Hybrids and Power Dividers 1/29/2016

1. Introduction

In many microwave applications, we need to divide the input/output signal into two branches. One example is a 3-port device which creates reference and thru signals (outputs) from the source signal (input). Some requirements of this device are

(1) all ports must have matched impedance (Z_o=50ohm).

(2) high isolation between two output ports

(3) no loss (no resistive element)

(4) wide bandwidth if possible

The simplest power divider is a T-junction power divider which is often used for low frequency applications. It may not be obvious but the T-junction does not satisfy (1) and (2).

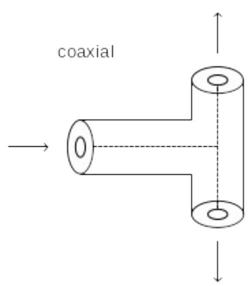


Fig.1: A simple coaxial T-junction

In order to satisfy all 4 conditions or top 3 conditions, we need to design new microwave circuits. In this section, we will study 90- and 180-degree hybrids as well as 3-port power dividers which are designed to satisfy 3 or 4 conditions.

Devices to be studied in this section.

Quadrature (90 degree) hybrid (4-port device but used as a 3-port device)

Can be used for creating sin(ωt) and cos(ωt), I-Q detection

Can be used as a feeding network to a circularly polarized antenna

180 degree hybrid (4-port device)

Sum and difference network

Moving target detection antenna

3-port power divider

T-section

Resistive type

Reactive type

2. Four-port network and hybrids

Hybrids are a 4-port device but they are often used as a 3-port device with one port terminated with a matched-impedance.

Desired characteristics are

- -Matched input ports (port 1, 2, 3 and 4)
- -Reciprocal device
- -Lossless
- -High isolation between two output ports
- -Magnitude and phase of two output ports can be designed for different values.

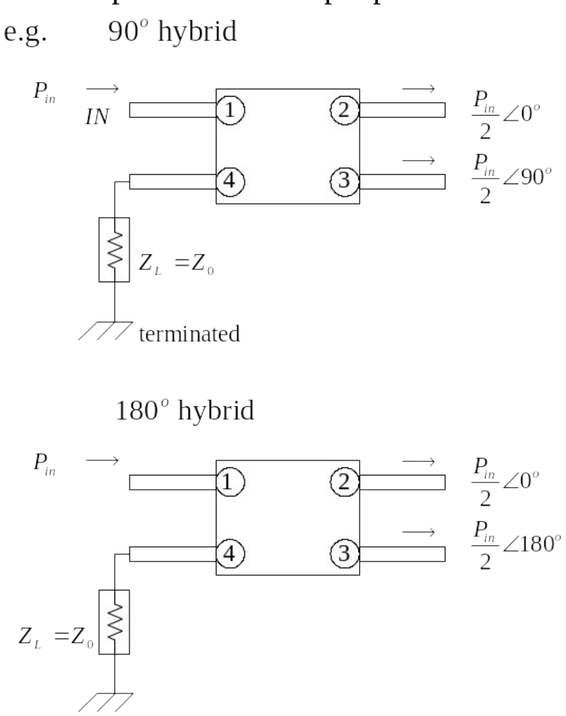


Fig. 2: Two hybrids used as 3-port devices.

2-1. Applications: I-Q detector

One application of a quadrature (90 degree) hybrid is an I-Q detection circuit which is used for demodulating a complex amplitude given by a magnitude A and phase θ. Figure 3 shows the diagram. The input signal is divided into 2 branches. One is multiplied by $\cos\omega t$ and the output of a mixer contains two signals (sum and difference frequencies). LPF is used for filtering out the $\cos(2\omega t)$ component. The desired output is $A\cos\omega t$. The other signal is multiplied by $\sin\omega t$ and after going thru a LPF, we get $A\sin\omega t$. From A $\cos\omega t$ and $A\sin\omega t$, we can find the desired the magnitude A and phase θ. This circuit requires one 90 degree hybrid and one power divider with equal phase outputs.

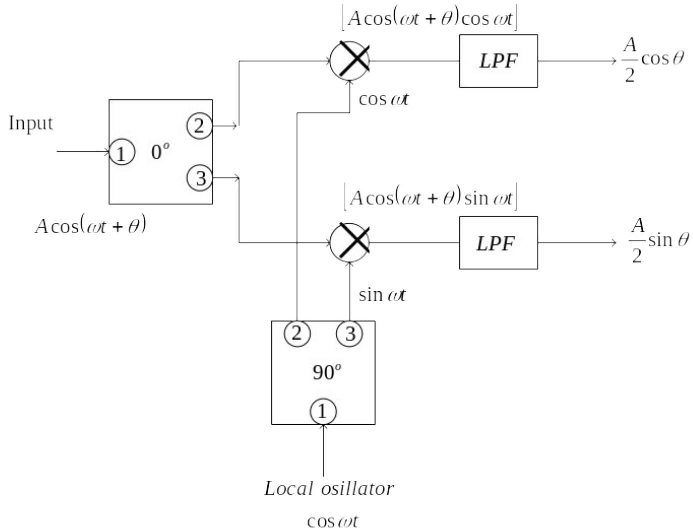
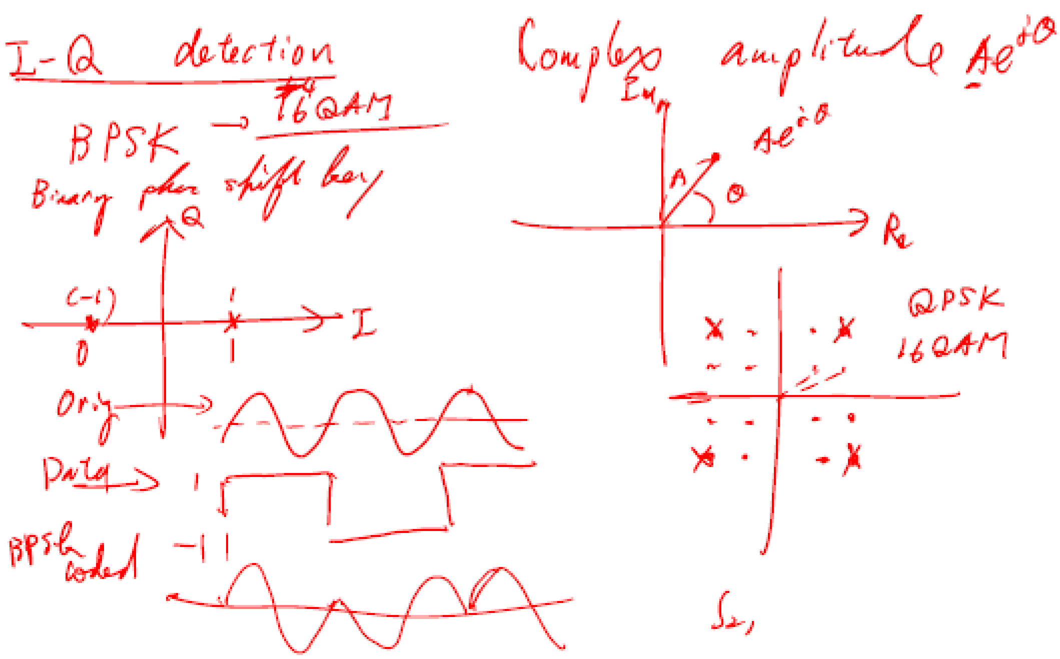


Fig.3: I-Q detection circuit



The requirements for the I-Q detection circuit are

Output phase difference: 0 or 90 degrees

Output power –3dB

High isolation between ports 2 and 3

Applications of I-Q circuits

Recovery of amplitude and phase in the communication system (old analog method)

Phase and amplitude modulations: BPSK, QPSK, 64QAM…

Another example is a feeding network to create a circularly polarized signal from a patch antenna

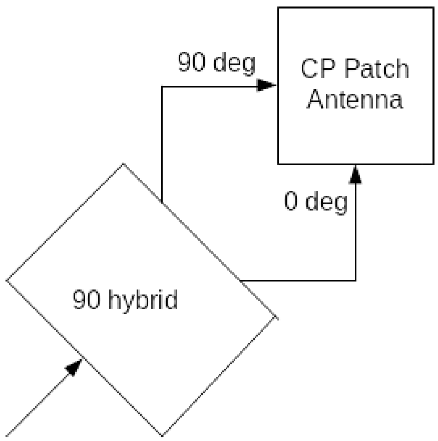


Fig. 4: Patch antenna to create a circular polarization

2-2. S-parameter representation of 4-port network

In order to design a circuit with desired characteristics, we need to specify it using S or similar parameters. Then we need to show if the circuit can be physically realizable. The second part is important. We can always specify all desired characteristics in S-parameters but there is no guarantee a circuit can be designed with these desired characteristics.

The 4-port device can be described as

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = [S] \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

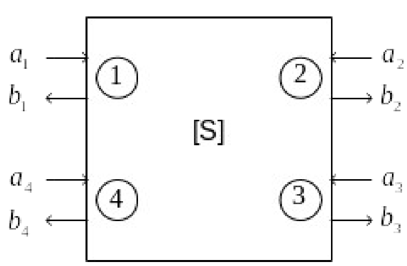


Fig. 5:

One of the desired characteristics is matched ports. Also because this is a passive device, we can add the reciprocity conditions. Then we can set

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

Input ports matched $S_{11}=S_{22}=S_{33}=S_{44}=0$

Reciprocal $[S]=[S]^t$

How about lossless or high isolation between two output ports?

Lossless condition needs to satisfy $[S]^t[S]^*=[U]$.

High isolation between two output ports means S_{23}=0 and S_{14}=0.

Now we need to see if we can satisfy these conditions.

To satisfy the lossless conditions, we need to get these conditions in terms of matrix elements

$$\sum_{k=1}^{N} S_{ki}S_{kj}^{*}=1 \qquad \mathrm{i}=j$$

$$\sum_{k=1}^{N} S_{ki}S_{kj}^{*}=0 \qquad \mathrm{i}\neq j$$

For a 4-port device, we get

$$S_{13}^{*}S_{23}+S_{14}^{*}S_{24}=0 \qquad (1)$$

$$S_{14}^{*}S_{13}+S_{24}^{*}S_{23}=0 \qquad (2)$$

$$S_{12}^{*}S_{23}+S_{14}^{*}S_{34}=0 \qquad (3)$$

$$S_{14}^{*}S_{12}+S_{34}^{*}S_{23}=0 \qquad (4)$$

From (1) and (2)

$$S_{14}^{*}\left(|S_{13}|^2-|S_{24}|^2\right)=0 \qquad \{(1)\mathrm{x}\, S_{24}^{*} \text{ and } (2)\mathrm{x}\, S_{13}^{*}\}$$

From (3) and (4)

$$S_{23}\left(|S_{12}|^2-|S_{34}|^2\right)=0 \qquad \{(3)\mathrm{x}\, S_{34}^{*} \text{ and } (4)\mathrm{x}\, S_{12}^{*}\}$$

To satisfy these two equations, choose

$S_{14}=0$ High isolation between ports 1 and 4

$S_{23}=0$ High isolation between ports 2 and 3

As we stated earlier, these are desired characteristics of a hybrid.

Similarly from the lossless condition

$$|S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = |S_{12}|^2 + |S_{13}|^2 = 1$$
$$|S_{12}|^2 + |S_{24}|^2 = 1$$
$$|S_{13}|^2 + |S_{34}|^2 = 1$$
$$|S_{24}|^2 + |S_{34}|^2 = 1$$

From these we can get

$$|S_{13}| = |S_{24}|$$
$$|S_{12}| = |S_{34}|$$

Although we have a condition for the magnitude, we don't know the phase relationship.

Let

$$S_{12} = S_{34} = \alpha$$
$$S_{13} = \beta e^{j\theta}$$
$$S_{24} = \beta e^{j\varphi}$$

We are satisfying $|S_{13}| = |S_{24}|$ and $|S_{12}| = |S_{34}|$.

From $|S_{12}|^2 + |S_{24}|^2 = 1$, we have

$$\therefore \alpha^2 + \beta^2 = 1$$

From

$$S_{12}^* S_{13} + S_{24}^* S_{34} = 0$$
$$\alpha\beta e^{j\theta} + \beta e^{-j\phi}\alpha = 0$$
$$e^{j\theta} + e^{-j\phi} = 0 \Rightarrow e^{j(\theta+\phi)} + 1 = 0$$
$$\therefore \theta + \phi = \pi \quad \text{or} \quad \pi \pm 2n\pi$$

We have two parameters to define the characteristics of a 4-port network. Those are α and θ . By choosing α and θ , we should be able to design a different hybrid.

Option 1: choose

$$\theta = \varphi = \frac{\pi}{2}$$
$$\alpha = \beta = \frac{1}{\sqrt{2}}$$

Then

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{j}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} S_{21} &= \frac{1}{\sqrt{2}} \\ S_{31} &= \frac{j}{\sqrt{2}} \end{aligned}\right\} \rightarrow \ 90^0 \ \text{ phase shift}$$

when we have an input signal at Port1.

This is a 90-degree hybrid.

Option 2: choose

$$\theta = 0 \qquad \varphi = \pi$$

$$\alpha = \beta = \frac{1}{\sqrt{2}}$$

Then

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

This circuit has two interesting characteristics depending on which port is used as an input.

Input from Port 1

$$\left.\begin{aligned} S_{21} &= \frac{1}{\sqrt{2}} \\ S_{31} &= \frac{1}{\sqrt{2}} \end{aligned}\right\}$$

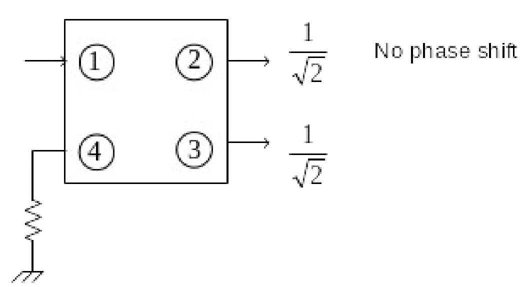


Fig. 6:

Input from Port 4

$$S_{24} = -\frac{1}{\sqrt{2}}$$

$$S_{34} = \frac{1}{\sqrt{2}}$$

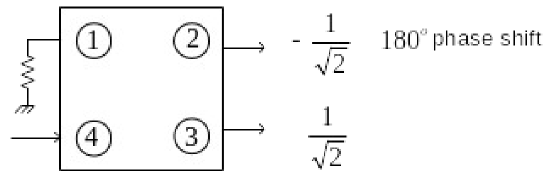


Fig. 7:

3. Quadrature hybrid (90° hybrid)

In the previous section, we obtain the desired S-parameters of a 90-degree hybrid. In principle, using the S-parameters, we should be able to design a microwave circuit. In practice, however, this process is not simple. In this section, therefore, instead of designing a new microwave circuit, we will analyze the existing microwave circuit which satisfies the conditions set for a 90-degree hybrid.

To design a practical hybrid using a microstrip PCB, we start with the analysis of 90^0 Hybrid.

Desired $[S]$

$$[S] = -\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

This S is different from the one in the previous section. But essentially they have the same characteristics.

The microstrip circuit implementation is given in Fig.8.

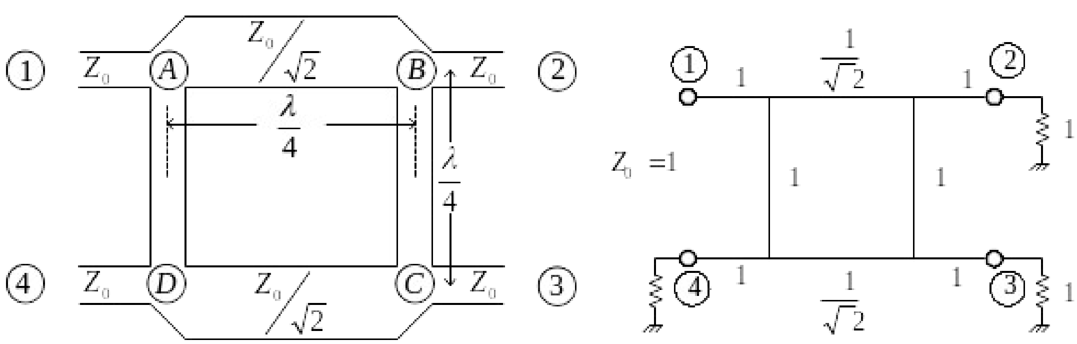


Fig. 8:

Before conducting a detailed analysis, we start with the intuitive analysis. The phase shift from input to output depends on the path length and can be estimated as.

Output at Port 4

$A \text{ to } D \quad \frac{\lambda}{4}$

$A \rightarrow B \rightarrow C \rightarrow D \quad \frac{\lambda}{4}+\frac{\lambda}{4}+\frac{\lambda}{4}=\frac{\lambda}{4}+\frac{\lambda}{2}$

They have a 180° phase difference: destructive interference (No output)

Output at Port 2 and 3

$A \text{ to } B \quad \frac{\lambda}{4}$ ¿}¿¿¿

90^{0} phase difference

3-1 Detailed Analysis Using Even-Odd mode Circuits

Many man-made structures are symmetric and the line of symmetry can be used for separating the circuit into Even and Odd mode excitations. The circuit shown in Fig. 9 has two lines of symmetry and the horizontal line will be used here.

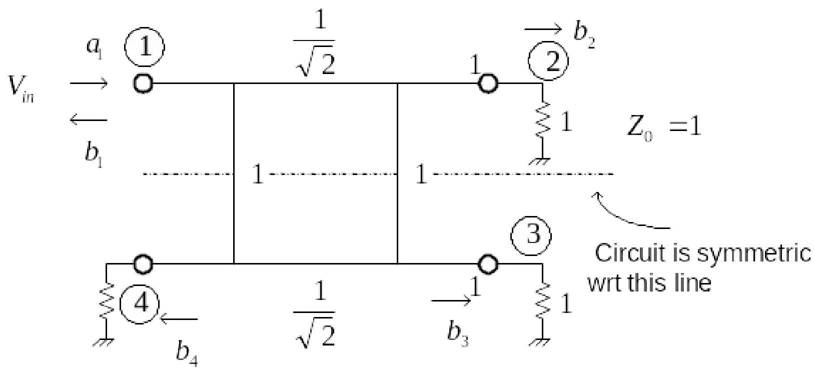


Fig.9:

Assume we have an input at port 1 and separate it into two components.

Port 1: $V_{\text{in}} = \frac{1}{2} V_{in} + \frac{1}{2} V_{in}$

Because there is no input at port 4, we can write

Port 4: $0 = \frac{1}{2} V_{in} - \frac{1}{2} V_{in}$

Notice that the first terms have the same signs (+) but the second terms have an opposite signs (+-). If the sign is the same, it is called the even signals. If the sign is opposite, it is called the odd signals.

Even mode excitation

If X = Y (even mode) in Fig.10, the signal at E due to X is the same as the signal due to Y (same phase). Therefore, effectively the point E can be replaced by an open circuit because the same phase signal is reflected (traveling in the reverse direction).

$$V_{\mathrm{ref}} = \Gamma V_{in} = V_{inc}$$

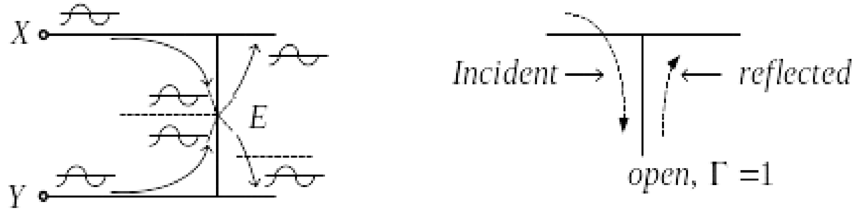


Fig.10:

Odd mode excitation

If X = -Y (odd mode) in Fig.11, the signals at E due to X and Y have a 180^0 phase difference. Therefore, the point E can be replaced by a short circuit which creates a phase shift of 180°.

$$V_{\mathrm{ref}} = \Gamma V_{inc} = -V_{inc}$$

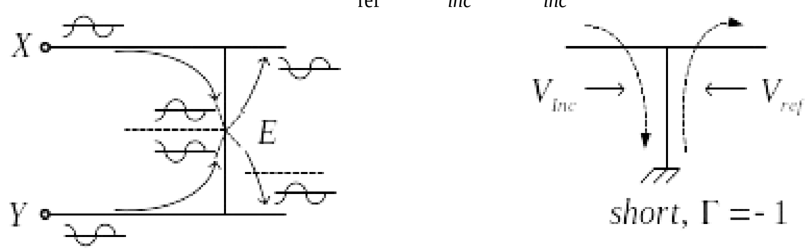


Fig.11:

Even mode case analysis

The use of even and odd mode excitations enables us to separate the 4-port network into two 2-port network as shown in Figs. 10 and 11. We will start the analysis with the even mode case.

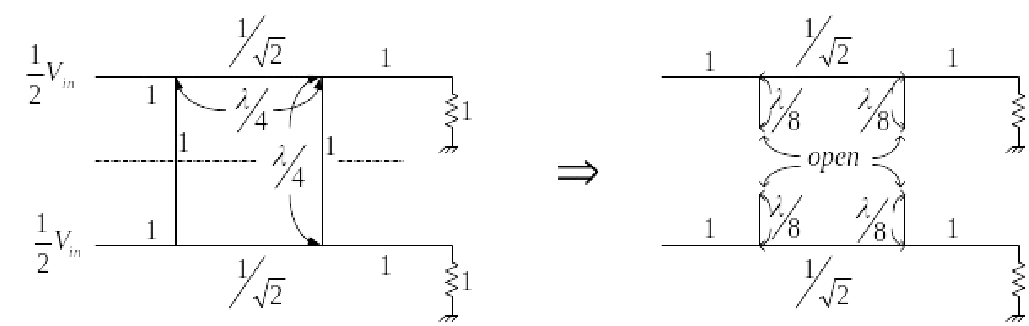


Fig. 12:

The $\frac{\lambda}{8}$ open stub is attached in parallel and it is an open-ended TL. The input impedance is given by

$$Z_{in} = Z_o\left(\frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}\right) = -jZ_0 \cot\beta\ell = -j$$

or
$$Y_{in} = \frac{1}{Z_{in}} = j$$

Each half contains several sections and it is a cascaded network. We will express the each section using ABCD parameters. The total ABCD parameter is the multiplications of these sections as shown below.

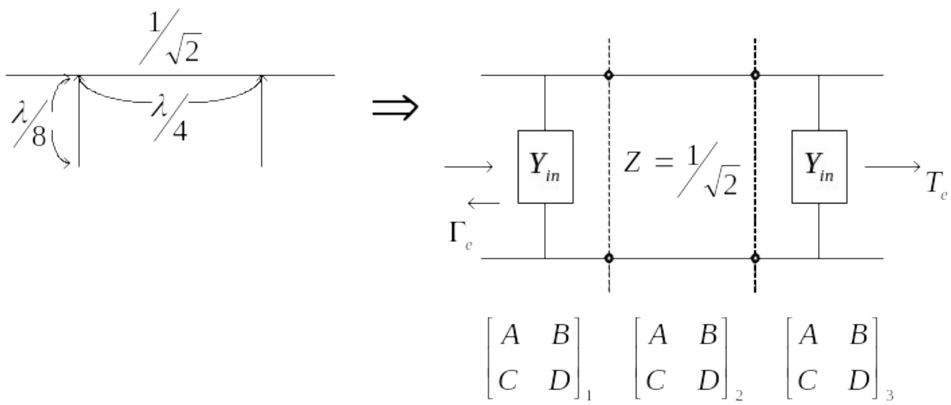
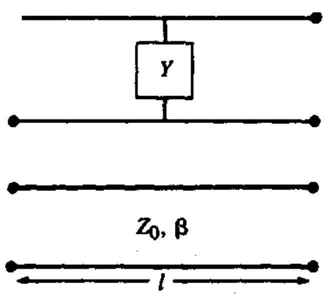


Fig. 13:

ABCD parameter of each can be found using the following figure.



$A = 1 \qquad B = 0$

$C = Y \qquad D = 1$

$A = \cos\beta\ell, \qquad B = jZ_0 \sin\beta\ell$

$C = jY_0 \sin\beta\ell \qquad D = \cos\beta\ell$

Total ABCD parameters are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_1 \begin{bmatrix} A & B \\ C & D \end{bmatrix}_2 \begin{bmatrix} A & B \\ C & D \end{bmatrix}_3$$

$$¿\begin{pmatrix} 1 & 0 \\ j & 1 \end{pmatrix}\begin{pmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ j & 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & j \\ j & -1 \end{pmatrix}$$

Because the hybrid is specified using S-parameters, we need to convert ABCD to S and obtain the reflection coefficient of the even mode. It is given by

$$\Gamma_e = \frac{A + B - C - D}{A + B + C + D} = 0 \qquad (\Gamma_e \text{ is } S_{11\text{-even}})$$

Similarly, the transmission coefficient of the even mode ($S_{21\text{-}even}$) is given by

$$T_e=\frac{2}{A+B+C+D}=\frac{-1}{\sqrt{2}}(1+j)$$

Here we used

$$S_{11}=\frac{A+B/Z_o-CZ_o-D}{A+B/Z_o+CZ_o+D},\quad S_{12}=\frac{2(AD-BC)}{A+B/Z_o+CZ_o+D}$$

$$S_{21}=\frac{2}{A+B/Z_o+CZ_o+D},\quad S_{22}=\frac{-A+B/Z_o-CZ_o+D}{A+B/Z_o+CZ_o+D}$$

Odd mode case analysis

The same approach can be used for the odd mode analysis.

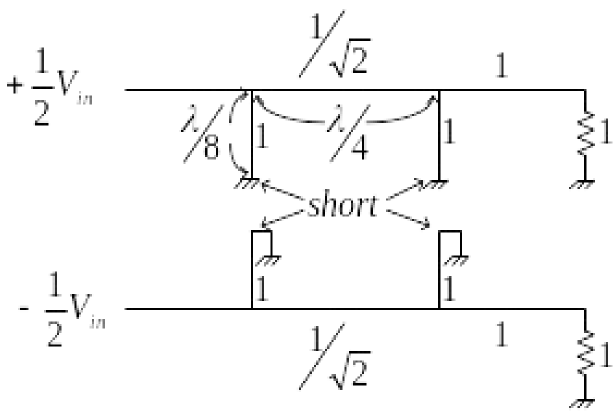


Fig. 14:

The $\frac{\lambda}{8}$ short stub has $Z_{in}=jZ_0\tan\beta\ell=j$.

or $Y_{in}=\frac{1}{Z_{in}}=-j$

Therefore, the total ABCD parameters are given by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o=\underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{\text{short stub}}\underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{\frac{\lambda}{4}\text{ TL}}\underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{\text{short stub}}$$

$$=\begin{pmatrix} 1 & 0 \\ -j & 1 \end{pmatrix}\begin{pmatrix} 0 & j/\sqrt{2} \\ j\sqrt{2} & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -j & 1 \end{pmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & j \\ j & 1 \end{pmatrix}$$

The reflection coefficient of the odd mode is (S_{11-odd})

$$\Gamma_o = \frac{A+B-C-D}{A+B+C+D} = 0$$

The transmission coefficient of the odd mode is (S_{21-odd})

$$T_o = \frac{2}{A+B+C+D} = \frac{1}{\sqrt{2}}(1-j)$$

In order to relate Γ_e , T_e , Γ_0 , and T_0 to the S-parameter of a 90^0 hybrid, we need to find the reflected signals from each port given by b_1, b_2, b_3, and b_4.

Since we know at port 1 $\Gamma_e = \frac{b_{1e}}{a_{1e}}$ and $\Gamma_o = \frac{b_{1o}}{a_{1o}}$

Also a_1 is the total input given by $a_1 = a_{1e} + a_{1o} = \frac{1}{2} + \frac{1}{2}$

Therefore,

$$b_1 = b_{1e} + b_{1o} = a_{1e}\Gamma_e + a_{1o}\Gamma_o = \frac{1}{2}(\Gamma_e + \Gamma_o) = 0$$

Similarly at port 4

$$\Gamma_e = \frac{b_{4e}}{a_{4e}} \quad \text{or} \quad \Gamma_o = \frac{b_{4o}}{a_{4o}}$$

$$a_4 = a_{4e} + a_{4o} = \frac{1}{2} - \frac{1}{2} = 0$$

Therefore

$$b_4 = b_{4e} + b_{4o} = \frac{1}{2}(\Gamma_e - \Gamma_o) = 0$$

The same process can be used for b_2 and b_3 but they are given in terms of the transmission coefficients.

$$b_2 = \frac{1}{2}T_e + \frac{1}{2}T_o = -\frac{j}{\sqrt{2}}$$

$$b_3 = \frac{1}{2}T_e - \frac{1}{2}T_o = -\frac{1}{\sqrt{2}}$$

These are related to S-parameters as

$$\therefore b_1 = 0 \quad \rightarrow \; S_{11} = 0$$

$$b_2 = -\frac{j}{\sqrt{2}} \rightarrow S_{21} = -\frac{j}{\sqrt{2}}$$

$$b_3 = -\frac{1}{\sqrt{2}} \rightarrow S_{31} = -\frac{1}{\sqrt{2}}$$

$$b_4 = 0 \quad \rightarrow \; S_{41} = 0$$

By symmetry, we can obtain the remaining elements of S-parameters and get

$$[S] = -\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

This shows the microstrip circuit shown in Fig.8 works as a 90-degree hybrid.

It is important to remember the length is specified in terms of wavelength and it is valid at the designed frequency. When the frequency is changed, the phase shift of $\lambda/4$ lines is no longer -90 degrees. The simple hybrid has a relatively narrow bandwidth. To obtain a wider bandwidth, a multi-section hybrid can be used.

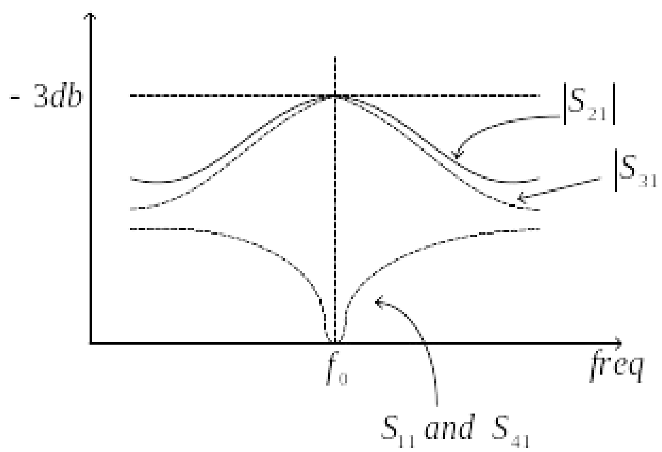


Fig. 15:

4. 180º Hybrid Circuit

The 180-degree hybrid can be specified by choosing

$$\begin{cases} \alpha=\beta=\dfrac{1}{\sqrt{2}} \\ \theta=0 \\ \varphi=\pi \end{cases}$$

Then [S] is given by

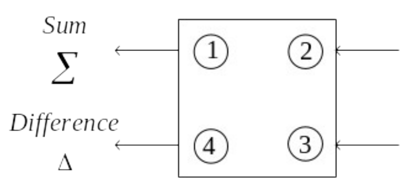
$$[S]=-\frac{j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

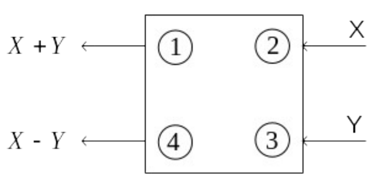
This circuit has

$S_{21}=S_{31}$ Same phase

$S_{42}=-S_{43}$ 180 phase difference

The 180° hybrid is known as a SUM and DIFFERENCE circuit. Suppose we have signal X at port 2 and signal Y at port 3. It is easy to show the output at port 1 contains X+Y and the output at port 4 contains X-Y. This type of circuit is useful for combining two antenna radiation patterns.





$$\begin{bmatrix} X+Y \\ 0 \\ 0 \\ -(X-Y) \end{bmatrix}=-\frac{j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ X \\ Y \\ 0 \end{bmatrix}$$

Fig. 16:

The common implementation of 180 degree hybrid using the microstrip circuit is shown in Fig. 17.

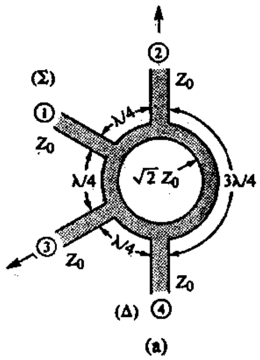


Fig. 17:

Intuitive analysis

Like a 90-degree hybrid case, we will start with an intuitive analysis based on the phase shift.

Input Port 1 — Outputs 2 and 3 in phase

Output 4 $\quad (\text{ccw})\frac{\lambda}{4}+\frac{\lambda}{4}$

) $\quad (\text{cw})\frac{\lambda}{4}+\frac{3\lambda}{4}\;) \rightarrow \frac{\lambda}{2}$ phase difference

¿

Output 2 $\quad 3\frac{\lambda}{4}$

Output 3 $\quad \frac{\lambda}{4}$ $\;\rightarrow \frac{\lambda}{2}$ phase difference

Input Port 4

Output 1 $\quad (\text{cw})\frac{\lambda}{4}+\frac{\lambda}{4}$

) $\quad (\text{ccw})\frac{\lambda}{4}+\frac{3\lambda}{4}\;) \rightarrow \frac{\lambda}{2}$ phase difference

¿

4-1. Even – odd mode analysis to get [S]-parameters

The process of analyzing the 180degree hybrid is the same as the 90-degree hybrid. However, there is only one line of symmetry in this circuit (horizontal line in Fig.18) and this requires the extra analysis as we discussed later.

Analysis with the input signal at Port 1

Assume we have an input at port 1. Port 1 and 4 can be used for the even and odd cases. The second subscript 1 denotes port 1 is used as input.

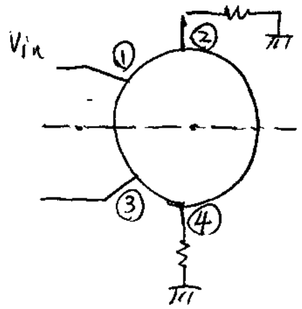
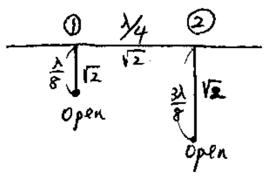


Fig. 18:

Even mode case

$$\begin{cases} \frac{1}{2}V_{\text{in}} \text{ at port 1} \\ \frac{1}{2}V_{\text{in}} \text{ at port 3} \end{cases}$$

The center point becomes an open circuit and the equivalent circuit is given by



$$Z_{\text{in}}(l=\frac{\lambda}{8})=-j\sqrt{2}$$

$$Z_{\text{in}}(l=\frac{3\lambda}{8})=j\sqrt{2}$$

Fig. 19:

The total ABCD parameters are given by

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{pmatrix} \begin{pmatrix} 0 & j\sqrt{2} \\ j\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{pmatrix} = \begin{pmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{pmatrix}$$

$$\therefore \Gamma_{e1} = \frac{-j}{\sqrt{2}}$$

$$T_{e1} = \frac{-j}{\sqrt{2}}$$

Odd mode case

$$\begin{cases} \frac{1}{2}\mathrm{V}_{\text{in}} \text{ at port 1} \\ -\frac{1}{2}\mathrm{V}_{\text{in}} \text{ at port 3} \end{cases}$$

The center point becomes a short circuit and the equivalent circuit is given by

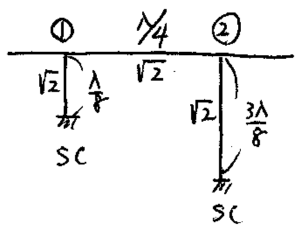


Fig. 20:

The total ABCD parameters are given by

Corrected

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -j/\sqrt{2} & 1 \end{pmatrix} \begin{pmatrix} 0 & j\sqrt{2} \\ j\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ j/\sqrt{2} & 1 \end{pmatrix} = \begin{pmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{pmatrix}$$

$$\therefore \Gamma_{o1} = \frac{j}{\sqrt{2}}$$

$$T_{o1} = \frac{-j}{\sqrt{2}}$$

To obtain S-parameters, we need to find b_{11}, b_{21}, b_{31}, and b_{41}. They are given by

$$b_{11} = \frac{1}{2}\Gamma_{e1} + \frac{1}{2}\Gamma_{o1} = 0$$

$$b_{21} = \frac{1}{2}T_{e1} + \frac{1}{2}T_{o1} = \frac{-j}{\sqrt{2}}$$

$$b_{31} = \frac{1}{2}\Gamma_{e1} - \frac{1}{2}\Gamma_{o1} = \frac{-j}{\sqrt{2}}$$

$$b_{41} = \frac{1}{2}T_{e1} - \frac{1}{2}T_{o1} = 0$$

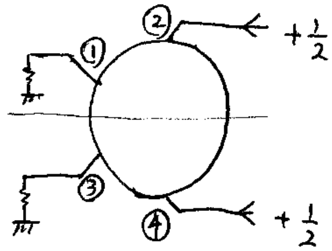
This shows when we have an input at Port 1 (a_1), outputs at Port 2 and 3 are in phase (no phase shift).

Unlike the 90-degree hybrid, when the input signal is at port 4, the response will be different from that of the previous case (port 1 is input). We need to conduct additional analysis using port 4 as input.

Even and odd mode analysis with the input signal at Port 4

The second subscript 4 denotes port 4 is used as input.

Even mode

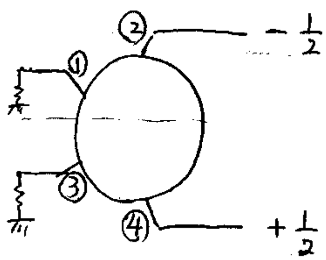


$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}_e = \begin{pmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{pmatrix}$$

$$\Gamma_{e4} = \frac{j}{\sqrt{2}}$$

$$T_{e4} = \frac{-j}{\sqrt{2}}$$

Odd mode



corrected $\begin{pmatrix} A & B \\ C & D \end{pmatrix}_o = \begin{pmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{pmatrix}$

$$\Gamma_{o4} = \frac{-j}{\sqrt{2}}$$

$$T_{o4} = \frac{-j}{\sqrt{2}}$$

From these we can find b_{14}, b_{24}, b_{34}, and b_{44}.

$$b_{14} = \frac{1}{2}T_{e4} - \frac{1}{2}T_{o4} = 0$$

$$b_{24} = \frac{1}{2}\Gamma_{e4} - \frac{1}{2}\Gamma_{o4} = \frac{j}{\sqrt{2}}$$

$$b_{34} = \frac{1}{2}T_{e4} + \frac{1}{2}T_{o4} = \frac{-j}{\sqrt{2}}$$

$$b_{44} = \frac{1}{2}\Gamma_{e4} + \frac{1}{2}\Gamma_{o4} = 0$$

When the input signal is from Port 4, outputs at Port 2 and 3 have a phase shift of 180 degree.

By combining these results, we can show the total S-parameters as

$$[S] = -\frac{j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

4-2. Applications of 180° hybrid

11.3.5 Mixer Design Examples

The following discussion illustrates the design procedure for a mixer with the following specifications:

$$f_s = 12.0 \text{ GHz}$$

$$f_p = 10.5 \text{ GHz}$$

$$\text{Conversion loss} < 6.5 \text{ dB}$$

$$\text{Noise figure} < 6 \text{ dB}$$

$$\text{LO/RF isolation} > 20 \text{ dB}$$

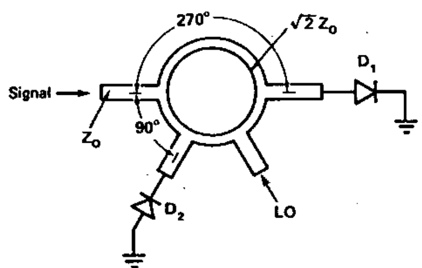


Figure 11.41 Mixer ring hybrid.

1. A single balanced mixer design using a 180-degree hybrid is selected from Table 11.4 to provide the 20-dB LO/RF isolation. A 3-dB ring hybrid shown in Fig. 11.41 is used to split the signal and LO power between the two diodes at the desired phase. The ring hybrid has the characteristic that the signal and LO are in phase at one diode and 180 degrees out of phase at the other diode. The electrical length between the signal and LO ports is 180 degrees going one way, and 360 degrees going the other way around the ring. This provides the cancellation required to achieve the 20-dB LO/RF isolation. This hybrid is easily implemented in microstrip. It will be used in the following mixer design.

2. *Diode Parameter Selection*. For a LO frequency of 10.5 GHz,

$$C_{j0} = \frac{1.6}{f_p} = 0.15 \text{ pF}$$

$$f_{c0} = 500 \text{ GHz} \quad \text{(using a GaAs Schottky-barrier diode)}$$

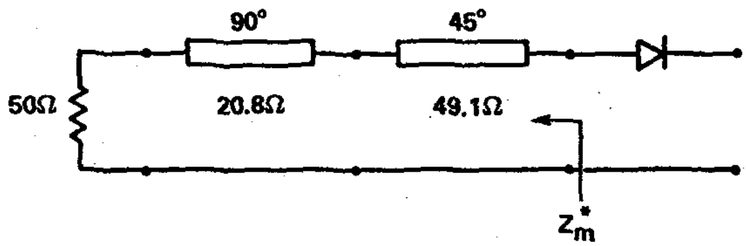
$$R_s = \frac{1}{2\pi f_{c0} C_{j0}} = 2.1\ \Omega$$

$$I_s = 1 \times 10^{-9} \text{ A} \quad \text{(for the selected diode)}$$

3. *Package Selection*. The package is selected that has parasitic capacitance and inductance that are resonant approximately at the sum frequency $f_{sum} = 22.5$ GHz. The package parasitics are

$$C_p = 0.15 \text{ pF}$$

$$L_p = 0.4 \text{ nH}$$



ALL ELECTRICAL LENGTHS AT 10.5 GHz

Figure 11.42 Initial diode-matching circuit.

4. *Determine the Two Diode Impedance States*. The two impedances for the diode were calculated from Fig. 11.26 to be

$$Z_{on} = 12.8 + j33.9\ \Omega$$

$$Z_{off} = 0.55 - j49.5\ \Omega$$

5. Calculate the Hyperbolic Mean from (11.60) and (11.61):

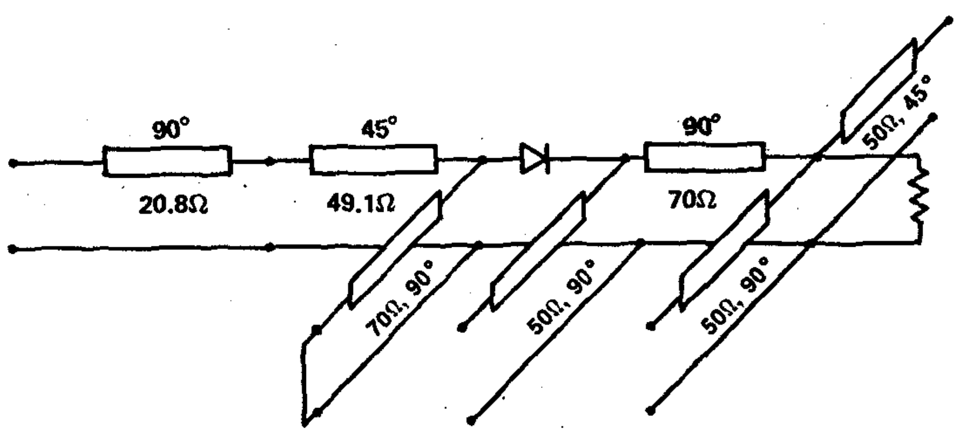
$$Z_m = 16.8 - j46.1\ \Omega$$

The two-section matching circuit shown in Fig. 11.42 transforms Z_m to 50 Ω. The resulting mixer reflection coefficients for the two diode states are

$$\Gamma_1 = 0.939\angle{-176.9°}$$

$$\Gamma_2 = 0.939\angle{+3.1°}$$

which are seen to be equal in amplitude and 180 degrees out of phase.



ALL ELECTRICAL LENGTHS AT 10.5 GHZ

Figure 11.43 *Preliminary mixer subcircuit design.*

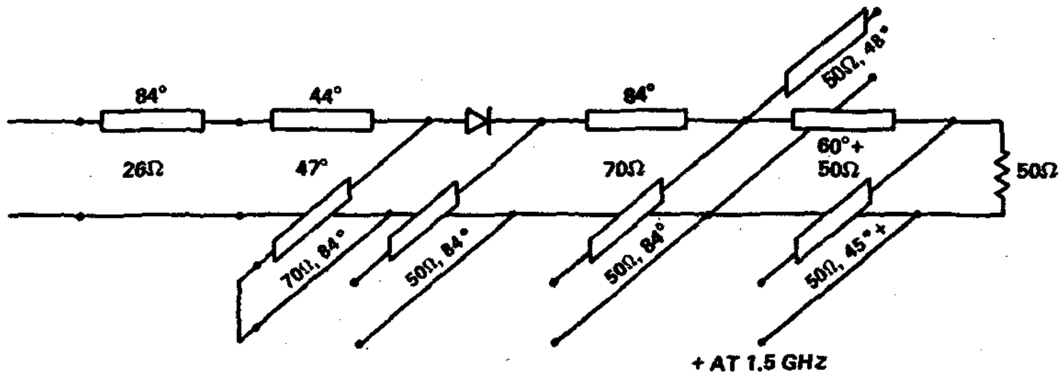


Figure 11.44 Final mixer subcircuit schematic.

6. The initial mixer subcircuit schematic (without the hybrid) including short- and open-circuited stubs, is shown in Fig. 11.43. The IF filter is realized with quarter-wavelength (at the LO frequency) open-circuited stubs separated by a quarter-wavelength transmission line. It uses the mixer open-circuited stub as the first element. Because of the periodic nature of this filter, an additional open-circuited stub (90 degrees at twice the LO frequency) is included to control the circuit impedance at the sum and second harmonic of the LO frequency. This circuit was analyzed using the Siegel, Kerr, and Hwang computer program previously described [22], and was found to have a high conversion loss of 9.3 dB due to a significant mismatch loss at the signal and IF frequencies. The signal-frequency mismatch is due to the high IF that separates the LO and signal frequencies. Based upon the computed diode impedance at these frequencies, the signal-matching circuit was modified to more closely match the diode at f_s and an IF-port matching circuit was added to reduce the IF mismatch. The modified schematic of the mixer subcircuit is shown in Fig. 11.44. The mixer subcircuit performance was again computed and is summarized in Table 11.7. This calculated performance must be modified to account for the hybrid loss that adds directly to the loss, noise figure, and required LO power. Assuming a hybrid loss of 0.5 dB, the mixer performance would be as listed in Table 11.8. A layout of the final mixer design is shown in Fig. 11.45. The linewidths and line lengths would be determined by the substrate material used. Discontinuity models for the steps, T junctions, and open-

TABLE 11.7 Mixer Subcircuit Performance

Conversion loss (L_{01})	5.7 dB
Noise temperature	296 K
Noise figure	3.1 dB
Required LO power (per diode)	1.6 mW

TABLE 11.8 Final Mixer Performance

Conversion loss (L_{01})	6.2 dB
Noise figure	3.6 dB
LO power	3.6 mW

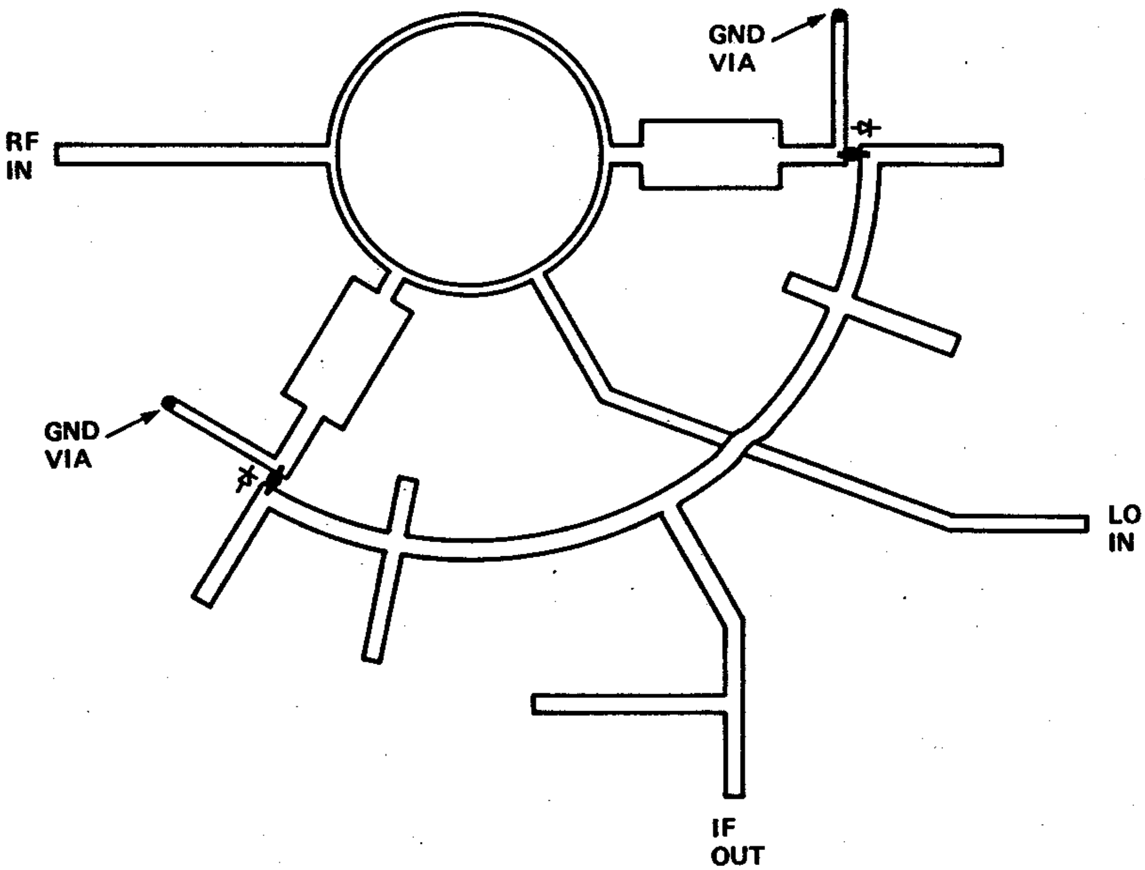


Figure 11.45 Microstrip mixer layout.

circuited stubs should be included in the analysis at this point. The desired embedding impedances are known from the analysis, and standard circuit optimization programs can be used to adjust the transmission-line lengths and impedances to compensate for the discontinuity effects.

5. 3-Port Power Divider

Although hybrids are 4-port devices, many of them are used as a 3-port device with an unused port terminated with a matched impedance. If we need a power divider as shown in Fig. 5-1, we should design a 3-port device rather than using a 4-port device as a 3-port device.

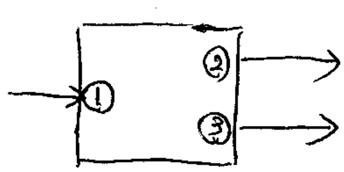


Fig. 5-1:

The desired characteristics of a power divider are

- Input power separated into 2 ports.
- High isolation between ports 2 and 3
- Matched ports
- $[S]=[S]^t$: Network is reciprocal
- Lossless

From these we can express the desired S-parameters as

$$[S]=\begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

Among the desired characteristics, we have not checked the lossless condition in [S]. The lossless conditions are given by

$$[S]^t[S]^*=[U]$$

$$|S_{12}|^2+|S_{13}|^2=1$$

$$\left.\begin{aligned} |S_{21}|^2 &= \frac{1}{2} \neq 1 \\ |S_{31}|^2 &= \frac{1}{2} \neq 1 \end{aligned}\right\} \text{Does not satisfy the lossless condition.}$$

It is apparent that [S] does not satisfy the lossless condition and we don't know if we can realize the desired [S]. Later we will discuss where the loss is coming from.

To investigate if we can design a 3-port device with desired characteristics, we start with less restrictive conditions.
Assume we want a lossless, matched and reciprocal circuit. Then we can write a general [S] as

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

We don't know the values of non-zero elements yet but [S] satisfy two conditions.

$[S]=[S]^t$: Reciprocal

$S_{11}=S_{22}=S_{33}$: Matched

How about the lossless condition?

We need to show

$$|S_{12}|^2+|S_{13}|^2=1$$

$$|S_{12}|^2+|S_{23}|^2=1$$

$$|S_{13}|^2+|S_{23}|^2=1$$

Also we need to satisfy

$$\left.\begin{aligned} S_{12}S_{13}^* &=0 \\ S_{12}S_{23}^* &=0 \\ S_{13}S_{23}^* &=0 \end{aligned}\right| \rightarrow (S_{12}=0 \text{ and } S_{23}=0) \text{ OR } (S_{12}=0 \text{ and } S_{13}=0)$$

But these conditions cannot be satisfied.

Therefore, there is no power divider which satisfies

Lossless
)Matched) → *Characteristics*
)Reciprocal
¿

5-1. T-junction Power Divider

To understand what [S] we can get from common power dividers, we will take a look at a simple and least expensive power divider called T-junctions. Two examples T-junctions are shown in Fig. 5-2.

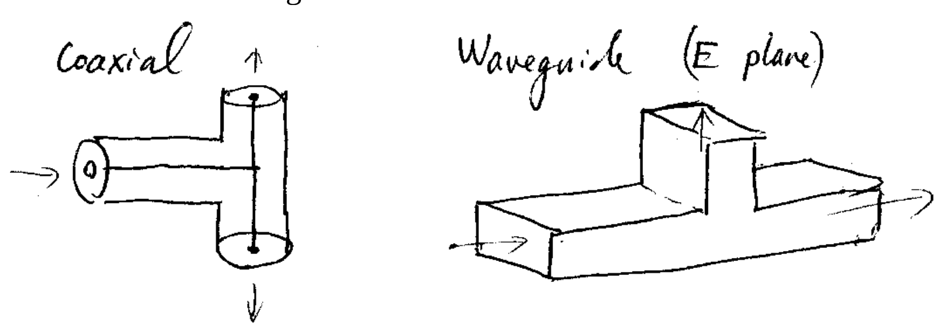
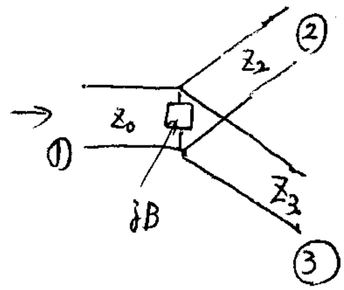


Fig. 5-2:

The equivalent circuit of a T-junctions is



jB: reactive element due to discontinuity

Fig. 5-3:

Assume there is no physical length (no phase shift). At Port 1, the input admittance is given by

$$Y_{in}=jB+\frac{1}{Z_2}+\frac{1}{Z_3}$$

To simplify the problem, let assume $jB=0$.

If we want to set $Y_{in}=\frac{1}{Z_0}=\frac{1}{50}$, then

Z_2 and Z_3 must have a characteristic impedance of 100 ohm

If $Z_2=Z_3=100\,\Omega$, Port 1 is matched.

However, Port 2 and 3 are not matched. Because at Port 2, $Y_{in}=\frac{1}{Z_0}+\frac{1}{Z_3}\neq\frac{1}{Z_2}$.

This shows the simple T-junction does not have matched ports.

Analysis of T – junction

Now we will take a look at the detailed analysis of a T-junction. Let assume port 2 and 3 are terminated with the matched impedance of Zo.

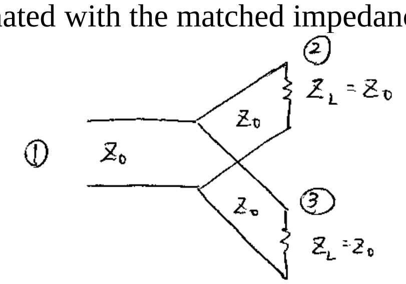


Fig. 5-4:

The reflection coefficient at port 1 is

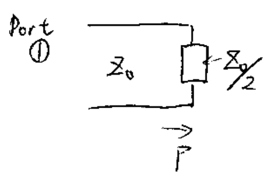


Fig. 5-5:

$$\Gamma = \frac{\frac{Z_0}{2} - Z_0}{\frac{Z_0}{2} + Z_0} = -\frac{1}{3}$$

Since the T-junction is the same whether we use port 1 as input or port 2 or 3 as input, we get

$$S_{11} = -\frac{1}{3}$$

$$)S_{22} = -\frac{1}{3}\,) \rightarrow \text{ Input ports: not matched}$$

$$)S_{33} = -\frac{1}{3}$$

¿

The transmission coefficient can be obtained by taking a look at Fig. 5-6. We need to find the voltage at port 2 to obtain the S_{21}.

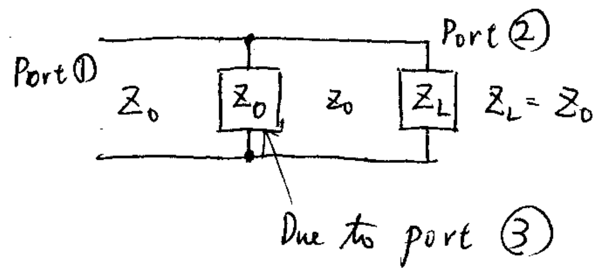


Fig. 5-6:

Since we used the ABCD parameters in the previous section, we will follow the same process. The ABCD of the shunt impedance is given by

$$\Rightarrow \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{Z_0} & 1 \end{pmatrix}$$

Fig. 5-7:

Using this we get

$$\Gamma = \frac{A + \frac{B}{Z_0} - CZ_0 - D}{A + \frac{B}{Z_0} + CZ_0 + D} = -\frac{1}{3}$$

$$T = \frac{2}{A + \frac{B}{Z_0} + CZ_0 + D} = \frac{2}{3}$$

Therefore, S21 and S31 are

$$S_{21} = \frac{2}{3}$$

$$S_{31} = \frac{2}{3}$$

Then we can get

$$[S] = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

This [S] is reciprocal because

$$[S] = [S]^t \quad :$$

This [S] is also lossless because

$$(|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$$

$$(S_{11} S_{12}^{*} + S_{21} S_{22}^{*} + S_{31} S_{32}^{*} = 0\ ($$

$$([S]^t [S]^{*} = [U]$$

Therefore, the T-junction is lossless and reciprocal but it does not have matched input ports. Also the isolation of two output ports (S_{32} if port 1 is input) is bad.

The well made T – junction has a wide bandwidth if the size is small compared to the wavelength and it can be used from DC up to ** GHz (** depends on the structure).

Another approach:

The total voltage on the load Z_L is V_L. The load reflection coefficient is Γ_L.

$\Gamma_L = (Z_L - Z_o)/(Z_L + Z_o)$

$V^- = \Gamma_L V^+$

$V_L = V^+ + V^-$

Define the transmission coefficient as

$V_L = TV^+$

$T V^+ = V^+ + V^- = V^+ (1+ \Gamma_L)$

Therefore

$T = 1+ \Gamma_L$

Γ_L of T-junction is -1/3. Then the transmission coefficient into Port 2 and 3 is T=2/3. We get

$$[S] = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

5-2. Resistive Power Divider

Although the T-junction is inexpensive, the unmatched ports create problems in many applications. To solve this problem, the resistive power divider was developed. The equivalent circuit is shown in Fig. 5-8. Each resister has a value of Zo/3 and it must be very small compare to the wavelength.

The resistive power divider has the characteristics

1. Matched input ports
2. Reciprocal
3. Not lossless (resistive)

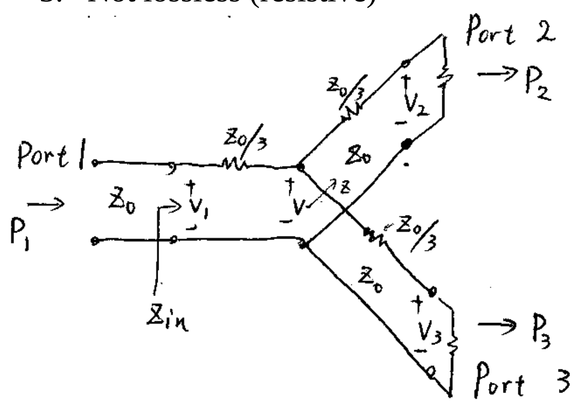


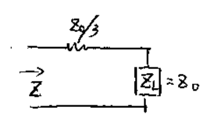
Fig. 5-8:

From Fig. 5-8, we can get the input impedance at port 1 as

$$Z_{in} = \frac{Z_0}{3} + (Z \,//\, Z) = Z_0$$

where

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}$$



Therefore, all ports are matched.

$$S_{11} = S_{22} = S_{33} = 0$$

The transmission coefficient can be obtained by using the voltage divider.
V is the voltage at the center due to the input V1.

$$V = V_1\left(\frac{Z_T}{Z_0/3 + Z_T}\right) = \frac{2}{3}V_1, \qquad Z_T = Z \,//\, Z = \frac{2}{3}Z_0$$

Then the output voltage is

$$V_2 = V_3 = V\left(\frac{Z_0}{Z_0/3 + Z_0}\right) = \frac{3}{4}V = \frac{1}{2}V_1$$

V_2/V_1 and V_3/V_1 are S_{21} and S_{31}, respectively.
Therefore, we get

$$\therefore S_{12} = S_{21} = S_{31} = S_{13} = S_{23} = S_{32} = \frac{1}{2}$$

Finally, we have

$$[S] = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

The input power can be expressed as

$$P_1 = \frac{1}{2}\frac{V_1^2}{Z_0}$$

Then the output power at port 2 and 3 is

$$P_2 = P_3 = \frac{1}{2}\frac{1}{Z_0}\left(\frac{1}{2}V_1^2\right) = \frac{1}{4}P_{in}$$

This shows the 3dB power loss due to resistive elements. It is easy to show the resistive power divider does not satisfy the lossless conditions.

Also the resistive power divider does not have a high isolation between two output ports (S_{32}=1/2). The resistive power divider has a wide bandwidth. It can be used from DC up to ** GHz (** depends on the structure).

5-3. Reactive Power Divider

At the beginning of this section, we show the desired characteristics and [S] of a power divider. We also show this [S] does not satisfy the lossless condition. In this section, we can realize the desired [S] using the reactive power divider. We will also show why the reactive power divider is not lossless.

The reactive power divider has the characteristics

1. Matched input ports
2. High isolation between ports 2 and 3
3. Lossless if three ports are matched

Fig. 5-9 shows an example of the Wilkinson power divider based on the microstrip circuit.

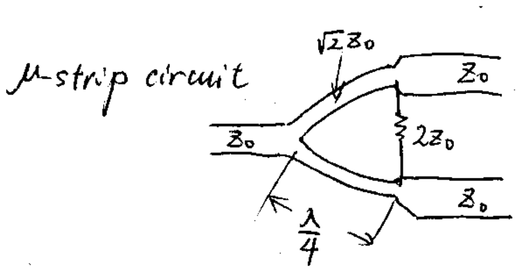


Fig. 5-9:

To analyze the reactive power divider, we express Fig.5-9 using the equivalent circuit shown in Fig.5-10.

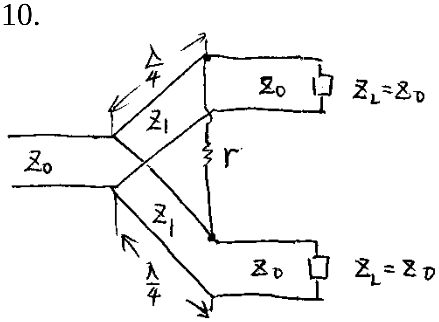


Fig. 5-10:

The characteristic impedance Z_1 and the value of a resistor ***r*** are still unknowns which must be defined to get the desired characteristics. The analysis will be based on the even and odd mode method. To identify the line of symmetry, we separate r into two ***r***/2 and the port 1 TL into two TLs with the characteristic impedance of $2Z_o$ in parallel.

The equivalent circuit using $Z_0 = 1$ is shown in Fig. 5-11. The line of symmetry is also shown.

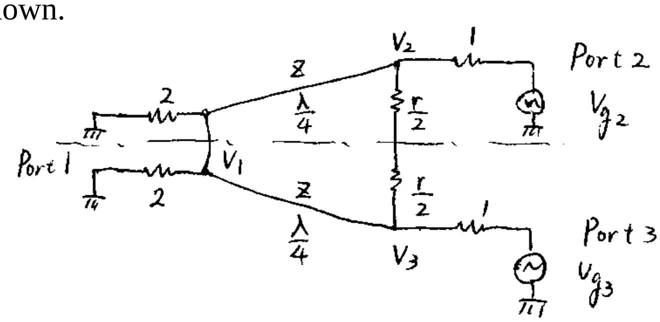


Fig. 5-11:

Assume Port 2 is excited and there is no input at port 3. We write

$V_{g2} = 1$ and $V_{g3} = 0$

This can be separated into even and odd cases as

$V_{g2} = V_{g3} = 1/2$ Even mode excitation

$V_{g2} = 1/2$ $V_{g3} = -1/2$ Odd mode excitation

Even mode case

First, we want to find the input impedance at Port 2. Because r is much smaller than the wavelength, the open ended resistor has an infinite value.

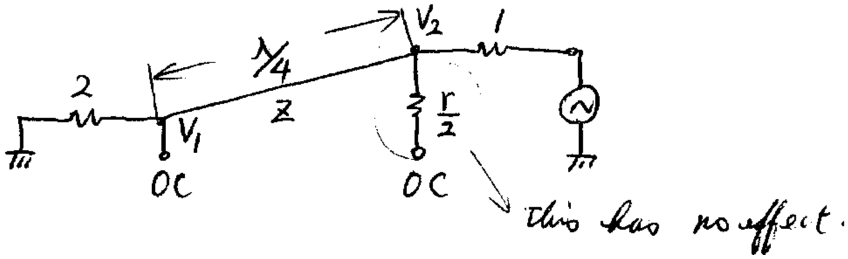


Fig. 5-12:

Then the input impedance at port 2 is

$$Z_{in} = Z_{01}\left(\frac{Z_L + jZ_{01}\tan\beta\ell}{Z_{01} + jZ_L\tan\beta\ell}\right) = \frac{Z^2}{2}, \rightarrow \left(\begin{array}{l} Z_L = 2 \\ Z_{01} = Z \\ \beta\ell = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2} \end{array}\right. \qquad \left(\therefore Z = \sqrt{Z_{in} * 2}\right.$$

If we want to set $Z_{in} = 1$, then $Z = \sqrt{2}$. This will create

$S_{22e} = 0$ matched port 2

Because of symmetry, we can show

$S_{33e} = 0$ matched port 3

However, we don't know S_{11e} yet.

Next we want to find the transmission coefficient from port 2 to port 1.

$$S_{12e} = \frac{V_1}{V_2}$$

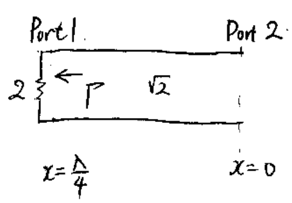


Fig. 5-13:

Let us define the position x = 0 at Port 2 and $x = \frac{\lambda}{4}$ at Port 1. Since the voltage on the TL can be expressed by $V(x=0) = V^+\left(e^{-j\beta x} + \Gamma e^{j\beta x}\right)$.
at x = 0, we get

$$V(x=0)=V_2=V^+(1+\Gamma)$$
$$V^+=\frac{V_2}{1+\Gamma}$$

at $x=\frac{\lambda}{4}$, we get

$$V\left(x=\frac{\lambda}{4}\right)=V_1=V^+(-j+\Gamma j)=jV_2\left(\frac{\Gamma-1}{\Gamma+1}\right)$$
$$\therefore V_1=jV_2\left(\frac{\Gamma-1}{\Gamma+1}\right)$$

Also

$$\Gamma=\frac{2-\sqrt{2}}{2+\sqrt{2}}$$

Finally we get

$$S_{12e}=\frac{V_1}{V_2}=-j\frac{1}{\sqrt{2}}=-j0.707$$

Similarly, we get

$$S_{13e}=-j0.707$$

Odd mode case

The equivalent circuit of the odd mode case is shown in Fig. 5-14.

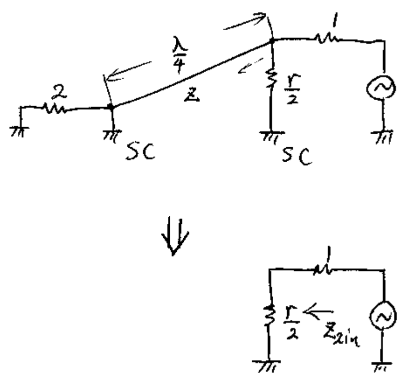


Fig. 5-14:

Because a TL is terminated with SC and $\ell=\frac{\lambda}{4}$, the input impedance of the $\frac{\lambda}{4}$ TL section has $Z_{in}=\infty$ (open circuit). Then the input impedance of port 2 is simply r/2 as shown on Fig. 5-14.

To match the input impedance to Zo (Zo=1), Z_{2in} must be

$$Z_{2\,in}=\frac{r}{2}=1$$
$$\therefore r=2$$

With this choice, we have $S_{22o}=0$ and $S_{33o}=0$.

Because the center point is either open or short, there is no coupling between ports 2 and 3 for both even and odd cases. We have

$$S_{23o}=S_{23e}=0$$
$$S_{32o}=S_{32e}=0$$

Finally we need to find S_{11}. Instead of using even and odd cases, we will consider the whole circuit to obtain S_{11}. The equivalent circuit is shown in Fig. 5-15.

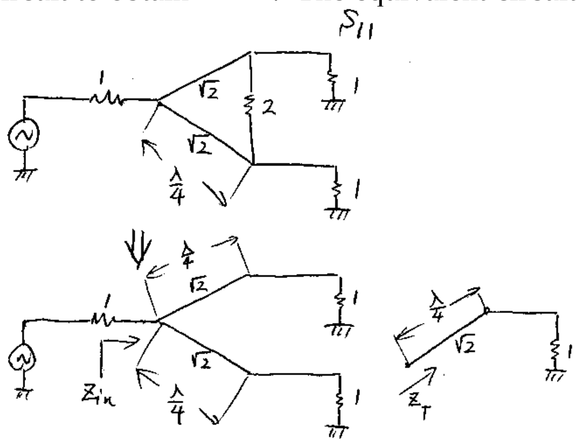


Fig. 5-15:

In this circuit, there will be no current flowing thru ***r***=2. Therefore, it is effectively open and can be neglected. We also get Z_T using the quarter wave transformer technique.

$$Z_T=\frac{\left(\sqrt{2}\right)^2}{1}, \qquad \left(Z_{01}=\sqrt{2}\right.,\ \left(Z_L=1\right.,\ \left(\beta\ell=\frac{\pi}{2}\right.$$

Then the input impedance is

$$Z_{in}=\left[\frac{\left(\sqrt{2}\right)^2}{1}\right]//\left[\frac{\left(\sqrt{2}\right)^2}{1}\right]=1$$
$S_{11}=0$ matched.

So far we have

$S_{22e}=S_{33e}=0$ and $S_{22o}=S_{33o}=0$

$S_{12e} = -j0.707$ and $S_{12o} = 0$
$S_{13e} = -j0.707$ and $S_{13o} = 0$
$S_{23o} = S_{23e} = 0$
$S_{32o} = S_{32e} = 0$
$S_{11} = 0$

The total S-parameter is the sum of even and odd cases. However, the nonzero elements are only S_{12e} and S_{13e}. Therefore, we get

$$[S] = \begin{bmatrix} 0 & -j0.707 & -j0.707 \\ -j0.707 & 0 & 0 \\ -j0.707 & 0 & 0 \end{bmatrix}$$

The input power is equally divided into two ports without the loss of energy. The isolation of port 2 and 3 are very good (S_{23}=0). Also all 3 ports have the matched impedance.

It is easy to show this [S] does not satisfy the lossless conditions. However, we also show the input power will be divided without loss. The loss of energy occurs if the loads connected to port 2 and 3 are not matched. The reflected power will be dissipated in $r = 2Z_0$. As long as all ports are terminated with the matched impedance, the reactive power divider does not have any loss.

The reactive power divider has TL sections specified by the wavelength. This mean the characteristics are sensitive to the frequency change. The typical frequency response is shown in Fig. 5-16.

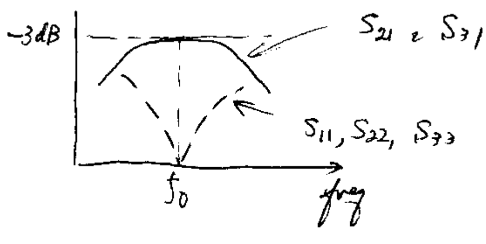


Fig. 5-16:

The common frequency band of the reactive power dividers are the octave-band such as 1-2 GHz, 2-4 GHz, and 4-8 GHz. The multi λ/4 sections usually used to increase the bandwidth.

6. Mitering a Right-Angle Bend

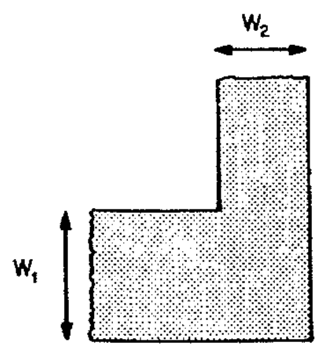
The hybrids and power dividers may have many bend and tee sections. The parasitic admittance due to the discontinuity must be included in the design. Fortunately, the modern simulation tools such as Ansoft Designer have this capability. If the simulation tool does not have an option to include the discontinuity, the simple TL length correction shown in this section can be used.

IN THIS CHAPTER the four dominant discontinuity effects in microstrip shall be considered: the excess capacitance of a corner, the capacitive end effect for an open circuit, the step change in width, and the length correction for the shunt arm of a tee. *Puff* will automatically miter corners in the artwork to reduce their effect. However, you must compensate for the other discontinuities yourself; they are neglected in the *Puff* analysis.

When a sharp right-angle bend occurs in a circuit (Fig. 7.1a), there will be a large reflection from the corner capacitance. *Puff* miters corners to reduce the capacitance and minimize this reflection, as shown in Fig. 7.1b. You can change the value of the miter fraction m defined to be

$$\mathtt{m} = 1 - b/\sqrt{W_1^2 + W_2^2}. \tag{7.1}$$

When the two line widths are equal, this formula reduces to the conventional definition [1]. In `setup.puf`, m is set to 0.6.



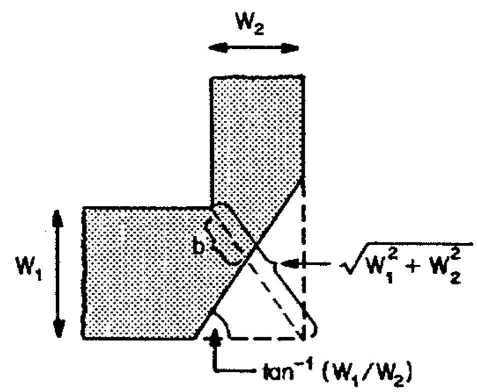


Figure 7.1 Mitering a right-angle bend.

In an open-circuit, the electric fields extend beyond the end of the line. This excess capacitance makes the electrical length longer than the nominal length, typically by a third to a half of the substrate thickness. This will cause the design frequencies of patch antennas and filters to be shifted. To compensate for this effect in the artwork, a negative length correction can be added to the parts list. Hammerstad and Bekkadal give an empirical formula for the length extension l in microstrip [2],

$$\frac{l}{h} = 0.412\left(\frac{\epsilon_{re}+0.3}{\epsilon_{re}-0.258}\right)\left(\frac{W/h+0.262}{W/h+0.813}\right), \tag{7.2}$$

where h is the thickness of the substrate. This formula is plotted in Fig. 7.2 for several different dielectric constants.

A similar method may be used to compensate for a step change in width between high and low impedance lines. The discontinuity capacitance at the end of the low impedance line will have the effect of increasing its electrical length. Assuming the wider low impedance line has width W_2, and the narrower high impedance line has width W_1, compensate using the expression [1]

$$\frac{l_s}{h} \approx \frac{l}{h}\left(1 - \frac{W_1}{W_2}\right) \tag{7.3}$$

where l_s is the step length correction for line W_2, and l/h is the value obtained from (7.2) and Fig. 7.2.

In the tee, shown in Fig. 7.3, the electrical length of the shunt arm is shortened by distance d_2. The currents effectively take a short cut, passing close to the corner. It is particularly noticeable in the branch-line coupler because there are four tees. Hammerstad and Bekkadal give an empirical formula for d_2 in microstrip [2],

$$\frac{d_2}{h} = \frac{120\pi}{Z_1\sqrt{\epsilon_{re}^1}}\left(0.5 - 0.16\frac{Z_1}{Z_2}\left[1 - 2\ln(Z_1/Z_2)\right]\right), \tag{7.4}$$

where ϵ_{re}^1 is the effective dielectric constant of the through arm. Equation (7.4) is plotted in Fig. 7.3 for a 50-Ω through line. This correction can also be used as a first estimate for compensating a four-way cross. For additional help on discontinuity modeling for both microstrip and stripline, see Gupta, et al. [3].

References

[1] T. C. Edwards, *Foundations for Microstrip Engineering*, John Wiley, New York, 1981.

[2] E. O. Hammerstad and F. Bekkadal, *A Microstrip Handbook*, ELAB Report, STF 44 A74169, N7034, University of Trondheim, Norway, 1975.

[3] K.C. Gupta, R. Garg, and R. Chadha, *Computer-Aided Design of Microwave Circuits*, Artech House, Dedham, Mass., 1981.

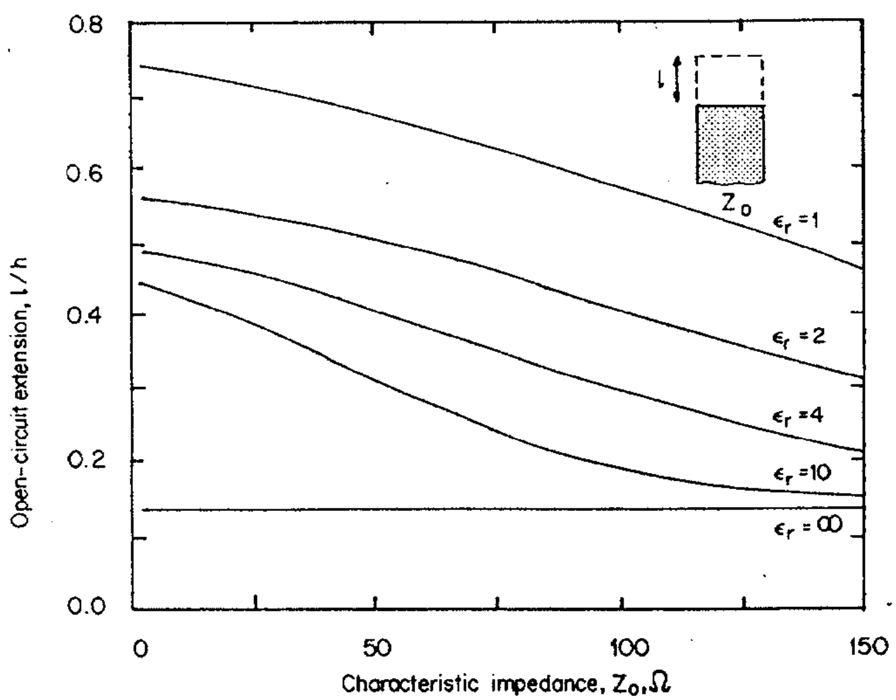


Figure 7.2 The open-circuit end correction in microstrip, plotted from (7.2). The artwork length correction in a parts list should be *negative.*

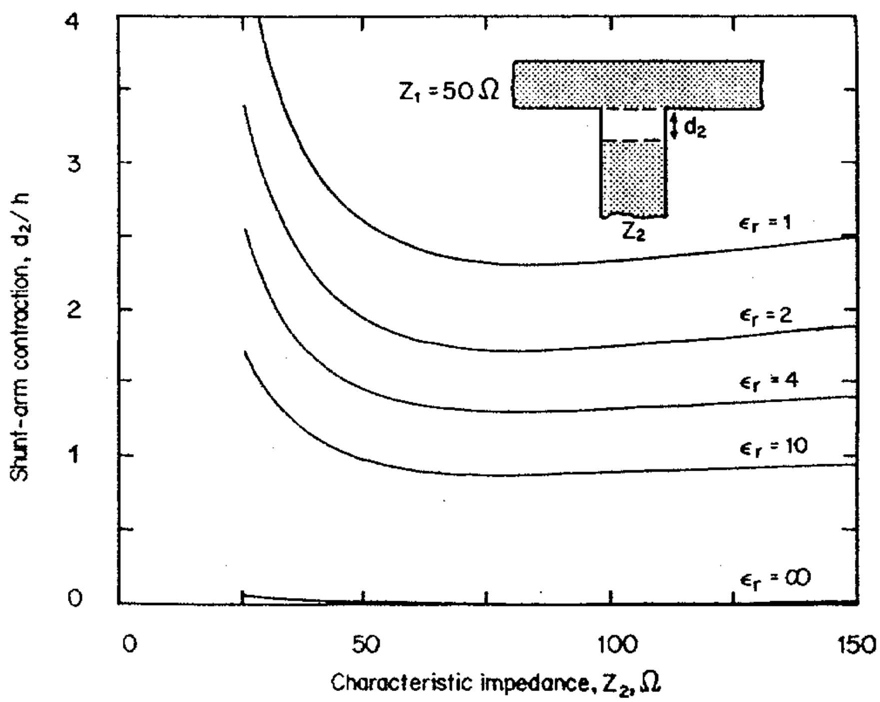
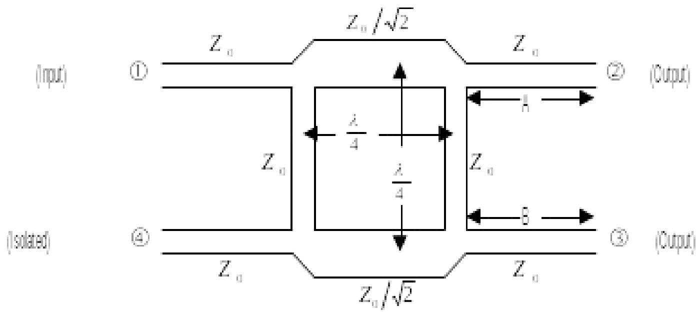


Figure 7.3 The shortening of the shunt arm of a microstrip tee, given by (7.4). The length correction for the shunt arm should be *positive.*

Two designs of 90 degree hybrid

1. Traditional layout



2. Different 90^{o} hybrid layout option

The sector section can be reduced to compensate the parasitic impedance.