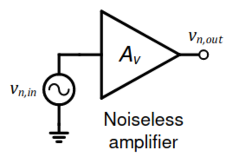
Assignment 06

EE 538 Spring 2020
Analog Circuits for Sensor Systems
University of Washington Electrical & Computer Engineering

Due: May 23, 2020
Author: Kevin Egedy

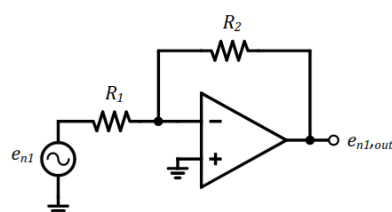
In [1]:
```
# Imports
import os
import sys
import cmath
import math
import matplotlib.pyplot as plt
import matplotlib
import numpy as np
import pandas as pd
import ltspice
import sympy as sp
from scipy import signal
%matplotlib inline
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
```

Input-Referred Voltage Noise



$$v_{n,in} = \frac{v_{n,out}}{A_v}$$

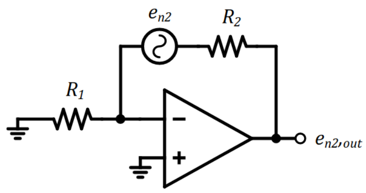
Input Resistor Noise



$$e_{n1,out} = \frac{-R_2}{R_1} e_{n1}$$

$$e^2_{n1,out} = (\frac{R_2}{R_1})^2 4kTR_1 \quad \frac{V^2}{\mathrm{Hz}}$$

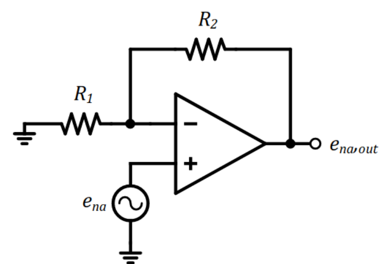
Feedback Resistor Noise



$$e_{n2,out} = e_{n2}$$

$$e^2_{n2,out} = 4kTR_2 \quad \frac{V^2}{\mathrm{Hz}}$$

Opamp Noise



$$e_{n,out} = (1 + \frac{R_2}{R_1})e_{na}$$

$$e^2_{na,out} = (1 + \frac{R_2}{R_1})^2 e^2_{na} \quad \frac{V^2}{\mathrm{Hz}}$$

Problem 1: Feedback amplifier noise

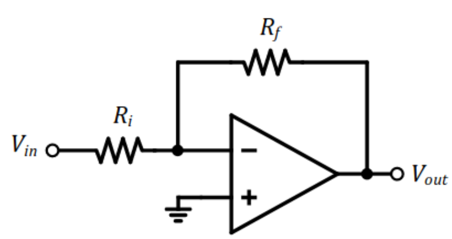


Figure 1. Inverting amplifier

R_f = 10kΩ and R_i = 1kΩ. The opamp has a gain-bandwidth product (f_T) of 10MHz, input voltage noise density (e_n) of 10nV/√Hz, and input current noise density of 1pA/√Hz.

a) (10 points) Determine the *input-referred* noise voltage density (in V/√Hz) of the amplifier, including contributions from all noise sources.
b) (5 points) Determine the equivalent noise bandwidth (f_{ENB}) and input-referred RMS noise voltage.
c) (5 points) Check your answers to a) and b) by simulating the circuit in Ltspice using the UniversalOpamp2. Be sure to set the noise voltage density 'en' and current noise density 'in' to the appropriate values. To verify the RMS noise voltage, you can ctrl-click on the plot title (or export the data to MATLAB/Python).

Part A

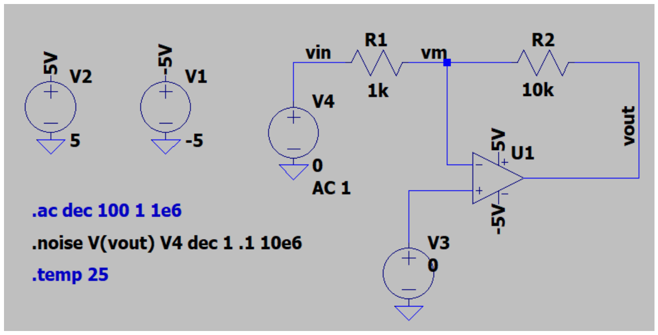
Given: $e_n = \frac{10nV}{\sqrt{\text{Hz}}}, i_n = \frac{1pA}{\sqrt{\text{Hz}}}, f_T = 10\text{MHz}, kT = 4.1145e-21$

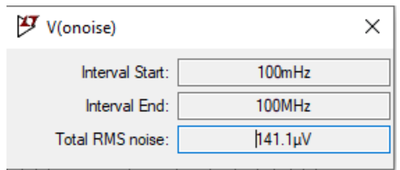
$$\begin{aligned}
A_V &= \frac{1}{\beta} = \frac{-R_f}{R_i} = 10 \\
e^2_{n,out} &= (\frac{R_f}{R_i})^2 4kTR_i + 4kTR_f + (1+\frac{R_f}{R_i})^2 e_n^2 + (i_n R_f)^2 \\
e_{n,out} &= \sqrt{(\frac{R_f}{R_i})^2 4kTR_i + 4kTR_f + (1+\frac{R_f}{R_i})^2 e_n^2 + (i_n R_f)^2} \\
e_{n,out} &= 1.18365 \cdot 10^{-7} = 118.4\frac{nV}{\sqrt{\text{Hz}}} \\
e_{n,in} &\approx \frac{e_{n,out}}{1/\beta} \\
e_{n,in} &= 1.18365 \cdot 10^{-8} = 11.84\frac{nV}{\sqrt{\text{Hz}}}
\end{aligned}$$

Part B

$$\begin{aligned}
A_V &= \frac{1}{\beta} = \frac{-R_f}{R_i} = 10 \\
f_{ENB} &= \frac{\pi}{2} f_{3dB} = \frac{\pi}{2}\beta f_T \approx 157 \cdot 10^6 \text{Hz} \\
v_{n,out(rms)} &= \sqrt{e^2_{n,out}\frac{\pi}{2}\beta f_T} = \sqrt{e^2_{n,out} f_{ENB}} \\
v_{n,in(rms)} &\approx \frac{v_{n,out(rms)}}{1/\beta} \\
&= \beta\sqrt{e^2_{n,out}\frac{\pi}{2} f_T} \\
&= \frac{1}{10}\sqrt{(1.18365 \cdot 10^{-7})^2 (157 \cdot 10^6)} = 0.0001483 = 148.3\mu V
\end{aligned}$$

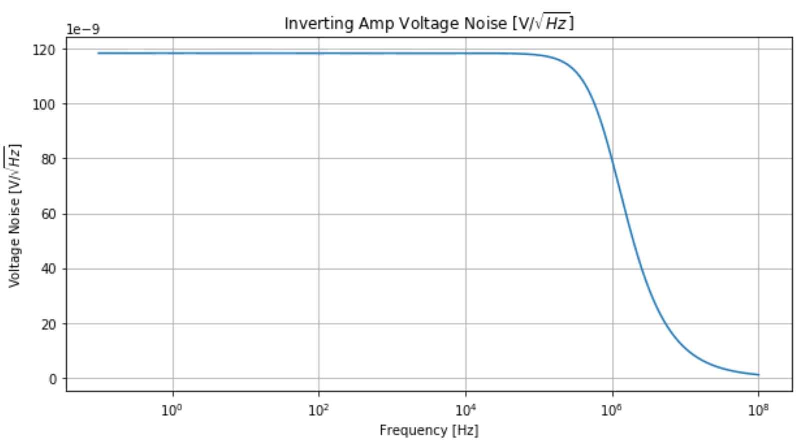
Part C





In [2]:
```
filepath = 'data/fb_amp_noise.txt'
df = pd.read_csv(filepath)
freq = df['frequency']
mag1 = df['V(inoise)']
mag2 = df['V(onoise)']
```

In [3]:
```
#Plot
fig, ax = plt.subplots(figsize=(10,5))
ax.semilogx(freq, mag2)
ax.set_xlabel('Frequency [Hz]')
ax.set_ylabel('Voltage Noise [V/$\sqrt{Hz}$]')
ax.set_title('Inverting Amp Voltage Noise [V/$\sqrt{Hz}$]')
ax.ticklabel_format(style='sci', axis='y', scilimits=(-9,-9))
ax.grid()
#ax.legend()
plt.show();
```



Problem 2: Opamp noise characterization

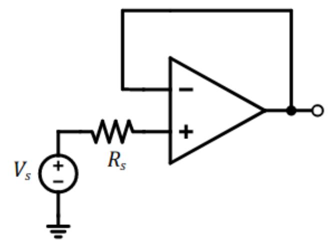


Figure 2. Opamp noise testbench

Characterize the noise performance of the AD8691 and ADA4898 opamps. For the AD8691, use a single supply of 5V. For the ADA4898, use a split supply of +/-5V. Be sure to observe any test conditions provided in the datasheet(s) when running your tests.

a) (5 points) Using the unity-gain feedback configuration shown, verify the 3dB bandwidth of each amplifier. Use this to calculate their equivalent noise bandwidths.
b) (5 points) Setting R_s = 0, perform noise simulations to verify the input voltage noise density of each opamp at the frequencies specified in their datasheets.
c) (5 points) For the ADA4898 only, set R_s to a value large enough such that the opamp's current noise density should be 10 times that of its voltage noise. Verify the current noise at the output of the amplifier using the relationship $e_{n,out} = i_n R_s$. Be sure to use the term 'noiseless' after the resistor value to ensure its noise doesn't affect the simulation results.
d) (5 points) Calculate the expected output RMS output noise in a 1MHz bandwidth for each amplifier and perform a noise simulation to verify. Note/explain any discrepancies.

Part A

ADA4898 (https://www.analog.com/media/en/technical-documentation/data-sheets/ADA4898-1_4898-2.pdf) AD8691 (https://www.analog.com/media/en/technical-documentation/data-sheets/AD8691_8692_8694.pdf)

	ADA4898	AD8691
β (V/V)	1	1
i_n (pA)	2.4	0.05
e_n (nA)	0.9	8
f_T (MHz)	65	10
$f_{3dB} = \beta f_T$	65	10

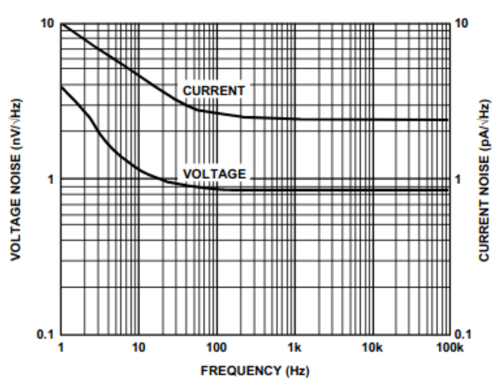


Figure 3. Input Voltage Noise and Current Noise vs. Frequency

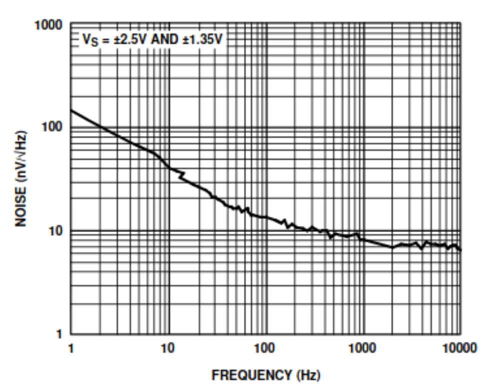


Figure 25. Voltage Noise Density

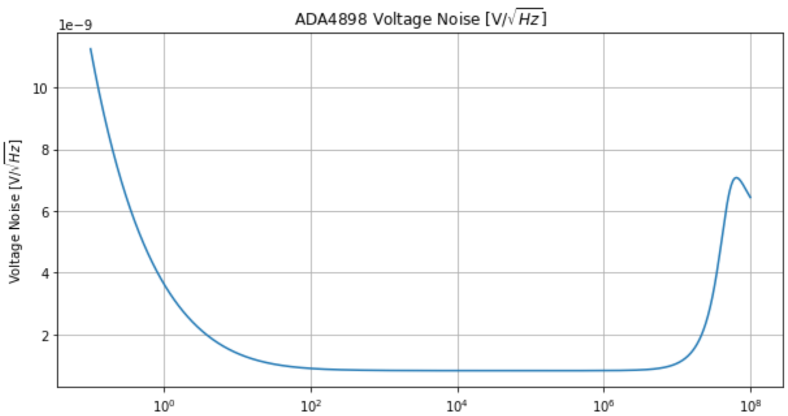
Noise Bandwidth

ADA4898: $f_{ENB} = \frac{\pi}{2} f_{3dB} = 102\text{MHz}$

AD8691: $f_{ENB} = \frac{\pi}{2} f_{3dB} = 15.7\text{MHz}$

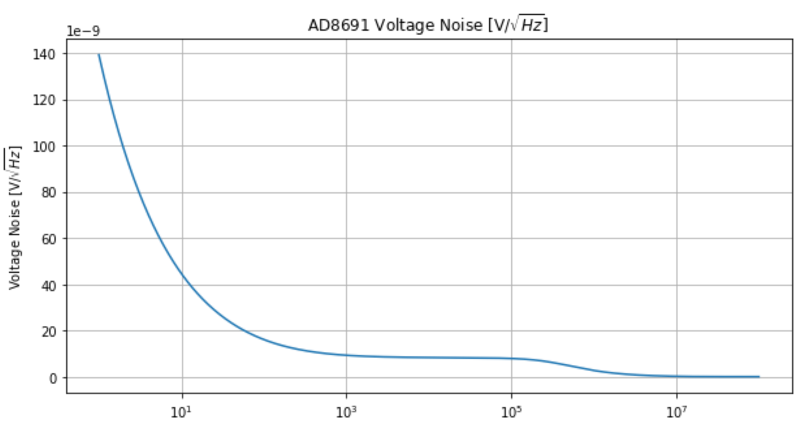
In [4]:
```
filepath = 'data/ADA4898.txt'
df = pd.read_csv(filepath)
freq = df['frequency']
mag1 = df['V(onoise)']
```

In [5]:
```
#Plot
fig, ax = plt.subplots(figsize=(10,5))
ax.set_title('ADA4898 Voltage Noise [V/$\sqrt{Hz}$]')
ax.semilogx(freq, mag1)
ax.set_ylabel('Voltage Noise [V/$\sqrt{Hz}$]')
ax.ticklabel_format(style='sci', axis='y', scilimits=(-9,-9))
ax.grid()
plt.show();
```



In [6]:
```
filepath = 'data/AD8691.txt'
df = pd.read_csv(filepath)
freq = df['frequency']
mag1 = df['V(onoise)']
mag2 = 20*np.log10(df['V(onoise)']/min(df['V(onoise)']))
```

In [7]:
```
#Plot
fig, ax = plt.subplots(figsize=(10,5))
ax.set_title('AD8691 Voltage Noise [V/$\sqrt{Hz}$]')
ax.semilogx(freq, mag1)
ax.set_ylabel('Voltage Noise [V/$\sqrt{Hz}$]')
ax.ticklabel_format(style='sci', axis='y', scilimits=(-9,-9))
ax.grid()
plt.show();
```



Part B

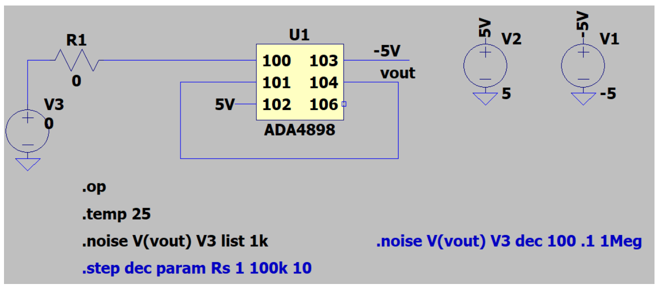
$$e_{n,out} = \sqrt{e_n^2 + R_S^2 i_n^2}\,\frac{V}{\sqrt{\text{Hz}}}$$

Since $R_S = 0$, then the voltage noise observed is e_n

ADA4898

NOISE/DISTORTION PERFORMANCE			
Harmonic Distortion SFDR	f = 100 kHz, V_{OUT} = 2 V p-p	−116	dBc
	f = 500 kHz, V_{OUT} = 2 V p-p	−93	dBc
	f = 1 MHz, V_{OUT} = 2 V p-p	−79	dBc
Input Voltage Noise	f = 1 kHz	0.9	nV/√Hz
Input Current Noise	f = 1 kHz	2.4	pA/√Hz

Ltspice

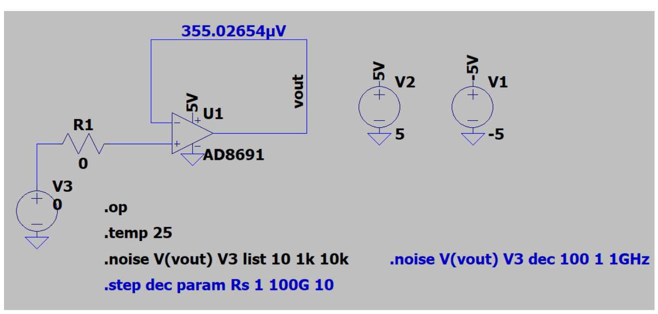


frequency	V(onoise)
1 kHz	0.841671 nV

AD8691

NOISE PERFORMANCE					
Voltage Noise	$e_{n\ p\text{-}p}$	f = 0.1 Hz to 10 Hz	1.6	3.0	μV p-p
Voltage Noise Density	e_n	f = 1 kHz	8	12	nV/√Hz
	e_n	f = 10 kHz	6.5		nV/√Hz
Current Noise Density	i_n	f = 1 kHz	0.05		pA/√Hz

Ltspice

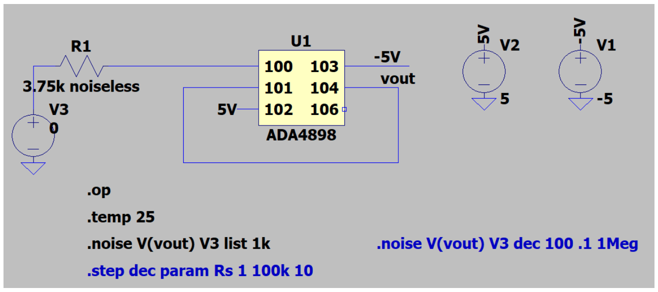


frequency	V(onoise)
10 Hz	44.68291 nV
1 kHz	9.387410 nV
10 kHz	8.409573 nV

Part C

ADA4898

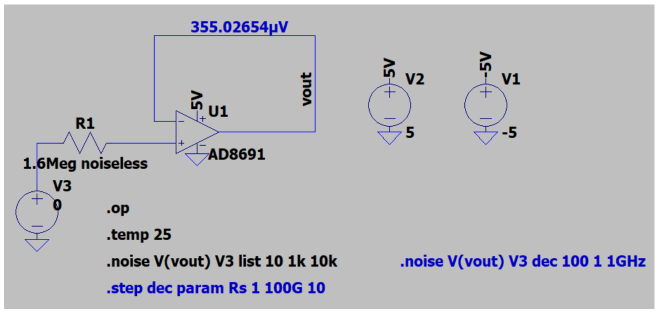
$$
\begin{aligned}
R_S &= 10 \cdot \frac{0.9\text{nV}}{2.4\text{pA}} = 3750\Omega \\
e_{n,out} &= \sqrt{e_n^2 + R_S^2 i_n^2}\,\frac{V}{\sqrt{\text{Hz}}} \\
&= \sqrt{(0.9\text{nV})^2 + 3750^2(2.4\text{pA})^2}\,\frac{V}{\sqrt{\text{Hz}}} \\
&= 9.405\frac{nV}{\sqrt{\text{Hz}}}
\end{aligned}
$$



frequency	V(onoise)
1 kHz	11.0425 nV

AD8691

$$
\begin{aligned}
R_S &= 10 \cdot \frac{8\mathrm{nV}}{0.05\mathrm{pA}} = 1.6 \cdot 10^6 \Omega \\
e_{n,out} &= \sqrt{e_n^2 + R_S^2 i_n^2} \frac{V}{\sqrt{\mathrm{Hz}}} \\
&= \sqrt{(8\mathrm{nV})^2 + 1.6\mathrm{MHz}^2 (0.05\mathrm{pA})^2} \frac{V}{\sqrt{\mathrm{Hz}}} \\
&= 80.4 \frac{nV}{\sqrt{\mathrm{Hz}}}
\end{aligned}
$$

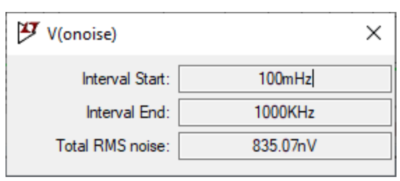


frequency	V(onoise)
10 Hz	446.8291 nV
1 kHz	93.87410 nV
10 kHz	84.09573 nV

Part D

$$
v_{n,out(rms)} = \sqrt{e_{n,out}^2 \frac{\pi}{2} \beta f_T} = \sqrt{e_{n,out}^2 f_{ENB}} \rightarrow \sqrt{e_{n,out}^2 \mathrm{BW}}
$$

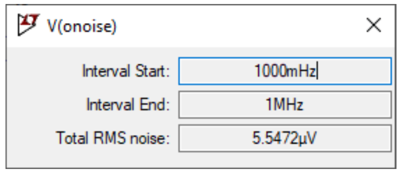
ADA4898



$$
\begin{aligned}
v_{n,out(rms)} &= \sqrt{e_{n,out}^2 1\mathrm{MHz}} \\
&= e_{n,out} \cdot 10^3, \quad e_{n,out} = 0.9\mathrm{nV} \\
&= 0.9\mu\mathrm{V}
\end{aligned}
$$

The difference between ideal and actual is $\frac{900-835}{900} = 7.22\%$.

AD8691



$$\begin{aligned} v_{n,out(rms)} &= \sqrt{e_{n,out}^2 1\mathrm{MHz}} \\ &= e_{n,out} \cdot 10^3, \quad e_{n,out} = 8\mathrm{nV} \\ &= 8\mu\mathrm{V} \end{aligned}$$

The difference between ideal and actual is $\frac{8-5.55}{8} = 30.63\%$. This is a large difference that is probably due to the fact that the current noise density is much smaller than the spec value.

```
In [ ]:
```