Assignment 03

EE 538 Spring 2020
Analog Circuits for Sensor Systems
University of Washington Electrical & Computer Engineering

Due: April 25, 2020
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In [1]:
```
# Imports
import os
import sys
import cmath
import math
import matplotlib.pyplot as plt
import matplotlib
import numpy as np
import pandas as pd
import ltspice
import sympy as sp
from scipy import signal
%matplotlib inline
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
```

In [2]:
```
def read_ltspice_tran(file_name):
    cols = []
    arrs = []
    with open(file_name, 'r',encoding='utf-8') as data:
        for i,line in enumerate(data):
            if i==0:
                cols = line.split()
                arrs = [[] for _ in cols]
                continue
            parts = line.split()
            for j,part in enumerate(parts):
                arrs[j].append(part)
    df = pd.DataFrame(arrs,dtype='float64')
    df = df.T
    df.columns = cols
    return df
```

In [3]:
```
def RoundNonZeroDecimal(num, place, rnd='ceil'):
    # Requires numpy library
    # Examples:
    #   RoundNonZeroDecimal(0.0004512,1,'floor') -> 0.0045
    #   RoundNonZeroDecimal(0.0004512,1,'ceil') -> 0.0046
    #
    tmp = num # implement so that num can be array
    mag = 0

    if rnd=='ceil':
        while(abs(tmp)<1):
            tmp*=10
            mag+=1
        for i in range(place):
            tmp*=10
            mag+=1
        return int(np.ceil([tmp])[0])/(10**(mag))

    if rnd=='floor':
        while(abs(tmp)<1):
            tmp*=10
            mag+=1
        for i in range(place):
            tmp*=10
            mag+=1
        return int(np.floor([tmp])[0])/(10**(mag))

    else:
        raise ValueError('Invalid argument')
        return None
```

Problem 1: Common-emitter versus common-source amplifier

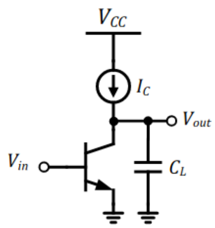


Figure 1a. Common-emitter (CE) amplifier

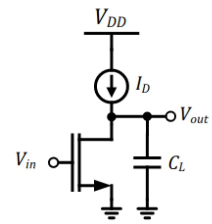


Figure 1b. Common-source (CS) amplifier

For the following, T = 300K, V_A = 100V, $V_{GS} - V_{th}$ = 500mV, λ = 0.1V^{-1}, C_L = 10pF and $I_C = I_D$ = 1mA.

a) (5 points) Calculate the DC voltage gain v_{out}/v_{in} for each structure. Determine the ratios g_m/I_C and g_m/I_D (transconductance efficiency).

b) (5 points) For each structure, determine the small-signal transfer function v_{out}/v_{in} as a function of frequency. Plot the Bode magnitude and phase (by hand or using MATLAB/Python). For each, calculate the transit frequency f_T, the frequency at which the magnitude of the transfer function is equal to 1V/V.

c) (5 points) The so-called "square-law" model of the FET incorrectly predicts that current becomes arbitrarily small (and g_m arbitrarily large) as $V_{GS} - V_{th}$ approaches zero. For values of V_{GS} smaller than V_{th} (subthreshold operation), the drain current is better described as

$$I_D = I_S e^{V_{GS}/nV_T},$$

where I_S and n are technology parameters related to the device structure. For n = 1.5, calculate the transconductance efficiency (g_m/I_D) of the FET assuming subthreshold operation. How does it compare to your answers in Part a)?

Part A

Structure A: $T = 300K, V_A = 100, C_L = 10\text{pF}, I_C = 1\text{mA}$

Given

$$I_{C0} = I_S(e^{V_{\text{BE}}/V_T} - 1)$$

$$I_C = I_{C0}(1 + \frac{V_{CE}}{V_A}), V_A \gg V_{CE} \rightarrow I_C \approx I_{C0}$$

Find $\frac{v_{\text{out}}}{v_{\text{in}}}$

$$\begin{aligned}
V_{in} &= V_{\text{BE}} \\
V_T &= \left.\frac{kT}{q}\right|_{T=300K} && = 25.86\text{mV} \\
g_m &= \frac{I_{CO}}{V_T} \\
r_0 &= \frac{V_A}{I_{C0}} \\
i_C &= g_m v_{\text{be}} \\
v_{\text{out}} &= -i_c r_0 = -g_m v_{\text{in}} r_0 \\
\frac{v_{\text{out}}}{v_{\text{in}}} &= -g_m r_0 \\
&= \frac{V_A}{V_T} = -3867
\end{aligned}$$

Find transconductance efficiency $\frac{g_m}{I_C}$

$$\begin{aligned} g_m &= \frac{I_{C0}}{V_T} \\ \frac{g_m}{I_C} &= \frac{I_{C0}}{I_C V_T} \approx \frac{1}{V_T} = 38.67\mathrm{V}^{-1} \end{aligned}$$

Structure B $T = 300K, C_L = 10\mathrm{pF}, I_D = 1\mathrm{mA}, V_{\mathrm{GS}-V_{\mathrm{th}}} = 500\mathrm{mV}, \lambda = 0.1\mathrm{V}^{-1}$
Given

$$I_D \approx I_{D0}$$

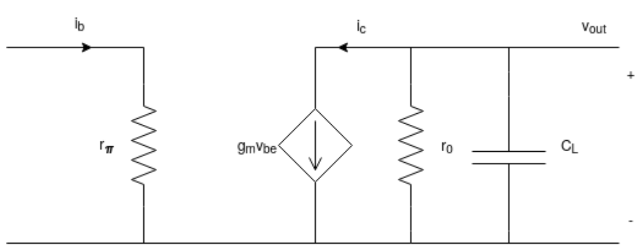
Find $\frac{v_{\mathrm{out}}}{v_{\mathrm{in}}}$

$$\begin{aligned} v_{\mathrm{in}} &= v_{\mathrm{GS}} \\ g_m &= \frac{2I_{D0}}{V_{\mathrm{GS}} - V_{\mathrm{th}}} \\ r_0 &= \frac{1}{\lambda I_{D0}} \\ i_D &= g_m v_{\mathrm{gs}} \\ v_{\mathrm{out}} &= -i_D r_0 = -g_m v_{\mathrm{in}} r_0 \\ \frac{v_{\mathrm{out}}}{v_{\mathrm{in}}} &= -g_m r_0 \\ &= \frac{-2I_{D0}}{\lambda I_{D0}(V_{\mathrm{GS}} - V_{\mathrm{th}})} \\ &= \frac{-2}{\lambda(V_{\mathrm{GS}} - V_{\mathrm{th}})} \\ &= -40 \end{aligned}$$

Find transconductance efficiency $\frac{g_m}{I_D}$

$$\begin{aligned} g_m &= \frac{2I_{D0}}{V_{\mathrm{GS}} - V_{\mathrm{th}}} \\ \frac{g_m}{I_D} &\approx \frac{2}{V_{\mathrm{GS}} - V_{\mathrm{th}}} = 4\mathrm{V}^{-1} \end{aligned}$$

Part B



In [4]:
```
def parallel(Z1,Z2):
    return 1/((1/Z1)+(1/Z2))
```

Structure A: $T = 300K, V_A = 100, C_L = 10\text{pF}, I_C = 1\text{mA}$

$$\begin{aligned}
V_{in} &= V_{\text{BE}} \\
i_C &= g_m v_{\text{be}} \\
g_m &= \frac{I_{CO}}{V_T} = 0.03867 \\
r_0 &= \frac{V_A}{I_{C0}} = 100000 \\
v_{\text{out}} &= -\, i_c r_{\text{eq}} = -g_m v_{\text{in}}(r_0 \| C_L) \\
\frac{v_{\text{out}}}{v_{\text{in}}} &= -\, g_m (r_0 \| C_L) \\
&= -\, g_m \frac{r_0}{1 + s r_0 C} \\
\left| \frac{v_{\text{out}}}{v_{\text{in}}} \right| &= g_m \frac{r_0}{\sqrt{1 + (2\pi f r_0 C)^2}} \rightarrow f_T = 6.15 \cdot 10^8 \text{Hz}
\end{aligned}$$

In [5]:

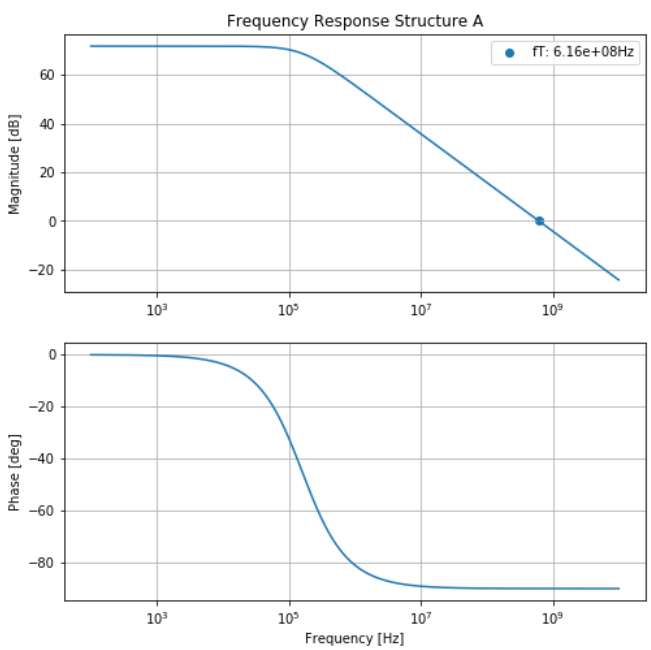
```
f1 = np.linspace(1e2,1e6,100000)
f2 = np.linspace(1e6,1e10,100000)
f = np.concatenate((f1,f2))
w = 2*np.pi*f
s = 1j*w
gm = 0.03867              # change
r0 = 100000               # change
C = 10e-12
Z_C = 1/(s*C)
H = gm*parallel(r0,Z_C)
#H = gm*r0/(1+s*r0*C)

#Find fT, frequency where magnitude is 1V/V.
mag = abs(H)
x0 = np.where(mag<=1)[0][0]
label = "{:.2e}".format(f[x0])
print(f"fT frequency at {label}")

# Plot the frequency response
fig, axs = plt.subplots(2,figsize=(8,8))
axs[0].set_title('Frequency Response Structure A')
axs[0].semilogx(f, 20*np.log10(abs(H)))
axs[0].scatter(f[x0], 20*np.log10(mag[x0]),label=f"fT: {label}Hz")
axs[0].set_ylabel('Magnitude [dB]')
axs[0].grid()
axs[0].legend()

axs[1].semilogx(f, np.angle(H,deg=True),label='Ideal')
axs[1].set_ylabel('Phase [deg]')
axs[1].set_xlabel('Frequency [Hz]')
axs[1].grid()
#axs[1].legend()
plt.show();
```

```
fT frequency at 6.16e+08
```



Structure B $T = 300K, C_L = 10\text{pF}, I_D = 1\text{mA}, V_{\text{GS}-V_{\text{th}}} = 500\text{mV}, \lambda = 0.1\text{V}^{-1}$

$$
\begin{aligned}
V_{\text{in}} &= V_{\text{GS}} \\
i_D &= g_m v_{\text{gs}} \\
g_m &= \frac{\partial I_D}{\partial V_{\text{GS}}} = \frac{2I_{D0}}{V_{\text{GS}} - V_{\text{th}}} = 0.004 \\
r_0 &= \frac{1}{\lambda I_{D0}} = 10000 \\
v_{\text{out}} &= -\, i_D r_{\text{eq}} = -g_m v_{\text{in}} (r_0 \| C_L) \\
\frac{v_{\text{out}}}{v_{\text{in}}} &= -\, g_m (r_0 \| C_L) \\
&= -\, g_m \frac{r_0}{1 + s r_0 C} \\
\left| \frac{v_{\text{out}}}{v_{\text{in}}} \right| &= g_m \frac{r_0}{\sqrt{1 + (2\pi f r_0 C)^2}} \rightarrow f_T \quad = 6.36 \cdot 10^7 \text{Hz}
\end{aligned}
$$

In [6]:

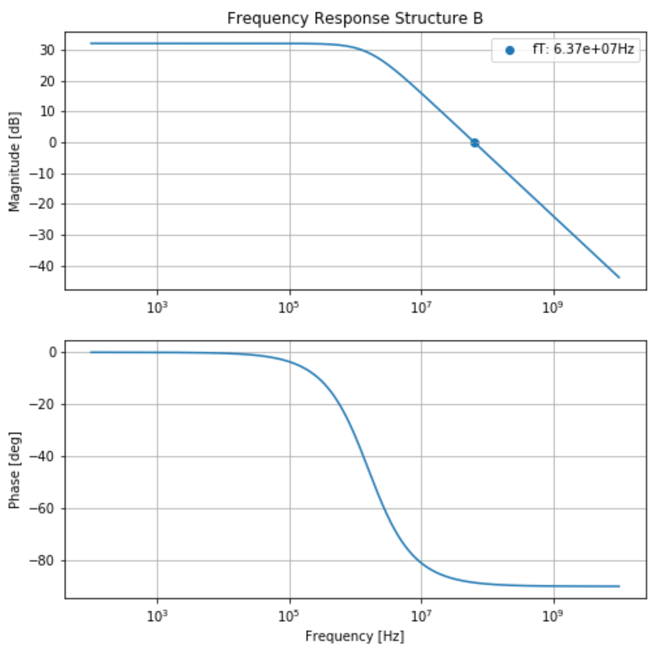
```
f1 = np.linspace(1e2,1e6,100000)
f2 = np.linspace(1e6,1e10,100000)
f = np.concatenate((f1,f2))
w = 2*np.pi*f
s = 1j*w
gm = 0.004                # change
r0 = 10000                # change
C = 10e-12
Z_C = 1/(s*C)
H = gm*parallel(r0,Z_C)
#H = gm*r0/(1+s*r0*C)

#Find fT, frequency where magnitude is 1V/V.
mag = abs(H)
x0 = np.where(mag<=1)[0][0]
label = "{:.2e}".format(f[x0])
print(f"fT frequency at {label}")

# Plot the frequency response
fig, axs = plt.subplots(2,figsize=(8,8))
axs[0].set_title('Frequency Response Structure B')
axs[0].semilogx(f, 20*np.log10(abs(H)))
axs[0].scatter(f[x0], 20*np.log10(mag[x0]),label=f"fT: {label}Hz")
axs[0].set_ylabel('Magnitude [dB]')
axs[0].grid()
axs[0].legend()

axs[1].semilogx(f, np.angle(H,deg=True))
axs[1].set_ylabel('Phase [deg]')
axs[1].set_xlabel('Frequency [Hz]')
axs[1].grid()
#axs[1].legend()
plt.show();
```

```
fT frequency at 6.37e+07
```



Part C

Given $I_D = 1\text{mA}, V_{\text{GS}} - V_{\text{th}} = 500\text{mV}$

$$\frac{g_m}{I_D} \approx \left.\frac{2I_{D0}}{I_D(V_{\text{GS}} - V_{\text{th}})}\right|_{I_D=1\text{mA}, V_{\text{GS}}-V_{\text{th}}=500\text{mV}} = 4\text{V}^{-1}$$

Given $\left. I_D = I_s e^{V_{\text{GS}}/nV_T} \right|_{n=1.5, V_T=25.86\text{mV}}$

$$g_m = \frac{\partial I_D}{\partial V_{\text{GS}}} = \frac{\partial}{\partial V_{\text{GS}}} I_s e^{V_{\text{GS}}/nV_T} = \frac{I_s}{nV_T} e^{V_{\text{GS}}/nV_T}$$

$$\begin{aligned} \frac{g_m}{I_D} &= \frac{\frac{I_s}{nV_T} e^{V_{\text{GS}}/nV_T}}{I_s e^{V_{\text{GS}}/nV_T}} \\ &= \frac{1}{nV_T} \\ &= 25.78\text{V}^{-1} \end{aligned}$$

This model is $\frac{25.78}{4} = 6.45$**x larger.**

Problem 2: Temperature-independent voltage reference (BJT DC analysis)

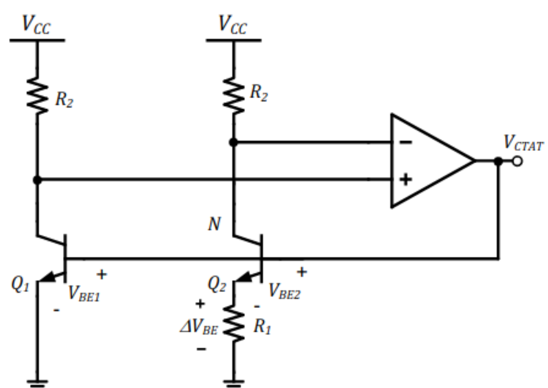


Figure 2. PTAT Voltage Generator

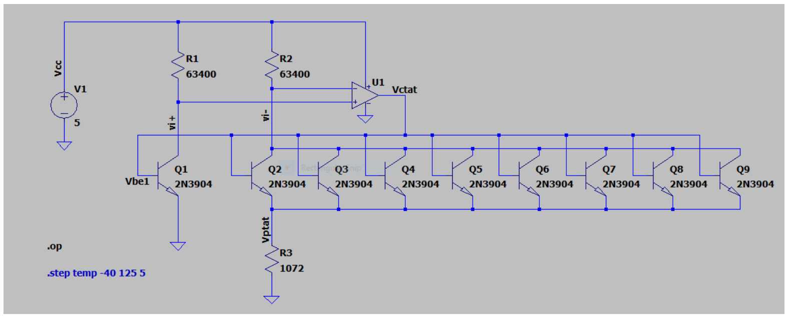
Temperature-insensitive voltage and current references are critical components of precision sensor systems. A temperature-independent reference is created by combining something (e.g. a voltage) that has a positive temperature coefficient (proportional-to-absolute-temperature, PTAT) with something that has a negative temperature coefficient (complementary-to-absolute-temperature, CTAT). When biased with a constant current, the V_{BE} of a BJT exhibits a slope of *approximately* –2mV/°C (CTAT). Combining this with the *difference* of the V_{BE}'s of two BJT's biased with different current densities (which is PTAT), properly scaled, will yield a voltage that is (approximately) independent of temperature:

$$V_{BG} = V_{CTAT} + V_{PTAT} = V_{BE}(T) + M \times \Delta V_{BE}(T)$$

Note that different current densities for Q_1 and Q_2 are achieved by connecting N transistors in parallel for Q_2.

For the following, use the 2N3904 npn transistor ($I_S = 10^{-14}$A, β = 300, $V_A = 100$) and the UniversalOpamp2 models in Ltspice. Use $V_{CC} = 5$V for the supply voltage.

a) (5 points) Determine values for N and R_1 such that $I_{C1} = I_{C2} = 50\mu$A at room temperature (27C).
b) (5 points) Determine the temperature slope of V_{BE1} via simulation and calculate the value of M that would satisfy the above equation.
c) (5 points) Verify the design of the PTAT generator in Ltspice, plotting the expression $V_{BE}(T) + M \times \Delta V_{BE}(T)$ as a function of temperature. Include your schematic in your submission, showing all relevant voltages and currents at room temperature. Evaluate
   1. the value of V_{BG} at room temperature, and
   2. the maximum deviation from this value over the temperature range –40C to 125C.



V(vcc)	5	voltage
V(vi+)	1.829	voltage
V(vi-)	1.82901	voltage
V(vbe1)	0.577333	voltage
V(vptat)	0.0537948	voltage
Ic(Q1)	5.00171e-05	device_current
Ib(Q1)	1.64682e-07	device_current
Ie(Q1)	-5.01817e-05	device_current
I(R3)	5.01817e-05	device_current

Part A

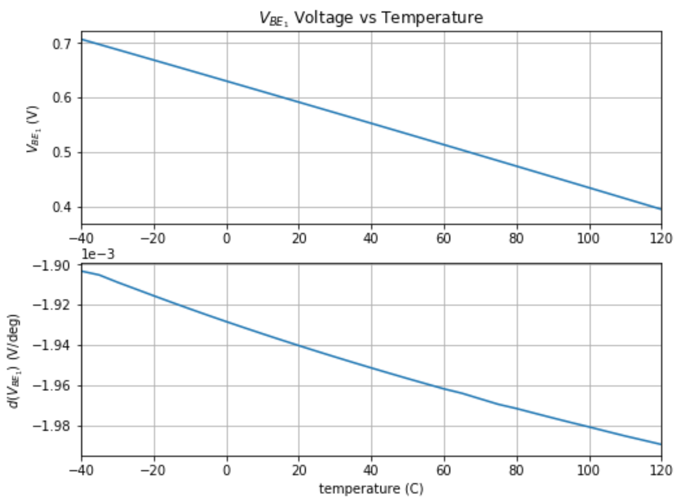
Given: $I_S = 10^{-14}\text{A}, \beta = 300, V_A = 100, V_{CC} = 5, N = 8$
Find: R_1 **such that** $I_{C1} = I_{C2} = 50\mu$**A**

$$
\begin{aligned}
V_T &= \frac{kT}{q}\bigg|_{T=27C=300K} && = 25.86\text{mV} \\
V_{\text{BE}_2} &= V_T \ln\left(\frac{I_{\text{bias}}}{NI_S}\right)\bigg|_{\text{N parallel BJTs}} \\
\Delta V_{\text{BE}} &= V_{\text{BE}_1} - V_{\text{BE}_2} \\
&= V_T[\ln\left(\frac{I_{C1}}{I_S}\right) - \ln\left(\frac{I_{C2}}{NI_S}\right)]\bigg|_{N=8} \\
&= 0.05377\text{V} \\
I_B &= \frac{I_C}{\beta} && = \frac{50\mu\text{A}}{300} \\
I_E &= (\beta+1)I_B && = 50.17\mu\text{A} \\
\Delta V_{\text{BE}} &= I_E R_1 \rightarrow R_1 = \frac{\Delta V_{\text{BE}}}{I_E} \\
R_1 &= \frac{\Delta V_{\text{BE}}}{I_E} && = 1072\Omega \\
I_C &= I_{C0}(1 + \frac{V_{CE}}{V_A}) \\
V_{CE_1} &= V_A(\frac{I_C}{I_{C0}} - 1) && = 1.83\text{V} \\
R_2 &= \frac{V_{CC} - V_{\text{CE}_1}}{I_C}\bigg|_{V_{\text{CE}_1}=1.83} && = 63400\Omega
\end{aligned}
$$

Part B

In [15]:
```
filepath = 'data/PTAT_voltage_generator.txt'
df = read_ltspice_tran(filepath)
```

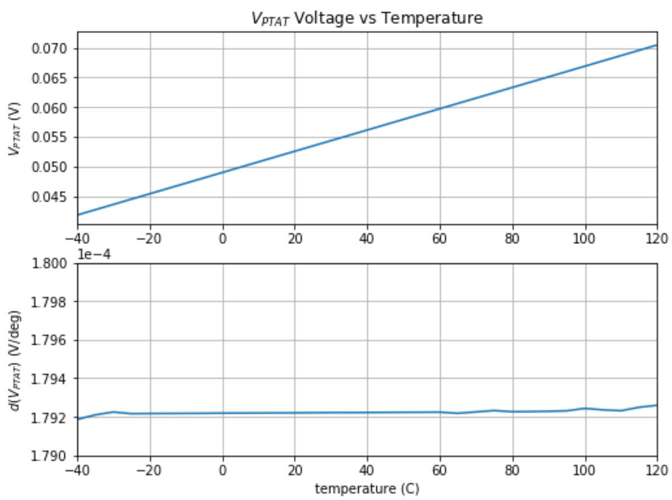
In [8]:
```
fig, axs = plt.subplots(2,figsize=(8,6))
axs[0].set_title('$V_{{BE}_1}$ Voltage vs Temperature')
axs[0].plot(df['temperature'], df['V(vbe1)'],label='$V_{{BE}_1}$')
axs[0].set_ylabel('$V_{{BE}_1}$ (V)')
axs[0].set_xlim(-40, 120)
axs[0].grid()
#xs[0].legend()
axs[1].plot(df['temperature'], df['d(V(vbe1))'],label='$d(V_{{BE}_1})$')
axs[1].set_ylabel('$d(V_{{BE}_1})$ (V/deg)')
axs[1].set_xlabel('temperature (C)')
axs[1].set_xlim(-40, 120)
axs[1].grid()
#xs[1].legend()
axs[1].ticklabel_format(style='sci', axis='y', scilimits=(-2,-3))
plt.show();
```



In [9]:
```
temp = df['temperature'].to_numpy()
x0 = np.where(temp<=27)[0][-1]
v1 = df.iloc[x0]['d(V(vbe1))']
print(f'Derivative value of Vbe1 at 27 Celcius is {round(1e3*v1,3)}mV')
```

```
Derivative value of Vbe1 at 27 Celcius is -1.943mV
```

In [10]:
```python
fig, axs = plt.subplots(2,figsize=(8,6))
axs[0].set_title('$V_{PTAT}$ Voltage vs Temperature')
axs[0].plot(df['temperature'], df['V(vptat)'],label='$V_{PTAT}$')
axs[0].set_ylabel('$V_{PTAT}$ (V)')
axs[0].set_xlim(-40, 120)
axs[0].grid()
#xs[0].legend()
axs[1].plot(df['temperature'], df['d(V(vptat))'],label='$d($V_{PTAT}$)$')
axs[1].set_ylabel('$d(V_{PTAT})$ (V/deg)')
axs[1].set_xlabel('temperature (C)')
axs[1].set_xlim(-40, 120)
axs[1].set_ylim(179*1e-6, 180*1e-6)
axs[1].grid()
#xs[1].legend()
axs[1].ticklabel_format(style='sci', axis='y', scilimits=(-4,-4))
plt.show();
```



In [11]:
```python
v2 = df.iloc[x0]['d(V(vptat))']
print(f'Derivative value of Vptat at 27 Celcius is {round(1e3*v2,3)}mV')
```

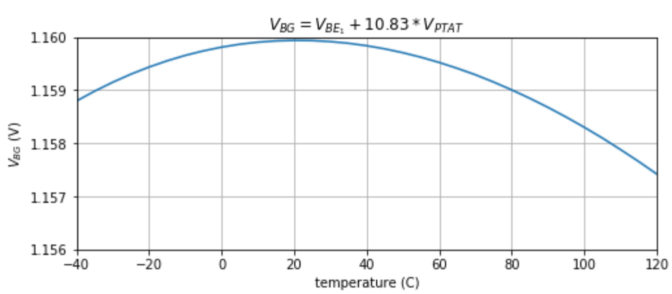
```
Derivative value of Vptat at 27 Celcius is 0.179mV
```

In [12]:
```python
print(f'M = 1.943/0.179 = {round(-v1/v2,3)}')
```

```
M = 1.943/0.179 = 10.843
```

Part C

In [13]:
```python
fig, ax = plt.subplots(figsize=(8,3))
ax.set_title('$V_{BG} = V_{{BE}_1}+10.83*V_{PTAT}$')
ax.plot(df['temperature'], df['V(vbe1)+10.83*V(vptat)'],label='$V_{{BE}_1}$')
ax.set_ylabel('$V_{BG}$ (V)')
ax.set_ylim(1.156, 1.16)
ax.set_xlim(-40, 120)
ax.set_xlabel('temperature (C)')
ax.grid()
#ax.legend()
plt.show();
```



In [14]:
```python
v3 = df.iloc[x0]['V(vbe1)+10.83*V(vptat)']
df['delta'] = df['V(vbe1)+10.83*V(vptat)']-v3
print(f'V_BG at 27 Celcius is {round(v3,4)}V')
print(f'Maximum deviation is {RoundNonZeroDecimal(max(df["delta"]),4)}V')
```

```
V_BG at 27 Celcius is 1.1599V
Maximum deviation is 4e-06V
```

Problem 3: Nonlinear distortion in a common-source amplifier

The output voltage of a resistor-loaded common-source amplifier is expressed (neglecting λ) as

$$V_{out} = V_{DD} - \kappa(V_{in} - V_{th})^2 R_D$$

a) (5 points) Assuming the amplifier is driven with a sinusoidal voltage $V_{in} = a_{in} \times \sin(2\pi \cdot f_0 \cdot t) + V_{DC}$, where $V_{DC} = V_{th} + 500\text{mV}$, determine expressions for the *amplitudes* of the fundamental (sinusoid at f_0 with amplitude a_1) and second harmonic (sinusoid at $2f_0$ with amplitude a_2) using the trigonometric relationship

$$\sin^2(x) = \frac{1}{2}[1 - \cos(2x)]$$

b) (5 points) Calculate the ratio of a_2/a_1 for $a_{in} = 1\text{mV}$ and $a_{in} = 10\text{mV}$.

Part A

$$\begin{aligned}
V_{\text{out}} &= V_{DD} - \kappa(V_{\text{in}} - V_{\text{th}})^2 R_D \\
V_{\text{in}} &= a_{\text{in}} \sin(2\pi f t) + V_{DC} \Big|_{V_{DC} = V_{\text{th}} + 500\text{mV}} \\
V_{\text{out}} &= V_{DD} - \kappa R_D [a_{\text{in}} \sin(2\pi f_0 t) + V_{\text{th}} + 500\text{mV} - V_{\text{th}}]^2 \\
V_{\text{out}} &= V_{DD} - \kappa R_D [a_{\text{in}} \sin(2\pi f t) + 0.5]^2 \\
&= V_{DD} - \kappa R_D [{a_{\text{in}}}^2 \sin^2(2\pi f_0 t) + a_{\text{in}} \sin(2\pi f_0 t) + 0.25] \Big|_{w = 2\pi f_0} \\
&= V_{DD} - \kappa R_D [{a_{\text{in}}}^2 \sin^2(wt) + a_{\text{in}} \sin(wt) + 0.25] \\
&= V_{DD} - \kappa R_D [\frac{{a_{\text{in}}}^2}{2}(1 - cos(2wt)) + a_{\text{in}} \sin(wt) + 0.25] \\
&= V_{DD} - \kappa R_D \frac{{a_{\text{in}}}^2}{2} + \kappa R_D \frac{{a_{\text{in}}}^2}{2} \cos(2wt)) - \kappa R_D a_{\text{in}} \sin(wt) - \kappa R_D 0.25 \\
&= (V_{DD} - \kappa R_D (\frac{{a_{\text{in}}}^2}{2} + 0.25)) - \kappa R_D a_{\text{in}} \sin(wt) + \kappa R_D \frac{{a_{\text{in}}}^2}{2} \cos(2wt)) \\
&= \text{DC} + \text{fundamental sinusioid} + \text{2nd harmonic} \\
a_1 &= -\kappa R_D a_{\text{in}} \\
a_2 &= \kappa R_D \frac{{a_{\text{in}}}^2}{2}
\end{aligned}$$

Part B

$$\begin{aligned}
\frac{a_2}{a_1} &= \frac{\kappa R_D \frac{{a_{\text{in}}}^2}{2}}{-\kappa R_D a_{\text{in}}} &&= \frac{-a_{\text{in}}/2}{1} \frac{(\text{V})}{(\text{V})} \\
&= \frac{-a_{\text{in}}/2}{1} \Big|_{1\text{mV}} &&= -0.5 \cdot 10^{-3} = 0.05\% \\
&= \frac{-a_{\text{in}}/2}{1} \Big|_{10\text{mV}} &&= -5 \cdot 10^{-3} \quad = 0.5\%
\end{aligned}$$

```
In [ ]:
```