03.02.25

Корректность метода двух {указателей/индексов}

Код:

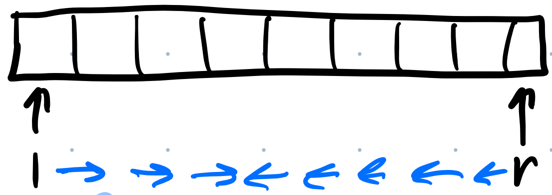
```
inp: arr, x
srt = sorted(arr)
l = 0
r = len(arr) - 1

while (l != r):
    s = srt[l] + srt[r]
    if (s == x): return True
    if (s > x): r -= 1
    if (s < x): l += 1
return False
```

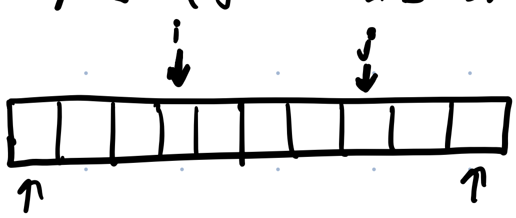
2	6	9	1	1	2	5	5	3

1	1	2	2	3	5	5	6	9

1) $\nexists\, i \neq j : srt[i] + srt[j] = x$



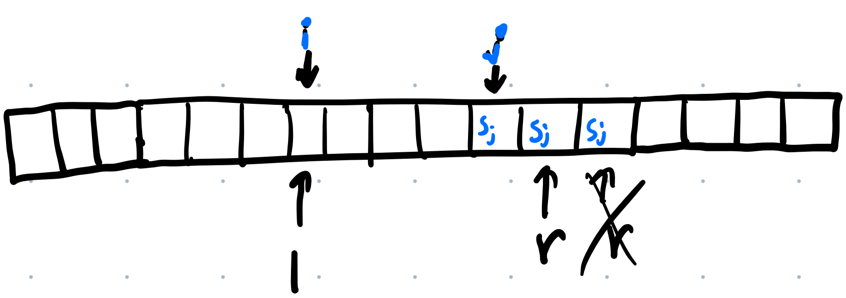
2) $\exists\, i \neq j : srt[i] + srt[j] = x$



Без огр. общности:

$i < j$

$l = i$



$s = srt[i] + srt[r]$

1) $s == x$

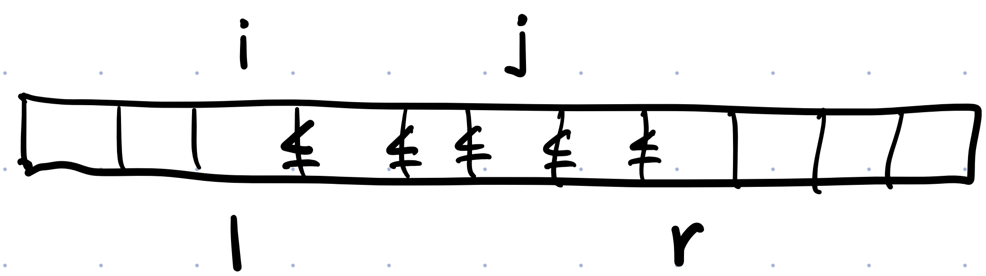
$s = srt[i] + srt[j] = srt[i] + srt[r]$

2) $s > x$

$r -= 1$

3) $s < x$ НЕВОЗМОЖНО

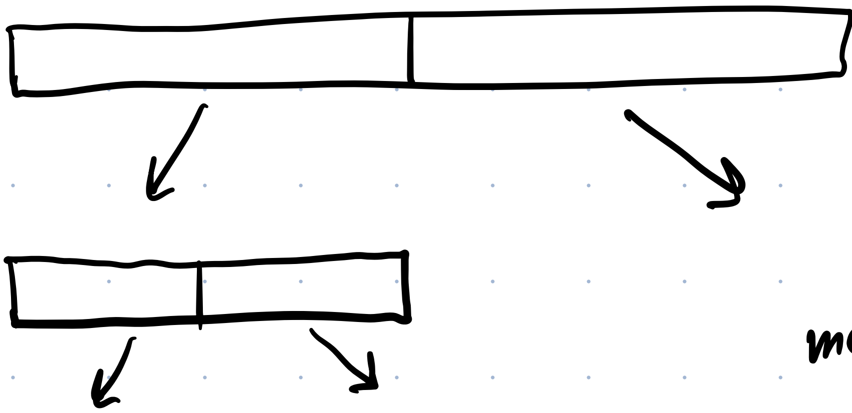
А ПОЧЕМУ?



УТВЕРЖ

$$\underbrace{srt[i] + srt[j]}_{x} > \underbrace{srt[i] + srt[r]}_{\text{ТЕКУЩАЯ СУММА } s}$$

СОРТ. СЛИЯНИЕМ (MERGE SORT)



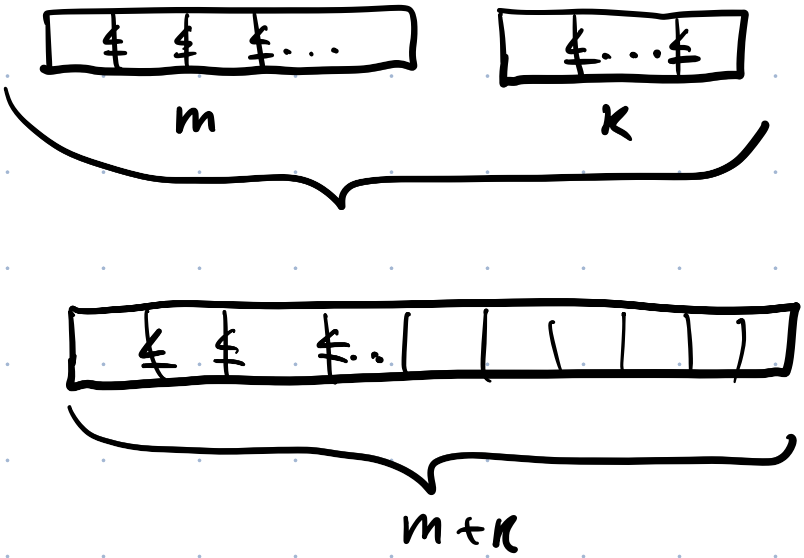
merge_sort(arr)

  merge_sort(arr[l, $\frac{l+r}{2}$])

  merge_sort(arr[$\frac{l+r}{2}$, r])

  merge($\genfrac{}{}{0pt}{}{\text{левая}}{\text{полов.}}$, $\genfrac{}{}{0pt}{}{\text{правая}}{\text{полов}}$)

СЛИЯНИЕ:

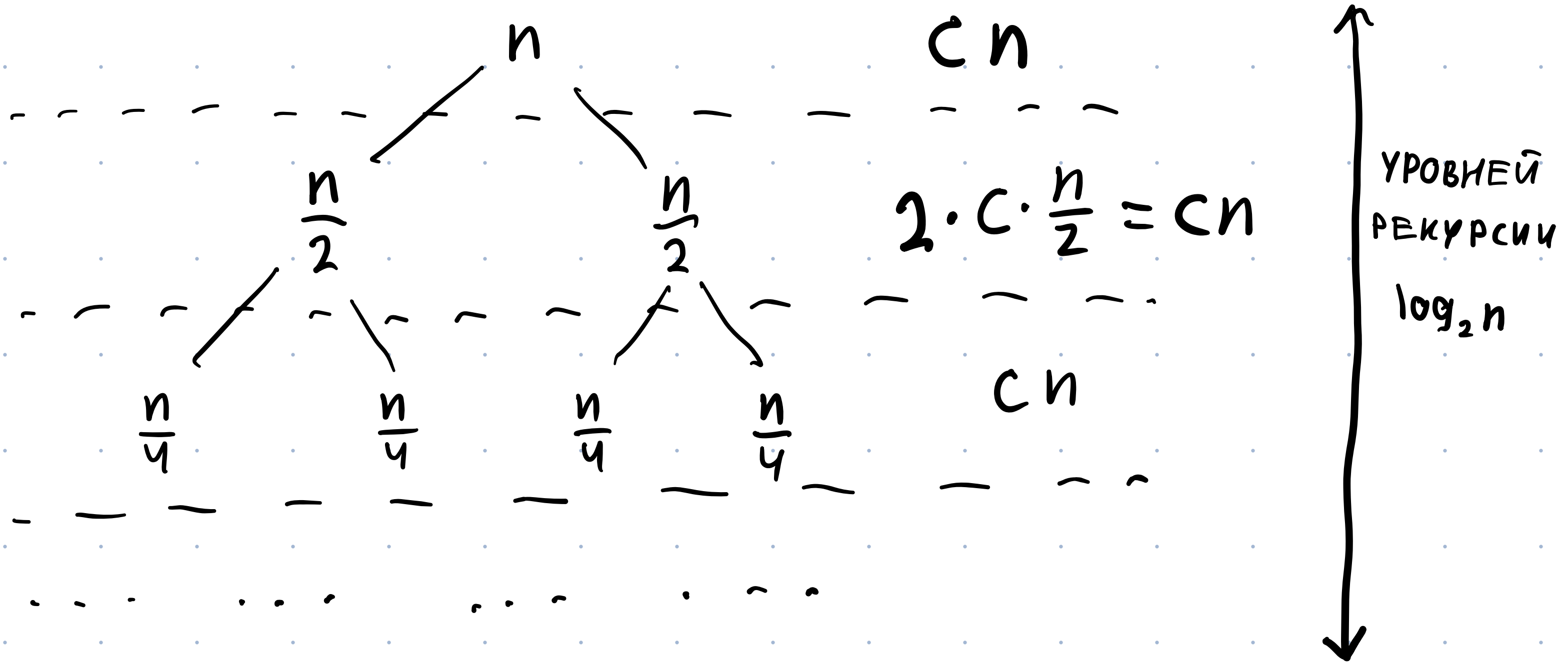


T(n) – время работы merge sort

$$T(n) = 2T\left(\frac{n}{2}\right) + f(n)$$

$$\boxed{T(n) = 2T\left(\frac{n}{2}\right) + cn}$$

$$\exists\, N \in \mathbb{N} : \forall n < N \quad T(n) = c$$



$O(1)$; "МАЛЕНЬКИЕ ЗНАЧЕНИЯ"

$$T(n) = \Theta(cn \cdot \log n)$$
$$T(n) = \Theta(n \log n)$$

ОСНОВНАЯ ТЕОРЕМА О РЕКУРРЕНТНЫХ СООТНОШЕНИЯХ (МАСТЕР-ТЕОРЕМА)

$$T(n) = aT\left(\left\lceil \frac{n}{b} \right\rceil\right) + f(n) =$$

$a \in \mathbb{N}$

$b \in \mathbb{R}_{>1}$

$f: \mathbb{N} \to \mathbb{R}_+$

$$\underbrace{T\left(\frac{n}{b}\right) = aT\left(\frac{n}{b^2}\right) + f\left(\frac{n}{b}\right)}$$

$$= a\left[T\left(\frac{n}{b}\right)\right] + f(n) = a^2 T\left(\frac{n}{b^2}\right) + a \cdot f\left(\frac{n}{b}\right) + f(n) =$$

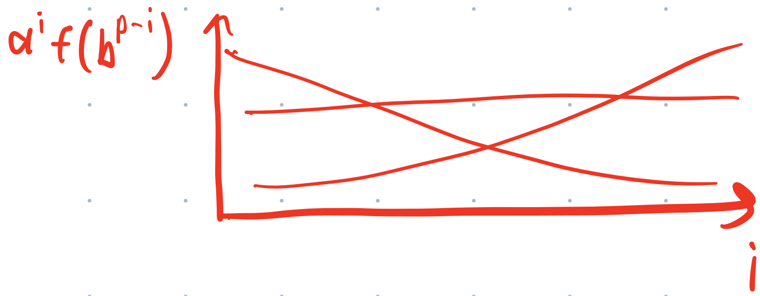
$$= a^3 T\left(\frac{n}{b^3}\right) + a^2 f\left(\frac{n}{b^2}\right) + a f\left(\frac{n}{b}\right) + f(n) =$$

$$= a^{\log_b n} T\left(\frac{n}{b^{\log_b n}}\right) + \sum_{i=0}^{\lfloor \log_b n \rfloor} a^i f\left(\frac{n}{b^i}\right) =$$

$$= a^{\log_b n} T(1) + \sum_{i=0}^{\lfloor \log_b n \rfloor} a^i f\left(\frac{n}{b^i}\right) =$$

⅃ $n = b^p$; $\log_b n = p$

$$= a^{\log_b n} T(1) + \boxed{\sum_{i=0}^{p} a^i f(b^{p-i})}$$



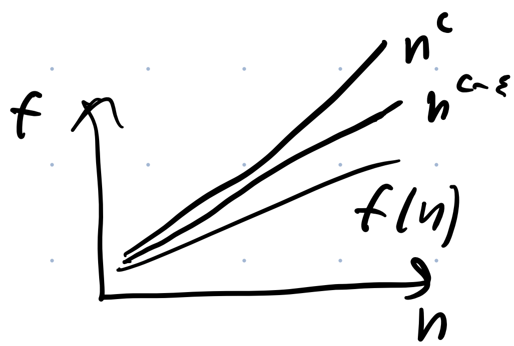
МАСТЕР-ТЕОРЕМА

$T(n) = a T\left(\frac{n}{b}\right) + f(n)$

$a \in \mathbb{N}$

$b \in \mathbb{R}_{>1}$

$f : \mathbb{N} \to \mathbb{R}_+$



$c = \log_b a$

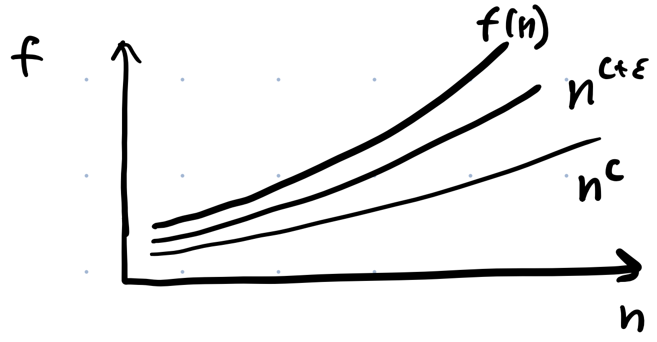
$\boxed{-\varepsilon}$ – "ПОЛИНОМИАЛЬНЫЙ ЗАЗОР"

n^{ε}

1) $\exists \varepsilon > 0 : f(n) = O(n^{c-\varepsilon})$

$T(n) = \Theta(n^{\log_b a})$

2) $\exists \varepsilon > 0: \ f(n) = \Omega(n^{c+\varepsilon})$



ПРИМЕР НЕПОЛИНОМИАЛЬНОГО ЗАЗОРА

$f(n) = n^2$

$g(n) = n^2 \log n$

$f(n) = O(g(n))$

$\nexists \varepsilon > 0: \ f(n) = O(g(n) \cdot n^{-\varepsilon})$

$n^2 \neq O(n^{2-\varepsilon} \log n)$

$1 \neq O(n^{-\varepsilon} \log n)$

$n^{\varepsilon} \neq O(\log n)$

УСЛОВИЕ РЕГУЛЯРНОСТИ

$\exists k < 1: \ a f\left(\frac{n}{b}\right) < k f(n)$

$$\boxed{\sum_{i=0}^{p} a^i f(b^{p-i})} < \underbrace{f(n)}_{0} + \underbrace{k f(n)}_{1} + \underbrace{k^2 f(n)}_{2} + \ldots \leq \frac{1}{1-k} f(n)$$

$$\underbrace{f(b^{p-0})}_{0} + \underbrace{a f(b^{p-1})}_{1} + \underbrace{a^2 f(b^{p-2})}_{2} + \ldots$$

$$T(n) = \Theta(f(n))$$

3) $f(n) = \Theta(n^c)$

$$T(n) = \Theta(n^c \log n)$$

НЕПРИМЕНИМА!

$T(n) = \sqrt{n}\, T(n!) + n^3$

ПЕРЕМЕННОЕ ЧИСЛО ПОДЗАДАЧ

$T(n) = T(n-2) + n$

$T\left(\frac{n}{b}\right)$

$n-2$ НЕ ПРЕДСТАВЛ. В ВИДЕ $\frac{n}{b}$

$T(n) = 2T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + n^2$

РАССМ. ПРИМЕРЫ

1) $T(n) = 2T\left(\frac{n}{2}\right) + \underbrace{Kn}_{f(n)}$

$c = \log_2 2 = 1$

$f(n) = \Theta(n^1)$ $\rightsquigarrow T(n) = \Theta(n \log n)$

2) $T(n) = 27\,T\left(\frac{n}{9}\right) + \sqrt{n}$

$c = \frac{3}{2} = \log_9 27$

$n^{\frac{1}{2}}$ $\quad n^{\frac{3}{2}} = n^c$ $\rightsquigarrow T(n) = \Theta\left(n^{\frac{3}{2}}\right)$

$\exists\, \varepsilon = 0{,}01 \quad f(n) = O\left(n^{\frac{3}{2}-\varepsilon}\right)$

$n^{0,5} = O\left(n^{1,49}\right)$

3) $T(n) = 100\,T\left(\frac{n}{10}\right) + n^3$ $\rightsquigarrow T(n) = \Theta(n^3)$

$c = \log_{10} 100 = 2$

$\exists\, \varepsilon = 0{,}01: \quad f(n) = \Omega\left(n^{c+\varepsilon}\right)$

$$n^3 = \Omega(n^{2+0,01})$$

УСЛ. РЕГ. ВЫПОЛНЯЕТСЯ

$K = 0,9$

$$100 \cdot f\left(\frac{n}{10}\right) \overset{?}{<} 0,9 \cdot f(n)$$

$$100 \cdot \frac{\cancel{n^3}}{1000} < 0,9 \cdot \cancel{n^3}$$

$$0,1 < 0,9$$