Lecture Note 1: Review of Transmission Lines 9/25/2018

1-1. What is microwave frequency?

Freq: 0.3 GHz (λ=1m) to 300 GHz (λ =1mm)

Narrowly defined

Below 1 GHz	RF (radio frequency)	
1 - 30 GHz	Microwave	cm waves
30 - 300 GHz	Millimeter wave	

Frequency designation:

L-band	1.1 - 1.7 GHz
S-band	2.6 - 3.95 GHz
C-band	5.85 - 8.2 GHz
X-band	8.2 - 12.4 GHz
Ku-band	12.4 - 18 GHz
K-band	18 - 26.6 GHz
Ka-band	26.5 - 40 GHz
V-band	50 - 75 GHz
W-band	75 - 110 GHz

Microwave applications:

Cellular phone	800/900 MHz, 2.4 GHz
Microwave oven	2.45 GHz
Police radar	X-band, K-band
Military radar	X, Ka,….
Communication links	S, Ku, Ka
IEEE 802b (Wireless)	2.4 GHz, linear Pol
GPS	1565-1585MHz Right-hand circular pol (RHCP)

1-2. Difference between audio and microwave amplifiers

Audio amplifier

Freq range:	10-20 KHz
Power:	10W more
Input voltage:	1mV
Output voltage:	~9V
Gain:	80dB
Noise:	Power line noise, white noise
Distortion:	Harmonic noise, intermodulation noise
Input impedance:	Usually high

Output impedance: low to drive 8, 16 ohm speakers
Stability: Usually stable

Microwave amplifier

Freq range: example 1 to 4 GHz
Power: mW
Noise: Specified in terms of noise temperature
Impedance: Must be matched to transmission line characteristic impedance
Usually 50 ohm
Gain: 10-30dB High gain amp is very expensive
Stability: Very important

<u>Wavelength vs size</u>

VLF region (Similar to the audio frequency range):

At 1 KHz λ =300m
Size of a device << λ

RF region:

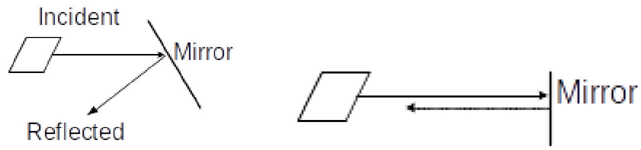
At 1 GHz λ =0.3 m
Size of a circuit board ~ λ

Microwave region:

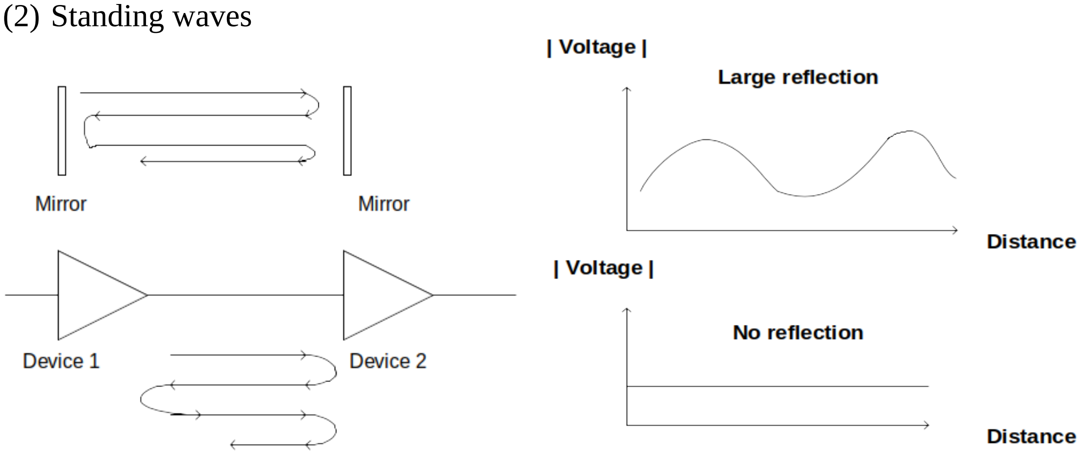
At 10 GHz λ =3 cm
Size of a device ~ λ

<u>1-3. Important phenomena at high frequency</u>

(1) Reflection from an unmatched load



(2) Standing waves



1-4. Units

Time:	nsec (nano second)	10^{-9}
	psec (pico second)	10^{-12}

Power:	dBm	dB milliwatts
	0 dBm	1mW
	20 dBm	100mW

10log(Power/1mW)

*** Important *** dBm is an absolute value.

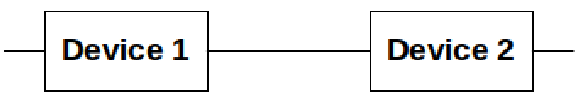
Voltage and S-parameters:

20log |S11|	S11=0.1 --->	-20 dB
	S21=10 --->	20 dB gain

*** Important *** dB is a relative value.

Length: meter, cm, millimeter

1-5. Transmission line and its effects



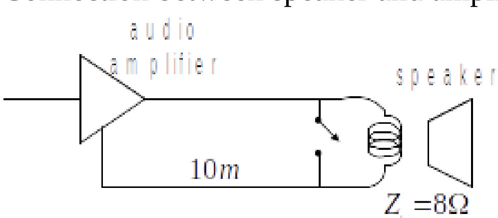
We will take a look at two examples. The first case shows that the length of the TL is not important. However, the second case shows that the low frequency approach is not applicable at high frequency.

Example 1: Audio System

Frequency: 20Hz-20 KHz
Power level: 1-100W
Output device: Speaker (convert audio [electric] signal to acoustic waves)
Impedance 4 - 8 ohm

Power Amplifier: Output impedance of power amplifiers ~ low

Connection between speaker and amplifier: copper wire (2 conductors)

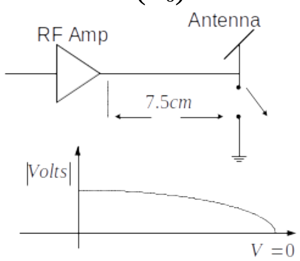


Example: at 1 KHz (wavelength=$3\times10^8/1000=3\times10^5$m)
The length of a copper wire is much less than that of the wavelength.

If the end of the copper wire (speaker input) is shorted, the voltage becomes zero everywhere on the copper wire. The voltmeter measures a total voltage which consists of incident and reflected voltages. The voltmeter reading will be zero because the length of the copper wire is much less than that of the wavelength. Therefore, we can neglect the effects due to the length of the copper wire.

Example 2: RF (radio frequency) amplifier
Frequency: 1 GHz (free apace wavelength=$3x10^8/10x10^9$=0.3m)
Power level: 100 mW
Output device: Antenna (radiate RF signal)
Output impedance is usually matched to a 50 ohm TL.

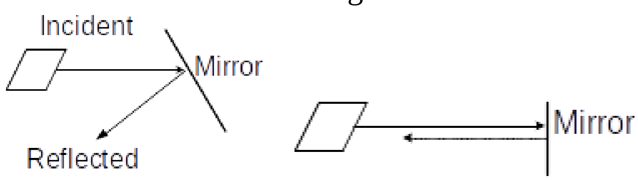
RF Amplifier: Output impedance of RF amplifier must be matched to the characteristic impedance of TL (Z_o).



Assume the cable connecting between RF amplifier and antenna is a 7.5 cm RG58 coaxial cable (Z_0=50 ohm)

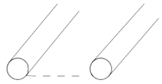
Inside the coaxial cable, the wavelength is given by $\lambda \sim \lambda_0/1.5$ where 1.5 is a square root of the dielectric constant of teflon (ε_r=2.2). The length of the TL is comparable to the wavelength.

If the TL is shorted at the antenna side, the total voltage on the TL is not zero. The incident and reflected voltages on the TL interfere and create a standing wave. This phenomenon is similar to a laser light reflected from a mirror.

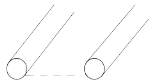


1-6. Different Types of Transmission Lines

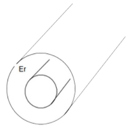
Power line (2 conductors)
120V 60 Hz λ=5000km
Carry energy
Transmit TEM wave



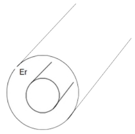
Speaker wire (2 conductors)
20-20,000 Hz
Carry audio signals
Transmit TEM wave



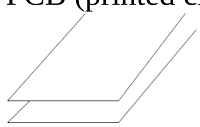
RG58 Coaxial cable (2 conductors)
DC-1GHz
Instrumentation, computer network
Transmit TEM waves



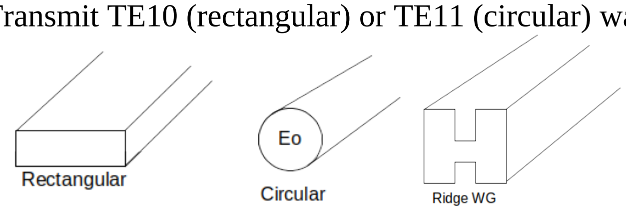
141” Semi-rigid coaxial cable (2 conductors)
Up to 26GHz
Diameter=0.141”
Transmit TEM waves



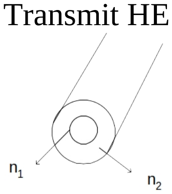
Parallel plates (2 conductors)
Microstrip lines
Strip lines
PCB (printed circuit board)



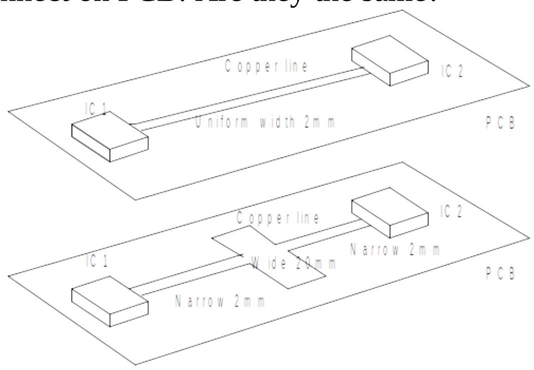
Waveguides (1 conductor)
Limited frequency range (High-pass)
Up to 300 GHz using different sizes
Transmit TE10 (rectangular) or TE11 (circular) wave



Optical fiber (no conductor)
Transmit light
Transmit HE and EH modes

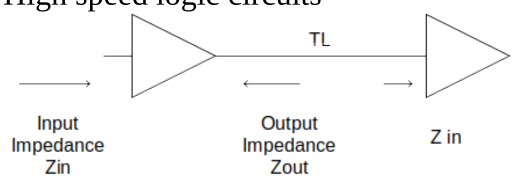


Interconnect on PCB: Are they the same?



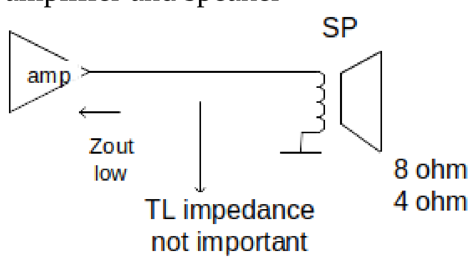
Low frequency: No difference
High frequency: Different because of impedance mismatch
High-speed digital circuits??

Why do we need to worry about TL?
Case 1: High speed logic circuits



TL characteristic impedance must be matched to Zin and Zout.

Case 2: Audio amplifier and speaker

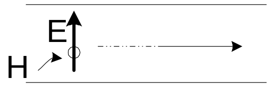


TL characteristic impedance is not important.

1-7. Modes in transmission lines

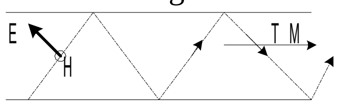
TEM and quasi-TEM wave

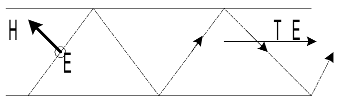
Transverse electromagnetic wave: Electric and magnetic fields are perpendicular to the direction of the signal propagation. The wave velocity is given by $v_p = c_o / \sqrt{\varepsilon_r}$.



TE and TM waves

Either electric or magnetic field is transverse to the direction of the signal propagation.





TL which use TEM or quasi-TEM mode

1. Coaxial line: Instrumentation
2. Two-wire line: TV and FM radio
3. Parallel-plate line:
4. Strip line: microwave circuits, high performance PCB (printed circuit board)
5. Microstrip line: microwave circuits, high performance PCB

TL which use TE or TM modes

1. Rectangular and circular waveguides
2. Coplanar waveguides

Effects due to TL

1. Signal delay (phase change)
2. Reflection due to unmatched impedance
2. Power loss (attenuation)
3. Dispersion (signal distortion)

1-8. Lumped-Element Model

Although TLs have different shapes and number of conductors, we will represent them with a two-wire configuration which is similar to a TV cable.

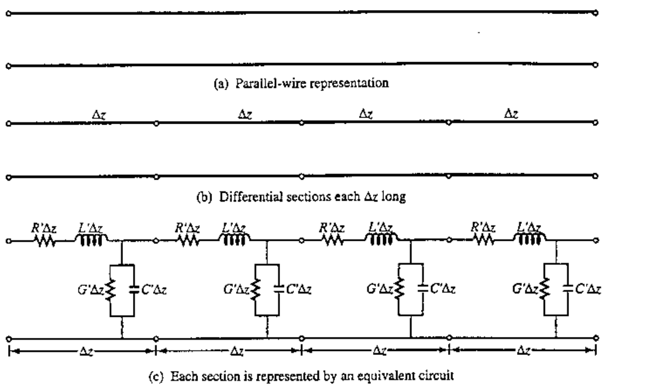


Figure 2-6: Regardless of its actual shape, a TEM transmission line is represented by the parallel-wire configuration shown in (a). To analyze the voltage and current relations, the line is subdivided into small differential sections (b), each of which is then represented by an equivalent circuit (c).

If we take a look at a unit length of this TL, we can express it in terms of an equivalent circuit (lumped elements).

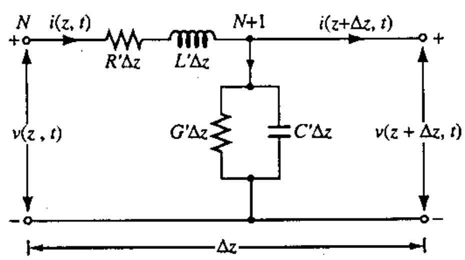


Figure 2-8: Equivalent circuit of a differential length Δz of a two-conductor transmission line.

R': series resistance (ohm/m)
Related to the surface resistivity of a conductor
L': series inductance (H/m)
G': parallel conductance (S/m)
Related to the dielectric loss of a dielectric material
C': parallel capacitance(C/m)

This is also equivalent to a circuit which consists of a lossy inductance and lossy capacitance as shown above.

If a TL is lossless (perfect inductor and capacitor), R'=0 and G'=0.

All TLs satisfy

$L'C' = \mu\varepsilon$

Velocity of TEM wave $v_p = 1/\sqrt{\mu\varepsilon} = c_o/\sqrt{\mu_r\varepsilon_r} = 1/\sqrt{LC}$

Transmission Line Equations

Using the difference equation and Kirckkoff voltage and current laws, we can obtain

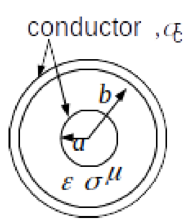
$$v(z,t) - R'\Delta z i(z,t) - L'\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

$$i(z,t) - G'\Delta z v(z+\Delta z,t) - C'\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

This can be expressed in terms of partial derivative ad obtain two differential equations.

$$-\frac{\partial v(z,t)}{\partial z} = R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = G' v(z,t) + C' \frac{\partial v(z,t)}{\partial t}$$



$R' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$, $R_s = \sqrt{\frac{\pi f \mu c}{\sigma_c}}$, $R_s \propto \sqrt{f}$

Table 2-1: Transmission-line parameters R', L', G', and C' for three types of lines.

Parameter	Coaxial	Two Wire	Parallel Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{R_s}{\pi a}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]$	$\frac{\mu d}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]}$	$\frac{\sigma w}{d}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right]}$	$\frac{\varepsilon w}{d}$	F/m

Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2) μ, ε, and σ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\pi f \mu_c/\sigma_c}$. (4) μ_c and σ_c pertain to the conductors. (5) If $(d/2a)^2 \gg 1$, then $\ln\left[(d/2a)+\sqrt{(d/2a)^2-1}\right] \simeq \ln(d/a)$.

Using the phasor notation, we can write the above equations as

$$-\frac{dV(z)}{dz}=(R'+j\omega L')I(z)$$

$$-\frac{dI(z)}{dz}=(G'+j\omega C')V(z)$$

where $v(z,t)=\mathrm{Re}[V(z)e^{j\omega t}]$, $i(z,t)=\mathrm{Re}[I(z)e^{j\omega t}]$

Two first-order differential equations in terms of V and I can be combined to get one 2nd-order differential equation in terms of either V or I as

$$\frac{d^2V(z)}{dz^2}-\gamma^2V(z)=0$$

$$\frac{d^2I(z)}{dz^2}-\gamma^2I(z)=0$$

These equations are called wave equations which describe the propagation characteristics of signal (V and I) on a TL.

γ is called a propagation constant of a signal on a TL. If a TL contains loss, γ becomes complex and given by

$$\gamma=\alpha+j\beta=\sqrt{(R'+j\omega L')(G'+j\omega C')}$$

where

α: attenuation constant

$$\alpha=\mathrm{Re}[\gamma]$$

$\alpha>0$. Related to $e^{-\alpha z}$

β: phase constant

$$\beta=\mathrm{Im}[\gamma]$$

Take $\beta>0$. Related to $e^{j\beta z}$

If a TL is lossless, $\gamma=j\beta$.

The solution for the wave equation is given by

$$\frac{d^2V(z)}{dz^2}-\gamma^2V(z)=0$$

General solutions

$$V(z) = Ae^{-\gamma z}$$
$$V(z) = Be^{\gamma z}$$
$$V(z) = Ae^{-\gamma z} + Be^{\gamma z}$$

If $\gamma = j\beta$, we can write the above equation as

$$\frac{d^2 V(z)}{dz^2} + \beta^2 V(z) = 0$$

Wave equation

The solutions are now given by $Ae^{-j\beta z},\ Be^{j\beta} z,\ Ae^{-j\beta z} + Be^{j\beta z}$.

Because

$$e^{-j\beta z} \rightarrow \cos\beta z - j\sin\beta z$$
$$e^{+j\beta z} \rightarrow \cos\beta z + j\sin\beta z$$

Another form of solution is

$$V(z) = C\cos\beta z + D\sin\beta z$$

We write voltage and current as

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$
$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z}$$

If we express, I in terms of Vo^+ and Vo^-,

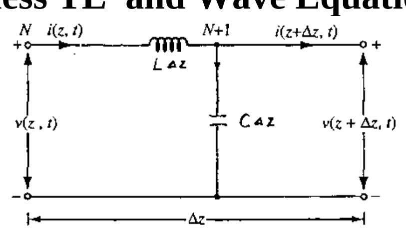
$$I(z) = \frac{\gamma}{R' + j\omega L'}\left[V_o^+ e^{-\gamma z} - V_o^- e^{\gamma z} \right]$$

(-) sign is due to the direction of current

$$-\frac{dV(z)}{dz} = (R' + j\omega L') I(z)$$

is used to get the above equation.

Lossless TL and Wave Equation (New)



L: series inductance (H/m)
C: parallel capacitance(C/m)
Δz : unit length

All TLs satisfy

$LC = \mu\varepsilon$

Velocity of TEM wave $v_p = 1/\sqrt{\mu\varepsilon} = c_o/\sqrt{\mu_r\varepsilon_r} = 1/\sqrt{LC}$

Characteristic impedance $Zo = \sqrt{\dfrac{L}{C}}$

Transmission Line Equations

Using the difference equation and Kirckkoff voltage and current laws, we can obtain

$$v(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

$$i(z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

This can be expressed in terms of partial derivative ad obtain two differential equations.

$$-\frac{\partial v(z,t)}{\partial z} = L\frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = C\frac{\partial v(z,t)}{\partial t}$$

Using the phasor notation, we can write the above equations as

$$-\frac{dV(z)}{dz} = j\omega L\ I(z)$$

$$-\frac{dI(z)}{dz} = j\omega C\ V(z)$$

where $v(z,t) = \mathrm{Re}[V(z)e^{j\omega t}],\ \ i(z,t) = \mathrm{Re}[I(z)e^{j\omega t}]$

Two first-order differential equations in terms of V and I can be combined to get one 2nd-order differential equation in terms of either V or I as

$$\frac{d^2V(z)}{dz^2} + \beta^2 V(z) = 0$$

$$\frac{d^2I(z)}{dz^2} + \beta^2 I(z) = 0$$

These equations are called **wave equations** which describe the propagation characteristics of signal (V and I) on a TL.

β is called a phase constant of a signal on a TL.

$$\beta = \omega\sqrt{LC}$$

The solution for the wave equation is given by one of these

$V(z) = A' e^{-j\theta z}$ Propagating in +z direction

$V(z) = B' e^{j\theta z}$ Propagating in -z direction

$V(z)=A' e^{-j\beta z}+B' e^{j\beta z}$ Wave propagating in +z and -z directions (Standing wave)

Because

$$e^{-j\beta z} \rightarrow \cos\beta z - j\sin\beta z$$
$$e^{+j\beta z} \rightarrow \cos\beta z + j\sin\beta z$$
$$\cos\beta z = \frac{1}{2}\left(e^{+j\beta z}+e^{-j\beta z}\right)$$
$$\sin\beta z = \frac{1}{2j}\left(e^{+j\beta z}-e^{-j\beta z}\right)$$

Another form of solution is

$$V(z)=C\cos\beta z$$
$$V(z)=D\sin\beta z$$
$$V(z)=C\cos\beta z + D\sin\beta z$$

Note: $e^{-j\theta z}$ and $e^{j\theta z}$ are used for describing propagating waves
$\cos\beta z$ and $\sin\beta z$ are used for describing standing waves

We write voltage and current as

$$V(z)=V_o^+ e^{-j\beta z}+V_o^- e^{j\beta z}$$
$$I(z)=I_o^+ e^{-j\beta z}+I_o^- e^{j\beta z}$$

where V_o^+ and V_o^- are complex amplitude of incident and reflected voltages.

If we express, *I* in terms of V_o^+ and V_o^- ,

$$I(z)=\frac{\beta}{\omega L}\left[V_o^+ e^{-j\beta z}-V_o^- e^{j\beta z}\right] \quad =\frac{1}{\sqrt{L/C}}\left[V_o^+ e^{-j\beta z}-V_o^- e^{j\beta z}\right]$$
$$=\frac{1}{Z_o}\left[V_o^+ e^{-j\beta z}-V_o^- e^{j\beta z}\right]$$

(-) sign is due to the direction of current

$-\frac{dV(z)}{dz}=j\omega L\ I(z)$ is used to get the above equation.

Z_o: characteristic impedance

1-9. Characteristic Impedance

The characteristic impedance is defined as the ratio of incident voltage and current (or reflected voltage and current).

$$Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-}$$

Lossless case $$Z_o = \frac{\omega L}{\beta} = \sqrt{\frac{L}{C}}$$

Including loss on a TL $$Z_o = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

In terms of Z_o, $I(z)$ becomes (lossless case)

$$I(z) = \frac{1}{Z_o}\left[V_o^+ e^{-j\beta z} - V_o^- e^{j\beta z} \right]$$

*The real part of Z_o must be positive.

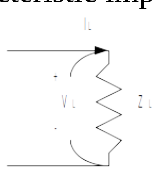
*The negative sign in $Z_o = -\frac{V_o^-}{I_o^-}$ is due to the direction of the current on a TL.

*The characteristic impedance does not depend on the position on a TL.

*Z_o depends only on the geometry of TL and materials.

*Z_o cannot be obtained by taking a ratio of the total voltage and current.

For example, Z_L in the following figure is defined in terms of the total voltage and current. It is not defined in terms of the incident voltage and current. Therefore, Z_L is not a characteristic impedance.



$$Z_L = \frac{V_L}{I_L}$$

V_L: total load voltage
I_L: total load current

Input impedance and characteristic impedance

Voltage, current → impedance

R=V/I

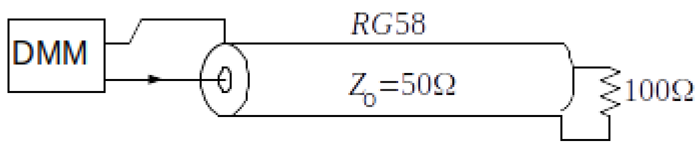
Electric field and magnetic field → impedance

Free space impedance $\eta_0 = E/H = 120\pi = 377\,\Omega = \sqrt{\mu_o/\varepsilon_o}$

units of E and H are V/m and A/m, respectively

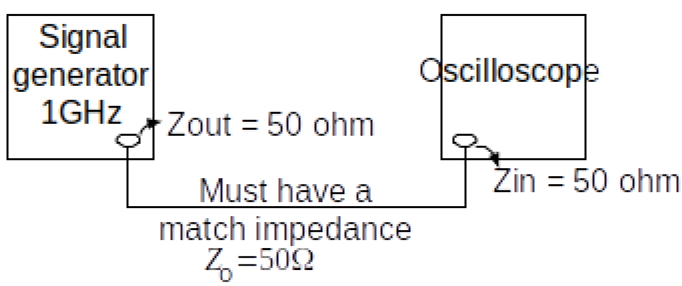
Materials impedance $\eta = E/H = \sqrt{\mu/\varepsilon} = 120\pi\sqrt{\mu_r/\varepsilon_r}$

We know a RG58 coaxial cable has the characteristic impedance of 50 ohm.
Can we measure this value with the digital multi-meter (DMM)?



DMM measures the voltage drop due to the current I at DC. Therefore, the RG58 cable does not have any effect.

All TLs have characteristic impedance



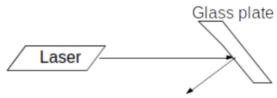
If the characteristic impedance of the TL is not matched to the load, the signal reflection occurs.

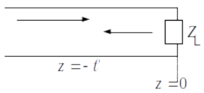
Examples: Light reflection from a glass plate (index of refraction n=1.5)

Index of refraction $n = \sqrt{\varepsilon_r}$

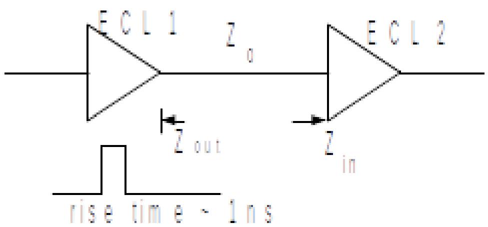
Free space impedance $\eta_0 = \sqrt{\mu_o/\varepsilon_o} = 120\pi$

Glass $\eta = 120\pi/\sqrt{\varepsilon_r} = 120\pi/1.5$

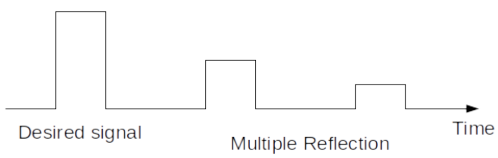




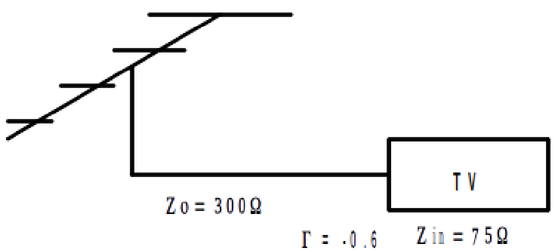
Example: Fast digital circuits
If Z_{in}<>Z_o and/or Z_{out}<>Z_o, the signal will be reflected.



Received signal at ECL2



Example: TV
If we connect a TV with 75Ω input impedance to a 300 TL, 36% of the incident power will be reflected from the TV set. This will be the waited power. If we want to get the best reception (optimum power transfer), the impedance of the TV set should be matched to that of a TL

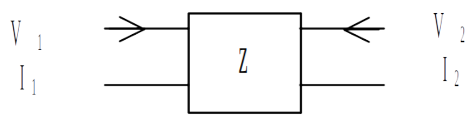


Power loss: $|\Gamma|^2$=0.36.

1-10. Two-port device and S-parameters

Any two-port device, such as a filter, can be described using different parameters. Most common types are Z-, Y-, and *h*-parameters, which are suited for low frequency devices. At high frequency, most devices are specified using S- or T-parameters.

Z-parameters



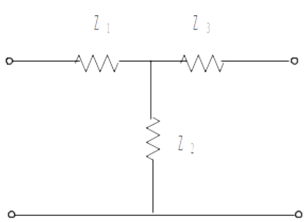
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad [Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

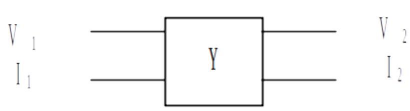
Output Open

Example: T network



$Z_{11} = Z_1 + Z_2$, $Z_{22} = Z_3 + Z_2$, $Z_{12} = Z_{21} = Z_2$

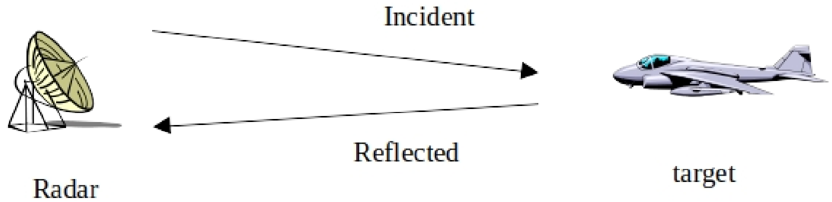
Y-parameters



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Z- and Y-parameters are specified in terms of total voltage and total current because it is difficult to separate voltage/current into incident and reflected components at low frequency. Also to obtain Z- and Y-parameters, one port must be either OPEN or SHORT. For example, $Z_{11}=V_1/I_1$ when $I_2=0$ (Output port OPEN). Similarly to get Y_{11}, V_2 must be 0 (output port SHORT).

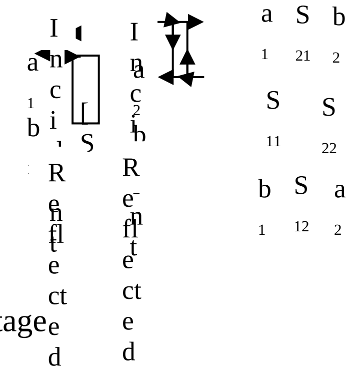
At microwave frequency, we consider the incident and reflected signal separately. One example is a radar system designed to detect a reflected signal.



One way to express the reflected and incident signals in a network is via S-parameters. The total signal is given by adding the incident and reflected signals. Usually we use a_1(from left side) and a_2(from right side) to denote incident signals. b_1 is the reflected (also transmitted if a_2 is non-zero) signal into the left side and b_2 is the transmitted (also reflected if a_2 is non-zero) signal into the right side.

Total = Incident + Reflected

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = [S] \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



Total Voltage

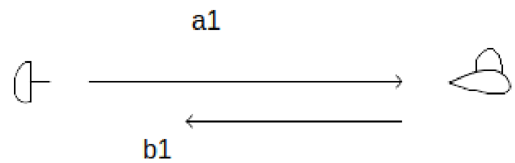
$$b_1 = S_{11} a_1 + S_{12} a_2$$
$$b_2 = S_{21} a_1 + S_{22} a_2$$
$$V_1 = (a_1 + b_1)\sqrt{Z_0}$$
$$V_2 = (a_2 + b_2)\sqrt{Z_0}$$
$$a_1 = \frac{V_1^+}{\sqrt{Z_0}} \qquad V_1^+ = incident\ \ voltage$$
$$b_1 = \frac{V_1^-}{\sqrt{Z_0}} \qquad V_1^- = reflected\ \ voltage$$
$$|a_1|^2 = a_1 a_1^{¿} = \frac{|V_1^+|^2}{Z_0} \ :\ incident\ \ power$$
$$|b_1|^2 = b_1 b_1^{¿} = \frac{|V_1^-|^2}{Z_0} \ :\ reflected\ \ power$$

S_{11} (reflection) is defined as:

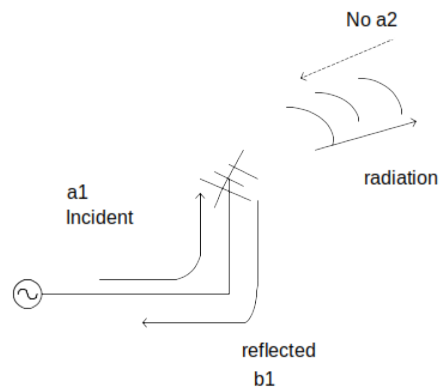
$$S_{11} = \frac{b_1}{a_1}|{¿}_{a_2=0}(No\ signal\ from\ the\ output\ side)$$

This is similar to the reflection coefficient Γ which is defined as $\Gamma = V^-/V^+$. However, S_{11} requires that a_2 must be zero. If S_{11} is large, most of the incoming signal will be reflected. On the other words, if S_{11} is small, most of the incoming signal will be transmitted into the next device. If you are designing an antenna, S_{11} must be small at the designed frequency.

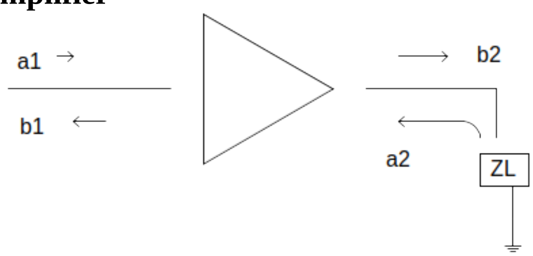
Radar

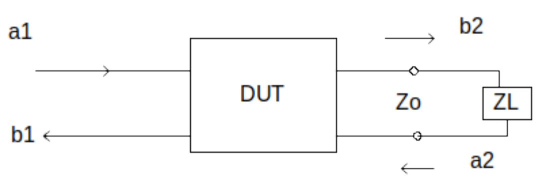


Antenna



Amplifier





If $Z_L=Z_o$, $\Gamma_L=0$. No reflection from output port.

Then the measured input reflection b_1/a_1 is S_{11}.

S_{21} (forward transmission) is defined as:

$$S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0}$$ Forward gain (e.g. gain of an amplifier)

For a passive device, if S_{21} is close to 1 (0dB), then there is no loss due to reflection and/or absorption. S_{21} is similar to the transfer function of low frequency devices. Like a transfer function, S_{21} can be a complex value (magnitude and phase). The phase term shows the propagation delay due to the length of a device and TL. The condition to get S_{11} and S_{21} is that a_2 must be 0 (no signal is coming into PORT2). This can be achieved by attaching a matched load to PORT2. If the characteristic impedance of TL is Z_o, the load impedance must be $Z_L=Z_o$. Microwave devices should not be measured with the un-used port OPEN or SHORT.

S_{12} (reverse transmission) is defined as:

$$S_{12} = \frac{b_1}{a_2}\Big|_{a_1=0}$$ Reverse gain (e.g. unwanted feedback in an amplifier)

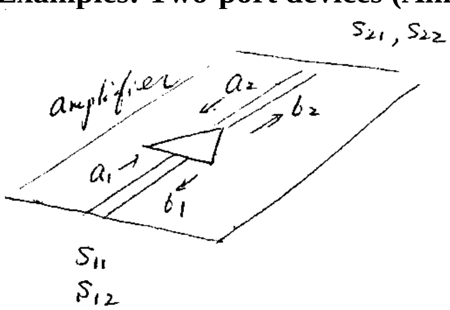
a_1=0: Input circuit (source side) is terminated with $Z_s=Z_o$.

S_{22} (output reflection) is defined as:

$$S_{22} = \frac{b_2}{a_2}\Big|_{a_1=0} \left(\text{No signal from the input side}\right)$$

Output matching condition

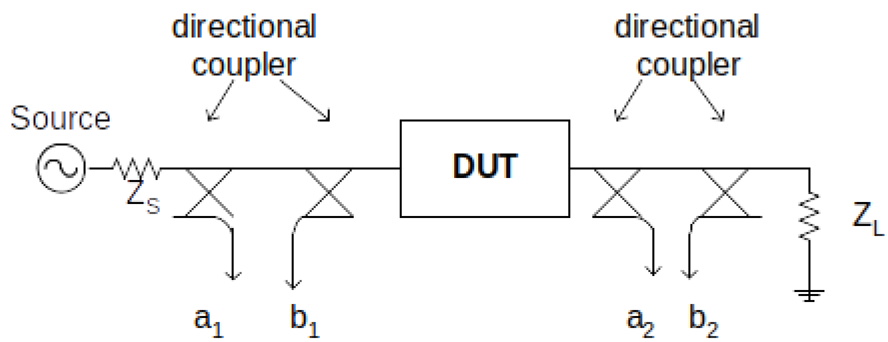
Examples: Two-port devices (Amplifier)



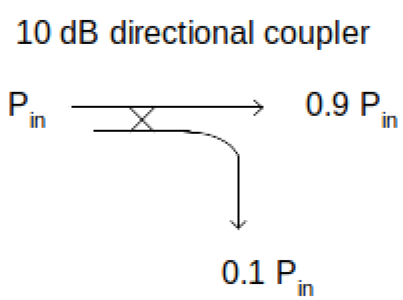
S_{11}: Should be small
S_{22}: Should be small
S_{21}: Should be large
S_{12}: Should be small

Important parameters: S_{11} and S_{21}

System to measure S-parameters



Z_S: source impedance
Z_L: load impedance



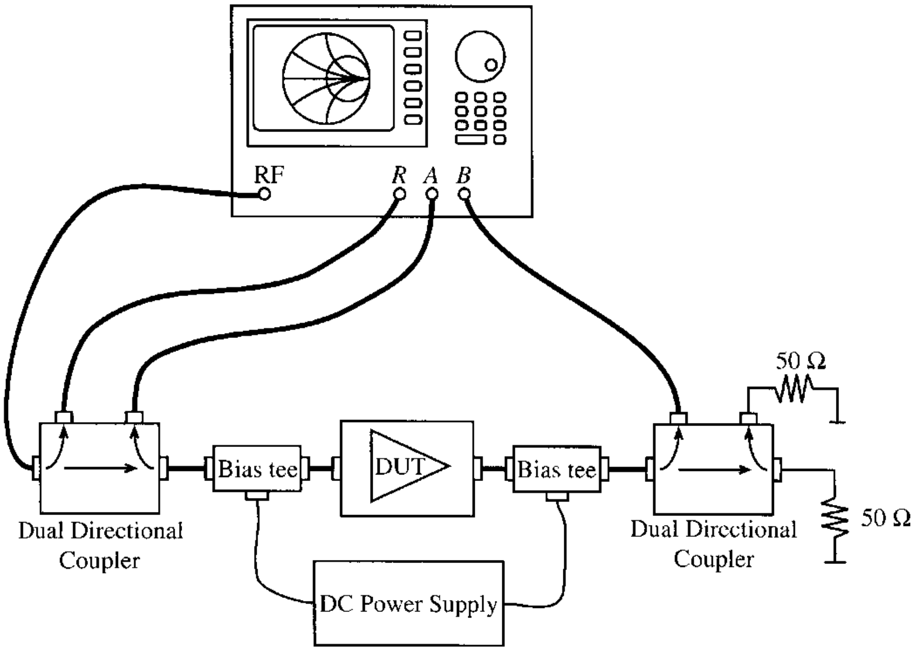


Figure 4-28 Measurement system for S_{11} and S_{21} parameters using a network analyzer.

1-11. Microstrip Transmission Lines

Microstrip and stripline TLs are used in all high-speed digital circuits such as computers and microwave circuits. There are many advantages, including (1) easy to fabricate, (2) easy to integrate with other components, (3) low cost [Substrate: FR4 (glass epoxy) cheap, low frequency; Teflon composite expensive, high frequency], and (4) couplers and power dividers can be fabricated with the same technique.

In terms of physical characteristics, the microstrip TL is specified by the line width (W), substrate height (*d*) and the material of a substrate (dielectric constant or permittivity ε_r) as shown in Fig.1. To model a microstrip TL circuit, you need to obtain (1) signal velocity (phase velocity) on a microstrip TL and (2) characteristic impedance in terms of *W*, *d* and ε_r. In many cases, however, you want to find the width *W* for a given characteristic impedance (Z_0).

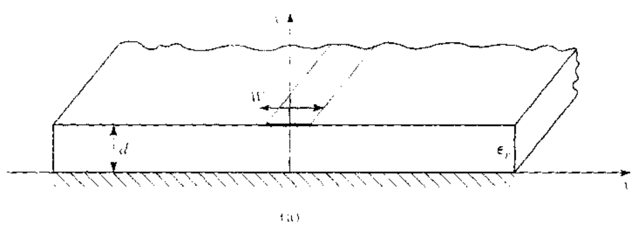


Figure 1: Microstrip TL.
W: line width, *d*: substrate thickness, ε_r: dielectric constant substrate.

The length of a TL is often specified in terms of a wavelength, which is (speed)/(frequency). Frequency does not change when the electromagnetic wave (signal) moves from air into a different material, but the speed depends on the material in which a wave (signal) is propagating. In free-space (air), the speed of light is $c_o=3x10^8$m/s. In water the speed of visible light is $c_o/1.33$ where 1.33 is the index of refraction of water at 633nm. Later we will study the velocity (phase velocity and group velocity) of different waves. In this lab, we will assume the speed of a signal on a microstrip TL is the "phase velocity" which is given in part by the effective permittivity (see Effective Permittivity and Phase Velocity).

Serenade SV has a tool (TRL) to calculate both effective permittivity and characteristic impedance of a given microstrip TL structure. TRL can also be used for obtaining *W* for a given characteristic impedance Zo (TL synthesis). Become familiar with this tool. Simple equations to estimate these values are also shown in the following section.

d-1. Effective Permittivity and Phase Velocity

The figure below shows the electric and magnetic field lines for a microstrip transmission line.

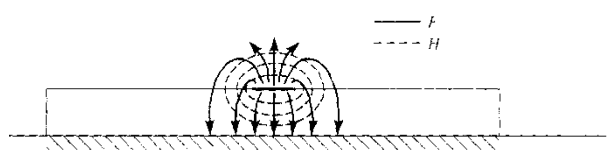


Figure 2: E and H field distribution

Since the field lines exist in media (air and a dielectric substrate) of differing permittivity, the effective permittivity (ε_{eff}) lies somewhere between 1 and the dielectric constant of the substrate (ε_r). The exact value of ε_{eff} depends on the width of the microstrip (*W*) and the thickness of the substrate (*d*).

$$\varepsilon_{eff}=\frac{\varepsilon_r+1}{2}+\frac{\varepsilon_r-1}{2}\left(\frac{1}{\sqrt{1+12\,d/W}}\right)$$

where *d* is much smaller than the wavelength of the input signal.

A good estimate of the phase velocity ("speed of light" for a particular medium) for that section of transmission line is:

$$v_p=\frac{c}{\sqrt{\varepsilon_{eff}}}$$

d-2. Characteristic Impedance

The characteristic impedance is determined by the geometry of TL and material in it. It does not depend on the position on a uniform TL. A simple equation to calculate the characteristic impedance Z_o of a microstrip TL is:

$$Z_0=\begin{cases} \dfrac{60}{\sqrt{\varepsilon_{eff}}}\ \ln\left(\dfrac{8d}{W}+\dfrac{W}{4d}\right) & for\ \ \dfrac{W}{d}\le 1 \\ \dfrac{120\pi}{\sqrt{\varepsilon_{eff}}\left[\dfrac{W}{d}+1.393+0.667\ln\left(\dfrac{W}{d}+1.444\right)\right]} & for\ \ \dfrac{W}{d}\ge 1 \end{cases}$$

A simple equation to calculate the width *W* for a given Z_o:

$$\frac{W}{d}=\begin{cases} \dfrac{8e^{A}}{e^{2A}-2} & for\ \ \dfrac{W}{d}<2 \\ \dfrac{2}{\pi}\left[B-1-\ln(2B-1)+\left(\dfrac{\varepsilon_r-1}{2\varepsilon_r}\right)\left\{\ln(B-1)+0.39-\dfrac{0.61}{\varepsilon_r}\right\}\right] & for\ \ \dfrac{W}{d}>2 \end{cases}$$

$$A=\frac{Z_0}{60}\sqrt{\frac{\varepsilon_r+1}{2}}+\frac{\varepsilon_r-1}{\varepsilon_r+1}\left(0.23+\frac{0.11}{\varepsilon_r}\right)$$

$$B=\frac{377\pi}{2Z_0\sqrt{\varepsilon_r}}$$

1-12. Physical constants and material properties

Appendix E PHYSICAL CONSTANTS

- Permittivity of free-space $= \epsilon_0 = 8.854 \times 10^{-12}$ F/m
- Permeability of free-space $= \mu_0 = 4\pi \times 10^{-7}$ H/m
- Impedance of free-space $= \eta_0 = 376.7\Omega$
- Velocity of light in free-space $= c = 2.998 \times 10^{8}$ m/s
- Charge of electron $= q = 1.602 \times 10^{-19}$ C
- Mass of electron $= m = 9.107 \times 10^{-31}$ kg
- Boltzmann's constant $= k = 1.380 \times 10^{-23}$ J/°K
- Planck's constant $= \hbar = 1.054 \times 10^{-34}$ J-s
- Gyromagnetic ratio $= \gamma = 1.759 \times 10^{11}$ C/Kg (for $g = 2$)

Appendix F CONDUCTIVITIES FOR SOME MATERIALS

Material	Conductivity S/m (20°C)	Material	Conductivity S/m (20°C)
Aluminum	3.816×10^{7}	Nichrome	1.0×10^{6}
Brass	2.564×10^{7}	Nickel	1.449×10^{7}
Bronze	1.00×10^{7}	Platinum	9.52×10^{6}
Chromium	3.846×10^{7}	Sea water	3–5
Copper	5.813×10^{7}	Silicon	4.4×10^{-4}
Distilled water	2×10^{-4}	Silver	6.173×10^{7}
Germanium	2.2×10^{6}	Steel (silicon)	2×10^{6}
Gold	4.098×10^{7}	Steel (stainless)	1.1×10^{6}
Graphite	7.0×10^{4}	Solder	7.0×10^{6}
Iron	1.03×10^{7}	Tungsten	1.825×10^{7}
Mercury	1.04×10^{6}	Zinc	1.67×10^{7}
Lead	4.56×10^{6}		

Appendix G DIELECTRIC CONSTANTS AND LOSS TANGENTS FOR SOME MATERIALS

Material	Frequency	ϵ_r	$\tan \delta$ (25°C)
Alumina (99.5%)	10 GHz	9.5–10.	0.0003
Barium tetratitanate	6 GHz	37±5%	0.0005
Beeswax	10 GHz	2.35	0.005
Beryllia	10 GHz	6.4	0.0003
Ceramic (A-35)	3 GHz	5.60	0.0041
Fused quartz	10 GHz	3.78	0.0001
Gallium arsenide	10 GHz	13.	0.006
Glass (pyrex)	3 GHz	4.82	0.0054
Glazed ceramic	10 GHz	7.2	0.008
Lucite	10 GHz	2.56	0.005
Nylon (610)	3 GHz	2.84	0.012
Parafin	10 GHz	2.24	0.0002
Plexiglass	3 GHz	2.60	0.0057
Polyethylene	10 GHz	2.25	0.0004
Polystyrene	10 GHz	2.54	0.00033
Porcelain (dry process)	100 MHz	5.04	0.0078
Rexolite (1422)	3 GHz	2.54	0.00048
Silicon	10 GHz	11.9	0.004
Styrofoam (103.7)	3 GHz	1.03	0.0001
Teflon	10 GHz	2.08	0.0004
Titania (D-100)	6 GHz	96±5%	0.001
Vaseline	10 GHz	2.16	0.001
Water (distilled)	3 GHz	76.7	0.157

Appendix H PROPERTIES OF SOME MICROWAVE FERRITE MATERIALS

Material	Trans-Tech Number	$4\pi Ms$ G	ΔH Oe	ϵ_r	$\tan \delta$	T_c °C	$4\pi Mr$ G
Magnesium ferrite	TT1-105	1750	225	12.2	0.00025	225	1220
Magnesium ferrite	TT1-390	2150	540	12.7	0.00025	320	1288
Magnesium ferrite	TT1-3000	3000	190	12.9	0.0005	240	2000
Nickel ferrite	TT2-101	3000	350	12.8	0.0025	585	1853
Nickel ferrite	TT2-113	500	150	9.0	0.0008	120	140
Nickel ferrite	TT2-125	2100	460	12.6	0.001	560	1426
Lithium ferrite	TT73-1700	1700	<400	16.1	0.0025	460	1139
Lithium ferrite	TT73-2200	2200	<450	15.8	0.0025	520	1474
Yttrium garnet	G-113	1780	45	15.0	0.0002	280	1277
Aluminum garnet	G-610	680	40	14.5	0.0002	185	515