29.10.25

ПРОБЛЕМЫ ФИЗИЧЕСКОГО МИРА:

- ДИСКР. ВРЕМЕНИ (+ ИЗМЕРЕНИЙ)
- ЗАДЕРЖКИ
- ШУМЫ
- ВОЗМУЩЕНИЯ
- НЕТОЧНОСТИ МОДЕЛИ $\dot{x} = f(x,u)$; $\dot{x} = f^{\mu}(x,u)$

АДАПТИВНОСТЬ vs РОБАСТНОСТЬ

АДАПТИВНОСТЬ:

$\dot{x} = f^{\mu}(x,u)$

$\hat{\mu}$ – ???

$\|f^{\mu}(x,u) - f^{\hat{\mu}}(x,u)\| \xrightarrow[\hat{\mu}]{} \min$

РОБАСТНОСТЬ:

$\dot{x} = f^{\mu}(x,u)$

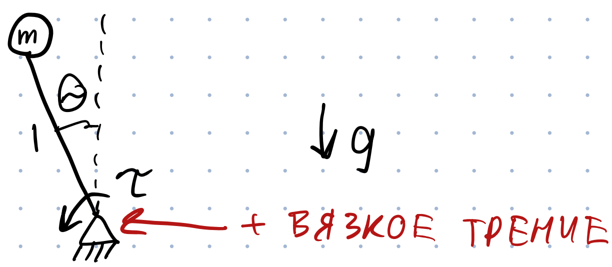
~~$u = u(x(t))$ т.ч. $\int_0^T \dots dt$~~ $\to \min$ (ОБЫЧН.)

$$\max_{\mu} \int_0^T \dots dt \xrightarrow[u]{} \min$$

МИНИМ. СТОИМ. В ХУДШ. СЛУЧАЕ

aCLF (adaptive CLF)

$$\ddot{\theta} = \frac{g}{l}\sin\theta + \frac{\tau}{ml^2} - \mu\dot{\theta}$$



$$x = \begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix} ; \; u = (\tau)$$

КАК ПРОИЗВ. КОМПЕНСАЦИЮ?

$\tau = -\tau_{max}\,\mathrm{sign}((E - E_d)\dot{\theta})$ (БЫЛО)

$\tau_b = -0{,}8\,\tau_{max}\,\mathrm{sign}((E - E_d)\dot{\theta})$ (СТАЛО)

τ baseline

compensatory

$$\tau = \tau_b + \tau_c$$

ЕСЛИ μ ИЗВЕСТНО $(\hat{\mu} = \mu)$, ТО

$$\frac{\tau}{ml^2} - \mu\dot{\theta} = \frac{\tau_b}{ml^2}$$

$$\cancel{\tau_b} + \tau_c - ml^2\mu\dot{\theta} = \cancel{\tau_b}$$

$$\tau_c = ml^2 \mu \dot{\theta}$$

ЕСЛИ $\hat{\mu} \neq \mu$, ТО

$$\tau_c = ml^2 \hat{\mu} \dot{\theta}$$

КАКИМ СТАНЕТ УР. ДИНАМИКИ ПРИ НЕИДЕАЛЬНОЙ КОМПЕНС.?

$$\tau = \tau_b + ml^2 \hat{\mu} \dot{\theta}$$

$$\ddot{\theta} = \frac{g}{l}\sin\theta + \frac{\tau_b}{ml^2} + \frac{ml^2 \hat{\mu}\dot{\theta}}{ml^2} - \mu\dot{\theta} = \frac{g}{l}\sin\theta + \frac{\tau_b}{ml^2} + (\hat{\mu} - \mu)\dot{\theta}$$

ВВЕДЁМ АДАПТИВН. Ф-Ю УПР. ЛЯПУНОВА:

$$V = \frac{(E - E_0)^2}{2} + \frac{(\hat{\mu} - \mu)^2}{2\alpha}$$

$$E = \frac{ml^2\dot{\theta}^2}{2} + mgl\cos\theta$$

ЕД. ИЗМ. КОЭФФ. ВЯЗ.ТР. ДЛЯ МАЯТН.:

$$\frac{Н \cdot м \cdot с}{РАД}$$

$$\frac{dV}{dt} = (E - E_0)(ml^2\dot{\theta}\ddot{\theta} - mgl\sin\theta\,\dot{\theta}) + \frac{(\hat{\mu} - \mu)}{\alpha}\dot{\hat{\mu}} =$$

$$= (E - E_0)\dot{\theta}\left(\cancel{ml^2\frac{g}{l}\sin\theta} + \cancel{ml^2}\cdot\frac{\tau_b}{\cancel{ml^2}} + ml^2(\hat{\mu} - \mu)\dot{\theta} - \cancel{mgl\sin\theta}\right) +$$

$$+ \frac{(\hat{\mu} - \mu)}{\alpha}\dot{\hat{\mu}} = (E - E_0)\dot{\theta}\,\tau_b + (E - E_0)\dot{\theta}\,ml^2(\hat{\mu} - \mu)\dot{\theta} + \frac{(\hat{\mu} - \mu)}{\alpha}\dot{\hat{\mu}} =$$

$$= \underbrace{(E - E_0)\dot{\theta}\,\tau_b}_{<0} + \underbrace{(\hat{\mu} - \mu)}_{\neq 0}\underbrace{\left((E - E_0)\dot{\theta}^2 ml^2 + \frac{\dot{\hat{\mu}}}{\alpha}\right)}_{=0} < 0$$

$$\Updownarrow$$

$$(E - E_0)\dot{\theta}^2 ml^2 + \frac{\dot{\hat{\mu}}}{\alpha} = 0$$

$$\dot{\hat{\mu}} = -\alpha(E - E_0)\dot{\theta}^2 ml^2$$

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mu_hat -= mu_hat_dot * dt
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ИДЕНТИФИКАЦИЯ ЛИНЕЙНОЙ СИСТЕМЫ

$$x_{k+1} = A x_k$$

$$\{x_i\}_{i=0}^{N}$$

$$x_1 = A x_k$$

$$\begin{pmatrix} \dot p \\ \ddot p \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{pmatrix} \begin{pmatrix} p \\ \dot p \end{pmatrix}$$

$$\begin{pmatrix} p_{k+1} \\ \dot p_{k+1} \end{pmatrix} = \begin{pmatrix} 1 & \Delta t \\ -\frac{k}{m}\Delta t & 1 \end{pmatrix} \begin{pmatrix} p \\ \dot p \end{pmatrix}$$

1)

$$\underbrace{\begin{bmatrix} x_1 & x_2 & \dots & \dots & x_N \end{bmatrix}}_{X_1} = \boxed{A} \underbrace{\begin{bmatrix} x_0 & x_1 & \dots \end{bmatrix}}_{X_0}$$

$$X_1 = A X_0 \quad \Big| \cdot X_0^T$$

$$X_1 X_0^T = A X_0 X_0^T \quad \Big| \cdot (X_0 X_0^T)^{-1}$$

$$A = X_1 X_0^T (X_0 X_0^T)^{-1}$$

$$x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$x_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$X_0 = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$X_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ 3 & 3 \end{pmatrix}$$

2)

$$x_{k+1} = A x_k \qquad \hat A$$

$$\hat x_{k+1} = \hat A x_k$$

$$e_{k+1} = x_{k+1} - \hat x_{k+1} = (A - \hat A) x_k = x_{k+1} - \hat A x_k$$

$$\frac{\partial\, e_{k+1}^T e_{k+1}}{\partial \hat A} = \frac{\partial (x_{k+1} - \hat A x_k)^T (x_{k+1} - \hat A x_k)}{\partial \hat A} =$$

$$= \frac{\partial}{\partial \hat{A}} \left(x_{k+1}^T x_{k+1} - 2 x_{k+1}^T \hat{A} x_k + x_k^T \hat{A}^T \hat{A} x_k \right) =$$

$$= -2 x_k x_{k+1}^T + 2 A x_k x_k^T$$

$$\hat{A} -= \alpha \frac{\partial e^T e}{\partial \hat{A}}$$

$$x_{k+1} = A x_k \rightsquigarrow x_{k+1} = A x_k + B u_k$$