Homework3 Solution

November 7, 2022

1 Problem 1

1.1 a

M1 at the edge of the triode region:

$$V_{DS} = V_{GS} - V_{TH}$$
$$V_{OUT} = V_{IN} - V_{TH}$$

Drain current for M1 and M2 are:

$$I_{D1} = \frac{1}{2}\mu_n c_{ox}\left(\frac{W}{L}\right)_1 V_{\text{out}}^2$$
$$I_{D2} = \frac{1}{2}\mu_n c_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{\text{out}} - V_{TH}\right)^2$$

Also, $I_{D1} = I_{D2}$ are same, hence:

$$\sqrt{\left(\frac{W}{L}\right)_1} V_{\text{out}} = \sqrt{\left(\frac{W}{L}\right)_2}\left(V_{DD} - V_{out} - V_{TH}\right)$$

Plug in $V_{OUT} - V_{IN} - V_{TH}$, we can obtains:

$$\sqrt{\left(\frac{W}{L}\right)_1}\left(V_{in} - V_{TH}\right) = \sqrt{\left(\frac{W}{L}\right)_2}\left(V_{DD} - V_{in} + V_{TH} - V_{TH}\right)$$

$\Rightarrow V_{IN} = 1.41V$ The small signal gain is

$$Av = -\frac{\sqrt{(W/L)_1}}{\sqrt{(W/L)_2}} = -2.2$$

2 Problem 2

2.1 a

For the upper PMOS, one has:

$$V_{DS} = V_{DD} - V_{out}$$
$$V_{GS} = V_{in} - V_{VDD}$$

We want both MOSFETs to work in saturation to have a larger gain. So one obtains the following equations for drain current:

$$
\begin{aligned}
I_{D1} &= \tfrac{1}{2}\mu_n C_{ox}\left(\tfrac{W}{L}\right)_n (V_{IN} - V_{THN})^2 \\
I_{D2} &= \tfrac{1}{2}\mu_p C_{ox}\left(\tfrac{W}{L}\right)_p (V_{IN} - V_{VDD} - |V_{THP}|)^2
\end{aligned}
$$

We can also obtain the following expression:

$$\frac{V_{OUT} - V_{IN}}{R_1} = I_{D2} - I_{D1}$$

The reason that this can help to determine the bias voltage is one can introduce a small impedance RD so that the left side of the equation has a larger slope in the I D -V DS curve, which makes the DC point less sensitive.

2.2 b

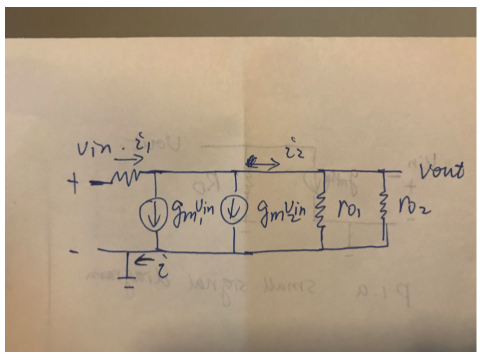

Figure 1: Small signal model

$$
\begin{aligned}
i &= i_1 - i_2 \\
i_1 &= (v_{\text{in}} - v_{\text{out}})/R_1 \\
i_2 &= v_{\text{out}}/r_o
\end{aligned}
$$

we can obtains that:

$$
\begin{aligned}
(g_{m1} + g_{m2})\, v_{in} &= \frac{v_{in} - v_{out}}{R_1} - \frac{v_{out}}{r_o} \\
A_v = \frac{v_{out}}{v_{in}} &= -\frac{g_{m1} + g_{m2} - 1/R_1}{1/r_o + 1/R_1}
\end{aligned}
$$

where $r_o = r_{o1}//r_{o2}$

2.3 c

$$
\begin{aligned}
i_{in} = i_1 &= \frac{v_{in} - v_{out}}{R_1} = \frac{v_{in}(1 - A_v)}{R_1} \\
Z_{in} = \frac{v_{in}}{i_{in}} &= \frac{R_1}{1 - A_v} \\
Z_{out} &= r_o//R_1 = r_{o1}//r_{o2}//R_1
\end{aligned}
$$

2.4 d

According to the Lemma, we can draw the following two figures to obtain the transconductance when output is shorted to ground and the output resistance when the input voltage is set to zero.

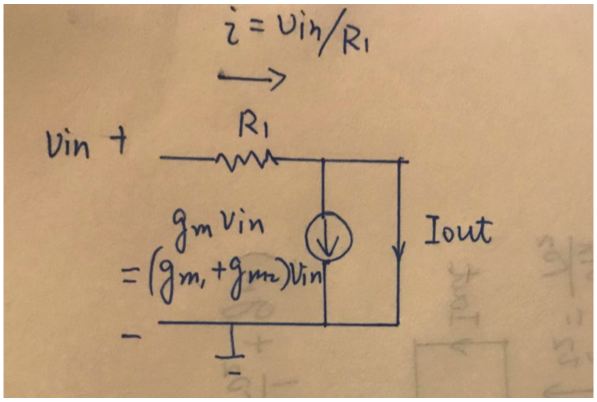

Figure 2: Lemma: Gm

$$I_{out} = -\left(g_{m1} + g_{m2}\right) v_{in} + \frac{v_{in}}{R_1}$$
$$G_m = \frac{I_{out}}{v_{in}} = -\left(g_{m1} + g_{m2}\right) + 1/R_1$$

Also,

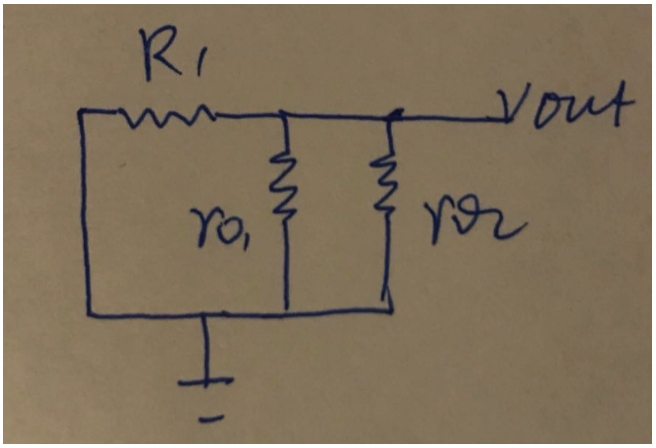

Figure 3: Lemma: R

$$R = r_{o1}//r_{o2}//R_1$$

⇒

$$A_v = -G_m R = -\frac{g_{m1} + g_{m2} - 1/R_1}{1/r_o + 1/R_1}$$

2.5 e)

The small signal model is shown below in this case:

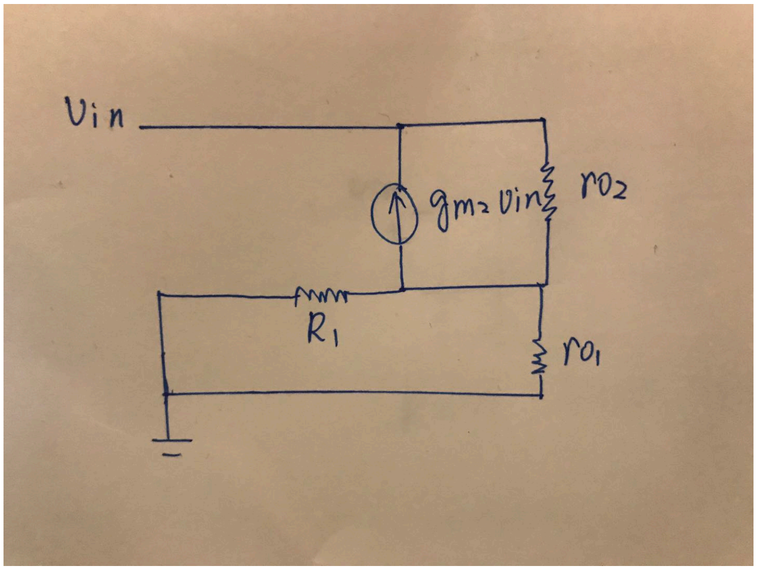

Figure 5: Small signal model for VDD as AC input

We can again use the Lemma to determine the gain, first we ground the V_{out} and determine the Gm

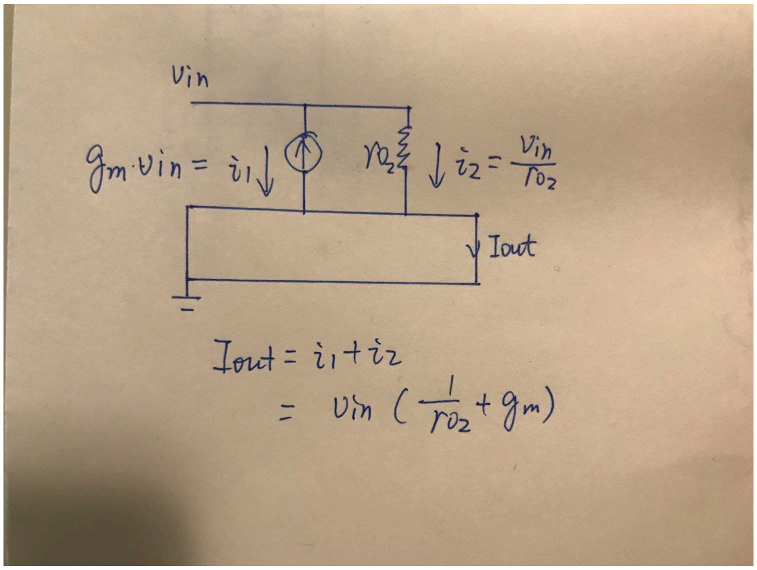

Figure 6: Determine Gm

$$i_{out} = \frac{v_{in}}{r_{o2}} + g_{m2}v_{in} \tag{33}$$

$$G_m = \frac{i_{out}}{v_{in}} = \frac{1}{r_{o2}} + g_{m2} \tag{34}$$

Then the effective resistance can be determined by set the input $v_{in} = 0$.

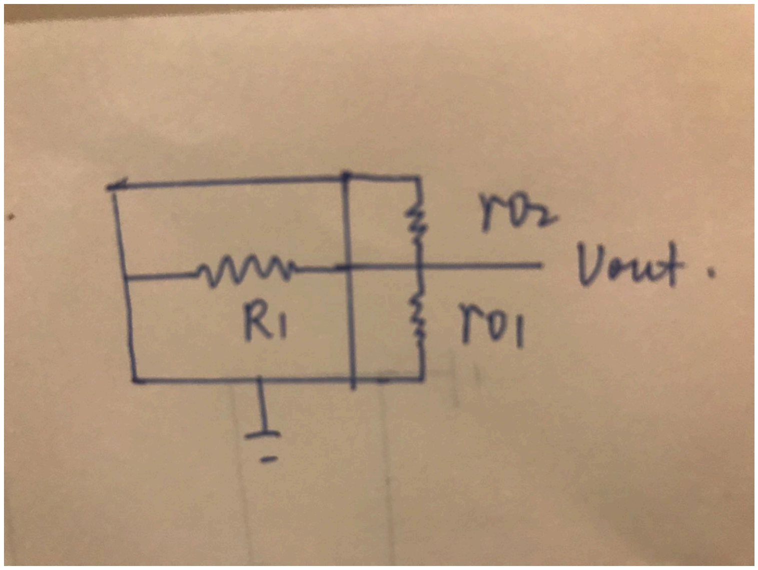

Figure 7: Determine Gm

The three resistance are parallel, so $R = r_o//R_1$. So the total gain is given by

$$A'_v = -G_m R = (\frac{1}{r_{o2}} + g_{m2}) \cdot r_o//R_1 \tag{35}$$

Compared with the previous result where $A_v = (g_{m1} + g_{m2} - 1/R_1) \cdot r_o//R_1$, this is smaller, because g_{m1} and g_{m2} are usually big.

Note: For the following questions, we used different $C_{ox} = 77.6fF/\mu m^2$ from the textbook's Table 1 (which corresponding to $C_{ox} = 3.8fF/\mu m^2$). People use either value should be correct considering people could refer to the textbook directly.

3 Problem 3

3.1 a)

$$\begin{aligned} g_{m1} &= \frac{2I_D}{V_{GS} - V_{TH}} && (36) \\ &= \sqrt{2C_{ox}\mu_n(\frac{W}{L})_1 I_D} && (37) \\ &= \sqrt{2 \times 77.6 \times 10^{-3}F/m^2 \times 350 \times 10^{-4}m^2/V/s \times 100 \times 0.5 \times 10^{-3}A} && (38) \\ &= 0.016\Omega^{-1} && (39) \end{aligned}$$

Note all the parameters given are defined at $L = 0.5\mu m$. In this problem, L_2 is $2\mu m$ and $lambda$ is inversely proportional to the length of the transistor. Thus in this case $\lambda_2 = 0.2/4 = 0.05V^{-1}$

$$r_{o1} = \frac{1}{\lambda_1 I_D} = 2 \times 10^4\Omega \tag{40}$$

$$r_{o2} = \frac{1}{\lambda_2 I_D} = 4 \times 10^4\Omega \tag{41}$$

$$r_{o1}//r_{o2} = \frac{r_{o1}r_{o2}}{r_{o1} + r_{o2}} = 13.3k\Omega \tag{42}$$

So the gain is

$$A_v = -g_{m1}(r_{o1}//r_{o2}) = -212.8 \tag{43}$$

3.2 b)

If we assume that M_1 is in the edge of the triode region, then we have:

$$V_{GS} - V_{TH1} = V_{DS1} = V_{OUT} \tag{44}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_1(V_{GS} - V_{TH1})^2(1 + \lambda_N V_{DS}) \tag{45}$$

$$= \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_1 V_{DS}^2(1 + \lambda_N V_{DS}) \tag{46}$$

Thus one can determine

$$V_{DS} = \sqrt{\frac{2I_D}{\mu_n C_{ox}(\frac{W}{L})_1(1 + 0.1V_{DS})}} = \sqrt{\frac{3.68 \times 10^{-3}}{1 + 0.1V_{DS}}} \tag{47}$$

Solve for V_{DS} using graphic method or numerical method, one obtains

$$V_{o,min} = V_{DS,min} = 0.06V \tag{48}$$

If we assume that M_2 is in the edge of the triode region, then one obtains:

$$V_{DS} = \sqrt{\frac{2I_D}{\mu_p C_{ox}(\frac{W}{L})_2(1 + 0.05V_{SD})}} = \sqrt{\frac{0.052}{1 + 0.05V_{SD}}} \tag{49}$$

One obtains that $V_{SD,min} = 0.23V$, so $V_{o,max} = V_{DD} - V_{SD,min} = V_{DD} - 0.23V$.
So the swing is $\frac{V_{DD} - 0.29}{2}$.