EE 538 Spring 2021
Low-Noise Analog Circuit Design
University of Washington Electrical & Computer Engineering

Instructor: Jason Silver
Assignment #4 (10 points)
Due Sunday, May 2 (Submit on Canvas as a Jupyter Notebook)

Please show your work

Problem 1: Common-source JFET amplifier

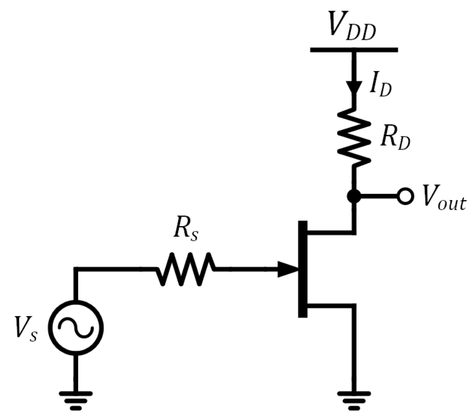


Figure 1. Common-source amplifier

The drain current of an n-channel JFET can be described as a function of V_{gs} by

$$I_d = \beta(V_{gs} - V_{th})^2$$

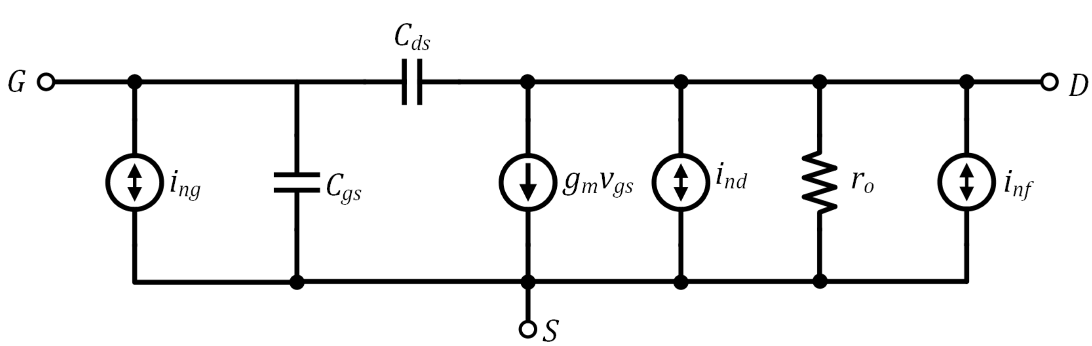
where $\beta = I_{DSS}/V_{th}^2$ (unrelated to the current gain parameter for BJTs). I_{DSS} represents the maximum drain current that is reached when $V_{gs} = 0$. Using this expression, the transconductance can be expressed as

$$g_m = 2 \cdot \sqrt{\beta I_D}$$

For the following, use $\beta = 24mA/V^2$ and $V_{gs} - V_{th} = 0.29V$

<u>*Analysis*</u>

a) Assuming $I_D = 2mA$, $I_G = 2pA$, and $R_D = 4k\Omega$, determine noise density values for i_n and e_n of the common-source amplifier in Figure 1, ignoring $1/f$ noise.



Given

$$
\begin{aligned}
\gamma &= \frac{2}{3} \\
g_m &= 2 \cdot \sqrt{\beta I_D} \\
&= 2 \cdot \sqrt{(24 \cdot 10^{-3}) \cdot (2 \cdot 10^{-3})} = 0.0139 \\
V_{gs} - V_{th} &= V_{ov} = 0.29V
\end{aligned}
$$

Current Noise Density

$$
\begin{aligned}
i_n^2 &= i_{ng}^2 \\
&= 2qI_G \\
i_n &= \sqrt{2q \cdot 2 \cdot 10^{-12}} \\
&= 0.8 \text{ fA}/\sqrt{Hz}
\end{aligned}
$$

Voltage Noise Density

$$
\begin{aligned}
e_n^2 &= \frac{i_{nd}^2}{g_m^2} \\
&= \frac{4kT\gamma g_m}{g_m^2} = \frac{4kT\gamma}{g_m} \\
e_n &= \sqrt{\frac{4kT\gamma}{g_m}} \\
&= \sqrt{\frac{4kT \cdot (2/3)}{0.0139}} \\
&= 0.893 \text{ nV}/\sqrt{Hz}
\end{aligned}
$$

---

b) The $1/f$ drain current noise of a JFET can be expressed as

$$
i_{nf}^2 = \frac{K_f \cdot I_D}{f}
$$

If f_c is defined as the frequency at which the thermal and flicker noise densities are equal, determine f_c if $K_f = 0.0021 fA$.

$$
\begin{aligned}
i_{nf}^2 &= n_{thermal}^2 \\
\frac{K_f \cdot I_D}{f} &= 4kT\gamma g_m \\
\frac{(0.0021 \cdot 10^{-15}) \cdot (2 \cdot 10^{-3})}{f_c} &= 4kT \cdot (2/3) \cdot (0.0139) \\
f_c &= 27.6 \text{Hz}
\end{aligned}
$$

---

c) Assuming $R_s = 100k\Omega$, what are the signal gain and noise figure of the amplifier at $100Hz$?

$$
\begin{aligned}
A_v &= -g_m(r_0 \parallel R_D) \\
&\approx -g_m R_D \\
&\approx -(0.0139) \cdot 4000 \\
&\approx 55.6 \text{ V/V} = 34.9 \text{ dB}
\end{aligned}
$$

$$
NF = 1 + \frac{e_n^2 + i_n^2 R_S^2}{4kTR_S}
$$

In [3]:

```
R_s,R_D,I_D,I_G,beta,gamma = sp.symbols('R_s,R_D,I_D,I_G,beta,gamma')
k = 1.38e-23
T = 300
q = 1.602e-19
V_T = k*T/q

gm = 2 * sp.sqrt(beta * I_D)
in_sq = 2*q*I_G
en_sq = 4*k*T*gamma/gm
NF = 1 + (en_sq + in_sq*R_s**2)/(4*k*T*R_s)

components = {
    R_s : 100*1e3,
    R_D : 4000,
    I_D : 2*1e-3,
    I_G : 2*1e-12,
    beta : 24*1e-3,
    gamma : 2/3
}
H = sp.Matrix([NF])
H = H.subs(components)
print(f'Noise Figure is {round(float(H[0]),3)}')
```

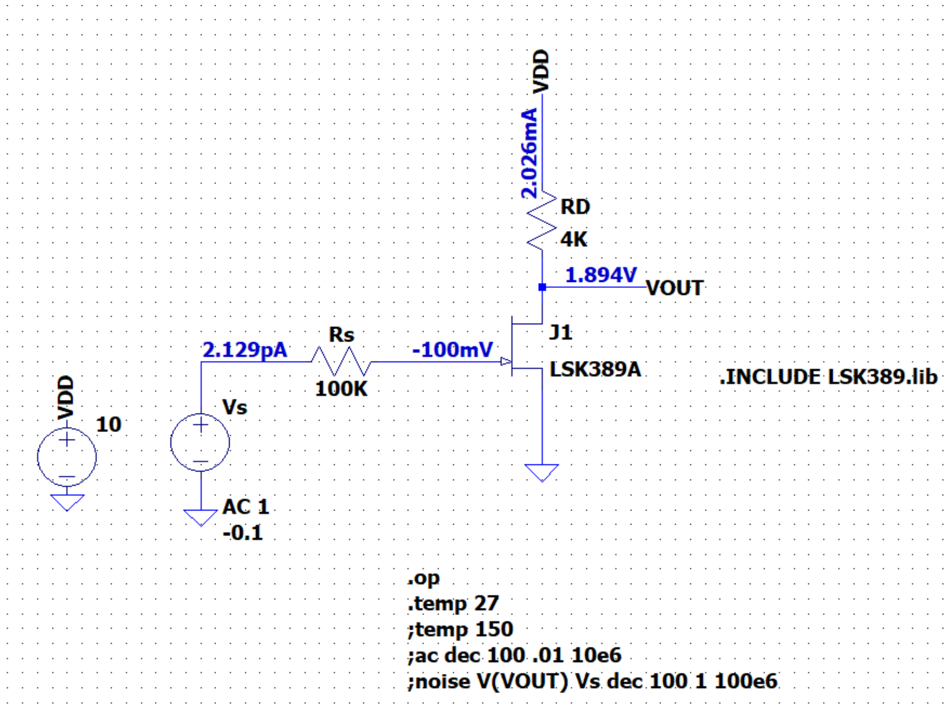
```
Noise Figure is 1.0
```

Verification

---

d) Verify your answers to a-c in Ltspice using the SPICE model of the LSK389 JFET transistor from Linear Systems. Set the DC value of V_s to $-0.1V$.

What happens to the noise figure if the temperature rises to $150C$? (this part only needs to be verified in SPICE, no calculation)



* C:\Users\keged\Documents\LTspiceXVII\EE538.1\HW04\HW04.asc

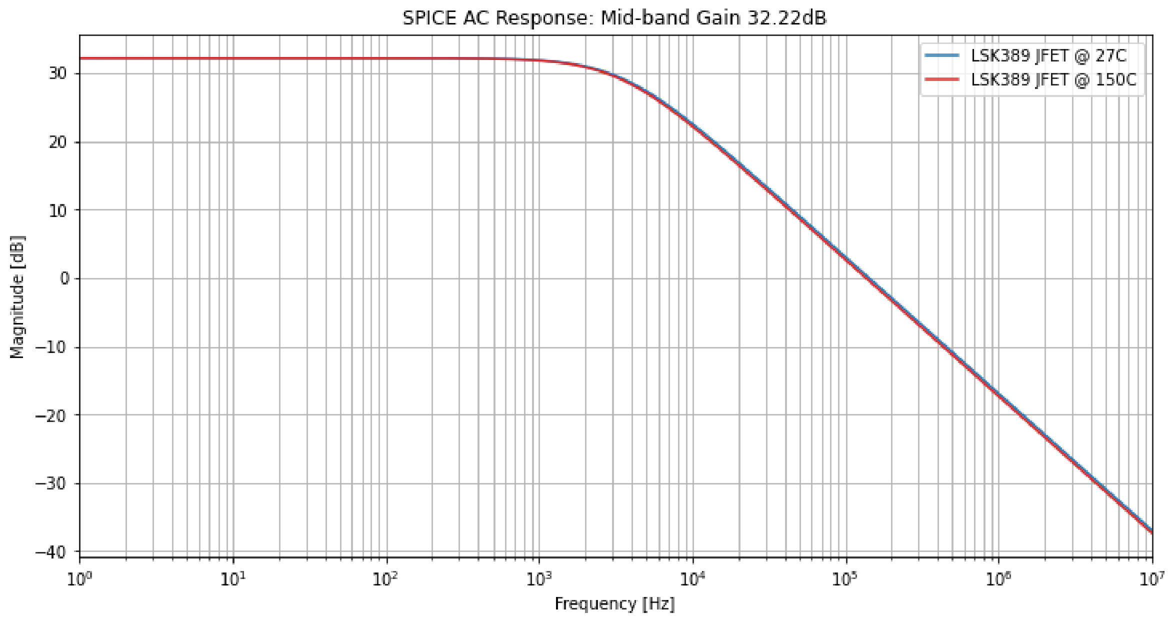
```
       --- Operating Point ---

V(vout):        1.89405         voltage
V(n002):        -0.0999998      voltage
V(n001):        -0.1            voltage
V(vdd):         10              voltage
Id(J1):         0.00202649      device_current
Ig(J1):         -2.12872e-012   device_current
Is(J1):         -0.00202649     device_current
I(Rd):          0.00202649      device_current
I(Rs):          2.12872e-012    device_current
I(V3):          -0.00202649     device_current
I(Vs):          2.12872e-012    device_current
```

In [4]:

```
filepath = 'data/HW04_27C.txt'
filepath2 = 'data/HW04_150C.txt'
df = read_ltspice(filepath,'ac')
df2 = read_ltspice(filepath2,'ac')
freq = df['Freq.']
freq2 = df2['Freq.']
mag = df['Mag_V(vout)']
mag2 = df2['Mag_V(vout)']
```

In [5]:

```python
fig, ax = plt.subplots(1,figsize=(12,6))

ax.semilogx(freq, mag, color='tab:blue',label='LSK389 JFET @ 27C')
ax.semilogx(freq2, mag2, color='tab:red',label='LSK389 JFET @ 150C')
ax.grid(True,which='both')
ax.set_xlabel('Frequency [Hz]')
ax.set_ylabel('Magnitude [dB]')
ax.set_title(f'SPICE AC Response: Mid-band Gain {round(max(mag),2)}dB')
ax.set_xlim(1e0,1e7)

ax.legend()
plt.show();
```

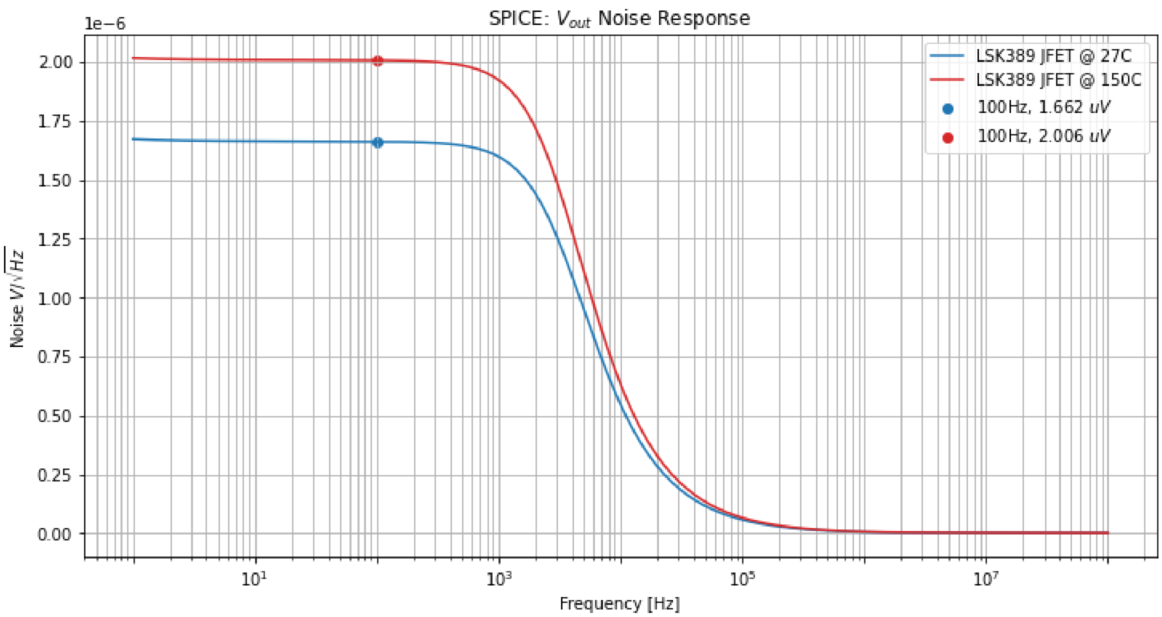


In [6]:

```python
filepath = 'data/HW04_noise_27C.txt'
filepath2 = 'data/HW04_noise_150C.txt'
df = pd.read_csv(filepath)
df2 = pd.read_csv(filepath2)
freq = df['frequency']
freq2 = df2['frequency']
onoise = df['V(onoise)']
onoise2 = df2['V(onoise)']
inoise = df['V(rs)']
inoise2 = df2['V(rs)']
```

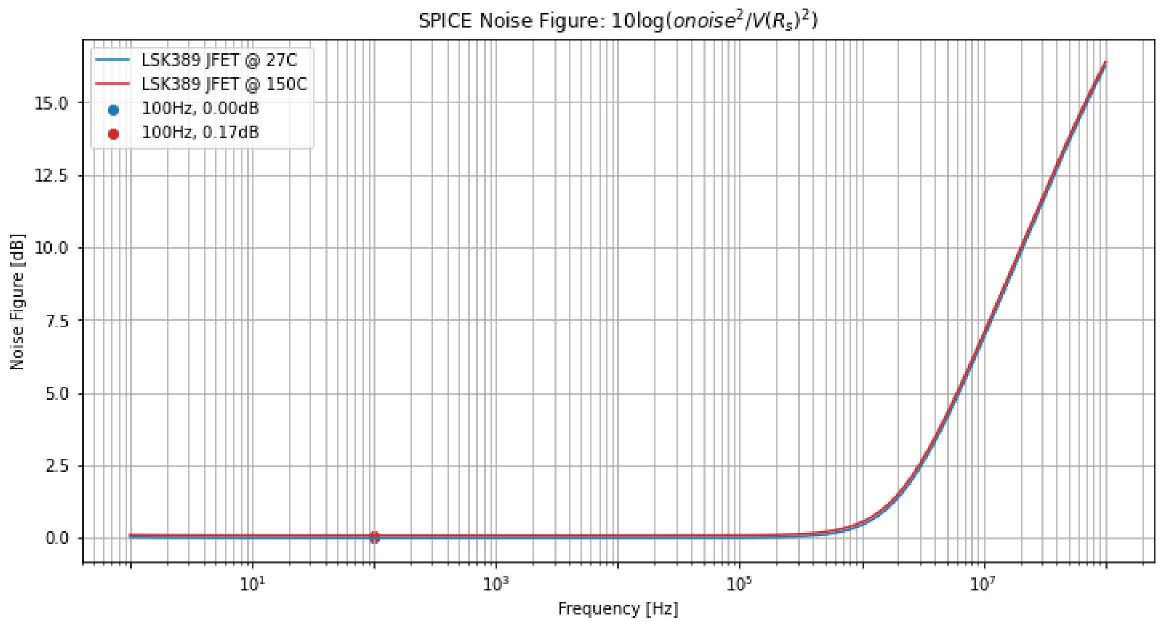
In [7]:

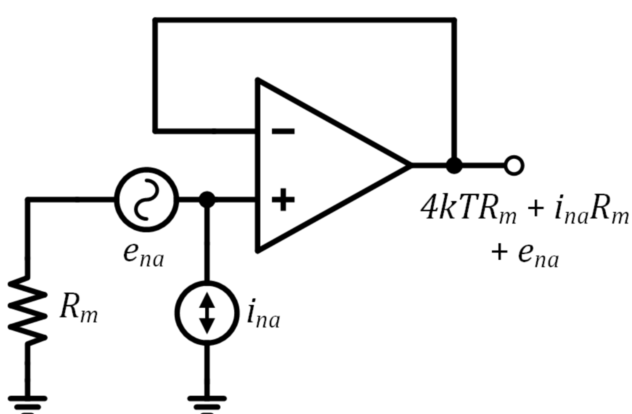
```
fig, ax = plt.subplots(1,figsize=(12,6))

x1 = np.where(freq<=100)[0][-1]
label1 = r"100Hz, {:.3f} $uV$".format(onoise[x1]*1e6)
x2 = np.where(freq2<=100)[0][-1]
label2 = r"100Hz, {:.3f} $uV$".format(onoise2[x2]*1e6)

ax.semilogx(freq, onoise, color='tab:blue',label='LSK389 JFET @ 27C')
ax.scatter(freq[x1],onoise[x1],label=label1,color='tab:blue')
ax.semilogx(freq2, onoise2, color='tab:red',label='LSK389 JFET @ 150C')
ax.scatter(freq2[x2],onoise2[x2],label=label2,color='tab:red')
ax.grid(True,which='both')
ax.set_xlabel('Frequency [Hz]')
ax.set_ylabel(r'Noise $V/\sqrt{Hz}$')
ax.set_title(r'SPICE: $V_{out}$ Noise Response')
ax.ticklabel_format(style='sci', axis='y', scilimits=(-6,-6))
#ax.set_ylim(10e-9,50e-9)

# manipulate x-axis ticks and labels
ax.xaxis.set_major_locator(LogLocator(numticks=15)) #(1)
ax.xaxis.set_minor_locator(LogLocator(numticks=15,subs=np.arange(2,10))) #(2)
for label in ax.xaxis.get_ticklabels()[::2]:
    label.set_visible(False) #(3)

ax.legend()
plt.show();
```



In [8]:

```python
fig, ax = plt.subplots(1,figsize=(12,6))

x1 = np.where(freq<=100)[0][-1]
label1 = r"100Hz, {:.2f}dB".format(10*np.log10(onoise[x1]**2/inoise[x1]**2))
x2 = np.where(freq<=100)[0][-1]
label2 = r"100Hz, {:.2f}dB".format(10*np.log10(onoise2[x2]**2/inoise2[x2]**2))

ax.semilogx(freq, 10*np.log10(onoise/inoise), color='tab:blue',label='LSK389 JFET @ 27C')
ax.scatter(freq[x1],10*np.log10(onoise[x1]/inoise[x1]),label=label1,color='tab:blue')
ax.semilogx(freq2, 10*np.log10(onoise2/inoise2), color='tab:red',label='LSK389 JFET @ 150
ax.scatter(freq[x2],10*np.log10(onoise2[x2]/inoise2[x2]),label=label2,color='tab:red')
ax.grid(True,which='both')
ax.set_xlabel('Frequency [Hz]')
ax.set_ylabel(r'Noise Figure [dB]')
ax.set_title(r'SPICE Noise Figure: $10 \log(onoise^2/V(R_s)^2)$')

# manipulate x-axis ticks and labels
ax.xaxis.set_major_locator(LogLocator(numticks=15)) #(1)
ax.xaxis.set_minor_locator(LogLocator(numticks=15,subs=np.arange(2,10))) #(2)
for label in ax.xaxis.get_ticklabels()[::2]:
    label.set_visible(False) #(3)

ax.legend()
plt.show();
```



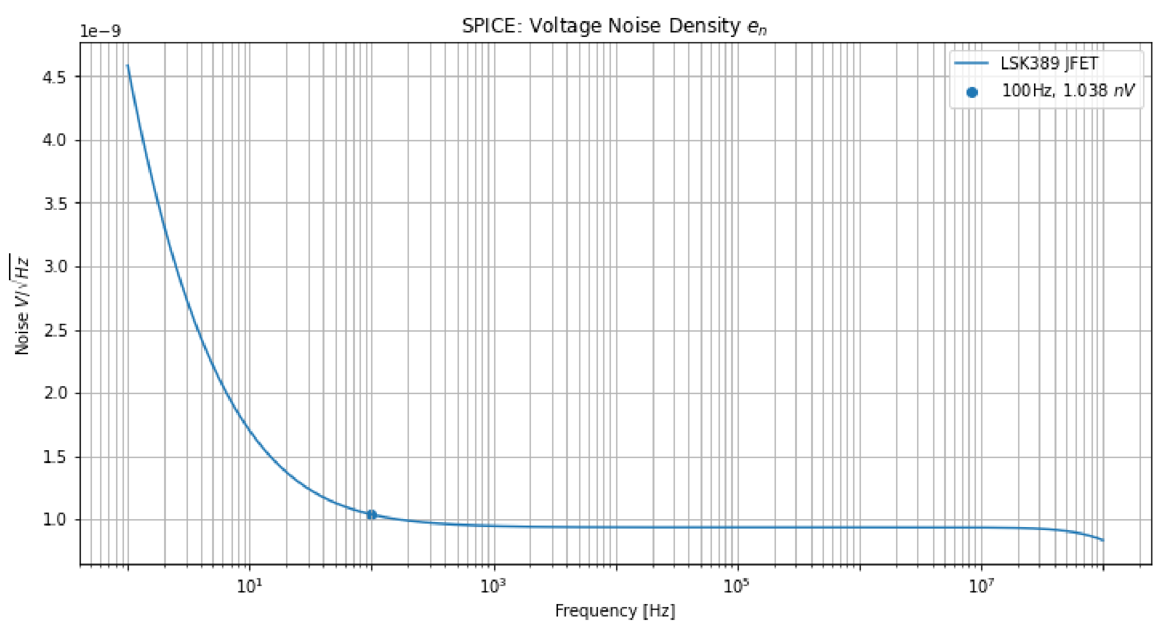


- The noise properties of an amplifier can be determined by two measurements: 1) with the input shorted to AC ground for e_n, and 2) with an resistor between the input and ground for i_n
- To measure i_n, the resistance R_m should be selected such that $i_n R_m >> e_n$. In this case, i_n can be calculated as

$$i_n = \frac{e_{out} - 4kTR_m}{R_m}$$

In [9]:
```
filepath = 'data/HW04_en.txt'
df = pd.read_csv(filepath)
freq = df['frequency']
e_n = df['V(onoise)/gain']
```

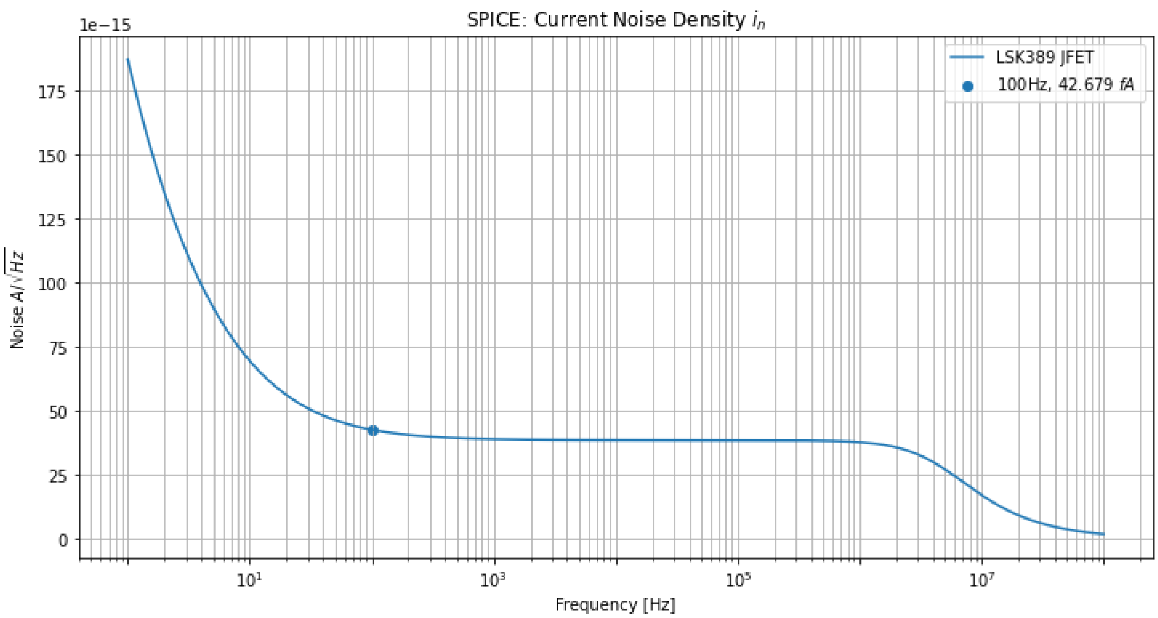
In [10]:
```
fig, ax = plt.subplots(1,figsize=(12,6))

x1 = np.where(freq<=100)[0][-1]
label1 = r"100Hz, {:.3f} $nV$".format(e_n[x1]*1e9)

ax.semilogx(freq, e_n, color='tab:blue',label='LSK389 JFET')
ax.scatter(freq[x1],e_n[x1],label=label1,color='tab:blue')
ax.grid(True,which='both')
ax.set_xlabel('Frequency [Hz]')
ax.set_ylabel(r'Noise $V/\sqrt{Hz}$')
ax.set_title(r'SPICE: Voltage Noise Density $e_n$')
ax.ticklabel_format(style='sci', axis='y', scilimits=(-9,-9))
#ax.set_ylim(10e-9,50e-9)

# manipulate x-axis ticks and labels
ax.xaxis.set_major_locator(LogLocator(numticks=15)) #(1)
ax.xaxis.set_minor_locator(LogLocator(numticks=15,subs=np.arange(2,10))) #(2)
for label in ax.xaxis.get_ticklabels()[::2]:
    label.set_visible(False) #(3)

ax.legend()
plt.show();
```



In [11]:
```
filepath = 'data/HW04_in.txt'
df = pd.read_csv(filepath)
freq = df['frequency']
i_n = df['(V(onoise)-4*4.11*1e(U+2212)21*1e6)/(1e6)']
```

In [12]:

```python
fig, ax = plt.subplots(1,figsize=(12,6))

x1 = np.where(freq<=100)[0][-1]
label1 = r"100Hz, {:.3f} $fA$".format(i_n[x1]*1e15)

ax.semilogx(freq, i_n, color='tab:blue',label='LSK389 JFET')
ax.scatter(freq[x1],i_n[x1],label=label1,color='tab:blue')
ax.grid(True,which='both')
ax.set_xlabel('Frequency [Hz]')
ax.set_ylabel(r'Noise $A/\sqrt{Hz}$')
ax.set_title(r'SPICE: Current Noise Density $i_n$')
ax.ticklabel_format(style='sci', axis='y', scilimits=(-15,-15))
#ax.set_ylim(10e-9,50e-9)

# manipulate x-axis ticks and labels
ax.xaxis.set_major_locator(LogLocator(numticks=15)) #(1)
ax.xaxis.set_minor_locator(LogLocator(numticks=15,subs=np.arange(2,10))) #(2)
for label in ax.xaxis.get_ticklabels()[::2]:
    label.set_visible(False) #(3)

ax.legend()
plt.show();
```



Reference Page

In [13]:

```
# Imports
import os
import sys
import cmath
import math
import matplotlib.pyplot as plt
import matplotlib
import numpy as np
import pandas as pd
import ltspice
import sympy as sp
from sympy.utilities.lambdify import lambdify
from scipy import signal
%matplotlib inline
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
from matplotlib.ticker import LogLocator
```

In [14]:

```
def read_ltspice(file_name,ftype='trans',units='db'):
    cols = []
    arrs = []
    with open(file_name, 'r',encoding='utf-8') as data:
        for i,line in enumerate(data):
            if i==0:
                cols = line.split()
                arrs = [[] for _ in cols]
                continue
            parts = line.split()
            for j,part in enumerate(parts):
                arrs[j].append(part)
    df = pd.DataFrame(arrs,dtype='float64')
    df = df.T
    df.columns = cols
    if ftype=='trans':
        return df
    elif ftype=='ac':
        if units=='db':
            for col in cols:
                if df[col].str.contains(',').all():
                    df[f'Mag_{col}'] = df[col].apply(lambda x: x.split(',')[0])
                    df[f'Mag_{col}'] = df[f'Mag_{col}'].apply(lambda x: x[1:-2])
                    df[f'Mag_{col}'] = df[f'Mag_{col}'].astype('float64')
                    df[f'Phase_{col}'] = df[col].apply(lambda x: x.split(',')[1])
                    df[f'Phase_{col}'] = df[f'Phase_{col}'].apply(lambda x: x[0:-2])
                    df[f'Phase_{col}'] = df[f'Phase_{col}'].astype('float64')
        if units=='cartesian':
            for col in cols:
                if df[col].str.contains(',').all():
                    df[f'Re_{col}'] = df[col].apply(lambda x: x.split(',')[0])
                    df[f'Re_{col}'] = df[f'Re_{col}'].astype('float64')
                    df[f'Im_{col}'] = df[col].apply(lambda x: x.split(',')[1])
                    df[f'Im_{col}'] = df[f'Im_{col}'].astype('float64')
        df['Freq.'] = df['Freq.'].astype('float64')
        return df
    else:
        print('invalid ftype')
```