Assignment 02

EE 538 Spring 2020
Analog Circuits for Sensor Systems
University of Washington Electrical & Computer Engineering

Due: April 18, 2020
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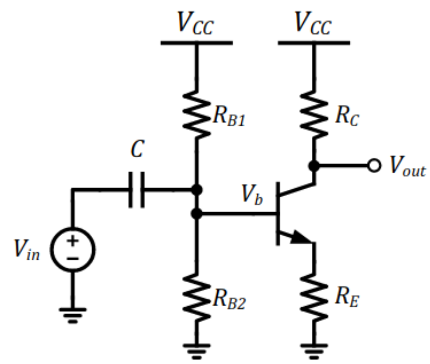
In [1]:
```
# Imports
import os
import sys
import cmath
import math
import matplotlib.pyplot as plt
import matplotlib
import numpy as np
import pandas as pd
import ltspice
import sympy as sp
from scipy import signal
%matplotlib inline
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
```

In [2]:
```
def read_ltspice_tran(file_name):
    cols = []
    arrs = []
    with open(file_name, 'r',encoding='utf-8') as data:
        for i,line in enumerate(data):
            if i==0:
                cols = line.split()
                arrs = [[] for _ in cols]
                continue
            parts = line.split()
            for j,part in enumerate(parts):
                arrs[j].append(part)
    df = pd.DataFrame(arrs,dtype='float64')
    df = df.T
    df.columns = cols
    return df
```

Problem 1: Common-emitter amplifier



For the following, use the figure and the simplified NPN model from Section 2.1.1 in AoE. V_{CC} = 5V, C = 1μF, and β = 100. Use the *npn* Ltspice component for your simulations.

a) Select values for R_{B1} and R_{B2} to achieve V_B = 1V and a high-pass corner frequency (f_{3dB}) of 10Hz. You may ignore base current for this step.
b) Choose R_C and R_E for a gain of –10V/V (at 10kHz) and a collector current of 1mA. What is the DC value of V_{CE}?
c) Derive the transfer function for the small-signal gain v_{out}/v_{in} and plot the frequency response (magnitude and phase) in MATLAB/Python. It may be convenient to separate the transfer function into two parts and compute their product:

$$\frac{v_{out}}{v_{in}} = \left(\frac{v_{out}}{v_b}\right)\left(\frac{v_b}{v_{in}}\right)$$

d) Perform an AC simulation of the circuit in Ltspice. Export the frequency response data for comparison to the transfer function you derived in Part c and plot them together. Explain any observed discrepancies between the two.

Part A

Given:
$f_{3\text{dB,HP}} = 10\text{Hz}$ and $V_B = 1V$.

Solve

$$\begin{aligned} f_{3\text{dB}} &= \frac{1}{2\pi RC} \\ 10\text{Hz} &= \left.\frac{1}{2\pi R_{\text{eq}} C}\right|_{C=1\text{pF}} \\ R_{\text{eq}} &= R_{B_2} // R_{B_1} = 15915\Omega \end{aligned}$$

and

$$\begin{aligned} V_{\text{out}} &= V_{\text{in}} \frac{R_2}{R_1 + R_2} \\ V_b &= V_{\text{in}} \frac{R_{B_2}}{R_{B_1} + R_{B_2}} \\ 1 &= 5 \frac{R_{B_2}}{R_{B_1} + R_{B_2}} \end{aligned}$$

then

$$R_{B_2} = 19893, R_{B_1} = 79572$$

Part B

Find $\frac{v_{\text{out}}}{v_B}$

$$\begin{aligned}
I_E &= \beta I_B &&= \frac{V_E}{R_E} \\
I_C &= (\beta+1)I_B &&= \frac{V_{CC}-V_{\text{out}}}{R_C} \\
V_E &= V_B - 0.6 &&\approx V_S - 0.6 \\
I_E &\approx I_C && \text{for } \beta >= 100 \text{ then} \\
\frac{V_S-0.6}{R_E} &= \frac{V_{CC}}{V_{\text{out}}} \\
V_{\text{out}} &= V_{CC} - \frac{R_C}{R_E}(V_S-0.6) \\
V_{\text{out}} &= -\frac{R_C}{R_E}V_S + V_{DC}
\end{aligned}$$

Given $V_B = 1\text{V}$ and $I_C = 1\text{mA}$, solve

$$\begin{aligned}
V_{\text{out}} &= V_{CC} - \frac{R_C}{R_E}(V_S-0.6) \\
V_{\text{out}} &= 5 - 10\cdot(1-0.6) \\
V_{\text{out}} &= 1\text{V} \\
\text{And} \\
I_C &= \frac{V_{CC}-V_{\text{out}}}{R_C} \\
1\text{mA} &= \frac{5-1}{R_C} \\
\text{then} \\
R_C &= 4k\Omega, R_E = 400\Omega
\end{aligned}$$

Find V_{CE}

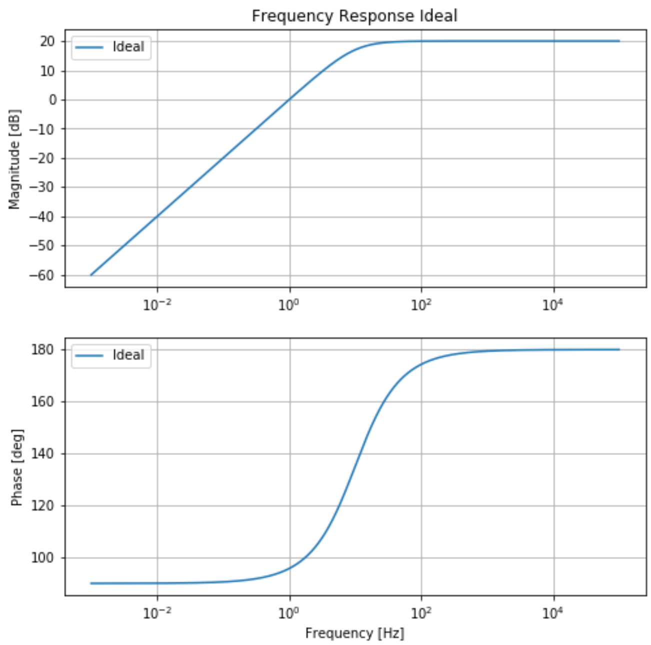
$V_C = V_{\text{out}} = 1$ and $V_E = 1 - 0.6 = 0.4$ so $V_{CE} = 0.6$.

Part C

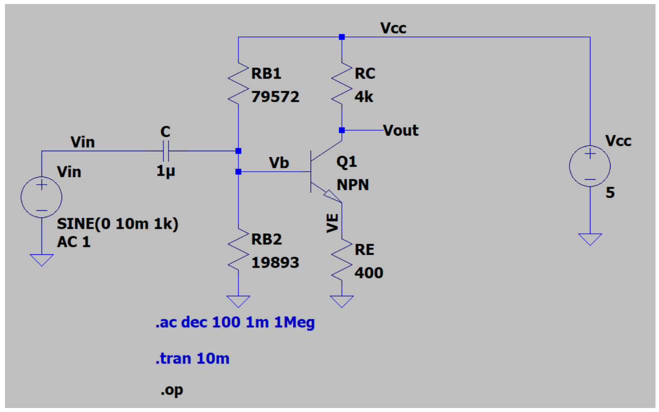
In [12]:
```
f1 = np.linspace(1e-3,1e2,100000)
f2 = np.linspace(1e2,1e5,100000)
f = np.concatenate((f1,f2))
w = 2*np.pi*f
s = 1j*w
R = 15915
C = 1e-6
tau = R*C
Vout_Vb = -10            # R_C/R_E
Vb_Vin = s*tau/(s*tau+1)
H = Vb_Vin * Vout_Vb

# Plot the frequency response
fig, axs = plt.subplots(2,figsize=(8,8))
axs[0].set_title('Frequency Response Ideal')
axs[0].semilogx(f, 20*np.log10(abs(H)),label='Ideal')
axs[0].set_ylabel('Magnitude [dB]')
axs[0].grid()
axs[0].legend()

axs[1].semilogx(f, -np.angle(H,deg=True),label='Ideal')
axs[1].set_ylabel('Phase [deg]')
axs[1].set_xlabel('Frequency [Hz]')
axs[1].grid()
axs[1].legend()
plt.show();
```



Part D



V(ve)	0.177253	voltage
V(vcc)	5	voltage
V(vout)	3.24502	voltage
V(vb)	0.930176	voltage
V(vin)	0	voltage
Ic(Q1)	0.000438746	device_current
Ib(Q1)	4.38746e-06	device_current
Ie(Q1)	-0.000443134	device_current
I(C)	9.30176e-19	device_current
I(Rb2)	4.6759e-05	device_current
I(Rb1)	5.11464e-05	device_current
I(Rc)	0.000438746	device_current
I(Re)	0.000443133	device_current
I(Vin)	9.30176e-19	device_current
I(Vcc)	-0.000489892	device_current

The DC operating point has key differences from the quick calculations.

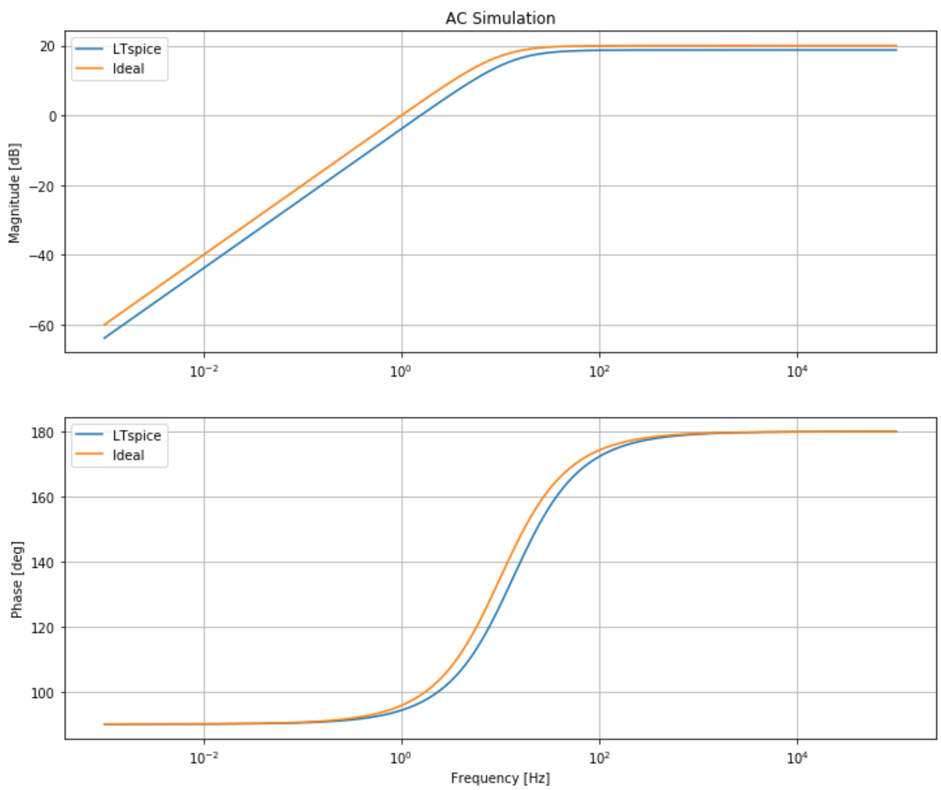
Simulated Results	Calculated Results
$V_{\text{out}} = 3.24$V	$V_{\text{out}} = 1$V
$V_{\text{BE}} = 0.75$V	$V_{\text{BE}} = 0.6$V
$V_{\text{B}} = 0.93$V	$V_{\text{B}} = 1$V
$I_C = 438\mu$A	$I_C = 1$mA

In [4]:
```
filepath = 'data/common_emitter.txt'
df = read_ltspice_tran(filepath)
df['Vout_Mag'] = df['V(vout)'].apply(lambda x: x.split(',')[0])
df['Vout_Mag'] = df['Vout_Mag'].apply(lambda x: x[1:-2])
df['Vout_Mag'] = df['Vout_Mag'].astype('float64')
df['Vout_Phase'] = df['V(vout)'].apply(lambda x: x.split(',')[1])
df['Vout_Phase'] = df['Vout_Phase'].apply(lambda x: x[1:-2])
df['Vout_Phase'] = df['Vout_Phase'].astype('float64')
df['Freq.'] = df['Freq.'].astype('float64')
df = df[df['Freq.']<=1e5]
```

In [5]:
```python
fig, axs = plt.subplots(2,figsize=(12,10))
freq = df['Freq.']
mag = df['Vout_Mag']
ang = df['Vout_Phase']

axs[0].set_title('AC Simulation')
axs[0].semilogx(freq, mag, label='LTspice')
axs[0].semilogx(f, 20*np.log10(abs(H)),label='Ideal')
axs[0].set_ylabel('Magnitude [dB]')
axs[0].grid()
axs[0].legend()
axs[1].semilogx(freq, ang, label='LTspice')
axs[1].semilogx(f, -np.angle(H,deg=True),label='Ideal')
axs[1].set_ylabel('Phase [deg]')
axs[1].set_xlabel('Frequency [Hz]')
axs[1].grid()
axs[1].legend()
plt.show();
```

```
/home/kegedy/anaconda3/lib/python3.7/site-packages/IPython/core/pylabtools.py:128: UserWarning: Creating
legend with loc="best" can be slow with large amounts of data.
  fig.canvas.print_figure(bytes_io, **kw)
```

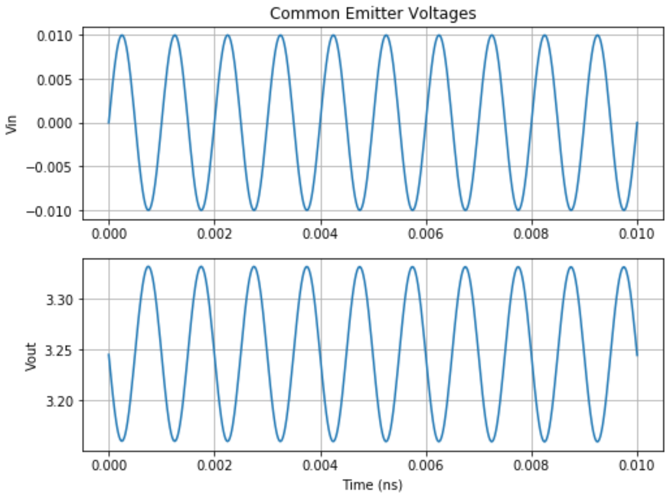


The AC analysis for the simplified calculation and simulation are very similar.

In [6]:
```python
filepath = 'data/common_emitter_trans.txt'
df = read_ltspice_tran(filepath)
print(f'Gain is {round((max(df["V(vout)"])-min(df["V(vout)"]))/(max(df["V(vin)"])-min(df["V(vin)"])),
4)}')
```

```
Gain is 8.6501
```

In [7]:
```
fig, axs = plt.subplots(2,figsize=(8,6))
axs[0].set_title('Common Emitter Voltages')
axs[0].plot(df['time'], df['V(vin)'],label='Vin')
axs[0].set_ylabel('Vin')
axs[0].grid()
#xs[0].legend()
axs[1].plot(df['time'], df['V(vout)'],label='Vout')
axs[1].set_ylabel('Vout')
axs[1].set_xlabel('Time (ns)')
axs[1].grid()
#xs[1].legend()
plt.show();
```



Problem 2: Emitter follower

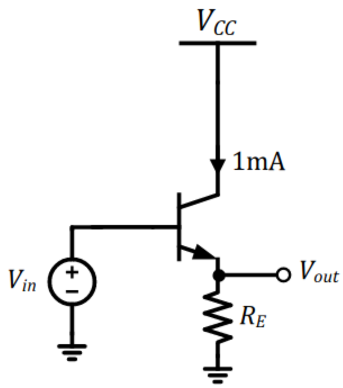


Figure 2a. Emitter follower

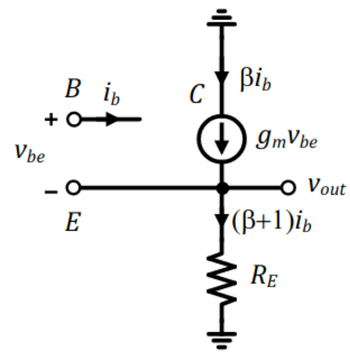


Figure 2b. Small-signal equivalent circuit

Use the Ebers-Moll model of the BJT and the figures to answer the following questions. V_{CC} = 5V, I_S = 10^{-16}, and β = 100. When determining input/output resistances, connect a test voltage to the small-signal circuit and determine the resistance as $r = v_{test}/i_{test}$. Use the *npn* Ltspice component for your simulations.

a) Design the biasing of the emitter follower (i.e. determine the DC value of V_{in} and the resistance value R_E) such that the collector current is 1mA and the DC level of V_{out} is 1V. You can do this by hand or use a MATLAB/Python script.
b) Use the small-signal model (Fig. 2b) to determine the *small-signal* input resistance of the circuit.
c) Use the small-signal model (Fig. 2b) to determine the *small-signal* output resistance of the circuit.
d) Verify your design in Ltspice and include all relevant SPICE schematics and results in your submission. At minimum, your submission should include:
    1. DC simulation results indicating the bias point
    2. AC simulation result for input resistance
    3. AC simulation result for output resistance

Part A

Small signal analysis:
$$V_{\text{out}} = V_{\text{out,DC}} + v_{\text{out}} = V_{\text{out,DC}} + A_v v_{\text{in}}$$

Ebers-Moll Model:
$$\begin{aligned}
I_C &= I_S(T)(e^{\frac{V_{BE}}{V_T}} - 1) &&\approx I_S e^{\frac{V_{BE}}{V_T}}\Big|_{V_T=\frac{kT}{q}=25.3\text{mV}} \\
V_{BE} &= \frac{kT}{q}\ln\left(\frac{I_C}{I_S(T)} + 1\right) &&\approx \frac{kT}{q}\ln\left(\frac{I_C}{I_S}\right) \\
V_{BE} &= 25.3\cdot 10^{-3}\cdot\ln\left(\frac{1\cdot 10^{-3}}{10^{-16}}\right) &&= 0.757V
\end{aligned}$$

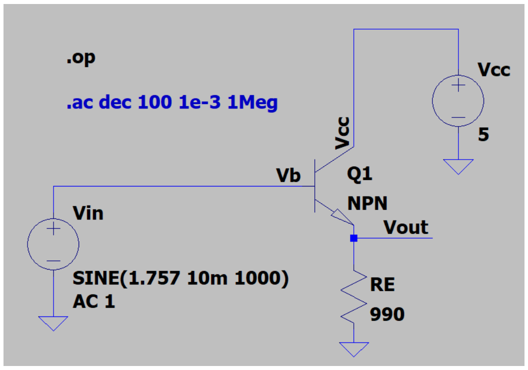
Solve:
$$\begin{aligned}
I_C &= \beta I_B\Big|_{I_C=10^{-3},\beta=100} &&\rightarrow I_B = 10^{-5} \\
I_E &= (\beta+1)I_B \\
V_{\text{out}} &= I_E\cdot R_E\Big|_{V_{\text{out}}=1} &&\rightarrow R_E = 990 \\
V_{\text{in}} &= V_{\text{out}} + V_{BE} &&= 1.757\text{V}
\end{aligned}$$

Part B

$$
\begin{aligned}
i_e &= \frac{v_e}{R_E} &&= \frac{v_{\text{test}}}{R_E} \\
i_{\text{test}} &= \frac{i_e}{\beta+1} &&= \frac{v_{\text{test}}}{R_E(\beta+1)} \\
r_{\text{in}} &= \frac{v_{\text{test}}}{i_{\text{test}}} &&= R_E(\beta+1)\Big|_{R_E=990} &= 99990\Omega
\end{aligned}
$$

Part C

$$
\begin{aligned}
i_e &= -i_{\text{test}} \\
v_{\text{test}} &= i_e \cdot R_E \\
r_{\text{out}} &= \frac{v_{\text{test}}}{i_{\text{test}}} &= R_E = 990\Omega
\end{aligned}
$$



DC Operating Point

V(vout)	0.983204	voltage
V(vcc)	5	voltage
V(vb)	1.757	voltage
Ic(Q1)	0.000983307	device_current
Ib(Q1)	9.83307e-06	device_current
Ie(Q1)	-0.00099314	device_current
I(Re)	0.000993135	device_current
I(Vin)	-9.83302e-06	device_current
I(Vcc)	-0.000983302	device_current

The DC Operating point closes reflects the calculations from Ebers-Moll model. The simulated value of $I_C = 0.98$mA is very close to the design of 1mA. Also simulated $V_{\text{out}} = 0.98$ is very close to 1V.

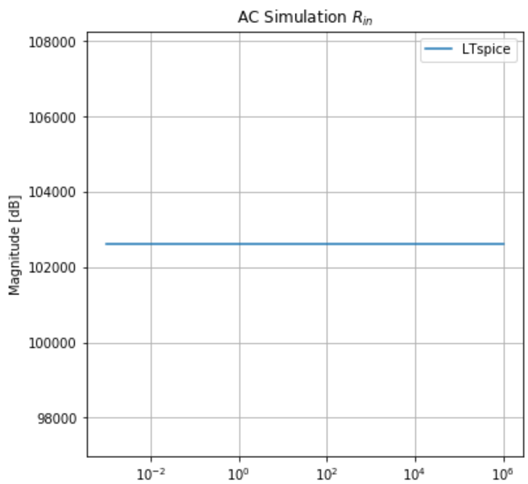
AC Simulation for Rin

In [8]:
```
filepath = 'data/emitter_follower_rin.txt'
df = read_ltspice_tran(filepath)
df['Rin'] = df['1/Ib(Q1)'].apply(lambda x: x.split(',')[0])
df['Rin'] = df['Rin'].astype('float64')
df['Freq.'] = df['Freq.'].astype('float64')
print(f'Rout is {round(max(df["Rin"]),2)}')
```

```
Rout is 102620.39
```

In [9]:
```
fig, ax = plt.subplots(1,figsize=(6,6))
freq = df['Freq.']
mag = df['Rin']

ax.set_title(r'AC Simulation $R_{in}$')
ax.semilogx(freq, mag, label='LTspice')
ax.set_ylabel('Magnitude [dB]')
ax.grid()
ax.legend()
plt.show();
```



The difference betweeen R_{in} calculations and the simulation is $\frac{(102620.39-99990)}{99990} = 2.6\%$.

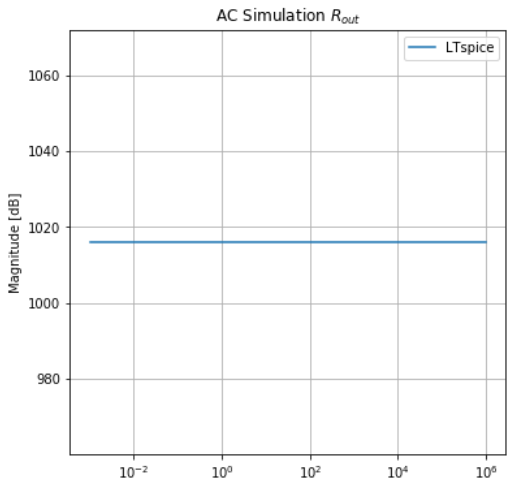
AC Simulation for Rout

In [10]:
```
filepath = 'data/emitter_follower_rout.txt'
df = read_ltspice_tran(filepath)
df['Rout'] = df['1/Ie(Q1)'].apply(lambda x: x.split(',')[0])
df['Rout'] = df['Rout'].astype('float64')
df['Freq.'] = df['Freq.'].astype('float64')
print(f'Rout is {round(-max(df["Rout"]),2)}')
```

```
Rout is 1016.04
```

In [11]:
```
fig, ax = plt.subplots(1,figsize=(6,6))
freq = df['Freq.']
mag = -df['Rout']

ax.set_title(r'AC Simulation $R_{out}$')
ax.semilogx(freq, mag, label='LTspice')
ax.set_ylabel('Magnitude [dB]')
ax.grid()
ax.legend()
plt.show();
```



The difference betweeen R_{out} calculations and the simulation is $\frac{(1016.04-990)}{990} = 2.6\%$

In [ ]:
```

```