24.09.25

Д3 зад. 2

$$\bar{x} = \begin{pmatrix} p \\ \dot{p} \end{pmatrix}$$

$$\cancel{\dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} p \\ \dot{p} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix}(F)} \qquad \dot{x} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{pmatrix}\begin{pmatrix} p \\ \dot{p} \end{pmatrix}$$

$$x_{k+1} = (\mathbb{I} + \Delta t A)x_k + \Delta t B u_k$$

$$\cancel{x_{k+1} = \begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix}\Big(} \qquad x_{k+1} = \begin{pmatrix} p_{k+1} \\ \dot{p}_{k+1} \end{pmatrix} = \begin{pmatrix} 1 & \Delta t \\ -\Delta t\frac{k}{m} & 1 \end{pmatrix}\begin{pmatrix} p_k \\ \dot{p}_k \end{pmatrix} = \begin{pmatrix} p_k + \dot{p}_k \Delta t \\ \dot{p}_k - \Delta t p_k \frac{k}{m} \end{pmatrix}$$

$$E_k^{kin} = \frac{m\dot{p}_k^2}{2} \ ; \quad E_k^{pot} = \frac{k p_k^2}{2}$$

$$\Delta E = E^{\Sigma}_{k+1} - E^{\Sigma}_k = \frac{m}{2}\left(\dot{p}_{k+1}^2 - \dot{p}_k^2\right) + \frac{k}{2}\left(p_{k+1}^2 - p_k^2\right) =$$

$$= \frac{m}{2}\left(\cancel{\dot{p}_k^2} - 2\dot{p}_k \Delta t p_k \frac{k}{m} + \Delta t^2 p_k^2 \frac{k^2}{m^2} - \cancel{\dot{p}_k^2}\right) +$$

$$+ \frac{k}{2}\left(\cancel{p_k^2} + 2p_k\dot{p}_k\Delta t + \dot{p}_k^2\Delta t^2 - \cancel{p_k^2}\right) =$$

$$= \cancel{-m\dot{p}_k\Delta t p_k \frac{k}{m}} + \frac{\cancel{m}}{2}\Delta t^2 p_k^2 \frac{k^2}{m^{\cancel{2}}} \cancel{+ k p_k \dot{p}_k \Delta t} + \frac{k}{2}\dot{p}_k^2 \Delta t^2 =$$

$$= \Delta t^2\left(\frac{k^2}{m 2}p_k^2 + \frac{k}{2}\dot{p}_k^2\right) = \Delta t^2\left(\frac{k p_k^2}{2}\cdot\frac{k}{m} + \frac{m\dot{p}_k^2}{2}\cdot\frac{k}{m}\right) =$$

$$= \Delta t^2 \frac{k}{m}\cdot E_k^{\Sigma}$$

"НАПОМИНАЕТ" $\frac{d}{dt}y = Cy$

$$\Delta t \to \frac{1}{q}\Delta t$$

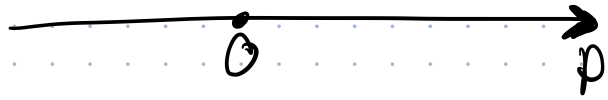
$$\Delta t^2 \frac{k}{m} E_k^{\Sigma} \longrightarrow \sim q\cdot\left(\frac{1}{q}\Delta t\right)^2 \frac{k}{m} E_k^{\Sigma}$$

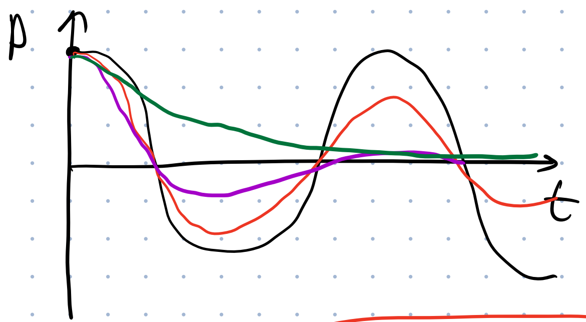
$$\frac{1}{q} \dots\dots$$

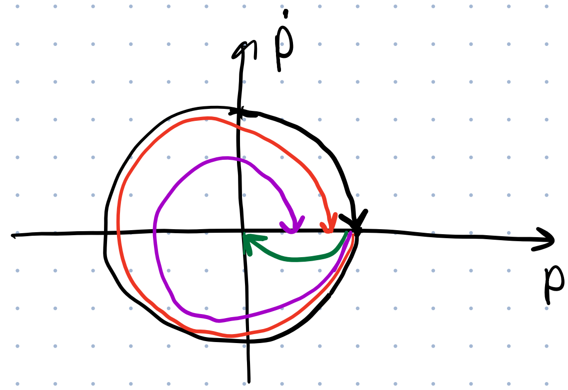
PD-controller

proportional - differential

$$\circ\bar{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$





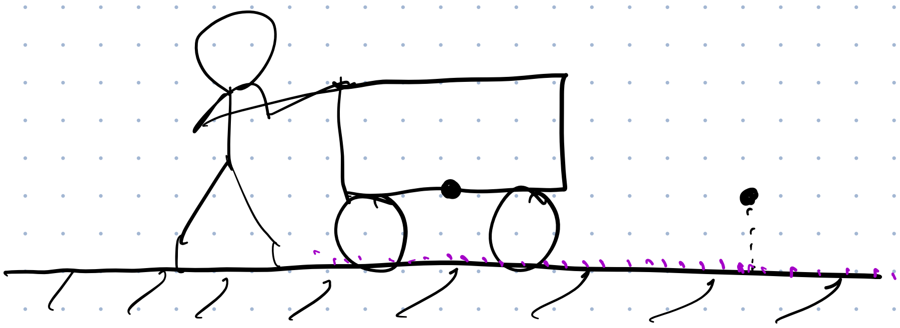


P-регулятор

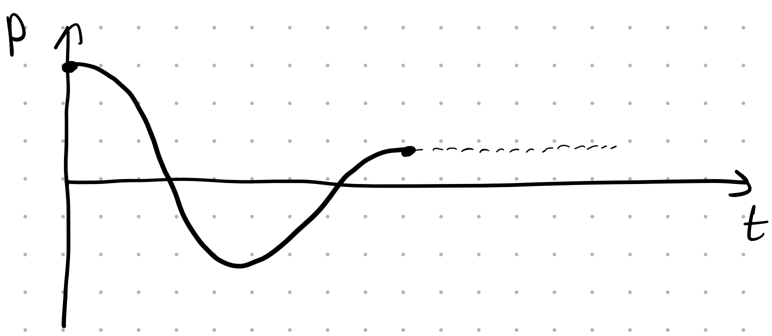
$$u = (F) = \boxed{-K_p p} - K_d \dot{p}$$

PD-регулятор

коэфф (proportional)



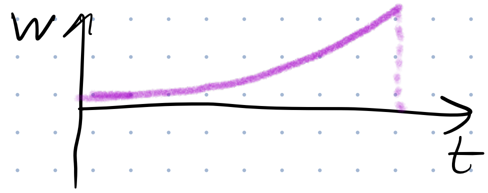
$$u = -K_p p = F_{тр}$$

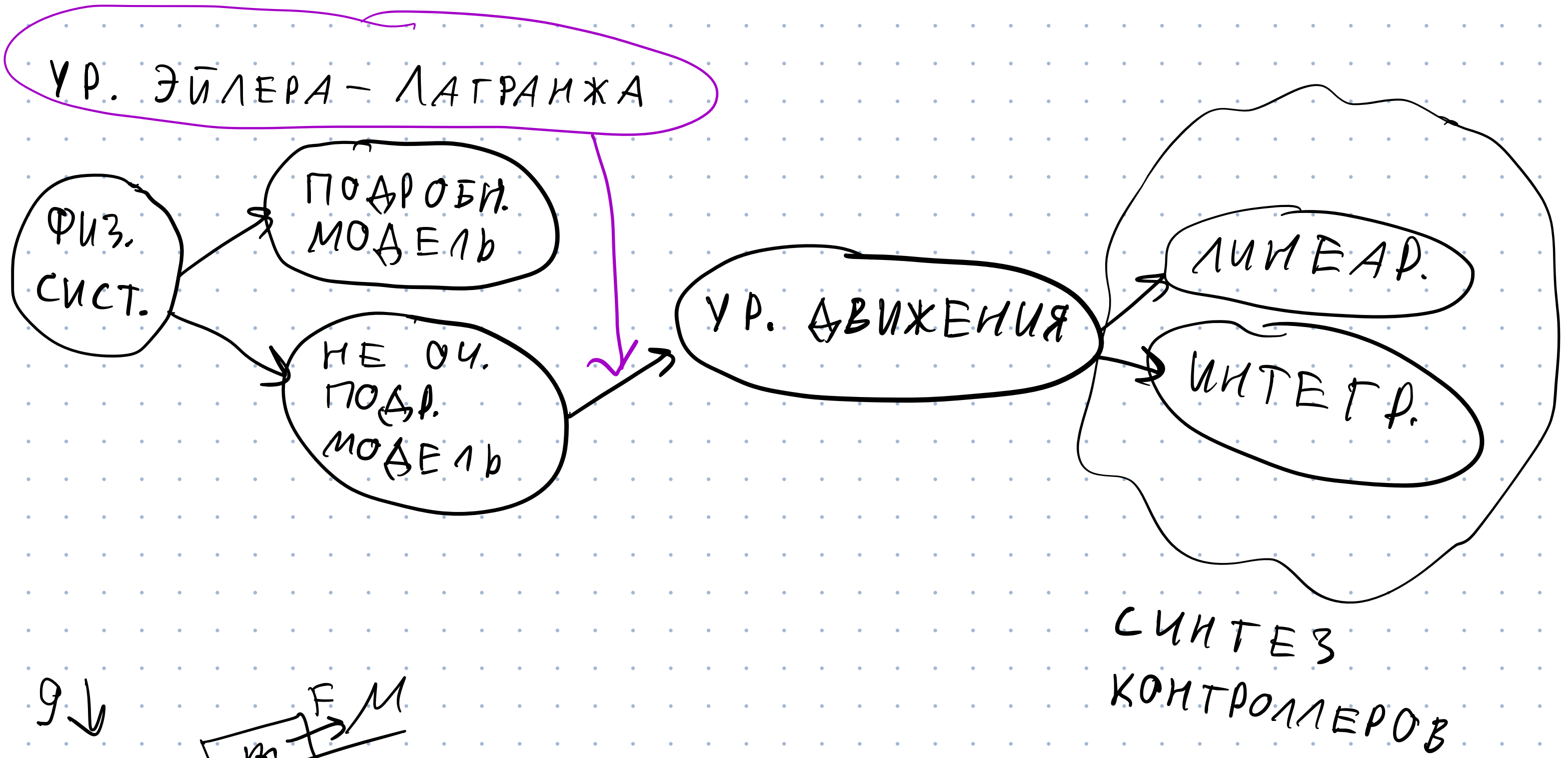


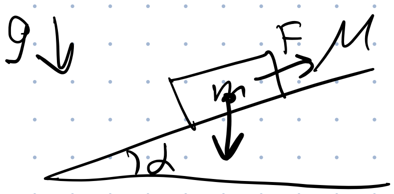
$$u = -K_p p - K_D \dot{p} - K_I \int_0^T p \, dt$$

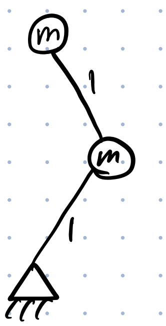
PID-регулятор

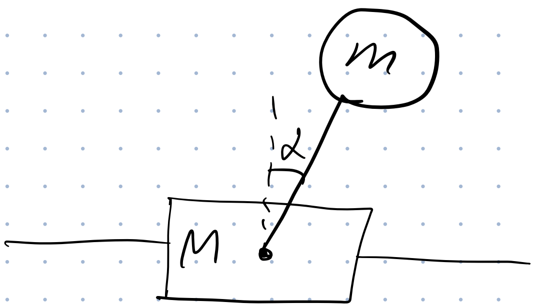
диск. инт. $\int_0^T e^{-\sigma(T-t)} p(t) \, dt$











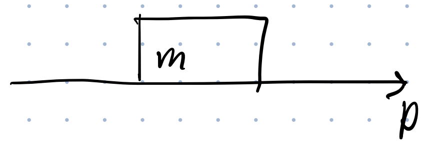
Лагранжиан → $L = \underbrace{T}_{\text{кин. эн.}} - \underbrace{П}_{\text{пот. эн.}}$

Для сист. с DoF $q_1, q_2, q_3 \ldots$ ур. движ. получаются из

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$

ПРИМЕР:

1 DoF

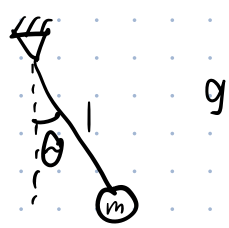


$$q_1 = p$$
$$\dot{q}_1 = \dot{p}$$
$$E_к = \frac{m\dot{p}^2}{2}$$
$$E_p = 0$$
$$L = E_к - E_p = \frac{m\dot{p}^2}{2}$$
$$\frac{\partial L}{\partial \dot{p}} = \frac{\partial}{\partial \dot{p}}\left[\frac{m\dot{p}^2}{2}\right] = m\dot{p}$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{p}} = \frac{d}{dt}\left[m\dot{p}\right] = m\ddot{p}$$
$$\frac{\partial L}{\partial p} = 0$$

УР. ДВИЖ.: $m\ddot{p} = \cancel{0}\ F$

$$\ddot{p} = 0$$
$$\dot{p} = const$$
$$p(t) = \dot{p}_0 t + p_0$$

ДЛЯ МАЯТН.



$$\bar{x} = \begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix}$$

$$L = \frac{m l^2 \dot{\theta}^2}{2} - (-mgl\cos\theta) = \frac{m l^2 \dot{\theta}^2}{2} + mgl\cos\theta$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = m l^2 \ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -mgl\sin\theta$$

УР. ДВИЖ.: $\cancel{m} l^{\cancel{2}} \ddot{\theta} + \cancel{m} g \cancel{l} \sin\theta = 0 \quad | : l$

$$\ddot{\theta} = -\frac{g}{l}\sin\theta$$

ЛИНЕАР.: $\ddot{\theta} = -\frac{g}{l}\theta$