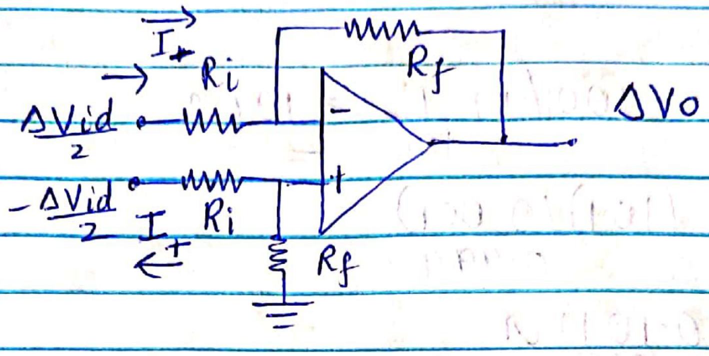
HW 5

(1) $R_f = 10k\Omega$, $R_i = 100\Omega$

a)



$$Z_{in} = \frac{\Delta V_{id}}{I_- - I_+} \quad \text{——(1)}$$

$$V_- = V_+ = -\frac{\Delta V_{id}}{2}\left(\frac{R_f}{R_i + R_f}\right)$$

$$I_- = \frac{\frac{\Delta V_{id}}{2} - V_-}{R_i} = \frac{\frac{\Delta V_{id}}{2}\left[1 + \frac{R_f}{R_i + R_f}\right]}{R_i} \quad \text{——(2)}$$

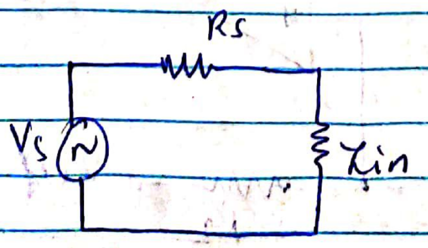
$$I_+ = \frac{+\frac{\Delta V_{id}}{2} + V_+}{R_i} = \frac{\frac{\Delta V_{id}}{2}\left[1 - \frac{R_f}{R_i + R_f}\right]}{R_i} \quad \text{——(3)}$$

from (1)

$$Z_{in} = \frac{\cancel{\Delta V_{id}} \cdot R_i}{\frac{\cancel{\Delta V_{id}}}{\cancel{2}}\left(\cancel{2}\frac{R_f}{R_i + R_f}\right)}$$

$$Z_{in} = R_i\left(1 + \frac{R_i}{R_f}\right)$$

$$\Rightarrow \boxed{Z_{in} = R_i\left(1 + \frac{1}{A_{CL}}\right)} \qquad \boxed{A_{CL} = \frac{R_f}{R_i}}$$



For 0.1% Attenuation

$$\frac{Z_{in}}{R_s + Z_{in}} = 0.999$$

$R_s(0.999) = Z_{in}(0.001)$ $A_{CL} = \frac{R_f}{R_1} = 10$

$Z_{in} = 100\left(1+\frac{1}{100}\right) = \underline{\underline{101\,\Omega}}$

$\Rightarrow R_s = \frac{(101)(0.001)}{0.999}$

$R_s = \underline{\underline{0.1011\,\Omega}}$

Proplum (2)

$A_{DC} = 120dB$ $R_G = 100\Omega$

$f_T = 1MHz$ $R_1 = R_2 = 10K\Omega$

$R_{fp} = R_{fm} = 4.95K\Omega$ $tol = 0.1\%$

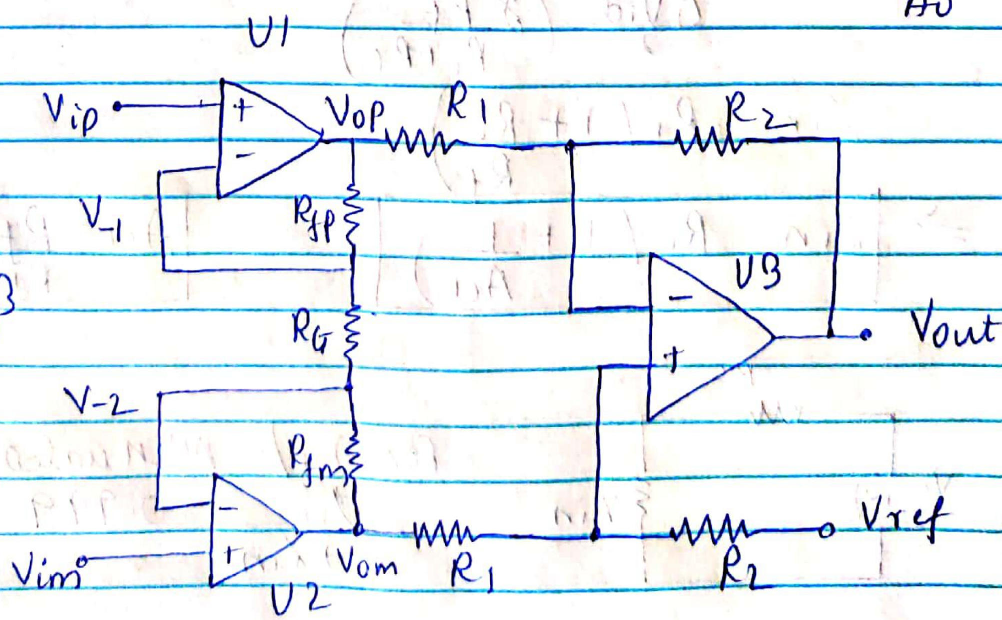
a) $V_{id} = V_{ip} - V_{im}$

$\Rightarrow V_{od} = V_{op} - V_{om}$

$A_{OL} = \frac{A_0}{1+s\tau}$ $A_0 = 10^6 V/V$

$f_{3dB} = \frac{f_T}{A_0} = 1Hz$

$\frac{1}{1+s\tau+A_0\beta}$



$$V_{-1} = V_{ip} - \frac{V_{op}}{A_{OL}} \quad \& \quad V_{-2} = V_{im} - \frac{V_{om}}{A_{OL}}$$

$$I_G = \frac{V_{-1} - V_{-2}}{R_G} \quad \text{——} \quad 1$$

$$I_G = \frac{\left(V_{ip} - \frac{V_{op}}{A_{OL}} - V_{im} + \frac{V_{om}}{A_{OL}}\right)}{R_G}$$

$$I_G R_G = (V_{ip} - V_{im}) - (V_{op} - V_{om}) A_{OL}^{-1} \quad \text{——} \quad 2$$

Also, $V_{op} - V_{om} = I_G (R_{fp} + R_G + R_{fm}) \quad \text{——} \quad 3$

$$(V_{op} - V_{om}) = \frac{(R_{fp} + R_G + R_{fm})}{R_G} \left[V_{ip} - V_{im} - (V_{op} - V_{om}) A_{OL}^{-1} \right]$$

$$(V_{op} - V_{om}) \left[1 + A_{OL}^{-1} \left(1 + \frac{R_{fp} + R_{fm}}{R_G} \right) \right] = (V_{ip} - V_{im}) \left(1 + \frac{R_{fp} + R_{fm}}{R_G} \right)$$

$$\frac{V_{op} - V_{om}}{V_{ip} - V_{im}} = \frac{\left(1 + \frac{R_{fp} + R_{fm}}{R_G} \right)}{1 + \frac{(1 + s\tau)}{A_0} \left(1 + \frac{R_{fp} + R_{fm}}{R_G} \right)}$$

$$= \frac{A_0 (R_G + R_{fp} + R_{fm})}{R_G A_0 + (1 + s\tau)(R_G + R_{fp} + R_{fm})}$$

$$= \frac{\left(\frac{R_G + R_{fp} + R_{fm}}{R_G} \right)}{\frac{R_G}{R_G} + \frac{(R_G + R_{fp} + R_{fm})}{R_G A_{OL}}}$$

$$\boxed{\frac{V_{op} - V_{om}}{V_{ip} - V_{im}} = \frac{A_{OL}}{1 + \frac{R_G}{R_G + R_{fp} + R_{fm}} \cdot A_{OL}}} \quad \text{——} \quad 4$$

$$\frac{V_{out}}{V_{op} - V_{om}} = -\frac{R_2}{R_1} = \frac{10K}{10K} = -1$$

$$\boxed{\frac{V_{out}}{V_{ip}-V_{im}} = \frac{-A_{OL}}{1+\frac{R_G}{R_G+R_{fp}+R_{fm}} \cdot A_{OL}}}$$

where $\beta = \frac{R_G}{R_G+R_{fp}+R_{fm}}$

& $A_{OL} = \frac{A_0}{1+s\tau}$ $\qquad A_0 = 10^6 \, V/V$

$\beta = 0.01$

$$\Rightarrow A_{DC} = \frac{-A_0}{1+\beta A_0} = \frac{10^6}{1+10^{-2} \cdot 10^6}$$

$$A_{DC,CL} = -99.99 \, V/V$$

$$f_{3dB,CL} = (1+\beta A_0) f_{3dB,OL}$$
$$= (1+10^{-2} \cdot 10^6) \frac{f_T}{A_0}$$

$$f_{3dB,CL} = (1+10^4) \frac{10^6}{10^6}$$

$$\boxed{f_{3dB,CL} = 10.001 \, KHz}$$

b) Gain error $= \frac{G_{ideal} - G_{actual}}{G_{ideal}}$

$$G_{ideal} = \left(1+\frac{R_{fp}+R_{fm}}{R_G}\right)(-1)$$

$$G_{ideal} = -100 \, V/V$$

At 100 Hz, $\quad G_{actual} = -\left|\frac{10^6}{1+j(2\pi \times 100)+10^4}\right|$

$$= -99.79325 \, V/V$$

$$\text{Gain error} = \frac{-100 + 99.79325}{-100}$$
$$= 0.20674\%$$

c) CMRR = ? Worst-case DC gain error = ?

$$CMRR = CMRR_1 \cdot CMRR_2$$
$$= \frac{1 + \left(\frac{R_{fp} + R_{fm}}{R_G}\right)}{1} \times \frac{A_{vd2} + 1}{4 \times \epsilon}$$
$$= \left(1 + \frac{9900}{100}\right) \times \frac{2}{4 \times 0.001}$$

$$\boxed{CMRR = 93.979\,dB}$$

$$\Rightarrow A_{DC} = -\left(1 + \frac{R_{fp} + R_{fm}}{R_G}\right)\left(\frac{R_2}{R_1}\right)$$

$$A_{ideal} = -100\,V/V$$

W.C Resistor values:

$$R_{fp} = R_{fm} = 4.95K(0.999) = 4945.05\,\Omega$$
$$R_G = 100(1 + 0.001) = 100.1\,\Omega$$
$$R_2 = 10K(1 - 0.001) = 9990\,\Omega$$
$$R_1 = 10K(1 + 0.001) = 10010\,\Omega$$

$$A_{worst\text{-}case} = -\left(1 + \frac{R_{fp} + R_{fm}}{R_G}\right)\left(\frac{R_2}{R_1}\right)$$
$$= -\left(1 + \frac{9890.1}{100.1}\right)\left(\frac{9.990K}{10.010K}\right)$$
$$= -99.6\,V/V$$

$$\boxed{\text{W.C Gain error} = 0.4\%}$$

(d)

$V_{os,U1} = +100\mu V$

$V_{os,U2} = -100\mu V$

$$V_{out1} = \left(1 + \frac{R_{fp} + R_{fm}}{R_G}\right)(V_{os,U1} - V_{os,U2})$$

$$= 20mV$$

$$V_{out} = -\frac{R_2}{R_1}(V_{os,U3} + V_{out1}) + \left(1 + \frac{R_2}{R_1}\right)V_{os,U3}$$

$$-250\mu V \times 100 = -1(\cancel{V_{os,U3}} + 20mV) + \left(1 + \frac{R_2}{R_1}\right)V_{os,U3}$$

$$V_{os,U3} = \frac{-5mV}{2}$$

$$\Rightarrow \boxed{V_{os,U3} = -2.5mV}$$

Problem 1) Part B)

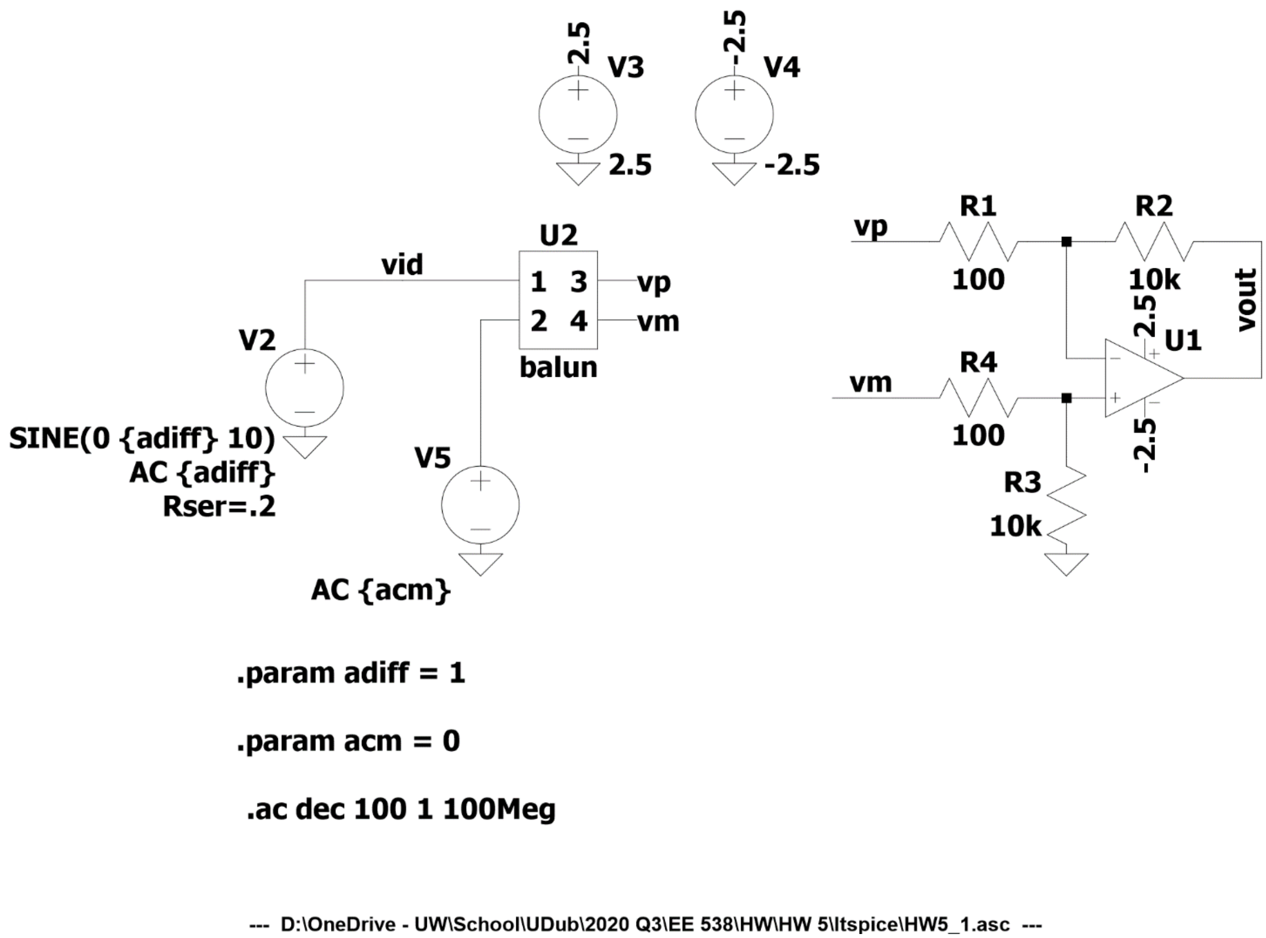


Fig 1) Schematic

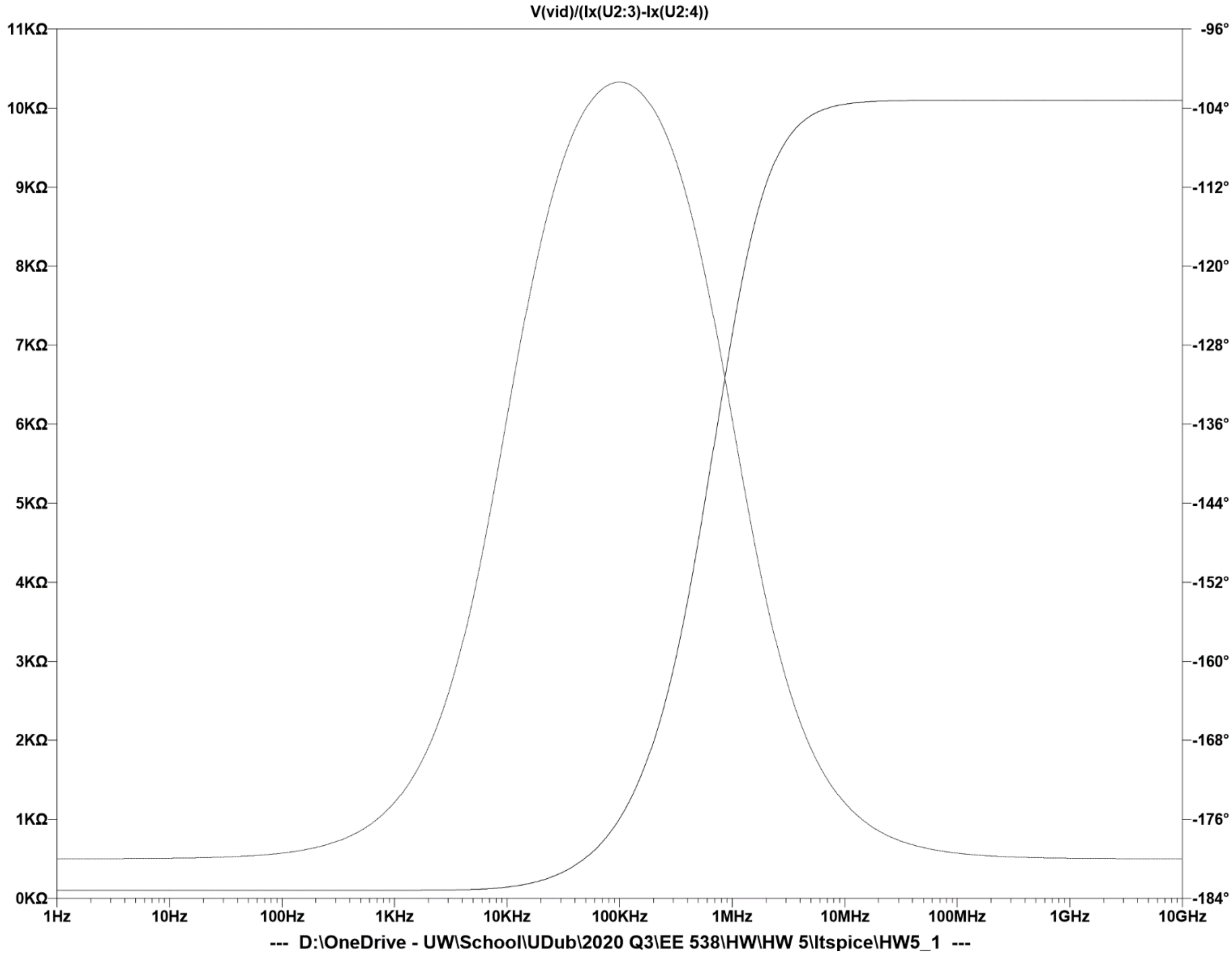


Fig 2) Zin vs Freq

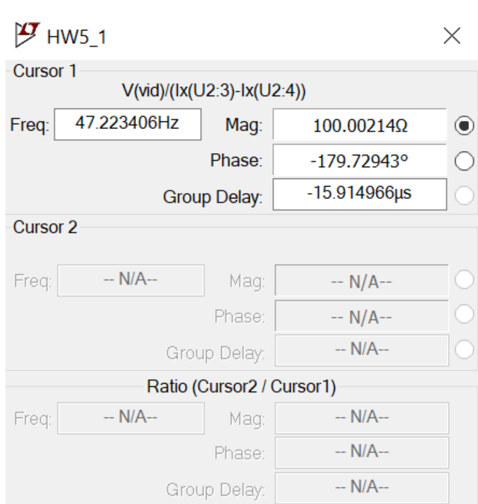


Fig 3) Zin value at low frequency

Problem 2) Part E)

1) DC Operating point

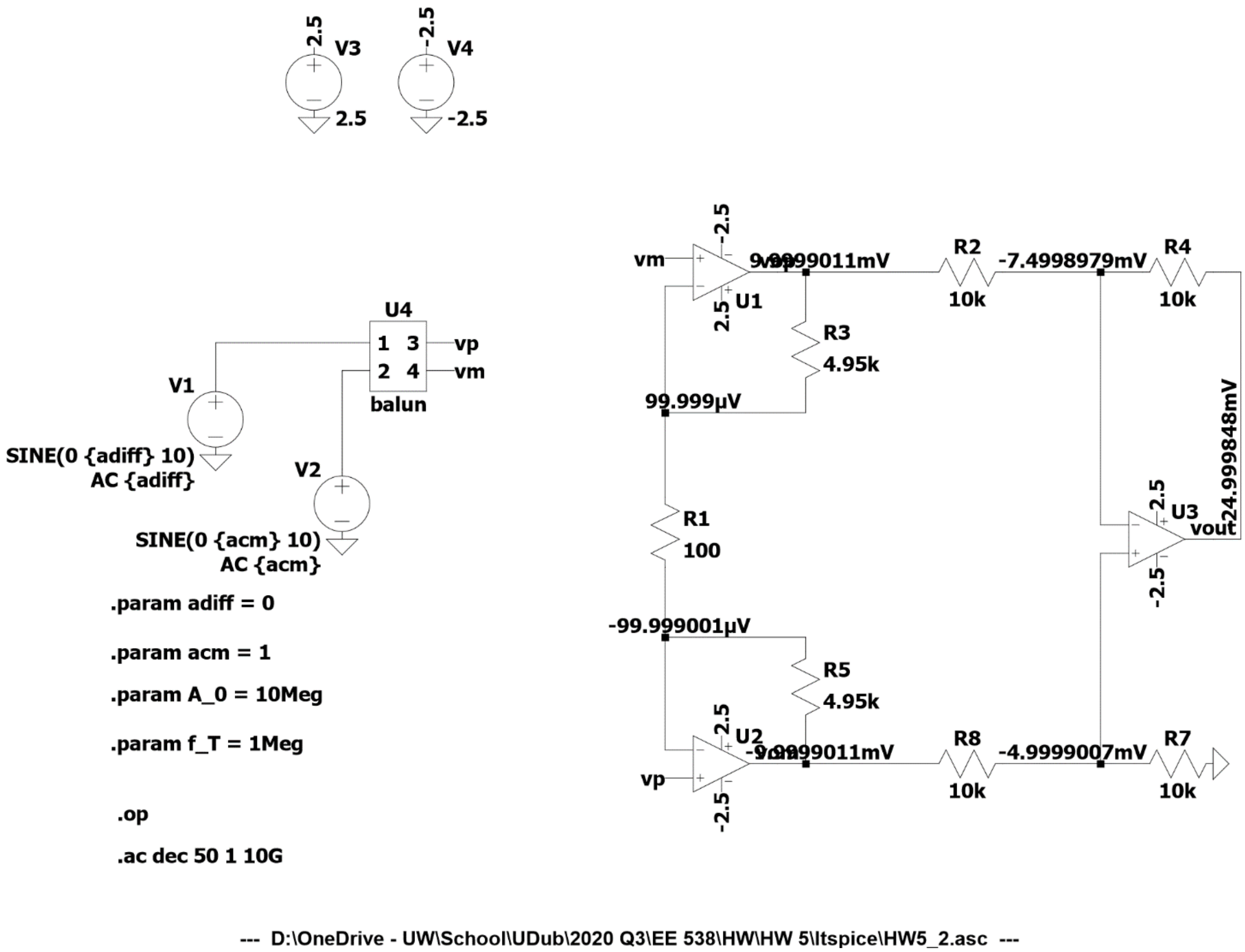


Fig 4) Schematic without resistor mismatch

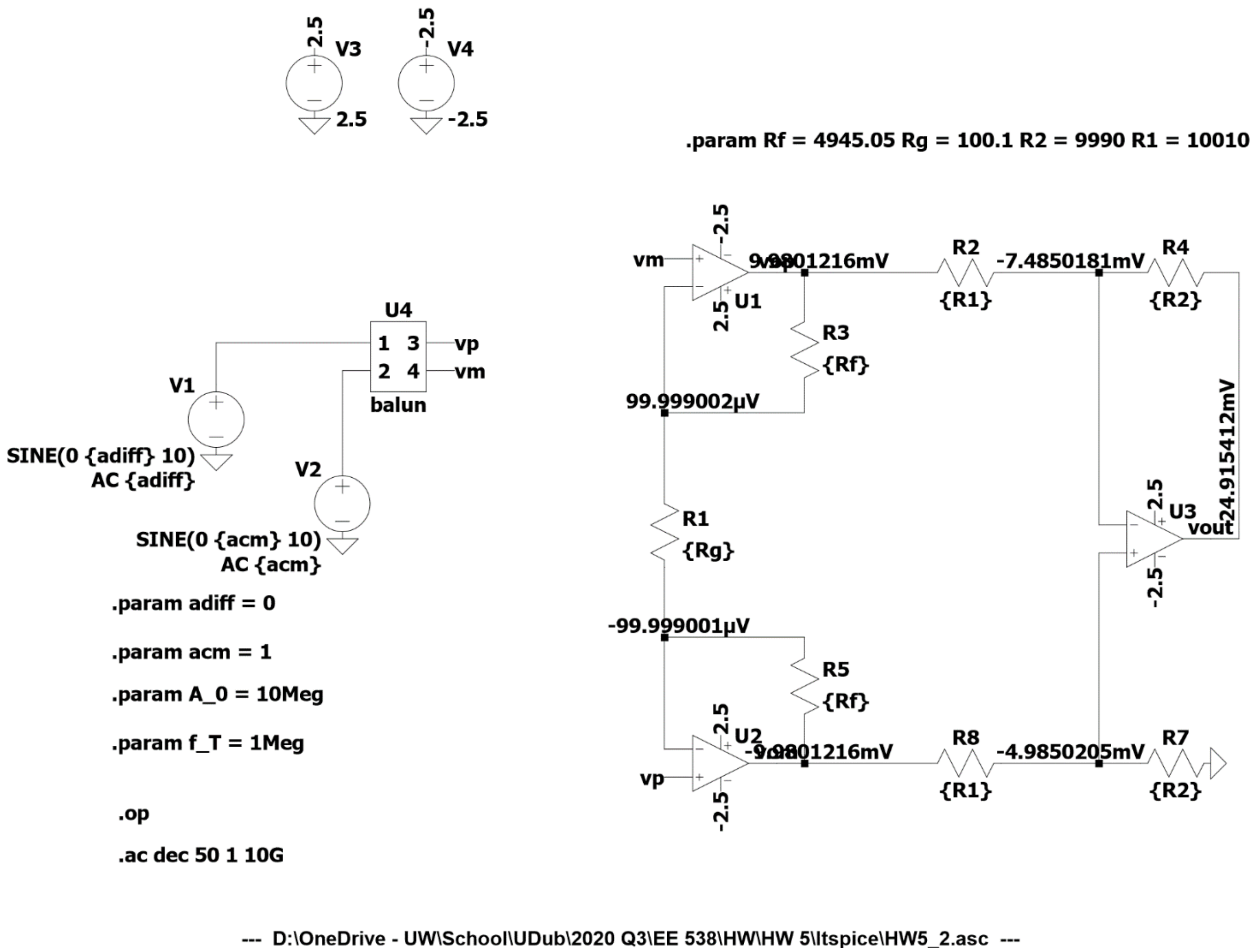


Fig 5) Schematic with resistor mismatch

Resistor mismatch causes an offset of about 85uV on V_{OUT}

2) Gain error

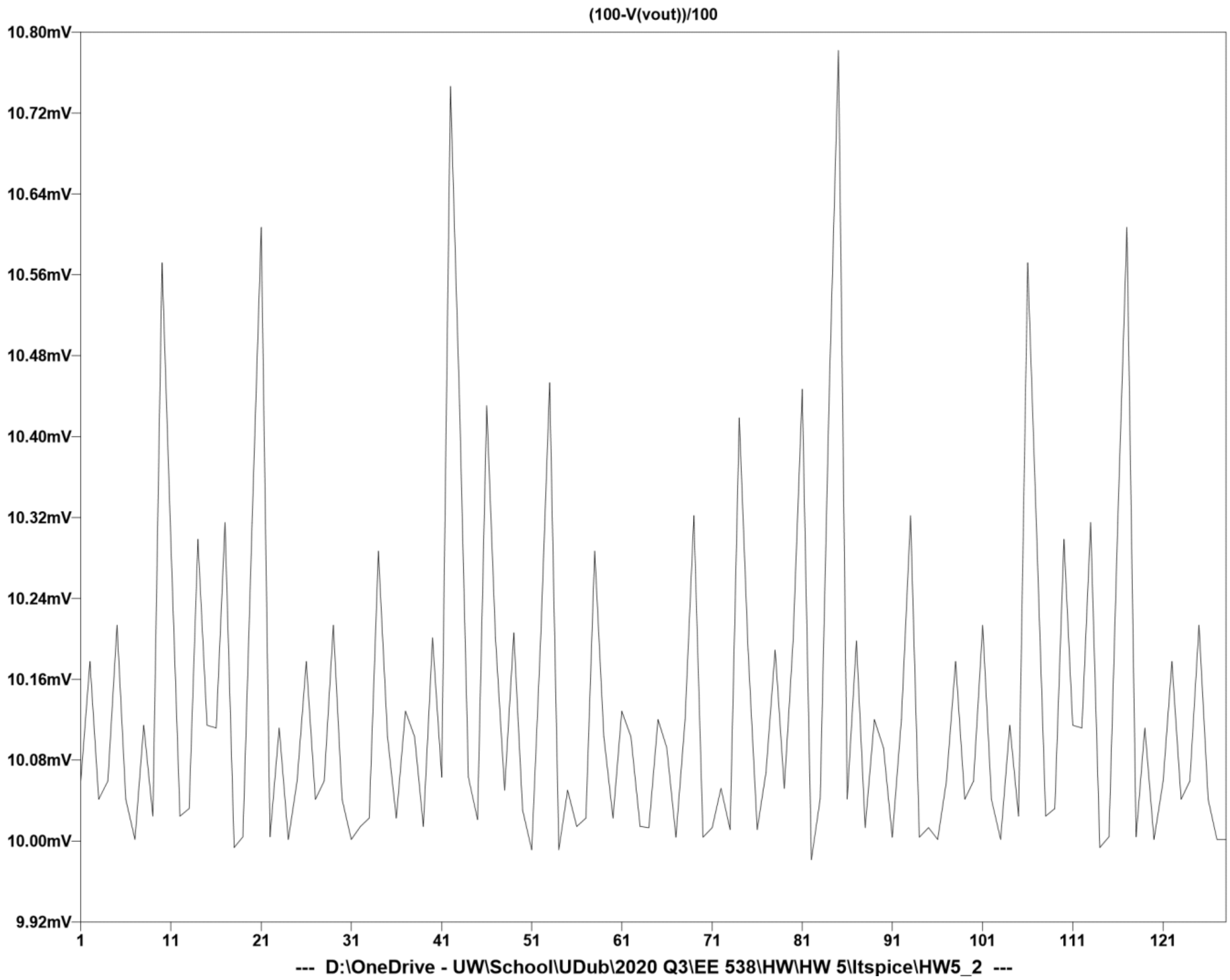


Fig 6) Gain error due to resistor mismatch

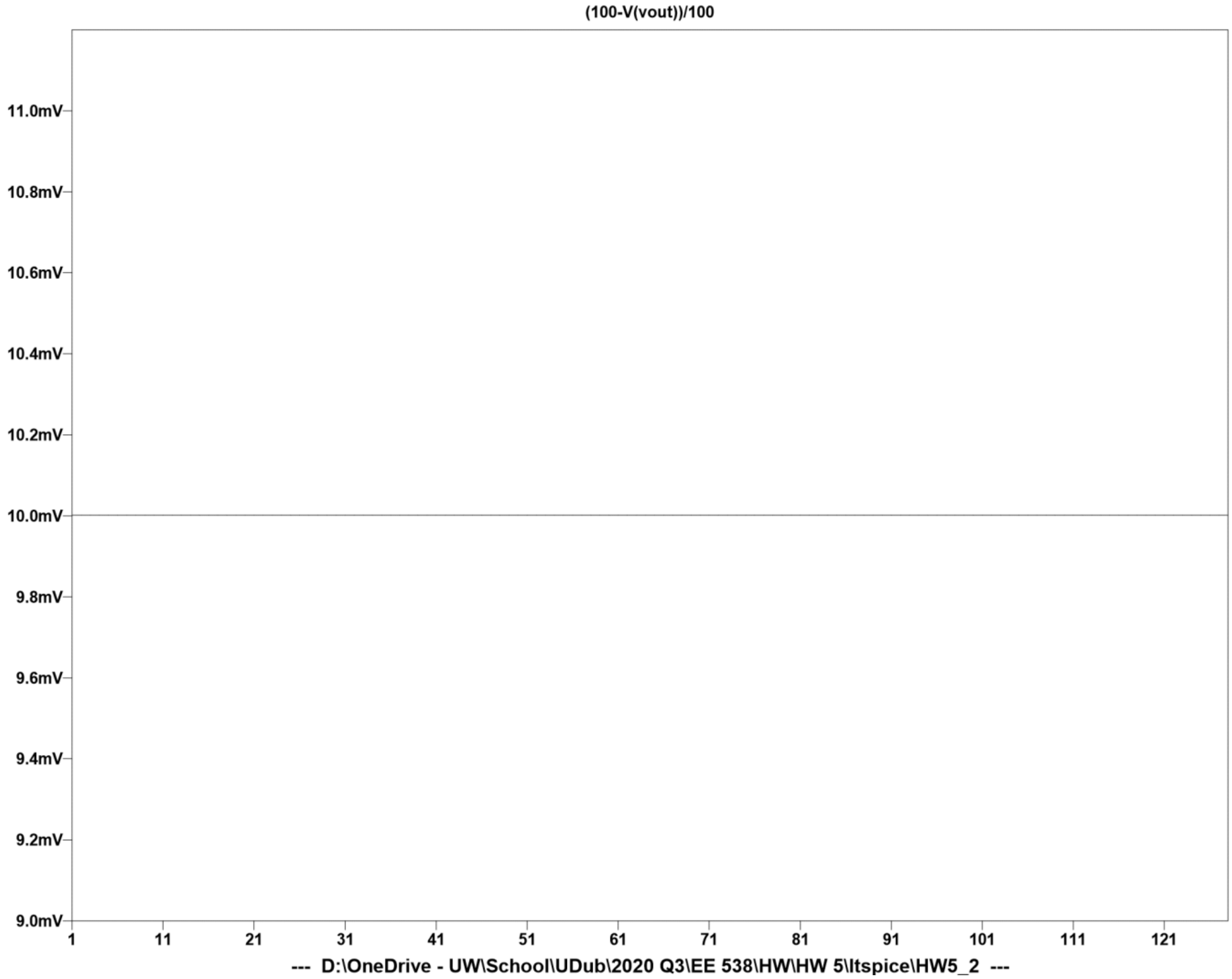


Fig 7) Gain error due to finite open loop gain of the Op-amp

From the plots, we can infer that gain error due to resistor mismatch is worse than the gain error due to finite open loop gain of the op-amp

3) Bode plots

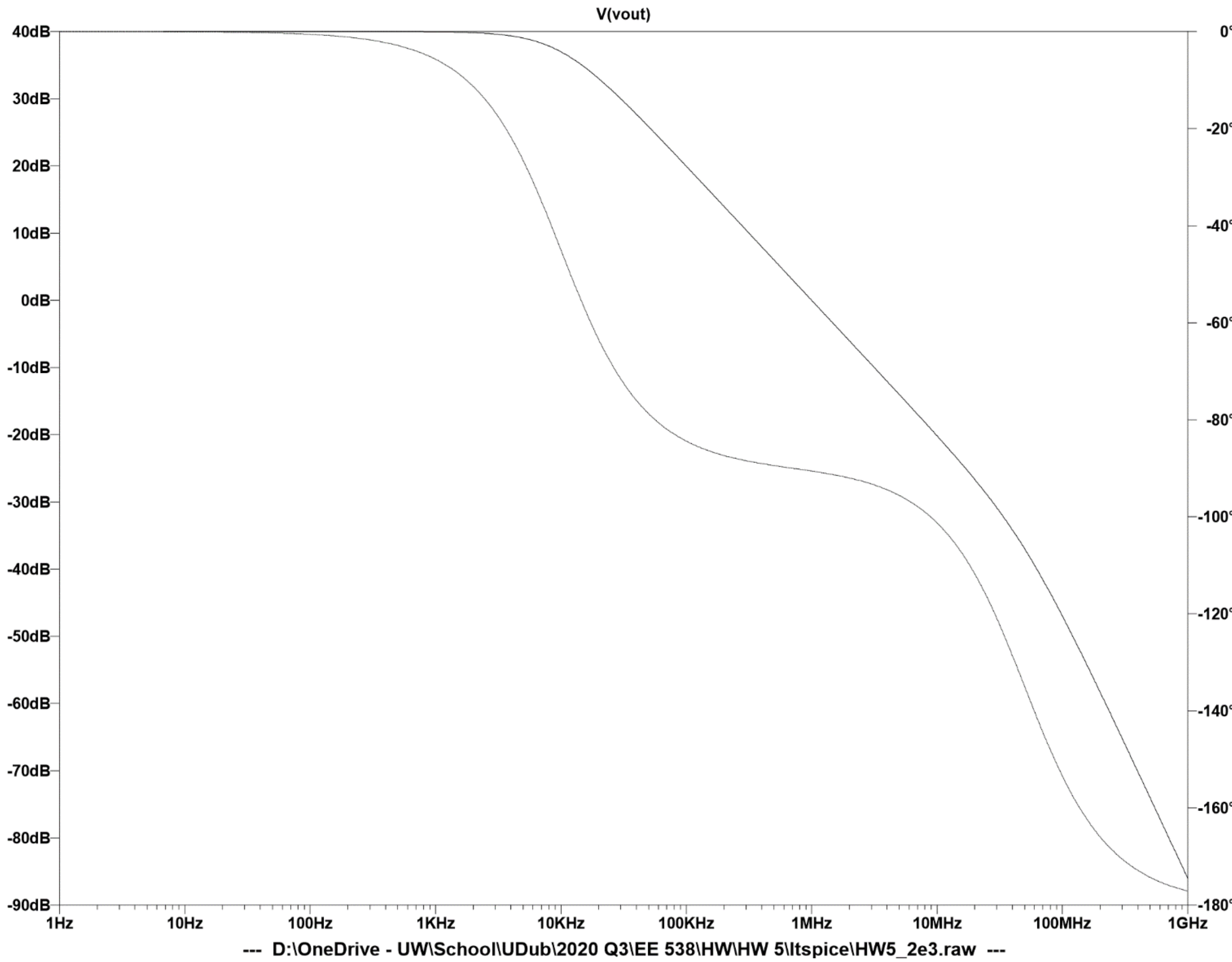


Fig 8) Closed loop differential gain (no resistor mismatch)

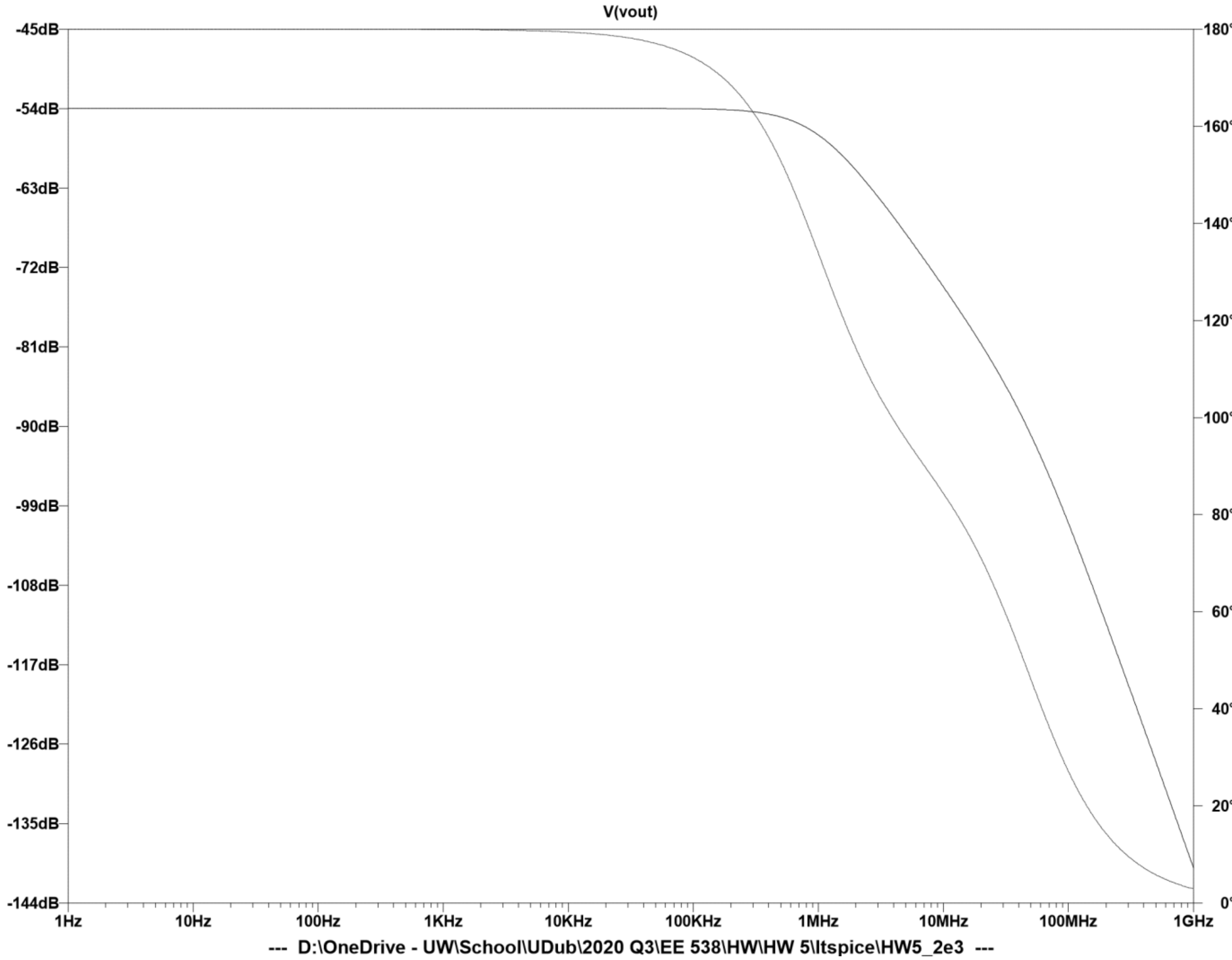


Fig 9s) Closed loop common mode gain (with resistor mismatch)