EE 538: Low-Noise Analog Circuit Design

Spring 2021

Instructor: Jason Silver

Announcements

- Midterm exam due Sunday, May 16
- Design project posted
  - In-class presentation Wednesday, June 2
  - Final report due Friday, June 11 at midnight

Week 7

- EE538 lecture notes
- Photodiode Front Ends — The REAL Story, *Opt. Photon. News, 12, 42-45 (April 2001).
- AoE Chapter 8, Section 8.11
- AoE — The x-Chapters, Section 4x.3

Overview

- Last time...
  - Nonlinear device characteristics
  - Distortion in amplifiers
  - Total harmonic distortion
  - Intermodulation distortion
  - Effect of feedback on nonlinearity
  - Dynamic range
- Today...
  - TIA frequency response and noise
  - Shot noise limit and minimizing added noise
  - Cascode isolation of photodiode capacitance
  - Regulated (i.e. gain-boosted) cascode
  - Boostrapping photodiode capacitance

Python packages/modules

In [5]:

```python
import matplotlib as mpl
from matplotlib import pyplot as plt
from matplotlib import ticker, cm
import numpy as np
from scipy import signal
from scipy import integrate
from scipy.fft import fft
#%matplotlib notebook

mpl.rcParams['font.size'] = 14
mpl.rcParams['legend.fontsize'] = 'large'

def plot_xy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5));
    ax.plot(x, y, 'b')
    ax.grid()
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)

def plot_2xy(x, y1, y2, xlabel, ylabel, y1label, y2label):
    fig, ax = plt.subplots(figsize=(10.0,7.5))

    ax.plot(x, y1, 'b', label=y1label)
    ax.plot(x, y2, 'r', label=y2label)
    ax.set_ylabel(ylabel)
    ax.set_xlabel(xlabel)
    ax.grid()

    ax.legend()
    ax.legend(loc='upper center', ncol=2, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_xy2(x1, y1, x1label, y1label, x2, y2, x2label, y2label):
    fig, ax = plt.subplots(2, figsize = (10.0, 7.5));
    ax[0].plot(x1, y1, 'b')
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].plot(x2, y2, 'b')
    ax[1].set_xlabel(x1label)
    ax[1].set_xlabel(x2label)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    fig.align_ylabels(ax[:])

def plot_xy3(x, y1, y2, y3, xlabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(3, figsize=(10.0,7.5))

    ax[0].plot(x, y1)
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].plot(x, y2)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    ax[2].plot(x, y3)
    ax[2].set_ylabel(y3label)
    ax[2].set_xlabel(xlabel)
    ax[2].grid()

def plot_logxy3(x, y1, y2, y3, xlabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(3, figsize=(10.0,7.5))

    ax[0].semilogx(x, y1)
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].semilogx(x, y2)
    ax[1].set_ylabel(y2label)
    ax[1].grid()

    ax[2].semilogx(x, y3)
    ax[2].set_ylabel(y3label)
    ax[2].set_xlabel(xlabel)
    ax[2].grid()

def plot_logxy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    ax.semilogx(x, y, 'b')
    ax.grid();
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)

def plot_log3xy(x, y1, y2, y3, xlabel, ylabel, y1label, y2label, y3label):
    fig, ax = plt.subplots(figsize=(10.0,7.5))

    ax.semilogx(x, y1, 'b', label=y1label)
    ax.semilogx(x, y2, 'r', label=y2label)
    ax.semilogx(x, y3, 'g', label=y3label)
    ax.set_ylabel(ylabel)
    ax.set_xlabel(xlabel)
    ax.grid()

    ax.legend()
    ax.legend(loc='upper center', ncol=3, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_log2xy(x, y1, y2, xlabel, ylabel, y1label, y2label):
    fig, ax = plt.subplots(figsize=(10.0,7.5))

    ax.semilogx(x, y1, 'b', label=y1label)
    ax.semilogx(x, y2, 'r', label=y2label)
    ax.set_ylabel(ylabel)
    ax.set_xlabel(xlabel)
    ax.grid()

    ax.legend()
    ax.legend(loc='upper center', ncol=2, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_loglog(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    ax.loglog(x, y, 'b')
    ax.grid();
    ax.set_xlabel(xlabel)
    ax.set_ylabel(ylabel)


def plot_loglog2(x, y1, y2, xlabel, ylabel, y1label, y2label):
    fig, ax = plt.subplots(figsize=(10.0,7.5))

    ax.loglog(x, y1, 'b', label=y1label)
    ax.loglog(x, y2, 'r', label=y2label)
    ax.set_ylabel(ylabel)
    ax.set_xlabel(xlabel)
    ax.grid()

    ax.legend()
    ax.legend(loc='upper center', ncol=2, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_xlogy(x, y, xlabel, ylabel):
    fig, ax = plt.subplots(figsize=(10.0, 7.5));
    ax.semilogy(x, y, 'b');
    ax.grid();
    ax.set_xlabel(xlabel);
    ax.set_ylabel(ylabel);

def read_ltspice_ac(file_name):
    with open(file_name, 'r') as data:
        x = []
        y = []
        z = []
        next(data) # skip header line
        for line in data:
            p = line.split()
            x.append(float(p[0]))
            complex = p[1].split(",")
            y.append(float(complex[0]))
            z.append(float(complex[1]))

    return x, y, z

def plot_logxy2(x1, y1, x2, y2, x1label, y1label, x2label, y2label):
    fig, ax = plt.subplots(2, figsize = (10.0, 7.5));
    ax[0].semilogx(x1, y1, 'b');
    ax[0].set_ylabel(y1label)
    ax[0].grid()

    ax[1].semilogx(x2, y2, 'b');
    ax[1].set_xlabel(x1label)
    ax[1].set_xlabel(x2label);
    ax[1].set_ylabel(y2label);
    ax[1].grid();

    fig.align_ylabels(ax[:])

def plot_noise_bandwidth(f, mag):
    fig, ax = plt.subplots(2, figsize=(10.0,7.5))
    ax[0].semilogx(f, RC_mag)
    ax[0].set_xscale("log")
    ax[0].set_xlim(f[0], f[-1])
    ax[0].set_xticks(np.logspace(0.1,4,5))
    ax[0].set_xticklabels([])
    ax[0].set_ylabel('Magnitude [V/V]')
    ax[0].set_title('Equivalent Noise Bandwidth')
    ax[0].grid()

    ax[1].hlines(1, 0, f_enb, color='tab:blue')
    ax[1].hlines(0, f_enb, f[-1], color='tab:blue')
    ax[1].vlines(f_enb, 0, 1, color='tab:blue')
    ax[1].set_xlim(f[0], f[-1])
    ax[1].set_xscale("log")
    ax[1].set_xticks(np.logspace(0.1,4,5))
    ax[1].set_xticklabels([r'$10^0$',r'$10^1$', r'$10^2$', r'$10^3$', r'$10^4$'])
    ax[1].set_ylabel('Magnitude [V/V]')
    ax[1].set_xlabel('Frequency [Hz]')
    ax[1].grid()

def noise_hist(vnoise, vn_rms, bins):
    fig = plt.figure( figsize=(10.0,7.5) )
    vn_norm = vnoise/ vn_rms
    ax = fig.add_subplot(111)
    n, bins, rectangles = ax.hist(vn_norm, bins, density=True, range=(-3, 3),
                                  color='b')
    ax.set_xlabel(r'Sample Voltage [$v_{n(rms)}$]')
    ax.set_ylabel('Probability Density')
    ax.grid()
    fig.canvas.draw()

def plot_NF_vs_Rs(en_vals, in_vals, Rs_min, Rs_max, T_in_K):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    k = 1.38e-23
    Rs = np.logspace(np.log10(Rs_min), np.log10(Rs_max), num=200)
    F1 = 1 + (en_vals[0]**2+Rs**2*in_vals[0]**2)/(4*k*T_in_K*Rs)
    F2 = 1 + (en_vals[1]**2+Rs**2*in_vals[1]**2)/(4*k*T_in_K*Rs)
    F3 = 1 + (en_vals[2]**2+Rs**2*in_vals[2]**2)/(4*k*T_in_K*Rs)
    ax.semilogx(Rs, 10*np.log10(F1), 'b', label=r'$e_{n1}$, $i_{n1}$')
    ax.semilogx(Rs, 10*np.log10(F2), 'r', label=r'$e_{n2}$, $i_{n2}$')
    ax.semilogx(Rs, 10*np.log10(F3), 'g', label=r'$e_{n3}$, $i_{n3}$')
    ax.grid();
    ax.set_xlabel(r'Source Resistance $R_s$ [\Omega]$')
    ax.set_ylabel(r'Noise Figure $NF$ [$dB$]')

    ax.legend()
    ax.legend(loc='upper center', ncol=3, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_noise_curve(e_n, i_n, Rs_min, Rs_max):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    Rs = np.logspace(np.log10(Rs_min), np.log10(Rs_max), num=200)
    e_ni_2 = 4*k*T*Rs + e_n**2 + i_n**2*Rs**2
    ax.loglog(Rs, np.sqrt(e_ni_2), 'b', label='Total Noise')
    ax.loglog(Rs, np.sqrt(4*k*T*Rs), 'r', label=r'$\sqrt{4kTR_s}$')
    ax.loglog(Rs, e_n*np.ones(np.size(Rs)), 'g', label=r'$e_n$')
    ax.loglog(Rs, i_n*Rs, 'y', label=r'$i_n R_s$')
    ax.grid();
    ax.set_xlabel(r'Source Resistance $R_s$ [\Omega]$')
    ax.set_ylabel(r'Equivalent Input Noise [$V/\sqrt{Hz}$]')

    ax.legend()
    ax.legend(loc='upper center', ncol=4, fancybox=True,
              shadow=True, bbox_to_anchor=(0.5,1.13) )

def plot_bjt_NF(beta, r_bb, Rmin, Rmax, Imin, Imax):
    fig, ax = plt.subplots(figsize=(10.0, 7.5))
    k = 1.38e-23
    T = 300
    q = 1.602e-19
    V_T = k*T/q
    rs = np.logspace(np.log10(Rmin), np.log10(Rmax), num = 100)
    ic = np.logspace(np.log10(Imin), np.log10(Imax), num = 100)
    I_C, R_S = np.meshgrid(ic, rs)
    e_n_2 = 4*k*T*(V_T/2/I_C + r_bb)
    i_n_2 = 2*q*I_C/beta_0
    NF = 1 + (e_n_2 + i_n_2*R_S**2)/(4*k*T*R_S)
    cp = ax.contourf(I_C, R_S, 10*np.log10(NF), levels=np.linspace(0,15, num=16))
    plt.xscale('log')
    plt.yscale('log')
    plt.ylabel(r'Source Resistance $R_s$ [$\Omega$]')
    plt.xlabel(r'Collector Current $I_C$ [A]')
    fig.colorbar(cp)

def fftnoise(f):
    f = np.array(f, dtype='complex')
    Np = (len(f) - 1) // 2
    phases = np.random.rand(Np) * 2 * np.pi
    phases = np.cos(phases) + 1j * np.sin(phases)
    f[1:Np+1] *= phases
    f[-1:-1-Np:-1] = np.conj(f[1:Np+1])
    return np.fft.ifft(f).real

def band_limited_noise(min_freq, max_freq, samples=1024, samplerate=1):
    freqs = np.abs(np.fft.fftfreq(samples, 1/samplerate))
    f = np.zeros(samples)
    idx = np.where(np.logical_and(freqs>=min_freq, freqs<=max_freq))[0]
    f[idx] = 1
    return fftnoise(f)

def fft_mag(x, N, T, t):
    fft_sig  = fft(x, N)
    freqs = np.linspace(0.0, 1.0/(2.0*T), N//2)
    mags = 2.0/N * np.abs(fft_sig[0:N//2]) # single-sided FFT

    return freqs, mags

def plot_fft_dB(freqs, mags, fmin, fmax):
    fig, ax = plt.subplots(figsize = (10.0,7.5))
    ax.plot(1e-3*freqs, 20*np.log10(mags), 'b')
    ax.set_xlim(fmin, fmax)
    ax.set_xlabel('Frequency [kHz]')
    ax.set_ylabel('Magnitude [dB]')
    ax.grid()
```

Lecture 7 - Transimpedance Amplifier Design

Photodiodes for wideband optical sensing

- Photodiodes produce current in response to incident light (i.e. photons absorbed by the diode)
- To convert this current to a voltage that can be processed by our signal conditional circuitry (e.g. filter, ADC, gain stage, etc.), we might consider using just a resistor, which produces an output voltage of $v_o = -i_d R_f$
- The value of R_f should be chosen such that its thermal noise current $4kT/R_f$ is neglible compared to the shot noise current of the diode $2qi_d$
- To put this in concrete terms, assume our application requires a bandwidth of $500kHz$, a minimum signal level of $10\mu A$, and our photodiode has a reverse-biased depletion capacitance of $100pF$
- The photodiode produces a shot noise current with power spectral density given by

$$i_{ns}^2 = 2qi_d \tag{1}$$

- To ensure the resistor contributes negligibly to the total noise, we might require the spectral density of its noise to be 10% of that of the shot noise (this is a 0.5% increase in the rms current noise density, which could admittedly be too extreme)
- Applying this constraint gives us an R_f value of

$$R_f = 10 \cdot \frac{4kT}{2qi_d} \tag{2}$$

- If we need to design for a signal current of $10\mu A$, this necessitates an R_f value of ~$50k\Omega$, which limits our $3dB$ bandwidth to $30kHz$, quite a bit lower than the target $500kHz$!
- If the minimum signal amplitude were further reduced, our bandwidth would be even lower
- How can we improve the situation?

Transimpedance amplifier (TIA)

- The transimpedance amplifier addresses this issue by using an opamp's virtual ground to present a low impedance to the capacitance of the photodiode
- The input pole is effectively eliminated by the amplifier feedback (kind of)
- The ideal transfer function of the TIA is given by

$$\frac{v_o}{i_d} = R_f \tag{3}$$

- Ignoring the opamp's noise (for the moment), the resistor value and its associated noise are identical to the above derivation

TIA stability

- Unfortunately, the required large value of R_f contributes a parasitic pole to the *open-loop response* of the amplifier, compromising stability
- C_{in} is the total capacitance connected to the feedback node, which includes the input capacitance of the opamp and is often dominated by the photodiode's depletion capacitance
- The addition of C_f contributes a zero to the transfer function that can be used to compensate for the phase lag due to $R_f C_{in}$
- C_f addresses the stability issue, but what about bandwidth?

TIA frequency response

- The closed-loop transfer function of the TIA is second order, and can be expressed as

$$\frac{v_o}{i_d} = R_f \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 + \omega_0^2} \tag{4}$$

- Assuming a single-pole response of the opamp given by the transfer function $A = A_0/(1 + s/s_p)$ and a unity-gain frequency of $\omega_t \approx A_0\omega_p$, where ω_p is the open-loop pole frequency, the closed-loop bandwidth is given by

$$\omega_0 \approx \sqrt{\frac{\omega_t}{R_f(C_{in}+C_f)}} = \sqrt{\omega_t \cdot \omega_{p,cl}} \tag{5}$$

- The damping factor (or, equivalently, the Q factor) is given by

$$2\zeta = \frac{1}{Q} \approx R_f C_f \cdot \sqrt{\frac{\omega_t}{R_f(C_{in}+C_f)}} \tag{6}$$

- The Q factor determines the "quality" of both the step and frequency responses, and it is typically desirable to keep Q below some critical value for a "well-behaved" response
- For a damping factor of $\zeta = 1/\sqrt{2}$ (or equivalently, $Q = 1/\sqrt{2}$, a maximally flat response), and assuming $C_{in} >> C_f$, the required value of C_f is approximately

$$C_f \approx \sqrt{\frac{2 \cdot C_{in}}{R_f \omega_t}} = \frac{1}{R_f}\sqrt{\frac{2 \cdot R_f C_{in}}{\omega_t}} \tag{7}$$

- That is, in order to achieve a "well-behaved response" the zero frequency $\omega_z = 1/R_f C_f$ should be equal to the geometric mean of the "parasitic" pole frequency $\omega_{p,cl} = 1/R_f(C_f + C_{in})$ and the opamp transit frequency ω_t ($\omega_z = \sqrt{\omega_{p,cl}\omega_t}$)
- So, in order to achieve loop stability the closed-loop bandwidth can only exceed the parasitic pole frequency $w_{p,cl}$ by the square root of the ratio of ω_t to $\omega_{p,cl}$

TIA noise

- The basic transimpedance amplifier contains four noise sources, including the shot noise of the photodiode
- $i_{naR_f} = 4kT/R_f$ should be minimized by choosing an appropriately large value of R_f
- i_{na} can be controlled by choosing an opamp with low input noise current (i.e. JFET- or CMOS-based, with the caveat that $1/f$ noise may be problematic with the latter, but generally only for narrow bandwidths)
- Assuming these first two noise sources are appropriately managed, this leaves the opamp's voltage noise e_{na}, which at low frequencies can be input-referred by dividing by R_f:

$$i_{ne_{na},LF} = \frac{e_{na}}{R_f} \tag{8}$$

- How does a typical "low-noise" opamp stack up for low-noise photodiode applications?
- Assuming $i_d = 10\mu A$, the diode's shot noise is given by

$$i_{ns} = \sqrt{2qi_d} = 1.8pA/\sqrt{Hz} \tag{9}$$

- To reduce R_f's contribution to noise, let's use $R_f = 100k\Omega$, which gives an input-referred noise current $i_{nR_f} = \sqrt{4kT/R_f} \approx .4pA/\sqrt{Hz}$
- Let's say $e_{na} = 5nV/\sqrt{Hz}$. This gives a low-frequency input-referred noise current due to e_{na} of

$$i_{ne_{na},LF} = \frac{e_{na}}{R_f} = \frac{5nV/\sqrt{Hz}}{100k\Omega} = 50fA\sqrt{Hz} \tag{10}$$

- Not bad! Even an opamp with moderate noise performance does well, at least at low frequencies
- What happens as we increase the bandwidth of the TIA?

Noise peaking

- Assuming an ideal (infinite-bandwidth) opamp, the noise transfer function is the input/output relationship for the amplifier voltage noise

$$\frac{e_{na,out}}{e_{na}} = \left(1 + \frac{Z_f}{Z_{in}}\right) \tag{11}$$

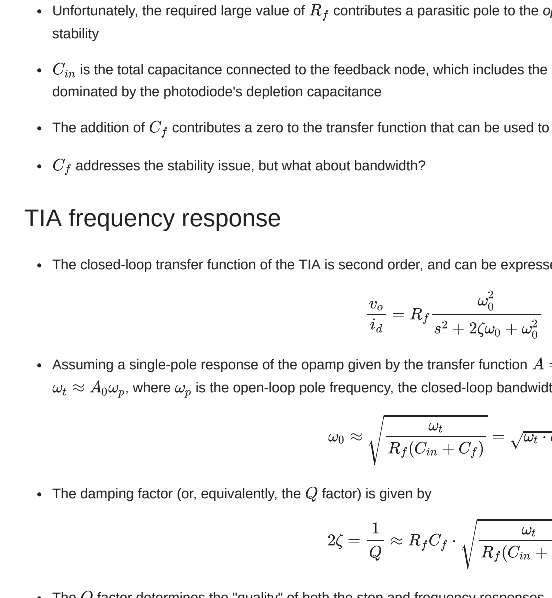
$$\tag{12}$$

$$= \frac{1 + s(C_{in}+C_f)R_f}{1 + sC_f R_f} \tag{13}$$

- This expression has a zero at $1/R_f(C_{in}+C_f) \approx 1/R_f C_{in}$, which results in "peaking" of the output noise due to e_{na} and significantly degrades the noise performance of the TIA
- The noise due to e_{na} rises with frequency, as the impedance of C_{in} decreases

In [52]:

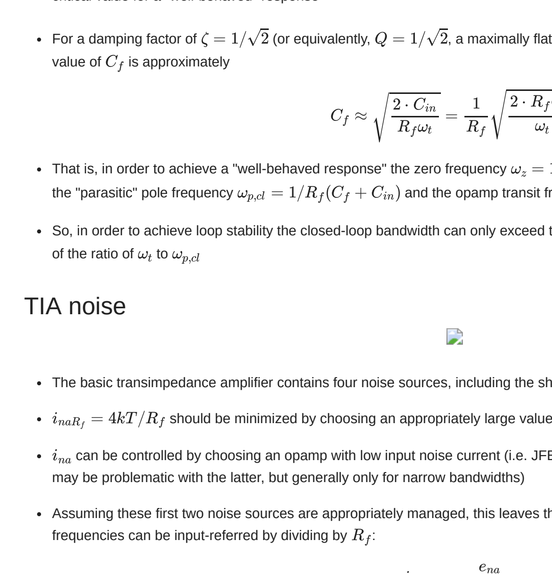
```python
w = np.logspace(1, 8, num=700)
Cin = 100e-12
Rf = 100e3
wt = 10e6
Cf = np.sqrt(2*Cin/Rf/wt)
ena = 5e-9
G_noise = signal.TransferFunction([(Cin+Cf)*Rf, 1], [Cf*Rf, 1])
w, mag, phase = signal.bode(G_noise, w=w)
plot_logxy(w/2/np.pi, 1e9*10**(mag/20)*ena, 'Frequency [Hz]', r'Output Noise [$nV/\sqrt{Hz}$]')
```



- The output noise begins rising at relatively low frequencies, substantially increasing the rms noise

In [53]:

```python
w0 = np.sqrt(wt/(Cin+Cf)/Rf)
zeta = Rf*Cf*w0/2
G_sig = signal.TransferFunction([Rf*w0**2], [1, 2*zeta*w0, w0**2])
w, mag, phase = signal.bode(G_sig, w=w)
plot_loglog(w/2/np.pi, 10**(mag/20), 'Frequency [Hz]', r'Transimpedance Gain [$\Omega$]')
```



- Even worse, the bandwidth of the TIA has been significantly hampered by the large input capacitance
- How can we (significantly) increase bandwidth while managing the rms noise?

Cascoded TIA

- A cascode transistor (Q_1) can be used to isolate the large photodiode capacitance from the voltage noise of the amplifier
- The opamp noise voltage at the feedback (inverting) node of the amplifier sees a high impedance, minimizing the resulting noise current
- What effect does the Q_1 have on the signal, and what noise does it contribute?

Cascode noise

- The impedance looking into the emitter of Q_1 is given by $r_e = 1/g_m$
- The diode is reverse-biased, so its resistance is high and i_{nc} is approximately split between r_e and C_d, resulting in a frequency-dependent output noise given by

$$i_{nc,out} = \frac{sC_d r_e}{sC_d r_e + 1} i_{nc} \tag{14}$$

- Below the frequency $g_m/2\pi C_d$, Q_1 contributes only base current shot noise, which is β times lower, in terms of power spectral density, than the shot noise current of the photodiode
- Beyond this bandwidth, Q_1 contributes full shot noise, as the shot noise of the photodiode becomes attenuated (resulting in full shot noise at all frequencies)

Common-base stage

- The cascode stage is often referred to as a *common-base* stage, since the base is common to both the input (emitter) and output (collector)
- As a voltage amplifier, its (midband) gain is non-inverting and given by

$$A_v = g_m R_C \tag{15}$$

- The input impedance of the cascode/common-base stage is critical in determining a) how much of the signal current makes it to the TIA input and b) the bandwidth limitations of the added cascode
- At signal frequencies (where C_{ac} constitutes a short) the common-base/cascode input impedance is given by $R_{in} = 1/g_m$

Cascode frequency response

- The photodiode signal current also sees an input impedance of $1/g_m$ (at low frequencies) due to the cascode device
- At high frequencies, the diode capacitance C_d shunts the signal current, reducing the current going into the TIA
- The transfer function seen by the photodiode current is

$$i_{in} = \frac{1}{1 + sC_d/g_m} i_d \tag{16}$$

- Assuming $i_d = 15\mu A$ and a collector current in Q_1 approximately equal to this value, the input impedance of the cascode device is approximately $1.7k\Omega$
- For a photodiode capacitance of $500pF$, this results in a pole at ~$190kHz$, severely limiting the frequency response of the amplifier if wide bandwidth is needed
- How can we reduce the input impedance of the cascode/common-gate stage and avoid significant bandwidth limitations?

Biased cascode

- The input impedance of the cascode stage can be reduced significantly by adding a DC bias to increase the frequency of the input pole, which is proportional to the value of Q_1's collector current
- R_E can be chosen to set the collector current of Q_1 such that the g_{m1}/C_d pole is placed outside of the TIA bandwidth
- For example, if I_C is set at $200\mu A$ the pole is moved from ~$190kHz$ to $2.5MHz$ due to the increase in g_{m1}
- How does biasing the cascode stage affect noise performance?

Biased cascode noise

- The bias current of the cascode stage is developed based on the value of R_E and the supply voltage, and is approximately given by $I_C = (V_{EE} - 0.6V)/R_E$
- For a bias current of, say, $200\mu A$ and a supply voltage of $\pm 15V$, this gives $R_E \approx 72k\Omega$ (R_C will be a similar value)
- Still trying to limit the PSD of the thermal noise to 10% of the shot noise limit sets a lower bound on the signal current:

$$i_d \geq 10 \cdot \frac{4kT}{2q \cdot (R_E + R_C)} \approx 3.6\mu A \tag{17}$$

- The output noise contributed by Q_1 is minimized by R_E, and given by

$$i_{nc1,out}^2 = \frac{2qI_C}{(1 + g_{m1}R_E)^2} \tag{18}$$

Base current noise

- Recall that the collector current of the BJT is the difference of the emitter and base currents: $I_C = I_E - I_B$
- This means that even though I_C is I_B *less* than I_E, I_B's shot noise of $2qI_B$ still manifests as a noise current at the collector and affects the noise performance of the TIA
- In the biased cascode we have driven the collector current (and hopefully the thermal noise currents) to a negligible level but in the process we have *increased* base current shot noise, such that (in the best case)

$$i_{n,out} \approx i_{nb1} \tag{19}$$

- If $I_C = 200\mu A$ and $\beta = 200$ the resulting shot noise is equivalent to that of a $1\mu A$ photodiode current!
- Thus, we have solved our bandwidth problem but have increased the noise (somewhat substantially)

Regulated cascode

- We can achieve both low noise and low input impedance by applying feedback and decreasing Q_1's collector current to reduce both its shot noise and the thermal noise from R_E and R_{C1}
- The regulated (or, gain-boosted) cascode works by amplifying Q_1's emitter voltage and feeding it back to its base, effectively multiplying the transconductance of Q_1 by the voltage gain of the common-emitter amplifer formed by Q_2, R_{C2}
- The input impedance of the regulated cascode is given by

$$R_{in} = \frac{1}{g_{m1}(1 + g_{m2}R_{C2})} \tag{20}$$

- The result is an increase in the input pole frequency by the factor $g_{m2}R_{C2}$ *without* a substantial increase in Q_1's collector current (or the resulting base current shot noise)

Regulated cascode noise

- Q_1's shot noise is reduced even further by the g_m-boosting action of the Q_2 common-emitter stage, and is given approximately by

$$i_{nc1,out}^2 \approx \frac{2qI_{C1}}{(1 + g_{m2}R_{C2}g_{m1}R_E)^2} \tag{21}$$

- Q_2's shot noise sees a gain of

$$i_{nc2,out} = \frac{g_{m1}R_{C2}}{1 + g_{m1}R_E} i_{nc2} \tag{22}$$

Bootstrapping

- Recall that the mechanism producing rising noise due to e_{na} with frequency is the resulting current through the photodiode capacitance, $i_{e_{na}} = e_{na}\omega C_d$
- If we can ensure that the current through the diode capacitance is zero, the input noise due to e_{na} will be limited to the nominal value e_{na}/R_f
- This can be accomplished by the *feedforward* amplifier G, which is used to ensure that the noise voltage across the capacitance is zero, thus eliminating the rising amplifier noise
- How do we implement the feedforward amplifier?

Source follower bootstrapping

- The unity-gain amplifier can be realized as a JFET source follower, the input current of which and resulting current noise are small
- At moderate frequencies the capacitor noise voltage due to the amplifier voltage noise is

$$e_{n,C_d} = e_{na} - \frac{g_m R_{eq}}{1 + g_m R_{eq}} e_{na} = \frac{1}{1 + g_m R_{eq}} e_{na} \tag{23}$$

- The resulting noise current magnitude is

$$|i_{n,C_d}| = \frac{e_{na}\omega C_d}{1 + g_m R_{eq}} \tag{24}$$

- This is accurate to approximately g_m/C_d, at which point the source follower gain rolls off at $-20dB/dec$

Effective capacitance

- The current through C_d determines the *effective* capacitance seen by the amplifier's voltage noise
- The effective impedance is given by

$$|Z_{eff}| = \frac{1 + g_m R_{eq}}{e_{na}\omega C_d} e_{na} = \frac{1 + g_m R_{eq}}{\omega C_d} \tag{25}$$

- From this the effective capacitance is determined to be $C_d/(1 + g_m R_{eq})$
- Depending on the values of g_m and R_S boostrapping can reduce the effective capacitance by an order of magnitude or mor, where the reduction depends on the gain accuracy of the source follower
- This effective capictance appears in parallel with the input capcitance of the opamp and the gate-drain capacitance of the source follower, the sum total of which may not be insigificant when seeking wide TIA bandwidths
- In this case, choosing an opamp with low input capacitance becomes critical

Source follower noise

- The source follower still suffers from noise peaking of its own voltage noise, resulting in a noise current of

$$|i'_{n,C_d}| \approx e_n \omega C_d \tag{26}$$

- However, e_n can be designed to be significantly lower than the opamp voltage noise, particularly high-speed opamps which aren't necessarily optimized for low noise

Summary

- High sensitivity requires a large value of R_f, producing a low-frequency pole that limits the bandwidth achievable while maintaining stability (i.e. damping factor ζ greater than some minimum value)
- Photodiode capacitance degrades both bandwidth/stability and noise performance of TIAs, requiring circuit techniques that increase bandwidth while controlling noise
- Some circuit techniques that can be used to alleviate this tradeoff are
  - Cascoding to isolate the diode capacitance from the noise and feedback path of the amplifier
  - Bootstrapping to reduce the effective capacitance seen by the amplifier
  - A combination of the two