Problem 2

Show that the subset of $\mathbb{R}^3$ consisting of all the vectors whose first component (coordinate) is 0 is a linear subspace of $\mathbb{R}^3$

In order to proove this, we have to show 2 Things

1) if $\vec{u}$ and $\vec{v}$ are in a linear subspace, then $\vec{u} + \vec{v}$ are also in the subset, and
2) also if c is a scalar and $\vec{u}$ is in the subspace, then $c\vec{u}$ is also in the subspace.

1) $\langle 0, a_1, a_2 \rangle$ and $\langle 0, b_1, b_2 \rangle$

$\langle 0, a_1, a_2 \rangle + \langle 0, b_1, b_2 \rangle = \langle 0, a_1+b_1, a_2+b_2 \rangle$

$\underline{\langle 0, a_1+b_1, a_2+b_2 \rangle}$ is in the subset

2) $c \cdot \langle 0, a_1, a_2 \rangle = \underline{\langle 0, ca_1, ca_2 \rangle}$

Both $\langle 0, a_1+b_1, a_2+b_2 \rangle$ and $\langle 0, ca_1, ca_2 \rangle$ are in the subset

Problem 3 Show that the subset of $\mathbb{R}^3$ consisting of all the vectors whose first component (coordinate) is ≥ 0 is not a linear subspace $\mathbb{R}^3$.

Must proove 1. $\vec{u} + \vec{v}$ is in subspace

and 2. $c \cdot \vec{u}$ is in subspace.

1.) Because $x > 0$ + $y > 0$ is always greater than 0, we know this part passes.

2). $c \cdot \vec{a}$ though does NOT pass.

If c is negative, the first coordinate will not be above 0.

$\vec{a} = \langle 5, 1, 2 \rangle$

$c = -1$

$\vec{a} \cdot c = \underline{\langle -5, -1, -2 \rangle}$

This is NOT in the subset, so

Problem 4

a) are <3,4> and <1, -3> linearly independent.

Must show for

$$c_1 \begin{bmatrix} 3 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Only viable options for this to be true are $c_1 = c_2 = 0$.

$$3c_1 + c_2 = 0 \quad \rightarrow \times 3$$
$$4c_1 - 3c_2 = 0$$
$$\downarrow \times 3$$
$$9c_1 + 3c_2 = 0$$
$$4c_1 - 3c_2 = 0$$
$$\downarrow$$
$$13c_1 = 0$$
$$\downarrow$$
$$c_1 = 0$$
$$\downarrow$$
$$3(c_1) + c_2 = 0$$
$$\downarrow$$
$$3(0) + c_2 = 0$$
$$c_2 = 0$$

$$\boxed{c_1 = c_2 = 0}$$

Because c_1 and c_2's only viable answer is 0, we know this is linearly independent

Problem 4

b) $\langle 2, -3 \rangle$ and $\langle 6, -9 \rangle$

$$c_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + c_2 \begin{bmatrix} 6 \\ -9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2c_1 + 6c_2 = 0$$

$$-3c_1 - 9c_2 = 0$$

$$2c_1 = -6c_2$$

$$\underline{c_1 = -3c_2}$$

$$2(-3c_2) + 6c_2 = 0$$

$$-6c_2 + 6c_2 = 0$$

$$0 = 0$$

We know $\underline{c_1 = -3c_2}$

Because there are more solutions than just 0, so it is not linearly independent.

Problem (5)

a) Show that vectors $\langle 1,2 \rangle$ and $\langle 0,1 \rangle$ form a basis for R^2. make sure you .....

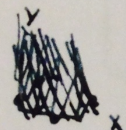
Must show:

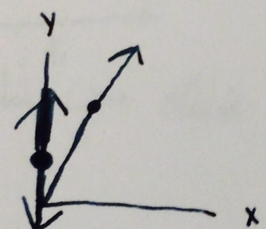
1) $\vec{u}$ and $\vec{v}$ are in $\mathbb{R}^2$

2) $\vec{u}$ and $\vec{v}$ are linearly independent.

3) any vector in $\mathbb{R}^2$ can be expressed as a linear combination of $\vec{u}$ and $\vec{v}$

1.





They both can be graphed on a 2-axis plane, they are in $\mathbb{R}^2$.

2.

$$c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$c_1 = 0$

$2c_1 + c_2 = 0$

$\downarrow$

$c_2 = 0$

$\downarrow$ only viable answers for c_1 and c_2 are 0, so they are linearly independent.

Problem 5

3. To show all ~~these~~ vectors in $\mathbb{R}^2$ can be expressed as a linear combination of $\vec{u}$ and $\vec{v}$, we set them equal to x and y.

$$c_1\begin{bmatrix}1\\2\end{bmatrix} + c_2\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}x\\y\end{bmatrix}$$

$$c_1 = x$$

$$2c_1 + c_2 = y$$

$$\boxed{y = 2x + c_2}$$

$\downarrow$

This can elicit all values in the $\mathbb{R}^2$ space.

b) Express the vector <4, 4> in this basis.

We just solved $y = 2x + c_2$

$$y = 2x + c_2$$

$$4 = 8 + c_2$$

$$\boxed{c_2 = -4}$$

This shows that <4, 4> is in this basis

Problem 5

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$$c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$c_1 = x$$

$$2c_1 + c_2 = y$$

$$\boxed{y = 2x + c_2}$$

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Problem 6

In $\mathbb{R}^3$ let two subspaces be

(a) What is $U \oplus V$? What is its dimension?

(b) What is $U \cap V$? What is its dimension?

$U = \{\langle a, b, 0\rangle \mid a, b \in R\}$ $V = \{\langle 0, b, c\rangle \mid b, c \in R\}$

$\vec{U} = \langle a, b, 0\rangle$

$\vec{V} = \langle 0, b, c\rangle$

$U \oplus V = \langle a, b, c\rangle$

$U \oplus V$ is the merge of the two. It is the entire subspace that these two create.

They create $\langle a, b, c\rangle$, so they are in $\mathbb{R}^3$.

$U \cap V$ is the interjection of U and V.

so

$\boxed{U \cap V = \langle 0, b, 0\rangle}$

This spans in $\mathbb{R}^1$.

Problem 7

In $\mathbb{R}^2$, let two subspaces be

$U = \{ \langle x, y \rangle \mid 4x = 0 \}$ and $V = \{ \langle x, y \rangle \mid x + y = 0 \}$

a) Dimension of U?

The dimension of U is 1.

b) Dimension of V?

The dimension of V is 1.

c) What is $U \oplus V$? What is it's dimension?

$U \oplus V$ is $\mathbb{R}^2$. It is 2 dimensions.

d) What is $U \cap V$? What is it's dimension?

$U \cap V$ is the intersection of the two lines, which is just the point they intersect at, which is just the origin, or $(0,0)$

Problem 8

In $\mathbb{R}^3$, let 2 subspaces be

$U = \{ \langle x, y, z \rangle \mid 4x = 0 \}$ and

$V = \{ \langle x, y, z \rangle \mid x + y = 0 \}$

a) Dimension of U?

U is a plane spanning ~~ulta~~ $y+z$ where $x = 0$. Dimension $(U) = 2$.

b) Dimension of V?

V is a plane spanning the z axis and the line $x = -y$.

$\text{Dim}(V) = 2$.

c) What is $U \oplus V$? What is its dimension?

U and V share the z axis and are two linearly independent Planes. These planes merge to encapsolute the ~~R~~ $\mathbb{R}^3$ Subspace. Also Have a dimension of 3.

d) What is $U \cap V$? What is its dimension?

~~The~~ $U \cap V$ is just a line. the z axis to be exact.

Its dimension 1