Keeyan Haghshenas March 7th, 2018 Probability and Random Processes HW2

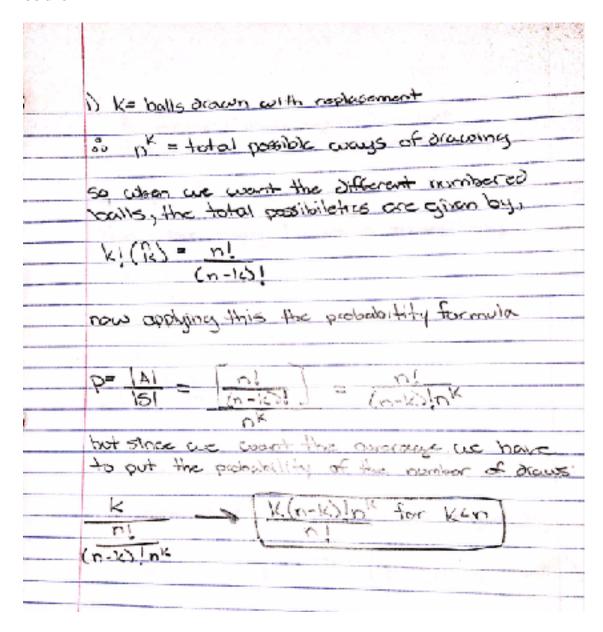
HW2: Coupon Collecting

Given:

There is an urn containing n balls numbered from 1 -> n. Sampling with replacement: A ball is drawn, its number is recorded and then the ball is thrown back in the urn.

1: How many draws does it take on average to record k different numbers? When (k<n)

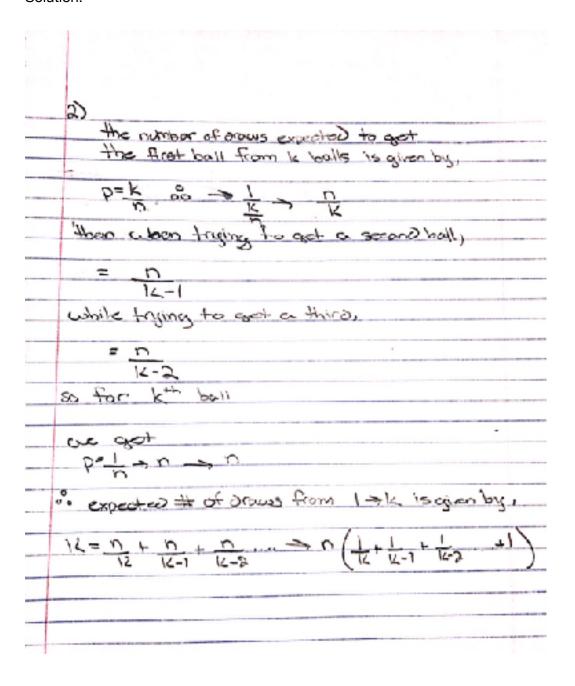
Solution:



Explanation:

To calculate this we have to start off by first calculating the total number of ways to draw k balls from n balls in the urn. We first do this by setting n balls in the urn to the power of k balls being drawn because the k balls are being drawn with replacement and we don't care about repetition of the samples. Then we have to move to calculating the number of ways of drawing k balls with different results (without repetition) rather than drawing the same ball over and over again, because again it is with replacement we get, the factorial of n balls divided by the the factorial of the difference of n-k. We then apply this to the probability formula using the non-repetition as our desired set and the repetition as our sample set. This yields the probability though because we want the average amount of draws to record k different numbers we must take k and divide it by the probability yielding the result for k<n.

2: How many draws does it take on average to record numbers 1 through k? Solution:



Explanation:

This is a geometric probability. Therefore the first ball being drawn's probability is given as k/n which we then flip. Then as we draw more balls with replacement we take this fraction and subtract the denominator by 1 less than the draw number. We then see that the result average is the expected values of k's summation.