Last time: Laplace transform for circuit analysis.

1. Find the Laplace transforms of an important functions and build a table for future reference.	ALMOST DONE
2. Develop technique to go from s-domain back to time-domain.	EXAMPLES WERE CONSIDERED
3. Develop circuit analysis techniques in s-domain.	STILL WORKING

Let's discuss again how to get back to time domain

Circuit in time-domain
V(t) - unknown

⇩

Circuit in s-domain
V(s) - found

⇩

Use partial fraction expansion to convert V(s) to V(t)

Conversion of the circuit from time- to s-domain is equivalent to application of the Laplace transform to differential equation describing the circuit in time-domain.

We have already noticed that the voltages and currents in s-domain are represented by ratios of polynomials – rational functions.

Partial fraction expansion of proper rational functions.

$$\tilde{V}(s) = Const \cdot \frac{s^m + a_{m-1}\cdot s^{m-1} + \ldots + a_1 \cdot s + a_0}{s^n + b_{n-1}\cdot s^{n-1} + \ldots + b_1 \cdot s + b_0} \quad ; m < n$$

1. Find the roots of the denominator (poles) and represent V(s) as:

$$\tilde{V}(s) = Const \cdot \frac{s^m + a_{m-1}\cdot s^{m-1} + \ldots + a_1 \cdot s + a_0}{(s-p_1)^{k_1} \ldots \cdot (s-p_\ell)^{k_\ell}} \quad ; n = \sum_{i=1}^{\ell} k_i$$

2. Find the coefficients in the partial fraction expansion for each pole:

$$\tilde{V}(s) = \frac{A_{k1}}{(s-p_1)^{k_1}} + \frac{A_{k1-1}}{(s-p_1)^{k_1-1}} + \ldots + \frac{A_1}{s-p_1} + \ldots$$

$$A_{k1} = \tilde{V}(s)(s-p_1)^{k_1}\Big|_{s=p_1}$$

$$A_{k1-1} = ? \qquad A_{k1-1} = \frac{d}{ds}\left(\tilde{V}(s)\cdot(s-p_1)^{k_1}\right)\Big/_{s=p_1}, \ etc.$$

3. Find the time-domain function corresponding to each term in partial fraction expansion:

$$L\left[e^{-\alpha t}\cdot h(t)\right] = \frac{1}{s+\alpha} \quad \textit{and} \quad L\left[\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}\right] = \frac{1}{(s+\alpha)^n}$$

Complex roots always come in conjugate pairs.

$p_2 = p_1^* \quad ; \; p_1 = \alpha + j\beta$

$$\tilde{V}(s) = \frac{1}{(s-p_1)(s-p_2)} = \frac{A}{s-p_1} + \frac{B}{s-p_2}$$

$$A = \tilde{V}(s)\cdot(s-p_1) - B\cdot\frac{s-p_1}{s-p_2} = \frac{1}{s-p_2} - B\cdot\frac{s-p_1}{s-p_2} = \frac{1}{p_1-p_2}$$

OR

$$\left.\begin{aligned} A &= \tilde{V}(s)(s-p_1)\Big|_{s=p_1} = \frac{1}{p_1-p_2} \\ B &= \tilde{V}(s)(s-p_2)\Big|_{s=p_2} = \frac{1}{p_2-p_1} = A^* \end{aligned}\right\} B = A^*$$

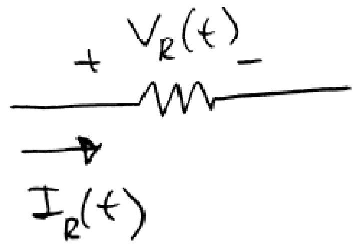
$$V(t) = A e^{p_1 t} + A^* e^{p_2 t} = A e^{\alpha t} e^{j\beta t} + A^* e^{\alpha t} e^{-j\beta t}$$

$$A = |A| e^{j\varphi} \quad ; \; A^* = |A| e^{-j\varphi}$$

$$V(t) = |A| e^{\alpha t}\left[e^{j(\beta t+\varphi)} + e^{-j(\beta t+\varphi)}\right] = 2|A| e^{\alpha t}\cdot\cos(\omega t+\varphi)$$

Resistor in s-domain.

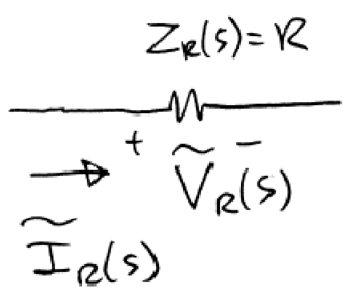
time-domain



$$I_R(t) = \frac{V_R(t)}{R}$$

$$L\left[I_R(t)\right] = L\left[\frac{V_R(t)}{R}\right] = \frac{L\left[V_R(t)\right]}{R}$$

s-domain

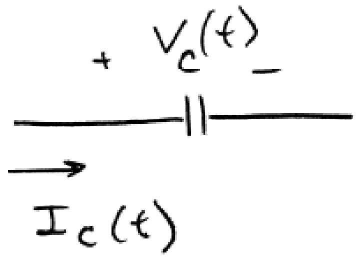


$$\tilde{I}_R(s) = \frac{\tilde{V}_R(s)}{R} = \frac{\tilde{V}_R(s)}{Z_R(s)}$$

$Z_R(s) = R$ – ***Impedance in s-domain***

Capacitor in s-domain.

time-domain

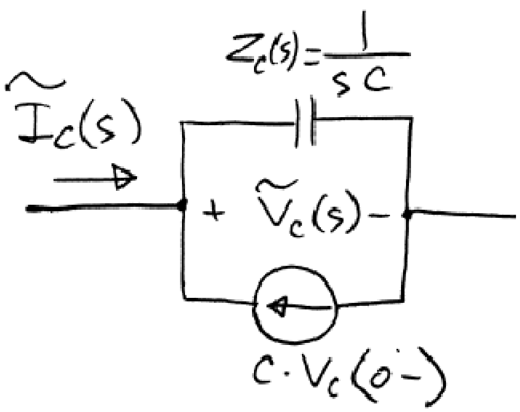


$$I_c(t) = C\,\frac{dV_c}{dt}$$

$$L\left[I_c(t)\right] = C\,L\left[\frac{d}{dt}V_c\right] = C\cdot\left(s\cdot L\left[V_c(t)\right] - V_c(0-)\right)$$

$$\tilde{I}_c(s) = (C\cdot s)\cdot\tilde{V}_c(s) - \underbrace{C\cdot V_c(0-)}_{\text{Initial condition}}$$

s-domain

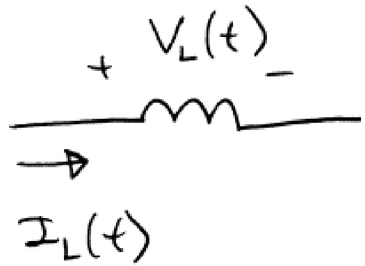


$$\tilde{I}_c(s) = \frac{\tilde{V}_c(s)}{Z_c(s)} - C\cdot V_c(0-)$$

$$Z_c(s) = \frac{1}{s\cdot C}$$ – *Impedance in s-domain*

Inductor in s-domain.

time-domain

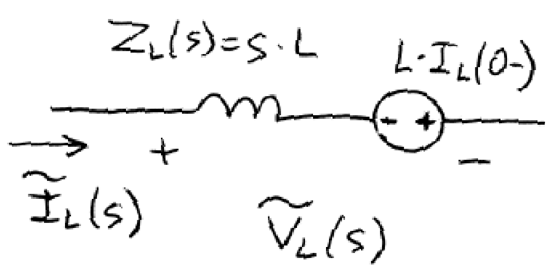


$$V_L(t) = L \cdot \frac{dI_L}{dt}$$

$$\mathcal{L}\left[V_L(t)\right] = \mathcal{L}\left[L \cdot \frac{dI_L}{dt}\right] = L \cdot \mathcal{L}\left[\frac{d}{dt} I_L(t)\right]$$

$$\tilde{V}_L(s) = L \cdot \left(s \cdot \tilde{I}_L(s) - I_L(0-)\right) = (s \cdot L) \cdot \tilde{I}_L(s) - \underbrace{L \cdot I_L(0-)}_{\textit{Initial condition}}$$

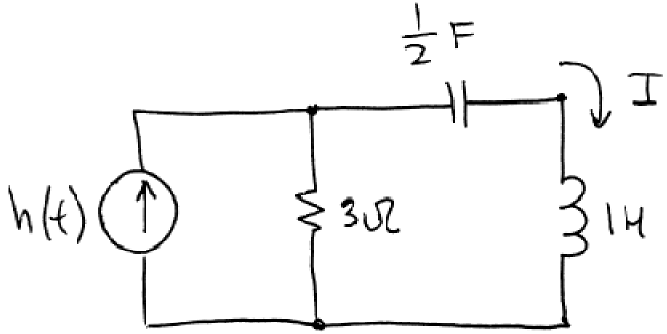
s-domain



$$\tilde{V}_L(s) = Z_L(s) \cdot \tilde{I}_L(s) - L \cdot I_L(0-)$$

$Z_L(s) = s \cdot L$ – ***Impedance in s-domain***

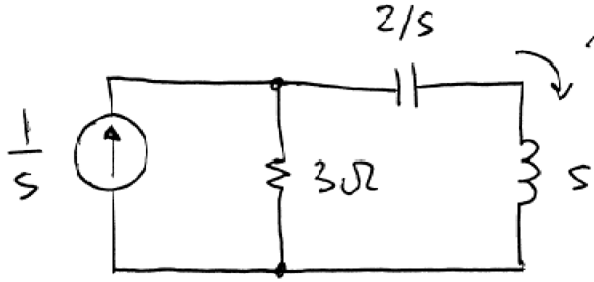
Example.



$I(t) = ?$ $\qquad L[h(t)] = \frac{1}{s}$

(ZERO INITIAL CONDITIONS)

↓ to S-DOMAIN



$$\tilde{I}(s) = \left(\frac{1}{s}\right) \cdot \frac{3}{3 + \left(\frac{2}{s} + s\right)} = \frac{3}{3s + 2 + s^2} =$$

$$= \frac{3}{s^2 + 3s + 2} = \frac{3}{(s - p_1)(s - p_2)}$$

$$p_{1,2} = -\frac{3}{2} \pm \sqrt{\frac{9}{4} - 2} = -\frac{3}{2} \pm \sqrt{\frac{1}{4}} = -\frac{3}{2} \pm \frac{1}{2} = \{-1; -2\}$$

$$\tilde{I}(s) = 3\left(\frac{A}{s+1} + \frac{B}{s+2}\right) = 3\,\frac{As + 2A + Bs + B}{(s+1)(s+2)} \Rightarrow \begin{matrix} A = -B \\ A = 1 \end{matrix}$$

$$\tilde{I}(s) = 3\left(\frac{1}{s+1} - \frac{1}{s+2}\right)$$

Example - cont.

$$\tilde{I}(s) = 3\left(\frac{1}{s+1} - \frac{1}{s+2}\right) \quad ; \quad L\left[e^{-\alpha t}\right] = \frac{1}{s+\alpha}$$

$$I(t) = 3\left(e^{-t} - e^{-2t}\right), \text{A}$$

$$V_c(t) = \frac{1}{C}\int_0^t I(\tau)d\tau = 2 \cdot 3 \cdot \left(\int_0^t e^{-\tau}d\tau - \int_0^t e^{-2\tau}d\tau\right) =$$

$$= 6 \cdot \left(e^{-\tau}\Big|_t^0 - \frac{1}{2}e^{-2t}\Big|_t^0\right) = 6 \cdot \left(1 - e^{-t} - \frac{1}{2}\left(1 - e^{-2t}\right)\right) =$$

$$= 6\left(\frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}\right) = 3 + 3 \cdot e^{-2t} - 6 \cdot e^{-t}, \text{V}$$

Example - cont.

$$\tilde{V}_c(s) = \tilde{I}(s)\cdot Z_c(s) = \frac{3}{(s+1)(s+2)}\cdot\frac{2}{s} = \frac{6}{s(s+1)(s+2)} =$$

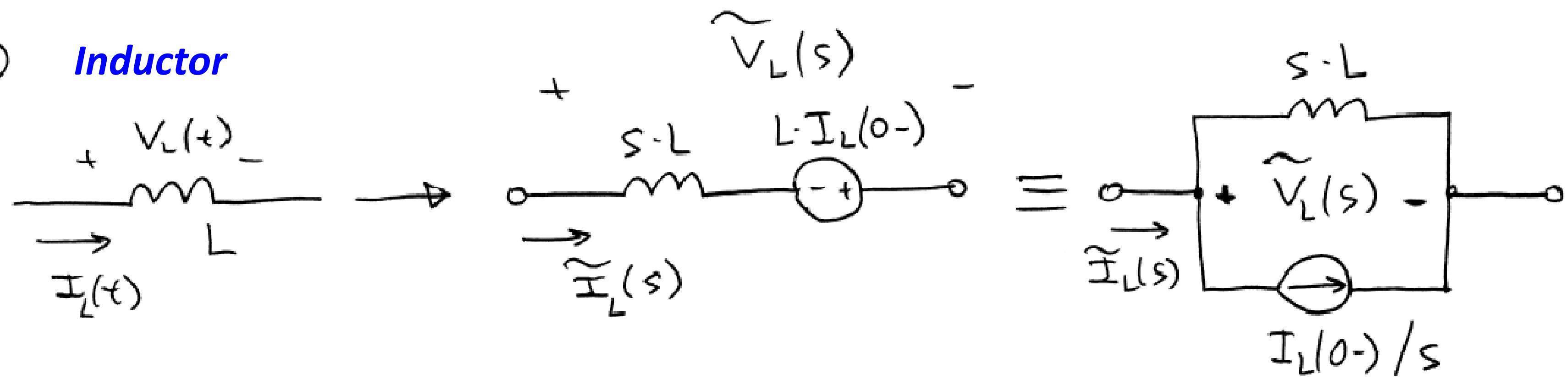
$$= 6\left(\frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}\right) \longrightarrow \left[\begin{aligned} A &= \frac{\tilde{V}_c(s)}{6}\cdot s\Big|_{s=0} = \frac{1}{2} \\ B &= \frac{\tilde{V}_c(s)}{6}(s+1)\Big|_{s=-1} = -1 \\ C &= \frac{\tilde{V}_c(s)}{6}(s+2)\Big|_{s=-2} = +\frac{1}{2} \end{aligned}\right.$$

$$V_c(t) = 6\left(A + Be^{-t} + Ce^{-2t}\right)$$

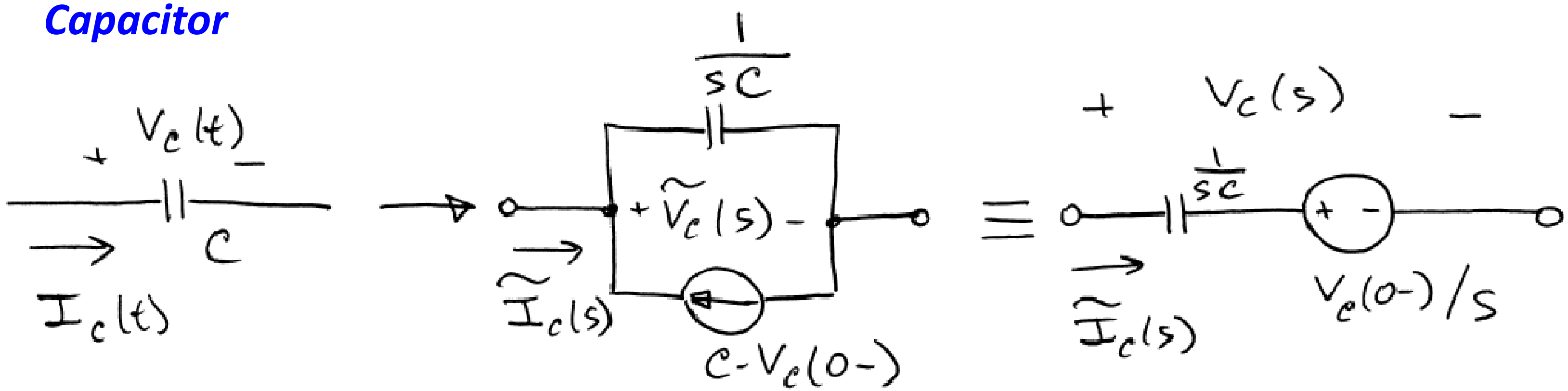
$$V_c(t) = 3 - 6\cdot e^{-t} + 3\cdot e^{-2t}, V$$

Thevenin and Norton forms for inductor and capacitor.

① *Inductor*

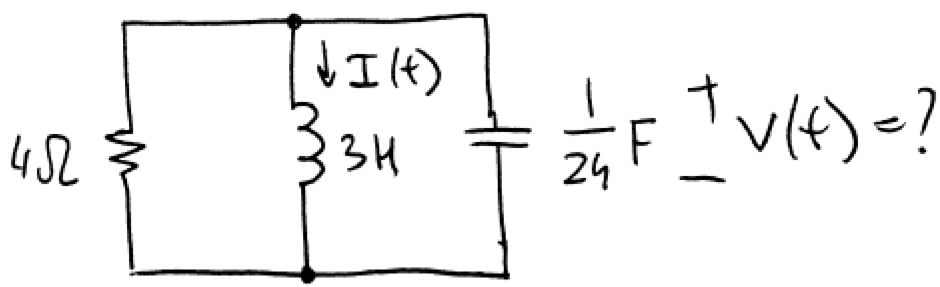


② *Capacitor*



Initial conditions are treated like independent sources in s-domain circuits

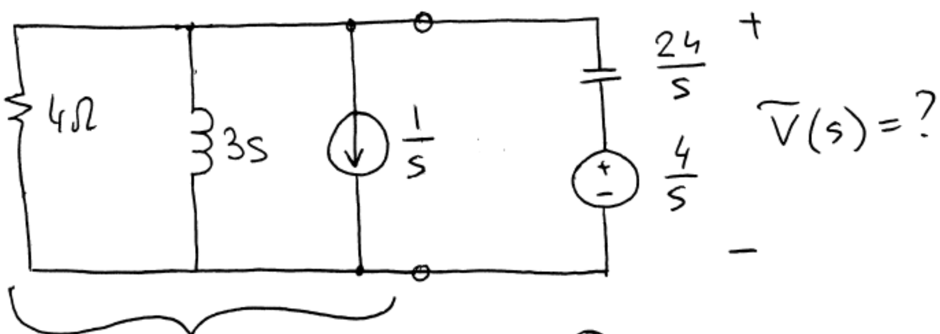
Example 1.

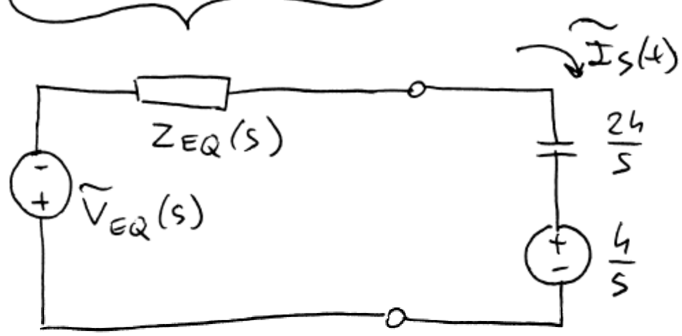


$I(0-) = 1\,A$

$V(0-) = 4\,V$

1. Construct s-domain circuit.



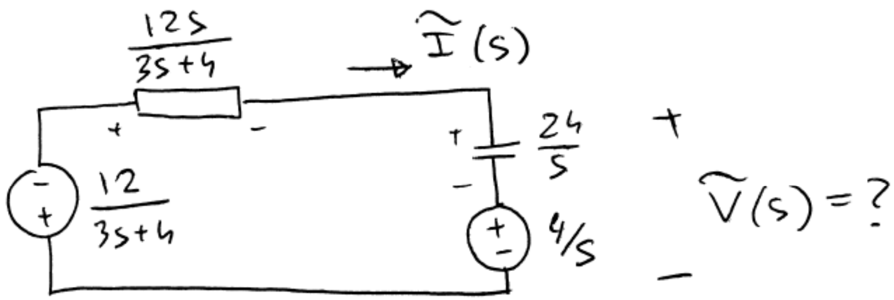


$$Z_{EQ}(s) = \frac{4\cdot 3s}{4+3s} = \frac{12s}{3s+4}$$

$$\tilde{V}_{EQ}(s) = \frac{1}{s}\cdot\frac{12s}{3s+4} = \frac{12}{3s+4}$$

Example 1 - cont.

2. Solve problem in s-domain.



$$\tilde{I}(s)\cdot\frac{12s}{3s+4} + \tilde{I}(s)\cdot\frac{24}{s} + \frac{4}{s} + \frac{12}{3s+4} = 0 \quad \leftarrow \text{s-KVL}$$

$$\tilde{I}(s) = -\frac{\frac{4}{s}+\frac{12}{3s+4}}{\frac{12s}{3s+4}+\frac{24}{s}} = -\frac{4}{12}\cdot\frac{\frac{1}{s}+\frac{3}{3s+4}}{\frac{s}{3s+4}+\frac{2}{s}} =$$

$$= -\frac{1}{3}\cdot\frac{\frac{3s+4+3s}{s(3s+4)}}{\frac{s^2+6s+8}{s(3s+4)}} = -\frac{1}{3}\cdot\frac{6s+4}{s^2+6s+8}$$

$$\tilde{V}(s) = \tilde{I}_s(s)\cdot\frac{24}{s} + \frac{4}{s} = \frac{4}{s}\left(6\cdot\tilde{I}(s)+1\right) =$$

$$= -\frac{4}{s}\left(2\,\frac{6s+4}{s^2+6s+8} - 1\right) = -\frac{4}{s}\left(\frac{12s+8-s^2-6s-8}{s^2+6s+8}\right)$$

Example 1 - cont.

3. Partial fraction expansion.

$$\tilde{V}(s) = -\frac{4}{s}\cdot\frac{12s+8-s^2-6s-8}{s^2+6s+8} = \frac{4}{s}\cdot\frac{s^2-6s}{s^2+6s+8} = \frac{4(s-6)}{s^2+6s+8}$$

Find poles: $s^2+6s+8=0$

$$s = -3 \pm \sqrt{9-8} = -3 \pm 1 = \{-2; -4\}$$

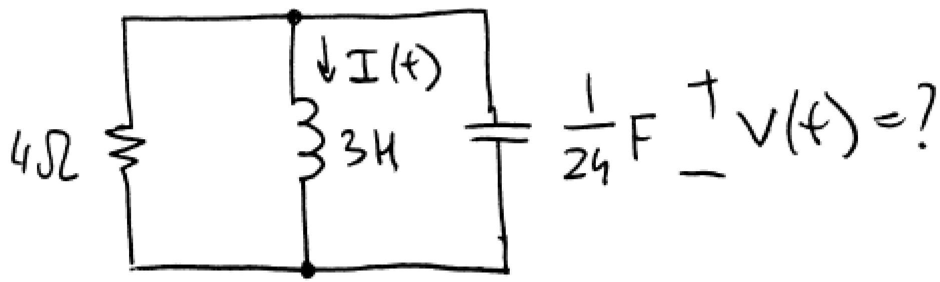
$$\frac{4(s-6)}{(s+2)(s+4)} = \frac{A}{s+2} + \frac{B}{s+4} = \tilde{V}(s)$$

$$A = \tilde{V}(s)(s+2)\Big|_{s=-2} = \frac{4(-8)}{-2+4} = -16$$

$$B = \tilde{V}(s)(s+4)\Big|_{s=-4} = \frac{4(-10)}{-4+2} = +20$$

$$\tilde{V}(s) = \frac{-16}{s+2} + \frac{20}{s+4}$$

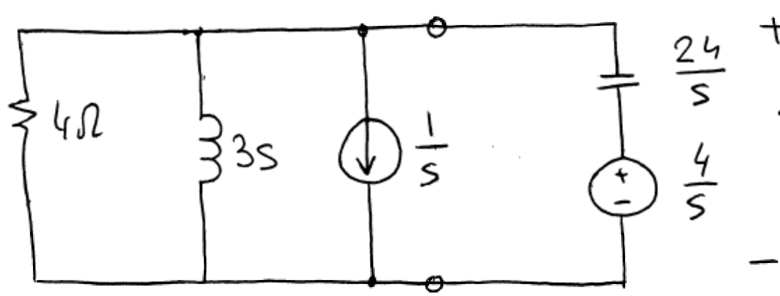
Example 1 - cont.



$I(0-) = 1\,A$

$V(0-) = 4\,V$

Solution in s-domain:



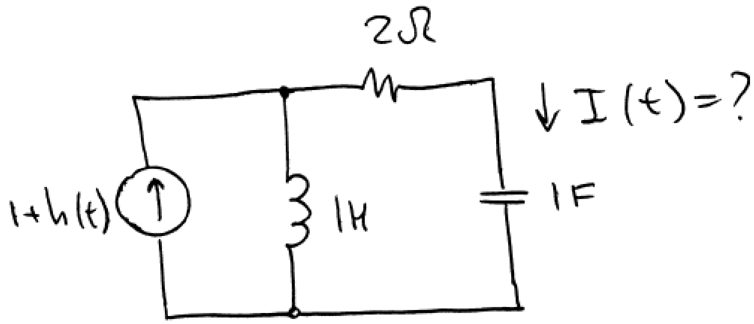
$$\tilde{V}(s) = \frac{-16}{s+2} + \frac{20}{s+4}$$

Solution in time-domain: $v(t) = -16e^{-2t} + 20e^{-4t}$

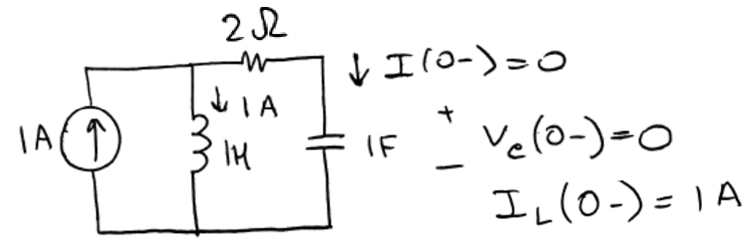
$t = 0 \Rightarrow v(0) = 4$

$t \to \infty \Rightarrow v \longrightarrow 0$ ***Circuit contains resistive element and no independent sources.***

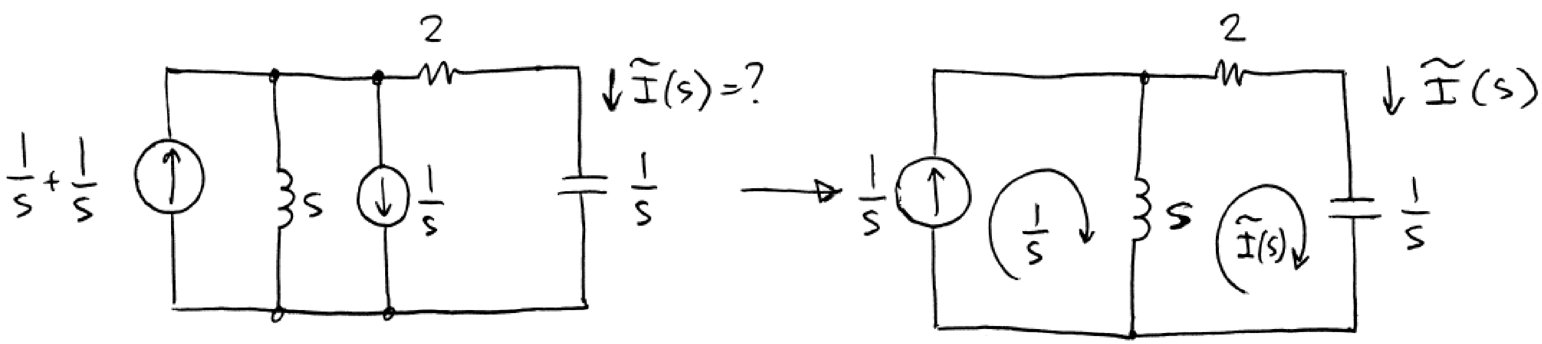
Example 2.



1. Find initial conditions: $t = 0-$

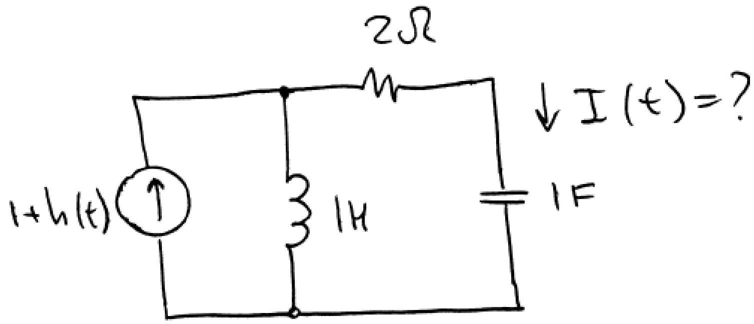


2. Construct s-domain circuit: $t > 0$

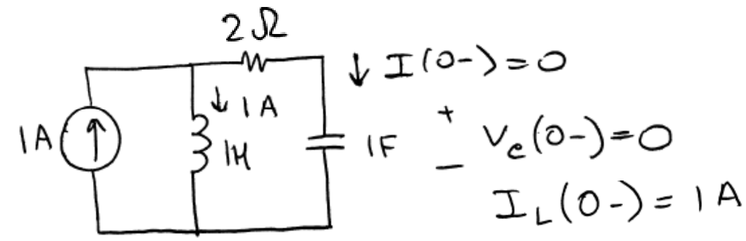


Use mesh analysis – KVL: $s\left(\tilde{I}(s) - \frac{1}{s}\right) + 2 \cdot \tilde{I}(s) + \tilde{I}(s)\frac{1}{s} = 0$

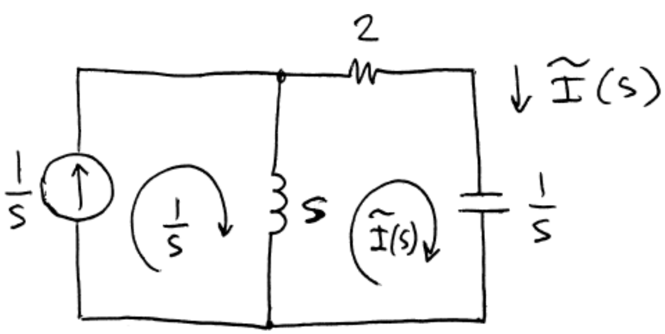
Example 2 - cont.



1. Find initial conditions: $t = 0-$



3. Solve in s-domain:

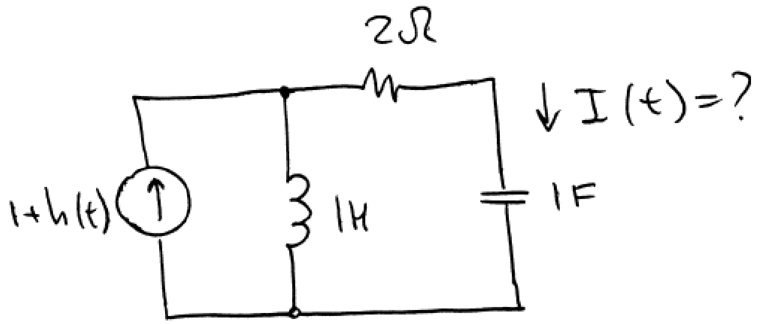


$$s\left(\tilde{I}(s) - \frac{1}{s}\right) + 2\cdot\tilde{I}(s) + \frac{1}{s}\cdot\tilde{I}(s) = 0$$

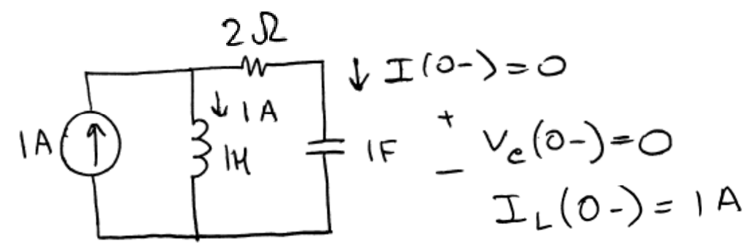
$$\tilde{I}(s)\left(s + 2 + \frac{1}{s}\right) = 1 \Rightarrow \tilde{I}(s) = \frac{1}{s + 2 + \frac{1}{s}}$$

$$\tilde{I}(s) = \frac{s}{s^2 + 2s + 1} = \frac{s}{(s+1)^2}$$

Example 2 - cont.



1. Find initial conditions: $t = 0-$



4. Convert to time-domain:

$$\tilde{I}(s) = \frac{s}{(s+1)^2} = \frac{A}{(s+1)^2} + \frac{B}{s+1}$$

$$\begin{cases} A = \left(I(s)\cdot(s+1)^2\right)\Big|_{s=-1} = -1 \\ B = \frac{d}{ds}\left(I(s)(s+1)^2\right)\Big|_{s=-1} = 1 \end{cases}$$

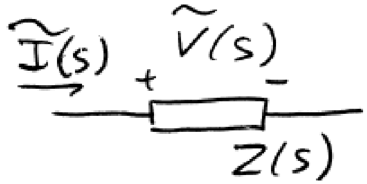
$$\tilde{I}(s) = -\frac{1}{(s+1)^2} + \frac{1}{s+1}$$

$$I(t) = -t\cdot e^{-t} + e^{-t} = (1-t)e^{-t}\ , A$$

$$I(0+) = 1\,A \quad \& \quad I(\infty) = 0$$

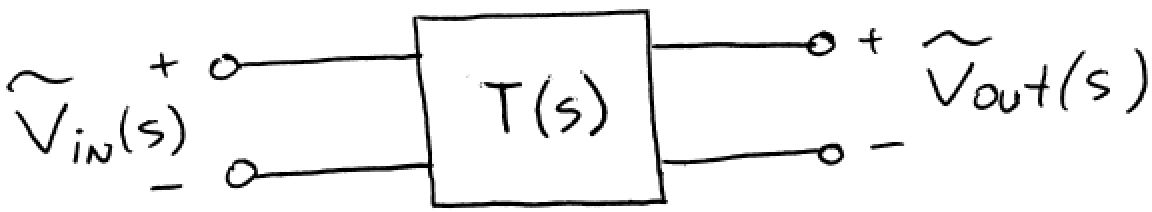
Transfer function.

We already considered:



where $Z(s) = \left.\dfrac{\tilde{V}(s)}{\tilde{I}(s)}\right|_{\text{ZERO IC}}$ *- impedance in s-domain.*

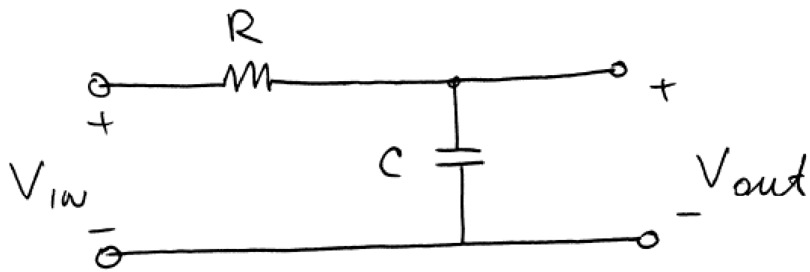
Now let's consider somewhat generalized case:



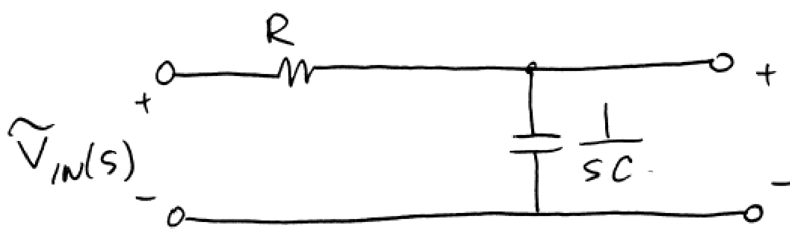
Transfer function: $T(s) = \left.\dfrac{\tilde{V}_{out}(s)}{\tilde{V}_{in}(s)}\right|_{\text{ZERO IC}}$

$$\boxed{\tilde{V}_{out}(s) = T(s)\cdot\tilde{V}_{in}(s)}$$

Transfer function of RC integrator.



$\downarrow$ S-DOMAIN + [IC = 0]

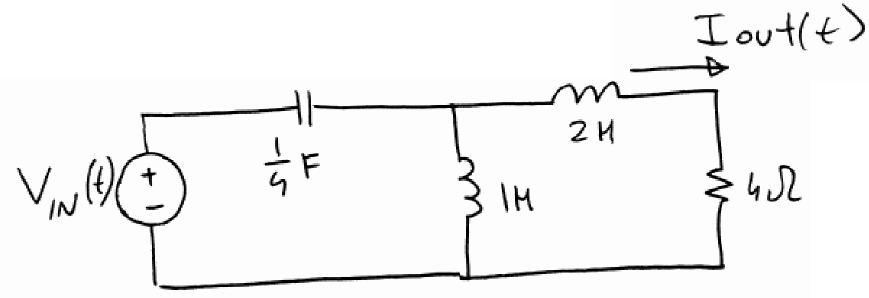


$$\tilde{V}_{out}(s) = \tilde{V}_{IN}(s)\,\frac{\frac{1}{sC}}{R+\frac{1}{sC}}$$

$$T(s) = \frac{\tilde{V}_{out}(s)}{\tilde{V}_{in}(s)} = \frac{1}{1+s\cdot RC}$$

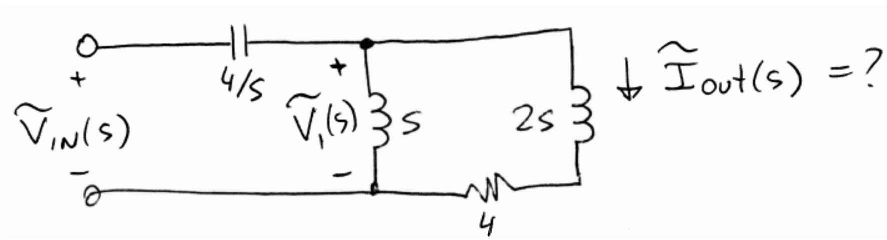
T-function has single negative real pole: $s = -\frac{1}{RC}$

Example.



T-function does not necessarily means ratio of voltages.

$$T(s) = \frac{\tilde{I}_{out}(s)}{\tilde{V}_{IN}(s)} = ?$$



$$\tilde{V}_1(s) = \tilde{V}_{IN}(s)\,\frac{s \,||\, (2s+4)}{\frac{4}{s} + s \,||\, (2s+4)} =$$

$$= \tilde{V}_{IN}(s) \cdot \frac{s^2(s+2)}{s^2(s+2) + 2(3s+4)}$$

$$\tilde{I}_{out}(s) = \tilde{V}_1(s) \,/ (2s+4) = \tilde{V}_{IN}(s)\,\frac{s^2/2}{s^3 + 2s^2 + 6s + 8}$$

$$T(s) = \frac{\tilde{I}_{out}(s)}{\tilde{V}_{IN}(s)} = \frac{s^2}{2(s^3 + 2s^2 + 6s + 8)}$$

T-function has two "zeros" s = 0 and three "poles".

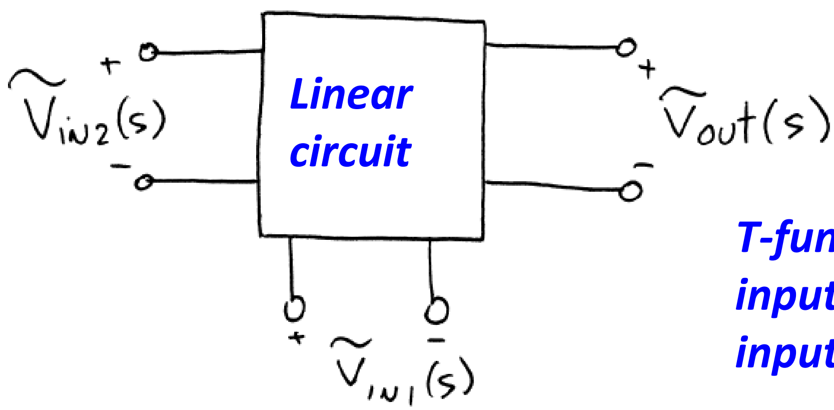
Circuits with multiple inputs.

For single input circuit:

$$\tilde{V}_{out}(s) = T(s) \cdot \tilde{V}_{IN}(s)$$

$$\tilde{V}_{IN}(s) = 0 \Rightarrow \tilde{V}_{out}(s) = 0$$

But if circuit has independent sources inside then: $\tilde{V}_{out}(s) \neq 0$



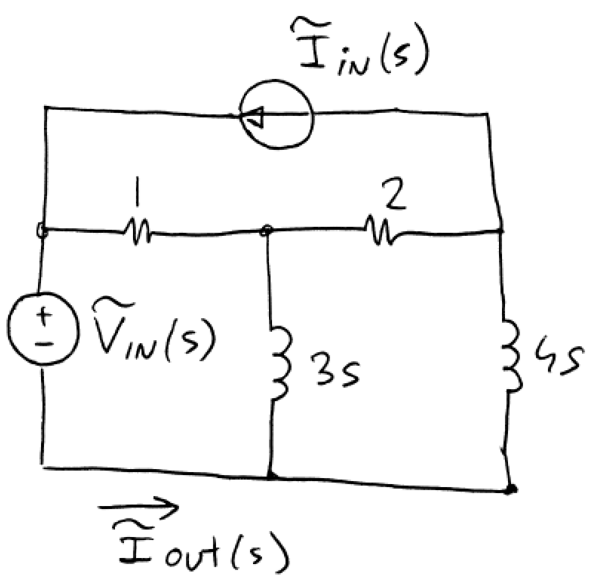
Each source is input.

T-function can be found for each input independently with other inputs "killed".

$$\tilde{V}_{out}(s) = T_1(s)\Big|_{V_{IN2}=0} \cdot \tilde{V}_{IN1}(s) + T_2(s)\Big|_{V_{IN1}=0} \cdot \tilde{V}_{IN2}(s)$$

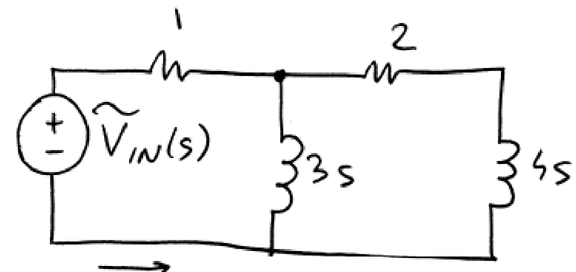
**Always use zero initial conditions to compute T-function.*

Example.



$$\tilde{I}_{out}(s) = T_1(s)\cdot\tilde{V}_{IN}(s) + T_2(s)\cdot\tilde{I}_{IN}(s)$$

① $T_1(s) = ?$ $\Rightarrow$ $\tilde{I}_{IN}(s) \to 0$

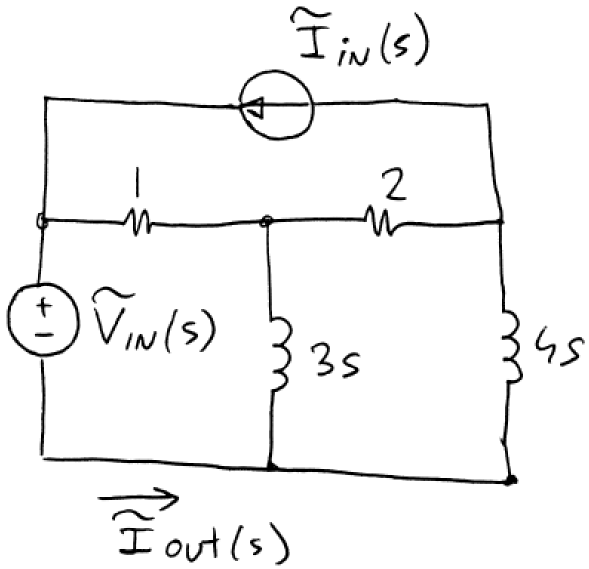


$$I'_{out}(s) = -\frac{\tilde{V}_{IN}(s)}{1 + 3s\,//\,(4s+2)}$$

$$T_1(s) = \frac{I'_{out}(s)}{\tilde{V}_{IN}(s)} = -\frac{1}{1 + \dfrac{3s(4s+2)}{7s+2}} = \frac{-(7s+2)}{7s+2+12s^2+6s}$$

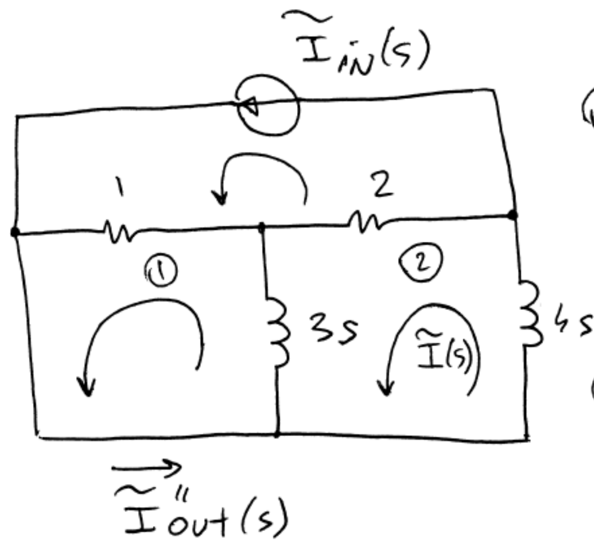
$$T_1(s) = \frac{-(7s+2)}{12s^2+13s+2}$$

Example - cont.



$$\tilde{I}_{out}(s) = T_1(s)\cdot\tilde{V}_{IN}(s) + T_2(s)\cdot\tilde{I}_{IN}(s)$$

(2) $T_2(s) = ? \Rightarrow \tilde{V}_{IN}(s) \to 0$

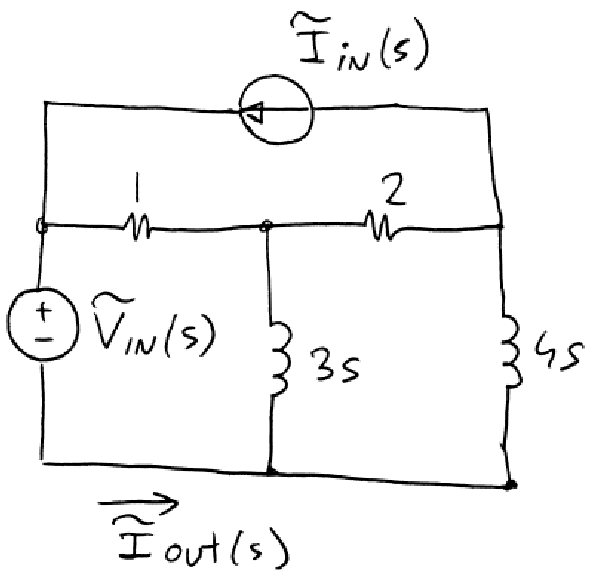


(1) $$\left(\tilde{I}''_{out}(s) - \tilde{I}_{IN}(s)\right)\cdot 1 + \left(\tilde{I}''_{out}(s) - \tilde{I}(s)\right)\cdot 3s = 0$$

(2) $$\tilde{I}(s)\cdot 4s + \left(\tilde{I}(s) - \tilde{I}_{IN}(s)\right)\cdot 2 + \left(\tilde{I}(s) - \tilde{I}''_{out}(s)\right)\cdot 3s = 0$$

$$T_2(s) = \frac{13s + 2}{12s^2 + 13s + 2}$$

Example - cont.



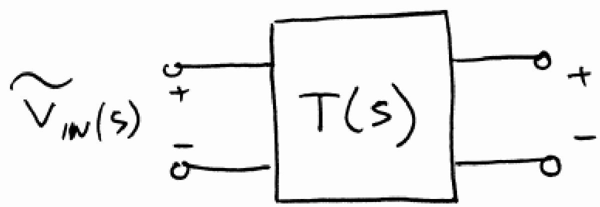
$$\tilde{I}_{out}(s) = T_1(s)\cdot\tilde{V}_{IN}(s) + T_2(s)\cdot\tilde{I}_{IN}(s)$$

$$\tilde{I}_{out}(s) = -\frac{7s+2}{12s^2+13s+2}\cdot\tilde{V}_{IN}(s) + \frac{13s+2}{12s^2+13s+2}\cdot\tilde{I}_{IN}(s)$$

$$\tilde{I}_{IN}(s) = \frac{1}{s} \quad \& \quad \tilde{V}_{IN}(s) = \frac{1}{s}$$

$$\tilde{I}_{out}(s) = \frac{6}{12s^2+13s+2} = \frac{1}{2}\cdot\frac{1}{s^2+\frac{13}{12}s+\frac{1}{6}} \quad \cdots$$

Impulse response

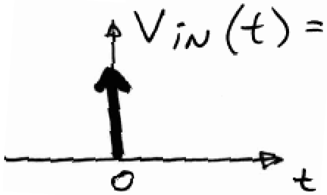


$$\tilde{V}_{out}(s) = T(s) \cdot \tilde{V}_{IN}(s)$$

★ ZERO INITIAL CONDITIONS

Now assume that

$V_{IN}(t) = \delta(t)$



$$L[\delta(t)] = 1 = \tilde{V}_{IN}(s)$$

$$\Rightarrow \tilde{V}_{out}(s) = T(s) \cdot 1 = T(s)$$

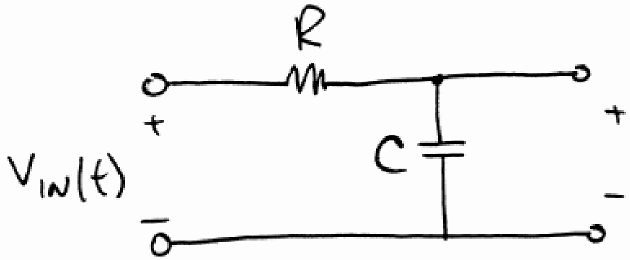
$$L^{-1}[\tilde{V}_{out}(s)] = V_{out}(t) = L^{-1}[T(s)] = imp(t)$$

Impulse response

$$T(s) = L[imp(t)]$$

★ $imp(t) = 0$ FOR $t < 0$

Impulse response of RC-integrator

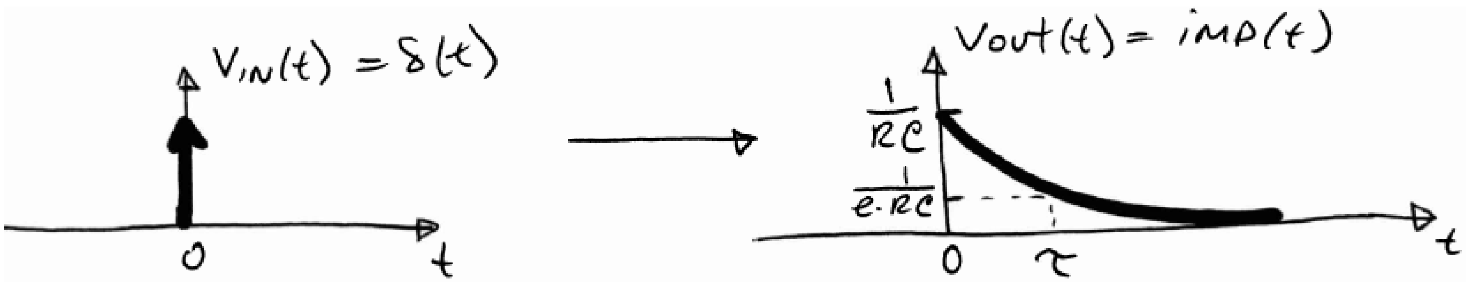


$$T(s) = \frac{\tilde{V}_{out}(s)}{\tilde{V}_{IN}(s)} = \frac{1}{1 + s \cdot RC}$$

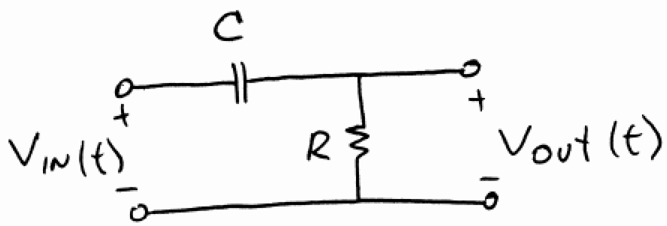
$$imp(t) = L^{-1}\left[T(s)\right] = L^{-1}\left[\frac{1}{1 + s \cdot RC}\right] = \frac{1}{RC} \cdot L^{-1}\left[\frac{1}{s + \frac{1}{RC}}\right]$$

$$\star \quad L\left[e^{-\alpha t}\right] = \frac{1}{s + \alpha}$$

$$imp(t) = \frac{h(t)}{RC} \cdot \exp\left(-\frac{t}{RC}\right)$$



Impulse response of RC-differentiator



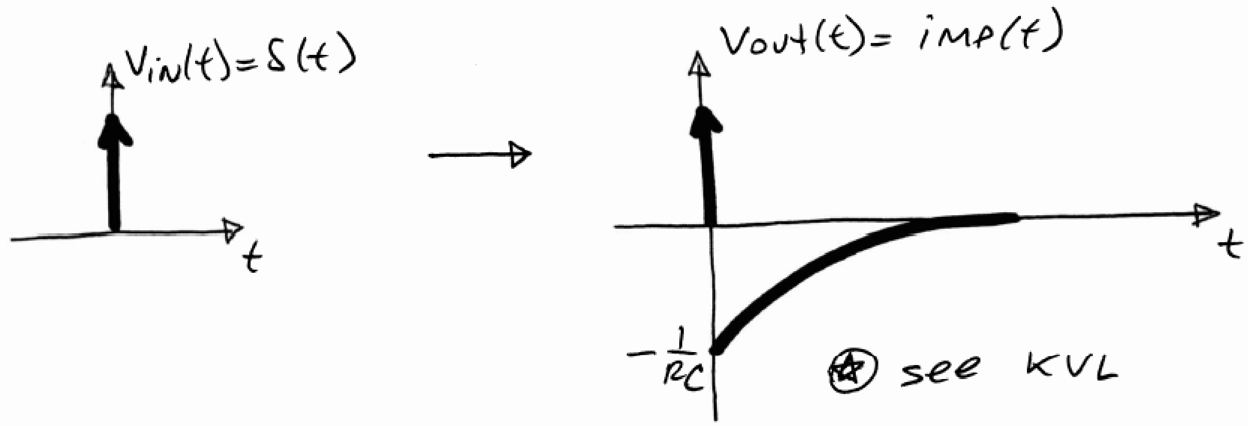
AGAIN $V_{in}(t) = \delta(t)$

$\Rightarrow V_{out}(t) = imp(t)$

$$imp(t) = L^{-1}[T(s)] \quad ; \quad T(s) = \frac{R}{R + \frac{1}{sC}} = \frac{s \cdot RC}{1 + s \cdot RC}$$

$$T(s) = \frac{s \cdot RC}{1 + s \cdot RC} = \frac{s \cdot RC + 1 - 1}{1 + s \cdot RC} = 1 - \frac{1}{1 + s \cdot RC}$$

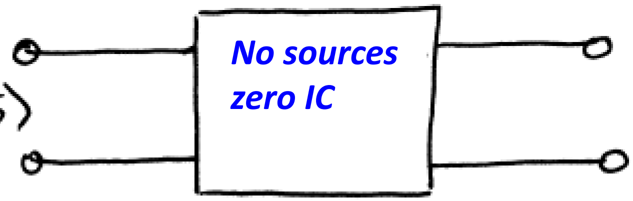
$$imp(t) = \delta(t) - \frac{1}{RC} \cdot exp\left(-\frac{t}{RC}\right) \cdot h(t)$$



Example.

Circuit response for the specific input

$3 \cdot \delta(t-5)$



$e^{-4t} \cdot \cos(7(t-9)) \cdot h(t-9)$

Find: $T(s) = ?$ & $imp(t) = ?$

① $L[3 \cdot \delta(t-5)] = 3 \cdot L[\delta(t-5)] = 3 \cdot e^{-5s} \cdot 1 = 3e^{-5s}$

$* \quad L[v(t-t_0) \cdot h(t-t_0)] = e^{-s \cdot t_0} \cdot \tilde{V}(s)$

② $L[e^{-4t} \cdot \cos(7(t-9)) \cdot h(t-9)] = L[e^{-4(t-9+9)} \cdot \cos(7 \cdot (t-9)) \cdot h(t-9)] =$

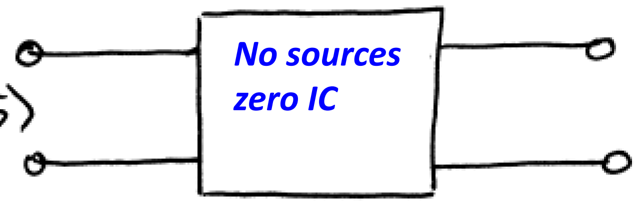
$= e^{-4 \cdot 9} \cdot L[e^{-4(t-9)} \cdot \cos(7 \cdot (t-9)) \cdot h(t-9)] =$

$= e^{-36} \cdot e^{-9 \cdot s} \cdot L[\underbrace{e^{-4t} \cdot \cos(7t)}_{\text{Damped cosine – use table}}] = e^{-36} \cdot e^{-9s} \cdot \frac{s+4}{(s+4)^2 + 7^2}$

Example - cont.

Circuit response for the specific input

$3 \cdot \delta(t-5)$



$e^{-4t} \cdot \cos(7(t-9)) \cdot h(t-9)$

Find: $T(s) = ?$ & $imp(t) = ?$

$$T(s) = \frac{\tilde{V}_{out}(s)}{\tilde{V}_{in}(s)} = \frac{e^{-36} \cdot e^{-9s} \cdot (s+4)}{(s+4)^2 + 49} \Big/ 3 \cdot e^{-5s}$$

$$T(s) = \frac{1}{3} \cdot e^{-36} \cdot \underbrace{e^{-4s} \cdot \frac{s+4}{(s+4)^2 + 49}}$$

Damped cosine time shifted by 4 seconds

$$\begin{cases} L\left[e^{-\alpha t} \cos \omega t\right] = \dfrac{s+\alpha}{(s+\alpha)^2 + \omega^2} \\ L\left[v(t-t_0) h(t-t_0)\right] = e^{-st_0} \cdot \tilde{V}(s) \end{cases}$$

$$imp(t) = L^{-1}\left[T(s)\right] = \frac{e^{-36}}{3} \cdot e^{-4(t-4)} \cdot \cos(7(t-4)) \cdot h(t-4) =$$

$$= \frac{1}{3} \cdot e^{-4(t+5)} \cdot \cos(7(t-4)) \cdot h(t-4)$$

Observe: $imp(t) = 0$ FOR $t < 4$