EE P 538
Analog Circuits for Sensor Systems

Spring 2020

Instructor: Jason Silver, PhD

Announcements

- Assignment 5 due Saturday, May 9 at midnight
- Final weekly assignment will be posted May 9
  - Due Saturday, May 16 at midnight
- Design project description will be posted May 9
  - Submissions in 2 phases
    - Phase 1: 5-10 slides due May 30
    - Phase 2: Final design report due June 13

Week 6

- AoE Chapter 8 – Low Noise Design
  - Sections 8.1, 8.2, 8.8, 8.9, 8.13.1,2,4

Overview

- Last time…
  - Error modeling
  - Opamp errors
  - Differential-input amplifiers
- Today…
  - Thermal, flicker, and shot noise
  - Analyzing circuits with noise
  - Noise bandwidth
  - BJT noise
  - Opamp noise

Lecture 6 – Noise

Noise

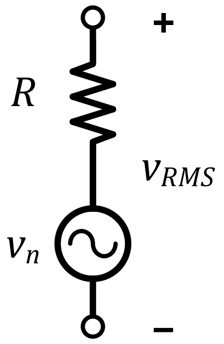
- For most applications, the ultimate limit of the detectability of weak signals is set by noise
- Noise is defined as any unwanted “signals” that obscured the desired signal
- Some forms of noise are unavoidable (e.g. thermal noise) and can only be overcome by averaging or bandwidth limiting
- Other forms can be reduced or eliminated by circuit/system architecture (e.g. ground loops and other common-mode interference)
- A significant amount of design effort in signal conditioning is devoted to reducing/managing noise

Physical Noise

- Although noise can take on many definitions, the term typically refers to “random” noise that has some physical origin
- Some examples are
  - **Johnson noise**: Random voltage noise caused by thermal fluctuations in a resistor
  - **Shot noise**: Random statistical fluctuations in current caused by the discrete nature of charge
  - **Flicker noise (1/f)**: Random noise, caused by a variety of factors, characterized by an increase in power at low frequencies
  - **Burst noise**: Low-frequency noise typically seen as random jumps between two signal levels

Johnson-Nyquist (Thermal) Noise

- A resistor unconnected to anything generates a noise voltage across its terminals due to the thermal energy of electrons
- Thermal noise has the same power in every 1-Hz bandwidth of frequency
- Another way of stating this is that the rms thermal noise-voltage density, e_n (rms voltage per square root bandwidth), is constant with frequency
- We often refer to the *mean square noise density*, which is the just the square of the noise density



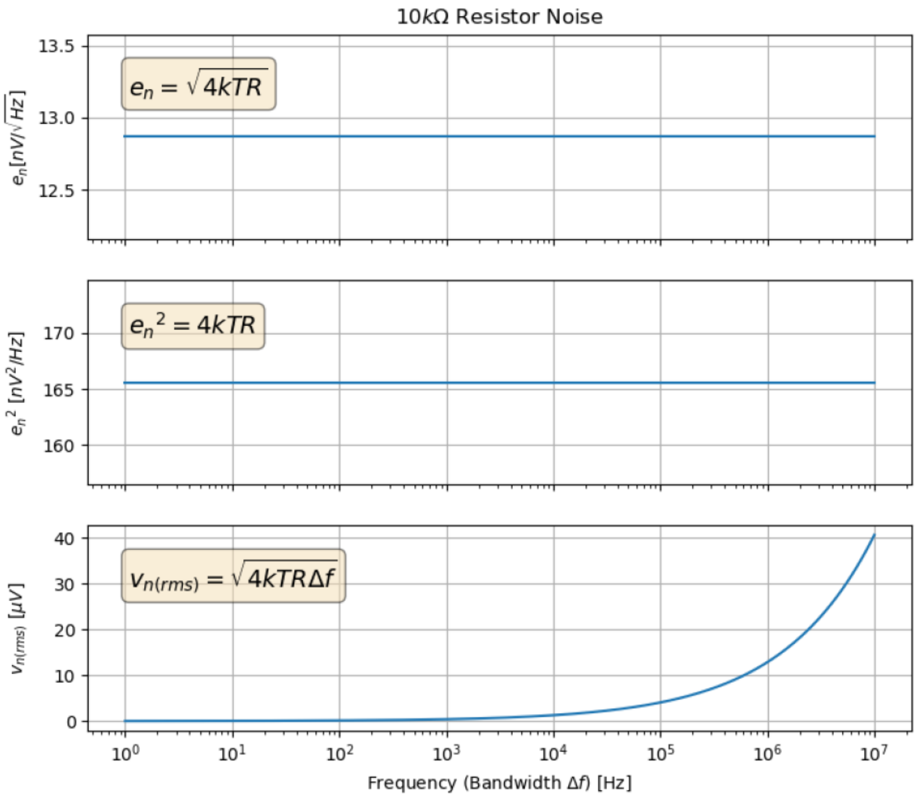
$$v_{n(rms)} = e_n\sqrt{\Delta f} = \sqrt{4kTR\Delta f}$$

$$e_n = \sqrt{4kTR}\,V/\sqrt{Hz}$$

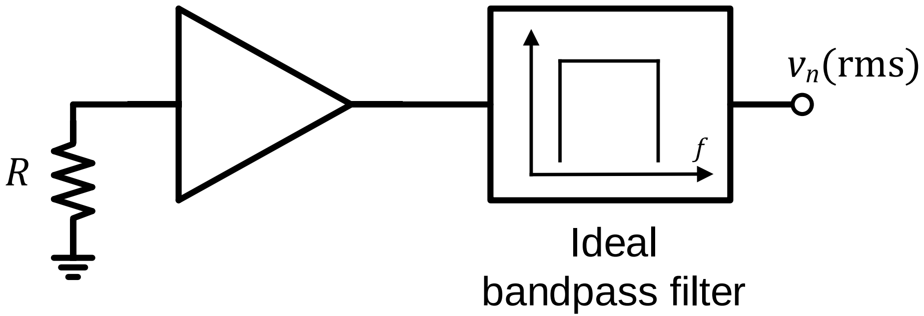
$$e_n^2 = 4kTR\,V^2/Hz$$

Thermal Noise Density

- For example, a 10k resistor at room temperature (25C) has an open-circuit noise density e_n = 13nV/√Hz (e_n^2 = 165nV²/Hz)
- To determine the rms noise over a given bandwidth Δf, we integrate the mean square noise e_n^2 over that bandwidth and take the square root
- Over a 10kHz bandwidth, this amounts to 1.3μV *rms voltage noise* for the 10k resistor
- For a 100kHz bandwidth, the rms voltage noise is 4μV



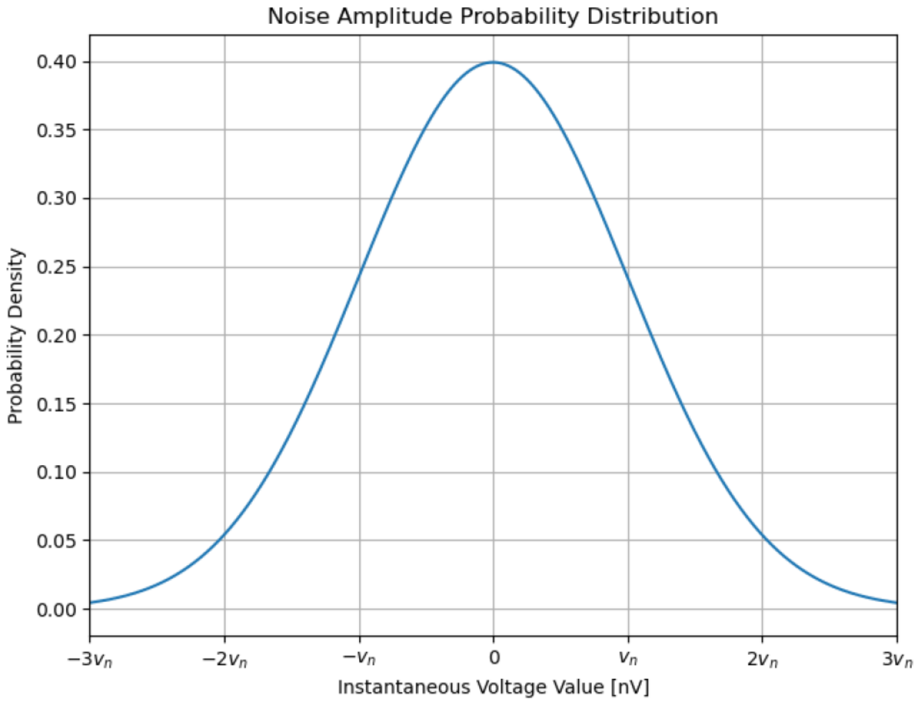
RMS Noise



- The rms noise is the voltage we would measure if we connected the resistor to an ideal (noiseless) amplifier and bandpass filter and looked at the output on a spectrum analyzer
- Note that this does not say anything about the voltage we measure at any given time instant, only its average (rms) value

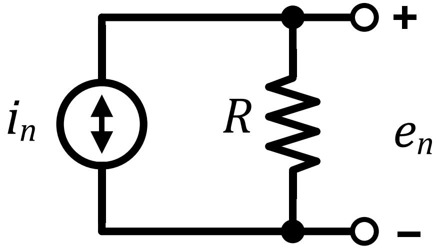
Noise Amplitude

- The amplitude of thermal noise at any given instant is unpredictable, but obeys a normal distribution
- The probability for a given range of amplitudes is given by the area under the curve over that range
- For example, if we integrate the probability density function over the range -3$v_{n(\mathrm{rms})}$ to 3$v_{n(\mathrm{rms})}$, the probability of the instantaneous voltage being in that range would be 99.73%



Short-Circuit Noise Current

- As with any linear circuit, we can convert a resistor with voltage noise to its Norton equivalent by shorting the two terminals
- The bidirectional current source just means that we're dealing with rms or mean-square current
- Thermal *current* noise density decreases with resistance magnitude, and is specified in units of A/√Hz



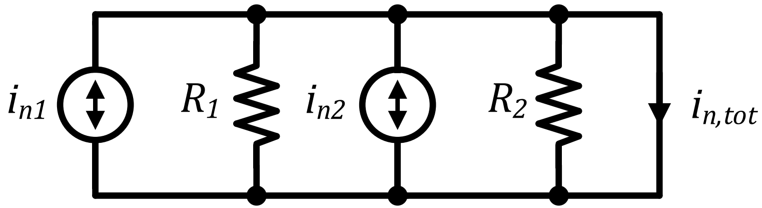
$$i_n = \sqrt{4kT/R}\, A/\sqrt{Hz}$$

$$i_n^2 = 4kT/R\, A^2/Hz$$

$$e_n = i_n R = \sqrt{4kTR}\, V/\sqrt{Hz}$$

Combining Noise Quantities

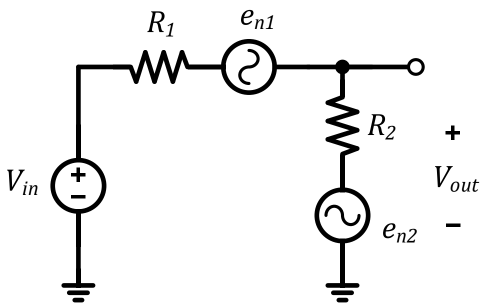
- Because we deal with rms values of noise voltages and currents, our treatment of them in circuits is slightly different from that of other sources
- RMS sources that are *uncorrelated* (i.e. independent of each other) cannot be added together directly, but must be added as *mean-square* quantities
- To combine two uncorrelated noise sources we compute the *square-root of the sum of their mean-square values*



$$i_{n1} = \sqrt{\frac{4kT}{R_1}}\,\mathrm{A}/\sqrt{\mathrm{Hz}} \qquad i_{n2} = \sqrt{\frac{4kT}{R_2}}\,\mathrm{A}/\sqrt{\mathrm{Hz}}$$

$$i_{n,tot} = \sqrt{i_{n1}^2 + i_{n2}^2}\,\mathrm{A}/\sqrt{\mathrm{Hz}}$$

Voltage Divider Noise



$$e_{n1,out} = \frac{R_2}{R_1 + R_2} e_{n1}$$

$$e_{n2,out} = \frac{R_1}{R_1 + R_2} v_{n2}$$

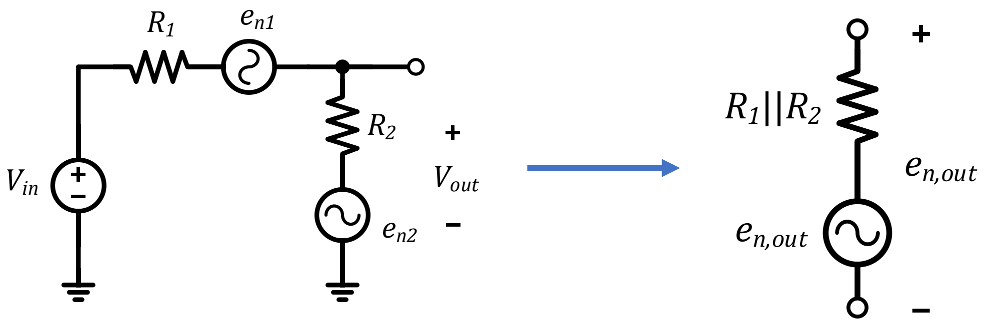
$$e_{n,out} = \sqrt{e_{n1,out}^2 + e_{n2,out}^2}$$

$$= \sqrt{4kT \frac{R_1 R_2}{R_1 + R_2}} \frac{V}{\sqrt{Hz}}$$

- We can assess the net effect of multiple noise sources by using superposition, keeping in mind that we only add mean-square quantities

- In the case of a voltage divider, the output noise is equal to that of a single resistor formed by the parallel combination of R_1 and R_2

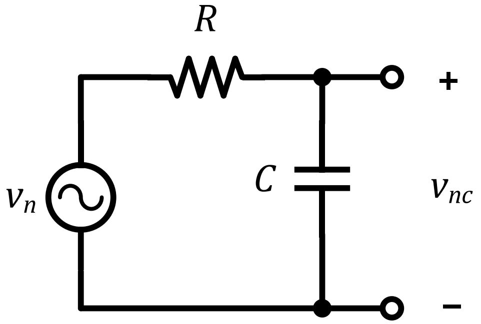
Noise-Equivalent Circuit



$$e_{n,out} = \sqrt{4kT(R_1||R_2)}\frac{V}{\sqrt{Hz}}$$

- We can combine resistors into their Thevenin equivalent representations, and the resulting noise voltage (or current) of the *equivalent resistance* is identical to that of the original circuit
- This is in general a more efficient approach than analyzing the effect of each noise source independently
- LTspice: divider noise density

Band-limited Noise



$$e_n^2(f) = 4kTR\,V^2/Hz$$

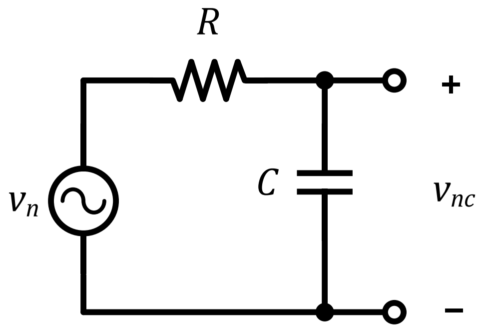
$$\left|\frac{v_{nc}}{v_n}\right|^2 = \frac{1}{1+(2\pi fRC)^2}$$

$$v_{nc(rms)} = \sqrt{\int_0^\infty \frac{4kTR}{1+(2\pi fRC)^2}\,df} = \sqrt{kT/_C}$$

- All circuits operate within some limited bandwidth, which ensures the total noise is finite
- For example, the voltage noise density above the 3dB bandwidth of an RC lowpass filter becomes increasingly small with frequency
- To determine the rms noise on the capacitor, we integrate the mean square noise density with respect to frequency and take the square root

kT/C Noise

- An interesting result of this is that the total noise of an RC lowpass filter depends only on the capacitance value
- If we increase the resistance value, the noise density goes up, but the bandwidth goes down proportionately, keeping the total noise constant
- Because thermal noise density is constant with frequency, it is convenient to define an effective “noise bandwidth,” the bandwidth of an ideal lowpass filter that would result in the same total noise
- [Ltspice: kT/C noise]



$$e_n^2(f) = 4kTR\, V^2/Hz$$

$$f_{3dB} = \frac{1}{2\pi RC}$$

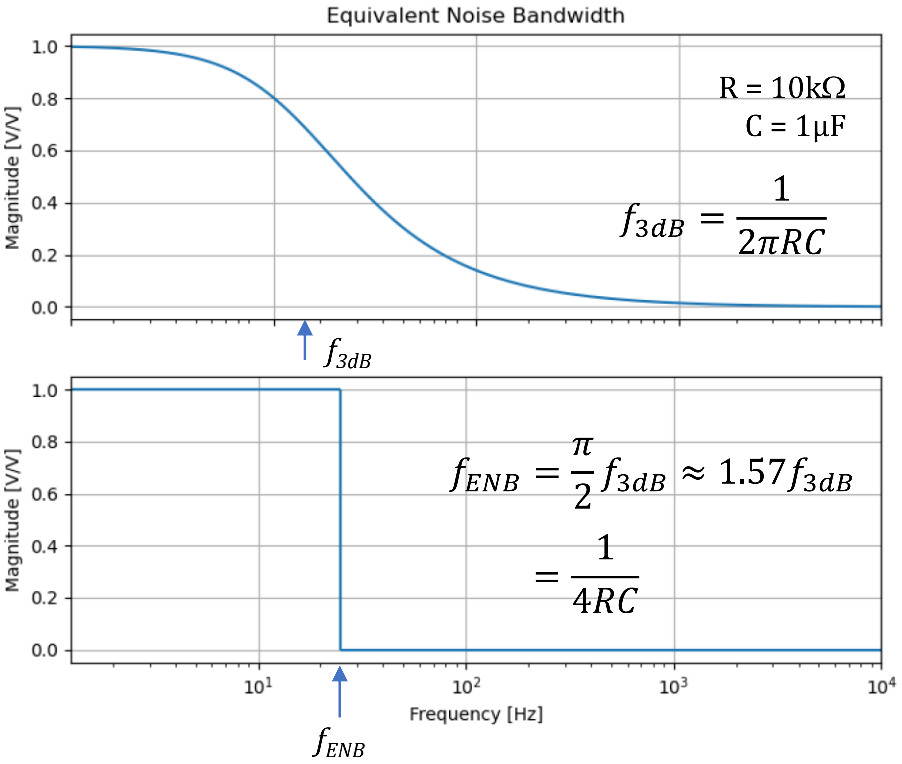
$$v_{nc(rms)} = \sqrt{kT/C}$$

Equivalent Noise Bandwidth

- The equivalent noise bandwidth (ENB) is the bandwidth of a "brick-wall" filter that would result in the same rms noise as the real filter
- The ENB enables easy evaluation of rms noise when white noise dominates:

$$v_{n(rms)} = \sqrt{e_n^2 f_{ENB}} = \sqrt{\frac{4kTR}{4RC}}$$

$$= \sqrt{kT/C}$$



Signal-to-Noise Ratio

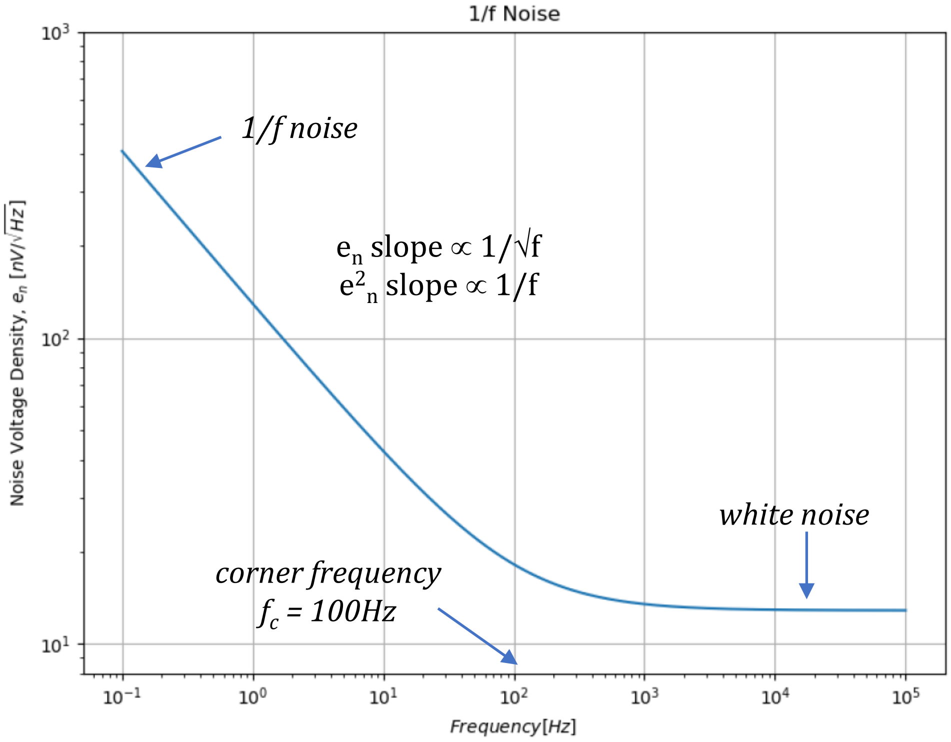
- Signal-to-noise ratio (SNR) is defined as

$$SNR = 10\log_{10}\left(v_s^2/v_n^2\right) = 20\log_{10}(v_s/v_n)$$

  where the voltages are rms values
- Bandwidth and center frequency should both be specified to assess the SNR, particularly if the signal itself has a narrow bandwidth (e.g. a sinusoid)
- This is because SNR will decrease as the circuit bandwidth is increased beyond that of the signal (circuit keeps adding noise power, while signal power remains constant)
- Bandwidth should only be as wide as the application dictates!

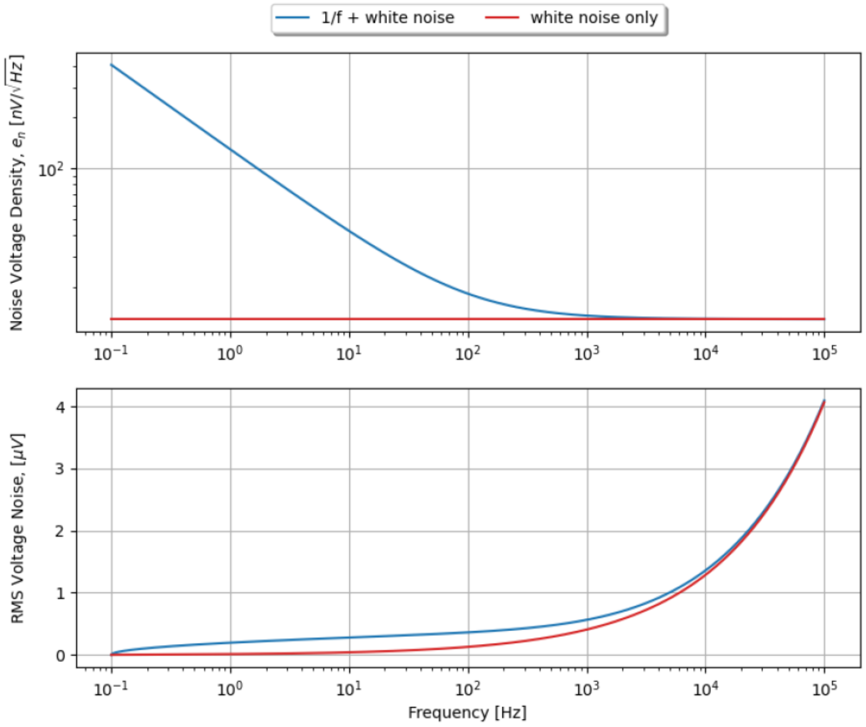
Flicker (1/*f*) Noise

- Flicker (1/f) noise arises in devices due to a variety of factors, including device (e.g. resistor, transistor) construction
- Flicker noise has approximately a 1/f *power* spectrum, and is sometimes called “pink noise”
- Many devices exhibit 1/f noise, but certain devices (such as MOSFET’s) exhibit high 1/f corner frequencies ($1/f_c$)

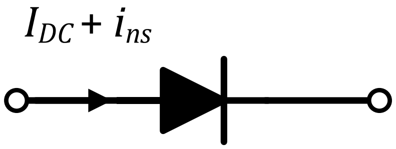


RMS Noise vs Bandwidth

- For narrow measurement bandwidths beginning at low frequencies, flicker noise dominates the RMS noise
- As bandwidth increases, wideband thermal/shot noise density is much higher than that of 1/f noise, so the total noise can be approximated by white noise alone
- For applications requiring high precision at very low frequencies, 1/f noise can have a substantial impact



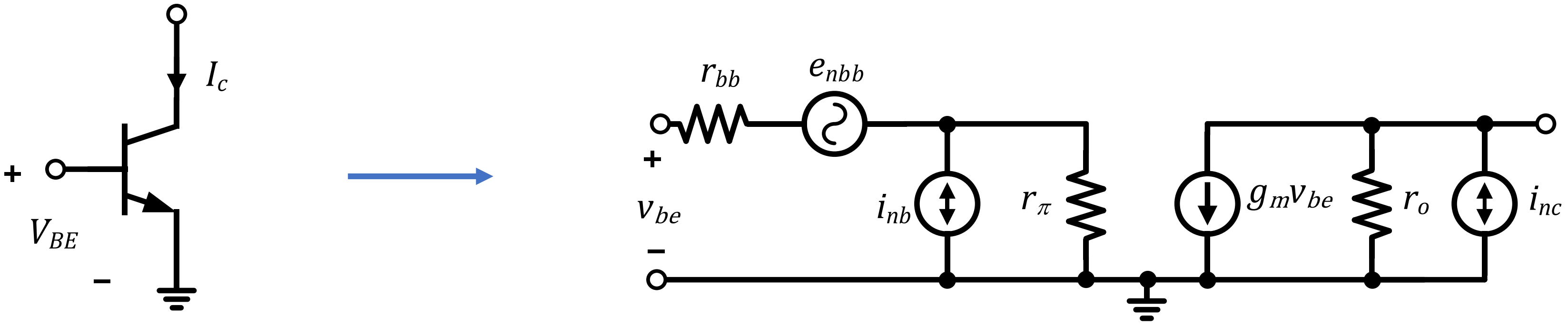
Shot Noise



$$i_{ns}^2 = 2qI_{DC}\Delta f$$

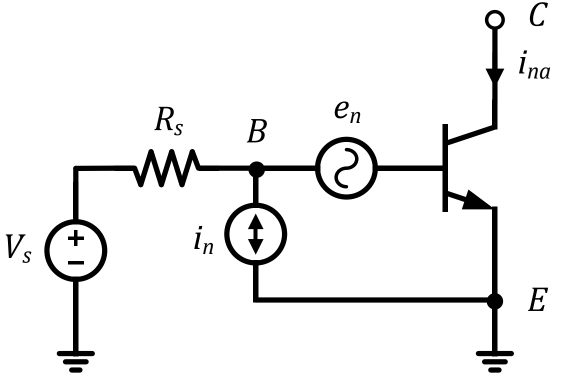
- Shot noise is random noise occurring as the result of charge carriers crossing a potential barrier (e.g. a diode potential)
- Shot noise density, like that of thermal noise, is constant with frequency (i.e. white)
- Unlike resistor thermal noise, shot noise is *always* associated with DC current flow, and its mean-square value is proportional to the DC current magnitude

BJT Noise Sources



- The standard BJT noise model includes thermal, flicker, burst, and shot noise voltages and currents
- r_{bb} is the base resistance of the BJT, typically 100’s of ohms
- Note that r_π and r_o are not real resistances, so they don’t exhibit thermal noise

BJT Noise Model

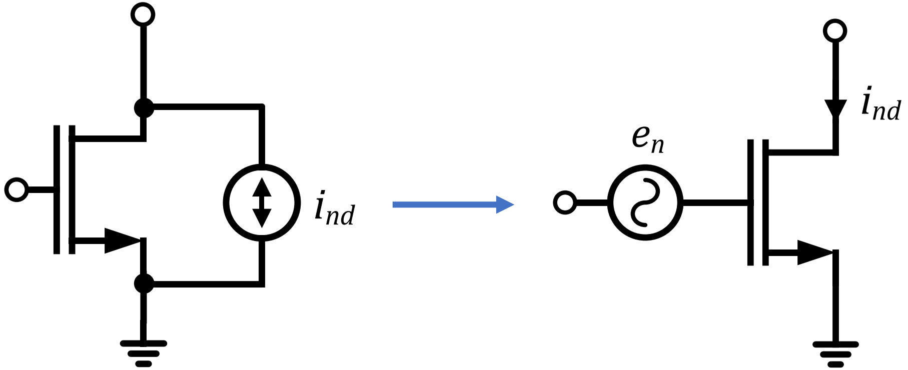


$$e_{na} = \sqrt{e_n^2 + (R_s i_n)^2}\, V/\sqrt{Hz}$$

$$i_{na} = g_m \sqrt{e_n^2 + (R_s i_n)^2}\, A/\sqrt{Hz}$$

- The noise generated by a transistor (or amplifier) is usually described by a simple noise model that is accurate enough for most purposes
- e_n and i_n represent the BJT's internal noise, modeled by a noise voltage e_n in series with the input and a noise current i_n injected at the input
- The transistor is assumed noiseless, and it amplifies (via its transconductance) the noise voltage that appears at the input

FET Noise Model

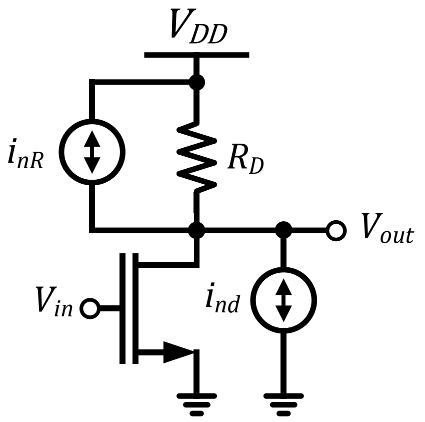


$$i_{nd} = \sqrt{\frac{2}{3}4kTg_m + i_{nf}}\ A/\sqrt{Hz}$$

$$e_n = \frac{i_{nd}}{g_m} V/\sqrt{Hz}$$

- The FET noise model is simpler, typically consisting of only thermal and 1/f drain current noise (i_{nd})
- Gate current noise is usually neglected at low frequencies, due to the high input impedance of the FET
- Because the *input-referred* noise decreases as $1/g_m$, it is generally better to operate FETs with high g_m to minimize voltage noise ($g_m \propto I_D$)

Common-Source Amplifier Noise



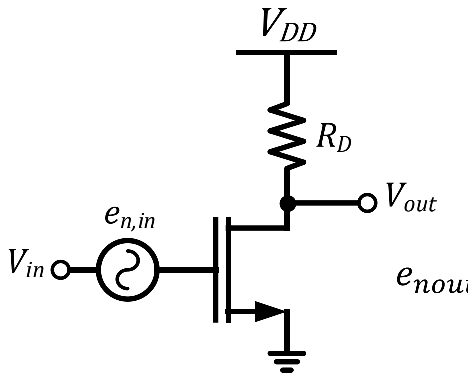
$$e_{nout} = \sqrt{i_{nR}^2 + i_{nd}^2} R_D$$

$$= \sqrt{4kT\left(\frac{1}{R_D} + \frac{2}{3} g_m\right)} R_D$$

- To determine the total output noise, we combine the mean-square current densities of both the FET and the drain resistor R_D
- Both noise currents flow through the drain resistor R_D, so the output voltage noise density is the product of the combined noise current and R_D
- However, we are typically more interested in the noise as seen at the *input* to the amplifier (note that we have omitted 1/f noise for simplicity)

Input-Referred Noise

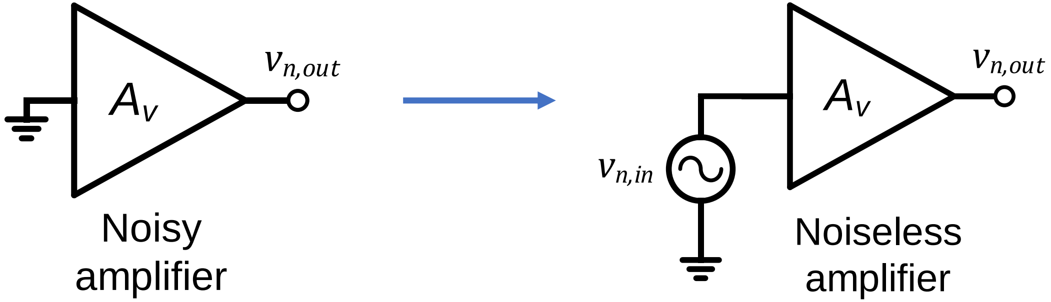
- By dividing by the gain we refer the noise to the input of the amplifier
- In doing so we are able to compare the magnitude of the noise with that of an input signal and thereby assess the SNR
- The input-referred noise can be reduced by increasing g_m, which in general requires an increase in drain current
- This reveals an inherent tradeoff between noise and power dissipation



$$e_{nout} = \sqrt{4kT\left(\frac{1}{R_D} + \frac{2}{3}g_m\right)R_D}$$

$$\boxed{e_{n,in} = \frac{e_{nout}}{|A_v|} = \frac{e_{nout}}{g_m R_D}}$$

Input-Referred Voltage Noise



$$v_{n,in} = \frac{v_{n,out}}{A_v}$$

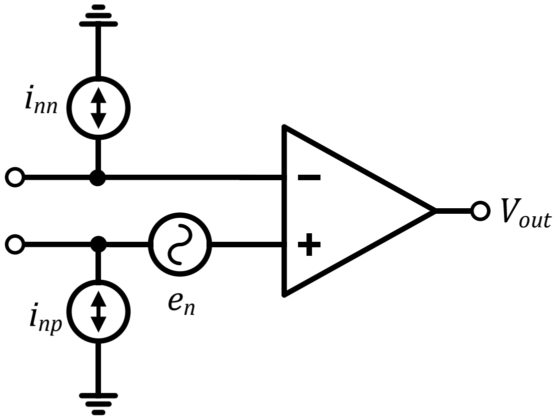
- Ultimately, we are concerned with the effect of all noise sources on a measured signal
- As such, we refer an amplifier's noise to its input to be able to compare it with the magnitude of the input signal
- To do this, we determine the output noise and divide by the gain

Opamp Noise

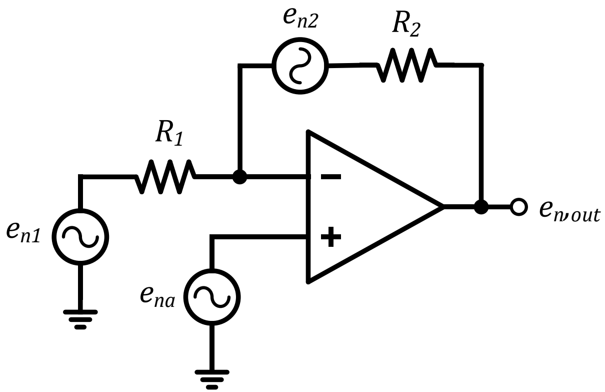
- Opamp noise is primarily due to the thermal, flicker, and shot noise of the transistors that make up the opamp
- Opamp data sheets specify noise in terms of e_n and i_n, just as with BJTs and FETs, often along with the 1/f corner frequency f_c
- Opamp manufacturers measure the noise characteristics of a large sampling of a given device to determine the typical characteristics
- As usual, there are inherent tradeoffs between accuracy, speed, input current, power dissipation, and cost
- Refer to the Tables 8.3a – 8.3c (pp 522 – 524) of AoE for detailed specifications of a wide range of opamps

Opamp Noise Model

- The noise model of the opamp includes both voltage (e_n) and current (i_n) noise sources
- When analyzing noise in opamp circuits, the effect of each uncorrelated noise source can be determined separately
- The total output noise *density* can be assessed by applying superposition of all mean square noise expressions
- The rms output noise can then be calculated by integrating over the equivalent noise bandwidth of the amplifier



Feedback Amplifier Noise



$$e_{n1} = \sqrt{4kTR_1}\, V/\sqrt{Hz}$$

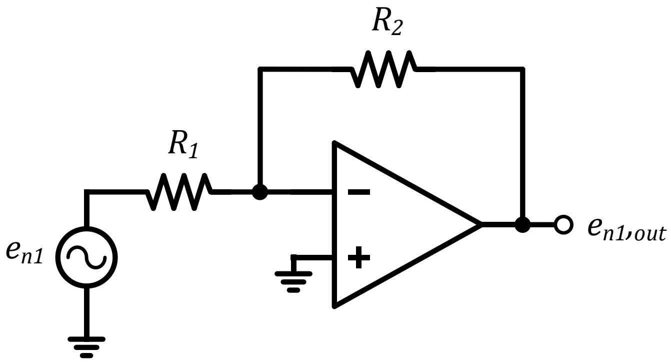
$$e_{n2} = \sqrt{4kTR_2}\, V/\sqrt{Hz}$$

$$2n\,V/\sqrt{Hz} \leq e_{na} \leq 100n\,V/\sqrt{Hz}$$

- To determine the total noise, we need to analyze the effect of each noise source independently, and combine the mean-square quantities
- Opamp voltage noise can vary widely, depending heavily on both the technology used (FET, BJT) and the opamp supply current
- Note that we have ignored the effect of opamp input current noise, which can be significant if the resistances in the feedback network are large

Noise from Input Resistor

- To determine the noise from the input resistance, we remove all other noise sources (superposition) and analyze the circuit
- The input voltage noise sees an inverting gain stage, so the result is straightforward:



$$e_{n1,out} = -\frac{R_2}{R_1} e_{n1}$$

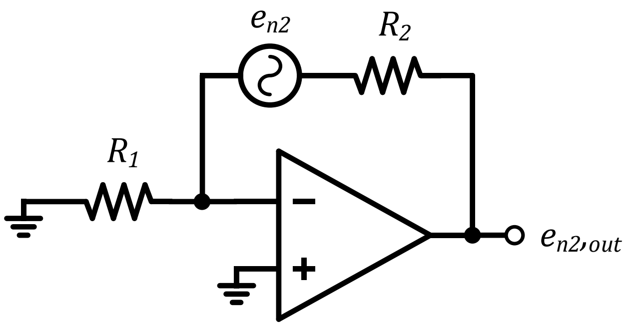
$$e_{n1,out}^2 = \left(\frac{R_2}{R_1}\right)^2 4kTR_1 \frac{V^2}{Hz}$$

Noise from Feedback Resistor

- Again, we remove other noise sources to determine the effect of R_2's noise
- In this case, the noise is seen directly at the output (no gain) due to the virtual ground:

$$e_{n2,out} = e_{n2}$$

- Because R_2's noise does see any gain, the output noise contribution from R_1 is greater



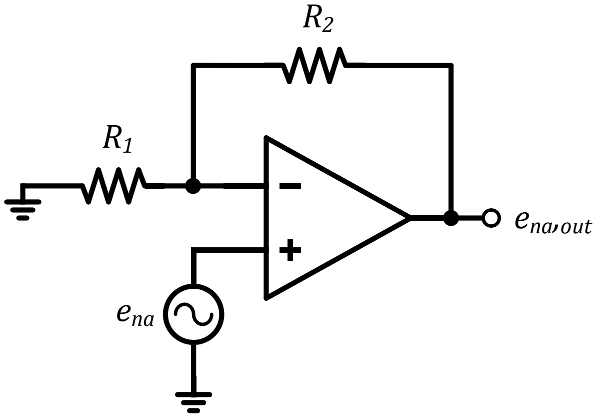
$$e_{n2,out}^2 = 4kTR_2 \frac{V^2}{Hz}$$

Opamp Noise

- The opamp noise as seen at the output can be determined in the same manner as the resistances
- The input voltage noise of the opamp sees the gain of a non-inverting amplifier:

$$e_{n,out} = \left(1 + \frac{R_2}{R_1}\right) e_{na}$$

- The relative magnitude of the opamp's noise as compared to the resistors depends on e_{na} and the *closed-loop* gain



$$e_{na,out}^2 = \left(1 + \frac{R_2}{R_1}\right)^2 e_{na}^2$$

Output Noise

Resistors:

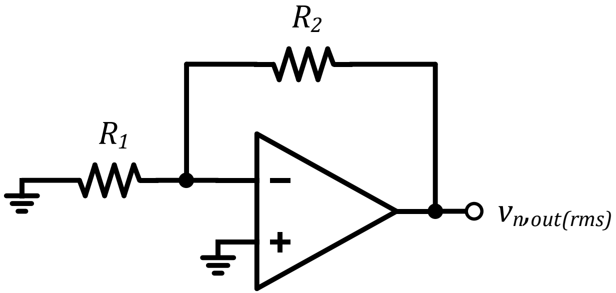
$$e_{n1out}^2 = \frac{{R_2}^2}{R_1^2}\, e_{n1}^2 \qquad e_{n2out}^2 = e_{n2}^2 \qquad e_{n,out} = \sqrt{e_{n1,out}^2 + e_{n2,out}^2 + e_{na,out}^2}$$

Opamp:

$$e_{naout}^2 = \left(1 + \frac{R_2}{R_1}\right)^2 e_{na}^2 \qquad = \sqrt{4kTR_2\left(\frac{R_1 + R_2}{R_1}\right) + \left(1 + \frac{R_2}{R_1}\right)^2 e_{na}^2}$$

- The total output noise is determined by computing the RMS value of all individual noise sources combined
- Depending on the opamp noise and the resistor values used, either the opamp noise or resistor noise may dominate
- Note that we have ignored the input current noise of the opamp, which can be significant (particularly for BJT-input opamps)

RMS Output Noise

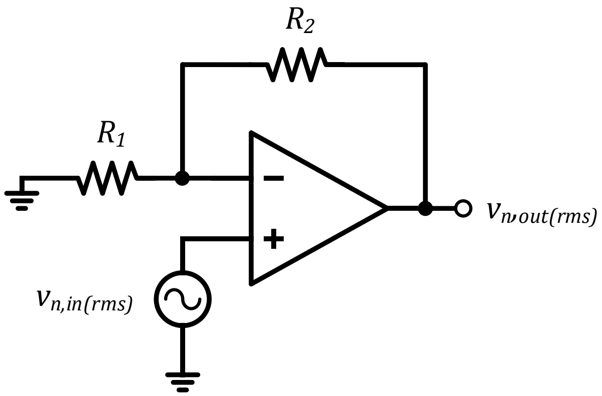


$$f_{3dB} \approx \beta f_T \qquad f_{ENB} = \frac{\pi}{2}\beta f_T$$

$$v_{n,out(rms)} = \sqrt{e_{nout}^2 f_{ENB}} = \sqrt{e_{nout}^2 \frac{\pi}{2}\beta f_T}$$

- To determine the total (rms) output noise, we integrate over the bandwidth of the amplifier
- The equivalent noise bandwidth (f_{ENB}) allows us to accomplish this by simple multiplication of the mean-square noise density
- Note that this is only the case if *all* noise sources (or at least the dominant noise source) have a flat spectrum (i.e. white)

RMS Input Noise

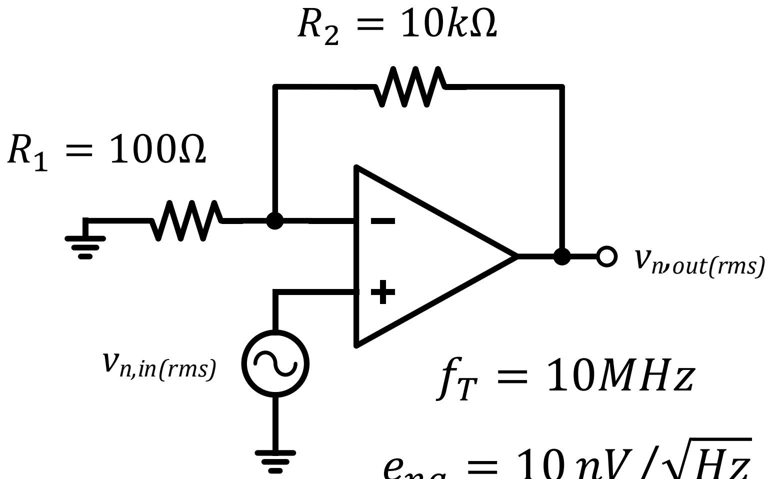


$$v_{n,out(rms)} = \sqrt{e_{nout}^2 \frac{\pi}{2} \beta f_T}$$

$$\boxed{v_{n,in(rms)} \approx \frac{v_{n,out(rms)}}{1/\beta} = \beta \sqrt{e_{nout}^2 \frac{\pi}{2} f_T}}$$

- Once we've determined the output rms noise, either by integration or using the equivalent noise bandwidth, we can calculate the rms input noise by simply dividing by the closed-loop gain of the amplifier
- Note that different amplifier structures (e.g. inverting vs non-inverting) may have the same output noise but different input-referred noise

Example Design



$f_T = 10MHz$

$e_{na} = 10\,nV/\sqrt{Hz}$

$i_n = 0$

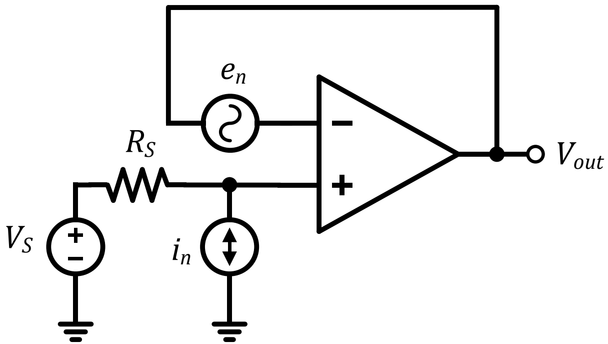
$$f_{3dB} \approx \beta f_T = 100kHz \qquad f_{ENB} = \frac{\pi}{2} f_{3dB} \approx 157kHz$$

$$v_{n,out(rms)} = \sqrt{\left(1\,\mu V/\sqrt{Hz}\right)^2 \cdot 157kHz} \approx 400\mu V$$

$$v_{n,in(rms)} = \frac{v_{n,out(rms)}}{A_{v,CL}} = \frac{400\mu V}{100V/V} = \mathbf{4\mu V}$$

Ltspice: FB Amp Noise

Input Current Noise



$$e_{n,out} = \sqrt{e_n^2 + R_S^2 i_n^2}\, V/\sqrt{Hz}$$

- The effect of opamp input current noise depends on the impedance(s) connected to the input of the opamp
- For low impedance sources and feedback networks (and opamps with low noise current), opamp noise is dominated by its input voltage noise
- For higher source/feedback resistances, current noise can have a substantial effect (Ltspice: opamp current noise)