EE 538: Low-Noise Analog Circuit Design

Spring 2021

Instructor: Jason Silver

Announcements

- Instructor office hours moved to Saturdays at 11am
- Assignment 3 due Monday, April 26 at midnight
- Assignment 4 will be posted Saturday, April 24

Week 4

- Motchenbacher Chapter 5
- Art of Electronics Chapter 8

Overview

- Last time...
  - Hybrid-π model
  - BJT noise model
  - BJT noise figure
  - Noise in common-emitter structures
- Today...
  - FET large/small-signal operation
  - Thermal noise in FET devices
  - 1/f noise and gate current noise
  - FET noise model
  - FET vs BJT

Python packages/modules

In [23]:
```
import matplotlib as mpl
from matplotlib import pyplot as plt
from matplotlib import ticker, cm
import numpy as np
from scipy import signal
from scipy import integrate
#%matplotlib notebook

mpl.rcParams['font.size'] = 14
mpl.rcParams['legend.fontsize'] = 'large'

def plot_xy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5));
    ax.plot(x, y, 'b')
    ax.grid()
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)

def plot_xy2(x1, y1, x1label, y1label, x2, y2, x2label, y2label):
    fig, ax = plt.subplots(2, figsize = (10.0, 7.5));
    ax[0].plot(x1, y1, 'b')
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].plot(x2, y2, 'b')
    ax[1].set_xlabel(x1label)
    ax[1].set_xlabel(x2label)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    fig.align_ylabels(ax[:])

def plot_xy3(x, y1, y2, y3, xlabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(3, figsize=(10.0,7.5))

    ax[0].plot(x, y1)
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].plot(x, y2)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    ax[2].plot(x, y3)
    ax[2].set_ylabel(y3label)
    ax[2].set_xlabel(xlabel)
    ax[2].grid()

def plot_logxy3(x, y1, y2, y3, xlabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(3, figsize=(10.0,7.5))

    ax[0].semilogx(x, y1)
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].semilogx(x, y2)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    ax[2].semilogx(x, y3)
    ax[2].set_ylabel(y3label)
    ax[2].set_xlabel(xlabel)
    ax[2].grid()

def plot_logxy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    ax.semilogx(x, y, 'b')
    ax.grid();
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)

def plot_log3xy(x, y1, y2, y3, xlabel, ylabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(figsize=(10.0,7.5))

    ax.semilogx(x, y1, 'b', label=y1label)
    ax.semilogx(x, y2, 'r', label=y2label)
    ax.semilogx(x, y3, 'g', label=y3label)
    ax.set_ylabel(ylabel)
    ax.set_xlabel(xlabel)
    ax.grid()

    ax.legend()
    ax.legend(loc='upper center', ncol=3, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_log2xy(x, y1, y2, xlabel, ylabel, y1label, y2label):
    fig, ax = plt.subplots(figsize=(10.0,7.5))

    ax.semilogx(x, y1, 'b', label=y1label)
    ax.semilogx(x, y2, 'r', label=y2label)
    ax.set_ylabel(ylabel)
    ax.set_xlabel(xlabel)
    ax.grid()

    ax.legend()
    ax.legend(loc='upper center', ncol=2, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_loglog(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    ax.loglog(x, y, 'b')
    ax.grid();
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)


def plot_loglog2(x, y1, y2, xlabel, ylabel, y1label, y2label):
    fig, ax = plt.subplots(figsize=(10.0,7.5))

    ax.loglog(x, y1, 'b', label=y1label)
    ax.loglog(x, y2, 'r', label=y2label)
    ax.set_ylabel(ylabel)
    ax.set_xlabel(xlabel)
    ax.grid()

    ax.legend()
    ax.legend(loc='upper center', ncol=2, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_xlogy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5));
    ax.semilogy(x, y, 'b');
    ax.grid();
    ax.set_xlabel(xlabel);
    ax.set_ylabel(ylabel);

def read_ltspice_ac(file_name):
    with open(file_name, 'r') as data:
        x = []
        y = []
        z = []
        next(data) # skip header line
        for line in data:
            p = line.split()
            x.append(float(p[0]))
            complex = p[1].split(",")
            y.append(float(complex[0]))
            z.append(float(complex[1]))

    return x, y, z

def plot_logxy2(x1, y1, x2, y2, x1label, y1label, x2label, y2label):
    fig, ax = plt.subplots(2, figsize = (10.0, 7.5));
    ax[0].semilogx(x1, y1, 'b');
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].semilogx(x2, y2, 'b');
    ax[1].set_xlabel(x1label)
    ax[1].set_xlabel(x2label);
    ax[1].set_ylabel(y2label);
    ax[1].grid();

    fig.align_ylabels(ax[:])

def plot_noise_bandwidth(f, mag):
    fig, ax = plt.subplots(2, figsize=(10.0,7.5))
    ax[0].semilogx(f, RC_mag)
    ax[0].set_xscale("log")
    ax[0].set_xlim(f[0], f[-1])
    ax[0].set_xticks(np.logspace(0.1,4,5))
    ax[0].set_xticklabels([])
    ax[0].set_ylabel('Magnitude [V/V]')
    ax[0].set_title('Equivalent Noise Bandwidth')
    ax[0].grid()

    ax[1].hlines(1, 0, f_enb, color='tab:blue')
    ax[1].hlines(0, f_enb, f[-1], color='tab:blue')
    ax[1].vlines(f_enb, 0, 1, color='tab:blue')
    ax[1].set_xlim(f[0], f[-1])
    ax[1].set_xscale("log")
    ax[1].set_xticks(np.logspace(0.1,4,5))
    ax[1].set_xticklabels([r'$10^0$',r'$10^1$', r'$10^2$', r'$10^3$', r'$10^4$'])
    ax[1].set_ylabel('Magnitude [V/V]')
    ax[1].set_xlabel('Frequency [Hz]')
    ax[1].grid()

def noise_hist(vnoise, vn_rms, bins):
    fig = plt.figure( figsize=(10.0,7.5) )
    vn_norm = vnoise/ vn_rms
    ax = fig.add_subplot(111)
    n, bins, rectangles = ax.hist(vn_norm, bins, density=True, range=(-3, 3),
                                  color='b')
    ax.set_xlabel(r'Sample Voltage [$v_{n(rms)}$]')
    ax.set_ylabel('Probability Density')
    ax.grid()
    fig.canvas.draw()

def plot_NF_vs_Rs(en_vals, in_vals, Rs_min, Rs_max, T_in_K):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    k = 1.38e-23
    Rs = np.logspace(np.log10(Rs_min), np.log10(Rs_max), num=200)
    F1 = 1 + (en_vals[0]**2+Rs**2*in_vals[0]**2)/(4*k*T_in_K*Rs)
    F2 = 1 + (en_vals[1]**2+Rs**2*in_vals[1]**2)/(4*k*T_in_K*Rs)
    F3 = 1 + (en_vals[2]**2+Rs**2*in_vals[2]**2)/(4*k*T_in_K*Rs)
    ax.semilogx(Rs, 10*np.log10(F1), 'b', label=r'$e_{n1}$, $i_{n1}$')
    ax.semilogx(Rs, 10*np.log10(F2), 'r', label=r'$e_{n2}$, $i_{n2}$')
    ax.semilogx(Rs, 10*np.log10(F3), 'g', label=r'$e_{n3}$, $i_{n3}$')
    ax.grid();
    ax.set_xlabel(r'Source Resistance $R_s [\Omega]$')
    ax.set_ylabel(r'Noise Figure $NF$ [$dB$]')

    ax.legend()
    ax.legend(loc='upper center', ncol=3, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_noise_curve(e_n, i_n, Rs_min, Rs_max):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    Rs = np.logspace(np.log10(Rs_min), np.log10(Rs_max), num=200)
    e_ni_2 = 4*k*T*Rs + e_n**2 + i_n**2*Rs**2
    ax.loglog(Rs, np.sqrt(e_ni_2), 'b', label='Total Noise')
    ax.loglog(Rs, np.sqrt(4*k*T*Rs), 'r', label=r'$\sqrt{4kTR_s}$')
    ax.loglog(Rs, e_n*np.ones(np.size(Rs)), 'g', label=r'$e_n$')
    ax.loglog(Rs, i_n*Rs, 'y', label=r'$i_n R_s$')
    ax.grid();
    ax.set_xlabel(r'Source Resistance $R_s [\Omega]$')
    ax.set_ylabel(r'Equivalent Input Noise [$V/\sqrt{Hz}$]')

    ax.legend()
    ax.legend(loc='upper center', ncol=4, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_bjt_NF(beta, r_bb, Rmin, Rmax, Imin, Imax):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    k = 1.38e-23
    T = 300
    q = 1.602e-19
    V_T = k*T/q
    rs = np.logspace(np.log10(Rmin), np.log10(Rmax), num = 100)
    ic = np.logspace(np.log10(Imin), np.log10(Imax), num = 100)
    I_C, R_S = np.meshgrid(ic, rs)
    e_n_2 = 4*k*T*(V_T/2/I_C + r_bb)
    i_n_2 = 2*q*I_C/beta_0
    NF = 1 + (e_n_2 + i_n_2*R_S**2)/(4*k*T*R_S)
    cp = ax.contourf(I_C, R_S, 10*np.log10(NF), levels=np.linspace(0,15, num=16))
    plt.xscale('log')
    plt.yscale('log')
    plt.ylabel(r'Source Resistance $R_s$ [$\Omega$]')
    plt.xlabel(r'Collector Current $I_C$ [A]')
    fig.colorbar(cp)
```

In [2]:
```
def fftnoise(f):
    f = np.array(f, dtype='complex')
    Np = (len(f) - 1) // 2
    phases = np.random.rand(Np) * 2 * np.pi
    phases = np.cos(phases) + 1j * np.sin(phases)
    f[1:Np+1] *= phases
    f[-1:-1-Np:-1] = np.conj(f[1:Np+1])
    return np.fft.ifft(f).real

def band_limited_noise(min_freq, max_freq, samples=1024, samplerate=1):
    freqs = np.abs(np.fft.fftfreq(samples, 1/samplerate))
    f = np.zeros(samples)
    idx = np.where(np.logical_and(freqs>=min_freq, freqs<=max_freq))[0]
    f[idx] = 1
    return fftnoise(f)
```

Lecture 4 - Noise in Field Effect Transistors

MOSFET operation (NMOS)

- $V_{gs} > V_{th}$ creates a vertical electric field attracting minority carriers from the source/drain to form a conductive channel in the bulk region (i.e. the conductivity of the channel is controlled by V_{gs})
- $V_{ds} > 0$ generates an lateral electric field causing electrons in the channel to drift from source to drain
- Saturation occurs for $V_d > V_{ov} = V_{gs} - V_{th}$, at which point the drain current magnitude is predominantly determined by V_{gs} and the effect of V_{ds} on I_d is reduced

MOSFET large-signal model

$$I_d = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs} - V_{th})^2(1 + \lambda V_{ds})$$
$$I_g = 0 \qquad I_s = I_d \tag{1}$$
$$V_{gs} = \sqrt{\frac{2I_d}{\mu C_{ox}\frac{W}{L}}} + V_{th}$$

- In saturation drain current (I_d) is a quadratic function of V_{gs} and (secondarily) a linear function of V_{ds}
- $I_g = 0$ at DC and increases with frequency due to gate oxide capacitance
- If I_d is the independent variable (assuming $\lambda = 0$), V_{gs} varies as the square root of I_d
- If derived from first principles (and neglecting channel-length modulation), the long-channel drain current expression of the MOSFET can be expressed as

$$I_d = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs} - V_{th})^2 \tag{2}$$

- This expression can be decomposed into the effects of 2 electric fields. The first is a vertical electric field that controls the channel charge:

$$C_{ox}W(V_{gs} - V_{th}) \tag{3}$$

- The second field is the lateral field that controls charge velocity ($v = \mu E$), assuming the potential at the edge of the channel is equal to $V_{gs} - V_{th}$

$$\mu\frac{(V_{gs} - V_{th})}{L} \tag{4}$$

FET small-signal model

$$g_m \equiv \frac{\partial I_D}{\partial V_{GS}} = \frac{2I_D}{V_{gs} - V_{th}} = \frac{2I_D}{V_{ov}} \tag{5}$$

$$r_o \equiv \frac{\partial V_{DS}}{\partial I_D} = \frac{1}{\lambda I_D} \tag{6}$$

- The transconductance g_m is the derivative of drain current with respect to gate-source voltage
- r_o is the small-signal output resistance of the transistor which captures the dependence of i_d on v_{gs}
- r_o is primarily associated with the phenomenon of *channel-length modulation*, the influence of the drain voltage on the effective length of the channel

FET capacitances

$$C_{gs} = \frac{2}{3}WLC_{ox} \tag{7}$$

$$C_{gd} = WC_{ov} \tag{8}$$

- The high frequency behavior of the FET, like that of the BJT, is governed by two primary capacitances C_{gs} and C_{gd}
- In strong inversion C_{gs} assumes the majority of the the total gate oxide capacitance (WLC_{ox}, where $[C_{ox}] = F/m^2$)
- C_{gd} comprises the "overlap" capacitance arising along the edge of the gate-drain interface ($[C_{ov}] = F/m$)
- In most practical cases of interest, $C_{gs} \gg C_{gd}$, though C_{gd} can play a significant role due to a phenomenon known as the "Miller effect" in which the apparent capacitance contributed by C_{gd} is "amplified" by the voltage gain of the transistor/amplifier

C_{gd}, not C_{ds}

FET transit frequency

- The FET transit frequency f_T is defined as the frequency at which the short-circuit current gain is unity:

$$f_T = \frac{g_m}{2\pi \cdot (C_{gs} + C_{gd})} \tag{9}$$

- f_T constitutes the maximum frequency at which the transistor is able to provide power gain - beyond this frequency, it loses its utility as a gain device
- As with the BJT, f_T increases with I_D, with more power being required to operate at higher speeds
- Transit frequency often serves as a technology characterization metric, and is a useful means of comparing different devices/technologies
- Assuming $C_{gs} \gg C_{gd}$, the transit frequency of the MOSFET in strong inversion can be expressed as

$$f_T = \frac{g_m}{2\pi \cdot (C_{gs} + C_{gd})} \approx \frac{\mu C_{ox}(W/L)(V_{gs} - V_{th})}{2\pi \cdot (2/3) \cdot WLC_{ox}} = \frac{3}{2}\frac{\mu \cdot (V_{gs} - V_{th})}{2\pi \cdot L^2} \tag{10}$$

- From this expression we gather that the speed of a MOS device depends significantly on the gate length, and to a lesser extent, on mobility and overdrive voltage

MOSFET as a resistance/switch

- In linear/triode operation, the drain current is heavily influenced by the modulation of the lateral electric field with V_{ds} and can be expressed as

$$I_d = \mu C_{ox}\left(\frac{W}{L}\right)\left[(V_{gs} - V_{th})V_{ds} - \frac{V_{ds}^2}{2}\right] \tag{11}$$

- For $V_{ds} \ll V_{gs} - V_{th}$, this can be approximated by a linear function in V_{ds}

$$I_d \approx \mu C_{ox}\left(\frac{W}{L}\right)(V_{gs} - V_{th})V_{ds} \tag{12}$$

- From this expression we can defined an effective resistance given by

$$r_{ds} = \frac{1}{\mu C_{ox}\left(\frac{W}{L}\right)(V_{gs} - V_{th})} \tag{13}$$

- This is the expression typically used to describe MOS behavior when operated as a switch

JFET operation (NFET)

- $V_g \le 0$ reverse-biases the gate-channel junction and modulates the conductivity of the channel by increasing/decreasing the width of the depletion region(s)
- $V_d > 0$ produces a lateral electric field and causes majority carriers (electrons in this case) to drift between source and drain, in addition to modulating the depletion region width near the drain
- Saturation occurs for $V_d > V_P$, where V_P is referred to as the "pinch-off" voltage, at which point V_d has diminished influence on the field experienced by mobile charge carriers in the non-depleted region of the channel

JFET model

$$I_d = I_{DSS}\left(1 - \frac{V_{gs}}{V_p}\right)^2$$
$$g_m = \frac{2I_{DSS}}{V_p}\left(1 - \frac{V_{gs}}{V_p}\right) \tag{14}$$
$$= \frac{2I_d}{V_p - V_{gs}} \tag{15}$$

- In saturation, drain current (I_d) is a quadratic function of V_{gs}
- $I_g \neq 0$ at DC, but because the junction is reverse-biased it is small (pA) at low frequencies
- V_p is referred to as the "pinch-off" voltage, the value of V_{gs} at which the channel thickness goes to zero

FET channel noise

- The primary noise mechanism in the MOSFET (and JFET) is the thermally-induced movement of mobile charge carriers in the channel
- In the MOSFET, the channel charge is modulated by an electric field that attracts minority carriers from the source and drain to the region of the bulk directly under the gate
- In the JFET, the applied gate voltage increases/decreases the width of the depletion layer(s) near the gate contact(s), decreasing/increasing the effective height of the channel and modulating the conductivity
- In this sense, prior to saturation, both devices can be viewed as resistances whose magnitude is controlled by the gate voltage

Thermal noise in a resistor

- The mean-square current spectral density of noise due to thermal fluctuation of charge carriers in a resistance R is expressed as

$$i_n^2 = \frac{4kT}{R}\ A^2/Hz \tag{16}$$

- Resistance can be expressed in terms of material parameters μ_c (carrier mobility) and n_c (carrier concentration), legnth L, and cross-sectional area A

$$R = \frac{L}{\sigma A} = \frac{L}{qn_c\mu_c A} \tag{17}$$

- The *total charge* in the volume defined by $A \cdot L$ is given by

$$Q_{tot} = qn_cAL \tag{18}$$

- The spectral density of the noise can be expressed in terms of the total charge as

$$i_n^2 = \frac{4kT}{R} = 4kT\frac{qn_c\mu_cA}{L} = 4kT\frac{\mu_cQ_{tot}}{L^2}\ A^2/Hz \tag{19}$$

MOSFET channel charge

- The inversion charge in the conducting channel of a MOSFET is a function of the gate capacitance C_{ox} and the applied voltage (above threshold) $V_{gs} - V_{th}$
- In the linear mode of operation, when $V_{ds} \ll V_{gs} - V_{th}$ the capacitance and charge are approximately uniform along the channel and the charge is given by $Q_{lin} = WLC_{ox}(V_{gs} - V_{th})$
- In saturation, the capacitance is non-uniform between the source and drain and the total charge can be approximated as $Q_{sat} = \frac{2}{3}WLC_{ox}(V_{gs} - V_{th})$

Channel thermal noise

- In triode/linear operation, the spectral density of the thermal noise can be expressed in terms of the channel charge as

$$i_{nd}^2 = 4kT\frac{\mu Q_{lin}}{L^2} = 4kT\mu C_{ox}\frac{W}{L}(V_{gs} - V_{th})\ A^2/Hz \tag{20}$$

- The noise is often expressed in terms of g_{d0}, the derivative of the drain current with respect to V_{ds} when $V_{ds} = 0$:

$$g_{d0} \equiv \left.\frac{dI_d}{dV_{ds}}\right|_{V_{ds}=0} = \mu C_{ox}\frac{W}{L}(V_{gs} - V_{th}) \tag{21}$$

- This gives

$$i_{nd}^2 = 4kT\mu C_{ox}\frac{W}{L}(V_{gs} - V_{th}) = 4kTg_{d0}\ A^2/Hz \tag{22}$$

- In saturation the drain current noise can be expressed as

$$i_{nd}^2 = 4kT\frac{\mu Q_{sat}}{L^2} = 4kT\gamma g_{d0}\ A^2/Hz \tag{23}$$

- It is often assumed that $g_m \approx g_{d0}$, resulting in the common expression

$$i_{nd}^2 = 4kT\gamma g_m\ A^2/Hz \tag{24}$$

- The factor γ is sometimes called the "excess noise factor" and is typically taken to be $\frac{2}{3}$ for long-channel devices
- Particularly for shorter channel lengths (i.e. $L \ll 10\mu m$), velocity saturation and other effects cause an increase in the observed noise, resulting in excess noise factors as high as $2 - 3$

JFET thermal noise

- Measured noise of discrete JFETs shows decent conformity to the to the $\gamma = \frac{2}{3}$ prediction at frequencies well above the $1/f$ corner
- *Figure source: Art of Electronics, Third Edition*

1/f noise

- MOSFET drain current exhibits $1/f$ noise due to the trapping/release of charge carriers primarily at the $\mathrm{Si} - \mathrm{SiO_2}$ interface, producing current noise with a $1/f$ characteristic

$$i_{nf}^2 = \frac{K_f}{f} \cdot \frac{g_m^2}{WLC_{ox}} \tag{25}$$

- $1/f$ noise in MOSFETs can be reduced by increasing gate area (as seen in the above expression), though this comes at the expense of increased gate capacitance and a reduced f_T
- $1/f$ noise corners for MOSFETs are substantially higher that those of BJT's, and this tends to be limiting factor when considering their use at low frequencies
- JFET's also exhibit $1/f$ noise, though this tends to be more subdued than in MOSFET's
- Many devices produce low-frequency noise that does not neatly conform to the $1/f$ model, and in these cases it is common to see noise reported as integrated or peak-to-peak noise within a given frequency range

BJT, MOSFET, JFET 1/f comparison

- At low to moderate frequencies, the high $1/f$ noise corners of MOSFETs makes them far inferior to both BJT and JFET devices
- *Figure source: Art of Electronics, Third Edition*

Gate current shot noise

- Gate current in MOSFETs is due to charge leakage through the silicon dioxide gate to the source region, and can be expressed as

$$i_{ng}^2 = 2qI_G \tag{26}$$

- MOSFET gate current is typically extremely small (measured in fA), so this noise current is in most cases negligible
- JFET gate current is due to the leakage current of the reverse-biased pn junction comprising the gate, and tends to be somewhat higher than that of the MOSFET
- Gate leakage in both devices increases with temperature, making gate current noise in FET devices highly temperature-dependent

Miller effect

- Another source of gate current noise can be explained with the help Miller's theorem, which decomposes the impedance between the input and output nodes of an inverting votlage amplifier (Z) into equivalent input and output impedances to ground (Z_1 and Z_2)
- For a given input voltage v_i, the gain of the amplifier increases the effective current through Z, lowering the impedance "seen" by the input voltage
- Using Miller's theorem it can be shown that $Z_1 = Z/(1 + A_v)$

Excess input current noise

- Assuming a feedback capacitance C_{fb} and a voltage gain of g_mZ_L, where Z_L is the parallel combination of R_L and C_L, it can be shown that the effective input admittance $Y_1 = 1/Z_1$ due to the Miller effect is given by

$$Y_1 = \frac{-\omega^2R_LC_LC_{fb} + \omega^2C_{fb}(1 + g_mR_L)R_LC_L + j\omega C_{fb}[1 + g_mR_L + \omega^2R_L^2C_L^2]}{1 + \omega^2R_L^2C_L^2} \tag{27}$$

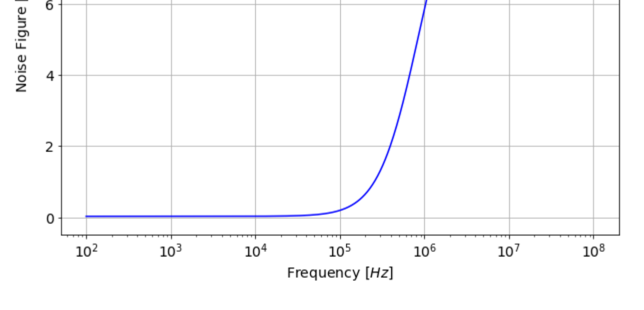
- The real portion of Y_1 is given by

$$G_1 = \frac{1}{R_1} = \frac{\omega^2g_mR_L^2C_{fb}C_L}{1 + \omega^2R_L^2C_L^2}\ [\Omega^{-1}] \tag{28}$$

- This conductance/resistance is real, and thus produces thermal noise given by

$$i_n^2 = 4kTG_1 = 4kT\frac{\omega^2g_mR_L^2C_{fb}C_L}{1 + \omega^2R_L^2C_L^2}\ A^2/Hz \tag{29}$$

In [181…
```
f = np.logspace(2,8,num=800)
w = 2*np.pi*f
gm = 10e-3
RL = 10e3
Rs = 10e3
CL = 10e-12
Cfb = 1e-12
G_1 = (w**2*gm*RL**2*Cfb*CL)/(1+w**2*RL**2*CL**2)
i_n = np.sqrt(2*q*10e-12 + 4*k*T*G_1)
e_n = 1e-9
e_ns = np.sqrt(4*k*T*Rs)
F = 1+(e_n**2+i_n**2*Rs**2)/(e_ns**2)
plot_logxy(f, 10*np.log10(F), r'Frequency [$Hz$]', r'Noise Figure [$dB$]')
```



- Using reasonable values for g_m, C_L, R_L, and C_{fb}, we see that the current noise starts to degrade the noise figure above $100kHz$, an effect which depends on the source resistance ($10k\Omega$ in this case)

FET noise sources

- FET drain current noise arises due to the resistivity of the channel and $1/f$ noise and is expressed as

$$i_{nd}^2 = 4kT\gamma g_m \quad i_{nf}^2\frac{K_f}{f} \cdot \frac{g_m^2}{WLC_{ox}} \tag{30}$$

- FET gate current noise is primarily due to shot noise and thermal noise due to the Miller effect (only significant at high frequencies)

$$i_{ng}^2 = 2qI_G + 4kT\frac{\omega^2g_mR_L^2C_{fb}C_L}{1 + \omega^2R_L^2C_L^2} \tag{31}$$

FET equivalent noise model

- The equivalent voltage noise is determined by reflecting the drain current noise through g_m:

$$e_n^2 = \frac{4kT\gamma}{g_m} + \frac{K_f}{f} \cdot \frac{1}{WLC_{ox}} \tag{32}$$

- The equivalent current noise is due both to gate leakage shot current and the Miller effect:

$$i_n^2 = 2qI_G + 4kT\frac{\omega^2g_mR_L^2C_{fb}C_L}{1 + \omega^2R_L^2C_L^2} \tag{33}$$

Voltage noise in FETs versus BJTs

- At moderate frequencies FET input voltage noise is dominated by the thermal noise of the channel

$$i_{nd}^2 = 4kT\gamma g_m \rightarrow e_n^2 = \frac{i_{nd}^2}{g_m^2} = \frac{4kT\gamma}{g_m} = \frac{2kT\gamma}{I_D} \cdot V_{OV} \tag{34}$$

- Similarly, assuming $r_e/2 \gg r_b$, BJT voltage noise is dominated by collector current shot noise

$$i_{nc}^2 = 2qI_C = 2kTg_m \rightarrow e_n^2 = \frac{i_{nc}^2}{g_m^2} = \frac{2kT}{g_m} = \frac{2kT}{I_C} \cdot V_T \tag{35}$$

- To compare the two device types, the concept of *transconductance efficiency* proves useful, the ratio of transconductance to device current
- For the MOSFET, the transconductance efficiency in strong inversion is given by

$$\frac{g_m}{I_D} = \frac{2I_D/V_{ov}}{I_D} = \frac{2}{V_{ov}}\ S/A \tag{36}$$

- Though V_{ov} varies based on device fabrication and sizing, a representative range of values for moderate drain currents (i.e. milliamps) is $0.2 - 1\mathrm{V}$, resulting in g_m/I_D values from $2 - 10\ \mathrm{S/A}$
- For the BJT, transconductance efficiency in the forward active mode is

$$\frac{g_m}{I_C} = \frac{1}{V_T} = \frac{q}{kT} \tag{37}$$

- At $300K$, $g_m/I_C \approx 40\mathrm{S/A}$, making BJT's far superior to FETs in terms of transconductance efficiency
- To achieve higher values of g_m/I_D, FETs must be operated with lower current densities, increasing gate capacitance and impacting bandwidth/stability
- Returning to the question of input voltage noise (still neglecting r_b), we have

$$e_{n,BJT}^2 = \frac{2kT}{g_m} \quad e_{n,FET}^2 = \frac{4kT\gamma}{g_m} \tag{38}$$

- Using the concept of transconductance efficiency we can rewrite these expressions as

$$e_{n,BJT}^2 = \frac{2kT}{(g_m/I_C) \cdot I_C} \quad e_{n,FET}^2 = \frac{4kT\gamma}{(g_m/I_D) \cdot I_D} \tag{39}$$

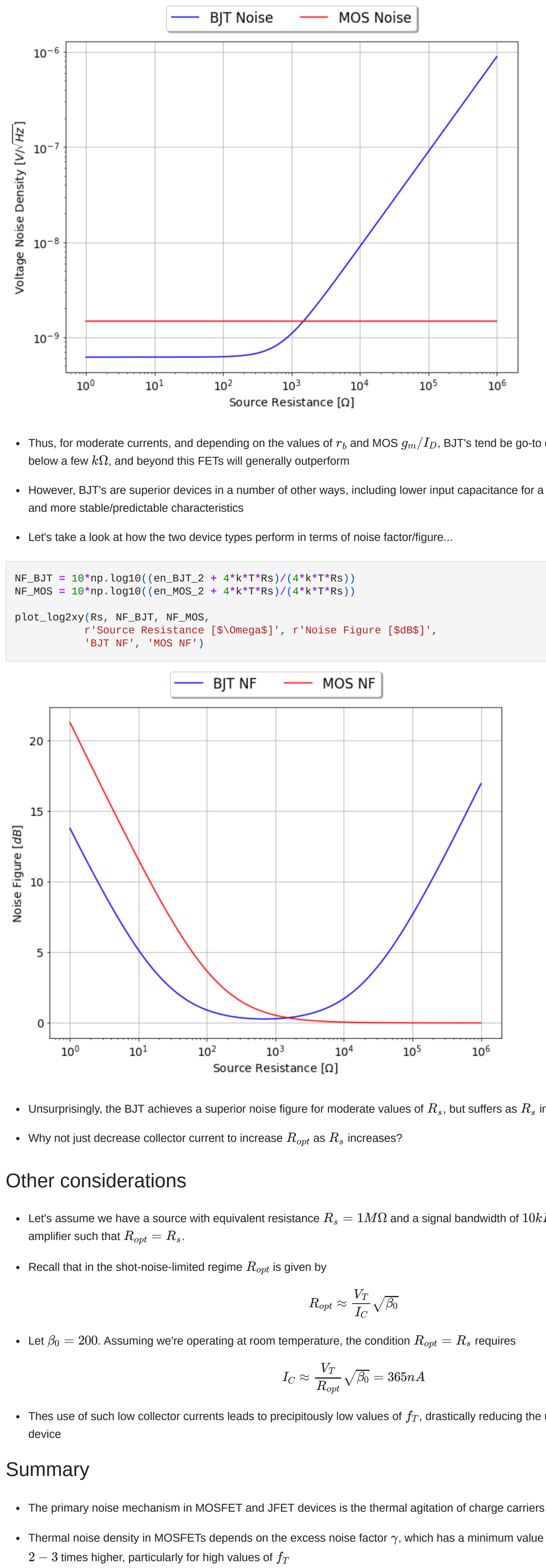
- As an example, assuming $I_D = I_C = 1mA$ and $g_m/I_D = 5\ \mathrm{S/A}$ results in $e_{n,BJT} \approx 0.5nV/\sqrt{Hz}$ and $e_{n,MOS} \approx 1.5nV/\sqrt{Hz}$
- For increased values of f_T, the required reduction in channel length results in an increase in the excess noise factor γ, further increasing the gap between the two types of devices
- As a result, for low to moderate source impedances (assuming small r_b), a BJTs typically significantly outperform MOSFETs

Effect of source impedance

- Assuming an ideal BJT (i.e. one with no base resistance or $1/f$ noise), we can compare the noise performance of BJT and FET devices

In [75]:
```
Rs = np.logspace(0, 6, num=600)
IC = 1e-3
ID = IC
beta = 400
r_b = 10
gm_over_ID = 5
gm_BJT = IC/(k*T/q)
gm_MOS = gm_over_ID*ID
in_BJT_2 = 2*q*IC/beta
en_BJT_2 = 4*k*T*r_b + 2*k*T/gm_BJT + in_BJT_2*Rs**2
en_MOS_2 = 4*k*T*gamma/gm_MOS*np.ones(np.size(Rs))
```

In [76]:
```
plot_loglog2(Rs, np.sqrt(en_BJT_2), np.sqrt(en_MOS_2),
             r'Source Resistance [$\Omega$]', r'Voltage Noise Density [$V/\sqrt{Hz}$]',
             'BJT Noise', 'MOS Noise')
```



- Thus, for moderate currents, and depending on the values of r_b and MOS g_m/I_D, BJT's tend be go-to devices for source resistances below a few $k\Omega$, and beyond this FETs will generally outperform
- However, BJT's are superior devices in a number of other ways, including lower input capacitance for a given g_m, lower $1/f$ noise, and more stable/predictable characteristics
- Let's take a look at how the two device types perform in terms of noise factor/figure...

In [77]:

```
NF_BJT = 10*np.log10((en_BJT_2 + 4*k*T*Rs)/(4*k*T*Rs))
NF_MOS = 10*np.log10((en_MOS_2 + 4*k*T*Rs)/(4*k*T*Rs))

plot_log2xy(Rs, NF_BJT, NF_MOS,
            r'Source Resistance [$\Omega$]', r'Noise Figure [$dB$]',
            'BJT NF', 'MOS NF')
```


- Unsurprisingly, the BJT achieves a superior noise figure for moderate values of R_s, but suffers as R_s increases
- Why not just decrease collector current to increase R_{opt} as R_s increases?

Other considerations

- Let's assume we have a source with equivalent resistance $R_s = 1M\Omega$ and a signal bandwidth of $10kHz$. We could design our BJT amplifier such that $R_{opt} = R_s$.
- Recall that in the shot-noise-limited regime R_{opt} is given by

$$R_{opt} \approx \frac{V_T}{I_C}\sqrt{\beta_0} \tag{40}$$

- Let $\beta_0 = 200$. Assuming we're operating at room temperature, the condition $R_{opt} = R_s$ requires

$$I_C \approx \frac{V_T}{R_{opt}}\sqrt{\beta_0} = 365nA \tag{41}$$

- Thes use of such low collector currents leads to precipitously low values of f_T, drastically reducing the usable frequency range of the device

Summary

- The primary noise mechanism in MOSFET and JFET devices is the thermal agitation of charge carriers in the channel
- Thermal noise density in MOSFETs depends on the excess noise factor γ, which has a minimum value of $\frac{2}{3}$ in saturation and can be $2-3$ times higher, particularly for high values of f_T
- $1/f$ noise is more prounounced in FET devices relative to BJTs, particularly in MOSFETs due to defects at the oxide-silicon interface
- Gate current noise in FETs is extremely low at low to moderate frequencies, but can increase substantially at high frequencies/temperatures
- BJTs outperform FETs in terms of thermal noise, in part due to their higher transconductance efficiency, but FETs are generally superior for applications involving high source resistances