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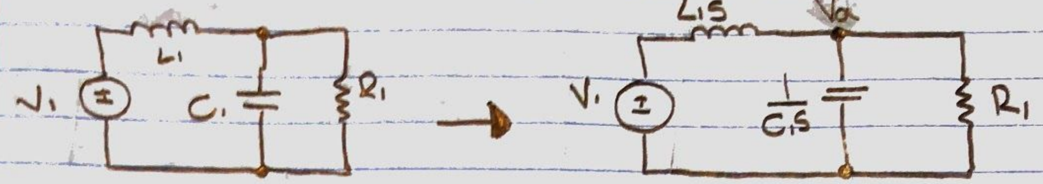
PEE II Design Project - 332:222 - Spring 2018

Design Type - A

Task 1:

A) Woofer [Low PASS Filter]



<u>NVM:</u>

$$0 = \frac{V_a}{R_1} + \frac{V_a}{\frac{1}{C_1 s}} + \frac{V_a - V_1}{L_1 s} = \frac{V_a}{R_1} + V_a C_1 s + \frac{V_a}{L_1 s} - \frac{V_1}{L_1 s} = 0$$

$$\frac{V_1}{L_1 s} = \frac{V_a}{R_1} + V_a C_1 s + \frac{V_a}{L_1 s} = V_a\left[\frac{1}{R_1} + C_1 s + \frac{1}{L_1 s}\right] = \frac{V_1}{L_1 s}$$

$$V_a\left[\frac{1}{R_1} + C_1 s + \frac{1}{L_1 s}\right] \cdot \left[\frac{R_1 L_1 s}{R_1 L_1 s}\right] = V_a\left[\frac{L_1 s}{R_1 L_1 s} + \frac{R_1 C_1 L_1 s^2}{R_1 L_1 s} + \frac{R_1}{R_1 L_1 s}\right]$$

$$= V_a\left[\frac{L_1 s + R_1 L_1 C_1 s^2 + R_1}{R_1 L_1 s}\right] = \frac{V_1}{s L_1} ; \quad V_1 = V_a\left[\frac{L_1 s + R_1 L_1 C_1 s^2 + R_1}{R_1}\right]$$

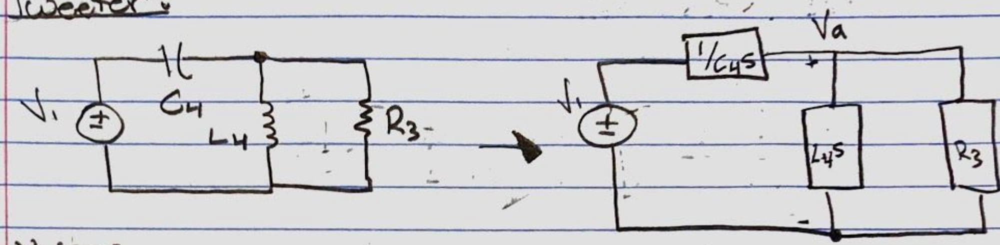
$$\frac{V_a}{V_1} = \frac{R_1}{L_1 s + R_1 L_1 C_1 s^2 + R_2} = H(s)$$

@ $R_1 = 16\,\Omega$

$$\boxed{H(s) = \frac{16}{16 L_1 C_1 s^2 + L_1 s + 16}}$$

← Low Pass

Tweeter:



NVM:

$$\frac{V_a - V_1}{\frac{1}{C_4 s}} + \frac{V_a}{L_4 s} + \frac{V_a}{R_3} = 0 = V_a C_4 s - V_1 C_4 s + \frac{V_a}{L_4 s} + \frac{V_a}{R_3} = 0$$

$$V_a\left[C_4 s + \frac{1}{L_4 s} + \frac{1}{R_3}\right] = V_1 C_4 s$$

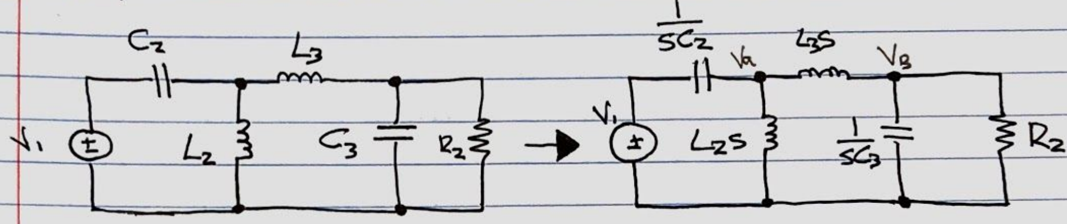
$$V_a\left[C_4 s + \frac{1}{L_4 s} + \frac{1}{R_3}\right]\left[\frac{R_3 L_4 s}{R_3 L_4 s}\right] = V_a\left[\frac{s^2 R_3 L_4 C_4 + R_3 + sL_4}{R_3 L_4 s}\right]$$

$$= V_1 C_4 s \longrightarrow \frac{V_a}{V_1} = \frac{s^2 R_3 L_4 C_4}{s^2 R_3 L_4 C_4 + sL_4 + R_3} = H(s)$$

@ $R_3 = 16\Omega$

$$\boxed{H(s) = \frac{s^2 16 L_4 C_4}{s^2 16 L_4 C_4 + sL_4 + 16}} \leftarrow \text{High Pass}$$

Bandpass: Transfer function:



@ V_a

$$\frac{V_a}{L_2 s} + \frac{V_a - V_1}{\frac{1}{sC_2}} + \frac{V_a - V_B}{L_3 s} = 0$$

@ V_B

$$\frac{V_B - V_a}{L_3 s} + \frac{V_B}{\frac{1}{sC_3}} + \frac{V_B}{R_3} = 0$$

$$\frac{V_a}{L_2 s} + (V_a - V_1)sC_2 + \frac{V_a - V_B}{L_3 s} = 0$$

$$\frac{V_B - V_a}{L_3 s} + V_B s C_3 + \frac{V_B}{R_3} = 0$$

plugged into matlab code:

Matlab Code To Solve Nodal Equations:

```
syms C2 C3 L2 L3 R s Va Vi Vb;


funcs = [
(Va-Vi)/(1/(C2*s))+Va/(L2*s)+(Va-Vb)/(L3*s);
(Vb-Va)/(L3*s)+(Vb)/(1/(s*C3))+Vb/(R);
];

sol = solve(funcs==0, [Vb,Va]);

transfer_function = sol.Vb/Vi;

disp(transfer_function);

```

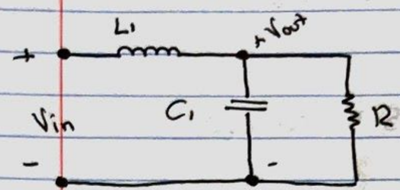
$$\frac{16 C_2 L_2 s^2}{(16 C_2 C_3 L_2 L_3 s^4 + C_2 L_2 L_3 s^2 + (16 s^2 (C_2 L_2 + C_3 L_2 + C_3 L_3)) + (s(L_3 + L_2)) + 16}$$

Bandpass Transfer Function:

Task 2:

Task 2:

Low Pass Filter:



$$H(s) = \left[\frac{16}{s^2(16L_1C_1) + L_1 s + 16}\right]$$

standard form

$$= \frac{1/L_1C_1}{s^2 + \left[\frac{1}{16C_1}\right]s + \frac{1}{L_1C_1}}$$

Assuming this is critically damped $\alpha^2 = \omega_0^2$

Lower crossover frequency (Hz) = 125 = $785.3981 \ \frac{rad}{s}$

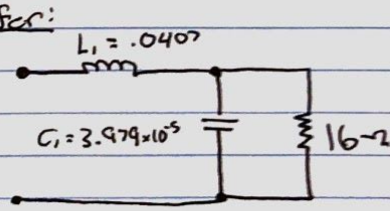
$$2\alpha = \frac{1}{16C_1} \rightarrow \alpha = \frac{1}{32C_1} \rightarrow \alpha^2 = \omega_0^2 \rightarrow \frac{1}{1024C_1} = 616850.2751$$

$$\boxed{C_1 = 3.979 \times 10^{-5} \text{ F}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow L_1 = \frac{1}{\omega_0^2 C_1} = \frac{1}{(616850.2751)(3.979 \times 10^{-5})}$$

$$\boxed{L_1 = 0.0407 \text{ H}}$$

Woofer:



High Pass Filter:

$$H(s) = \frac{s^2}{s^2 + s\left(\frac{1}{16C_4}\right) + \frac{1}{L_4C_4}}$$

Assuming critically damped

$\alpha^2 = \omega_0^2$

upper crossover frequency = 2500Hz = 15707.96 $\frac{rad}{s}$

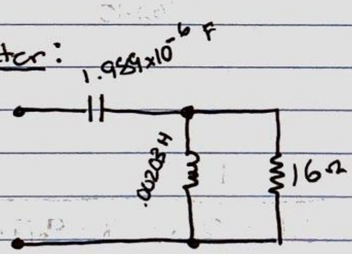
$$(15707.96)^2 = \frac{1}{(32C)^2} \rightarrow 246740007.4 = \frac{1}{1024C^2}$$

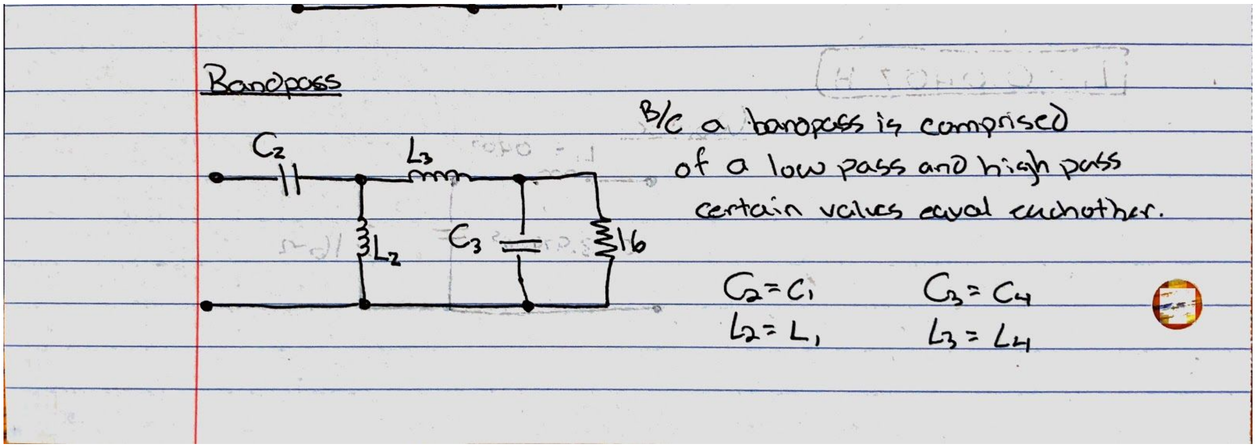
$$\frac{1}{C^2} = 2.5266 \cdot 10^{11} \rightarrow \boxed{C = 1.989 \times 10^{-6}\,F}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow L_1 = \frac{1}{\omega_0^2 C_1} = \frac{1}{(15707.96)^2 (1.989 \times 10^{-6})}$$

$$\boxed{L_1 = 0.00203H}$$

Tweeter:

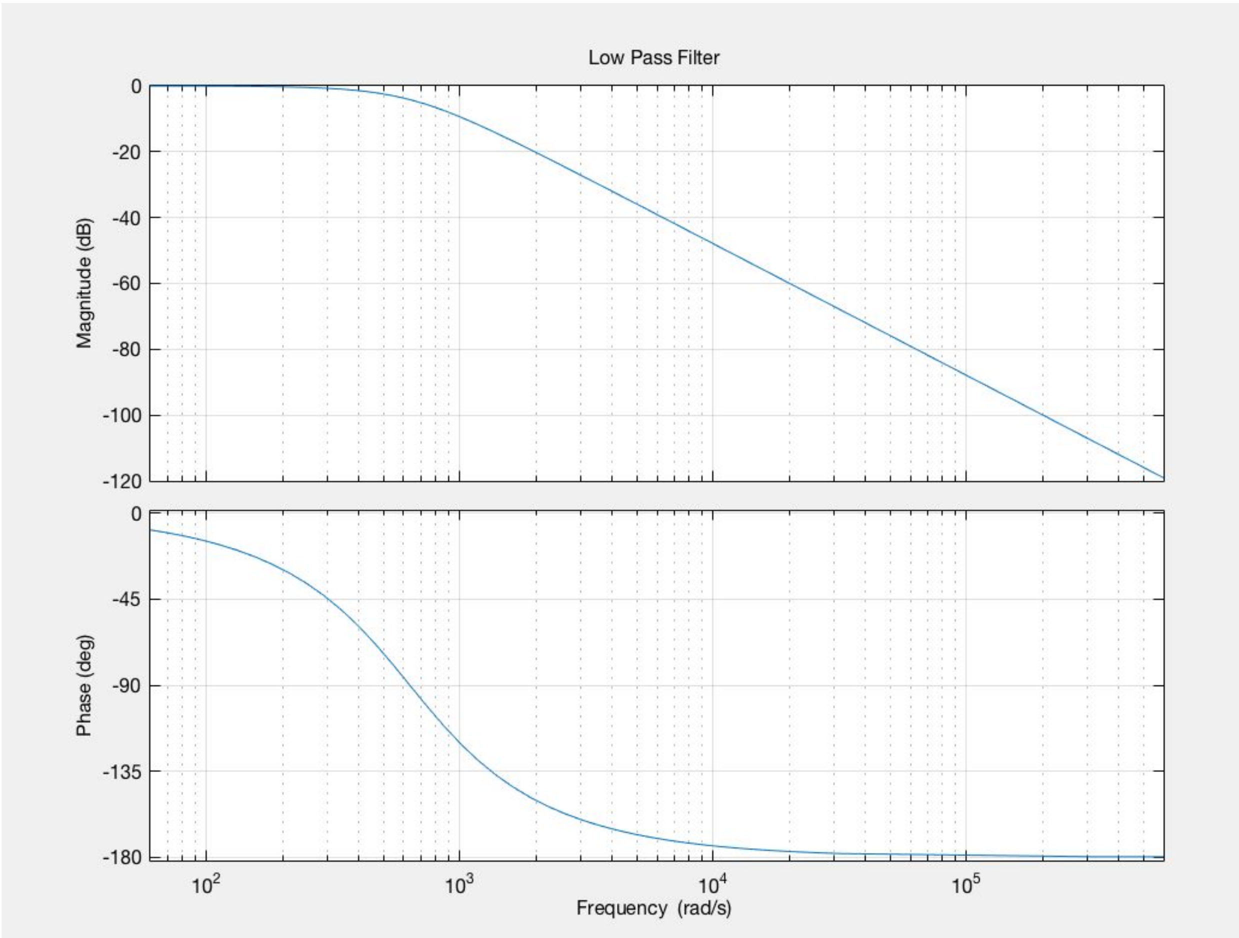




<u>Explanation</u>:

The process to finding the L and C values was a relatively difficult process to figure out but once I did, it all made sense. The first step is taking the transfer function for either Woofer or Tweeter and putting it in standard form. From here we know that $\alpha^2 = \omega_0^2$ because the circuit is critically damped. Once we know this relationship we go to our crossover frequencies, lower for low pass and higher for high pass and converted them into rad/s by multiplying by 2pi. Now that we have the crossover frequency and the relationship between alpha and omega, we can apply the relationship that $2\alpha = \frac{1}{RC}$. From here you solve for alpha making sure to square all terms in the fraction leaving you with $\alpha = \frac{1}{4R^2C^2}$ and equate this to ω_0^2 making sure you square your converted crossover frequency. From there you simply solve for C knowing that R is equal to 16 ohms. Once you have found C finding L is easy, because of the relationship $\omega_0 = \frac{1}{\sqrt{LC}}$. Simply plug in the crossover frequency and the C value that was just calculated and manipulate algebraically to find L. Once these values are found for both high and low pass we can draw various conclusion about the LC values for bandpass. The bandpass filter is comprised of a high pass and then a low pass cascaded together, therefore it is same to assume that C2 = C1, L2 = L1 and C3 = C4 and L3 = L4.

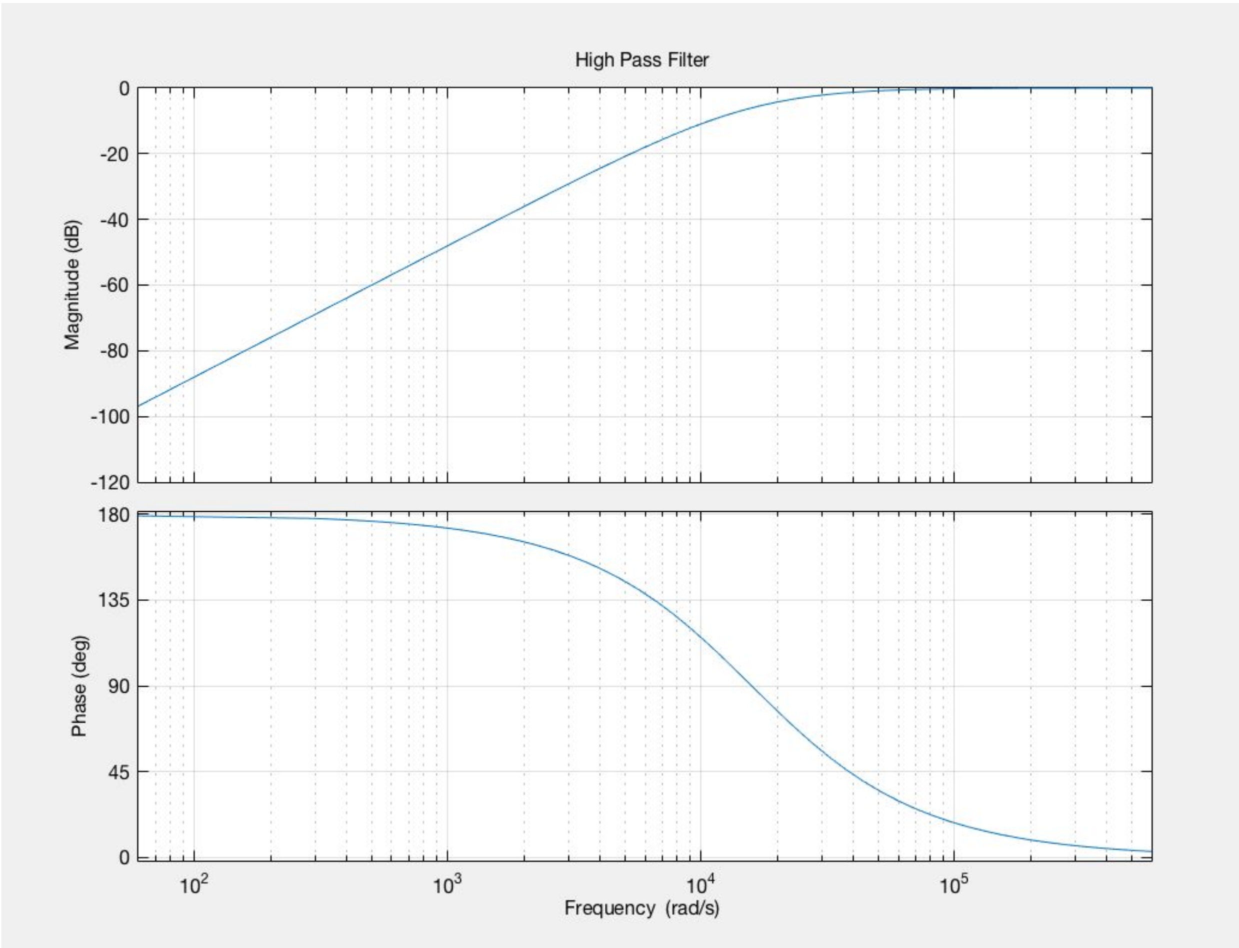
Task 3: Matlab Bode Plots:
Low Pass Filter:



<u>Explanation</u>: C1= 3.979* $10^{-5}F$ L1 = .0407 H

From immediate first glance the graph represents a low pass filter because of the shape of the line. Moreover is was checked again by looking at -6dB where the frequency is 763 rad/s. This is within 3% error of 2pi times my lower crossover frequency.

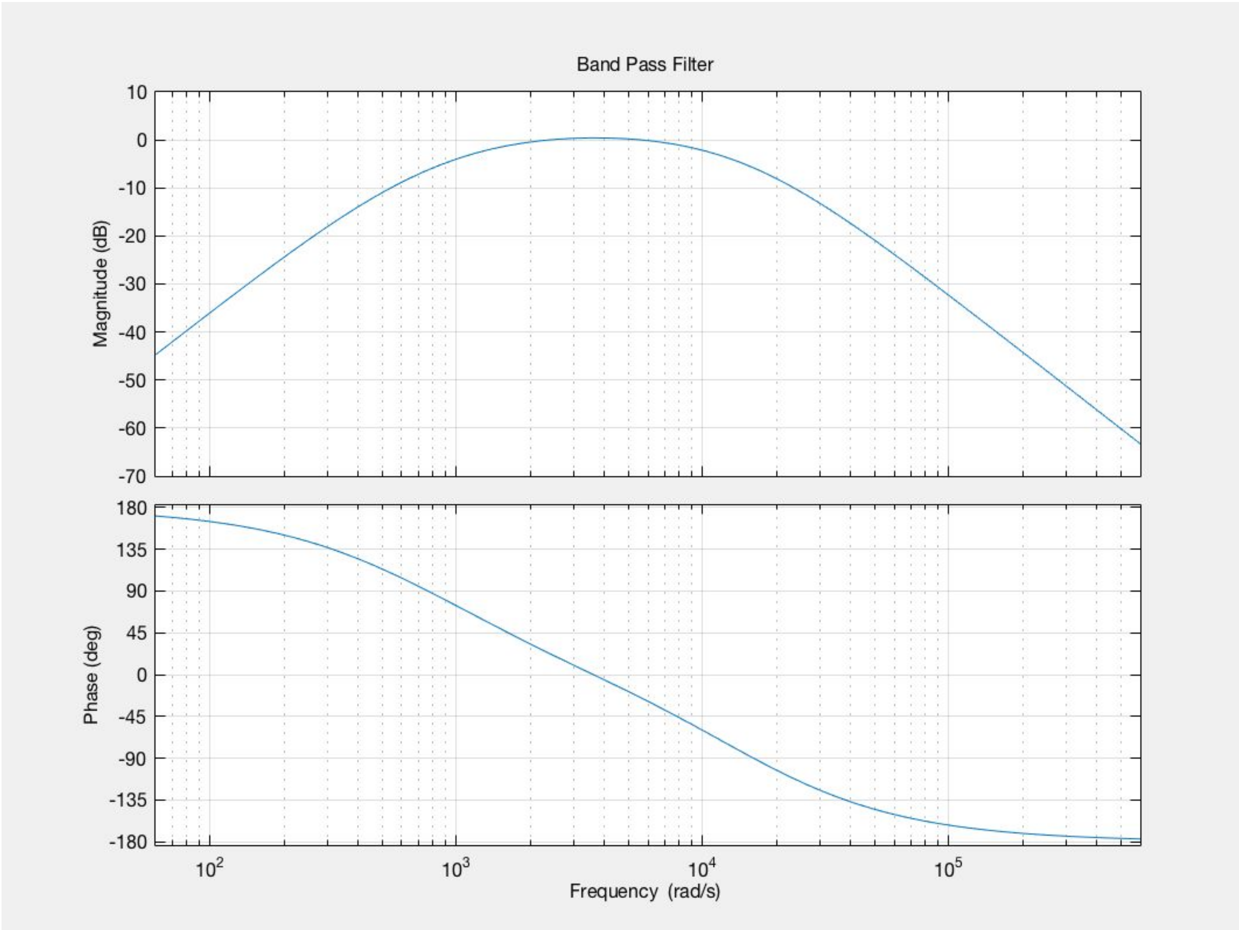
High Pass Filter:



Explanation: C4= 1.989* $10^{-6}F$ L4 = .00203 H

From immediate first glance the graph represents a high pass filter because of the shape of the line. Moreover is was checked again by looking at -6dB where the frequency is 15900 rad/s. This is within 1.2% error of 2pi times my higher crossover frequency.

BandPass Filter:

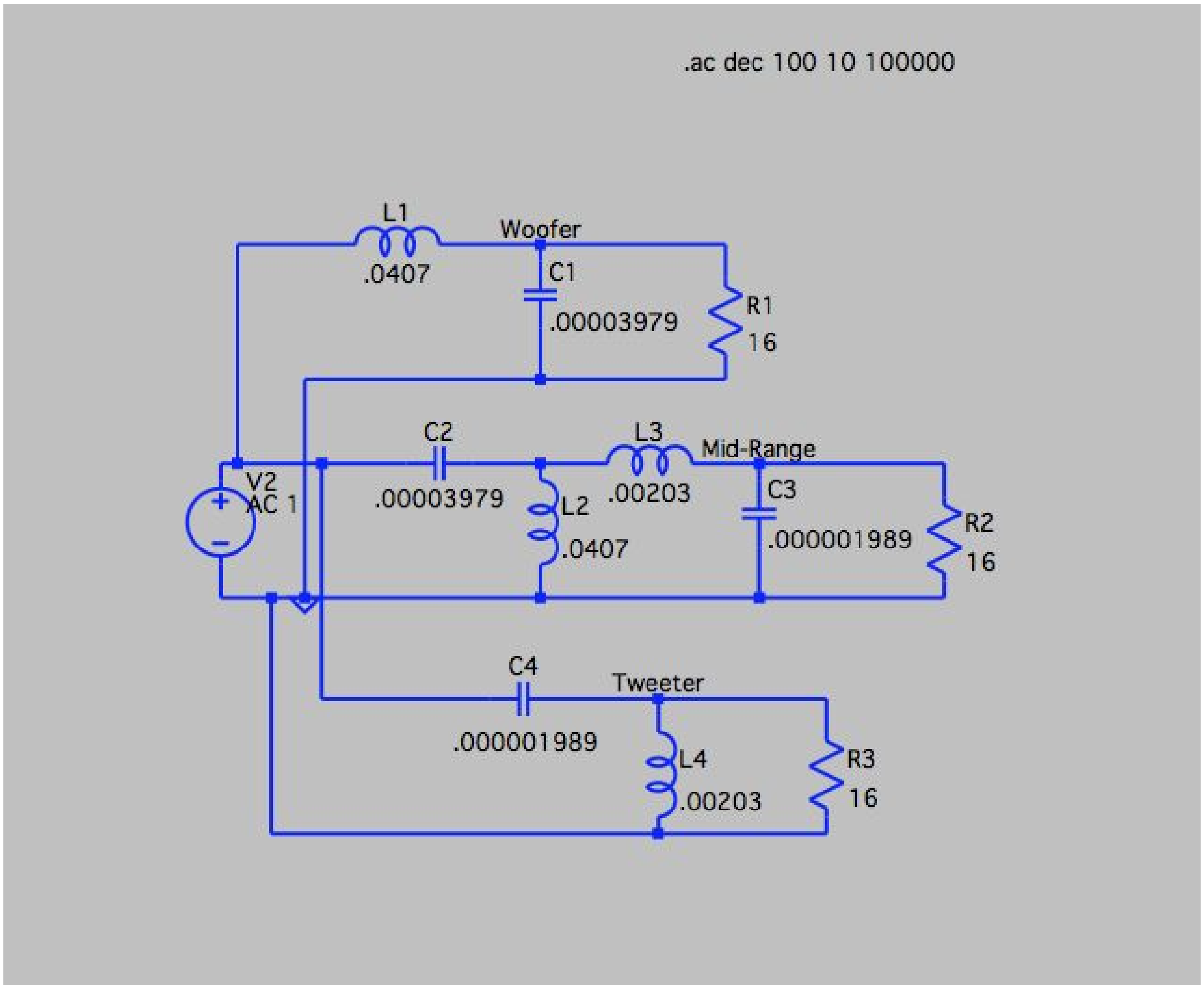


Explanation:

From immediate first glance the graph represents a band pass filter because of the shape of the line. Moreover is was checked again by looking at -6dB where the frequency is 789 rad/s on the left side of the peak and 16500 on the right side of the peak. These values represent the frequencies and they both fall under 5% error, 0.4% error for the lower frequency and 4.7% error for the upper frequency.

Task 4:

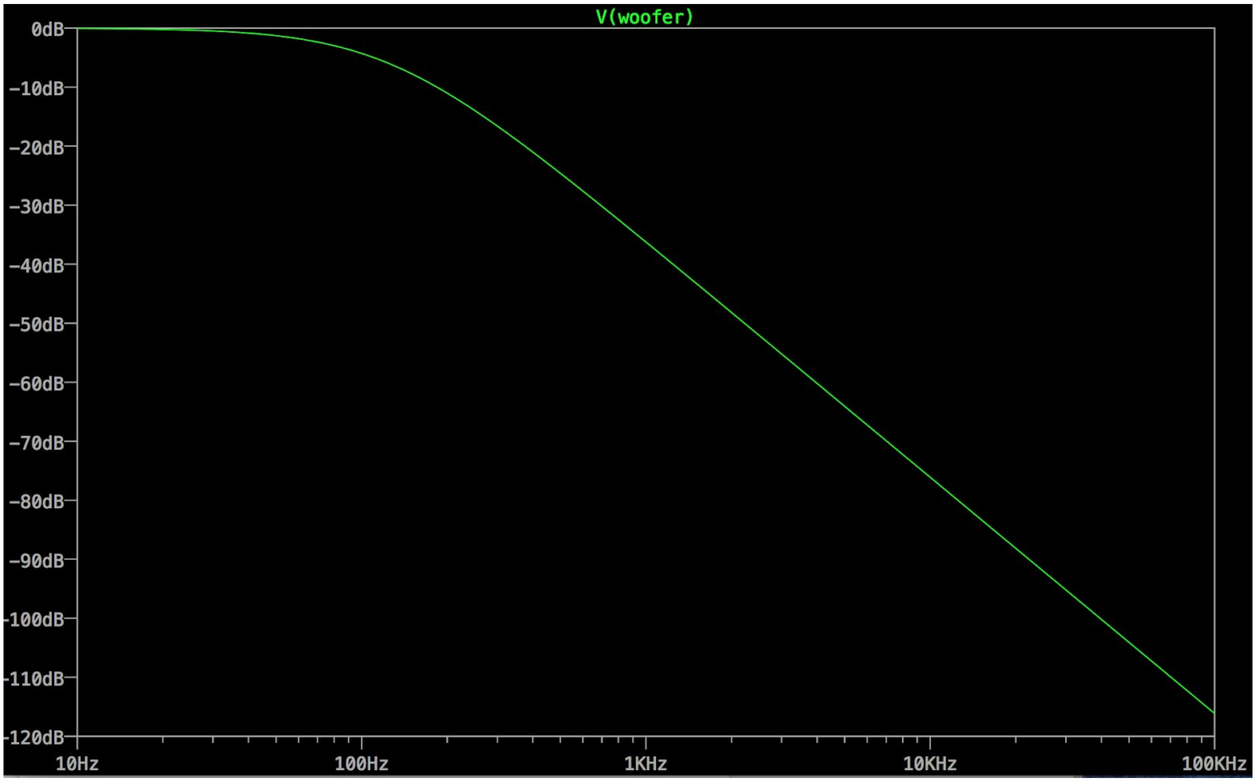
Schematic:



Explanation:

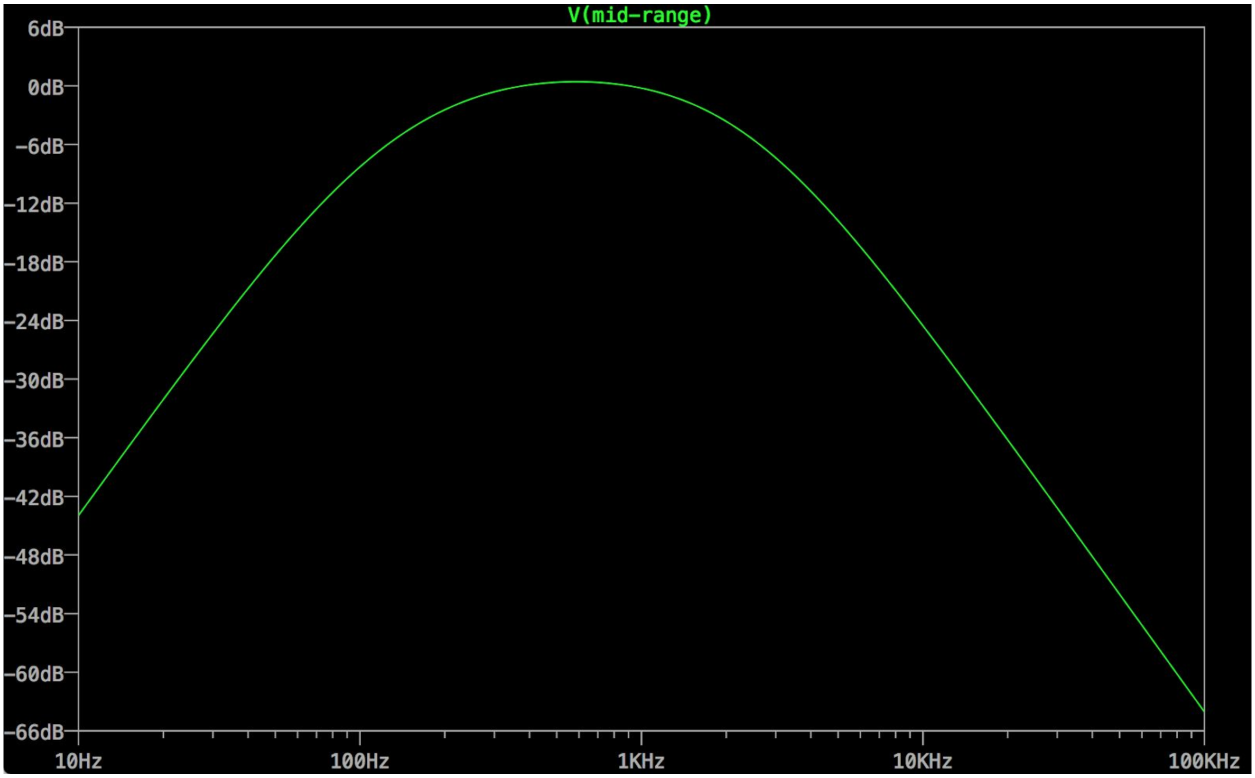
This is the schematic of the circuit with all the values calculated plugged in, it is this schematic that yielded the following plots.

Woofer:



Explanation: Immediately again we see the plot is a low pass filter plot, however because it is plotted on a x axis that is Hz, the -6db frequency is 125 Hz my lower crossover frequency.

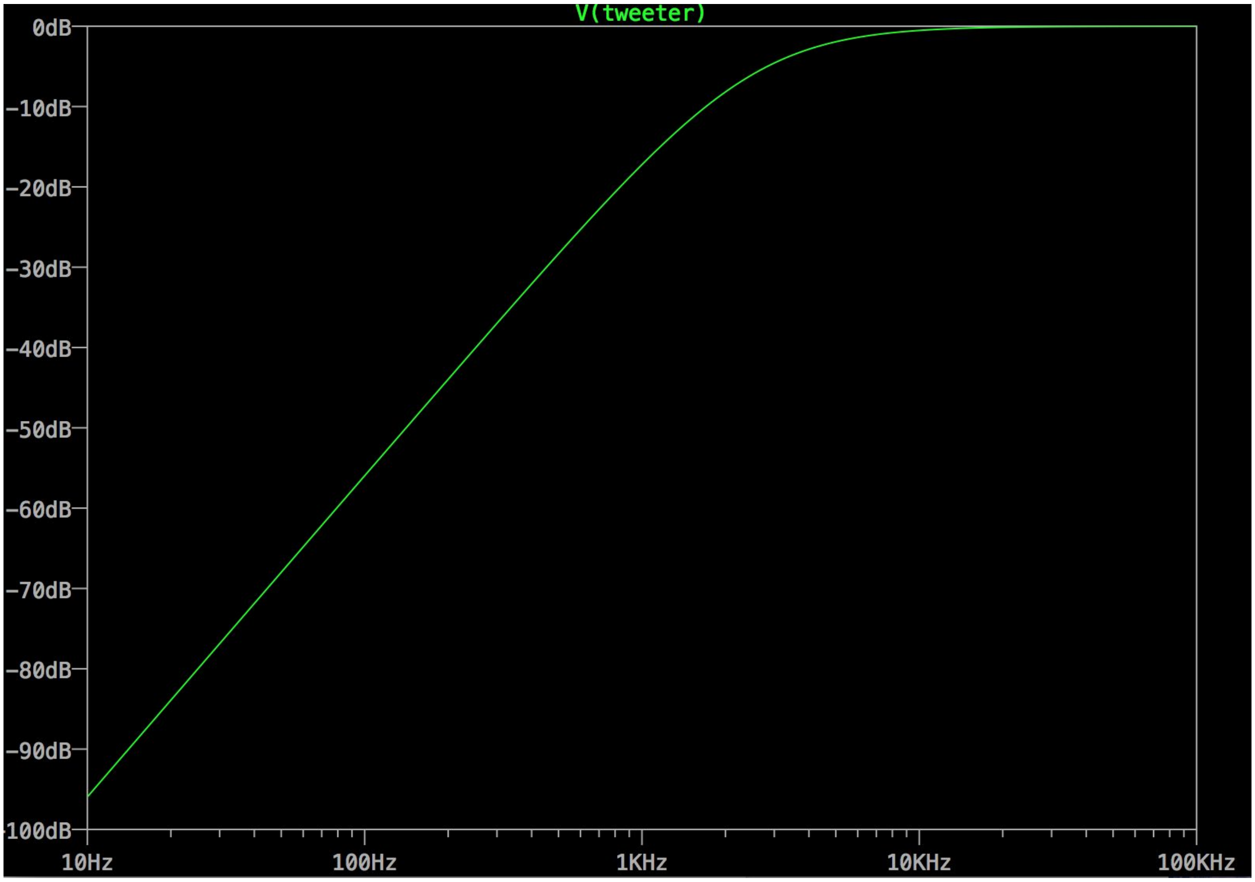
Bandpass:



Explanation:

Immediately again we see the plot is a Band pass filter plot, however because it is plotted on a x axis that is Hz, the -6db frequencies are 125 Hz my lower crossover frequency and 2500 Hz my upper crossover frequency.

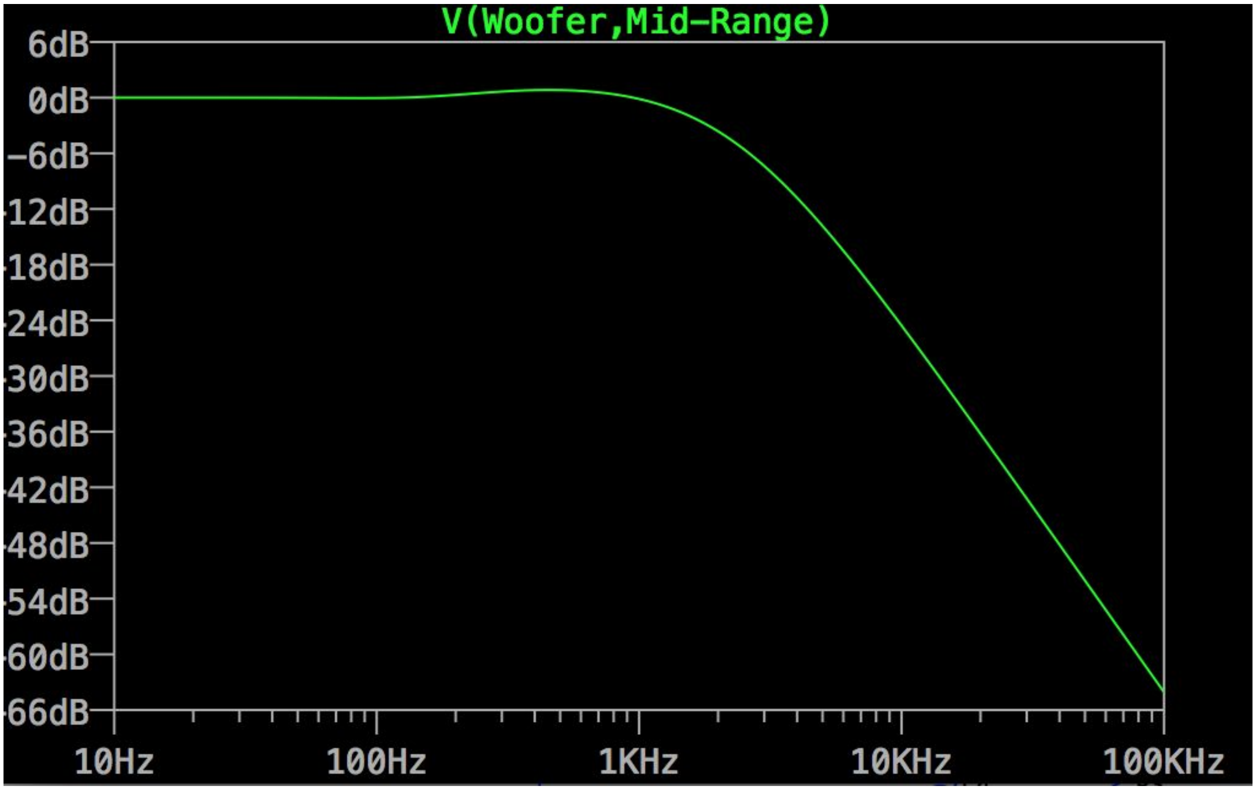
Tweeter



Explanation:

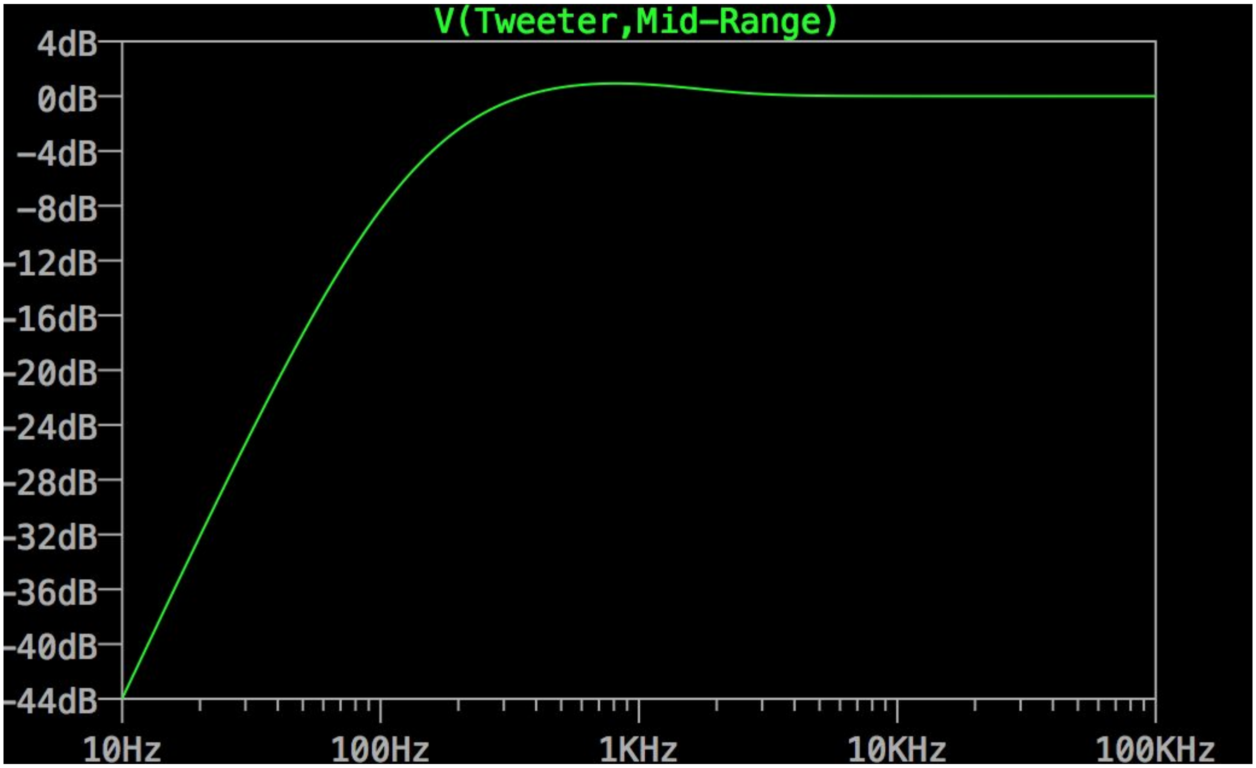
Immediately again we see the plot is a high pass filter plot, however because it is plotted on a x axis that is Hz, the -6db frequency is 2500 Hz my upper crossover frequency.

Differential, Woofer to Midrange



Explanation:
This shows the upper frequency while still plotting the low pass filter.

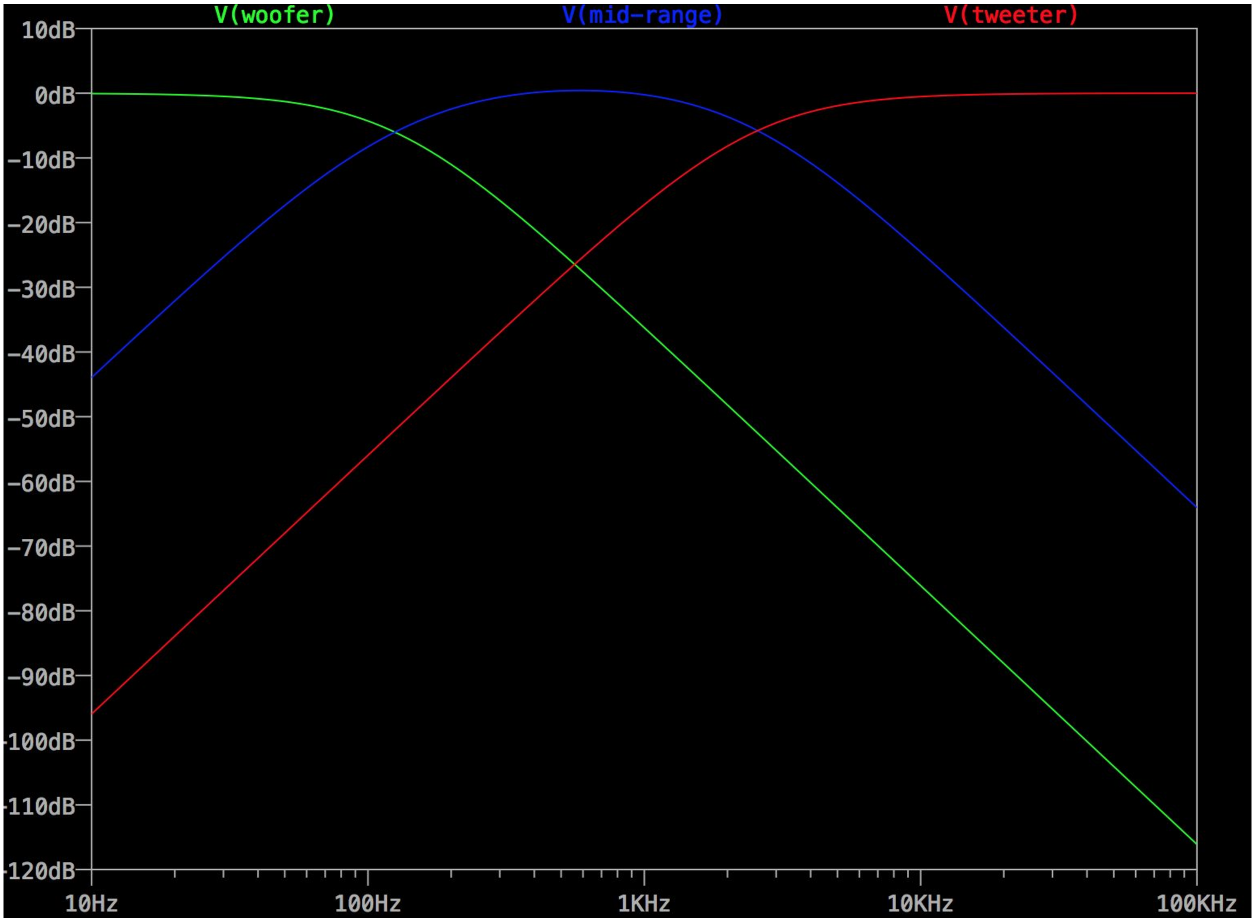
Differential, Tweeter to Midrange



Explanation:

This shows the lower frequency while still plotting the high pass filter.

All Graphs on Same Plot



Explanation:

The intersection of the plots represents the crossover frequencies.