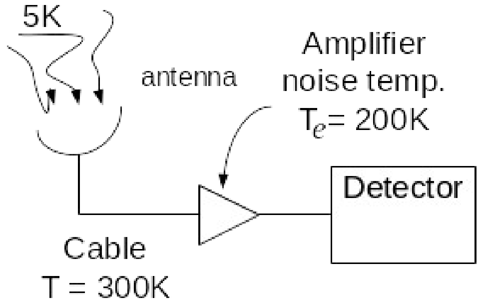
Noise in Microwave Systems 2/8/2017

1. Introduction:

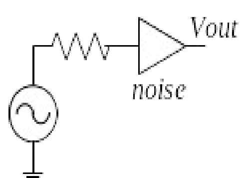
Example: Noise temperature of the sky (Known as the background microwave radiation. The temperature depends on the frequency).

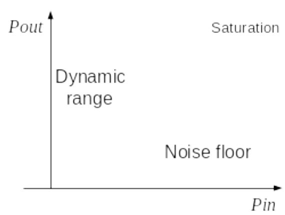


It is possible to measure the sky temperature of 5K with cables and antenna which have a temperature of 300K (room temperature)

- What is "**noise temperature"?**
- How do we measure it? Particularly very small noise temperature.
- How about the effects of cable loss and temperature.
- What is the total noise of the cascaded system?

Amplifier characteristics





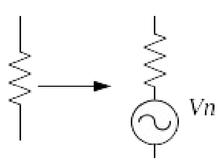
Noise will limit the dynamic range.

1-1 Type of noise

(a) Thermal noise (Johnson noise)

$v_n = \sqrt{4kTBR}$ Rayleigh-Jean approximation

Good for microwave but not applicable for optical wavelength



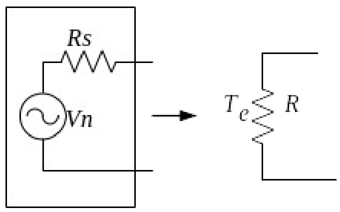
(b) Shot noise (Diode & Transistor)

$$i_n = \sqrt{2q\,I\,B}$$

(c) Flicker noise ($1/f$ noise)

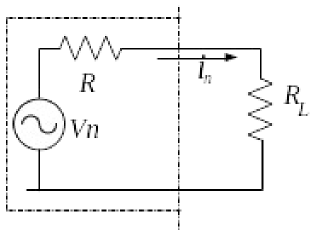
Not important at microwave region

1-2 Equivalent Noise Temperature



Need to define T_e in terms of v_n

Max power transfer if $R_L = R$



$$i_{in} = \frac{v_n}{2R} \qquad if \quad R_L = R$$

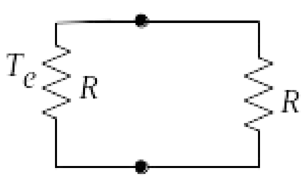
Then

$$P_S = \left(\frac{v_n}{2R}\right)^2 R = k\,T\,B$$

You can also show $v_n = \sqrt{4kTBR}$ from this.

Therefore, the power delivered to R_L is

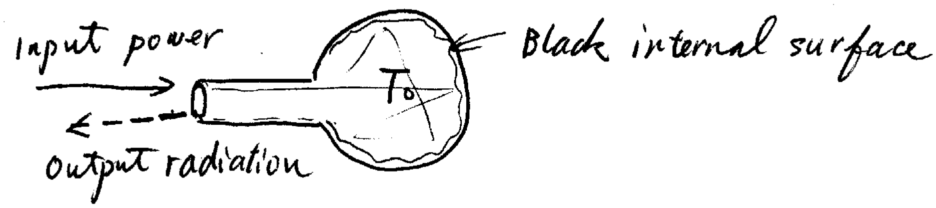
$$P_S = k\,T_e\,B \quad or \quad T_e = \frac{P_S}{kB}$$



1-3 Plank's Blackbody Radiation Law

$$B_f = \frac{2hf^3}{c^2}\left(\frac{1}{e^{hf/kT} - 1}\right)$$

Blackbody: perfect absorber and perfect emitter



Input power will be completely absorbed by the black surface.

Black surface at temperature T^o will radiate. If the object is a perfect blackbody, the input power is equal to the output radiation.

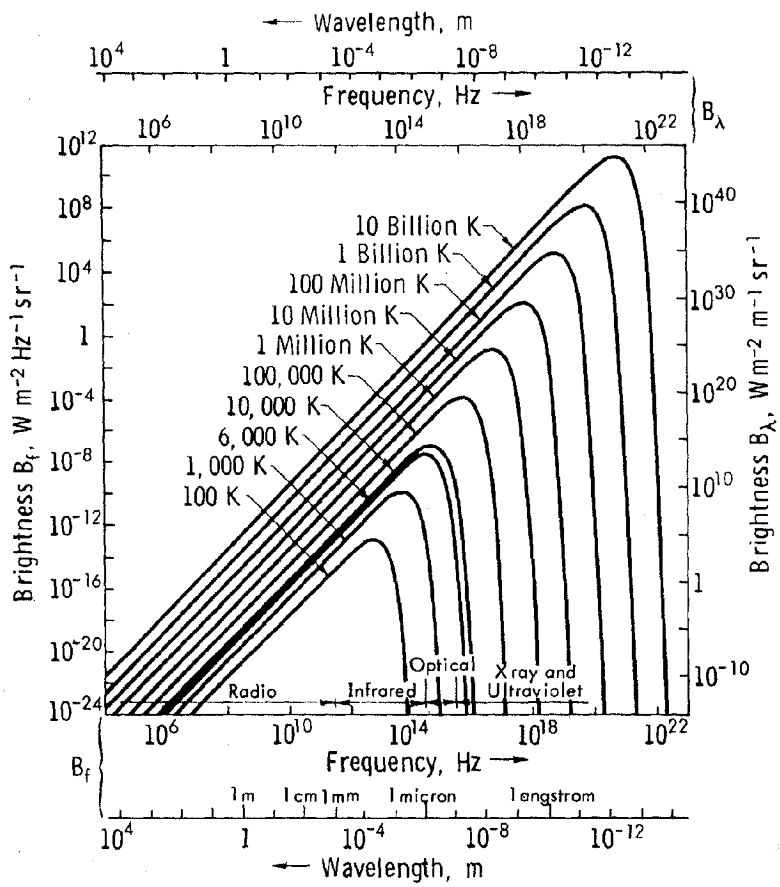


Fig. 4.4 Planck radiation-law curves (adapted from Kraus, 1966).

f = frequency, Hz,
k = Boltzmann's constant = 1.38×10^{-23} joule K^{-1}
T = absolute temperature, K,*
c = velocity of light = 3×10^8 m s^{-1}.

Total Brightness

$$B = \int_0^\infty B_f \, df$$
$$= \frac{\sigma T^4}{\pi}$$

where $\sigma = 5.67 \times 10^{-8} W m^{-2} K^4 sr^{-1}$

Stefan-Boltzmann Law

High and Low Frequency Approximations

High freq. limit

$$\frac{hf}{kT} >> 1$$

$$\frac{1}{e^{\frac{hf}{kT}} - 1} \sim e^{-\frac{hf}{kT}}$$

$$B_f = \frac{2h}{c^2} f^3 e^{-\frac{hf}{kT}}$$

Wien Radiation Law (good for optical limit)

Low freq limit

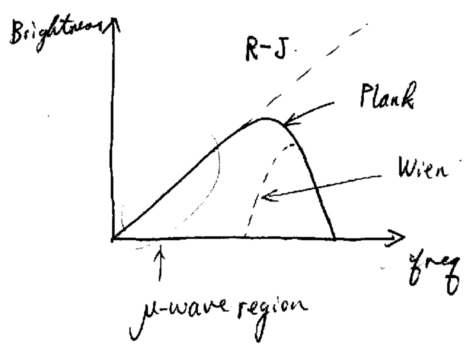
$$\frac{hf}{kT} << 1$$

$$B_f = \frac{2kT}{\lambda^2}$$

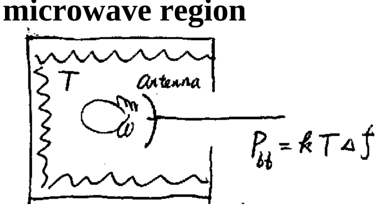
good for $\lambda T > 0.77 mK$

Here we use exp(x)=1+x if |x| is small

Rayleigh – Jeans approximation (good for microwave)



Power output from the perfect absorber (blackbody)
In the microwave region



$$P_{bb} = kT\Delta f$$

$$P_{bb} = \frac{1}{2} A_r \int_f^{f+\Delta f} df \iint \left(\frac{2kT}{\lambda^2} \right) F_n(\theta,\phi) d\Omega$$

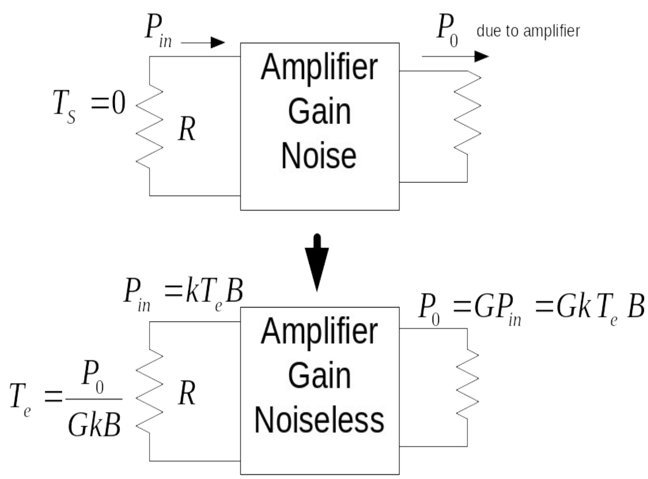
$$= kT\Delta f \left(\frac{A_r}{\lambda^2} \right) \iint F_n(\theta,\phi) d\Omega, \quad \text{where} \quad \iint F_n d\Omega = \Omega_p = \frac{\lambda^2}{A_r}$$

Then

$$\boxed{P_{bb} = kT\Delta f}$$

1-4 Active Devices

A noisy device can be replaced by a resistor with an equivalent noise temperature.



Example

Suppose we want to express the $1\ \mu W$ signal source at $1GHz$ and $BW = 1Hz$ using the equivalent noise temperature. Then it will be given by

$$T_e = \frac{P_S}{k\,B} = \frac{10^{-6}}{1.38 \times 10^{-23} \ J^0/K^0} = 7.24 \times 10^{16}\ K$$

This example shows that the noise temperature is not suited for expressing a narrowband signal source. The noise temperature of the source becomes too large.

Another example:

G=100, T_e=1000K, B=1GHz: $P_o = GKT_eB$=100*1000*1.38x10^{-23}*10^9=**1.38x10^{-9} *W***

We can measure this value.

Noise temperature should be used for a wide band noise source such as white noise.

2. Noise Temperature Measurement Techniques

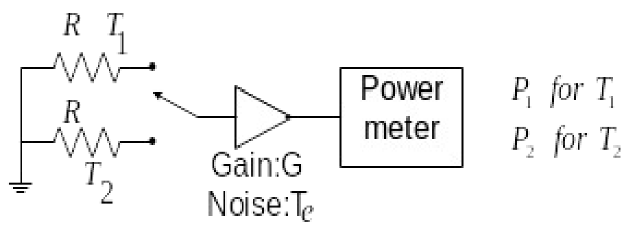
The easiest way to measure the noise temperature of DUT is the use of two noise sources as shown below. The noise source is usually the zener diode operating in two states and the value is given by **ENR (excess noise ratio)** in dB. When the DC voltage (28V) is ON, the noise source has a high temperature given by T_1. When there is no DC voltage, the noise source has low temperature and given by T_2. Both T_1 and T_2 must be known in the frequency range in which DUT is measured. The noise meter is essentially a microwave power meter. ENR is related to T_1 and T_2 as

ENR = $(T_1-T_2)/T_o$

where T_o is the ambient temperature given by T_o=290K.

Noise source ON T_1 $T_1 > T_2$

OFF T_2



Before the measurement, the system calibration is usually done without DUT. This will eliminate the system response from the measured data. When two noise temperature states T_1 and T_2 are measured with DUT (amplifier with gain G and noise temperature T_e), we get P_1 and P_2, respectively. In terms of gain, bandwidth, and temperature, we can write them as

$$P_1 = GkT_1B + GkT_eB$$
$$P_2 = GkT_2B + GkT_eB$$

By defining the Y-factor, we can express the DUT noise temperature as

$$Y = \frac{P_1}{P_2} = \frac{T_1 + T_e}{T_2 + T_e} > 1 \qquad if \quad T_1 > T_2$$

Then the unknown T_e of DUT is given by

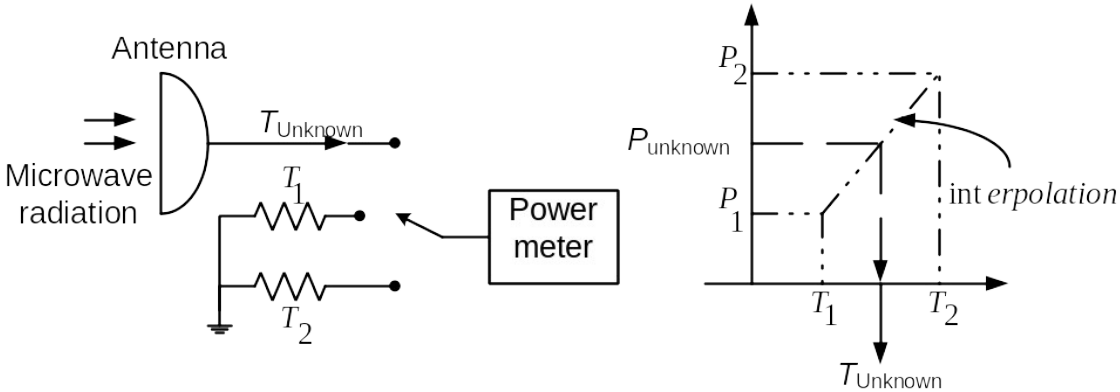
$$T_e = \frac{T_1 - Y\,T_2}{Y - 1}$$

Assume the noise source has ENR_{dB}= 20dB (ENR=100) at 1GHz when DC voltage is applied. When DC voltage is off, the noise source temperature is T_2=290K. The ambient temperature is given by T_o=290K. T_1 is

T_1 = ENR*T_o+T_2 = 100*290+290=29290K

If the measured powers are P_1=1mW at T_1 and P_2=0.1mW at T_2, then Y=10.
From this we get T_e=2932K.

The similar noise temperature measurement technique can be applied for microwave passive remote sensing, and it is known as microwave Radiometry. The microwave noise powers of 3 cases are measured and the unknown noise temperature is estimated with the interpolation techniques as shown below.



For the passive microwave remote sensing, the common calibrated noise sources are

$$\left.\begin{array}{ll} Liquid\ \ Nitrogen & T = 77K \\ Liquid\ \ Helium & T = 4K \end{array}\right\} \text{Low temperature devices}$$

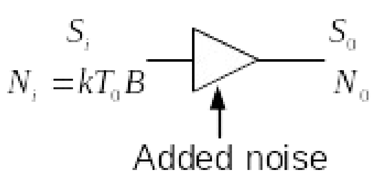
and a blackbody (microwave absorber) at room temperature (300K). The unknown temperature must be between T_{low} and T_{high}.

3. Noise Figure

In addition to " noise temperature", "noise figure" is often used for specifying the noise characteristics. The noise figure is defined as the ratio of input S/N (numerator) and output S/N (denominator). This value is always greater than 1 due to the added noise of DUT.

Note: If NF is expressed in dB, it is 10log(NF). [Signal Power, Noise Power]

$$F = \frac{S_i / N_i}{S_0 / N_0} > 1$$



Noise figure F can be expressed in terms of noise temperature T_e as

$$F = \frac{\dfrac{S_i}{kT_0B}}{\dfrac{S_iG}{kGB(T_0 + T_e)}} = \frac{T_0 + T_e}{T_0} = 1 + \frac{T_e}{T_0} > 1$$

or $T_e = (F - 1) T_0$

For example, the noise figure of Minicircuits amplifier ZRON-8G is specified as 6 dB. Then the corresponding noise temperature is

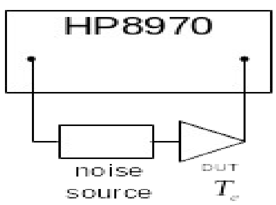
$$NF \; : \; 6\; dB \rightarrow \; F = 4$$

$$T_0 = 300\, K$$

$$\therefore T_e = (F - 1)\, T_0 = (4 - 1)300 = 900 K$$

4. Noise Figure Measurement Using the Y-factor Method

The easiest way to measure the noise figure is the noise figure (NF) meter which consists of a calibrated noise source and noise meter such as HP/Agilent 8970 as shown below.



This is called Y-factor method and it is basically the same method as we discussed for the noise temperature measurements. The noise source is usually the Zener diode operating in two states and the value is given by ENR (excess noise ratio) in dB. When the DC voltage (28V) is ON, the noise source has a high temperature given by T_1. When there is no DC voltage, the noise source has low temperature and given by T_2. Both T_1 and T_2 must be known in the frequency range in which DUT is measured. The noise meter is essentially a microwave power meter. ENR is related to T_1 and T_2 as

ENR = $(T_1\text{-}T_2)/T_o$

where T_o is the ambient temperature given by T_o=290K.

Assume $T_2=T_o$=290K. Y is given by

$$Y = \frac{P_1}{P_2} = \frac{T_1 + T_e}{T_2 + T_e} > 1 \qquad\qquad if \quad T_1 > T_2$$

Then the noise figure is

$$F = \frac{T_0 + T_e}{T_0} = \frac{ENR}{Y - 1}$$

It is common to express the noise figure in dB,

$NF_{dB}=10 \log_{10}(F)$ or

$$NF_{dB} = 10 \times \log_{10}\left(\frac{10^{ENR_{dB}/10}}{10^{Y_{dB}/10} - 1}\right)$$

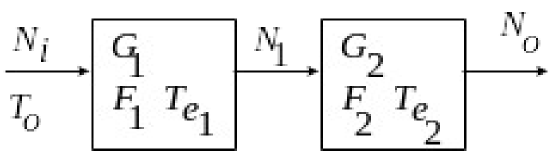
Suppose the noise source has ENR_{dB}=12.86 dB at 1 GHz, P_1=-75 dBm, and P_2=-87 dBm. This gives $Y_{dB}=P_1-P_2$=12 dB. Then

NF_{dB}=1.14 dB.

The above technique is applicable if DUT has a high gain. If not, the measurement system must be calibrated. This is discussed in 5-1 (also in the Agilent Tech note).

5. Noise Figure of a Cascaded System

Most microwave systems have several devices in series as shown below. Now we are interested in expressing the noise temperature of the cascaded system.



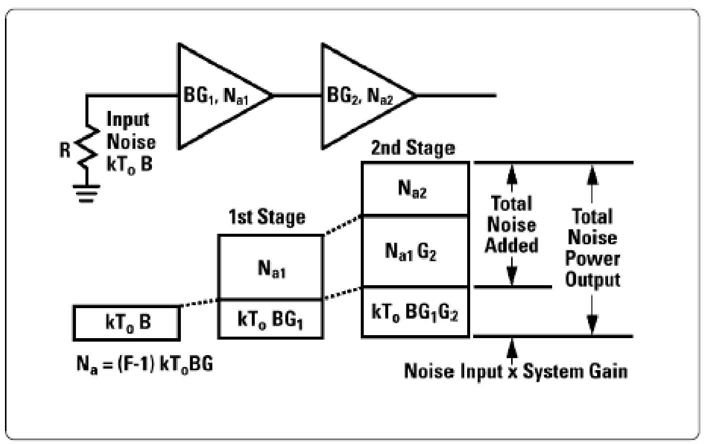


Figure 2-3. How noise builds up in a two-stage system.

We can write the noise power at each stage as

$$N_1 = G_1 k T_0 B + G_1 k T_{e_1} B$$

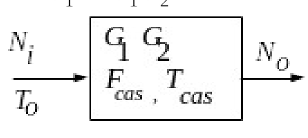
$$N_0 = G_2 N_1 + G_2 T_{e_2} k B$$

$$N_0 = G_1 G_2 k B \left(T_{cas} + T_0\right)$$

$$\text{where } T_{cas} = T_{e_1} + \frac{1}{G_1} T_{e_2}$$

When we have many cascaded devices, we can combine them and write the noise temperature as

$$T_{cas} = T_{e_1} + \frac{T_{e_2}}{G_1} + \frac{T_{e_3}}{G_1 G_2} + ...$$



$$F_{cas} = F_1 + \frac{1}{G_1}\left(F_2 - 1\right)$$

This expression shows that the noise characteristics of the cascaded system will be strongly influenced by the first stage gain G1 and noise temperature Te1. If the gain G1 is high, the effects of the 2nd stage noise will be small as shown above. When we design a sensitive microwave receiver, it is very important to have a high gain low noise amplifier (LNA) in the first stage. It is not wise to cut the cost by replacing LNA with a noisy amp.

5-1. Corrected Noise Figure

In section 4, we discussed the noise figure measurements. The noise figure obtained in Section 4 is that of the total system which contains the effect of the second stage (SA) as shown above and given by

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1)$$

If we can estimate G_1 (gain of DUT) and F_2 (NF of SA), we should be able to calculate the noise figure of DUT which is given by F_1. To do so, we need to measure the noise power with and without DUT.

Without DUT

N_{ON}: Noise power with noise source ON

N_{OFF}: Noise power with source OFF

With DUT

N_{ON}': Noise power with noise source ON

N_{OFF}': Noise power with source OFF

Using the linear scale.

$$G_1 = \frac{N_{ON}' - N_{OFF}'}{N_{ON} - N_{OFF}}$$

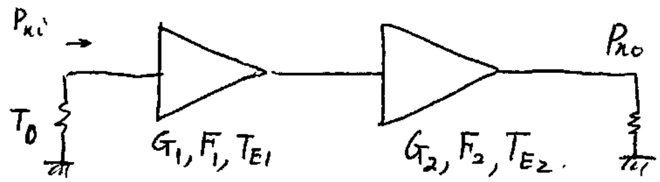
$Y_{SA}=N_{ON}/N_{OFF}$ (or $Y_{dB}=N_{ON\_dB} - N_{OFF\_dB}$)

$$F_2 = \frac{ENR}{Y_{SA} - 1}$$

From this we can find the corrected noise figure as

$$F_1 = F_{cas} - \frac{1}{G_1}(F_2 - 1)$$

Cascaded system and cable loss



Output noise

$$\begin{aligned} P_{no} &= P_{ni}G_1G_2 + P_{E_1}G_1G_2 + P_{E_2}G_2 \\ &= G_1G_2kB\left[T_0 + T_{E_1} + \frac{T_{E_2}}{G_1}\right] \\ &= G_1G_2kB\left[T_0 + T_E\right] \end{aligned}$$

Therefore, the noise equivalent temperature of the cascaded system is

$$T_E = T_{E1} + \frac{T_{E_2}}{G_1}$$

Since $(F-1)T_0 = T_E$

$$F = F_1 + \frac{F_2 - 1}{G_1}$$

In general

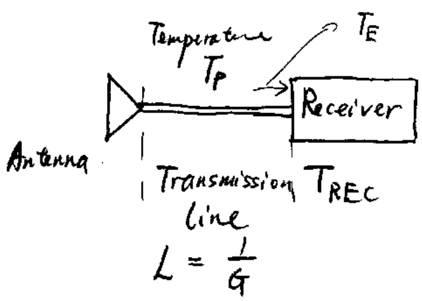
$$T_E = T_{E_1} + \frac{T_{E_2}}{G_1} + \frac{T_{E_3}}{G_1G_2}\ldots\ldots$$

To minimize T_E, T_{E_1} should be small (low noise) and G_1 should be high.

$\therefore$ High gain & low noise amp in the 1st stage.

Lossy Transmission Line

e.g.



$$L = \frac{1}{G}$$

Effective input noise temperature to the receiver due to TL.

$$T_E = (L-1)T_p$$

If $L = 1$ (lossless), then TL has no effect.

Total effective input noise temperature of the receiver.

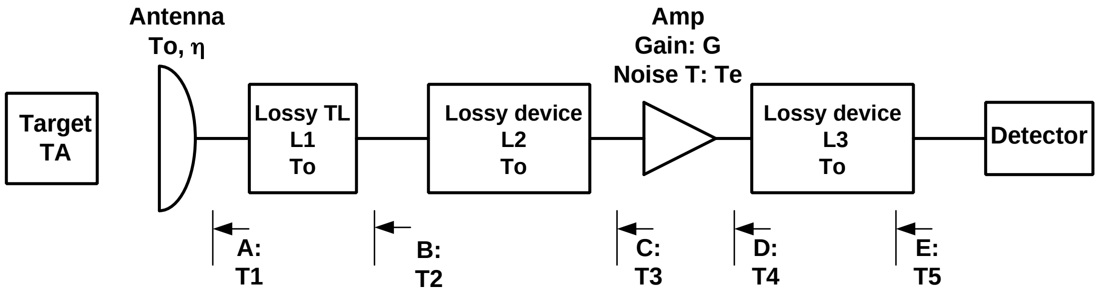
$$T'_{REC} = \underbrace{(L-1)T_p}_{\text{Due to TL}} + \underbrace{LT_{REC}}_{\text{Receiver}}$$

If $L = 1$, then $T'_{REC} = T_{REC}$

If $L = 10$ (10dB loss) , $T_p = 290K$ & $T_{REC} = 200K$, then

$$T'_{REC} = 9 \times 290 + 10 \times 200 = 4610K$$

How to calculate the effective temperature in a cascaded system



Assume we have these parameters:

TA=50 K
To: 300 K
L1=2 (3dB)
L2=4 (6dB)
L3=2 (3dB)
G=10 (10 dB)
Te=1000 K
η=0.9 (90%)

Note:
The conversion loss of mixer is basically loss (L1, L2, L3 in this case).
Noise temperature of an amp can be found from the noise figure.

At the end of each device, we get the following noise temperature by assuming antenna gain is 1.

At A we have T1 and the total noise power P_{nA} is

$$P_{nA} = \text{Input noise power + Noise generated by antenna} = \eta kBT_A + (1-\eta)kBT_0$$

$$Since\ P_{nA} = kBT_1$$

$$T_1 = \frac{P_{nA}}{kB} = \eta T_A + (1-\eta)T_0 = 0.9 * 50 + (1-0.9) * 300 = 75 \text{ K}$$

At B we have T2 and the total noise power P_{n1} is

$$P_{n1} = G_1 P_{nA} + \Delta P_{n1} = G_1 kBT_1 + \Delta P_{n1} = G_1 kBT_1 + (F_1 - 1)G_1 kT_0 B = G_1 kB(T_1 + T_0(L_1 - 1)),$$

$$\text{where } \Delta P_{n1} = (F_1 - 1)G_1 kT_0 B, \text{ and } G_1 = \frac{1}{L_1}, F_1 = L_1$$

$$\text{Since } P_{n1} = G_1 kBT_2$$

$$T_2 = \frac{P_{n1}}{G_1 kB} = T_1 + T_0(L_1 - 1) = 75 + (2-1)*300 = 375 \text{ K}$$

At C we have T3 and the total noise power P_{n2} is

$$P_{n2} = G_2 P_{n1} + \Delta P_{n2} = G_2 G_1 kBT_2 + (F_2 - 1)G_2 kBT_0 . \text{where } G_2 = \frac{1}{L_2}, F_2 = L_2$$

$$\text{Since } P_{n2} = G_1 G_2 kBT_3$$

$$T_3 = \frac{P_{n2}}{G_1 G_2 kB} = T_2 + L_1(L_2 - 1)T_0 = T_1 + T_0(L_1 - 1) + L_1(L_2 - 1)T_0 = 2175K$$

At D we have T4 and the total noise power P_{n3} is

$$P_{n3} = GP_{n2} + \Delta P_{n3} = GG_1 G_2 kBT_3 + GkBT_e .$$

$$\text{Since } P_{n3} = GG_1 G_2 kBT_4$$

$$T_4 = \frac{P_{n3}}{GG_1 G_2 kB} = T_3 + \frac{T_e}{G_1 G_2} = T_1 + T_0(L_1 - 1) + L_1(L_2 - 1)T_0 + L_1 L_2 T_e = 10175K$$

At E we have T5 and the total noise power P_{n4} is

$$P_{n4} = G_3 P_{n3} + \Delta P_{n4} = G_3 G_1 G_2 GkBT_4 + (F_3 - 1)G_3 kBT_0 \quad \text{where } G_3 = \frac{1}{L_3}, F_3 = L_3$$

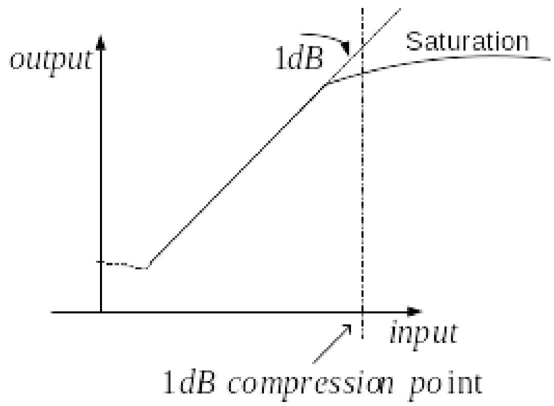
$$\text{Since } P_{n4} = G_3 G_1 G_2 GkBT_5$$

$$T_5 = \frac{P_{n4}}{G_3 G_1 G_2 GkB} = T_4 + (L_3 - 1)\frac{T_0}{G_1 G_2 G} = T_1 + T_0(L_1 - 1) + L_1(L_2 - 1)T_0 + L_1 L_2 T_e + (L_3 - 1)L_1 L_2 \frac{T_0}{G} = 10415K$$

Note: After the amplifier the added noise becomes very small as expected.

6. Amplifier in non-linear region

A microwave amplifier becomes a non-linear device when it is operated in the saturation region. Similar to a low frequency audio amplifier, the output of a microwave amplifier will be heavily distorted in this region. The commonly used term to describe the saturation input level is "1dB compression point". At this input the actual output power and the expected output power of the linear amplifier shows the difference of 1 dB as shown below.

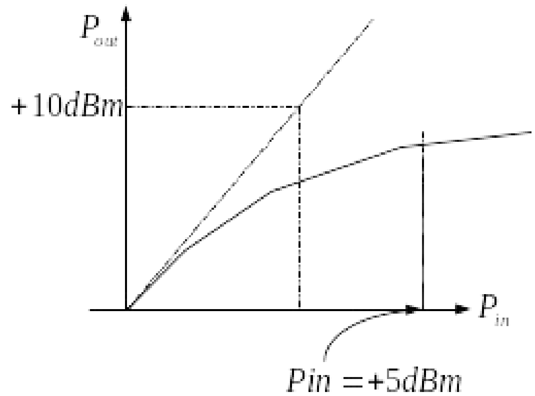


When we increase the input power level beyond the 1 dB compression point, the output power still increases. However, the gain of an amplifier becomes less than that of the linear region. The maximum output power is usually defined when the amplifier is completely saturated. Unless the system is operating at the saturated condition, the maximum power cannot be achieved in a linear region. Also the input power level should be well below the maximum input power level in order to avoid the damage.

Example: When the spec of amplifier shows the gain and output, they are

$$10\,dB\ gain \rightarrow\ linear\ \ gain$$

$$+10\,dBm\ \ output \rightarrow\ \ saturated$$



6-1. Non-linear responses due to a single input signal

When the amplifier is saturated, the output voltage is given by the large signal model as shown below. When we have a sinusoidal input signal, the output will contain 2nd, 3rd,.. harmonic signals as shown below.

$$v_{out} = a_0 + \underbrace{a_1 v_{in}}_{\text{linear term } \omega} + \underbrace{a_2 v_{in}^2}_{2\omega} + \underbrace{a_3 v_{in}^3}_{3\omega}$$

If

$$v_{in} = A\cos\omega t$$

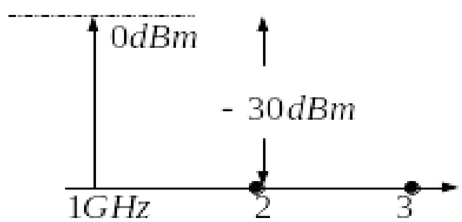
$$v_{in}^2 = A^2 \cos^2 \omega t \quad \rightarrow \quad \frac{1}{2}(1+\cos 2\omega t)$$

The power level of these harmonic signals will be substantially different for linear and non-linear cases as shown below.

Amp: gain 10 dB, max output 10dBm

(1) $v_{in} = -10\,dBm$ (Linear region) at 1 GHz

This amp may show a small 2nd harmonic noise at 2 GHz (-30dBm).

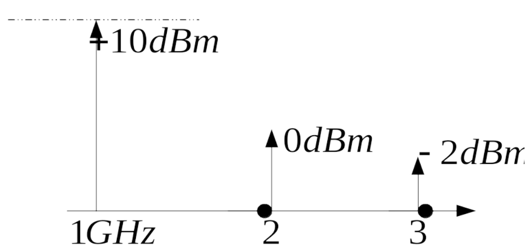


(2) Increase input to $v_{in} = +5\,dBm$.

The amp will be saturated and 1GHz output is at +10dBm.

Now 2nd and 3rd harmonics will be much larger.

0dBm and –2dBm in this example



6-2. Non-linear responses due to 2 or more signals (Signal Intermodulation)

The harmonic noise in non-linear amplifier may cause interference to other devices but 2nd and 3rd harmonic frequencies are much higher than the desired signal and they may not interfere the performance of the amplifier. However, when we have 2 or more closely spaced signals in the saturated amplifier, we get much more complicated responses known as signal intermodulation.

Suppose we have two signals given by

$$v_{in}(t) = A(\cos\omega_1 t + \cos\omega_2 t) \qquad\qquad \omega_1 \neq \omega_2$$

The output of a non-linear device is expressed as

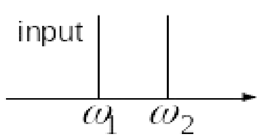
$$v_{out}(t) = a\, v_{in}(t) + a_2\, v_{in}^2(t) + a_3\, v_{in}^3(t)$$

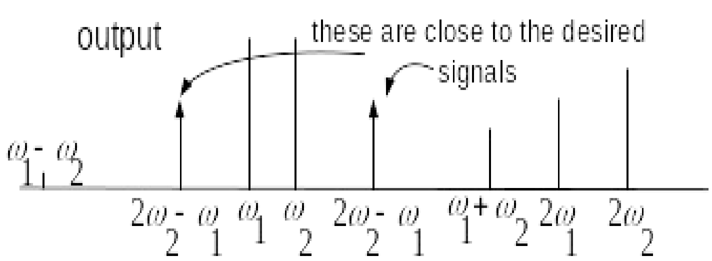
1st term: Linear term, ω_1, ω_2

2nd term: $\left.\begin{matrix}\omega_1 + \omega_2 \\ \omega_1 - \omega_2\end{matrix}\right)$ Second order intermodulation due to $v_{in}^2(t)$

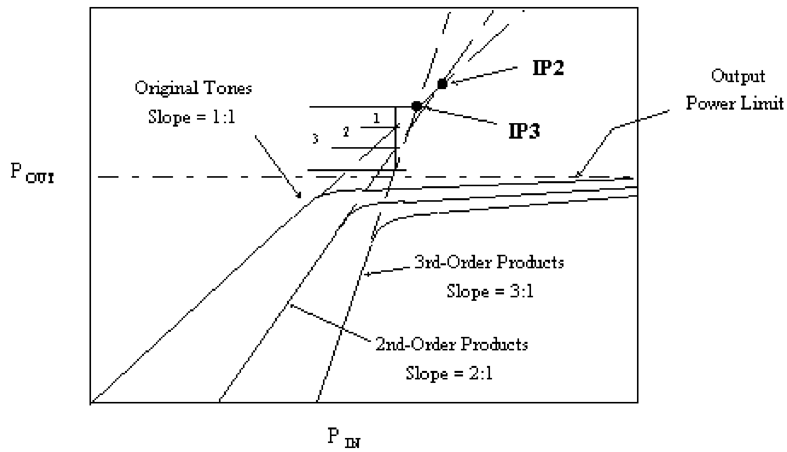
3rd term: $\left.\begin{matrix}2\omega_1 - \omega_2 \\ 2\omega_2 - \omega_1\end{matrix}\right)$ **3rd order intermodulation** due to $v_{in}^3(t)$

We can show the 2nd term contain sum and difference frequencies which are usually either much higher or lower than the desired signals. The most important frequencies are the 3rd order terms. Suppose we have f1=1GHz and f2=1.01 GHz as two inputs. The 3rd-order terms are given by 0.99 GHz and 1.02 GHz, respectively. These are very close to the desired frequency and they will pass through a BPF as shown below.





In addition, the coefficient of the 3rd-order term is proportional to A^3 where A is the input signal level. Although the 3rd-order term is small in the linear region, the increase will be much more rapid in the non-linear region as shown below.



Linear term: proportional to A

2nd–order: proportional to A^2

3rd–order: proportional to A^3

The **3rd-order intercept point** is defined as the intersection of the linear response and that of 3rd-order noise. Both of them are extrapolated using straight lines.

Note:
For many systems such as cellular phones, the 3rd-order intermodulation is one of the most important problems because these noise signals often fall into the signal band. The amplifiers for communication systems must be designed to minimize the 3rd-order intermodulation.

Example: Two cell phones are operating at user 1: 825MHz and user 2: 825.03 MHz.
The frequencies of 3rd-order intermodulation are at 824.7 and 825.06 MHz.
These frequencies are within the cell phone band.

How to get IP3 without measuring many data points
Assume all power levels are given in dBm.

$P_{\text{input tones@output}}$: Output power of the fundamental signal in dBm
$P_{\text{3rd-order products}}$: Output power of the 3^{rd} order products in dBm
IP3: Third-order intercept point in dBm

From the figure we can write

$P_{\text{3rd-order products}} = P_{\text{input tones@output}} - 2 \cdot (\text{IP3} - P_{\text{input tones@output}})$ {dBm}

Or $P_{\text{3rd-order products}} + 3*(\text{IP3} - P_{\text{input tones@output}}) = \text{IP3}$ {dBm}

$P_{\text{3rd-order products}} = 3 \cdot P_{\text{input tones@output}} - 2 \cdot \text{IP3}$ {dBm}

Note: When $P_{\text{3rd-order products}} = \text{IP3}$, $P_{\text{input tones@output}} = \text{IP3}$. This makes sense.

(IP3 - $P_{\text{input tones@output}}$) is shown as (1) in the following figure. The level of the 3rd order product $P_{\text{3rd-order products}}$ is 3 times less than that of $P_{\text{input tones@output}}$. The term 2・(IP3 - $P_{\text{input tones@output}}$) brings $P_{\text{input tones@output}}$ to $P_{\text{3rd-order products}}$ shown as (3) in the figure.

Finally

$$IP3 = 3/2 \cdot P_{\text{input tones@output}} - 1/2\ P_{\text{3rd-order products}}\ \{\text{dBm}\}$$

For example, when $P_{\text{input tones@output}}$ is set to 0 dBm and if $P_{\text{3rd-order products}}$ is -50 dBm, IP3 is given by +25 dBm [IP3 = - 1/2 $P_{\text{3rd-order products}}$ {dBm}].