Journal of Forecasting
J. Forecast. **23**, 237–250 (2004)
Published online in Wiley InterScience (www.interscience.wiley.com). **DOI**: 10.1002/for.909

Intrinsic Bubbles Revisited: Evidence from Nonlinear Cointegration and Forecasting

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ABSTRACT

This paper offers strong further empirical evidence to support the intrinsic bubble model of stock prices, developed by Froot and Obstfeld (*American Economic Review*, 1991), in two ways. First, our results suggest that there is a long-run nonlinear relationship between stock prices and dividends for the US stock market during the period 1871–1996. Second, we find that the out-of-sample forecasting performance of the intrinsic bubbles model is significantly better than the performance of two alternatives, namely the random walk and the rational bubbles model. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS intrinsic bubbles; nonlinear cointegration; forecasting; Kalman filter; random walk

INTRODUCTION

It has been well documented that there exist persistent deviations of the stock price from the present value (PV) model. Since the rejection of the simple PV model by LeRoy and Porter (1981) and Shiller (1981), there has been a substantial body of research in finding alternative models to explain the failure of the PV model. One approach is to allow a variable discount rate (see Campbell and Shiller, 1988a,b; West, 1987, 1988). There is little positive evidence, however, that a variable discount rate alone can explain these failures (see, for example, Flood and Hodrick, 1986; Campbell and Shiller, 1988a). Another approach is to incorporate speculative bubbles into the PV model (Blanchard and Watson, 1982; Blanchard, 1979; Flood and Garber, 1980). Rational bubbles are generated by extraneous events or rumours and driven by self-fulfilling expectations which have nothing to do with the fundamentals. The empirical results on this model are inconclusive. West (1987) and Wu (1997) rejected the null hypothesis of no bubbles, whilst Diba and Grossman (1988) and Barsky and DeLong (1993) reached the opposite conclusion.

Froot and Obstfeld (1991) introduced another type of bubbles, namely intrinsic bubbles. Unlike rational bubbles, intrinsic bubbles are driven by fundamentals alone in a *nonlinear* way, thereby entailing a nonlinear relationship between stock prices and dividends.[1] Thus, this approach provides

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[1] For a recent review of nonlinear relationships in the foreign exchange market, see Ma and Kanas (2000).

a potential alternative to explain the excessive volatility of stock prices. Empirically, Froot and Obstfeld (1991) find significant evidence to support their model based on linear and nonlinear least-square estimations for the US market.[2] However, they did not examine the *long-run* relationship between stock prices and dividends, and the out-of-sample forecasting performance of their model.

In this paper, we seek to extend Froot and Obstfeld's empirical findings in two ways. First, we examine whether there is a long-run nonlinear relationship between stock prices and dividends for the US stock market. In other words, we explore whether the intrinsic bubbles-driven nonlinearity in the stock price–dividend relationship is relevant in the long run. A finding of a long-run nonlinear stock price–dividend relationship is interpreted as evidence that intrinsic bubbles are relevant in the long run and, hence, are important in explaining the long-run excessive volatility of stock prices. To test for a nonlinear long-run stock price–dividend relationship, we test for cointegration between stock prices and a *specific* nonlinear transformation of dividends which satisfies the theoretical solution to the intrinsic bubbles model. Second, we examine and evaluate the forecasting performance of the estimated intrinsic bubbles model in out-of-sample forecasting exercises against two competitors, namely the rational bubbles model and the random walk.

Extending the empirical findings of Froot and Obstfeld (1991), we present further strong empirical evidence to support the intrinsic bubbles hypothesis. Our findings suggest that there is a significant long-run nonlinear relationship between the US stock price index and dividends, which can be interpreted as evidence of the validity of the intrinsic bubbles model in the long run. Further, the forecasting performance of the intrinsic bubbles model is significantly superior to that of the random walk and the rational bubbles model, highlighting the relevance of this model to forecasting.[3]

The structure of the rest of the paper is as follows. The next section outlines the nonlinear relationship between stock prices and dividends entailed by the intrinsic bubbles model. The following section discusses the methodology used for testing for a nonlinear long-run relationship, and presents the results. Next we compare the out-of-sample forecasting performance of the intrinsic bubbles model with that of the random walk and the rational bubbles model. A final section provides a summary and concludes.

NONLINEARITY IN THE STOCK PRICE–DIVIDENDS RELATIONSHIP: THE INTRINSIC BUBBLES MODEL

The PV model describing the stock price is given by:

$$P_t = e^{-r} E_t (D_t + P_{t+1}) \tag{1}$$

[2] Most of the literature surveyed here is focused on the US market. However, see Salge (1997) for a comprehensive study on the German stock market.

[3] It is beyond the scope of this paper to explore the issue of policy implications in terms of monetary policy reaction to asset bubbles. Currently, there is a debate on the policy implications of asset bubbles. For example, both stock market crashes of 1927 and 1987 were apparently anticipated. Based on option markets data, Rappoport and White (1994) and Bates (1991) found evidence that the appearance of a rational bubble and the fear of its burst were anticipated in advance of the two events in 1927 and 1987, respectively. However, the reactions of the Fed were totally different in these two events. In 1928, the Fed tightened monetary policy in order to curb stock market speculation. Although that policy was able to control the rise in stock prices, it also might have pushed the economy into recession. When the bubble was actually burst, the economy went into the Great Depression. In contrast, the Fed did not contract monetary supply in an effort to deflate a bubble prior to October 1987. As a result, the economy was better positioned to withstand the effects of the crash. As Cogley (1999) argued, any deliberate attempt to puncture asset price bubbles may generate destabilizing effects due to the uncertainty of both the existence of a bubble and the exact timing of its burst.

where P_t is the real stock price at the beginning of period t, D_t is the real dividend paid out over period t, r is the constant, instantaneous real interest rate, and $E_t(\cdot)$ is the market's expectation conditional on information known at the start of period t. The equilibrium PV solution to equation (1), denoted by P_t^{PV}, is given by:

$$P_t^{\mathrm{PV}} = \sum_{s=t}^{\infty} e_t^{-r(s-t+1)} E_t(D_s) \tag{2}$$

Equation (2) assumes that the transversality condition holds, i.e.

$$\lim_{s\to\infty} e^{-rs} E_t(P_s) = 0 \tag{2$'$}$$

The rational bubbles solution to equation (1) can be written as follows:[4]

$$P_t = P_t^{\mathrm{PV}} + B_t \tag{3}$$

where:[5]

$$B_t = e^{-r} E_t(B_{t+1}) \tag{4}$$

As shown in equation (3), rational bubbles are exogenous to the fundamental determinants of stock prices.

Froot and Obstfeld (1991) consider intrinsic bubbles that are formed by a nonlinear function of dividends that satisfies equation (4). Froot and Obstfeld considered the nonlinear function given by equation (5):[6]

$$B(D_t) = cD_t^{\lambda} \tag{5}$$

where c and λ are constants. By summing the PV solution given by equation (1) and the intrinsic bubble in equation (5), the Froot and Obstfeld (1991) intrinsic bubbles model can be written as follows:

$$P_t = P_t^{\mathrm{PV}} + cD_t^{\lambda} \tag{6}$$

In equation (6), P_t^{PV} is the solution from the simple PV model which satisfies the transversality condition, and cD_t^{λ} is the explosive intrinsic bubbles which satisfy condition (4) but not the transversality condition (2′). Equation (1) can be regarded as the simple PV model augmented with an intrinsic bubble. Under the assumption that log dividends follow the geometric martingale:[7]

[4] The rational bubbles solution to (1) does not satisfy the transversality condition given in (2′).
[5] Equation (4) is similar to the bubbles equation derived in the Appendix of Flood and Garber (1980).
[6] Froot and Obstfeld (1991, p. 1192) show that equation (5) satisfies equation (4).
[7] In deriving the closed-form solution of the intrinsic bubbles model, Froot and Obstfeld (1991) assumed that annual dividends follow a geometric martingale with drift, and that the discount rate has been constant for over a century. Both assumptions are theoretical approximations to the real time series in order to get the insights of the fundamental structure of the PV model. If either of these assumptions is violated, then the model solution will be different. In such a case, one needs to employ some nonparametric, nonlinear procedures such as the ARMA-ARCH-Artificial Neural Network model (see

$$\log(D) = \mu + \log(D_{t-1}) + \xi_{t+1} \tag{7}$$

where μ is a constant and ξ_{t+1} is a normal random variable with conditional mean zero and variance σ^2. We have:

$$\kappa = I\Big/\left(e^r - e^{\mu+\sigma^2/2}\right) \tag{8}$$

and λ is the positive root of the quadratic equation:[8]

$$\lambda^2\,\sigma^2/2 + \lambda\mu - r = 0 \tag{9}$$

Based on the assumption given by equation (7), we can write the statistical model of the theoretical intrinsic bubbles model as follows:[9]

$$P_t = \kappa D_t + cD_t^\lambda + \varepsilon_t \tag{10}$$

Hence, as shown in equation (10), intrinsic bubbles entail that the stock price–dividend relationship is nonlinear. In the next section, we explore the validity of the intrinsic bubbles approach as a long-run equilibrium model.

TESTING FOR A LONG-RUN NONLINEAR STOCK PRICE–DIVIDEND RELATIONSHIP

We examine the long-run validity of the intrinsic bubbles model by focusing on the US stock market over the period from 1871 to 1996. The stock price index used in this study is the January values of the Standard and Poor's Composite annual stock price index, and dividends are annual averages of nominal data for the calendar year. Stock prices and dividends are deflated by the Producer Price Index (PPI). All data are obtained from Shiller (1989).[10,11] A plot of the stock price–dividend ratios over time is provided in Figure 1.

We first estimate the parameters of equation (10). Due to collinearity, equation (10) cannot be freely estimated without restricting some of its coefficients. In order to be consistent with the theoretical intrinsic bubbles model, we impose in equation (10) the value of κ implied from equation (8).[12] To examine the long-run validity of equation (10), we test for cointegration between the non-

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Donaldson and Kamstra, 1996) to discrminate the two rival models. We believe this is beyond the scope of this paper. However, we have provided some empirical evidence to justify these two crucial assumptions of the model in the next section (see footnote 15).

[8] Our equation (9) is derived from Froot and Obstfeld (1991, eq. (9), p. 1192). It shows that if λ satisfies (9), then our equation (5) satisfies the bubble definition (4) in our paper and can therefore be defined as an intrinsic bubble.

[9] This is equation (12) in Froot and Obstfeld (1991, p. 1198).

[10] Shiller's (1989) data ended at 1988. The updated series from 1989 to 1996 can be found on Robert Shiller's web page (http://www.econ.yale.edu/~shiller/data/chapt26.html). We used the January 1982 series of PPI.

[11] We did not align the dividends following Froot and Obstfeld (1991). Besides the two reasons they give to justify this (see their footnote 18, p. 1198), they also found that this assumption does not affect their results statistically. We followed Froot and Obstfeld's approach and repeated our estimations with aligned dividends. In line with Froot and Obstfeld, we did not find statistically different results.

[12] In the subsequent analysis, we also relaxed this restriction $\kappa = \bar{\kappa}$ and freely estimated κ. Our findings suggest that the freely estimated κ is not significantly different from $\bar{\kappa}$.

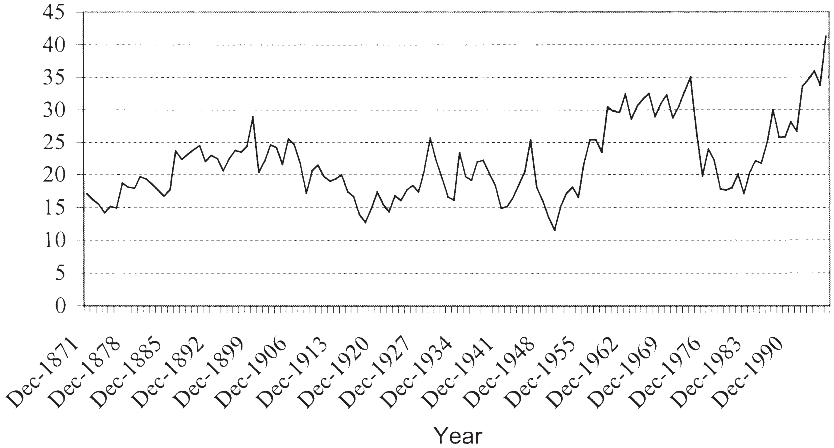

Figure 1. Price–dividend ratios (1871–1996)

linear transformation of the dividend series and the transformation of the stock price series implied by the theoretical intrinsic bubbles model. According to Granger and Hallman (1991), this procedure can be considered as nonlinear cointegration. In particular, the dividend series is nonlinearly transformed as indicated by the intrinsic bubble function given by equation (5) above:

$$D_t^* = D_t^{\bar{\lambda}} \tag{11}$$

where $\bar{\lambda}$ is the point estimate of λ from the data, and D_t^* is the nonlinearly transformed dividend series. Hence, as shown in equation (11), we impose the value of λ implied by the data.[13] We also transform the original stock price series on the basis of equation (10) as follows:

$$P_t^* = P_t - \bar{\kappa} D_t \tag{12}$$

where $\bar{\kappa}$ is the point estimate of κ from the data. Evidence of cointegration between P_t^* and D_t^* would suggest that P_t, D_t and $D_t^{\bar{\lambda}}$ are cointegrated.

To obtain the values of $\bar{\kappa}$ and $\bar{\lambda}$ needed for the transformations of the dividends and the stock prices, we first need to obtain the point estimates of the constants μ, σ and r denoted by $\bar{\mu}$, $\bar{\sigma}$ and $\bar{r}$, respectively. From the data, we get $\bar{\mu} = 0.0155$, $\bar{\sigma} = 0.129$, $\bar{r} = 0.083$.[14,15] Utilizing these point estimates and equation (8), we obtain $\bar{\kappa} = 16.0$. Next, solving for the positive root of (9) and using

[13] We relaxed this restriction $\lambda = \bar{\lambda}$ later and re-estimated λ. It turns out that there is no significant difference between the freely estimated λ and $\bar{\lambda}$.

[14] $\bar{r}$ is determined as 0.083 by taking the point estimates, i.e. the geometric mean, of the time series of r (see Froot and Obstfeld, 1991, p. 1199 for details). The exact formula is: $R = \exp(r) = (P+D_{-1})/P_{-1}$, $\bar{r} = \log(\text{arithmetic mean of } R)$.

[15] To justify the two crucial assumptions of the model mentioned in footnote 7, we have tested two hypotheses: (a) the residual of (7), ξ_{t+1}, is i.i.d. and (b) the discount rate is stationary. The BDS test (Brock *et al*., 1996) on the series ξ_t gives the test statistic of –0.00005 with p-value of 0.9999, which implies the null hypothesis that ξ_t is i.i.d. cannot be rejected at any significance level. The Phillips–Perron unit root test on the discount rates gives a test statistic of –3.05 and –10.68 for r_t and $R_t = \exp(r_t)$, respectively, thereby rejecting the hypothesis that r_t and R_t have unit roots at the 5% level. This gives some empirical support for the two assumptions made in the model.

Table I. Linear cointegration testing results for $P_t = \kappa_0 + \kappa D_t + u_t$ (sample period 1871–1996)

OLS estimation			
	Estimated coefficient	t-Value	
κ_0	−49.42	−8.30	
κ	33.97	31.83	
$R^2 = 0.89$			
Phillips–Perron unit root test			
	H_0: I(1)	H_0: I(2)	Conclusion
		With trend	
P	−3.23 (6)	−138.87*(5)	I(1)
D	−11.72 (3)	−100.89* (3)	I(1)
Residual	−19.02 (2) [−21.47]$^+$	−127.40* (3) [−22.65]$^+$	I(1)
		Without trend	
P	3.55 (6)	−140.09* (5)	I(1)
D	0.47 (4)	−101.29* (3)	I(1)
Residual	−20.84 (6) [−23.12]$^+$	−128.15* (3) [−16.86]$^+$	I(1)

Notes:
* Indicates significant at the 5% level.
$^+$ Indicates the bootstrapped critical value is applied. 5% bootstrapped critical values in squared brackets (Mandeno and Giles, 1995).
Optimum lags are in parentheses.
A constant term κ_0 is included in the estimation to capture the mean of u_t.

the point estimates of the constants κ, μ, λ and r yields a value of $\bar{\lambda} = 2.36$. These values of $\bar{\kappa}$ and $\bar{\lambda}$ will be used in testing for nonlinear cointegration.

As an initial step in our testing procedure, we examine whether there is linear cointegration between stock prices and dividends as implied by the simple PV model given by equation (13):

$$P_t = \kappa D_t + u_t \tag{13}$$

If the PV model holds in the long run, one would expect that cointegration is found, and also that the estimated value of κ is the same as the value implied by the data ($\bar{\kappa} = 16.0$). The cointegration results of the simple PV model are reported in Table I. This table reports the Phillips–Perron (PP) unit root test with trend and without trend on the fitted u_t together with the 5% bootstrapped critical values. The PP test with trend is −19.02 with a bootstrapped critical value of −21.47, suggesting that the fitted u_t is I(1) and hence that there is no linear cointegration between stock prices and dividends. A similar result is obtained on the basis of the PP test without trend. Moreover, the value of the estimated κ is 33.97, which is significantly higher[16] than the implied value $\kappa = \bar{\kappa} = 16$. These results suggest that the simple PV model fails to provide a long-run relationship between stock prices and dividends.

We next turn to testing for nonlinear cointegration between stock prices and dividends. Table II(a) reports the results from OLS estimation of equation (13) augmented with an intrinsic bubble given by (10) under the restrictions that $\kappa = \bar{\kappa} = 16$ and $\lambda = \bar{\lambda} = 2.36$. Based on the PP test with and

[16] This is based on the bootstrapped critical value at the 5% level. The results can be supplied on request.

Table II. Nonlinear cointegration testing results for $P_t = \kappa D_t + cD_t^{\lambda} + \varepsilon_t$ with restriction $\kappa = \bar{\kappa} = 16$ (sample period 1871–1996)

(a) OLS estimation, impose $\lambda = \bar{\lambda} = 2.36$

	Estimated coefficient	t-Value	
c	0.65	29.5*	
$R^2 = 0.81$			

Phillips–Perron unit root test

	H_0: I(1)	H_0: I(2)	Conclusion
	With trend		
$P - 16D$	−8.87 (6)	−145.6* (6)	I(1)
$D^{2.36}$	1.24 (3)	−92.6* (3)	I(1)
Residual	−28.76* (2) [−25.15]+		I(0)
	Without trend		
$P - 16D$	3.32 (6)	−147.2* (6)	I(1)
$D^{2.36}$	4.8 (3)	−88.2* (3)	I(1)
Residual	−28.95* (2) [−17.02]+		I(0)

(b) Nonlinear OLS estimation, with both c and λ being freely estimated

	Estimated coefficients	t-Value	
c	0.26	2.72*	
λ	2.77	17.0*	
$R^2 = 0.82$			

Phillips–Perron unit root test

	H_0: I(1)	H_0: I(2)	Conclusion
	With trend		
$P - 16D$	same as (a)		
$D^{2.77}$	3.63 (3) [−11.29]+	−89.2* (2) [−25.36]+	I(1)
Residual	−31.33 (2) [−23.72]+		I(0)
	Without trend		
$P - 16D$	same as (a)		
$D^{2.77}$	6.23+ (3) [−7.54]+	−83.0* (2) [−16.70]+	I(1)
Residual	−30.8 (2) [−16.56]+		I(0)

Notes:
* Indicates significant at the 1% level.
+ Indicates the bootstrapped critical value is applied. 5% bootstrapped critical values in squared brackets (Mandeno and Giles, 1995).
Optimum lags are in parentheses.

without trend and bootstrapped 5% critical values, the hypothesis that the fitted u_t is I(1) is rejected. In particular, the PP statistic with trend is −28.76 with a bootstrapped critical value of −25.15. Similarly, the PP test without trend is −28.95 with a corresponding critical value of −17.02. Rejection of the hypothesis that the fitted u_t is I(1) suggests that P_t, D_t and $D^{\bar{\lambda}}$ are cointegrated. This establishes that stock prices and dividends are nonlinearly cointegrated, providing support for the intrinsic bubbles model in the long run.

Table II(b) reports the results from nonlinear cointegration based on a nonlinear OLS estimation of equation (10) in which the restriction that $\lambda = \bar{\lambda} = 2.36$ is not imposed, and λ is freely estimated.

The estimated λ is 2.77, which is not significantly different from $\lambda = \bar{\lambda} = 2.36$ but is significantly higher than 1, thereby supporting the nonlinear (explosive) behaviour of intrinsic bubbles.[17] Furthermore, based on the PP test and bootstrapped critical values, the hypothesis that the fitted u_t is I(1) is rejected, suggesting that stock price and dividends are nonlinearly cointegrated. Overall, the results from Tables II(a) and II(b) are consistent and provide empirical support to the intrinsic bubbles model as a model of a long-run nonlinear relationship between stock prices and dividends.

OUT-OF-SAMPLE FORECASTING

The proof of the pudding is in the eating. This section compares the out-of-sample forecasting performance of the intrinsic bubbles model against two alternatives, namely the random walk and the rational bubbles model.

Having established the long-run nonlinear cointegration relationship between stock prices and dividends, we next consider an error-correction model (ECM) to forecast the stock prices and dividends. To simplify the econometrics, we focus on a *linear* ECM for P_t^* and D_t^*, the transformed series. The corresponding vector autoregressive (VAR) model is:

$$\mathbf{Y}_t = \mathbf{A} + \mathbf{C}\mathbf{Y}_{t-1} + \mathbf{U}_t \tag{14}$$

where $\mathbf{Y} = [P_t^*, D_t^*]'$, $\mathbf{A}$ and $\mathbf{C}$ are 2×1 and 2×2 coefficient matrices, respectively, $\mathbf{U}_t$ is a 2×1 vector of error terms. A parsimonious representation of (14) suggests that one lag should be chosen. Based on forecast values of P_t^* and D_t^*, we can recover the original series of P_t and D_t from (11) and (12). In the forecasting exercise, the period 1871–1974 is treated as the estimation sample period and the period 1975–1996 is the out-of-sample forecasting period.

It is well known that a good forecasting model requires necessarily constant coefficients *within* the estimation sample period. However, we find that the coefficients of the ECM of the intrinsic bubbles model are not constant over at least the 1965–1974 period. A Chow test for the stock price equation gives an F-statistic of 3.87 ($F(10,86) = 3.87$), which rejects the null hypothesis of constant coefficients at the 1% level. This leads us to apply a time-varying coefficient approach based on a Kalman filter in order to estimate the intrinsic bubbles model recursively for forecasting purposes. Specifically, we estimate the following forecasting model:

$$\mathbf{Y}_t = \mathbf{A}_t + \mathbf{C}_t \cdot \mathbf{Y}_{t-1} + \mathbf{U}_t \tag{15}$$

where $\mathbf{A}_t$ and C_t are assumed to follow a random walk for simplicity, namely $\mathbf{A}_t = \mathbf{A}_{t-1} + \mathbf{U}_{\mathbf{A}t}$, $\mathbf{C}_t = \mathbf{C}_{t-1} + \mathbf{U}_{\mathbf{C}t}$, and $\mathbf{U}_{\mathbf{A}t}$ and $\mathbf{U}_{\mathbf{C}t}$ are error terms in the coefficient equations.

As an alternative to the intrinsic bubbles model for forecasting purposes, a special version of the rational bubbles model is also estimated:

$$P_t = \alpha D_t + B_t + \varepsilon_t \tag{16}$$

where α is a coefficient, ε_t is the error term, and B_t is the extraneous rational bubble which is given by the following equation:

[17] Footnote 16 applies here too.

$$B_t = e^{\bar{r}} B_{t-1} + \varepsilon_{B1} \quad (\text{where } \bar{r} = 0.083) \tag{17}$$

such that B_t satisfies (4) but not the transversality condition (2′). Allowing for time-varying coefficients in the rational bubbles model as well leads to the following equation for the rational bubbles model to be estimated recursively using a Kalman filter:

$$P_t = \alpha_t D_t + B_t + \varepsilon_t \tag{18}$$

where α_t is assumed to follow a random walk for simplicity: $\alpha_t = \alpha_{t-1} + \varepsilon_{\alpha_t}$, ε_{α_t} is the error term in the coefficient equation.

The second alternative to the intrinsic bubbles model is the random walk model of $(P_t + D_t)$ given by:

$$P_t + D_t = \theta + P_{t-1} + D_{t-1} + e_t \tag{19}$$

where θ is a constant and e_t is the error term. The random walk forecast is based on the system of (19) and (7).

Table III reports the results on the forecasting performance of the time-varying coefficients (TVC) intrinsic bubbles model compared to the forecasting performance of the TVC rational bubbles model and the random walk model. The forecasting performance is measured by the root-mean-squared-errors (RMSE) of 1-year, 3-year and 5-year ahead out-of-sample forecasts of stock price for the period 1975–1996. Table III shows that the TVC intrinsic bubbles model gives the smallest RMSE among the three models across all three time horizons. The relative forecasting performance of the intrinsic bubbles model seems to be higher as the forecasting horizon lengthens. Specifically, as the horizon gets longer from 1 year to 5 years, the incremental RMSE of the intrinsic model is only 3.5 approximately (i.e. it increases from 39.87 for 1-year to 43.35 for 5-year horizons) compared to an incremental RMSE of approximately 14 of the rational bubbles model and 57 of the random walk model. This result is consistent with the finding of the previous section regarding the long-run validity of the intrinsic bubbles model, thereby offering further support to the intrinsic bubbles model. A pictorial exposition of this result is provided in Figures 2, 3 and 4. Figure 2 plots the 1-year-ahead forecasted stock price–dividend ratios obtained from the intrinsic bubbles model, the rational bubbles model and the random walk against the actual stock price–dividend ratios; Figure 3 plots the 3-year-ahead forecasts against the actual stock price–dividend ratios and Figure 4 plots the 5-year-ahead forecasts. In all three figures, the intrinsic bubbles model appears to provide more accurate forecasts.

Table III. Root-mean-squared errors of out-of-sample forecasting: 1975–1996

	TVC intrinsic bubbles model	TVC rational bubbles model	Random walk model
1-year ahead	39.87	57.17	45.69
3-year ahead	41.50	70.38	69.80
5-year ahead	43.35	71.30	101.89

Note: These are the root-mean-squared errors (RMSE) of 1-year, 3-year and 5-year ahead out-of-sample forecasts of stock price for the period 1975–1996.

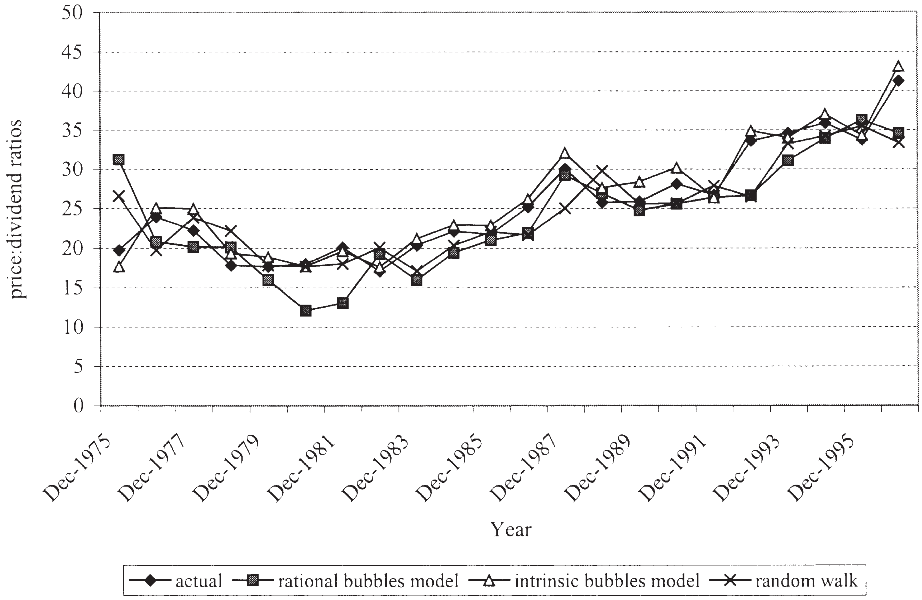

Figure 2. One-year-ahead forecasts

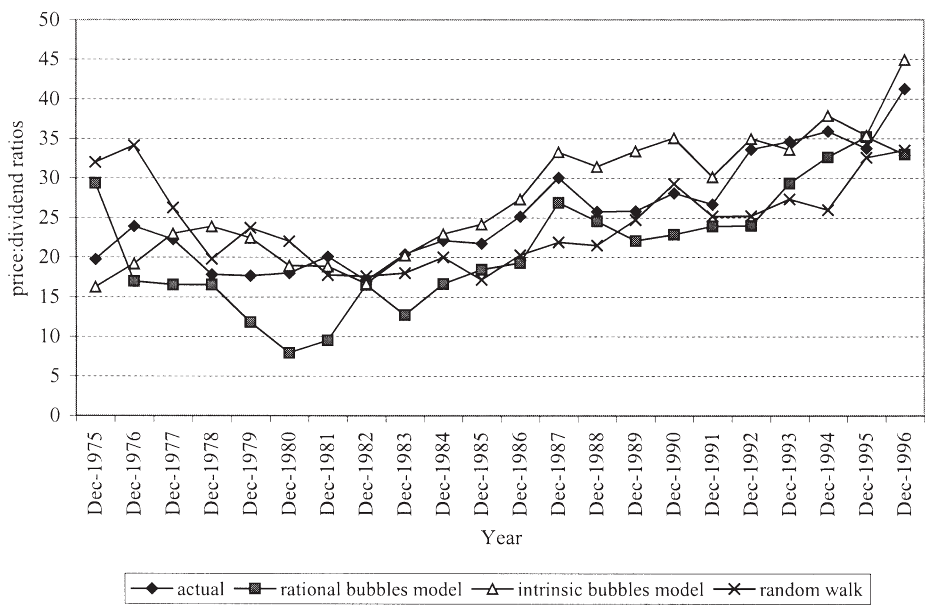

Figure 3. Three-year-ahead forecasts

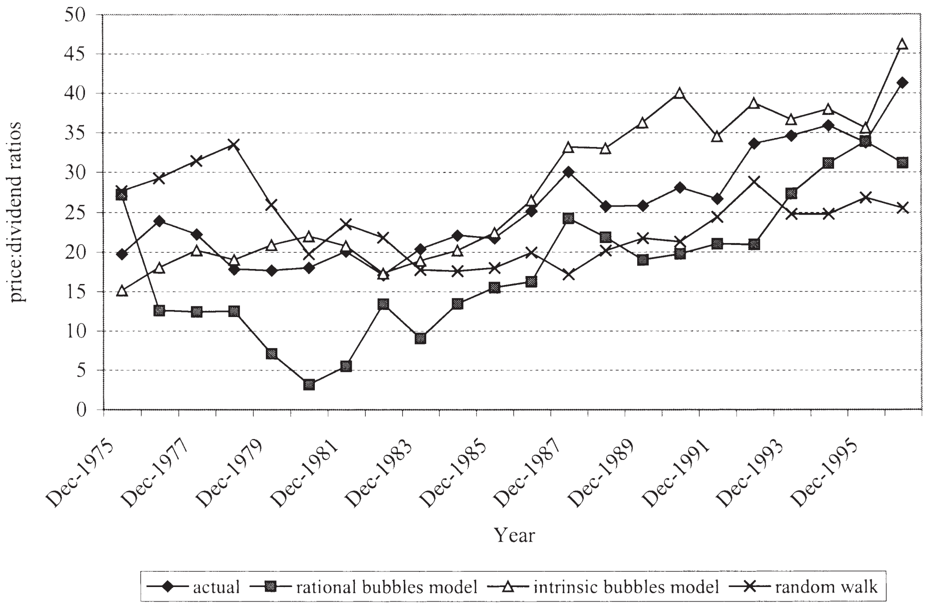

Figure 4. Five-year-ahead forecasts

Another point which emerges from Table III refers to the stock price forecasting performance of the rational bubbles model in comparison with that of the random walk. The rational bubbles model is outperformed by the random walk model at the 1-year and 3-year forecasting horizons. In contrast, for the 5-year horizons, the rational bubbles model seems to yield superior forecasts to those of the random walk model.

Finally, to test that the RMSE of the intrinsic model is significantly smaller than that of its rivals, we employ two nonparametric exact finite-sample tests, namely the binomial test and the Wilcoxon signed-rank test. Both tests were suggested by Diebold and Mariano (1995) for forecasting accuracy testing. For both tests, the null hypothesis is that the mean-squared-error of the intrinsic bubbles model is equal to that of its rivals, i.e. the RMSEs of the two models are the same. Table IV reports the results from these tests. As shown in this table, the null hypothesis is rejected at the 5% significance level in each case, thereby implying that the RMSE of the intrinsic bubbles model is significantly smaller than that of the rational bubbles model and the random walk.

CONCLUSION

Extending the empirical findings of Froot and Obstfeld (1991), this paper has provided further strong empirical evidence to support the intrinsic bubble model in two ways. First, we find evidence supporting the long-run validity of the model for the US stock market during the period 1871–1996, on the basis of cointegration tests between a nonlinear transformation of dividends and a linear transformation of stock prices. These transformations are specifically implied by the theoretical intrinsic bubbles model. Our results suggest that there is a long-run nonlinear relationship between stock

Table IV. Exact finite-sample tests of out-of-sample forecast accuracy: 1975–1996

	H_0: the mean-squared errors of the two models are equal H_1: the mean-squared errors of the two models are *not* equal			
	TVC intrinsic bubbles model vs. TVC rational bubbles model		TVC intrinsic bubbles model vs. random walk model	
	Binomial test statistics	Wilcoxon signed-rank test statistics	Binomial test statistics	Wilcoxon signed-rank test statistics
1-year ahead	22	251	18	240
p-value	[0.001]	[0.001]	[0.023]	[0.002]
3-year ahead	19	242	17	233
p-value	[0.010]	[0.002]	[0.043]	[0.003]
5-year ahead	20	249	20	249
p-value	[0.001]	[0.001]	[0.001]	[0.001]

Note: Both the binomial and Wilcoxon tests are based on the difference of the two series of squared errors (SE) of out-of-sample forecasts of stock price from the two rival models. The implied assumptions are that this differential series is i.i.d. and symmetric. The i.i.d. assumption can be relaxed by a slightly different procedure if we had a longer time series (see Diebold and Mariano, 1995, p. 255).

prices and dividends. Second, we find that the forecasting performance of the intrinsic bubbles model is superior to the performance of two rival models, namely that of rational bubbles and the random walk. On the basis of the finding of cointegration between the transformed variables, we construct an error-correction model for the transformed stock price and dividend series, and estimate this model by a time-varying Kalman filter. To compare the forecasting performance of this model, we also estimate a time-varying rational bubbles model by Kalman filter and a simple random walk model. Our results on out-of-sample forecasting suggest that the intrinsic bubbles model significantly outperforms the other two models, thereby offering further support to the intrinsic bubbles model.

APPENDIX: EMPIRICAL ESTIMATION OF κ

Froot and Obstfeld (1991) estimate a different specification of their intrinsic bubbles model from our specification (10). Dividing (10) by D_t they have:

$$P_t / D_t = \kappa + cD_t^{\lambda-1} + \varepsilon_t / D_t \tag{A.1}$$

They estimate (A.1) instead of our model (10). The problem with (A.1) is that it is difficult to establish the long-run relationship between price and dividends due to the complication of the new error term ε_t/D_t which contains the fundamental variable of D_t. This is the main reason we did not follow Froot and Obstfeld's specification of (A.1) in our paper. However, the advantage of (A.1) is that one may estimate the coefficient κ together with the other two coefficients of c and λ. In this Appendix, we try to estimate κ by this approach and then compare it with our point-estimate value $\bar{\kappa} = 16$ in (8).

Following Froot and Obstfeld (1991), we first estimate the restricted version of (A.1) by imposing $\lambda = \bar{\lambda} = 2.36$ from (9). Employing linear OLS to (A.1) for our full sample period of 1871 to 1996, we have:

$$P_t/D_t = \underset{(24.9)}{16.26} + \underset{(10.68)}{0.60}\,D_t^{1.36} + \varepsilon_t^*, \quad R^2 = 0.48, \quad \text{sample period: 1871–1996} \tag{A.2}$$

where t-values are in parentheses and ε_t^* is the residuals.

Clearly, the estimated value of $\kappa = 16.26$ is not significantly different from the theoretical point estimate $\bar{\kappa} = 16$.

Next, we relax the restriction $\lambda = \bar{\lambda} = 2.36$ and let λ be a freely estimated parameter in (A.1). Applying the nonlinear OLS to (A.1) for the same sample period, we have:

$$P_t/D_t = \underset{(26.11)}{18.69} + \underset{(0.85)}{0.032}\,D_t^{2.59(5.23)} + \varepsilon_t^*, \quad R^2 = 0.51, \quad \text{sample period: 1871–1996} \tag{A.3}$$

where t-values are in parentheses.

Similar to the finding by Froot and Obstfeld (1991), (A.3) does not improve the estimation of (A.1) very much in terms of R^2 and the estimated value of $\kappa = 18.69$ is still not significantly different from the theoretical point estimate, $\bar{\kappa} = 16$, although the estimated value of the coefficients c and λ changed quite a lot.

Therefore, we can conclude that our theoretical point estimate of $\bar{\kappa} = 16$ is a plausible and robust estimate for κ.

ACKNOWLEDGEMENTS

The authors wish to thank anonymous referees for insightful comments and suggestions.

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