Learning-aided 3D Occupancy Mapping with Mobile Robots

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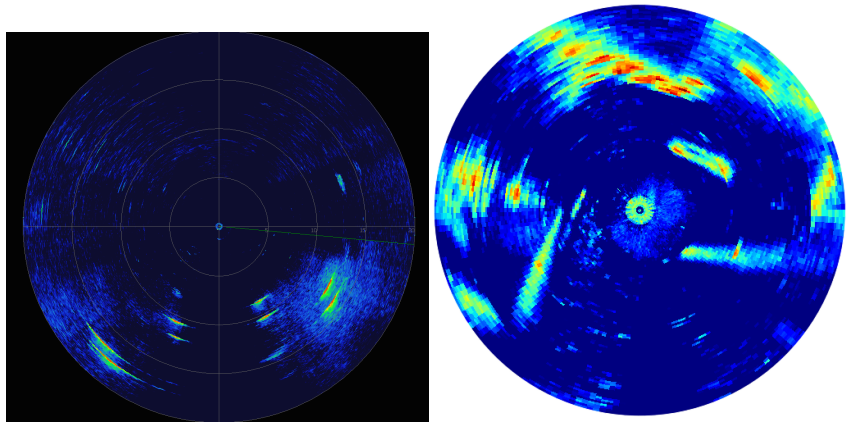
  - Environmental disturbances
  - Low resolution
  - Multiple returns



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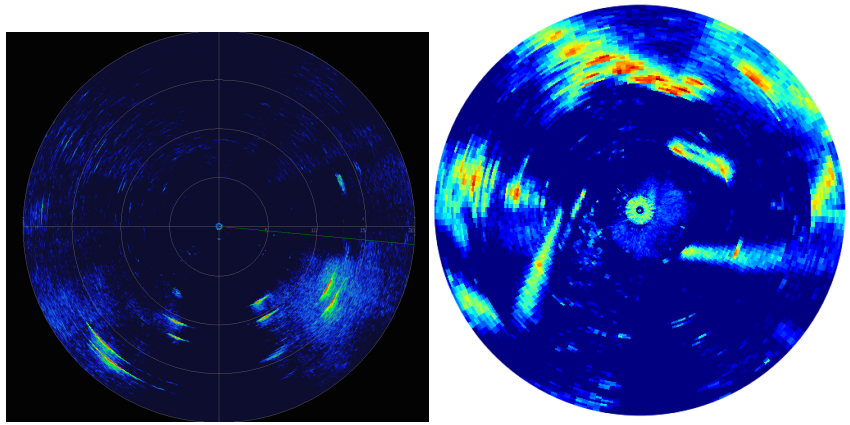

- Traditional occupancy grid mapping is limited by strong spatial independence assumptions

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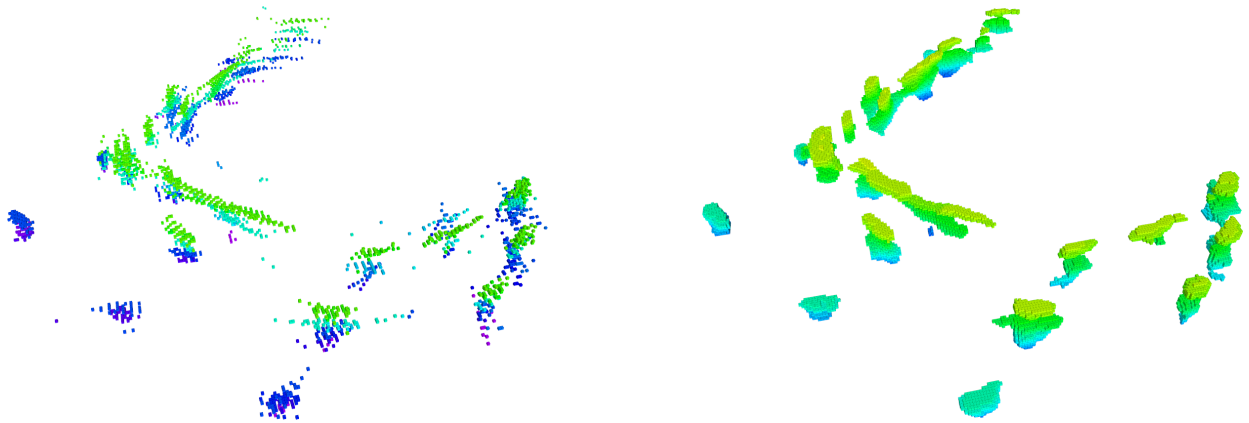

(a) Pier 84 OctoMap (b) Pier 84 GP OctoMap

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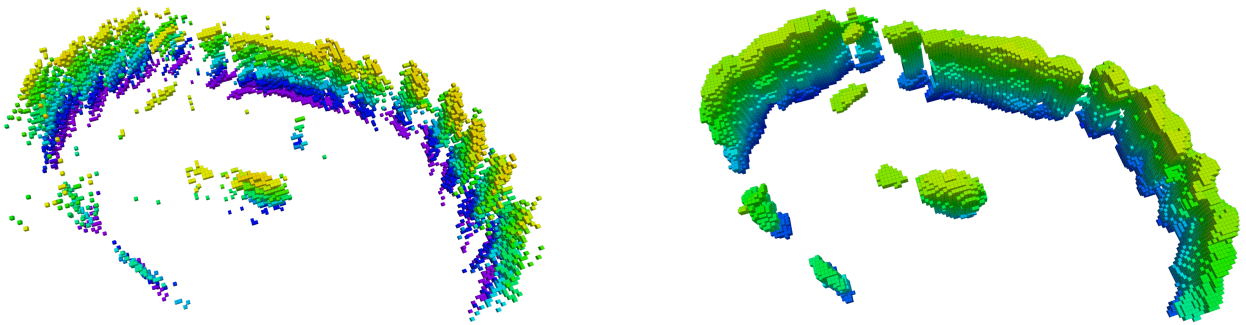

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- **Assumptions:**
  - We will assume that the map is static
  - We relax the assumption in standard occupancy grid mapping that cells of a grid map are independent

Our Contributions

We present two learning-aided mapping methods:

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- **Overlapping Hilbert Maps**
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- **Overlapping Hilbert Maps**
  - Approximate approach to Hilbert Maps, proposed by Ramos and Ott [22]
  - Train an estimator for each robot pose and fuse predictions
- **Bayesian Generalized Kernel Occupancy Maps**
  - Perform Bayesian inference with conjugate prior
  - Exact recursive updates
  - Introduce spatial correlations with kernel smoothing

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Mapping with Known Poses

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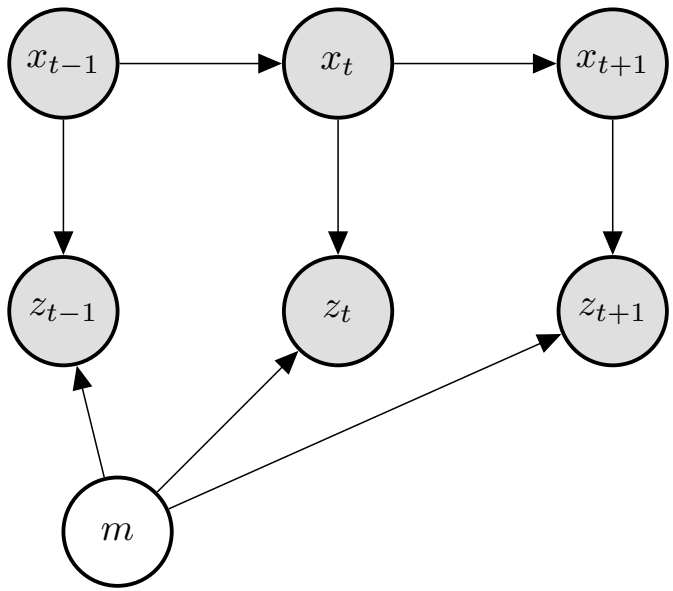

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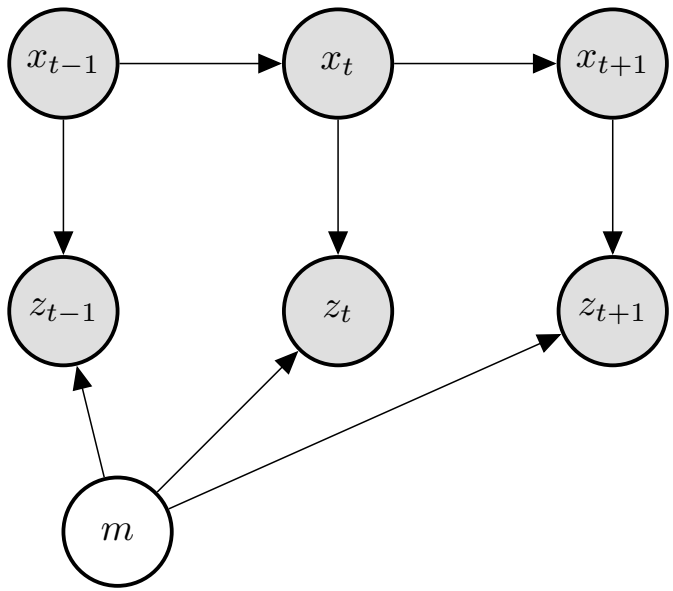

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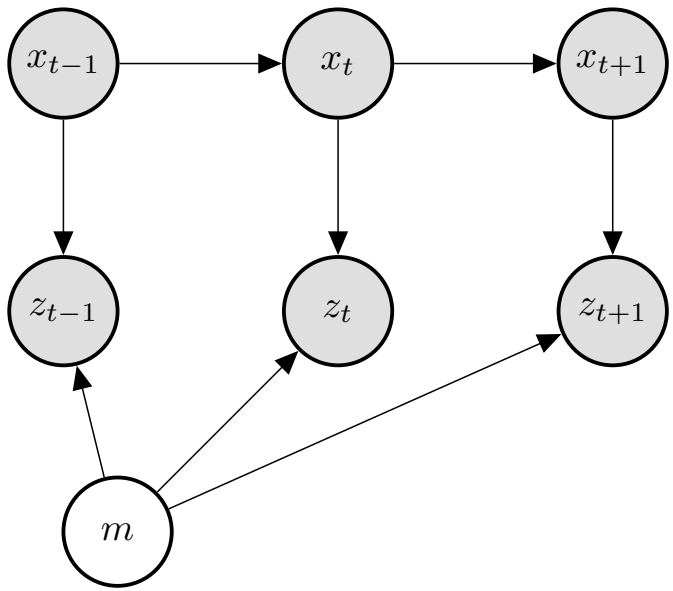

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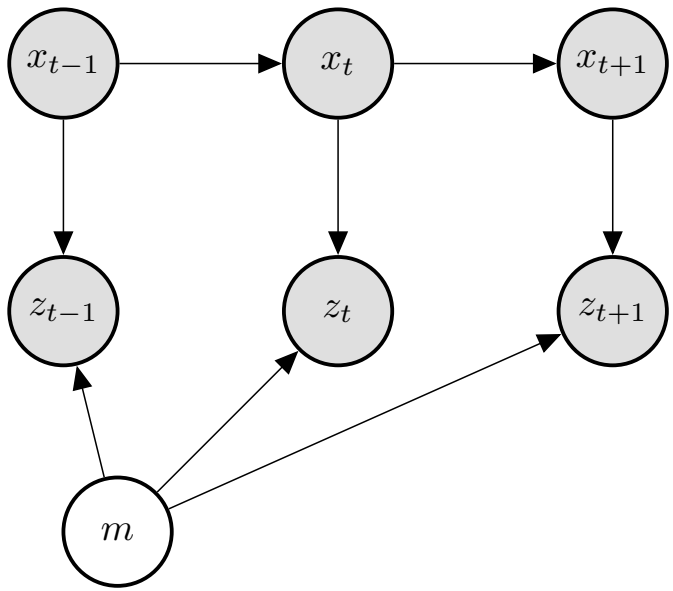

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$$\log \frac{p(m_i|z_t, x_t)}{1 - p(m_i|z_t, x_t)} + \log \frac{p(m_i|z_{1:t-1}, x_{1:t-1})}{1 - p(m_i|z_{1:t-1}, x_{1:t-1})} - \log \frac{p(m_i)}{1 - p(m_i)}$$
$$= l_{t,i} + l_{1:t-1,i} - l_0$$

Mapping with Known Poses

Occupancy Grid Mapping Algorithm

Given new (state, measurement) pair (x_t, z_t):

- Traverse every cell in the grid
  - If a cell is visible, update its log-odds
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Learning Occupancy Maps

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- Learning the inverse sensor model [30]
  - Avoids heuristics
  - Learn the characteristics of particular sensor
  - Doesn't consider readings outside the measurement cone
- Considering correlations between all cells [31]

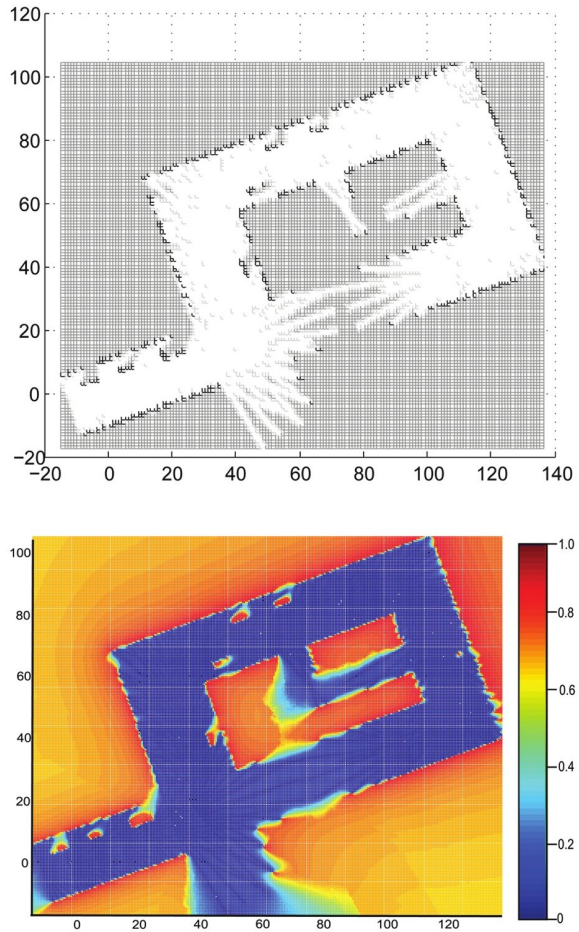
$$m^* = \arg\max_m \ \log p(m|z_{1:t}, x_{1:t})$$

  - Iterative cell flipping approach
  - Intractable for expansive or finely-discretized 3-dimensional grids

Gaussian Process Occupancy Maps

Proposed by O'Callaghan and Ramos [17], learn a model from measurement data, abandon discretization

- Formulate map inference as spatial interpolation between measurements
- Reason about correlations between measurements, not cells
- Map is *continuous*
- Provides uncertainty estimates



Gaussian Process Occupancy Maps

Given:

- Training data $X = \{x_1, x_2, \ldots, x_N \mid x_i \in \mathbb{R}^3\}$ comprised of the 3D coordinates of our "occupied" (sensor hits) and "free" points (sensor ray samples)
- Corresponding labels $y = \{y_1, y_2, \ldots, y_N \mid y_i \in \{-1, 1\}\}$

A Gaussian process is a distribution over functions f:

$$f(x) \sim \mathcal{GP}(m(x),\ k(x, x'))$$

Where we have:

- Mean function $m(x)$, typically taken to be 0
- Covariance function, or kernel, $k(x, x')$, a measure of similarity between x and x'

Goal: Infer latent function values f_* for query points X_*:

$$f_* | X, y, X_* \sim \mathcal{N}(\bar{f}_*, \mathrm{cov}(f_*))$$

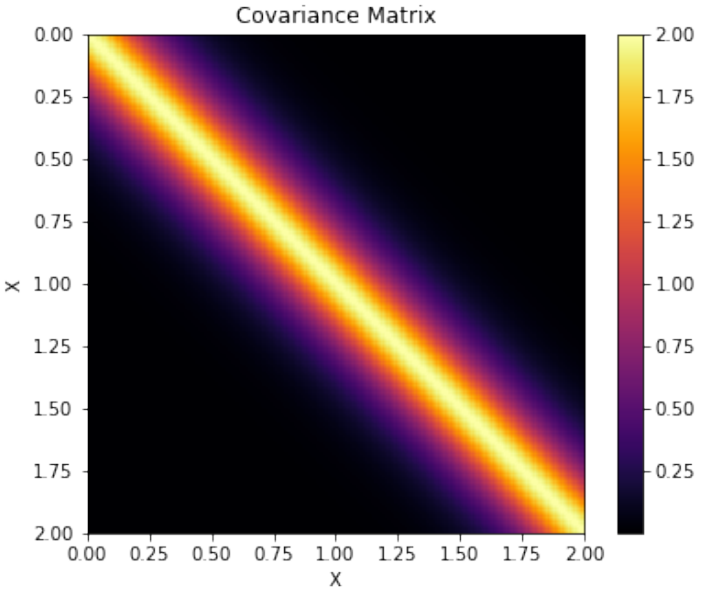
Gaussian Process Occupancy Maps

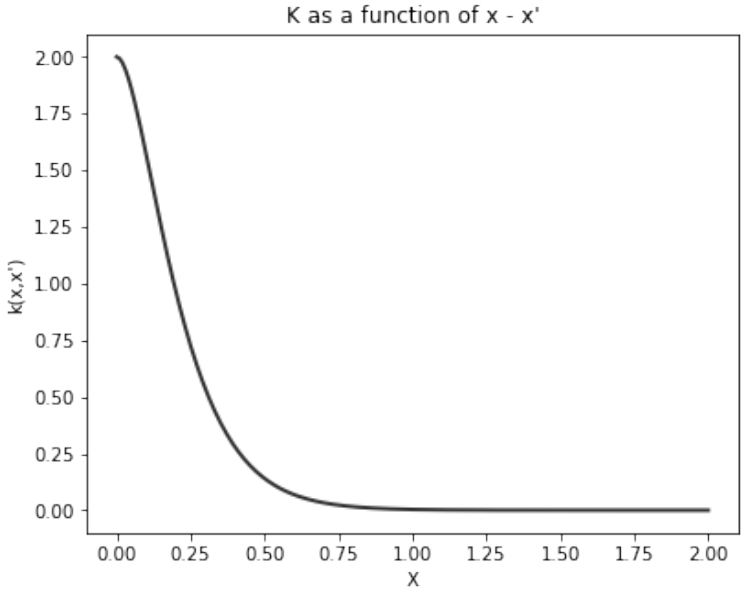
The mean and covariance of the latent function values can be computed as follows:

$$\bar{f}_* = K(X, X_*)^T (K(X, X) + \sigma_n^2 I)^{-1} y$$
$$\text{cov}(f_*) = K(X_*, X_*) - K(X, X_*)^T (K(X, X) + \sigma_n^2 I)^{-1} K(X, X_*).$$

What's the kernel?

Gaussian Process Occupancy Maps





What's the kernel? Matérn $\nu = 3/2$ is a popular choice:

$$k(x, x') = \sigma_f^2 \left(1 + \frac{\sqrt{3}\|x - x'\|}{l}\right) \exp\left(\frac{-\sqrt{3}\|x - x'\|}{l}\right)$$

σ_f^2 represents the prior signal variance and l controls the length-scale of the kernel

Gaussian Process Occupancy Maps

Obtains a mean and covariance; mean not guaranteed to be in the range [0,1], "squash" GP output to obtain something we can interpret as a probability

$$p(y_{*i} = 1|X, y) = \frac{1}{1 + \exp(-\gamma\omega_{*i})}$$

where γ is a positive constant, $\omega_{*i} = \sigma^2_{min}\mu_{*i}/\sigma^2_{*i}$, and σ^2_{min} is the minimum predicted variance

Gaussian Process Occupancy Maps

We can update the Gaussian process regression model using the Bayesian committee machine [33] as follows:

$$\hat{m}(f_*|\mathcal{D}) = \widehat{\text{cov}}(f_*|\mathcal{D}) \sum_{i=1}^{K} \text{cov}(f_*|\mathcal{D}_i)^{-1} m(f_*|\mathcal{D}_i)$$

$$\widehat{\text{cov}}(f_*|\mathcal{D})^{-1} = -(K-1)\sigma_f^{-2} + \sum_{i=1}^{K} \text{cov}(f_*|\mathcal{D}_i)^{-1}$$

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Gaussian Process Occupancy Maps

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- Prediction requires inversion of a matrix the size of the training data; in $\mathcal{O}(N^3)$
- Too slow for real-time mapping scenarios

Hilbert Maps

Proposed by Ramos and Ott, same idea as Gaussian process regression, but some modifications

- Nonlinear logistic regression

$$p(y_* = 1|x_*, x, w) = \frac{1}{1 + \exp(w^T k(x_*, x))}$$

- Approximate radial basis function kernel (Nyström method)

$$k(x, x') = \exp(-\gamma \|x - x'\|^2)$$

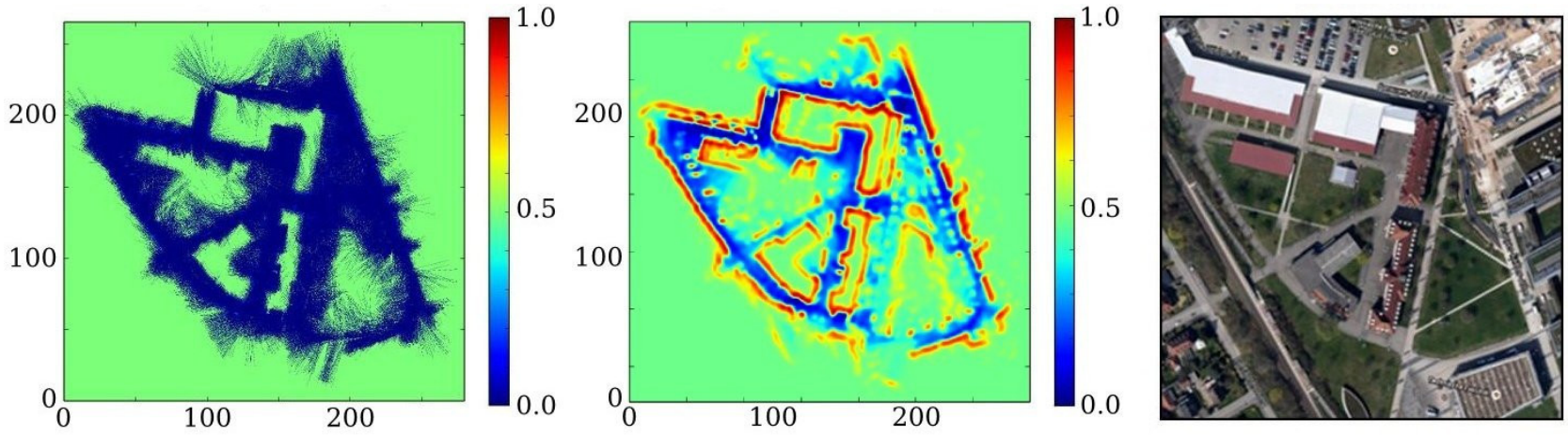
$$\phi_{\text{Nystöm}}(x) = D^{-1/2} V^T (k(x, \hat{x}_1), \ldots, k(x, \hat{x}_m))$$

- Train iteratively with stochastic gradient descent: minimize

$$E(w, b) = \frac{1}{n} \sum_{i=1}^{n} L(y_i, f(x_i)) + \alpha \|w\|_1$$

$$L(y_i, f(x_i)) = \log(\exp(-y_i(x_i^T w + b)) + 1)$$

Hilbert Maps



Pros:

- Faster than GPOM (Linear time complexity, but big coefficient attached)
- Comparable accuracy

Cons:

- No uncertainty estimates
- Still intractable for real-time 3D mapping

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  - In practice, the kernel produces values that are nearly zero for this data
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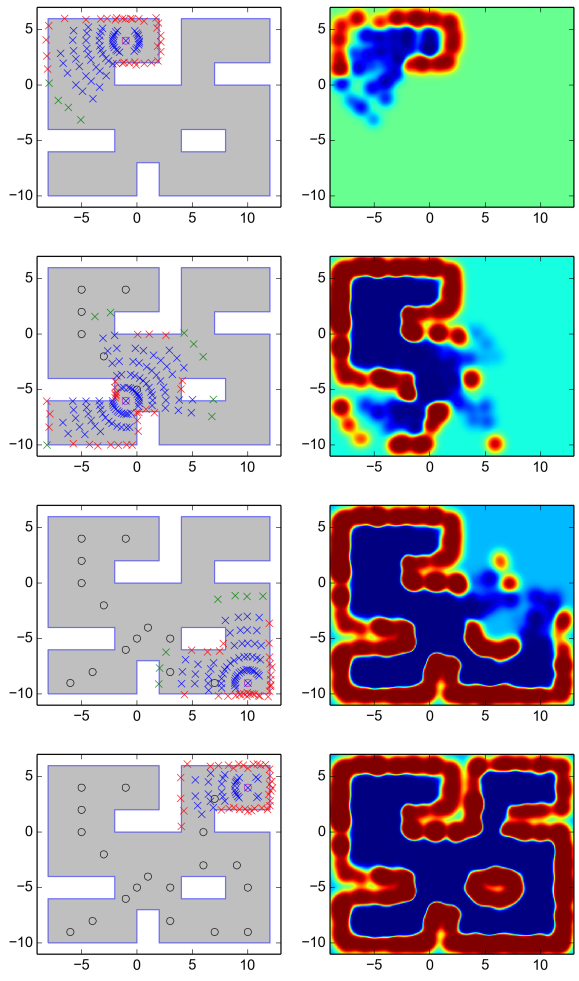
- Train different classifier on each range scan
  - **What do we do when a query falls within distance R of multiple classifiers?**



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$$p(m_i|\mathcal{D}_{1:t}) = \left[1 + \frac{1 - p(m_i|\mathcal{D}_{1:t-1})}{p(m_i|\mathcal{D}_{1:t-1})} \frac{1 - p(m_i|\mathcal{D}_t)}{p(m_i|\mathcal{D}_t)} \frac{p(m_i)}{1 - p(m_i)}\right]^{-1}$$

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$$p(m_i|\mathcal{D}_{1:t}) = \left[1 + \frac{1 - p(m_i|\mathcal{D}_{1:t-1})}{p(m_i|\mathcal{D}_{1:t-1})} \frac{1 - p(m_i|\mathcal{D}_t)}{p(m_i|\mathcal{D}_t)} \frac{p(m_i)}{1 - p(m_i)}\right]^{-1}$$

- Doesn't this assume grid cell independence?

Probabilistic Local Map Fusion

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- Doesn't this assume grid cell independence?
- Spatial dependence accounted for by logistic regression classifier
- We don't *need* to discretize this; treat the logistic regression as spatial interpolation, combine predictions at query time

Combining Logistic Regression Classifiers

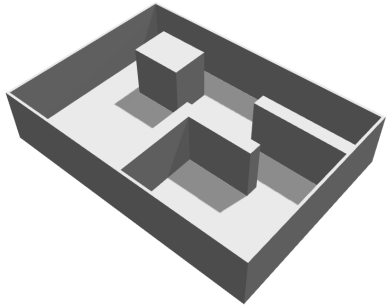
$$
\begin{aligned}
p(m_i|\mathcal{D}_1) &= \frac{1}{1+\exp(w_1^T k(X_1, x_*))} \\
p(\neg m_i|\mathcal{D}_1) &= 1 - p(m_i|\mathcal{D}) = \frac{\exp(w_1^T k(X_1, x_*))}{1+\exp(w_1^T k(X_1, x_*))} \\
p(m_i|\mathcal{D}_{1:2}) &= \left[1 + \frac{1-p(m_i|\mathcal{D}_1)}{p(m_i|\mathcal{D}_1)} \frac{1-p(m_i|\mathcal{D}_2)}{p(m_i|\mathcal{D}_2)} \frac{p(m_i)}{1-p(m_i)}\right]^{-1} \\
&= \left[1 + \exp(w_1^T k(X_1, x_*)) \exp(w_2^T k(X_2, x_*))\right]^{-1} \\
&= \frac{1}{1+\exp(w_{new}^T k(X_{new}, x_*))}
\end{aligned}
$$

where $w_{new} = [w_1, w_2]$ and $X_{new} = [X_1; X_2]$.

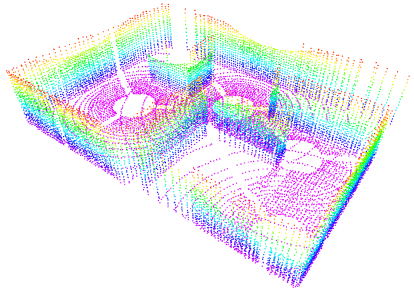
Computational Results

- Tested on 2 simulated datasets: "structured" and "unstructured" in Gazebo [12]
- Time and qualitative map evaluation performed for Freiburg Corridor dataset [34]
- Quantitatively compared our proposed method to our own implementation of Hilbert maps (as it was originally proposed) and OctoMap [7]

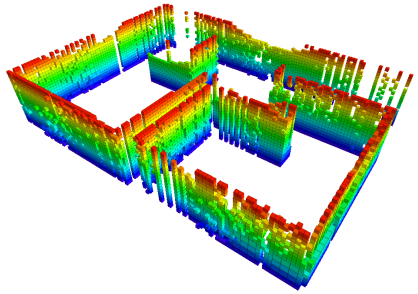
Computational Results



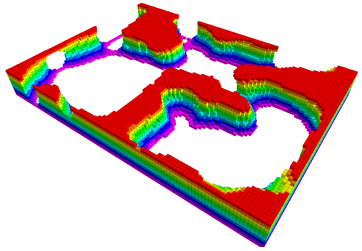
(a) The ground truth map



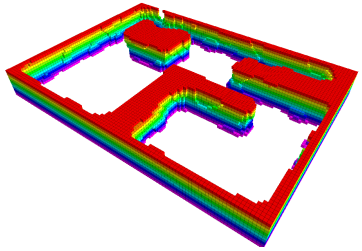
(b) The raw sensor data



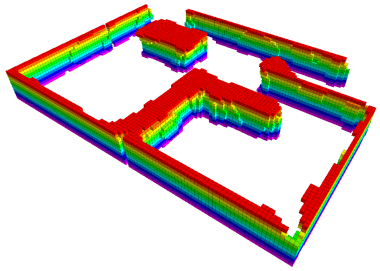
(c) The result of OctoMap



(d) Global HM ($m = 100$)

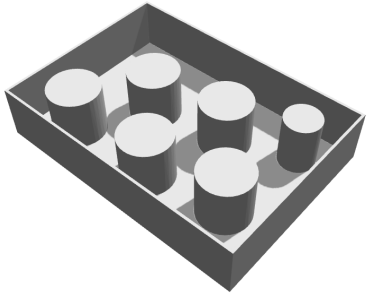


(e) Global HM ($m = 1000$)



(f) Overlapping HM

Computational Results



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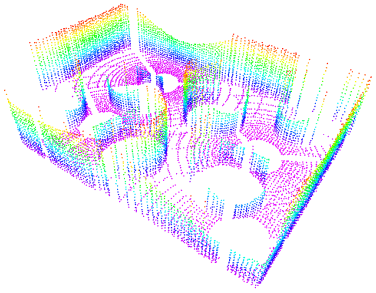

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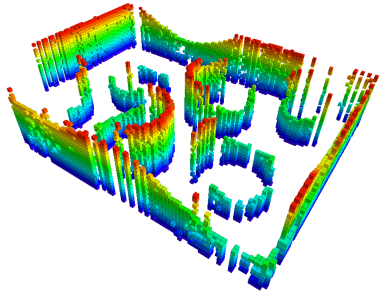

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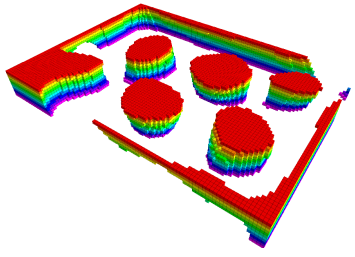
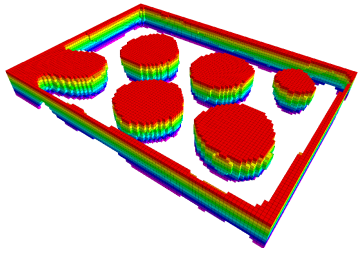


(d) Global Hilbert maps ($m = 100$)



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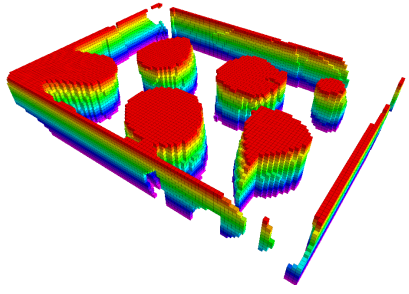

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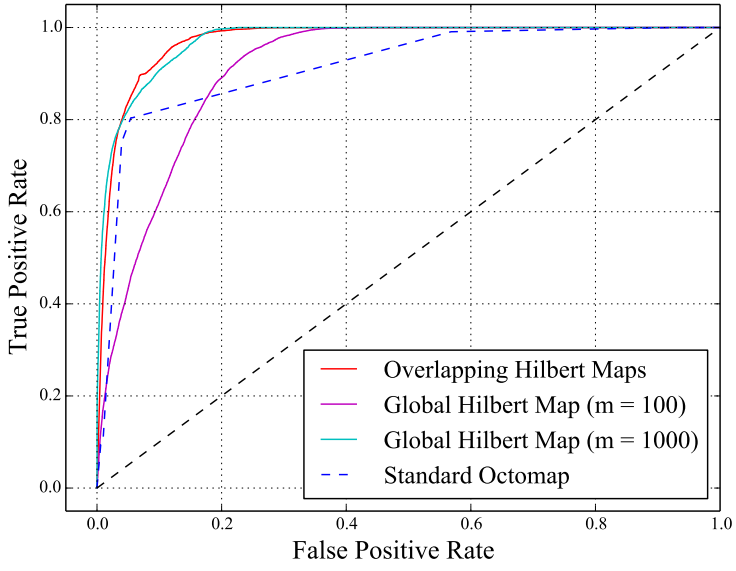
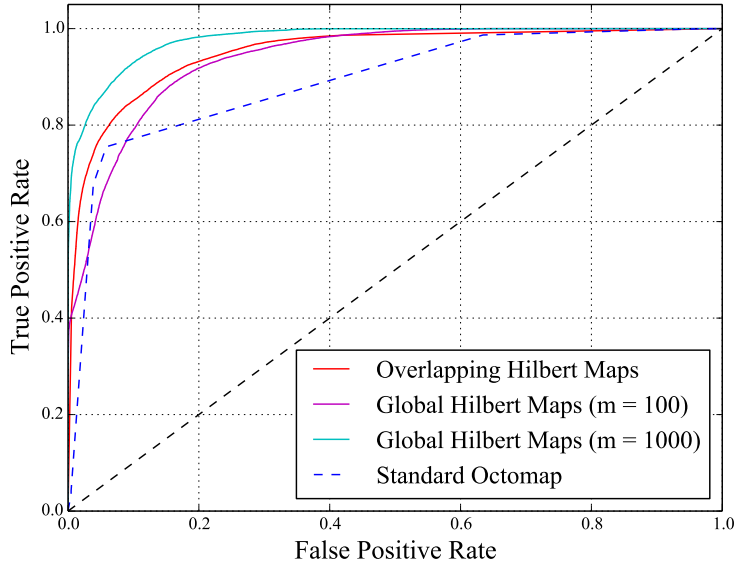
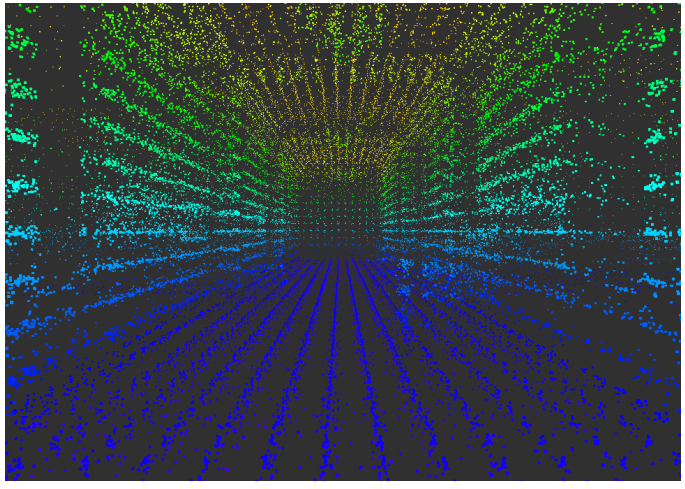
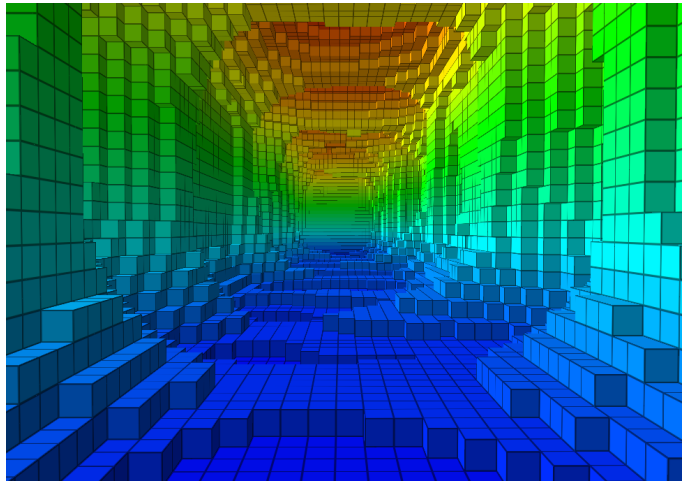
Computational Results









Computational Results

Dataset	Scans	Pts/Scan	Test Pts/Scan	Method	Time/Scan	Time (s)
Structured	12	3500	29892	OHM	1.89	22.68
				GHM-100	N/A	8.70
				GHM-1000	N/A	331.64
				OctoMap	0.02	0.2
Unstructured	12	3500	29892	OHM	1.83	21.96
				GHM-100	N/A	8.52
				GHM-1000	N/A	332.82
				OctoMap	0.01	0.14
FR-079	66	4943	371170	OHM	14.6	963.6
				GHM-100	N/A	1656.95
				GHM-1000	N/A	11914.96
				OctoMap	0.1	6.7

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- Do we *really* need to *train* all these models, or are all the features pretty much equally useful?
- Can we better take advantage of the idea of *local* spatial correlations?
- What about prediction uncertainty?

Table of Contents

Overview

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- Sparse kernel by Melkumyan and Ramos [13]
  - Enables exact inference with efficient spatial data structures like k-d trees
- Test data octrees by Wang et al. [36]
  - Fast query-time performance
  - Prune away redundant map cells to reduce memory consumption

Nonparametric Bayesian Kernel Inference

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where α and β are hyperparameters.We apply the recursive updates:

$$\alpha_t = \alpha_{t-1} + \bar{y}_t$$
$$\beta_t = \beta_{t-1} + \bar{k}_t - \bar{y}_t$$

$$\bar{y}_t = \sum_{i=1}^{N} k(x_i, x_*)y_i, \;\; \bar{k}_t = \sum_{i=1}^{N} k(x_i, x_*)$$

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- Bonus: We get variances associated with our predictions!

Sparse Kernel

The sparse kernel we used [13] is defined as

$$k(x, x') = \begin{cases} \sigma_0 \left[\frac{2+\cos(2\pi \frac{d}{l})}{3} (1 - \frac{d}{l}) + \frac{1}{2\pi} \sin(2\pi \frac{d}{l}) \right] & \text{if } d < l \\ 0 & \text{if } d \geq l \end{cases}$$

where $\sigma_0 > 0$ is a constant parameter of the kernel, $l > 0$ is the scale, and d is the distance between x and x'.

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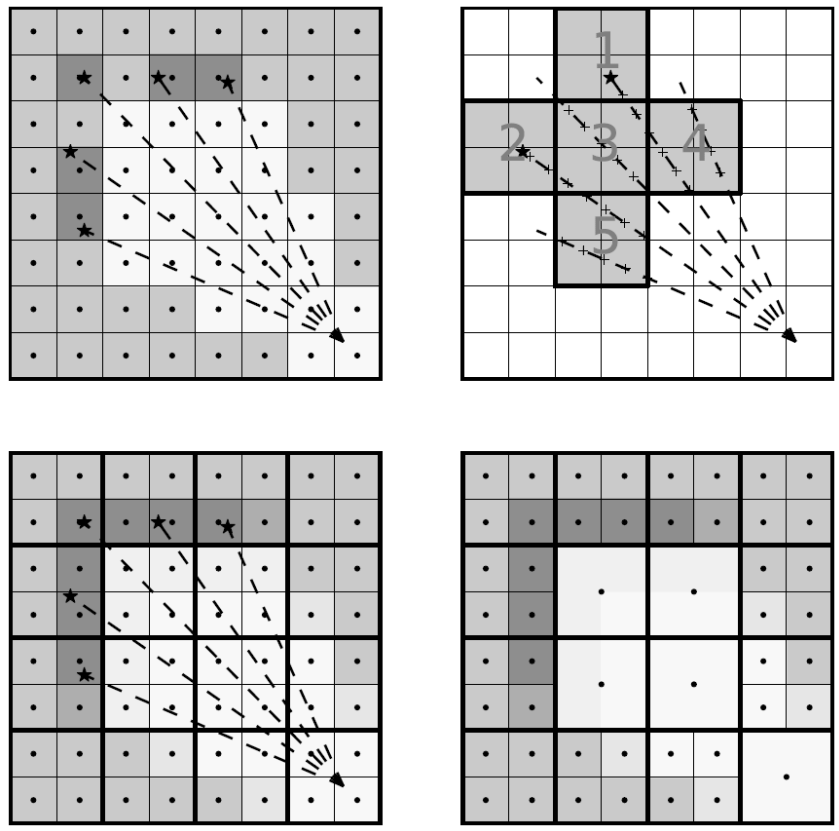
where $\sigma_0 > 0$ is a constant parameter of the kernel, $l > 0$ is the scale, and d is the distance between x and x'.

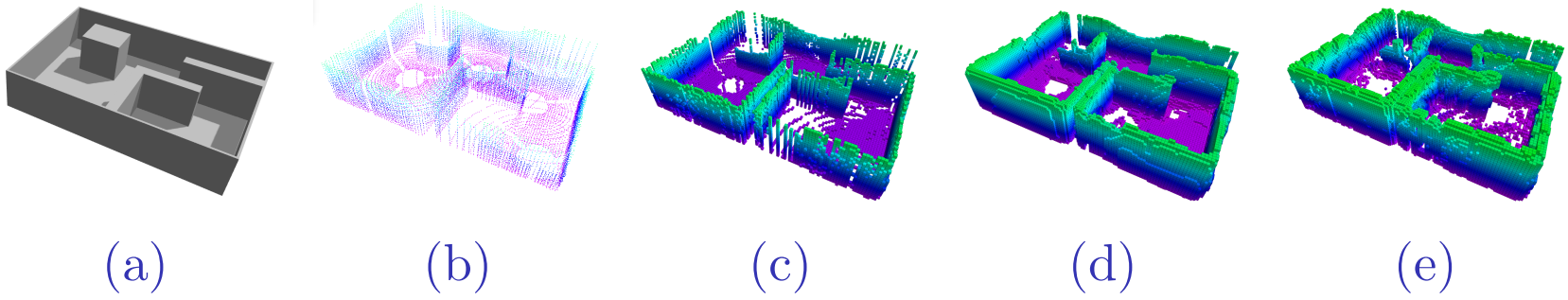
- Efficiently and exactly compute $\bar{y}$ and $\bar{k}$ in $\mathcal{O}(\log N)$ time using a k-d tree
- Overall computational complexity is $\mathcal{O}(M \log N)$, where M is the number of test points and N is the number of training points

Test-data Octrees

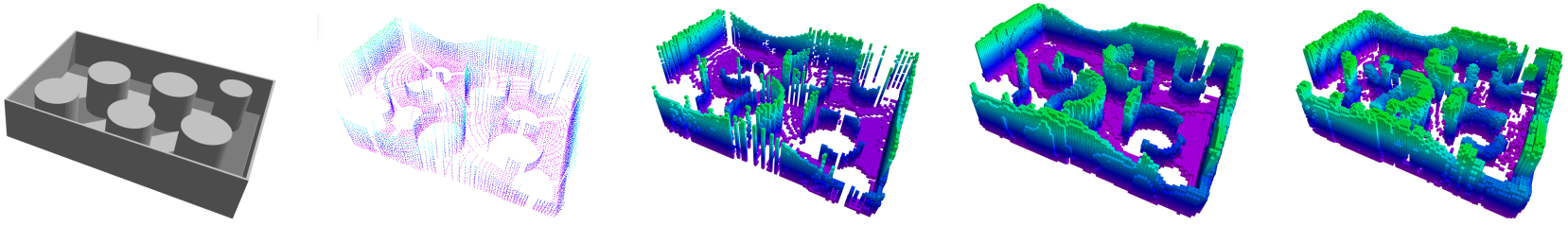
- Consider only points in "extended block"
- Quickly retrieve all relevant grid cells
  - and all relevant training data!
- Prune neighboring grid cells sharing the same state



Computational Results



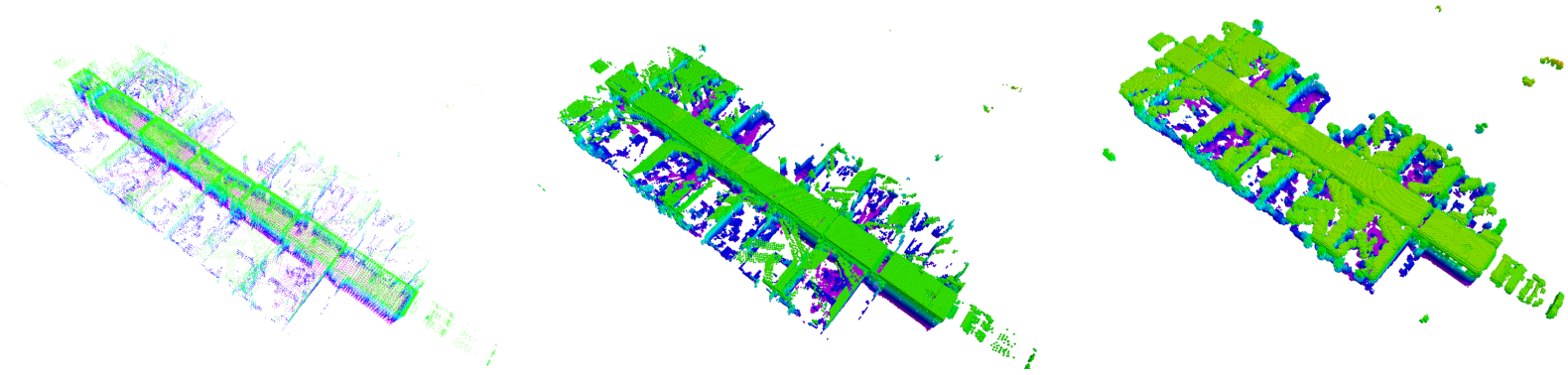
(a) (b) (c) (d) (e)



(a) (b) (c) (d) (e)

(a) Gazebo model, (b) Raw sensor data (“hit” points),
(c) OctoMap, (d) GPOctoMap, (e) Our method, BGKOctoMap

Computational Results



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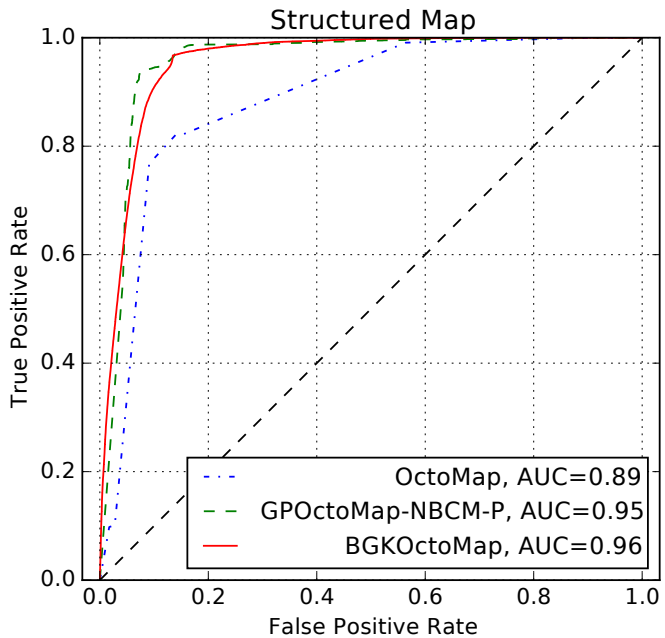
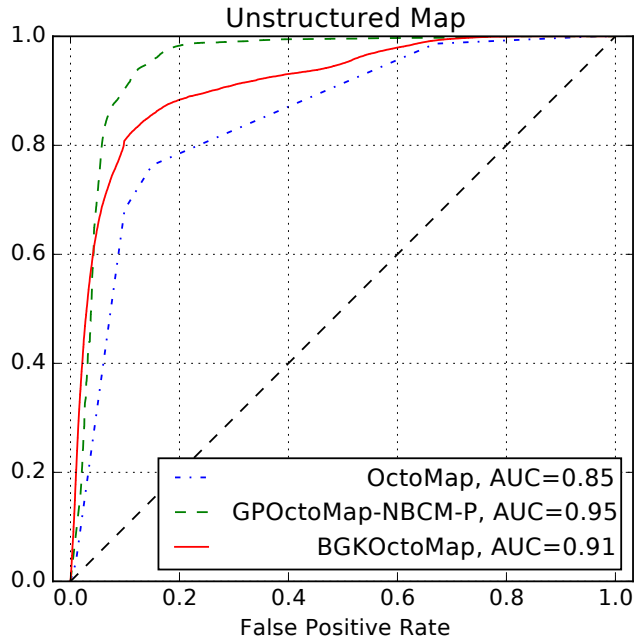


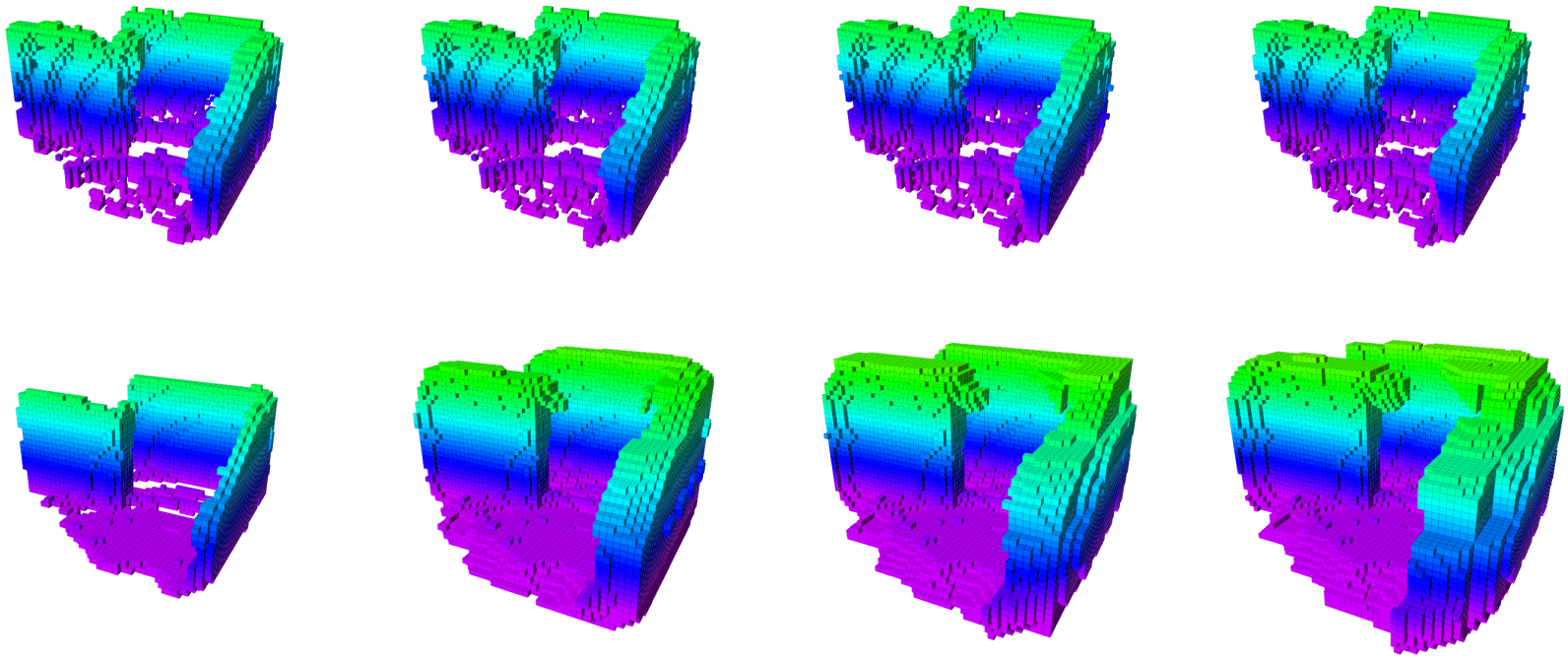


Computational Results

Dataset	Scans	Pts/Scan	Sampled Pts/Scan	Method	Time/Scan	Time (s)
Structured	12	3500	1506	BGK	**0.021**	**0.25**
				GPOM	0.091	1.1
				OctoMap	0.027	0.32
Unstructured	12	3500	1506	BGK	0.019	0.23
				GPOM	0.075	0.90
				OctoMap	**0.018**	**0.22**
FR-079	66	89445	7601	BGK	**0.15**	**9.6**
				GPOM	0.28	18.4
				OctoMap	0.15	10.1

Computational Results

Comparison to BCM updates:



Results of updating both BGKOctoMap (Top) and GPOctoMap-NBCM (Bottom) after the introduction of 1, 15, 30, and 60 scans containing the same data (Left to Right).

Long-term Mapping

BGKOctoMap is stable when mapping the same region

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- Why isn't the BCM?

Long-term Mapping

BGKOctoMap is stable when mapping the same region

- Why isn't the BCM?
  - Provably:

$$\lim_{K \to \infty} p(y_* = 1 | X, y) = \begin{cases} 1, & \text{if } \mu_1 > 0 \\ 0.5, & \text{if } \mu_1 = 0 \\ 0, & \text{if } \mu_1 < 0 \end{cases}$$

in the case of the same data (X, y) input to the BCM K times and "squashed" (derivation in thesis, for the curious)

Recent Work

BGKOctoMap is comparable in speed to GPOctoMap, has attractive long-term mapping benefits

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Recent Work

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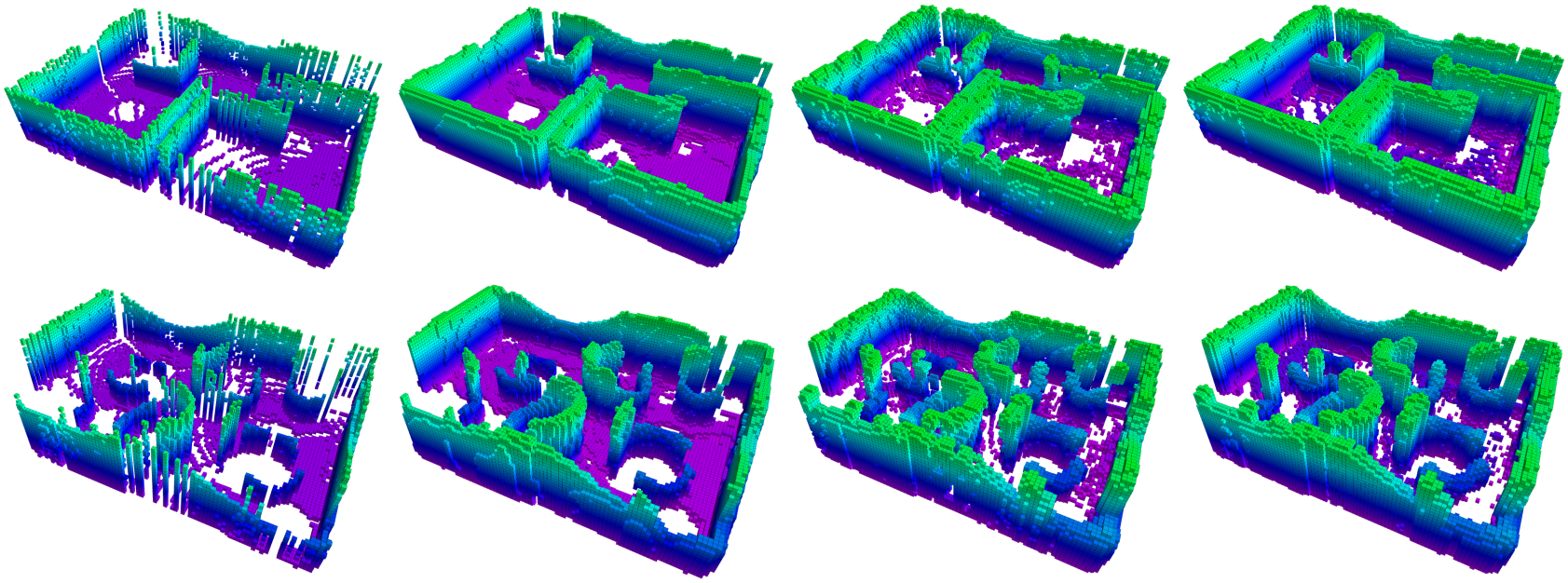
- Prediction of GPOctoMap is more accurate
- How can we improve accuracy of BGKOctoMap?
- Previous work used free space samples linearly interpolated along sensor rays
- Our recent work has been done with the point-to-line distance used by O'Callaghan and Ramos [17]:

$$x_{free} = \begin{cases} P, & \text{if } d < 0 \\ P + d\frac{PQ}{|PQ|}, & \text{if } 0 \leq d \leq 1 \\ Q, & \text{if } d > 1 \end{cases}$$

$$d = \frac{PQ \cdot Px_*}{|PQ|}.$$

Recent Work



Preliminary results using point-to-line distance for free-space representation. Left to Right: OctoMap, GPOctoMap, BGKOctoMap, BGKOctoMap using point-to-line distance. Top: Structured environment; Bottom: Unstructured environment.

Summary

Summary of BGKOctoMap

- Uses nonparametric Bayesian kernel inference
  - No explicit training step
  - Uncertainty estimates provided
- Sparse kernels
- Test data octrees
- Reliable long-term performance
- Predictably transitions to a prior “unknown” state of 0.5 mean with high variance when there is no training data

Table of Contents

Summary of Contributions

Two approaches to learning=aided 3D occupancy mapping have been presented:

- Overlapping Hilbert maps
  - Substantial time improvement over Hilbert maps
  - Comparable accuracy to HM
  - Still not real-time ready
  - No uncertainty estimates
- BGK occupancy maps
  - Real-time viable
  - Comparable accuracy to GPOM
  - Provides uncertainty estimates
  - Improved stability mapping same region long-term, compared to GPOM+BCM

Learning-aided 3D Mapping Library (LA3DM) on Github:
`https://github.com/RobustFieldAutonomyLab/la3dm`

Avenues for Future Work

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- Learning the kernel length scale
  - Making the kernel length scale dependent on local data

Questions

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