EE 538 Spring 2021 Low-Noise Analog Circuit Design University of Washington Electrical & Computer Engineering

Instructor: Jason Silver
Assignment #2 (10 points)
Due Sunday, April 18 (Submit on Canvas as a Jupyter Notebook)

Please show your work

Problem 1: Photodiode amplifier design

Transimpedance amplifiers (TIAs) are frequently used as current-input amplifiers, wherein a feedback resistance R_f determines the TIA gain. Unfortunately, R_f forms a pole with both the photodiode capacitance and the input capacitance of the opamp, degrading phase margin and leading to peaking in the closed-loop amplifier response.

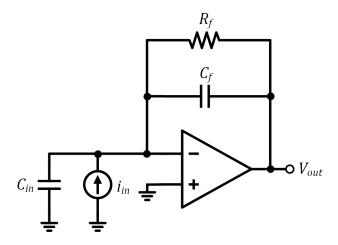


Figure 1. Current-input amplifier (TIA)

<u>Analysis</u>

The closed-loop transfer function of the uncompensated ($C_f=0$) TIA can be expressed as

$$\frac{V_{out}}{i_{in}} = R_f \cdot \frac{A_{ol}(f)}{1 + A_{ol}(f) + sC_{in}R_f}$$

a) Determine an expression for the closed-loop transfer function of the compensated TIA (including C_f) assuming an opamp with a first-order response ($A_{ol}(f) = A_0/(1+s/s_p)$, where A_0 is the DC gain of the opamp). What is the closed-loop resonant frequency (ω_0) of the TIA in terms of R_f , C_f , C_{in} and ω_u ?

$$|s_p| = \omega_p = \omega_0$$

 $\omega_u = A_0 \omega_p$
 $C_{in} \gg C_f$ so $C_f + C_{in} \approx C_{in}$

1

$$A_{OL}(f) = \frac{A_0}{1 + \frac{s}{s_p}}$$

2.

$$V_{out}$$
 = $(V^+ - V^-) \cdot A_{OL}(f) \Big|_{V^+=0} = -V^- \cdot A_{OL}(f)$

3.

$$Z_f = R_f \parallel C_f$$

$$= (\frac{1}{R_f} + sC_f)^{-1}$$

$$= \frac{R_f}{1 + sR_fC_f}$$

4.

$$\begin{split} i_{in} &= \frac{V^{-}}{Z_{in}} + \frac{V^{-} - V_{out}}{Z_{f}} \\ &= V^{-} s C_{in} + (V^{-} - V_{out}) (\frac{1}{R_{f}} + s C_{f}) \\ &= V^{-} (s C_{in} + \frac{1}{R_{f}} + s C_{f}) - V_{out} (\frac{1}{R_{f}} + s C_{f}) \\ &= V^{-} (1 + \frac{s R_{f} (C_{in} + C_{f})}{R_{f}}) - V_{out} (\frac{1 + s R_{f} C_{f}}{R_{f}}) \\ &= \frac{-V_{out}}{A_{OL}(f)} (1 + \frac{s R_{f} (C_{in} + C_{f})}{R_{f}}) - V_{out} (\frac{1 + s R_{f} C_{f}}{R_{f}}) \\ &= \frac{-V_{out}}{A_{OL}(f) R_{f}} (1 + s R_{f} (C_{in} + C_{f})) - \frac{V_{out}}{R_{f}} (1 + s R_{f} C_{f}) \\ &= \frac{-V_{out}}{A_{OL}(f) R_{f}} (1 + s R_{f} (C_{in} + C_{f})) - \frac{V_{out}}{A_{OL}(f) R_{f}} A_{OL}(f) (1 + s R_{f} C_{f}) \end{split}$$

5.

$$\begin{split} \frac{V_{out}}{i_{in}} &= \frac{-A_{OL}(f)R_{f}}{1 + sR_{f}(C_{in} + C_{f}) + A_{OL}(f)(1 + sR_{f}C_{f})} \\ \frac{V_{out}}{i_{in}} &= \frac{-A_{OL}(f)R_{f}}{1 + sR_{f}C_{in} + A_{OL}(f)(1 + sR_{f}C_{f})} \\ &= \frac{-\frac{A_{O}}{1 + \frac{s}{s_{p}}}R_{f}}{1 + sR_{f}C_{in} + \frac{A_{O}}{1 + \frac{s}{s_{p}}}(1 + sR_{f}C_{f})} \\ &= \frac{-A_{O}R_{f}}{(1 + \frac{s}{s_{p}})(1 + sR_{f}C_{in}) + \frac{A_{O}}{1 + \frac{s}{s_{p}}}(1 + sR_{f}C_{f})} \\ &= \frac{-A_{O}R_{f}}{(1 + \frac{s}{s_{p}})(1 + sR_{f}C_{in}) + A_{O}(1 + sR_{f}C_{f})} \\ &= \frac{-A_{O}R_{f}}{1 + sR_{f}C_{in} + \frac{s}{s_{p}} + \frac{s^{2}}{s_{p}}R_{f}C_{in} + A_{O} + sA_{O}R_{f}C_{f}} \\ &= \frac{-A_{O}R_{f}}{(1 + A_{O}) + s(\frac{1}{s_{p}} + R_{f}C_{in} + A_{O}R_{f}C_{f}) + \frac{s^{2}}{s_{p}}R_{f}C_{in}} \\ &\approx \frac{-R_{f}}{1 + s(\frac{1}{A_{O}s_{p}} + \frac{R_{f}C_{in}}{A_{O}} + R_{f}C_{f}) + \frac{s^{2}}{A_{O}s_{p}}R_{f}C_{in}} \\ &\approx \frac{-R_{f}}{1 + sR_{f}C_{f} + \frac{s^{2}}{s_{o}}R_{f}C_{in}} \\ &\approx \frac{-R_{f}}{1 + sR_{f}C_{f} + \frac{s^{2}}{s_{o}}R_{f}C_{in}} \end{split}$$

$$H(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 \cdot s + \omega_0^2}$$

$$= \frac{\omega_0^2}{s^2 + \frac{s \cdot \omega_0}{Q} + \omega_0^2}$$

$$= \frac{1}{\frac{s^2}{\omega_0^2} + \frac{s \cdot \omega_0}{Q} + 1}$$

$$\frac{V_{out}}{i_{in}} \approx -R_f \frac{1}{1 + sR_fC_f + \frac{s^2}{\omega_u}R_fC_{in}}$$

$$\frac{1}{\omega_0^2} = \frac{R_fC_{in}}{\omega_u}$$

$$\omega_0^2 = \frac{\omega_u}{R_fC_{in}}$$

$$\omega_0 = \sqrt{\frac{\omega_u}{R_fC_{in}}}$$

b) Show that choosing C_f such that $\frac{1}{R_f C_f} \approx \sqrt{\frac{\omega_u}{2R_f C_{in}}}$ (assuming $C_{in} >> C_f$) results in a maximally flat (i.e. Butterworth) response.

$$H(s) = \frac{1}{\frac{s^2}{\omega_0^2} + \frac{s}{Q\omega_0} + 1}$$

$$\omega_0 = \sqrt{\frac{\omega_u}{R_f C_{in}}}$$

$$\frac{V_{out}}{i_{in}} \approx -R_f \frac{1}{1 + sR_f C_f + \frac{s^2}{\omega_v^2} R_f C_{in}}$$

$$\text{then} \qquad R_f C_f = \frac{1}{Q\omega_0} = \frac{\sqrt{2}}{\omega_0}$$

$$\sqrt{2} = \sqrt{\frac{\omega_u}{R_f C_{in}}} R_f C_f$$

$$R_f C_f = \frac{\sqrt{2}}{\sqrt{\frac{\kappa_u}{R_f C_{in}}}}$$

$$= \sqrt{2} \sqrt{\frac{R_f C_{in}}{\omega_u}}$$

$$= \sqrt{\frac{2R_f C_{in}}{\omega_u}}$$

$$= \sqrt{\frac{2R_f C_{in}}{\omega_u}}$$

$$C_f = \frac{1}{R_f} \sqrt{\frac{2R_f C_{in}}{\omega_u}}$$

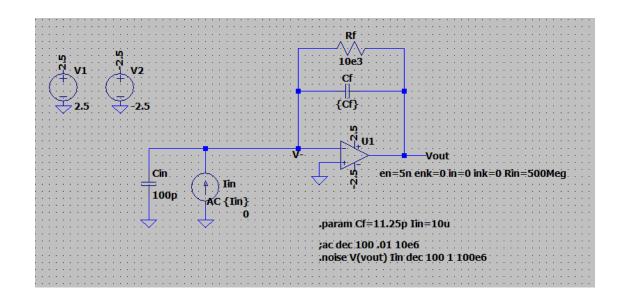
$$= \sqrt{\frac{2C_{in}}{R_f A_0 \omega_p}}$$

$$= \sqrt{\frac{2C_{in}}{2\pi R_f A_0 f_p}}$$

$$C_f = \sqrt{\frac{C_{in}}{\pi R_f A_0 f_p}}$$

<u>Design</u>

c) Design the TIA in Figure 1 for a gain of $10k\Omega$ and a Butterworth response with a 3dB bandwidth of 2MHz. Assuming $C_{in} = 100pF$, what are the required values of ω_u (opamp GBW) and C_f ? Verify your design in Ltspice.



1.

$$H(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 \cdot s + \omega_0^2}$$

$$= \frac{1}{\frac{s^2}{\omega_0^2} + \frac{s}{Q\omega_0} + 1} \quad \text{and Butterworth } Q = \frac{1}{\sqrt{2}}$$

$$|H(s)| = \frac{1}{\sqrt{1 + \frac{\omega^4}{\omega_0^4}}}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \frac{\omega^4}{\omega_0^4}}}$$

2.

$$\omega_{3dB_{CL}} = \omega_0$$

$$\omega_{3dB_{CL}} = \sqrt{\frac{\omega_u}{R_f C_{in}}}$$

$$= \sqrt{\frac{A_0 \omega_p}{R_f C_{in}}}$$

$$= \sqrt{\frac{2\pi A_0 \cdot f_{3dB_{OL}}}{R_f C_{in}}}$$

3.

$$2\pi f_{3dB_{CL}} = \sqrt{\frac{2\pi A_0 \cdot f_{3dB_{OL}}}{R_f C_{in}}}$$

$$f_{3dB_{CL}} = \sqrt{\frac{A_0 \cdot f_{3dB_{OL}}}{2\pi R_f C_{in}}} \quad \text{and similarly}$$

4.

$$f_{3dB_{OL}} = 2\pi (f_{3dB_{CL}})^2 C_{in} \bigg|_{A_0 = R_f}$$

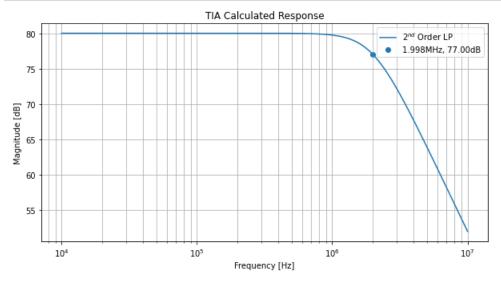
$$\omega_u = A_0 \omega_p$$

$$\omega_u = A_0 \cdot 2\pi f_{3dB_{OL}}$$

```
omega_u = 157.91 MHz
omega_0 = 12.57 MHz
C_f = 1.12539539519638E-11 F
```

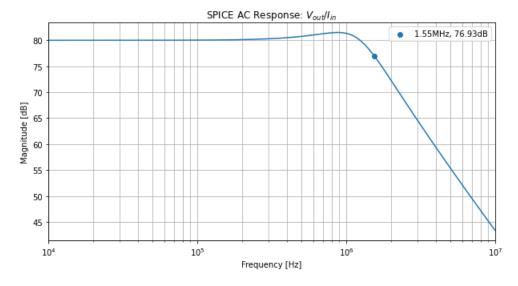
Calculated

```
In [5]:
                                                     1 \mid s, R_f, C_f, C_{in}, A_0, omega_p, omega_0, omega_u, f3db = sp.symbols('s, R_f, C_f, C_{in}, A_0, omega_p, omega_b, omega_b
                                                     2 \mid f = np.logspace(4, 7, 100000)
                                                     3 w = 2*np.pi*f
                                                                  num = R f
                                                                   den = (R_f*C_in/(omega_u))*s**2 + (R_f*C_f)*s + 1
                                                                   components = {
                                                                                         R f : 10*1e3,
                                                 10
                                                                                          C in: 100*1e-12,
                                                11
                                                                                         C_f : 1.1254e-11,
                                                12
                                                                                         omega_u: 157.91*1e6,
                                                13 }
                                                14 H = sp.Matrix([num/den])
                                                15 H1 = H = H.subs(components)
                                                16 | H = lambdify(s,H,modules='numpy')
                                                17 H = H(1j*w)
                                                18 H = H[0][0]
```



LTspice

```
In [7]: 1 filepath = 'data/HW02.txt'
2 df = read_ltspice(filepath, 'ac')
3 freq = df['Freq.']
4 mag = df['Mag_V(vout)/I(Iin)']
```



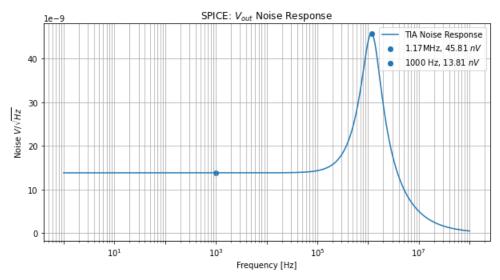
d) Assuming an opamp with $e_{na} = 5nV/\sqrt{Hz}$ and $i_{na} = 0$, determine an expression for the mean-squre input-referred noise current density i_n^2 of the TIA as a function of frequency (ignore bandwidth limitations of the TIA). Taking the TIA bandwidth into account, what is the maximum value of i_n ? At what frequency do we see the maximum noise density? Verify in Ltspice.

Maximum noise density occurs at the 3dB cutoff frequency..

$$\begin{split} i_{na} &= 0 \\ e_{na} &= 5nV/\sqrt{Hz} \\ i_{nC} &= e_{na} \cdot 2\pi f C_{in} = \frac{5nV}{\sqrt{Hz}} \cdot 2\pi f \cdot 100 \text{pf} \\ i_{sh}^2 &= 2qI_D \Delta f \\ i_n(f) &= \sqrt{(i_{sh})^2 + (\frac{e_{na}}{R_f})^2 + (\frac{\sqrt{4kTR_f}}{R_f})^2 + (i_{nC})^2} \\ &= \sqrt{(2qI_D)\Delta f + (\frac{e_{na}}{R_f})^2 + (\frac{\sqrt{4kTR_f}}{R_f})^2 + (e_{na} \cdot 2\pi f C_{in})^2} \\ &= \sqrt{(2qI_D)\Delta f + (\frac{e_{na}}{R_f})^2 + (\frac{4kT}{R_f}) + (e_{na} \cdot 2\pi f C_{in})^2} \\ i_n(f)^2 &= (2qI_D)\Delta f + (\frac{e_{na}}{R_f})^2 + (\frac{4kT}{R_f}) + (e_{na} \cdot 2\pi f C_{in})^2 \Big|_{f=f_{3dB}} \end{split}$$

LTspice

```
1 filepath = 'data/HW02_noise.txt'
 In [9]:
          2 df = pd.read_csv(filepath)
          3 freq = df['frequency']
          4 mag = df['V(onoise)']
In [10]:
          1 fig, ax = plt.subplots(1,figsize=(10,5))
             x1 = np.where(freq <= 1000)[0][-1]
             label1 = r"1000 Hz, {:.2f} snVs".format(mag[x1]*1e9)
            x2 = mag.idxmax()
          7
             label2 = r"{:.2f}MHz, {:.2f} $nV$".format(rnd(freq[x2],2,"MHz"),mag[x2]*1e9)
          8
          9 ax.semilogx(freq, mag, color='tab:blue',label='TIA Noise Response')
          10 | ax.scatter(freq[x2],mag[x2],label=label2,color='tab:blue')
         11 | ax.scatter(freq[x1],mag[x1],label=label1,color='tab:blue')
          12 ax.grid(True, which='both')
         13 ax.set_xlabel('Frequency [Hz]')
         14 ax.set_ylabel(r'Noise $V/\sqrt{Hz}$')
         15 ax.set_title(r'SPICE: $V_{out}$ Noise Response')
         16 ax.ticklabel_format(style='sci', axis='y', scilimits=(-9,-9))
         17 #ax.set ylim(10e-9,50e-9)
         18
         19 # manipulate x-axis ticks and labels
         20 ax.xaxis.set major locator(LogLocator(numticks=15)) #(1)
         21
             ax.xaxis.set_minor_locator(LogLocator(numticks=15,subs=np.arange(2,10))) #(2)
         22
             for label in ax.xaxis.get_ticklabels()[::2]:
         23
                 label.set_visible(False) #(3)
         24
         25
             ax.legend()
         26 plt.show();
```



e) If the TIA is driven by a photodiode with a DC current of $10\mu A$, what is the noise figure of the amplifier, in terms of noise density, at 1kHz?

```
NF = 10 \log F

F \equiv \frac{\text{total output noise power}}{\text{output noise due to input source}}

total output noise power = (2qI_D)\Delta f + (\frac{e_{na}}{R_f})^2 + (\frac{4kT}{R_f}) + (e_{na} \cdot 2\pi f C_{in})^2
input source noise power = (2qI_D)

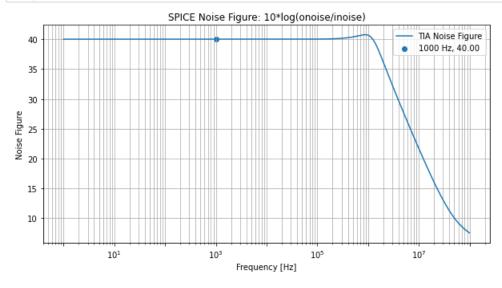
Noise Figure = 38.37
```

```
In [11]:
           1 T = 298
           2 k = 1.381 * 1e-23
           3 q = 1.38 * 1e-23
           4 \mid I \mid D = 10*1e-6
           5 | f = 1000
           6 e_na = 5*1e-9
           7 | R\overline{f} = 10*1e3
           8 Cin = 100*1e-12
          10 | shot = (2*q*I D) |
          11 amp = (e_na/Rf)**2
          12 therm = 4*k*T/Rf
          13 cap = (e_na *2*np.pi*f*Cin)**2
          14
          15
          16 total_output = shot + amp + therm + cap
          17
             input_source = shot
          18
          19 NF = 10* np.log10(total output/input source)
          20 print(f'Noise Figure = {round(NF,2)}')
```

Noise Figure = 38.37

```
In [12]: 1 filepath = 'data/HW02_noise2.txt'
2 df = pd.read_csv(filepath)
3 freq = df['frequency']
4 onoise = df['V(onoise)']
5 inoise = df['inoise']
```

```
In [17]:
          1 | fig, ax = plt.subplots(1, figsize=(10,5))
             x1 = np.where(freq <= 1000)[0][-1]
          3
             label1 = r"1000 Hz, \{:.2f\}".format(10*np.log10(onoise[x1]/inoise[x1]))
             ax.semilogx(freq, 10*np.log10(onoise/inoise), color='tab:blue',label='TIA Noise Figure')
          6
             ax.scatter(freg[x1],10*np.log10(onoise[x1]/inoise[x1]),label=label1,color='tab:blue')
          8
             ax.grid(True,which='both')
             ax.set xlabel('Frequency [Hz]')
          10 | ax.set_ylabel(r'Noise Figure')
          11 ax.set_title(r'SPICE Noise Figure: 10*log(onoise/inoise)')
          12 #ax.ticklabel_format(style='sci', axis='y', scilimits=(-9,-9))
          13
             #ax.set ylim(10e-9,50e-9)
          14
          15
             # manipulate x-axis ticks and labels
             ax.xaxis.set major locator(LogLocator(numticks=15)) #(1)
          16
             ax.xaxis.set_minor_locator(LogLocator(numticks=15, subs=np.arange(2,10))) #(2)
          17
             for label in ax.xaxis.get_ticklabels()[::2]:
          18
                 label.set visible(False) #(3)
          19
          20
          21
             ax.legend()
          22
             plt.show();
```



Refence Page

Resources:

https://www.analog.com/en/analog-dialogue/articles/compensating-current-feedback-amplifiers.html (https://www.analog.com/en/analog-dialogue/articles/compensating-current-feedback-amplifiers.html)

https://www.analog.com/en/technical-articles/transimpedance-amplifier-noise-considerations.html (https://www.analog.com/en/technical-articles/transimpedance-amplifier-noise-considerations.html)

https://e2e.ti.com/support/amplifiers/f/amplifiers-forum/379953/open-loop-gain-vs-closed-loop-gain-in-tia (https://e2e.ti.com/support/amplifiers/f/amplifiers-forum/379953/open-loop-gain-vs-closed-loop-gain-in-tia)

https://www.allaboutcircuits.com/technical-articles/negative-feedback-part-8-analyzing-transimpedance-amplifier-stability/ (https://www.allaboutcircuits.com/technical-articles/negative-feedback-part-8-analyzing-transimpedance-amplifier-stability/)

https://www.embeddedcomputing.com/technology/analog-and-power/batteries-power-supplies/tia-fundamentals-the-noise-transfer-function-part-4 (https://www.embeddedcomputing.com/technology/analog-and-power/batteries-power-supplies/tia-fundamentals-the-noise-transfer-function-part-4)

https://www.tij.co.jp/jp/lit/an/snoa942a/snoa942a.pdf?ts=1618760264451&ref_url=https%253A%252F%252Fwww.google.com%252F (https://www.tij.co.jp/jp/lit/an/snoa942a/snoa942a.pdf?ts=1618760264451&ref_url=https%253A%252F%252Fwww.google.com%252F)

https://e2e.ti.com/support/amplifiers/f/amplifiers-forum/504360/how-to-maintain-stability-of-a-tia-when-exceeding-the-open-loop-gain (https://e2e.ti.com/support/amplifiers-forum/504360/how-to-maintain-stability-of-a-tia-when-exceeding-the-open-loop-gain)

https://people.engr.tamu.edu/spalermo/ecen689 oi/lecture5 ee689 tias.pdf (https://people.engr.tamu.edu/spalermo/ecen689 oi/lecture5 ee689 tias.pdf)

https://2n3904blog.com/trans-impedance-amplifier-transfer-function/ (https://2n3904blog.com/trans-impedance-amplifier-transfer-function/) function/)

http://www.seas.ucla.edu/brweb/papers/Journals/BR SSCM 1 2019.pdf (http://www.seas.ucla.edu/brweb/papers/Journals/BR SSCM 1 2019.pdf)

https://www.ti.com/lit/an/sboa122/sboa122.pdf?ts=1618932133586 (https://www.ti.com/lit/an/sboa122/sboa122.pdf? ts=1618932133586)

https://www.ti.com/lit/an/snoa515a/snoa515a.pdf?ts=1618912072084&ref_url=https%253A%252F%252Fwww.google.com%252F (https://www.ti.com/lit/an/snoa515a/snoa515a/snoa515a.pdf?ts=1618912072084&ref_url=https%253A%252F%252Fwww.google.com%252F)

$$\begin{split} \frac{V_{out}}{i_{in}} &= \frac{Z_f A_{OL}(f)}{1 + \frac{Z_f}{Z_{C_{in}}} + A_{OL}(f)} \\ &= Z_f \frac{A_{OL}(f)}{1 + A_{OL}(f) + \frac{Z_f}{Z_{C_{in}}}} \\ &= Z_f \frac{1}{1 + \frac{1 + \frac{Z_f}{Z_{C_{in}}}}{A_{OL}(f)}} \\ &= Z_f \frac{1}{1 + \frac{1 + \frac{Z_f}{Z_{C_{in}}}}{A_{OL}(f)}} \quad \text{where } \frac{1}{\beta} = 1 + \frac{Z_f}{Z_{C_{in}}} = 1 + \frac{(1 + sR_fC_f)sC_{in}}{R_f} = \frac{C_fC_{in}s^2}{R_f} + \frac{C_{in}s}{R_f} + 1 \end{split}$$

```
In [14]:
          1 # Imports
           2 import os
3 import sys
4 import cmath
           5 import math
           6 import matplotlib.pyplot as plt
           7 import matplotlib
           8 import numpy as np
          9 import pandas as pd
          10 import ltspice
          11 import sympy as sp
          12 | from sympy.utilities.lambdify import lambdify
          13 from scipy import signal
          14 %matplotlib inline
          15 | from IPython.core.interactiveshell import InteractiveShell
          16 | InteractiveShell.ast_node_interactivity = "all"
          17 from matplotlib.ticker import LogLocator
```

```
In [15]:
             def read ltspice(file name,ftype='trans',units='db'):
                 cols = []
                 arrs = []
           3
           4
                 with open(file name, 'r', encoding='utf-8') as data:
           5
                      for i,line in enumerate(data):
           6
                          if i==0:
           7
                              cols = line.split()
           8
                              arrs = [[] for in cols]
           9
                              continue
          10
                          parts = line.split()
          11
                          for j,part in enumerate(parts):
          12
                              arrs[j].append(part)
          13
                 df = pd.DataFrame(arrs,dtype='float64')
          14
                 df = df.T
                 df.columns = cols
          15
                 if ftype=='trans':
          16
          17
                      return df
          18
                 elif ftype=='ac':
          19
                      if units=='db':
          20
                          for col in cols:
          21
                              if df[col].str.contains(',').all():
                                  df[f'Mag {col}'] = df[col].apply(lambda x: x.split(',')[0])
          22
          23
                                  df[f'Mag {col}'] = df[f'Mag {col}'].apply(lambda x: x[1:-2])
                                  df[f'Mag_{col}'] = df[f'Mag_{col}'].astype('float64')
          24
                                  df[f'Phase_{col}'] = df[col].apply(lambda x: x.split(',')[1])
          25
          26
                                  df[f'Phase_{col}'] = df[f'Phase_{col}'].apply(lambda x: x[0:-2])
                                  df[f'Phase_{col}'] = df[f'Phase_{col}'].astype('float64')
          27
          28
                      if units=='cartesian':
                          for col in cols:
          29
          30
                              if df[col].str.contains(',').all():
                                  df[f'Re {col}'] = df[col].apply(lambda x: x.split(',')[0])
          31
                                  df[f'Re_{col}'] = df[f'Re_{col}'].astype('float64')
          32
          33
                                  df[f'Im_{col}'] = df[col].apply(lambda x: x.split(',')[1])
          34
                                  df[f'Im_{col}'] = df[f'Im_{col}'].astype('float64')
                      df['Freq.'] = df['Freq.'].astype('float64')
          35
          36
                      return df
          37
                 else:
                      print('invalid ftype')
          38
```

In []: 1