EE 538 Spring 2021
Low-Noise Analog Circuit Design
University of Washington Electrical & Computer Engineering

Instructor: Jason Silver
Assignment #3 (10 points)
Due Monday, April 26 (Submit on Canvas as a Jupyter Notebook)

Please show your work

Problem 1: Low-noise common-emitter amplifier design

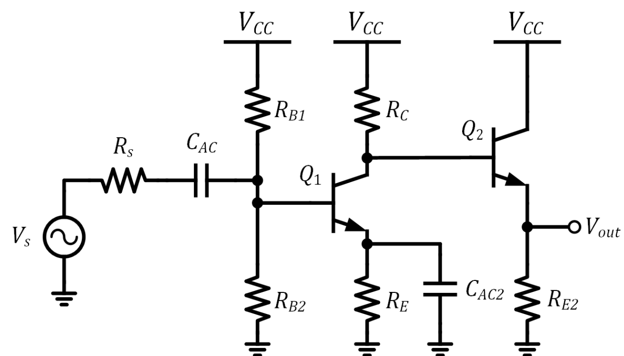


Figure 1a. Common-emitter amplifier with bias

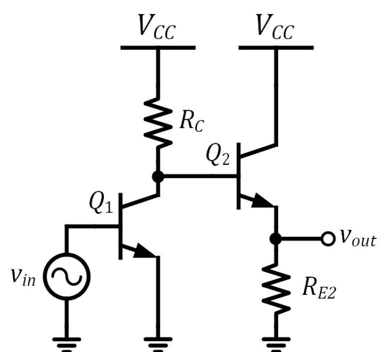


Figure 1b. Equivalent "mid-band" circuit

The common-emitter amplifier is often biased using the configuration shown in Figure 1a, where $R_{B1,2}$ are used to set the DC base voltage and R_E is needed to stabilize the DC bias condition against variations in temperature and transistor parameters. Q_2 is added to form an emitter-follower for low output impedance. The DC emitter current of the common-emitter stage (I_{E1} is determined by

$$I_E = \frac{V_B - V_{BE}}{R_E + \frac{R_B}{\beta_0+1}}$$

where R_B is the parallel combination of R_{B1} and R_{B2} and component values satisfy the conditions $V_B >> V_{BE}$ and $R_E >> R_B/(\beta_0 + 1)$.

C_{AC} and C_{AC2} are used to realize low impedances at signal frequencies at the base and emitter nodes, resulting in the equivalent "mid-band" circuit shown in Figure 1b.

<u>*Analysis*</u>

a) Assuming $R_s = 0$ and ignoring transistor capacitances C_π and C_μ, use the small-signal equivalent circuit of the amplifier in Figure 1a to derive its transfer function in terms of R's, C's, and transistor parameters g_m, r_o, and β_0.

$$
\begin{aligned}
R_B &= R_{B_1} \parallel R_{B_2} && \text{(short DC voltages in AC analysis)} \\
Z_s &= R_s + \frac{1}{sC_{AC}} \\
Z_E &= R_E \parallel \frac{1}{sC_{AC_2}} \\
r_b &= 0 && \rightarrow v_\pi = v_{be} \\
r_\pi &= \frac{v_\pi}{i_b} \approx \frac{v_{be}}{(g_m v_{be})/\beta} = \frac{\beta_0}{g_m}
\end{aligned}
$$

Find v'_{in} voltage divider $(R_{in} \parallel R_B)$ and R_s

$$
\begin{aligned}
v'_{in} &= \frac{(R_{in} \parallel R_B)}{(R_{in} \parallel R_B) + Z_s} V_s \\
\frac{v'_{in}}{V_s} &= \frac{(R_{in} \parallel R_B)}{(R_{in} \parallel R_B) + Z_s}
\end{aligned}
$$

Find R_{in} Common Emitter

$$
\begin{aligned}
i_t &= \frac{v_{be}}{r_\pi} \text{ (current from test voltage)} \\
v_t &= v_{be} + Z_E(\frac{v_{be}}{r_\pi} + g_m v_{be}) \\
R_{in} &= \frac{v_t}{i_t} \\
&= (v_{be} + Z_E(\frac{v_{be}}{r_\pi} + g_m v_{be}))\frac{r_\pi}{v_{be}} \\
&= r_\pi + Z_E + Z_E g_m r_\pi \\
&\approx r_\pi(1 + g_m Z_E) \\
\lim_{s\to\infty} R_{in} &= r_\pi
\end{aligned}
$$

Find R_{out_1} Common Emitter

$$
\begin{aligned}
R_{out_1} &= R_C \parallel r_0(1 + g_m R_E) \\
&\approx R_C
\end{aligned}
$$

Find V_{out_1} Common Emitter

$$\begin{aligned}
v'_{in} &= v_{be} + Z_E(g_m v_{be} + \frac{v_{be}}{r_\pi}) \\
&= v_{be}(1 + Z_E(g_m + \frac{1}{r_\pi})) \\
&\approx v_{be}(1 + g_m Z_E) \\
V_{out_1} &= -g_m v_{be} R_C \\
\frac{V_{out_1}}{v'_{in}} &= \frac{-g_m R_C}{1 + g_m Z_E}
\end{aligned}$$

Find v'_{in_2} Emitter Follower

$$\begin{aligned}
v'_{in_2} &= \frac{R_{in_2}}{R_{in_2} + R_{out_1}} V_{out_1} \\
\frac{v'_{in_2}}{V_{out_1}} &= \frac{R_{in_2}}{R_{in_2} + R_{out_1}}
\end{aligned}$$

Find R_{in_2} Emitter Follower

$$R_{in_2} = r_\pi(1 + g_m R_{E_2})$$

Find V_{out_2} Emitter Follower

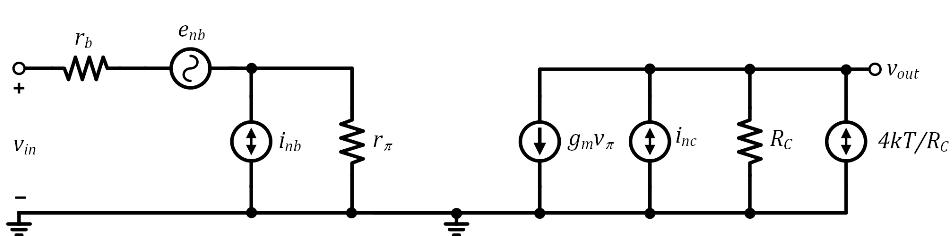
$$\frac{V_{out_2}}{v'_{in_2}} = \frac{g_m R_{E_2}}{1 + g_m R_{E_2}}$$

Find V_{out_2}

$$\begin{aligned}
\frac{V_{out_2}}{V_s} &= \frac{v'_{in}}{V_s} \cdot \frac{V_{out_1}}{v'_{in}} \cdot \frac{v'_{in_2}}{V_{out_1}} \cdot \frac{V_{out_2}}{v'_{in_2}} \\
\frac{V_{out_1}}{V_s} &= \frac{(R_{in} \parallel R_B)}{(R_{in} \parallel R_B) + Z_s} \cdot \frac{-g_m R_C}{1 + g_m Z_E} \cdot \frac{R_{in_2}}{R_{in_2} + R_{out_1}} \cdot \frac{g_m R_{E_2}}{1 + g_m R_{E_2}} \\
&= \frac{(r_\pi(1 + g_m Z_E) \parallel R_B)}{(r_\pi(1 + g_m Z_E) \parallel R_B) + Z_s} \cdot \frac{-g_m R_C}{1 + g_m Z_E} \cdot \frac{r_\pi(1 + g_m R_{E_2})}{r_\pi(1 + g_m R_{E_2}) + R_C} \cdot \frac{g_m R_{E_2}}{1 + g_m R_{E_2}} \quad \text{where } r_\pi = \frac{\beta_0}{g_m}
\end{aligned}$$

---

b) Determine expressions for the input-referred noise sources e_n and i_n of the amplifier in Figure 1b assuming only thermal and shot noise generators (no $1/f$ noise or f_T-dependent shot noise).



$$
\begin{aligned}
r_e &= \frac{1}{g_m} = \frac{V_T}{I_C} \\
e_n^2 &= 4kT\left(r_b + \frac{r_e}{2} + \frac{r_e^2}{R_C} + \frac{r_e^2}{R_{in_2}}\right) = 4kT\left(r_b + \frac{1}{2g_m} + \frac{1}{g_m^2 R_C} + \frac{1}{g_m^2 R_{in_2}}\right) \\
e_n &= \sqrt{4kT\left(r_b + \frac{r_e}{2} + \frac{r_e^2}{R_C} + \frac{r_e^2}{R_{in_2}}\right)} = \sqrt{4kT\left(r_b + \frac{1}{2g_m} + \frac{1}{g_m^2 R_C} + \frac{1}{g_m^2 R_{in_2}}\right)} \\
i_n^2 &= 2q\frac{I_C}{\beta_0} \\
i_n &= \sqrt{2q\frac{I_C}{\beta_0}} \\
NF &= 1 + \frac{e_n^2 + i_n^2 R_S^2}{4kTR_S}
\end{aligned}
$$

<u>*Design*</u>

c) Assuming $R_s = 100\Omega$, $r_b = 2\Omega$ and $\beta_0 = 250$, what is the minimum theoretical noise figure of the amplifier? What value of I_{C1} does this correspond to? Assume a mid-band gain of the common-emitter stage of 40dB and an output impedance of the emitter-follower of 50Ω.

Minimum theoretical noise figure of the amplifier is 1.07

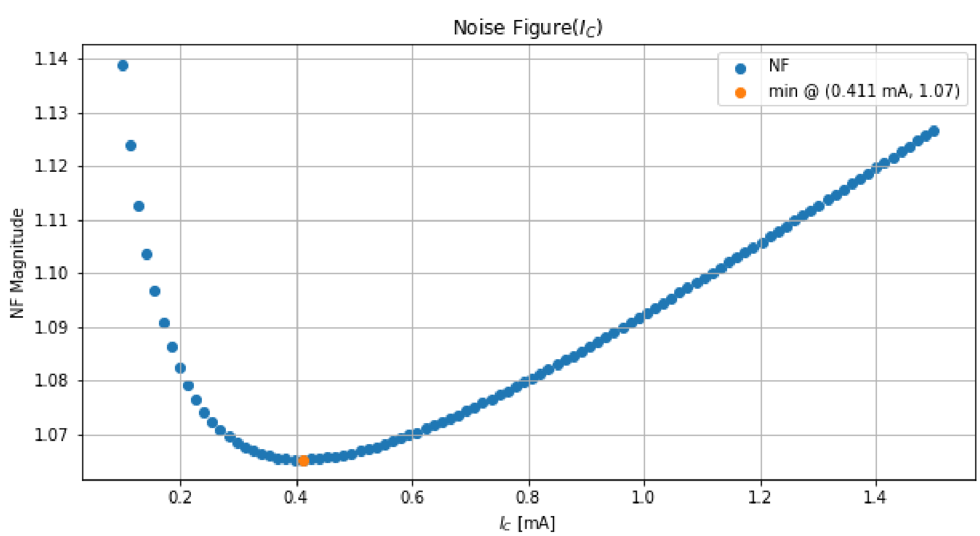
$$
\begin{aligned}
NF &= 1 + \frac{e_n^2 + i_n^2 R_S^2}{4kTR_S} \\
&= 1 + \frac{4kT(r_b + \frac{V_T}{2I_C}) + 2q\frac{I_c}{\beta_0}R_S^2}{4kTR_S} \\
&= 1 + \frac{4kT \cdot r_b}{4kTR_S} + \frac{4kT \cdot V_T}{4kTR_S \cdot 2I_C} + \frac{2qR_S I_c}{4kT \cdot \beta_0} \\
\frac{\partial NF}{\partial I_C} &= \frac{\partial}{\partial I_C}(1 + \frac{4kT \cdot r_b}{4kTR_S} + \frac{4kT \cdot V_T}{4kTR_S \cdot 2I_C} + \frac{2qR_S I_c}{4kT \cdot \beta_0}) \\
0 &= \frac{\partial}{\partial I_C}(\frac{V_T}{2R_S}\frac{1}{I_C} + \frac{R_S}{2V_T\beta_0}I_c) \\
0 &= \frac{-V_T}{2R_S}\frac{1}{I_C^2} + \frac{R_S}{2V_T\beta_0} \\
\frac{V_T}{2R_S}\frac{1}{I_C^2} &= \frac{R_S}{2V_T\beta_0} \\
\frac{2V_T^2\beta_0}{2R_S^2} &= I_C^2 \\
\beta_0\frac{V_T^2}{R_S^2} &= I_C^2 \\
I_C &= \sqrt{\beta_0}\frac{V_T}{R_S} \\
&= \sqrt{250}\frac{26 \cdot 10^{-3}}{1000} = 411\mu A
\end{aligned}
$$

In [4]:

```python
R_s,r_b,beta_0,I_C = sp.symbols('R_s,r_b,beta_0,I_C')
k = 1.38e-23
T = 300
q = 1.602e-19
V_T = k*T/q

I_Cs = np.linspace(100*1e-6,1.5*1e-3, num=100)

en_sq = 4*k*T*(V_T/(2*I_C) + r_b)
in_sq = 2*q*I_C/beta_0
NF = 1 + (en_sq + in_sq*R_s**2)/(4*k*T*R_s)

components = {
    R_s : 1000,
    r_b : 2,
    beta_0 : 250,
}
H = sp.Matrix([NF])
H = H.subs(components)
H = lambdify(I_C,H,modules='numpy')
H = H(I_Cs)
H = H[0][0]
```

In [5]:

```python
fig, ax = plt.subplots(figsize=(10,5))

label = f'min @ ({rnd(I_Cs[np.argmin(H)],3,"mA")} mA, {round(min(H),2)})'

ax.set_title(r'Noise Figure($I_C$)')
ax.scatter(I_Cs*1e3, H,label=r'NF')
ax.scatter(I_Cs[np.argmin(H)]*1e3, min(H), label=f'{label}')
ax.set_ylabel('NF Magnitude')
ax.set_xlabel(r'$I_C$ [mA]')
ax.grid(which='both', axis='both')
ax.legend()
plt.show();
```



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d) Design the amplifier (i.e. determine resistor values $R_{B1,2}$, R_E, R_C, and R_{E2}) for a mid-band gain of 40dB and the noise figure determined in part c. Use a supply voltage $V_{CC} = 9V$. Determine values for C_{AC} and C_{AC2} that ensure a 3dB highpass corner lower than 100Hz.

Verify your design in Ltspice using the SPICE model of the 2SC3324 npn transistor from Toshiba. Include an image of your schematic (with DC node voltages and branch currents annotated) and plots of the frequency response, output noise, and noise figure as a function of frequency. Indicate the noise figure at 10kHz.

DC Analysis

Given

$$I_E = \frac{V_B - V_{BE}}{R_E + \frac{R_B}{\beta_0+1}}$$

$$g_m \equiv \frac{\partial I_C}{\partial V_{BE}} = \frac{qI_C}{kT} = \frac{I_C}{V_T}$$

$$r_o \equiv \frac{\partial V_{CE}}{\partial I_C} = \frac{V_A}{I_C}$$

$$r_\pi = \beta_0 \frac{V_T}{I_C} = \frac{\beta_0}{g_m}$$

$$I_B = \frac{I_C}{\beta}$$

$$I_E = I_B + I_C$$

$$I_E = (1+\beta)I_B$$

Set I_C to minimize noise. I'm setting $I_C = 0.5$ mA. Will update if have time.
Choose V_C to be midpoint of V_{CC} and ground. $V_C = 4.5$
Find R_C

$$I_C = \frac{V_{CC} - V_C}{R_C}$$

$$R_C = \frac{V_{CC} - V_C}{I_C}$$

$$= \frac{9 - 4.5}{0.5mA}$$

$$= 9000\Omega$$

Find transconductance g_m

$$V_T \approx 26mV$$

$$g_m = \frac{I_C}{V_T}$$

$$g_m = \frac{0.5mA}{26mV}$$

$$= 0.019$$

Find V_{th} and R_{th}

$$V_{th} = V_{CC}\frac{R_{B_2}}{R_{B_2} + R_{B_1}}$$

$$R_{th} = R_{B_1} \parallel R_{B_2}$$

Choose Z_{in} to be large; 5K

$$
\begin{aligned}
Z_{in} &= 5K\Omega \\
Z_{in} &= R_{th} \parallel r_\pi \\
5000 &= R_{th} \parallel \frac{\beta_0}{g_m} \\
5000 &= R_{th} \parallel \frac{250}{0.019} \\
5000 &= \frac{R_{th} \cdot 13157}{R_{th} + 13157} \\
R_{th} &= 8065\Omega
\end{aligned}
$$

Apply KVL

$$-V_{th} + I_B R_{th} + V_{BE} + I_E R_E$$

$$I_B(R_{th} + R_E(1+\beta)) = V_{th} - V_{BE}$$

$$I_B = \frac{V_{th} - V_{BE}}{R_{th} + R_E(1+\beta)}$$

$$I_E = (\beta_0 + 1) I_B$$

$$I_E = \frac{V_B - V_{BE}}{R_E + \frac{R_B}{\beta_0+1}} \approx \frac{V_B}{R_E}$$

$$I_C \approx I_E$$

$$\text{then}$$

$$\frac{V_B}{R_E} = 0.5mA$$

Choose $V_B = 6V$, then R_E = $12K\Omega$

Solving V_{th} and R_{th} simultaneously results

$$R_{B_1} = 16130$$

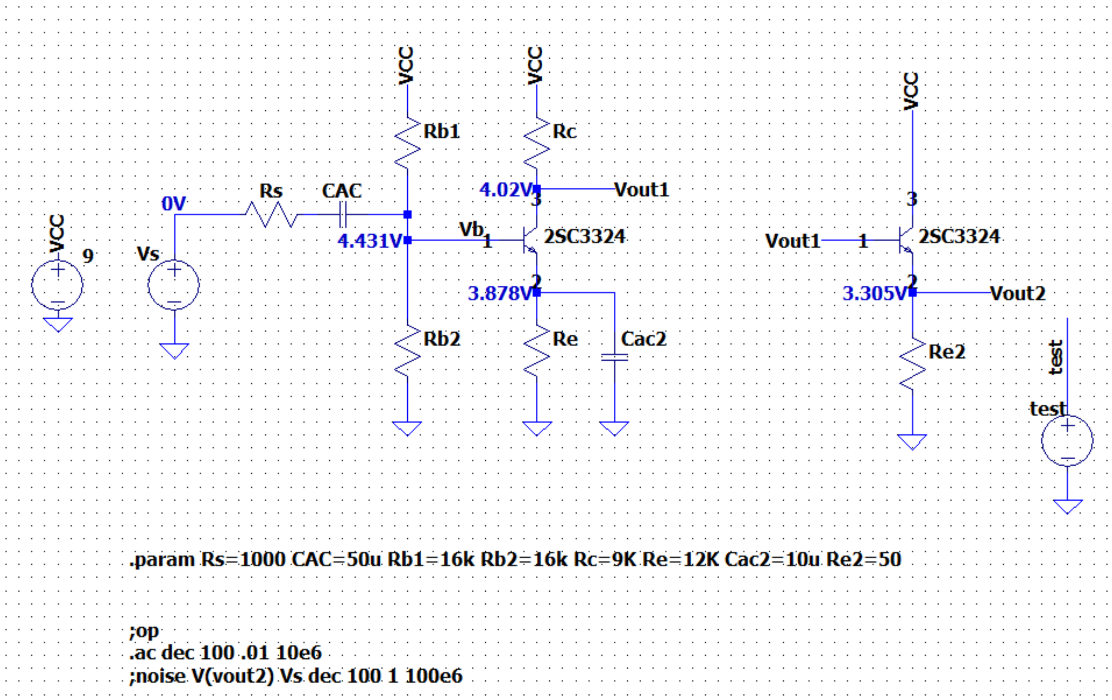
$$R_{B_2} = 16130$$

AC Analysis

$$
\begin{aligned}
\frac{V_{out}}{V_s} &= 40dB \\
V_{be} &= \frac{V_s R_{th}}{R_s + R_{th}} \\
\frac{V_{out}}{V_s} &= \frac{V_{be}}{V_s} \cdot \frac{V_{out}}{V_{be}} \cdot \frac{R_{in_2}}{R_{out_1} + R_{in_2}} \\
40dB &= \frac{R_{th}}{R_s + R_{th}} \cdot -g_m R_C \cdot \frac{R_{in_2}}{R_C + R_{in_2}} \\
10000 &= \frac{8065}{1000 + 8065} \cdot -0.019 \cdot 9000 \cdot \frac{R_{in_2}}{9000 + R_{in_2}} \\
R_{in_2} &= 8865
\end{aligned}
$$

Find R_{in_2}

$$
\begin{aligned}
R_{in_2} &= r_\pi(1 + g_m R_{E_2}) \approx \beta_0 R_{E_2} \\
8865 &= 250 R_{E_2} \\
R_{E_2} &= 35\Omega
\end{aligned}
$$



* C:\Users\keged\Documents\LTspiceXVII\EE538.1\HW03\HW03.asc

```
       --- Operating Point ---

V(vb):          4.43098         voltage
V(vcc):         9               voltage
V(n003):        3.87788         voltage
V(vout1):       4.02026         voltage
V(n002):        2.21549e-013    voltage
V(n001):        0               voltage
V(vout2):       3.30545         voltage
V(test):        0               voltage
I(Cac2):        3.87788e-017    device_current
I(Cac):         2.21549e-016    device_current
I(Re2):         0.066109        device_current
I(Rs):          2.21549e-016    device_current
I(Rc):          0.000553304     device_current
I(Re):          0.000323157     device_current
I(Rb1):         0.000285564     device_current
I(Rb2):         0.000276936     device_current
I(Test):        0               device_current
I(V1):          -0.0667091      device_current
I(Vs):          2.21549e-016    device_current
Ix(u1:1):       0.00031453      subckt_current
Ix(u1:2):       8.62703e-006    subckt_current
Ix(u1:3):       -0.000323157    subckt_current
Ix(u2:1):       0.0658703       subckt_current
Ix(u2:2):       0.000238774     subckt_current
Ix(u2:3):       -0.066109       subckt_current
```

In [6]:

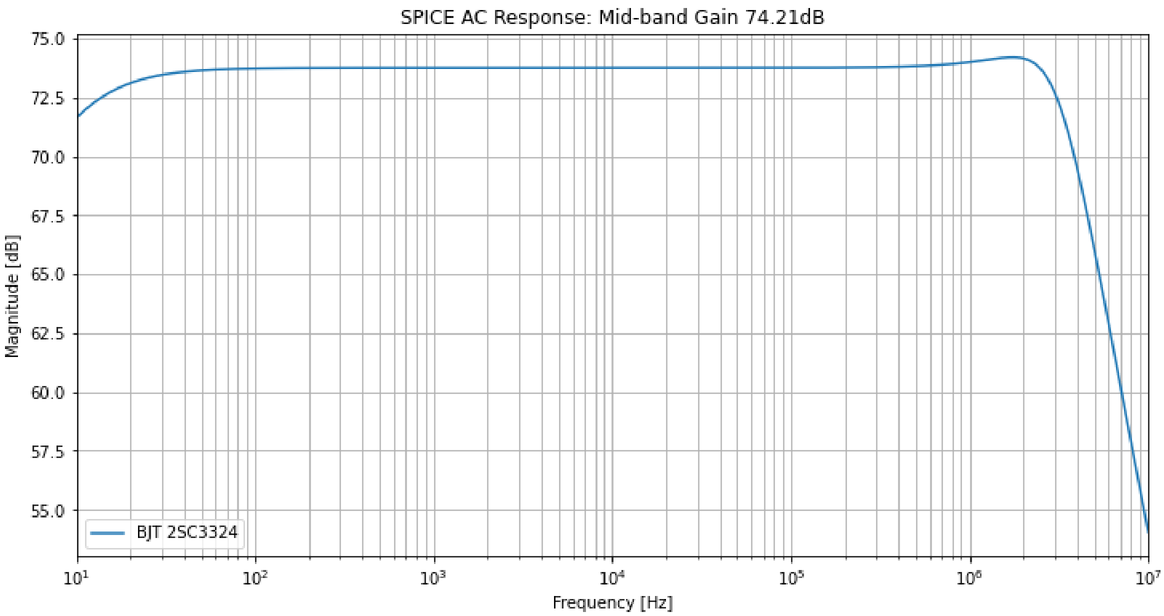
```
filepath = 'data/HW03.txt'
df = read_ltspice(filepath,'ac')
freq = df['Freq.']
mag = df['Mag_V(vout2)/I(vs)']
```

In [7]:

```
fig, ax = plt.subplots(1,figsize=(12,6))

ax.semilogx(freq, mag, color='tab:blue',label='BJT 2SC3324')
ax.grid(True,which='both')
ax.set_xlabel('Frequency [Hz]')
ax.set_ylabel('Magnitude [dB]')
ax.set_title(f'SPICE AC Response: Mid-band Gain {round(max(mag),2)}dB')
ax.set_xlim(1e1,1e7)

ax.legend()
plt.show();
```



In [8]:

```
filepath = 'data/HW03_noise.txt'
df = pd.read_csv(filepath)
freq = df['frequency']
onoise = df['V(onoise)']
inoise = df['V(inoise)']
```

In [9]:

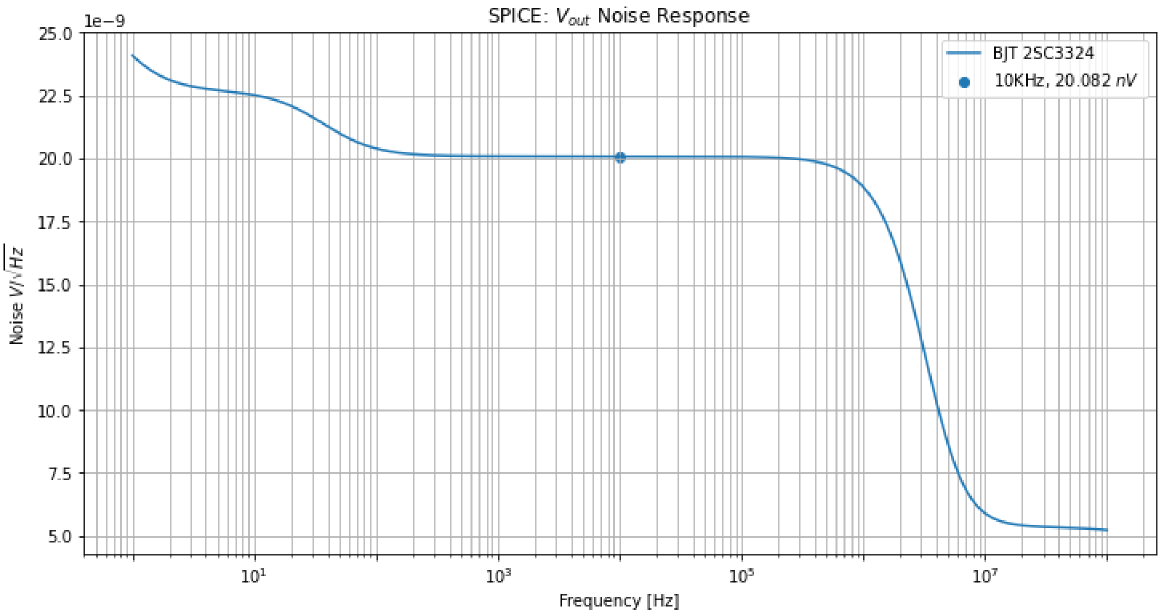
```python
fig, ax = plt.subplots(1,figsize=(12,6))

x1 = np.where(freq<=10000)[0][-1]
label1 = r"10KHz, {:.3f} $nV$".format(onoise[x1]*1e9)

ax.semilogx(freq, onoise, color='tab:blue',label='BJT 2SC3324')
ax.scatter(freq[x1],onoise[x1],label=label1,color='tab:blue')
ax.grid(True,which='both')
ax.set_xlabel('Frequency [Hz]')
ax.set_ylabel(r'Noise $V/\sqrt{Hz}$')
ax.set_title(r'SPICE: $V_{out}$ Noise Response')
ax.ticklabel_format(style='sci', axis='y', scilimits=(-9,-9))
#ax.set_ylim(10e-9,50e-9)

# manipulate x-axis ticks and labels
ax.xaxis.set_major_locator(LogLocator(numticks=15)) #(1)
ax.xaxis.set_minor_locator(LogLocator(numticks=15,subs=np.arange(2,10))) #(2)
for label in ax.xaxis.get_ticklabels()[::2]:
    label.set_visible(False) #(3)

ax.legend()
plt.show();
```



In [10]:

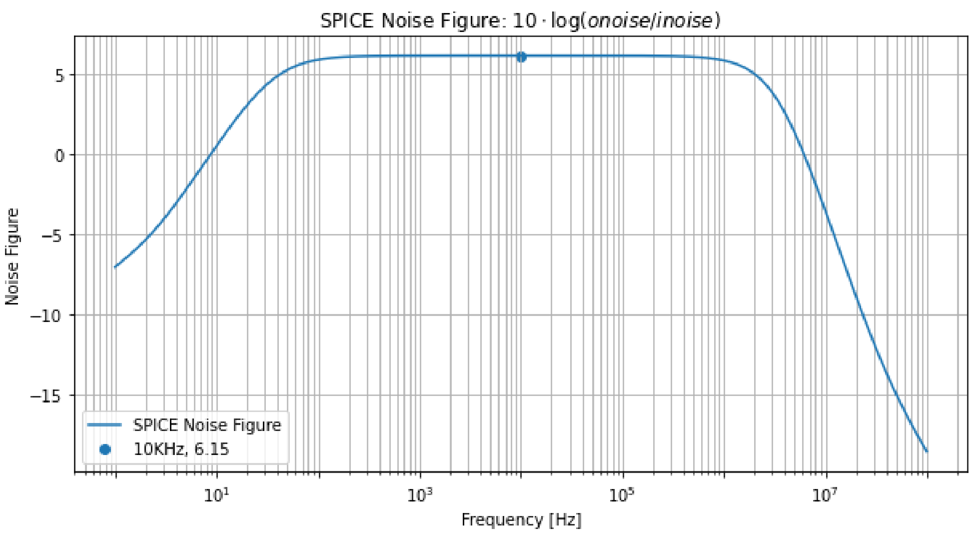
```python
fig, ax = plt.subplots(1,figsize=(10,5))

x1 = np.where(freq<=10000)[0][-1]
label1 = r"10KHz, {:.2f}".format(10*np.log10(onoise[x1]/inoise[x1]))

ax.semilogx(freq, 10*np.log10(onoise/inoise), color='tab:blue',label='SPICE Noise Figure')
ax.scatter(freq[x1],10*np.log10(onoise[x1]/inoise[x1]),label=label1,color='tab:blue')
ax.grid(True,which='both')
ax.set_xlabel('Frequency [Hz]')
ax.set_ylabel(r'Noise Figure')
ax.set_title(r'SPICE Noise Figure: $10\cdot \log(onoise/inoise)$')
#ax.ticklabel_format(style='sci', axis='y', scilimits=(-9,-9))
#ax.set_ylim(10e-9,50e-9)

# manipulate x-axis ticks and labels
ax.xaxis.set_major_locator(LogLocator(numticks=15)) #(1)
ax.xaxis.set_minor_locator(LogLocator(numticks=15,subs=np.arange(2,10))) #(2)
for label in ax.xaxis.get_ticklabels()[::2]:
    label.set_visible(False) #(3)

ax.legend()
plt.show();
```



Reference Page

In [11]:

```python
# Imports
import os
import sys
import cmath
import math
import matplotlib.pyplot as plt
import matplotlib
import numpy as np
import pandas as pd
import ltspice
import sympy as sp
from sympy.utilities.lambdify import lambdify
from scipy import signal
%matplotlib inline
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
from matplotlib.ticker import LogLocator
```

In [12]:
```python
def read_ltspice(file_name,ftype='trans',units='db'):
    cols = []
    arrs = []
    with open(file_name, 'r',encoding='utf-8') as data:
        for i,line in enumerate(data):
            if i==0:
                cols = line.split()
                arrs = [[] for _ in cols]
                continue
            parts = line.split()
            for j,part in enumerate(parts):
                arrs[j].append(part)
    df = pd.DataFrame(arrs,dtype='float64')
    df = df.T
    df.columns = cols
    if ftype=='trans':
        return df
    elif ftype=='ac':
        if units=='db':
            for col in cols:
                if df[col].str.contains(',').all():
                    df[f'Mag_{col}'] = df[col].apply(lambda x: x.split(',')[0])
                    df[f'Mag_{col}'] = df[f'Mag_{col}'].apply(lambda x: x[1:-2])
                    df[f'Mag_{col}'] = df[f'Mag_{col}'].astype('float64')
                    df[f'Phase_{col}'] = df[col].apply(lambda x: x.split(',')[1])
                    df[f'Phase_{col}'] = df[f'Phase_{col}'].apply(lambda x: x[0:-2])
                    df[f'Phase_{col}'] = df[f'Phase_{col}'].astype('float64')
        if units=='cartesian':
            for col in cols:
                if df[col].str.contains(',').all():
                    df[f'Re_{col}'] = df[col].apply(lambda x: x.split(',')[0])
                    df[f'Re_{col}'] = df[f'Re_{col}'].astype('float64')
                    df[f'Im_{col}'] = df[col].apply(lambda x: x.split(',')[1])
                    df[f'Im_{col}'] = df[f'Im_{col}'].astype('float64')
        df['Freq.'] = df['Freq.'].astype('float64')
        return df
    else:
        print('invalid ftype')
```

In [13]:
```python
def rnd(num,places,unit):
    if unit.lower()=='mhz':
        return round(num/(1e6),places)
    if unit.lower()=='ma':
        return round(num/(1e-3),places)
```

In [ ]:
```python

```