# Coupled Noise on Digital Circuits 10/15/16

## 1. Introduction

Assume we have two TLs as shown below and a high-speed digital signal is transmitted thru the left TL. Ideally, the energy propagation of high-speed digital signal should be confined solely to the left TL. In practice, however, energy leakage through capacitive and inductive paths occurs. This phenomenon is called ***coupling*** (also known as **crosstalk**). In this section, we will study what causes coupling, how to model it, and how to detect and analyze it.

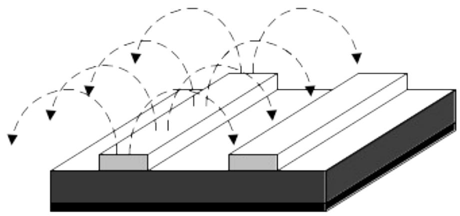


Fig.1: Two TL and coupling.

## Capacitive and inductive coupling

Before we start the detailed analysis, we need to visualize what causes the coupling. There are two types of coupling. The first one is called capacitive coupling. When we put two conducting lines (or plate) close together, there is always a parasitic capacitance between two conductors. When the fast changing signal (high frequency field) is excited on the left TL, the field extends beyond the left TL and some of it will be leaked into the right TL. The amount of leakage depends on the line separation and TL structures. In terms of lumped elements, the capacitive coupling is related to the self (between the signal line and ground) and mutual (between two signal lines) capacitances of TLs.

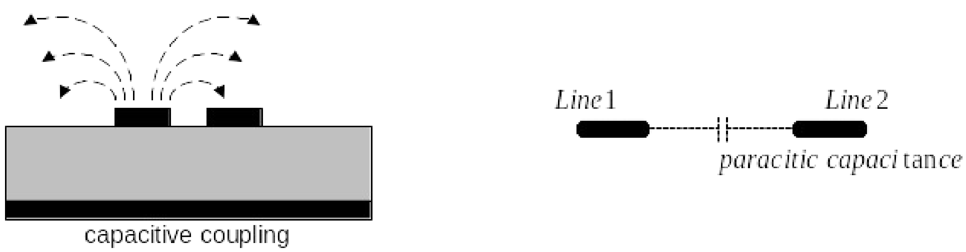



Fig. 2: Capacitive coupling due to *parasitic capacitance.* Energy leakage through spatial fringing of electric field.

The second type of coupling is called inductive coupling. This phenomenon is similar to the power coupling in an electric transformer. As shown below, the magnetic field due to the current going thru the left TL can be coupled into the right TL and induce the voltage.

Because any conducting line has a self inductance, we can view this type of coupling due to the mutual inductance between the self inductances of the left TL and that of right TL.

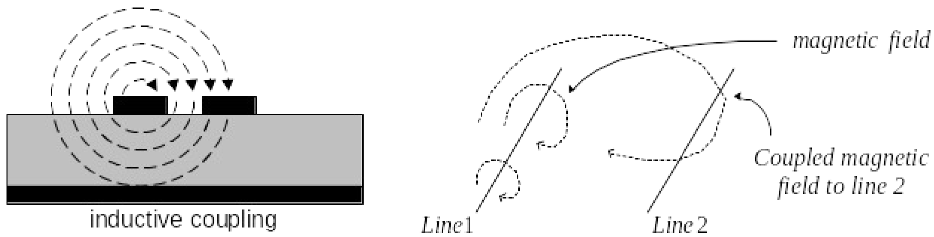



Fig. 3: Inductive coupling due to *mutual inductance.* Energy leakage through spatial fringing of magnetic field.

As the figures indicates, the amount of coupling is related to the line separation and it can be reduced by increasing the TL separation. The coupling can also be reduced by inserting a ground line between two signal lines. However, these techniques will increase the PCB area (material cost) and it is not desirable. Modern digital circuits have many data and address lines which are often placed in parallel on PCB. If the traces are not carefully designed, the coupling causes interference between the active and quiet lines. This phenomenon becomes especially important as clock frequency (rise-time) increases.

So far we have discussed the coupled signal as undesired noise. However, the coupled signal can have a very desirable effect for many microwave devices. One example is the directional coupler which is used for creating $a_1$, $b_1$, $a_2$, *and* $b_2$ signals in NWA. When a CW signal is transmitted thru line1 in Fig.1, the part of this input appears on line 2 if the TL length is set properly (usually $\lambda/4$). In other word, we can monitor the signal going thru the main TL using the coupled lines.

## 2. Adverse Effects Due to Coupling

Suppose we have two TLs and the 10 cm section is coupled as shown below. The fast switching digital signal will create coupled noises along this 10 cm section and the coupling occurs only during the rise- and fall-time. The coupled noise at each point will travel in both forward (same direction as the input signal) and backward (in the opposite direction) directions. What we can observe at the terminal (observation point) is the collection of all coupled noises as a function of time. If all coupled noises are arriving at the same time, we should expect one large short pulse at the observation point. On the other hand if the coupled signals are arriving successively over a period determined by the signal travel time, we should expect a long rectangular pulse. These two coupled signals are known as forward and backward coupled noise as shown in Fig.4.

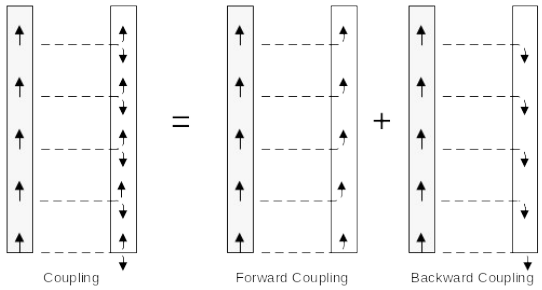


Fig. 4: Forward and backward coupled signals.

## Forward coupling

If the coupled signal travels in the same direction as the source signal, this is called forward coupled noise. Fig. 5 shows two forward coupled signals. If these signals are observed at the end of TL2, it is clear that the distance for them to travel is the same. In other word, they arrive at the same time which will create one large short pulse. We can also expect that the magnitude of this pulse should be related to the length of the coupled section. Although we have not studied the forward coupling in details yet, we will list some characteristics of the forward coupling here.

(a) Magnitude of the forward coupled noise is linearly proportional to the coupled length.
(b) Pulse width is related to the rise-time (fall-time).
(c) Rise-time (positive slope) will create a negative pulse and fall-time (negative slope) will create a positive pulse **in this example**. **This may not be correct for other TL geometry.**
(d) Forward coupled noise exists for a microstrip TL but not for a stripline TL. This is because the effects due to capacitive and inductive couplings will be cancelled in the stripline TL.

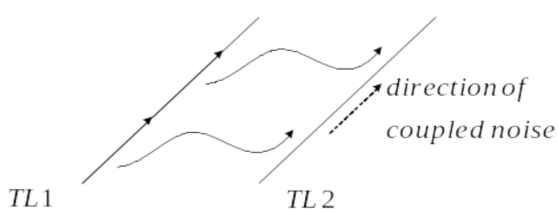


Fig.5: Forward coupling

## Backward coupling

If the coupled signal travels in the opposite direction as the source signal, this is called backward coupled noise. Fig. 6 shows three backward coupled signals. If these signals are observed at the front end of TL2, it is clear that they don't arrive at the same time because the travel distance is not the same. The observed signal will be a long rectangular pulse and the pulse width should be related to the round travel time of the coupled section. We can also expect that the magnitude of this pulse should not be related to the length of the coupled section because they arrive at different time. Some characteristics of the forward coupling are.

(a) Pulse width is related to the length of the coupled section.
(b) Magnitude of the backward coupled noise is not proportional to the coupled length.
(c) Rise-time (positive slope) will create a positive pulse and fall-time (negative slope) will create a negative pulse**.**
(d) Backward coupled noise exists for both microstrip and stripline TLs.

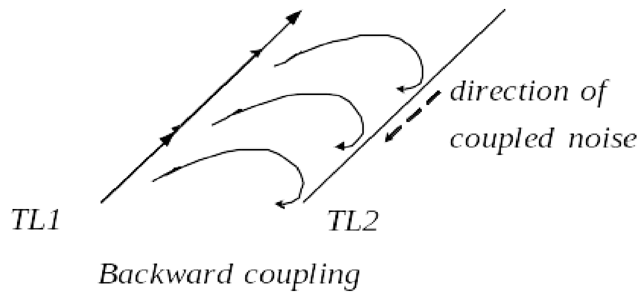


Fig.6: Backward coupling

**The important parameters to determine the coupled noises are**

(1) Input signal rise-time (or fall-time)

(2) Coupling coefficients which are related to
Coupled line length
Line separation
TL structures

## 3. Examples of Couples Noises

Before going thru the analysis, we will study the characteristics of the experimental data. The coupled section consists of the two microstrip TLs separated by *S*. The coupled length *d* is fixed in all results.

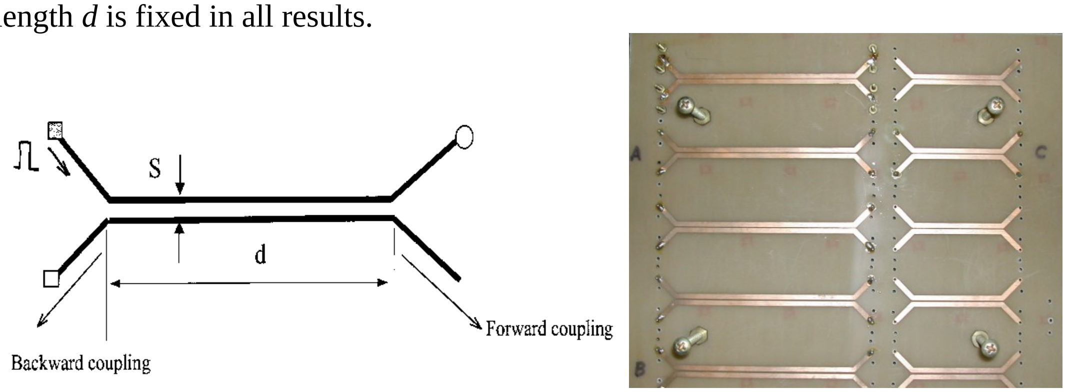



Fig.7: Coupled lines fabricated on PCB (microstrip TL). S: separation, d: coupled length

Figure 8 shows both forward and backward coupled noises for the 0.25ns rise-time (fall-time) input signal. The pulse duration is 10ns. The 0.25ns rise-time creates the positive backward coupled noise of approximately 1ns duration. The forward coupled noise is much narrower and the duration is close to 0.25ns. The rise-time creates the negative forward coupled noise whereas the fall-time creates the positive forward coupled noise.

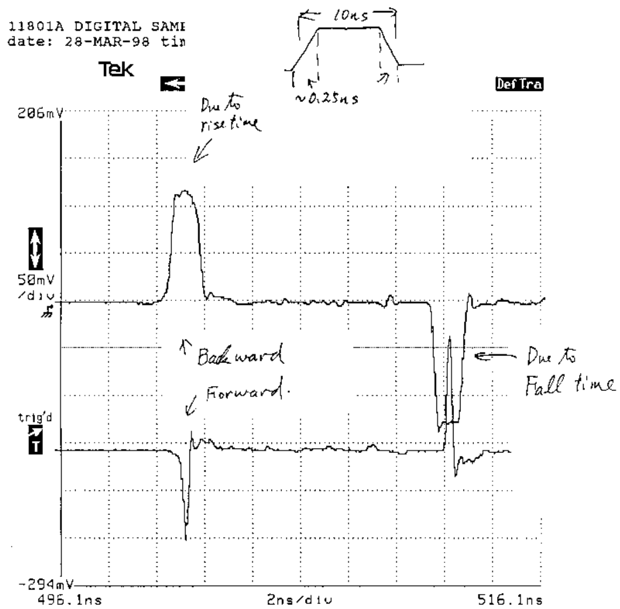


Fig.8: Rise-time=0.25ns, S=0.5 mm and d=100 mm.

## Effects of the coupled line separation

Next we will take a look at the effects of the coupled line separation *S*. Figures 8 and 9 show the forward and backward coupled noise for the same input signal. The difference is the line separation *S*. From these it is clear that when the line separation is increased, the magnitude of the backward coupled noise decreases but not the magnitude of the forward coupled noise. It is important to note that the forward coupled noise IS NOT independent of the TL separation. Figures 8 and 9 show the change is small when the TL separation is changed. The line separation has almost no effect on a pulse duration.

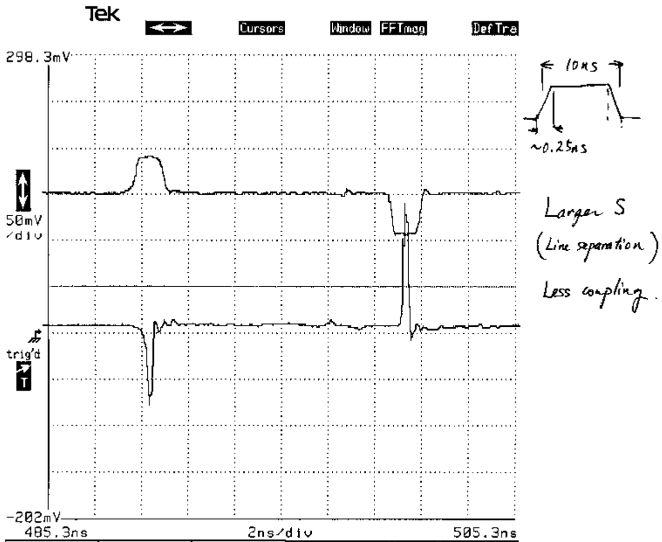



Fig. 9: Fig.8: Rise-time=0.25ns, S=1 mm and d=100 mm.

## Effects of rise-time (fall-time)

The effects of the rise-time (fall-time) are created by changing the input signal and using the same coupled lines. Figures 8 and 10 shows the coupled noises for the rise-time of 0.25ns, 0.8ns and 1.8ns for the same coupled TLs. The forward coupled noise is very sensitive to the rise-time and it almost disappears when the rise-time is increase to 1.8ns. The shape of the backward coupled noise changed from a clear rectangular shape to a smooth bell shape as the rise-time increases but the decrease of the magnitude is much less than that of the forward coupled noise.

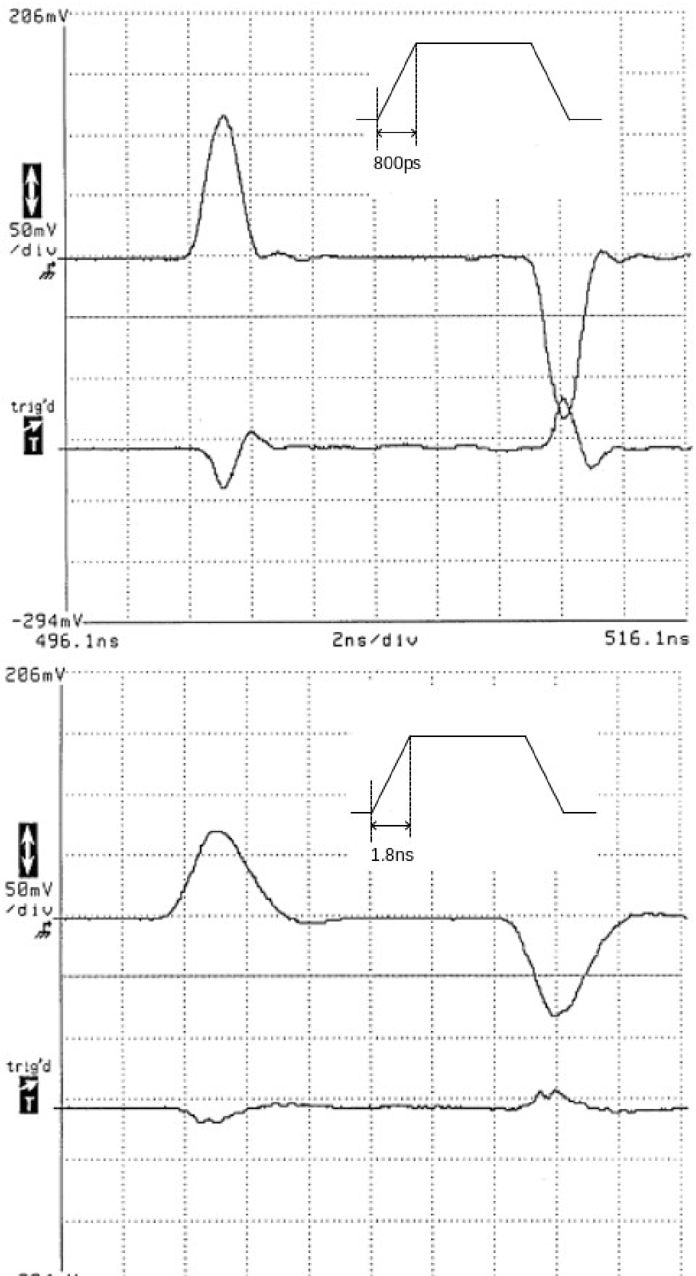



Fig. 10: Fig.8: rise-time=0.8ns and 1.8ns, S=0.5 mm and d=100 mm.

# 4. Characteristic impedance and velocity of signals on coupled lines

Previously we defined the characteristic impedance and signal velocity of an isolated TL in terms of L and C. When two or more TLs are in parallel, the values of L and C must be modified to include the mutual capacitance and mutual inductance.

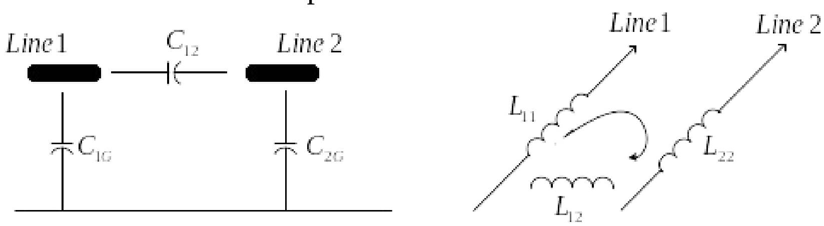



Fig.11: Lumped element model of two TLs. $L_{11}$ and $L_{22}$ are self inductances. $L_{12}$ is a mutual inductance. $C_{1G}$ and $C_{2G}$ are self capacitances. $C_{12}$ is a mutual capacitance.

For TL 1, we define $Z_o$ and signal velocity as

$$C_1 = C_{1G} + C_{12}$$

$$Z_{01} = \sqrt{\frac{L_{11}}{C_1}} = \sqrt{\frac{L_{11}}{C_{1G} + C_{12}}}$$

$$v_1 = \frac{1}{\sqrt{L_{11}C_1}} = \frac{1}{\sqrt{L_{11}(C_{1G} + C_{12})}}$$

Similarly for TL 2, we have

$$C_2 = C_{2G} + C_{12}$$

$$Z_{02} = \sqrt{\frac{L_{22}}{C_2}} = \sqrt{\frac{L_{22}}{C_{2G} + C_{12}}}$$

$$v_2 = \frac{1}{\sqrt{L_{22}C_2}} = \frac{1}{\sqrt{L_{22}(C_{2G} + C_{12})}}$$

Notice that the mutual inductance $L_{12}$ is not included in $Z_o$ and $v$ because $L_{12}$ has no effect without a signal on the other line (non-active line is effectively grounded).

# L and C of different TL structures

The values of C and L depend on the TL structures and they are usually obtained numerically. Figures 12 and 13 show self and mutual capacitances of different TL structures. The dielectric constant is listed as K in the figure. The self and mutual inductances of the same TL are shown in Fig. 14.

The two coupled line for microstrip TL is **(e)** in Fig. 12 and that of stripline TL is **(b)** in Fig. 13. These values will be used for the analysis of the coupled noises.

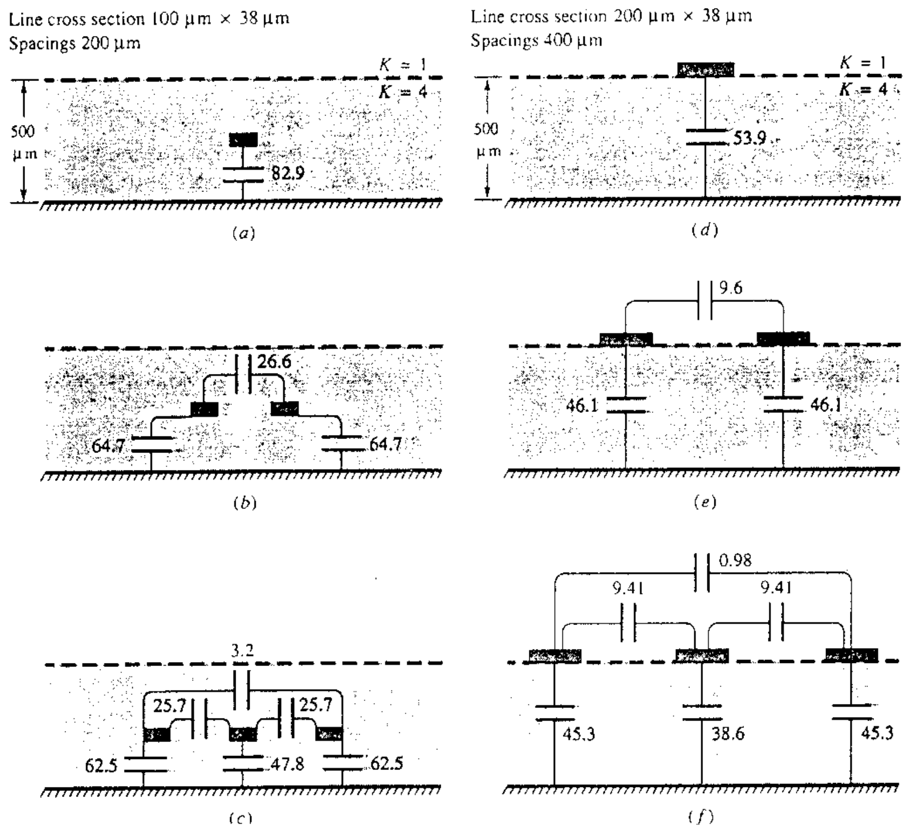



**FIGURE 3-5**
Examples of microstrip of infinite dielectric width and length showing lumped capacitance calculated numerically. Dielectric constant = 4. The width and spacing of cases (*d*), (*e*), and (*f*) is 2 times the width and spacing of (*a*), (*b*), and (*c*). Capacitances per unit length in units of fF/mm (fF/mm = $10^{-15}$ F/mm).

Fig. 12: Self and mutual capacitances of microstrip and embedded microstrip TLs.

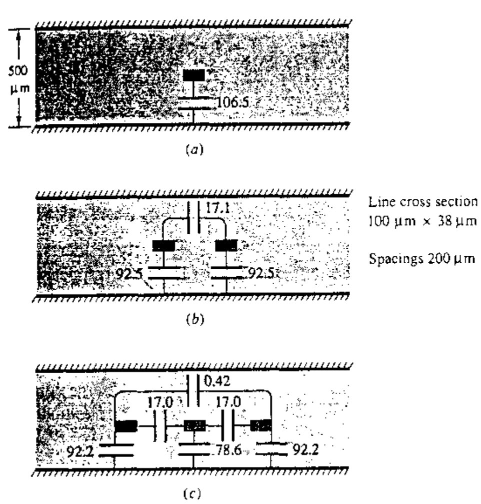



**FIGURE 3-6**
Examples of triplate of infinite dielectric width and length showing lumped capacitance calculated numerically. Dielectric constant = 4. Width and spacing is the same as that of Figure 3-5 (*a*), (*b*), and (*c*).

Fig. 13: Self and mutual capacitances of stripline TLs.

**Inductance coefficients for three stripline systems**
**(Units: pH/mm)**

| Figure | $L_{11}$ or $L_{33}$ | $L_{22}$ | $L_{12}$ or $L_{23}$ | $L_{13}$ | |
|---|---|---|---|---|---|
| 3-5(*a*) | 506 | | | | |
| 3-5(*b*) | 503 | | 135 | | Imbedded wires |
| 3-5(*c*) | 503 | 500 | 134 | 55.3 | in microstrip |
| 3-5(*d*) | 550 | | | | |
| 3-5(*e*) | 548 | | 142 | | Top lines in |
| 3-5(*f*) | 548 | 545 | 141 | 58.6 | microstrip |
| 3-6(*a*) | 417 | | | | |
| 3-6(*b*) | 416 | | 64.7 | | Line in midplane |
| 3-6(*c*) | 415 | 414 | 64.4 | 11.6 | of triplate |

See Example 3.7 for results from a somewhat different line geometry.

Fig. 14: Self and mutual inductances. The TL structures are shown in Fig. 12 and 13.

## Definition of coupling coefficients

As we discussed in the introduction, the coupling is due to both capacitive and inductive couplings on TL. In the previous section, we obtained the mutual capacitance and inductance of typical TL structures. We can define the coupling coefficients using L and C.

$K_C$ is the capacitive coupling coefficient and defined as

$$K_C = \frac{C_{12}}{C_{1G} + C_{12}}$$

$K_L$ is the inductive coupling coefficient and defined as

$$K_L = \frac{L_{12}}{L_{11}} \quad (L_{22} = L_{11})$$

Let us take a look at $K_C$ and $K_L$ of microstrip TL (Fig. 3.5(e)) and stripline TL (Fig. 3.6 (b)).

Microstrip TL

$$K_C = \frac{9.6}{46.1 + 9.6} = 0.172$$

$$K_L = \frac{142}{548} = 0.259$$

Stripline TL

$$K_C = \frac{17.1}{92.5 + 17.1} = 0.156$$

$$K_L = \frac{64.9}{416} = 0.156$$

Notice that $K_C = K_L$ for the stripline TL and $K_C \neq K_L$ for the microstrip TL. These relationships become important for the coupled noise analysis which will be discussed in the next section.

## 5. Equivalent Circuit Model and Analysis

In order to estimate the coupled noise for a given TL structure, we need to model the coupled lines. In this section, we will use a simple equivalent circuit model to analyze the forward and backward coupled noises. Although this is a simple analytical model, the important characteristics of both forward and backward coupled noises can be obtained with this model. If a more accurate analysis is needed, the numerical methods such as FDTD (finite-difference time-domain) and FEM (finite element method) must be used.

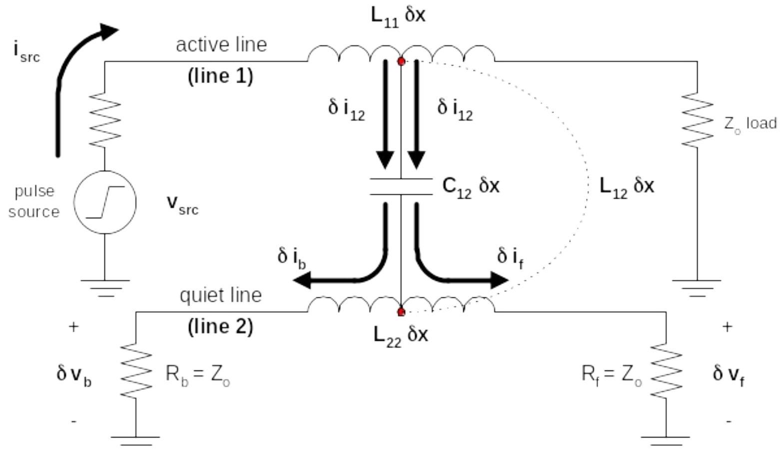



Fig. 15: An equivalent circuit of coupled lines.

$\Delta i_{12}$ in Fig. 15 is the coupled current due to the mutual capacitor and it can be expressed as

$$\Delta i_{12} = C_{12} \Delta x \frac{dV_{src}}{dt}$$

where $C_{12}\Delta x$ is a mutual capacitance per unit length.

The conservation of current dictates that $\Delta i_{12}$ is also the sum of backward and forward currents on TL2.

$$\Delta i_{12} = \Delta i_b + \Delta i_f = \frac{\Delta V_b + \Delta V_f}{Z_o}$$

By combining these two equations, we have

$$\Delta V_b + \Delta V_f = Z_o C_{12} \Delta x \frac{dV_{src}}{dt}$$

or

$$\frac{\Delta V_b}{\Delta x}+\frac{\Delta V_f}{\Delta x}=Z_o C_{12}\frac{dV_{src}}{dt}=\sqrt{\frac{L_{11}}{C_1}}C_{12}\sqrt{\frac{C_1}{C_1}}\frac{dV_{src}}{dt}$$

Changing the finite difference to the derivative and rearranging $C_1$, we get

$$\frac{dV_b}{dx}+\frac{dV_f}{dx}=\frac{C_{12}}{C_1}\sqrt{L_{11}C_1}\frac{dV_{src}}{dt}$$

This can be expressed using the coupling coefficient as

$$\frac{dV_b}{dx}+\frac{dV_f}{dx}=\frac{K_C}{v_o}\frac{dV_{src}}{dt}$$

where $K_C = C_{12}/C_1$ is the *capacitive coupling coefficient* and $v_o = 1/\sqrt{L_{11}C_1}$ is the signal propagation speed on TL1.

Now we take a look at the induced voltage due to the mutual inductance. The induced voltage on $L_{22}$ is given by the difference between backward and forward coupled voltages. The induced voltage can also be expressed using $L_{12}$ and the change of current on TL1 as

$$\Delta V_b - \Delta V_f = L_{12}\Delta x\frac{di_{src}}{dt}=\frac{L_{12}}{Z_o}\Delta x\frac{dV_{src}}{dt}$$

Changing the finite difference to the derivative and rewriting $Z_o$, we get

$$\frac{dV_b}{dx}-\frac{dV_f}{dx}=L_{12}\sqrt{\frac{C_1}{L_{11}}}\sqrt{\frac{L_{11}}{L_{11}}}\frac{dV_{src}}{dt}$$

This is also

$$\frac{dV_b}{dx}-\frac{dV_f}{dx}=\frac{L_{12}}{L_{11}}\sqrt{L_{11}C_1}\frac{dV_{src}}{dt}$$

Finally we get

$$\frac{dV_b}{dx}-\frac{dV_f}{dx}=\frac{K_L}{v_o}\frac{dV_{src}}{dt}$$

where $K_L = L_{12}/L_{11}$ is the *inductive coupling coefficient*.

Now we have two equations in terms of the forward coupled voltage $V_f$ and the backward coupled voltage $V_b$ as

$$\frac{dV_f}{dx}=\left(\frac{K_C - K_L}{2v_o}\right)*\frac{dV_{src}}{dt}$$

$$\frac{dV_b}{dx}=\left(\frac{K_C + K_L}{2v_o}\right)*\frac{dV_{src}}{dt}$$

By integrating $V_f$ and $V_b$ with respect to $x$, we can express the solution as

$$V_f = \int_0^d \left( \frac{K_C - K_L}{2v_o} \right) \frac{dV_{src}}{dt} dx$$

$$V_b = \int_0^d \left( \frac{K_C + K_L}{2v_o} \right) \frac{dV_{src}}{dt} dx$$

The forward coupling voltage $V_f$ can be computed easily by noting that the forward propagating component of $dV_{src}/dt$ doesn't change with $x$. This can be seen in Fig.4. Assume we have a finite rise-time source voltage as shown below.

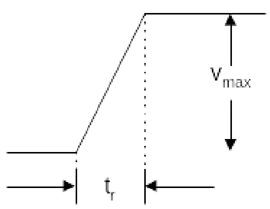



Fig. 16: Input waveform.

Then the integral simply becomes the coupled length $d$ and the derivative becomes the slope $V_{max}/t_r$. We can express the forward coupling voltage as

$$V_f = V_{max} \left( \frac{K_C - K_L}{2v_o} \right) \left( \frac{d}{t_r} \right)$$

$$V_f = V_{max} \left( \frac{K_C - K_L}{2v_o} \right) \left( \frac{d}{t_r} \right) = \left( \frac{K_C - K_L}{2} \right) \frac{d}{v_o} \left( \frac{V_{max}}{t_r} \right)$$

Notice that the forward couple noise is linearly proportional to the coupled length and (1/ rise-time) or $V_f \propto d \propto 1/t_r$. The coefficient contains the difference ($K_C$-$K_L$). If the TL structure has $K_C = K_L$, then the forward coupled noise will not be generated. This is the case for the stripline TLs. Because $K_C \neq K_L$ for the microstrip TLs, the forward coupled noise always exists for the microstrip coupled TLs. If $K_C<K_L$, $V_f$ is negative for a positive slope (rise-time). On the other hand, if $K_C>K_L$, $V_f$ is positive for a positive slope (rise-time). The values of $K_C$ and $K_L$ depend on the TL geometry as shown in Table 3.4. Our measured data is for a particular TL geometry. Please do not assume $V_f$ is always negative for a positive slope (rise-time).

The calculation of the backward coupling voltage $V_b$ requires consideration of the backward propagating component of $dV_{src}/dt$. Let us examine the solution of $V_b$. We can take the constant out of the integral but we cannot assume $dV_{src}/dt$ is constant with respect to $x$.

$$V_b = \left( \frac{K_C + K_L}{2v_o} \right) \int_0^d \frac{dV_{src}}{dt} dx$$

From Fig. 4, we can see $V_b$ is simply a time-delayed (by a factor of 2 $\ast$ traveled distance $x$ / propagation speed $v_o$) version of $dV_{src}/dt$.
Let us define

$$\frac{dV_{src}}{dt} = D(t - \frac{2x}{v_o})$$

where

$$\frac{dV_{src}(p)}{dp} = D(p)$$

If $V_{src}$ has a linear slope as shown in Fig. 16, $D$(p) is simply constant at p.

Using $D$(p), $V_b$ can be written as

$$V_b = \left( \frac{K_C + K_L}{2v_o} \right) \ast \int_0^d D(t - \frac{2x}{v_o}) dx$$

Now we perform the following variable transformation

$$p = t - \frac{2x}{v_o}$$

$$dp = -\frac{2}{v_o} dx$$

Using these, we can write $V_b$ as

$$V_b = \left( \frac{K_C + K_L}{2v_o} \right) \ast \int_t^{t-\frac{2d}{v_o}} \left( -\frac{v_o}{2} \right) \ast D(p) dp$$

Make use of $\frac{dV_{src}(p)}{dp} = D(p)$, we have

$$V_b = \left( -\frac{K_C + K_L}{4} \right) \ast \int_t^{t-\frac{2d}{v_o}} \frac{dV_{src}(p)}{dp} dp = \left( -\frac{K_C + K_L}{4} \right) \ast V_{src}(p) \Big|_t^{t-\frac{2d}{v_o}}$$

The integral can be expressed as

$$V_b = \left( \frac{K_C + K_L}{4} \right) \ast \left[ V_{src}(t) - V_{src}(t - \frac{2d}{v_o}) \right]$$

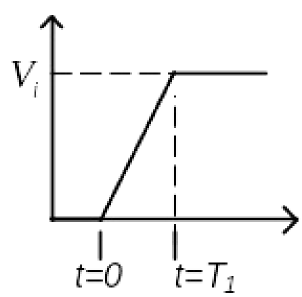
The term in the bracket is close to a rectangular pulse if a rise-time is short compare to the round trip time. The pulse duration is related to the signal round trip time of the coupled section. The coefficient contains ($K_C$+$K_L$) which is always non-zero. Therefore, the backward coupled noise exists for all TL structures including the stripline TLs. Unlike the forward coupled noise, the magnitude of $V_b$ does not depend on the coupled length *d* or the slope of the rise-time. Also $V_b$ is always positive for a positive slope (rise-time).

These characteristics of $V_f$ and $V_b$ are very close to what we observed from the experimental results. Although we modeled the coupled noises using a simple equivalent circuit in this section, we can effectively describe the important characteristics of forward and backward coupled noises with this model.

Another way to visualize the coupled noise is in terms of the distributed TL elements as shown in Fig. 17. This figure shows the coupled noises due to rise- and fall-time on TLs. Notice the change of the direction of coupled currents.

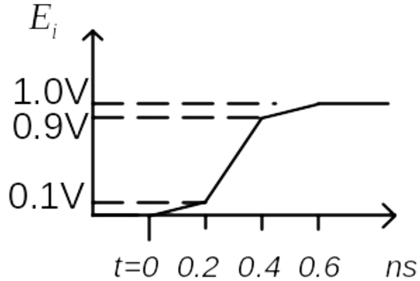
## Shapes of rise-time and function to represent them

### (1) Linear slope




$dV_{in}/dt$= a rectangular pulse.

### (2) Piece-wise linear



$dV_{in}/dt$= a series of rectangular pulses.

**(3) Square function (Non-linear)**

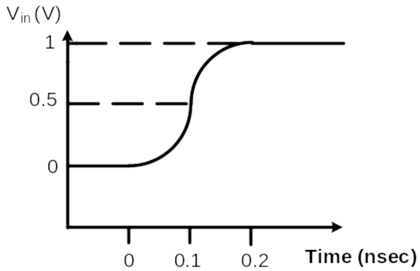
The input voltage can be expressed as

$V_{in} = At^2$ for $0 =< t =< 0.1$ns

$V_{in} = -At^2 + Bt + C$ for $0.1$ns $=< t =< 0.2$ns

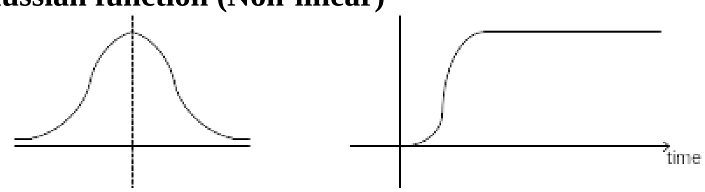
$V_{in} = 1$V for $t >= 0.2$ns

where $A=0.5\text{x}10^{20}$, $B=2\text{x}10^{10}$, $C=-1$




$dV_{in}/dt$= a triangular pulse.

**(4) Gaussian function (Non-linear)**




$$g_{step}(t)=\frac{1}{2}\left[1+erf\left(\frac{t}{2\tau_3}\right)\right]$$

$$erf(z)=\frac{2}{\sqrt{\pi}}\int_0^z e^{-\lambda^2}d\lambda$$

$dV_{in}/dt$=

**(5) Exponential function (Non-linear)**

<u>Single pole exponential</u>

$$h_{step}(t)=u(t)\left(1-e^{-\left(\frac{t}{\tau_1}\right)}\right)$$

$dV_{in}/dt$=

## 6. Estimation of $K_C$ and $K_L$

We can use the measured $V_f$ and $V_b$ to estimate $K_C$ and $K_L$ for different coupled lines. Using the simple analysis, we have the following relationships.

$$V_f = \frac{K_C - K_L}{2v_o} * d * \frac{\partial V_{in}}{\partial t}$$

$$V_f = V_{peak}\left(\frac{K_C - K_L}{2v_o}\right)\left(\frac{d}{t_{rise}}\right)$$

Applicable if the slope of $V_{in}$ has a linear slope. $V_{peak}$: Peak value of $V_{in}$.
Also $v_o$ is the signal speed.

$$V_b = \frac{K_C + K_L}{4}[V_{in}(t) - V_{in}(t - \frac{2d}{v_o})]$$

Step 1: Obtain the signal speed $v_o$ on the TL using $V_b$ waveform. Assume it is a square wave and the signal duration is given by $[V_{in}(t) - V_{in}(t - \frac{2d}{v_o})]$. Since $d$ is known, $v_o$ can be obtained from this.

Step 2: Using the peak values of $V_f$ and $V_b$ (voltage on a flat section), estimate $K_C$ and $K_L$. This can be done by solving two equations for $K_C$ and $K_L$. $[V_{in}(t) - V_{in}(t - \frac{2d}{v_o})]$ simply becomes $V_{peak}$.

Another approach is to use the time integration method. This may be more accurate if the measured data is noisy. First we need to integrate $V_f$ and $V_b$ over time. We get the following two equations.

$$\int_0^\infty V_f\,dt = \int_0^\infty [\frac{K_C - K_L}{2v_o} * d * \frac{dV_{in}}{dt}]dt \approx [\frac{K_C - K_L}{2v_o} * d * \frac{V_{peak}}{t_{rise}}]\int_0^{t_{rise}} dt = \frac{K_C - K_L}{2v_o} * d * V_{peak}$$

$$\int_0^\infty V_b\,dt = \int_0^\infty \frac{K_C + K_L}{4}[V_{in}(t) - V_{in}(t - \frac{2d}{v_o})]dt = \frac{K_C + K_L}{4} * V_{peak} * \frac{2d}{v_o}$$

The left-hand side will be the integration of the measured data. The right-hand side contains $K_C$ and $K_L$. Since we know $V_{peak}$, $v_o$ and $d$, we should be able to find $K_C$ and $K_L$.

$$K_C - K_L = \frac{2v_o}{d * V_{peak}} area(V_f)$$

$$K_C + K_L = \frac{4v_o}{2dV_{peak}} area(V_b)$$

## 7. Estimation of $C_{1G}$, $C_{2G}$, $C_{12}$, $L_{11}$, $L_{22}$, and $L_{12}$

We know the coupled lines can be expressed using the self-capacitance ($C_{1G}$=$C_{2G}$), mutual capacitance ($C_{12}$), self-inductance ($L_{11}$=$L_{22}$), and mutual inductance ($L_{12}$). We can also define the velocity and impedance. Since $TL_1$ and $TL_2$ are the same, we have

$$C_1 = C_{1G} + C_{12} = C_2$$

$$Z_{01} = \sqrt{\frac{L_{11}}{C_1}} = \sqrt{\frac{L_{11}}{C_{1G} + C_{12}}} = Z_{02}$$

$$v_1 = \frac{1}{\sqrt{L_{11}C_1}} = \frac{1}{\sqrt{L_{11}(C_{1G} + C_{12})}} = v_2$$

The coupling coefficient $K_C$ and $K_L$ are defined as

$$K_C = \frac{C_{12}}{C_{1G} + C_{12}}$$

$$K_L = \frac{L_{12}}{L_{11}} \quad (L_{22} = L_{11})$$

As shown above, using $K_C$, $K_L$, velocity, and impedance, we should be able to find the values of 6 lumped elements.

Description of coupled noise on TL

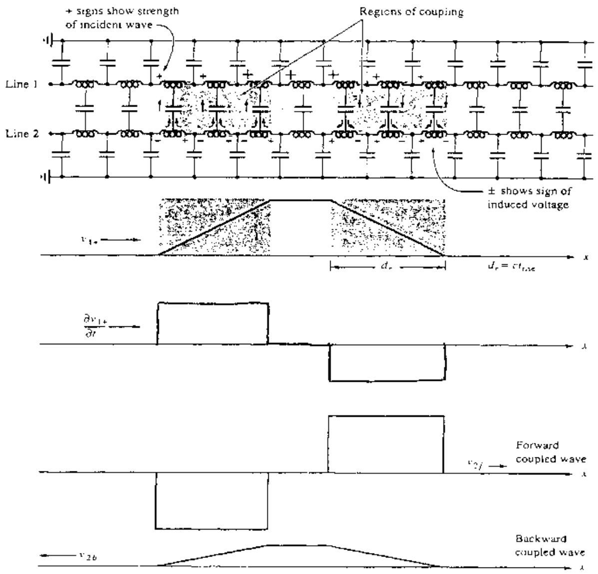



FIGURE 3-13
Circuit element representation of two coupled transmission lines showing coupling by means of mutual capacitances and inductances. Coupling currents flow and inductive voltages are induced only along the rise and fall regions of a trapezoidal impulse.

Fig. 17: Description of coupled noised due to a short pulse.

**TABLE 3.3**
**Comparison of results from computer circuit simulation and coupled noise equations**

| | Computer simulation | Coupled noise equation | $a$ or $b$ coefficient |
|---|---|---|---|
| Forward coupled wave peak voltage $v_{2-}$ | 50.8 mV | 47.6 mV | $2a_{12}$ = 95.2 mV/V |
| Backward coupled wave peak voltage $v_{2-}$ | 71.4 mV | 70 mV | $b_{12}$ = 140 mV/V |

Geometry is microstrip line pair, Figure 3-5($b$), line length = 1280 mm. Incident wave used is trapezoid of 0.5 volt peak amplitude, rise and fall times of 1 ns, and plateau width of 20 ns.

# Coupled TLs characteristics

**TABLE 3.4**
**Comparison of properties of three microstrip and triplate line pairs designed for 80 Ω characteristic impedance**

| Line spacing, µm | Dielectric thickness, µm | $Z_0$, single line | $Z_0$, line pair | $C_{1G}$, fF/mm | $C_{12}$, fF/mm | $L_{11}$, pH/mm | $L_{12}$, pH/mm | Propagation delay, ps/mm | $K_C$ | $K_L$ | $2a_{12}$, mV/V | $b_{12}$, mV/V |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Microstrip: | | | | | | | | | | | | |
| 200 | 155 | 80.26 | 79.52 | 61.2 | 7.05 | 431.9 | 86.8 | 5.43 | 0.103 | 0.201 | −53.1 | 76 |
| 300 | 155 | 80.26 | 80.04 | 64.0 | 3.65 | 433.3 | 56.6 | 5.41 | 0.054 | 0.131 | −41.6 | 46 |
| 500 | 155 | 80.26 | 80.23 | 66.1 | 1.32 | 434.0 | 28.7 | 5.41 | 0.020 | 0.066 | −25.3 | 22 |
| Triplate: | | | | | | | | | | | | |
| 200 | 1050 | 84.68 | 79.79 | 60.3 | 27.7 | 560.4 | 176.5 | 7.02 | 0.315 | 0.315 | 0 | 158 |
| 300 | 950 | 81.69 | 79.71 | 67.9 | 17.5 | 542.9 | 111.1 | 6.81 | 0.205 | 0.205 | 0 | 102 |
| 500 | 900 | 80.04 | 79.66 | 76.1 | 7.93 | 533.5 | 50.3 | 6.70 | 0.094 | 0.094 | 0 | 48 |

Line width fixed at 100 µm for all cases, line length = 200 mm, dielectric thickness for triplate structures indicates distance between ground planes. Dielectric constant = 4. Geometries follow Figures 3-5($e$) and 3-6($b$), except for differing physical parameters. Note that the microstrip lines are on top of the dielectric in this example, i.e., not imbedded as in Table 3.3 and Example 3.6.

## Same table (enlarged)

TABLE 3.3

**Comparison of results from computer circuit simulation and coupled noise equations**

| | Computer simulation | Coupled noise equation | $a$ or $b$ coefficient |
|---|---|---|---|
| Forward coupled wave peak voltage $v_{2-}$ | 50.8 mV | 47.6 mV | $2a_{12}$ = 95.2 mV/V |
| Backward coupled wave peak voltage $v_{2-}$ | 71.4 mV | 70 mV | $b_{12}$ = 140 mV/V |

Geometry is microstrip line pair, Figure 3-5($b$), line length = 1280 mm. Incident wave used is trapezoid of 0.5 volt peak amplitude, rise and fall times of 1 ns, and plateau width of 20 ns.

TABLE 3.4

**Comparison of properties of three microstrip and triplate line pairs designed for 80 Ω characteristic impedance**

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|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Microstrip: | | | | | | | | | | | | |
| 200 | 155 | 80.26 | 79.52 | 61.2 | 7.05 | 431.9 | 86.8 | 5.43 | 0.103 | 0.201 | -53.1 | 76 |
| 300 | 155 | 80.26 | 80.04 | 64.0 | 3.65 | 433.3 | 56.6 | 5.41 | 0.054 | 0.131 | -41.6 | 46 |
| 500 | 155 | 80.26 | 80.23 | 66.1 | 1.32 | 434.0 | 28.7 | 5.41 | 0.020 | 0.066 | -25.3 | 22 |
| Triplate: | | | | | | | | | | | | |
| 200 | 1050 | 84.68 | 79.79 | 60.3 | 27.7 | 560.4 | 176.5 | 7.02 | 0.315 | 0.315 | 0 | 158 |
| 300 | 950 | 81.69 | 79.71 | 67.9 | 17.5 | 542.9 | 111.1 | 6.81 | 0.205 | 0.205 | 0 | 102 |
| 500 | 900 | 80.04 | 79.66 | 76.1 | 7.93 | 533.5 | 50.3 | 6.70 | 0.094 | 0.094 | 0 | 48 |

Line width fixed at 100 μm for all cases, line length = 200 mm, dielectric thickness for triplate structures indicates distance between ground planes. Dielectric constant = 4. Geometries follow Figures 3-5($e$) and 3-6($b$), except for differing physical parameters. Note that the microstrip lines are on top of the dielectric in this example, i.e., not imbedded as in Table 3.3 and Example 3.6.

## 6. Effects on digital circuits due to coupled noise

Both forward and backward coupled noises can be viewed as the spurious noise on digital circuits. In Fig. 18, we have parallel and anti-parallel data lines which are creating coupled noises. If the output impedance is not matched, the reflected forward coupled noise becomes an input to the gate for the anti-parallel data lines.

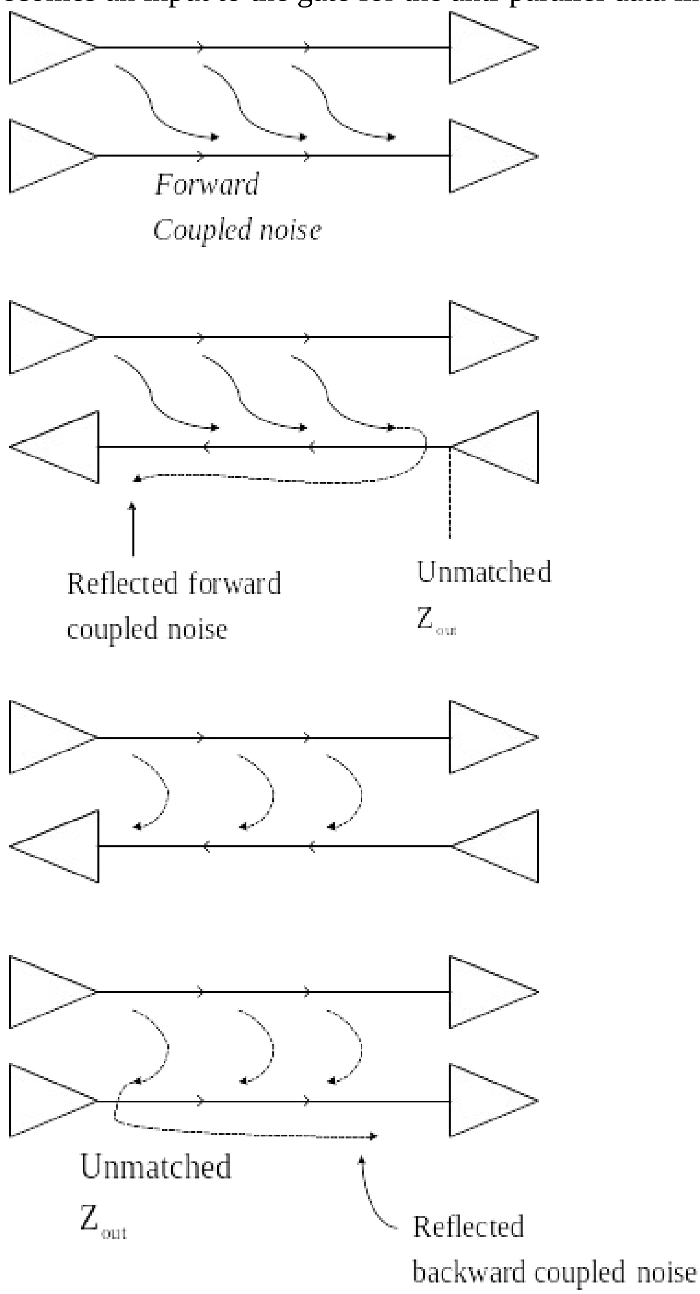
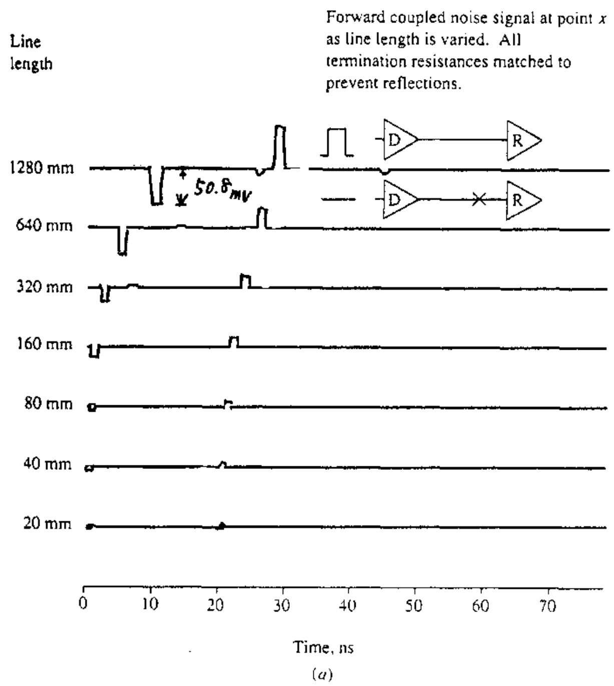
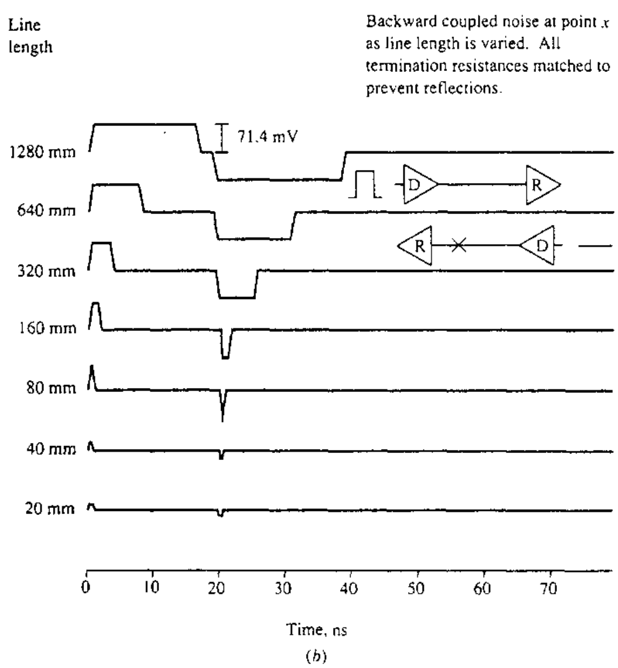



Fig. 18: Parallel and anti-parallel data line and coupled noises.




(a)




(b)

**FIGURE 3-14**
Simulated forward coupled signal (a) and backward coupled signal (b) on two coupled transmission lines of varying lengths terminated with $R_T = Z_0$. Microstrip pair of Figure 3-5(b) is used. Driving waveform is a 20-ns-wide trapezoidal pulse with 1 ns rise and fall times, and 0.5 V peak amplitude. The coupling length is the same as the line length. *The small secondary impulses on the longer lines are due to higher order effects not discussed here.*

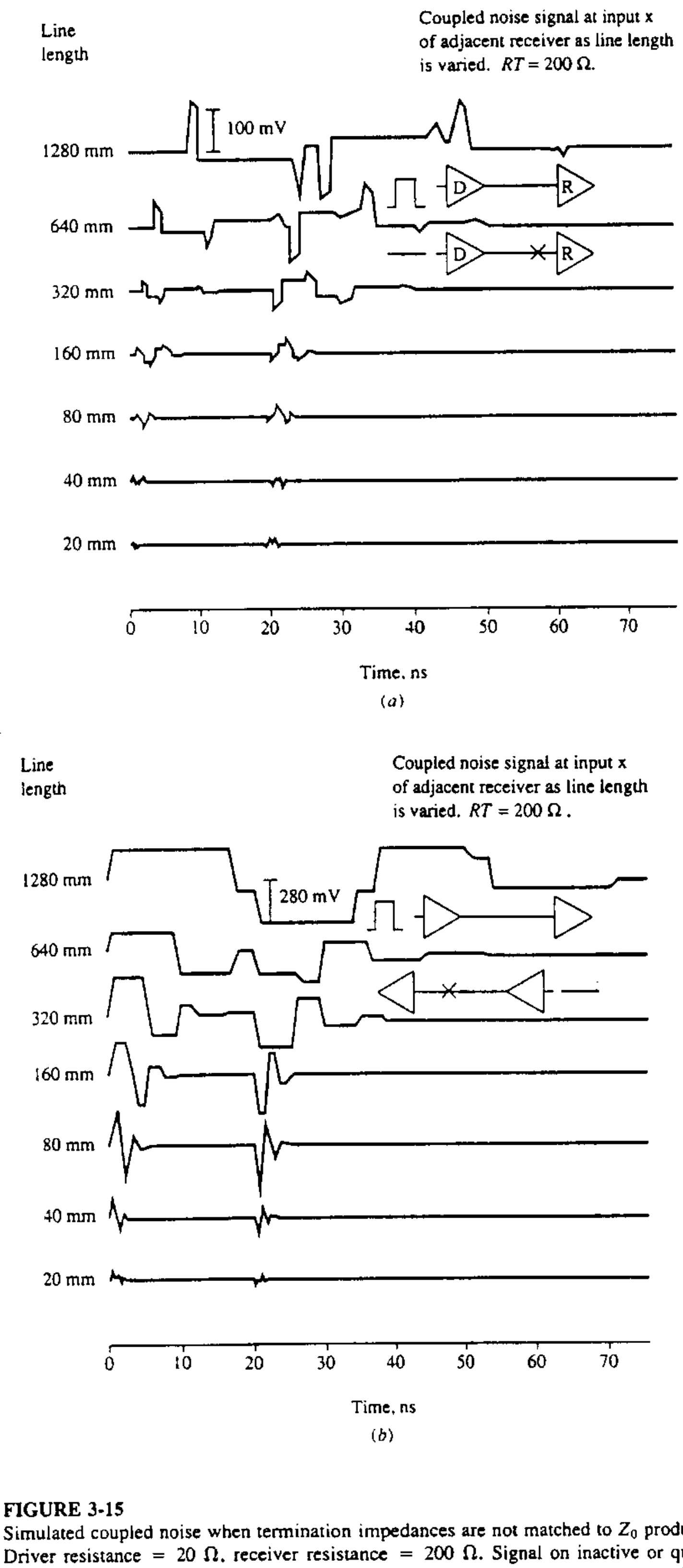



**FIGURE 3-15**
Simulated coupled noise when termination impedances are not matched to $Z_0$ producing reflections. Driver resistance = 20 $\Omega$, receiver resistance = 200 $\Omega$. Signal on inactive or quiet line adjacent to an active line, (a) far end, (b) near end. Driving waveform and line geometry is same as Figure 3-14.