EE 538: Low-Noise Analog Circuit Design

Spring 2021

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Announcements

- Assignment 1 due Sunday, April 11 at midnight
  - Jupyter Notebook (.ipynb) submission on Canvas
  - For LTspice problems, include image (e.g. screen capture) of schematic(s)
- Assignment 2 will be posted Saturday, April 17

Week 2

- Motchenbacher Chapters 2 and 3
- Art of Electronics Chapter 8

Overview

- Last time...
  - Random variables
  - Thermal noise
  - Noise bandwidth
  - Shot noise
  - Flicker noise
- Today
  - 2-port noise theory
  - Noise figure/noise factor
  - Amplifier noise model
  - Correlated noise sources
  - Feedback amplifier noise analysis

Python packages/modules

In [139…

```
import matplotlib as mpl
from matplotlib import pyplot as plt
import numpy as np
from scipy import signal
from scipy import integrate
#%matplotlib notebook

mpl.rcParams['font.size'] = 12
mpl.rcParams['legend.fontsize'] = 'large'

def plot_xy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5));
    ax.plot(x, y, 'b')
    ax.grid()
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)

def plot_xy2(x1, y1, x1label, y1label, x2, y2, x2label, y2label):
    fig, ax = plt.subplots(2, figsize = (10.0, 7.5));
    ax[0].plot(x1, y1, 'b')
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].plot(x2, y2, 'b')
    ax[1].set_xlabel(x1label)
    ax[1].set_xlabel(x2label)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    fig.align_ylabels(ax[:])

def plot_xy3(x, y1, y2, y3, xlabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(3, figsize=(10.0,7.5))

    ax[0].plot(x, y1)
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].plot(x, y2)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    ax[2].plot(x, y3)
    ax[2].set_ylabel(y3label)
    ax[2].set_xlabel(xlabel)
    ax[2].grid()

def plot_logxy3(x, y1, y2, y3, xlabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(3, figsize=(10.0,7.5))

    ax[0].semilogx(x, y1)
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].semilogx(x, y2)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    ax[2].semilogx(x, y3)
    ax[2].set_ylabel(y3label)
    ax[2].set_xlabel(xlabel)
    ax[2].grid()

def plot_log3xy(x, y1, y2, y3, xlabel, ylabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(figsize=(10.0,7.5))

    ax.semilogx(x, y1, 'b', label=y1label)
    ax.semilogx(x, y2, 'r', label=y2label)
    ax.semilogx(x, y3, 'g', label=y3label)
    ax.set_ylabel(ylabel)
    ax.set_xlabel(xlabel)
    ax.grid()

    ax.legend()
    ax.legend(loc='upper center', ncol=3, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_logxy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    ax.semilogx(x, y, 'b')
    ax.grid();
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)

def plot_loglog(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    ax.loglog(x, y, 'b')
    ax.grid();
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)

def plot_xlogy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5));
    ax.semilogy(x, y, 'b');
    ax.grid();
    ax.set_xlabel(xlabel);
    ax.set_ylabel(ylabel);

def read_ltspice_ac(file_name):
    with open(file_name, 'r') as data:
        x = []
        y = []
        z = []
        next(data) # skip header line
        for line in data:
            p = line.split()
            x.append(float(p[0]))
            complex = p[1].split(",")
            y.append(float(complex[0]))
            z.append(float(complex[1]))

    return x, y, z

def plot_logxy2(x1, y1, x2, y2, x1label, y1label, x2label, y2label):
    fig, ax = plt.subplots(2, figsize = (10.0, 7.5));
    ax[0].semilogx(x1, y1, 'b');
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].semilogx(x2, y2, 'b');
    ax[1].set_xlabel(x1label)
    ax[1].set_xlabel(x2label);
    ax[1].set_ylabel(y2label);
    ax[1].grid();

    fig.align_ylabels(ax[:])

def plot_noise_bandwidth(f, mag):
    fig, ax = plt.subplots(2, figsize=(10.0,7.5))
    ax[0].semilogx(f, RC_mag)
    ax[0].set_xscale("log")
    ax[0].set_xlim(f[0], f[-1])
    ax[0].set_xticks(np.logspace(0.1,4,5))
    ax[0].set_xticklabels([])
    ax[0].set_ylabel('Magnitude [V/V]')
    ax[0].set_title('Equivalent Noise Bandwidth')
    ax[0].grid()

    ax[1].hlines(1, 0, f_enb, color='tab:blue')
    ax[1].hlines(0, f_enb, f[-1], color='tab:blue')
    ax[1].vlines(f_enb, 0, 1, color='tab:blue')
    ax[1].set_xlim(f[0], f[-1])
    ax[1].set_xscale("log")
    ax[1].set_xticks(np.logspace(0.1,4,5))
    ax[1].set_xticklabels([r'$10^0$',r'$10^1$', r'$10^2$', r'$10^3$', r'$10^4$'])
    ax[1].set_ylabel('Magnitude [V/V]')
    ax[1].set_xlabel('Frequency [Hz]')
    ax[1].grid()

def noise_hist(vnoise, vn_rms, bins):
    fig = plt.figure( figsize=(10.0,7.5) )
    vn_norm = vnoise/ vn_rms
    ax = fig.add_subplot(111)
    n, bins, rectangles = ax.hist(vn_norm, bins, density=True, range=(-3, 3),
                                  color='b')
    ax.set_xlabel(r'Sample Voltage [$v_{n(rms)}$]')
    ax.set_ylabel('Probability Density')
    ax.grid()
    fig.canvas.draw()

def plot_NF_vs_Rs(en_vals, in_vals, Rs_min, Rs_max, T_in_K):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    k = 1.38e-23
    Rs = np.logspace(np.log10(Rs_min), np.log10(Rs_max), num=200)
    F1 = 1 + (en_vals[0]**2+Rs**2*in_vals[0]**2)/(4*k*T_in_K*Rs)
    F2 = 1 + (en_vals[1]**2+Rs**2*in_vals[1]**2)/(4*k*T_in_K*Rs)
    F3 = 1 + (en_vals[2]**2+Rs**2*in_vals[2]**2)/(4*k*T_in_K*Rs)
    ax.semilogx(Rs, 10*np.log10(F1), 'b', label=r'$e_{n1}$, $i_{n1}$')
    ax.semilogx(Rs, 10*np.log10(F2), 'r', label=r'$e_{n2}$, $i_{n2}$')
    ax.semilogx(Rs, 10*np.log10(F3), 'g', label=r'$e_{n3}$, $i_{n3}$')
    ax.grid();
    ax.set_xlabel(r'Source Resistance $R_s$ [\Omega]$')
    ax.set_ylabel(r'Noise Figure $NF$ [$dB$]')

    ax.legend()
    ax.legend(loc='upper center', ncol=3, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_noise_curve(e_n, i_n, Rs_min, Rs_max):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    Rs = np.logspace(np.log10(Rs_min), np.log10(Rs_max), num=200)
    e_ni_2 = 4*k*T*Rs + e_n**2 + i_n**2*Rs**2
    ax.loglog(Rs, np.sqrt(e_ni_2), 'b', label='Total Noise')
    ax.loglog(Rs, np.sqrt(4*k*T*Rs), 'r', label=r'$\sqrt{4kTR_s}$')
    ax.loglog(Rs, e_n*np.ones(np.size(Rs)), 'g', label=r'$e_n$')
    ax.loglog(Rs, i_n*Rs, 'y', label=r'$i_n R_s$')
    ax.grid();
    ax.set_xlabel(r'Source Resistance $R_s$ [\Omega]$')
    ax.set_ylabel(r'Equivalent Input Noise [$V/\sqrt{Hz}$]')

    ax.legend()
    ax.legend(loc='upper center', ncol=4, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )
```

In [107…

```
def fftnoise(f):
    f = np.array(f, dtype='complex')
    Np = (len(f) - 1) // 2
    phases = np.random.rand(Np) * 2 * np.pi
    phases = np.cos(phases) + 1j * np.sin(phases)
    f[1:Np+1] *= phases
    f[-1:-1-Np:-1] = np.conj(f[1:Np+1])
    return np.fft.ifft(f).real

def band_limited_noise(min_freq, max_freq, samples=1024, samplerate=1):
    freqs = np.abs(np.fft.fftfreq(samples, 1/samplerate))
    f = np.zeros(samples)
    idx = np.where(np.logical_and(freqs>=min_freq, freqs<=max_freq))[0]
    f[idx] = 1
    return fftnoise(f)
```

Lecture 2 - Noise in Amplifiers

Two-port noise model

- In the model, all noise appears as inputs to a noiseless network, but arises from the thermal, flicker, and shot noise of the transistors and resistors that make up the amplifier/system
- This noise model allows us to focus on system-level considerations rather than needing to keep track of all the internal noise sources
- The net effect of all internal noise sources can be represented by a single pair of extrenal sources: a noise voltage $\overline{e_n}$ and a noise current $\overline{i_n}$
- Note that both $\overline{e_n}$ and $\overline{i_n}$ are required to represent the noise characteristics of a system/amplifier
- Assuming the noise processes of the source and two-port system are uncorrelated (which would be the typical case), the short-circuit mean-square noise current is given by

$$\overline{i_{ni}^2} = \overline{i_{ns}^2} + \overline{|i_n + Y_s e_n|^2} \tag{1}$$

- Note that this expression does not assume that i_n and e_n are uncorrelated. In general, i_n contains components that are correlated (i_c) and uncorrelated (i_u) with e_n, such that

$$i_n = i_c + i_u \tag{2}$$

- i_c can be related to e_n through a *correlation admittance* Y_c, such that $i_c = Y_c e_n$
- Expanding the above expression gives

$$\overline{i_{ni}^2} = \overline{i_{ns}^2} + \overline{i_u^2} + |Y_s + Y_c|^2 \overline{e_n^2} \tag{3}$$

Correlation between noise sources

- Two noise sources are partially correlated when each contains noise that arises from a common phenomenon, in addition to independently generated noise
- As an example, to combine partially correlated voltage noise sources, we can use the general expression

$$e_n^2 = (e_{n1} + e_{n2})^2 = e_{n1}^2 + e_{n2}^2 + 2C_c e_{n1} e_{n2} \tag{4}$$

- where C_c is the correlation coefficient
- For completely independent noise sources, $C_c = 0$ and we combine the mean-square quantities directly:

$$e_n^2 = e_{n1}^2 + e_{n2}^2 \tag{5}$$

Noise factor

- For optimum noise performance, we want to minimize the degradation in the signal-to-noise ratio (SNR) that a system introduces
- The noise factor is defined as

$$F \equiv \frac{\text{total output noise power}}{\text{output noise due to input source}} \tag{6}$$

- F can also be expressed as the ratio of input SNR to output SNR. The larger the degradation in SNR, the larger the noise factor
- If a system adds no noise of its own (the ideal case), then the total output noise is due entirely to the source and the noise factor is unity
- Noise factor is a handy figure of merit to compare amplifiers (or transistors) for a signal source with a given impedance

Two-port noise factor

- With the input-referred noise current is given by

$$\overline{i_{ni}^2} = \overline{i_{ns}^2} + \overline{i_u^2} + |Y_s + Y_c|^2 \overline{e_n^2} \tag{7}$$

- We can define each noise source as an equivalent resistance/conductance (even if the noise isn't actually due a resistance/conductance):

$$R_n \equiv \frac{\overline{e_n^2}}{4kT\Delta f} \quad G_u \equiv \frac{\overline{i_u^2}}{4kT\Delta f} \quad G_s \equiv \frac{\overline{i_s^2}}{4kT\Delta f} \tag{8}$$

- From these definitions we can express the noise factor purely in terms of impedances and admittances

$$F = 1 + \frac{G_u + |Y_c + Y_s|^2 R_n}{G_s} = 1 + \frac{G_u + [(G_c + G_s)^2 + (B_c + B_s)^2] R_n}{G_s} \tag{9}$$

- where each admittance has been decomposed into a sum of conductance G and susceptance B
- Going forward we'll drop the overbars for convenience, but we need to remember that we're still dealing with random voltage and current noise sources expressed in terms of rms and mean-square values
- For the common (low-frequency) case where $Y_c = 0$, and assuming $Z_s = Y_s^{-1} \approx R_s$, the input-referred mean-square voltage noise density can be expressed as

$$e_{ni}^2 = e_{ns}^2 + e_n^2 + R_s^2 i_n^2 \tag{10}$$

- The two-port noise factor is thus

$$F = \frac{e_{ns}^2 + e_n^2 + R_s^2 i_n^2}{e_{ns}^2} = 1 + \frac{e_n^2 + R_s^2 i_n^2}{e_{ns}^2} = 1 + N_e \tag{11}$$

- N_e is an excess noise power ratio, equal to $F - 1$, that expresses the *excess* noise contributed by the amplifier
- The *noise figure* NF is the noise factor in decibels:

$$\mathrm{NF} = 10 \log F = 10 \log \left[1 + \frac{e_n^2 + R_s^2 i_n^2}{e_{ns}^2} \right] \tag{12}$$

Noise resistance

- The noise resistance of an amplifier is the (possibly fictitious) value of resistance that would generate thermal noise equal to that of the amplifier:

$$e_n^2 + i_n^2 R_s^2 = 4kTR_e \Delta f \tag{13}$$

$$R_e = \frac{e_n^2 + i_n^2 R_s^2}{4kT\Delta f} \tag{14}$$

- Using the concept of noise resistance, the noise factor can be expressed as

$$F = \frac{R_s + R_e}{R_s} = 1 + N_e \tag{15}$$

- The utility of R_e is in comparing amplifier noise to that of resistors used in, for example, amplifier feedback networks
- As a reference, neglecting i_n, a relatively low e_n of $1nV/\sqrt{Hz}$ equates to an R_e of approximately 60Ω

Noise temperature

- Noise temperature is an alternative way of expressing the effect of an amplifier's noise contribution, and is defined as the increase in temperature required for the source resistance to generate the entirety of the amplifier output noise

$$4kT_n R_s \Delta f = e_n^2 + i_n^2 R_s^2 \tag{16}$$

- This yields

$$T_n = \frac{e_n^2 + i_n^2 R_s^2}{4kT_{ref} R_s \Delta f} \tag{17}$$

$$F = 1 + \frac{T_n}{T_{ref}} \tag{18}$$

- T_{ref} is the reference/ambient temperature, typically taken to be $290K$
- Good low-noise designs have noise temperatures far below room temperature (or, equivalently, noise figures far less than $3\mathrm{dB}$)

Maximum power transfer

- Given a fixed source impedance Z_s, what load impedance Z_L (where Z_L could represent an amplifier's input impedance) maximizes the power delivered to the load?
- The power delivered to the load is given by

$$\frac{|V_R|^2}{R_L} = \frac{R_L |V_S|^2}{(R_L + R_S)^2 + (X_L + X_S)^2} \tag{19}$$

- To maximize the power delivered to R_L, we need $X_L = -X_S$ and $R_L = R_S$
- Thus, the optimum value of R_L for power transfer is R_S

Optimum source resistance

- To minimize the noise factor/figure, we can take the first derivative of F with respect to the source impedance and set it equal to zero. Assuming $Z_s = R_s$ (no reactive components), this yields

$$-\frac{e_n^2}{R_{opt}^2} + i_n^2 = 0 \tag{20}$$

- From which we find the optimum source resistance to be

$$R_{opt} = \frac{e_n}{i_n} \tag{21}$$

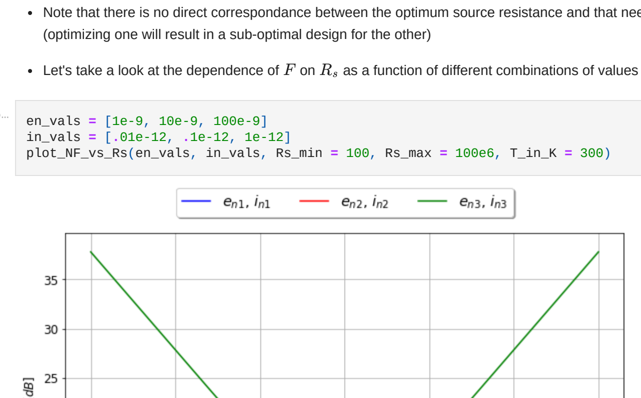
- The noise factor corresponding to R_{opt} is thus

$$F_{opt} = 1 + \frac{e_n^2 + R_{opt}^2 i_n^2}{4kTR_{opt}\Delta f} = 1 + \frac{2e_n^2}{4kT\Delta f e_n / i_n} = 1 + \frac{e_n \cdot i_n}{2kT\Delta f} \tag{22}$$

- Note that there is no direct correspondance between the optimum source resistance and that needed for maximum power transfer (optimizing one will result in a sub-optimal design for the other)
- Let's take a look at the dependence of F on R_s as a function of different combinations of values of e_n and i_n...

In [125…

```
en_vals = [1e-9, 10e-9, 100e-9]
in_vals = [.01e-12, .1e-12, 1e-12]
plot_NF_vs_Rs(en_vals, in_vals, Rs_min = 100, Rs_max = 100e6, T_in_K = 300)
```



- For values of R_S below R_{opt}, the noise factor is high due to the high value of e_n^2/e_{ns}^2
- As R_S increases beyond R_{opt}, the noise factor/figure degrades due to the passage of i_n through R_S
- *Question: Why not just increase R_S by adding resistance until $R_S = R_{opt}$?*

Equivalent input noise

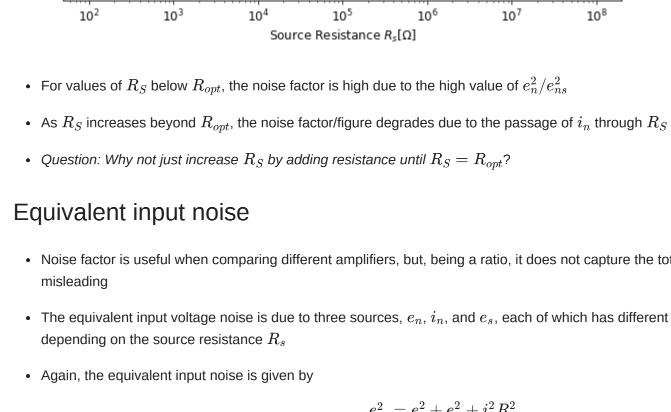
- Noise factor is useful when comparing different amplifiers, but, being a ratio, it does not capture the total noise and as such may be misleading
- The equivalent input voltage noise is due to three sources, e_n, i_n, and e_s, each of which has different contributions to the total noise depending on the source resistance R_s
- Again, the equivalent input noise is given by

$$e_{ni}^2 = e_s^2 + e_n^2 + i_n^2 R_s^2 \tag{23}$$

- How does each term contribute to the total noise?
- For low values of R_s, both e_s and i_n contribute minimally to the total noise and e_n dominates
- e_s's contribution to e_{ni}^2 depends linearly on R_s, while i_n's is quadtratic

In [126…

```
e_n = 5e-9
i_n = 1e-12
plot_noise_curve(e_n, i_n, 100, 10e6)
```



- Minimizing F for low values of R_s is a challenge, due to the low value of thermal noise due to the source resistance and the relatively high typical values of R_e

Transformer coupling

- When dealing with very low source impedances (say, $< 100\Omega$) transformer coupling can be used to realize a better noise factor than achievable by amplifier selection alone
- The transformer allows matching of the source resistance to R_{opt} (to within the precision of the windings ratio)
- The turns ratio n of the transformer is selected to transform R_s to R_{opt}

$$n^2 = \frac{R_{opt}}{R_s} = \frac{e_n/i_n}{R_s} \tag{24}$$

- *Caveats*: 1) resistance in the transformer's windings will contribute noise and limit the achievable noise figure, and 2) only useful for AC signals

Noise in cascaded systems

- The excess noise contribution of a single amplifier can be expressed as

$$N_e = F - 1 = \frac{R_e}{R_s} \tag{25}$$

- The noise factor of a cascaded system is determined by reflecting the excess noise contribution of each given stage all the way back to the input and dividing by the total power gain between that stage and the input

$$F = 1 + F_1 - 1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_N - 1}{\prod_{n=1}^{N-1} G_n} \tag{26}$$

- From this it is clear that the noise performance of a cascaded system is dominated by the first few stages, and that we should realize as much gain as possible early in the signal path

Opamp noise model

- The opamp noise model comprises two noise currents i_{na} (one at each input) and a single noise voltage e_{na}
- e_{na} represents contributions from all devices (transistors and resistors) internal to the opamp, referred to a single input
- i_{na} is associated with the input devices of the differential pair comprising the first stage of the opamp
- Note that the position of the i_{na}'s relative to e_{na} doesn't change the model

Opamp noise versus source resistance

- *Figure source: Art of Electronics, Third Edition*
- The lowest noise is achieved using BJT opamps and low source resistances (voltage noise regime)
- For higher values of R_S, BJT current noise dominates, making JFET (and some CMOS) amplifiers more attractive
- Due to the number of transistors used in opamps, specifically active-load devices, the lowest achievable noise density (which is limited by e_n), is typically higher that what is possible with discrete designs
- Nonetheless, due to their simplicity, cost, and size, opamps should be the preferred solution for most applications

Opamp voltage noise as a function of current

- Both FET- and BJT-based opamps require high quiescent currents to minimize input voltage noise
- Achieving high performance (i.e. low noise) becomes more difficult as the power budget tightens
- Low-power opamps are, by nature, noisy
- *Figure source: Art of Electronics, Third Edition*

Opamp noise measurement

- The noise properties of an amplifier can be determined by two measurements: 1) with the input shorted to AC ground for e_n, and 2) with an resistor between the input and ground for i_n
- To measure i_n, the resistance R_m should be selected such that $i_n R_m >> e_n$. In this case, i_n can be calculated as

$$i_n = \frac{e_{out} - 4kTR_m}{R_m} \tag{27}$$

Effect of feedback on noise

- The effect that feedback has on noise depends on the location of the noise source in the feedback loop (i.e. where the noise is "injected")
- We can assess the effect of each noise source separately by determining its *noise transfer function* (NTF)
- In the case of e_{ni}, it is obvious that the noise adds directly to the input, hence the feedback has no affect on this source
- What about e_{no}?
- To determine the effect of e_{no} on the output noise, we'll set v_{in} and e_{ni} to zero to find the NTF
- The output in this case is given by

$$v_{o,e_{no}} = e_{no} - H(s) \cdot \beta \cdot v_{o,e_{no}} \tag{28}$$

- This can be rearranged as

$$v_{o,eno} = \frac{e_{no}}{1 + H(s) \cdot \beta} \tag{29}$$

- The noise transfer function is thus

$$NTF_{e_{no}} = \frac{1}{1 + H(s) \cdot \beta} \tag{30}$$

- We see that the use of feedback reduces noise injected at the *output* of the amplifier by the loop gain. This property is exploited in both phase-locked loops and delta-sigma modulators to minimize certain noise sources (though it has minimal impact on feedback amplifiers)

Feedback amplifier noise model

- To utilize to tools/concepts previously developed, such as noise factor and optimum source resistance, the feedback amplifier's noise sources should be converted to equivalent noise sources e_n and i_n
- e_n is determined by referring e_{na}, e_{n1}, e_{n2}, and i_{na} at the opamp's inverting terminal to the output and dividing by the *closed-loop* gain
- In this case i_n is equal to i_{na} at the non-inverting terminal
- The original noise sources e_{na}, i_{na}, e_{n1}, and e_{n2} are due to the opamp's internal noise processes and the thermal noise of the feedback resistances
- The equivalent noise sources are given by

$$e_n^2 = e_{na}^2 + \frac{e_{n2}^2 + (R_2/R_1)^2 \cdot e_{n1}^2 + i_{na}^2 \cdot R_2^2}{G^2} \tag{31}$$

$$i_n^2 = i_{na}^2 \tag{32}$$

- G is the closed-loop gain of the non-inverting amplifier

Amplifier noise gain

- For both inverting an non-inverting amplifiers, the opamp + feedback resistor noise can be referred to the non-inverting terminal of the opamp as e_{ni}
- Assuming $R_s = 0$, the output noise for both inverting and non-inverting configurations is given by

$$e_{no}^2 = e_{ni}^2 \cdot \left(1 + \frac{R_2}{R_1}\right)^2 \tag{33}$$

- $G = \left(1 + \frac{R_2}{R_1}\right)$ is referred to as the *noise gain* of the amplifier, which is identical for both structures
- However, the *signal gain* is different for the two configurations (R_2/R_1 versus $1 + R_2/R_1$), making the inverting amplifier (slightly) worse from a noise perspective

Amplifier noise bandwidth

- The input-referred voltage-noise density is given by

$$e_{ni}^2 = e_{ns}^2 + e_n^2 + i_n^2 R_s^2 \tag{34}$$

- The bandwidth of the non-inverting amplifier is

$$f_{3dB} = \beta \cdot f_u \tag{35}$$

$$\beta = \frac{R_1}{R_1 + R_2} \tag{36}$$

- We're generally more interested in the integrated or rms noise over a given bandwidth than we are in the noise density alone
- The total integrated noise of the amplifier is equal to the equivalent input voltage noise multiplied by the noise bandwidth
- Assuming a first-order response, this can be estimated as: $v_{ni,rms} = \sqrt{e_{ni}^2 \cdot f_{3dB} \cdot \frac{\pi}{2}} = \sqrt{e_{ni}^2 \cdot \beta f_u \cdot \frac{\pi}{2}}$
- Since general-purpose opamps have high values of f_u ($1\mathrm{MHz}$ or higher) filtering is typically needed

Integrated noise

- We are generally more interested in the noise integrated over a (typically narrower) bandwidth dictated by the application
- Even for amplifiers that have comparable *spot noise* performance at a given frequency, integrated noise can vary appreciably due to $1/f$ noise
- Thus when comparing different amplifiers it is more instructive/useful to look at the amplifier's noise integrated over the bandwidth of interest
- The variable width of the "flat" region at low frequencies in the plots relates to the varying amounts of $1/f$ noise present at low frequencies
- For CMOS opamps in particular, which have high levels of $1/f$ noise, the flat region extends to relatively high frequencies
- *Figure source: Art of Electronics, Third Edition*

Opamp 1/f corner frequency

- $1/f$ noise can have an extremely detrimental effect on performance for low-frequency, low-bandwidth applications
- The $1/f$ performance of an opamp is typically reported in datasheets either as a corner frequency (f_c) or peak-to-peak noise over a given frequency range below f_c
- Let's take a look at the noise performance of three imaginary amplifiers with equivalent white noise density but different values of f_c

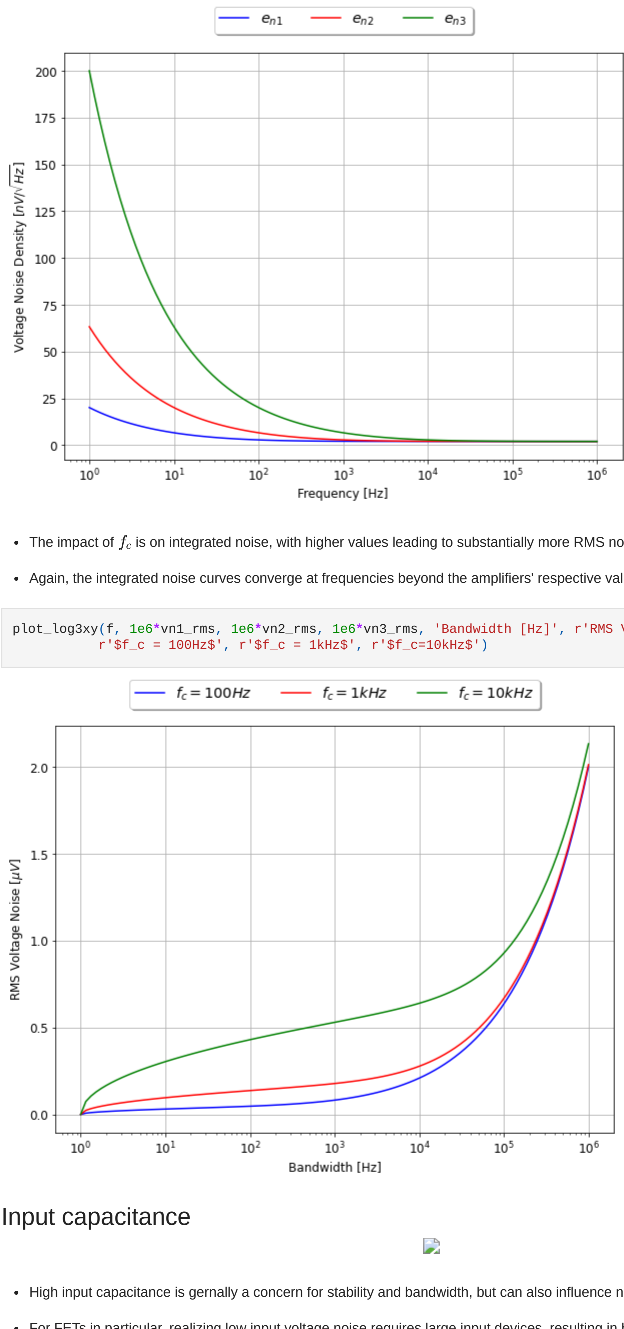
In [155…

```
e_nw = 2e-9
fc1 = 100
fc2 = 1e3
fc3 = 10e3
f = np.logspace(0, 6, num=100)
e_n1 = np.sqrt(e_nw**2*(1+fc1/f))
e_n2 = np.sqrt(e_nw**2*(1+fc2/f))
e_n3 = np.sqrt(e_nw**2*(1+fc3/f))
vn1_rms = np.sqrt(integrate.cumtrapz(e_n1**2, f, initial=0))
vn2_rms = np.sqrt(integrate.cumtrapz(e_n2**2, f, initial=0))
vn3_rms = np.sqrt(integrate.cumtrapz(e_n3**2, f, initial=0) )
```

- We see that the noise density curves converage at higher frequencies, but diverge substantially below f_c

In [156…

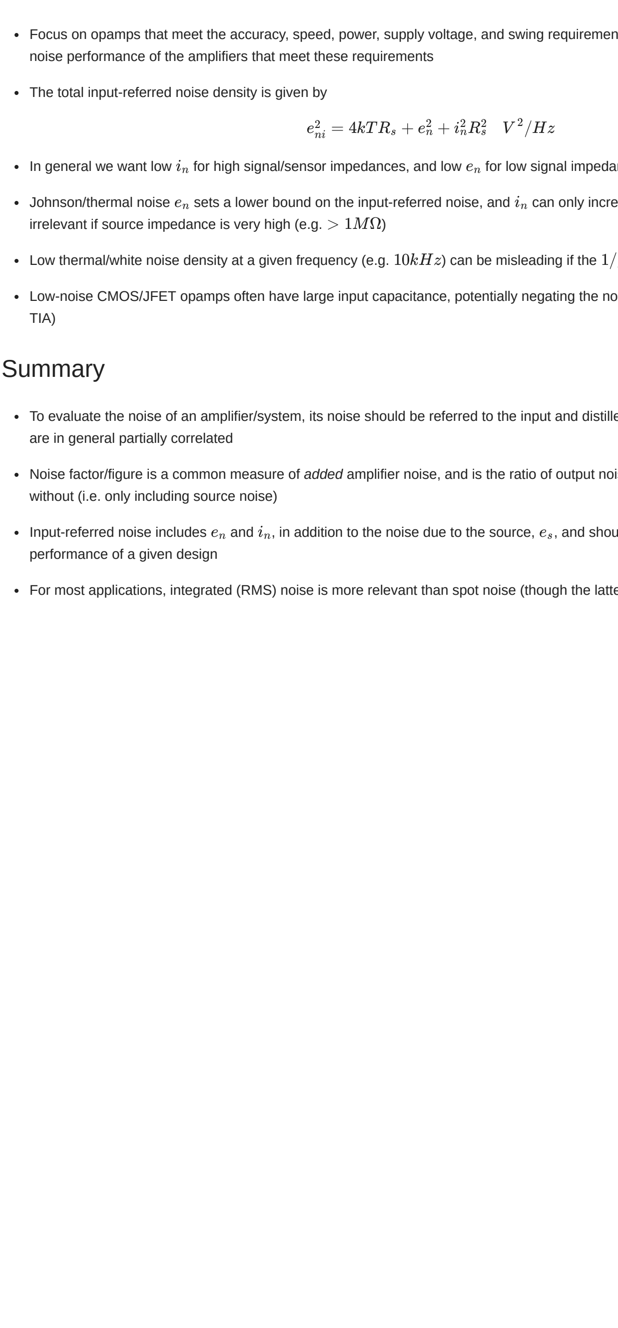
```
plot_log3xy(f, 1e9*e_n1, 1e9*e_n2, 1e9*e_n3, 'Frequency [Hz]', r'Voltage Noise Density [$nV/\sqrt{Hz}$]',
            r'$e_{n1}$', r'$e_{n2}$', r'$e_{n3}$')
```



- The impact of f_c is on integrated noise, with higher values leading to substantially more RMS noise and lower frequencies
- Again, the integrated noise curves converge at frequencies beyond the amplifiers' respective values of f_c

In [158…

```
plot_log3xy(f, 1e6*vn1_rms, 1e6*vn2_rms, 1e6*vn3_rms, 'Bandwidth [Hz]', r'RMS Voltage Noise [$\mu V$]',
           r'$f_c = 100Hz$', r'$f_c = 1kHz$', r'$f_c=10kHz$')
```



Input capacitance

- High input capacitance is gernally a concern for stability and bandwidth, but can also influence noise performance
- For FETs in particular, realizing low input voltage noise requires large input devices, resulting in high values of input capacitance C_{in}
- Opamp input capacitance is a major concern for transimpedance amplifiers and high-impedance sensor applications
- *Figure source: Art of Electronics, Third Edition*

Example: transimpedance amplifier

- With the opamp's noise voltage modeled as e_n in series with the non-inverting terminal, feedback forces e_n to appear at the inverting terminal
- The resulting noise current density through C_{in} is $i_{nC} = e_n \cdot 2\pi f C_{in}$
- That is, the amplifier's voltage noise creates a noise current proportional to C_{in} that increases with frequency
- For example, at $1\ \mathrm{MHz}$, a noise voltage of $10\mathrm{nV}/\sqrt{\mathrm{Hz}}$ and an input capacitance of $5\mathrm{pF}$ amount to a noise current density of $0.4\mathrm{pA}/\sqrt{\mathrm{Hz}}$

Choosing a low-noise opamp

- Focus on opamps that meet the accuracy, speed, power, supply voltage, and swing requirements of the application, and evaluate the noise performance of the amplifiers that meet these requirements
- The total input-referred noise density is given by

$$e_{ni}^2 = 4kTR_s + e_n^2 + i_n^2 R_s^2 \quad V^2/Hz \tag{37}$$

- In general we want low i_n for high signal/sensor impedances, and low e_n for low signal impedances
- Johnson/thermal noise e_n sets a lower bound on the input-referred noise, and i_n can only increase it. Low voltage noise is often irrelevant if source impedance is very high (e.g. $> 1M\Omega$)
- Low thermal/white noise density at a given frequency (e.g. $10kHz$) can be misleading if the $1/f$ corner is high
- Low-noise CMOS/JFET opamps often have large input capacitance, potentially negating the noise benefit for certain applications (e.g. TIA)

Summary

- To evaluate the noise of an amplifier/system, its noise should be referred to the input and distilled into two sources, e_n and i_n, which are in general partially correlated
- Noise factor/figure is a common measure of *added* amplifier noise, and is the ratio of output noise with the opamp's internal noise and without (i.e. only including source noise)
- Input-referred noise includes e_n and i_n, in addition to the noise due to the source, e_s, and should be used to evaluate the "raw" noise performance of a given design
- For most applications, integrated (RMS) noise is more relevant than spot noise (though the latter will inform the former)