Noise in Junction Transistors*

A. VAN DER ZIEL†, FELLOW, IRE

Summary—This paper gives a survey of the problem of shot noise and flicker noise in junction diodes and junction transistors. After a short introduction in Section I, the theory of shot effect is presented in Section II. First a simplified low-frequency theory is given and the close correspondence with earlier (heuristic) equivalent circuits is indicated. Then the theory is given in a more rigorous form, both from a collective point of view (Petritz, North, and van der Ziel) and from a corpuscular point of view (Uhlir, van der Ziel and Becking). Finally the conditions under which the theory holds are summed up and the possibility of deviations is discussed. Section III gives Fonger's theory of flicker noise in diodes and transistors and incorporates his discussion of base modulation effects into the equivalent noise circuit in a manner that differs somewhat from Fonger's original presentation. Section IV gives the experimental verification of the theory by Guggenbuehl and Strutt, Nielsen, Hanson and van der Ziel and others and also discusses some further experimental material. Finally, the problem of low-noise circuits, the choice of the operating point of the transistor, and the design criteria for low-noise transistors is discussed. Section V extends Fonger's theory of base modulation effects to shot effect and discusses possible consequences of this effect.

I. Introduction

Let an active four-terminal network be connected to a signal source of internal impedance $Z_s = R_s + jX_s$ or internal admittance $Y_s = 1/Z_s = g_s + jb_s$. The noisiness of the network may then be characterized in many ways. One may, *e.g.*, represent the noise by an equivalent emf e_n in series with the source or by an equivalent noise current generator i_n in parallel to the source; these quantities are defined such that the output noise power of the network is doubled if the noise emf e_n or the noise current generator i_n are introduced. One may then define the *equivalent noise resistance* R_n or the *input equivalent saturated diode current* I_n of the network by the equations

$$\overline{e_n^2} = 4kTR_n\Delta f; \qquad \overline{i_n^2} = 2eI_n\Delta f \tag{1}$$

where T is room temperature, k is Boltzmann's constant, e is the electron charge, and Δf a small frequency interval. Both quantities R_n and I_n may depend upon the internal impedance of the source.

It is more common to introduce the *noise figure* F of the network as the ratio of the total output noise power over the output noise power due to the thermal noise of the source. The latter can be represented by a noise emf $\sqrt{4kTR_s\Delta f}$ in series with the source or by a noise current generator $\sqrt{4kTg_s\Delta f}$ in parallel to the source. Then, according to (1)

$$F = \frac{R_n}{R_s} = \frac{e}{2kT}\,\frac{I_n}{g_s}\,. \tag{2}$$

The noise figure always shows a parabolic dependence on R_s and has a minimum value $F_{\min}$ for $R_s = (R_s)_{\min}$.

The smallest available signal power that can be detected against the noise background of an amplifier of noise figure F and bandwidth B is about $FkTB$. One thus wants to make the noise figure F as small as possible under the existing operating conditions.

In many cases it is possible to change the source impedance, as viewed from the input of the amplifier, within a wide range with the help of a lossless matching network. In such cases the amplifier with the lowest value of $F_{\min}$ is the best one. In other cases it is necessary to connect the signal source directly to the amplifier without the benefit of a lossless matching network; in that case the amplifier with the lowest value of $F_{\min}$ may be a rather poor choice. In the case of a low-impedance signal source the amplifier with the lowest noise resistance R_n is the best one. Whereas, with a high-impedance signal source the amplifier with the lowest input equivalent saturated diode current I_n is preferred.

The first step in characterizing the noisiness of an amplifier stage consists of finding the noise sources in their active element and locating the proper positions of these sources in their equivalent circuit. It is then possible to determine the most suitable operating conditions of a given active element, or to design the active element so that it gives the lowest noise figure F under the existing operating conditions.

The representation of the noise properties of an active network by an equivalent circuit is not unique, since a given circuit can be transformed into another one by applying certain network theorems. Usually one tries to find the equivalent circuit that fits closest to the physics of the device.

Since junction transistors have found many applications in amplifier circuits, it is important to have a good understanding of the noise properties of these devices. The noise properties of junction diodes also are of interest for two reasons: their noise properties are closely related to those of junction transistors and certain transistor equations follow directly from the corresponding diode equations. They also are of intrinsic interest because of their use as low-level radiation detectors.

It was found that the noise in these devices consists of two parts, a flicker noise part with a low-frequency noise spectrum and a shot noise part with, at least at low frequencies, a flat spectrum. Flicker noise probably is caused by a modulation mechanism located at the surface of the devices; it can be considerably reduced by appropriate surface treatments and therefore is not a basic limitation. Shot noise is due to the corpuscular character of the current flow and thus represents a basic

* Original manuscript received by the IRE, February 28, 1958; revised manuscript received, April 17, 1958.
† Elec. Eng. Dept., University of Minnesota, Minneapolis, Minn.

limitation, so that it is important to have a good understanding of the phenomenon.

II. Theory of Shot Noise in Junction Diodes and Transistors

The problem may be treated theoretically in two equivalent ways.

1) The collective approach, where the noise is attributed to the random diffusion of minority carriers and to the random recombination and generation of hole-electron pairs.
2) The corpuscular approach, where the shot noise is attributed to a series of random and independent events, *viz.*, the crossing of the emitter and/or the collector junction by the individual current carriers.

Petritz published the first paper on the collective approach, using a lumped-parameter approximation [44]. Later he solved the one-dimensional diode problem more accurately [44a] and obtained a result that is nearly identical with (16) of this paper; unfortunately, this result was not given in an easily applicable form and no detailed account of this work was published. North [42] showed that the mathematical difficulties could be greatly simplified by representing the diffusion and recombination of minority carriers by a distributed *RC* network.[1] Van der Ziel [55] treated both the one-dimensional diode and the one-dimensional transistor in this manner. Solow [49a] extended the theory to two- and three-dimensional geometries in his thesis; the thesis also gives a summary of Petritz's unpublished work. The one-dimensional problem was also solved independently by Becking with the help of the collective method,[2] but his results were not published.

Weisskopf [60] applied the corpuscular approach to crystal diodes; a similar approach is also the (hidden) basis of the earlier heuristic theories of diode and transistor noise [15], [40], [53]. Uhlir [52] extended the method to high frequencies for diodes; van der Ziel and Becking [56] generalized his approach and extended it to transistors.

A. The Low-Frequency Corpuscular Approach

First consider noise in junction diodes. Let the diode have a characteristic:

$$I = I_o(e^{eV/kT} - 1). \tag{3}$$

At low frequencies its admittance Y is a conductance G_o:

$$Y = G = G_o = \frac{dI}{dV} = \frac{e(I + I_o)}{kT}. \tag{4}$$

One may now consider the diode current I to consist of two parts, a part $(I+I_o)$ and a part $-I_o$; the minus sign indicates that the currents flow in opposite directions. Both currents should fluctuate independently and each should show full shot noise (see Section II-E). Hence, if the total noise of the junction is represented by a current generator i in parallel to the junction admittance $Y=G=G_o$, we have

$$\overline{i^2} = 2e(I + I_o)\Delta f + 2eI_0\Delta f. \tag{5}$$

This result should be valid for arbitrary diodes at low frequencies.

Application to point contact diodes showed that reasonable agreement could be obtained between theory and experiment for the case of forward bias, provided that the thermal noise of the contact resistance r of the diode was taken into account; this leads to the equivalent circuit of Fig. 1(a) [51,] [54]. Anderson and van der Ziel applied the equivalent circuit for low frequencies but could not explain their high-frequency data [2]. This is discussed later.

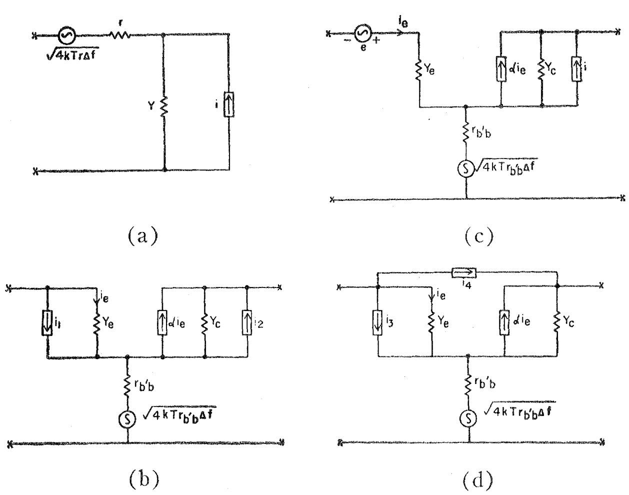

Fig. 1—Equivalent circuits for shot noise. (a) Equivalent circuit of a junction diode. (b) Equivalent circuit of a transistor. (c) Equivalent circuit of Montgomery, Clark, and van der Ziel. (d) Equivalent circuit of Giacoletto.

The above theory of the junction diode is easily extended to transistors. Consider, for example a *p-n-p* transistor; for sake of simplicity it is assumed at first that all current is carried by holes. Let I_e be the emitter current and I_c the collector current and let the collector be biased so that it does not inject holes into the base region. The current I_e can now be considered as consisting of a part (I_e+I_{ee}) due to holes flowing from the emitter to the base and a part $(-I_{ee})$ due to holes flowing from the base to the emitter. Both currents should fluctuate independently and each should show full shot noise (Section II-E). Hence, if the emitter noise is represented by a current generator i_1 in parallel to the emitter junction,

$$\overline{i_1^2} = 2e(I_e + I_{ee})\Delta f + 2eI_{ee}\Delta f. \tag{6}$$

In the collector junction all holes move in the same direction. One would thus expect full shot noise for the

[1] Petritz has used this method in the derivation of his 1953 diode formula [44a]. (R. L. Petritz, private communication.)
[2] A. G. T. Becking, private communication.

collector current I_c. Representing the collector noise by a current generator i_2 in parallel to the collector junction, we have

$$\overline{i_2^2} = 2eI_c\Delta f. \tag{7}$$

These equations follow directly from (5). The emitter noise is obtained by substituting $I = I_e$ and $I_o = I_{ee}$; the collector noise is obtained by putting $(I+I_o)=0$ and $I_o = I_c$. The latter seems strange at first, but it should be remembered that the collector is biased in the back direction and that $(I+I_o)$ corresponds to the hole current injected from the collector into the base, which is zero because of the existing bias conditions. Since (5) is valid for an arbitrary diode, (6) and (7) should also be valid if part of the current is carried by electrons (see below).

In addition, one would expect thermal noise for the series resistance of junctions. The most important thermal noise source is the true base resistance $r_{b'b}$.[3]

The full equivalent circuit thus is as shown in Fig. 1(b). The emitter admittance Y_e shown in this circuit is actually a conductance G_{eo} at low frequencies:

$$Y_e = G_e = G_{eo} = \frac{\partial I_e}{\partial V_e} = \frac{e(I_e + I_{ee})}{kT}. \tag{8}$$

The current generators i_1 and i_2 are strongly correlated.

To calculate the cross correlation $\overline{i_1^* i_2}$ (the asterisk denotes the conjugate complex quantity), we observe that the part $\beta_o(I_e+I_{ee})$ of the hole current (I_e+I_{ee}) injected into the base region by the emitter is collected by the collector; the quantity β_o is the *dc collector efficiency* of the collector junction. If the holes that are generated in the base region and are collected by the collector give a contribution I_{cc} to I_c, then

$$I_c = \beta_0(I_e + I_{ee}) + I_{cc} = \beta_0 I_e + I_{co}; \; I_{co} = \beta_o I_{ee} + I_{cc}. \tag{9}$$

The quantity I_{co} is called the *collector saturated current*; it is the collector current for open emitter.[4] The emitter and collector thus have the current $\beta_o(I_e+I_{ee})$ in common.

The cross correlation $\overline{i_1^* i_2}$ is caused by fluctuations in this current. It should have full shot noise, hence

$$\overline{i_1^* i_2} = 2e\beta_o(I_e + I_{ee})\Delta f. \tag{10}$$

The current $\beta_o(I_e+I_{ee})$ is also responsible for the signal transfer properties. The transfer admittance Y_{ce} of the transistor is actually a transfer conductance G_{ceo} at low frequencies:

$$Y_{ce} = G_{ceo} = \frac{\partial I_c}{\partial V_e} = \frac{\beta_o e(I_e + I_{ee})}{kT}. \tag{11}$$

The signal transfer can be represented by a current generator $Y_{ce}v_e$ in parallel to the collector junction, where v_e is the ac emitter voltage. This current generator is also shown in the equivalent circuit of Fig. 1(b)

Next, drop the assumption that all current is carried by holes. There are then also electrons going from emitter to base, from base to emitter, and from collector to base. Each of these electrons contributes either to I_e or to I_c, but no electrons contribute to *both* I_e and I_c. The conditions for full shot noise again exist and hence (6) through (8) remain valid, provided that the currents (I_e+I_{ee}) and $(-I_{ee})$ are now properly redefined; for example, (I_e+I_{ee}) is now partly caused by holes injected from the emitter into the base and partly by electrons injected from the base into the emitter.

In this case let the part $\gamma_o(I_e+I_{ee})$ of the emitter current be due to holes injected into the base by the emitter; γ_o is known as the *dc emitter efficiency*. The part β_o of these holes is collected by the collector; the emitter and collector junctions thus have the current $\gamma_o\beta_o(I_e+I_{ee})$ in common and this current should show full shot noise.

We define the dc current amplification factor α_o

$$\alpha_o = \gamma_o\beta_o \tag{12}$$

of the transistor and observe that the quantity β_o in (9) through (11) should be replaced by α_o in this case. These equations thus become:

$$\overline{i_1^* i_2} = 2e\alpha_0(I_e + I_{ee})\Delta f \tag{13}$$

$$Y_{ce} = G_{ceo} = \alpha_o G_{eo} = \alpha_o \frac{e(I_e + I_{ee})}{kT} \tag{14}$$

$$I_c = \alpha_o I_e + I_{co}; \qquad I_{co} = \alpha_o I_{ee} + I_{cc}. \tag{15}$$

B. Extension to High Frequencies

Eqs. (4) through (6), (8), (10) through (12), (14), and (15) cease to be valid at higher frequencies. For example, the diode admittance Y becomes complex and its real part G is no longer equal to G_o at high frequencies. The emitter admittance Y_e of a transistor also becomes complex and its real part G_e is no longer equal to G_{eo}. Finally, the transfer admittance Y_{ce} becomes complex and $|Y_{ce}|$ decreases with increasing frequency. The noise equations (5), (6), and (13) should thus be extended to high frequencies; in addition, the current amplification factor should be redefined. The extension of the noise equations to high frequencies is carried out in Sections II-D and II-E; here we only quote the results and show that they are compatible with the equations derived for low frequencies.

For diodes, the following equation is of general validity:

$$\overline{i^2} = 4kTG\Delta f - 2eI\Delta f \tag{16}$$

where I is taken positive for forward bias and negative

[3] The "true" base resistance $r_{b'b}$ is the base resistance found when the contribution of Early's feedback emf $\mu_{ec}v_c$ (v_c is the ac voltage across the emitter junction) to the "measured" base resistance is subtracted [11]. Early's feedback emf is omitted from the equivalent noise circuit given here, since it does not affect the noise figure of the device [55].

[4] The definition of I_{co} shows that I_{cc} is *not* the collector saturated current but that it is related to it. In the same way I_{ee} is *not* the emitter saturated current.

for back bias. For low frequencies $G=G_o$; substituting (4) for G_o, we obtain (5), so that (5) and (16) are compatible. The new equation fits well with the experimental data, as was shown by Champlin [7], [8] (Section IV-A).

For transistors, the following equations are of general validity:

$$\overline{i_1^2} = 4kTG_e\Delta f - 2eI_e\Delta f \tag{17}$$

$$\overline{i_2^2} = 2eI_c\Delta f \tag{18}$$

$$\overline{i_1^* i_2} = 2kTY_{ce}\Delta f. \tag{19}$$

At low frequencies, $G_e=G_{eo}$; substituting (8) into (17) gives (6) back again. Moreover, $Y_{ce}=G_{ceo}$ at low frequencies; substituting (14) into (19) gives (13) back again. Eqs. (17) through (19) thus seem to be the proper extensions of (6), (7), and (10) [or (13)] for higher frequencies.

We observe with Guggenbuehl and Strutt [23] that (17) and (18) are a direct consequence of (16). For if (16) holds for arbitrary diodes, it will also hold for the emitter diode and for the collector diode. The emitter diode has a conductance G_e and a current I_e; substituting this into (16) yields (17). The collector diode has $G=0$ and $I=-I_c$ (because the collector diode is biased in the back direction, we have to use the minus sign; a diode biased in the back direction has practically zero conductance). Substituting this into (16) yields (18).

Eqs. (16) through (19) reduce to thermal noise if the proper amounts of shot noise power are *added* for forward bias and *subtracted* for back bias. Guggenbuehl and Strutt [23] have given arguments in favor of such a procedure and have used it to derive these equations from thermal noise considerations only. One may consider this as a first attempt towards a thermodynamical derivation of the equations. A rigorous derivation of these equations along these lines should be based upon the principles of irreversible thermodynamics.

At low frequencies, the current amplification factor α_o may be defined as $\alpha_o=G_{ceo}/G_{eo}$, according to (14). It thus seems logical to define the high-frequency amplification factor α as

$$\alpha = \frac{Y_{ce}}{Y_e}. \tag{20}$$

The value $|\alpha|$ decreases with increasing frequency; generally

$$\alpha = \frac{\alpha_o}{1 + jf/f_o} \tag{20a}$$

where f_o is the α-cutoff frequency.

C. Other Equivalent Circuits

The equivalent circuit of Fig. 1(b) is nearly identical with two other equivalent circuits that were developed earlier on a more or less heuristic basis by Montgomery, Clark, and van der Ziel [40], [53] and by Giacoletto [15].

The first proposed the equivalent circuit of Fig. 1(c). It contains two independent noise sources, a noise emf e_e in series with the emitter and a noise current generator i in parallel to the collector junction. Both noises were assumed to be uncorrelated, and[5]

$$\overline{e_2^2} = 2kTR_{eo}\Delta f\left(\frac{I_e + 2I_{ee}}{I_e + I_{ee}}\right);$$

$$\overline{i^2} = 2e\alpha_0(1 - \alpha_0)I_e\Delta f + 2eI_{co}\Delta f, \tag{21}$$

where $R_{eo}=1/G_{eo}$. The signal transfer properties of the transistor are represented in this circuit by the current generator $\alpha_o i_e$, where i_e is the current passing through the emitter junction.

The circuit of Fig. 1(b) is easily transformed into the one of Fig. 1(c); this leads to

$$i = (i_2 - \alpha_0 i_1); \qquad e_e = i_1 R_{eo}, \tag{21a}$$

according to (14). The quantity $\overline{e_e^2}$ is calculated from (6) and (8) and the expression for $\overline{i^2}$ follows from (6), (7), and (13). Finally, it is indeed true that e_e and i are practically uncorrelated, for

$$\overline{e_e^* i} = R_{eo}\overline{i_1^*(i_2 - \alpha i_1)} = 2e\alpha_o I_{ee}R_{eo}\Delta f \tag{21b}$$

for $I_e \gg I_{ee}$ this is small in comparison with $\sqrt{\overline{e_e^2}\cdot\overline{i^2}}$.

Giacoletto [15] represented the noise by two uncorrelated current generators i_3 and i_4; i_3 was connected in parallel to the emitter junction and i_4 was connected between the emitter and the collector junction, whereas

$$\overline{i_3^2} = 2eI_b\Delta f; \qquad \overline{i_4^2} = 2eI_c\Delta f, \tag{22}$$

where $I_b=(I_e-I_c)$ is the base current. This equivalent circuit is shown in Fig. 1(d).

The circuit of Fig. 1(b) is easily transformed into the one of Fig. 1(d); this leads to

$$i_1 = i_3 + i_4; \qquad i_2 = i_4 \tag{23}$$

so that $\overline{i_4^2}$ follows directly from (7), whereas

$$\overline{i_3^2} = \overline{(i_1 - i_2)^2} = 2eI_b\Delta f + 4e(I_{ee} + I_{cc})\Delta f \tag{24}$$

according to (6), (7), (13), and (15). This corresponds to (22), if $2(I_{ee}+I_{cc}) \ll I_b$. Moreover, i_3 and i_4 are practically uncorrelated, since

$$\overline{i_3^* i_4} = \overline{(i_1^* - i_2^*)i_2} = -2eI_{cc}\Delta f \tag{25}$$

according to (7), (13), and (15), which is small in comparison with $\sqrt{\overline{i_3^2}\cdot\overline{i_4^2}}$ as long as I_{cc} is small.

These discussions show that the three equivalent circuits are interchangeable, except for the minor details just mentioned.

[5] Montgomery and Clark [40] gave the expression for $\overline{e_e^2}$ except for the (usually unimportant) factor $(I_e+2I_{ee})/(I_e+I_{ee})$ that was added by van der Ziel. They also gave the second term in $\overline{i^2}$; the first term was added by van der Ziel [53].

D. The Collective Approach

The operation of the junction diode and the junction transistor is based upon the injection and extraction of minority carriers. The flow of these carriers is by diffusion; they disappear sooner or later by recombination. These two processes are studied in detail in the collective approach. As mentioned in the beginning of Section II the diffusion and recombination of the minority carriers can be represented by a distributed RC network; for a one-dimensional model this corresponds to a distributed line without distributed inductance. That this is indeed the case is most easily seen by comparing the differential equation for a distributed line of series resistance R, parallel conductance G, and parallel capacitance C (all per unit length) with the differential equations describing a one-dimensional diffusion problem in which drift is negligible in comparison with diffusion.[6]

Let the minority carriers be holes with a charge e, a diffusion constant D_p, and a lifetime τ_p. Let p_n be the equilibrium hole concentration, p the total hole concentration, and $p' = p - p_n$ the excess hole concentration (all per unit length) and let i_p be the hole current. In the transmission line let E be the voltage on the line and I the current. Then, according to van der Ziel, the following correspondence holds between the diffusion problem and the transmission line problem: E corresponds to p', I corresponds to i_p, R corresponds to $1/(eD_p)$, G corresponds to (e/τ_p), and C corresponds to e. A diffusion problem can be solved now by first translating it into a transmission line problem, solving that by standard methods, and then translating back to the diffusion problem.

In this model the noise is caused by recombination fluctuations and diffusion fluctuations. In a section of length Δx the first effect can be represented by a fluctuating current Δi_{px} disappearing in the section Δx, whereas the diffusion fluctuations give rise to a fluctuating hole density in that section. Van der Ziel [55] showed that

$$\overline{\Delta i_{px}^2} = 2e^2\Delta f(p + p_n)\Delta x/\tau_p \tag{26}$$

$$\overline{\Delta p_x^2} = 4p\Delta f\Delta x/D_p. \tag{27}$$

Van der Ziel proved (26) from shot noise considerations and showed in an indirect manner that the expression for $\overline{\Delta p_x^2}$ had to have the form (27); otherwise a junction diode at zero bias would not give full thermal noise at all frequencies. Petritz [44a], [49a] derived expressions for these noise sources with the help of the Kolmogaroff-Fokker-Planck equation. He was able to give a rigorous proof of (27), whereas he obtained

$$\overline{\Delta i_{px}^2} = 4_e{}^2\Delta f p\Delta x/\tau_p \tag{26a}$$

instead of (26).[7] In the opinion of this author, the discrepancy between (26) and (26a) is due to the fact that the Kolmogaroff-Fokker-Planck equation has to be applied with caution to the recombination fluctuations if $p \neq p_n$, whereas the shot noise method remains fully applicable in that case.

The quantities Δi_{px} and Δp_x are, of course, independent, since they represent independent fluctuations. In the transmission line analogy Δp_x corresponds to a distributed series noise emf and Δi_{px} corresponds to a distributed parallel noise current generator. The fluctuations in the sections Δx can be treated as independent as long as Δx is large in comparison with the free path length of the carriers. The final result is obtained by adding the contributions of all sections Δx quadratically.

Solow [49a] has extended this treatment to two and three dimensions; his theory includes the effects of surface recombination velocity. He uses Petritz's bulk noise generators and derives surface generators with the help of the Kolmogaroff-Fokker-Planck equation.

A simple proof of (26) may be given with the help of the shot noise method; it starts from the diffusion equation

$$\frac{\partial p}{\partial t} = -\frac{p'}{\tau_p} - \frac{1}{e}\frac{\partial i_p}{\partial x}. \tag{28}$$

For stationary current flow $\partial p/\partial t = 0$ and hence, according to (28), a current $ep'\Delta x/\tau_p$ disappears between x and $(x+\Delta x)$ by recombination. Because of the equilibrium concentration p_n an additional current $ep_n\Delta x/\tau_p$ disappears for the same reason, which is balanced by the appearance of a current $ep_n\Delta x/\tau_p$ by pair generation. The total current *disappearing* in the section Δx is thus $ep\Delta x/\tau_p$ and the total current *appearing* in that section is $ep_n\Delta x/\tau_p$. Both currents should fluctuate independently and each should show full shot noise. The Fourier component Δi_{px} of this fluctuation therefore is given by

$$\overline{\Delta i_{px}^2} = 2e\left(\frac{ep\Delta x}{\tau_p}\right)\Delta f + 2e\left(\frac{ep_n\Delta x}{\tau_p}\right)\Delta f$$

which corresponds to (26).

Knowing the noise sources, it is not difficult to calculate the noise currents in the leads short-circuiting the electrodes and to prove (16) through (19). The equations thus are valid at all frequencies for a one-dimensional model in which all current is carried by holes.

Eqs. (16) through (19) do not give any reference to the model; therefore it was expected that they should be

[6] The fact that the problem is one-dimensional implies that the recombination must be *volume* recombination, not *surface* recombination.

[7] Petritz used (26a) and (27) for noise generators in his derivation of the diode result [44a]. By a slight rearrangement of terms his expression can be written as

$$\overline{i^2} = 4kTG\Delta f - 2eI\Delta f\left(\frac{2G}{2G + G_0}\right). \tag{16a}$$

The difference between (16a) and (16) arises entirely from the difference in the expression for the recombination source [(26a) instead of (26)].

of general validity. The discussion of Section II-E shows that this is indeed the case.

E. Extension of the Corpuscular Theory [56]

The basic assumptions underlying the theory of Sections II-A and II-B will now be verified and the results of those sections extended to higher frequencies. To do so, the problem of current flow in p-n junctions has to be investigated in greater detail.

Consider first an n-type semiconductor sample with an ohmic contact. If electrons are injected into the material, space-charge neutrality will be re-established in a very short time, of the order of magnitude of the dielectric relaxation time of the material (about 10^{-12} seconds for germanium). This is achieved by a small displacement of the other electrons; at the same time electrons will leave through the ohmic contact to make the material externally neutral. If holes are injected into the material, space-charge neutrality will again be established in a very short time by a slight rearrangement of the electrons; at the same time electrons will enter through the ohmic contact to make the material externally neutral. The injected holes now spread out slowly by diffusion and disappear by recombination, but this does not cause any current in the external lead to the ohmic contact. In both cases, therefore, current occurs only at the instant that the carriers are injected into the material.

Now consider a p-n junction with two ohmic contacts. A very short current pulse will occur in the external circuit if a hole *enters* into the n region through the space-charge region or when a hole leaves the n region through that region. The two current pulses have opposite polarity and the displaced charge per pulse is $\pm e$. The duration of the pulse is determined by the diffusion time of the carriers through the space-charge region and is, in general, very short. Similar considerations hold for electrons entering or leaving the n region.

It is interesting to note that the individual current pulses are independent and that each pulse transfers a charge $\pm e$ in the external circuit. It is thus allowed to assign full shot noise to the various currents and, because the individual current pulses are so short, this should be the case for all frequencies of practical interest.

If the applied voltage is changed, then the minority carrier concentration at the boundaries of the transition region follows the applied voltage practically instantaneously, since the diffusion time through the transition region is so small. However, the subsequent diffusion of the minority carriers is a very slow process; it is responsible for the high-frequency behavior of the diodes and the transistors. To understand this high-frequency behavior, consider devices in which all current is carried by holes and split the carriers into different groups.

In a p-n junction diode one has to split the carriers into three groups[8] (Fig. 2).

Group 1: Holes flowing from the p region into the n region and recombining there. They give very short, random, and independent single current pulses and carry a current $(I+I_o)$; their contribution to $\overline{i^2}$ is therefore equal to the first term in (5) for all frequencies of practical interest. Moreover, because the rate of diffusion of the holes of Group 1 across the space-charge region follows the applied voltage practically instantaneously, this group gives a contribution $e(I+I_o)/kT$ to the junction admittance Y at all frequencies.

Group 2: Holes flowing from the p region into the n region and returning to the p region before having recombined. They give independent and random *double* current pulses, each consisting of two single, short current pulses of opposite polarity with the second one being delayed by a random delay time with respect to the first one. This group of holes is responsible for the high-frequency behavior.

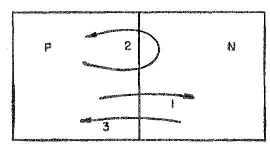

Fig. 2—The holes taking part in the conduction process in a junction diode are divided into three groups.

Group 3: Holes generated in the n region and diffusing into the p region; they give rise to very short, random, independent, single current pulses carrying a total current $(-I_o)$; thus they give a contribution equal to the second term of (5) for all frequencies of practical interest. This group of holes does not contribute to the diode admittance Y, since the current $(-I_o)$ is independent of the applied voltage.

The admittance Y thus consists of a part G_o caused by the holes of Group 1, an unknown part Y_2 due to the holes of Group 2, and a part $j\omega C_T$ due to the capacitance C_T of the space-charge region.[9] Putting

$$Y = G + jB = G_0 + Y_2 + j\omega C_T \tag{29}$$

we have

$$Y_2 = (G - G_0) + j(B - \omega C_T). \tag{29a}$$

[8] One might object that it is not known in advance whether a hole will belong to Group 1 or Group 2. It is sufficient for our argument that it will *either* belong to Group 1 *or* to Group 2. It is also unnecessary to describe processes in which the hole under discussion crosses the space-charge region several times. For, if a hole enters (or re-enters) the p region, another hole will leave through the ohmic contact to maintain space-charge neutrality; the hole can then no longer be distinguished from the other holes in that region.

[9] The applied ac voltage changes the width of the space-charge region periodically with time and the charge stored in that region varies in the same rhythm; the region thus acts as a capacitance. Because the charge transfer follows the applied voltage practically instantaneously, this effect gives a contribution $j\omega C_T$ (with constant C_T) to the admittance Y for all frequencies of practical interest.

The holes of Group 2 thus give a contribution $(G-G_o)$ to the diode conductance G. Without going into a detailed calculation, we see that the high-frequency behavior of the admittance is due to the holes of Group 2. These holes return to the p region by diffusion, which is a thermal process; the noise caused by these holes should thus be thermal noise of the conductance $(G-G_o)$, so that Group 2 gives a contribution $4kT(G-G_o)\Delta f$ to $\overline{i^2}$. Adding this to (5) we obtain (16) after substituting (4). By making a Fourier analysis of the random, independent current pulses of the individual holes of Group 2, van der Ziel and Becking proved in a rigorous manner that the holes of Group 2 give indeed the contribution $4kT(G-G_o)\Delta f$ to $\overline{i^2}$.

This result holds for all geometrical configurations and its validity does not depend upon the mode of recombination of the injected carriers.

The condition that the current is carried by holes may now be dropped; since the current carriers give independent pulses, (16) remains valid if part of the current is carried by electrons.

In p-n-p transistors one has to split the holes into 5 groups (Fig. 3):

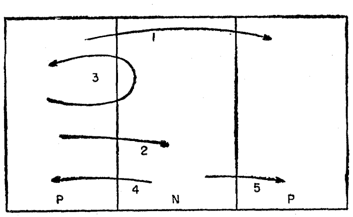

Fig. 3—The holes taking part in the conduction process in a junction transistor are divided into five groups.

1) Holes injected into the base region and collected by the collector.
2) Holes injected into the base region and recombining in that region with a free electron.
3) Holes injected into the base region and returning to the emitter.
4) Holes generated in the base region and collected by the emitter.
5) Holes generated in the base region and collected by the collector.

The validity of (17) and (18) already follows from the general validity of (16). A similar proof may also be given by making a careful analysis of the contributions of the holes of groups 1) to 5) to $\overline{i_1^2}$ and $\overline{i_2^2}$. A similar analysis also shows that the equations remain true if part of the current is carried by electrons.

Finally (19) has to be proved. We observe that only the holes of group 1) contribute to both the emitter current I_e and the collector current I_c and that no electrons contribute to both I_e and I_c. Hence only the holes of group 1) contribute to $\overline{i_1^*i_2}$ and to the signal transfer admittance Y_{ce}.

If a small ac voltage v_e is applied to the emitter, then the ac current in the short-circuited collector is $Y_{ce}v_e$ and the ac emitter current is $Y_e v_e$. The part $\alpha_o G_{eo} v_e$ of this emitter current comes from the holes of group 1); since the rate of diffusion of the holes of group 1) follows the ac emitter voltage practically instantaneously, this contribution to $Y_e v_e$ is the same for all frequencies of practical interest. At low frequencies the ac collector current follows the emitter voltage practically instantaneously, so that $Y_{ce}=G_{ceo}=\alpha_o G_{eo}$ as mentioned before. If all holes of group 1) had the same diffusion time τ through the base region, one would have:

$$Y_{ce} = G_{ceo}e^{-j\omega\tau}. \tag{30}$$

Because the diffusion of holes through the base region is a random process, there will be a distribution $h(\tau)d\tau$ in diffusion times and, as a consequence

$$Y_{ce} = \int_0^\infty G_{ceo}e^{-j\omega\tau}h(\tau)d\tau. \tag{30a}$$

This explains the decrease in $|Y_{ce}|$ at high frequencies.

Now turn to the cross-correlation $\overline{i_1^*i_2}$. Let i_{11} and i_{21} be the contributions of the holes of group 1) to i_1 and i_2, then $\overline{i_1^*i_2}=\overline{i_{11}^*i_{21}}$. Since individual current pulses are independent and since all holes of group 1) will ultimately pass *both* the emitter junction *and* the collector junction we have in analogy with (13)

$$\overline{i_{11}^2} = \overline{i_{21}^2} = 2e\alpha_0(I_e + I_{ee})\Delta f = 2kTG_{ceo}\Delta f. \tag{31}$$

If all holes had the same diffusion time through the base region, one would thus expect

$$\overline{i_1^*i_2} = \overline{i_{11}^*i_{21}} = \overline{i_{11}^2}e^{-j\omega\tau} = 2kTG_{ceo}\Delta f e^{-j\omega\tau}. \tag{32}$$

Introducing again the distribution $h(\tau)d\tau$ in diffusion times τ we thus obtain, by substituting (30a)

$$\overline{i_1^*i_2} = \int_0^\infty 2kTG_{ceo}\Delta f e^{-j\omega\tau}h(\tau)d\tau = 2kTY_{ce}\Delta f \tag{32a}$$

which is identical with (19).

F. *Validity of the Theory*

The above general proof of the validity of (16) through (19) is based upon the following (implicit and explicit) assumptions:

Assumption 1: It was explicitly assumed that the individual current pulses were independent and occurred at random. At high injection levels the injected carriers give rise to an appreciable space charge; this means that the above assumption is not satisfied under that condition, because space charge implies interaction between individual carriers. A violation of this assumption does not necessarily lead to a large deviation from (16) through (19).

Assumption 2: It was explicitly assumed that a single current pulse displaced a charge $\pm e$ in the external circuit. This cannot be correct if the space-charge region(s) of the junction(s) have an appreciable trap density. At low injection levels these traps are only partly filled so that part of the carriers diffusing through the space-charge region will get trapped there. Those that get trapped will not displace the full charge $\pm e$, so that the average charge displaced per pulse is less than this amount and the low-frequency noise is less than full shot noise. At higher injection levels practically all the traps are permanently filled and the noise equals full shot noise. Champlin has detected this effect in silicon diodes [7].

Assumption 3: The series resistance of the junctions, partly caused by the finite conductivity of the bulk material and partly due to the contact resistance of the junctions, was neglected. This effect is usually taken into account by introducing these resistances into the equivalent circuit and ascribing full noise to them [Fig. 1(a)–1(d)]. Moreover, these series resistances are strongly current dependent and this gives rise to interesting modulation effects that were discovered by Fonger [13]. The influence of these effects upon flicker noise is discussed in Section III, and upon shot noise in Section V.

As far as shot noise is concerned, we shall first neglect modulation effects altogether for two reasons. The first is that these effects have not been taken into account before, so that it is difficult to discuss earlier work when this effect is taken into account. Moreover, it will be shown that it is often warranted to neglect these effects for shot noise, because of peculiar coincidences.

To understand most of the earlier work, represent the shot noise by an emf e_e in series with the emitter and a current generator i in parallel with the collector junction (Fig. 4). As is easily seen,

$$i = i_2 - \alpha i_1 \qquad e_e = i_1 Z_e \tag{34}$$

which is the proper extension of (21a). The signal transfer properties of the transistor are now represented by the current generator αi_e, where $i_e = v_e/Z_e$ is the current flowing in the emitter junction. Furthermore, introduce the emitter impedance $Z_e = 1/Y_e$, the collector impedance $Z_c = 1/Y_c$, and the base impedance $Z_{b'b} = r_{b'b} + jX_{b'b}$ (to take into account that the base impedance may be complex at high frequencies); $r_{b'b}$ should show thermal noise. Fig. 4 is the high-frequency extension of Fig. 1(c).

Substituting (17) through (19), we have

$$\begin{aligned}\overline{i^2} = \overline{(i_2^* - \alpha^* i_1^*)(i_2 - \alpha i_1)} &= 2e(I_c - |\alpha|^2 I_e)\Delta f \\ &= 2e[(\alpha_0 - |\alpha|^2)I_e + I_{co}]\Delta f\end{aligned} \tag{35}$$

and, if $I_e \gg I_{ee}$, so that $G_{eo} \simeq eI_e/kT$:

$$\overline{e_e^2} = \overline{i_1^2}\,|Z_e|^2 = 2kT(2G_e - G_{eo})\Delta f\,|Z_e|^2 \tag{36}$$

which are the high-frequency extensions of (21). Finally, if $I_e \gg I_{ee}$, we also have

$$\overline{e_e^* i} = 2kT\alpha(G_{eo} - Y_e^*)\Delta f. \tag{37}$$

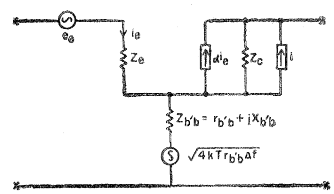

Fig. 4—Extension of Montgomery, Clark, and van der Ziel's circuit to higher frequencies.

The correlation is thus practically zero at lower frequencies but may have an appreciable value at higher frequencies.

III. Flicker Noise in Diodes and Transistors

We now turn to the problem of flicker noise. Its causes are not yet fully understood, though several investigators have worked on the problem. Fonger has discovered several noise sources and has found their proper place in the equivalent circuit. This allows a discussion of its effect in circuit applications. Here we follow Fonger's theory with some slight modifications.

A. General Characteristics of Flicker Noise [13]

According to Fonger there are two types of flicker noise, both with a low-frequency spectrum: surface noise and leakage noise.

We discuss surface noise first. It is now known that there are two types of energy levels at the surface of a semiconductor: "slow" states and "fast" states; the first act mainly as traps for the majority carriers and the latter as recombination centers for minority carriers [33]–[35]. The fluctuating occupancy of the slow states modulates the conductivity; this is the cause of flicker noise in bulk material. In addition, it modulates the capture cross section of the recombination centers; this is the cause of surface noise in diodes and transistors. The fluctuating current of minority carriers disappearing at the surface causes a fluctuating current to flow through the junction (or junctions) and modulates the series resistance of the junction (or junctions).

Leakage is caused by a thin conducting film bypassing the junction; it occurs at the perimeter of the junction and gives rise to a dc leakage current I_L and a leakage conductance g_L, which increase strongly with increasing bias. Spontaneous fluctuations in g_L cause leakage noise.

Surface noise is very sensitive to the ambient atmosphere; it is, *e.g.*, quite large in a humid atmosphere. It may be considerably reduced by proper surface treatment, it increases strongly with increasing current, and it is most prominent for junctions biased in the forward direction. Leakage noise is also very sensitive to the ambient atmosphere. Proper heat treatment can reduce it to such an extent that it becomes negligible for bias voltages less than a few volts; for that reason leakage noise is usually negligible for forward bias but may become quite important for large back bias.

Other studies, especially on diodes biased in the back direction, were carried out by Kennedy [31] and Mc-

Whorter [35], [38]; their results agree in general with Fonger's. McWhorter distinguishes between flicker noise and channel noise. A "channel" is a surface layer having a conductivity opposite to that of the bulk material. Channels formed in reverse bias diodes cause excess back current and a considerable increase in noise. An increased channel length increases the effective junction area (and hence the back current) and the length of its perimeter (and hence the leakage current); the back current increases linearly with the channel length due to both effects. Channel effects can also be diminished by proper surface treatments.

B. Flicker Noise in Diodes

The influence of modulation effects on the series resistance is discussed first. Consider a junction diode carrying a dc current I; let R be the dc series resistance of the junction. Because of the current dependence of R, the ac series impedance Z_s of the junction differs from the dc resistance R, since the ac current flowing through R will modulate R. We may thus split Z_s into a dc part R and a modulation part Z_{mb}. At low frequencies $Z_{mb}=R_{mb}$ is real and negative and

$$R_{mb} = I\frac{\partial R}{\partial I}; \qquad Z_s = r = R + I\frac{\partial R}{\partial I}, \tag{38}$$

where r is the ac resistance of the junction. R_{mb} is negative, since R decreases with increasing current; hence r is smaller than R. At high frequencies Z_{mb} (and hence Z_s) becomes complex; there are strong indications that Z_{mb} becomes inductive at high frequencies. At the frequencies where flicker noise is important, (38) may be used.

Now turn to the surface noise. Since the series resistance of the junction is strongly current dependent, fluctuations in the rate of generation and recombination of hole electron pairs at the surface will randomly modulate this resistance; because of the flow of dc current this modulation will show up as noise. Let this noise be described by a current generator i_s across the junction. In addition, fluctuations in the surface recombination rate will modulate the dc resistance in two ways:[10]

1) Directly. This is described by a noise emf e_b in series with R; e_b should be partly correlated with i_s.
2) Indirectly, through the current generator i_s. Fonger describes this by an additional emf i_sR_{mb} in series with R_{mb}. An equivalent representation connects the current generator i_s across both the junction impedance R_0 and the modulation impedance R_{mb}, as shown in Fig. 5(a); this demonstrates more clearly how the current generator i_s modulates the dc resistance R.

[10] This splitting of the modulation effect is, of course, somewhat arbitrary. It means that we try to take the modulation effect into account by relocating the current generator i_s. That part of the modulation effect that is not taken into account by this procedure is incorporated into the emf e_b. The sole justification of this noise schematic is that e_b happens to be very small for flicker noise.

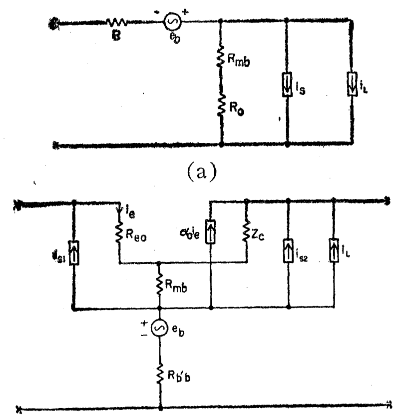

Fig. 5—Equivalent circuits for flicker noise. (a) Equivalent flicker noise circuit for a junction diode. (b) Equivalent flicker noise circuit of a transistor.

Since R_{mb} is negative at the frequencies of interest for flicker noise, the influence of the current generator i_s will be zero if $(R_o+R_{mb})=0$. Fonger found indeed that the low-frequency noise of a diode went through a deep minimum at a certain current I, in agreement with the above prediction; his results also indicated that the effect of the noise emf e_b was quite small and usually negligible. To enhance the modulation effect, Fonger used transistors with a large base resistance in diode connection (emitter and collector in parallel); then the effect became easily observable. For normal diodes, the effect is much less pronounced and may only be noticeable at very large currents [2].

The leakage noise can be described by a current generator i_L. It is not immediately clear where this current generator should be located. If the leakage noise modulates the dc resistance R as effectively as the surface noise, i_L should be connected across both R_o and the modulation resistance R_{mb}; if the leakage noise does not modulate the dc resistance R appreciably, the current generator i_L should be connected across R_o only. Fonger has tried to discriminate between these two possibilities by measuring leakage noise in transistors (see the following section) but his experiments were inconclusive. This means that it makes little difference either way which one of the two possibilities is chosen in practice. Fonger has chosen the second possibility and his approach is followed here. He finds that $\overline{i_L^2}$ is proportional to the square of the dc leakage current I_L; since I_L increases strongly with increasing back bias, the leakage noise can be considerably reduced by bringing the back bias closer to zero.

Fig. 5(a) gives the full equivalent circuit of the diode. As was said before, surface noise predominates strongly over leakage noise for forward bias; the current generator i_L may then be neglected. The leakage noise is only observable for diodes biased in the back direction. In that condition i_s is quite small (because the dc current is so small) and i_L usually predominates over i_s. The junction impedance for back bias is so high that resistance modulation effects may be neglected. In that case the modulation resistance R_{mb} may be eliminated from the equivalent circuit, which completely removes the uncertainty in the location of the current generator i_L.

C. Flicker Noise in Transistors

The influence of modulation effects on the series resistances of the junctions is discussed first. Here the base resistance is strongly current dependent. Because of the resistance modulation, the ac base impedance $Z_{b'b}$ may be split into a dc part $R_{b'b}$ and a modulation part Z_{mb}. At low frequencies $Z_{mb}=R_{mb}$ is real and

$$Z_{mb} = R_{mb} = I_b \frac{\partial R_{b'b}}{\partial I_b};$$

$$Z_{b'b} = r_{b'b} = R_{b'b} + I_b \frac{\partial R_{b'b}}{\partial I_b}, \tag{39}$$

where $r_{b'b}$ is the ac base resistance. Since $R_{b'b}$ decreases with increasing I_b, $r_{b'b}<R_{b'b}$. At high frequencies Z_{mb}, and hence $Z_{b'b}$ becomes complex; in analogy with the diode case one would expect Z_{mb} to become inductive. For frequencies at which flicker noise is important, however, one may safely assume that $Z_{mb}=R_{mb}$.

We now turn to the noise and discuss first the surface noise component. The minority carriers disappearing at the surface give rise to two current generators i_{s1} and i_{s2} connected across the emitter and the collector junction. This noise source also modulates the dc base resistance $R_{b'b}$ in two ways: 1) directly, as described by the noise emf e_b; 2) indirectly, by means of the current generators i_{s1} and i_{s2}. Fonger describes this with the help of two noise emf's $i_{s1}R_{mb}$ and $i_{s2}R_{mb}$, but it is better represented by connecting i_{s1} across R_{eo} and R_{mb} and the current generator i_{s2} across Z_c and R_{mb}. The noise emf e_b is, of course, correlated with i_{s1} and i_{s2}; its influence is usually so small that it can be neglected.

The leakage noise can be described by a current generator i_L connected across the collector. If base resistance modulation is important for this noise source, i_L should be connected across Z_c and R_{mb}; if it is unimportant, i_L should be connected across Z_c only. We follow Fonger who has chosen the first possibility. The best transistors show negligible leakage noise for $|V_c|<10$ volts; to avoid leakage noise in poorer units, it is recommended that $|V_c|$ be kept considerably smaller.

The full equivalent circuit for flicker noise in transistors thus is as shown in Fig. 5(b). In this equivalent circuit the current generator $\alpha_o i_e$ is also connected across Z_c and R_{mb} and not across Z_c only, since this current generator should also modulate the base resistance; one would expect the current generators i_{s2}, i_L and $\alpha_o i_e$ to be connected in the same manner.

Now represent the flicker noise by an emf e_c in series with the emitter and a current generator i in parallel with the collector impedance Z_c. This corresponds to the equivalent circuit of Fig. 4, but with different values for i and e_e:

$$i = i_{s2} + \alpha_0 i_{s1} + i_L; \quad e_e = - i_{s1}(R_{e0} + R_{mb}) + e_b. \tag{40}$$

If e_b and i_L are negligible, i and e_e will be practically fully correlated. We return to this later.

In one respect there is a considerable difference between shot noise and flicker noise. In shot noise, even though the current generators i_1 and i_2 are strongly correlated, e_e and i are nearly uncorrelated, but in flicker noise e_e and i are strongly correlated. This difference comes about because i_1 and i_2 on the one hand, and i_{s1} and i_{s2} on the other hand, have an opposite phase relationship. One may also put it as follows. Shot noise mainly *circulates* through the transistor and the current generator i is the *difference* between i_2 and αi_1; flicker noise generated at the surface flows *from* the base surface towards both junctions and the current generator i is the *sum* of i_{s2} and $\alpha_o i_{s1}$.

IV. Experimental Verification of the Theory and Circuit Applications

Here the experimental data on diode noise and transistor noise are reviewed and the theory is applied to transistor circuits.

A. Semiconductor Diodes [2], [7], [8], [17], [18], [30], [35a], [43], [49]

We have already dealt with the work on low-frequency noise in some detail; now shot noise and application of the theory to photodiodes is discussed.

For shot noise we define the noise ratio n of the junction conductance as

$$\overline{i^2} = n \cdot 4kTG\Delta f. \tag{41}$$

Let $G_{oo}=eI_o/kT$ be the low-frequency junction conductance for zero bias and $G_o=e(I+I_o)/kT$, the low-frequency conductance of the biased diode. Substituting (16) into (41), we obtain

$$n = 1 - \frac{G_o}{2G} + \frac{G_{oo}}{2G}. \tag{41a}$$

For forward bias ($G_o>G_{oo}$) the junction conductance has a noise ratio n varying between $\frac{1}{2}$ and 1, going to unity for very high frequencies, since $G>G_o$ in that case. Assuming full thermal noise of the ac series resistance r thus gives that the noise ratio of the whole device is between $\frac{1}{2}$ and 1; this agrees roughly with Anderson and van der Ziel's early data [2].

Probably the most accurate measurements were made by Champlin who used an ac bridge circuit with the junction in one arm, a variable RC network in the other arm, and noise diodes connected across the junction and across the RC network [7], [8]. First, the RC network was adjusted so that the bridge circuit was balanced at the frequency at which the noise measurement was to be performed and then the saturated current of one of the noise diodes was adjusted so that the two arms of the bridge circuit gave equal amounts of noise power.

In most cases he observed full thermal noise of the ac series resistance plus full shot noise of the junction. The frequency dependence of the junction noise agreed well with (41a). In some silicon junction diodes at low currents the noise was less than full shot noise; the cause of that result was mentioned previously (see Section II-F).

In *p-n* junctions used as photoelectric cells the junction is biased in the back direction. The light then generates hole-electron pairs; the holes are collected by the *p* region and the electrons by the *n* region. Full shot noise should be associated with this photocurrent; in addition, some low-frequency noise will be generated, but this may not be very large if the surface is properly treated. Shot noise in semiconductor photoelectric cells has been observed by Slocum and Shive [49] and by Pearson, Montgomery, and Feldmann [43]. The latter group found shot noise down to 80 cycles in a dry atmosphere. In a humid atmosphere the noise was low-frequency noise and the noise power at 100 cycles was a factor 3×10^5 above shot noise. This shows the importance of the ambient atmosphere.

In *p-n* junction photocells operated under open-circuited condition the photovoltage biases the junction in forward direction to such an extent that the forward current exactly balances the photocurrent I. Gianola [16] investigated silicon photovoltaic cells and found low-frequency noise at low frequencies; the mean square value $\overline{e^2}$ of the open-circuit noise voltage was proportional to the photocurrent I for small values of I and inversely proportional to I for large values of I. His result is understandable if for the flicker noise the current generator i (i_s or i_L, probably i_s) has a mean square value $\overline{i^2}$ that is proportional to I. If R_o is the internal resistance of the cell, then $\overline{e^2}=\overline{i^2}R_o^2$. For small I the resistance R_o is independent of I and hence $\overline{e^2}$ is proportional to I in that case; for larger values of I the resistance R_o varies as $1/I$ (since $R_o \simeq kT/eI$) and hence $\overline{e^2}$ varies as $1/I$. It is shown in the next section that the proportionality of $\overline{i^2}$ to the current I occurs more often.

Hyde [29] has found spectra of the form const $/(1+\omega^2\tau^2)$ in point contact diodes under reverse bias conditions. Such spectra are probably due to traps; they might also be expected for some types of junction diodes.

An interesting noise phenomenon associated with avalanche breakdown is found in silicon *p-n* junctions biased in the back direction. The breakdown seems to occur at very tiny discharge spots (microplasmas) that emit light. The noise is generated as pulses, many millivolts high, occurring at random and at a rate that depends very strongly upon the current [9], [10], [36], [37], [45].

B. Shot Noise in Transistors

The validity of the shot noise theory may be tested by verifying the equivalent circuit of Fig. 4, that is, by determining $\overline{i^2}$, $\overline{e_e^2}$, and the cross correlation $\overline{e_e^* i}$. Extensive tests have been carried out by Nielsen, by Guggenbuehl and Strutt, and by Hanson and van der Ziel and others [6], [19]–[21], [23]–[28], [41], [50].

One way consists in expressing the noise figure F in terms of $\overline{i_1^2}$, $\overline{i_2^2}$, and $\overline{i_1^* i_2}$. This was done by Guggenbuehl and Strutt [23], who found

$$F = \frac{e}{2kTR_s}\left[\frac{I_c}{|\alpha|^2}\,|Z_s + r_{b'b} + Z_e|^2 - I_e\,|Z_s + r_{b'b}|^2\right]. \quad (42)$$

The merit of this equation is that it expresses the noise in terms of the macroscopic parameters of the transistor. However, if one finds deviations between theory and experiment, detection of the source of the discrepancy is not so easy. In that case it is better to introduce with Hanson and van der Ziel [28] the noise conductance g_{s1}, the noise resistance R_{s1}, and the correlation impedance $Z_{sc}=(R_{sc}+jX_{sc})$ as follows. First e_e is split into a part e_e' that is uncorrelated with i and a part e_e' that is fully correlated with i. One then defines

$$\frac{\overline{i^2}}{|\alpha|^2} = 4kTg_{s1}\Delta f; \qquad \overline{e_e''^2} = 4kTR_{s1}\Delta f;$$
$$Z_{sc} = \frac{\alpha e_e'}{i} = \frac{\overline{\alpha e_e i^*}}{\overline{i^2}}, \quad (43)$$

according to (34). Substituting (17) through (19) into (35) one obtains

$$g_{s1} = \frac{e}{2kT}\left[\frac{(\alpha_0 - |\alpha|^2)I_e + I_{co}}{|\alpha|^2}\right]. \quad (43a)$$

At low frequencies, the noise resistance R_{s1} for $I_e \gg I_{ee}$ is

$$R_{s1} = \tfrac{1}{2}R_{eo}, \quad (43b)$$

where R_{eo} is the low-frequency emitter resistance.

Calculating the noise figure F and expressing it in terms of g_{s1}, R_{s1}, Z_{sc}, and the other transistor parameters, we obtain

$$F = 1 + \frac{(r_{b'b} + R_{s1})}{R_s} + \frac{g_{s1}}{R_s}\,|Z_s + Z_e + Z_{b'b} + Z_{sc}|^2, \quad (44)$$

where R_s is the resistive part of the source impedance Z_s.

Nielsen neglects the correlation between e_e and i, which amounts to putting $Z_{sc}=0$, and further assumes that $R_{s1} \simeq \frac{1}{2}R_{eo}$ at all frequencies. One then obtains, if $Z_{b'b}=r_{b'b}$,

$$F = 1 + \frac{(r_{b'b} + \frac{1}{2}R_{eo})}{R_s} + \frac{g_{s1}}{R_s}\,|Z_s + Z_e + r_{b'b}|^2. \quad (44a)$$

Nielsen found reasonable agreement between theory and experiment and showed how the theory could be used for the design of low-noise transistor circuits. This indicates that his approximations were warranted; we shall see later why that should be the case.

Expression (44a) shows that the frequency dependence of the noise figure comes mainly from the frequency dependence of g_{s1}. To understand this, we substitute $\alpha=\alpha_0/(1+jf/f_0)$; (43a) then becomes

$$g_{s1} = \frac{e}{2kT}\left[\frac{(\alpha_0 I_e + I_{co})(1+f^2/f_0^2) - \alpha_0^2 I_e}{\alpha_o^2}\right] \tag{45}$$

from which it follows that g_{s1} has increased by a factor 2 at the frequency $f=f_o\sqrt{1-\alpha_o}$. The noise figure of the transistor is thus reasonably constant for $f<f_o\sqrt{1-\alpha_o}$ and increases rapidly with increasing frequency for $f>f_o\sqrt{1-\alpha_o}$. For frequencies below the α-cutoff frequency f_o, the quantity g_{s1} increases with increasing frequency mainly because of the frequency dependence of the term $(\alpha_o-|\alpha|^2)$ in (43a); for frequencies above f_o, the quantity g_{s1} increases mainly because the factor $|\alpha|^2$ in the denominator of (43a) goes to zero. In order to obtain good noise figures at high frequencies, it is important to use transistors with a high α-cutoff frequency.

Guggenbuehl and Strutt's (42) takes the correlation between e_e and i into account. The source reactance X_s should now be chosen such that F is a minimum. In (44a) this is the case if $X_s=-X_e$; in (44) the expression

$$\frac{I_c}{|\alpha|^2}(X_s+X_e)^2 - I_e X_s^2 \tag{46}$$

should be made a minimum. Differentiation shows this to be the case if

$$X_s = -\frac{I_c}{(I_c - |\alpha|^2 I_e)} X_e. \tag{46a}$$

Substitution of (46a) into (42) shows that the correlation should have an appreciable effect on the noise figure at higher frequencies. This is discussed later.

Guggenbuehl and Strutt also found reasonable agreement between theory and experiment [23]. They noticed the strong increase in noise figure with increasing frequency and found, for constant source resistance R_s, that they had to go to very low values of I_e to attain minimum noise figures at high frequencies. At least a major part[11] of their results can be explained by the frequency dependence of g_{s1} and by the facts that g_{s1} decreases with decreasing I_e whereas R_e and R_{s1} increase with decreasing I_e. The minimum noise figure thus occurs at the value of I_e where any further decrease in F due to g_{s1} is offset by the increase in F due to R_e and R_{s1}.

We now turn to Hanson and van der Ziel's results [28]. Starting with (44) and considering F as a function of the source reactance X_s, F is a minimum if

$$X_s + X_e + X_{b'b} + X_{sc} = 0 \tag{47}$$

[11] Guggenbuehl and Strutt maintain that the frequency dependence of g_{s1} cannot *fully* explain the observations at high injection levels. According to Section II-F, deviations between theory and experiment are not impossible at high injection levels, but no quantitative theory exists at present.

in which case

$$F = 1 + \frac{(r_{b'b}+R_{s1})}{R_s} + \frac{g_{s1}}{R_s}(R_s+R_e+r_{b'b}+R_{sc})^2$$
$$= A + \frac{B}{R_s} + CR_s. \tag{48}$$

The quantities A, B, and C may be determined experimentally by measuring F as a function of R_s; calculating from (48) we also have

$$A = 1 + 2g_{s1}(R_e + r_{b'b} + R_{sc});$$
$$B = r_{b'b} + R_{s1} + g_{s1}(R_e + r_{b'b} + R_{sc})^2; \quad C = g_{s1}. \tag{48a}$$

Having deduced the values of A, B, and C from the measurements, we may express the following quantities in terms of A, B, and C:

$$(R_e + r_{b'b} + R_{sc}) = \frac{(A-1)}{2C}$$
$$(r_{b'b} + R_{s1}) = \frac{4BC-(A-1)^2}{4C}$$
$$g_{s1} = C. \tag{49}$$

Unfortunately, it often happens that $(A-1)$ is only small and/or that $4BC$ and $(A-1)^2$ differ relatively little. In that case the quantities $(R_e+r_{b'b}+R_{sc})$ and $(r_{b'b}+R_{s1})$ are only inaccurately known and hence the values of R_{s1} and R_{sc} can only be determined inaccurately too.

Theoretically, [6] R_{sc} is zero at low frequencies and passes through a maximum for higher frequencies. The quantity X_{sc} should have a rather broad maximum around $f=f_o\sqrt{1-\alpha_o}$ and should have an appreciable value at those frequencies, so that tuning for minimum noise figure should give a marked noise figure improvement. Finally, R_{s1} should be of the order of $\frac{1}{2}R_e$ at low frequencies and should decrease rapidly with increasing frequency.

Hanson and van der Ziel [28] found that the experimental values of g_{s1} agreed very well with the theoretical expectations. In most transistors the experimental values of R_{s1} and R_{sc} agreed with the theoretical values within the (rather large) limits of experimental error. In some transistors with a low value of α_o the value of R_{sc} differed markedly from zero; this effect might possibly be attributed to base modulation (Section V). Moreover, they found that only little improvement in noise figure could be obtained by properly adjusting the source reactance X_s; this probably indicates that the experimental value of X_{sc} was considerably smaller than the expected theoretical value. This might either be caused by the reactive component of the base impedance $Z_{b'b}$ or by base modulation effects (Section V).

Now we may also understand why Nielsen could ignore the correlation effect. If X_s is adjusted for minimum noise figure, then the quantity R_{sc} in A and B may be omitted if R_{sc} is small in comparison with $(R_e+r_{b'b})$.

This is especially true at low frequencies, where the terms $2g_{s1}(R_e+r_{b'b}+R_{sc})$ and $g_{s1}(R_e+r_{b'b}+R_{sc})^2$ are often quite small; even at higher frequencies, where these two terms are larger, the omission of R_{sc} from the equations may not cause too large an error. Moreover, if $R_{s1}<r_{b'b}$, it does not make much difference whether one puts $R_{s1}=\frac{1}{2}R_{eo}$ or a smaller value. For good information on R_{sc} and R_{s1} we thus need accurate measurements of A, B, and C.

These can be achieved as follows. The quantities B and C may be determined accurately by a direct method. Since the total noise resistance R_n of the whole circuit is equal to FR_s, the quantity B corresponds to the noise resistance R_{no} for zero source impedance

$$B = R_{no} = r_{b'b} + R_{s1} + g_{s1}(R_e + r_{b'b} + R_{sc})^2. \tag{50}$$

The quantity C is related to the equivalent input saturated diode current I_n for large source impedance R_s. If we define the equivalent input saturated diode current I_n by an equivalent current generator $\sqrt{\overline{i_n^2}}=\sqrt{2eI_n\Delta f}$ in parallel to R_s, then obviously

$$\overline{i_n^2} = 2eI_n\Delta f = F\cdot 4kT\Delta f/R_s;$$

or

$$I_n = \frac{2kT}{e}\left[\frac{A}{R_s} + \frac{B}{R_s^2} + C\right]. \tag{51}$$

For large values of R_s, we have $I_n=I_{n\infty}$ independent of R_s, or

$$I_{n\infty} = \frac{2kT}{e}C, \quad \text{or} \quad C = \frac{e}{2kT}I_{n\infty}. \tag{51a}$$

The quantity $(A-1)$ may finally be determined from the minimum noise figure $F_{\min}$. According to (48) the minimum noise figure $F_{\min}$ is attained if $R_s=\sqrt{B/C}$, in which case

$$F=F_{\min}=A+2\sqrt{BC}; \text{or } (A-1)=(F_{\min}-1-2\sqrt{BC}). \tag{52}$$

The accuracy with which $(A-1)$ can be determined, depends upon the difference between $F_{\min}$ and $(1+2\sqrt{BC})$.

As mentioned before, the measurement of the noise conductance g_{s1} allows the determination of whether (35) is correct. This equation can also be checked very accurately by inserting a large impedance in series with the emitter (input open) and determining the equivalent output saturated diode current I_{eq} of this circuit. To do so, one connects a noise diode in parallel to the output and determines the diode current for which the output noise power is doubled. According to Fig. 4 we have

$$\overline{i^2}|Z_c|^2 + 4kTr_{b'b}\Delta f = 2eI_{\text{eq}}\Delta f|Z_c + r_{b'b}|^2. \tag{53}$$

Unless the frequency is very high, it may be assumed that $|Z_c|$ is large in comparison with $r_{b'b}$. Retaining only the terms in Z_c^2 in that case, we have

$$\overline{i^2} = 2eI_{\text{eq}}\Delta f \tag{54}$$

or, substituting (35)

$$I_{\text{eq}} = (\alpha_0 - |\alpha|^2)I_e + I_{co}. \tag{55}$$

If I_{co} is very small, then this equation tells us that I_{eq} is equal to $(1-\alpha_o)I_c$ for small frequencies, practically twice as large for $f=f_o\sqrt{1-\alpha_o}$, equal to $\frac{1}{2}I_c$ at the α-cutoff frequency f_o, and equal to I_c above the cutoff frequency. If I_c is quite small, I_{eq} should be equal to I_{co} at all frequencies. All these predictions were well verified by Hanson and van der Ziel's measurements; their results indicated that the measurement of I_{eq} as a function of frequency might be used to determine the α-cutoff frequency f_o [28].

We note that the quantities I_{eq} and $I_{n\infty}$ are closely related, since[12]

$$g_{s1} = C = \frac{e}{2kT}I_{n\infty} = \frac{e}{2kT}\frac{I_{\text{eq}}}{|\alpha|^2},$$

we have

$$I_{\text{eq}} = I_{n\infty}|\alpha|^2. \tag{56}$$

Hanson and van der Ziel found good agreement between the experimental values of g_{s1} and I_{eq} at relatively low frequencies.

At high frequencies, Hanson and van der Ziel sometimes found values of I_{eq} that differed from these predictions. Some transistors, for example, had $I_{\text{eq}}<I_c$ for large currents and $I_{\text{eq}}>I_c$ for small current at frequencies at which one would expect $\overline{i^2}=2eI_c\Delta f$. This could be attributed to the fact that $|Z_c|$ was no longer large in comparison with $r_{b'b}$; calculating $\overline{i^2}$ from (53), using the observed value of I_{eq}, it was found that $\overline{i^2}\simeq 2eI_c\Delta f$ even in this case. This apparent deviation between theory and experiment is thus caused by the fact that the output terminals are not connected directly to Z_c but are connected through the base resistance $r_{b'b}$ that also has noise associated with it.

In the case discussed by Guggenbuehl and Strutt [23] it was important to go to low emitter current. Hanson [27] has reported a few cases in which the capacitive feedback between emitter and collector gave a considerable increase in the apparent current amplification factor of the device.[13] This resulted in a considerable decrease in the noise figure that could be greatly reduced by going to *larger* emitter currents.

A similar condition occurs in drift transistors. Here the α-cutoff frequency increases with increasing emitter current; for high-frequency applications of drift transistors, the emitter current therefore should not be chosen too small.

[12] This means that the simultaneous measurement of I_n and I_{eq} might be used to determine the value of $|\alpha|^2$ under the exact operating conditions of the circuit; this sometimes may be useful.

[13] It is thus important to screen input and output of transistor amplifiers at high frequencies.

C. Flicker Noise in Transistors [1], [3], [5], [13], [39], [58], [61]

As was already mentioned in Section III, one has to discriminate between surface noise and leakage noise. Both have a low-frequency spectrum. We saw that leakage noise was most easily reduced by bringing the collector bias closer to zero; by giving the device the proper treatment, the effect can be reduced still further. Surface noise could also be reduced by proper surface treatment.

Another interesting feature of surface noise is the difference between *p-n-p* and *n-p-n* transistors; the latter show considerably more flicker noise than the former [5], [6a]. This reflects differences in the physical characteristics of the surface layer of the base regions of the two types of transistors. The reduction of surface noise in *p-n-p* transistors has apparently proceeded farther than in *n-p-n* transistors.

Another interesting point is that the current dependence of the surface noise resembles the current dependence of shot noise. Measuring the equivalent output saturated diode current I_{eq} with open input, one has, for frequencies where shot noise predominates,

$$I_{\text{eq}} = (I_{\text{eq}})_s = 2I_e\alpha_0(1 - \alpha_0)\Delta f \tag{57}$$

if the collector saturated diode current I_{co} is negligible. This dependence is especially characteristic for tetrode transistors where α_o can be changed over a wide range by changing the current bias of the base region. Yajima [61] showed that a similar expression holds for these transistors at frequencies where flicker noise predominates

$$(I_{\text{eq}})_f = \frac{\text{const}}{f} I_e\alpha_0(1 - \alpha_0)\Delta f. \tag{58}$$

He concluded from this result that flicker noise in junction transistors arises at least partly from the recombination process of injected carriers in the base and nearby surface. This agrees with Fonger's ideas.

For triode transistors for which (58) is also valid, $(I_{\text{eq}})_f$ should be proportional to I_e, since α_o is practically independent of I_e in that case. Many transistors indeed show such a relationship, but deviations do occur in some types.

In (40) the flicker noise was represented by an input emf e_e and an output current generator i. Neglecting leakage noise, which is allowed in good units, and ignoring the noise emf e_b, which is also allowed according to Fonger, gives that e_e and i are practically fully correlated. Splitting e_e into a part e_e'' that is uncorrelated with i and a part e_e' that is fully correlated with i, and representing the flicker noise by an equivalent emf in series with the input yields:

$$\begin{aligned} e_{nf} &= e_e + \frac{1}{\alpha_o}(Z_s + R_{eo} + r_{b'b}) \\ &= e_e'' + \frac{i}{\alpha_o}\left(Z_s + R_{eo} + r_{b'b} + \frac{\alpha_o i_e'}{i}\right). \end{aligned} \tag{59}$$

We now define the flicker noise resistance R_{nf} by the equation $\overline{e_{nf}^2} = 4kTR_{nf}\Delta f$; furthermore we introduce three constants, the emitter noise resistance R_{f1}, the flicker noise conductance g_{f1}, and the correlation resistance R_{fc} by

$$\overline{e_e''^2} = 4kTR_{f1}\Delta f; \quad \frac{\overline{i^2}}{\alpha_o^2} = 4kTg_{f1}\Delta f; \quad R_{fc} = \frac{\alpha_o e_e'}{i} \tag{60}$$

as a characterization of the flicker noise properties of the transistor; R_{f1} and g_{f1} should vary as $1/f$ and, in view of what was said above, g_{f1} is proportional to I_e. According to (40)

$$R_{fc} = -(R_{eo} + R_{mb})\frac{\alpha_o i_{s1}}{(i_{s2} + \alpha_0 i_{s1})}\cdot \tag{60a}$$

Substituting into (59), we have

$$R_{nf} = R_{f1} + g_{f1}\,|\,Z_s + R_{eo} + r_{b'b} + R_{fc}\,|^2. \tag{61}$$

For minimum flicker noise resistance, the source reactance X_s should be chosen such that

$$X_s^2 \ll (R_{eo} + r_{b'b} + R_{fc})^2. \tag{61a}$$

It is thus not sufficient that X_s is small in comparison with the input resistance of the transistor circuit, which may be quite large if feedback is applied; one has to satisfy (61a). If a source is thus capacitively coupled to the input of a low-noise (audio) transistor amplifier, the coupling capacitance should be chosen sufficiently large.

The emitter noise resistance R_{f1} is zero if e_e and i are fully correlated for flicker noise. If $\sqrt{R_{nf}}$ is then plotted as a function of R_s, one should obtain a straight line. Chenette [5] has carried out the experiment. Measuring R_{nf} as a function of the source resistance R_s, he found that the linear relationship was well satisfied; this indicates that e_e and i are indeed practically fully correlated and that $R_{f1} \simeq 0$. The straight line intercepts the zero axis at the point

$$\begin{aligned} R_s &= -(R_{eo} + r_{b'b} + R_{fc}) \\ &= -\left[R_{b'b} + (R_{eo} + R_{mb})\frac{i_{s2}}{(i_{s2} + \alpha_o i_{s1})}\right] \end{aligned} \tag{62}$$

since R_{fc} is given by (60a) and $r_{b'b} = (R_{b'b} + R_{mb})$ according to Section III. The values for R_s observed by Chenette roughly agree with the theoretical expectations.

D. Applications to Low-Noise Circuits [3], [12], [19], [32], [39], [47], [50], [58], [59]

Most of our circuit discussions held for a grounded base circuit. It is easily shown, however, that *a grounded base and a grounded emitter circuit have the same noise figure F if the source impedances used in the two circuits are identical* [23]. The grounded emitter circuit usually is recommended, since it allows a much simpler interstage coupling and a much higher gain per stage.

As mentioned before, the noise figure due to shot noise is reasonably flat for frequencies up to $f_o\sqrt{1-\alpha_o}$, where f_o is the α-cutoff frequency. It therefore is important to use transistors with high cutoff frequency in low-noise applications.

Let us first consider the low-frequency case and let us further assume that $R_{s1}=\frac{1}{2}R_{eo}$, $R_{sc}\simeq 0$, and $R_{f1}\simeq 0$. The total noise resistance of the circuit, according to (48) and (61), is then

$$R_n = R_s + r_{b'b} + \tfrac{1}{2}R_{eo} + g_{s1}(R_s + R_{eo} + r_{b'b})^2 + g_{f1}(R_s + R_{eo} + r_{b'b} + R_{fc})^2. \quad (63)$$

In these equations g_{f1} and g_{s1} usually depend linearly on I_e, whereas R_{eo} is inversely proportional to I_e.

First consider the case where flicker noise predominates. For minimum noise resistance at a given value of the source impedance R_s, the last term in (63) should then be made a minimum. For $R_s \gg (R_{eo}+r_{b'b}+R_{fc})$ this leads to the condition that g_{f1} must be as small as possible, which is the case for very small emitter currents. For $R_s \ll (R_{eo}+r_{b'b}+R_{fc})$ the quantity $g_{f1}(R_{eo}+r_{b'b}+R_{fc})^2$ should be made a minimum; experimentally it is found that this also leads to quite small emitter currents.

If shot noise predominates and R_s is given, one should make $[(R_s+r_{b'b}+\frac{1}{2}R_{eo})+g_{s1}(R_s+r_{b'b}+R_{eo})^2]$ a minimum. We observe that the first term predominates over the second one if $g_{s1}(R_s+r_{b'b}+R_{eo})<1$. For that reason R_n does not depend so strongly upon R_s as in the previous case, unless R_s is large. For very large R_s one should make g_{s1} as small as possible; this leads again to very small values of I_e. If $g_{s1}(R_s+r_{b'b}+R_{eo}) \ll 1$, one makes R_{eo} as small as possible; this leads to relatively large values of I_e. For low source impedances R_s the requirements for low flicker noise resistances and low shot noise resistances are thus opposite.

Low-noise *p-n-p* transistors, operating at emitter currents of about 0.5 ma and fed from a source with low source resistance $R_s(R_s<50$ ohms), may have noise resistances as low as 1000 ohms at 10 cycles. At higher frequencies, where shot noise predominates, the noise resistance may be as low as 100–150 ohms. This is considerably better than vacuum pentodes, which have noise resistances of about 10^5–10^6 ohms at 10 cycles and about 10^3 ohms at frequencies where shot noise predominates. Low-noise audio and subaudio amplifiers, operating from a source of low impedance, should thus use low-noise transistors instead of vacuum tubes.

For signal sources with very large source impedances R_s, the vacuum tube is much better. The reason is that the noise resistance R_{nt} of the tube is independent of the source impedance, whereas the noise resistance of the transistors varies as R_s^2 for large R_s. The total noise resistance of the vacuum tube circuit is thus (R_s+R_{nt}), whereas the total noise resistance of the transistor circuit is approximately $[R_s+(g_{s1}+g_{f1})R_s^2]$ for large R_s. Hence if:

$$R_s > \sqrt{\frac{R_{nt}}{g_{s1}+g_{f1}}}$$

the vacuum tube is better than a transistor. Nevertheless, in some applications it may happen that the transistor amplifier, though inferior to the vacuum tube amplifier, is still adequate for the purpose for which it is used.

Volkers and Pedersen [58] were the first to notice the low noise resistance of the transistor for low source impedances. The units with which they worked had considerable leakage noise; to eliminate this, they had to operate the transistor at nearly zero bias. They called this mode of operation the "hushed" operation. In modern transistors the leakage noise is much smaller so that the collector bias does not have to be chosen so close to zero; it is still important, however, not to make $|V_c|$ too large.

We now consider the case where the circuit can be adjusted for minimum noise figure. If flicker noise predominates strongly and if F_{shot} is the noise figure due to shot noise,

$$F = F_{\text{shot}} + \frac{R_{nf}}{R_s} = F_{\text{shot}} + g_{f1}\frac{(R_s + R_{eo} + r_{b'b} + R_{fc})^2}{R_s} \quad (64)$$

which has its minimum value if the last term is a minimum,

$$F_{\min} = F_{\text{shot}} + 4g_{f1}(R_{eo} + r_{b'b} + R_{fc})^2$$

for

$$(R_s)_{\min} = (R_{eo} + r_{b'b} + R_{fc}. \quad (64a)$$

In Section IV-C the plot showing R_{nf} as a function of R_s intercepted the zero axis at $R_s=-(R_s)_{\min}$. Considered as a function of I_e, the minimum noise figure is smallest if I_e is quite small.

For frequencies where shot noise predominates, the value of F was given by (48) and its minimum value $F_{\min}$ was shown in (52). Substituting the values of A, B, and C given by (48a), putting $R_{s1}=\frac{1}{2}R_{eo}$, and neglecting R_{sc} yields

$$F_{\min} = 1 + 2g_{s1}(R_{eo} + r_{b'b}) + 2\sqrt{g_{s1}(\tfrac{1}{2}R_{eo} + r_{b'b}) + g_{s1}^2(R_{eo} + r_{b'b})^2} \quad (65)$$

for

$$R_s = \frac{1}{g_s}\sqrt{g_{s1}(\tfrac{1}{2}R_{eo} + r_{b'b}) + g_{s1}^2\ (R_{eo} + r_{b'b})^2}. \quad (65a)$$

Depending on the relative magnitude, one thus has to minimize $g_{s1}(\frac{1}{2}R_{eo}+r_{b'b})$ or $g_{s1}(R_{eo}+r_{b'b})$; since these conditions do not differ so strongly, we shall here minimize $g_{s1}(\frac{1}{2}R_{eo}+r_{b'b})$. This is easily done, for g_{s1} and R_{eo} depend upon the emitter current T_e in an opposite manner; g_{s1} decreases with decreasing I_e, whereas $R_{eo}\simeq kT/eI_e$ increases. We have from (43a)

$$g_{s1}(\tfrac{1}{2}R_{eo} + r_b{'}_b) = \frac{1}{4}\left(\frac{1-\alpha_o}{\alpha_o}\right)\left[I_e + \frac{I_{co}}{\alpha_o(1-\alpha_o)}\right] \cdot \left[\frac{1}{I_e} + \frac{2er_{b'b}}{kT}\right]. \quad (66)$$

Assuming α_o to be independent of I_e, this has a minimum value.

$$\frac{1}{4}\left(\frac{1-\alpha_o}{\alpha_o}\right)\left[1+\sqrt{\frac{I_{co}}{\alpha_o(1-\alpha_o)}\cdot\frac{2er_{b'b}}{kT}}\right]^2 \tag{66a}$$

for

$$I_e = \sqrt{\frac{I_{co}}{\alpha_0(1-\alpha_0)}\cdot\frac{kT}{2er_{b'b}}}. \tag{66b}$$

Substituting into (65) yields

$$F_{\min} \simeq 1 + 2\sqrt{g_{s1}(\tfrac{1}{2}R_{eo}+r_{b'b})}$$

$$= 1 + \sqrt{\frac{1-\alpha_o}{\alpha_o}} + \sqrt{\frac{I_{co}}{\alpha_0^2}\cdot\frac{2er_{b'b}}{kT}}. \tag{67}$$

Taking, for example, $I_{co}\simeq 1$ μa, $\alpha_o=0.98$, and $r_{b'b}=100$ ohms, we have a minimum noise figure of about 1.25 at $I_e\simeq 80$ μa. This shows that rather low noise figures can be obtained by proper choice of the source impedance and the emitter current.[14]

The design conditions for low-noise transistors are that the quantities $(1-\alpha_o)$ and $I_{co}r_{b'b}$ should be made as small as possible and that $(2eI_{co}r_{b'b}/kT)<(1-\alpha_o)$.

Next we investigate the noise figure close to the cutoff frequency and determine how R_s and I_e should be chosen in order to make the noise figure F a minimum. The minimum noise figure is again given by (65), if we neglect the correlation resistance R_{sc} (which is probably allowed). Furthermore, we assume that $g_{s1}(R_{s1}+r_{b'b}) \ll g_{s1}^2(R_e+r_{b'b})^2$ and take into account that $R_e\simeq R_{eo}$ up to the cutoff frequency. In that case

$$F_{\min} \simeq 1 + 4g_{s1}(R_{eo}+r_{b'b}) \text{ for } R_s \simeq (R_{eo}+r_{b'b}). \tag{68}$$

We now minimize this expression as a function of the emitter current I_e. In a good transistor $\alpha_o\simeq 1$ so that $|\alpha|^2\simeq\frac{1}{2}$ at the cutoff frequency. One then has at that frequency

$$g_{s1} = \frac{e}{2kT}(I_e+2I_{co}); \qquad R_{eo}\simeq\frac{kT}{eI_e}$$

so that

$$F_{\min} \simeq 1 + 2(I_e+2I_{co})\left(\frac{1}{I_e}+\frac{er_{b'b}}{kT}\right) \tag{68a}$$

at that frequency. This has to be minimized as a function of I_e. The minimum value is:

$$F_{\min} \simeq 1 + 2\left(1+\sqrt{2I_{co}\cdot\frac{er_{b'b}}{kT}}\right)^2$$

for

$$I_e = \sqrt{2I_{co}\cdot\frac{kT}{er_{b'b}}}. \tag{69}$$

As in our previous example, take $I_{co}=1$ μa and $r_{b'b}=100$ ohms, then $F_{\min}\simeq 3.4$ at $I_e\simeq 20$ μa. The minimum noise figure is now obtained for a much smaller emitter current than in the previous case, in agreement with Guggenbuehl and Strutt's results [23]. Moreover, the example shows that the task of designing transistors that have a reasonable noise figure at the cutoff frequency is not a hopeless one, though it is impossible to have $F_{\min}<3$ in this case (corresponding to about 5 db).

The design condition for transistors with a low noise figure at the α-cutoff frequency is thus that $I_{co}r_{b'b}$ should be made small. The minimum obtainable noise figure at the cutoff frequency will not change very much if α_o depends upon I_e.

The condition which must be satisfied for $I_{co}r_{b'b}$ is actually less stringent than in the previous case. For we now have to require only that $(2eI_{co}r_{b'b}/kT) \ll 1$, whereas it had to be $<(1-\alpha_o)$ in the previous case. Viewed in that light, the requirement of a low noise figure at the cutoff frequency does not pose any additional restrictions upon the product $I_{co}r_{b'b}$.

If we look at the circuit from a noise resistance point of view, (63) has to be modified somewhat. If we now put $R_{s1}\simeq 0$ and $R_{sc}\simeq 0$, which may not be too far from the truth, we have for the cutoff frequency

$$R_n \simeq R_s + r_{b'b} + g_{s1}(R_{eo}+R_s+r_{b'b})^2$$

$$= (R_s+r_{b'b}) + \frac{1}{2R_{eo}}(R_{eo}+R_s+r_{b'b})^2 \tag{70}$$

since

$$R_e \simeq R_{eo} = \frac{kT}{eI_e} \quad\text{and}\quad g_{s1}\simeq\frac{eI_e}{2kT}=\frac{1}{2R_{eo}}. \tag{70a}$$

Considered as a function of R_{eo}, this has a minimum value

$$R_n \simeq 3(R_s+r_{b'b}) \text{ if } R_{eo}\simeq(R_s+r_{b'b}), \text{ or}$$

$$I_e = \frac{kT}{e(R_s+r_{b'b})}. \tag{70b}$$

Putting $R_s=0$ and $r_{b'b}=100$ ohms, this corresponds to $I_e=0.25$ ma and $R_n\simeq 300$ ohms. At low frequencies at the same current, g_{s1} is quite small, hence

$$R_n \simeq (R_s+r_{b'b}+\tfrac{1}{2}R_{eo}) = \tfrac{3}{2}(R_s+r_{b'b}) \tag{70c}$$

so that $R_n\simeq 150$ ohms if $R_s=0$. This shows that at the frequency f_o there is an optimum noise resistance for small source impedance R_s and that the noise resistance only increases by a factor 2 by going to the cutoff frequency. *The low-noise properties of a transistor operating from a low impedance source thus hold quite well for frequencies up to the cutoff frequency.*

[14] This calculation is incorrect if α_o depends on I_e. This is, for example, the case in silicon transistors, where α_o decreases strongly with decreasing I_e for small emitter currents. This probably will not change the design conditions very much.

At very large source impedances R_s, the noise resistance $R_n \simeq g_s R_s^2$; close to the cutoff frequency R_n thus increases rapidly with increasing frequency unless I_e is of the order of I_{co}, the collector saturated current. Whether or not this noise performance can be tolerated in practical cases depends upon the required value of the equivalent input saturated diode current I_n [see (56)].

We finally turn to the grounded collector [6], [41] circuit (Fig. 6). In order to calculate its noise figure, we use open-circuit output and obtain after some calculation, if the correlation impedance Z_{sc} is neglected,

$$F = 1 + \frac{r_{b'b} + R_{s1}}{R_s} + \frac{|\alpha|^2 g_{s1}}{R_s}(R_s + r_{b'b})^2. \qquad (71)$$

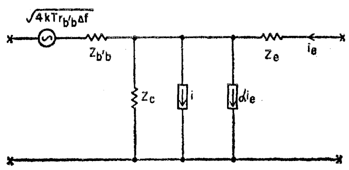

Fig. 6—Equivalent circuit of a transistor in grounded collector connection.

For low frequencies, $|\alpha|^2 \simeq 1$ and this is only slightly better than the other two circuits. For very high frequencies ($f > f_o$) the last term is smaller and the grounded collector circuit has a much lower noise figure than the other two circuits; unfortunately, this is not very useful, since the circuit gives a power gain less than unity at those frequencies.

To investigate how much improvement can be obtained at lower frequencies, we determine $F_{\min}$.

$$F_{\min} = 1 + 2|\alpha|^2 g_{s1} r_{b'b} + 2\sqrt{|\alpha|^2 g_{s1}(r_{b'b} + R_{s1}) + |\alpha|^4 g_{s1}^2 r_{b'b}^2}. \qquad (72)$$

If $|\alpha|^2 g_{s1} r_{b'b} \ll 1$, this may be written, if $R_{s1} = \frac{1}{2}R_{eo}$:

$$F_{\min} \simeq 1 + 2\sqrt{|\alpha|^2 g_{s1}(r_{b'b} + \tfrac{1}{2}R_{eo})}. \qquad (72a)$$

If $|\alpha| \simeq 1$, this is practically identical with the value

$$F_{\min} \simeq 1 + 2\sqrt{g_{s1}(r_{b'b} + \tfrac{1}{2}R_{eo})}$$

obtained for the other two circuits from (65) under corresponding conditions.

As far as the noise figure is concerned, the grounded collector circuit is thus practically identical with the other circuits. Since its power gain is much smaller, it should be checked carefully whether the noise of the following stage becomes important. In most cases the other two circuits will be preferred.

V. Modulation Noise Caused by Shot Effect

In view of Fonger's successful treatment of flicker noise, it seems logical to apply his approach also to the modulation noise caused by shot effect. This has not been done before; but, since the various noise sources cannot be expected to behave in different manners, modulation noise caused by shot effect should also exist.

Fonger assumed that *not the ac but the dc* series resistance of the junctions should show thermal noise. This means that the bulk material, quite apart from the fluctuations in the rate of injection, recombination, and escape of the carriers will also show noise because of the random motion of the carriers; this corresponds to thermal noise of the dc resistance. We thus follow Fonger's suggestion.

First turn to the junction diode. An ac current passing through the diode will change the rate of injection, recombination, and escape of the carriers; this is the cause of the modulation impedance Z_{mb}. Fluctuations in these rates will cause the shot noise described by the current generator i [Fig. 1(a)] connected in parallel to the junction; they will also modulate the dc series resistance R of the diode in two ways.[15]

1) Directly, as indicated by the noise emf e_b in series with R.
2) Indirectly, through the current generator i; this is taken into account (as in the flicker noise case) by connecting i across both the impedances Z and Z_{mb}.

The full equivalent circuit is thus as shown in Fig. 7(a). In the case of low-frequency noise Fonger found that the noise emf e_b had negligible influence; this does not necessarily have to be true for the shot noise case. Moreover, e_b and i should be partly correlated. The correlation does not necessarily have to be a complete one, since part of e_b may be caused by hole-electron pairs that never cross the junction; they only modulate the dc resistance, but do not contribute to i.

If e_b is neglected, the open-circuit noise emf is

$$e = i(Z + Z_{mb}) + \sqrt{4kTR_{b'b}\Delta f}. \qquad (73)$$

At low frequencies, $Z_{mb} = R_{mb}$ and $R_{mb} = I(\partial R/\partial I)$ is negative, whereas $Z = R_o = 1/G_o$; the shot noise term thus should be zero if $(R_o + R_{mb}) = 0$. This effect has not yet been detected for shot noise.

If we now introduce the noise resistance R_n by equating $\overline{e^2} = 4kTR_n\Delta f$ and bear in mind that $\overline{i^2} = 2kT\Delta f/R_o$ for $I \gg I_o$ at relatively low frequencies, we obtain

$$R_n = R + \frac{1}{2}\frac{(R_0 + R_{mb})^2}{R_o} = \frac{1}{2}R_o + r + \frac{R_{mb}^2}{2R_o}. \qquad (73a)$$

In most diodes the last term is negligible in comparison with the other two. The total noise power is then practically equal to the full shot noise power of the junction plus full thermal noise of the ac resistance; this agrees with Champlin's experimental data [7], [8]. Deviations should be expected for diodes with a large

[15] For the meaning of these two modes of modulation see footnote 10.

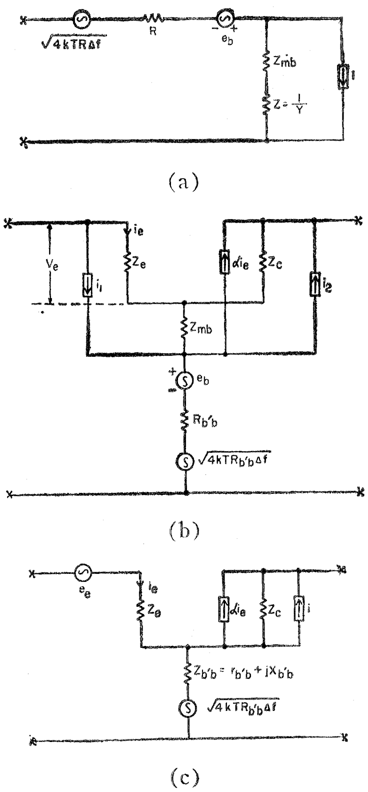

Fig. 7—Modulation noise caused by shot effect. (a) Equivalent circuit of a junction diode. (b) Equivalent circuit of a transistor. (c) Modification of the equivalent circuit of Fig. 4.

dc resistance R operating at large currents (small R_o). Attempts are being made at the University of Minnesota to detect such deviations.

The same ideas can be directly applied to transistors and lead to the equivalent circuit of Fig. 7(b). Here the current generator i is connected in parallel to Z_e and Z_{mb}, and the current generator i_2 is connected in parallel to Z_c and Z_{mb}. The noise emf e_b in series with the dc base resistance $R_{b'b}$ should be partly correlated with i_1 and i_2; it is probably not always warranted to ignore e_b (contrary to the flicker noise case).

As far as the noise figure is concerned,[16] we may replace the circuit of Fig. 7(b) by the one of Fig. 7(c) with

$$i = (i_2 - \alpha i_1); \quad e_e = i_1(Z_e + Z_{mb}) - e_b. \tag{74}$$

This circuit is nearly equivalent to the one shown in Fig. 4. The only differences are that the dc base resistance now shows thermal noise and that e_e has a different value. The current generator i apparently is not affected by the base modulation. The circuit of Fig. 7(c) should only be used if it may be assumed that $|Z_c| \gg |Z_{b'b}|$; the same condition was imposed upon Fig. 4.

Defining again the quantities g_{s1}, R_{s1}, and R_{sc} as in (43), we may write the noise figure F as in (48), but with values for A, B, and C different from the ones given in (48a)

$$A = 1 + 2g_{s1}(R_e + r_{b'b} + R_{sc});$$
$$B = R_{b'b} + R_{s1} + g_{s1}(R_e + r_{b'b} + R_{sc})^2; \quad C = g_{s1}; \tag{75}$$

where $r_{b'b}$ is again the ac base resistance. In these equations R_{s1} and R_{sc} are slightly different, whereas in the first term of B the ac resistance $r_{b'b}$ has been replaced by the dc resistance $R_{b'b}$.

The quantity g_{s1}, which gives the most important contribution to the noise figure F, has not changed, and the other terms in A, B, and C have not changed very much; it may thus be assumed that the theory of Section IV-B remains essentially correct even if base modulation noise is taken into account.

Now consider some of the changes expected at relatively low frequencies. First let e_b be assumed negligible, then

$$e_e'' = i_1(R_{eo} + R_{mb});$$

or

$$R_{s1} = \frac{\overline{i^2}(R_{eo} + R_{mb})^2}{4kTf} = \frac{1}{2} R_{eo}\left(\frac{R_{eo} + R_{mb}}{R_{eo}}\right)^2. \tag{76}$$

Substituting into the expression for B and putting $(R_{b'b} + R_{mb}) = r_{b'b}$, we have

$$B = R_{no} = \frac{1}{2} R_{eo} + r_{b'b} + \frac{1}{2}\frac{R_{mb}^2}{R_{eo}}$$
$$+ g_{s1}(R_{eo} + r_{b'b} + R_{sc})^2. \tag{77}$$

Without base modulation we would have had, since $R_{s1} = \frac{1}{2}R_{eo}$,

$$B = R_{no} = \tfrac{1}{2}R_{eo} + r_{b'b} + g_{s1}(R_{eo} + r_{b'b} + R_{sc})^2. \tag{77a}$$

Admittedly, the R_{sc} values may be slightly different in the two cases, but since the last terms in (77) and (77a) are small in comparison with the other ones, this will not cause a considerable difference. In that case the two B values differ mainly by the amount $\frac{1}{2}R_{mb}^2/R_{eo}$, which should be noticeable for transistors with a large dc base resistance $R_{b'b}$ at large emitter currents (small R_{eo}). If e_b is not negligible, the difference may be either larger or smaller, depending upon the correlation between e_b'' and i_1.[17] Coffey [6a] seems to have found indications that the measured noise resistance R_{no} for zero source impedance is somewhat larger than would be expected from (77a).

Now investigate the correlation resistance R_{sc} at relatively low frequencies. In that case

$$R_{sc} = \alpha_0 \frac{\overline{e_e i^*}}{\overline{i^2}} = \alpha_0(R_{eo} + R_{mb}) \frac{\overline{i_1 i^*}}{\overline{i^2}} + \alpha_0 \frac{\overline{e_b i^*}}{\overline{i^2}}$$
$$= \alpha_0 \frac{\overline{e_b i^*}}{\overline{i^2}} \tag{78}$$

since $\overline{i_1 i^*} = 0$ at relatively low frequencies, according to

[16] The circuits of Fig. 7(b) and 7(c) do not have equal response to the input signal, because of the different location of the current generator αi_e in the two cases. But since this change affects the signal and the noise in the same manner, the two circuits have identical noise figures. The effects of the base modulation are thus incorporated into the noise sources e_e and i.

[17] e_b'' is that part of e_b that is uncorrelated with i.

(17) and (19). At low frequencies one would thus expect a measurable correlation resistance only if e_b is not negligible and if a considerable correlation exists between i and e_b. Hanson and van der Ziel [28] found a measurable correlation resistance in transistors with a low value of α_o (and hence with a large base current). This is probably a base modulation effect; if it is, it would indicate that e_b is not always negligible.

The influence of shot noise base modulation requires further study. Work in progress at the University of Minnesota is aimed at obtaining a better understanding of the effect.

Bibliography

[1] Amakasu, K., and Asano, M. "Temperature Dependence of Flicker Noise in *N-P-N* Junction Transistors," *Journal of Applied Physics*, Vol. 27 (October, 1956), p. 1249.

[2] Anderson, R. L., and van der Ziel, A. "On the Shot Effect of *P-N* Junctions," IRE Transactions on Electron Devices, Vol. ED-1 (November, 1952), pp. 20–24.

[3] Bargellini, P. M., and Herscher, M. B. "Investigations of Noise in Audio Frequency Amplifiers Using Junction Transistors," Proceedings of the IRE, Vol. 43 (February, 1955), pp. 217–226.

[4] Becking, A. G. T., Groendijk, H., and Knol, K. S. "The Noise Factor of Four-Terminal Networks, *Philips Research Reports*, Vol. 10 (October, 1955), pp. 349–357.

[5] Chenette, E. R. "Measurement of Correlation between Flicker Noise Sources," this issue, p. 1304.

[6] Coffey, W. N. "Behavior of Noise Figure in Junction Transistors," Proceedings of the IRE, Vol. 46 (February, 1958), pp. 495–496.

[6a] ——. Private communication.

[7] Champlin, K. S. "A Study of Shot and Thermal Noise in Silicon *P-N* Junction Diodes." Unpublished Master of Science Thesis, University of Minnesota; September, 1955.

[8] ——. "Bridge Method of Measuring Noise in Low-Noise Devices at Radio Frequencies," Proceedings of the IRE, Vol. 46 (April, 1958), p. 779.

[9] Chynoweth, A. G., and McKay, K. G. "Photon Emission from Avalanche Breakdown in Silicon," *Physical Review*, Vol. 102 (April, 1956), pp. 369–376.

[10] ——. "Internal Field Emission in Silicon *P-N* Junction," *Physical Review*, Vol. 106 (May, 1957), pp. 418–426.

[11] Early, J. M. "Effects of Space-Charge Layer Widening in Junction Transistors," Proceedings of the IRE, Vol. 40 (November, 1952), pp. 1401–1406.

[12] Englund, J. W. "Noise Considerations for *P-N-P* Junction Transistors," in *Transistors I*. Princeton: RCA Laboratories, 1956, pp. 309–321.

[13] Fonger, W. H. "A Determination of *L/F* Noise Sources in Semiconductor Diodes and Transistors," in *Transistors I*. Princeton: RCA Laboratories, 1956, pp. 239–297.

[14] Freedman, L. A. "Design Considerations of the First Stage of Transistor Receivers," *RCA Review*, Vol. 18 (June, 1957), pp. 145–162.

[15] Giacoletto, L. J. "The Noise Factor of Junction Transistors," in *Transistors I*. Princeton: RCA Laboratories, 1956, pp. 296–308.

[16] Gianola, U. F. "Photovoltaic Noise in Silicon Broad Area *P-N* Junctions," *Journal of Applied Physics*, Vol. 27 (January, 1956), pp. 51–54.

[17] Guggenbuehl, W., and Strutt, M. J. O. "Messungen der spontanen Schwankungen bei Stromen mit Verschiedenen Trägern in Halbleitersperrschichten," *Helvetica Physica Acta*, Vol. 28, No. 7 (1955), pp. 694–704.

[18] ——, ——. "Experimentelle Bestätigung der Schottky'schen Rauschformeln an neueren Halbleiterflächendioden im Gebiet des weissen Rauschspektrums," *Archiv der elecktrischen Übertragung*, Vol. 9 (March, 1955), pp. 103–108.

[19] ——, ——. "Experimentelle Untersuchung und Trennung der Rauschursachen in Flächentransistoren," *Archiv der electrischen Übertragung*, Vol. 9 (June, 1955) pp. 259–269.

[20] ——, ——. "Theorie des Hochfrequenzrauschens von Transistoren bei kleinen Stromdicten," *Nachrichtentech, Fachberichte, Beihefte der N.T.Z.*, Vol. 5, 1956, pp. 30–33.

[21] ——, Schneider, B., and Strutt, M. J. O. "Messungen über das Hochfrequenzrauschen von Transistoren," *Nachrichtentech, Fachberichte, Beiheft der N.T.Z.*, Vol. 5, (1956), pp. 34–36.

[22] ——. "Theoretische Ueberlegungen zur physikalischen Begründung des Ersatzschaltbildes von Halbleiterflächendioden bei hohen Stromdichten," *Archiv der elecktrischen Übertragung*, Vol. 10 (November, 1956), pp. 433–435.

[23] ——, and Strutt, M. J. O. "Theory and Experiments of Shot Noise in Semiconductor Junction Diodes and Transistors," Proceedings of the IRE, Vol. 45 (June, 1957), pp. 839–857.

[24] ——, ——. "Transistors in high-frequency amplifiers," *Electronic and Radio Engineer*, Vol. 34 (July, 1957), pp. 258–267.

[25] ——, "Beiträge zur Kenntnis des Halbleiterrauschens mit besondere Berücksichtigung für Kristaldioden und Transistoren." Unpublished Ph.D. dissertation, Eidgenössischen Technischen Hochschule, Zurich, 1955.

[26] Hanson, G. H. "Shot Noise in *P-N-P* Transistors," *Journal of Applied Physics*, Vol. 26 (November, 1955), pp. 1338–1339.

[27] ——. "An Experimental Investigation of Noise in Transistors." Unpublished Ph.D. dissertation, University of Minnesota, 1957.

[28] ——, and van der Ziel, A. "Shot Noise in Transistors," Proceedings of the IRE, Vol. 45 (November, 1957), 1538–1542.

[29] Hyde, F. J. "Measurements of Noise Spectra of a Point Contact Germanium Rectifier," *Proceedings of the Physical Society B*, Vol. 66 (December, 1953), pp. 1017–1024.

[30] ——. "Measurement of Noise Spectra of a Germanium *P-N* Junction Diode," *Proceedings of the Physical Society B*, Vol. 69 (February, 1956), Pt. 2, pp. 231–241.

[31] Kennedy, D. "Gaseous Ambients and Diode Noise." Unpublished paper presented at the 1954 IRE-AIEE Conference on Semiconductor Device Research, University of Minnesota.

[32] Keonjian, E., and Schaffner, J. S. "An Experimental Investigation of Transistor Noise," Proceedings of the IRE, Vol. 40 (November, 1952), pp. 1456–1460.

[33] Kingston, R. H. "Review of Germanium Surface Phenomena," *Journal of Applied Physics*, Vol. 27 (February, 1956), pp. 101–114.

[34] ——, and McWhorter, A. L. "Relaxation Time of Surface States on Ge," *Physical Review*, Vol. 103 (August, 1956), pp. 534–540.

[35] ——, *et al. Surface Physics*. Philadelphia: University of Pennsylvania Press, 1957.

[35a] Lummis, F. L., and Petritz, R. L. "On Noise in *P-N* Junction Rectifiers: II Experiment," *Physical Review*, Vol. 91 (July, 1953) p. 231.

[36] McKay, K. G., and McAfee, K. B. "Electron Multiplication in Silicon and Germanium," *Physical Review*, Vol. 91 (September, 1953), pp. 1079–1084.

[37] ——. "Avalanche Breakdown in Silicon," *Physical Review*, Vol. 94 (May, 1954), pp. 877–884.

[38] McWhorter, A. L., and Kingston, R. L. "Channels and Excess Reverse Current in Grown Germanium *P-N* Junction Diodes," Proceedings of the IRE, Vol. 42 (September, 1954), pp. 1376–1380.

[39] Montgomery, H. C. "Transistor Noise in Circuit Applications," Proceedings of the IRE, Vol. 40 (November, 1952), pp. 1461–1471.

[40] ——, and Clark, M. A., "Shot Noise in Junction Transistors," *Journal of Applied Physics*, Vol. 24 (October, 1953), 1337–1338.

[41] Nielsen, E. C. "Behavior of Noise Figure in Junction Transistors," Proceedings of the IRE, Vol. 45 (July, 1957), pp. 957–963.

[42] North, D. O. "A Physical Theory of Noise in Transistors." Unpublished paper presented at the 1955 IRE-AIEE Conference on Semiconductor Device Research, University of Pennsylvania. Philadelphia, Pa.

[43] Pearson, G. L., Montgomery, H. C., and Feldmann, W. L. "Noise in Silicon *P-N* Junction Photocells," *Journal of Applied Physics*, Vol. 27 (January, 1956), pp. 91–92.

[44] Petritz, R. L. "On the Theory of Noise in *P-N* Junctions and Related Devices," Proceedings of the IRE, Vol. 40 (November, 1952), pp. 1440–1456.

[44a] ——. "On Noise in *P-N* Junction Rectifiers and Transistors: I Theory," *Physical Review*, Vol. 91 (July, 1953), pp. 204, 231. See especially the correction of p. 231 shown on p. 204.

[45] Rose, D. J. "Microplasmas in Silicon," *Physical Review*, Vol. 105 (January, 1957), pp. 413–418.

[46] Rothe, H., and Dahlke, W. "Theory of Noisy Four-Poles," Proceedings of the IRE, Vol. 44 (June, 1956), pp. 811–818.

[47] Ryder, R. M., and Kircher, R. J. "Some Circuit Aspects of the Transistor," *Bell System Technical Journal*, Vol. 28 (July, 1949), pp. 367–400.

[48] Shockley, W. *Electrons and Holes in Semiconductors*. New York: D. van Nostrand Co., Inc., 1950, p. 342.

[49] Slocum, A., and Shive, J. N. "Shot Dependence of *P-N* Junction Phototransistor Noise," *Journal of Applied Physics*, Vol. 25 (March, 1954), p. 406.

[49a] Solow, M. "Theory of Noise in a Multidimensional Semiconductor with a *P-N* Junction." Thesis, Catholic University of America, 1957. Also, Silver Spring: U. S. Naval Ordnance Laboratory, Navord 5762.

[50] Stephanson, W. L. "Measurements of Junction Transistor Noise in the Frequency Range 7–50 KC/S," *Proceedings of the IEE*, Vol. 102B (November, 1955), pp. 753–756.

[51] Torrey, H. C., and Whitmer, C. A. *Crystal Rectifiers*. New York: McGraw-Hill Book Co., Inc., 1948.

[52] Uhlir, A. "High Frequency Shot Noise in *P-N* Junctions," PROCEEDINGS OF THE IRE, Vol. 44 (April, 1956), pp. 557–558. Erratum, Vol. 44 (November, 1956), p. 1541.

[53] van der Ziel, A. "Note on Shot and Partition Noise in Junction Transistors," *Journal of Applied Physics*, Vol. 25 (June, 1954), pp. 815–816.

[54] ——. *Noise*. Englewood Cliffs: Prentice-Hall, Inc., 1954, ch. 8.

[55] ——. "Shot Noise in Junction Diodes and Transistors," PROCEEDINGS OF THE IRE, Vol. 43 (November, 1955), pp. 1639–1646; and Vol. 45 (July, 1957), p. 1011.

[56] ——, and Becking, A. G. T. "Theory of Junction Diode and Junction Transistor Noise," PROCEEDINGS OF THE IRE, Vol. 46 (March, 1958), pp. 589–594.

[57] ——. *Fluctuation Phenomena in Semiconductors*. London: Thornton Butterworth, Ltd., to be published.

[58] Volkers, W. K., and Pedersen, N. E. "The 'Hushed' Transistor Amplifier," *Tele-Tech*. Vol. 14 (December, 1955), pp. 82–84, 156–158; and Vol. 15 (January, 1956), p. 70.

[59] Wallace, R. L., and Pietenpol, W. J. "Some Circuit Properties and Applications of *N-P-N* Transistors," *Bell System Technical Journal*, Vol. 30 (July, 1951), pp. 530–563. Also, PROCEEDINGS OF THE IRE, Vol. 39 (July, 1951), pp. 753–757.

[60] Weisskopf, V. F. "On the Theory of Noise in Conductors, Semiconductors and Crystal Rectifiers." NDRC No. 14-133, May, 15, 1953, unpublished.

[61] Yajima, T. "Emitter Current Noise in Junction Transistors," *Journal of the Physical Society, Japan*, Vol. 11 (October, 1956), pp. 1126–1127.

The Effects of Neutron Irradiation on Germanium and Silicon*

G. C. MESSENGER†, ASSOCIATE MEMBER, IRE, AND J. P. SPRATT†

***Summary*—The known effects of neutron irradiation upon majority and minority carrier properties of germanium and silicon are reviewed, and used as a basis to derive a theoretical expression for the dependence of grounded-emitter current gain of a transistor upon accumulated neutron dose. This theoretical expression assumes a Shockley-Read recombination mechanism in the base of the transistor; the crystal defects introduced by bombardment act as recombination sites. A number of germanium and silicon transistors were irradiated at different facilities; the observed changes in transistor parameters are explained in terms of the theory. This explanation enables determination for germanium of certain basic quantities in recombination theory, *viz.*, the position in the forbidden band of the recombination site ($E_C - E_t = 0.23$ ev), and the capture cross section of the site for hole and electron capture ($\sigma_p = 1.0 \times 10^{-15}$ cm^2, and $\sigma_n \simeq 4 \times 10^{-15}$ cm^2).**

INTRODUCTION

THE electrical properties of semiconducting materials are extremely dependent upon disorder in lattice structure. Nuclear bombardment of these materials greatly increases this disorder through the creation of Frenkel defects, *i.e.*, regions in which atoms have been knocked from their sites in the lattice and placed in interstitial positions. Such defects produce energy levels in the forbidden band of a semiconductor and thus cause changes in the electrical properties. James and Lark-Horovitz[1] have proposed a model for the energy levels associated with Frenkel defects that provides a qualitative, and to some extent a quantitative, explanation of the observed changes in germanium and silicon as a result of bombardment. This model predicts that the interstitial atom will act as a donor, and the vacancy as an acceptor. It predicts further that for semiconductors with high dielectric constants, such as germanium and silicon, there will occur in the forbidden band two levels (*viz.*, the first and second ionization potentials) corresponding to the interstitial atom, and also two levels corresponding to the vacancy. When Frenkel defects are formed, the electrons donated by the interstitial atom are redistributed among the states of lower energy, so that the interstitial may be single or even doubly ionized. Correspondingly, the vacancy may have one or two electrons in it.

Cleland[2,3] and co-workers at Oak Ridge National Laboratories have attempted to explain radiation-induced changes in the majority-carrier properties of n and p-type germanium in terms of the James-Lark-Horovitz model. Agreement between the theory and these experiments was good at room temperatures; all four levels were located and identified. The results of these experiments show that the interstitial atom introduces one level approximately 0.2 ev below the conduction band and one level 0.18 ev above the valence band,

* Original manuscript received by the IRE, March 14, 1958. The work described was carried out under a subcontract with Boeing Airplane Co., sponsored by the Air Force under Prime Contract No. AF-33 (600)-35050.

† Res. Div., Philco Corp., Philadelphia, Pa.

[1] H. M. James and K. Lark-Horovitz, "Localized electronic states in bombarded semiconductors," *Z. Physik Chem.*, vol. 198, nos. 1–4, pp. 107–126; 1956.

[2] J. W. Cleland, J. H. Crawford, and J. C. Pigg, "Fast-neutron bombardment of n-type Ge," *Phys. Rev.*, vol. 98, pp. 1742–1750; June 1955.

[3] J. W. Cleland, J. H. Crawford, and J. C. Pigg, "Fast neutron bombardment of p-type germanium," *Phys. Rev.*, vol. 99, pp. 1170–1181; August, 1955.