CS/ECE/ISyE 524 Final Exam Preparation

Details!

Final Exam

- 07:45AM-09:45AM, May 10, 2024.
- Van Vleck B102
- McBurney Students: 7:45AM-10:45AM, 3126 Mechanical Engineering Building

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- Closed Book, Closed Notes, Closed Everything!
- Calculators are allowed, but not your phones.
- I will provide the Slide of Trix (and other notes)—final standard "cheat sheet" posted by next Tuesday

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Other Announcements

- Prof. L Office Hours: May 7, 9AM-10AM

Today's Docket

1. Final Details
2. Review Course Topics
3. Questions
4. Practice Problems?

Final Exam

- Coverage: Everything in Lecture Notes (Lecture 1-Lecture 18)
- Probably Six Problems
  1. True/False
  2. Short Answer
  3. Duality
  4. Modeling
  5. Modeling
  6. Modeling

Final Exam

- No Note Sheet Allowed
- No calculators or other electronic devices
- Bring your ID to the exam and place it out. We will be checking IDs during the exam.
    - Also remember to take it with you
- Spread out in the room! (We may ask you to move)

Final Exam

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- Bring your ID to the exam and place it out. We will be checking IDs during the exam.
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When Exam is Over...

- I will ask everyone to put their pencils down
- You are not to pick up your pencil again, even to write your name.
- If you pick up your pencil again, I will take off 10% of grade.

Math Modeling—Being More Strict

- You *must* write proper mathematical notation and syntax to get credit. We will give less partial credit.
  - Writing JuMP code `@constraint(m, sum(x) == 1)` is *not* mathematical syntax
- You have to write specifically the set of entities over which you are doing a sum.
  - $\sum_i x_i$ is incorrect. i is an index, not a set
  - $\sum x_i \forall i \in S$ is incorrect
  - $\sum_{i=1}^4 x_i$ or $\sum_{i \in S} x_i$ is proper notation
- $x_k \geq 42 \forall k$ is also incorrect notation. k is an index. For all k *in what set?*

Lecture 2

- Geometeric properties of LP Feasible Region
- Graphing and Solving Linear Programs

Lecture 3

- Linear Algebra, Matrices, and Definitions
  - Linear Combinations, Convex combinations, Polyhedra
- Converting LP to Standard Form

Lecture 4

- Input-Output Linear Programs
- General Modeling:
  - Sets, Parameters, Vars
  - When to use Σ, when to use $\forall$
- Infeasible, Unbounded, Optimal Solution

Lecture 5

- Blending Constraints
- Multiperiod Planning
  - Backlogging

Lecture 6

- Modeling epigraph of convex piecewise linear-functions
  - $|\cdot|$
  - Mini-max (Maxi-Min)
- CPM/Project Scheduling

Lecture 7

- (More) Multiperiod
- Networks
- Min-Cost Network Flow
  - Transportation Problem, Assignment Problem
  - Shortest Path, Longest Path (DAG)
  - Max FLow
- Integer Properties of Solutions

Lecture 8

- Duality
  - Taking dual
  - Weak duality
  - Strong Duality
  - Complementary Slackness
- Economic Interpretation: Shadow Prices

Lecture 9

- Duality / Complementary Slackness
- Network Duality (and Interpretation)
- Max-Flow Min Cut

Lecture 10

- Quadratic Functions/Quadratic Forms
- Spectral Theorem
- PSD Matrices
- Definition of Convex and Concave Functions
- Examples of convex and concave functions

Lecture 11

- Least Squares is a (Convex) Optimization Problem
- Geometry of Least Sqaures
- Regression and Curve fitting
- Multiobjectve and Tradeoffs
- Pareto Cuirves
- Regularization
- Norm balls
- Hierarchical Optimization

Lecture 12

- Ellipsoids
- Degenerate ellipsoids
- Shifted Ellipdoids
- Quadraic Programs: Solution properties (Convex/Nonconvex)
- Where do Quadratic occur?
- Portfolio Optimization

Lecture 13

- Definition of Cone
- Types of Cones

1. Second Order Cone
2. Rotated Second Order Cone
3. Modeling them wuth Julia/JuMP
4. Rational Powers
5. Semidefinite Cone

Lectures 14 and 15

- Geometry of Feasible Region
- Relaxations and LP relaxation and Convex hull
- IP Modeling: Fixed Costs. Big M
- IP Modeling Variable Lower Bounds
- IP Modeling: "Simple" Logic

Lecture 16

- Logic Constraints and "Slide of Trix"
- Modeling restricted set of values (like SOS1)
- Sudoku

Lecture 17

- Set Cover Formulations
  - Enumerate/encode "feasible" patterns
- Cutting stock (and column generation)
- Traveling Salesman Problem
  - Subtour elimination and cutting planes
  - Miller-Tucker Zemlin Constraints

lecture 18

- Quadratic Assignment Problem
- Linearize (0,1) quadratic term:

$$x, y \in \{0, 1\}, z = xy \Leftrightarrow z \geq x + y - 1, z \leq x, z \leq y$$

- Piecewise Linear Functions
  - SOS2
  - Convex Combination Model
  - Multiple Choice Model

Problem 1

Consider the following LP:

$$
\begin{array}{lrcrcl}
\text{maximize } Z = & x_1 & + & 2x_2 & & \\
\text{s.t.} & -x_1 & + & x_2 & \leq & 4 \\
 & x_1 & - & 2x_2 & \leq & 2 \\
 & 4x_1 & + & a x_2 & \leq & 16 \\
 & x_1 & , & x_2 & \geq & 0
\end{array}
$$

Problem 1, cont'd

(a) Find a value of a such that there is a single optimal solution.

(b) Find a value of a such that there are an infinite number of optimal solutions.

(c) Find a value of a such that the problem is feasible but there are no optimal solutions.

Problem 2

Consider the following LP:

$$\begin{array}{lrcrcrcl}
\text{maximize } Z = & 2x_1 & + & 5x_2 & + & 3x_3 & & \\
\text{s.t.} & x_1 & - & 2x_2 & + & x_3 & \geq & 20 \\
 & 2x_1 & + & 4x_2 & + & x_3 & = & 50 \\
 & x_1 & , & x_2 & , & x_3 & \geq & 0
\end{array}$$

(a) Write the dual LP.

(b) Find the complementary dual solution to the primal solution $x_1 = 25, x_2 = 0, x_3 = 0$

(c) Is $x_1 = 25, x_2 = 0, x_3 = 0$ an optimal solution to the problem?

Problem 1

(a) $a = 0$

(b) $a = 8$

(c) $a = -8$

There are many possible answers for each. To see why these answers are valid, plot the lines.

Example

- Use a 0-1 variable δ to indicate whether *or not* the constraint $2x_1 + 3x_2 \le 1$ is satisfied.
  - x_1, x_2 are nonnegative continuous variables that cannot exceed 1.
- $\delta = 1 \Leftrightarrow 2x_1 + 3x_2 \le 1$
- M : Upper Bound on $2x_1 + 3x_2 - 1$. 4 works
- m : Lower Bound on $2x_1 + 3x_2 - 1$. -1 works.
- ε : 0.1

Example, Cont.

- $(\Rightarrow)$ Recall the trick.
  - $z = 1 \Rightarrow \sum_{j \in N} a_j x_j \leq b \Leftrightarrow \sum_{j \in N} a_j x_j \leq b + M(1 - z)$
- $2x_1 + 3x_2 + 4z \leq 5$
- $(\Leftarrow)$. Recall the trick.
  - $\sum_{j \in N} a_j x_j \leq b \Rightarrow z = 1 \Leftrightarrow \sum_{j \in N} a_j x_j \geq b + mz + \varepsilon(1 - z)$
- $2x_1 + 3x_2 + 1.1z \geq 1.1$

$$
\begin{aligned}
2x_1 + 3x_2 + 4z &\leq 5 \\
2x_1 + 3x_2 + 1.1z &\geq 1.1
\end{aligned}
$$

Slide o' Trix: Drink Production

- Facilities $F = B \cup V \cup G$
  - $B \cap V = B \cap G = V \cap G = \emptyset$
- $B \subset F$: Beer production
- $V \subset F$: Vodka production
- $G \subset F$: Gin production
- Drinks $D = \{\text{Beer, Vodka, Gin}\}$
- Colleges $C = T \cup I$
- $T \subset C$: Big 10 Colleges
- $I \subset C$: Ivy league college

- a_{cd}: Amount of drink $d \in D$ required by college $c \in C$
- b_f: Maximum amount of production at facility F
- α_f: Per unit production cost at $f \in F$
- β_{fc}: Per unit tranportation cost from $f \in F$ to $c \in C$

A Linear Program

Meet college requirements for each drink at minimum cost

Some Problems of Varying Degrees of Difficulty

- You can open at most 4 Vodka Production Facilites
- If you open a vodka production facility, you must ship at least Δ units out of the facility
- If you open at least 3 vodka facilities, then you must open at least 6 Gin facilities
- You must open at least 2 beer facilities or 2 vodka facilities or 2 gin facilities
- If you ship more Gin to Big Ten College than to Ivy colleges, then you must ship less Beer to Ivy Colleges than Big 10 Colleges
- If Ivy League colleges drink more vodka and more Gin than Big Ten students, then the Big Ten students GET NO BEER!