

Homework 1

Due Friday, 27 Jan, at the beginning of class.

Reading Assignment: Read Wall & Jenkins, Chapters 1 & 2. Optional additional reading: For an excellent in-depth discussion of Bayesian vs. Frequentist methods, see Loredó, “From Laplace to Supernova SN 1987A: Bayesian Inference in Astrophysics,” 1990, posted under the “papers” link on the course homepage.

1 A Hunt for Galaxy Clusters

An astronomer is hunting for a new galaxy cluster to study. She knows from previous surveys of a different part of the sky that the probability of finding a galaxy cluster is 30% in a single square-degree image (the field of view of her instrument). The astronomer’s plan is to take images until she finds a single cluster, then stop.

a) Find an expression for the probability $P[k]$ of the astronomer finding the first cluster (and stopping) after the k^{th} image is taken.

b) Show that, in your solution from a), $\sum_{k=1}^{\infty} P[k] = 1$

c) Write a simulation of the astronomer’s observations in Python/IDL/Matlab, run $N = 1000$ trials of her observing scheme, and plot a histogram of frequency vs. number of images k taken in a given trial. For comparison purposes, on your histogram, overplot your analytical solution for $P[k]$ from a), multiplying by a factor of 1000 to account for the fact that you have generated $N = 1000$ trials. Does your histogram agree with your analytical calculation?

2 Bayesian Analysis of Tainted Beef Patties

Two different suppliers, A and B , provide a supermarket with ground beef patties. In the past, 0.01% of the patties supplied by A and 0.03% of the patties supplied by B have been found to be contaminated by E. Coli. A supplies five times as many patties to the supermarket as B .

An unsuspecting consumer is found to be infected by E. Coli due to eating a burger purchased at this supermarket. Using Bayes’s Theorem, what is the probability that the contaminated burger was supplied by A ?

3 PDF, CDF, and Moments

Consider the following probability density function (PDF):

$$f(x) = \begin{cases} xe^{-x} & x \geq 0, \\ 0 & x < 0. \end{cases}$$

- a) Plot the PDF, and verify that $\int_{-\infty}^{\infty} f(x)dx = 1$.
- b) Find the cumulative distribution function (CDF), $F(x)$.
- c) Find the mean, $\mu \equiv E[x]$ of the distribution. What fraction of the total probability lies below the mean?
- d) Find the variance $\sigma^2 \equiv V[x]$ of the distribution. What fraction of the total probability lies within the interval $\mu \pm \sigma$, where $\sigma = \sqrt{\sigma^2}$ is the standard deviation?