Noise in Solid-State Devices and Lasers

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Invited Paper

Abstract—A survey is given of the most important noise problems in solid-state devices. Section II discusses shot noise in metal-semiconductor diodes, p-n junctions, and transistors at low injection; noise due to recombination and generation in the junction space-charge region; high-level injection effects; noise in photodiodes, avalanche diodes, and diode particle detectors, and shot noise in the leakage currents in field-effect transistors (FETs). Section III discusses thermal noise and induced gate noise in FETs; generation–recombination noise in FETs and transistors at low temperatures; noise due to recombination centers in the space-charge region(s) of FETs, and noise in space-charge-limited solid-state diodes. Section IV attempts to give a unified account of 1/f noise in solid-state devices in terms of the fluctuating occupancy of traps in the surface oxide; discusses the kinetics of these traps; applies this to flicker noise in junction diodes, transistors, and FETs; and briefly discusses flicker noise in Gunn diodes and burst noise in junction diodes and transistors. Section V discusses shot noise in the light emission of luminescent diodes and lasers, and noise in optical heterodyning. Section VI discusses circuit applications. It deals with the noise figure of negative conductance amplifiers (tunnel diodes and parametric amplifiers), and of FET, transistor, and mixer circuits. In the latter discussion capacitive up-converters, and diode, FET, and transistor mixers are dealt with.

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I. INTRODUCTION

THERE are five main sources of noise that play a part in solid-state devices. The first source is *shot noise*, caused by the random emission of electrons or photons, or the random passage of carriers across potential barriers. The second source is *thermal noise*, usually caused by the random collision of carriers with the lattice, but generally found in conditions of thermal equilibrium. The third source is *partition noise*, caused when a carrier current is split into two parts that flow to different electrodes, or a photon current that is split into two parts, e.g., a part that emits zero electrons per photon and a part that emits one electron per photon in photoemission processes. The fourth source is *generation–recombination noise* (GR noise) that is caused by the random generation and recombination of hole–electron pairs, or the random generation of carriers from traps, or the random recombination of carriers with empty traps. Finally there is *flicker noise or $1/f$ noise*, characterized by a $1/f^{\alpha}$ spectrum with $\alpha \simeq 1$.

It is the aim of this paper to review the most important effects of these noise sources in lasers and solid-state circuits and devices. It should be clear to the reader that even a rather lengthy review paper cannot cover *all* the detailed aspects of noise in electron devices. For this the reader is referred to the original literature.

II. Shot Noise in Diodes and Transistors [155], [170]

In discussing noise in p-n junction diodes and transistors one must distinguish between low injection and high injection. Low injection in an abrupt p-n junction means that $p \ll N_d$ everywhere in the n-region and $n \ll N_a$ everywhere in the p-region; here p and n are the hole and electron concentrations and N_d and N_a are the donor and acceptor concentrations, respectively. At high-injection levels $p > N_d$ in part of the n-region of a p^+n diode and $n > N_a$ in part of the p-region of a pn^+ diode.

At low injection the passage of carriers across barriers can be considered as a series of independent random events [147], [152]. As a consequence full shot noise should be associated with each current flowing in the device. At high frequencies the noise may be modified by transit time or diffusion time effects. This approach is called the "corpuscular approach"; it breaks down at high injection, since the carriers are no longer independent in that case. For many cases, however, it should be a good first approximation.

Another approach considers the noise as being caused by the random processes of diffusion and/or recombination [114], [120], [121], [151]. The noise sources to be introduced here are diffusion and generation–recombination noise sources. This is called the "collective approach."

It has been shown that the two approaches are equal at low-injection levels [157]. At high-injection levels only the collective approach can be used and the diffusion and recombination noise sources must be properly modified (Section II-E) [171].

A. Shot Noise in Metal-Semiconductor (Schottky Barrier) Diodes

The current in a metal-semiconductor diode[1]

$$I = I_0(V)\left[\exp\left(\frac{qV}{mkT}\right) - 1\right] \tag{1}$$

consists of two parts, a current $I_0(V)\exp(qV/mkT) = (I + I_0)$ due to carriers flowing from the semiconductor into the metal, and a current $-I_0(V)$ due to carriers flowing from the metal into the semiconductor. Both currents should fluctuate independently and each should show full shot noise. Hence one would expect

$$S_i(f) = 2q[(I + I_0) + I_0] = 2q(I + 2I_0). \tag{2}$$

This agrees essentially with Weisskopf's early formula [175]. Since the low-frequency (LF) conductance g_0 may be written

$$g_0 = \frac{dI}{dV} \simeq \frac{q}{mkT} I_0 \exp\left(\frac{qV}{mkT}\right) = \frac{q}{mkT}(I + I_0), \tag{3}$$

(2) may be written

$$S_i(f) = 2mkTg_0\left(\frac{I + 2I_0}{I + I_0}\right) \tag{4}$$

corresponding to full thermal noise at zero current ($I = 0$) and to half thermal noise at large currents ($I \gg I_0$) if $m = 1$. In addition the series resistance r of the diode should show full thermal noise:

$$S_e(f) = 4kTr. \tag{5}$$

In metal-semiconductor mixer diodes one should make the series resistance r small in comparison with the LF diode resistance $R_0 = 1/g_0$. This can be done by proper diode design. For large forward bias (R_0 small) the effect of the series resistance r can be quite significant. The spectral intensity $S_{sc}(f)$ of the short-circuit noise current is then, if $R_0 = 1/g_0$,

$$S_{sc}(f) = \frac{2mkTR_0 + 4kTr}{R^2} \tag{6}$$

where $R = R_0 + r$. This increases from the value $2mkT/R_0$, for $R_0 \gg r$, to the value $4kT/r$, for $R_0 \ll r$.

In point contact diodes the current I flows through a very small cross section at the contact and hence considerable heating occurs. As a consequence T can be larger than the temperature T_0 of the environment. Therefore, if (6) is written

$$S_{sc}(f) = n_1 \cdot 4kT_0/R, \tag{6a}$$

it may sometimes happen that n_1 is somewhat larger than unity for forward bias. For a well designed Schottky barrier diode, n_1 can be as low as 1/2 if $m \simeq 1$. This is one of the reasons why Schottky barrier diodes are gradually replacing point contact diodes as microwave mixers. One of the other reasons is low $1/f$ noise.

At high frequencies, transit-time effects across the barrier should be taken into account. This problem, which should be significant in the high gigahertz range, has not yet been solved. Equations (2) through (6a) have been well verified by experiment [87].

B. Shot Noise in p-n Junction Diodes at Low Injection

The theory of the previous section can also be applied to p-n junction diodes at low injection. For the sake of simplification we assume that the diode is a p^+n diode so that practically all current is carried by holes. The current

$$I = I_0\left[\exp\left(\frac{qV}{kT}\right) - 1\right] \tag{7}$$

then consists of two parts, a part $(I + I_0)$ due to holes injected into the n-region and recombining there, and a part $-I_0$ due to holes generated in the n-region and being collected by the p-region. Both currents should show full shot noise and

[1] In this equation the factor m, unity at zero current, increases slowly with current. Also, $I_0(V)$ is slowly voltage dependent.

hence (2) through (4) should be valid with $m=1$. Again the series resistance r of the diode should show thermal noise.

At higher frequencies an additional source of noise occurs. It is due to holes injected into the n-region and returning to the p-region by back diffusion before having recombined. This changes the junction admittance from the low-frequency value $Y_0=g_0$ to the high-frequency (HF) value $Y=g+jb$, where g increases with increasing frequency. The increase $(g-g_0)$ in g comes from these back-diffused holes. Since diffusion is a thermal process, one would expect the conductance $g-g_0$ to have thermal noise. Consequently, at higher frequencies (2) should be written

$$S_i(f) = 2q(I + 2I_0) + 4kT(g - g_0). \tag{8}$$

Van der Ziel and Becking proved this in greater detail [152] from the corpuscular point of view.

In long p^+n junctions, that is, junctions where the width of the n-region is larger than a few times the diffusion length $L_p=(D_p\tau_p)^{1/2}$ of holes, the junction admittance Y is

$$Y = g_0(1 + j\omega\tau_p)^{1/2} \tag{9}$$

where τ_p is the lifetime of the holes in the n-region, so that

$$g = g_0[\tfrac{1}{2}(1 + \omega^2\tau_p^2)^{1/2} + \tfrac{1}{2}]^{1/2}. \tag{9a}$$

These equations have been verified by experiment [4], [57], [58], [88].

For p-i-n diodes a somewhat different behavior is found [119]. Here the characteristic is of the form

$$I = I_0\left[\exp\left(\frac{qV}{mkT}\right) - 1\right] \tag{10}$$

where m is unity for low currents and increases slowly with increasing current [43a]. In that case (8) must be replaced by

$$S_i(f) = 2q(I + 2I_0) + 4mkT(g - g_0). \tag{11}$$

This has been discovered by experiment for $I \gg I_0$. The occurrence of the factor m in (11) has been understood from a slight extension of the Becking–van der Ziel theory [119].

The same shot noise approach also applies to tunnel diodes. Here there are two opposing currents flowing that depend on bias in a different way and that are equal at zero bias. For forward bias [2], [125], [145]

$$S_i(f) = 2qI \coth(qV/2kT) \tag{12}$$

which reduces to full shot noise of the current I for $qV/kT \gg 1$:

$$S_i(f) = 2qI. \tag{12a}$$

This is the case in the negative conductance region. Equation (12) has been verified by experiment [2].

In the valley region the noise is sometimes larger than shot noise [2]. This behavior is not yet fully understood; it seems that trapping states in the space-charge region are involved. It would be worthwhile to investigate whether this condition still exists for modern tunnel diodes.

C. Shot Noise in Transistors at Low Injection

For the sake of simplicity we assume the transistor to be a p-n-p transistor in which practically all the current is carried by holes. The emitter current I_E then consists of a part $I_{ES}\exp(qV_{EB}/kT)$ due to holes injected into the base, where V_{EB} is the emitter-base voltage, and a part $-I_{BE}$ due to holes generated in the base region and collected by the emitter. The emitter current I_C consists of a part $\alpha_F I_{ES}\exp(qV_{EB}/kT)$ due to the collection of holes injected by the emitter, where α_F is the forward-current amplification factor, and a part I_{BC} due to holes generated in the base region and collected by the collector. Therefore,

$$I_E = I_{ES}\exp\left(\frac{qV_{EB}}{kT}\right) - I_{BE} \tag{13}$$

$$I_C = \alpha_F I_{ES}\exp\left(\frac{qV_{EB}}{kT}\right) + I_{BC} = \alpha_F I_E + I_{CO} \tag{14}$$

where

$$I_{CO} = \alpha_F I_{BE} + I_{BC} \tag{14a}$$

is the collector saturated current. The LF emitter conductance g_{eb0} is

$$g_{eb0} = \frac{\partial I_E}{\partial V_{EB}} = \frac{q}{kT} I_{ES}\exp\left(\frac{qV_{EB}}{kT}\right) = \frac{q}{kT}(I_E + I_{EB}), \tag{15}$$

the LF transconductance g_{m0} is

$$g_{m0} = g_{ec0} = \frac{\partial I_c}{\partial V_{EB}} = \frac{q}{kT}\alpha_F I_{ES}\exp\left(\frac{qV_{EB}}{kT}\right) = \alpha_F g_{eb0}, \tag{16}$$

and the LF current amplification factor is

$$\alpha_0 = \frac{\partial I_C}{\partial I_E} = \frac{\partial I_C/\partial V_{EB}}{\partial I_E/\partial V_{EB}} = \alpha_F. \tag{17}$$

All these currents should show full shot noise. Consequently, at low frequencies

$$S_{I_E}(f) = 2qI_{ES}\exp\left(\frac{qV_{EB}}{kT}\right) + 2qI_{BE} = 2q(I_E + 2I_{BE}) \tag{18}$$

$$S_{I_C}(f) = 2q\alpha_F I_{ES}\exp\left(\frac{qV_{EB}}{kT}\right) + 2qI_{BC} = 2qI_C. \tag{19}$$

Since the current I_E and I_C have the component $\alpha_F I_{ES}\exp(qV_{EB}/kT)$ in common, and this current should show full shot noise, the cross-correlation current should be

$$S_{I_C,I_E}(f) = 2q\alpha_F I_{ES}\exp\left(\frac{qV_{EB}}{kT}\right) = 2kTg_{ec0}. \tag{20}$$

At high frequencies the emitter admittance Y_{eb} becomes complex and its real part g_{eb} increases with increasing frequency. This effect is due to holes injected by the emitter and returning by back-diffusion before having been collected. The noise associated with these holes corresponds to thermal noise of the increment $(g_{eb}-g_{eb0})$ in the emitter

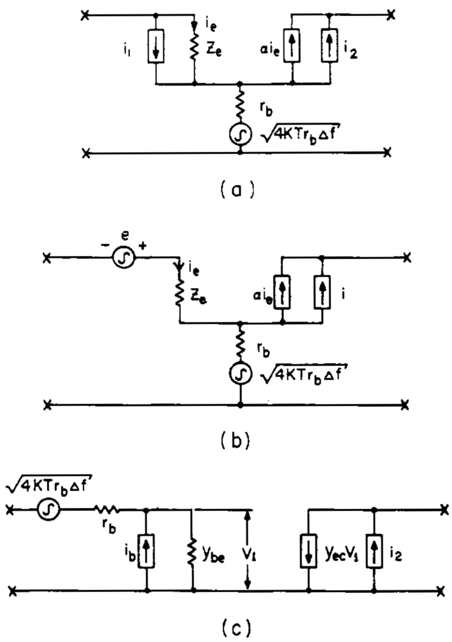

Fig. 1. Equivalent transistor circuits. (a) General equivalent circuit. (b) Common base equivalent circuit. (c) Common emitter equivalent circuit.

conductance since diffusion is a thermal process. Hence

$$S_{I_E}(f) = 2q(I_E + 2I_{BE}) + 4kT(g_{eb} - g_{eb0}). \tag{18a}$$

In addition the HF transfer admittance changes from the LF value g_{ec0} to the complex HF value Y_{ec}, because diffusion gives rise to a random delay time between emission and collection so that the coherence of the ac collector current deteriorates. Since one would expect the noise to behave in the same way as the signal, (20) should be written

$$S_{I_C,I_E}(f) = \frac{Y_{ec}}{g_{ec0}} \cdot 2kTg_{ec0} = 2kTY_{ec} = 2kT\alpha Y_{eb}. \tag{20a}$$

Finally, since the passage of carriers across the collector junction still consists of a series of independent random events, (19) should remain valid. Van der Ziel and Becking have proven (18a) and (20a) in greater detail [152]. The HF current amplification factor α is here defined as

$$\alpha = \frac{Y_{ec}}{Y_{eb}} \simeq \frac{\alpha_0}{1 + jf/f_\alpha} \tag{21}$$

where f_α is the alpha cutoff frequency of the transistor. Especially in cases where drift (e.g., due to built-in fields) and diffusion in the base region are comparable, it is found that (21) must be replaced by

$$\alpha = \frac{\alpha_0 \exp(-j\omega\tau)}{1 + jf/f_\alpha} \tag{21a}$$

where τ is a time constant associated with the drift effect. This does not alter most noise effects, however, since $\overline{i^2}$ depends on $|\alpha|^2$ only. (See (23) for the definition of i.)

The noise can now be represented by two current generators i_1 and i_2 in parallel with the emitter and the collector junctions (Fig. 1(a)), respectively, where

$$\overline{i_1^2} = S_{I_E}(f)\Delta f; \quad \overline{i_2^2} = S_{I_C}(f)\Delta f; \quad \overline{i_1^* i_2} = S_{I_C,I_E}(f)\Delta f. \tag{22}$$

In Fig. 1(a) the thermal noise of the base resistance r_b has been included.[2]

The noise can also be represented by an electromotive force (EMF) e in series with the emitter junction and a current generator i in parallel with the collector junction (Fig. 1(b)), where

$$e = i_1 Z_e; \quad i = (i_2 - \alpha i_1), \tag{23}$$

where $Z_e = 1/Y_e$ is the emitter impedance. The advantage of this representation is that e and i are only weakly correlated. At low frequencies

$$\begin{aligned} \overline{e^2} &= \overline{i_1^2}|Z_e|^2 = 2kTg_{ebo}\left[\frac{I_E + 2I_{BE}}{I_E + I_{BE}}\right]\Delta f; \\ \overline{i^2} &= 2q[I_E\alpha_F(1 - \alpha_F) + I_{CO}]\Delta f; \ \overline{ei^*} \simeq 0 \end{aligned} \tag{24}$$

since $\alpha_0 = \alpha_F$ in this approximation. This representation corresponds to the LF equivalent circuit developed by Montgomery and Clark [111] and van der Ziel [150]. It has been well verified by experiment, even in cases where α_0 differs from α_F (see Section II-D). It is obvious that the equivalent circuit of Fig. 1(b) is best adapted to the common-base circuit.

At high frequencies (24) must be modified [30]. A detailed calculation shows that $\overline{e^2}$ decreases with increasing frequency, whereas $\overline{ei^*}$ can have a significant reactive component [29], [34]. Finally, $\overline{i^2}$ depends on $|\alpha|^2$ and may be written

$$\overline{i^2} = 2q\Delta f\left[\alpha_F I_E \frac{(1 - \alpha_F + f^2/f_\alpha^2)}{1 + f^2/f_\alpha^2} + I_{CO}\right] \tag{24a}$$

so that it reaches the limiting value $2qI_C\Delta f$, for $f \gg f_\alpha$.

To measure $\overline{i^2}$, a large resistance R is inserted in series with the emitter and the output noise of the transistor is measured. The resistance R eliminates the effect of the noise EMF e. The only remaining noise source is then the current source i as long as $4kT\Delta f/R \ll \overline{i^2}$. The measurements have verified (24a) very accurately even at relatively high injection [146], and even when recombination in the emitter space-charge region is significant [22].

To measure $\overline{e^2}$ and the cross correlation $\overline{ei^*}$ one must use the transistor as an HF amplifier. This problem will be discussed in Section VI-C.

In common emitter connection another representation, suggested by Giacoletto, is often used [50]. It consists of a current generator $i_b = i_1 - i_2$ in parallel with the base–

[2] In a more accurate description of the equivalent circuit one sometimes splits the base resistance into two parts. This should have little effect on the noise properties. It should be noted that the values of r_b deduced from the noise data and those obtained by other methods often do not agree too well [107].

emitter admittance $Y_{be} = Y_{eb} - Y_{ce}$ and the current generator i_2 between emitter and collector (Fig. 1(c)). It is easily evaluated that

$$\overline{i_b^2} = 2q(I_B + 2I_{BE} + 2I_{BC})\Delta f + 4kT[g_{eco} - \mathrm{Re}(Y_{ec})]\Delta f + 4kT(g_{eb} - g_{ebo})\Delta f \quad (25)$$

$$\overline{i_b^* i_c} = 2kT(Y_{ec} - g_{eco})\Delta f - 2qI_{BC}\Delta f. \quad (26)$$

At low frequencies this can be approximated as

$$\overline{i_b^2} = 2qI_B\Delta f; \quad \overline{i_b^* i_c} = 0. \quad (27)$$

At high frequencies this circuit leads to very complicated calculations and hence a transformation back to the common-base circuit is advisable (Section VI-C).

D. Noise Due to Recombination in the Space-Charge Region of a Junction Diode

This problem was discussed by van der Ziel [156], by Scott and Strutt [136], and by Lauritzen [98]. The effect is especially significant in silicon devices. Since Lauritzen has given the most detailed discussion, we state essentially his results.

The generation and recombination of carriers in the space-charge region comes about because of the presence of recombination centers, the so-called Shockley–Hall–Read (SHR) centers. For back-biased diodes these centers alternately generate an electron and a hole with the relaxation time τ_t of the centers relatively large. For forward-biased diodes these centers alternately capture an electron and a hole with the relaxation time τ_t of the centers relatively small.

To calculate the noise, the space-charge region is split into small sections Δx, the contribution of each section Δx to the noise is evaluated, and then the result is integrated over the width of the space-charge region. It must be taken into account in the calculation that an emission or capture event displaces a total charge that depends on the position of the center and that is only a fraction of the electronic charge q.

For back-biased diodes and low frequencies ($\omega\tau_t \ll 1$), Lauritzen finds

$$S_i(f) = 2qI\frac{e_p^2 + e_n^2}{(e_p + e_n)^2} \quad (28)$$

where e_p and e_n are the hole and electron emission rates of the center. For $e_p \gg e_n$ or $e_p \ll e_n$ this corresponds to full shot noise, for $e_p = e_n$ it corresponds to half shot noise. For high frequencies ($\omega\tau_t \gg 1$),

$$S_i(f) = \tfrac{2}{3} \cdot 2qI. \quad (29)$$

This expression has been verified by Scott and Strutt [136].

It is interesting to note that at low frequencies ($\omega\tau_t \ll 1$), the alternate emission of a hole and an electron appears as a single event and any possible reduction comes about because the SHR center tends to "smooth" the emission of carriers if $e_p \simeq e_n$. At high frequencies ($\omega\tau_t \gg 1$), the carriers appear to be generated independently; the shot noise reduction in this case comes about because an emission event displaces a charge that is only a fraction of the electronic charge q.

For forward-biased diodes the current I is partly due to carriers injected into the space-charge region; the part I_R is due to recombination in the space-charge region. The first part gives full shot noise

$$S(f) = 2q(I - I_R). \quad (30)$$

According to Lauritzen [98] the spectral intensity of the recombination noise for $\omega\tau_t \ll 1$ is

$$S_{I_R}(f) = 2qI_R \quad (31)$$

for relatively small forward bias, and

$$S_{I_R}(f) = \tfrac{3}{4} \cdot 2qI_R \quad (32)$$

for larger forward bias. Since the relaxation time τ_t in these cases is much smaller than in the back-biased condition, the frequency range $\omega\tau_t \gg 1$ is not very important. For $I \gg I_R$ little error is made by assuming full shot noise of the current I. It is doubtful that (31) and (32) can be verified experimentally.

In p-n-p transistors recombination can occur in the emitter space-charge region, giving rise to a component I_R in the emitter current I_E. Since the transistor has a low current-amplification factor α_F if I_R is comparable to I_E, modern transistors are so designed that in the useful range of operation I_R is small in comparison with the emitter current I_E.

Nevertheless it has a significant effect. For since I_R increases slower with increasing emitter–base voltage V_{EB} than I_E, $\alpha_F = I_C/I_E$ increases with increasing V_{EB}, or α_F increases with increasing I_E. Consequently, the LF current amplification factor α_0 is

$$\alpha_0 = \frac{\partial I_C}{\partial I_E} = \alpha_F + \frac{\partial \alpha_F}{\partial I_E} I_E \quad (33)$$

so that $\alpha_0 > \alpha_F$ and $\beta_0 = \alpha_0/(1-\alpha_0) > \beta_F = \alpha_F/(1-\alpha_F)$. The effect of the recombination current on β_F and β_0 can be appreciable.

As far as the noise is concerned, (18) through (20a) remain essentially correct. But instead of (15), one now has

$$g_{ebo} \simeq \frac{\alpha_F}{\alpha_0}\frac{q(I_E + I_{BE})}{kT}. \quad (34)$$

Finally, if one calculates $\overline{i^2}$, one finds that (24a) is essentially correct as long as $(\alpha_0 - \alpha_F)^2 \ll (1-\alpha_F)$; usually this is the case [22], [31].

E. High-Level Injection Effects

At high currents so much hole charge can be stored in the n-region of a p$^+$n diode that $p > N_d$ in the part of the n-region adjacent to the space-charge region. This effect is readily observable in Ge diodes. It has an interesting effect on the input admittance. According to Schneider and Strutt [134], [135], the equivalent circuit of the diode for that case

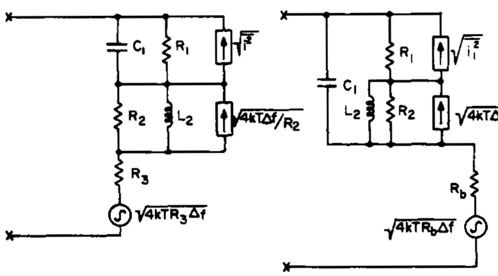

Fig. 2. (a) Equivalent junction diode circuit at high injection. (b) Partial equivalent transistor circuit at high injection.

is shown in Fig. 2(a). Their impedance measurements seem to indicate that the device can be well represented by the equivalent circuit; their noise measurements indicate that thermal noise is associated with the impedances $Z_2 = R_2 + j\omega L_2$ and the series resistance R_3.

Similar effects were found in p-n-p and n-p-n transistors. Here there is an HF impedance associated with the emitter junction; the equivalent circuit is shown in Fig. 2(b). This effect gives rise to a current dependence of the HF current-amplification factor α. Shot noise is associated with the diode proper and thermal noise is associated with R_2 and the base resistance R_b. This circuit has also been well verified by Schneider and Strutt [134], [135].[3]

Nevertheless, there is something peculiar about these circuits, for the storage capacitance C_1 is in parallel with R_1 in Fig. 2(a) and in parallel with the series connection of R_1 and R_2, and L_2 in Fig. 2(b). It is difficult to account for this difference. For that reason a careful investigation of this problem would seem appropriate.

The effect does not seem to be present in modern transistors. In modern transistors other high-level injection effects have been found. Agouridis and van der Ziel [3] found a saturation effect in the collection region of microwave p-n-p transistors. It comes about as follows. Most microwave transistors have a narrow p^- collector region adjacent to the base and a wider p^+ collector region adjacent to the contact. At high current the p^- collector region becomes space-charge limited; as a consequence the effective path length of carriers in the base region becomes larger, resulting in a considerable drop in the alpha cutoff frequency. This effect has been studied with the help of noise figure measurements.

Another effect was found by Tong and van der Ziel [146]. They found an appreciable correlation between the emitter noise EMF e and the collector noise current generator i and they demonstrated that this was a high-level injection effect. The magnitude of the effect is somewhat disturbing and, therefore, it would be worthwhile to study it in greater detail.

[3] Johnson and van der Ziel [91] have reported that noise measurements in 2N332 transistors indicated that a slight modification in Fig. 2(b) was necessary. Later measurements on this transistor type did not verify the need for such a modification.

The theory of high-level injection is presently being developed. Van Vliet [171] has recently evaluated the noise sources. Since electrons and holes are present in about equal numbers, the ambipolar diffusion equation must be used. One then obtains for hole injection into n-type material

$$\frac{\partial p}{\partial t} = g - r - \mu_a \boldsymbol{E} \cdot \nabla p + \gamma(\boldsymbol{r}, t) + \nabla \cdot \boldsymbol{\eta}_a(\boldsymbol{r}, t) + D_a \nabla^2 p \quad (35)$$

where g and r are the generation and recombination rates, $\boldsymbol{E}$ is the field strength, μ_a the ambipolar mobility, D_a the ambipolar diffusion constant, $\gamma(\boldsymbol{r}, t)$ the generation–recombination noise source, and $\nabla \cdot \boldsymbol{\eta}_a(\boldsymbol{r}, t)$ the diffusion noise source. Van Vliet has shown that these sources have the following spatial cross-correlation spectra:

$$S_\gamma(\boldsymbol{r}, \boldsymbol{r}', f) = \frac{2p}{\tau_H} \frac{\kappa^2 + \delta^2}{(\kappa + \delta)^2} \delta(\boldsymbol{r} - \boldsymbol{r}'), \quad (36)$$

for recombination by SRH centers [compare (28)], and

$$s_{\Delta \cdot \eta_a}(\boldsymbol{r}, \boldsymbol{r}', f) = 4\nabla\nabla' \left[D_a \frac{np}{n + p} \delta(\boldsymbol{r} - \boldsymbol{r}') \right] : \boldsymbol{i}. \quad (37)$$

Here τ_H is the high-injection lifetime, δ and κ are transition constants, ∇ means differentiation with respect to $\boldsymbol{r}$, and ∇' differentiation with respect to $\boldsymbol{r}'$, whereas $\delta(\boldsymbol{r} - \boldsymbol{r}')$ is the Dirac delta function and $\boldsymbol{i}$ is the unit tensor.[4] Usually $\kappa \neq \delta$ and $(\kappa^2 + \delta^2)/(\kappa + \delta)^2 \simeq 1$.

Tarng [141] evaluated the noise spectrum for a p^+n diode in the high-injection limit and obtained:

1) at low frequencies ($\omega\tau_H \ll 1$)

$$S_I(f) = \frac{2}{3}\left(1 + \frac{1}{b}\right) \cdot 2eI \quad (38)$$

2) at high frequencies ($\omega\tau_H \gg 1$)

$$S_I(f) = \frac{1}{2}\left(1 + \frac{1}{b}\right) \cdot 2eI(2\omega\tau_H)^{1/2}, \quad (39)$$

whereas the low-injection values are, if τ is the low-injection lifetime,

$$S_I(f) = 2eI \qquad (\text{for} \quad \omega\tau \ll 1) \quad (38a)$$

$$S_I(f) = 2eI(2\omega\tau)^{1/2} \qquad (\text{for} \quad \omega\tau \gg 1). \quad (39a)$$

Here $b = \mu_n/\mu_p$ is the mobility ratio. For n^+p diodes the term $1/b$ must be replaced by b.

This theory has not yet been verified experimentally, and for that reason a further study seems warranted.

In addition the effect of the stored charge on the noise must be evaluated. Van der Ziel [153], [155] calls this effect a *modulation effect* and has suggested that the diode noise current generator i should extend not only over the junction proper but also over the impedance due to the stored charge. This problem requires a much more detailed analysis and a

[4] For the one-dimensional case the tensor notation can be dropped, $\nabla = \partial/\partial x$ and $\nabla' = \partial/\partial x'$.

careful experimental study that should verify and/or extend Schneider and Strutt's results [134], [135].

F. Noise in Photodiodes and Avalanche Diodes

Junction diodes are often used as photodetectors. If the photocurrent generated in the photodiode by amplitude-modulated light is

$$I_s(t) = I_{s0} + I_{s1}\cos\omega_p t \tag{40}$$

where ω_p is the modulation frequency and I_0 is the dark current of the diode, then the noise is [116]

$$S_I(f) = 2q(I_{s0} + I_0). \tag{41}$$

Another way of using the diode as a photodetector is the photovoltaic mode of operation. In that case the photodiode is used open circuited and the dc current is zero, or

$$I = -I_{s0} + I_0\left[\exp\left(\frac{eV}{kT}\right) - 1\right] = 0. \tag{42}$$

The noise is now shot noise of the independent currents I_{s0}, I_0, and $I_0 \exp(eV/kT)$ and hence one obtains [51], instead of (41),

$$S_I(f) = 4q(I_{s0} + I_0). \tag{43}$$

It often happens that the signal coming out of a photodiode drowns in the noise background of the associated receivers and amplifiers. In that case it is convenient to use a multiplication process before the signal is fed into the receiver. One way of doing so is to use avalanche multiplication.

Early avalanche diodes (or Zener diodes) showed breakdown spots, so-called "microplasmas," in the space-charge region. These microplasmas switched on and off at random and thus produced large amounts of noise. This noise process was first studied in detail by Champlin [26] and has since been used for noise generators [67]–[69]. It has been possible, however, to design avalanche diodes that show uniform breakdown. Such diodes can be used for avalanche multiplication [13]–[15] in the prebreakdown region and as oscillators and negative resistance amplifiers in the breakdown region [83].

To calculate the multiplied signal and the noise, we assume first for the sake of simplicity that the electron and the hole have equal ionizing power. If $I_s(t)$ is again the photocurrent generated in the diode and I_0 the dark current of the diode, and if p hole–electron pairs are generated when an individual hole–electron pair traverses the space-charge region once, then the multiplied ac current is

$$\begin{aligned} I_{sc}\cos\omega_p t &= I_{s1}\cos\omega_p t[1 + p + p^2 + \cdots] \\ &= MI_s\cos\omega_p t \end{aligned} \tag{44}$$

so that

$$M = \frac{1}{1-p} \tag{44a}$$

is the multiplication factor; it can be quite large if p is close to unity.

For the noise we start with the shot noise of $I_{s0}+I_0$, and this is multiplied. In addition, since each generated hole–electron pair is generated at random and traverses the space-charge region, all the generated currents should show full shot noise before being multiplied. One would thus expect

$$\begin{aligned} S_I(f) &= 2[(I_{s0} + I_0) + p(I_{s0} + I_0) + p^2(I_{s0} + I_0)\cdots]M^2 \\ &= 2q(I_{s0} + I_0)M^3. \end{aligned} \tag{45}$$

The case in which the electrons and holes have different ionizing power has been evaluated by McIntyre [105] and by Gummel and Blue [65]. McIntyre finds that the noise can be considerably better or considerably worse than given by (45), depending on the situation. If α and β are the ionization coefficients for electrons and holes, respectively, and $\beta = k\alpha$, then

$$S_I(f) = 2eI_{in}M^3\left[1 + \left(\frac{1-k}{k}\right)\left(\frac{M-1}{M}\right)^2\right] \tag{45a}$$

if the injected current I_{in} consists of holes and

$$S_I(f) = 2eI_{in}M^3\left[1 - (1-k)\left(\frac{M-1}{M}\right)^2\right] \tag{45b}$$

if the injected current I_{in} consists of electrons. The first expression is quite large when $k \ll 1$ and much smaller if $k \gg 1$, so that the best results are obtained if the holes have the largest ionizing power. The second expression is quite large when $k \gg 1$ and much smaller if $k \ll 1$, so that the best results are obtained if the electrons have the largest ionizing power. These predictions have been well verified by experiment [13]–[15].

If the devices are used in the breakdown region as microwave negative-conductance amplifiers, one obtains large noise figures. It seems that silicon avalanche amplifiers have the highest noise figure, germanium avalanche amplifiers are better, and gallium arsenide avalanche amplifiers are still better [96]. This seems to be associated with (45a) and (45b). If the avalanche devices are used as oscillators, the microwave signals thus generated have amplitude noise modulation and frequency noise modulation. Again, silicon avalanche oscillators are the noisiest, germanium avalanche oscillators are quieter, and gallium arsenide oscillators are much quieter.

G. p-n Diodes as Particle Detectors [46]

When p-n junctions are bombarded with high-energy particles a considerable number of hole–electron pairs will be generated in the space-charge region. Most of these will be collected, and as a consequence a large number of random current pulses will pass through the device. This generates a considerable amount of noise. In analogy with the noise generated in secondary electron multiplication, the spectral intensity of the noise is

$$S_I(f) = 2e^2\overline{N_B}\,\overline{m^2} \tag{46}$$

where $\overline{N_B}$ is the rate of incidence of bombarding particles and m the number of collected hole–electron pairs per primary particle. This has been verified experimentally.

H. Shot Noise in FETs [167]

In junction FETs thermal generation of carriers gives rise to a leakage current flowing from gate to channel, which contributes to the gate noise and to some extent to the drain noise. In metal-oxide-semiconductor (MOS) FETs thermal generation of carriers gives rise to a leakage current flowing from substrate to channel, resulting in an additional term in the drain noise. In modern silicon FETs both effects are insignificant at room temperature but may become important at elevated temperatures.

According to van der Ziel [167] a gate current δi_g flowing into the gate between x_0 and $x_0+\Delta x_0$ gives rise to a drain current δi_d such that

$$\delta i_d = \frac{x_0}{L}\,\delta i_g. \tag{47}$$

If the gate is slightly back biased, δi_g will show full shot noise. One can thus evaluate the self-spectral intensities $\Delta S_{gg}(f)$, $\Delta S_{dd}(f)$, and the cross-spectral intensity $\Delta S_{gd}(f)$ of δi_g and δi_d. Integrating over the length of the channel, assuming that the gate current density $J_g(x_0)$ is independent of x_0, yields the spectral intensities

$$S_{gg}(f) = 2qI_g; \qquad S_{dd} = \tfrac{1}{3}\cdot 2qI_g; \qquad S_{gd} = \tfrac{1}{2}\cdot 2qI_g \tag{48}$$

so that the correlation coefficient is

$$C = \frac{S_{gd}}{\sqrt{S_{gg}\cdot S_{dd}}} = \tfrac{1}{2}\sqrt{3}. \tag{48a}$$

For junction FETs the device is generally used at a high input impedance level. Therefore even a relatively small gate leakage current I_g can be quite harmful, so that the gate leakage sets a serious limit to the high temperature operation of the device.

For MOS FETs there is no gate noise, but the drain noise has a component due to the current I_s flowing to the substrate. It consists of three parts:

1) a leakage current I_{ss} from source to substrate, that does not contribute to $S_{dd}(f)$;
2) a leakage current I_{cs} from channel to substrate, that contributes $\frac{1}{3}\cdot 2qI_{cs}$ to $S_{dd}(f)$;
3) a leakage current I_{ds} from drain to substrate, that contributes $2qI_{ds}$ to $S_{dd}(f)$.

The total contribution of the substrate leakage current to $S_{dd}(f)$ is, therefore,

$$2q(\tfrac{1}{3}I_{cs} + I_{ds}) = 2q\beta I_s \tag{49}$$

where β is a factor somewhat smaller than unity. This must be compared with the term due to the thermal noise of the channel, which may be written as

$$\tfrac{2}{3}\cdot 4kTg_{\max}, \tag{50}$$

which is usually much larger. It may thus be concluded that the substrate leakage current gives only a small contribution to $S_{dd}(f)$ and therefore hardly sets a high temperature limitation to the operation of the device.

I. Summary

We have seen how a simple shot noise picture can give a reasonable description of most noise phenomena in diodes and transistors, and of some noise phenomena in FETs. It was shown that noise in avalanche diodes could also be described in terms of an "amplified shot noise" picture.

Only in the case of recombination–generation in the space-charge region of a junction is it necessary to develop a more detailed picture. While recombination–generation in the emitter space-charge region of a transistor has a profound effect on the current amplification factor, the noise description in terms of shot noise mechanisms remains practically valid.

It is not fully clear how this picture must be modified at high injection, since the theory for this case has not yet been developed. Also the effect of the stored charge on the noise has not been explored sufficiently. Experiments seem to indicate that high-level injection effects affect the noise in long junction diodes much more than in transistors with a narrow base region. More work is needed before a unified picture can be developed.

III. Thermal and Generation–Recombination Noise in FETs and Other Devices [170]

Junction FETs operate on the principle that the width of a conducting channel is modulated in the rhythm of the applied gate voltage. Since the conducting channel should show thermal noise, one would expect the limiting noise of junction FETs to be thermal noise, modified perhaps by the modulation effect.

MOS FETs operate on the principle that the mobile charge present under the gate is modulated in the rhythm of the applied gate voltage. Since the conducting channel is only present when the gate voltage is properly chosen, it is not a priori certain that the device should have thermal noise. Rather, one would expect the noise to be diffusion noise, caused by the collisions of carriers with the lattice. It has been shown, however, that this diffusion noise corresponds to thermal noise as long as the Einstein relation $qD_n = kT\mu_n$ holds [164]. Since this is usually the case, one may assume that the limiting noise in the channel is thermal noise. Deviations may occur, of course, near saturation, where the electric field strength near the drain may be so high that the mobility becomes field dependent. In that case it is not certain that the Einstein relation holds, and even if it does, the electron temperature will not be equal to the device temperature.

A similar situation occurs in single- and double-injection space-charge-limited solid-state diodes. Here the carriers are injected and the steady-state situation is again a nonequilibrium situation. The limiting noise should again be diffusion noise, but as long as the Einstein relation holds this corresponds to thermal noise. Again, in some instances, especially in single-injection diodes at high currents, the electric field strength in the device can be so high that the mobility becomes field dependent. It has been shown experimentally that this results in a higher "effective noise temperature" of the device.

At room temperature practically all the impurity atoms in the channel of a junction FET are ionized, but at lower temperatures part of the atoms are neutral. As a consequence the carrier density will fluctuate by processes of the type

neutral donor + energy $\rightleftarrows$ ionized donor + electron,

etc. This corresponds to a local change in the conductance of the channel, which shows up as noise in the external circuit if dc current is passed through the device. The effect becomes more and more pronounced at lower temperatures. It should not occur in MOS FETs, but may be significant in the base region of transistors at low temperatures.

In the space-charge region of the junction FET and in the space-charge region between channel and substrate of the MOS FET recombination centers will be present. Due to the fluctuating occupancy of these centers, the channel conductance will be modulated, resulting in a noise current in the external circuit.

A. Operation of the FET

In order to understand the noise behavior of the FET, one has to know something about the operation of the device. We first introduce some concepts associated with the FET.

Since the device operates on the principle of modulation of a conducting channel, we introduce the conductance per unit length $g(x)$, at a point at a distance x from the source end of the channel. If $V_0(x)$ is the channel potential at the point, taken with respect to the source, we may write $g(V_0)$ instead of $g(x)$. For an n-type channel, we may then write for the current I_d

$$I_d = g(V_0)\frac{dV_0}{dx}; \quad \text{or} \quad I_d dx = g(V_0)dV_0 \tag{51}$$

so that, since I_d is independent of x,

$$I_d L = \int_0^{V_d} g(V_0)dV_0; \quad \text{or} \quad I_d = \frac{1}{L}\int_0^{V_d} g(V_0)dV_0 \tag{52}$$

where V_d is the drain voltage. Here I_d flows from drain to source.

For a junction FET with an n-type channel

$$g(V_0) = g_{00}\left\{1 - \left[\frac{-V_g + V_{\rm dif} + V_0}{V_{00}}\right]^{1/2}\right\} \tag{53}$$

where g_{00} is the conductance of the fully open channel, V_g the gate potential, $V_{\rm dif}$ the diffusion potential of the junction, and V_{00} the potential difference between gate and channel for zero channel conductance. The channel is thus cut off at the drain (that is, it has zero conductance at the drain) if

$$V_d + V_{\rm dif} - V_g = V_{00}, \text{ or } V_d = V_{d0} = V_{00} - V_{\rm dif} + V_g. \tag{53a}$$

For a MOS FET with n-type channel and a low-conductivity substrate

$$g(V_0) = \mu w C_{\rm ox}(V_g + V_{g0} - V_0) \tag{54}$$

where V_g is the gate potential, V_{g0} the offset potential between gate and channel, $C_{\rm ox}$ the capacitance per unit area of the gate-oxide-channel system, w the width of the channel, and μ the carrier mobility. In this case the channel is cut off at the drain if

$$V_d = V_{d0} = V_g + V_{g0}. \tag{54a}$$

For the case of arbitrary substrate conductivity the expression for $g(V_0)$ is more complicated.

It is thus possible to calculate I_d as a function of V_g and V_d in each case, and to introduce the transconductance g_m and the drain conductance g_d by the definitions

$$g_m = \frac{\partial I_d}{\partial V_g}; \qquad g_d = \frac{\partial I_d}{\partial V_d} = \frac{g(V_d)}{L}. \tag{55}$$

A calculation shows that g_d has its maximum value

$$g_{d0} = \frac{g_0}{L} \tag{55a}$$

at zero drain bias and zero value if the channel is cut off at the drain, whereas g_m has zero value at zero drain bias and its maximum value $g_{\max}$ if the channel is cut off at the drain. The device is said to be *saturated* if the channel is cut off at the drain $[g(V_d)=0]$. Beyond saturation, I_d and g_m are practically independent of V_d. The device is usually operated under that condition.

To understand the high-frequency behavior of the FET, the device must be treated as an active distributed line. One can derive the wave equation for this line and find solutions in terms of special transcendental functions [49]. For the junction FET, the wave equation for the small signal voltage distribution $\Delta V(x)$ along the channel is

$$\frac{d^2}{dx^2}[g(W_0)\Delta V(x)] = \frac{j\omega\rho_0 wa\Delta V(x)}{V_{00}^{1/2}W_{00}^{1/2}(x)} \tag{56}$$

where ρ_0 is the charge density in the space-charge region, w the width of the channel, $2a$ the height of the open channel, $W_0(x) = -V_g + V_{\rm dif} + V_0$ is the bias between channel and gate, ω is the frequency, and $j = \sqrt{-1}$. For the MOS FET one has instead

$$\frac{d^2}{dx^2}[V_1(x)\Delta V(x)] = \frac{j\omega}{\mu}\Delta V(x) \tag{57}$$

where μ is the mobility of the carriers, and $V_1(x) = V_g + V_{\rm dif} - V_0(x)$.

When these equations are solved [49], it is found that the transconductance becomes complex and may be approximated as

$$Y_m = \frac{g_{m0}}{1 + jf/f_0} \tag{58}$$

where g_{m0} is the transconductance at low frequencies and f_0 a kind of cutoff frequency which is a few times larger than $g_m/(2\pi C_{gs})$. Furthermore one finds for the gate-source admittance Y_{gs},

$$Y_{gs} = j\omega C_{gs} + g_{gs} \tag{59}$$

where C_{gs} is independent of frequency and g_{gs} varies as ω^2, both over a wide frequency range.[5]

B. Thermal Noise in FETs [158]

We follow here the treatment given by Klaassen and Prins [95]. To that end we extend (51) as follows:

$$I_d + \Delta I_d(t) = g(V_0 + \Delta V)d\left(\frac{V_0 + \Delta V}{dx}\right) + h(x, t). \quad (60)$$

Here $h(x, t)$ is the thermal noise source operating on the channel, $\Delta I_d(t)$ the noise current flowing in the external circuit and $\Delta V(x, t)$ the noise voltage along the channel, both caused by the noise source $h(x, t)$. Making a Taylor expansion of $g(V_0 + \Delta V)$, ignoring higher order terms in ΔV, and bearing in mind that

$$I_d = g(V_0)\frac{dV_0}{dx}, \quad (60a)$$

yields

$$\Delta I_d(t) = \frac{d}{dx}\left[g(V_0)\Delta V\right] + h(x, t). \quad (61)$$

Here $\Delta I_d(t)$ flows from drain to source.

We now integrate this equation with respect to x between the limits 0 and L for the case that the drain is HF short-circuited to the source, so that $\Delta V = 0$ at $x = 0$ and $x = L$. This yields

$$\Delta I_d(t)L = \int_0^L h(x, t)dx \quad (62)$$

or

$$\overline{\Delta I_d(t)\Delta I_d(t + s)} = \frac{1}{L^2}\int_0^L \int_0^L S_h(x, x', f)dxdx'. \quad (63)$$

But since $h(x, t)$ is a thermal noise source one would expect for the spatial cross-correlation spectrum of $h(x, t)$

$$S_h(x, x', f) = 4kTg(x')\delta(x' - x) \quad (64)$$

where $\delta(x' - x)$ is the Dirac delta function. Equation (64) expresses the fact that the thermal noise source at the points x and x' at a given instant t are uncorrelated.

Carrying out the calculation yields

$$S_i(f) = \frac{4kTg_0}{L}\frac{\int_0^{V_d}[g(V_0)/g_0]^2 dV_0}{\int_0^{V_d}[g(V_0)/g_0]dV_0} = \gamma 4kTg_{d0}. \quad (65)$$

It is easily seen that $\gamma = 1$ for $V_d = 0$, since in that case $g(V_0) = g_0$ everywhere. Also $\gamma < 1$ for $V_d > 0$, since in that condition

$$g(V_0) \leqq g_0 \quad \text{or} \quad \left[\frac{g(V_0)}{g_0}\right]^2 \leqq \frac{g(V_0)}{g_0}$$

so that the denominator in γ is larger than the numerator. Carrying out the calculation one finds that γ decreases monotonically with increasing V_d and reaches a minimum value γ_{sat} at saturation. For a junction FET one finds

$$\gamma_{sat} = \frac{1}{2}\frac{(1 + 3z^{1/2})}{1 + 2z^{1/2}}$$

where

$$z = \frac{-V_g + V_{dif}}{V_{00}} \quad (66)$$

so that $\gamma_{sat} = \frac{1}{2}$ for $z = 0$ (channel fully open at the source) and $\gamma_{sat} = \frac{2}{3}$ for $z = 1$ (channel fully cut off at the source) [158]. For an MOS FET with a low-conductivity substrate one finds $\gamma = \frac{2}{3}$ for all values of V_g [92]. For a MOS FET with a channel of higher substrate conductivity γ_{sat} lies slightly lower.

If one plots γ versus V_d one should thus obtain a curve that decreases monotonically with increasing V_d. Experimentally one finds that this is not true for a junction FET; the reason is not that the preceding theory is incorrect, but rather that it is incomplete, due to the presence of series resistances at the end of the channel. When this series resistance effect was taken into account, Bruncke found good agreement between theory and experiment [21].

For MOS FETs there are also deviations in some instances. Often γ decreases first with increasing V_d and then rises rapidly near saturation [73]. Halladay and van der Ziel [74] attributed this to mobility fluctuations, and Takagi and van der Ziel [140] showed that such fluctuations, if present, should have the strongest effect for substrates of larger conductivity. Recent experiments [179] have indicated, however, that MOS FETs can be built on substrates of higher conductivity that show only thermal noise. While this eliminates the idea of noise caused by mobility fluctuations, it leaves open the question of the source of the excess noise found in earlier devices. While the tail end of the flicker noise could perhaps explain some of the data, it cannot explain all. A further investigation in this problem would thus be worthwhile.

Another way of representing the noise is to write

$$S_i(f) = \alpha_{sat}4kTg_{max} \quad (67)$$

where g_{max} is the maximum transconductance at saturation. A comparision with (65) indicates that

$$\alpha_{sat} = \gamma_{sat}\frac{g_{d0}}{g_{max}}. \quad (67a)$$

MOS FETs with low-conductivity substrates have $g_{d0} = g_{max}$ so that $\alpha_{sat} = \gamma_{sat}$. For substrates of higher conductivity [95] $g_{max} < g_{d0}$ so that $\alpha_{sat} > \gamma_{sat}$.

[5] Van der Ziel and Ero [162] have developed an approximation method in which one writes

$$\Delta V(x) = \Delta V_0(x) + j\omega\Delta V_1(x) + (j\omega)^2\Delta V_2(x) + \cdots.$$

By collecting equal terms in $j\omega$ one then obtains differential equations in $\Delta V_0(x)$, $\Delta V_1(x)$, $\Delta V_2(x) \cdots$, that can be solved successively. This method is especially useful in the case of substrates of arbitrary conductivity [125a].

Introducting the noise resistance R_n of the device by equating $S_i(f)=4kTR_n g_{\max}^2$ yields

$$R_n = \alpha_{\text{sat}}/g_{\max},$$

or

$$\alpha_{\text{sat}} = R_n g_{\max} \tag{68}$$

so that α_{sat} can be determined from noise resistance data. Since the noise resistance R_n is a useful quantity for the design engineers, the quantity α_{sat} is of greater practical significance than γ_{sat}. The latter parameter, however, gives more physical insight.

C. Induced Gate Noise in FETs

At high frequencies the noise voltage distribution along the channel gives, by capacitive coupling, rise to a noise current i_g flowing to the gate that is partially correlated with the drain noise i_d. One must thus calculate the mean square values of these current generators and their cross correlation, i.e., one needs the quantities $\overline{i_g^2}$, $\overline{i_d^2}$, and $\overline{i_g i_d^*}$.

The rigorous way of solving this problem is to introduce localized noise EMFs ΔV_{x0} between x_0 and $x_0+\Delta x_0$, calculate the contributions Δi_g and Δi_d by solving the wave equation, evaluate $\overline{\Delta i_g^2}$, $\overline{\Delta i_d^2}$, and $\overline{\Delta i_g \Delta i_d^*}$, and then sum over all the sections Δx_0, which corresponds to integration over the length of the channel. This procedure is quite tedious and often not very rewarding. (For the rigorous solution of the junction FET problem, see Klaassen [94a].)

If one wants Δi_g and Δi_d up to first-order terms in $j\omega$, one can apply a procedure developed by van der Ziel [125a], [159]. The starting point is now (61), slightly modified by omitting the distributed noise source and by changing the direction of positive current flow. We thus have

$$\frac{d}{dx}\left[g(V_0)\Delta V(x)\right] = -\Delta I_d(t). \tag{69}$$

where $I_d(t)$ now flows *out of* the drain. The initial condition is that $\Delta V(x)$ shows a jump ΔV_{x0} between x_0 and $x_0+\Delta x_0$. Since the drain is HF connected to the source, if we want the short-circuit noise current, we must require $\Delta V(0)=\Delta V(L)=0$. Integration yields

$$\begin{aligned} g(V_0)\Delta V &= -\Delta I_d x, && \text{for } 0 < x < x_0,\\ g(V_0)\Delta V &= -\Delta I_d(x-L), && \text{for } x_0+\Delta x_0 < x < L. \end{aligned} \tag{70}$$

Bearing in mind the condition at $x=x_0$, yields immediately

$$\Delta I_d(t) = \frac{g(V_0)}{L}\,\Delta V_{x0}. \tag{71}$$

Moreover, if $C(x)$ is the capacitance per unit area between channel and gate and w is the channel width, then the potential distribution $\Delta V(x)$ induces a charge $d\Delta Q_g = -wC(x)dx\Delta V(x)$ in the gate. The total charge ΔQ_g is obtained by integrating over the channel length,

$$\Delta Q_g = -w\int_0^L C(x)\Delta V(x)dx. \tag{72}$$

The current flowing into the gate is therefore $\Delta I_g(t) = d\Delta Q_g/dt$.

Making a Fourier analysis of $\Delta V_{x0}(t)$, $\Delta I_d(t)$, and $\Delta I_g(t)$, introducing the Fourier components Δv_{x0}, Δi_d, and Δi_g, and bearing in mind that

$$\overline{\Delta v_{x0}^2} = \frac{4kT\Delta x\Delta f}{g(V_0)}, \tag{73}$$

one can calculate $\overline{\Delta i_g^2}$, $\overline{\Delta i_d^2}$ and $\overline{\Delta i_g\Delta i_d^*}$. By integrating over the channel length L one obtains $\overline{i_g^2}$, $\overline{i_d^2}$, and $\overline{i_g i_d^*}$. The value of $\overline{i_d^2}$ corresponds to (65) but the other results are new. Since d/dt becomes $j\omega$ in the Fourier analysis it is obvious that $\overline{i_g^2}$ varies as ω^2 and $\overline{i_g i_d^*}$ as $j\omega$.

One may now introduce the correlation coefficient

$$c = \frac{\overline{i_g i_d^*}}{\sqrt{\overline{i_g^2}\cdot\overline{i_d^2}}}. \tag{74}$$

In the approximation used here c is imaginary. Evaluation of $|c|$ indicates that for a junction FET the factor $|c|$ decreases monotonically with increasing $z=(-V_g+V_{\text{dif}})/V_{00}$, starting from the value $|c|=0.445$ for $z=0$ to the value [159] $|c|=0.395$ for $z=1$. For a MOS FET with a low-conductivity substrate [74] $|c|=0.395$, for a MOS FET with substrates of higher conductivity $|c|$ is about the same.

The reason why $|c|$ is so much smaller than unity is that the sign of $\Delta Q_g(x_0, t)$ changes with increasing x_0, whereas the sign of $\Delta I_d(x_0, t)$ does not depend on x_0. Consequently the contributions of different sections Δx_0 to $\overline{i_g i_d^*}$ partly cancel each other. The value of c can be determined from noise figure measurements of the device (see Section VI), but not very accurately since $|c|$ is so small.

Since the input conductance g_{gs} and $\overline{i_g^2}$ both vary as ω^2 over a wide frequency range, it is convenient to introduce the equivalent noise temperature T_{ng} of g_{gs}, by putting

$$(\overline{i_g^2})_{\text{sat}} = 4kT_{ng}(g_{gs})_{\text{sat}}\Delta f. \tag{75}$$

A long calculation shows that for a junction FET[6] at temperature T

$$T_{ng} = T\frac{1+9z^{1/2}+15z+7z^{3/2}}{1+8z^{1/2}+10\frac{5}{7}z+4\frac{2}{7}z^{3/2}} \tag{76}$$

where $z=(-V_g+V_{\text{dif}})/V_{00}$ as before, so that $T_{ng}=T$ for $z=0$ (channel fully open at the source) and $T_{ng}=\frac{4}{3}T$ for $z=1$ (channel fully cut off at the source). This agrees roughly with the measurements [23].

For a MOS FET with a low-conductivity substrate the

[6] This result can be evaluated from van der Ziel and Ero's paper if a misprint in that paper is corrected. The expression $(\frac{1}{6}z^{1/2}-\frac{1}{8}z^2+\frac{1}{10}z^{5/2})$ in (37a) of that paper should have a minus sign.

value of T_{ng}/T is evaluated [74], [138] at 4/3. Experimentally one sometimes finds a much higher value [74]. The reason for this is not quite clear; it is unlikely that this is a substrate effect, however [179].

D. Generation–Recombination (GR) Noise in FETs and Transistors [161]

If we consider a section Δx of the channel of a junction FET then the carrier density in that section will show fluctuations due to processes of the type:

free electron + ionized donor ⇄ neutral donor + energy.

Hence the conductance

$$g(W_0) = q\mu \frac{\Delta N}{\Delta x} \tag{77}$$

where ΔN is the number of carriers in the section Δx, will show fluctuations around the equilibrium value $g(W_0)$ because ΔN shows fluctuations $\delta\Delta N$. As a consequence the resistance $\Delta R = \Delta x/g(W)$ will show fluctuations $\delta\Delta R$,

$$\delta\Delta R = -\frac{\Delta x}{[g(W_0)]^2}\delta g = -\frac{\Delta x}{g(W_0)}\frac{\delta\Delta N}{\Delta N}, \tag{78}$$

which in turn gives rise to a fluctuating EMF

$$\Delta V_{x0} = -I_d\delta\Delta R = \frac{I_d\Delta x}{g(W_0)}\frac{\delta\Delta N}{\Delta N}. \tag{79}$$

According to (71) this gives rise to a drain current fluctuation

$$\delta\Delta I_d(t) = \frac{I_d\Delta x}{L\Delta N}\delta\Delta N(t) \tag{80}$$

so that

$$\overline{\delta\Delta I_d(t)\delta\Delta I_d(t+s)} = \frac{I_d^2\Delta x^2}{L^2\Delta N^2}\overline{\delta\Delta N(t)\delta\Delta N(t+s)}. \tag{80a}$$

But if $\overline{\delta\Delta N^2} = \alpha\Delta N$, and if $\delta N(t)$ decays exponentially,

$$\overline{\delta\Delta N(t)\delta\Delta N(t+s)} = \alpha\Delta N \exp\left(-\frac{s}{\tau}\right). \tag{81}$$

Substituting into (80a) and applying the Wiener–Khintchine theorem, yields for the spectrum $S_i(f)$ of $\delta\Delta I_d(t)$, since $I_d\Delta x = g(W_0)\Delta W_0$ and $g(W_0) = q\mu\Delta N/\Delta x$,

$$\Delta S_i(f) = \frac{4q\mu I_d\alpha}{L^2}\frac{\tau}{1+\omega^2\tau^2}\Delta W_0. \tag{82}$$

By integrating over the length of the channel, one obtains for the spectrum of the drain noise

$$S_i(f) = \frac{4q\mu I_d(W_d - W_s)}{L^2}\frac{\tau}{1+\omega^2\tau^2} \tag{83}$$

where $W_s = -V_g + V_{\text{dif}}$ and $W_d = V_d - V_g + V_{\text{dif}}$, if α and τ are independent of x. In saturation W_d must be replaced by V_{00} and I_d by its saturated value.

If we apply this to the case of deep lying donors in the channel, where the generation rate $g(N)$ and the recombination rate $r(N)$ are given by

$$g(N) = \gamma(N_d - N), \qquad r(N) = \rho N^2, \tag{84}$$

we find after some calculations [24], [170], if N_0 is the equilibrium carrier concentration,

$$\tau = \frac{1}{dr/dN - dg/dN}\bigg|_{N=N_0} = \frac{N_d - N_0}{\rho N_0(2N_d - N_0)}$$

and (85)

$$\alpha = \frac{N_d - N_0}{2N_d - N_0}.$$

In silicon junction FETs the effect begins to set in around 100°K and it increases rapidly with decreasing temperature. At the onset of the effect $N_d - N_0$ is still small, and hence τ will be so small that the noise is practically white. This effect has been studied by Shoji [137]. Since germanium has much shallower donor levels than silicon, one will have to go to very low temperatures to see the effect in germanium junction FETs. This has been verified experimentally.

Another effect studied by Shoji [137] is due to deep lying traps in the channel. It should not occur in well-designed units. The effect also should not occur in MOS FETs. To understand this, consider an MOS FET with an n-type channel on a p-type substrate. The electrons in the channel now come from the source and drain, which are usually made of n^+ material, so that the donors lie very close to the bottom of the conduction band. The only effect that could occur is that part of the acceptors in the channel and in the space-charge region between channel and substrate may not be ionized. It is presently unknown what this would do to the noise.

GR noise of this type may also occur in the base of transistors at low temperature. The generation–recombination processes of the type

free electron + ionized donor ⇄ neutral donor + energy

gives rise to a fluctuation δr_b in the base resistance r_b, which is converted into a noise EMF $I_B\delta r_b$ due to the flow of base current. This results, therefore, in a noise EMF in series with r_b that should have a spectral intensity proportional to I_B^2. This effect is presently under study at the University of Florida, Gainesville, Fla.

E. Noise Due to Recombination Centers in the Space-Charge Region of an FET [126]

We follow here essentially Sah's treatment of the problem [126].

Consider an n-type channel of length L, width W, and height $2a$. Let the x axis be in the direction of the channel and the y axis perpendicular to it. Let n_T be the density of centers, then a small volume element $\Delta V = \Delta x\Delta y w$ around the point (x_1, y_1) in the space-charge region contains $\Delta N_T = n_T\Delta V$ centers. Let ΔN_t of these centers be occupied, and let $\delta\Delta N_t$ be the fluctuation in N_t, then

$$\overline{\delta\Delta N_t^2} = \Delta N_T f_t(1 - f_t) = n_T \Delta V f_t(1 - f_t). \tag{86}$$

The reader will recognize this as a partition noise term. Hence the spectral intensity of $\delta\Delta N_t$ will be

$$S_{\delta\Delta_t(f)} = 4n_T f_t(1 - f_t)\Delta V \frac{\tau_t}{1 + \omega^2\tau_t^2}. \tag{87}$$

The fluctuating occupancy of the centers modulates the width $2b$ of the conducting channel by an amount $2\delta b$, where[7]

$$\delta b = -\frac{\delta\Delta N_t}{N_d \Delta x_1 W} \frac{(a - y_1)}{(a - b)}. \tag{88}$$

The fluctuation δb gives rise to a fluctuation $\delta\Delta R$ in the resistance $\Delta R = \Delta x_1/[2q\mu_n N_d bw]$ of the section Δx_1, where μ_n is the carrier mobility. This gives, for $\delta\Delta R$,

$$\delta\Delta R = -\Delta R \frac{\delta b}{b} = \frac{\delta\Delta N_t}{2q\mu_n N_d^2 w^2} \frac{a - y_1}{(a - b)b^2}. \tag{89}$$

The flow of the dc current I_d gives a fluctuating EMF $\delta\Delta V_{x0} = I_d \delta\Delta R$ in the section Δx, and, by virtue of (71), to a fluctuating current $\delta\Delta I_d = g(W_0)\delta\Delta V_{x0}/L$ in the external circuit, so that, substituting for $g(W_0)$

$$\delta\Delta I_d = \frac{I_d}{N_d wL} \frac{a - y_1}{(a - b)b} \delta\Delta N_t. \tag{90}$$

Making a Fourier analysis one obtains for the spectrum

$$\Delta S_{I_t}(f) = \frac{I_d^2}{N_d^2 w^2 L^2} 4n_T f_t(1 - f_t) \cdot \frac{\tau_t}{1 + \omega^2\tau_t^2} \frac{(a - y_1)^2}{(a - b)^2 b^2} \Delta x_1 \Delta y_1. \tag{91}$$

Assuming that $f_t(1 - f_t)$ and τ_t are independent of y_1 and x_1 and integrating over the full space-charge region yields

$$S_{I_d}(f) = \frac{8q\mu_n V_{00} I_d}{3L^2 N_d} n_T f_t(1 - f_t) \frac{\tau_t}{1 + \omega^2\tau_t^2} \cdot \left[-(y - z) - 2\,(y^{1/2} - z^{1/2}) + 2\ln\left(\frac{1 - y^{1/2}}{1 - z^{1/2}}\right) \right] \tag{92}$$

[7] δb is evaluated by solving Poisson's equation

$$\nabla^2\psi = -\frac{q}{\varepsilon\varepsilon_0}\left[N_d - \frac{\delta\Delta N_t}{\Delta x_1 \Delta y_1} f(x - x_1) f(y - y_1)\right]$$

where N_d is the donor concentration, and $f(x - x_1)$ and $f(y - y_1)$ are unity in the volume element ΔV and zero elsewhere. This two-dimensional problem can be reduced approximately to a one-dimensional problem by assuming that $|d\psi/dx| \ll |d\psi/dy|$ and $|d^2\psi/dx^2| \ll |d^2\psi/dy^2|$ and that $\delta b(x) = \delta b$ inside the interval $x_1 < x < x_1 + \Delta x_1$ and zero outside that interval. Poisson's equation between x_1 and $x_1 + \Delta x_1$ may be written

$$\frac{d^2\psi}{dy^2} = -\frac{q}{\varepsilon\varepsilon_0}\left[N_d - \frac{\delta\Delta N_t}{\Delta x_1 \Delta y_1 w} f(y - y_1)\right]$$

with $\psi = 0$ and $d\psi/dy = 0$ at $y = b + \delta b$ and $\psi = V_g - V_{\text{dif}}$ at $y = a$. The solution of this equation corresponds to (88).

where $y = (-V_g + V_{\text{dif}} + V_d)/V_{00}$ and $z = (-V_g + V_{\text{dif}})/V_{00}$ and V_{00} is the cutoff voltage.

This shows a logarithmic divergence for $y \to 1$ (saturation). Actually $S_{I_d}(f)$ converges to a finite value because the mobility becomes field dependent near the drain and hence the equations for ΔR and $\delta\Delta R$ must be modified accordingly [72]. Beyond saturation $S_{I_d}(f)$ is practically independent of V_d.

We now discuss a few additional corrections [170] of this result.

1) The fluctuation δb of (88) is calculated for one side of the channel. Since both sides of the channel fluctuate independently with fluctuations δb_1 and δb_2 and the total width is $2b_1$, we have

$$2\delta b = \delta b_1 + \delta b_2, \quad \text{or } \overline{\delta b^2} = \tfrac{1}{4}(\overline{\delta b_1^2} + \overline{\delta b_2^2}). \tag{93}$$

Therefore, (92) must be divided by a factor 4 if $\overline{\delta b_1^2} \ll \overline{\delta b_2^2}$ and by a factor 2 if $\overline{\delta b_1^2} = \overline{\delta b_2^2}$.

2) $f_t(1 - f_t)$ is not independent of y_1, as assumed, but is very small except near the point where the quasi-Fermi level of the traps crosses the trapping level, at which point $f_t(1 - f_t)$ has its maximum value $\frac{1}{4}$. It is, in principle, not difficult to take this effect into account, and it would be worthwhile doing it.

3) Often there is a distribution in time constants τ_t. One then obtains a somewhat "smeared-out" $1/(1 + \omega^2\tau_t^2)$ spectrum.

A similar effect also occurs in the space-charge region between channel and substrate of a MOS FET. This problem has been studied by Yau and Sah [178].

F. Noise in Space-Charge-Limited Solid-State Diodes

In space-charge-limited diodes the limiting noise is diffusion noise. In single-injection diodes only one type of carrier is present, in double-injection diodes both electrons and holes are present in about equal numbers.

In the case where the carriers are electrons, the noise in a section Δx can be represented by a current generator $\overline{i^2}$ in parallel with Δx. As van der Ziel and van Vliet [163], [164] indicated,

$$\overline{i^2} = 4\frac{q^2 D_n n(x) A}{\Delta x}\Delta f = 4kT\left[\frac{q\mu_n n(x) A}{\Delta x}\right]\Delta f = \frac{4kT\Delta f}{\Delta R} \tag{94}$$

where ΔR is the resistance of the section Δx, $n(x)$ the carrier density, A is the cross-sectional area of the device, and D_n the diffusion constant for electrons. We have here made use of the Einstein relation $qD_n = kT\mu_n$.

In the case of double injection the electrons and holes fluctuate independently and hence [169]

$$\overline{i^2} = 4\frac{q^2 D_n n(x) A}{\Delta x}\Delta f + \frac{4q^2 D_p p(x) A}{\Delta x}\Delta f = 4kT\left[\frac{q\mu_n n(x) + q\mu_p p(x)}{\Delta x}\right] A\Delta f = \frac{4kT\Delta f}{\Delta R}. \tag{95}$$

Here D_p is the hole diffusion constant, $p(x)$ the hole concentration, and ΔR the resistance of the section Δx. We have here again made use of the Einstein relation.

The open-circuit EMF in series with the section Δx is thus, in either case,

$$\overline{\Delta v^2} = \overline{i^2}\Delta R^2 = 4kT\Delta R\Delta f = 4kT\frac{\Delta V}{I_a}\Delta f \qquad (96)$$

where I_a is the device current and ΔV the dc voltage developed across the section Δx. Summing over all sections Δx, bearing in mind that the fluctuations in individual sections are independent, yields for the mean square open-circuit voltage

$$\overline{v_a^2} = 4kT\frac{V_a}{I_a}\Delta f. \qquad (97)$$

This should hold for both single and double injection.

For single injection, at relatively low frequencies where transit-time effects do not play a part, $g = 2I_a/V_a$ is the ac conductance so that the mean square short-circuit current is

$$\overline{i_a^2} = g^2\ \overline{v_a^2} = 8kTg\Delta f. \qquad (98)$$

This equation remains valid when transit-time effects become important. This has been verified experimentally [85], [103].

For double-injection diodes, at frequencies for which $\omega\tau \gg 1$, where τ is the carrier lifetime, $g = I_a/V_a$ and, hence [169],

$$\overline{i_a^2} = \overline{v_a^2}g^2 = 4kTg\Delta f. \qquad (99)$$

This equation has also been verified experimentally [18], [37], [104]. Note that the last result is half the previous one.

G. Summary

We have seen that a simple thermal noise picture can give a reasonable description of most noise phenomena in FETs and in space-charge-limited solid-state diodes.

Moreover, a detailed application of the theory of GR noise can explain the low-temperature noise behavior of junction FETs. The same theory, when applied to recombination centers in the space-charge regions of junction and MOS FETs, can describe some low-frequency noise phenomena in these devices. In both cases one has to know the statistics of recombination centers and traps, and one must evaluate how the fluctuating occupancy of recombination centers or traps modulates the local channel conductance. The theory is well understood in principle, but some details need further clarification.

It is not clear at present how a field-dependent mobility affects the noise. A very likely possibility is that the noise remains thermal noise but at a temperature corresponding to the effective temperature of the "hot" electrons. This seems to be a fruitful field of further study.

IV. Flicker Noise in Solid-State Devices

Besides shot noise and/or thermal noise all solid-state devices show a noise component with a $1/f^\alpha$ spectrum, where $\alpha \simeq 1$. This type of noise is known as "flicker noise" or "$1/f$-noise." It has been demonstrated in many instances that this $1/f$ noise spectrum holds down to extremely low frequencies [108], [109].

General discussions about $1/f$ noise have been given in the past and several formal theories have been presented. Recent progress has come from a more detailed study of the mechanisms operating in devices showing $1/f$ noise.

It was first suggested that a wide distribution in time constants might be responsible for the $1/f$ spectrum. This idea was specified by McWhorter who demonstrated that a tunneling mechanism in the surface oxide of the material was a very likely cause for such a distribution in time constants [94], [106]. This was reinforced by field-effect experiments which indicated for the first time that a wide distribution of time constants was actually present at the surface of many semiconductors.

The idea behind this $1/f$ noise source is that the carriers in the material communicate with trapping levels at some depth in the surface oxide by tunneling. The time constants in the process are of the form

$$\tau = \tau_0 \exp(\alpha x) \qquad (100)$$

where x is the distance between the trap and the semiconductor-oxide interface, α is of the order of $10^8\ \text{cm}^{-1}$, and τ_0 is the time constant for a trap at the surface. If x varies between 0 and 40 Å, τ will vary over many orders of magnitude, from very small time constants (less than 10^{-6} seconds) to very large time constants (more than 10^6 seconds). The traps that are most effective in the process lie near the Fermi level in the oxide, for those more than a few kT above it are permanently empty and those more than a few kT below it are permanently filled.

This is directly applicable to FETs and to semiconductor filaments. But in junction diodes and transistors the current flow is caused by minority carriers, and holes and electrons are generated in pairs and recombine in pairs, mostly at the surface. Since the work of McWhorter, Fonger [45], and Watkins [174] it is held that the traps in the oxide, just mentioned, modulate the generation and recombination processes, and that this gives rise to flicker noise in these devices. This has recently been challenged by new experiments, but we shall see that these experiments can be incorporated with the theory if it is further developed.

A. Kinetics of the Oxide Traps [155], [170] and Explanation of the 1/f Noise

Let $\Delta N_T = n_T\Delta S\Delta x$ be the number of traps in a small surface element ΔS at a distance between x and $x+\Delta x$ from the semiconductor–oxide interface and let ΔN_t be the number of electrons trapped in that volume element. If E_t is the depth of the trap below the bottom of the conduction band in the semiconductor, and $g(\Delta N_t)$ and $r(\Delta N_t)$ are the generation and release rates of the trapped electrons, respectively, then

$$g(\Delta N_t) = C_1 n(\Delta N_T - \Delta N_t)\exp(-\alpha x);$$
$$r(\Delta N_t) = C_2 \Delta N_t \exp\left(-\frac{E_t}{kT}\right)\exp(-\alpha x) \qquad (101)$$

where C_1 and C_2 are constants and n is the electron density in the semiconductor. Putting $g(\Delta N_t)=r(\Delta N_t)$ yields the average number of trapped electrons as

$$\overline{\Delta N_t} = \lambda \Delta N_T; \quad \lambda = \frac{C_1 n}{C_1 n + C_2 \exp(-E_t/kT)}. \tag{102}$$

ΔN_t fluctuates by an amount $\delta\Delta N_t$, and this amount and the time constant τ of trapping processes are given by [24]

$$\overline{\delta\Delta N_t^2} = \Delta N_T \lambda(1-\lambda) \tag{103}$$

$$\tau = \cfrac{1}{\cfrac{dr(\Delta N_t)}{dt} - \cfrac{dg(\Delta N_t)}{dt}} = \frac{\exp(\alpha x)}{C_2 \exp(-E_t/kT) + C_1 n} = \frac{\lambda \exp(\alpha x)}{C_1 n}. \tag{104}$$

Only those traps are effective which lie near the Fermi level in the oxide; for those, $\lambda \simeq 1/2$ and, hence,

$$\overline{\Delta N_t^2} = \tfrac{1}{4}\Delta N_T; \qquad \tau = \frac{\exp(\alpha x)}{2C_1 n}. \tag{105}$$

We thus see that (100) is valid and that $\tau_0 = 1/(2C_1 n)$.

The spectral intensity $S_{\Delta N_t}(f)$ of the fluctuation $\delta\Delta N_t$ is [155]

$$S_{\Delta N_t}(f) = 4\overline{\delta N_t^2}\frac{\tau}{1+\omega^2\tau^2} = n_T \Delta S \Delta x \frac{\tau}{1+\omega^2\tau^2}. \tag{106}$$

Since the corresponding fluctuation δN in N equals $-\delta\Delta N_t$, the spectral intensity of δN equals (106). Here $N = nV$ is the average number of carriers in the sample and V is the sample volume. The fluctuation in N thus has a spectral intensity, obtained by integrating over the surface S,

$$S_N(f) = n_T S x_1 \int_0^{x_1} \frac{\tau}{1+\omega^2\tau^2}\frac{dx}{x_1} \tag{106a}$$

if n_T is constant for $0<x<x_1$ and zero otherwise. But in view of (100), if $\tau_1 = \tau_0 \exp(\alpha x_1)$,

$$\frac{dx}{x_1} = \frac{d\tau/\tau}{\ln \tau_1/\tau_0}. \tag{106b}$$

Carrying out the integration yields

$$S_N(f) = \frac{n_T S x_1}{\omega \ln(\tau_1/\tau_0)}[\tan^{-1}(\omega\tau_1) - \tan^{-1}(\omega\tau_0)] \tag{107}$$

which has a $1/f$ spectrum for $1/\tau_1 \ll \omega \ll 1/\tau_0$. The model thus explains the occurrence of $1/f$ noise in a very natural manner.

B. Flicker Noise in Junction Diodes

As mentioned in the introduction of this section, flicker noise occurs here due to fluctuations in the surface recombination processes, since these processes are modulated by the fluctuating occupancy of the traps in the oxide. Since the surface recombination is measured by the surface recombination velocity s, the flicker noise occurs because of fluctuations δs in s.

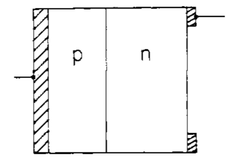

Fig. 3. Cross section of a p-n diode with surface recombination at the exposed face of the n-region.

Fonger [45] and Watkins [174] found that the flicker noise in diodes could be zero at a particular current; earlier indications of such an effect are already found in measurements by Anderson and van der Ziel [4]. Fonger attributed it to a modulation effect, but Watkins demonstrated that it was caused by the fact that $dV/ds = 0$ at that point; here V is the total diode voltage.

We shall now expand [168] Watkins' theory, using the geometry of Fig. 3; this geometry is chosen because it reduces the problem to a one-dimensional one. It is further assumed that the diode is a p^+n diode and that the surface recombination occurs at the exposed side of the n-region. The boundaries of the n-region are located at $x=0$ and $x=w$.

The current density equations are

$$J_p = q\mu_p pF - qD_p \frac{dp}{dx} \tag{108}$$

$$J_n = q\mu_n nF + qD_n \frac{dn}{dx} = 0 \tag{109}$$

so that the electric field strength F is

$$F = -\frac{D_p}{\mu_p}\frac{1}{(N_d+p)}\frac{dp}{dx}. \tag{110}$$

We have here made use of the Einstein relation and have assumed that space-charge neutrality prevails, so that $n = N_d + p$, where N_d is the donor concentration.

Substituting (110) into (108) yields

$$J_p = qD_p\left(1 + \frac{p}{N_d+p}\right) = qsp(w). \tag{111}$$

Integration gives

$$2p(0) - 2p(x) - N_d \ln\left[\frac{N_d + p(0)}{N_d + p(x)}\right] = \frac{s}{D_p}p(w)x \tag{112}$$

so that at $x=w$

$$2p(0) - 2p(w) - N_d \ln\left[\frac{N_d + p(0)}{N_d + p(w)}\right] = \frac{sw}{D_p}p(w). \tag{112a}$$

If p_n is the equilibrium hole concentration in the n-region, then the junction voltage

$$V_g = \frac{kT}{q}\ln\left[\frac{p(0)}{p_n}\right], \tag{113}$$

and the voltage V_b in the bulk n-region is

$$V_b = -\int_w^0 F dx = \frac{kT}{q} \ln\left[\frac{N_d + p(0)}{N_d + p(w)}\right], \quad (114)$$

so that the total diode voltage V is

$$V = V_g + V_b = \frac{kT}{q}\left\{\ln\frac{p(0)}{p_n} + \ln\left[\frac{N_d + p(0)}{N_d + p(w)}\right]\right\}. \quad (115)$$

Differentiating (112a) with respect to s for constant J_p, that is for a constant product $sp(w)$, yields

$$\left[\frac{2p(0) + N_d}{p(0) + N_d}\right]\frac{dp(0)}{ds} - \left[\frac{2p(w) + N_d}{p(w) + N_d}\right]\frac{dp(w)}{ds} = 0. \quad (116)$$

Evaluating dV/ds and eliminating $dp(0)/ds$ with the help of (116) yields

$$\frac{dV}{ds} = \frac{kT}{q}\left[\frac{2p(w) + N_d - p(0)}{p(0)}\right]\frac{dp(w)/ds}{N_d + p(w)} \quad (117)$$

which is zero for

$$2p(w) + N_d - p(0) = 0. \quad (118)$$

A fluctuation in s will not give any fluctuation in V in that case.

To see whether this leads to a realizable value of $p(w)$, we substitute into (112a). This yields

$$2p(w) + N_d(2 - \ln 2) = \frac{sw}{D_p}\,p(w) \quad (119)$$

which gives a positive solution for $p(w)$ only if $sw/D_p > 2$. Consequently the minimum in the flicker noise occurs only if the surface recombination velocity is sufficiently large.

Some time ago Guttkov [66] published flicker noise measurements on germanium p-n junctions. He found that the spectrum of the current fluctuation could be represented as

$$S_I(f) \simeq \frac{I^2}{s^2}\,B(f) \quad (120)$$

where $B(f)$ depends on frequency as $1/f^\alpha$ with $\alpha \simeq 1$; he claimed that this was incompatible with Fonger's surface recombination model.

We shall see that this claim is invalid if the surface recombination model is properly extended [168]. For small injection, $p(0) \ll N_d$ and $p(w) \ll N_d$, and hence (117) may be written

$$\frac{dV}{ds} = \frac{kT}{q}\,\frac{dp(w)/ds}{p(0)} = -\frac{kT}{q}\,\frac{p(w)}{p(0)}\,\frac{1}{s} \quad (121)$$

since, according to (111),

$$\frac{dp(w)}{ds} = -\frac{J_p/q}{s^2} = -\frac{p(w)}{s}. \quad (121a)$$

Making a Taylor expansion of (112a) for $p(0) \ll N_d$ and $p(w) \ll N_d$ yields

$$p(0) = \left(1 + \frac{sw}{D_p}\right)p(w) \quad (122)$$

so that

$$\frac{dV}{ds} = -\frac{kT}{q}\,\frac{1}{s(1 + sw/D_p)}. \quad (123)$$

Therefore a fluctuation δs in s gives a fluctuation δV in V

$$\delta V = -\frac{kT}{q}\,\frac{\delta s}{s(1 + sw/D_p)} \quad (123a)$$

so that the spectral intensities $S_v(f)$ and $S_s(f)$ are related as

$$S_v(f) = \left(\frac{kT}{q}\right)^2 \frac{S_s(f)}{s^2(1 + sw/D_p)^2}. \quad (124)$$

Now $S_s(f)$ should be independent of current at low injection and, according to Section IV-A, it should be proportional to the trap density in the oxide which, in turn, would be proportional to s. Putting $S_s(f) = sB(f)$ yields

$$S_v(f) = \left(\frac{kT}{q}\right)^2 \frac{B(f)}{s(1 + sw/D_p)^2} \quad (124a)$$

and, hence,

$$S_I(f) = \left(\frac{qI}{kT}\right)^2 S_v(f) = \frac{I^2 B(f)}{s(1 + sw/D_p)^2}. \quad (125)$$

This varies as $1/s$ for $sw/D_p \ll 1$ and as $1/s^3$ for $sw/D_p \gg 1$. Intermediately there will be a region where $S_I(f)$ varies approximately as $1/s^2$, as found by Guttkov.

Guttkov mentioned that in his diodes s was fairly large. If this geometry was not one-dimensional, which most likely will have been the case, $S_I(f)$ must be averaged over a wide range of values of w. This has the tendency of extending the $1/s^2$-region. We thus conclude that Guttkov's results can be brought into agreement with the surface recombination model.

Hsu and his coworkers [86] found that the noise spectrum $S_I(f)$ in silicon diodes at constant current was proportional to the surface recombination velocity s. This seems to be in disagreement with Guttkov's results and with the surface recombination model. But we must now bear in mind that in silicon diodes a significant carrier recombination occurs in the space-charge region, especially at the surface of that region. If the flicker noise is due to fluctuations in the recombination current produced in that region, as likely is the case, the discrepancy disappears [168].

The recombination occurs in a well defined part of the space-charge region. Let this region be characterized by the coordinate x_1 and let it have an effective area A_{eff}.

Let for an applied potential V the change in potential at x_1 be V_1, and let the hole concentration $p(x)$ at x_1 be denoted by $p(x_1)$. It varies with V_1 as

$$p(x_1) = p_1(x_1)\exp\left(\frac{qV_1}{kT}\right) \quad (126)$$

where $p_1(x_1)$ is the value of $p(x_1)$ for $V_1=0$. The recombination current I_R is then

$$I_R = qsp(x_1)A_{\text{eff}} = qp_1A_{\text{eff}}s \exp\left(\frac{eV_1}{kT}\right). \tag{127}$$

If now s fluctuates by an amount δs, the fluctuation δI_R in I_R is

$$\delta I_R = qp_1A_{\text{eff}} \exp\left(\frac{qV_1}{kT}\right)\delta s \tag{128}$$

so that

$$S_{I_R}(f) = [qp_1A_{\text{eff}}]^2 \exp\left(\frac{2qV_1}{kT}\right)S_s(f). \tag{129}$$

If all flicker noise is associated with δI_R, the spectrum $S_I(f)$ of the diode current I is equal to (129). Putting

$$I = I_0 \exp\left(\frac{qV}{mkT}\right) \tag{130}$$

with $m \geqq 1$ and substituting $S_s(f)=sB(f)$ yields for $S_I(f)$

$$S_I(f) = \text{const}\ I^{2mV_1/V}B(f)s, \tag{130a}$$

so that $S_I(f)$ at constant current is indeed proportional to s, as required by the experiments.

For a symmetrical junction $V_1 \simeq \frac{1}{2}V$ and, hence, for $m \simeq 1$, $S_I(f)$ in a given diode is proportional to I. In an asymmetric diode, such as a p^+n diode, V_1 may be different and, hence, $S_I(f)$ may have a somewhat different current dependence. This is significant for understanding the current dependence of flicker noise in transistors.

We thus conclude that the surface recombination model of flicker noise, if properly applied, can explain the experiments on flicker noise in germanium and silicon diodes.

C. Flicker Noise in Transistors

In analogy with the shot noise case one would like to represent flicker noise in transistors by two current generators i_{f_1} and i_{f_2} in parallel with the emitter and collector junction, respectively. Chenette [28] demonstrated that i_{f_1} and i_{f_2} were almost fully correlated, but his experiments are equally compatible with the idea that most of the $1/f$ noise comes from the current generator i_{f_1} only. In addition he showed that the base resistance r_b could be determined from flicker noise measurements. This was further substantiated by the work of Gibbons [52].

The clearest demonstration is found in a paper by Plumb and Chennette [122] who showed that i_{f_2}/i_{f_1} was quite small. In their experiments the collector was HF connected to ground, a large resistance R_E was inserted in the emitter lead, and a variable resistor R_B was inserted between base and ground. The noise was measured between emitter and ground and R_B was so adjusted that the measured noise was a minimum. As seen from Fig. 4, the noise voltage appearing at the emitter terminal is

$$v = i_{f_1}r_{e0} - (i'_{f_2} + \alpha_0 i_{f_1})(R_B + r_b) - i''_{f_2}(R_B + r_b) \tag{131}$$

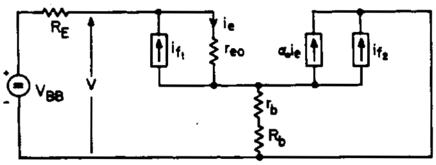

Fig. 4. Circuit for locating flicker noise sources in transistors.

where $r_{e0}=kT/qI_E$ and i_{f_2} has been split into two parts i'_{f_2} and i''_{f_2}. Here i'_{f_2} is fully correlated with i_{f_1}, and i''_{f_2} is uncorrelated. Therefore, $\overline{v^2}$ will go through a deep minimum if

$$r_{e0} - \left(\alpha_0 + \frac{i'_{f_2}}{i_{f_1}}\right)(R_b + r_b) = 0$$

or

$$R_B = (R_B)_{\min} = \frac{1}{1 + i'_{f_2}/(\alpha_0 i_{f_1})} \cdot \frac{r}{\alpha_0} - r_b. \tag{131a}$$

Hence, by plotting $(R_B)_{\min}$ versus $r_{e0}/\alpha_0 = kT/qI_C$, one should obtain a straight line intercepting the vertical axes at $-r_b$, whereas the slope of the line gives a measure for $[1+i'_{f_2}/(\alpha_0 i_{f_1})]$. This agreed very well with the experimental data and the values found for i'_{f_2} were only a few percent of i_{f_1}, so that one can neglect i'_{f_2} for all practical purposes. Measurements of $\overline{i^2_{f_1}}$ were obtained for small values of I_E by omitting R_B altogether; they indicated that $\overline{i^2_{f_1}}$ varied as I_E^β with β somewhat smaller than unity. This is compatible with (130a). Other units give $\beta > 1$.

Measurements by Viner [172] on silicon transistors operated at elevated temperatures indicated another interesting feature. At elevated temperatures the base current I_B may be written as $-I_{CBO}+I'_B$ where $-I_{CBO}$ is the base current for zero emitter current and I'_B the injected base current. Viner showed that the currents I_{CBO} and I'_B fluctuated independently and that each current showed flicker noise.

Flicker noise in silicon transistors must be interpreted in terms of a fluctuation in the recombination current I_R in the emitter-junction space-charge region, presumably mostly at the surface. Therefore the surface model of Section IV-B should be applicable.

The effect can be reduced by reducing I_R; this has been achieved by surface pacification techniques. It is not quite clear where the remaining noise comes from. Is it generated at the surface or in the interior of the space-charge region? A careful comparison of r_b data obtained from flicker noise and HF noise measurements may give the answer. Also a further comparison between p-n-p and n-p-n transistors would be useful; it seems that the latter show more $1/f$ noise.

D. Flicker Noise in FETs [32], [112]

The theory of flicker noise in silicon junction FETs should be similar to the theory for transistors. The flicker noise will be generated in the space-charge region between gate and channel, presumably mostly at the surface. It can be reduced by surface pacification techniques and is presently not very large in well-designed units.

In MOS FETs the $1/f$ noise is often much larger than in junction FETs. The reason for this is that the MOS FET is a surface device; the fluctuating occupancy of traps in the oxide can modulate the conducting surface channel all along the channel.

The fact that the $1/f$ noise in MOS FETs was a surface effect was first demonstrated by Sah and Hielscher [129], who showed that quantitative correlation existed between the $1/f$ noise power spectrum and the lossy part of the gate impedance due to carrier recombination in the surface states at the oxide–silicon interface. The $1/f$ noise is proportional to the surface-state density [1]; this is important, for it means that the $1/f$ noise can be reduced by proper choice of the crystal orientation of the substrate. Since (100) surfaces have a much smaller surface-state density than the more commonly used (111) surfaces, the $1/f$ noise can be lowered by making the MOS FETs on (100) surfaces. Other effects also seem to play a part, however.

The theory of Section III-E, if properly modified, can be applied to this case [32]. Let $\Delta N_T = n_T w \Delta x \Delta y$ be the number of traps in a volume element $w\Delta x \Delta y$ of the oxide, at a point $P(x, y)$ in the oxide; here n_T is the trap density, w is the channel width, and the x axis is directed along the channel. Let ΔN_t electrons be trapped and let $\delta\Delta N_t$ be the fluctuation in ΔN_t due to the random processes of trapping and detrapping. Then, if f_t is the fractional occupancy of the traps,

$$\overline{(\delta\Delta N_t)^2} = n_T f_t(1 - f_t) w \Delta x \Delta y;$$
$$\overline{\delta\Delta N_t(t)\delta\Delta N_t(t + s)} = \overline{(\delta\Delta N_t)^2} \exp\left(-\frac{s}{\tau}\right) \tag{132}$$

where τ is the time constant of the trapping process. A fluctuation $\delta\Delta N_t$ in the number of trapped electrons gives rise to a fluctuation $\delta\Delta N = -\delta\Delta N_t$ in the section Δx of the channel.

We now apply (80a), replace W_0 by V_0, the dc potential of the section Δx of the channel, and write

$$\frac{I_d \Delta x}{L \Delta N} = \frac{q\mu I_d}{L g(V_0)}$$

where $g(V_0)$ is the conductance per unit length of the channel. Then (80a) may be written

$$\overline{\delta\Delta I_d(t)\delta\Delta I_d(t + s)} = \left[\frac{q\mu I_d}{L g(V_0)}\right]^2 n_T f_t(1 - f_t) w \Delta x \Delta y \cdot \exp\left(-\frac{s}{\tau}\right). \tag{133}$$

Applying the Wiener–Khintchine theorem yields

$$\Delta S_{I_d}(f) = 4\left[\frac{q\mu I_d}{L g(V_0)}\right]^2 n_T f_t(1 - f_t) w \Delta x \Delta y \frac{\tau}{1 + \omega^2\tau^2}. \tag{134}$$

By integrating over the oxide one obtains the total spectrum $S_{I_d}(f)$,

$$S_{I_d}(f) = 4\left(\frac{q I_d}{L}\right)^2 n_T w \int_0^L dx \int_0^d \frac{f_t(1 - f_t)}{[g(V_0)]^2} \frac{\tau}{1 + \omega^2\tau^2} dy \tag{134a}$$

where d is the thickness of the oxide. Note that τ depends on y, since the time constant is governed by a tunneling process; integration with respect to y then gives the $1/f$ spectrum, as before. Integration with respect to x gives the dependence of the noise in the operating conditions.

It is here not allowed to put $f_t = 1/2$, but it is necessary to evaluate the dependence of f_t on the operating conditions explicitly. It is thus necessary to introduce a distribution in the energies of the trapping level and to integrate over that distribution. Only if one does that, does one obtain a finite spectrum. If one merely puts $f_t = 1/2$ and integrates, one obtains a logarithmic divergence at saturation. See Christenson *et al.* [32] for details.

An alternate theory of flicker noise in MOS FETs by Leventhal [100] assumes that the carriers move in a surface band. He then obtains a $1/f$ frequency dependence and a linear dependence on the surface-state density. Moreover, for a given sample the noise should be inversely proportional to the absolute temperature. It would seem, however, that a tunnel mechanism is a more likely cause of $1/f$ noise.

E. *Flicker Noise in Gunn Diodes* [43], [110]

Flicker noise has also been observed in Gunn diodes. The flicker noise, measured between the terminals of the device is almost independent of whether or not the device is oscillating. This $1/f$ noise can contribute heavily to the FM noise of the oscillator.

F. *Burst Noise* [25], [53], [90], [131], [176]

Burst noise has been observed in planar silicon and germanium diodes and transistors. The phenomenon consists of a random turning on and off of a current pulse; this can be described by the random telegraph signal approach. This is not properly flicker noise, since it has a const/$(1 + \omega^2\tau^2)$ spectrum. It is believed that a current pulse is caused by a single trapping center in the space-charge region.

G. *Summary*

We have seen how the model of distributed traps in the surface oxide layer of a semiconductor can give a natural explanation of $1/f$ noise. The fluctuating occupany of the traps corresponds to a fluctuating carrier density in the material, which is detected as noise when dc current is passed through the material. The communication of carriers inside the semiconductor with the traps comes about by tunnel effect; this gives the wide distribution in time constants needed for the $1/f$ spectrum.

This picture is directly applicable to noise in FETs, but for noise in junction diodes and transistors one must bear in mind that the current flow is by *minority* carriers instead of by *majority* carriers. One must now take into account that the fluctuating occupancy of the traps in the oxide modulates the surface recombination velocity. We have seen how this model can explain a variety of seemingly contradictory experimental data. The model, when applied to transistors, indicates how the flicker noise source is located in the equivalent circuit.

Further work to clarify these flicker noise models would be useful.

V. Light Noise in Luminescent Solid-State Diodes and Solid-State Lasers

The general noise problem of lasers is too complicated to be treated in this survey. We shall restrict ourselves chiefly to light diodes and laser diodes. It will be seen that the photons always show shot noise, but in addition, especially near threshold, there is also spontaneous emission noise. Finally we discuss optical heterodyning as a means of receiving very weak optical signals.

A. Luminescent Diodes

Luminescent diodes should show shot noise. The reason for that is a very simple one: the carriers crossing the barrier and recombining after crossing show shot noise.

To discuss this problem we need a property of shot noise that has not directly concerned us up to now. Let a certain series of events occur at the average rate $\bar{n}$ and let n be the fluctuating number of events occurring during a given second. We can then make up

$$\operatorname{var} n = \overline{n^2} - (\bar{n})^2 \tag{135}$$

for this fluctuating process. If the events are independent and occur at random (Poisson process or shot noise process), one finds

$$\operatorname{var} n = \bar{n}. \tag{136}$$

A process satisfying condition (136) gives *shot noise*. We can now apply this to photon emission and see when the photon emission process is a shot noise process.

Let us for the sake of simplicity assume that the injected carriers are electrons. According to Section II-B, those electrons give shot noise; i.e.,

$$\operatorname{var} n = \bar{n}. \tag{137}$$

Let it further be assumed that all injected electrons recombine under the generation of a photon and that all generated photons are actually emitted. Then the rate n_p of emission of photons equals the rate n of injection of electrons so that

$$n_p = n, \qquad \operatorname{var} n_p = \operatorname{var} n = \overline{n_p}. \tag{138}$$

In other words the emitted photons show shot noise in this case.

Let it now be assumed that each injected electron has a probability λ of giving rise to an emitted photon. Then the partition noise theorem applies according to which

$$\overline{n_p} = \lambda\bar{n}; \qquad \overline{\Delta n_p^2} = \operatorname{var} n_p = \lambda^2 \operatorname{var} n + \bar{n}\lambda(1-\lambda). \tag{139}$$

But since $\operatorname{var} n = \bar{n}$ this may be written

$$\overline{\Delta n_p^2} = \operatorname{var} n_p = \lambda\bar{n} = \overline{n_p} \tag{139a}$$

so that again the photons show shot noise.

This is not altered if optical feedback is applied so that the luminescent diode becomes a laser, since the emitted photons still obey Poisson statistics (see the following section). Near threshold there is perhaps some additional noise due to spontaneous emission, but far away from threshold, that is, at large emitted powers of the laser diode, the diode should show shot noise (see Section V-B).

This is quite different from a black-body radiator. If n quanta of frequency between ν and $\nu+\Delta\nu$ are received per second from a blackbody radiator at the temperature T, then according to statistical mechanics

$$\operatorname{var} n = \frac{\bar{n}}{1-\exp(-h\nu/kT)}. \tag{140}$$

This reduces to $\operatorname{var} n = \bar{n}$ for large $h\nu/kT$ but $\operatorname{var} n \gg \bar{n}$ for small $h\nu/kT$. In the latter case the noise of the received photons is thus much larger than would be expected from shot noise considerations. Equation (140) is one of the oldest formulas of quantum theory.

The fluctuations in the diode current (in n) and in the emitted radiation (in n_p) are correlated. This is easily seen as follows. Let during a given second n electrons be injected. We then write $n=\bar{n}+\Delta n$. The Δn electrons will give rise to $\lambda\Delta n$ emitted photons, so that

$$\Delta n_p = \lambda\Delta n + \Delta n_p' \tag{141}$$

is the fluctuation in n_p. Here $\Delta n_p'$ is the noise caused by the randomness in the recombination of the carriers and/or the emission of the photons. It is of course independent of Δn; consequently, $\overline{\Delta n\Delta n_p'}=0$ and, hence,

$$\overline{\Delta n\Delta n_p} = \lambda\overline{\Delta n^2} + \overline{\Delta n\Delta n_p'} = \lambda\bar{n} \tag{142}$$

so that the correlation coefficient

$$c = \frac{\overline{\Delta n\Delta n_p}}{\sqrt{\overline{\Delta n^2}\,\overline{\Delta n_p^2}}} = \frac{\lambda\bar{n}}{\sqrt{\bar{n}\,\lambda\bar{n}}} = \sqrt{\lambda}. \tag{142a}$$

According to Guekos and Strutt the pattern of light noise fluctuations and voltage fluctuations across GaAs laser diodes closely parallel each other [55], which suggests a strong correlation between the two noise phenomena. Actual correlation measurements indicate that the correlation coefficient fluctuates strongly with varying current [56].

B. Shot Noise and Spontaneous Emission Noise in Lasers

A laser far above threshold can be considered as a generator of a stable amplitude. In that case the emitted radiation obeys Poissonian statistics, that is,

$$\operatorname{var} n = \bar{n}$$

and the device shows shot noise of the emitted photons.

We can give two reasons why a laser should show shot noise. First consider an ideal laser with 100 percent efficiency and zero optical loss. Then the rate n of emission of photons is equal to the pumping rate W. Since the pumping can be represented as a series of independent random events, we have $\operatorname{var} W = \overline{W}$ and, hence, $\operatorname{var} n=\bar{n}$. If there are optical losses, so that the part λ of the produced quanta is actually emitted, then the partition noise theorem gives

$$\bar{n} = \overline{W}\lambda; \ \operatorname{var} n = \lambda^2 \operatorname{var} W + \overline{W}\lambda(1-\lambda) = \overline{W}\lambda = \bar{n}$$

since var $W = \overline{W}$, as stated before. Hence, n shows full shot noise.

One can also use the following reasoning. An arbitrary spectral line always gives at least shot noise of the quanta contained in the line. In addition beats occur between the frequencies within the line (photon bunching); this is called *wave interaction noise* [3a] and gives an additional contribution to var n. A single-mode laser sufficiently far above threshold is a single-frequency oscillator that produces no beats and, hence, no wave interaction noise. The spontaneous emission noise observed near threshold can be interpreted as wave interaction noise. Another interpretation, to be given in the following, provides additional insight.

Closer to threshold there is excess noise caused by spontaneous emission. The laser can then be considered a van der Pol oscillator driven by spontaneous emission noise [80]. The noise was first observed by Prescott and van der Ziel [124] in a gas laser and by Smith and Armstrong in a GaAs laser [139]. According to Freed and Haus [47] the bandwidth B of the spontaneous emission noise spectrum is inversely proportional to the average laser power $\bar{P}$ below threshold and proportional to $\bar{P}$ above threshold, having a minimum value at threshold. The low-frequency ratio $S_e(0)/S_s(0)$, where $S_e(f)$ is the spontaneous emission noise power and $S_s(f)$ the shot noise power of the laser, is proportional to $(\bar{P})^2$ below threshold and inversely proportional to $(\bar{P})^2$ above threshold, with a maximum value near threshold. The spectrum $S_e(f)$ is of the form

$$S_e(f) = \frac{S_e(0)}{1 + (f/B)^2} \tag{143}$$

where B is the bandwidth of the spectrum.[8] For gas lasers the minimum bandwidth B is relatively small and, hence, $S_e(0)/S_s(0)$ near threshold is reasonably large. For GaAs lasers the bandwidth B is very much larger and, hence, the observed value of $S_e(0)/S_s(0)$ near threshold observed by Armstrong and Smith was only 0.017. This small value was determined with the help of a counting technique. Elaborate photon counting experiments on gas lasers were reported by Arecchi *et al.* [5].

Because of the dependence of $S_e(0)/S_s(0)$ on average laser power, $S_e(0)$ becomes negligible sufficiently far above threshold. If the laser radiation is then detected by a photodetector, the noise observed in the detector current is full shot noise. The same is true for noise measurements for low laser powers at sufficiently high frequencies ($f \gg B$) [124].

Additional noise has been observed in a multimode gas laser when the modes are not locked [84]. The effect disappears when the modes are locked. This effect has not been observed in GaAs lasers, but Smith and Armstrong [139] have found two additional interaction effects in these devices:

1) low-frequency nonstationary noise which occurs when a weak mode is lasing in competition with a strong mode,
2) broad-band stationary noise which occurs when two modes are about equal in intensity.

The authors believe that the first type of noise arises from heat transfer processes in the diodes and dewar, while the second is thought to be partition noise which must occur when a photon can be stimulated into one of a number of lasing modes. The total noise for all modes is very small, being comparable to that of a single mode with the same total power.

According to Haug the junction current noise is larger than shot noise [77], [78]. He finds two additional noise terms:

1) a term due to the fluctuations in emission and absorption rates,
2) a term due to light-field fluctuations.

It is interesting to note that Guekos and Strutt [54] found that the current noise levels of GaAs laser diodes was much higher than expected from shot noise considerations. The same is true for the voltage noise observed in the lasing mode. This may have some connection with Haug's prediction.

In their lasing diodes the ratio of excess noise over shot noise in the light output and in the diode voltage was quite large and fluctuated wildly with current. The authors suggest that this may be associated with the switching of lasing filaments in the junction.

In conclusion it would seem that a further study of these noise phenomena would be worthwhile.

C. Optical Heterodyning

When photomultipliers are used in the reception of weak laser signals, enough built-in gain is present in the photomultiplier to make the output noise of the multiplier large in comparison with the noise of the associated amplifier used for processing the detected signal. If direct photodetectors, either photodiodes or photoconductors, are used, the detected signal may drown in the receiver noise. If an optical heterodyning method is used, whereby one beats the incoming signal with a local oscillator laser (pump) signal, one obtains a large power gain so that the noise of the amplifier can be overcome. It has the disadvantage, however, of requiring accurate alignment of the two beams.

Teich *et al.* [142] did an optical heterodyning experiment at 10.6 micron in photoconductive copper-doped germanium. They measured the signal-to-noise power ratio as a function of the incident power and obtained an experimental curve that was about a factor two below the theoretical curve, well within the experimental error, however.[9]

[8] Since $S_e(0)/S_s(0)$ varies as $1/B^2$ below threshold, $S_e(0)/S_s(0)$ is very small if the minimum bandwidth is large, as is the case in GaAs lasers.

[9] Even this discrepancy disappears when one corrects the theoretical (signal-to-noise) power ratio used by the authors. The correct expression is $\eta P_s/(2h\nu\Delta f)$ for a photodiode and $\eta P_s/(4h\nu\Delta f)$ for a photoconductive detector [see (145)].

Lee and van der Ziel [99] measured the noise at the output of a photodiode detector carrying a current I_p due to the local oscillator signal and an additional current I_1 due to the incoming radiation. The power gain, defined as the ratio[10]

$$\frac{\text{output signal power of optical heterodyne detector}}{\text{output signal power of direct detector}}$$

turns out to be $2I_p/I_1$, as long as I_1 is not too small, whereas the spectral intensity $S_m(f)$ of the optical heterodyne detector is

$$S_m(f) = 2eI_{\text{eq}} = 2eI_p + 4eI_p\eta. \tag{144}$$

Here the first term is the shot noise due to the pump signal and the second term is the amplified shot noise of the incoming radiation. By measuring I_{eq} versus I_p they could demonstrate the presence of the second term. They also showed that I_{eq} was independent of I_1.

The noise measurements indicate that the optical heterodyne detector is operating closely to the limit set by the noise of the incoming radiation. The signal-to-noise ratio of the optical heterodyne detector is found to be

$$\left(\frac{S}{N}\right)_m = \left(\frac{2}{1+2\eta}\right)\frac{I_1}{2eB} = \left(\frac{2}{1+2\eta}\right)\frac{\eta P_s}{2h\nu B}, \tag{145}$$

where η is the quantum efficiency of the detector, $I_1 = \eta eP_s/h\nu$ the detected current for direct detection, and B the bandwidth. The reader will recognize $P_s/(2h\nu B)$ as the signal-to-noise power ratio of the incoming radiation for the bandwidth B. (See footnote nine.)

D. Summary

We have seen that GaAs light diodes and laser diodes should always show at least shot noise of the emitted photons. In other lasers there may also be spontaneous emission noise near threshold, but in GaAs diodes this type of noise is very small. There should be additional terms in the current noise of the GaAs laser. These, together with the correlation between current noise and laser light noise, should be studied further.

In a good optical mixer one can see the contribution of the noise of the incoming radiation if the quantum efficiency of the detector is sufficiently large. Such a mixer therefore operates closely to the theoretical limit.

VI. Solid-State Devices in Circuits

It is here assumed that the reader is familiar with the concepts of noise figure F and noise measure M. He should also be familiar with Friiss' formula for the noise figure of a combination of stages [79], [149], [170]. Using these concepts we now discuss the noise figure of various solid-state amplifiers.

[10] The reason for the large power gain is that the direct detector is quadratic, whereas the optical mixer is linear. Therefore, at low input power the mixer is much better than the direct detector.

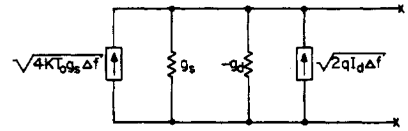

Fig. 5. Equivalent circuit of tunnel diode amplifiers.

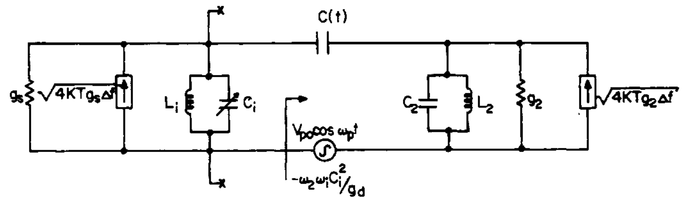

Fig. 6. Equivalent circuit of parametric amplifier.

A. Noise Figure of Negative Conductance Amplifiers

As a first example we consider a tunnel diode amplifier [27], [118], [145], [170]. The equivalent circuit is shown in Fig. 5, the output circuit is considered to belong to the next stage. We assume that the circuit has been so designed that $g_s > g_d$, where $-g_d$ is the negative conductance of the tunnel diode. It is then seen by inspection that the available gain is

$$G_{\text{av}} = \frac{g_s}{g_s - g_d} \tag{146}$$

which approaches infinity if g_s approaches g_d. It is also seen by inspection that

$$F = \frac{4kT_0g_s\Delta f + 2qI_d\Delta f}{4kT_0g_s\Delta f} = 1 + \frac{q}{2kT_0}\frac{I_d}{g_s}. \tag{147}$$

It would thus seem at first sight that a low noise figure would be obtained for large values of g_s. However, since G_{av} approaches unity if g_s approaches infinity, one must be careful. One finds the noise measure of the stage

$$M = \frac{F-1}{1-1/G_{\text{av}}} = \frac{q}{2kT_0}\frac{I_d}{g_d} \tag{148}$$

independent of g_s, so that it is cheapest to use a single stage and choose g_s relatively close to g_d. In that case

$$F \simeq F_\infty = 1 + \frac{q}{2kT_0}\frac{I_d}{g_d}. \tag{148a}$$

One should choose the operating point of the diode in such a way that I_d/g_d attains its minimum value. Noise figures of the order of 2 (3 dB) are achievable in this manner.

As a second example we consider a parametric amplifier [81], [82], [154] made by connecting a varactor diode driven by a pump signal of frequency ω_p and with a tuned output (idler) circuit of tuned conductance g_2 and tuning frequency $\omega_2 = (\omega_p - \omega_i)$ in series with the pump, in parallel with a signal source and a tuned circuit tuned at the frequency ω_i (Fig. 6). Let the time dependent capacitance $C(t)$ of the varactor diode have a Fourier expansion

$$C(t) = C_0 + 2C_1\cos\omega_p t + 2C_2\cos 2\omega_p t \cdots. \tag{149}$$

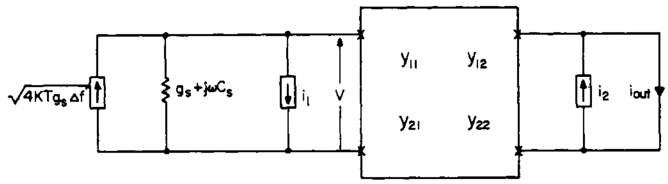

Fig. 7. Equivalent circuit of FET amplifier for determining noise figure.

Then the tuned idler circuit with tuned circuit conductance g_2 produces a negative conductance

$$-g_d = -\frac{\omega_2 \omega_i C_1^2}{g_2} \tag{150}$$

in parallel with the source and the input circuit. The theory of the tunnel-diode amplifier can thus be applied, provided that the noise $\sqrt{\overline{i_d^2}}$ associated with $-g_d$ is known. Since this represents the converted thermal noise EMF $4kT\Delta f/g_2$ of the tuned idler circuit, we have

$$\overline{i_d^2} = \frac{4kT\Delta f}{g_2}\omega_i^2 C_1^2 = 4kT\Delta f g_d \frac{\omega_i}{\omega_2} \tag{151}$$

and, hence,

$$F = \frac{4kTg_s\Delta f + \overline{i_d^2}}{4kTg_s\Delta f} \simeq 1 + \frac{\omega_i}{\omega_2} \tag{152}$$

if g_s approaches g_d. Very low noise figures can thus be obtained by choosing $\omega_2 \gg \omega_i$.

It is beyond the scope of this review to include the many practical realizations of this principle or the evaluation of the effect of the shot noise and of the series resistance of the varactor diode on the performance of the circuit [118a].

B. Field-Effect Transistor Circuits [102], [159], [170]

The equivalent circuit of the field-effect transistor in common source connection is shown in Fig. 7. This equivalent circuit is purposely chosen different from the circuit recommended by the IRE Standards Committee on Noise [180] because the noise parameters hereby introduced relate closely to the physics of the device.[11] Fig. 7 shows the equivalent circuit of the FET in common source connection. Let the source-free two-port in this circuit be represented by the admittance matrix

$$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}, \tag{153}$$

and let i_1 be split into a part i_1', fully correlated with i_2, and a part i_1'' correlated with i_2. Let a source, consisting of a current generator $\sqrt{4kT_0 g_s \Delta f}$ in parallel with the source admittance $Y_s = g_s + j\omega C_s$, be connected to the input and let the output be short-circuited. Collecting the various contributions to the short-circuited current i_{out}, one easily obtains for the noise figure

$$F = 1 + \frac{\overline{i_1''^2}}{4kT_0 g_s \Delta f} + \frac{\overline{i_2^2}}{4kT_0 g_s |Y_{21}|^2}\left|Y_s + Y_{11} + \frac{i_1'}{i_d} Y_{21}\right|^2. \tag{154}$$

Introducing the noise parameters g_n, R_n, and Y_{cor} by the definitions

$$\overline{i_1''^2} = 4kT_0 g_n \Delta f;$$
$$\overline{i_2^2} = 4kT_0 R_n \Delta f |Y_{21}|^2;$$
$$Y_{cor} = Y_{21}\left|\frac{i_1'}{i_2}\right| = Y_{21}\frac{\overline{i_1 i_2^*}}{\overline{i_2^2}}, \tag{155}$$

one obtains

$$F = 1 + \frac{g_n}{g_s} + \frac{R_n}{g_s}|Y_s + Y_{11} + Y_{cor}|^2 \tag{156}$$

which is easily optimized as a function of b_s and g_s. Note that in the representation g_n, R_n, and Y_{cor} are directly related to the physics of the device; R_n measures the output noise, g_n the uncorrelated part of the input noise, and the correlation admittance Y_{cor} measures the correlated part of the input noise.

Substituting $Y_{11} = g_{11} + j\omega C_{11}$; $Y_{cor} = g_{cor} + j\omega C_{cor}$, one obtains that F as a function of C_s has a minimum value

$$F_t = 1 + \frac{g_n}{g_s} + \frac{R_n}{g_s}(g_s + g_{11} + g_{cor})^2 \tag{157}$$

for

$$C_s + C_{11} + C_{cor} = 0. \tag{157a}$$

It turns out that C_{cor} is not very large if Y_{12} is neutralized, so that tuning for maximum signal then increases the noise figure only slightly. By measuring F_t as a function of g_s one can determine the other noise parameters. F_t as a function of g_s has a minimum value

$$F_{min} = 1 + 2R_n(g_{11} + g_{cor}) + 2\sqrt{g_n R_n + (g_{11} + g_{cor})^2 R_n^2} \tag{158}$$

for

$$g_s = \sqrt{(g_{11} + g_{cor})^2 + \frac{g_n}{R_n}}. \tag{158a}$$

For a junction FET $g_{cor} \ll g_{11}$ and $g_n \simeq g_{11}$, so that F_{min} can be simplified to

$$F_{min} = 1 + 2R_n g_{11} + 2\sqrt{g_{11}R_n + (g_{11}R_n)^2}. \tag{159}$$

In this case only a simple noise parameter $g_{11}R_n$ is needed

[11] In my opionion the settling down to one particular equivalent circuit representation was a mistake. The IRE Standards Committee should have made several options available, so that the device and circuit engineer could choose which option should be used in a particular case. We demonstrate this freedom of choice in our treatment of the FET and the transistor problem.

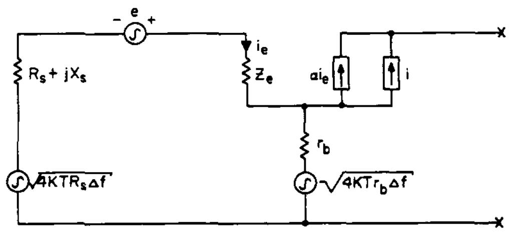

Fig. 8. Equivalent circuit of common base transistor for determining noise figure.

to describe the minimum noise figure over a wide range of frequencies. This agrees very well with experiment. In MOS FETs, however, it is not always true that $g_n \simeq g_{11}$. It is often necessary to neutralize the feedback admittance Y_{12} for greater stability.

We see that $F_{\min} \simeq 6$, for $g_{11}R_n = 1$. One can consider this condition to be the cutoff condition for low-noise operation. To achieve low-noise operation over a wide frequency range one should therefore make $g_{11}R_n$ as small as possible.

If one investigates how the product $g_{11}R_n$ depends on the device parameters, one finds that it decreases sharply with decreasing channel length. To achieve microwave operation of FETs one should make the channel as short as feasible. Channel lengths of two microns bring the operation well into the microwave range [95a].

Since g_{11} varies as f^2 and R_n is practically frequency independent for the frequency range where the device is useful, it is seen from (159) that $F_{\min}$ increases rapidly for $g_{11}R_n > 1$.

It can be demonstrated easily that the noise figure of the common gate circuit can be transformed back to that of the common source circuit, if it is assumed that the admittance Y_{12} is neutralized in each case [170]. It is therefore not worthwhile to give that circuit a separate treatment.

Another popular circuit is the *cascode* circuit. It consists of a common source FET as the first stage and a common gate FET as the second stage with the drain of the first stage directly connected to the source of the second stage. Units in which the two devices are manufactured on a single chip are known as *tetrode FETs*.

The noise figure of this circuit has been calculated by van der Ziel and Takagi [166], who have shown that improvement in noise figure can be obtained by neutralizing the first (equal to common source) stage and by tuning the interstage network. This agrees with experiment.

C. Transistor Circuits [155]

We first consider the common base circuit of Fig. 8 which incorporates the transistor equivalent circuit of Fig. 1(b).[12] Open-circuiting the output, and neglecting the noise developed across r_b in comparison with the noise voltage across the collector junction, one easily obtains for the noise figure, if $Z_s = R_s + jX_s$,

[12] Again, we deviate here from the equivalent circuit recommended by the IRE Standards Committee on Noise [180].

$$F = 1 + \frac{r_b}{R_s} + \frac{\overline{e''^2}}{4kT_0R_s\Delta f} + \frac{\overline{i^2}/|\alpha|^2}{4kT_0R_s\Delta f}\left|Z_s + r_b + Z_e + \frac{e'\alpha}{i}\right|^2 \tag{160}$$

where the EMF e has been split into a part e' fully correlated with the current generator i and a part e'' uncorrelated with i. We now introduce the parameters R_n, g_n, and Z_{cor} by the definitions

$$\overline{e''^2} = 4kT_0R_n\Delta f; \quad \frac{\overline{i^2}}{|\alpha|^2} = 4kT_0g_n\Delta f; \qquad Z_{\text{cor}} = \alpha\frac{e'}{i} = \alpha\frac{\overline{ei^*}}{\overline{i^2}}. \tag{161}$$

These noise parameters again relate directly to the physics of the device. Equation (160) may now be written

$$F = 1 + \frac{r_b + R_n}{R_s} + \frac{g_n}{R_s}|Z_s + r_b + Z_{\text{cor}}|^2. \tag{162}$$

If we put $Z_e = R_e + jX_e$, the correlation impedance $Z_{\text{cor}} = R_{\text{cor}} + jX_{\text{cor}}$, then (162), when considered as a function of X_s, has a minimum value

$$F = F_t = 1 + \frac{r_b + R_n}{R_s} + \frac{g_n}{R_s}(R_s + r_b + R_e + R_{\text{cor}})^2 \tag{163}$$

if

$$(X_s + X_e + X_{\text{cor}}) = 0. \tag{163a}$$

Usually X_{cor} is quite small so that its effect is small, little deterioration in noise figure will occur if the input is tuned for maximum signal transfer. By measuring F_t as a function of R_s, one can determine the other noise parameters. Considered as a function of R_s, F_t has a minimum value

$$F_{\min} = 1 + 2g_n(r_b + R_e + R_{\text{cor}}) + 2\sqrt{g_n(r_b + R_n) + g_n^2(r_b + R_e + R_{\text{cor}})^2} \tag{164}$$

if

$$R_s = (R_s)_{\text{opt}} = \sqrt{(r_b + R_e + R_{\text{cor}})^2 + \frac{(r_b + R_n)}{g_n}}. \tag{164a}$$

Usually the correlation resistance R_{cor} is small so that it can be neglected [113]. Moreover, it is often assumed [113] that $R_e \simeq \frac{1}{2}R_{e0}$, whereas g_n follows from (24a) as

$$g_n = \frac{\alpha_F}{R_{e0}\alpha_0^2}\left(1 - \alpha_F + \frac{f^2}{f_\alpha^2}\right). \tag{165}$$

Another approximation [48], [107] which may be useful at higher frequencies, is to neglect $R_e + R_{\text{cor}}$ and R_n with respect to r_b, so that

$$F_{\min} = 1 + 2g_nr_b + 2\sqrt{g_nr_b + g_n^2r_b^2}. \tag{166}$$

Then only one parameter $g_n r_b$ is needed to characterize the noise over a wide frequency range. Both approximations give reasonable results up to near the cutoff frequency of the transistor.

If one substitutes $g_n r_b = 1$, $F_{\min} = 6$ is obtained. We can thus define this condition as the cutoff condition for low-noise operation. To achieve low-noise operation one should therefore make $g_n r_b$ as small as possible. This can be achieved by making the alpha cutoff frequency f_α of the transistor as large as possible. Essentially this amounts to making the base region very thin. Operation up to 10 GHz is feasible with present-day techniques.

Since g_n varies as $(f/f_\alpha)^2$ over the frequency range of interest, and r_b is practically frequency independent, a raising of the alpha cutoff frequency has the additional benefit that the frequency range of very low noise figure (say $F_{\min} < 1.5$) is materially extended.

Next we consider the common emitter transistor circuit of Fig. 9. We now revert back to the current generators i_1 and i_2 by substituting $i_b = i_1 - i_2$. After some manipulating whereby one takes the feedback through the capacitance C_{cb} into account and puts

$$e = i_1 Z_e; \quad i' = i_2 - \alpha' i_1; \quad \alpha' = \alpha - j\omega C_{cb} Z_e. \tag{167}$$

one obtains a result similar to (160) with the only difference being α must be replaced by α' and i by i'. Noise parameters can now be introduced by putting

$$\overline{e''^2} = 4kT_0 R_n \Delta f; \quad \frac{\overline{i'^2}}{|\alpha|^2} = 4kT_0 g_n' \Delta f; \qquad Z'_{\text{cor}} = \frac{\alpha' e'}{i'} = \alpha' \frac{\overline{ei'^*}}{\overline{i'^2}}. \tag{168}$$

One then obtains (162) except that g_n and Z_{cor} are replaced by g_n' and Z'_{cor}. Here there is quite a difference between the tuning for minimum noise figure and the tuning for maximum signal transfer.

When comparing theory and experiments one finds good agreement for $F_{\min}$ up to near the cutoff frequency, even at current levels where one expects high-level injection effects in the base. This is especially true for the parameter g_n. In some units a collector saturation effect occurs at the collector [3], as mentioned in Section II. The only high-level injection effect that has not been explained is that in some units R_{cor} is much larger than anticipated [146].

At very high frequencies one gets into trouble because of the header parasitics. At most frequencies these parasitics act as lossless transformers that affect $(R_s)_{\text{opt}}$ but do not affect $F_{\min}$. At the highest frequencies, however, the parasitics act as attenuators; that increases the noise figure to a value larger than expected from (164). Malaviya [107] found good agreement between theory and experiment up to 3 GHz for a 4-GHz transistor, but at 4 GHz, where the transistor was near the end of its operating range, the experimental value of $F_{\min}$ was larger than the theoretical one, as expected from the preceding discussion.

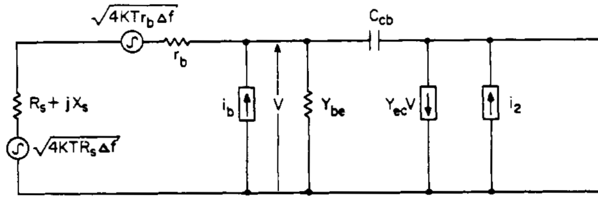

Fig. 9. Equivalent circuit for common emitter transistor for determining noise figure.

D. Mixer Circuits [149], [170]

In a mixer a large pump (or local oscillator) signal of frequency ω_p is applied to a nonlinear device, an input signal of frequency ω_i is applied to the input and an output signal of frequency, ω_o is taken from the output where either $\omega_p = |\omega_i - \omega_o|$, or $\omega_p = \omega_i + \omega_o$. In the latter case the phase angle ϕ_i of the input signal gives rise to a phase angle $-\phi_i$ in the converted output signal, whereas there is no phase reversal in the first case.

We first turn to the diode mixer [149]. If the instantaneous conductance $g(t)$ of the diode has a Fourier representation

$$g(t) = g_0 + 2g_1 \cos \omega_p t + 2g_2 \cos 2\omega_p t + \cdots, \tag{169}$$

then the diode mixer can be represented by a two-port active network with an admittance matrix

$$\begin{pmatrix} g_0 & -g_1 \\ -g_1 & g_0 \end{pmatrix}. \tag{169a}$$

If a source current generator i_s in parallel with a conductance g_s is applied to the input of this network, the output is connected to a load conductance g_L, and the maximum power gain, obtained by matching the source to the input and the load to output, is evaluated, one obtains

$$G_{\max} = \frac{\beta}{[1 + (1-\beta)^{1/2}]^2}, \text{ for } g_s = g_L = g_0(1-\beta)^{1/2} \tag{170}$$

where $\beta = (g_1/g_0)^2$. We note that $G_{\max}$ approaches unity if β approaches unity. We shall see that a large value of $G_{\max}$ is beneficial for the noise figure.

In order to evaluate the noise figure one must bear in mind that the noise has a component of frequency ω_o and a component of frequency ω_i. The latter passes through the input circuit, gives rise to a noise voltage of frequency ω_i at the input, and by mixing gives rise to a converted noise signal of frequency ω_o that is correlated with the original component of frequency ω_o. If one now puts for the diode noise[13]

$$\overline{i^2} = n \cdot 4kTg_0 \Delta f \tag{171}$$

where $n \simeq 1$ for a point contact diode and $n \simeq \frac{1}{2}$ for a Schottky

[13] The n-value here represents an averaging over the periodic local oscillator waveform. It can be as low as $\frac{1}{2}$ in devices with a low series resistance and a low back current, about unity in devices with an appreciable series resistance and a low back current, and much higher than unity in devices with a large back current.

barrier diode, one obtains for the minimum noise figure of the mixer

$$F_{\min} = 1 + n\left(1 - \frac{1}{G_{\max}}\right), \quad \text{for } g_s = g_0(1-\beta)^{1/2} \tag{172}$$

so that the noise figure approaches unity if $G_{\max}$ approaches unity.

If the mixer is followed by an intermediate frequency (IF) amplifier of noise figure F_2, then according to Friiss' formula the noise figure of the combination is

$$F = F_{\min} + \frac{F_2 - 1}{G_{\max}} = 1 + \frac{n(1 - G_{\max}) + F_2 - 1}{G_{\max}} \tag{173}$$

which approaches F_2 if $G_{\max}$ approaches unity. Recent decreases in the noise figure of diode mixers have come about by lowering n (by going to Schottky barrier diodes), by raising $G_{\max}$ (by going to diodes with low series resistance), and by lowering F_2 (by going to low-noise transistor IF amplifiers) [16].

In Doppler radar receivers one usually starts with a mixer, chooses $\omega_o = 0$, and obtains beat frequencies. One then wants to avoid that the beat frequencies drown in the noise background of the receiver. To that end one must require that the mixer diodes have low $1/f$ noise. Well constructed Schottky barrier diodes and backward diodes seem to meet this requirement best [38]–[40], [44].

The impedance of the input circuit of a mixer diode is usually so low that it not only responds to the wanted frequency $\omega_i = \omega_p - \omega_o$ but also to the frequency $\omega_i' = \omega_p + \omega_o$, which is called the *image* frequency. A lower value of the noise figure $F_{\min}$ is now obtainable by proper choice of the admittance of the input circuit for the frequency ω_i'. The best results are obtained if the input circuit has an infinite impedance (i.e., is open-circuited) for the frequency ω_i' [170].

Since a point contact or Schottky barrier diode also has a voltage-dependent capacitance, the net result is that the diode mixer is a combination of a variable conductance and a variable capacitance mixer. This may alter the above considerations to some extent in some cases.

Next we consider the variable capacitance mixer [148], where the diode is replaced by a voltage-dependent capacitor. If a large pump voltage of frequency ω_p is applied, the instantaneous capacitance of the diode has a Fourier representation

$$C(t) = C_0 + 2C_1 \cos \omega_p t + 2C_2 \cos 2\omega_p t + \cdots. \tag{174}$$

Then for $\omega_p = |\omega_i - \omega_o|$ the mixer can be represented by a two-port active network with an admittance matrix

$$\begin{pmatrix} j\omega_i C_0 & -j\omega_i C_1 \\ -j\omega_o C_1 & j\omega_o C_0 \end{pmatrix}. \tag{174a}$$

If a source current generator i_s in parallel with a conductance g_s is applied to the input of this network, the output is connected to the load conductance g_L, and the maximum power gain, obtained by matching the source to the input and the load to the output, is evaluated, one obtains

$$G_{\max} = \frac{\omega_o}{\omega_i} \tag{175}$$

so that the up-converter ($\omega_o > \omega_i$) has power gain.

For the case $\omega_p = \omega_i + \omega_o$, the load conductance g_L gives rise to a negative conductance $-\omega_o \omega_i C_1^2 / g_L$ seen at the input side of the circuit. This is the basis of the parametric amplifier (Section VI-A).

An ideal voltage-dependent capacitor has no noise sources associated with it, and as a consequence an ideal variable capacitance mixer should have a noise figure of unity [148]. Because of the unavoidable losses in the tuned circuits and in the nonlinear device, the noise figure of actual circuits will be somewhat larger than unity [82], [118a], [154]; in practice, however, very low noise figures have been obtained.

We now turn to the FET and the transistor mixer [170]. The best results are obtained for the common source FET and for the common emitter transistor circuits. The common gate FET and the common base transistor mixer have too much current flowing through the input of the device; the noise due to these currents seriously deteriorates the noise figure of the device unless the current flows in very short pulses.

The general approach to noise problems in FET and transistor mixers in the two recommended circuits is the same as before. One constructs an equivalent circuit with an input noise current generator i in parallel with the input admittance for the frequency ω_i, and an output noise current generator i_2 in parallel with the output admittance for the frequency ω_o. The correlation between the two noise sources is usually so small that it can be neglected. The theory of Section VI-B is then applicable, with $Y_{\mathrm{cor}} = 0$, for the FET mixer [170]. The transistor mixer is slightly more complicated because of the base resistance r_b [170], [173].

The noise figure of these devices when used as a mixer is somewhat larger than when used as amplifiers. This is seen as follows. If the instantaneous transconductance $g_m(t)$ of the device is represented by its Fourier series

$$g_m(t) = g_{m0} + 2g_{m1} \cos \omega_p t + 2g_{m2} \cos 2\omega_p t + \cdots, \tag{176}$$

then the output noise of the mixer can be represented as

$$\overline{i_2^2} = n_2 \cdot 4kT_0 g_{m0} \Delta f = 4kT_0 R_{nm} \Delta f g_{m1}^2 \tag{177}$$

where n_2 is of the order of unity, so that the mixer noise resistance R_{nm} is

$$R_{nm} = n_2 \frac{g_{m0}}{g_{m1}^2}, \tag{177a}$$

whereas the noise resistance R_{na} of the amplifier is

$$R_{na} = \frac{n_2}{g_{m0}}. \tag{177b}$$

Since $g_{m1} < g_{m0}$, $R_{nm} > R_{na}$ and, hence, the noise figure of the mixer is somewhat higher than the noise figure of the corresponding amplifier.

The effect can be eliminated by HF feedback from output to input. If so much feedback is applied that the circuit is at its limit of stability, the noise figure of the mixer becomes equal to that of the HF amplifier [115], [165].

E. Summary

The noise figure of various amplifier and mixer circuits is discussed. The tunnel diode amplifier and the parametric amplifier have quite acceptable noise figures at microwave frequencies.

In FETs the noise figure is determined by the parameter $g_{11}R_n$, where g_{11} is the HF input conductance and R_n is the noise resistance of the device. For low-noise operation $g_{11}R_n \leqq 1$. The cutoff condition $g_{11}R_n = 1$ leads to the requirement that the channel length should be made very small. Low-noise microwave FETs are feasible.

In transistors the noise figure is essentially determined by the parameter $g_n r_b$, where g_n is defined in the text and r_b is the base resistance. Again, for low-noise operation $g_n r_b \leqq 1$. The cutoff condition $g_n r_b = 1$ leads to devices with a very high alpha cutoff frequency. Microwave operation is quite feasible and the frequency range of low-noise operation is increasing steadily.

Diode mixers and variable capacitance mixers are also discussed. Lossless capacitance mixers should have a noise figure of unity. In diode mixers the noise of the IF amplifier gives a contribution to the noise figure of the system.

In common source FET mixers and in common emitter transistor mixers, the noise figure is somewhat larger than in the corresponding HF amplifier.

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