EE 538: Low-Noise Analog Circuit Design

Spring 2021

Instructor: Jason Silver

Announcements

- Jason's office hours moved to Saturdays at 11am
- Assignment 4 due Sunday, May 2 at midnight
- Assignment 5 will be posted Saturday, May 1
- Midterm exam will be posted Saturday, May 1
    - Online format administered as a quiz on Canvas

Week 5

- Motchenbacher Chapter 5
- Art of Electronics Chapter 8

Overview

- Last time...
    - FET large/small-signal operation
    - Thermal noise in FET devices
    - 1/f noise and gate current noise
    - FET noise model
    - FET vs BJT
- Today...
    - Noise in common-emitter stages
    - Active load and current mirror noise
    - Cascode and parallel stages
    - Differential amplifier noise

Python packages/modules

In [38]:

```
import matplotlib as mpl
from matplotlib import pyplot as plt
from matplotlib import ticker, cm
import numpy as np
from scipy import signal
from scipy import integrate
#%matplotlib notebook

mpl.rcParams['font.size'] = 14
mpl.rcParams['legend.fontsize'] = 'large'

def plot_xy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5));
    ax.plot(x, y, 'b')
    ax.grid()
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)

def plot_2xy(x, y1, y2, xlabel, ylabel, y1label, y2label):
    fig, ax = plt.subplots(figsize=(10.0,7.5))

    ax.plot(x, y1, 'b', label=y1label)
    ax.plot(x, y2, 'r', label=y2label)
    ax.set_ylabel(ylabel)
    ax.set_xlabel(xlabel)
    ax.grid()

    ax.legend()
    ax.legend(loc='upper center', ncol=2, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_xy2(x1, y1, x1label, y1label, x2, y2, x2label, y2label):
    fig, ax = plt.subplots(2, figsize = (10.0, 7.5));
    ax[0].plot(x1, y1, 'b')
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].plot(x2, y2, 'b')
    ax[1].set_xlabel(x1label)
    ax[1].set_xlabel(x2label)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    fig.align_ylabels(ax[:])

def plot_xy3(x, y1, y2, y3, xlabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(3, figsize=(10.0,7.5))

    ax[0].plot(x, y1)
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].plot(x, y2)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    ax[2].plot(x, y3)
    ax[2].set_ylabel(y3label)
    ax[2].set_xlabel(xlabel)
    ax[2].grid()

def plot_logxy3(x, y1, y2, y3, xlabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(3, figsize=(10.0,7.5))

    ax[0].semilogx(x, y1)
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].semilogx(x, y2)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    ax[2].semilogx(x, y3)
    ax[2].set_ylabel(y3label)
    ax[2].set_xlabel(xlabel)
    ax[2].grid()

def plot_logxy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    ax.semilogx(x, y, 'b')
    ax.grid();
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)

def plot_log3xy(x, y1, y2, y3, xlabel, ylabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(figsize=(10.0,7.5))

    ax.semilogx(x, y1, 'b', label=y1label)
    ax.semilogx(x, y2, 'r', label=y2label)
    ax.semilogx(x, y3, 'g', label=y3label)
    ax.set_ylabel(ylabel)
    ax.set_xlabel(xlabel)
    ax.grid()

    ax.legend()
    ax.legend(loc='upper center', ncol=3, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_log2xy(x, y1, y2, xlabel, ylabel, y1label, y2label):
    fig, ax = plt.subplots(figsize=(10.0,7.5))

    ax.semilogx(x, y1, 'b', label=y1label)
    ax.semilogx(x, y2, 'r', label=y2label)
    ax.set_ylabel(ylabel)
    ax.set_xlabel(xlabel)
    ax.grid()

    ax.legend()
    ax.legend(loc='upper center', ncol=2, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_loglog(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    ax.loglog(x, y, 'b')
    ax.grid();
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)


def plot_loglog2(x, y1, y2, xlabel, ylabel, y1label, y2label):
    fig, ax = plt.subplots(figsize=(10.0,7.5))

    ax.loglog(x, y1, 'b', label=y1label)
    ax.loglog(x, y2, 'r', label=y2label)
    ax.set_ylabel(ylabel)
    ax.set_xlabel(xlabel)
    ax.grid()

    ax.legend()
    ax.legend(loc='upper center', ncol=2, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_xlogy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5));
    ax.semilogy(x, y, 'b');
    ax.grid();
    ax.set_xlabel(xlabel);
    ax.set_ylabel(ylabel);

def read_ltspice_ac(file_name):
    with open(file_name, 'r') as data:
        x = []
        y = []
        z = []
        next(data) # skip header line
        for line in data:
            p = line.split()
            x.append(float(p[0]))
            complex = p[1].split(",")
            y.append(float(complex[0]))
            z.append(float(complex[1]))

    return x, y, z

def plot_logxy2(x1, y1, x2, y2, x1label, y1label, x2label, y2label):
    fig, ax = plt.subplots(2, figsize = (10.0, 7.5));
    ax[0].semilogx(x1, y1, 'b');
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].semilogx(x2, y2, 'b');
    ax[1].set_xlabel(x1label)
    ax[1].set_xlabel(x2label);
    ax[1].set_ylabel(y2label);
    ax[1].grid();

    fig.align_ylabels(ax[:])

def plot_noise_bandwidth(f, mag):
    fig, ax = plt.subplots(2, figsize=(10.0,7.5))
    ax[0].semilogx(f, RC_mag)
    ax[0].set_xscale("log")
    ax[0].set_xlim(f[0], f[-1])
    ax[0].set_xticks(np.logspace(0.1,4,5))
    ax[0].set_xticklabels([])
    ax[0].set_ylabel('Magnitude [V/V]')
    ax[0].set_title('Equivalent Noise Bandwidth')
    ax[0].grid()

    ax[1].hlines(1, 0, f_enb, color='tab:blue')
    ax[1].hlines(0, f_enb, f[-1], color='tab:blue')
    ax[1].vlines(f_enb, 0, 1, color='tab:blue')
    ax[1].set_xlim(f[0], f[-1])
    ax[1].set_xscale("log")
    ax[1].set_xticks(np.logspace(0.1,4,5))
    ax[1].set_xticklabels([r'$10^0$',r'$10^1$', r'$10^2$', r'$10^3$', r'$10^4$'])
    ax[1].set_ylabel('Magnitude [V/V]')
    ax[1].set_xlabel('Frequency [Hz]')
    ax[1].grid()

def noise_hist(vnoise, vn_rms, bins):
    fig = plt.figure( figsize=(10.0,7.5) )
    vn_norm = vnoise/ vn_rms
    ax = fig.add_subplot(111)
    n, bins, rectangles = ax.hist(vn_norm, bins, density=True, range=(-3, 3),
                                  color='b')
    ax.set_xlabel(r'Sample Voltage [$v_{n(rms)}$]')
    ax.set_ylabel('Probability Density')
    ax.grid()
    fig.canvas.draw()

def plot_NF_vs_Rs(en_vals, in_vals, Rs_min, Rs_max, T_in_K):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    k = 1.38e-23
    Rs = np.logspace(np.log10(Rs_min), np.log10(Rs_max), num=200)
    F1 = 1 + (en_vals[0]**2+Rs**2*in_vals[0]**2)/(4*k*T_in_K*Rs)
    F2 = 1 + (en_vals[1]**2+Rs**2*in_vals[1]**2)/(4*k*T_in_K*Rs)
    F3 = 1 + (en_vals[2]**2+Rs**2*in_vals[2]**2)/(4*k*T_in_K*Rs)
    ax.semilogx(Rs, 10*np.log10(F1), 'b', label=r'$e_{n1}$, $i_{n1}$')
    ax.semilogx(Rs, 10*np.log10(F2), 'r', label=r'$e_{n2}$, $i_{n2}$')
    ax.semilogx(Rs, 10*np.log10(F3), 'g', label=r'$e_{n3}$, $i_{n3}$')
    ax.grid();
    ax.set_xlabel(r'Source Resistance $R_s$ [\Omega]$')
    ax.set_ylabel(r'Noise Figure $NF$ [$dB$]')

    ax.legend()
    ax.legend(loc='upper center', ncol=3, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_noise_curve(e_n, i_n, Rs_min, Rs_max):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    Rs = np.logspace(np.log10(Rs_min), np.log10(Rs_max), num=200)
    e_ni_2 = 4*k*T*Rs + e_n**2 + i_n**2*Rs**2
    ax.loglog(Rs, np.sqrt(e_ni_2), 'b', label='Total Noise')
    ax.loglog(Rs, np.sqrt(4*k*T*Rs), 'r', label=r'$\sqrt{4kTR_s}$')
    ax.loglog(Rs, e_n*np.ones(np.size(Rs)), 'g', label=r'$e_n$')
    ax.loglog(Rs, i_n*Rs, 'y', label=r'$i_n R_s$')
    ax.grid();
    ax.set_xlabel(r'Source Resistance $R_s$ [\Omega]$')
    ax.set_ylabel(r'Equivalent Input Noise [$V/\sqrt{Hz}$]')

    ax.legend()
    ax.legend(loc='upper center', ncol=4, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_bjt_NF(beta, r_bb, Rmin, Rmax, Imin, Imax):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    k = 1.38e-23
    T = 300
    q = 1.602e-19
    V_T = k*T/q
    rs = np.logspace(np.log10(Rmin), np.log10(Rmax), num = 100)
    ic = np.logspace(np.log10(Imin), np.log10(Imax), num = 100)
    I_C, R_S = np.meshgrid(ic, rs)
    e_n_2 = 4*k*T*(V_T/2/I_C + r_bb)
    i_n_2 = 2*q*I_C/beta_0
    NF = 1 + (e_n_2 + i_n_2*R_S**2)/(4*k*T*R_S)
    cp = ax.contourf(I_C, R_S, 10*np.log10(NF), levels=np.linspace(0,15, num=16))
    plt.xscale('log')
    plt.yscale('log')
    plt.ylabel(r'Source Resistance $R_s$ [$\Omega$]')
    plt.xlabel(r'Collector Current $I_C$ [A]')
    fig.colorbar(cp)
```

In [4]:

```
def fftnoise(f):
    f = np.array(f, dtype='complex')
    Np = (len(f) - 1) // 2
    phases = np.random.rand(Np) * 2 * np.pi
    phases = np.cos(phases) + 1j * np.sin(phases)
    f[1:Np+1] *= phases
    f[-1:-1-Np:-1] = np.conj(f[1:Np+1])
    return np.fft.ifft(f).real

def band_limited_noise(min_freq, max_freq, samples=1024, samplerate=1):
    freqs = np.abs(np.fft.fftfreq(samples, 1/samplerate))
    f = np.zeros(samples)
    idx = np.where(np.logical_and(freqs>=min_freq, freqs<=max_freq))[0]
    f[idx] = 1
    return fftnoise(f)
```

Lecture 5 - Low-Noise Amplifier Design

Common-emitter stage noise

- Considering only the shot noise from the input device (which is often the dominant source), the output current noise of a common-emitter stage is

$$i_{nc}^2 = 2qI_C = 2kTg_m \tag{1}$$

- As discussed previously, this can be considered to be the thermal noise of a resistance with a value of $2r_e = 2/g_m$
- This constitutes the minimum noise achievable with a gain stage, assuming zero source and base resistances

Gain of a CE stage

- The voltage gain of a resistively-loaded common-emitter stage is limited by the supply voltage
- Assuming $r_o \gg R_C$, the gain is given by

$$|A_v| = g_m \cdot R_C = \frac{I_C}{V_T} \cdot R_C \tag{2}$$

- If the output is biased at $V_{CC}/2$, the gain can be expressed as

$$|A_v| = \frac{I_C}{V_T} \cdot R_C = \frac{I_C}{V_T} \cdot \frac{V_{CC}}{2I_C} = \frac{V_{CC}}{2V_T} \tag{3}$$

- If $V_{CC} = 5V$ (for example), the maximum gain at room temperature is approximately $100V/V$ (slightly higher if the bias point is set lower than $V_{CC}/2$)

Source/emitter degeneration

- Source/emitter degeneration provides several benefits:

  1) reduction of bias sensitivity to temperature, process, and voltage and

  2) gain linearization characteristic when $g_m R_E \gg 1$

- The cost is a reduction in voltage gain, which can attributed to a reduced effective transconductance, i_c/v_b
- The input-referred voltage noise of a single stage remains the same, but sensitivity to the noise of subsequent stages is increased relative to a CE stage without degeneration

- The passage of i_c through R_E causes a reduction in the base-emitter voltage v_{be}, reducing the sensitivity of i_c to v_b
- The collector current is thus

$$i_c = g_m v_{be} = g_m(v_b - i_c R_E) \rightarrow i_c = \frac{g_m}{1+g_m R_E} \cdot v_b \tag{4}$$

- The effective transconductance, i_c/v_b can be expressed as

$$G_m = \frac{i_c}{v_b} = \frac{g_m}{1+g_m R_E} \tag{5}$$

- If $g_m R_E \gg 1$, $G_m \approx 1/R_E$

CE with active load

- Primarily used in integrated circuit implementations, "active" load increases the maximum gain
- In this case, the load resistance is the parallel combination of the small-signal output resistances of Q_1 and Q_2
- The gain can be expressed as

$$|A_v| = \left|\frac{v_o}{v_i}\right| = g_{m1} \cdot r_{o1}||r_{o2} \tag{6}$$

- Assuming $V_{A1} = V_{A2} = V_A$, this becomes

$$|A_v| = \left|\frac{v_o}{v_i}\right| = g_{m1} \cdot \frac{r_o}{2} = \frac{I_C}{V_T} \cdot \frac{V_A}{2I_C} = \frac{V_A}{2V_T} \tag{7}$$

- If $V_A = 100V$, this is an order of magnitude greater than the gain of a resistively-loaded amplifier!

Active load noise

- The output current noise from Q_2 is given by

$$i_{n2}^2 = 4kTr_{b2} \cdot g_{m2}^2 + 2qI_{C2} \tag{8}$$

- The total output current noise is thus

$$\begin{align} i_{n,out}^2 &= 4kTr_{b2} \cdot g_{m2}^2 + 2qI_{C2} + 2qI_{C1} \tag{9} \\ &= 4kTr_{b2} \cdot g_{m2}^2 + 4qI_C \tag{10} \end{align}$$

- The equivalent input voltage noise is

$$e_n^2 = 4kTr_{b1} + \frac{4KTr_{b2} \cdot g_{m2}^2 + 4qI_C}{g_{m1}^2} \tag{11}$$

- Q_2 effectively doubles the input noise power of the amplifier

- To determine the total input-referred voltage noise, first determine the short-circuit output current
- The short-circuit current is the "unloaded" output current, analogous to the open-circuit output voltage
- $i_{n,out}$ is referred to the input by dividing by the input transconductance g_{m1}

Current mirrors

- Generally used in integrated circuits due to superior device matching, current mirrors are used to provide voltage biasing for current sources through a *translinear* function
- Ignoring base current, Q_1 converts I_{BIAS} into a voltage V_{BE1} through the relation

$$V_{BE1} = V_T \ln\frac{I_{BIAS}}{I_{S1}} \tag{12}$$

- Assuming matched devices ($I_{S1} = I_{S2} = I_S$), the output current is given by

$$I_{OUT} = I_S \exp\left(\frac{V_{BE1}}{V_T}\right) = I_{BIAS} \tag{13}$$

Current mirror noise

- Considering only shot noise from Q_1 and Q_2 (and still ignoring base current for simplicity), the output current noise can be determined through superposition
- First, we can refer i_{nc1} to the base node by dividing by g_{m2}

$$e_{n1}^2 = \frac{2qI_{BIAS}}{g_{m1}^2} \tag{14}$$

- The output current noise is the mean-square sum of i_{nc2} and $g_{m1}e_{n1}$ (assume $g_{m1} = g_{m1}$):

$$\begin{align} i_{n,out}^2 &= 2qI_{OUT} + g_{m2}^2 e_{n1}^2 \tag{15} \\ &= \boxed{4qI_{OUT}} \tag{16} \end{align}$$

- What if the emitter areas of Q_1 and Q_2 are ratioed such that $I_{OUT} = KI_{BIAS}$?
- In this case, $g_{m2} = Kg_{m1}$, and Q_1's voltage noise is $2qI_{BIAS}/g_{m1}^2 = K \cdot 2qI_{OUT}/g_{m2}^2$
- The output current noise due to Q_1 is therefore $K \cdot 2qI_{OUT}$ and the total noise is given by

$$i_{n,out}^2 = (K+1) \cdot 2qI_{OUT} \tag{17}$$

- Thus, Q_1's collector shot noise sees the gain of the current mirror, $\sqrt{K+1}$, and we see a tradeoff between noise and power when biasing a current mirror
- An RC filter may be added between the base nodes of Q_1 and Q_2 to limit Q_1's noise bandwidth

Current source degeneration

- In the active CE amp, Q_2 functioned as a current source/active load to increase gain, but at the expense of an increase of (at least) $3dB$ in the input-referred voltage noise of the amplifier
- As discussed previously, current source noise can be reduced by degeneration, i.e. the addition of a resistor R_E between the emitter of Q_2 and V_{CC}
- The inclusion of R_E serves to "degenerate" Q_2's transconductance and, as a result, its noise
- This comes at the expense of noise due to R_E and reduced voltage headroom
- Is there an optimal value of R_E from a noise perspective?

- First, let's asses the noise due only to Q_2. Ignoring noise due to r_b, Q_2's noise source, i_{n2}, is given by

$$i_{n2}^2 = 2qI_C \tag{18}$$

- KCL at Q_2's emitter (v_{e2}) is (assuming i_{n2} flows away from v_{e2})

$$i_{n2} + g_{m2}v_{e2} + v_{e2}/R_E = 0 \tag{19}$$

- The sum of the noise current through R_E (from only Q_2) determines the noise voltage v_{e2}

$$v_{e2} = (-i_{n2} - g_{m2}v_{e2}) \cdot R_E \rightarrow v_{e2} = -i_{n2}R_E/(1+g_{m2}R_E) \tag{20}$$

- The total output noise due to Q_2 is thus

$$i_{n2,out} = i_{n2} - \frac{g_{m2}i_{n2}R_E}{1+g_{m2}R_E} = \boxed{\frac{i_{n2}}{1+g_{m2}R_E} \approx \frac{\sqrt{2kT/r_{e2}}}{1+g_{m2}R_E}} \tag{21}$$

- In other words, R_E reduces the output noise due to Q_2 by a factor $1+g_{m2}R_E$

- The output current noise due to R_E is given by

$$i_{nR_E,out} = i_{nR_E} \cdot \frac{R_E}{1/g_{m2} + R_E} = i_{nR_E} \cdot \frac{g_{m2}R_E}{1+g_{m2}R_E} = \sqrt{\frac{4kT}{R_E}} \cdot \frac{g_{m2}R_E}{1+g_{m2}R_E} \tag{22}$$

- Assuming $g_{m2}R_E \gg 1$ (a measure of the amount of degeneration), the majority of R_E's noise (which is equal to $4kT/R_E$) flows through the output
- The total output noise of the current source is thus

$$\begin{align} i_n^2 &= \frac{2kT/r_e}{(1+g_{m2}R_E)^2} + \frac{4kT}{R_E} \cdot \left(\frac{g_{m2}R_E}{1+g_{m2}R_E}\right)^2 \tag{23} \\ \tag{24} \\ &= \frac{2kT/r_e}{(1+g_{m2}R_E)^2} + \frac{4kT}{r_e} \cdot \frac{g_{m2}R_E}{(1+g_{m2}R_E)^2} = \frac{2kT}{r_e} \cdot \frac{1+2g_{m2}R_E}{(1+g_{m2}R_E)^2} \tag{25} \end{align}$$

- As $g_m R_E$ becomes $\gg 1$, the output noise approaches

$$\lim_{g_m R_E \rightarrow \infty} i_n^2 = \frac{2kT}{r_e} \cdot \frac{2}{g_{m2}R_E} = \frac{4kT}{R_E} \tag{26}$$

- The appropriate value of R_E can be expressed in terms of the voltage drop across it (let's call this V_{RE})
- The gain factor $g_{m2}R_E$ can be expressed as

$$g_{m2}R_E = \frac{I_C}{V_T}\frac{V_{RE}}{I_C} = \frac{V_{RE}}{V_T} \tag{27}$$

- For $g_{m2}R_E >> 1$ the total noise is approximately $i_{nc}^2 \cdot (2/g_{m2}R_E) = i_{nc}^2 \cdot 2V_T/V_{RE}$
- Thus, the RMS current noise reduces by the factor $\sqrt{2V_T/V_{RE}}$ as $g_{m2}R_E$ increases
- That is, with a voltage drop across R_E equal to approximately $8V_T$ (~$200mV$ at room temperature), the total noise is reduced by ~$6dB$

Miller effect

- As discussed previously in the context of high-frequency current noise, the Miller effect occurs due to the presence of a feedback capacitance between the input and output of voltage gain stage
- The Miller effect reduces the *effective* impedance (to ground) of C_μ, from the perspective of the input, by a factor $1+A_v$, where A_v is the voltage gain v_o/v_i
- The reduction in impedance of C_μ constitutes an increase in the effective capacitance looking into the input given by $(1+A_v)C_\mu$
- The resulting pole at $1/R_s(1+A_v)C_\mu$ can be a bandwidth limitation for high-frequency (e.g. RF) applications

Adding a cascode

- The addition of a common-base stage (Q_2) between the collector of Q_1 and v_o serves to "shield" C_μ from the high-impedance output node, thereby alleviating the consequences of the Miller effect
- Assuming $r_o \gg 1/g_{m2}, R_C$, the impedance looking into the emitter of Q_2 is *approximately*

$$R_{up} \approx \frac{1}{g_{m2}} \tag{28}$$

- The effective gain between v_i and v_{c1} is thus

$$\left|\frac{v_{c1}}{v_i}\right| \approx \frac{g_{m1}}{g_{m2}} = 1 \tag{29}$$

- This results in only a small increase 2 in the effective input capacitance due to C_μ, and an increase in the bandwidth of the common-emitter stage

Cascode noise

- The noise due to the cascode device Q_2 can be evaluated using the concept of degeneration previously discussed
- The output noise current due to Q_2 is given by

$$i_{n2,out}^2 = \frac{2qI_C + 4kTr_{b2} \cdot g_{m2}^2}{(1+g_{m2}r_{o1})^2} \tag{30}$$

- Recall that the intrinsic gain of a BJT transistor is a (temperature-dependent) quantity given by $g_m r_o = V_A/V_T$
- Assuming that for any reasonable transistor $g_m r_o \gg 1$, the noise due to Q_2 is effectively eliminated by the output impedance of Q_1
- Again assuming $g_m r_o \gg 1$, the noise due to Q_1 is largely unaffected by the presence of Q_2

Parallel input devices

- Using parallel input devices can improve the noise performance of an amplifier
- Assuming $I_{C1} = I_{C2} = I_C$ and identical devices, the output current noise can be expressed as

$$i_{n,out}^2 = 4qI_C + 8kT \cdot r_b \cdot g_m^2 \tag{31}$$

- The input-referred voltage noise is determined by dividing by the *effective* transconductance $G_m = 2I_C/V_T = 2 \cdot g_m$

$$e_n^2 = \frac{4qI_C + 8kT \cdot r_b \cdot g_m^2}{4 \cdot g_m^2} = \frac{qI_C}{g_m^2} + 2kTr_b \tag{32}$$

- Compared to the input-referred voltage noise of a single BJT with collector current I_C, this constitutes a $\sqrt{2}$ reduction
- The input noise current is increased by a factor of $\sqrt{2}$

Differential amplifier

- A differential pair, comprising a common-emitter (or common-source) forms the core of any operational amplifier
- Differential amplifiers can be driven with complementary inputs, or one input can be held fixed
- One major advantage of the differential amplifier is its tolerance to the common-mode input voltage, including DC inputs
- The DC current (which is common-mode) is degenerated by R_{EE}, while the small-signal *differential* current is unaffected by R_{EE}
- This makes it possible to use the the differential amplifier as a single-ended DC-coupled amplifier by connecting one of the inputs to a constant voltage

Large-signal characteristic

- Using a current source instead of a resistor for biasing improves the common-mode rejection of the differential amplifier, and is the typical biasing approach in integrated circuits
- Assuming $R_{TAIL} \rightarrow \infty$ the base-emitter voltages are given by

$$V_{be1} = V_T \ln\frac{I_{c1}}{I_{S1}} \quad V_{be2} = V_T \ln\frac{I_{c2}}{I_{S2}} \tag{33}$$

- Assuming identical transistors ($I_{S1} = I_{S2}$) gives

$$\frac{I_{c1}}{I_{c2}} = \exp\left(\frac{V_{ip} - V_{im}}{V_T}\right) = \exp\left(\frac{V_{id}}{V_T}\right) \tag{34}$$

- Defining the emitter current gain α as

$$\alpha = \frac{I_C}{I_E} = \frac{\beta}{1+\beta} \tag{35}$$

- We can express I_{c1} and I_{c2} as

$$I_{c1} = \frac{\alpha I_{TAIL}}{1+\exp\left(-\frac{V_{id}}{V_T}\right)} \quad I_{c2} = \frac{\alpha I_{TAIL}}{1+\exp\left(\frac{V_{id}}{V_T}\right)} \tag{36}$$

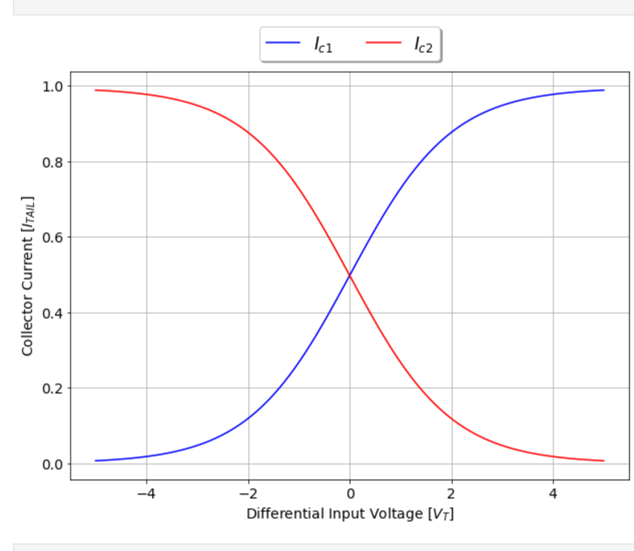
- The differential output voltage is a *hyperbolic tangent* function of the input voltage

$$V_{od} = V_{op} - V_{om} = \alpha I_{TAIL} R_C \tanh\left(\frac{-V_{id}}{2V_T}\right) \tag{37}$$

- The collector currents reach their min/max values for $V_{id} \approx 3V_T$

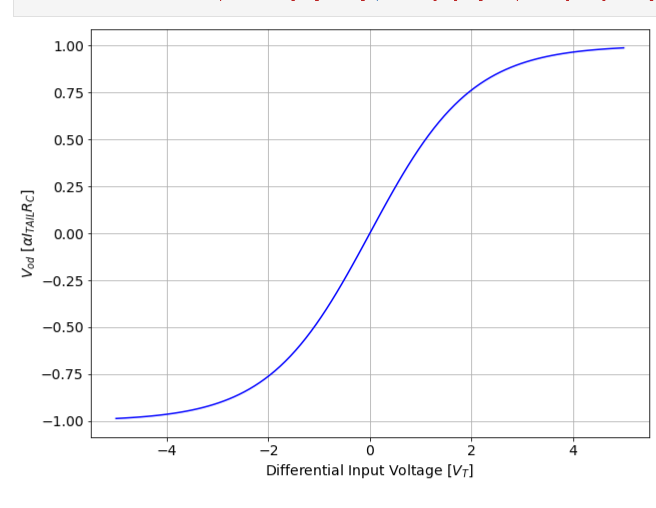
In [45]:

```
beta = 200
alpha = beta/(beta+1)
I_TAIL = 1e-3
V_id = np.linspace(-5*k*T/q, 5*k*T/q, 100)
I_c1 = alpha*I_TAIL/(1+np.exp(-V_id/(k*T/q)))
I_c2 = alpha*I_TAIL/(1+np.exp(V_id/(k*T/q)))
plot_2xy(V_id/(k*T/q), I_c1/I_TAIL, I_c2/I_TAIL,
         r'Differential Input Voltage [$V_T$]', r'Collector Current [$I_{TAIL}$]',
        r'$I_{c1}$', r'$I_{c2}$')
```



In [53]:

```
V_CC = 5
R_C = 1e3
V_om = V_CC - I_c1*R_C
V_op = V_CC - I_c2*R_C
plot_xy(V_id/(k*T/q), (V_op - V_om)/alpha/I_TAIL/R_C,
        r'Differential Input Voltage [$V_T$]', r'$V_{od}$ [$\alpha I_{TAIL} R_C$]')
```



Differential gain

- Analysis of the differential amplifier is simplified by the assumption that v_{ip} and v_{im} are complementary and $V_{id} \ll 3V_T$
- Assuming this is the case, the small-signal current through R_{EE} consists of complementary currents, making the net change in V_P zero
- In this sense, V_P is treated as a small-signal ground, enabling straightforward analysis of the differential half-circuit
- Large input signals (greater than $3V_T$) violate the *small-signal assumption* and cause significant movement of V_P

- As with the single-ended common-emitter amplifier, the small-signal gain is given by

$$|A_v| = \frac{g_m}{1+g_m R_E} \cdot R_C \tag{38}$$

- If $g_m R_E \gg 1$, this is approximated as R_C/R_E
- If $R_E = 0$, we see the full gain of the common-emitter amplifier, $g_m R_C$
- Assuming $v_{ip} = -v_{im} = v_{id}/2$, the differential gain is given by

$$\begin{align} A_{vd} &= \frac{v_{op} - v_{om}}{v_{ip} - v_{im}} \tag{39} \\ &= \frac{\frac{1}{2}g_m R_C - (-\frac{1}{2}g_m R_C)}{v_{id}} \cdot v_{id} \tag{40} \\ &= \boxed{g_m R_C} \tag{41} \end{align}$$

Common-mode gain

- The common-mode gain is a measure of the differential amplifier's ability to reject common-mode (including DC) inputs
- Common-mode inputs and outputs are, by definition, equal to each other, allowing us to treat them as virtual shorts
- Due to the absence of a virtual ground the common-mode gain is reduced by dengeration
- Assuming $2g_m R_{EE} \gg 1$, we have

$$\frac{v_{ocm}}{v_{icm}} \approx \frac{R_C}{R_E + 2R_{EE}} \tag{42}$$

- For current-source-biased designs, R_{EE} is the output impedance of a transistor (or cascode current source)

Single-ended input

- A differential amplifier can be used for single-ended applications where DC-coupling is desired/needed
- In this case, the small-signal current of Q_1 flows through the emitter of Q_2
- Assuming $1/g_{m2} \ll R_{TAIL}$, $g_{m1} = g_{m2}$, and $R_s = 0$ the small-signal current of Q_1 is given by

$$i_{c1} = \frac{g_{m1} v_i}{2} \tag{43}$$

- The small-signal output voltage is thus

$$v_o = \frac{g_{m1} R_C}{2} \tag{44}$$

Noise analysis

- The noise analysis of the differential amplifier can't employ the differential half-circuit, since the noise does not constitute a differential excitation
- The single-ended output current noise at the collector of Q_2 includes contributions from Q_1, Q_2, and R_{C2} (but not R_{C1}, why?)

$$i^2_{n2,out} = 4kTr_{b1} \cdot g^2_{m1}/4 + 4kTr_{b2} \cdot g^2_{m2}/4 + 2qI_{C1}/4 + 2qI_{C2}/4 + 4kT/R_{C2} \tag{45}$$

- The single-ended output noise voltage is the product of this noise current and R_{C2},

$$e_{n2,out} = i_{n2,out} \cdot R_{C2} \tag{46}$$

- Each input has a noise current due to shot noise given by

$$i_n = qI_{TAIL}/\beta \tag{47}$$

Noise discussion

- Note that for differential operation, the single-ended output noise voltages are correlated, with the exception of the noise due to $R_{C1,2}$
- As a result, the RMS differential output noise is approximately twice the single-ended RMS noise (instead of twice the mean-square noise), and given by

$$e^2_{nd,out} = R^2_C \cdot (4kTr_{b1} \cdot g^2_{m1} + 4kTr_{b2} \cdot g^2_{m2} + 2qI_{C1} + 2qI_{C2} + 4kT/R_{C1} + 4kT/R_{C2}) \tag{48}$$

- Compare this to the noise of a single-ended common-emitter amplifier:

$$e^2_{nse,out} = R^2_C \cdot (4kTr_b \cdot g^2_m + 2qI_C + 4kT/R_C) \tag{49}$$

Operational amplifiers

- An opamp is essentially a differential pair with an active current-mirror load, typically followed by a low-impedance output stage (emitter-follower in this case)
- The equivalent input voltage noise primarily contains contributions from $Q_{4,5}$ and $Q_{6,7}$, while the input current noise is the base current shot noise from $Q_{4,5}$
- Due to the large number of active devices, opamps are generally less noise-efficient than the resistively-loaded stages we've been discussing
- However, other desirable properties of opamps, such as high gain and common-mode rejection, make them attractive for many applications
- An opamp can be combined with a discrete differential pair for the best of both worlds, including ultra-low noise

Summary

- The common-emitter stage provides substantial gain, particularly with an active load, but DC-coupling is challenging due to the (exponential) sensitivity of bias conditions to the DC input voltage
- An active load increases gain of the CE stage by an order of magnitude, but doubles the mean-square equivalent input voltage noise
- Degeneration decreases the RMS shot noise from an active load by the ratio $\sqrt{V_{RE}/2V_T}$, where V_{RE} is the voltage drop across the degeneration resistor
- The Miller effect, which degrades high-frequency performance of high-gain stages like the CE amplifier, can be alleviated by cascoding, which ntroduces very little additional noise
- Differential amplifiers offer a reasonable tradeoff between DC input range, gain, and noise