Discontinuities and Parameter Extraction Nov. 1, 2012

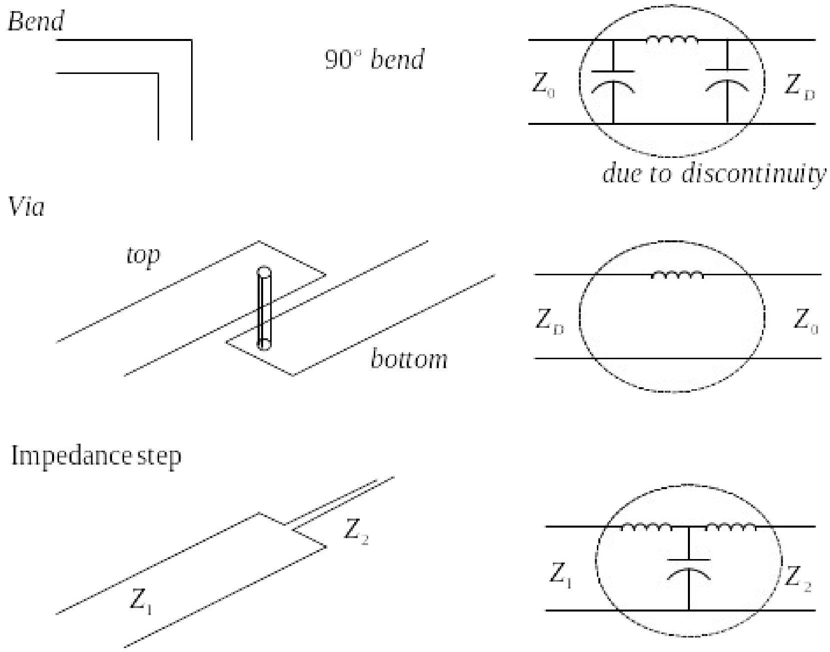
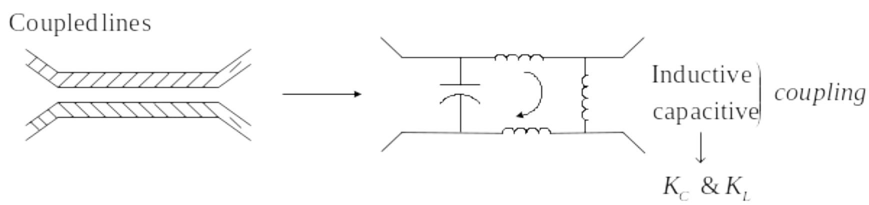
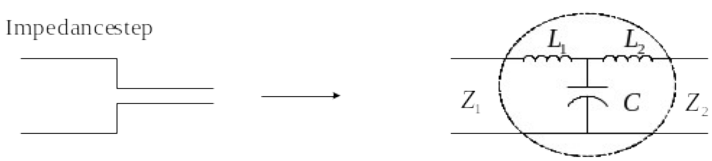


Fig. 1-1: Examples of discontinuities

2. Parameter Extraction Techniques

Lumped element approach





Distributed TL approach

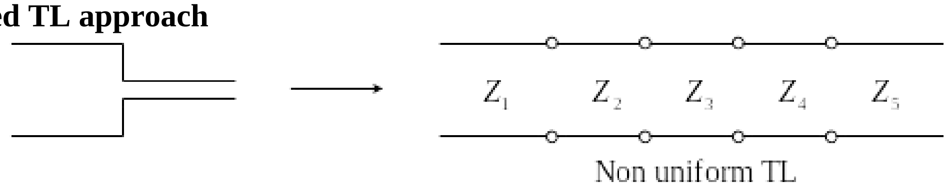
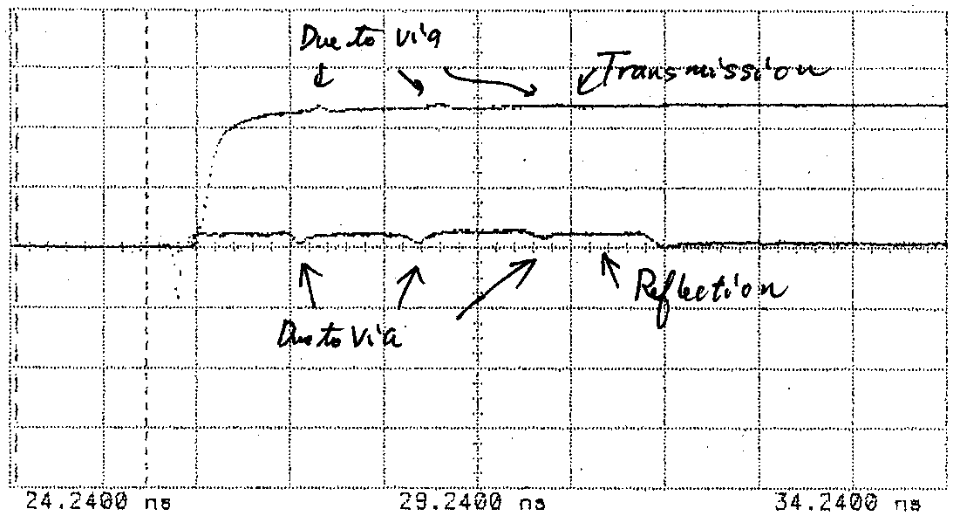


Fig. 2-1: Lumped element and distributed TL approaches

Cascaded VIA on PCB



Ch. 1	=	80.00 mVolts/div	Offset	=	200.0 mVolts
Ch. 2	=	80.00 mVolts/div	Offset	=	0.000 Volts
Timebase	=	1.00 ns/div	Delay	=	24.2400 ns
Delta T	=	1.4000 ns			
Start	=	24.3000 ns	Stop	=	25.7000 ns

Trigger is Freerunning at 500 kHz with Step on

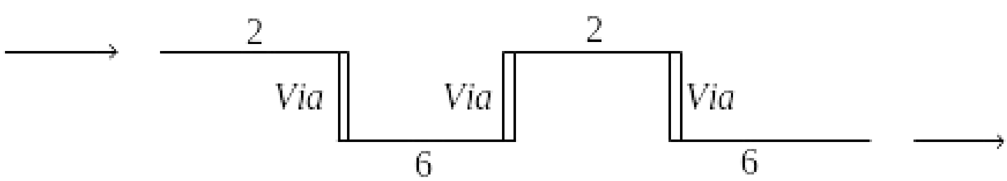


Fig. 2-2: TDR time-domain responses of VIAs (top). The physical layout of FR-4 PCB (bottom). 2 and 6 stand for layer 2 and layer 6 in the multilayer PCB. The input signal has a rise-time of ~0.2 ns.

3. Useful methods and formula for network analysis

3-1. Shift in Reference Planes

The modern microwave network analyzer (NWA) such as Agilent 8720 has the PORT EXTENSION (one-way phase shift) and ELECTRICAL DELAY (two-way phase shift) capabilities. These are signal processing functions performed on the measured data. NWA must be calibrated at a certain location (usually the end of the test cable) to obtain the accurate data. This location, however, is often different from the measurement plane at which the device must be characterized. The process of moving the calibration position to the desired position is called port extension. Figure 4 shows one example of the port extension applied for the desired impedance measurement technique. If the TL is lossless and the distance *l* is known, the port extension is essentially the phase shift given by $\theta=\beta l$ where β is the propagation constant. In Fig. 3-1, the input impedance Z_{in} can be expressed by Z_L, Z_o, β, and l. Rather than calculating Z_L from Z_{in}, if we can remove the phase shift caused by βl, we should be able to get Z_L directly from the instrument. The port extension on NWA does this function.

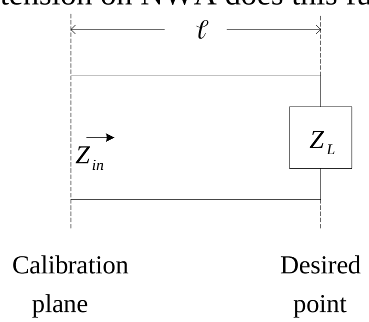
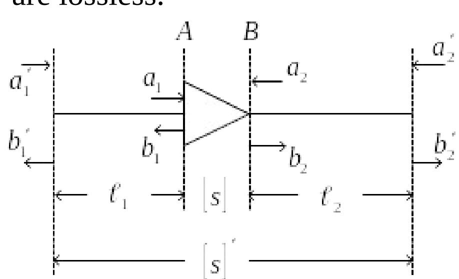


Fig. 3-1: NWA uses the port extension to convert Z_{in} to Z_L.

In this section, we will derive the formula used for the port extension. Assume NWA is calibrated at the plane specified by [S]'. The desired device is given by [S]. We want to relate the measured [S]' to the desired [S]. We assume the TL connecting between [S] and [S]' are lossless.



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = [S] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\begin{bmatrix} b_1' \\ b_2' \end{bmatrix} = [S'] \begin{bmatrix} a_1' \\ a_2' \end{bmatrix}$$

Measured S-parameter: [S]’
Desired S-parameter: [S]
Assume A and B are reference planes.

Fig. 3-2: Relationship between [S]' and [S].

We can relate [S]' to [S] using

$$a_1^{'} = a_1 e^{+j\beta\ell_1} = a_1 e^{j\theta_1}$$
$$a_2^{'} = a_2 e^{+j\beta\ell_2} = a_2 e^{j\theta_2}$$
$$b_1^{'} = b_1 e^{-j\theta_1}$$
$$b_2^{'} = b_2 e^{-j\theta_2}$$

Then we have

$$\begin{bmatrix} b_1^{'} e^{j\theta_1} \\ b_2^{'} e^{j\theta_2} \end{bmatrix} = [S] \begin{bmatrix} a_1^{'} e^{-j\theta_1} \\ a_2^{'} e^{-j\theta_2} \end{bmatrix}$$

$$\begin{bmatrix} e^{j\theta_1} & 0 \\ 0 & e^{j\theta_2} \end{bmatrix} \begin{bmatrix} b_1^{'} \\ b_2^{'} \end{bmatrix} = [S] \begin{bmatrix} e^{-j\theta_1} & 0 \\ 0 & e^{-j\theta_2} \end{bmatrix} \begin{bmatrix} a_1^{'} \\ a_2^{'} \end{bmatrix}$$

$$\begin{bmatrix} b_1^{'} \\ b_2^{'} \end{bmatrix} = \underbrace{\begin{bmatrix} e^{-j\theta_1} & 0 \\ 0 & e^{-j\theta_2} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & 0 \\ 0 & e^{-j\theta_2} \end{bmatrix}}_{[S]^{'}} \begin{bmatrix} a_1^{'} \\ a_2^{'} \end{bmatrix}$$

Finally, we obtain

$$\therefore \quad [S] = \begin{bmatrix} e^{j\theta_1} & 0 \\ 0 & e^{j\theta_2} \end{bmatrix} [S^{'}] \begin{bmatrix} e^{j\theta_1} & 0 \\ 0 & e^{j\theta_2} \end{bmatrix}$$

3-2. Cascaded Network

It is common to find a system consisting of many devices in series connection. This is called the cascaded network and one example is shown in Fig. 3-3. Assume we know the S-parameters of all devices. What we need to find is the S-parameter of the cascaded network given by [S_{total}] in Fig. 3-3.

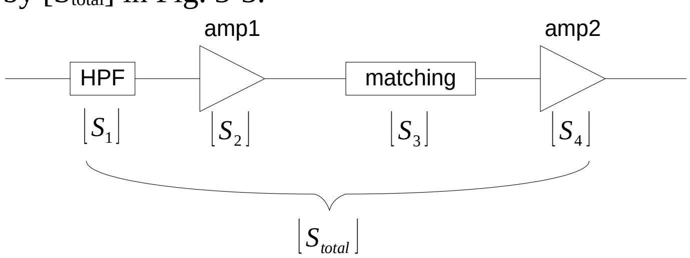


Fig. 3-3: Cascaded network

What is $[S_{total}]$? W cannot obtain the total S-parameter by taking a matrix multiplication of cascaded [S].

$$[S_{total}] \neq [S_1][S_2][S_3][S_4]$$

This is because the definition of S-parameter contains both input and output on the left side as shown below. $[S]$ is defined as

Input side ↖

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = [S] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

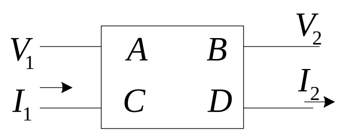
Output side

To analyze the cascaded network, we usually use ABCD or T-parameters. ABCD parameter is defined in terms of total voltage and current and it is similar to Z- and Y-parameters.

We write the two-port network using ABCD-parameter as

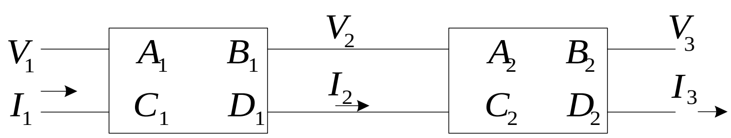
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Note: direction of I2



It is important to remind that V_1 and V_2 are total voltages and contain both incident and reflected voltages. Similarly, I_1 and I_2 are total currents.

When two devices given by ABCD parameters are cascaded, we have



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

↓

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{total} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

ABCD parameters of common impedance network are shown in the following table.

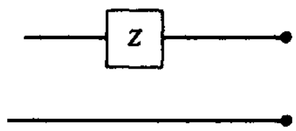
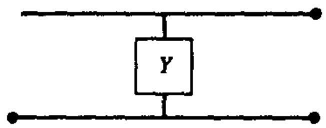
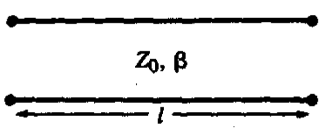
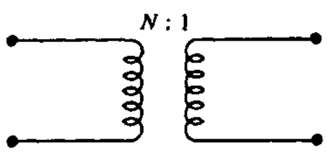
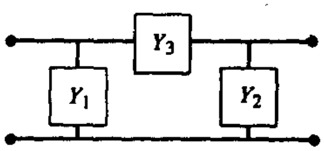
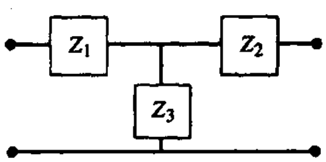
Since we are describing the same device using different parameters, we should be able to relate ABCD to any other parameters including S-parameter. The conversion table for S, Z, Y, and ABCD is shown in the following page.

For example, S-parameters are given by

$$S_{11} = \frac{A + B/Z_o - CZ_o - D}{A + B/Z_o + CZ_o + D}, \quad S_{12} = \frac{2(AD - BC)}{A + B/Z_o + CZ_o + D}$$

$$S_{21} = \frac{2}{A + B/Z_o + CZ_o + D}, \quad S_{22} = \frac{-A + B/Z_o - CZ_o + D}{A + B/Z_o + CZ_o + D}$$

TABLE 5.1 The *ABCD* Parameters of Some Useful Two-Port Circuits

Circuit	*ABCD* Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta \ell$ $C = jY_0 \sin \beta \ell$	$B = jZ_0 \sin \beta \ell$ $D = \cos \beta \ell$
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $C = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

which indicates that A is found by applying a voltage V_1 at port 1, and measuring the open-circuit voltage V_2 at port 2. Thus, $A = 1$. Similarly,

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{V_1/Z} = Z,$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0,$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{I_1}{I_1} = 1.$$

○

Table 1:

	S	Z	Y	$ABCD$
S_{11}	S_{11}	$\frac{(Z_{11}-Z_0)(Z_{22}+Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0-Y_{11})(Y_0+Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	S_{12}	$\frac{2Z_{12}Z_0}{\Delta Z}$	$\frac{-2Y_{12}Y_0}{\Delta Y}$	$\frac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	S_{21}	$\frac{2Z_{21}Z_0}{\Delta Z}$	$\frac{-2Y_{21}Y_0}{\Delta Y}$	$\frac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	S_{22}	$\frac{(Z_{11}+Z_0)(Z_{22}-Z_0)-Z_{12}Z_{21}}{\Delta Z}$	$\frac{(Y_0+Y_{11})(Y_0-Y_{22})+Y_{12}Y_{21}}{\Delta Y}$	$\frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$
Z_{11}	$Z_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{11}	$\frac{Y_{22}}{\|Y\|}$	$\frac{A}{C}$
Z_{12}	$Z_0\frac{2S_{12}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{12}	$\frac{-Y_{12}}{\|Y\|}$	$\frac{AD-BC}{C}$
Z_{21}	$Z_0\frac{2S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{21}	$\frac{-Y_{21}}{\|Y\|}$	$\frac{1}{C}$
Z_{22}	$Z_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1-S_{11})(1-S_{22})-S_{12}S_{21}}$	Z_{22}	$\frac{Y_{11}}{\|Y\|}$	$\frac{D}{C}$
Y_{11}	$Y_0\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{22}}{\|Z\|}$	Y_{11}	$\frac{D}{B}$
Y_{12}	$Y_0\frac{-2S_{12}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{12}}{\|Z\|}$	Y_{12}	$\frac{BC-AD}{B}$
Y_{21}	$Y_0\frac{-2S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{-Z_{21}}{\|Z\|}$	Y_{21}	$\frac{-1}{B}$
Y_{22}	$Y_0\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{(1+S_{11})(1+S_{22})-S_{12}S_{21}}$	$\frac{Z_{11}}{\|Z\|}$	Y_{22}	$\frac{A}{B}$
A	$\frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{11}}{Z_{21}}$	$\frac{-Y_{22}}{Y_{21}}$	A
B	$Z_0\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{\|Z\|}{Z_{21}}$	$\frac{-1}{Y_{21}}$	B
C	$\frac{1}{Z_0}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$	$\frac{1}{Z_{21}}$	$\frac{-\|Y\|}{Y_{21}}$	C
D	$\frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$	$\frac{Z_{22}}{Z_{21}}$	$\frac{-Y_{11}}{Y_{21}}$	D

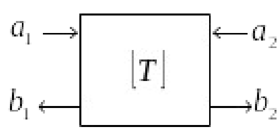
$|Z| = Z_{11}Z_{22} - Z_{12}Z_{21}$; $|Y| = Y_{11}Y_{22} - Y_{12}Y_{21}$; $\Delta Y = (Y_{11}+Y_0)(Y_{22}+Y_0) - Y_{12}Y_{21}$; $\Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0) - Z_{12}Z_{21}$; $Y_0 = 1/Z_0$

Table 2:

3-3. T-Parameters

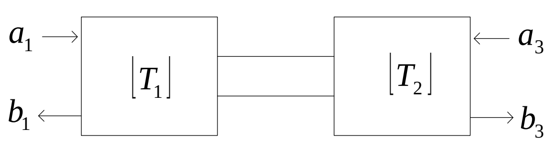
ABCD parameter is useful for many applications but we often need to express the device in terms of incident and reflected signal as we do with S-parameters. If we want to express the cascaded network using the incident and reflected signal, the device description based on T-parameter is better suited. We will show that T-parameters can be cascaded.

The two-port network is given by the incident and reflected signals. Similar to the S-parameter case, a_1 and a_2 are incident and b_1 and b_2 are reflected signals. To express T-parameter, we put the input signals on the left side and they are given by the output signals and T-parameters as shown below.



$$\underbrace{\begin{bmatrix} b_1 \\ a_1 \end{bmatrix}}_{\text{input}} = [T] \underbrace{\begin{bmatrix} a_2 \\ b_2 \end{bmatrix}}_{\text{output}}$$

When two devices are cascaded, the total T-parameter can be expressed as



$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = [T_1][T_2] \begin{bmatrix} a_3 \\ b_3 \end{bmatrix}$$

$$[T]_{total} = [T_1][T_2]$$

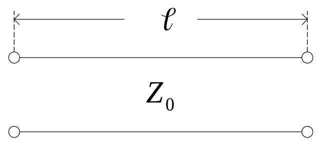
The conversions between $[T]$ and $[S]$ are given by

$$[T] = \begin{bmatrix} -\left(\dfrac{S_{11}S_{22} - S_{12}S_{21}}{S_{21}}\right) & \dfrac{S_{11}}{S_{21}} \\ -\dfrac{S_{22}}{S_{21}} & \dfrac{1}{S_{21}} \end{bmatrix}$$

$$[S] = \begin{bmatrix} \dfrac{T_{12}}{T_{22}} & \dfrac{T_{11}T_{22} - T_{12}T_{21}}{T_{22}} \\ \dfrac{1}{T_{22}} & -\dfrac{T_{21}}{T_{22}} \end{bmatrix}$$

Notice that [T] and [S] cannot be obtained if S_{21}=0 or T_{22}=0, respectively.

As an example of [S] and [T] parameters, a matched TL with the length ℓ is shown below.



$$\theta = \beta\ell$$

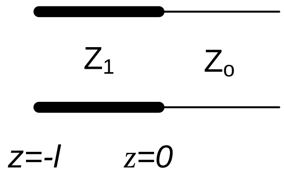
$$[S] = \begin{bmatrix} 0 & e^{-j\theta} \\ e^{-j\theta} & 0 \end{bmatrix}$$

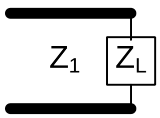
$$[T] = \begin{bmatrix} e^{-j\theta} & 0 \\ 0 & e^{j\theta} \end{bmatrix}$$

If ℓ=0 in the above example, [T] becomes a unit matrix. [T] also becomes a unit matrix when $\theta = m\pi$ where *m* is an integer.

ABCD parameter of an ideal TL

The ABCD parameters of an ideal TL is shown in Table 1. We will show the method to get those from the TL model. Assume we have a TL with 3 sections as shown below.





Top: Two TL sections, Bottom: equivalent circuit.

In Z_1 section, we get

$$V(z=-l)=V_0^+(e^{+j\beta l}+\Gamma_L e^{-j\beta l})$$

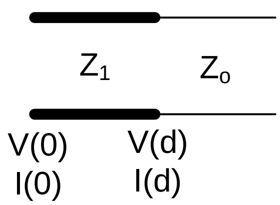
$$I(z=-l)=\frac{V_0^+}{Z_0}(e^{+j\beta l}-\Gamma_L e^{-j\beta l})$$

$$V_L=V_0^+ + V_0^- = V_0^+(1+\Gamma_L)$$

$$V_0^+ = V_L/(1+\Gamma_L)$$

$$\Gamma_L=\frac{V_0^-}{V_0^+}=\frac{Z_L-Z_1}{Z_L+Z_1}$$

Consider the same case with ABCD. From Table 1, this should be



$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V(d) \\ I(d) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & jZ_1\sin\theta \\ jY_1\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V(d) \\ I(d) \end{bmatrix}$$

$$\theta=\beta d, \quad Y_1=\frac{1}{Z_1}$$

To show these two cases are the same, we write the TL equations as

$$V(z=-l)=\frac{V_L}{1+\Gamma_L}(e^{+j\theta}+\Gamma_L e^{-j\theta})$$

$$I(z=-l)=\frac{1}{Z_1}\frac{V_L}{1+\Gamma_L}(e^{+j\theta}-\Gamma_L e^{-j\theta})$$

Rearranging the voltage equation, we get

$$\begin{aligned} V(z=-l) &= \frac{Z_0+Z_1}{2Z_0}(e^{+j\theta}+\frac{Z_0-Z_1}{Z_0+Z_1}e^{-j\theta})V_L \\ &= \frac{1}{2Z_0}(2Z_0\cos\theta+j2Z_1\sin\theta)V_L \\ &= \cos\theta V_L + jZ_1\sin\theta I_L \end{aligned}$$

where $I_L=\frac{V_L}{Z_L}=\frac{V_L}{Z_0}$

Similarly for the current, we get

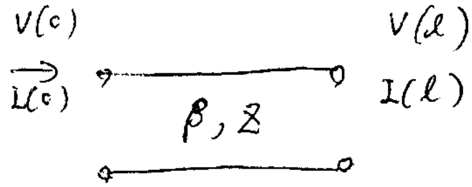
$$\begin{aligned} I(z=-l) &= \frac{Z_0+Z_1}{2Z_0}(e^{+j\theta}-\frac{Z_0-Z_1}{Z_0+Z_1}e^{-j\theta})\frac{V_L}{Z_1} \\ &= \frac{1}{2Z_0}(Z_0(e^{+j\theta}-e^{-j\theta})+Z_1(e^{+j\theta}+e^{-j\theta}))\frac{V_L}{Z_1} \\ &= (j\sin\theta+\frac{Z_1}{Z_0}\cos\theta)\frac{V_L}{Z_1} \\ &= jY_1\sin\theta V_L+\cos\theta I_L \end{aligned}$$

Therefore, the voltage and current equations can be expressed as

$$\begin{bmatrix} V(z=-l) \\ I(z=-l) \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_1\sin\theta \\ jY_1\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_L \\ I_L \end{bmatrix}$$

This is the same as

$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_1\sin\theta \\ jY_1\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V(d) \\ I(d) \end{bmatrix}$$



$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V(\ell) \\ I(\ell) \end{bmatrix}$$

$$\begin{cases} A = \cos\beta\ell \\ B = jZ\sin\beta\ell \\ C = \frac{j}{Z}\sin\beta\ell \\ D = \cos\beta\ell \end{cases}$$

$$V(z) = V^{+}e^{-j\beta z} + V^{-}e^{+j\beta z}$$

$$I(z) = \frac{V^{+}}{Z}e^{-j\beta z} - \frac{V^{-}}{Z}e^{+j\beta z}$$

$$\begin{bmatrix} V(z) \\ I(z) \end{bmatrix} = \begin{bmatrix} e^{-j\beta z} & e^{+j\beta z} \\ \frac{1}{Z}e^{-j\beta z} & -\frac{1}{Z}e^{+j\beta z} \end{bmatrix} \begin{bmatrix} V^{+} \\ V^{-} \end{bmatrix}$$

at $z = 0$

$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{1}{Z} & -\frac{1}{Z} \end{bmatrix} \begin{bmatrix} V^{+} \\ V^{-} \end{bmatrix}$$

$$\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{Z} & -\frac{1}{Z} \end{pmatrix} \begin{bmatrix} \quad & \quad \end{bmatrix}^{-1} \begin{bmatrix} V(\ell) \\ I(\ell) \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V(\ell) \\ I(\ell) \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{1}{Z} & -\frac{1}{Z} \end{bmatrix} \left(\frac{-Z}{2}\right) \begin{bmatrix} -\frac{1}{Z}e^{+j\beta\ell} & -e^{j\beta\ell} \\ -\frac{1}{Z}e^{-j\beta\ell} & e^{-j\beta\ell} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\beta\ell & jZ\sin\beta\ell \\ \frac{j}{Z}\sin\beta\ell & \cos\beta\ell \end{bmatrix} \qquad AD - BC = 1$$

4.

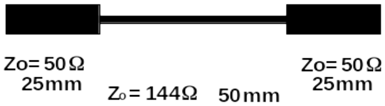


Fig. 4-1: Non-uniform TL test PCB.

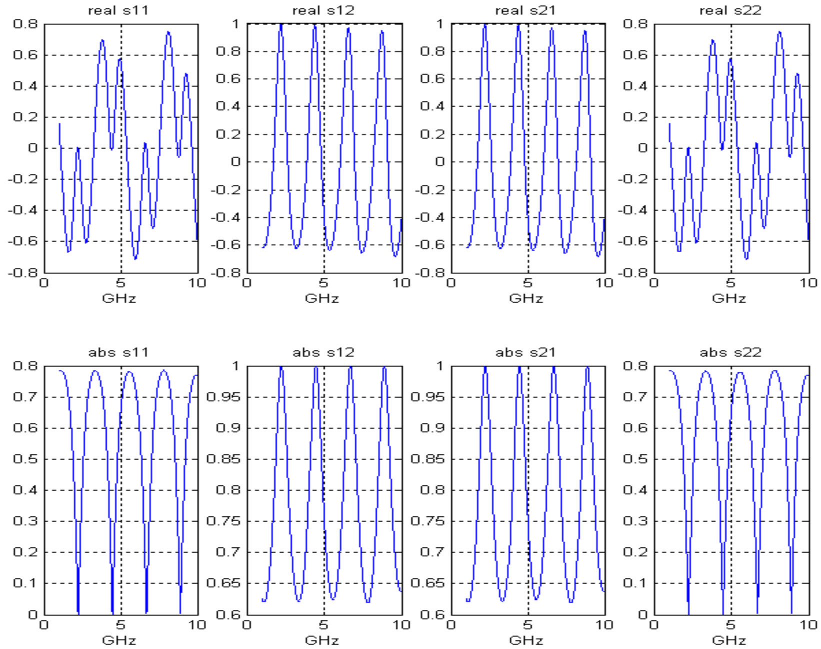


Fig. 4-2: S-parameters based on the ideal transmission line model (50-144-50)

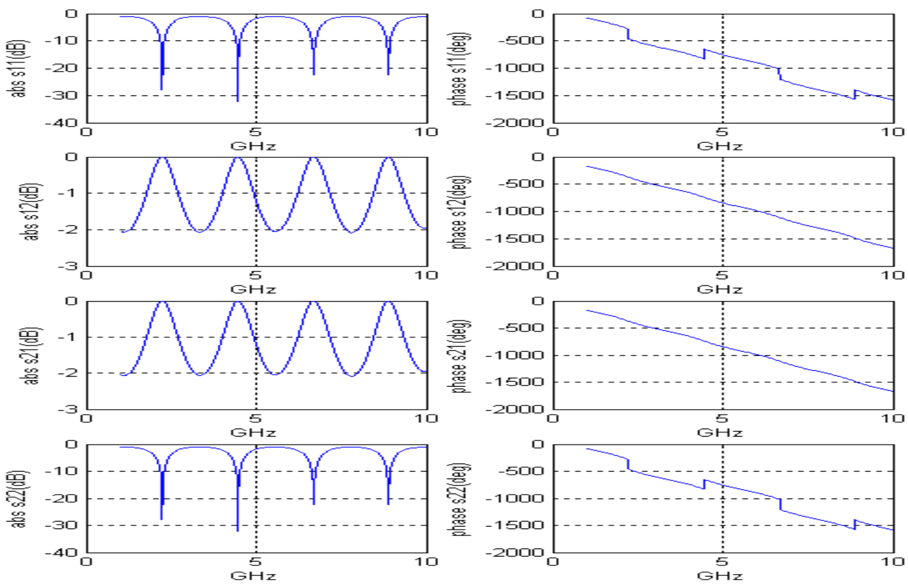


Fig 4-3: S-parameters of ideal transmission line (Mag in dB and phase)

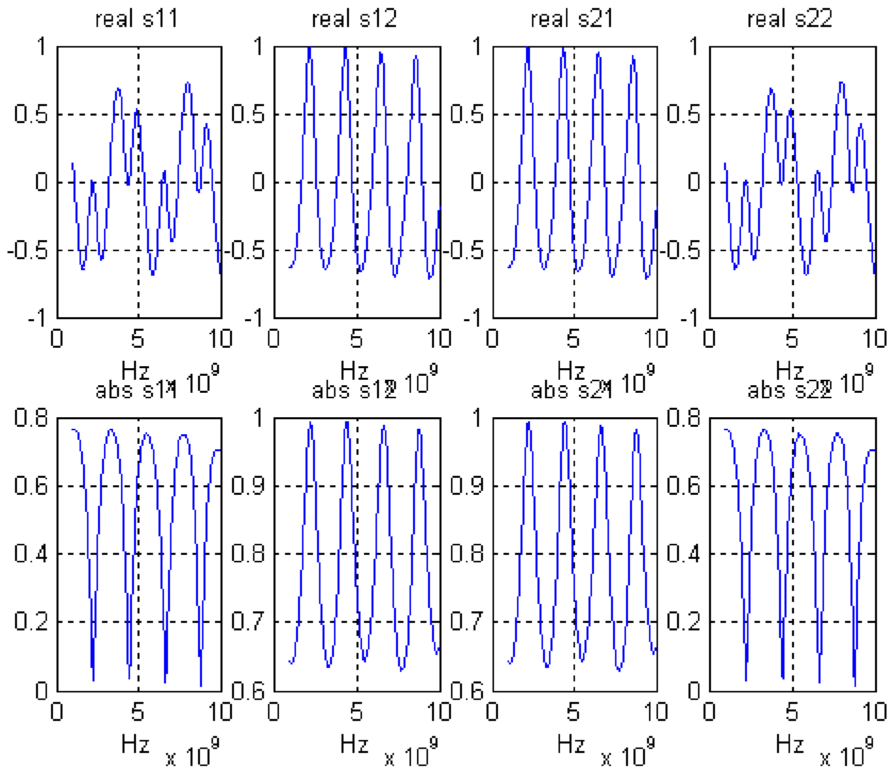


Fig 4-4: S-parameters of test PCB with HSFF

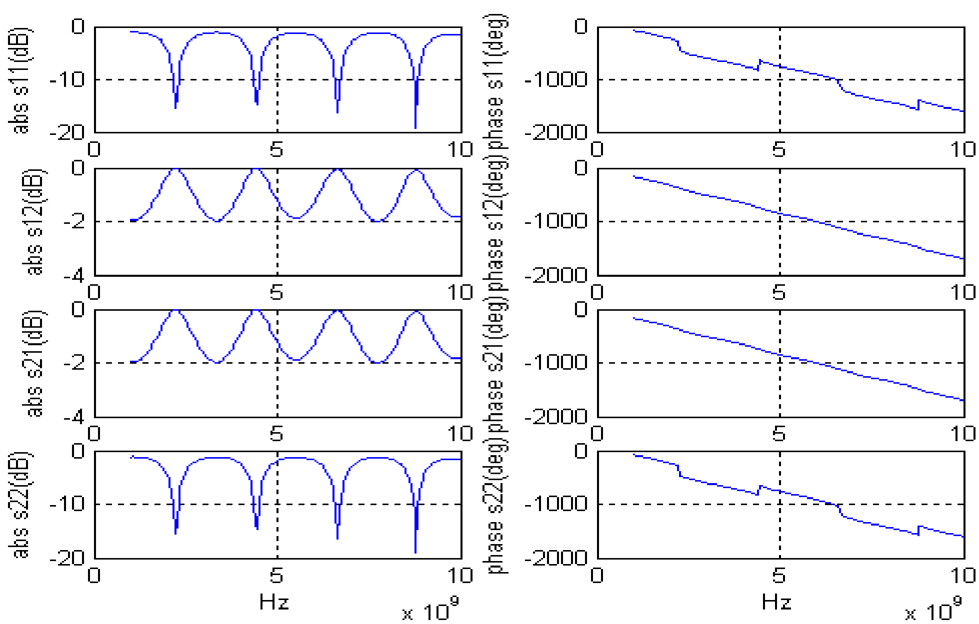


Fig 4-5: S-parameters of the test PCB with HFSS (Mag in dB and phase)

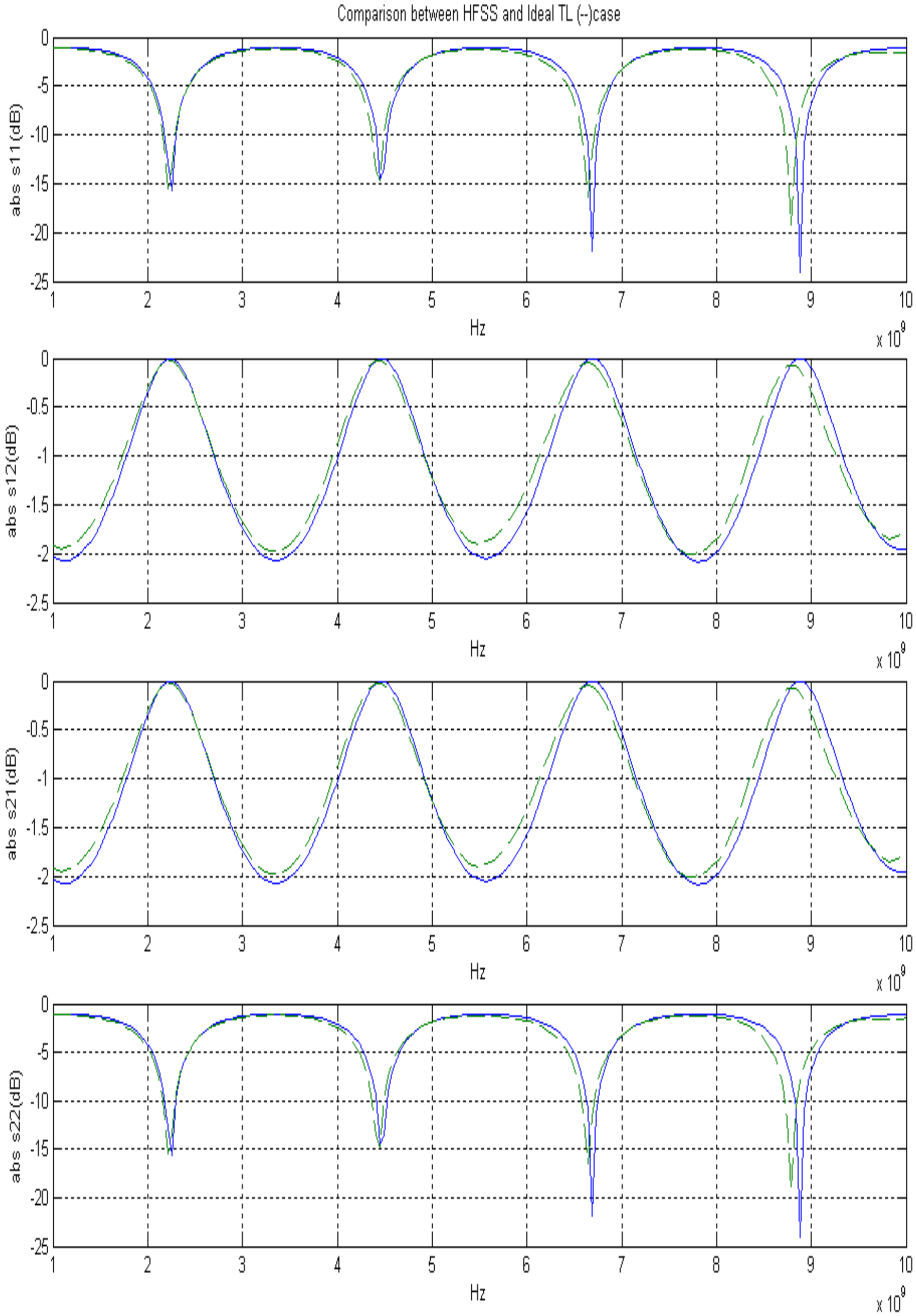


Fig. 4-6: Comparison between HFSS (with discontinuity effect) and Ideal TL

The difference between the ideal TL model and HFSS simulations is small at low frequency but increases at high frequency as shown in Fig. 4-6.

5. Parameter Extraction

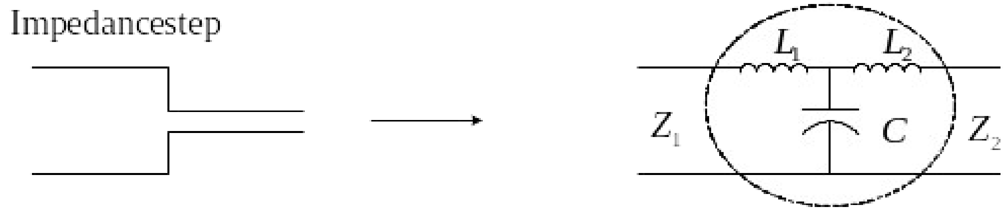


Fig. 5-1: Equivalent circuit representation of a step

5-1. Parameter extraction based on the "Piecewise approach"

Process

The test circuit is given by the cascaded 3 TLs as shown in Fig. 5-2. The simulation results are shown in Section 4.

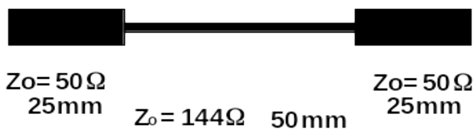


Fig. 5-2: Test PCB

Note: The following process use ABCD parameters but it can be done with T-parameters.

(1) Divide the structure into uniform TL sections and discontinuities

(a) Uniform 50 ohm section L=24.5mm

(b) Step from 0.5mm 50 ohm to 0.5 mm 144 ohm

(c) Uniform 144 ohm section L=49mm

(d) Step from 0.5mm 144 ohm to 0.5 mm 50 ohm

(e) Uniform 50 ohm section L=24.5mm

(2) EM (in this case HFSS) simulations of discontinuity up to 10 GHz

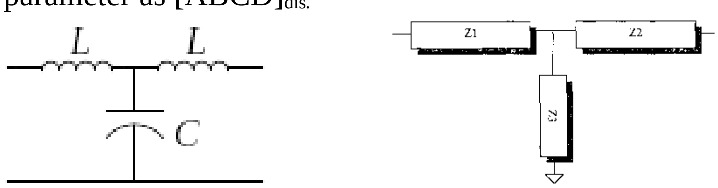
Model the step (b) and (d) using HSFF and obtain S-parameters. If HFSS data is not smooth, use the smoothing method to pre-process data.

Convert S-parameter data into ABCD.

$[S]_{HFSS}$ --> $[ABCD]_{HFSS}$

(3) Lumped element model based on T-network

Assume the step can be expressed using the T-network as shown below. Express it using ABCD parameter as $[ABCD]_{dis.}$



$Z_1=Z_{11}-Z_{12}=j\omega L$

$Z_2=Z_{22}-Z_{12}=j\omega L$

$Z_3=Z_{12}=1/j\omega C$

Fig. 5-3: T-network representation

(4) Simplified lumped element model with only C

Assume $[ABCD]_{dis}$ can be expressed using only a shunt *C* (set *L*=0). Obtain the ABCD parameter of this case.

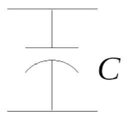


Fig. 5-4: Single C representation

(5) Estimation of lumped element values
Calculate the difference between HFSS and the lumped element model and minimize the difference. When the values of L and C are changed, $[ABCD]_{model}$ changes. The difference should be minimized over the frequency range. This will give us the optimum values of L and C.

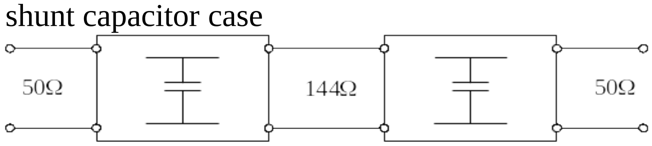
$$diff = [ABCD]_{HFSS} - [ABCD]_{model}$$

(6) Describe the non-uniform TL using ABCD parameters. Assume the uniform TL is an ideal lossless section.

$$[ABCD]_{model}=[ABCD]_{50}*[ABCD]_{dis}*[ABCD]_{144}**[ABCD]_{dis}*[ABCD]_{50}$$

$[ABCD]_{50}$ and $[ABCD]_{144}$ are ideal TLs.

Simple shunt capacitor case



T-network case

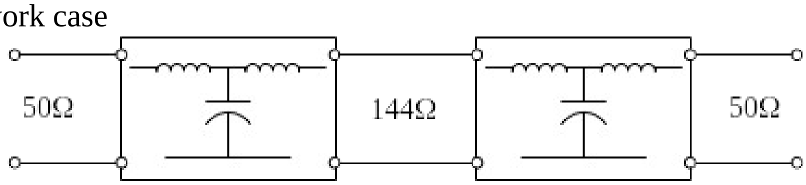


Fig. 5-3: Cascaded network model including discontinuities.

(7) Obtain the frequency-domain response and compare it with the HFSS simulations
This can be done with the circuit simulator such as Ansoft Designer. Or a simple Matlab code can be written for this.

(8) Obtain the time-domain response.
This can be done with PSPICE.

Unit step function response

Finite slope response

5-2. Parameter extraction using the whole structure
Process
(1) EM (in this case HFSS) simulations of the total PCB (Fig.5-2) up to 10 GHz
This result is shown in Section 4. If HFSS data is not smooth, use the smoothing method to pre-process data.

Convert S-parameter data into ABCD.
$[S]_{HFSS}$ --> $[ABCD]_{HFSS}$

(2) Create the lumped element model based on T-network and ideal TL
Assume the step can be expressed using the T-network as shown below. Express it using ABCD parameter as $[ABCD]_{dis}$. Then the test PCB becomes

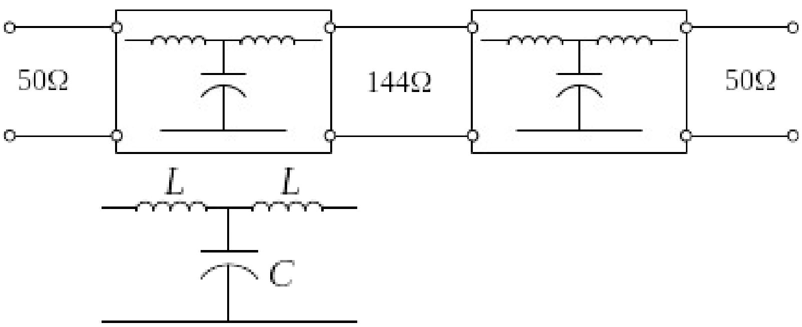


Fig. 5-4: T-network representation

The ABCD parameter of this network can be expressed as

$$[ABCD]_{model}=[ABCD]_{50}*[ABCD]_{dis}*[ABCD]_{144}**[ABCD]_{dis}*[ABCD]_{50}$$

$[ABCD]_{50}$ and $[ABCD]_{144}$ are ideal TLs.

If the discontinuity is expressed using a shunt capacitor, the model becomes

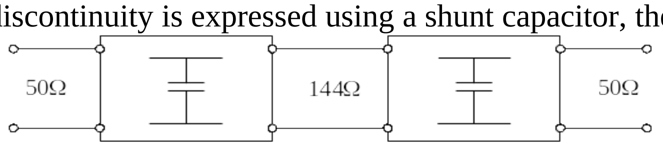


Fig. 5-5: Model using the simple shunt capacitor

(3) Obtain the frequency-domain response of the whole network and compare it with the HFSS simulations. The network simulations can be done with the circuit simulator such as SPICE, Ansoft Designer or a simple Matlab code can be written for this case. If a T-network is used, we have two unknowns L and C in this example. The global optimization over the frequency range must be done to minimize the difference of all ABCD parameters. If the physical insight is needed, it may be better to work with S-parameters rather than ABCD.

(4) Obtain the time-domain response.
This can be done with PSPICE or HSPICE.

Unit step function response
Finite slope response

6. Examples of Parameter Extraction Using a Distributed T-network Model: Frequency- and time-domain EM analysis of a Via in a multilayer PCB

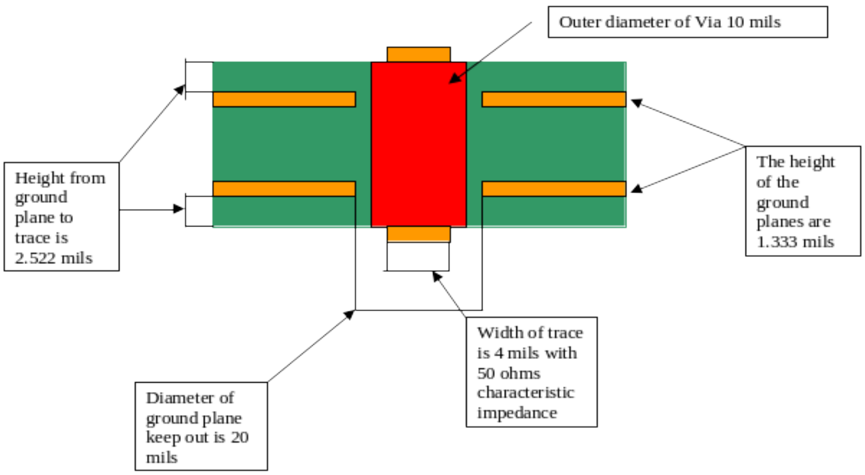


Fig. 6-1: Cross section of Via structure with typical dimensions.

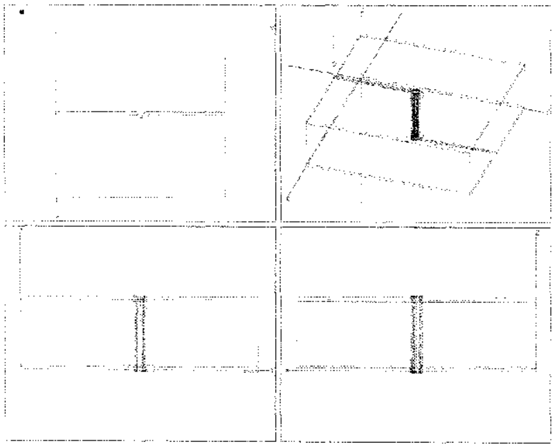


Figure 1 3-D view of the modeled via

Table 1 Geometries of the modeled via

Via	Height	62 mils
	Diameter	10 mils
	Thickness	1.378 mils
Antipad	Diameter	20 mils
Trace	Width	4 mils
	Thickness	1.378 mils
	Distance from Ground	2.522 mils
Ground	Thickness	1.378 mils

Fig. 6-2:

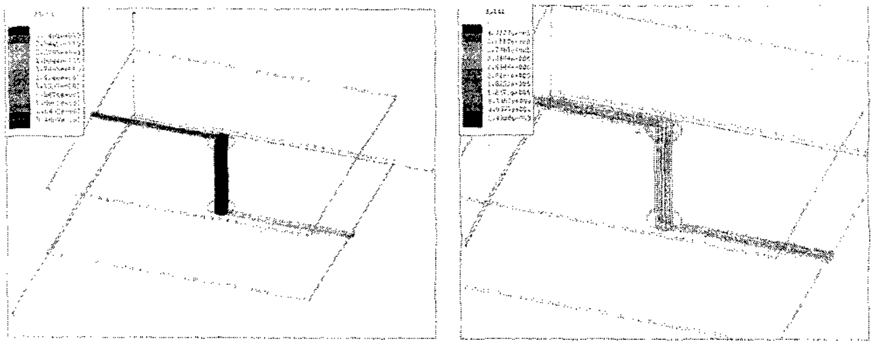


Figure 2 and 3 H/E Field distribution

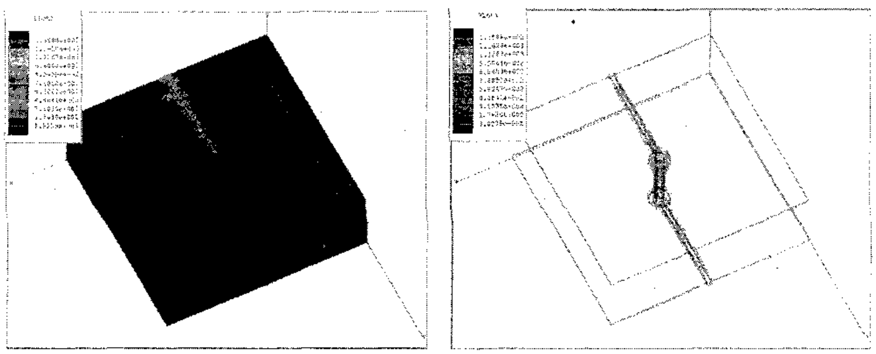


Figure 4 and 5 Surface and Volume J distribution

Fig. 6-3:

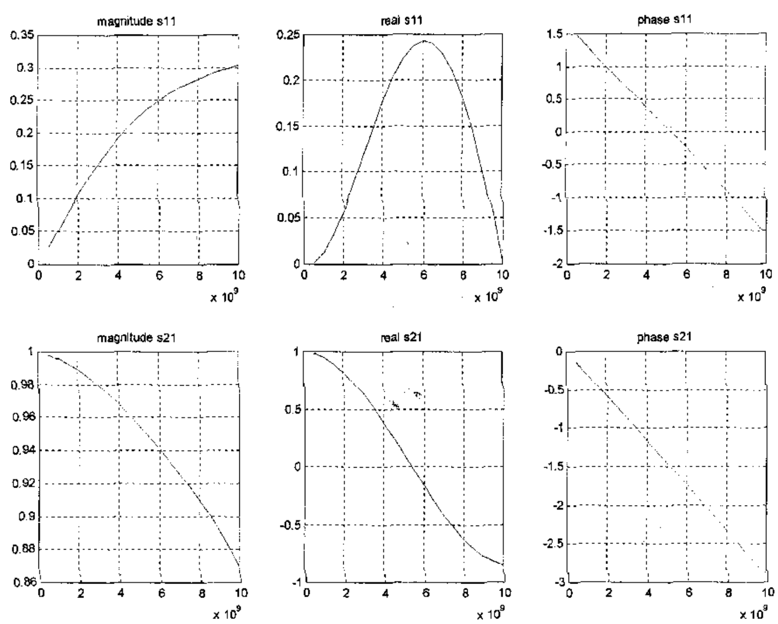


Figure 6 S parameters extracted from HFSS simulation

Fig. 6-4:

Convert from S- to Z-parameters.

$$Z11 = Z0 \cdot \frac{(1+S11)\cdot(1-S22)+S12\cdot S21}{(1+S11)\cdot(1-S22)-S12\cdot S21}$$

$$Z12 = 2\cdot Z0 \cdot \frac{S12}{(1+S11)\cdot(1-S22)-S12\cdot S21}$$

$$Z21 = 2\cdot Z0 \frac{S21}{(1+S11)\cdot(1-S22)-S12\cdot S21}$$

$$Z22 = Z0 \cdot \frac{(1-S11)\cdot(1+S22)+S12\cdot S21}{(1+S11)\cdot(1-S22)-S12\cdot S21}$$

Relate Z-parameter to T-network.

$$Z1 = Z11 - Z12$$

$$Z2 = Z22 - Z12$$

$$Z3 = Z12$$

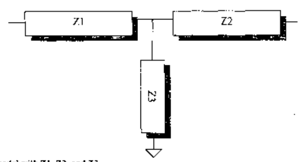


Figure 7 T model with Z1, Z2, and Z3

Fig. 6-5:

Discontinuity modeled as a distributed T.

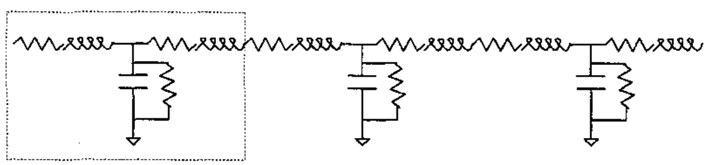


Figure 8 Distributed T model

Fig. 6-6:

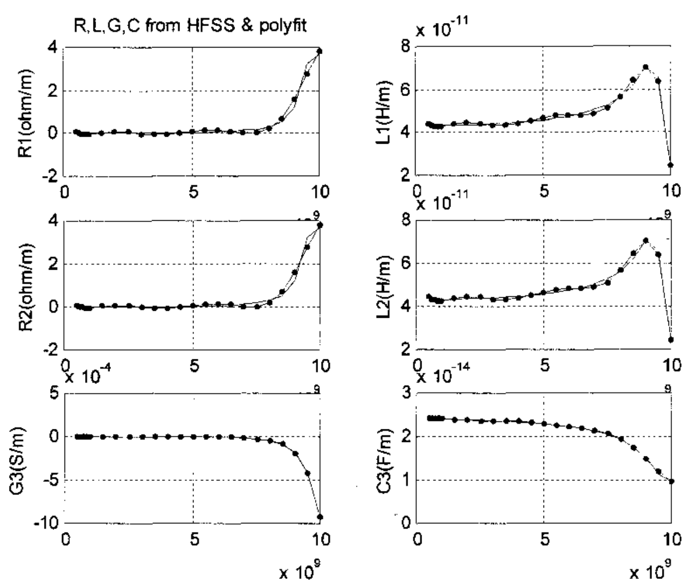


Figure 9 R,L,G,C values from HFSS and Polyfit

Fig. 6-7:

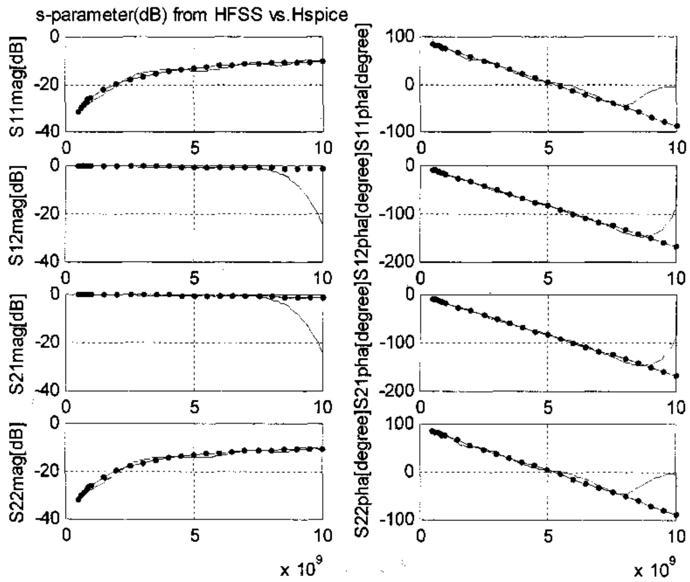


Figure 10 Final HSPICE results compared to HFSS S parameters in dB

Fig. 6-8:

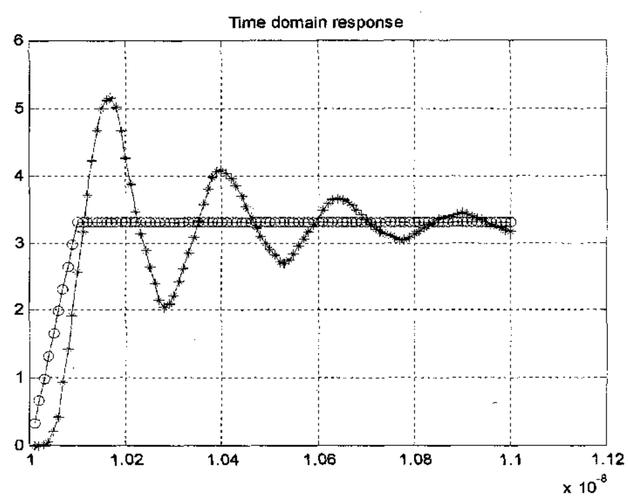


Figure 11 Time domain response of via (HSPICE)

Fig. 6-9:

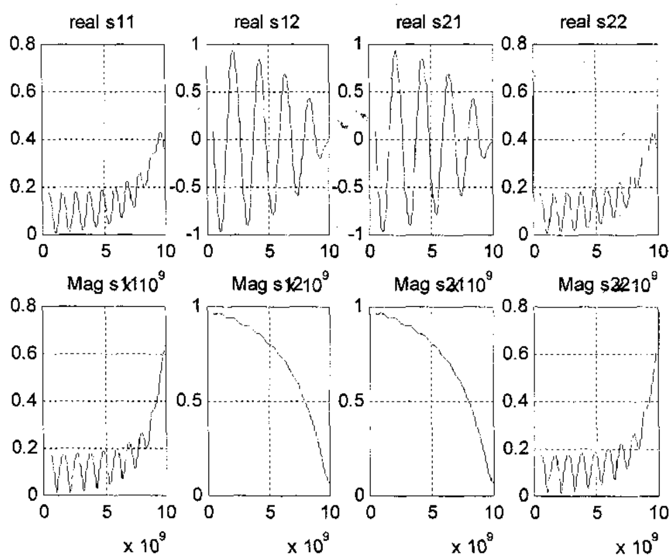


Figure 12 10 cascaded vias S parameters

Fig. 6-10:

7. Models of discontinuity based on cascaded T-parameters

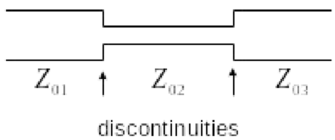


Fig. 7-1:

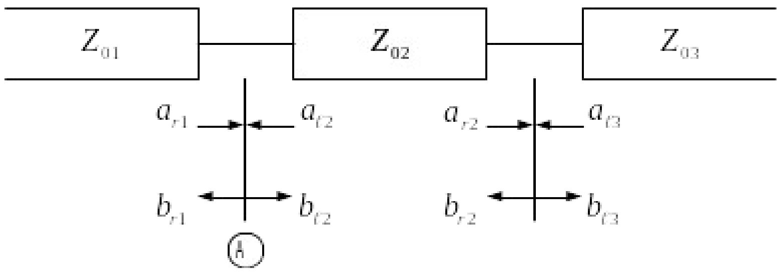


Fig. 7-2:

To express the impedance discontinuity at Ⓐ, we use

$$\begin{bmatrix} b_r(i) \\ a_r(i) \end{bmatrix} = \underset{\substack{\uparrow \\ T\text{ - } parameter}}{\begin{bmatrix} T_{11}(i) & T_{12}(i) \\ T_{21}(i) & T_{22}(i) \end{bmatrix}} \begin{bmatrix} a_t(i+1) \\ b_t(i+1) \end{bmatrix}$$

If we define

$$\rho_{12} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

or

$$\rho_{i,i+1} = \frac{Z_0(i+1) - Z_0(i)}{Z_0(i+1) + Z_0(i)}$$

$$[T(i)] = \frac{1}{\sqrt{1-\rho^2_{i,\,i+1}}} \text{¿} \left[1 \;\; \underset{\text{¿}}{\rho_{i,i+1}} \text{¿} \right] \text{¿} \text{¿}$$

This is the same as

$$[T] = \text{¿} \left[-\underset{\text{¿}}{\left(\frac{S_{11} S_{22} - S_{12} S_{21}}{S_{21}} \right)} \;\; \frac{S_{11}}{S_{21}} \text{¿} \right] \text{¿} \text{¿} \text{¿}$$

For a lossless two-port network

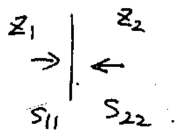


Fig. 7-3:

$$s_{11}=\frac{Z_2-Z_1}{Z_2+Z_1}=-\left(\frac{Z_1-Z_2}{Z_1+Z_2}\right)=-s_{22}$$

$$s_{22}=-s_{11}\ \ also\ \ s_{12}=s_{21}$$

$$[s]^{¿}[s]^{¿}=[u]$$

lossless condition

In terms of T.

$$T_{11}=T_{22}^{¿}$$

$$T_{12}=T_{21}^{¿}$$

$$[T]=\frac{1}{s_{21}}\begin{bmatrix} s_{12}^2+s_{11}^2 & s_{11} \\ s_{11} & 1 \end{bmatrix}$$

$$=\frac{1}{s_{21}}\begin{bmatrix} 1 & s_{11} \\ s_{11} & 1 \end{bmatrix}\ \ because\ s_{12}^2+s_{11}^2=1,\ from\ \left[T_{11}=T_{22}^{*}\right]$$

$$s_{12}^2=1-s_{11}^2$$

or

$$s_{12}=\sqrt{1-s_{11}^2}$$

TL of length ℓ (matched impedance)

$$\theta=\beta\ell=\left(\frac{\omega}{c}\right)\ell=\omega\left(\frac{\ell}{c}\right)$$

$$\uparrow$$

$$T_{pd}$$

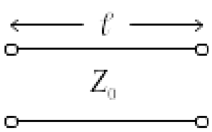


Fig. 7-4:

$$[T]=¿\left[e^{-j\theta}\ \ 0¿\right]¿¿¿$$

$$¿$$

Therefore, using $a_\ell, b_\ell, a_r, b_r,$ we get

$$\left[a_\ell(i+1)\ ¿\right]¿¿¿¿$$

$$¿$$

$$¿$$

$$\left[b_r(i)\ ¿\right]¿\ ¿\ ¿\ ¿$$
¿
¿

Model for *n* sections

$$\left[b_r(1)\ ¿\right]¿\ ¿\ ¿\ ¿$$
¿
¿

Recursive Relationship

$$\left[b_r(1)\ ¿\right]¿\ ¿\ ¿\ ¿$$
¿
¿

To obtain $T(2)$, we need to know $T(1)$.

In general, to obtain $T(n)$, we need to find $T(1)$ .... $T(n-1)$.

$$[D(s)]^{-1} = ¿\left[D^{-1}\quad 0\ ¿\right]¿\ ¿\ ¿$$
¿

Recursive formula.

$$[T(s)]_{n-i} = [D(s)]^{-1}[T(i)]^{-1}[T(s)]_{n-(i-1)}$$

$$[D(s)]^{-1} = ¿\left[e^{sT_{pd}}\quad 0\ ¿\right]¿\ ¿ \qquad ¿$$
¿
¿

Let $D = e^{-sT_{pd}}$ *and* $D^{-1} = e^{sT_{pd}}$

delay operator

$$[T(i)]^{-1} = \left(\frac{1}{\sqrt{1-\rho_{i,\,i+1}^2}}\right)¿\left[1\quad -\rho_{i,\,i+1}\ ¿\right]¿\ ¿$$
¿

Explicitly,

$$\left[T_{22}(D)_{n-i} \quad T_{12}\left(D^{-1}\right)_{n-i} \text{¿}\right] \text{¿¿¿¿}$$
¿
¿

Non-uniform TL

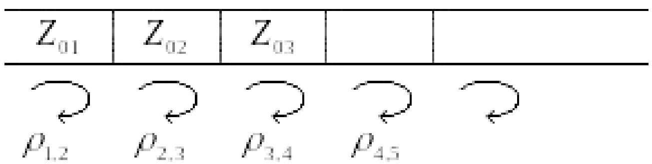


Fig. 7-5:

Elements of $[T(s)]$ can be written as

$$T_{12}(D)_n = B_n(1) + B_n(2) D^2 + \ldots + B_n(i) D^{2i}$$
$$T_{22}(D)_n = A_n(1) + A_n(2) D^2 + \ldots + A_n(i) D^{2i}$$

Reflection coefficient of the first section

$$\rho_{i,i+1} = \frac{T_{12\ n-i}}{T_{22\ n-i}} \sim \frac{B_{n-i}(1)}{A_{n-i}(1)}$$
$$\textit{keep only the } 1^{st} \textit{ term}$$
$$\rho_{12} = \frac{B_{n-1}(1)}{A_{n-1}(1)}$$
$$\rho_{23} = \frac{B_{n-2}(1)}{A_{n-2}(1)}$$

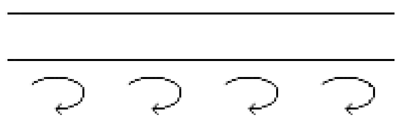
Once ρ_{12} is found, we can get $[T(D)]_{n-2}$.
From $[T(D)]_{n-2}$ we can get ρ_{23}.

This process continues up to $\rho_{n-2,n-1}$.
From $\rho_{i,i+1}$, we can obtain $Z_o(t)$

Lumped element model

How to get *L* and *C* from the distributed circuits

I. Obtain $\rho_{i,\,i+1}$



II. Convert to $Z_o(t)$

III. Characteristic impedance: $Z_o=\sqrt{\dfrac{L}{C}}$

Velocity: $v=\dfrac{1}{\sqrt{LC}}=\dfrac{1}{T_{pd}}$

From these we get

$$L=Z_o T_{pd}$$

$$C=\frac{T_{pd}}{Z_o}$$

For the distributed Z_0 , we use

$$L=\frac{1}{2}\int_{t_a}^{t_b} Z_o(t)\,dt=\frac{1}{2}\sum_{t=0}^{n} Z_{oi}\delta t_i$$

$$C=\frac{1}{2}\int_{t_a}^{t_b}\frac{1}{Z_o(t)}\,dt=\frac{1}{2}\sum_{t=0}^{n}\left(\frac{\delta t_i}{Z_{oi}}\right)$$