10] Observe that although this constrained four-state specification encompasses our general two-state specification, it does not encompass our general three-state specification.
[11] Note that Kandel and Stambaugh (1990) do not use a maximum likelihood method and their period of estimation is different from ours (1929–1983, instead of 1871–1985).
[12] The transition probability estimates of the four-state model for the GNP series indicate that the fourth state is practically never reached. This explains why the likelihood value is less with the four-state model than with the three-state model, since the four-state model has only two means and two variances instead of three means and three variances for the three-state model.

Table II. Maximum likelihood estimates for the four-state models

$$y_t = \alpha_0 + \alpha_1(S_{2t} + S_{3t}) + (\omega_0 + \omega_1[S_{1t} + S_{2t}])e_t$$
$$S_{it} = 1 \text{ if } S_t = i, \text{ 0 otherwise}$$

	Consumption	Dividends	GNP
α_0	0·0054	−0·0048	−0·0397
	(0·0028)	(0·0072)	(0·0247)
α_1	0·0302	0·0217	0·0587
	(0·0034)	(0·0152)	(0·0247)
ω_0	0·0141	0·0445	0·0312
	(0·0016)	(0·0051)	(0·0028)
ω_1	0·0369	0·1479	0·0769
	(0·0051)	(0·0200)	(0·0191)
p_{01}	0·0000	0·0131	0·0160
	(0·0000)	(0·0163)	(0·0201)
p_{02}	0·0000	0·0000	0·0250
	(0·0000)	(0·0000)	(0·0305)
p_{03}	0·4162	0·0000	0·9586
	(0·1263)	(0·0000)	(0·0425)
p_{11}	0·8708	0·7111	0·6501
	(0·0030)	(0·0379)	(0·1929)
p_{12}	0·0996	0·0119	0·2366
	(0·0006)	(0·0035)	(0·2221)
p_{13}	0·02963	0·2769	0·0164
	(0·0024)	(0·0355)	(0·0486)
p_{21}	0·3918	0·0000	0·0320
	(0·1020)	(0·0000)	(0·0717)
p_{22}	0·6076	0·3236	0·9085
	(0·0975)	(0·6999)	(0·0927)
p_{23}	0·0005	0·0000	0·0001
	(0·0050)	(0·0000)	(0·0006)
p_{31}	0·0000	0·5153	0·0105
	(0·0000)	(0·1476)	(0·0000)
p_{32}	0·0352	0·0000	0·0002
	(0·0365)	(0·0002)	(0·0000)
p_{33}	0·6255	0·4846	0·9894
	(0·1138)	(0·1479)	(0·0000)
L	285·67	196·69	312·62
LR24	5·78	9·86	1·92

Notes:
(1) Asymptotic standard errors in parentheses.
(2) LR24 refers to the likelihood ratio statistic for the one-mean two-variance two-state model as the null and the four-state model with two means and two variances as the alternative.

in favour of C3, D3, G3. For reasons of parsimony, we retain C2, D2 and G2[13] as our model for the endowment process and use the corresponding parameter estimates of this model to generate stock returns series. In the next section, we investigate if this model produces negative serial correlation as do the models of CLM (1990) and Kandel and Stambaugh (1990).

[13] An additional reason for retaining specification 2 is that the estimate of α_1 in model 3 is numerically very close to zero. As a consequence, the implications for mean reversion of models 2 and 3 are very similar notwithstanding the significance of α_1.

3.2. Implications for Negative Autocorrelation in Returns

We use as measures of autocorrelation the variance ratios at various lags ($V(m) = \mathrm{Var}(R_{t,t+m})/(m\ \mathrm{Var}(R_{t,t+1}))$), for m equal 2 to 10.[14] To assess the implications of the selected specification, we generate the corresponding variance ratio distributions[15] to infer the p-value of the variance ratios for the actual returns.

Table III reports the results of the Monte Carlo experiment for the consumption variance ratios.[16] We report the results obtained with the selected specification along with the CLM specification (two means and one variance—2M1V) and the Kandel and Stambaugh specification (two means and two variances with four states—4S2M2V). With a concave utility function, both the 2M1V and 4S2M2V specifications produce negative serial correlation in the magnitude observed in the actual data.[17] In contrast, the negative serial correlation produced by the selected 1M2V specification is much less pronounced and of the same order than the one produced by the same specification with a linear utility function ($\gamma = 0$). The large sample results in the bottom part of the table reinforce the conclusions one can infer from the top part: both the 2M1V and 2M2V4S specifications combined with a concave utility function generate large sample variance ratios substantially smaller than one, but not the 1M2V. In other words, the variance ratio values smaller than unity obtained from the 1M2V model in the upper part of the table are due to small-sample bias.[18] These results suggest that finding the best model for the endowment process in the class of Markov switching models that have been used in the literature is indeed important, since it changes the implications for the behaviour of stock returns in terms of negative serial correlation.

3.3. Interpretation of the results

To investigate why the 2M1V and 4S2M2V models generate negative autocorrelation in returns while the best specification, 1M2V, does not, we focus our analysis on the 2M1V specification and contrast it with the 1M2V model. We then argue that the features identified as sources of negative autocorrelation of returns in the 2M1V model are also present in 4S2M2V.

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[14] Another measure is given by the multiperiod regression coefficients of returns. The results for this measure are reported only in Bonomo and Garcia (1991a) because the two statistics give similar results. This is explained by the fact that the population regression coefficients for k periods returns ($b(k)$) and variance ratios (V) are linked by the following formula: $b(k) = V(2k)/V(k) - 1$.

[15] The Monte Carlo distributions for the variance ratio statistics are generated in the following way. Given a randomly drawn vector of $N(0, 1)$ errors ε_{t+1} and a randomly drawn vector of S_t according to the transition probabilities estimated in Section 3.1, we generate series of returns according to formula (6) for R^e_{t+1}, with the estimates obtained in Section 3.1 for the α and ω parameters of the endowment models for the respective series. We replicate the procedure 10,000 times and compute each time the variance ratios at various lags to obtain the corresponding distributions.

[16] Since the results obtained with the dividend and GNP series are basically the same, we report and discuss only the consumption results for space considerations. See Bonomo and Garcia (1991a) for full details.

[17] For comparison purposes, we choose 1·7 for the parameter γ, which is the value chosen by CLM for the consumption series. It is interesting to note that a value of γ of 2·3 is sufficient to reproduce the negative serial correlation in real returns with the four state specification of Kandel and Stambaugh (1990). For high values of γ in the order of 55 used by Kandel and Stambaugh (1990), the price function will in fact not exist.

[18] It should be emphasized that the conclusion holds for all three series, which are different in terms of parameter estimates for the means, variances, and transition probabilities. This fact is of importance since, given that there is only one switch from the high variance state to the low variance state for the consumption growth series (in 1950), one might think that the high variance state in the selected model for consumption is a figment of the data (see Romer 1986). For the GNP growth series, the low variance state is present from the beginning of the sample to the 1930's. The dividend series shows many more switches between states than the two other series, yet no value of the relative risk aversion parameter can be found that produces mean reversion with positive prices. If we increase γ, the price function no longer exists. The same pattern is observed for the consumption and the GNP series.

Table III. Median of distribution of variance ratios of returns for MS models estimated for consumption

k	Actual	$y=0$ 2M1V	$y=0$ 1M2V	$y=0$ 4S2M2V	$y=1\cdot7$ 1·7 2M1V	$y=1\cdot7$ 1·7 2M1V	$y=2\cdot3$ 4S2M2V
				$T=116$			
2	1·0137	0·9886	0·9897	0·9972	0·9528	0·9900	0·8995
		(0·57)	(0·59)	(0·56)	(0·66)	(0·59)	(0·87)
3	0·8664	0·9622	0·9736	0·9852	0·9023	0·9734	0·8423
		(0·31)	(0·24)	(0·23)	(0·43)	(0·24)	(0·56)
4	0·8351	0·9346	0·9559	0·9693	0·8563	0·9558	0·8095
		(0·33)	(0·26)	(0·24)	0·47)	(0·26)	(0·56)
5	0·7978	0·9090	0·9415	0·9546	0·8209	0·9418	0·7864
		(0·33)	(0·25)	(0·24)	(0·47)	(0·25)	(0·52)
6	0·7459	0·8851	0·9256	0·9422	0·7928	0·9253	0·7626
		(0·29)	(0·22)	(0·21)	(0·44)	(0·22)	(0·47)
7	0·7259	0·8679	0·9064	0·9289	0·7652	0·9074	0·7446
		(0·31)	(0·23)	(0·22)	(0·44)	(0·23)	(0·47)
8	0·7363	0·8527	0·8923	0·9141	0·7441	0·8915	0·7303
		(0·35)	(0·28)	(0·26)	(0·49)	(0·28)	(0·51)
9	0·7102	0·8342	0·8756	0·8974	0·7239	0·8750	0·7143
		(0·34)	(0·27)	(0·26)	(0·48)	(0·27)	(0·49)
10	0·7242	0·8203	0·8596	0·8798	0·7084	0·8584	0·7004
		(0·38)	(0·32)	(0·31)	(0·52)	(0·32)	(0·53)
				$T=1160$			
2	1·0137	0·9985	0·9988	1·7561	0·9582	0·9987	0·9128
3	0·8664	0·9961	0·9970	0·9980	0·9262	0·9973	0·8710
4	0·8351	0·9928	0·9957	0·9962	0·9041	0·9962	0·8499
5	0·7978	0·9902	0·9932	0·9947	0·8870	0·9934	0·8382
6	0·7459	0·9874	0·9920	0·9936	0·8736	0·9924	0·8315
7	0·7259	0·9849	0·9903	0·9915	0·8630	0·9904	0·8277
8	0·7363	0·9826	0·9880	0·9902	0·8546	0·9887	0·8262
9	0·7102	0·9804	0·9862	0·9884	0·8477	0·9871	0·8253
10	0·7242	0·9777	0·9845	0·9871	0·8419	0·9852	0·8255

Notes:

(1) The figures between parentheses give the percentage of the Monte Carlo distribution below the actual value.
(2) 2M1V stands for the two-state Markov switching model with two means and one variance.
(3) 1M2V stands for the two-state Markov switching model with two variances and one mean.
(4) 4S2M2V stands for the four-state Markov switching model with two means and two variances.
(5) For the length of the series, we choose 116 observations (the number of observations for the actual returns) to generate the small sample distributions, and 1160 observations for the large sample ones. For the small sample results, we report the medians of the distributions as well as the percentage of the distribution below the actuals. This percentage is to be interpreted as a p-value for the hypothesis that the actuals are produced by the model. The large sample statistics are produced to evaluate whether the model is capable of generating some negative autocorrelation even in large samples. In other words, we want to assess the magnitude of the small-sample bias present in the small-sample results.

To understand how the characteristics of the Markov switching model for the endowment process affect the theoretical autocorrelation of returns, one has to look at equation (6) which defines the equilibrium returns. The return is seen to depend on the Markov state in two adjacent periods and on the realization of the i.i.d. term ε in the second period. In the case of two states, if both states are persistent (high p_{00} and p_{11}), the state observed in period t is likely to remain unchanged and the successive returns are likely to differ only through ε_{t+1}. This tends to generate positive autocorrelation in long-horizon returns. Conversely, when both states show little persistence (low p_{00} and p_{11}), the observed state is likely to change, producing negative autocorrelation in returns. The fact that the estimated p_{00} and p_{11} are both very high for the 1M2V model is at least part of the explanation for the slightly positive value found for the large sample autocorrelation of returns.

When one state is persistent and the other is not, as in the 2M1V specification, the pattern of autocorrelation depends on the magnitude of the change in the price dividend ratio when changing states. For the specific values chosen or estimated for the parameters, $\rho(0)$ is less than $\rho(1)$. In the explanation that follows, we neglect the effect of the i.i.d. term ε.

Suppose the economy has been in state 0 for some periods. When there is a change from state 0 to state 1, equation (6) will read as follows:

$$R^e_{t+1} = \frac{\rho(1)+1}{\rho(0)} \exp(\alpha_0) \tag{7}$$

Since $\rho(1)$ is greater than $\rho(0)$, the part of the equation which multiplies the exponential term will increase. For the next period, if we stay in state 1 the return tends to be lower than in state 0 (since the negative coefficient α_1 appears in the exponential term and $\rho(1)$ is greater than $\rho(0)$), but there is still a good chance of returning to state 0, since state 1 is not very persistent ($p_{11} = 0{\cdot}5115$). In this case, the return equation will be given by:

$$R^e_{t+1} = \frac{\rho(0)+1}{\rho(1)} \exp(\alpha_0 + \alpha_1) \tag{8}$$

The return will now be very low because $\rho(1)$ appears in the denominator, $\rho(0)$ in the numerator, and α_1 in the exponential term. This sequence of positive and negative spikes in returns is what generates the negative autocorrelation pattern associated with the 2M1V specification. For the 1M2V specification, the same effect will not appear since p_{00} and p_{11} are high and α_1 is 0.

As for the negative autocorrelation in returns found with the 4S2M2V specification, observe that the four states can be divided into two groups of two states: one of low variance composed of states 0 and 3, and one of high variance composed of states 1 and 2. These two groups are very persistent, in the sense that if the economy is in one state of a group, it will be in one of the states of the same group next period with very high probability. However, within each group, there is a considerable probability of changing states. Since changing states within a group denotes changes in means, the dynamics produced inside a group is similar to the one generated by the 2M1V model.[19]

4. IMPLICATIONS OF THE MS CLASS OF EQUILIBRIUM MODELS FOR EXCESS RETURNS

In this section we want to investigate if the previous class of equilibrium asset pricing models, with a MS endowment process, are able to reproduce empirical facts related to excess return dynamics. First, although there is as much evidence of negative serial correlation in excess returns as in real returns in actual data, none of the previous endowment models are able to generate it.[20] Second, Pesaran and Potter (1993) test the non-negativity property of the

[19] In principle, the mean reversion effect in equilibrium asset-pricing models based on a regime-switching endowment can be produced by two kinds of dynamics. A high-variance state of some persistence increases risk in returns and implies that the required expected return is higher than in a low-variance state. Switching from a high- to a low-variance state could produce mean reversion in asset prices. The main reason why the two-variance and one-mean estimated model cannot produce mean reversion is that the states are too persistent. The other kind of dynamics is generated by the intertemporal substitution of the agent when faced with alternating high and low growth rates in her endowment. The fact that the mean growth rates in the estimated model are very far apart explains the success of the CLM model in producing mean reversion.

[20] For details, see Bonomo and Garcia (1991a).

expected excess returns and find statistically significant evidence against it. Since Mehra and Prescott (1985), we know that simple Lucas tree models with power utility are unable to match the unconditional mean of excess returns. We show below[21] that such models are also unable to produce predictions of negative excess returns, thereby adding evidence against such simple equilibrium asset pricing models.

Following Abel (1992), the theoretical expected excess return function is equal to:

$$E_t[R^e_{t+1} - R^f_t] = \frac{E_t[(D_{t+1}/D_t)}{\beta E_t\{((D_{t+1}/D_t))^{1-\gamma}\}} - \frac{1}{\beta}\left\{E_t\left[\left(\frac{D_{t+1}}{D_t}\right)^{-\gamma}\right]\right\}^{-1}$$

where R^f_t denotes the return on the risk-free asset.

Given equation (3), we obtain after some manipulations:

$$E_t[R^e_{t+1} - R^f_t] = \frac{1}{\beta}\exp[\gamma(\alpha_0 + \alpha_1 S_{1,t} + \cdots + \alpha_{k-1}S_{k-1,t}) - \tfrac{1}{2}\gamma^2(\omega_0 + \omega_1 S_{1,t} + \cdots + \omega_{k-1}S_{k-1,t})^2]$$
$$\{\exp[\gamma(\omega_0 + \omega_1 S_{1,t} + \cdots + \omega_{k-1}S_{k-1,t})^2] - 1\}$$

This expression is always non-negative since γ is positive.[22] All the equilibrium asset-pricing models based on (3) and a power utility function are therefore unable to predict negative excess returns, independently of the value of the preference parameter. Pesaran and Potter (1993) show that for an intertemporal asset pricing model to predict negative excess returns, it is necessary for the market portfolio to serve as a hedge (for example, against inflation).

5. CONCLUSION

In this paper we showed that in the class of Markov models for the endowment process, mean reversion of the magnitude detected in the data could be obtained only by a misspecification of the process. In particular, the two-state two-mean and one-variance model was selected by CLM on the grounds of being on the tradeoff frontier between a model that completely matches the data and one that is tractable. We showed that when the best tractable specification is selected in the class of Markov switching models, the amount of negative autocorrelation generated is substantially smaller than is found in the data. Moreover, the small sample bias stands as its only explanation. We showed also that such simple Lucas models are unable to reproduce facts related to excess returns, such as predictions of negative excess returns.

APPENDIX: DATA SOURCES (SAME AS IN CLM, 1990)

Nominal dividends and stock prices: Campbell and Shiller (1987) data set.

Price index: CPI	1871–1926:	Wilson and Jones (1987)
	1930–1985:	Ibbotson and Sinquefield (1988)
Consumption:	1889–1928:	Kendrick Consumption series (Balke and Gordon, 1986)
	1929–1985:	NIPD Accounts
Population:	1869–1938:	Historical Statistics of the United States (Series A7 for 1869–1928, Series A6 1929–1938)
	1938–1985:	Economic Report of the President (1989), Table B-31.

[21] We thank the editor for conjecturing this shortcoming of the model.

[22] It should be noted that the information assumption implicit in equation (3) and discussed in footnote 7 is crucial to obtain this non-negativity result.

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