Noise Analysis of Switched Capacitor Networks

CLAUDE-ALAIN GOBET AND ALEXANDER KNOB, MEMBER, IEEE

Abstract —Noise generated in switched capacitor (SC) networks has its origin in the thermal fluctuations of charged particles in the channels of the MOS switch transistors on one hand, in the operational amplifiers on the other hand. Using a SC integrator as vehicle, it is shown that the output noise spectrum consists in general of a broad-band component due to a continuous-time noise signal and of a narrow-band contribution predominating in the baseband of the SC network resulting from a sampled-data noise signal. The ratio of undersampling is introduced and it is shown that the latter noise contribution can be evaluated by sampled-data techniques using the z-transform transfer function. These results are applied to the SC integrator and excellent concordance to the measurements made on a laboratory model is established.

I. Introduction

A SERIOUS PROBLEM arises when designing low-noise switched capacitor (SC) circuits since they produce in general higher noise levels than their RC-active counterparts. There is hence a need to understand the phenomena which lead to generation of noise in SC networks.

Although a great variety of SC analysis methods was presented in the last few years, just in a few papers [6], [7] the problem of noise has been mentioned. This is essentially due to the fact that for most methods SC networks without resistive components (i.e., transients) are assumed. For such networks, the aliasing of higher noise spectral components would result in a infinite power in the baseband. Also the fact that the bandwidth of noise signals present in SC networks exceeds in most practical cases the sampling frequency by several orders of magnitude implies that the required approach of analysis be essentially different from the case where signals satisfying the Nyquist criterion are present. The first noise calculations on SC networks have been performed by Allstot [1] for a parallel SC integrator. The output noise was attributed to the thermal noise associated with the on-resistance of the MOS switch transistors and the equivalent input noise of the MOS op amp. This latter was assumed to have a flicker component dominating in the low-frequency range. Recently, the same SC configuration has been reinvestigated independently by the authors [3] and some more accurate evaluations have been set up, without considering however the flicker noise component of the op amp. Similar results have been obtained by Furrer and Guggenbühl [4] for a SC inverting amplifier using the correlation technique and by Maloberti *et al.* [5] for a bilinear SC integrator. In the latter paper however, just the op amp's noise has been considered.

In this paper, the SC integrator investigated in [3] will be used as a vehicle to set up more general statements on SC noise analysis. In Section II it will be outlined that the output noise of a SC network can be split into a direct broad-band component which is in most practical cases negligible for the interesting frequency range up to the Nyquist rate, and into a sampled-and-held contribution due to the sampling of broad-band components. While the evaluation of the "direct noise" contribution (due to noise sources with direct output coupling in at least one phase) can be carried out by classical analysis techniques, for the determination of the latter component sampled-data techniques can be used. It will be shown furthermore, that the evaluation of the "sampled noise" component, predominating up to the Nyquist rate, can be made by considering the SC network as transientless (without resistors and op amp poles). The band limitation of the white noise sources, caused by the transients present in SC networks, is implicitely contained in the "ratio of undersampling" of the sampled broad-band noise spectrum (Section III). This method can be of great use in SC network noise analysis: it permits the noise analysis of SC networks by "classical" analysis programs which do not take into account the transients present in SC networks. Finally, the noise spectral density of a SC integrator is measured (Section IV), and its good correspondence with the analytical results confirms the noise model established.

II. SC Integrator's Noise Equivalent

The simple parallel switched integrator, as it has been implemented in the first SC realizations, is shown in Fig. 1, with the gates of the MOS transistors (MOST) controlled by two nonoverlapping clocks Φ_1, Φ_2 of period T and duty cycle Δ/T. We propose to evaluate the spectral distribution of the output noise voltage $S_n(\omega)$ for a shunted input. The output noise is assumed to have two origins: the thermal noise created in the resistance of the MOS switches and the noise generated in the operational amplifier. Since the different noise sources are uncorrelated, their contribution to the output noise spectrum can be evaluated separately.

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C.-A. Gobet is with the Electronics Laboratory of the Swiss Federal Institute of Technology Lausanne, av. de Cour 33, 1007 Lausanne, Switzerland.
A. Knob is with the Institut de Recherches Robert Bosch S.A., post-box 18, 1027 Lonay, Switzerland.

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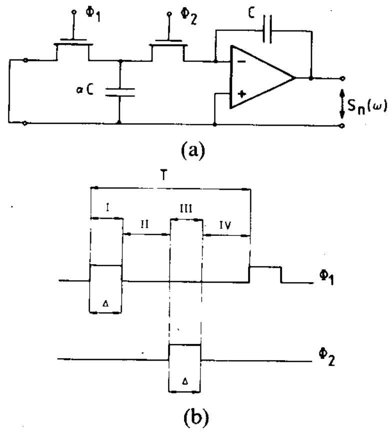

Fig. 1. Switched capacitor integrator with (a) input shunted, controlled by (b) two nonoverlapping clock signals Φ_1 and Φ_2 (transistors conducting for high level) of period T, duty cycle Δ/T and corresponding time-slots I–IV.

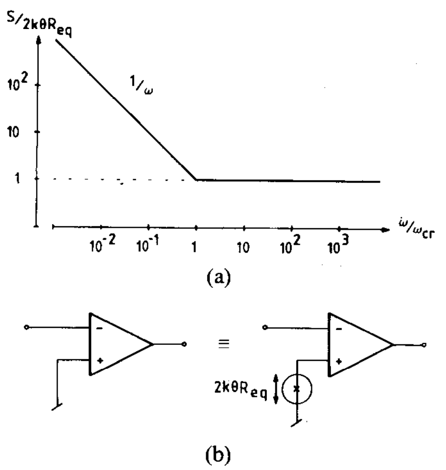

Fig. 2. Typical plot of (1) equivalent MOS op amp input noise spectral density and (b) noise model (white noise voltage source with noiseless op amp) for grounded op amp with one pole rolloff characteristic (R_{eq} is the equivalent input noise resistance, k is the Boltzmann's constant, θ is the absolute temperature).

A. Noise Model for Op Amp and MOS Switch Transistors

The equivalent input noise spectral density of a MOS operational amplifier is frequency dependent as illustrated in Fig. 2(a). The power spectral density at low frequencies is dominated by the flicker component which decreases as $1/\omega$, determined by the area of the input transistors. The spectral density at higher frequencies is constant (typically 0,1 $\mu V/\sqrt{Hz}$ for C-MOS low-power devices) and depends on the bias current of the input stage. The latter noise contribution can be represented as the thermal noise $2k\theta R_{eq}$ (bilateral representation) produced in a pseudoresistance at one input of the op amp (with k the Boltzmann's constant and θ the absolute temperature).

Depending on the passband frequency and the sampling rate of the filter, one or both components may be important in determining the dynamic range at the filter's output. However, in this paper, for the sake of simplicity, only the white noise component will be considered. This assumption is realistic in many practical cases, since the flicker contribution is submerged, as it will be stated later, by the aliased broadband components. It will be assumed furthermore that the op amp has a low-frequency gain of

Fig. 3. Noise model of the MOSFET switch transistor with $R_\Phi(t) = R_{on}$ (Φ high) and $R_\Phi(t) = R_{off}$ (Φ low), the on and off-resistances of the switch transistors, respectively.

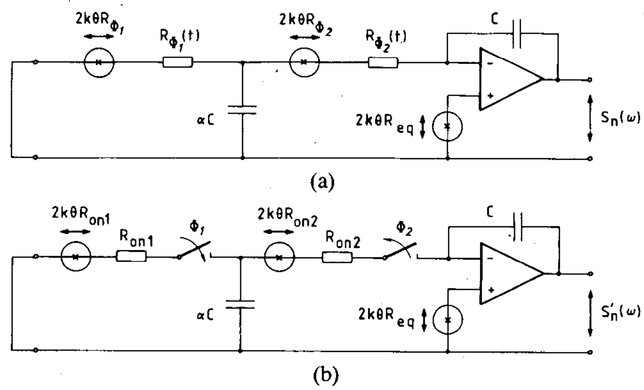

Fig. 4. SC integrator's exact time-variant noise equivalent (a) and (b) simplified noise model (noise contributions of the MOSFET transistors in the off-state neglected).

A_0 with a one pole rolloff

$$|A(\omega)| = \frac{A_0}{\sqrt{1+(A_0\omega/\omega_u)^2}} \tag{1}$$

where ω_u is the unity gain bandwidth. The noise model adopted for the op amp is shown in Fig. 2(b).

The noise contributions from the MOS switch transistors also need to be considered. The derivation of their noise model is straightforward and yields a series connected noiseless resistor R_Φ with a noise voltage source of uniform spectral density $2k\theta R_\Phi$ (see Fig. 3). The noise model depends thus on the clock signal Φ, with $R_\Phi = R_{on}$ (Φ high) and $R_\Phi = R_{off}$ (Φ low), the on and off resistances of the MOS switch transistors, respectively.

B. Complete Noise Model of the SC Integrator

By means of the results derived in the foregoing section, the noise equivalent of the SC integrator can be established in a straightforward manner, as shown in Fig. 4(a). Note that the noise equivalent is a time-variant four-phase network, depending on the time intervals I, II, III, and IV of Fig. 1. The noise signals issued from the white noise voltage sources are lowpass filtered by the different RC time constants and/or by the op amp's rolloff characteristic. Since in current MOS technologies the on and off resistances of the MOS switches differ up to ten orders of magnitude, the noise power due to the switch transistor in the off state is concentrated at extremely low frequencies. This contribution can be considered as a slowly varying offset voltage and will henceforth be ignored. Each nonconducting MOS switch transistor is therefore replaced by an open circuit, the four-phase noise equivalent is hence reduced to a resistive SC network with internal noise sources, as illustrated in Fig. 4(b). Note that the output noise spectral density $S_n'(\omega)$ consists of two components: a broadband noise contribution (due to the sources $2k\theta R_{on2}$ and $2k\theta R_{eq}$ while Φ_2 is active and only to the latter when

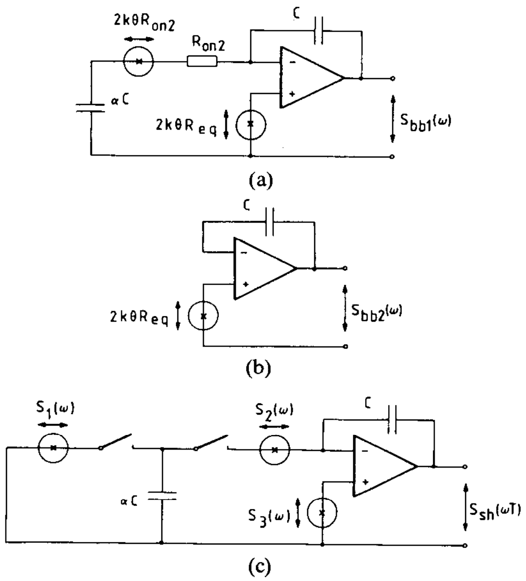

Fig. 5. Decomposition of simplified noise equivalent of Fig. 4(b) into two time-invariant subcircuits, valid for the intervals (a) Φ_2 high and (b) Φ_2 low, and (c) a two-phase transientless sampled-data network with band-limited noise sources.

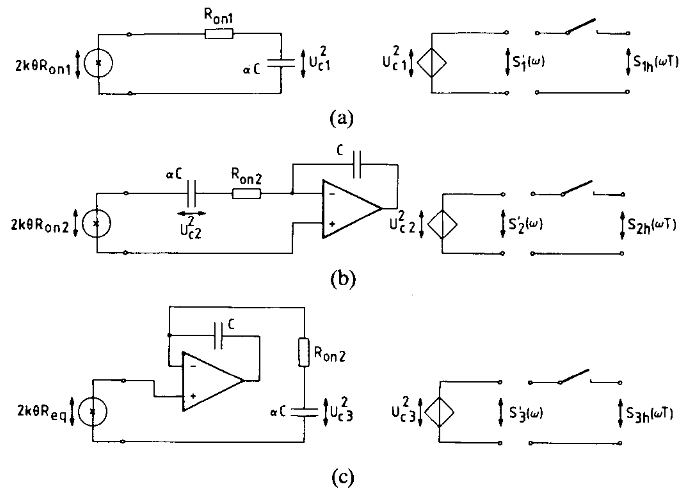

Fig. 6. Possible representation of the continuous-time broad-band contributions $S_j'(\omega)$ (for $j=1,2,3$) of the corresponding noise source $S_j(\omega)$ on the capacitor αC and theirs sampled spectra $S_{jh}(\omega T)$.

Φ_2 is inactive) and a sampled-and-held noise component due to all three noise sources at the end of phase Φ_2.

In SC networks, all time constants must be much smaller than the clock period, in order to enable a total charge transfer. This condition implies that the noise cutoff (determined by the RC time constants and the unity gain bandwidth of the op amp) is situated at higher frequencies than the sampling rate. These broad-band noise components are in consequence undersampled. This means in terms of the noise autocorrelation function that the correlation time constant is much smaller than the sampling period T. It can be assumed in consequence that the broadband noise components in the different time slots and also the sampled-and-held noise are decorrelated and hence their contributions can be evaluated separately. The SC integrator's total output noise spectral density $S_n'(\omega)$ can be determined by means of the three subcircuits shown in Fig. 5, namely

$$S_n'(\omega)=S_{\mathrm{bb}1}(\omega)+S_{\mathrm{bb}2}(\omega)+S_{\mathrm{sh}}(\omega T). \tag{2}$$

The broad-band noise components can be calculated in a straightforward manner and are given

$$S_{\mathrm{bb}1}(\omega)=\frac{\Delta}{T}\cdot\frac{k\theta}{\alpha C}\cdot\frac{\dfrac{1}{\omega_{\mathrm{on}}}\alpha^2+\dfrac{1}{\omega_{\mathrm{eq}}}\cdot\left[(1+\alpha)^2+\left(\dfrac{\omega}{\omega_u}\right)^2\left(\dfrac{\omega_u}{2\omega_{\mathrm{on}}}\right)^2\right]}{1+\left(\dfrac{\omega}{\omega_u}\right)^2\left[(1+\alpha)^2+\alpha\left(\dfrac{\omega_u}{\omega_{\mathrm{on}}}\right)+\left(\dfrac{\omega_u}{2\omega_{\mathrm{on}}}\right)^2\right]+\left(\dfrac{\omega}{\omega_u}\right)^4\left(\dfrac{\omega_u}{2\omega_{\mathrm{on}}}\right)^2} \tag{3}$$

$$S_{\mathrm{bb}2}(\omega)=\frac{T-\Delta}{T}\cdot\frac{k\theta}{\alpha C}\cdot\frac{\dfrac{1}{\omega_{\mathrm{eq}}}}{1+\left(\dfrac{\omega}{\omega_u}\right)^2} \tag{4}$$

with $\omega_{\mathrm{on}}=(2R_{\mathrm{on}2}\alpha C)^{-1}$, $\omega_{\mathrm{eq}}=(2R_{\mathrm{eq}}\alpha C)^{-1}$, and $A_0\gg 1$.

While the derivation of both continuous-time subcircuits is straightforward, the network shown in Fig. 5(c) needs some explanation: it can be depicted of Fig. 4(b) that every time a phase Φ is active, a conductive path is created by the corresponding MOS switch transistor, and the spectra of the white noise sources present in this path are low-pass filtered by the corresponding RC time constants and/or by the op amp rolloff. The resulting circulating current is creating a noise voltage across the capacitor αC. At the moment of the switch transistor's transition from the conducting to the blocked state, the instantaneous noise voltage is "frozen" on the capacitor and transferred later, as a sampled-data signal, to the output. The resulting output noise spectrum $S_{\mathrm{sh}}(\omega T)$ is hence periodic and can be evaluated with sampled-data techniques. The sampled-and-held noise S_{sh} can thus be considered as the output of the transientless SC network shown in Fig. 5(c) with three internal noise sources (S_1, S_2, S_3) of band-limited spectra. Note that a nonideal charge transfer is merely due to the finite low-frequency op amp gain A_0. The above-mentioned band-limitation of the white noise source outputs is accomplished here by the two-ports of Fig. 6. They represent between their input terminals the original SC network "seen" by the white noise source in the corresponding phases. A linear voltage source at the output, controlled by the resulting band-limited noise voltage on the capacitor αC, represents the continuous-time broad-band contribution $S_j'(\omega)$ (for $j=1,2,3$) of the corresponding noise source $S_j(\omega)$ on the capacitor, wherefrom it is transferred as a sampled-data signal to the output. Thus the output noise spectrum $S_{\mathrm{sh}}(\omega T)$ is the sum of all sampled broad-band spectra $S_{jh}(\omega T)$ multiplied by the z-transform transfer function (from capacitor αC to the output) evaluated on the unit circle. As it has been stated above, the inherent condition of total charge transfer in SC networks results in an undersampling of the different broad-band noise components. In order to evaluate $S_{jh}(\omega T)$, the spectrum of the continuous-time noise component $S_j'(\omega)$ has to be convoluted by a Dirac comb whose period represents a small fraction of the noise bandwidth. Because of this rather uncommon procedure, the next section has been devoted to highlight the undersampling effect.

III. Undersampled Broad-Band Noise Spectrum

The results presented in this section are taken from [8], where they have been derived in an extensive form. We limit us here to recall the essential steps. Let assume that $r_n(\tau)$ is the autocorrelation function of a continuous-time, stationary random signal of finite energy. By sampling this signal with period T, its autocorrelation function at time-lag $\tau = kT$ yields

$$r_{ns}(k) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta(k-n)\cdot r_n(kT). \quad (5)$$

The spectral density of the sampled random signal is given by the Fourier-transformed of (5), namely

$$FT\{r_{ns}(k)\} = \frac{1}{T} \sum_{n=-\infty}^{\infty} r_n(nT)\cdot e^{-j\omega nT} = S_{ns}(\omega T) \quad (6)$$

and is identical to the bilateral z-transform of $r_n(nT)$ evaluated on the unit circle. Let be $r_n(\tau)$ now the autocorrelation function of a first-order low-pass filtered noise with spectral density S_{n0} at the origin and with cutoff frequency ω_c. Then

$$r_n(\tau) = (\omega_c/2)\cdot S_{n0}\cdot \exp(-|\tau|\omega_c). \quad (7)$$

By introducing this expression into (6) and with the supplementary assumption that the noise signal is sampled and held with duty-cycle Δ/T, we get

$$S_{ns}(\omega T) = \omega_c \frac{T}{2}\cdot S_{n0}\cdot \frac{\sinh(\omega_c T)}{\cosh(\omega_c T) - \cos(\omega T)} \cdot (\Delta/T)^2 \operatorname{sinc}^2(\Delta\omega/2) \quad (8)$$

the spectrum of the sampled first-order low-pass filtered noise. In what follows, the value $\omega_c T$ will be called ratio of undersampling. Note that equation (8) holds for all ratios of undersampling (i.e., for the over- and the undersampled case). For $\omega_c T \gg 1$, the effect of undersampling appears clearly: the spectral density of the sampled noise at the origin is increased with respect to the continuous-time noise spectrum by the ratio of undersampling. In fact, the aliasing of the periodical spectrum occurs due to spectral components at the multiples of the sampling frequency in the initial spectrum. This is the explication why a supplementary flicker component is submerged by the aliased broad-band noise if a sufficiently strong undersampling can be assumed. Furthermore, for already low ratios of undersampling the power density of the sampled noise becomes uniform [8]. Hence for $\omega_c T > \pi$, the sampled noise spectrum is approximately white.

IV. Comparison of Theoretical Results and Measurements

A. Theoretical Results

In the foregoing two sections, all the elements have been treaten which enable the derivation of the output noise generated in a SC integrator in a closed analytical form. As it has been stated in Section II, this output noise is composed of two continuous-time broad-band components whose spectra are given by (3) and (4) and of a sampled-and-held noise contribution whose spectral density will now be evaluated.

First, the ratios of undersampling for the different broad-band noise contributions sampled on the capacitor αC have to be derived. It is evident from inspection of Fig. 6 that we are just in the case (a) in presence of a first-order low-pass filtered white noise. Hence only in this case the equations derived in Section III can be applied in a strict sense. Assuming however the cutoff frequency, formed by the RC time constants, to be several times higher than the op amp's bandwidth, the spectra in the cases (b) and (c) can be considered as approximately first-order low-pass filtered. This assumption is realistic, since in SC design the general trend goes in diminishing the capacitor values (thus the RC time constants) in order to save chip area and to achieve low-power consumption, and hence the frequency limiting element is the MOS op amp. With the above assumptions, the cutoff frequencies are

$$\begin{aligned}\omega_{ca} &= (R_{\mathrm{on}1}\alpha C)^{-1}\\ \omega_{cb} &= \omega_{cc}\\ &= \left(2R_{\mathrm{on}2}\alpha C\sqrt{1/4+\alpha(\omega_{\mathrm{on}}/\omega_u)+(\alpha+1)^2(\omega_{\mathrm{on}}/\omega_u)^2}\right)^{-1}.\end{aligned} \quad (9)$$

The square of the z-transform transfer function from capacitor αC to output evaluated on the unit circle yields (for ideal op amp)

$$|H(\omega T)|^2 = \left(\frac{\alpha}{2}\frac{1}{\sin(\omega T/2)}\right)^2. \quad (10)$$

Finally, the sampled broad-band noise spectrum can be considered as white, since for all realistic cases a sufficiently strong undersampling can be assumed. The sampled-and-held noise at the output of the SC integrator is then the sum of the three white noise spectral densities multiplied by the different ratios of undersampling, weighted by (10) and by a $\sin x/x$ due to the hold function

$$\begin{aligned}S_{\mathrm{sh}}(\omega T) &= \frac{T}{2}\cdot\left[\omega_{ca}\cdot 2k\theta R_{\mathrm{on}1} + \omega_{cb}\cdot 2k\theta(R_{\mathrm{on}2}+R_{\mathrm{eq}})\right]\cdot|H(\omega T)|^2\cdot \sin c^2(\omega T/2)\\ &= T\frac{k\theta}{\alpha C}\cdot\left[1+\frac{1+R_{\mathrm{eq}}/R_{\mathrm{on}2}}{2\sqrt{1/4+\alpha(\omega_{\mathrm{on}}/\omega_u)+(\alpha+1)^2\cdot(\omega_{\mathrm{on}}/\omega_u)^2}}\right]\cdot(\alpha/\omega T)^2.\end{aligned} \quad (11)$$

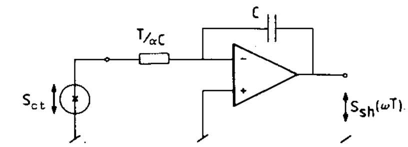

Fig. 7. Continuous-time noise model for the sampled-and-held noise contribution generated in the SC integrator of Fig. 1.

TABLE I
VALUES AND CHARACTERISTICS OF THE COMPONENTS USED FOR THE BREADBOARD REALIZATION

α	=	1
C	=	10 pF
T	=	0.1 msec
Δ/T	=	0.25
ω_u	=	$2\Pi\cdot700$ kHz
Θ	=	293 °K
R_{on2}	=	3.5 kΩ
R_{eq}	=	1.55 MΩ

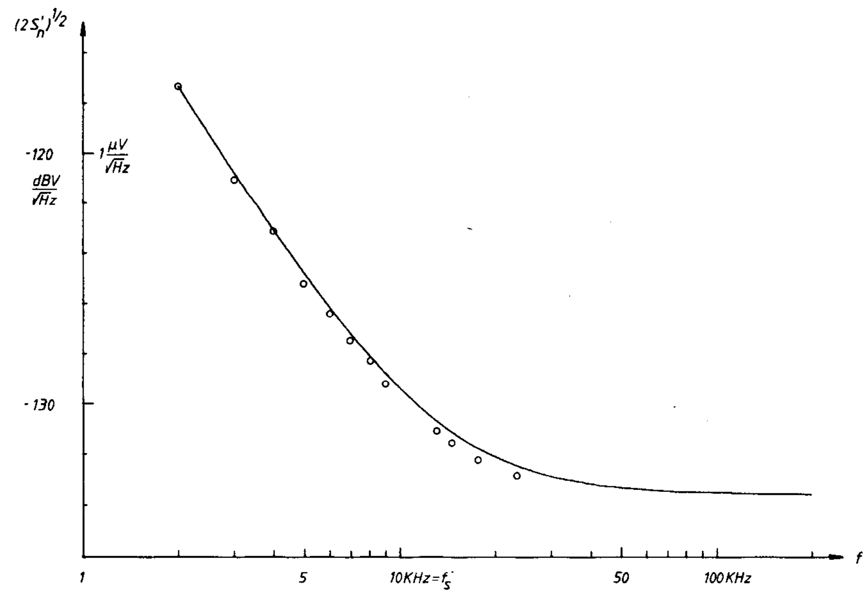

Fig. 8. Noise spectrum of the switched capacitor integrator shown in Fig. 1. —— calculated. O measured.

The expression (11) can be considered as the output noise of an ideal (noiseless) continuous-time integrator with T/α time constant and a white noise source S_{ct} at the input (Fig. 7), where

$$S_{ct}=T\frac{k\theta}{\alpha C}\cdot\left[1+\frac{1+R_{eq}/R_{on2}}{2\sqrt{1/4+\alpha(\omega_{on}/\omega_u)+(\alpha+1)^2\cdot(\omega_{on}/\omega_u)^2}}\right]. \tag{12}$$

Note that the integrator's resistor value is identical to the low-frequency ($\omega T\ll 1$) equivalent of the switched capacitor αC [9]. Comparing the broad-band noise spectra (3) and (4) with (11), it is evident that the sampled-and-held noise contribution is predominating in the interesting frequency range up to the Nyquist rate, since $R_{on2}\alpha C$, $R_{eq}\alpha C \ll T$. However, if the integrator's output is sampled, the broad-band noise component has to be considered as well.

B. Measurements on a Laboratory Model

In order to verify the noise calculations presented above, the SC integrator shown in Fig. 1 has been implemented using discrete elements. This procedure allows on the one hand to determine the characteristics of the components (switch resistances, op amp bandwidth and noise) in an easy manner separately. On the other hand, the clock feedthrough could be diminuated in this way by capacitive compensation circuits and hence the instrument sensitivities increased. The component values and characteristics are shown in Table I. The plot of $\sqrt{2S_n'}$ versus frequency (solid line) with the above values is shown in Fig. 8. Excellent correlation with the measurements made on the laboratory model can be observed in spite of the neglected op amp's flicker noise component and the small ω_{on}/ω_u ratio. Note that the switch noise contribution is 3 percent of the total output noise.

V. Conclusion

In this section, some general statements on SC noise analysis will be established on base of the specific results derived in the foregoing chapters.

Noise generated in a SC network has two origins: the thermal fluctuations associated with the channel-resistances of the MOS switch transistors and the noise created in the op amp. In a first step, two assumptions have been made.

1. Since the off resistances of the MOS switches present in conjunction with the capacitors extremely great time constants, the resulting "off noise" is considered as a slowly varying offset voltage and is hence neglected in the noise spectrum. In consequence, the noise equivalent of each MOS switch transistor consists of a white noise source, a noiseless resistor (both corresponding to the on resistance of the MOST) and an ideal switch. The initial four-phase noise equivalent of the SC circuit can in this way be reduced to a resistive two-phase SC network with internal noise sources.

2. By neglecting at the same time the op amp's flicker noise component and the noise of the output stage, all the internal noise sources can be considered as white and the op amp is assumed to have a rolloff characteristic. This assumption is justified in case that the noise corner frequency ω_{cr} (see Fig. 2(a)) is smaller than the sampling rate. In this case, the flicker noise is "submerged" by the aliased broad-band noise of the op amp (cutoff frequency approximately at the op amp's unity-gain bandwidth) and ω_{cr} shifted towards lower frequencies.

The noise at the output of the SC network consists of a continuous-time broad-band component and of a sampled-and-held noise contribution whose spectral power is concentrated in the range up to the sampling rate. The broad-band component is due to all noise sources which are coupled directly to the output in at least one phase. Since the noise sources are independent, their contribution can be evaluated separately. The cutoff frequencies of the different contributions are determined by the transients ($R_{on}C$ time constants and op amp rolloff) of the SC network valid in the corresponding phases. In order to enable total charge transfer, these transients have to be much smaller than the clock period. A first consequence is that the broad-band components in adjacent time slots can be considered decorrelated, their noise contributions can simply be added. With these assumptions, the broad-band output noise spectrum of SC networks can be evaluated by classical continuous-time analysis methods. At the moment of a MOS switch transistor's transition from the conducting to the blocked state, the instantaneous broad-band noise voltage is "frozen" on the capacitor which is in parallel to the noise source, then transferred as a sampled-data signal to the output. The SC network's output noise spectral density due to this sampled-and-held noise contribution is in consequence the spectrum of the sampled broad-band noise present on the above mentioned capacitor multiplied by the z-transform transfer function from the capacitor to the output. The condition of total charge transfer implies on one hand that the sampled-and-held noise can be considered as approximately decorrelated from the broad-band noise which it is generated from. The total noise spectral density at the output of the SC network is hence approximatively the sum of the broad-band noise spectrum and the sampled-and-held noise spectral density. The total charge transfer condition in SC networks results on the other hand in the fact that the broad-band noise sources generating the sampled-and-held noise contribution are strongly undersampled. The spectrum of a sampled first-order low-pass filtered noise has been evaluated and two important results have been established in the case of undersampling.

1) The initial continuous-time broad-band noise spectrum is increased by the factor of undersampling due to aliased broadband noise component.

2) For relatively low ratios of undersampling, the spectrum of the sampled broad-band noise becomes approximately white.

As a result of the first statement, the sampled-and-held noise is predominating in the baseband of the SC networks. The second statement implies that the shape of the sampled-and-held noise spectrum is determined by the z-transform transfer function from the capacitor in parallel with the noise source to the output, weighted by a $\sin x/x$ due to the hold function. As a result, the predominating sampled-and-held noise can be determined by SC analysis programs which do not take into account the transients in SC networks, since their band-limiting role is implicitely content in the ratio of undersampling.

As a conclusion, for SC low noise design, the following points have to be considered: (a) for a given sampling rate, the bandwidth of the op amps has to be reduced to a minimum in order to prevent excessive undersampling; (b) in either of the phases, no small valued capacitors should be present in a path which is not lowpass filtered by the rolloff characteristic of an op amp.

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Claude-Alain Gobet was born in Fribourg, Switzerland, on February 25, 1954. He received the M.S. degree in electrical engineering from the Swiss Federal Institute of Technology, Lausanne, Switzerland, in 1980.

He is presently with the Electronics Laboratory of the Swiss Federal Institute of Technology. His field of research is noise analysis of switched capacitor networks.

Alexander Knob (M'80) was born in Budapest, Hungary, in 1948. He received the Dipl. Ing. degree in electrical engineering from the Swiss Federal Institute of Technology, Zurich, Switzerland, in 1972 and the Dr. Ing. degree in signal processing from the National Institute of Technology, Grenoble, France, in 1977.

He joined the Electronics Laboratory of the Swiss Federal Institute of Technology, Lausanne, Switzerland, in 1977, where he has been involved in the analysis and synthesis of switched capacitor filters. Presently, he is working on digital TV signals processing at the Institut de Recherches Robert Bosch S.A., in Lonay, Switzerland.

Analysis of Switched-Capacitor Networks Using General-Purpose Circuit Simulation Programs

BERT D. NELIN, MEMBER, IEEE

***Abstract*—This paper presents a method for the analysis of switched-capacitor (SC) networks in the frequency and time domains by the use of general-purpose circuit simulation programs such as SPICE. It is shown how the z-domain equivalent circuit can be modeled *exactly* in the continuous-time domain using the lossless transmission line element of the simulation program to implement the required one-port storage element (storistor). The resulting continuous-time equivalent circuit retains the sample-data characteristics of the SC networks and is not restricted to frequencies below the Nyquist rate.**

It is also shown that the transmission line element can be used to simulate the $(\sin x)/x$ effect on the frequency response of the SC network due to the sample-and-hold process. Examples of the simulation of simple SC networks with and without op amps using SPICE are given.

I. Introduction

In [1], a method for analyzing switched-capacitor (SC) networks has been reported which develops continuous-time equivalent circuits of basic SC structures. These equivalent circuits contain voltage controlled current sources, resistors, capacitors, and frequency dependent resistors. An approximate model of the frequency dependent resistor allows the SC network to be simulated by a general-purpose circuit simulation program. However, in the process of obtaining the equivalent structures, the sample-data characteristics of the SC network are lost. Thus this method is only useful for frequencies below the Nyquist rate and for those cases where the sampling rate is so great, that errors in the obtained frequency response at frequencies close to the Nyquist rate can be neglected.

Reference [2] introduces a bilinear transformation from the z-domain to the s-domain which results in continuous-time equivalent circuits of SC structures. However, due to the transformation, the frequency scale is warped, and analysis in the time domain is not possible.

In this paper, we start with the z-domain equivalent circuits presented in [3], [4], [5] and used extensively in [5], [6]. These equivalent circuits contain voltage sources, ideal op amps, capacitors, resistors, and storage elements called

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The author is with Sensormatic Electronics Corporation, Deerfield Beach, FL 33441.

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