EE P 538
Analog Circuits for Sensor Systems

Spring 2020

Instructor: Jason Silver, PhD

Announcements

- Assignment 2 will be posted on Canvas soon
  - Due April 18 at midnight

Week 2

- AoE Chapter 2 – Bipolar Transistors
  - 2.1, 2.2.3 – 2.2.9, 2.3

Overview

- Last time…
  - Passive devices: resistors and capacitors
  - Thevenin/Norton equivalent circuits
  - Circuit loading
  - Basic RC filters: LPF and HPF
- Today…
  - Simple model of a BJT
  - Emitter follower and common emitter amplifier
  - Ebers–Moll exponential model
  - Small-signal approximation of a BJT

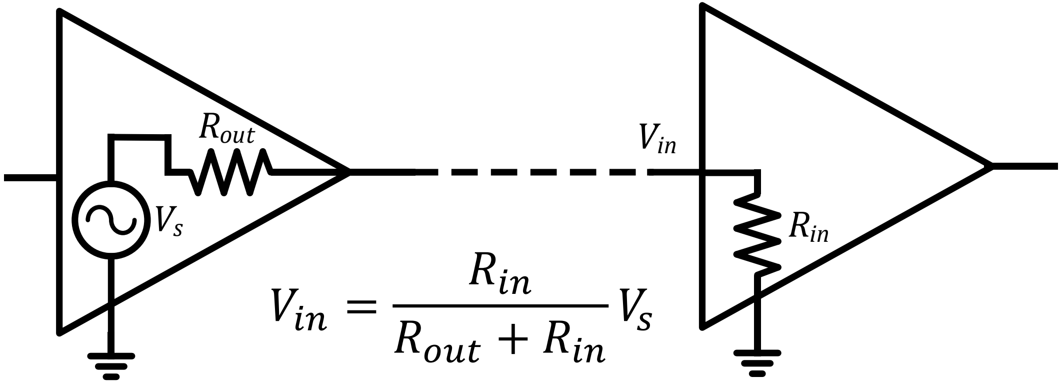
Lecture 2 – Transistors (BJTs)

Sage Advice

"A good understanding of transistors is very important, even if most of your circuits are made from ICs, because you need to understand the input and output properties of the IC in order to connect it to the rest of your circuit and to the outside world."

– *The Art of Electronics*

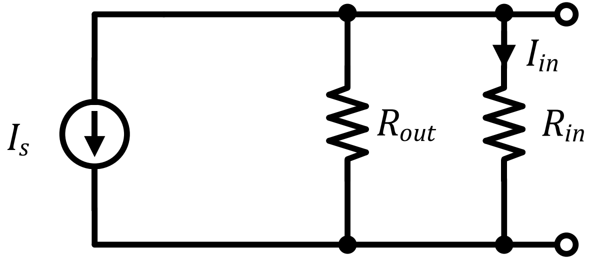
Loading Revisited



$$V_{in} = \frac{R_{in}}{R_{out} + R_{in}} V_s$$

- In electronic circuits, we're always connecting the output of "something" to the input of "something else"
- The loading effect of the following stage causes a reduction of signal, dependent on the ratio of R_{out} to R_{in} (voltage divider)
- It is usually best to keep R_{out} << R_{in} (a good rule of thumb is a factor of 10)

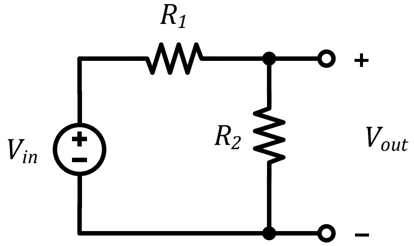
Current Source Loading

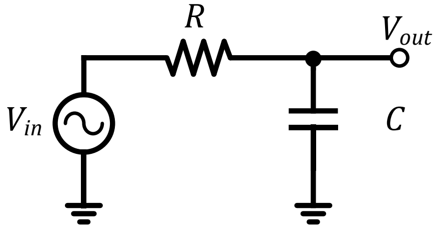


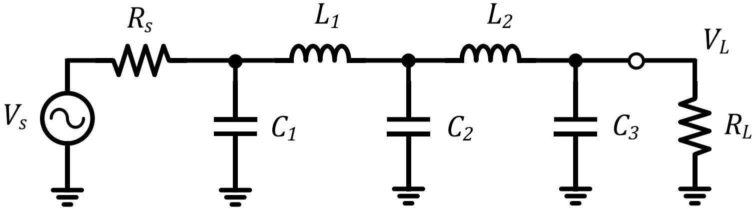
$$I_{in} = \frac{R_{out}}{R_{in} + R_{out}} I_s$$

- Current-based circuits also exhibit loading effects (current divider)
- In contrast to voltage-based circuits, output currents are attenuated when the load resistance is *large* relative to the source
- The rule of thumb is opposite that for voltage-based circuits: one should ensure R_{in} << R_{out} (a factor of 10 is a safe minimum)

Passive Devices





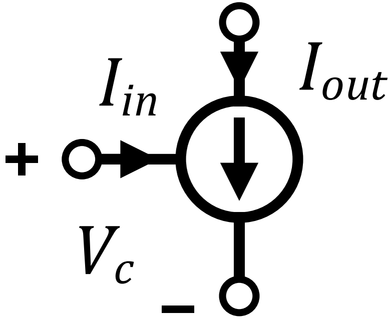


- In circuits containing only R’s, L’s, and C’s, the output voltage (current) is typically lower than (or equal to) the input voltage (current)
- Such circuits are referred to as “passive circuits,” and are not capable of (power) gain
- In order to realize gain, we need active devices

Active Devices

- An active device is one that can amplify, producing an output signal with more *power* in it than the input signal
- The transistor is the most significant example of an active component
- The two main “flavors” of transistor are bipolar junction transistors (BJT’s) and field effect transistors (FET’s)
- Some coarse generalizations about the two:
  - BJT’s excel in accuracy, low noise, and power efficiency
  - FET’s excel in low power, high impedance, and switching
- Noteworthy applications of each:
  - BJT’s: power amplifiers, opamps
  - FET’s: digital circuits/systems (CMOS), opamps

Ideal Transistor



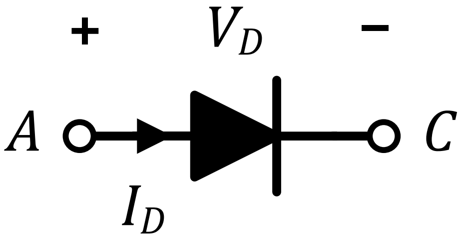
$$I_{in} \ll I_{out}$$

$$I_{out} = GV_c \quad \text{or} \quad I_{out} = \beta I_{in}$$

$$R_{out} \to \infty$$

- Ideal transistors are essentially voltage- or current-controlled current sources with high gain and high output resistance
- I_{out} is supplied by the circuit's power supply, and not the driving circuit
- This is what makes amplification (i.e. power gain) possible

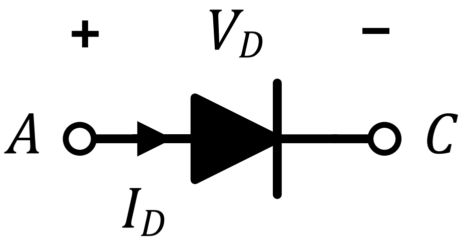
A Step Back: Diodes



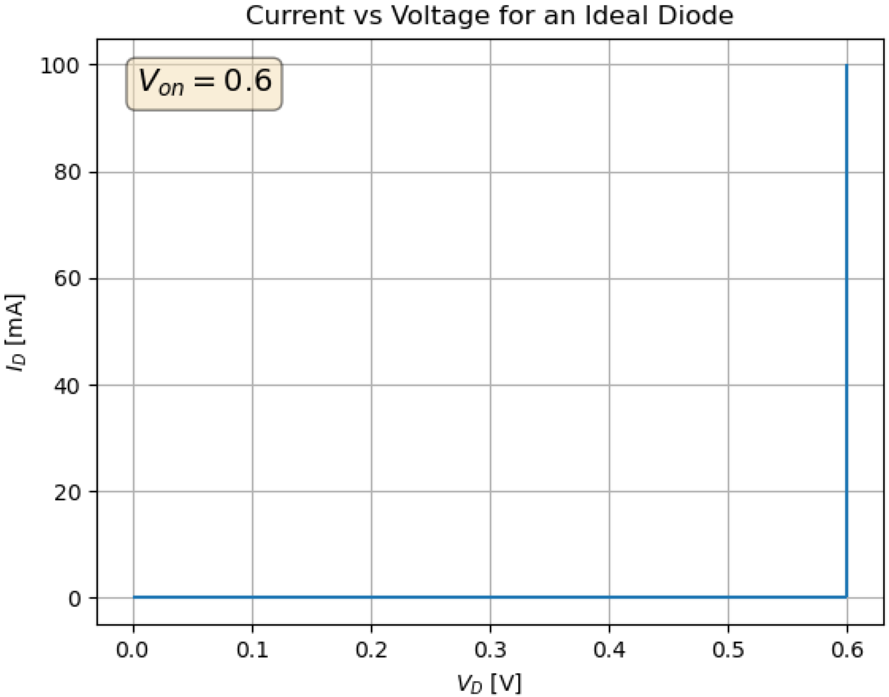
I_D	V_D
0	0
> 0	V_{on}

- Diodes are *passive semiconductor devices* with *nonlinear* V-I relationships
- For most practical purposes, diodes can be treated as exhibiting a constant voltage V_{on} for any positive current value
- That is, *any nonzero* I_D will result in $V_D = V_{on}$

Ideal Diode

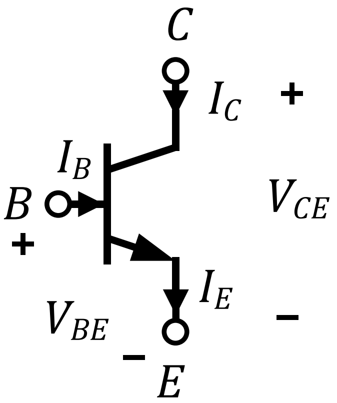


+
V_D
−
A
C
I_D



Current vs Voltage for an Ideal Diode
$V_{on} = 0.6$
100
80
60
40
20
0
I_D [mA]
0.0
0.1
0.2
0.3
0.4
0.5
0.6
V_D [V]

Bipolar Junction Transistor (BJT)



Simplified BJT Model

$$V_{CE} > 0$$

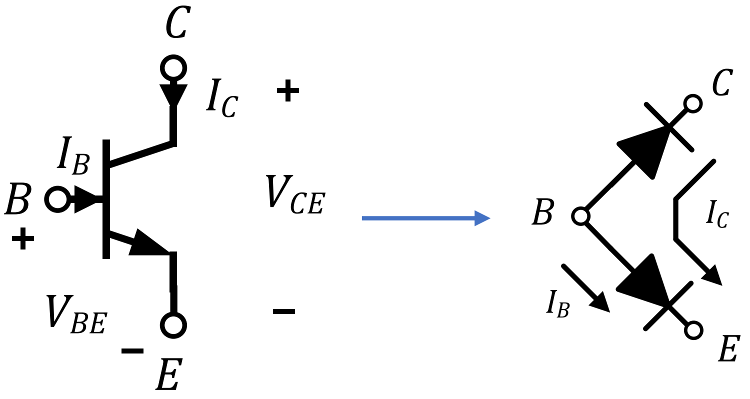
$$V_{BE} \approx 0.6V$$

$$I_C = \beta I_B \quad \text{where} \quad \beta \approx 100$$

$$I_E = (\beta + 1) I_B$$

- In the simple model, we can view the BJT as a current amplifier with current gain of β
- The base-emitter junction behaves like a forward-biased diode, with a voltage of approximately 600 mV (constant V_{BE} approximation)
- The collector current I_C is independent of $V_{CE,}$ depending only on I_B

NPN Transistor



Simplified NPN Model

$$V_{CE} > 0$$

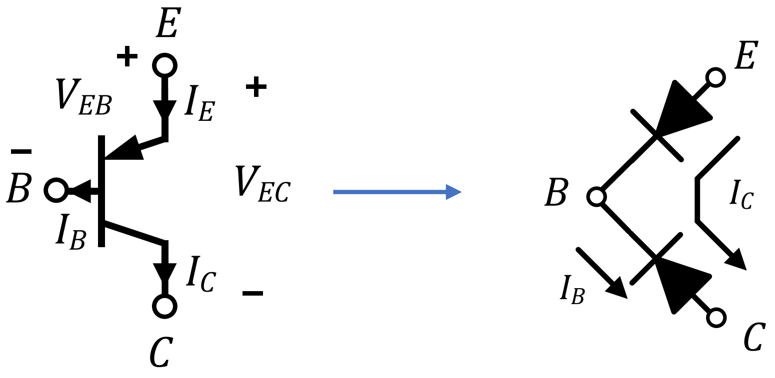
$$V_{BE} \approx 0.6V$$

$$I_C = \beta I_B \quad \text{where} \quad \beta \approx 100$$

$$I_E = (\beta + 1) I_B$$

- Collector voltage must be higher (more positive) than emitter
- Base-emitter junction behaves like a forward-biased diode, with voltage V_{BE} = 600 mV
- BJT is essentially a current amplifier with a current gain of β

PNP Transistor



Simplified PNP Model

$$V_{EC} > 0$$

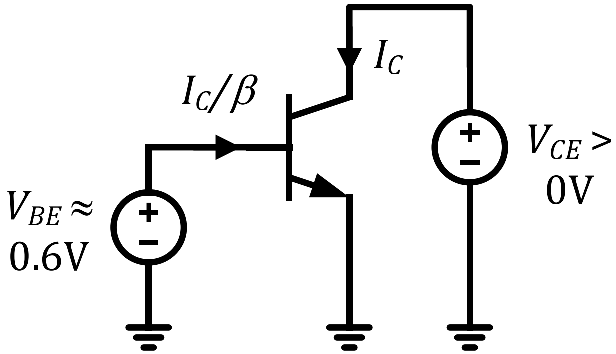
$$V_{EB} \approx 0.6V$$

$$I_C = \beta I_B \quad \text{where} \quad \beta \approx 50$$

$$I_E = (\beta + 1) I_B$$

- Emitter voltage must be higher (more positive) than collector
- Emitter-base junction behaves like a forward-biased diode, with voltage V_{EB} = 600 mV
- Current gain for PNP's is typically lower than that of NPN's

BJT "Biasing"



Simplified BJT Model

$$V_{CE} > 0$$

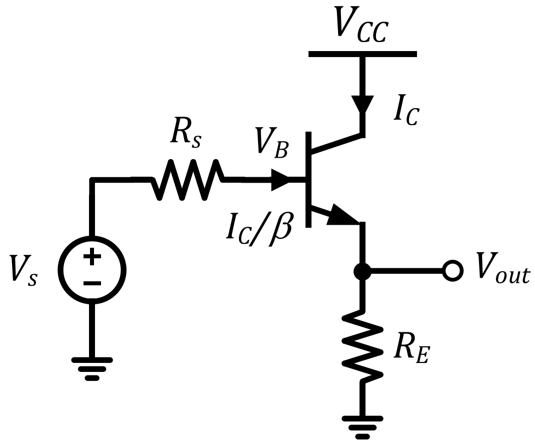
$$V_{BE} \approx 0.6V$$

$$I_C = \beta I_B \quad \text{where} \quad \beta \approx 100$$

$$I_E = (\beta + 1)I_B$$

- Biasing is the process of connecting voltages and currents with appropriate DC values to transistors to set them up for correct operation
- This can either be accomplished by explicit "biasing networks," which use resistors, or by ensuring that the previous stage meets the biasing needs of the current stage

Emitter Follower (1)



Simplified NPN Model

$$V_{CE} > 0$$

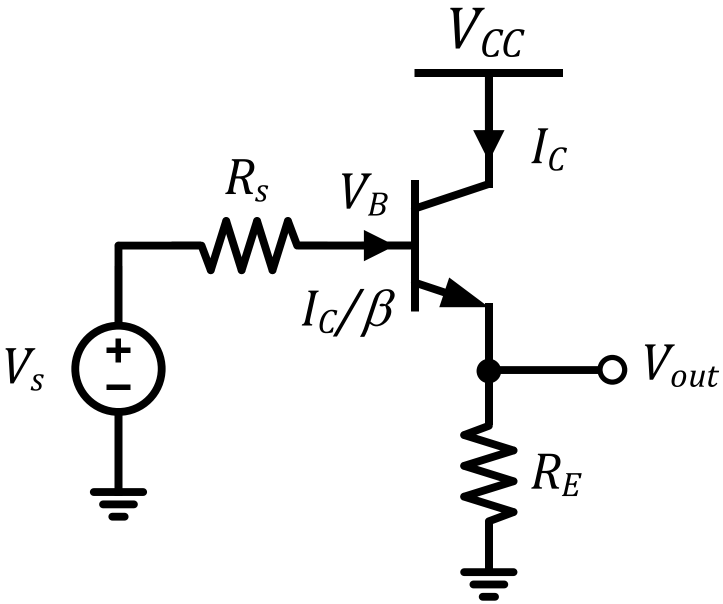
$$V_{BE} \approx 0.6V$$

$$I_C = \beta I_B \quad \text{where} \quad \beta \approx 100$$

$$I_E = (\beta + 1) I_B$$

- If β is large (high current gain), I_C/β is small and $V_B \approx V_s$
- Using the constant V_{BE} approximation (V_{BE} = 0.6V), $V_{out} = V_B - 0.6\text{V}$
- Thus, we say the emitter voltage *follows* the input/base voltage (follower)

Emitter Follower (2)



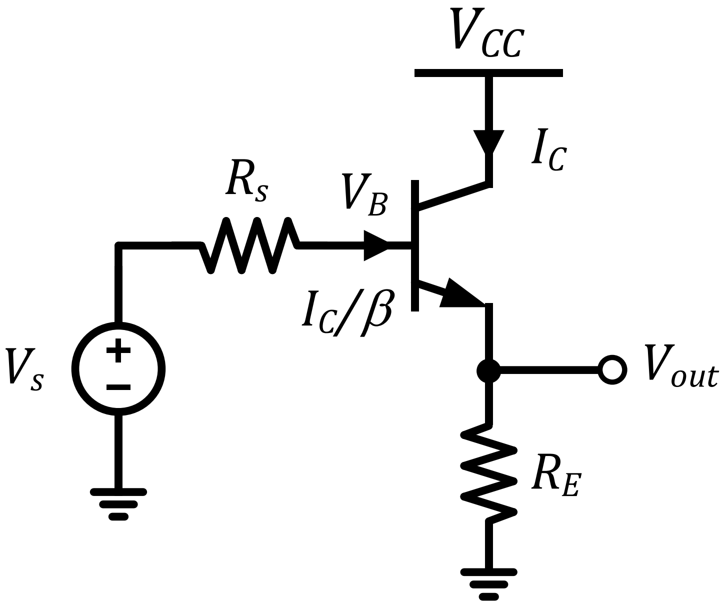
$$I_C \approx \frac{V_{out}}{R_E} \tag{1}$$

$$V_B = V_{out} + 0.6V = V_s - \frac{I_C}{\beta} R_s \tag{2}$$

$$V_{out} = V_s - \frac{V_{out}}{\beta R_E} R_s - 0.6V \tag{3}$$

$$V_{out}\left(1 + \frac{R_s}{\beta R_E}\right) = V_s - 0.6V \tag{4}$$

Emitter Follower (3)

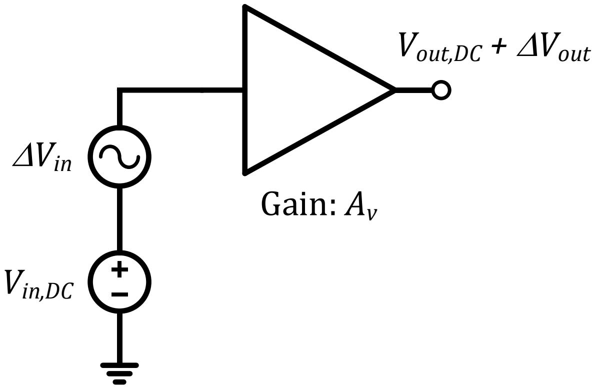


$$V_{out}\left(1+\frac{R_s}{\beta R_E}\right)=V_s-0.6V$$

$$if\ \frac{R_s}{\beta R_E}\ll 1,$$

$$\boxed{V_{out}\approx V_s-0.6V}$$

Small Signal Analysis (1)



$$V_{in} = V_{in,DC} + \Delta V_{in}$$

$$V_{out} = V_{out,DC} + \Delta V_{out} = V_{out,DC} + A_v \Delta V_{in}$$

$$\Delta V_{out} = A_v \Delta V_{in}$$

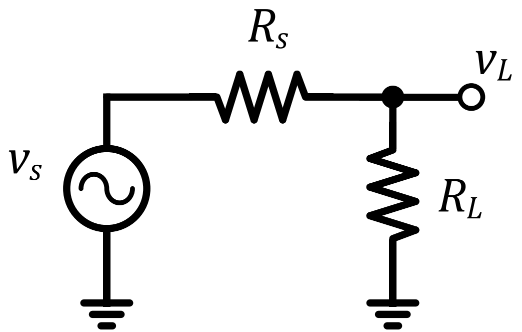
- In analog design, we often differentiate voltages and currents that are changing (AC) from those that aren't (DC)
- One way of representing a voltage (or current) that contains both is to use the Greek letter "delta" to indicate the changing quantity
- This technique allows us to separate the two and focus on the changing quantity

Small Signal Analysis (2)

- The technique of separating AC and DC signals is called “small-signal analysis,” because the former are typically much smaller in magnitude
- This “small-signal assumption” enables the analysis of nonlinear circuits (e.g. those with transistors) using linear techniques
- In small-signal analysis, we treat DC voltages as ground and DC currents as open circuits, since DC quantities, by definition, do not change
- Typical shorthand notation for small signals uses lowercase letters:

$$V_{out} = V_{out,DC} + v_{out} = V_{out,DC} + A_v v_{in}$$

Loading Revisited (again)



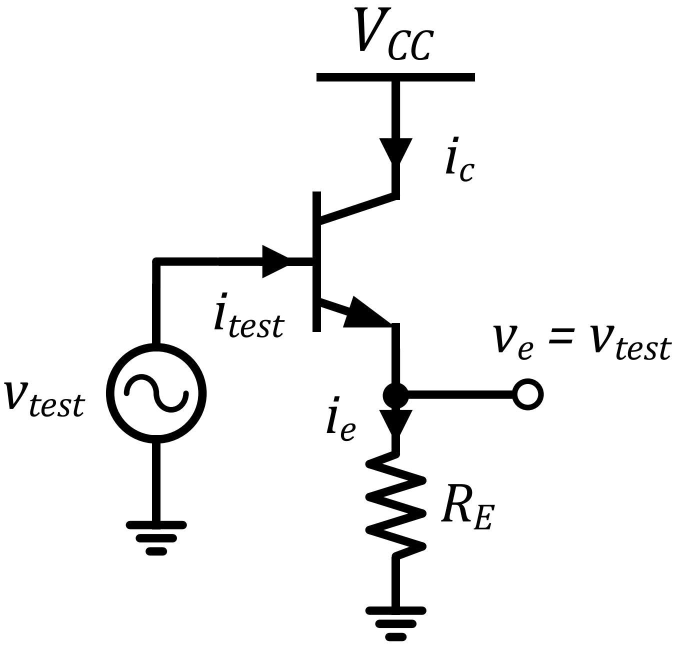
$$v_L = \frac{R_L}{R_s + R_L} v_s$$

$$R_s = 1k\Omega \qquad R_L = 100\Omega$$

$$\boxed{v_L = 0.09 v_s}$$

- Many design scenarios arise in which both source and load resistances are fixed (i.e. non-negotiable)
- To avoid significant attenuation in the signal path, we need a means of "buffering" the source against the resistance of the load
- It just so happens that an emitter follower can do just that!

Follower Input Impedance

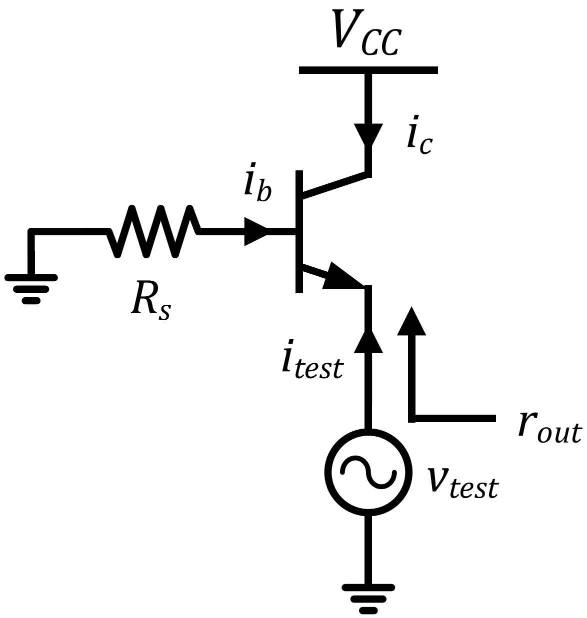


$$i_e = \frac{v_e}{R_E} = \frac{v_{test}}{R_E}$$

$$i_{test} = \frac{i_e}{\beta + 1} = \frac{v_{test}}{R_E(\beta + 1)}$$

$$\boxed{r_{in} = \frac{v_{test}}{i_{test}} = R_E(\beta + 1)}$$

Follower Output Impedance

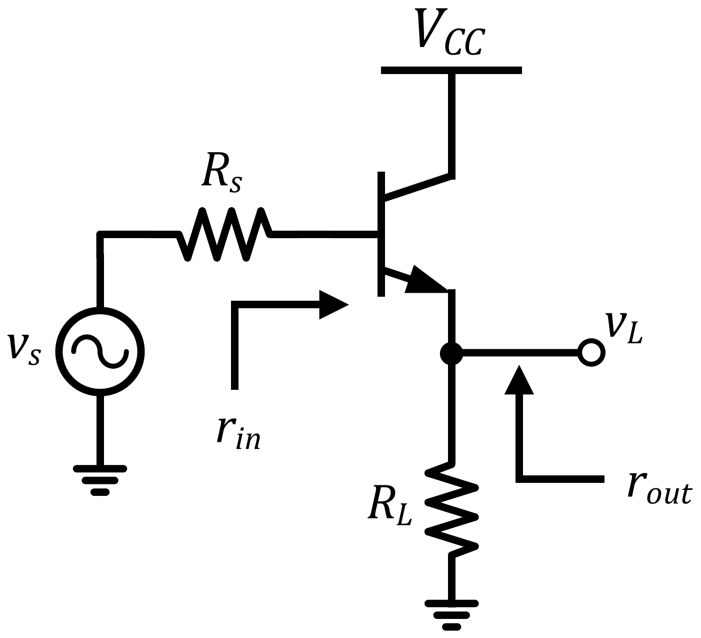


$$i_e = -i_{test} = -i_b(\beta + 1)$$

$$i_b = \frac{v_{test}}{R_s} = \frac{i_{test}}{(\beta + 1)}$$

$$\boxed{r_{out} = \frac{v_{test}}{i_{test}} = \frac{R_s}{(\beta + 1)}}$$

Emitter Follower Voltage Buffer



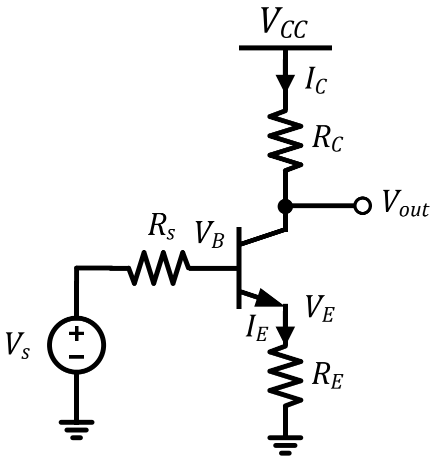
$$r_{in} = R_L(\beta + 1) = 10.1k\Omega$$

$$r_{out} = \frac{R_s}{(\beta + 1)} \approx 9.9\Omega$$

$$\frac{v_L}{v_s} = \left(\frac{r_{in}}{R_s + r_{in}}\right)\left(\frac{R_L}{r_{out} + R_L}\right)$$

$$\boxed{v_L = \frac{10.1k\Omega}{11.1k\Omega} \cdot \frac{100\Omega}{110\Omega} v_s = 0.8 v_s}$$

Common-Emitter Amplifier



Simplified NPN Model

$$V_{CE} > 0$$

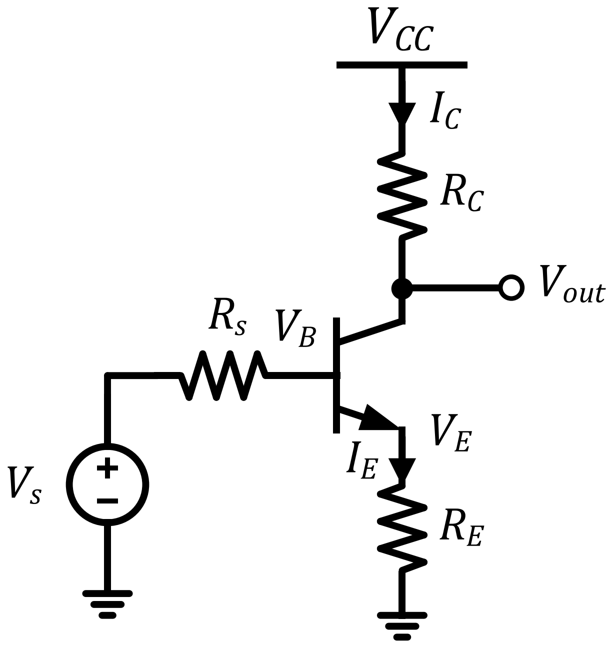
$$V_{BE} \approx 0.6V$$

$$I_C = \beta I_B \quad \text{where} \quad \beta \approx 100$$

$$I_E = (\beta + 1) I_B$$

- Again, $V_B \approx V_s$ and $V_E = V_s$ – 0.6V for large β and constant V_{BE}
- V_{out} is defined relative to V_{CC}, rather than ground
- $I_E \approx I_C$, so as V_E increases, V_{out} decreases proportional to R_C/R_E

Common-Emitter Amplifier



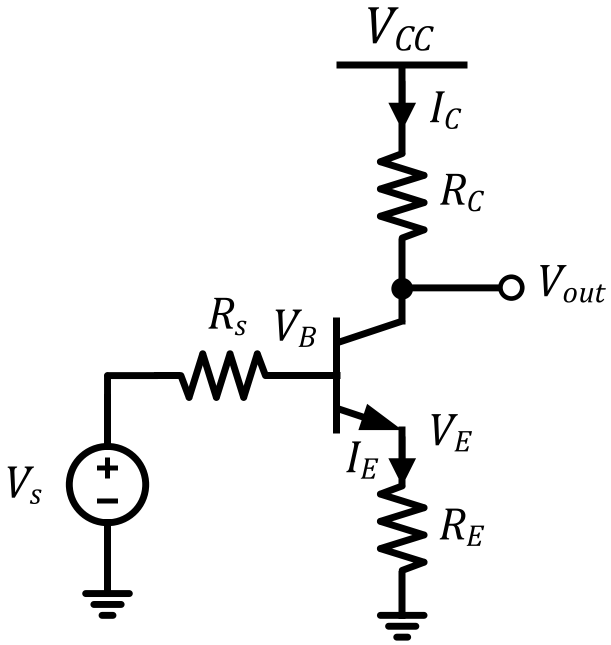
$$V_E = V_B - 0.6V \approx V_s - 0.6V \qquad (1)$$

$$I_E \approx I_C \rightarrow \frac{V_E}{R_E} = \frac{V_{CC} - V_{out}}{R_C} \qquad (2)$$

$$\frac{V_s - 0.6V}{R_E} = \frac{V_{CC} - V_{out}}{R_C} \qquad (3)$$

$$\boxed{V_{out} = V_{CC} - \frac{R_C}{R_E}(V_s - 0.6V)} \qquad (4)$$

Common-Emitter Gain

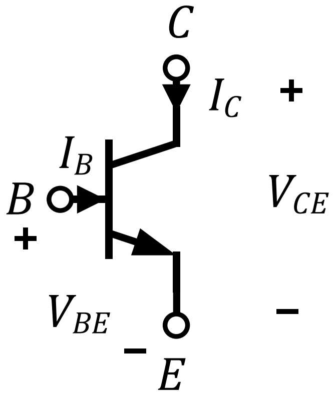


- Combining the constant terms, we get

$$V_{out} = -\frac{R_C}{R_E} V_s + V_{DC}$$

- We call $-R_C/R_E$ the *gain* of the common-emitter amplifier (CE amp)
- Because the gain depends on the *ratio* of resistances (rather than their absolute values), it can be very accurate (more on this later)

A Better Model (Ebers–Moll)



Collector current: $I_C = I_S(T)\left(e^{V_{BE}/V_T} - 1\right) \approx I_S e^{V_{BE}/V_T}$

Base-emitter voltage: $V_{BE} = \frac{kT}{q}\ln\left(\frac{I_C}{I_S(T)} + 1\right) \approx \frac{kT}{q}\ln\frac{I_C}{I_S(T)}$

- A more accurate (and powerful) model of the BJT views it as a *transconductance* device, rather than a current amplifier
- The BJT transconductance is nonlinear, since the collector current is an exponential function of the base-emitter voltage
- The exponential model is accurate over a large range of currents, from nanoamps to milliamps

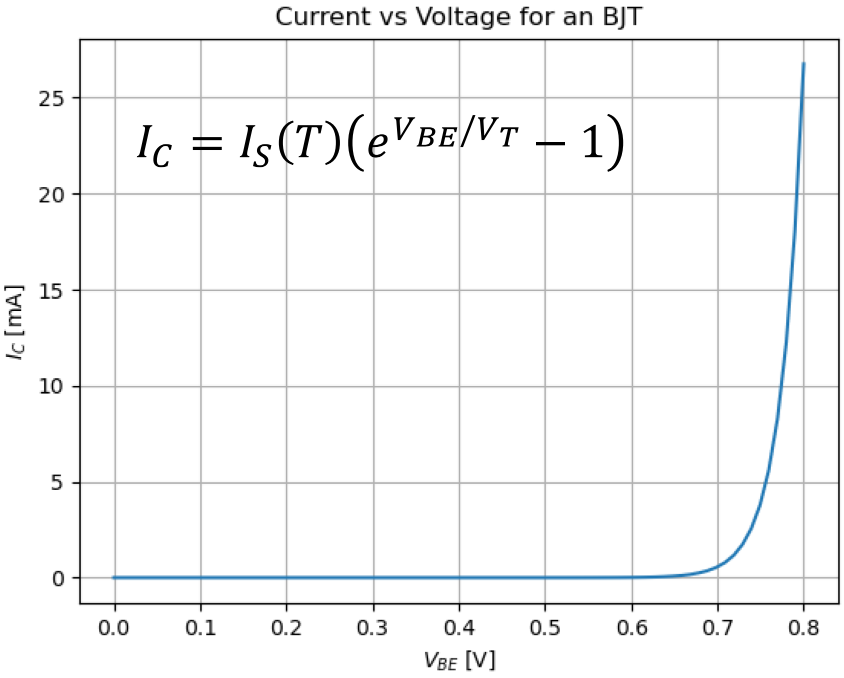
Ebers–Moll Model Parameters

Collector current: $I_C \approx I_S(T)e^{V_{BE}/V_T}$

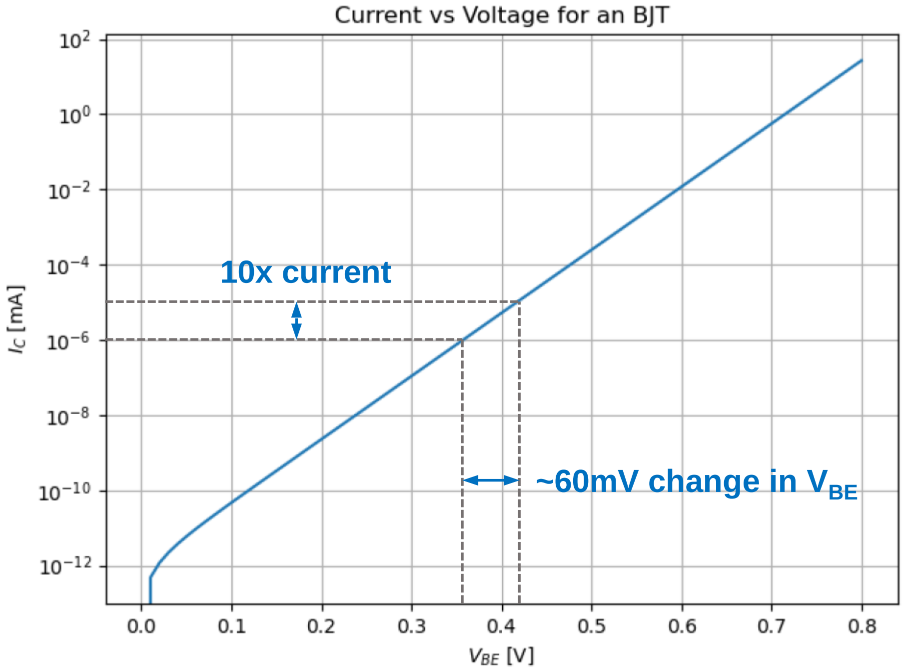
Base-emitter voltage: $V_{BE} \approx \frac{kT}{q} \ln \frac{I_C}{I_S(T)}$

- I_S is called the saturation current, which depends on temperature
- $V_T = kT/q$ is called the *thermal voltage*, where k is Boltzmann's constant, T is absolute temperature, and q is the charge of an electron in coulombs

BJT V-I Relationship

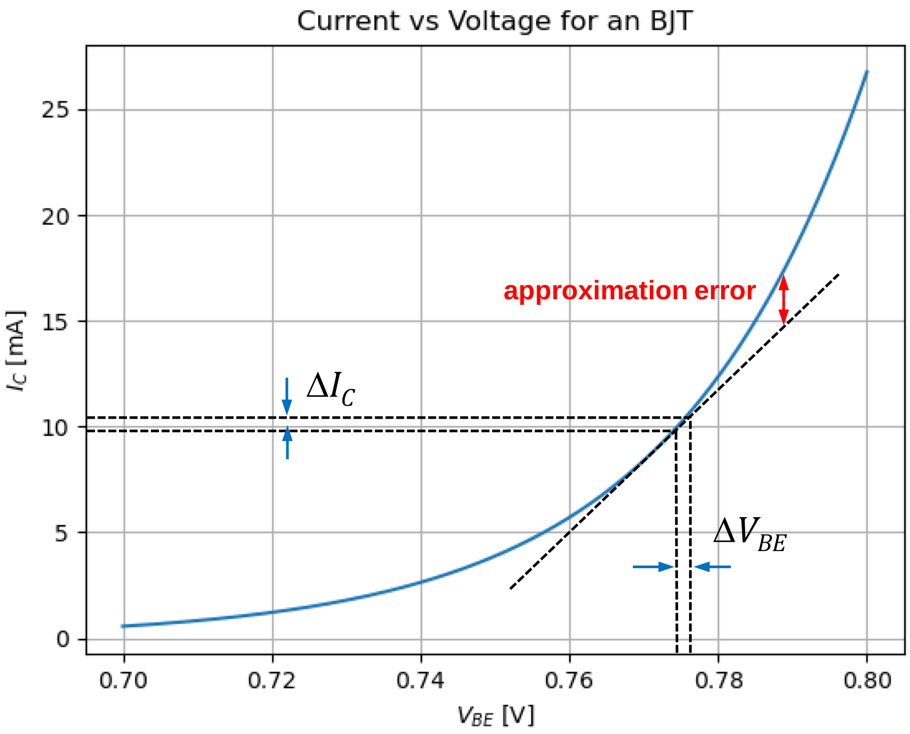


Linear Scale



Log Scale

Small-Signal Transconductance



- While it is accurate, the exponential model makes circuit analysis "by hand" challenging, due to the nonlinear relationship between voltage and current
- For very small changes in V_{BE}, we can approximate the exponential characteristic with a tangent line
- This linear approximation allows us to apply linear circuit analysis techniques to BJT's

Small-Signal Transconductance

- To assess how the collector current I_C responds to *small* changes in V_{BE}, we use the derivative of I_C with respect to V_{BE}:

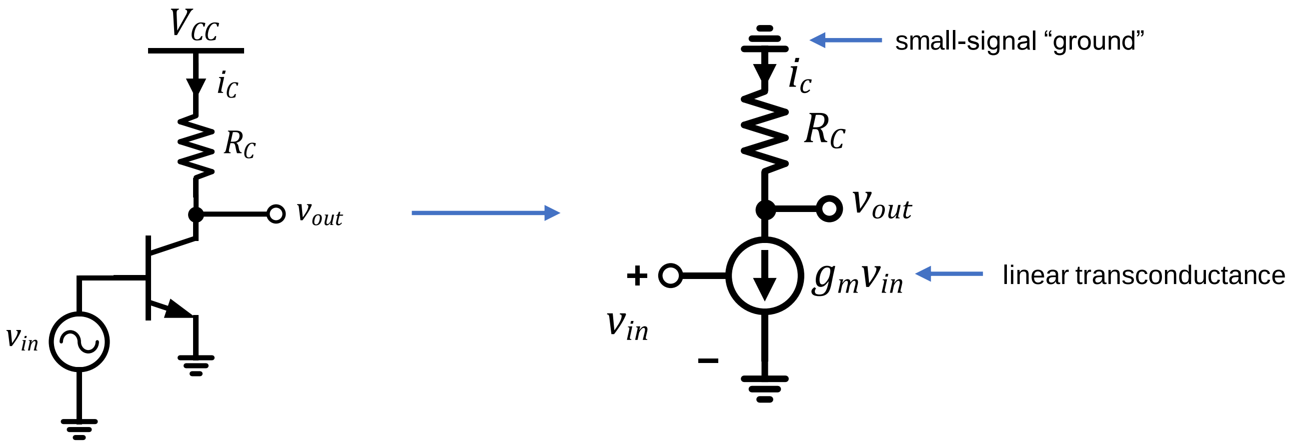
$$\frac{\partial}{\partial V_{BE}} I_S(T)\left(e^{V_{BE}/V_T} - 1\right) = \frac{1}{V_T} I_S(T) e^{V_{BE}/V_T}$$

- This is called the *transconductance* g_m, and it has a linear dependence on the DC collector current:

$$g_m = \frac{i_c}{v_{be}} = \frac{I_C}{V_T}$$

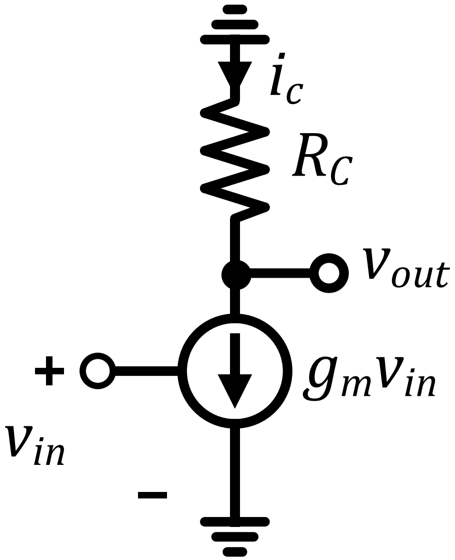
- Put another way, g_m is the ratio of the *small-signal* collector current to the *small-signal* base-emitter voltage

Common Emitter Revisited



- To analyze the small-signal response of the common-emitter amplifier, we convert it into its *small-signal equivalent circuit*
- The NPN becomes a linear transconductance, and all DC voltages/currents become zero
- Now we can analyze the circuit using familiar linear techniques

Small Signal Gain



$$i_c = g_m v_{in}$$

$$v_{out} = 0 - i_c R_C = -g_m v_{in} R_C$$

$$\boxed{A_v = \frac{v_{out}}{v_{in}} = -g_m R_C}$$

CE Amp2