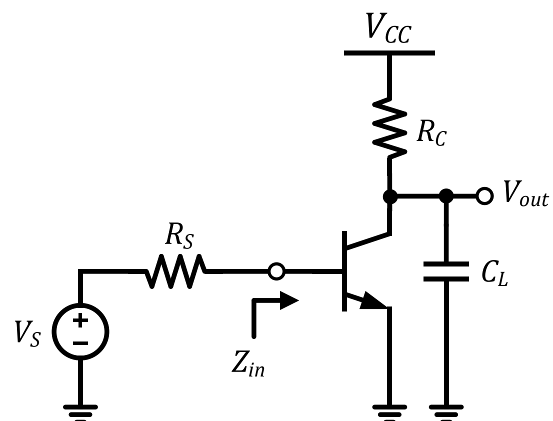
**EE 538 Spring 2020**
**Analog Circuits for Sensor Systems**
**University of Washington Electrical & Computer Engineering**

**Instructor: Jason Silver**
**Midterm**

*Please show your work.*

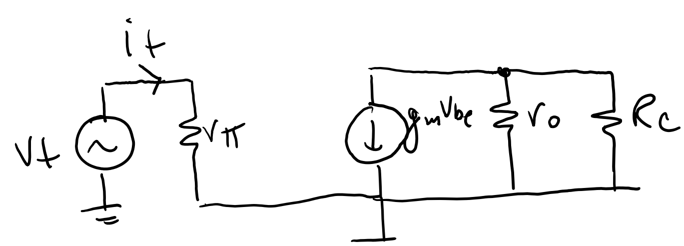
**Problem 1: Common-emitter amplifier**




For the following, $V_{CC}$ = 5V, $V_S$ = 1V, $R_S$ = 1kΩ, $I_{bias}$ = 100µA, $R_C$ = 10kΩ, $C_L$ = 10pF, β = 100, $V_A$ = 100V, $V_T$ = 25mV, and $I_S = 10^{-16}$A.

a) (10 points) Calculate the input impedance of the amplifier, $Z_{in}$. *Note: This does not include $R_S$.*
b) (10 points) Find an expression for the transfer function, $V_{out}/V_S$. Be sure to account for attenuation due to $Z_{in}$.
c) (10 points) Calculate the DC gain and transit frequency $f_T$.

a.




DC small-signal model

$$Z_{in} = \frac{v_t}{i_t} = r_\pi = \frac{\beta}{g_m}$$

$$g_m = \frac{100\mu A}{25mV} = 4mS$$

$$\boxed{Z_{in} = \frac{100}{4mS} = 25k\Omega}$$

b. $\frac{V_{in}}{V_S} = \frac{Z_{in}}{R_S + Z_{in}}$ ; $\frac{V_{out}}{V_{in}} = -g_m \left(R_C \| r_o \| \frac{1}{sC_L}\right)$

$$= \frac{-g_m R_C \| r_o \cdot \frac{1}{sC_L}}{R_C \| r_o + \frac{1}{sC_L}}$$

$$= \frac{-g_m R_C \| r_o}{sC_L R_C \| r_o + 1}$$

$$\boxed{\frac{V_o}{V_S} = \frac{Z_{in}}{Z_{in} + R_S} \cdot \frac{-g_m R_C \| r_o}{sC_L R_C \| r_o + 1}}$$

$Z_{in} = 25k\Omega$

c. $\frac{V_o}{V_S}(s=0) = \frac{Z_{in}}{Z_{in}+R_S}\left(-g_m R_C \| r_o\right)$

$r_o = \frac{V_A}{I_C} = 1M\Omega$

$R_C = 10k\Omega$

$$\approx \frac{Z_{in}}{Z_{in}+R_S}\left(-g_m R_C\right)$$

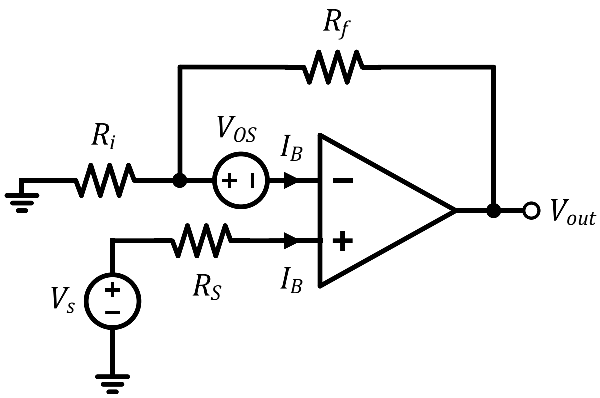
$$= \boxed{-38.5 \ V/V}$$

$$f_T \approx \frac{Z_{in}}{Z_{in}+R_S}\left(g_m \cdot R_C\right) \cdot \frac{1}{2\pi R_C \cdot C_L}$$

$$= \frac{25}{26} \cdot (40) \cdot \frac{1}{2\pi \cdot 10k\Omega \cdot 10pF}$$

$$\approx \boxed{61.2 \ MHz}$$

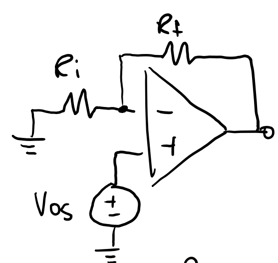
**Problem 2: Opamp nonidealities**




Assume the opamp has infinite gain and bandwidth. $V_{OS}$ = 1mV, and $I_B = -1\text{nA}$. $R_f = 10R_i$.

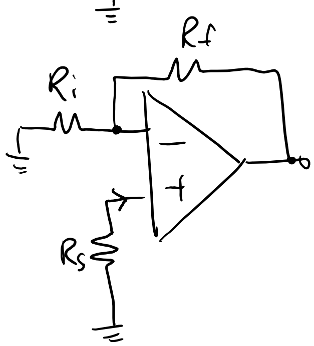
a) (15 points) Determine an expression for the output offset voltage, including the contributions from both $V_{OS}$ and $I_B$.
b) (10 points) Assuming $R_S$ = 0, calculate values for $R_i$ and $R_f$ that result in zero output offset.
c) (10 points) Assuming $R_S$ = 1k and $V_{OS}$ = 0, calculate values for $R_i$ and $R_f$ that result in zero output offset.

a. $V_{os}$:




$V_{out,vos} = \left(1+\frac{R_f}{R_i}\right)V_{os}$

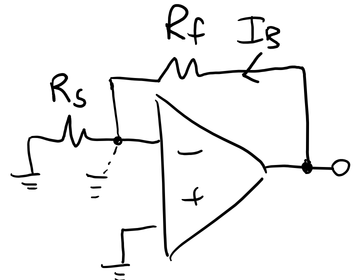
$I_B^+$:




$V^+ = 0V - I_B \cdot R_s$

$V_{out,I_B^+} = -I_B R_s\left(1+\frac{R_f}{R_s}\right)$

$I_B^-$:




$V_{out,I_B^-} = 0V + I_B \cdot R_f$

$$\boxed{V_{out,os} = \left(1+\frac{R_f}{R_i}\right)V_{os} + I_B\left[R_f - R_s\left(1+\frac{R_f}{R_i}\right)\right]}$$

b. $R_s = 0$

$$V_{out,os} = \left(1 + \frac{R_f}{R_i}\right) V_{os} + I_B \cdot R_f = 0$$

$$= 11 \cdot V_{os} - 1nA \cdot R_f = 0$$

$$11 V_{os} = 1nA \cdot R_f$$

$$R_f = \frac{11 \cdot 1mV}{1nA} = \boxed{11 M\Omega}$$

$$R_i = \frac{R_f}{10} = \boxed{1.1 M\Omega}$$

c. $$V_{out,os} = I_B \left[ R_f - R_s \left(1 + \frac{R_f}{R_i}\right)\right] = 0$$

$$R_f = R_s \left(1 + \frac{R_f}{R_i}\right)$$

$$R_s = \frac{R_f \cdot R_i}{R_f + R_i} = \frac{10 R_i^2}{11 R_i} = \frac{10}{11} R_i$$

$$\boxed{\begin{aligned} R_i &= 1.1 k\Omega \\ R_f &= 11 k\Omega \end{aligned}}$$

**Problem 3. Opamp AC and transient analysis**

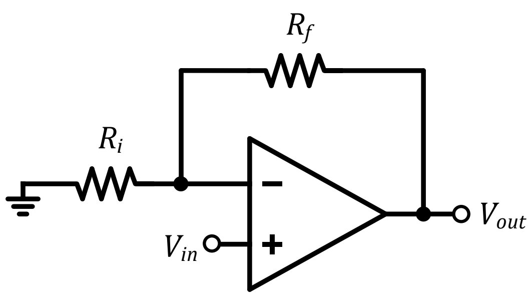



**Figure 3a. Non-inverting amplifier**

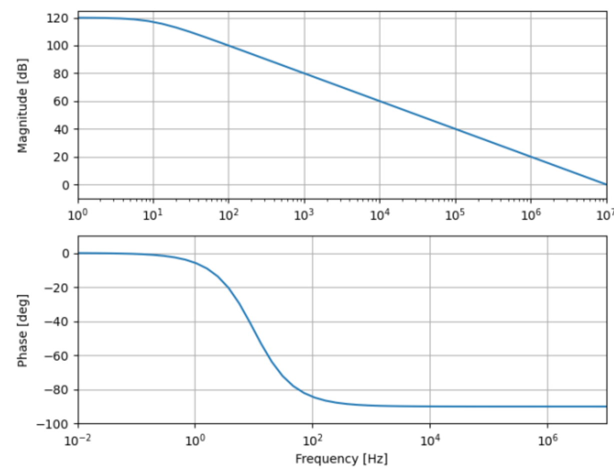



**Figure 3b. Opamp open-loop frequency response**

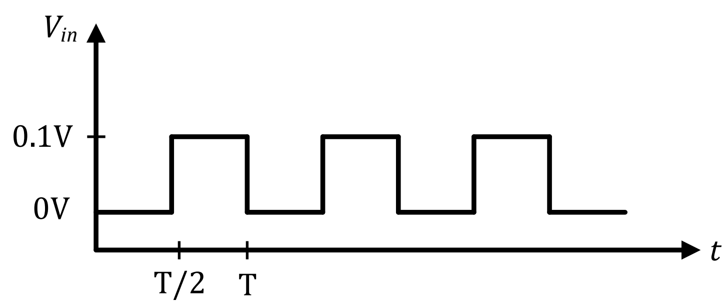



**Figure 3. Input waveform for c) and d)**

Assume ideal input/output resistances ($R_{in}$ and $R_o$) for the opamp. Let $R_f = 10R_{in}$.

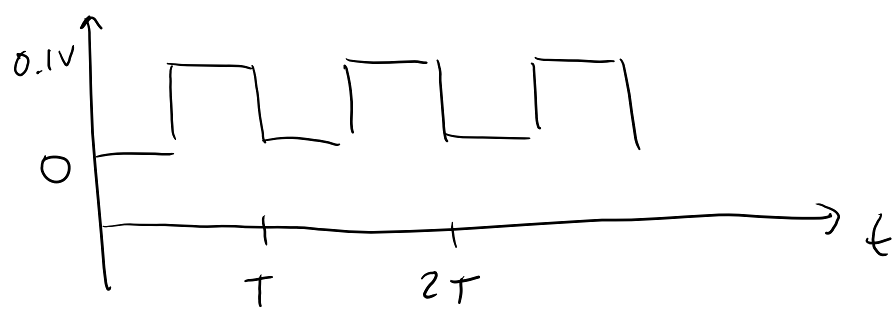
a) (10 points) Determine the DC gain and the 3dB frequency of the closed loop transfer function ($V_{out}/V_{in}$).
b) (7.5 points) Provide an expression for the transient response of the amplifier for a voltage step input of 0 to 100mV. Sketch the response and label all relevant times/voltages.
c) (10 points) Assume the amplifier is driven by the input waveform shown in Fig 3c. Determine the minimum period T for which 0.1% settling is achieved during each half-period (integer multiples of T/2). Sketch the output waveform.
d) (7.5 points) Calculate the total worst-case error in the output voltage (at the end of each half-period) if the resistors have a tolerance of 0.1%.

a. $\beta = \frac{R_i}{R_f + R_i} = \frac{1}{11}$ ; $f_{3dB,CL} = \beta \cdot f_T = \beta \cdot 10\,MHz = \boxed{909\,kHz}$

$A_{CL} = \frac{A_{OL}}{1 + \beta A_{OL}} \approx \boxed{10.9999\,V/V}$

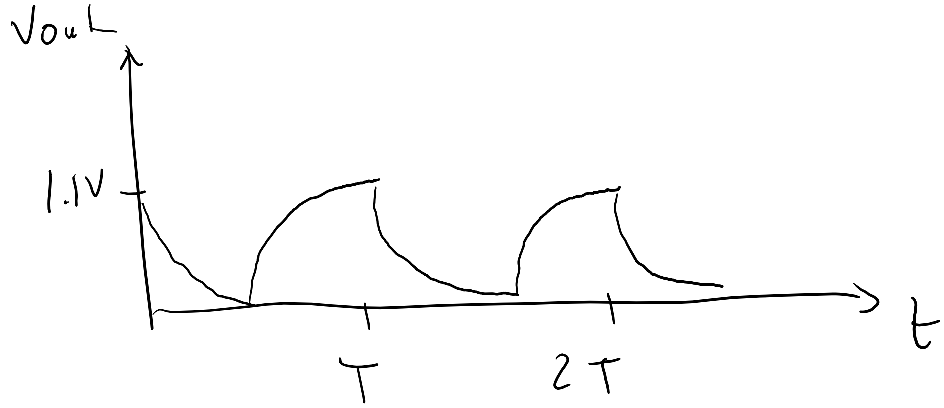
b. $V_{out} = 0.1V \cdot \left(1 - e^{-t/\tau}\right)$, where $\tau = \frac{1}{2\pi f_{3dB}}$

c. $V_{in}$




$$T/2 = -\ln(0.001)\cdot\tau \approx 1.21\mu s$$

$$\boxed{T = 2.4\mu s}$$




d. Worst-case error for $R_f(1-0.001)$ and $R_i(1+0.001)$

$$A_{CL}' = 1 + \frac{10(1-0.001)}{1+0.001} \approx 10.98\ V/V$$

@ $t = T/2$, $V_{out}(T/2) = 10.98\cdot\left(1-e^{-\frac{T/2}{\tau}}\right)\cdot 0.1V$

$$\approx 1.097V$$

$$\text{Error} = \frac{1.1-1.097}{1.1}\times 100\% = \boxed{0.27\%}$$