EE 538 Spring 2020
Analog Circuits for Sensor Systems
University of Washington Electrical & Computer Engineering

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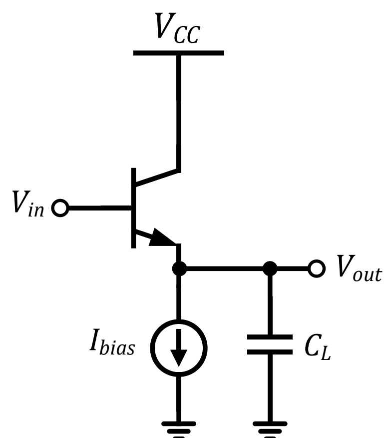
Practice Midterm

Please show your work.

Problem 1: Emitter-follower analysis



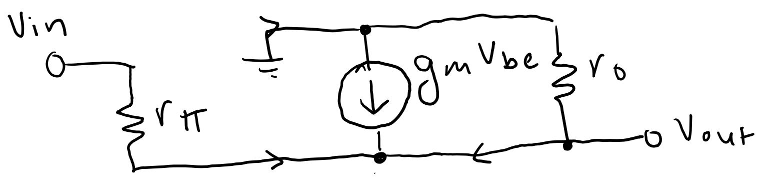
For the following, V_{CC} = 5V, V_{IN} = 1V, I_{bias} = 1mA, C_L = 30pF, and $I_S = 10^{-16}$A.

a) Calculate the DC value of V_{out}. For this step, assume $V_A = \infty$.
b) Calculate the small-signal DC gain (v_{out}/v_{in}) if V_A = 100V.
c) Calculate the small-signal output resistance of the emitter follower if V_A = 100V.
d) Calculate the transit frequency (f_T) of the emitter follower.
e) Suppose we replace the BJT with a MOSFET with $V_{GS} - V_{TH}$ = 0.25V to construct a source follower. Ignoring r_o of the MOSFET, what is the new transit frequency?

a. $V_{BE} = V_T \ln(I_C / I_S)$

$$V_{out,DC} = V_{in,DC} - V_{BE}$$
$$= 1V - V_T \ln(I_C / I_S) \approx \boxed{225mV}$$

b. Small-signal model:



$$\boxed{\begin{aligned} r_\pi &= \beta / g_m \\ r_o &= V_A / I_C \\ g_m &= I_C / V_T \end{aligned}}$$

$$\frac{V_{in} - V_{out}}{r_\pi} + g_m (V_{in} - V_{out}) - V_{out} / r_o = 0$$

$$V_{in}\left(\frac{1}{r_\pi} + g_m\right) = V_{out}\left(\frac{1}{r_\pi} + g_m + \frac{1}{r_o}\right)$$

$$r_\pi = \beta / g_m \Rightarrow g_m V_{in}\left(1 + \frac{1}{\beta}\right) = V_{out}\left(g_m\left(1 + \frac{1}{\beta}\right) + \frac{1}{r_o}\right)$$

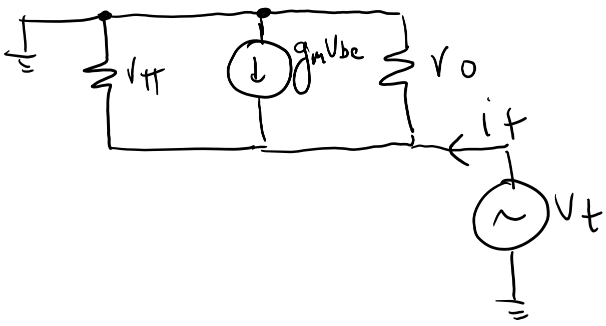
$$\frac{V_{out}}{V_{in}} = \frac{g_m\left(1 + \frac{1}{\beta}\right)}{g_m\left(1 + \frac{1}{\beta}\right) + 1/r_o}$$

$$= \frac{g_m\left(1 + \frac{1}{\beta}\right) r_o}{g_m\left(1 + \frac{1}{\beta}\right) r_o + 1}$$

assuming β is "large",

$$\boxed{A_{v,DC} = \frac{g_m r_o}{g_m r_o + 1} = 0.997 \text{ V/V}}$$

c.



$$i_t = \frac{v_t}{r_o} + g_m v_t + \frac{v_t}{r_\pi}$$

$$\boxed{Z_{out} = \frac{v_t}{i_t} = \frac{1}{\frac{1}{r_o} + g_m + \frac{1}{r_\pi}} \approx \frac{1}{g_m}}$$

d. $$A_v(s) = \frac{g_m r_o || C_L}{g_m r_o || C_L + 1} = \frac{g_m r_o}{g_m r_o + sC_L r_o + 1}$$

f_T : $$g_m r_o + s_o C_L r_o + 1 = 0$$

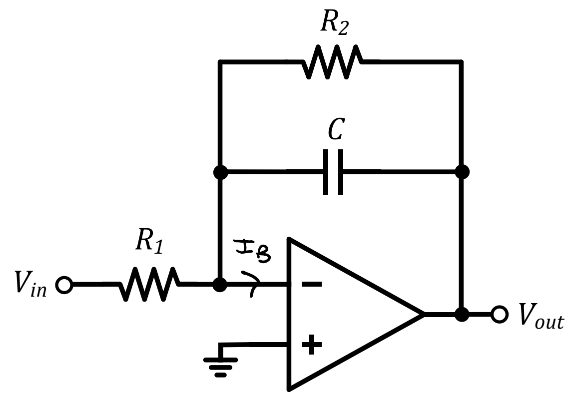
$$s_o C_L r_o = -1 - g_m r_o$$

$$s_o = \frac{-1 - g_m r_o}{C_L r_o} \approx -\frac{g_m}{C_L}$$

$$\boxed{f_T = \frac{|s_o|}{2\pi} = \frac{g_m}{2\pi C_L} \approx 205 \text{ MHz}}$$

e. $g_m = \frac{2I_D}{V_{ov}} = 8 \text{ mS}$, $f_T = \frac{g_m}{2\pi C_L} \approx \boxed{42 \text{ MHz}}$

Problem 2: Filter analysis and design



Assume the opamp has infinite gain and bandwidth, with input bias current I_B = 1nA.

a) Ignoring bias current, derive an expression for the closed-loop transfer function of the filter.
b) Design the filter (choose R_1, R_2, and C) to have a DC gain of 20dB and a 0.1% settling time of 10μs.
c) Still ignoring bias current, derive an expression for the closed-loop step response. Sketch the response for an input step of 0 to 1V and label all relevant times/voltages.
d) Re-sketch the closed-loop step response, accounting for the effect of input bias current.
e) Modify the design to reduce/eliminate the effect of the input bias current.

a. $$\frac{V_{out}}{V_{in}} = \frac{-R_2 \| \frac{1}{sC}}{R_1} = \boxed{-\frac{R_2}{R_1} \cdot \frac{1}{sCR_2 + 1}}$$

b. $$\tau_{CL} = CR_2 ; \quad \ln(0.001) \times \tau_{CL} = 10\mu s$$
$$\tau_{CL} = 10\mu s / 6.9$$
$$\approx 1.45\mu s = CR_2$$

let $\boxed{C = 10pF}$ => $\boxed{R_2 = 145k\Omega}$

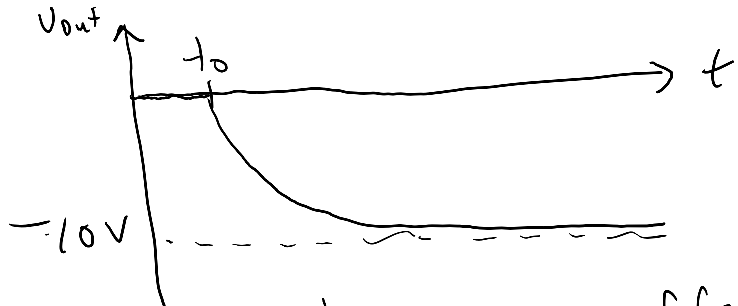
$R_1 = R_2 / 10 = \boxed{14.5k\Omega}$

Many combinations possible

c. $V_{in} = V_1 u(t)$

$$V_{out} = -V_1 u(t)\left(1 - e^{-t/\tau_{cc}}\right) \cdot \frac{R_2}{R_1}$$

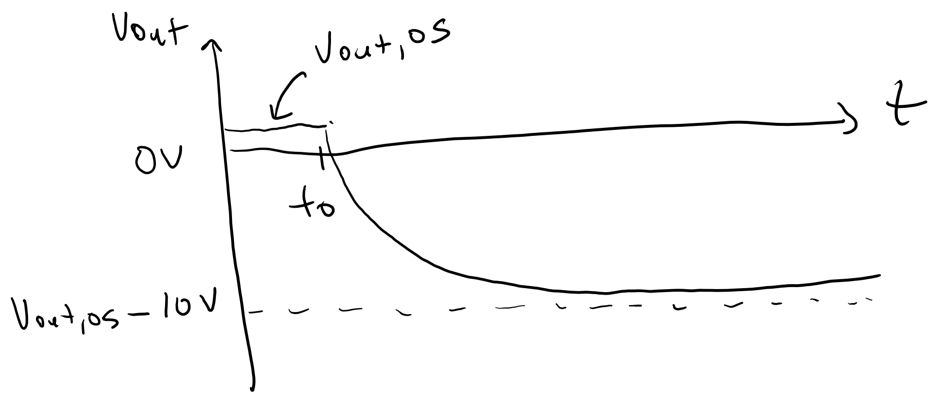
Step @ $t = t_0$



d. With bias current, we have an offset

$V_{out,os} = 0V + I_B \cdot R_2$

for $R_2 = 1nA$, $V_{out,os} = 145\mu V$



e. We could decrease R_1, R_2 to reduce the offset. E.g. $R_2 = 14.5k\Omega$, $R_1 = 1.45k\Omega$

$C = 100pF$

This would reduce the offset to $14.5\mu V$

Problem 3. Opamp circuit design

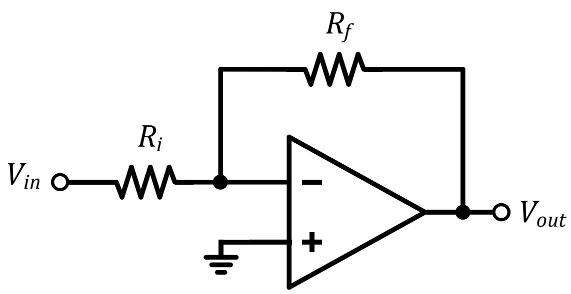


Figure 3a. Inverting amplifier

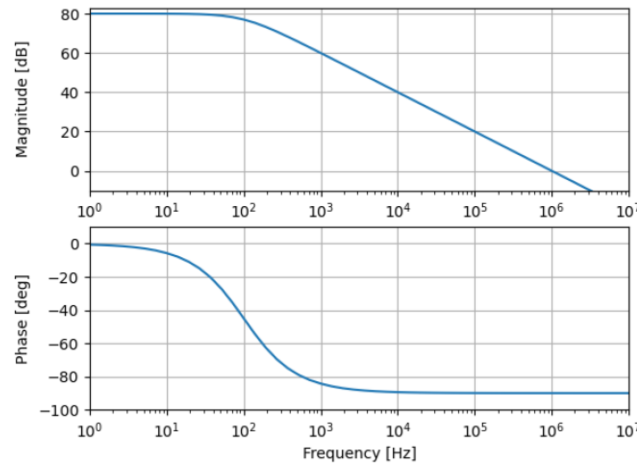


Figure 3b. Opamp open-loop frequency response

Assume ideal input/output resistances (R_{in} and R_o) for the opamp. Let $R_f = 10R_{in}$.

a) Determine the gain and the 3dB frequency of the closed loop transfer function.
b) Calculate the closed-loop gain error at DC and 100Hz.
c) Suppose you want to use this amplifier to amplify the voltage of a sensor with an equivalent source resistance of 1kΩ. Determine the values of R_i and R_f to achieve a DC gain of 10V/V and less than 0.1% input attenuation due to loading (*you can ignore finite gain for this step).*
d) Based on your answer to part c), is this a good choice of circuit for the application? How could it be improved?

a. 80dB $\Rightarrow$ $10^4\ V/V = A_0$; $\beta = R_i/(R_i + R_f) = \frac{1}{11}$

$$\frac{V_{out}}{V_{in}} = -(1-\beta)\frac{A_0}{1+\beta A_0} = \frac{-10}{11}\cdot\frac{A_0}{1+A_0/11} = \frac{-10A_0}{11+A_0}$$

$$= \boxed{-9.989\ V/V}$$

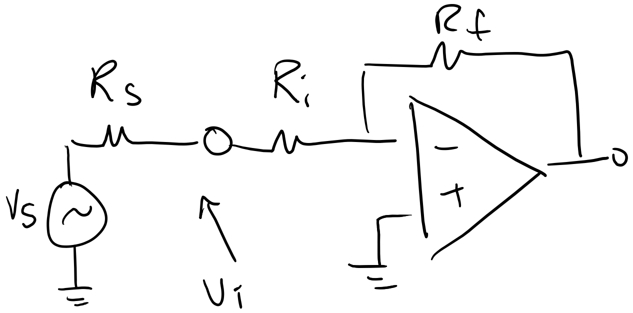
$$f_{3dB} \approx \beta f_T \approx \boxed{91\,kHz}$$

b. $A_{OL}(f=0) = 10^4\,V/V$

$A_{OL}(f=100\,Hz) = 10^4\,V/V / \sqrt{2} \approx 7071\,V/V$

DC: $\dfrac{10 - 9.989}{10} = \boxed{0.11\%}$; 100 Hz: $\dfrac{10 \cdot A_{OL}(100)}{11 + A_{OL}(100)} \approx \boxed{0.16\%}$

c. $R_{in} = R_i$; $R_f = 10 \cdot R_i$



$$V_i = \frac{R_i}{R_s + R_i} V_s = 0.999\,V_s$$

$$\frac{R_i}{R_s + R_i} = 0.999$$

$$0.001 R_i = 0.999 R_s$$

$$\boxed{R_i = 999\,k\Omega \qquad R_f = 9.99\,M\Omega}$$

d. $R_i \approx 1\,M\Omega$ is quite large. The input bias current of the opamp flowing through R_i will result in a large offset voltage. R_i will also generate a substantial amount of noise.

$\Rightarrow$ Use a non-inverting stage instead.