Assignment 04

EE 538 Spring 2020
Analog Circuits for Sensor Systems
University of Washington Electrical & Computer Engineering

Due: May 2, 2020
Author: Kevin Egedy

In [1]:
```python
# Imports
import os
import sys
import cmath
import math
import matplotlib.pyplot as plt
import matplotlib
import numpy as np
import pandas as pd
import ltspice
import sympy as sp
from scipy import signal
%matplotlib inline
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
```

In [2]:
```python
def read_ltspice_tran(file_name):
    cols = []
    arrs = []
    with open(file_name, 'r',encoding='utf-8') as data:
        for i,line in enumerate(data):
            if i==0:
                cols = line.split()
                arrs = [[] for _ in cols]
                continue
            parts = line.split()
            for j,part in enumerate(parts):
                arrs[j].append(part)
    df = pd.DataFrame(arrs,dtype='float64')
    df = df.T
    df.columns = cols
    return df
```

In [3]:
```python
def RoundNonZero(num, place, rnd='ceil'):
    # Requires numpy library
    # Examples:
    #   RoundNonZero(0.0004512,1,'floor') -> 0.0045
    #   RoundNonZero(0.0004512,1,'ceil') -> 0.0046
    #
    tmp = num
    mag = 0

    if rnd=='ceil':
        while(abs(tmp)<1):
            tmp*=10
            mag+=1
        for i in range(place):
            tmp*=10
            mag+=1
        return int(np.ceil([tmp])[0])/(10**(mag))

    if rnd=='floor':
        while(abs(tmp)<1):
            tmp*=10
            mag+=1
        for i in range(place):
            tmp*=10
            mag+=1
        return int(np.floor([tmp])[0])/(10**(mag))

    else:
        raise ValueError('Invalid argument')
        return None
```

Reference: https://inst.eecs.berkeley.edu/~ee105/fa14/lectures/Lecture04-Non-ideal%20Op%20Amps%20(Feedback%20circuit).pdf (https://inst.eecs.berkeley.edu/~ee105/fa14/lectures/Lecture04-Non-ideal%20Op%20Amps%20(Feedback%20circuit).pdf)

Problem 1: DC analysis of inverting and non-inverting amplifiers

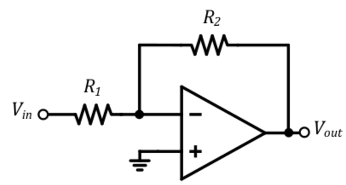


Figure 1a. Inverting amplifier

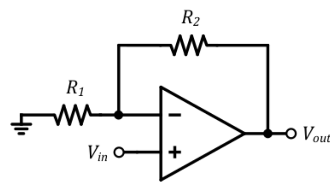


Figure 1b. Non-inverting amplifier

For the two amplifiers shown above, the opamp has open-loop DC gain A_0, input resistance R_{in}, and output resistance R_{out}. For the Ltspice parts, use the UniversalOpamp2 (SpiceModel level.1), with R_1 = 1kΩ and R_2 = 10kΩ. The default open-loop output resistance for the opamp model is 0.1Ω. You can use the 'DC Transfer' analysis.

a) (5 points) For the inverting and non-inverting amplifiers shown in Fig 1a and 1b, determine *expressions* for each of the following assuming $A_0 \to \infty$ (infinite open-loop gain). Provide comments on how each closed-loop parameter compares to its open-loop counterpart.
   1. Closed-loop gain (V_{out}/V_{in}).
   2. Closed-loop output resistance.
   3. Closed-loop input resistance.

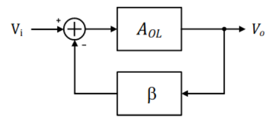
b) (5 points) Repeat Part a assuming A_0 is finite. Try to develop some intuition regarding how each parameter depends on A_0 and the feedback factor β. Check your answer by setting $A_0 \to \infty$ and comparing to your answer in Part a.

c) (2.5 points) Assuming the opamp has a voltage offset v_{OS}, what is the resulting output offset for each structure? Assume $A_0 \to \infty$. Check your answer in Ltspice.

d) (2.5 points) Assuming the opamp has input bias current I_B, what is the resulting output offset for each structure? Assume $A_0 \to \infty$.

Feedback

Opamp inputs are only exactly equal if the open loop gain is infinite

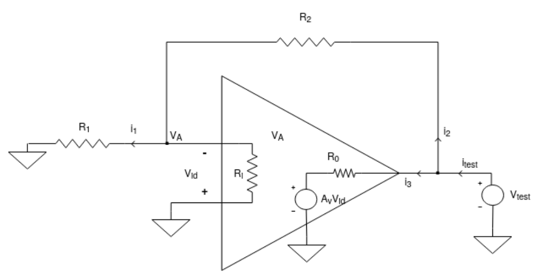


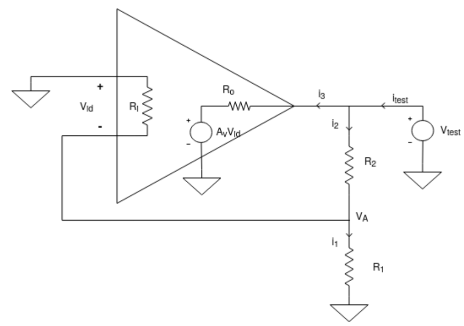
$$V_{\text{out}} = A_v v_{\text{in}}\bigg|_{v_{\text{in}}=v^+-v^-}$$

$$A_{CL} = \frac{V_o}{V_i} = \frac{A_{OL}}{1+\beta A_{OL}}$$

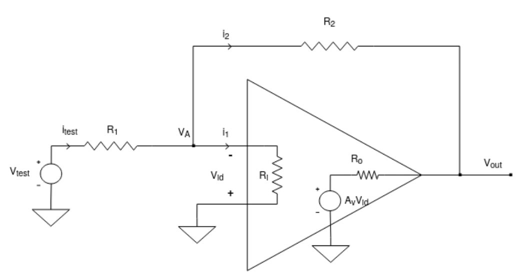
Output Resistance

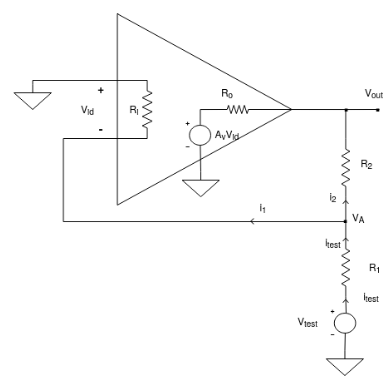
Same for Inverting and Non-inverting



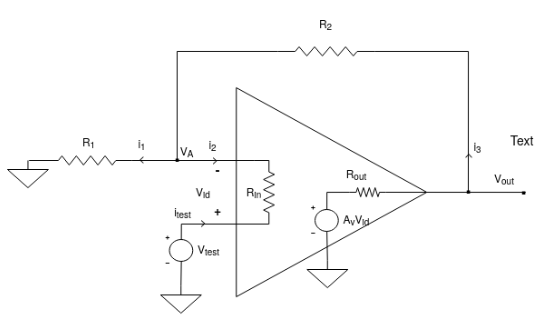


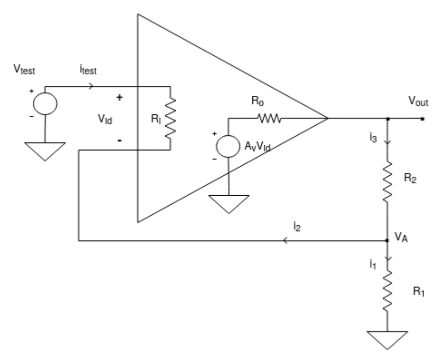
Input Resistance: Inverting





Input Resistance: Non-inverting





Part A

Inverting

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \left.\frac{A_{OL}}{1+\beta A_{OL}}\right|_{A_{OL}\to\infty} = \frac{1}{\beta} = \frac{-R_2}{R_1}$$

$$R_{\text{out}} = 0$$

$$R_{\text{in}} = \infty$$

Non-inverting

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \left.\frac{A_{OL}}{1+\beta A_{OL}}\right|_{A_{OL}\to\infty} = \frac{1}{\beta} = 1+\frac{R_2}{R_1}$$

$$R_{\text{out}} = 0$$

$$R_{\text{in}} = \infty$$

Part B

R_{out} Calculations

$$
\begin{aligned}
i_{\text{test}} &= i_2 + i_3 \\
i_2 &= \frac{v_{\text{test}}}{R_1 + R_2} \\
i_3 &= \frac{v_{\text{test}} - A v_{id}}{R_0} \\
v_{id} &= -v_A = \frac{-R_1}{R_1 + R_2} v_{\text{test}} = -\beta v_{\text{test}} \\
i_{\text{test}} &= \frac{v_{\text{test}}}{R_1 + R_2} + \frac{v_{\text{test}} + A\beta v_{\text{test}}}{R_0} \qquad = v_{\text{test}} \left(\frac{1}{R_1 + R_2} + \frac{1 + A\beta}{R_0}\right) \\
Z_1 \parallel Z_2 &= \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} \\
R_{\text{out}} = \frac{v_{\text{test}}}{i_{\text{test}}} &= \frac{1}{\frac{1}{R_1 + R_2} + \frac{1 + A\beta}{R_0}} \qquad = (R_1 + R_2) \parallel \frac{R_0}{1 + A\beta} \\
R_{\text{out}} &= (R_1 + R_2) \parallel \frac{R_0}{1 + A\beta} \\
& (R_1 + R_2) \gg \left. \frac{R_0}{1 + A\beta} \right|_{A \to \infty} \\
R_{\text{out}} &\approx \left. \frac{R_0}{1 + A\beta} \right|_{A \to \infty} = 0
\end{aligned}
$$

Non-inverting R_{in} Calculations

$$
\begin{aligned}
v_{id} &= v_{\text{test}} - v_A \\
i_{\text{test}} &= \frac{v_{\text{test}} - v_A}{R_i} \\
\frac{V_{\text{out}} - v_A}{R_2} &= \frac{v_A}{R_1} \\
v_A &= \frac{R_1}{R_1 + R_2} V_{\text{out}} = \beta V_{\text{out}} \\
&= A\beta V_{id} \\
&= A\beta (v_{\text{test}} - v_A) \\
v_A + A\beta v_A &= A\beta v_{\text{test}} \\
v_A (1 + A\beta) &= A\beta v_{\text{test}} \\
v_A &= \frac{A\beta}{1 + A\beta} v_{\text{test}} \\
i_{\text{test}} &= \frac{v_{\text{test}} - v_A}{R_i} \\
&= \frac{v_{\text{test}} - \frac{A\beta}{1+A\beta} v_{\text{test}}}{R_i} \\
&= \frac{v_{\text{test}} (1 - \frac{A\beta}{1+A\beta})}{R_i} \\
&= \frac{v_{\text{test}} \frac{1}{1+A\beta}}{R_i} \\
&= \frac{v_{\text{test}}}{R_i (1 + A\beta)}
\end{aligned}
$$

$$
R \;=\; v_{\text{test}} \;=\; R_i(1 + A\beta)\Big| \;=\; \infty
$$

Inverting Summary

$$
\begin{aligned}
\frac{V_{\text{out}}}{V_{\text{in}}} &= \\
R_{\text{out}} &= (R_1 + R_2) \parallel \left. \frac{R_0}{1 + A\beta} \right|_{A\to\infty} \approx 0 \\
R_{\text{in}} &= R_i(1 + A\beta) \Big|_{A\to\infty} = \infty
\end{aligned}
$$

Non-inverting Summary

$$
\begin{aligned}
\frac{V_{\text{out}}}{V_{\text{in}}} &= \frac{A_V}{\frac{R_{\text{out}} + R_S}{R_S} + \beta A_V} \\
R_{\text{out}} &= (R_1 + R_2) \parallel \left. \frac{R_0}{1 + A\beta} \right|_{A\to\infty} \approx 0 \\
R_{\text{in}} &= \infty
\end{aligned}
$$

In [ ]:

Problem 2: Opamp circuit transient response

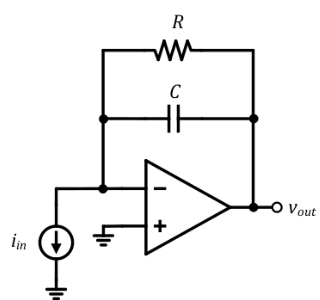


Figure 2a. Current-input integrator

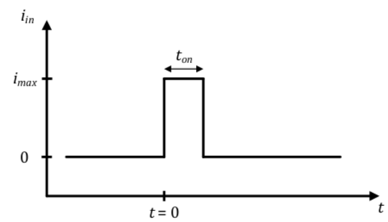


Figure 2b. Input current pulse

For the following, assume ideal opamp behavior.

a) (2.5 points) Determine an expression for the transfer function v_{out}/i_{in}.
b) (5 points) Determine an expression for the transient response of the circuit. What is the value of v_{out} (in terms of R, C, i_{max}, and t_{on}) at time $t = t_{on}$?

Bonus (2 points): Design the circuit (i.e. determine R and C) to function as an integrator, such that $v_{out}(t_{on}) = i_{max}/C$ with less than 0.1% error. Use $i_{max} = 10\mu A$ and ensure v_{out} doesn't exceed a bipolar supply voltage of $\pm 2.5V$. Verify your design in Ltspice.

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	t^p, $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^{\infty} F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

Part A

$$\left.\frac{V_{\text{out}}-V^-}{R \parallel C}\right|_{V^-=0} = i_{\text{in}}$$

$$\frac{V_{\text{out}}}{i_{\text{in}}} = R \parallel C$$

$$\frac{V_{\text{out}}}{i_{\text{in}}} = \frac{R}{1+sRC}$$

$$\frac{V_{\text{out}}}{i_{\text{in}}} = \frac{R}{1+s\tau}, \tau = RC$$

Part B

$$
\begin{aligned}
\mathcal{L}\{\frac{V_{\text{out}}}{i_{\text{in}}}\} &= \frac{R}{1+s\tau} \\
&= R\frac{1}{\frac{\tau+s}{\tau}} \\
&= R\frac{\tau}{\tau+s} \\
&= R\tau \cdot \frac{1}{\tau+s} \\
\mathcal{L}^{-1}\{\frac{V_{\text{out}}}{i_{\text{in}}}\} &= R\tau \cdot e^{-\tau t} \\
i_{\text{in}} &= i_{\max}(u(t)-u(t-t_{\text{on}})) \\
\mathcal{L}\{i_{\text{in}}\} &= i_{\max} \cdot \frac{1}{s}(1-e^{-t_{\text{on}}s}) \\
V_{\text{out}} &= \mathcal{L}\{\frac{V_{\text{out}}}{i_{\text{in}}}\} \cdot \mathcal{L}\{i_{\text{in}}\} \\
&= R\tau i_{\max} \cdot \frac{1}{s}(1-e^{-t_{\text{on}}s})\frac{1}{\tau+s} \\
\mathcal{L}^{-1}\{V_{\text{out}}\} &= R\tau i_{\max} \cdot (u(t)-u(t-t_{\text{on}})) \cdot e^{-\tau t} \\
\mathcal{L}^{-1}\{V_{\text{out}}\}\Big|_{t=t_{\text{on}}} &= R\tau i_{\max} e^{-\tau t}, \tau = RC
\end{aligned}
$$

In [ ]:

Problem 3. Difference amplifier

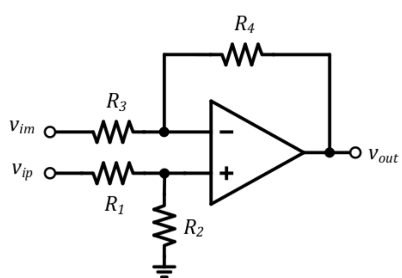


Figure 3. Difference amplifier

For the following, the opamp has a DC gain (A_0) of 100 dB and a unity-gain bandwidth (f_T) of 10MHz but is otherwise ideal ($R_{in} = \infty$ and $R_{out} = 0$). $R_1 = R_2 = R_3 = R_4 = 10\text{k}\Omega$.

a) (2.5 points) Sketch the Bode magnitude and use the graph to approximate the 3dB bandwidth. Sketch the Bode phase plot.
b) (5 points) Calculate the DC gain and 3dB bandwidth of the closed-loop transfer function $v_{out}/(v_{ip} - v_{im})$. Sketch the Bode magnitude and phase of the closed-loop transfer function.
c) (5 points) What is the resistance "looking into" each input (v_{im} and v_{ip})?
d) (5 points) Check your answers to Parts b and c in Ltspice using the Analog Devices opamp model for the AD8691.

Reference: https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-071j-introduction-to-electronics-signals-and-measurement-spring-2006/lecture-notes/23_op_amps2.pdf (https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-071j-introduction-to-electronics-signals-and-measurement-spring-2006/lecture-notes/23_op_amps2.pdf)

Part A

$$\begin{aligned}
V_{\text{out2}} &= V_{\text{ip}} \frac{R_2}{R_1 + R_2}(1 + \frac{R_4}{R_3}) \\
V_{\text{out1}} &= -V_{\text{im}} \frac{R_4}{R_3} \\
V_{\text{out}} &= V_{\text{out2}} + V_{\text{out1}} \\
&= V_{\text{ip}} \frac{R_2}{R_1 + R_2}(1 + \frac{R_4}{R_3}) - V_{\text{im}} \frac{R_4}{R_3} \\
\text{Note:} & \quad \text{weight of each signal must be the same} \\
& \quad \frac{R_2}{R_1 + R_2}(1 + \frac{R_4}{R_3}) = \frac{R_4}{R_3} \rightarrow \frac{R_2}{R_1} = \frac{R_4}{R_3} \\
V_{\text{out}} &= \frac{R_2}{R_1}(V_{\text{ip}} - V_{\text{ipm}})
\end{aligned}$$

Find β

$$\begin{aligned}
\frac{V_{\text{out}}}{V_{\text{in}}} &= \left.\frac{A_{OL}}{1 + \beta A_{OL}}\right|_{A_{OL} \rightarrow \infty} \\
&\approx \frac{1}{\beta} = \left.\frac{R_2}{R_1}\right|_{R_2 = 10K, R_1 = 10K} = 1
\end{aligned}$$

Frequency Response

$$A_{OL}(s) = \frac{A_0}{1 + s\tau}$$

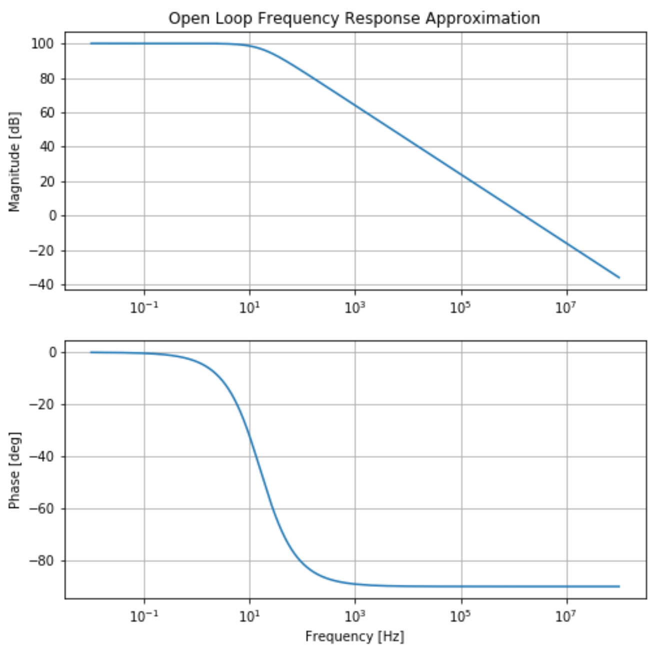
$$f_{\text{3dB,OL}} = \frac{1}{\tau} = \frac{f_T}{A_0} = \frac{10 \cdot 10^6}{10^5} = 100\text{Hz}$$

In [14]:

```
f1 = np.linspace(1e-2,1e3,100000)
f2 = np.linspace(1e3,1e8,100000)
f = np.concatenate((f1,f2))
w = 2*np.pi*f
s = 1j*w
A0 = 1e5
beta = 1
fT = 10e6
tau = A0/fT
H = A0/(1+s*tau)

#Find 3dB
mag = 20*np.log10(abs(H))
x0 = np.where(mag<=(max(mag)-3))[0][0]
label0 = "{:.2f}".format(f[x0])
x1 = np.where(mag<=0)[0][0]
label1 = "{:.2e}".format(f[x1])
#print(f"3dB frequency at {label0}")
#print(f"fT frequency at {label1}")

#Plot
fig, axs = plt.subplots(2,figsize=(8,8))
axs[0].set_title('Open Loop Frequency Response Approximation')
axs[0].semilogx(f, 20*np.log10(abs(H)))
#axs[0].scatter(f[x0], mag[x0],label=f"3dB: {label0}Hz")
#axs[0].scatter(f[x1], mag[x1],label=f"fT: {label1}Hz")
axs[0].set_ylabel('Magnitude [dB]')
axs[0].grid()
#axs[0].legend()
axs[1].semilogx(f, np.angle(H,deg=True))
axs[1].set_ylabel('Phase [deg]')
axs[1].set_xlabel('Frequency [Hz]')
axs[1].grid()
#axs[1].legend()
plt.show();
```



Part B

Frequency Response

$$A_{CL}(s) = \frac{A_0}{1 + s\tau + \beta A_0}$$

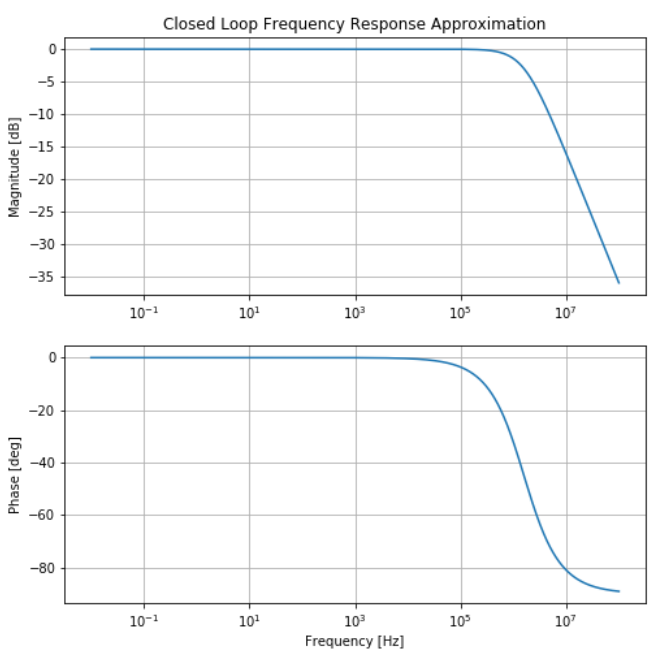
$$f_{\text{3dB,CL}} = \frac{\beta A_0}{\tau} = \beta f_T$$

$$\beta f_T \Big|_{\beta=1} = 10 \cdot 10^6 = 10^7 \text{Hz}$$

In [12]:
```
f1 = np.linspace(1e-2,1e3,100000)
f2 = np.linspace(1e3,1e8,100000)
f = np.concatenate((f1,f2))
s = 2j*np.pi*f
A0 = 1e5
beta = 1
fT = 10e6
tau = A0/fT
H = A0/(1+s*tau + beta*A0)

#Find 3dB
mag = 20*np.log10(abs(H))
x0 = np.where(mag<=(max(mag)-3))[0][0]
label = "{:.2e}".format(f[x0])
#print(f"3dB frequency at {label}")

#Plot
fig, axs = plt.subplots(2,figsize=(8,8))
axs[0].set_title('Closed Loop Frequency Response Approximation')
axs[0].semilogx(f, 20*np.log10(abs(H)))
#//axs[0].scatter(f[x0], mag[x0],label=f"3dB: {label}Hz")
axs[0].set_ylabel('Magnitude [dB]')
axs[0].grid()
#axs[0].legend()
axs[1].semilogx(f, np.angle(H,deg=True))
axs[1].set_ylabel('Phase [deg]')
axs[1].set_xlabel('Frequency [Hz]')
axs[1].grid()
#axs[1].legend()
plt.show();
```



Part C

$$R_{im} = R_3 + (R_i \parallel \frac{R_4}{1+A}) \approx R_3 = 10K\Omega$$

$$R_{ip} = R_1 + (R_2 \parallel \infty) = R_1 + R_2 = 20K\Omega$$

Part D

```
In [ ]:
```