Microwave Filter Design 1/27/2017

1. Introduction

The filter is often an essential part of microwave system. The received or transmitted signal conditioning using different types of filters can be found in almost all microwave systems.

Typical applications of filters are

For communications systems:
    Limit a bandwidth of a transmitted signal
    Eliminate unwanted signal in RF
    Eliminate RF and LO signals after the mixer

For radars:
    Reject jamming signals

    Limit a bandwidth of a transmitted signal
    Eliminate unwanted signal in RF
    Eliminate RF and LO signals after the mixer

Similar to the low-frequency applications, there are four different types of microwave filters. They are
    Low Pass Filter (LPF)
    High Pass Filter (HPF)
    Band Pass Filter (BPF)
    Band Reject Filter (BRF)
        Simple stub

In EE463/579, the main focus will be on LPF and BPF because you will be fabricating these filters in your projects.

The examples of the previous design projects are shown in Figs. 1 and 2.

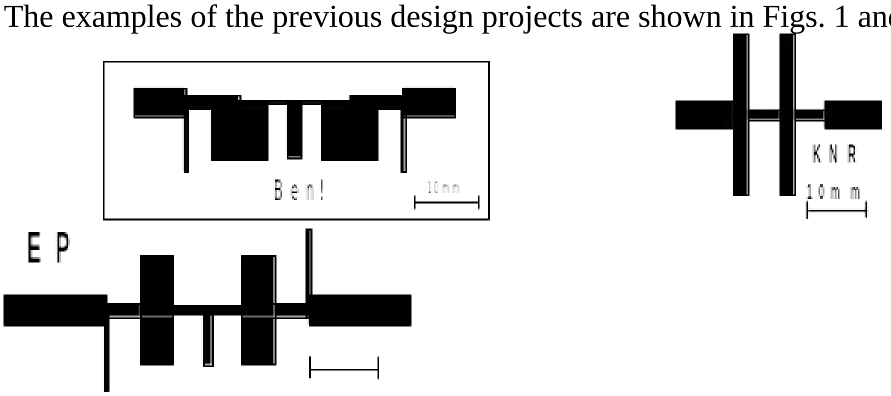


Fig. 1: Low pass filter (Cutoff=3 GHz)

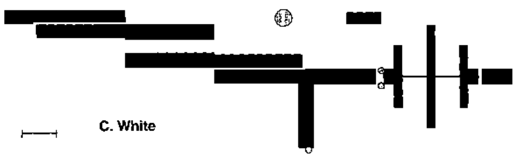


Figure 8. Printed circuit board layout.

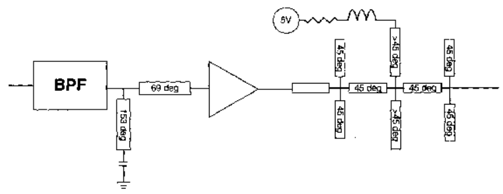


Figure 9. Block diagram/schematic.

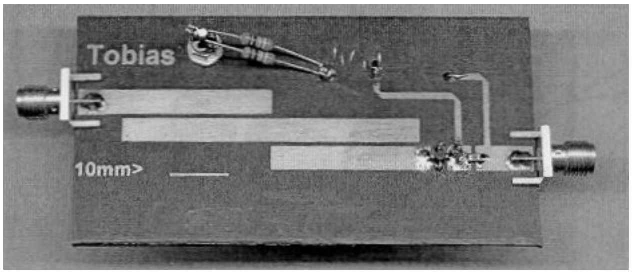


Fig. 2: Narrowband amplifier design and fabrication at 2 GHz (BW=100MHz)

2. Filter Design

There are many ways to design filters and some of them are discussed in the textbook such as Pozar. Rather than studying different techniques, we are interested in designing, fabricating, and testing a microwave filter in EE463/579. Therefore, we will focus on two methods namely the insertion loss method for LPF and stepped impedance method for LPF.

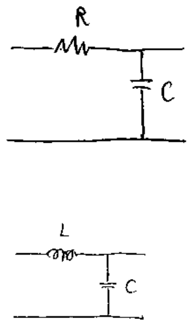
-LPF design by the insertion loss method involves
- (a) Lumped element model
- (b) Desired polynomial
    - -Maximally flat filter (binomial)
    - -Equal ripple filter (Chebyshev)
- (c) Filter transformation to get TL model
- (d) Realizable design

-LPF design by the stepped impedance involves
- (a) Lumped element model
- (b) Desired polynomial
    - -Maximally flat filter (binomial)
    - -Equal ripple filter (Chebyshev)
- (c) Conversion to TL model

Before discussing these two methods, we will take a look at simple microwave filters based on the knowledge learned in basic EE and EM courses.

2-1. Simple LPF with the lumped elements

A simple LPF can be designed using a RC or LC network as shown in Fig. 3.



$$T(\omega)=\frac{1}{1+\omega RC}$$

$$\omega_o=\frac{1}{RC}$$

$$T(\omega)=\frac{1}{1-\omega^2 LC}$$

$$\omega_o=\frac{1}{\sqrt{LC}}$$

Fig. 3: LPF based on lumped elements

The decay characteristics such as -20dB/dec and -40 dB/dec are determined by the number of reactive elements. The cutoff frequency is determined by the element values. It may not be clear but the passband and stopband frequency responses are also determined by the element values.

If we implement this simple LPF using the lumped elements with microstrip TL, we may see the circuit shown in Fig. 1. If we assume the cufoff frequency is 10 GHz and C=1pF, then the value of L must be 0.25nH. The circuit may look like the one shown in Fig.4.

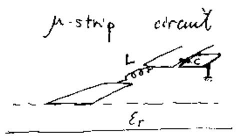


Fig. 4: LPF using lumped element and microstrip TL

Although there are many high quality microwave lumped elements these day, the problem of using the lumped elements is the inherent resonant frequency due to parasitic elements. For example, the equivalent circuit of a capacitor is L and C in series where as that of inductor is L and C in parallel. It is easy to show the inductor behaves like a capacitor beyond the resonant frequency. Then the circuit is no longer a LPF.

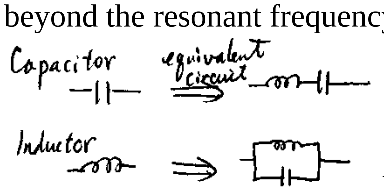
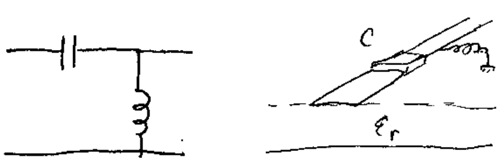


Fig. 5: Equivalent models of C and L.

To show this effect, let assume a HPF is designed using lumped element as shown in Fig. 6.



$$T(\omega)=\frac{\omega^2 LC}{\omega^2 LC-1}$$

Fig. 6: HPF and microstrip circuit implementation

Let us assume the value of C is 1pF and a resonant frequency due to parasitic L is 5 GHz. The equivalent circuit of C is

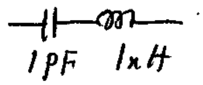


Fig. 7: Equivalent circuit of C. L is due to a parasitic inductance.

The total impedance is $Z = \frac{1}{j\omega C} + j\omega L$. At low frequency, this is obviously capacitive. Beyond the cutoff frequency, however, Z becomes positive or inductive. It may not be easy to construct microwave filters with lumped elements at very high frequency.

2-2. Microwave HPF with Waveguide

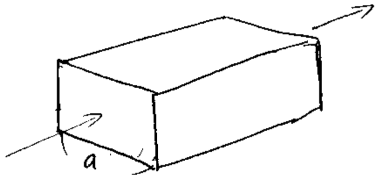


Fig. 8: Propagating mode TE_{10} mode

The rectangular waveguide is a HPF with the cutoff frequency

$$f_c = \frac{C_o}{2a} \qquad C_0 : 3*10^8 m/s$$

e.g X-band waveguide

a = 2.3 cm

then $f_c = \frac{C_o}{4.6} = 6.5\,GHz$

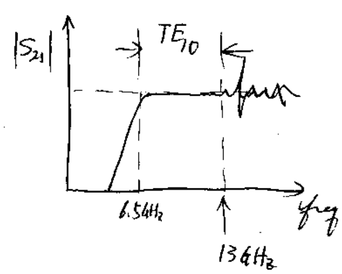


Fig. 9: frequency response of a X-band waveguide

2-3. Coupled Lines

e.g directional coupler

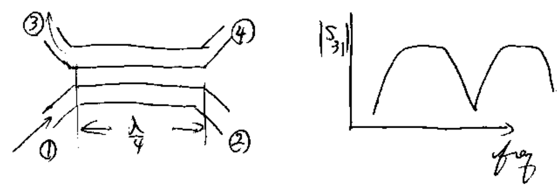


Fig. 10: Coupled lines

BPF response of λ/4 line

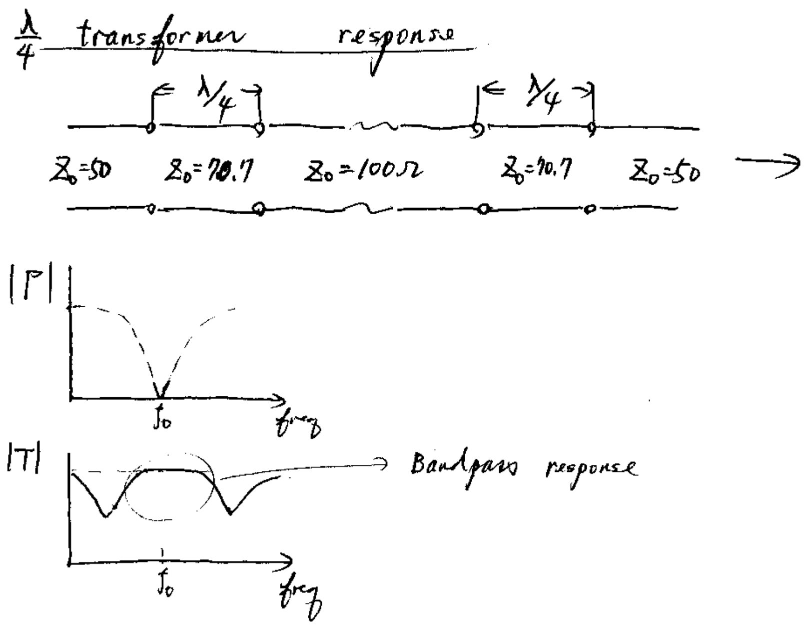


Fig. 11: Frequency response of λ/4 section

3. Filter Design by Insertion Loss Method

A simple RC LPF design process can be stated as.

1. Find the transfer function for a given circuit.
   Frequency response 20dB/dec, 40dB/dec → order
2. Determine the cutoff frequency
3. Find values of R and C

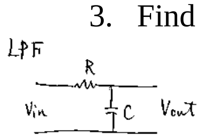


Fig. 12: RC circuit

$$\frac{Vout}{Vin} = \frac{1}{1 + j\omega RC}$$

$$|\frac{Vout}{Vin}|^2 = \frac{1}{1+\omega^2 RC}$$: Power

If we define $|V_{in}/V_{out}|^2$, this becomes a polynomial of order ω^2.

$$\left|\frac{Vin}{Vout}\right|^2 = 1 + \omega^2 RC$$

The order of polynomial will be determined by the number of reactive elements.

polynomial of ω^2		
1st order	ω^2	one reactive element
2nd order	ω^4	two reactive elements

3-1. Define Power Loss Ratio

Using the transfer function, we define the power loss ratio which is the same as $|V_{in}/V_{out}|^2$.

$$P_{LR} = \frac{P_{incident}}{P_{load}(delivered\ \text{power})}$$

Because in the lossless circuit, the power must be conserved.
The total incident power = Reflected Power + Transmitted Power
We can write this using the reflection and transmission coefficients as

$$1 = |\Gamma(\omega)|^2 + |T(\omega)|^2$$

The delivered power into the load is

$$P_{load} = |T(\omega)|^2 = 1 - |\Gamma(\omega)|^2$$

The power loss ratio is

$$P_{LR} = \frac{1}{1 - |\Gamma(\omega)|^2} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

M and N are polynomials of ω^2.

Now we need to relate the power loss ratio to a LPF.

Process to convert the power loss ratio to the circuit can be summarized as

1. Get the desired P_{LR}
2. Find $\Gamma(\omega)$ in terms of circuit elements
3. Calculate the values of circuit elements
4. Convert the lumped element model into microwave circuit

LPF is specified by the pass-band and stop-band characteristics. This can be related to the power loss ratio P_{LR}

P_{LR} **responses of different filter types**

1. Maximally flat (also known as butterworth or binomial)
   The passband has a flat response without ripple. The decay may not be as sharp as other types.

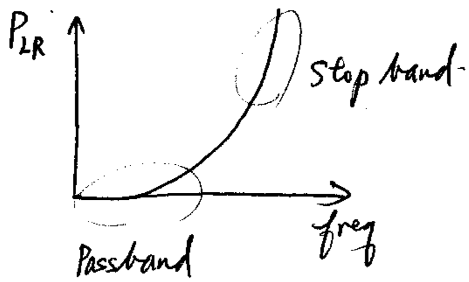


Fig. 13: Maximally flat

2. Equal ripple (Chebyshev)
   The passband has a small amount of ripple. A very sharp decay can be obtained. The stopband also has a ripple.

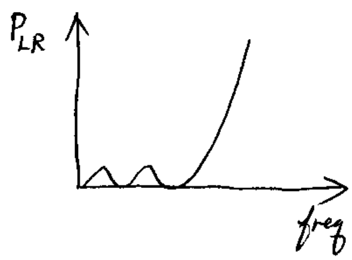


Fig. 14: Equal ripple (Chebyshev)

3. Linear phase
   The phase within the passband changes linearly wrt frequency.

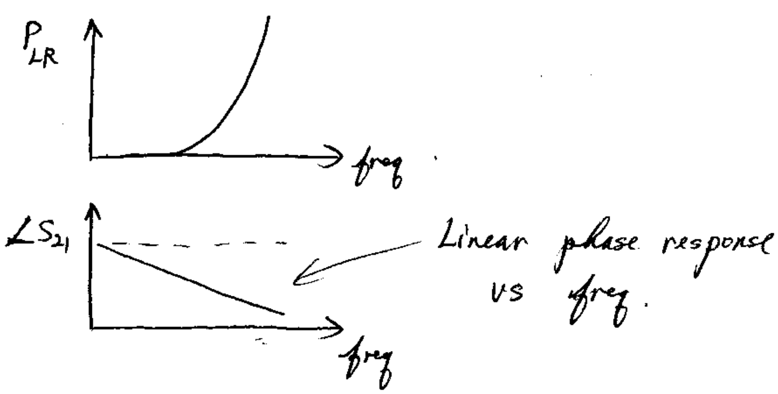


Fig. 15: Linear phase

3-2. Maximally Flat Filter (also known as butterworth or binomial)

The maximally flat LPF has a smooth (flat) response in the pass-band. It can eliminate the unwanted signal without changing the pass-band response.

The power loss ratio is a binomial polynomial (not correct, we are getting binomial coefficients) and it can be expressed as

$$P_{LR} = 1 + k^2 \left(\frac{\omega}{\omega_c} \right)^{2N}$$

ω_c : cutoff frequency

k^2 : constant to be determined

N : filter order

At the cutoff frequency $\omega = \omega_c$, the response must be -3dB. Therefore, we can find k.

$$P_{LR} = 1 + k^2 = 2$$

$$\therefore k = 1$$

When $\omega >> \omega_c$, we can simplify it as

$$P_{LR} \sim \left(\frac{\omega}{\omega_c} \right)^{2N}$$

$$10 \log P_{LR} = 20 N \log \left(\frac{\omega}{\omega_c} \right)$$

Therefore, the frequency response beyond the cutoff is -20N dB/dec.

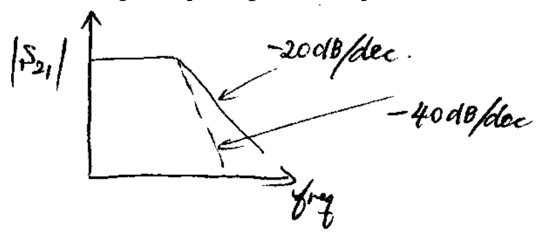


Fig. 16: Expected frequency response

Design Example

This example shows the process of determining the element values.

Let N = 2 2nd order filter

Also set ω_c = 1. This means the LPF is normalized to ω_c.

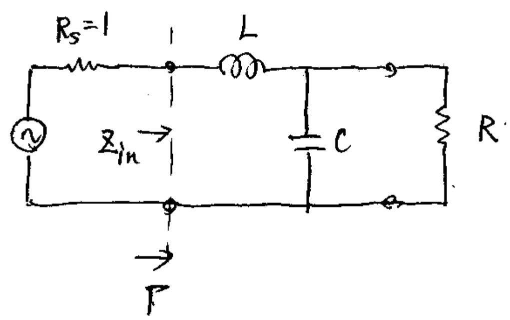


Fig. 17: Lumped element model

Although we have only 2 reactive elements, this circuit contains 3 parameters to decide (*L*, *C*, *R*). The value is normalized to Z_o=1. The desired value of *R* is *R* = 1 (matched to Z_o).
The polynomial is

$$P_{LR} = 1 + \omega^4$$

In terms of the circuit elements, we have

$$Z_{in} = j\omega L + \frac{R(1 - j\omega RC)}{1 + \omega^2 R^2 C^2}$$

$$\Gamma = \frac{Z_{in} - 1}{Z_{in} + 1}$$

Therefore, the power loss ratio in terms of *R*, *C* and *L* is

$$P_{LR} = \frac{1}{1 - |\Gamma|^2} = \frac{|Z_{in} + 1|^2}{2(Z_{in} + Z_{in}^*)}$$

$$= 1 + \frac{1}{4R}\left[(1 - R)^2 + (R^2C^2 + L^2 - 2LCR^2)\omega^2 + L^2C^2R^2\omega^4\right] = 1 + \omega^4$$

From this we can find

$$\begin{cases} 1 - R = 0 \\ R^2C^2 + L^2 - 2LCR^2 = 0 \\ \dfrac{1}{4R} L^2C^2R^2 = 1 \end{cases}$$

Because the desired value of *R* is 1, set *R*=1. Then

$$C^2 + L^2 - 2LC = (C - L)^2 = 0$$

$$\therefore C = L$$

$$\frac{1}{4}L^2C^2 = \frac{1}{4}C^4 = 1$$

Therefore

$$\therefore L = C = \sqrt{2}$$

Now we know all element values. The lumped element circuit is

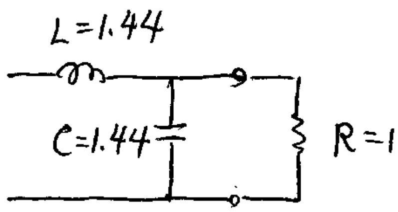
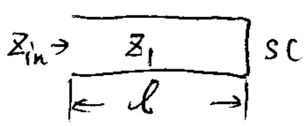


Fig. 18:

The next question is how to make capacitors and inductors with TL?

As we studied in TL section, both open-ended and short-ended TLs have a purely reactive input impedance. The value can be either inductive (positive) or capacitive (negative) depending on the TL length. These can be used as C and L.

For example, TL with short can be used as an inductor. If we set the TL length to be λ/8, then $\tan\beta\ell = 1$ and we get $Z_{in} = jZ_1$.



$$Z_{in} = jZ_1 \tan\beta\ell$$
$$\text{if } \ell = \frac{\lambda}{8}, \quad Z_{in} = jZ_1$$

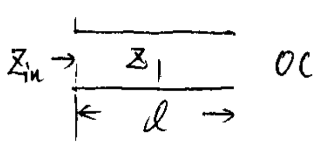
If we change Z_1, we can create different Z_{in} values

We need L=1.44 and at the designed frequency ω=1. Using TL with short, we can find the characteristic impedance of this TL to be

$$Z_{in} = jZ_1 = j\omega L = jL$$
$$\therefore Z_1 = L = 1.414$$

Let consider the capacitance. A short open-ended TL is capacitive. Assume the TL length *l*=λ/8.

$$Z_{in} = -\,jZ_1 \cot\beta\ell$$
$$\text{If } \ell = \frac{\lambda}{8}, \quad Z_{in} = -\,jZ_1 \quad (capacitive)$$



$$Y = \frac{1}{Z_{in}} = j\frac{1}{Z_1} = j\omega C$$

Since $\omega = 1$, Y = *jC*, we get

$$\therefore Z_1 = \frac{1}{C} = \frac{1}{1.414} = 0.707$$

The TL representation of our LPF is

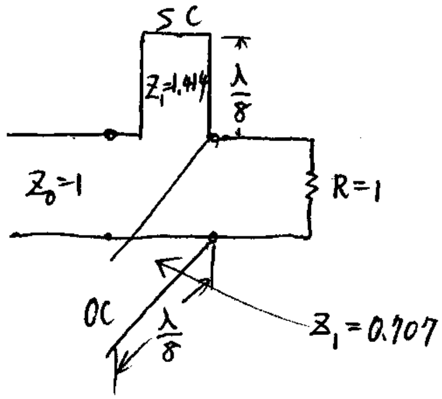


Fig. 19

We want to implement this circuit using microstrip TL. However, we have some problems.

1. Series TL is difficult to implement.
2. Shunt TL is easy to implement.
3. There is no space between series and shunt TLs (see Fig.19).

We need to change series TLs to parallel TLs. Also we need to add space between shunt TLs.

3-3. Kuroda’s identities

To convert the series TL to shunt, we use the Kuroda's identities. One of the Kuroda's identities states the following two TL circuits are identical. If we add an extra TL with a series TL (right figure), we can convert it into a shunt with an extra TL on the right side. The length of TLs are the same but the impedance values must be changed.

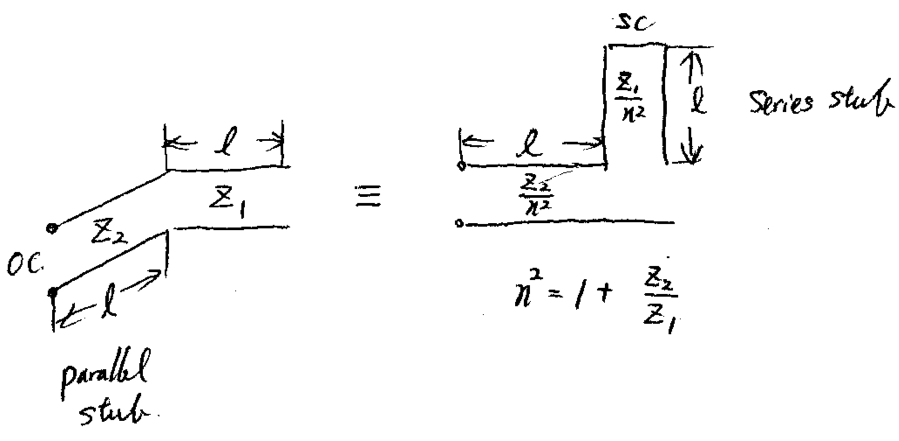
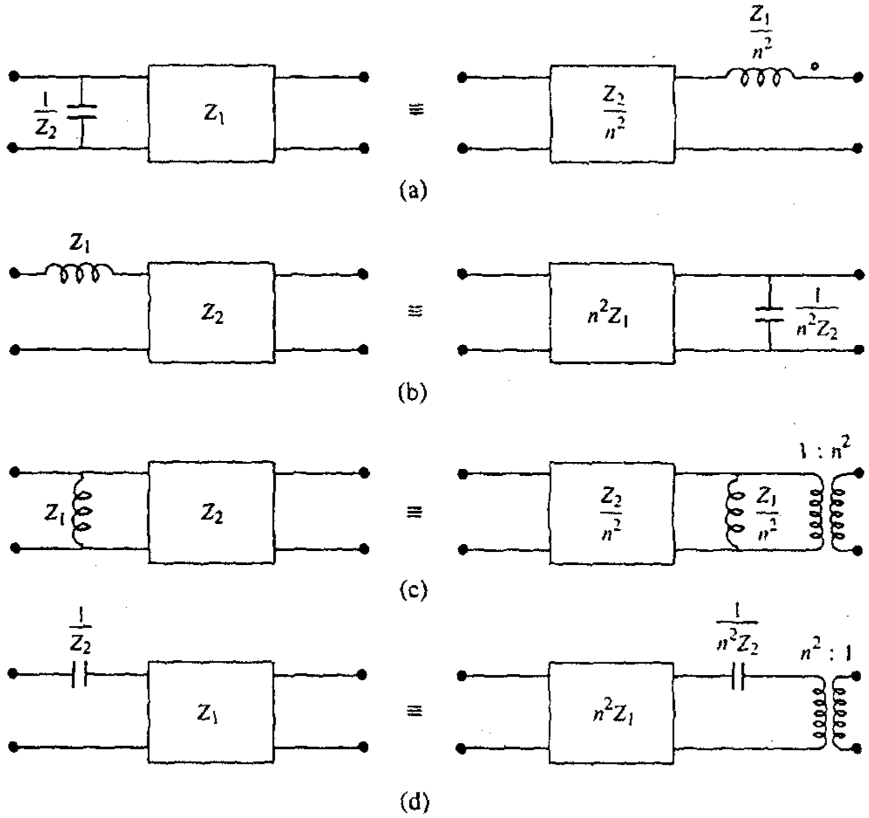


Fig. 20:

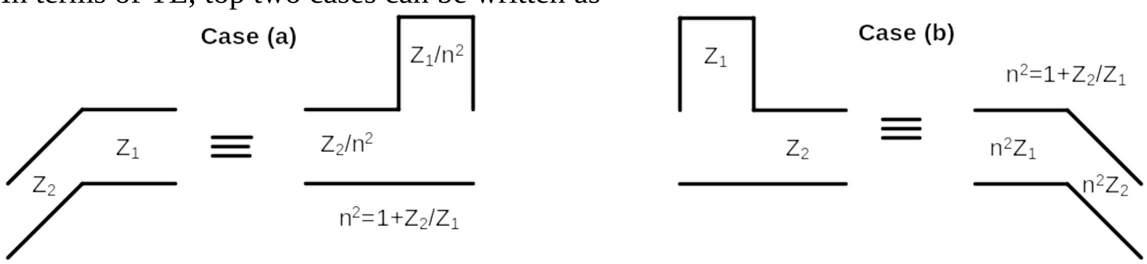
Other Kuroda's identities are shown below.

The Four Kuroda Identities



where $n^2 = 1 + Z_2/Z_1$

In terms of TL, top two cases can be written as



Note: All TL lengths are $\lambda/8$.

In our design we use the main TL section as an extra TL as shown in Fig. 21. These two TLs will be converted into a shunt TL and an extra TL.

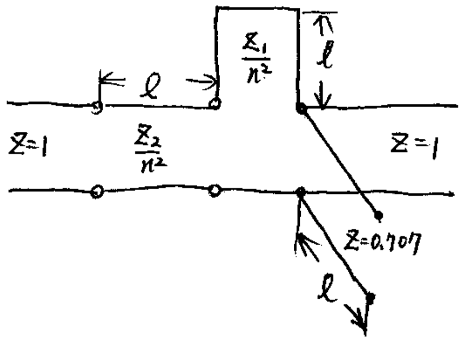


Fig. 21:

To obtain the values of new TLs, we use

$$\frac{Z_1}{n^2}=1.414$$

$$\frac{Z_2}{n^2}=1$$

Then, we get

$$n^2=1+\frac{Z_2}{Z_1}=1+\frac{1}{1.414}=1.707$$

$$Z_1=2.41$$

$$Z_2=1.707$$

Finally, we get a new TL circuit as

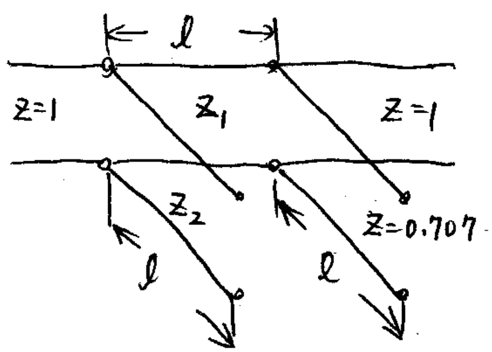


Fig. 22:

This circuit can be simulated with Designer (Serenade). First we use the ideal TL model and the TL discontinuities are not included. The length of 3 TL sections is set to $\lambda/8$. The simulation shows the -3dB point occurs at the designed frequency of 2 GHz. However, unlike the lumped element case, the decay characteristics is not -40dB/dec. Also there is another pass-band around 6 GHz. It is important to realize that although we started with the lumped element model, the response obtained with the LPF implemented with TL will not be the same as the lumped element model. The only two characteristics are the same. Those are the cutoff frequency and pass-band characteristics.

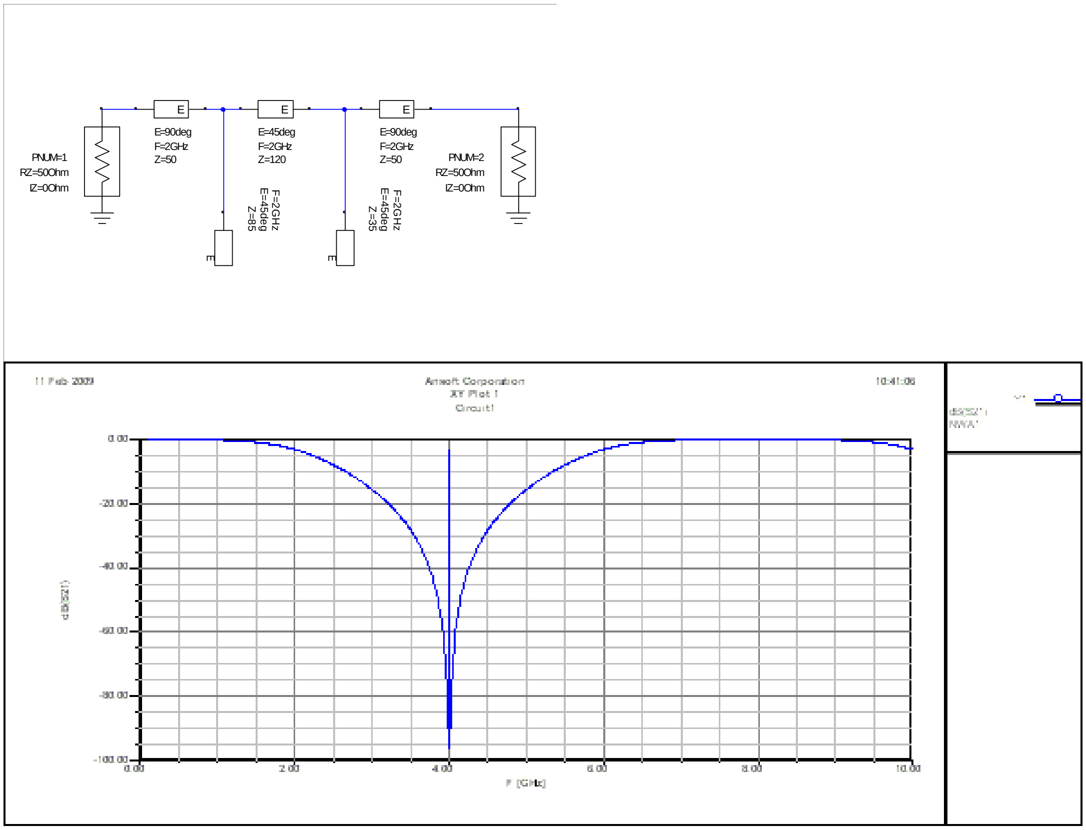
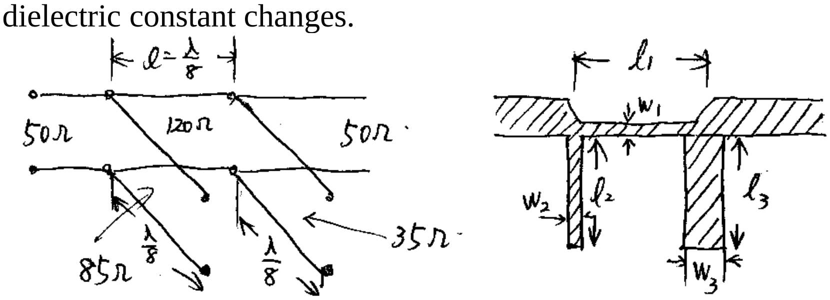


Fig. 23: ideal LPF using PLR method

3-4. Microstrip circuit implementation of LPF

The next step is to obtain the physical TL structure. Although the each section is given by λ/8, the actual length depends on the characteristic impedance because the effective dielectric constant changes.

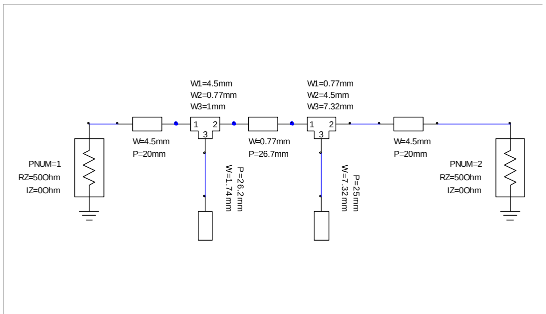


$Z_0 = 120\,\Omega \quad W_1 = 0.77\,mm, \qquad \varepsilon_{eff} = 1.97, \quad \ell_1 = 26.7\,mm$

$Z_0 = 85\,\Omega \quad W_2 = 1.74\,mm, \qquad \varepsilon_{eff} = 2.05, \quad \ell_2 = 26.2\,mm$

$Z_0 = 35\,\Omega \quad W_3 = 7.32\,mm, \qquad \varepsilon_{eff} = 2.25, \quad \ell_1 = 25\,mm$

Fig. 24: Example at 2 GHz. $\varepsilon_r = 2.6, \quad d = 1.59mm$



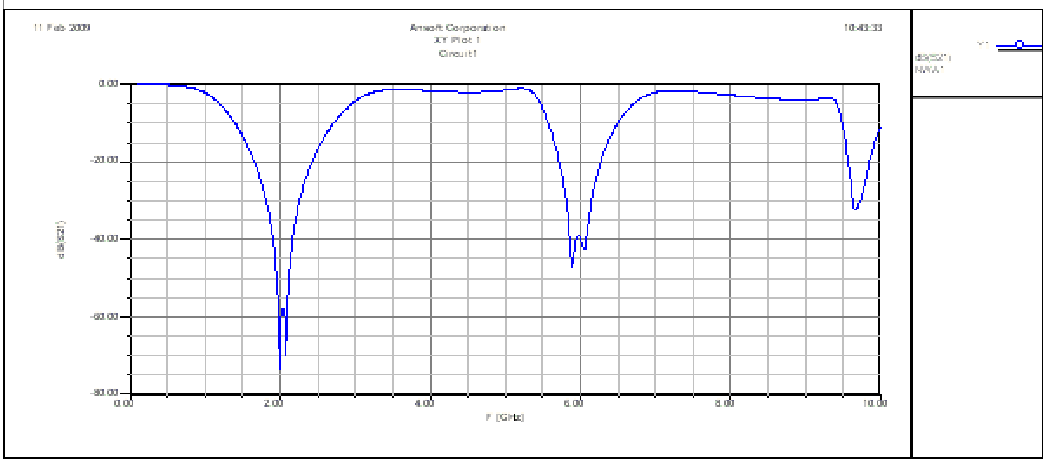


Fig.**: Simulation of LPF using the physical TL with T and OPEN. The cutoff frequency is no longer at 2 GHz.

The layout can be rearranged to create a symmetric circuit. The TL length should be determined so that the total admittance of stub will be the same as the single sided one. The length of new TL is not λ/16. The TL length should be determined such that a sum of two should have the same admittance as the original one.

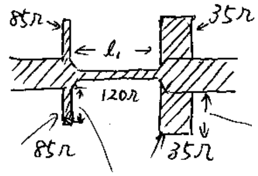
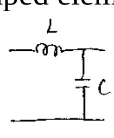
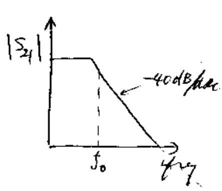


Fig. 25:

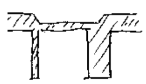
Here, we will briefly summarize the difference between the lumped element LPF and microwave LPF using the frequency response.

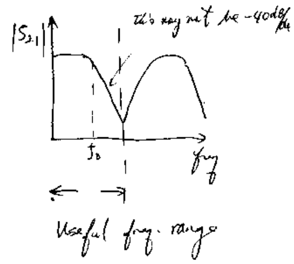
(i) Lumped element LPF





(ii) Microstrip LPF





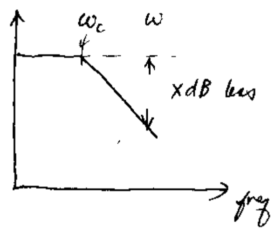
3-5. Summary of Filter Design Processes

(i) Specify filter characteristics

(1) Filter type

(2) Cutoff ω_c

(3) Minimum attenuation level at ω



Example:

f_c=2 GHz

-40dB attenuation at 4 GHz

Maximally flat type (Butterworth)

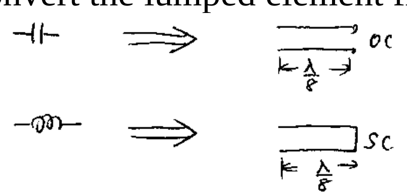
(ii) Using Fig 8.26 in the next page, find the required number of elements.

Normalized freq: $|\omega/\omega_c|-1=1$

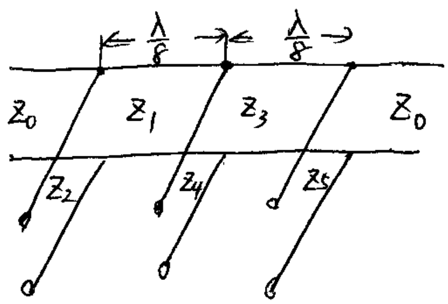
To get –40dB attenuation, 7 reactive elements are required.

(iii) Using Table 8.3 in the next page, find L and C element values.

(iv) Convert the lumped element filter to a TL circuit.



(v) Using Kuroda's identities, convert the circuit to a realizable microstrip circuit form.



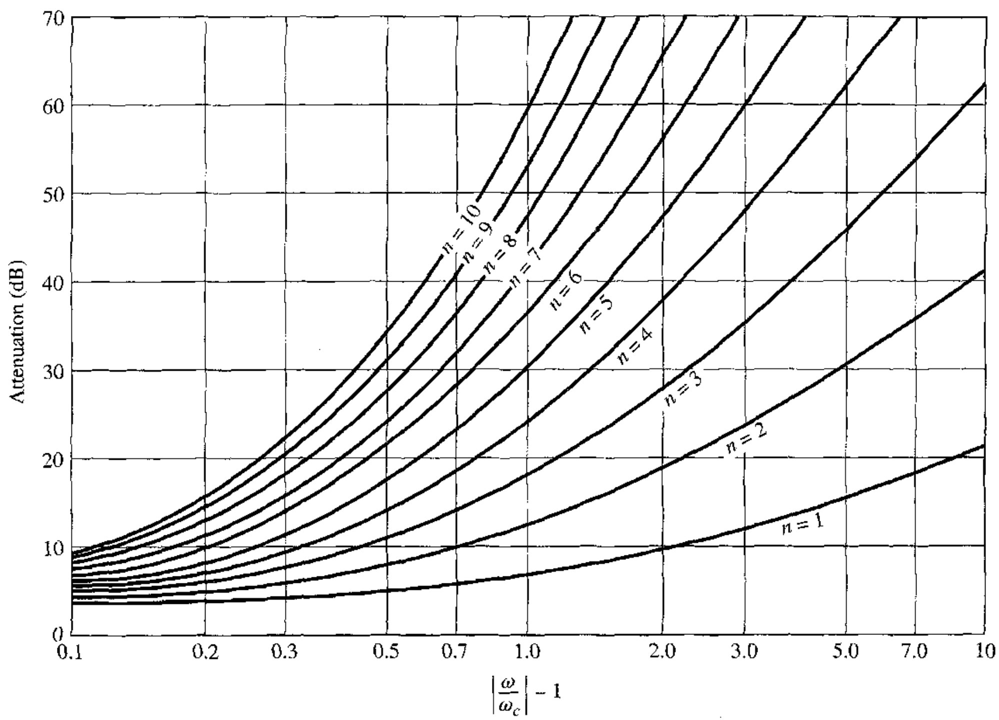


FIGURE 8.26 Attenuation versus normalized frequency for maximally flat filter prototypes. Adapted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Dedham, Mass.: Artech House, 1980) with permission.

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

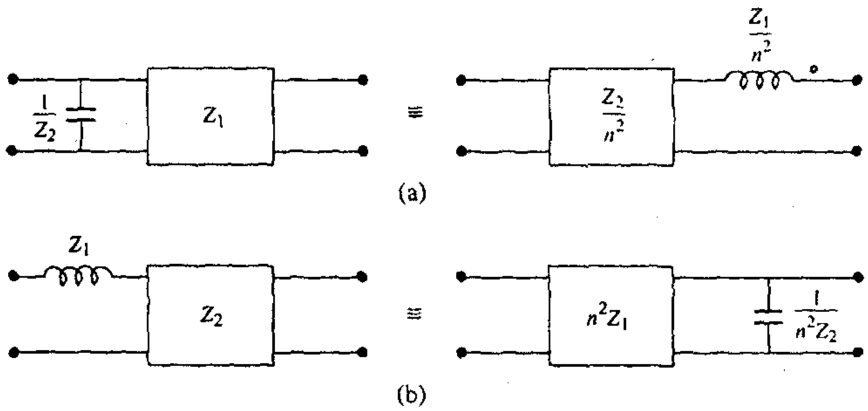
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Dedham, Mass.: Artech House, 1980) with permission.

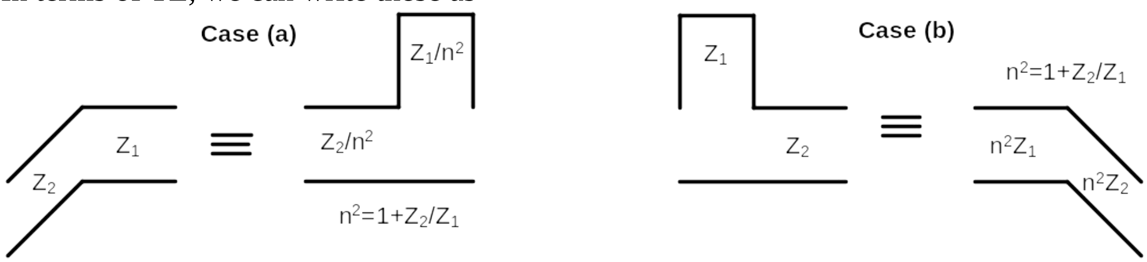
3-6. Design of the multi-element LPF

Converting the 2-element LPF into a realizable TL form is simple. However, when the number of elements becomes more than 4, we need to use the Koroda's identity repeatedly. The example is shown here.

Two of the Kuroda's identities are



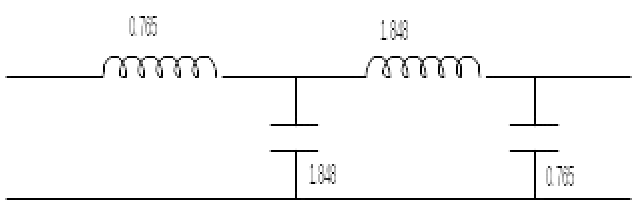
In terms of TL, we can write these as



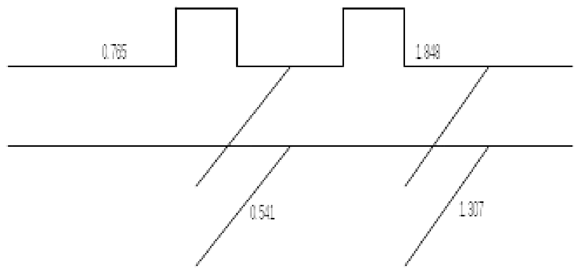
Filter type:

Butterworth LPF; N = 4; MF; f_0 = 8GHz; Z_0 = 50Ω

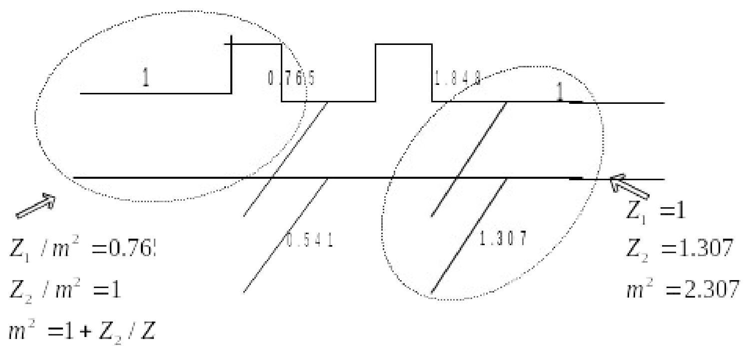
From the LPF design Table , the LP prototype is:



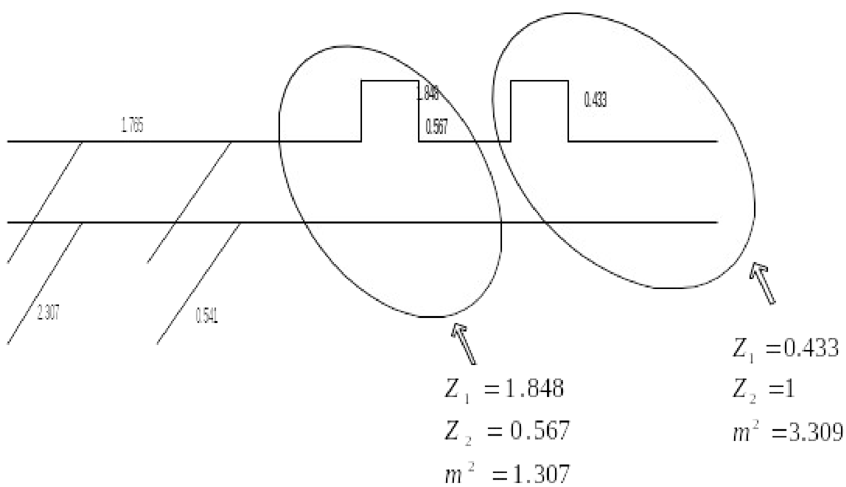
Applying Richard's transform (TL model), we get



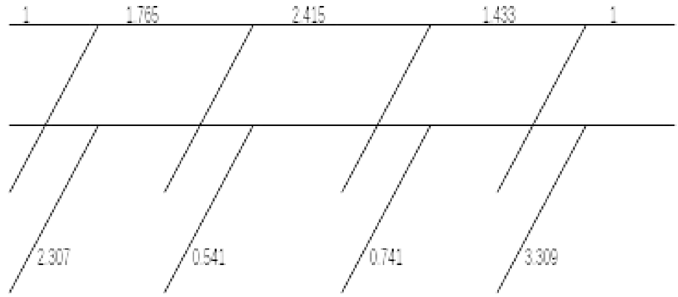
The above TL circuit has two problems. The first problem is 2 series TLs which must be converted to shunt. The second problem is the space between TLs. Currently, there is no space. We move from both ends to add spaces within the TL circuit.
Add unit *thru* elements to the left most series TL and also the right most shunt.
Use case (a) for both left and right.



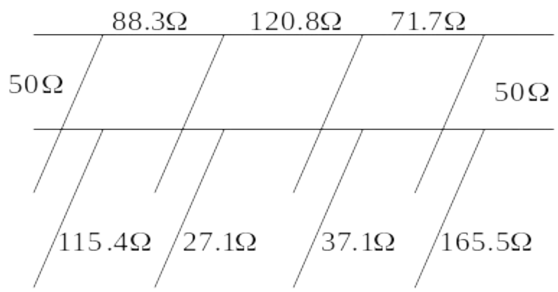
Use the second Kuroda identity on left which becomes shunt TL. Then apply the first Kuroda identity on right which becomes series TL. This process is needed to include space between TLs. Now we wave space everywhere except 1.848 series TL and 0.541 shunt. **Use case (b) for these**.



We combine 1.848 series with 0.567 *thru* and change them to shunt and *thru*. This process will move *thru* to the left side next to 0.541 shunt. After using second Kuroda identity twice to convert all series TL to shunt, we get.



After scaling all TLs to 50Ω, we have a final circuit. This circuit can be implemented with microstrip TL.



3-7. Equal Ripple LPF (Chebyshev)

The Chebyshev LPF will have a much sharper cutoff (decay) than that of Butterworth for the same number of elements. The disadvantage, however, is a small ripple within the passband. The Chebyshev LPF is usually specified in terms of allowable ripple within the passband such as 0.5dB or 3 dB. Another disadvantage is that the output impedance will not be Zo if the number of reactive elements is even. In this case an impedance matching section will be required.

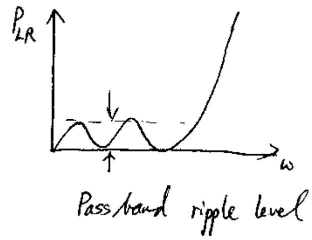
Power loss ratio

Similar to the maximally flat case, we express the power loss ratio using the Chebyshev polynomial. The constant *k* is related to the ripple level and must be determined.

$$P_{LR} = 1 + k^2 T_N^2(\omega)$$

$1 + k^2$: determines the ripple level

$T_N(\omega)$: Chebyshev polynomials



The Chebyshev polynomials of N=1 to N=3 are

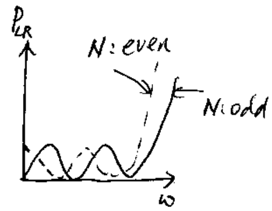
$$N=1 \quad T_1(\omega)=1$$

$$N=2 \quad T_2(\omega)=2\omega^2-1$$

$$N=3 \quad T_3(\omega)=4\omega^3-3\omega$$

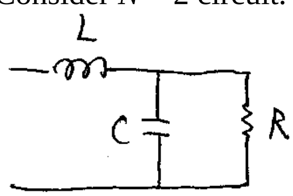
At ω = 0, the ideal value of P_{LR} is 1. However, for N even, we get $P_{LR} = 1 + k^2$.

$T_N(0)=$ ¿0 N: odd¿¿¿¿;¿ ¿



Example

Consider N = 2 circuit. We need to find L, C and R.



For N=2, we have

$T_2(\omega) = 2\omega^2 - 1$,

Then by relating P_{LR} to the circuit, we get

$$1 + k^2(4\omega^4 - 4\omega^2 + 1) = 1 + \frac{1}{4R}\left[(1-R)^2 + \left(R^2C^2 + L^2 - 2LCR^2\right)\omega^2 + L^2C^2R^2\omega^4\right]$$

At ω = 0

$$k^2 = \frac{(1-R)^2}{4R}$$

$$\therefore R = 1 + 2k^2 \pm 2k\sqrt{1+k^2}$$

This is valid only for N: even. If N is odd, R=1.

For $N = 3 \quad T_3(\omega) = 4\omega^3 - 3\omega$, $P_{LR} = 1 + k^2 T_N^2(\omega)$ becomes 1 at ω = 0.

The term for N=2

$k^2 = \frac{1}{4R}(1 - R)^2$ cannot be 0 because k becomes 0. Then R will not be 1.
******This is not correct for N=3.**** Need to calculate the term for N=3.**

Once k is given, R can be found from this. It is import to note that R is not 1 if N is even (output not matched).

L and C can be found by checking ω^2 and ω^4 terms as

$$4k^2 = \frac{1}{4R} L^2 C^2 R^2$$

$$-4k^2 = \frac{1}{4R}\left(R^2 C^2 + L^2 - 2LCR^2\right)$$

For N = even, $R \neq 1$
For N = odd, $R = 1$
If N = even, the output circuit must have an impedance transformer such as a $\lambda/4$ transformer.

Rather than specifying k, the maximum ripple level such as 0.5dB or 3 dB is used for finding element values. The figures and tables to find element values for a given attenuation characteristics are shown in the next pages.
For example, a 3dB ripple case gives us L=3.1013, C=0.5339 and R=5.8095. The corresponding value of k is k=0.9954.

Example

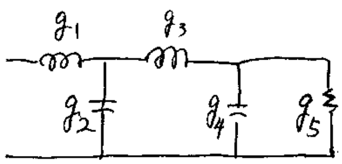
Assume we want f_c=2 GHz, 3dB ripple and -40 dB attenuation at 4 GHz.

Then the number of elements must be N=4 from Fig. **.

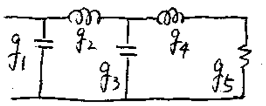
From Table 8.4 in page **, we can find these 5 values including the load impedance.

$$g_1 = 3.4389$$
$$g_2 = 0.7483$$
$$g_3 = 4.3471$$
$$g_4 = 0.5920$$
$$g_5 = 5.8095 = R$$

Notice that the output impedance R=5.8 (or 290 ohm if Zo=50 ohm).
This can be realized using either LCLC or CLCL as shown below.



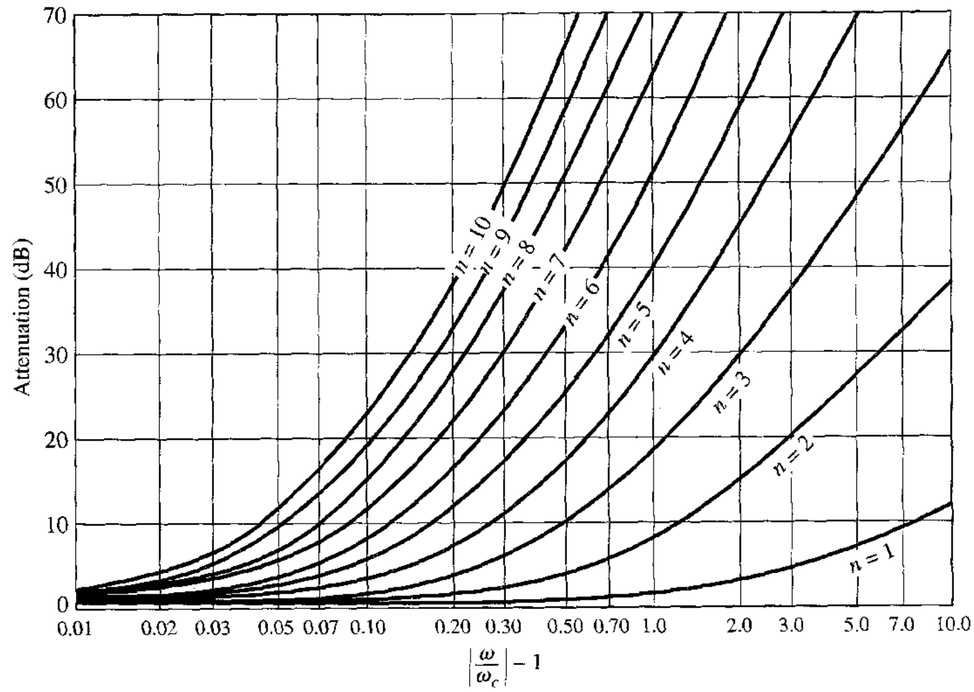
or



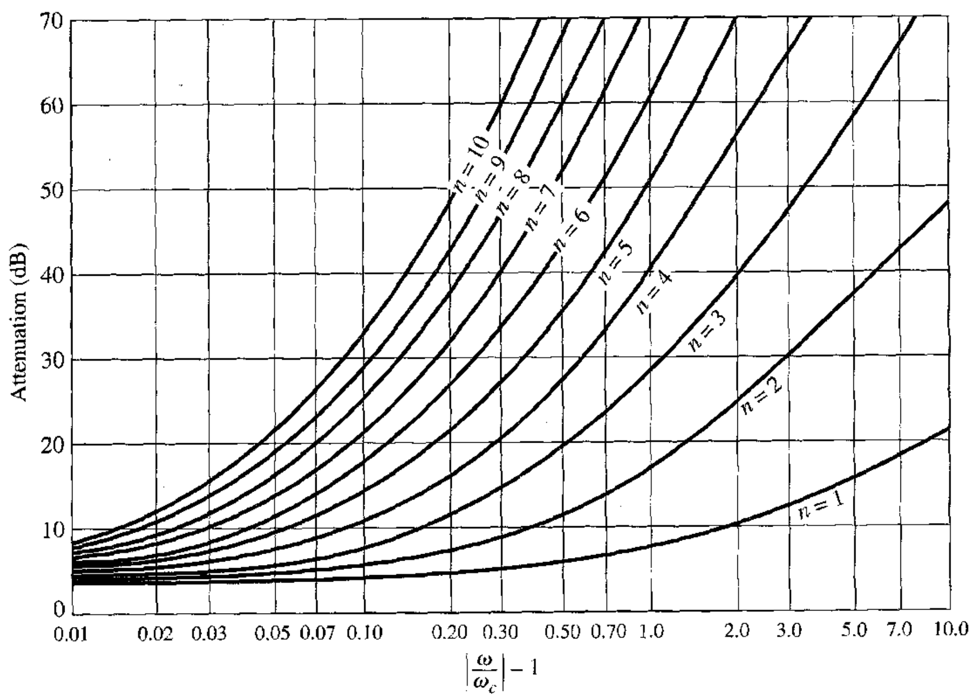
The simplest way to avoid the unmatched load condition is to increase *N*. If we choose N=5, we get

$$g_1 = 3.4817$$
$$g_2 = 0.7618$$
$$g_3 = 4.5381$$
$$g_4 = 0.7618$$
$$g_5 = 3.4817$$
$$g_6 = 1 = R$$

Once the element values are found, the transforming into TL is the same as before. Repeated use of Kuroda's identify may be required if *N* becomes greater than 4 as discussed before.



(a)



(b)

FIGURE 8.27 Attenuation versus normalized frequency for equal-ripple filter prototypes. (a) 0.5 dB ripple level. (b) 3.0 dB ripple level.

Adapted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Dedham, Mass.: Artech House, 1980) with permission.

TABLE 8.4 Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, N = 1 to 10, 0.5 dB and 3.0 dB ripple)

					0.5 dB Ripple						
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

					3.0 dB Ripple						
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

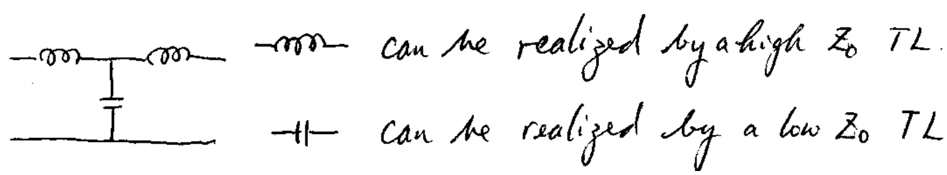
Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Dedham, Mass.: Artech House, 1980) with permission.

4. Stepped Impedance Filter Design (LPF only)

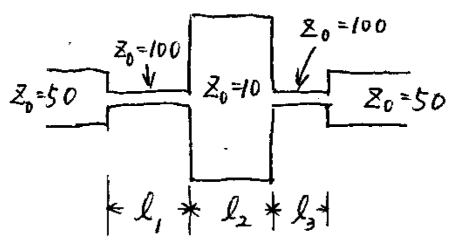
4-1. Introduction

The stepped impedance LPF is easy to analyze and simple to design. However, the filter performance is not as good as others. In particular, when the cutoff frequency becomes more than 2 GHz, the actual frequency response will be substantially different from that of calculated. In addition, the design process is applicable only to LPF.

Suppose we want to design a LPF given by L and C, we will show that L can be realized using a high-Zo TL and C can be realized using a low-Zo TL.



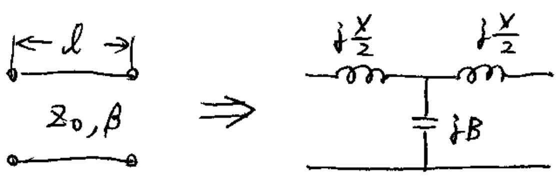
For example, if we have a microstrip circuit with 3 TL sections given by $Z_0 = 100\,\Omega$ and $Z_0 = 10\,\Omega$ as shown below, this effectively represent a LPF. The only question is what should be the length of 3 TL sections.



We need to determine ℓ_1, ℓ_2, and ℓ_3 to get desired LPF characteristics.

4-2. Equivalent circuit of a short TL

We know the equivalent circuit of TL is a cascaded L and C. The short section of TL given by an equivalent T circuit, therefore, can be expressed as a series inductance X/2 and a shunt B. This is a lumped element representation of a TL.

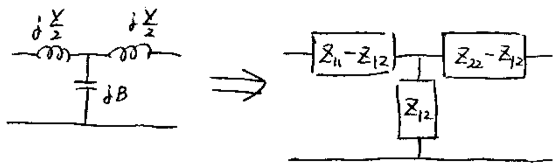


The same short TL can be represented using the ABCD parameters.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta\ell & jZ_0\sin\beta\ell \\ jY_0\sin\beta\ell & \cos\beta\ell \end{bmatrix}$$

This is a transmission line representation. Since we are describing the same structure, we should be able to relate the values of X and B to the TL length *l*.

Using the Z-parameters, the lumped element model of TL can be expressed as



$$[Z]=\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$j\frac{X}{2}=Z_{11}-Z_{12}=Z_{22}-Z_{12}$$

$$jB=\frac{1}{Z_{12}}$$

As we studied previously, $[Z]$ and ABCD parameters are related. The elements of $[Z]$ in terms of ABCD parameters are

$$Z_{11}=Z_{22}=\frac{A}{C}=\frac{D}{C}$$

$$Z_{12}=\frac{AD-BC}{C}$$

$$Z_{21}=\frac{1}{C}$$

Therefore

$$Z_{11}=\frac{\cos\beta\ell}{j\mathrm{Y}_0\sin\beta\ell}=-jZ_0\cot\beta\ell$$

$$Z_{22}=-jZ_0\cot\beta\ell$$

$$Z_{12}=\frac{\cos\beta\ell\cos\beta\ell-jZ_0\sin\beta\ell j\mathrm{Y}_0\sin\beta\ell}{j\mathrm{Y}_0\sin\beta\ell}=-jZ_0\csc\beta\ell$$

$$Z_{21}=-jZ_0\csc\beta\ell$$

Then

$$j\frac{X}{2}=Z_{11}-Z_{12}$$

$$j\frac{X}{2}=-jZ_0\left[\cot\beta\ell-\csc\beta\ell\right]=-jZ_0\left[\frac{\cos\beta\ell-1}{\sin\beta\ell}\right]=jZ_0\tan\frac{\beta\ell}{2}$$

$$jB=\frac{1}{Z_{12}}=j\frac{1}{Z_0}\sin\beta\ell$$

These relations reveal that the values of X and B can be chosen by adjusting the TL impedance and length *l*. In addition, by assuming the argument of tan(β*l*) and sin(β*l*) is small, we can create a simpler formula for X and B.

Case 1: Short high-Zo TL

$$Z_0 : large$$

$$\beta\ell < \frac{\pi}{4}$$

then

$$\tan\frac{\beta\ell}{2} \sim \frac{\beta\ell}{2}$$

$$\sin\beta\ell \sim \beta\ell$$

$$\frac{1}{Z_0} \sim 0$$

$$\left.\begin{aligned} \therefore X &\sim Z_0\beta\ell \\ B &\sim 0 \end{aligned}\right\}$$

This TL is an effective inductance.

The value of the inductance is related to the length of the section as

$$\beta\ell = \frac{X}{Z_0} = \frac{LR_0}{Z_h}$$

L: Normalized element value

R_0 : Characteristic impedance $R_0 = 50\,\Omega$

Z_h : Characteristic impedance of high Z sections.

Case II: Short low-Zo TL

$$Z_0 : small$$

$$\beta\ell < \frac{\pi}{4}$$

then

$$\tan\frac{\beta\ell}{2} \sim \frac{\beta\ell}{2}$$

$$\sin\beta\ell \sim \beta\ell$$

$$Z_0 \sim 0$$

$$\left.\begin{aligned} \therefore X &\sim 0 \\ B &\sim Y_0\beta\ell \end{aligned}\right\}$$

This TL is an effective capacitance.

The value of the capacitance is related to the length of TL as

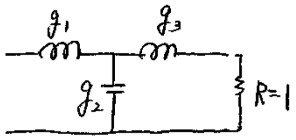
$$\beta\ell = \frac{B}{Y_0} = \left(\frac{C}{R_0}\right) Z_l$$

C: Normalized element value

Z_l : Characteristic impedance of low Z sections.

Example:

Suppose we want to design a 3 element maximally flat LPF using the step impedance method. For a given filter characteristics, we can obtain the values of normalized L and C from the table. They are $g_1 = 1, g_2 = 2, g_3 = 1$.



We also use $Z_h = 100\Omega$ and $Z_l = 10\Omega$.

For a given value of $g_1=1$, we need to find the length l_1. This section is an effective inductance and we use

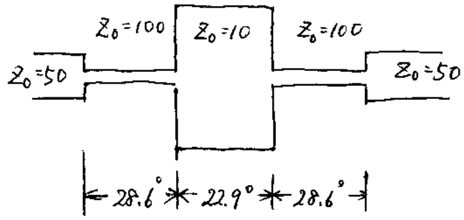
$$\beta\ell_1 = \frac{g_1 R_0}{Z_h} = \frac{1*50}{100} = 0.5\,radian = 28.6^0$$

For a given value of $g_2=2$, we need to find the length l_2. This section is an effective capacitance and we use

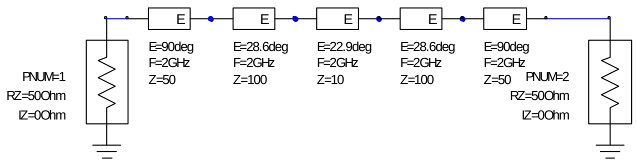
$$\beta\ell_2 = \frac{g_2 Z_l}{R_0} = \frac{2*10}{50} = 0.4\,radian = 22.9^0$$

Similarly for g_3, we get

$$\beta\ell_3 = \frac{g_3 R_0}{Z_h} = \frac{1*50}{100} = 0.5\,radian = 28.6^0$$



These values are normalized with respect to the cutoff wavelength. Once we decide the cutoff frequency, we can obtain the physical dimension of TLs.



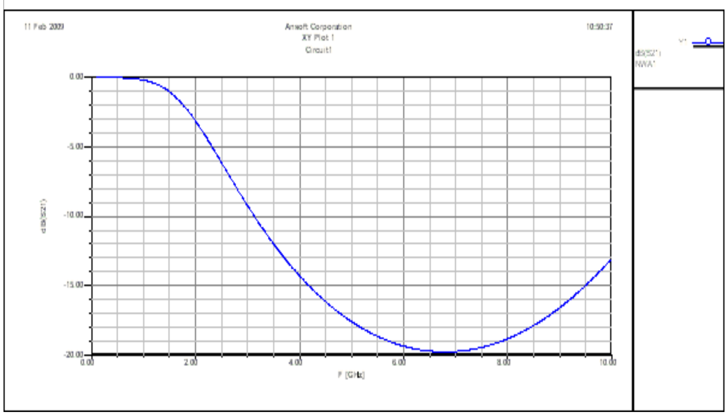
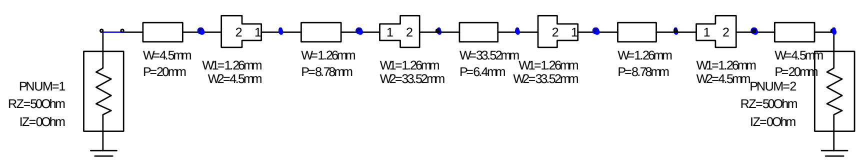


Fig. **: Simulations of ideal stepped impedance LPF



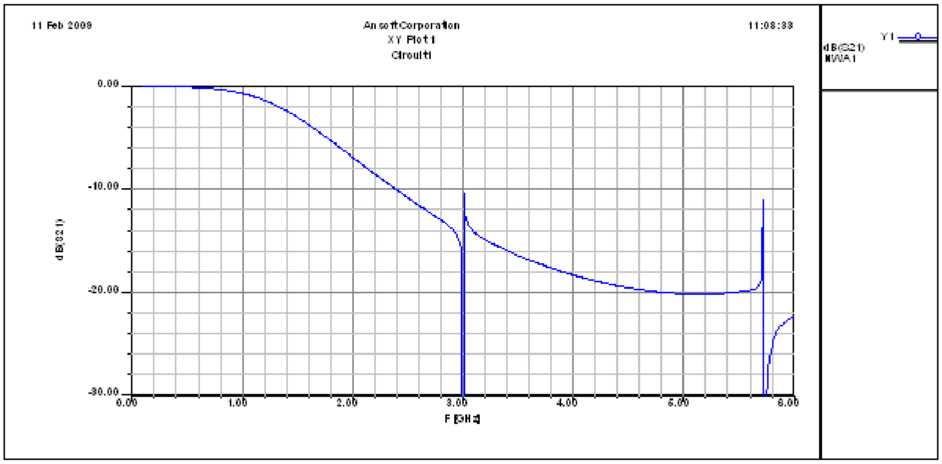


Fig.**: Simulations of stepped impedance LPF using physical TL

Some problems in derivation

Too many approximations are used in the derivation. The results may not be accurate

In many cases $\beta \ell$ is close to or larger than $\pi / 4$.

5. Filter Transformation

5-1. Impedance and frequency scaling

In the previous section, LPF is designed for $\omega_c = 1$ and $R_s = 1$. The element values are normalized with respect to these values. To obtain the actual value, we need to unnormalize L and C as

<u>Impedance scaling to obtain actual values</u>

$$L' = R_0 L$$
$$C' = \frac{C}{R_0}$$
$$R_s' = R_0$$
$$R_L' = R_0 R_L$$

where R_0 is characteristic impedance of the system.

<u>Frequency scaling</u>

$$L_k' = \frac{L_k}{\omega_c}$$
$$C_k' = \frac{C_k}{\omega_c}$$

where ω_c is a cutoff frequency.

If we combine these, we get L_k' and C_k' as

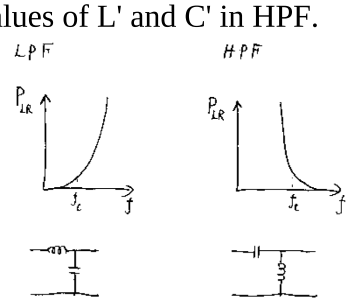
$$L_k' = \frac{R_0 L_k}{\omega_c}$$
$$C_k' = \frac{C_k}{R_0 \omega_c}$$

5-2. Filter Transformation and HPF Design

If we replace C by L and L by C in LPF, we can transform LPF to HPF as shown below. We need to find the values of L' and C' in HPF.

$L \to C'$

$C \to L'$



Assume ω_c is the cutoff frequency. We transform frequency by taking an inverse.

$$-\frac{\omega_c}{\omega} \to \omega \qquad \left(or\ \omega \to -\frac{\omega_c}{\omega}\right)$$

Then we get new L and C values as

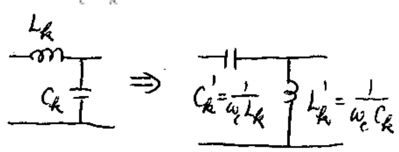
$$jX_k = j\omega L_k = j\left(-\frac{\omega_c}{\omega}\right)L_k = \frac{1}{j\omega C_k^{'}}$$

$$jB_k = j\omega C_k = j\left(-\frac{\omega_c}{\omega}\right)C_k = \frac{1}{j\omega L_k^{'}}$$

The values of $L_k^{'}$ and $C_k^{'}$ are given by

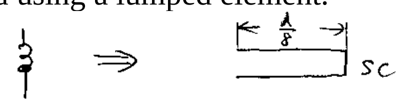
$$C_k^{'} = \frac{1}{\omega_c L_k}$$

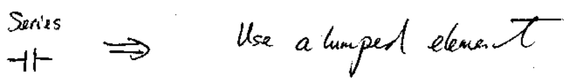
$$L_k^{'} = \frac{1}{\omega_c C_k}$$

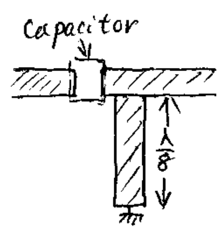


Microstrip TL implementation

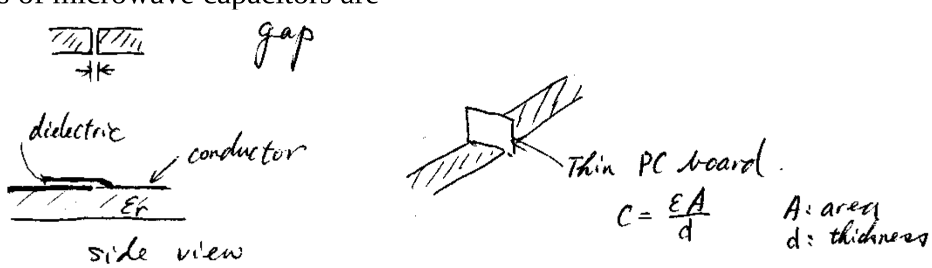
The shunt L can be realized using a TL as shown below. However, a series C must be designed using a lumped element.





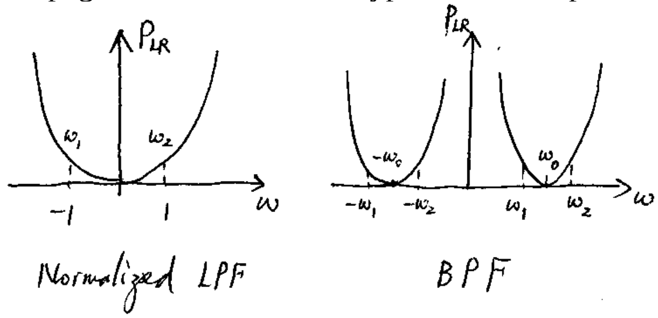


Examples of microwave capacitors are



5-3. Band Pass Filter (BPF) Design

Similarly LPF can be transformed into BPF using the frequency transformation. L must be replaced by series L and C whereas C must be replaced by parallel L and C. The table in the next page shows the element types and corresponding filter types.



To create a desired frequency response, we will replace ω of LPF by

$$\omega \leftarrow \left(\frac{\omega_0}{\omega_2-\omega_1}\right)\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)$$

$$\Delta=\frac{\omega_2-\omega_1}{\omega_0}$$: fractional bandwidth

at $\omega=\omega_0$ $\quad \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=0$

at $\omega=\omega_1$ $\quad \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=-1$

at $\omega=\omega_2$ $\quad \frac{1}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=1$

Now we change the original ω in X and B with a new frequency. The new filter elements are given by

$$jX_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)L_k = j\omega L_k' - j\frac{1}{\omega C_K'}$$

$$L_k' = \frac{L_k}{\Delta\omega_0}$$

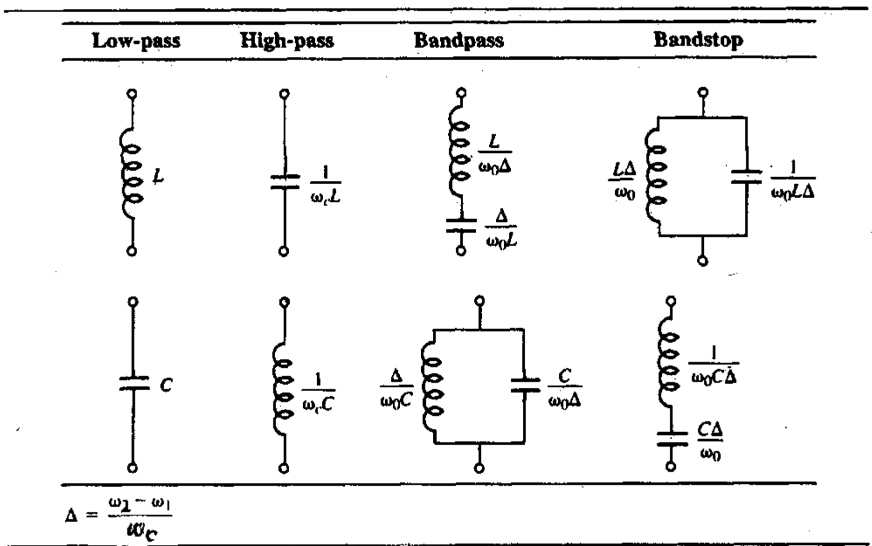
$$C_k' = \frac{\Delta}{\omega_0 L_k}$$

$$jB_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)C_k = j\omega C_K' - j\frac{1}{\omega L_K'}$$

$$L_k' = \frac{\Delta}{\omega_0 C_k}$$

$$C_k' = \frac{C_k}{\Delta\omega_0}$$

Table 8.6 Summary of Prototype Filter Transformation



Example:

BPF Specifications

$\left(0.5dB\right.$ ripple response

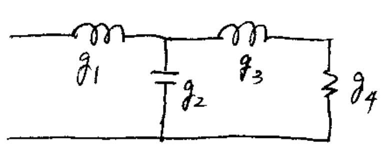
$\left(N=3\right.$

$\left(f_0=1GHz\right.$ (

$\left(R_0=50\Omega\right.$ (Z_0)

$\left(\Delta=10\%\right.$

<u>Initial LPF circuit implementation</u>



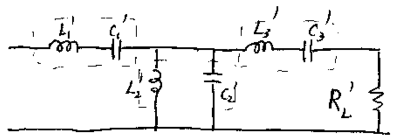
$g_1 = 1.5963 = L_1$

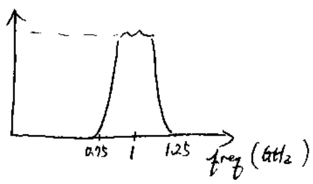
$g_2 = 1.0967 = C_1$

$g_3 = 1.5963 = L_2$

$g_4 = 1.00 = R_L$

<u>BPF circuit transformation</u>





$$L_1^{'}=\frac{L_1 Z_0}{\omega_0 \Delta}=127\,nH$$

$$C_1^{'}=\frac{\Delta}{\omega_0 L_1 Z_0}=0.199\,pF$$

$$L_2^{'}=\frac{\Delta Z_0}{\omega_0 C_2}=0.726\,nH$$

$$C_2^{'}=\frac{C_2}{\omega_0 \Delta Z_0}=34.91\,pF$$

$$L_3^{'}=\frac{L_3 Z_0}{\omega_0 \Delta}=127\,nH$$

$$C_3^{'}=\frac{\Delta}{\omega_0 L_3 Z_0}=0.199\,pF$$