Microwave Amplifier Design 2/27/2018

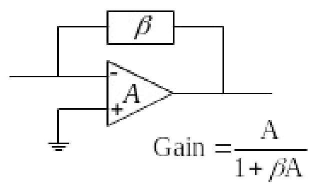
1. Introduction

The design of microwave amplifier depends on the type of applications. For example, if the receiver front-end requires a low noise amp (LNA), the main goal will be high gain and low noise. The wide frequency bandwidth may or may not be important. The nonlinear response may become important if it is used for communication system. The power amplifier used for a radar system, on the other hand, has the totally different requirements. The main differences between the microwave amplifier design and the low frequency audio amplifier design is the stability consideration and matching circuits. In this chapter, we will consider how to design practical microwave amplifiers.

The microwave amplifier design

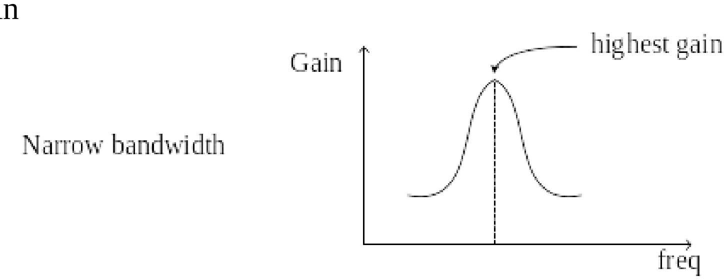
(1) Stability

e.g. Feedback loop



If $1+\beta A \to 0$, unstable.

(2) Maximum gain

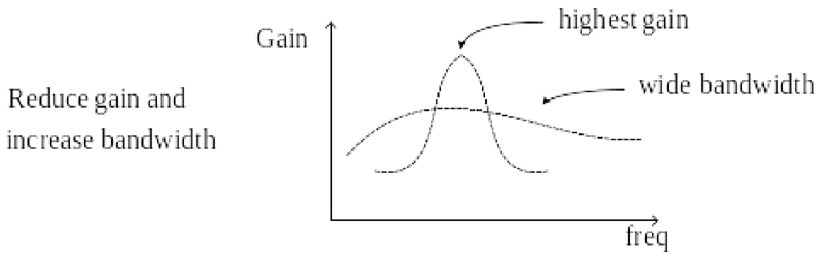


e.g. NEC 71000

$$[S] = \begin{bmatrix} 0.826\angle -116^o & 0.17\angle 23^o \\ 2.12\angle 86^o & 0.53\angle -64^o \end{bmatrix}$$

What will be the max gain of NEC 71000?

(3) Maximum bandwidth

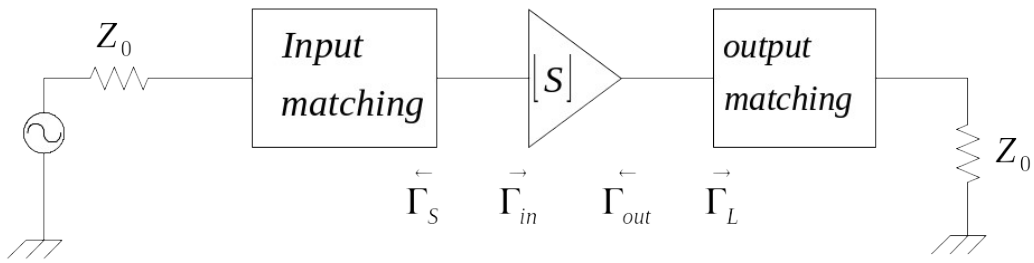


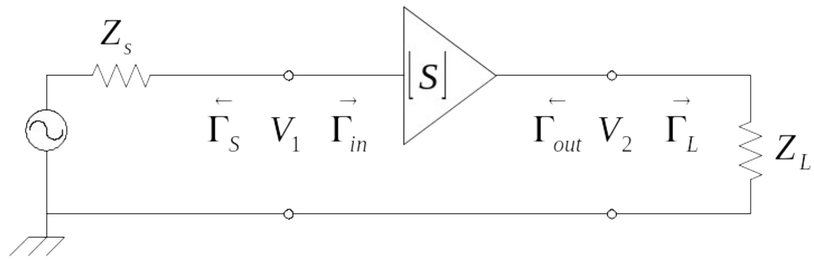
(4) Low noise

Usually the lowest noise operating point is not the same as the highest gain point.

2. Two-port Power Gain

First, we will define the power gain of a circuit which includes the matching circuits and the gain device such as a transistor.





We need to find Γ_s *and* Γ_L to design matching circuits.
First, we define the transducer power gain in terms of $\lfloor S \rfloor, \Gamma_s, \Gamma_L, \Gamma_{in}$ *and* Γ_{out}

$$G_T = \frac{P_\ell}{P_{ave}} = \frac{Power\ delivered\ to\ the\ load}{Power\ availabe\ from\ the\ source}$$

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}\ S_{21}\,\Gamma_L}{1 - S_{22}\,\Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}\ S_{21}\,\Gamma_S}{1 - S_{11}\,\Gamma_S}$$

If $S_{12} = 0$ (Unilateral case), then

$\Gamma_{in} = S_{11}$ Independent of Γ_L

$\Gamma_{out} = S_{22}$ Independent of Γ_S

Available power delivered to the network.

$$P_{in} = \frac{1}{2Z_0}\left|V_1^+\right|^2\left(1 - \left|\Gamma_{in}\right|^2\right)$$

$$V_1 = V_S\left(\frac{Z_{in}}{Z_S + Z_{in}}\right) = V_1^+ + V_1^- = V_1^+\left(1 + \Gamma_{in}\right)$$

$$V_1^+ = \frac{V_S}{2}\left(\frac{1 - \Gamma_S}{1 - \Gamma_S\Gamma_{in}}\right)$$

Then

$$P_{avs} = P_{in}\,|_{\Gamma_{in}=\Gamma_S^*} = \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{|1-\Gamma_S\Gamma_{in}|^2}\left(1-|\Gamma_{in}|^2\right)$$

$$= \frac{|V_S|^2}{8Z_0}\frac{|1-\Gamma_S|^2}{\left(1-|\Gamma_S|^2\right)}$$

$\Gamma_{in} = \Gamma_S^*$ is the conjugate matching condition.

$$Z_{in} = Z_S^*$$

Now we can obtain the power delivered into the load in terms of matching circuits.

$$P_\ell = \frac{|V_2^-|^2}{2Z_0}\left(1-|\Gamma_L|^2\right) = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2\left(1-|\Gamma_L|^2\right)|1-\Gamma_S|^2}{|1-S_{22}\Gamma_L|^2|1-\Gamma_S\Gamma_{in}|^2}$$

Then

$$G_T = \frac{P_\ell}{P_{avs}} = \underbrace{\left\{\frac{1-|\Gamma_S|^2}{|1-\Gamma_S\Gamma_{in}|^2}\right\}}_{G_S}\underbrace{|S_{21}|^2}_{G_0}\underbrace{\left\{\frac{\left(1-|\Gamma_L|^2\right)}{|1-S_{22}\Gamma_L|^2}\right\}}_{G_L}$$

Here we use $V_2^- = S_{21}V_1^+ + S_{22}V_2^+ = S_{21}V_1^+ + S_{22}\Gamma_L V_2^-$

Don’t use the following expression. **** Need to check.****

$$G_T = \frac{P_\ell}{P_{avs}} = \underbrace{\left\{\frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_{in}|^2}\right\}}_{G_S}\underbrace{|S_{21}|^2}_{G_0}\underbrace{\left\{\frac{\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_{out}\Gamma_L|^2}\right\}}_{G_L}$$

The special case is when $S_{12} = 0$ (no reverse gain). This case is called a **unilateral amplifier.**

$$G_{TU} = \left\{\frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}\right\}|S_{21}|^2\left\{\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}\right\}$$

Example: Assume we have

[illegible]

$Z_S = 20\Omega$ $\quad\quad$ $\Gamma_S = -0.429$

$Z_L = 30\Omega$ $\quad\quad$ $\Gamma_L = -0.25$

Then ** Corrected**

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.455\angle 150^o$$

$$G_{TU} = \left\{ \underbrace{\frac{1-(0.429)^2}{\left|1-(-0.429)(0.455\angle 150^o)\right|^2}}_{1.13} \right\} (2.05)^2 \left\{ \underbrace{\frac{1-(0.25)^2}{\left|1-(0.4\angle -150^o)(-0.25)\right|^2}}_{1.16} \right\}$$

$$= 5.49$$

This is larger than $|S_{21}|^2 = 4.2$.
The difference is due to input and output matching circuits.

3. Stability of an amplifier

Now consider the stability condition. If we send $|V_{inc}|=1$ into a passive network, the reflected voltage is always less than 1 or the magnitude of the reflection coefficient $|\Gamma|$ is always less than 1. If $|\Gamma|$ becomes greater than 1, then we know the network is generating power (oscillating). Therefore, we can say the **amplifier becomes unstable if** $|\Gamma_{in}| > 1 \quad or \quad |\Gamma_{out}| > 1$ **.**

Since

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

$\Gamma_L \; and \; \Gamma_S$ will determine the stability condition.

(1) Unconditionally stable amplifier
$|\Gamma_{in}| < 1 \; and \; |\Gamma_{out}| < 1$ for all load and source impedances.
This must be satisfied at all frequency.

(2) Conditionally stable amplifier
$|\Gamma_{in}| < 1 \; and \; |\Gamma_{out}| < 1$ only for all certain range of load and source impedances.
This must be satisfied at all frequency.

Stability conditions

3-1. Output stability circle

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}\, S_{21}\, \Gamma_L}{1 - S_{22}\, \Gamma_L} \right| < 1$$ If this is less than 1, the circuit is stable.

This condition can also be expressed using a circle with center C_L and radius R_L. This can be described as $|\Gamma_L - C_L| = R_L$ in a complex plane.

Set $|\Gamma_{in}| = 1$ and obtain a circle with

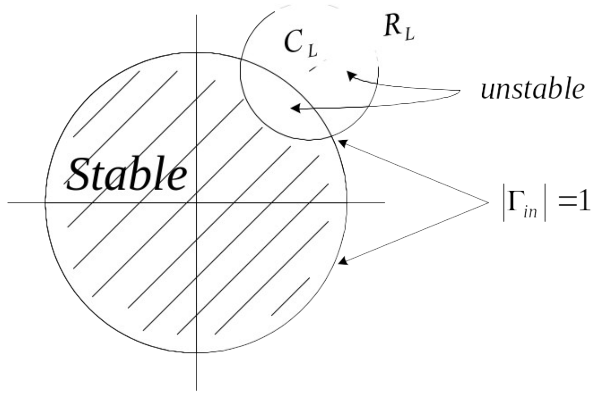
$$\begin{cases} Center \quad C_L = \dfrac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}| - |\Delta|^2} \quad , \quad \left| \Gamma_L - \dfrac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \right| = \left| \dfrac{S_{12} S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \\ \qquad\qquad\qquad\qquad\qquad\qquad \Delta = S_{11} S_{22} - S_{12} S_{21} \\ Radius \quad R_L = \left| \dfrac{S_{12} S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \end{cases}$$

Stability circle

Depending on the value of S11, we get two conditions for the stable region.

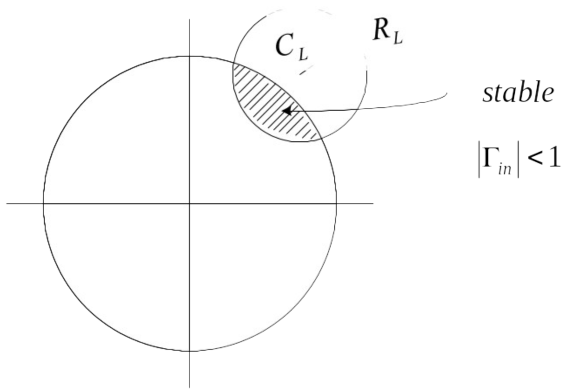
Most practical active devices (transistors)

$For\ |S_{11}| < 1$



Some special active devices (transistors)

$For\ |S_{11}| > 1$
(Not important)



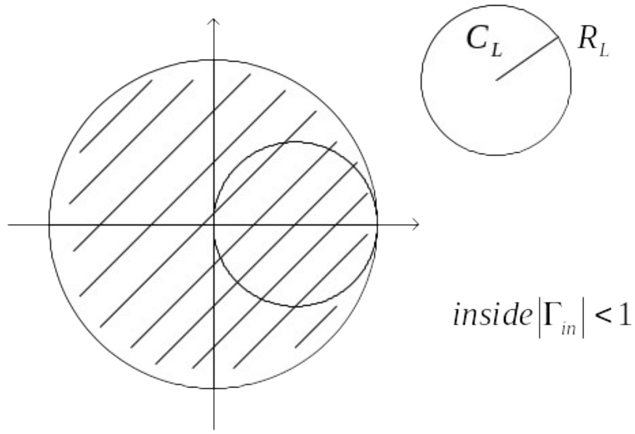
The amplifier must operate within the stable region. That means we need to choose the operating point Γ_L to be within the stable region. This will determine the output matching circuit.

We have two special unconditionally stable cases.

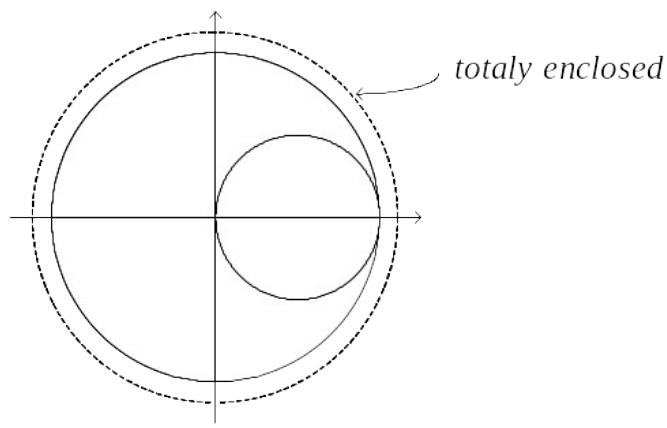
If we satisfy one of these conditions, the amplifier is unconditionally stable and we don't need to design a matching circuit to make it stable. We can still design a matching circuit to obtain higher gain, lower noise, or wider bandwidth**.**

$$||C_L| - R_L| > 1 \qquad if \quad |S_{11}| < 1$$

Case1



Case 2



3-2. Input stability circle

Similar to the output case, we can obtain the stability circuit for the input matching circuit by taking a look at Γ_{out}.

$$\left|\Gamma_{out}\right| = \left| S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S} \right| < 1$$

From this we get

$$\begin{cases} C_S = \dfrac{\left(S_{11} - \Delta S_{22}^*\right)^*}{\left|S_{11}\right|^2 - \left|\Delta\right|^2} & ,center \\ R_S = \dfrac{S_{12} S_{21}}{\left|S_{11}\right|^2 - \left|\Delta\right|^2} & ,radius \end{cases}$$

From this, we can find the operating point Γ_S .

$K > 1 \text{ and} \left|\Delta\right| < 1$

For a given active device (transistor), the simplest way to check if it is stable is $K \text{ and} \left|\Delta\right|$ which can be obtain from S-parameters. If the device has $K > 1 \text{ and} \left|\Delta\right| < 1$ at all frequency, then the device is unconditionally stable.

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

If $K > 1 \text{ and } |\Delta| < 1$, then the device/amplifier is unconditionally stable.

Example

MAR-3 at 3.5GHz

$$\begin{cases} S_{11} = 0.32\angle 140^o \\ S_{21} = 2.5\angle 21^o \\ S_{12} = 0.21\angle -6^o \\ S_{22} = 0.25\angle 152^o \end{cases}$$

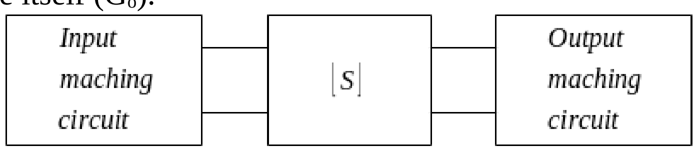
$$\Delta = -0.45 - j0.2$$

$$\begin{cases} |\Delta| = 0.5 & < 1 \\ K = 1.085 & > 1 \end{cases}$$

This shows MAR-3 is unconditionally stable at 3.5 GHz.

4. Amplifier Design for Maximum Gain

Let assume that the device is stable and we want to obtain the highest gain from the device by designing input and output matching circuits. The total gain is given by the product of 3 gain sections as shown here. First, we need to estimate if we can improve the total gain much higher than that of the device itself (G_o).



$G_T = G_S G_0 G_L$ | G_S Depends on the matching circuit | G_0 Determined by the device | G_L Depends on the matching circuit

The total gain is the product of 3 sections (G_S, G_o, G_L).

Control G_S & G_L to get a highest gain.
Control G_S & G_L to make a circuit stable.
Most transistors are not well matched to Z_0 (50Ω).

Example

HP HFET 102

$$|S_{11}| = 0.894 \quad , \quad |S_{21}| = 3.1 \quad , \quad |S_{22}| = 0.781 \quad at\ 2\,GHz$$

G_S *and* G_L can be much larger than 1
because $|S_{11}|$ *and* $|S_{22}|$ are large

This shows that G_T can be much larger than G_0.

MAR-3 amplifier

$$|S_{11}| = 0.06 \quad , \quad |S_{21}| = 3.3 \quad , \quad |S_{22}| = 0.25 \quad at\ 2GHz$$

$$G_S\ and\ G_L \text{ will be close to1}$$

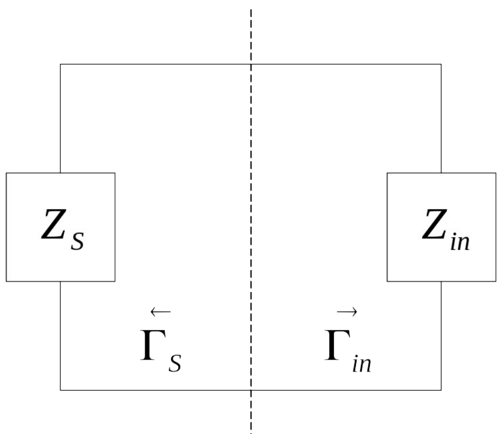
$$G_T \sim G_0$$

This shows G_T will not be much greater than G_o.

Conjugate matching condition

Highest gain can be obtained if the conjugate matching condition is satisfied.

$$Z_S = R_S + jX_S$$
$$Z_{in} = R_{in} + jX_{in}$$



If $Z_S = Z^*_{in}$, we can transfer maximum amount of energy from Z_S to Z_{in}.

$$R_S = R_{in}$$
$$jX_S = -jX_{in}$$

This is also

$$\Gamma_S = \Gamma^*_{in}$$

Then the maximum gain is given by

$$G_{T\max} = \left(\frac{1}{1 - |\Gamma_S|^2} \right) |S_{21}|^2 \left\{ \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \right\}$$

We want to find $\Gamma_S \ and \ \Gamma_L$ which give a highest gain.

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} = \Gamma_S^*$$

$$\Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S} = \Gamma_L^*$$

Solve these two equations and find $\Gamma_S \ and \ \Gamma_L$.

$$\begin{cases} \Gamma_S = \dfrac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \\ \Gamma_L = \dfrac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \end{cases}$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$C_1 = S_{11} - \Delta S_{22}^*$$

$$C_2 = S_{22} - \Delta S_{11}^*$$

If we design the matching circuits using these two values, we get the highest gain out of a device.

Now we compare G_{Tmax} with that of the unilateral amp ($S_{12} = 0$) assumption.

If we set S_{12}=0, then we get

$$G_{TU\max} = \left(\frac{1}{1 - |S_{11}|^2}\right)|S_{21}|^2\left(\frac{1}{1 - |S_{22}|^2}\right)$$

Example: Assume the device has

$$\text{at 4GHz} \quad \begin{cases} S_{11} = 0.72\angle -116^o \\ S_{21} = 2.6\angle 76^o \\ S_{12} = 0.03\angle 57^o \\ S_{22} = 0.73\angle -53^o \end{cases}$$

If we assume $S_{12} = 0$

$$G_S = \frac{1}{1 - |S_{11}|^2} = 2.08$$

$$G_S = \frac{1}{1 - |S_{22}|^2} = 2.17$$

$$G_{TU\max} = (2.07)(2.6)^2(2.17) = 30.5$$

If $S_{12} \neq 0$

$$G_S = 4.17 \ , \quad G_L = 1.67$$

$$G_{T\max} = (4.17)(2.6)^2(1.67) = 47$$

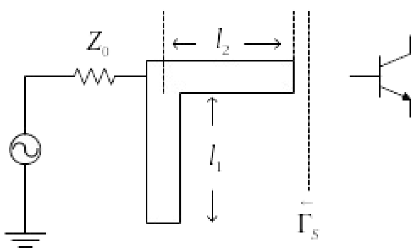
The highest gain available from this device, G_{Tmax} , is much higher than G_{TUmax}.

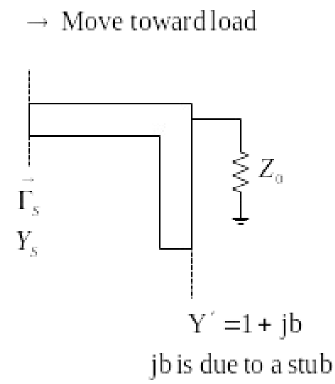
5. Actual matching circuit using microstrip TL

Once we find desired Γ_S and Γ_L, we need to design a practical circuit using the microstrip transmission line. This process is similar to the design of the stub matching circuit but there is a major difference shown below.

Assume we found $\Gamma_S = 0.872\angle 123^o$ to get a highest gain. How to find this value will be discussed later.

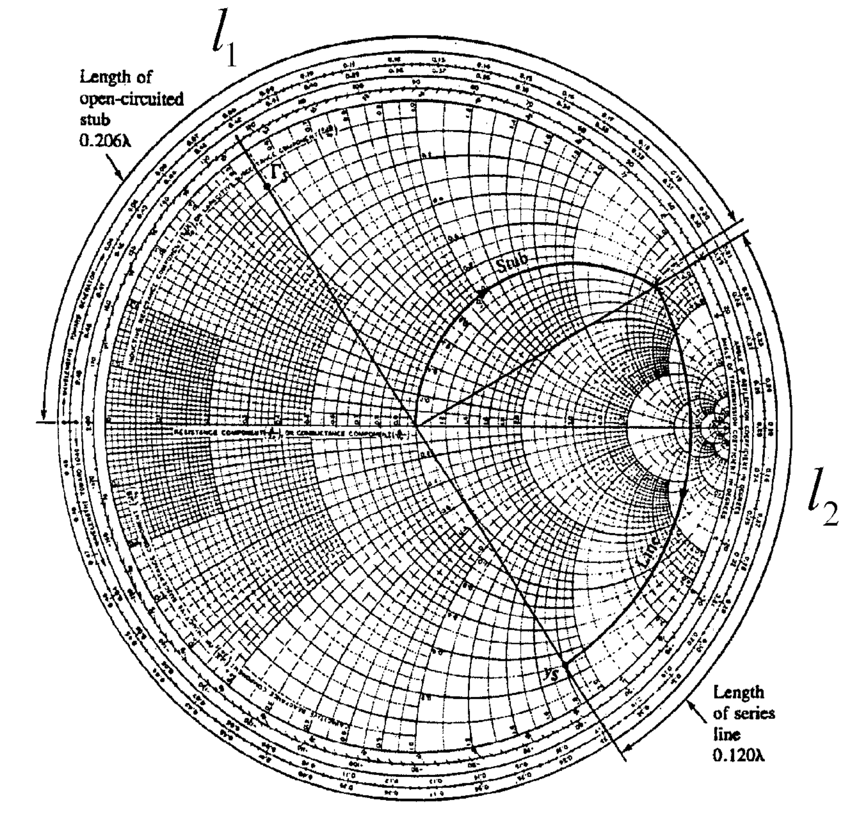
Input matching circuit is given by





Using the Smith Chart, we get l_1=0.206λ and l_2=0.12λ.

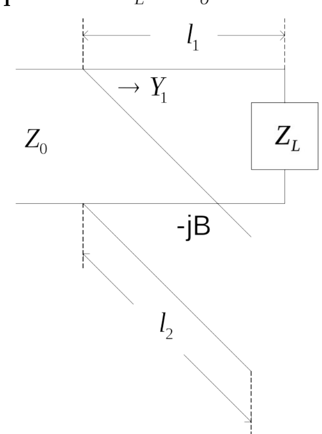
The important point is we need to start from the center of the chart (Y_o=1). First, add the reactive element to get Y'=1+jb. Then rotate a certain angle which corresponds to a length l_2, to change the admittance to a desired value Y_s.



(a)

FIGURE 11.28 Circuit design and frequency response for the transistor amplifier of Example 11.6. (a) Smith chart for the design of the input matching network.

This matching method is different from the Stub matching circuit which uses the following technique. $Z_L \neq Z_o$

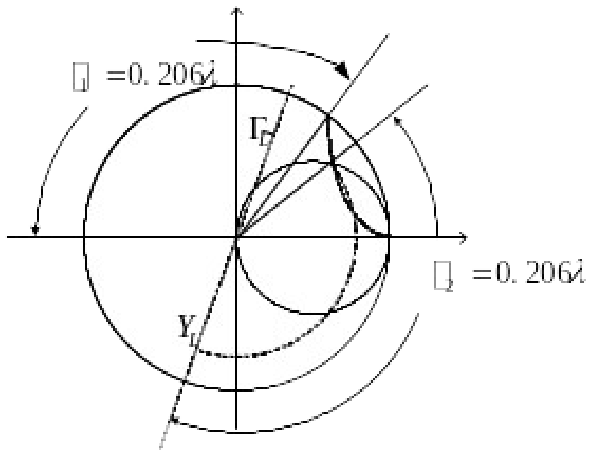


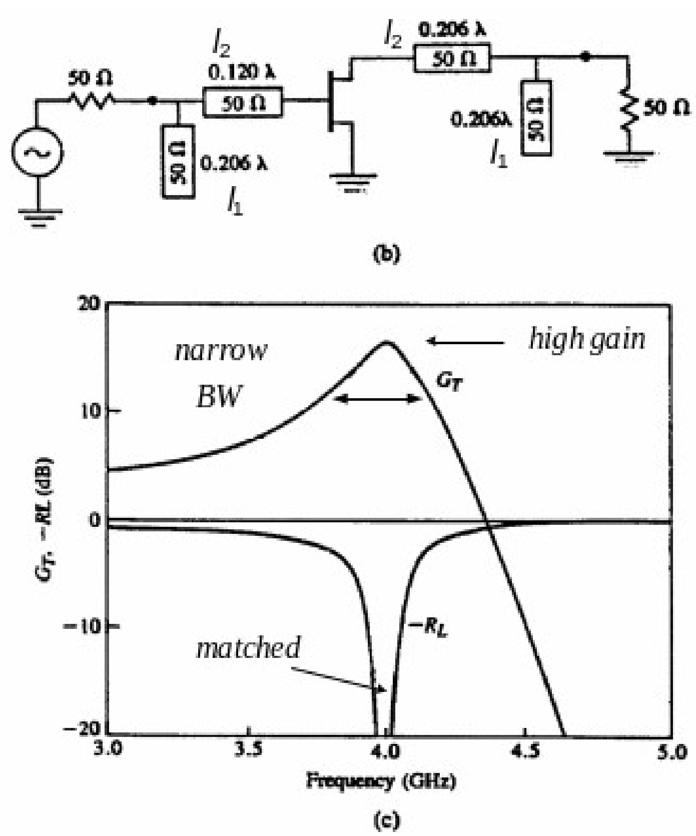
$Y_1 = 1 + jB$

Stub gives $-jB$

Then $Y_{total} = Y_1 - jB = 1$

Similarly the output matching circuit can be designed for Γ_L=*.**





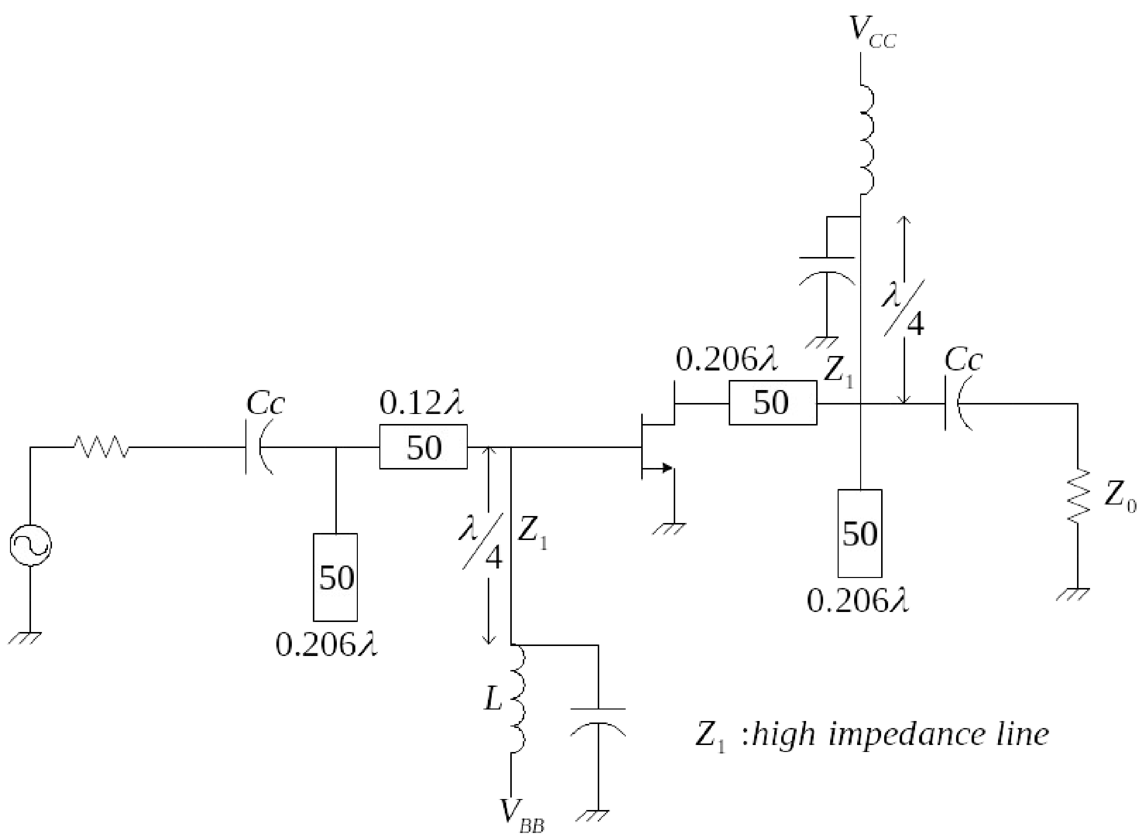
(b)

(c)

FIGURE 11.28 Continued. (b) RF circuit. (c) Frequency response.

6. Complete circuit including a DC bias circuit

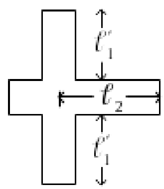
The matching circuits are not sufficient to design a practical amplifier. We need to add decoupling capacitors and the DC bias circuits as shown below.
The RF signal goes through the decoupling capacitors and the frequency response and gain of the amplifier will be affected by them. The value of the DC decoupling capacitors should be chosen to minimize the effects. As we discussed before an equivalent circuit of a capacitor at high frequency is a series L and C. This is a series resonance circuit and beyond the resonance frequency, the capacitor becomes an inductor which will reduce gain. The resonance frequency must be much higher than the desired frequency range. The value of L depends on the material and structure of a capacitor and we need to use the high quality capacitor at microwave frequency. The typical value of Cc is between 1 and 100pF and it should not have a lead (wire) which is an inductor.
Another required circuit is the DC bias. The RF signal should not go through the DC bias. This means that the input impedance of the DC bias circuit must be very high. The simplest way to get a high input impedance is a $\lambda/4$ TL terminated with a short. In principle, the effective impedance of this circuit will be infinite at the designed frequency. In practice, however, it is much less and we usually use the high impedance TL (Z_o>100 ohm) to ensure the high input impedance. Because the TL cannot be DC shorted to supply the DC voltage, we use a capacitor (or capacitors) to obtain the RF short circuit. The impedance of this capacitor should be small at the desired frequency.



The matching circuits can also be designed with different techniques as shown below.

Symmetric design

Input side only



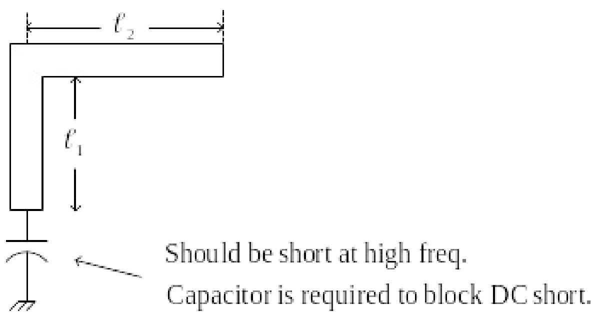
$\ell_1' = 0.165\lambda$

$\ell_2 = 0.12$

0.206λ *corresponds to* $j3.5$

each 0.165λ *stub has* $j1.75$

Using a short stub



6. Examples of Amplifier Matching Techniques

Stub matching and λ/4 matching techniques

Example 2.5.2

Design two microstrip matching networks for the amplifier shown in Fig. 2.5.12 whose reflection coefficients for a good match, in a 50-Ω system, are $\Gamma_s = 0.614\angle 160^\circ$ and $\Gamma_L = 0.682\angle 97^\circ$.

Solution. *Design 1:* The amplifier block diagram is shown in Fig. 2.5.12. The normalized impedances and admittances associated with Γ_s and Γ_L can be read, to reasonable accuracy, from the *ZY* chart—namely,

$$y_s = \frac{1}{z_s} = \frac{1}{0.245 + j0.165} = 2.8 - j1.9$$

and

$$y_L = \frac{1}{z_L} = \frac{1}{0.325 + j0.83} = 0.4 - j1.05$$

In order to design the input matching network, we locate y_s in the *Y* Smith chart shown in Fig. 2.5.13a. The shortest length of microstrip line plus stub is obtained by using an open-circuited shunt stub of length 0.159λ to move from the origin (i.e., 50 Ω) to point *A* on the Smith chart, and then using a transmission line length of 0.099λ to move from *A* to y_s.

Next, we locate y_L in Fig. 2.5.13b and follow a similar procedure. In this case, the shortest length of microstrip line plus stub is obtained by using a short-circuited shunt

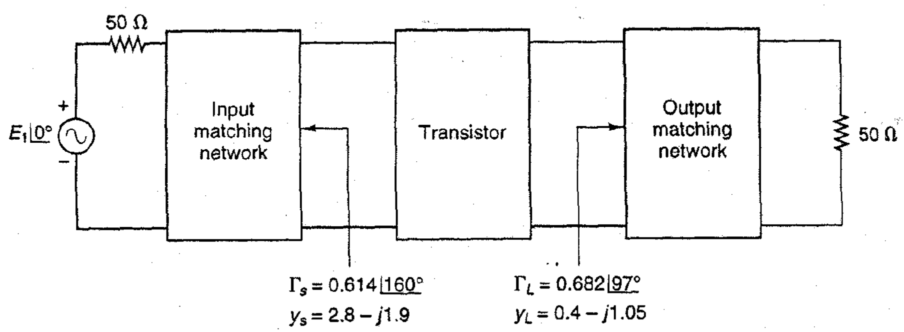


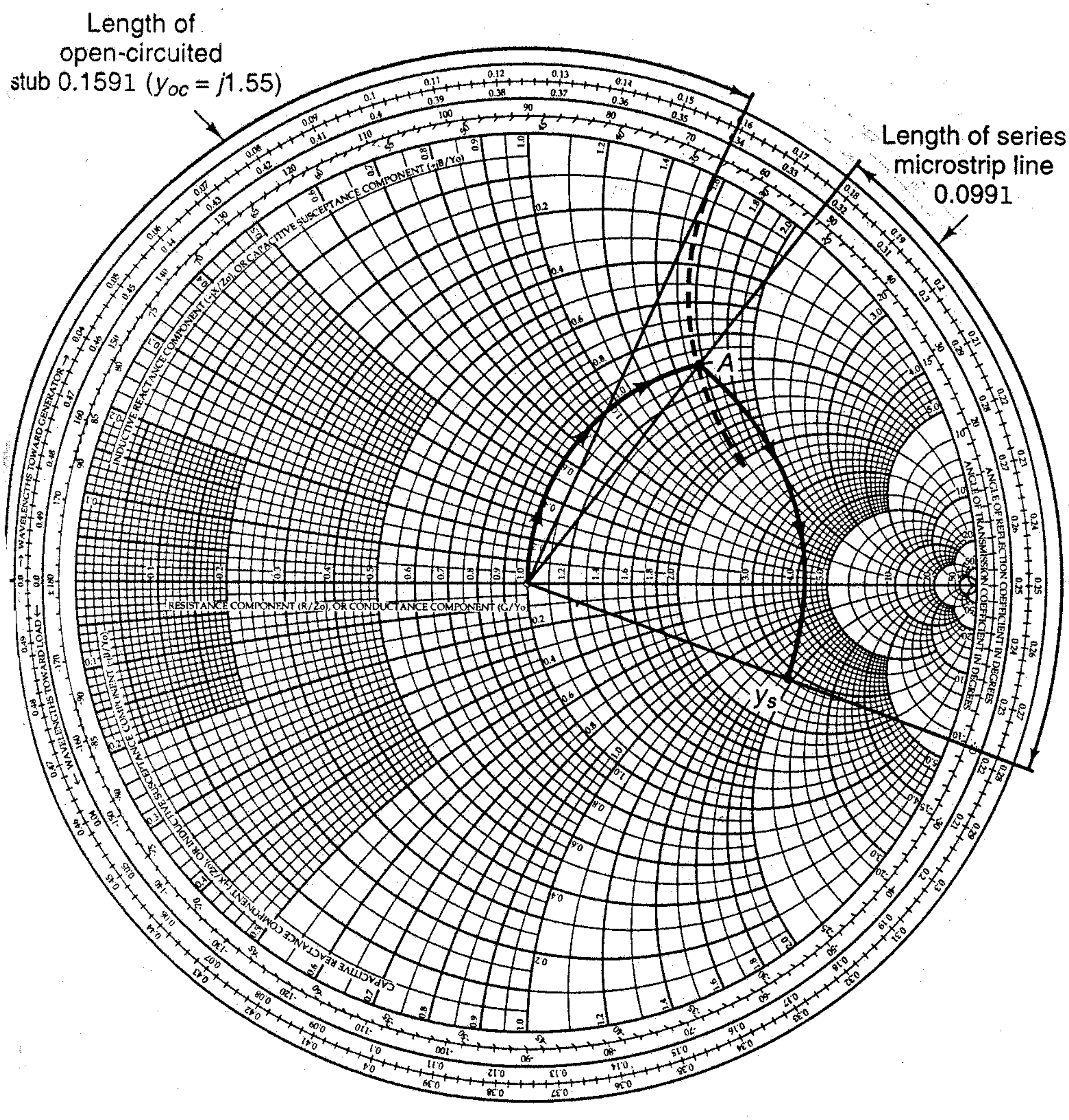
Figure 2.5.12 Amplifier block diagram.

stub of length 0.077λ to move from the origin to point *B*. Then a series transmission line of length 0.051λ is used to move from *B* to y_L.

The complete design, showing the transistor, the microstrip matching network, and the dc supply, is shown in Fig. 2.5.14. The characteristic impedance of all microstrip lines is 50 Ω.

The capacitors C_A are coupling capacitors. Typical values for the chip capacitors C_A are 200 to 1000 pF, high-*Q* capacitor. The bypass capacitors C_B (i.e., chip capacitors, 50 to 500 pF) provide the ac short circuits for the 0.077λ and $\lambda/4$ short-circuited stubs. The $\lambda/4$ short-circuited stub, high-impedance line (denoted by $Z_o \gg$), provides the dc path for the base supply voltage. It also presents an open circuit to the ac signal at the base of the transistor. The narrowest practical line (i.e., large Z_o) should be used for the $\lambda/4$ short-circuited stub to avoid unwanted ac coupling. Typical dc bias circuits are shown in Figs. 3.9.2 and 3.9.4.

To minimize transition interaction between the shunt stubs and the series transmission lines, the shunt stubs are usually balanced along the series transmission line. A schematic of the amplifier using balanced shunt stubs is shown in Fig. 2.5.15. The

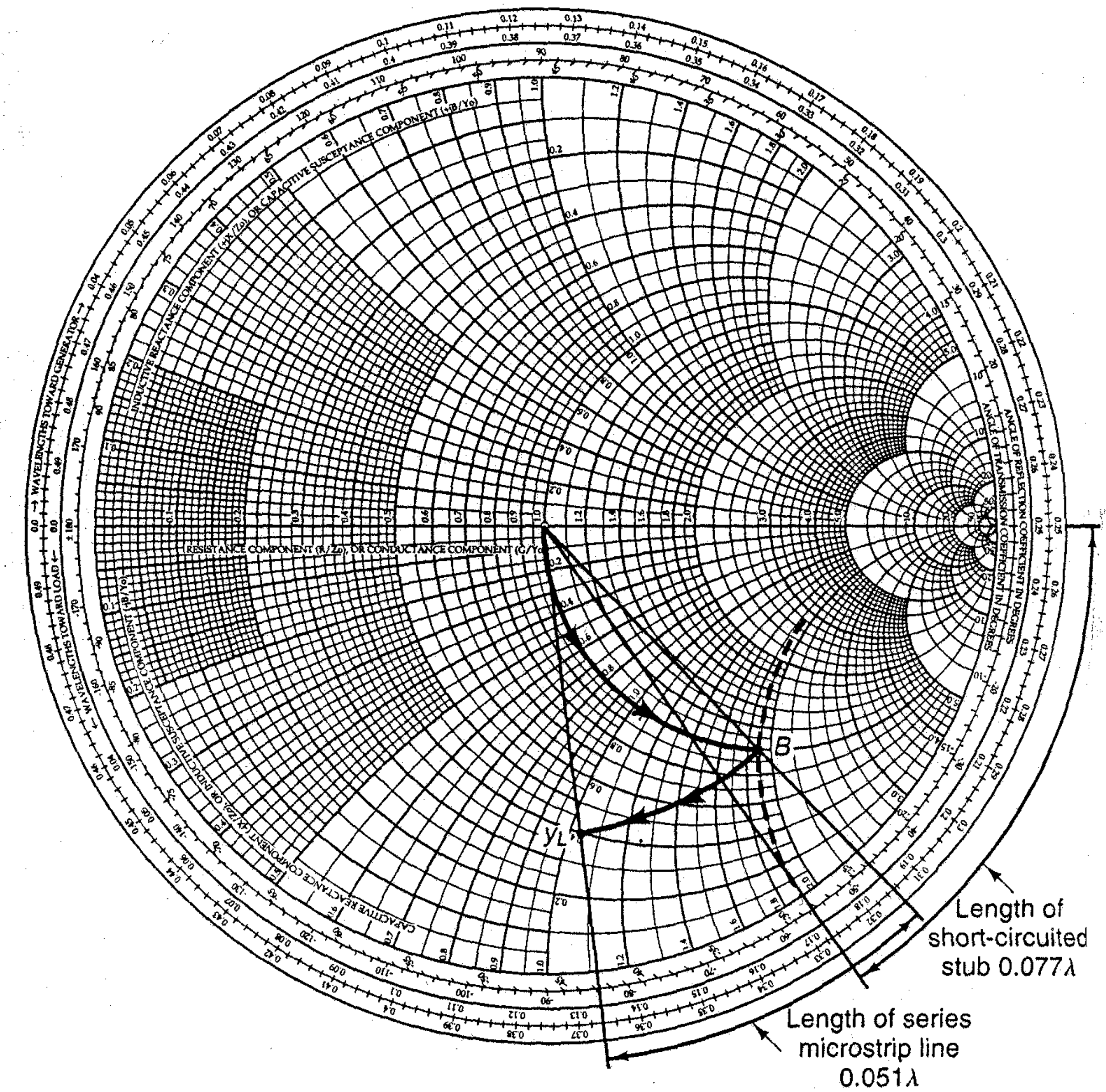


(a)

Figure 2.5.13 (a) Input matching network design; (b) output matching network design.

schematic also shows that 50-Ω lines were added on both sides of C_A to provide a soldering area.

In Fig. 2.5.15, two parallel shunt stubs must provide the same admittance as the single stub in Fig. 2.5.14. Therefore, the admittance of each side of the balanced stub must be equal to half of the total admittance. For example, each side of the input balanced shunt stubs must have an admittance of $y = j1.55/2 = j0.775$. Using the Smith



(b)

Figure 2.5.13 Continued

chart, we obtain that the length of each side must be 0.105λ. Observe that the length of the shunt stubs in Fig. 2.5.14 is not equal to the total length of the balance stubs in Fig. 2.5.15. Of course, a simple check will show that the admittance seen by the series transmission line is the same in both cases.

If we use RT/Duroid® with $\varepsilon_r = 2.23$ and $h = 0.7874$ mm to build the amplifier, we find from (2.5.8) to (2.5.11) (or from Figs. 2.5.2 and 2.5.3) that a characteristic impedance of 50 Ω is obtained with $W = 2.42$ mm and $\varepsilon_{ff} = 1.91$. The microstrip wave-

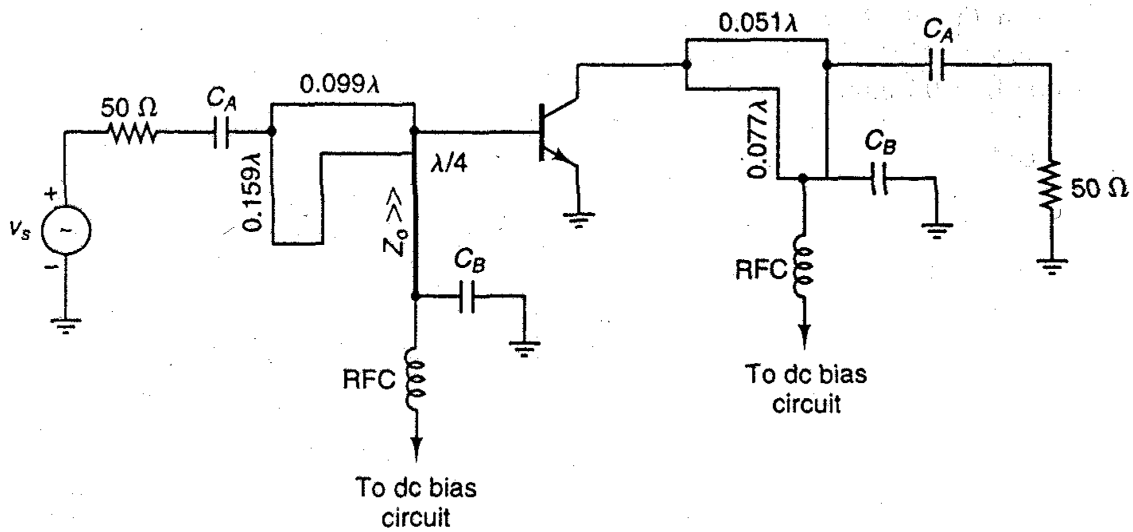


Figure 2.5.14 Complete amplifier schematic. The characteristic impedance of the microstrip lines is 50 Ω.

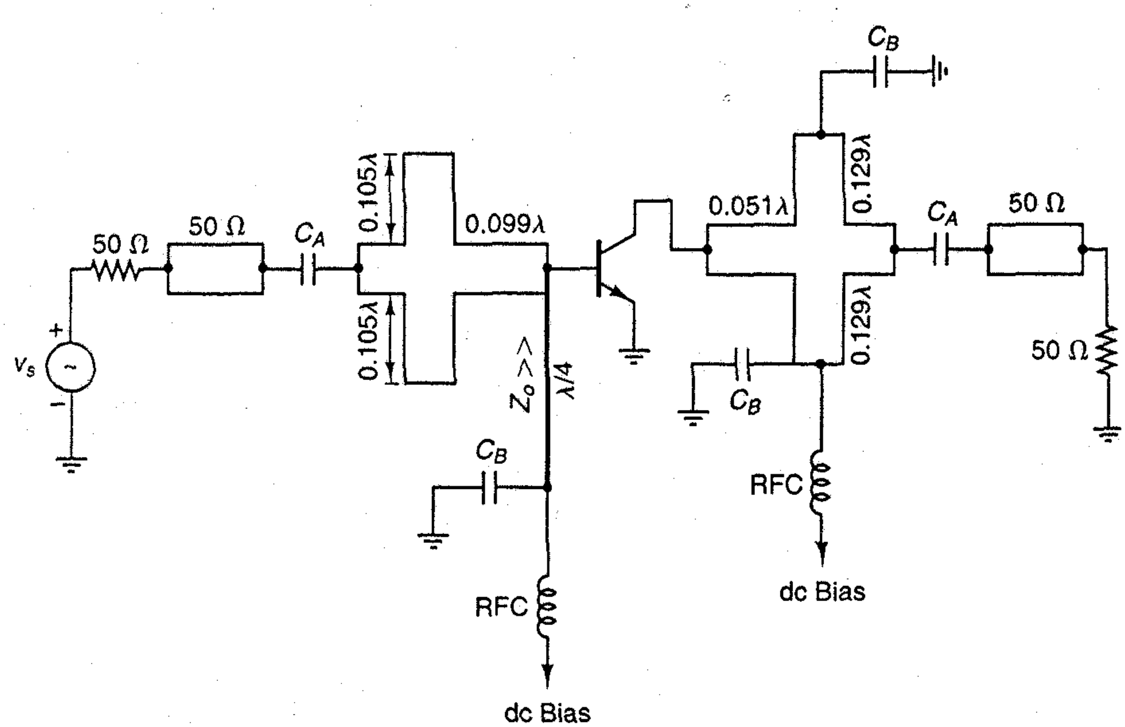


Figure 2.5.15 Complete amplifier schematic using balanced shunt stubs. The characteristic impedance of the microstrip lines is 50 Ω.

length in the 50-Ω Duroid microstrip line is $\lambda = \lambda_0/\sqrt{1.91} = 0.7236\lambda_0$, where $\lambda_0 =$ 30 cm at $f = 1$ GHz. For a characteristic impedance of 100 Ω in the $\lambda/4$ line, the width must be $W = 0.7$ mm. The line lengths in Fig. 2.5.15 are

$$0.105\lambda = 2.28 \text{ cm}$$

$$0.099\lambda = 2.15 \text{ cm}$$

$$0.051\lambda = 1.10 \text{ cm}$$

$$0.129\lambda = 2.80 \text{ cm}$$

$$\lambda/4 = 5.43 \text{ cm}$$

Design 2: This method uses microstrip lines with different characteristic impedances, as shown in Fig. 2.5.10a. The design requires the transformation of 50 Ω to $Y_s = (2.8 - j1.9)/50 = 0.056 - j0.038$ S. A quarter-wave transformer can be used to transform the source impedance of 50 Ω to the resistance $1/0.056 = 17.86$ Ω. The characteristic impedance of the quarter-wave transformer is

$$Z_{o1} = \sqrt{50(17.86)} = 29.9\ \Omega$$

An open-circuited shunt stub can be used to obtain the admittance $-j0.038$ S. Therefore, as shown in Fig. 2.5.10b, an open-circuited shunt stub of length $3\lambda/8$ looks like a shunt inductor having the admittance $-jY_{o2}$. Equating $-jY_{o2}$ to $-j0.038$ S, we find the characteristic impedance Z_{o2} to be

$$Z_{o2} = \frac{1}{Y_{o2}} = \frac{1}{0.038} = 26.32\ \Omega$$

If the design is done using a short-circuited shunt stub (see Fig. 2.5.10a), its length would be $\lambda/8$ and $Z_{o2} = 26.3$ Ω.

Similarly, for the output matching network $[Y_L = (0.4 - j1.05)/50 = 0.008 - j0.021$ S$]$, a quarter-wave line of characteristic impedance

$$Z_{o1} = \sqrt{50(125)} = 79.1\ \Omega$$

transforms the 50-Ω load to a resistance of value $1/0.008 = 125$ Ω. An open-circuited shunt stub of length $3\lambda/8$ and characteristic impedance $Z_{o2} = 1/Y_{o2} = 1/0.021 = 47.6$ Ω produces the required admittance of $-j0.021$ S.

The complete amplifier is shown in Fig. 2.5.16a. Figure 2.5.16b shows the amplifier using balanced shunt stubs of length $3\lambda/8$ to minimize the microstrip transition interaction. Observe that in the balance stubs the lengths were kept at $3\lambda/8$, but the characteristic impedance was doubled. For example, in Fig. 2.5.16b each half of the input balance stub must provide the admittance $-j0.038/2$ (since each half must contribute half of the total admittance). Therefore, the value of Z_{o2} for the balanced stubs at the input is $Z_{o2} = 2/0.038 = 52.6$ Ω.

Example 2.5.3

Design a microstrip matching network to transform the load $Z_L = 75 - j60$ Ω to an input impedance of value $Z_{IN} = 15 + j30$ Ω.

Solution. In this design, let us select a Z_o different from 50 Ω—for example, $Z_o = 75$ Ω. With $Z_o = 75$ Ω, the design consists of transforming a normalized load $z_L = Z_L/Z_o = 1 - j0.8$ (or $y_L = 0.61 + j0.49$) to the normalized input impedance $z_{IN} =$

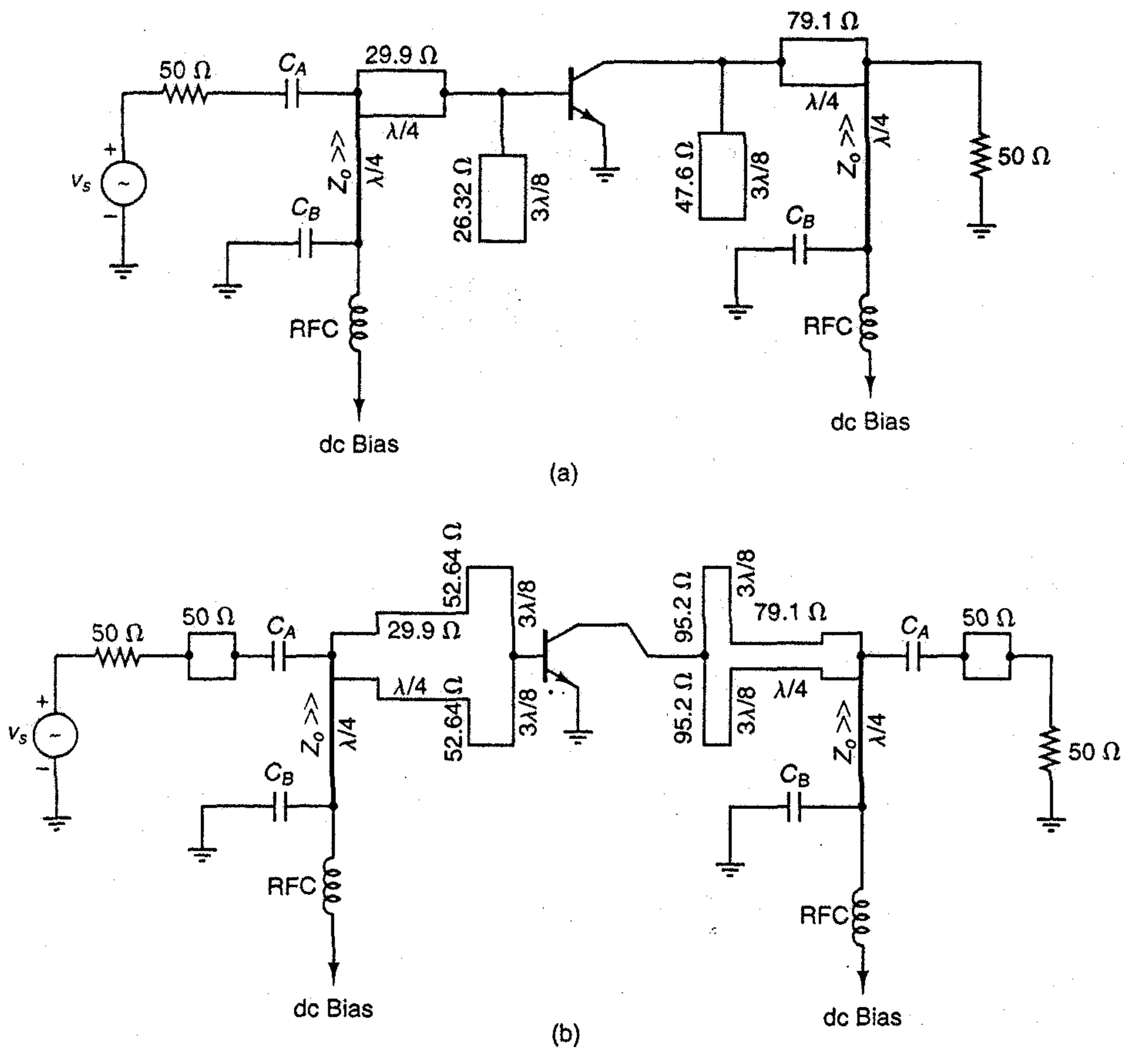
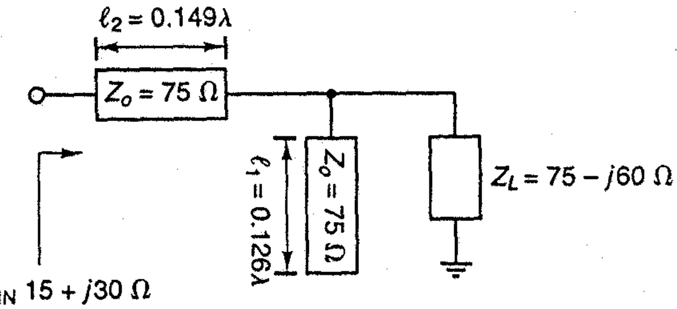


Figure 2.5.16 Matching network design using microstrip lines with different characteristic impedances.

$Z_{IN}/Z_o = 0.2 + j0.4\ \Omega$ (or $y_{IN} = 1 - j2$). The matching topology selected is shown in Fig. 2.5.17a, and the design in the Y Smith chart is shown in Fig. 2.5.17b. From Fig. 2.5.17b, the shunt admittance required to move from y_L to point A is $j1.5 - j0.49 = j1.01$. The admittance at point A is $y_A = 0.61 + j1.5$. An open-circuited shunt stub of length $l_1 = 0.126\lambda$ provides the admittance of $j1.01$. Then a series transmission line of length $l_2 = 0.313\lambda - 0.164\lambda = 0.149\lambda$ moves the admittance value, along a constant $|\Gamma|$ circle, from that at point A to y_{IN}.

A microstrip matching network which can be easily designed using a Z Smith chart is shown in Fig. 2.5.18a. This matching network uses a $\lambda/4$ line with characteristic impedance Z_{o1} to transform the 50-Ω load ($z_L = 1$) to a resistance R_x ($r_x = R_x/50$) that lies on the constant $|\Gamma|$ circle that passes through $z_{IN} = Z_{IN}/50$. The value of Z_{o1} is given by

$$Z_{o1} = \sqrt{50 R_x}$$



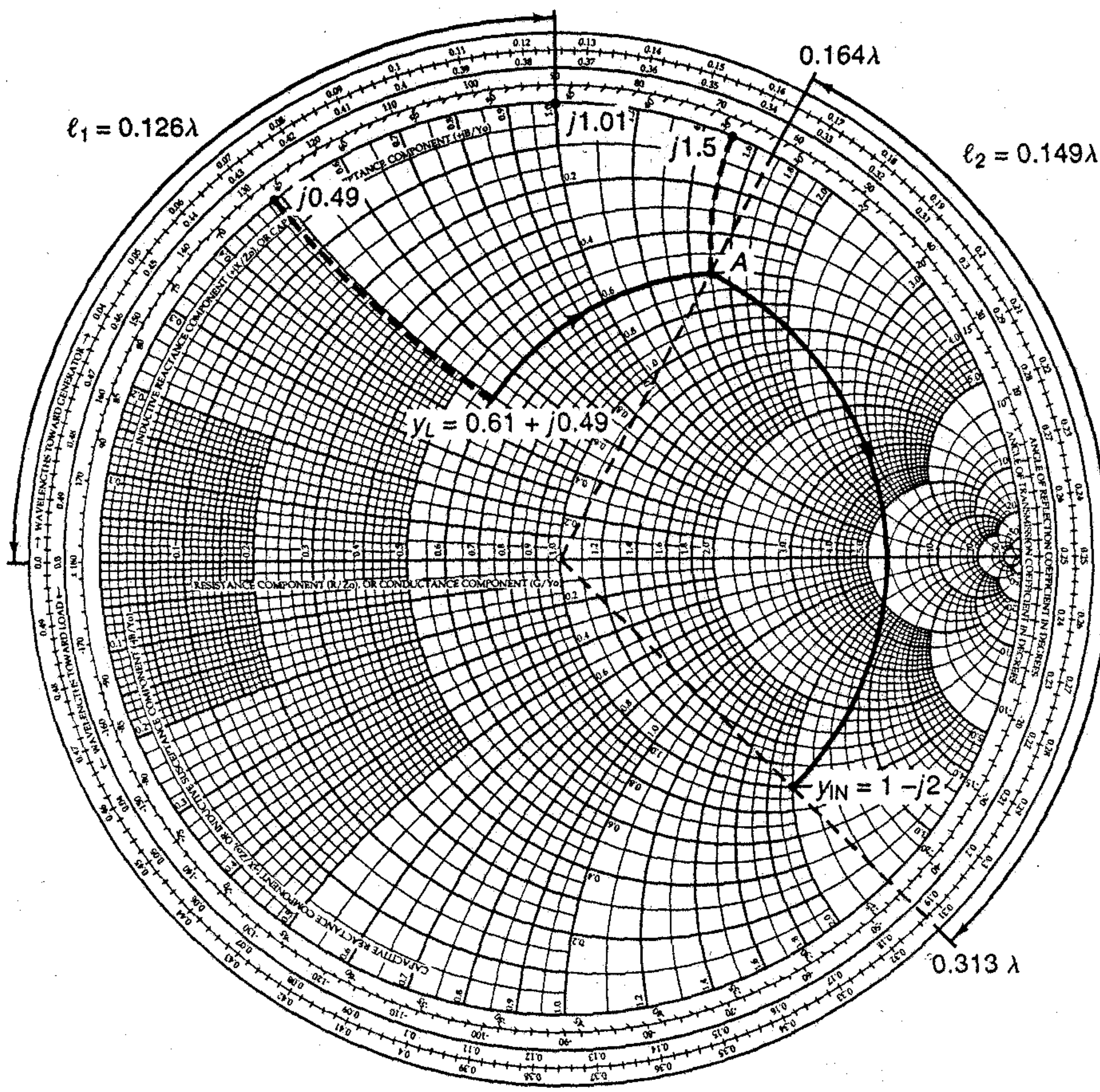


Figure 2.5.17 (a) Matching network for Example 2.5.3; (b) design in the Y Smith chart using $Z_o = 75\ \Omega$.

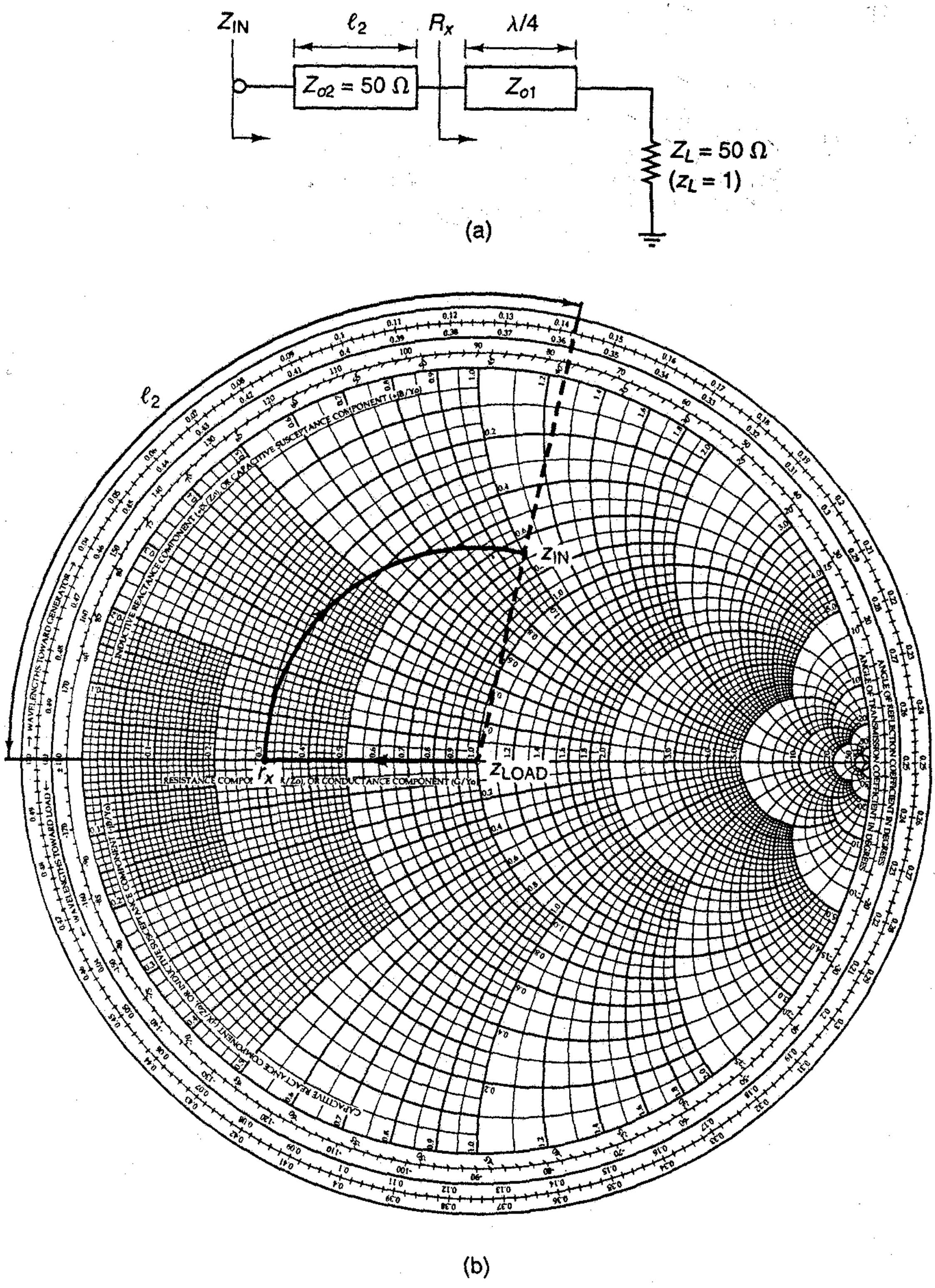
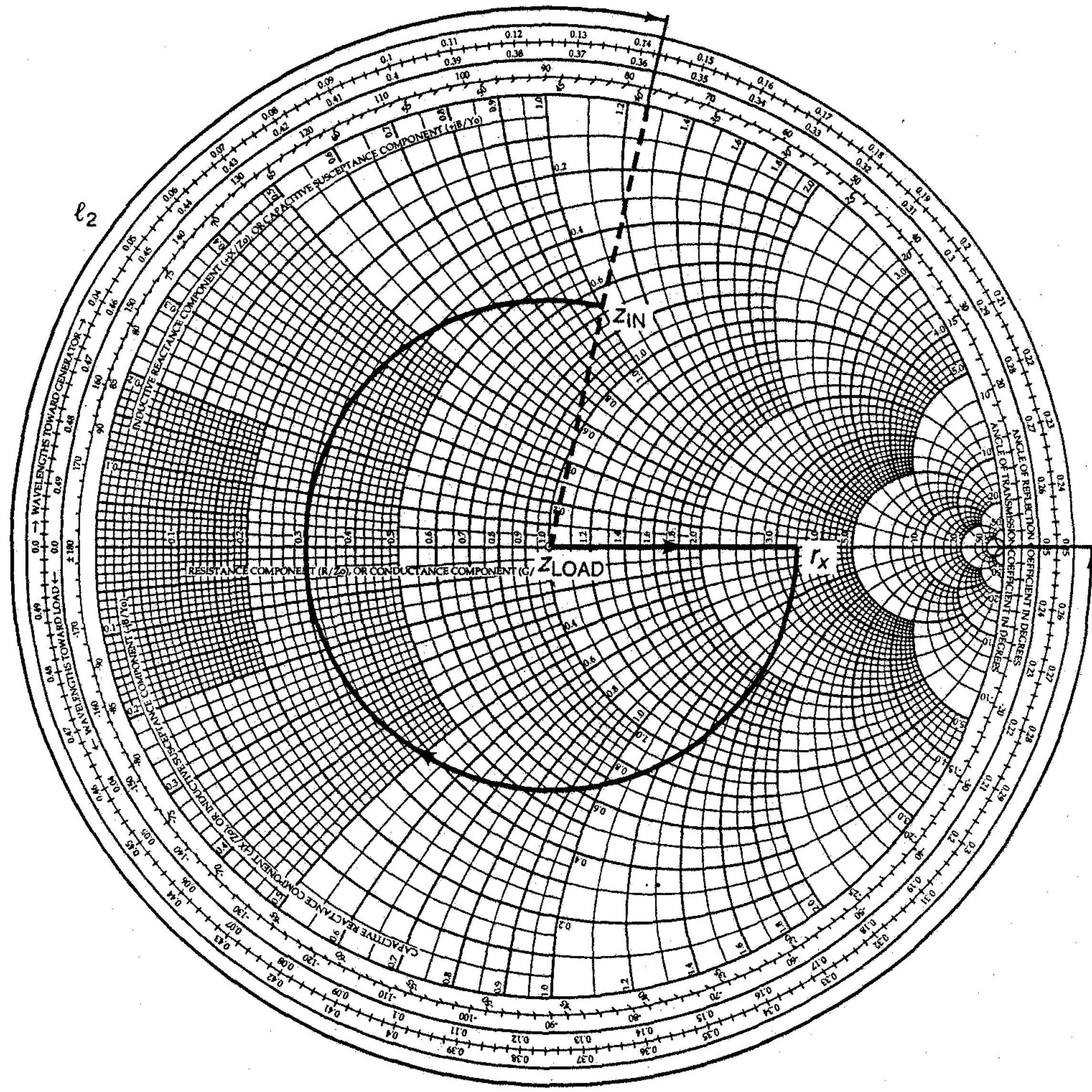


Figure 2.5.18 (a) A microstrip matching circuit; (b) design in the Z Smith chart that results in Z_{o1} smaller than 50 Ω; (c) design in the Z Smith chart that results in Z_{o1} greater than 50 Ω.



(c)

Figure 2.5.18 Continued

Then, the 50-Ω line of length l_2 changes the normalized resistance r_x to the input impedance z_{IN}. The design procedure in the Z Smith chart for an arbitrary value of Z_{IN} is shown in Fig. 2.5.18b. An alternate solution is shown in Fig. 2.5.18c. The solution described in Fig. 2.5.18b produces a Z_{o1} smaller than 50 Ω, while the solution in Fig. 2.5.18c produces a Z_{o1} greater than 50 Ω.

Example 2.5.4

Design the microstrip matching network in Fig. 2.5.18a to transform a 50-Ω load to the input impedance $Z_{IN} = 33 + j50\ \Omega$.

Solution. The location of $z_{IN} = Z_{IN}/50 = 0.66 + j1$ corresponds to the z_{IN} shown in Fig. 2.5.18b. From the constant $|\Gamma|$ circle through z_{IN}, we observe that $r_x = 0.3$, or $R_x = 50(0.3) = 15\ \Omega$. The $\lambda/4$ line is designed to transform $Z_L = 50\ \Omega$ to $R_x = 15\ \Omega$. Hence,

$$Z_{o1} = \sqrt{50(15)} = 27.4\ \Omega$$

Then, the 50-Ω series transmission line of length $l_2 = 0.143\lambda$ produces an input impedance equal to $z_{IN} = 0.66 + j1$, or $Z_{IN} = 50(z_{IN}) = 33 + j50\ \Omega$.

The design of matching networks containing lumped components and microstrip transmission lines can also be done using the various Smith charts. The following example illustrates one such design.

Example 2.5.5

(a) An oscillator is designed at 2.5 GHz using the output matching topology shown in Fig. 2.5.19a. The length of the microstrips is shown for $\varepsilon_{ff} = 1$ (i.e., for $v = c = 3 \times 10^{10}$ cm/s). The matching network uses a varactor diode as a voltage-variable capacitor for the control of the oscillator frequency. Determine the value of the load reflection coefficient.

(b) Specify the width, height, and length of the microstrip lines if they are constructed using an alumina substrate ($\varepsilon_r = 9.6$).

Solution. (a) The wavelength in free space is

$$\lambda_0 = \frac{c}{f} = \frac{3 \times 10^{10}}{2.5 \times 10^9} = 12 \text{ cm}$$

Hence, the shunt microstrip of length $l_1 = 17$ mm (or $l_1 = 0.142\lambda_0$) has an open-circuited admittance of $y_{oc} = -j1.25$. This shunt microstrip acts like a shunt inductor. Hence, the admittance value of 50 Ω in parallel with the shunt microstrip produces the admittance $y_A = 1 - j1.25$, shown as point A in the Y Smith chart in Fig. 2.5.19b.

The series microstrip of length $l_2 = 10$ mm (or $l_2 = 0.083\lambda_0$) produces the matching from point A to point B (see Fig. 2.5.19b). This motion is along a constant $|\Gamma|$ circle. The admittance at B is $y_B = 0.41 - j0.53$. Then, the shunt capacitance of the varactor diode (3 pF or $y_C = j2.36$) produces the motion from point B to point C. At point C the admittance is $0.41 + j1.83$, which corresponds to $\Gamma_L = 0.83\underline{|-124.5^\circ}$.

In practice, the capacitance of the varactor diode is 3 pF when a specific dc voltage is applied to it. Let us assume that 3 pF occurs when the dc voltage is 4 V. A practical dc bias circuit for the varactor diode and for the transistor is shown in Fig. 2.5.19c.

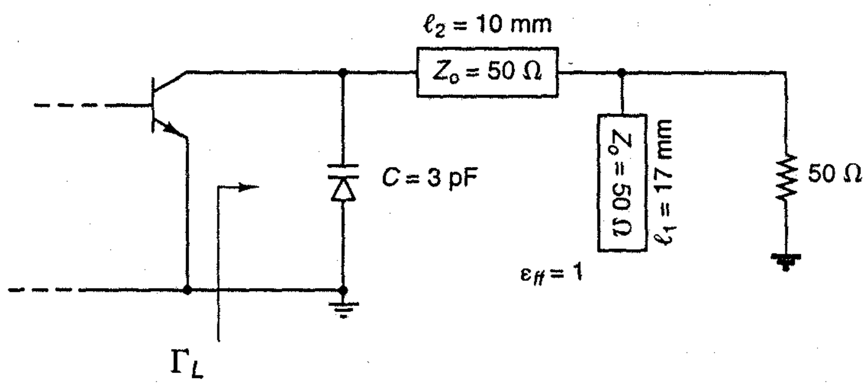
(b) If the microstrip lines are built using alumina with $\varepsilon_r = 9.6$, then from Fig. 2.5.4 a characteristic impedance of 50 Ω can be obtained with $W = 24.7$ mils and $h = 25$ mils.

It also follows that $\varepsilon_{ff} = 6.46$ (or $\lambda = \lambda_0/\sqrt{\varepsilon_{ff}} = 12/\sqrt{6.46} = 4.72$ cm) in the alumina. Hence, the length of the shunt stub, denoted by l_1', is

$$l_1' = \frac{l_1}{\sqrt{\varepsilon_{ff}}} = \frac{17}{\sqrt{6.46}} = 6.69 \text{ mm}$$

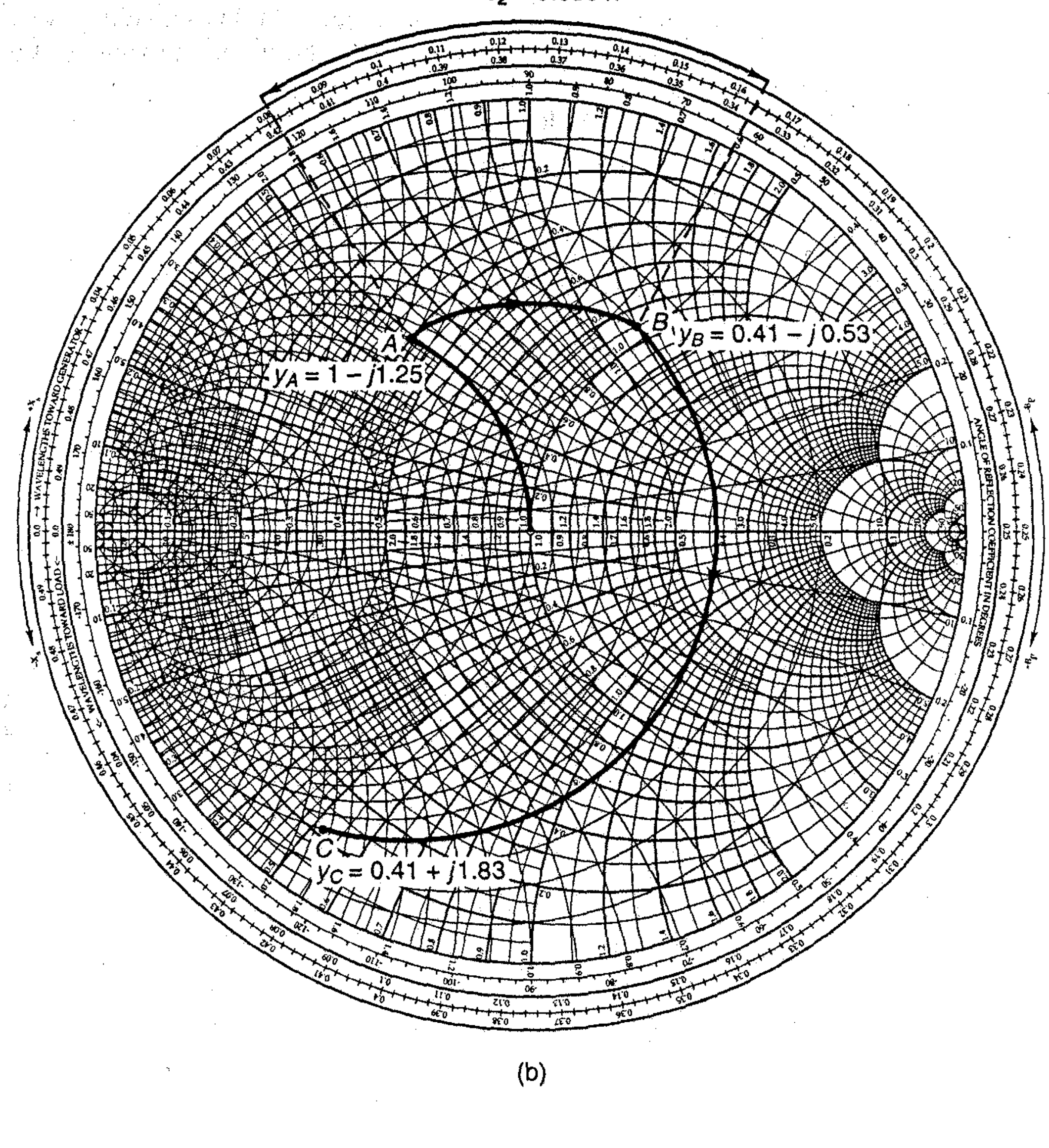
and that of the series microstrip line, denoted by, l_2' is

$$l_2' = \frac{l_2}{\sqrt{\varepsilon_{ff}}} = \frac{10}{\sqrt{6.46}} = 3.93 \text{ mm}$$



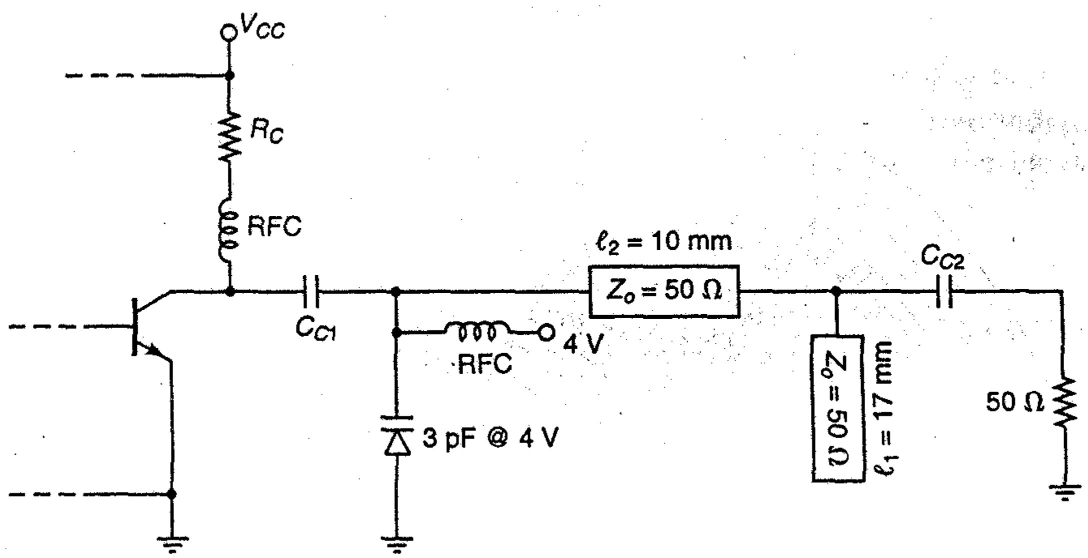
$\ell_2 = 10$ mm
$Z_o = 50\ \Omega$
$C = 3$ pF
$Z_o = 50\ \Omega$
$\ell_1 = 17$ mm
$50\ \Omega$
$\varepsilon_{ff} = 1$
Γ_L

(a)



$\ell_2 = 0.083\ \lambda$
A
$y_A = 1 - j1.25$
B
$y_B = 0.41 - j0.53$
C
$y_C = 0.41 + j1.83$

(b)



(c)

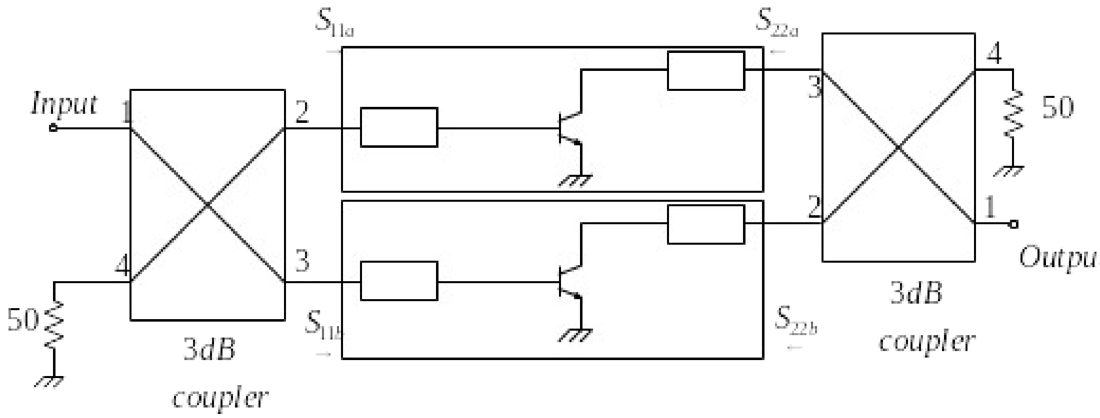
Figure 2.5.19 (a) Circuit schematic for Example 2.5.5; (b) calculation of Γ_L using the Y Smith chart; (c) an implementation of the dc bias circuit.

Example 2.5.6

Design a three-element microstrip matching network to transform a 50-Ω termination to a load reflection coefficient given by $\Gamma_L = 0.48\underline{|72^\circ}$.

Solution. We will solve this problem using the ZY Smith chart. The solution using this chart should be of interest to the reader (it is recommended that the reader also work this problem using the Y Smith chart). Figure 2.5.20a shows Γ_L in the ZY Smith chart, as well as the path selected for the matching network, using $Z_o = 50\ \Omega$. The first element transforms the normalized admittance from point A to point B in Fig. 2.5.20a. The normalized admittance at B is $y_B = 1 - j0.82$. As shown in Fig. 2.5.20b, this can be implemented with a short-circuited shunt stub of length $l_1 = 0.141\lambda$. The second element produces the admittance $y_C = 0.5 - j0.3$ at point C. This element is implemented using a series microstrip line of length $l_2 = 0.099\lambda$. Finally, the third element changes the susceptance along a constant conductance circle of 0.5 from $-j0.3$ to $-j0.6$. This element can be implemented using a short-circuited shunt stub of length $l_3 = 0.203\lambda$ (i.e., having an admittance of $-j0.3$). The matching network is shown in Fig. 2.5.20b.

8. Balanced Amplifier



90 deg hybrid and Input signal

$$[S] = -\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}, \quad \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}_{1st} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

After the first 90 deg hybrid

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}_{1st} = -\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-j}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \\ 0 \end{bmatrix}$$

After the second 90 deg hybrid
Note b2 becomes a3 and b3 becomes a2.

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}_{2nd} = (G_{amplifier})*(-\frac{1}{\sqrt{2}})\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}\begin{bmatrix} 0 \\ \frac{-1}{\sqrt{2}} \\ \frac{-j}{\sqrt{2}} \\ 0 \end{bmatrix} = (G_{amplifier})*\begin{bmatrix} j \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Examples of 3dB coupler

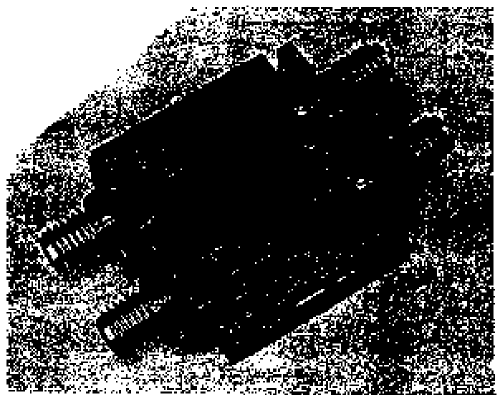
(1) 90o hybrid

Single section

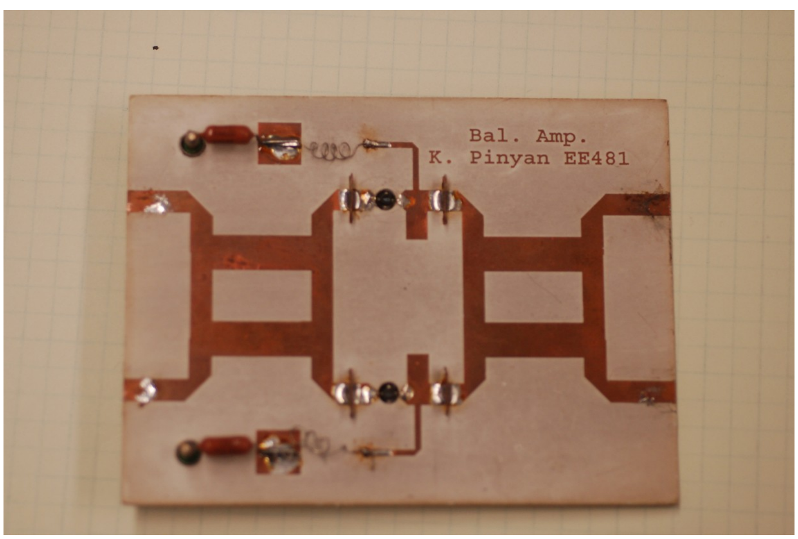
Narrow band

Multi section

Wider bandwidth



4 Photograph of a three-section microstrip quadrature hybrid. Courtesy of Harlan Howe, Jr., M/A-COM Inc.



(2) Lange Coupler

Wide bandwidth

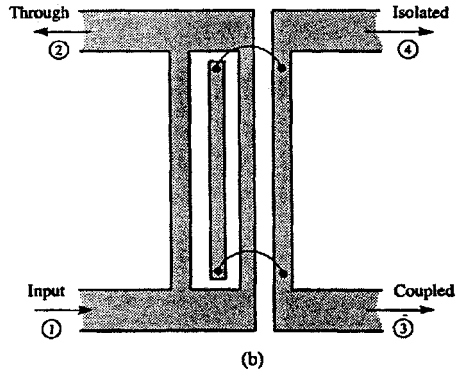


FIGURE 8.38 The Lange coupler. (a) Layout in microstrip form. (b) The unfolded Lange coupler.

Total response of the balanced amp.

$$|S_{11}| = 0.5|S_{11a} - S_{11b}|$$
$$|S_{21}| = 0.5|S_{21a} + S_{21b}|$$
$$|S_{12}| = 0.5|S_{12a} + S_{12b}|$$
$$|S_{22}| = 0.5|S_{22a} - S_{22b}|$$

- If transistors are matched, $S_{11} = 0$ and $S_{22} = 0$.
  S_{11a} and S_{22a} can be large.
  (S_{11b}) (S_{22b})
- Bandwidth is limited by the coupler.

Advantages:

(1) Input & output VSWR depend on the coupler

S_{11a} and S_{22a} can be large.

Design matching circuits to get $\left\{\begin{array}{ll} & \text{highest gain} \\ \text{or} & \text{best BW} \\ \text{or} & \text{lowest noise} \end{array}\right.$

(2) Output power is combined

(3) If one transistor fails, the balanced amp still works with a reduced gain.

(4) Easily cascadable

With unit is isolated by the couplers

Disadvantages

(1) Complex circuit

(2) Requires 2 transistors (cost, power, PCB space)