EE P 538
Analog Circuits for Sensor Systems

Spring 2020

Instructor: Jason Silver, PhD

ELECTRICAL ENGINEERING
UNIVERSITY of WASHINGTON

Announcements

- Assignment 6 due Saturday, May 23 at midnight
- Phase 1 of Design Project due Saturday, May 30 at midnight

Week 8

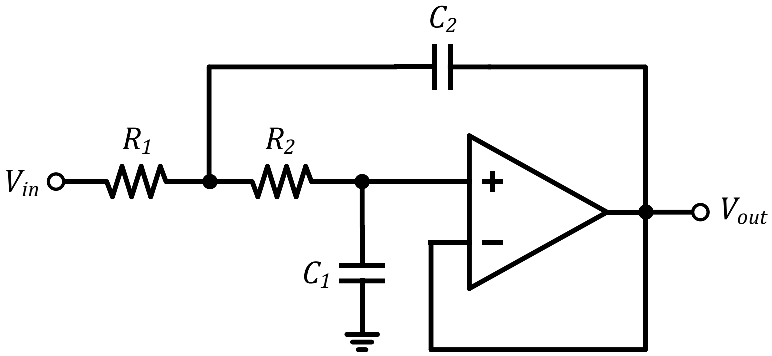
- AoE Chapter 6 – Filters
  - Sections 6.1, 6.2
- James Karkin – Active Low-Pass Filter Design
  - TI Application Note SLOA049B
- James Karkin – Sallen–Key Filter Analysis
  - TI Application Note SLOA024B

Overview

- Last time…
  - Noise filtering
  - Butterworth, Chebyshev, and Bessel filters
  - Sallen–Key filter architecture
- Today…
  - Higher order filters
  - Highpass filters
  - Bandpass filters
  - Multiple-feedback architecture

Lecture 8 – Filters 2

2^{nd} Order Sallen–Key Design



Specifications

$$\omega_c = 2\pi f_c$$

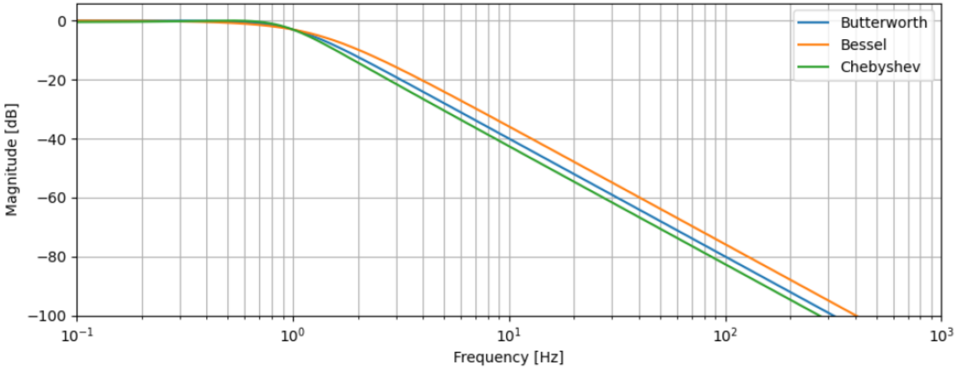
Response Type

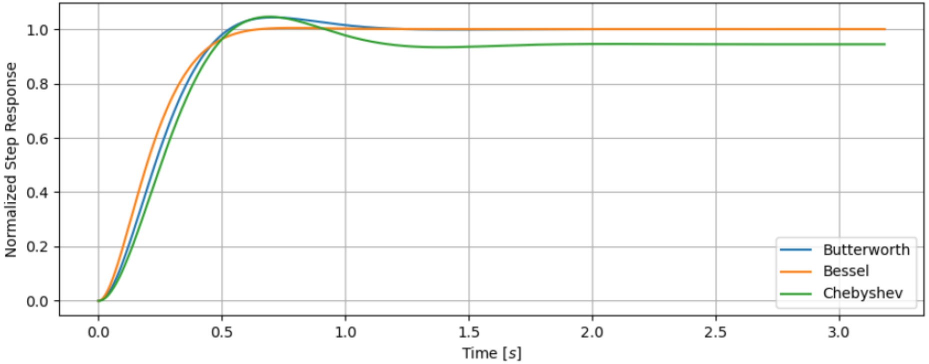
Butterworth, Chebyshev, Bessel

- Only parameters for tuning are the filter cutoff and the response type
- The choice of response type is based on desired passband variability, stopband attenuation, and target transient response
- For Butterworth and Bessel filters, ω_c is the frequency at which the magnitude is 3dB lower than the DC gain
- For Chebyshev filters, ω_c is the end of the ripple band

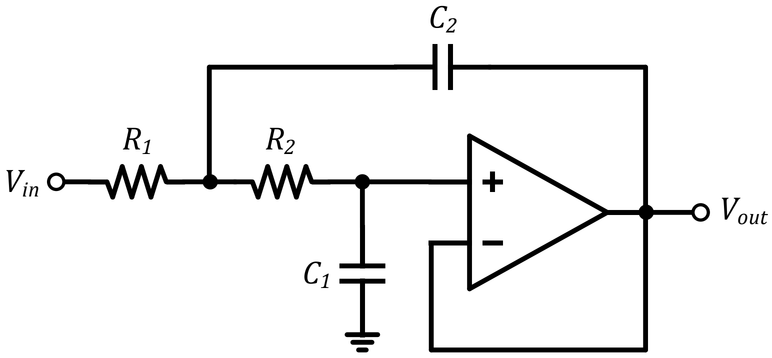
Filter Response Comparison

- If we compare filter types with identical 3dB frequencies, there are clear pros and cons to each type
- Chebyshev filters have the sharpest transition and best stopband attenuation
- Bessel filters have the most “well-behaved” step response, with almost no overshoot or ringing
- Butterworth filters constitute the middle ground in terms of both magnitude and step response





Ratio–Based Approach



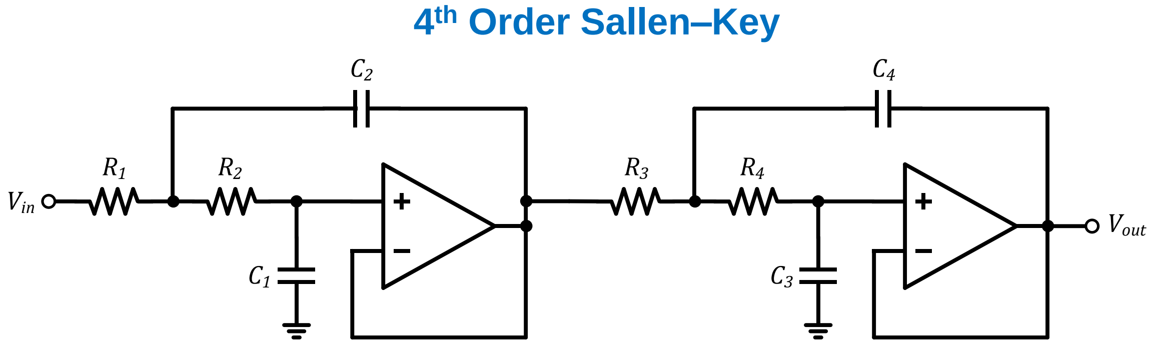
$$\omega_0 = \frac{1}{\tau\sqrt{mn}} = c_n\omega_c \qquad Q = \frac{\sqrt{mn}}{1+m}$$

$$m = \frac{R_1}{R_2} \qquad n = \frac{C_2}{C_1} \qquad \tau = R_2C_1$$

1. Determine the values of ω_0 and Q needed for the desired response; c_n depends on the type of filter (e.g. Butterworth, Chebyshev)
2. Choose either m or n and calculate the other based on the target Q
3. Choose either R_2 or C_1, and calculate the other based on τ
4. Calculate the remaining component values from m and n

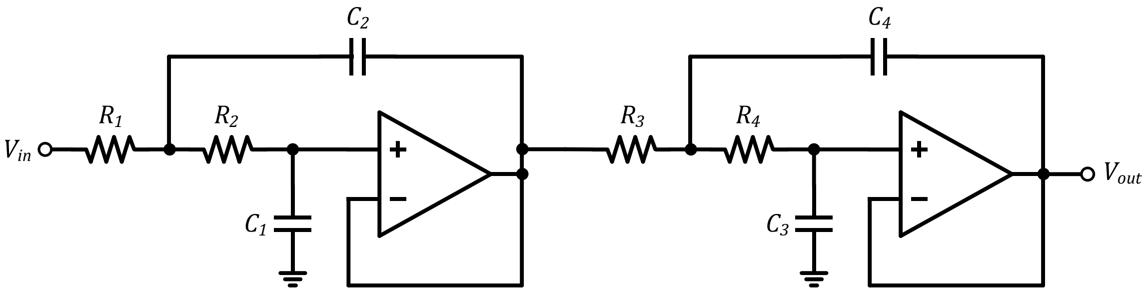
- Design example: [Ltspice: Sallen-Key Lowpass]

Higher-Order Filters



- Higher-order filters can be constructed by cascading 1st and 2nd order stages
- Use the ratio-based design approach for each stage, with ω_c and Q values taken from a filter design table
- Frequency scaling factors will differ from those associated with a single 2nd order filter, as each 2nd order section will have a different response

Design Example: 4th Order Chebyshev



Specifications

$f_c = 1kHz$

$1dB\ ripple$

- To design the 4th order filter, we can use the same approach to design two 2nd order stages and cascade them
- The Q factors will be different from a 1-dB ripple 2nd order Chebyshev, since the magnitude response of the 4th order cascade is the *product* of the 2nd order response

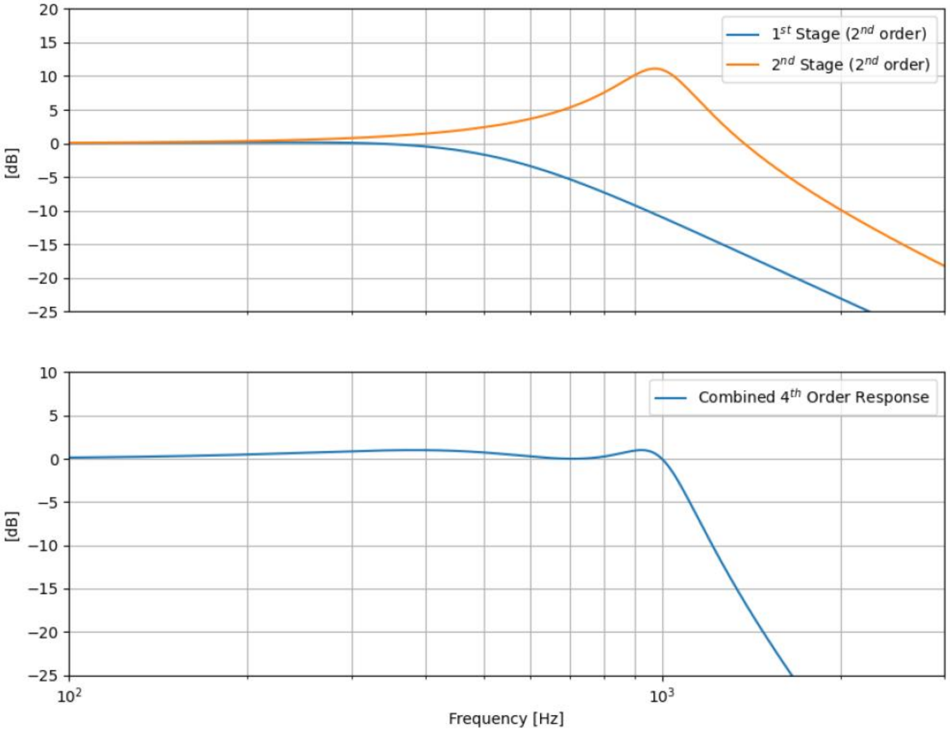
1-dB Chebyshev Filter Design Table

FILTER ORDER	Stage 1		Stage 2		Stage 3		Stage 4		Stage 5	
	FSF	Q	FSF	Q	FSF	Q	FSF	Q	FSF	Q
2	1.0500	0.9565								
3	0.9971	2.0176	0.4942							
4	0.5286	0.7845	0.9932	3.5600						
5	0.6552	1.3988	0.9941	5.5538	0.2895					
6	0.3532	0.7608	0.7468	2.1977	0.9953	8.0012				
7	0.4800	1.2967	0.8084	3.1554	0.9963	10.9010	0.2054			
8	0.2651	0.7530	0.5838	1.9564	0.5538	2.7776	0.9971	14.2445		
9	0.3812	1.1964	0.6623	2.7119	0.8805	5.5239	0.9976	18.0069	0.1593	
10	0.2121	0.7495	0.4760	1.8639	0.7214	3.5609	0.9024	6.9419	0.9981	22.2779

Source: SLOA049B: Active Low-Pass Filter Design. Texas Instruments, September 2002. http://www.ti.com/lit/an/sloa049b/sloa049b.pdf?&ts=1589465107385. Accessed 14 May 2020.

Magnitude Response

- For the 4^{th} order Chebyshev, a low-Q stage is combined with a high-Q stage to obtain the desired response
- f_c for each stage is different, determined by the frequency scaling factor for that stage
- As the high-Q stage's gain starts to rise, the low-Q stage attenuates to control the ripple
- To avoid saturation, the stages are typically arranged with the lowest-Q stage at the input and the highest-Q stage at the output



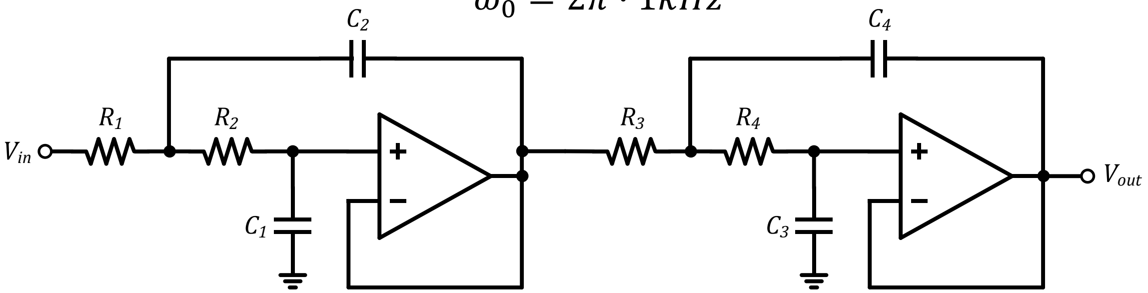
4^{th} Order Chebyshev Design

$$\omega_0 = 2\pi \cdot 1kHz$$

$c_{n1} = 0.5286$

$\omega_{c1} = \omega_0 / c_{n1}$

$Q_1 = 0.7845$



$c_{n2} = 0.9932$

$\omega_{c2} = \omega_0 / c_{n2}$

$Q_2 = 3.56$

Stage 1

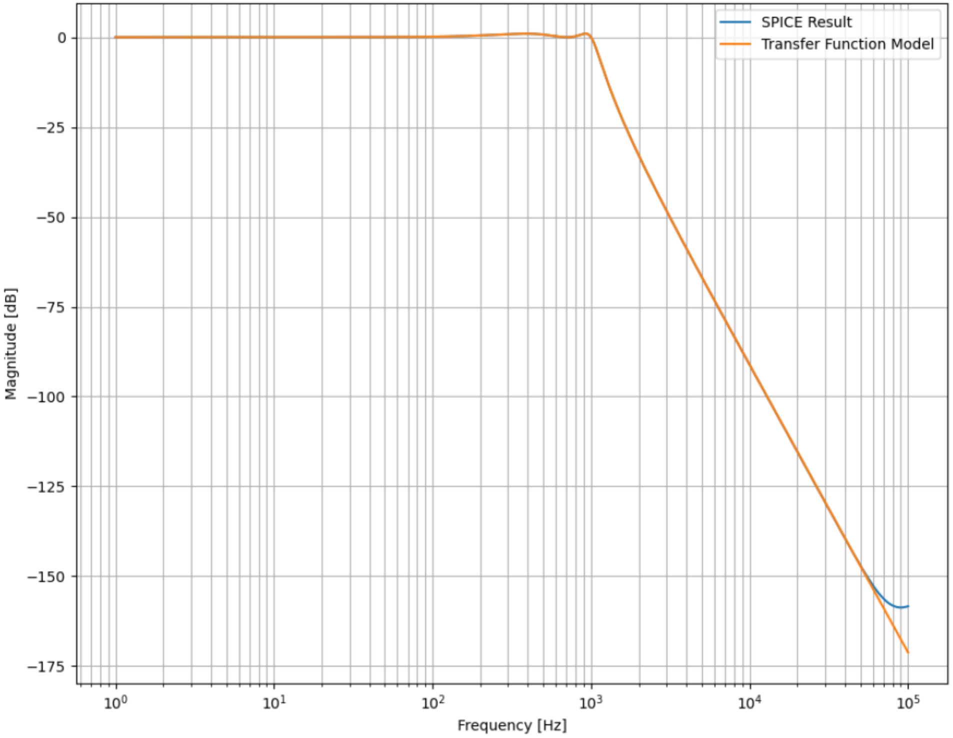
n	m	C_1	C_2	R_1	R_2
3.3	0.329	10nF	33nF	9.5k	28.9k

Stage 2

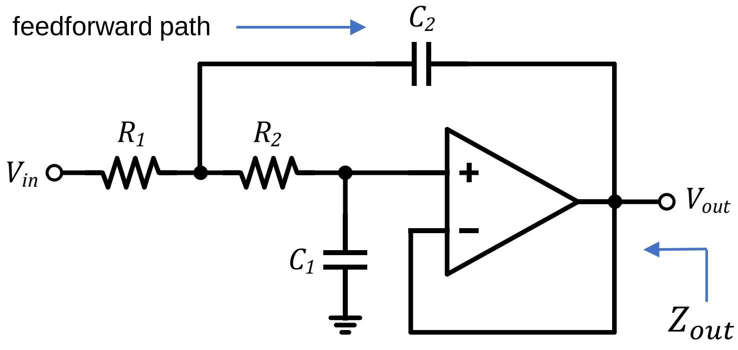
n	m	C_3	C_4	R_3	R_4
50.7	1	10nF	507nF	2.25k	2.25k

Verifying the Response

- The SPICE simulation result and transfer function model show excellent agreement up to about 50kHz ([Ltspice: 4th Order Cheby]())
- The availability of specific component values/ratios will affect the response somewhat
- At high frequencies, the SPICE response begins to rise while the model continues its roll-off
- What causes this discrepancy?



Feed-Forward Path



$$V_{out,ff} \approx \frac{Z_{out}}{R_{eq} + Z_{out}} V_{th}$$

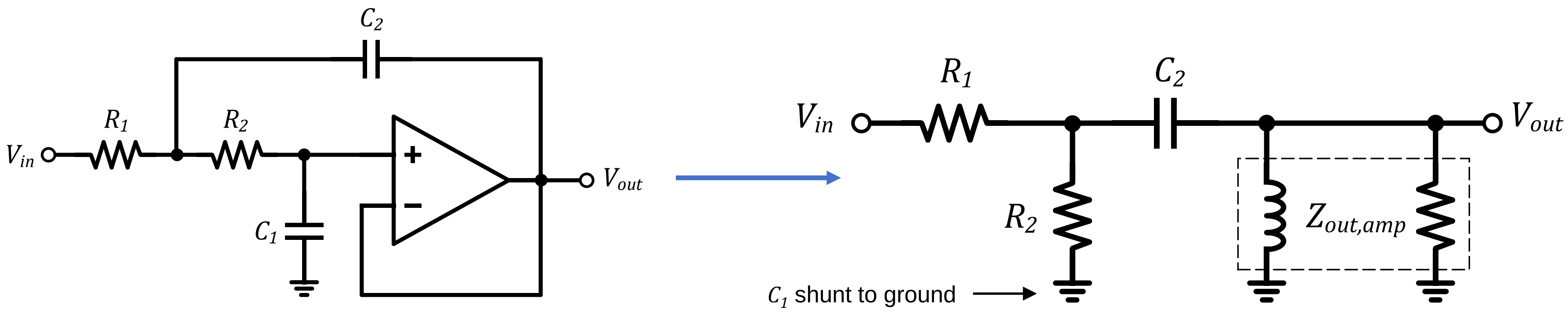
$$R_{eq} = R_1 || R_2$$

$$V_{th} = V_{in} \times R_2/(R_1 + R_2)$$

$$V_{out} = H(s)V_{in} + \frac{Z_{out}}{R_{eq} + Z_{out}} V_{th}$$

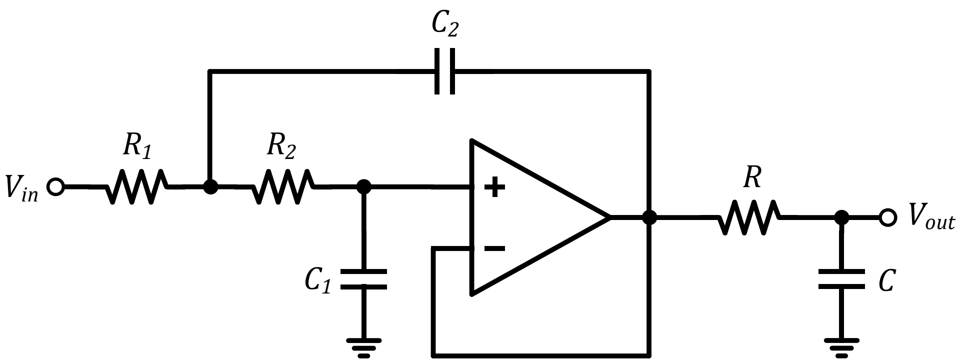
- The location of C_2 creates a feed-forward path for the input voltage at high frequencies
- At low frequencies, the filter transfer function $H(s)$ dominates due to the small closed loop output impedance, Z_{out}
- At high frequencies, C_1 appears as a short-circuit and Z_{out} increases with decreasing open-loop opamp gain

High-Frequency Feed-Through



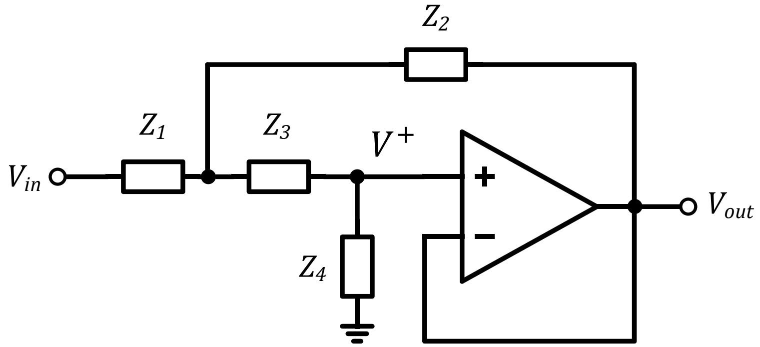
- The increasing output impedance of the amplifier can be modeled with an inductance, which has an impedance magnitude given by ωL
- The limit of the effect is determined by the ratio of $R_1||R_2$ to the open-loop output impedance of the opamp (Ltspice: HF Sallen–Key)

Feed-Through Reduction



- A passive, single-pole filter at the output of the filter can alleviate the effect of rising output resistance
- R and C should be chosen such that they don't affect the low-frequency response
- A pole placed at the "inflection" of the filter magnitude response is a good choice (depends on the opamp f_{3dB}) Ltspice: HF Sallen–Key

Generalized Sallen–Key Structure

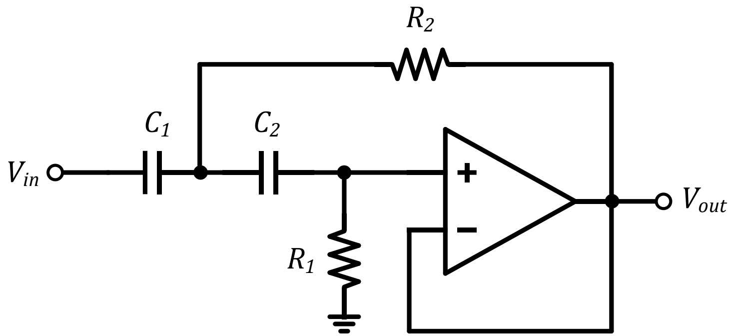


$$V_{out} = V^+ = dV_{out} + cV_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{Z_2 Z_4}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3 + Z_2 Z_4}$$

- The generalized Sallen–Key structure can be used to realize 2nd order lowpass, highpass, and bandpass functions
- Z_{1-4} are chosen based on the target filter type, and component values are selected based on the desired response (e.g. using the ratio-based design approach)

Sallen–Key Highpass

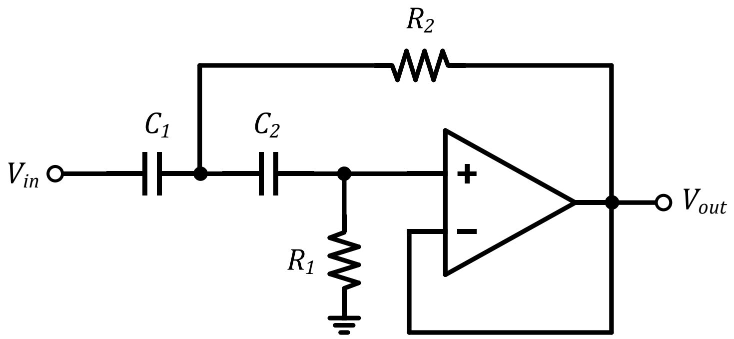


$$\frac{V_{out}}{V_{in}} = \frac{R_1 R_2}{\frac{1}{sC_1}R_2 + \frac{1}{s^2 C_1 C_2} + \frac{1}{sC_2}R_2 + R_1 R_2}$$

$$= \frac{s^2}{s^2 + s\frac{1}{R_1}\left(\frac{C_1 + C_2}{C_1 C_2}\right) + \omega_0^2}$$

- Highpass structure is identical to that of the lowpass with the R's and C's swapped
- Like the lowpass, the 2nd order highpass is fully described by ω_0 and Q
- A similar design approach can be followed as for the lowpass employing modified expressions for ω_c and Q

2^{nd} Order Highpass Design



$$\omega_0 = \frac{1}{\tau\sqrt{mn}} = \frac{\omega_c}{c_n} \qquad Q = \frac{\sqrt{mn}}{1+n}$$

$$m = \frac{R_2}{R_1} \qquad n = \frac{C_1}{C_2} \qquad \tau = R_2 C_2$$

- Lowpass filter tables can be used, with the understanding that the ratio ω_0/ω_c is changed
- Component ratios are determined using the target value of Q, noting that the denominator expression should be changed from $1+m$ to $1+n$
- Design example: [Ltspice: Sallen–Key Highpass]

0.1dB 4th Order Chebyshev HPF

Table 3.

Order	Section	Real Part	Imaginary Part	F_0	α	Q
2	1	0.6104	0.7106	0.9368	1.3032	0.7673
3	1	0.3490	0.8684	0.9359	0.7458	1.3408
	2	0.6970		0.6970		
4	1	0.2177	0.9254	$\boxed{0.9507}$	0.4580	$\boxed{2.1834}$
	2	0.5257	0.3833	$\boxed{0.6506}$	1.6160	$\boxed{0.6188}$

Table 1. 3 dB Bandwidth to Ripple Bandwidth

Order	0.01 dB	0.1 dB	0.25 dB	0.5 dB	1 dB
2	3.30362	1.93432	1.59814	1.38974	1.21763
3	1.87718	1.38899	1.25289	1.16749	1.09487
4	1.46690	$\boxed{1.21310}$	1.13977	1.09310	1.05300
5	1.29122	1.13472	1.08872	1.05926	1.03381
6	1.19941	1.09293	1.06134	1.04103	1.02344
7	1.14527	1.06800	1.04495	1.03009	1.01721
8	1.11061	1.05193	1.03435	1.02301	1.01316
9	1.08706	1.04095	1.02711	1.01817	1.01040
10	1.07033	1.03313	1.02194	1.01471	1.00842

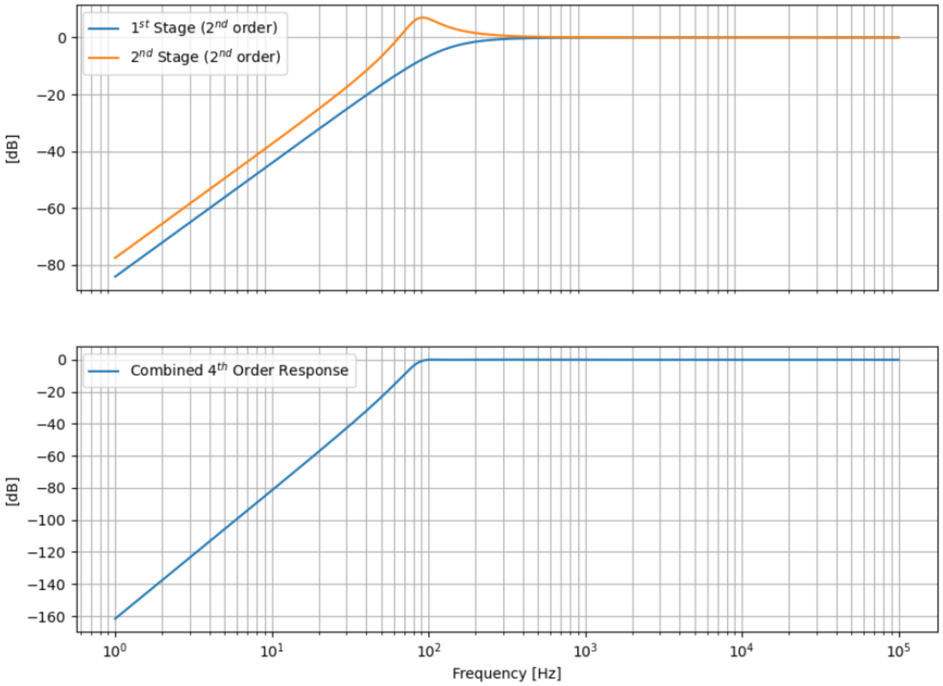
- Table 3 gives the value of Q and F_0, the ratio of the 3dB bandwidth (ω_{3dB}) to ω_0, for each stage
- Table 1 provides the ratio ω_{3dB}/ω_c, such that for each highpass stage

$$\omega_0 = \frac{\omega_{3dB}}{F_0} = \left(\frac{\omega_{3dB}}{\omega_c}\right)\frac{\omega_c}{F_0}$$

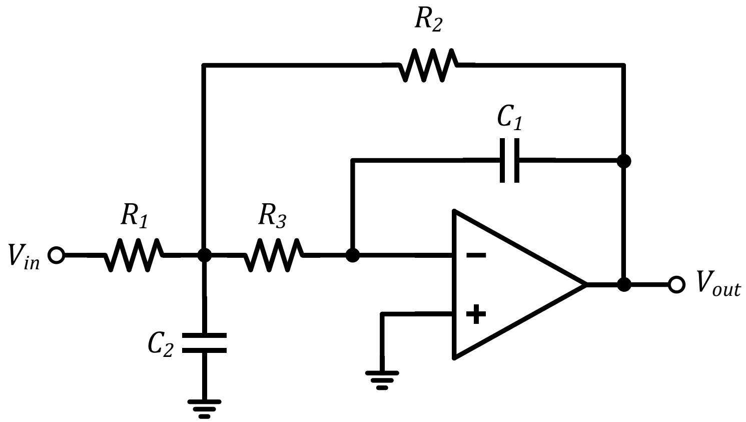
Source: MT-206: The Chebyshev Response. Analog Devices, 2012. https://www.analog.com/media/en/training-seminars/tutorials/MT-206.pdf. Accessed 16 May 2020.

0.1dB Chebyshev HPF Response

- The highpass response is symmetric with that of the lowpass filter, with two zeros at the origin for a second order filter
- As with the 4th order lowpass, a high Q stage is combined with a low Q stage with a different ω_0
- For the highpass Chebyshev filter, the cutoff frequency is *higher* than ω_0, since it marks the beginning of the ripple band rather than the end



Multi-Feedback Architecture

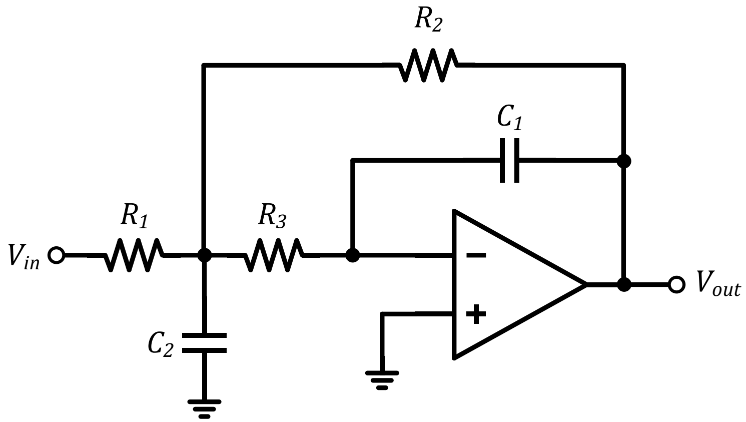


$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}\frac{\omega_0^2}{s^2 + s\frac{1}{C_2}\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_1}\right)+\omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{R_2 R_3 C_1 C_2}} \qquad Q = \frac{R_1 + R_2 + R_3}{C_2 R_1 R_2 R_3}\omega_0$$

- It is possible to realize a given response using a number of different active filter architectures
- The Multi-feedback (MFB) is an inverting gain structure with multiple feedback paths (hence the name)
- The design procedure is almost identical to that of a Sallen–Key Filter

Ratio-Based MFB Design



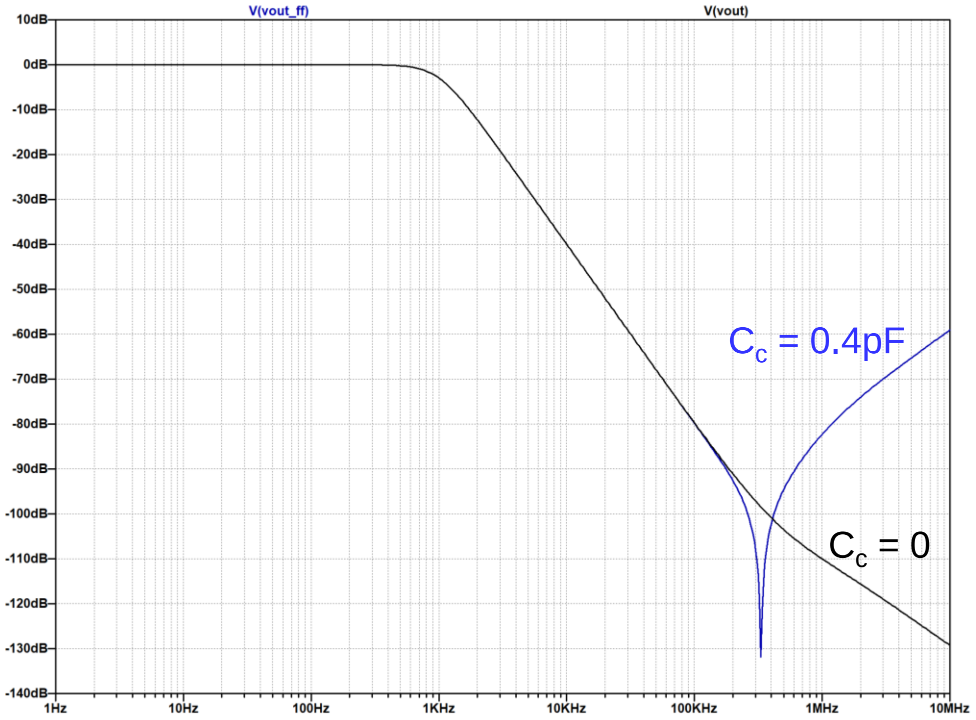
$$\omega_0 = \frac{1}{\tau\sqrt{mn}} = c_n\omega_c \qquad Q = \frac{\sqrt{mn}}{1 + m(1-K)}$$

$$m = \frac{R_3}{R_2} \qquad n = \frac{C_2}{C_1} \qquad \tau = R_2C_1$$

1. Determine the values of ω_0 and Q (e.g., using a table) needed for the desired response
2. Determine the ratios m and n required for the gain and Q of the filter, by starting with a reasonable target for m (or n) and calculating the other
3. Determine R_2 and C_1 to realize the realize the target cutoff frequency, then calculate the corresponding values of R_1 and C_2

MFB High Frequency Response

- Due to the absence of an explicit capacitance between input and output of the MFB stage, its high-frequency response is superior to that of the Sallen–Key
- This gives it a slight advantage over the Sallen–Key, but both are commonly used
- Care must be taken in circuit layout to minimize coupling due to stray capacitance, as this can degrade the high-frequency response
- [Ltspice: MFB Lowpass Filter]



Bandpass Transfer Function

- A lowpass transfer function can be transformed to a bandpass by replacing s in the transfer function with

$$s \rightarrow Q\left(s+\frac{1}{s}\right)=\left(\frac{f_0}{f_{c2}-f_{c1}}\right)\left(s+\frac{1}{s}\right)$$

where Q is the *normalized bandwidth* of the filter, f_0 is the center frequency, and f_{c2} and f_{c1} are the upper and lower cutoffs

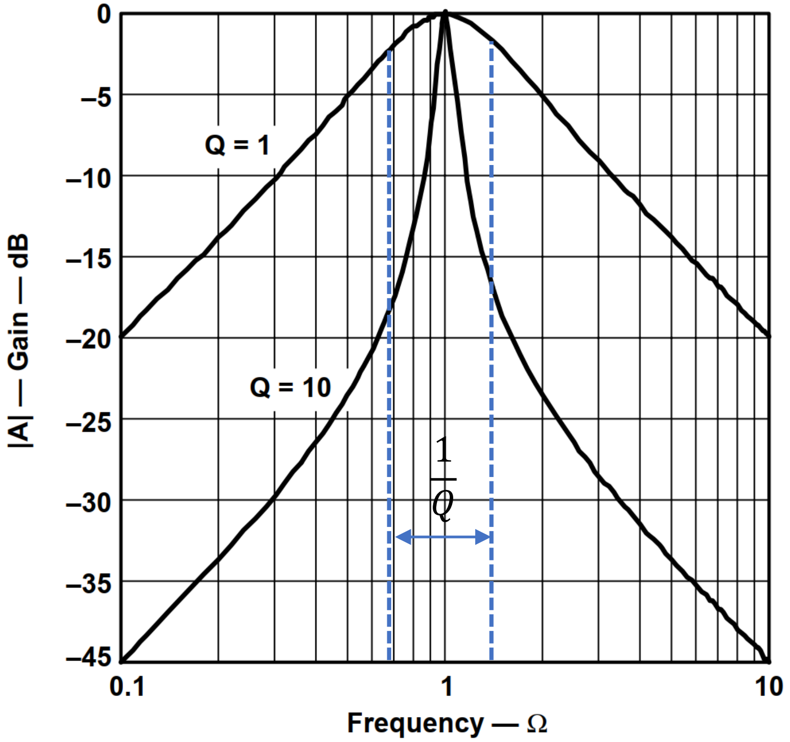
- This yields the general normalized bandpass transfer function

$$H(s)=\frac{A_m\, s/Q}{s^2+s/Q+1}$$

where A_m is the midband gain of the filter

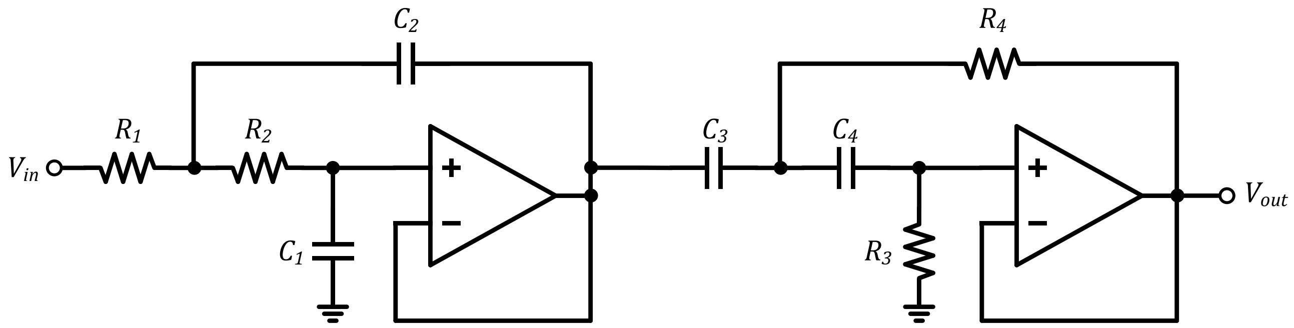
2^{nd} Order BPF Magnitude Response

- The magnitude response gets steeper with higher values of Q, increasing the "out-of-band" attenuation and making the filter more selective
- The choice of Q comes directly from the application requirements and the ratio of f_0 to bandwidth
- For wide-bandwidth applications, the typical choice is a higher-order filter with a lower Q value



Source: SLOA88: Active Filter Design Techniques. TI. https://www.ece.ucsb.edu/~ilan/Classes/ECE2A_F2010/Tutorials/App%20notes/sloa088.pdf. Accessed 16 May 2020.

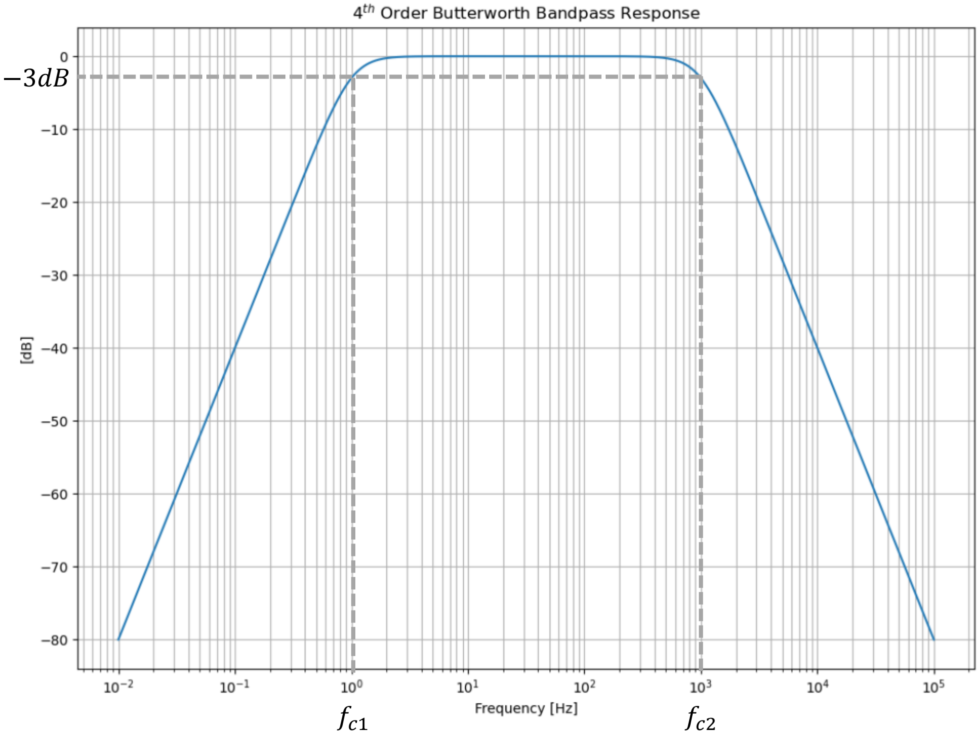
Cascade Bandpass Filter



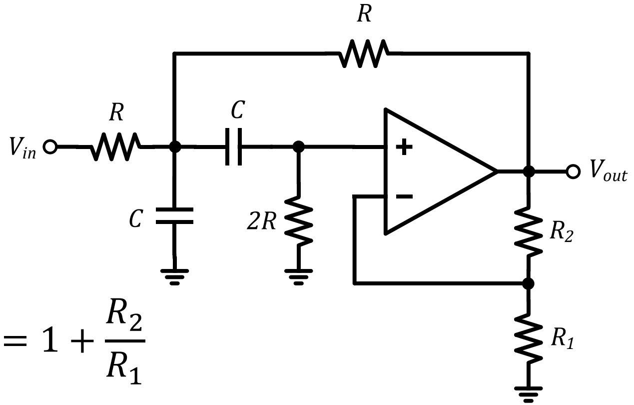
- For wide-bandwidth applications, low-Q bandpass filters can be constructed by cascading lowpass and highpass sections
- In this case, the individual stages can be designed using the approaches previously described (e.g. ratio-based design)
- To ensure accurate passband gain, the transition regions of the filters should be sharp enough (or the cutoff frequencies wide enough apart) to avoid significant attenuation

Bandpass Frequency Response

- The bandwidth of the bandpass filter is defined as the difference between –3dB frequencies, and is given by f_0/Q
- In this case, the bandwidth is relatively wide ($Q \approx 0.5$), so a cascade can suffice
- If a high-Q response is needed, a single-stage bandpass circuit, such as a Sallen–Key, MFB, or biquad bandpass is preferred
- [Ltspice: BP cascade]



Sallen–Key Bandpass Filter



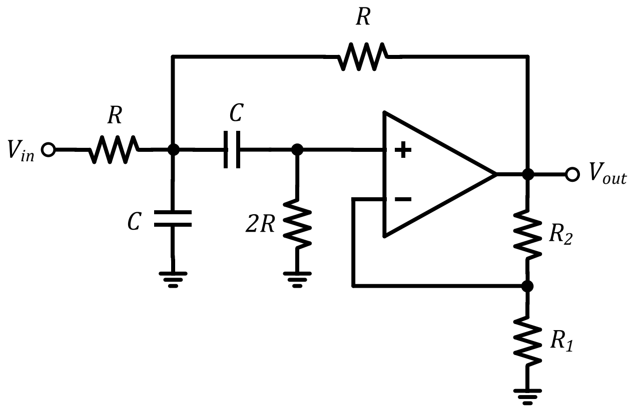
$$G = 1 + \frac{R_2}{R_1}$$

$$H(s) = \frac{V_{out}}{V_{in}} = G \frac{s/(RC\omega_0)}{s^2 + s\left(\frac{3-G}{RC\omega_0}\right) + \omega_0^2}$$

$$f_0 = \frac{1}{2\pi RC} \qquad Q = \frac{1}{3-G} \qquad A_m = \frac{G}{3-G}$$

- Center frequency f_0, bandwidth f_0/Q, and midband gain A_m
- Simplified design approach, similar to setting R's and C's equal in the lowpass filter
- Q can be varied by changing the gain factor G without affecting f_0

Sallen–Key BP Design Example



$f_0 = 1kHz$

$BW = 500Hz$

$C = 10nF$

$R_1 = 10k\Omega$

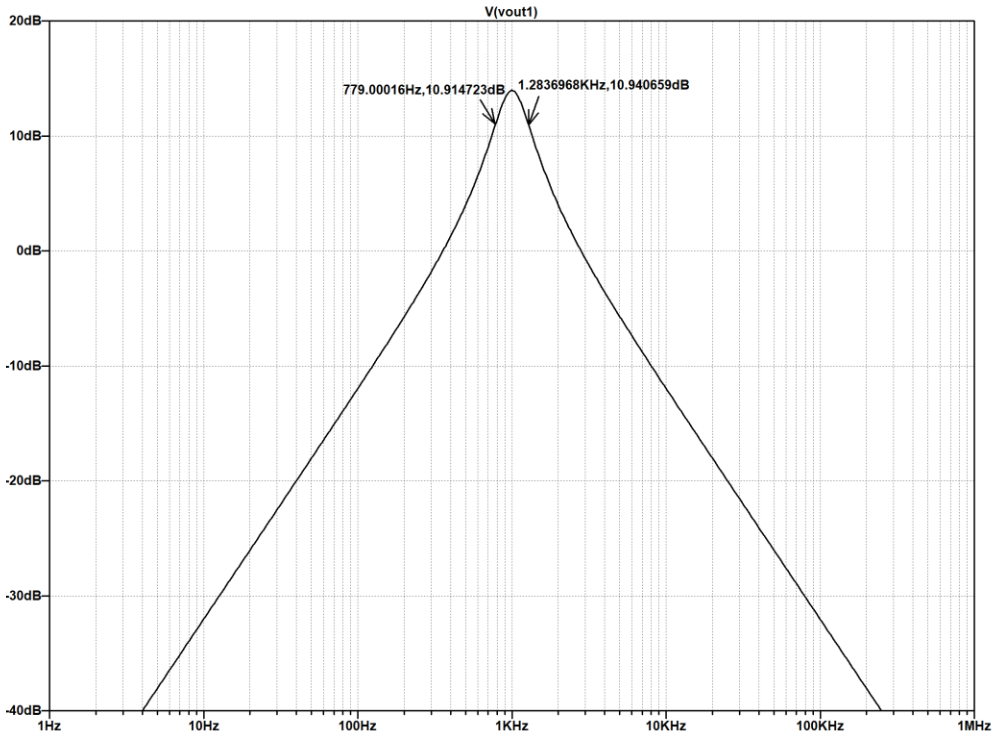
→

$Q = 2$

$R = 15.9k\Omega$

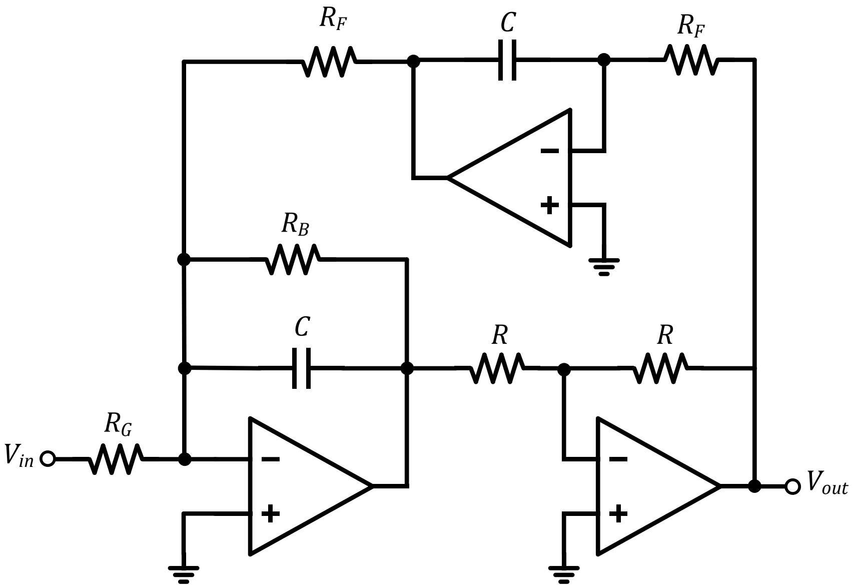
$R_2 = 15k\Omega$

$A_m = 5V/V$

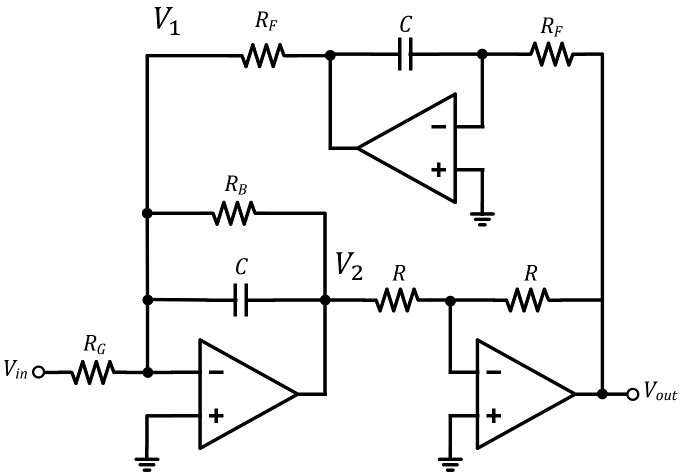


Biquad Bandpass Filter

- The biquad active filter combines a feed-forward lowpass with an integrator in the feedback path to realize a bandpass transfer function
- The name “biquad” comes from the fact that the transfer function is a quadratic function in the numerator and denominator
- The center frequency of the biquad can be tuned by adjusting R_F while keeping the bandwidth constant



Biquad Transfer Function



$$V_1 = -\frac{1}{sCR_F} V_{out} \qquad V_{out} = -V_2$$

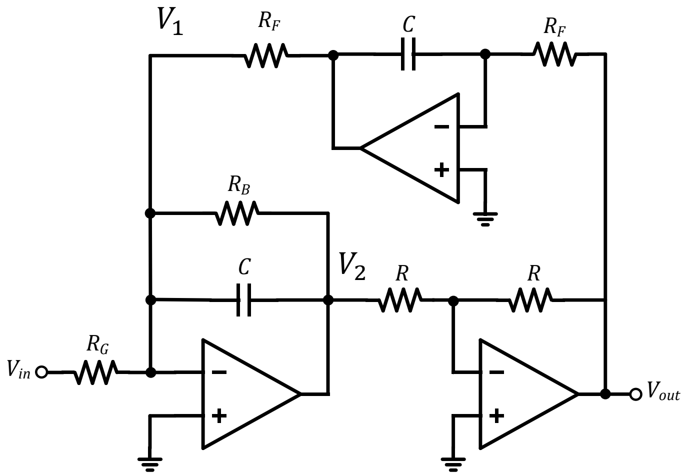
$$V_2 = -\left(\frac{R_B}{1 + sCR_B}\right)\left(\frac{V_{in}}{R_G} - \frac{V_{out}}{sCR_F^2}\right) = -V_{out}$$

rearranging gives →

$$V_{out}\left(1 + \frac{R_B}{R_F}\frac{1}{1 + sCR_B}\frac{1}{sCR_F}\right) = -\left(\frac{R_B/R_G}{1 + sCR_B}\right)V_{in}$$

$$\boxed{H(s) = \frac{V_{out}}{V_{in}} = -\frac{R_B}{R_G}\frac{sCR_F}{s^2C^2R_FR_B + sCR_F + R_B/R_F} = -\frac{R_B}{R_G}\frac{s/R_BC}{s^2 + s/R_B\,C + 1/(R_FC)^2}}$$

Biquad Transfer Function



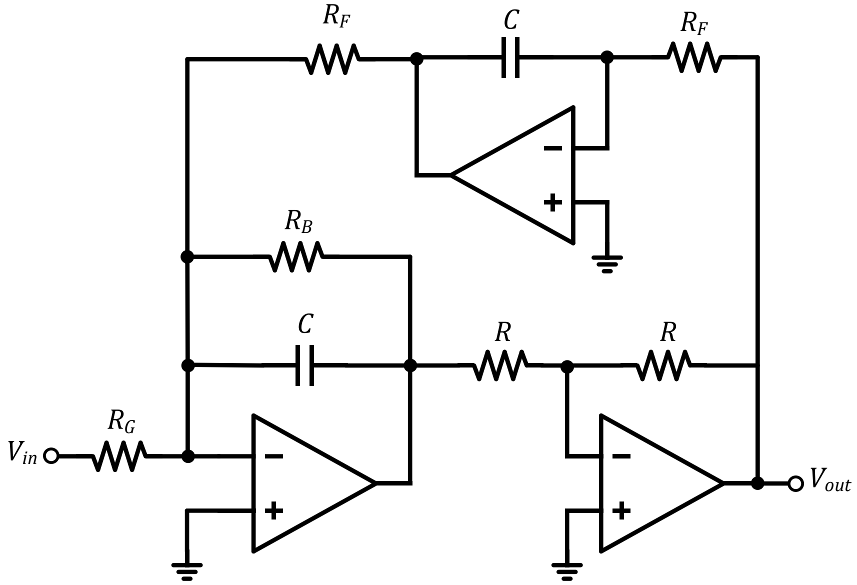
$$H(s) = \frac{V_{out}}{V_{in}} = -\frac{R_B}{R_G}\frac{s/R_BC}{s^2 + s/R_B\,C + 1/(R_FC)^2}$$

$$\omega_0 = \frac{1}{R_FC} \qquad Q = \frac{R_B}{R_F} \qquad BW = \frac{\omega_0}{Q} = \frac{1}{R_BC}$$

- As R_F is varied, the Q changes proportionately, so the bandwidth remains constant
- In switched-capacitor implementations, R_F can be modified electronically by changing the clock frequency of a “switched-capacitor resistor,” resulting in an easily re-configurable bandpass filter

Biquad Filter Design

1. Use an opamp with bandwidth at least 10 times $\mathrm{G}\omega_0$
2. Select a capacitance value according to $\mathrm{C} = 10/f_0$
3. Use the target center frequency to calculate the corresponding value of R_F
4. Calculate R_B based on the bandwidth required
5. Use R_G to set the passband gain (or set it equal to R_B for unity gain)



$$\omega_0 = \frac{1}{R_F C} \qquad BW = \frac{1}{R_B C} \qquad G = \frac{R_B}{R_G}$$