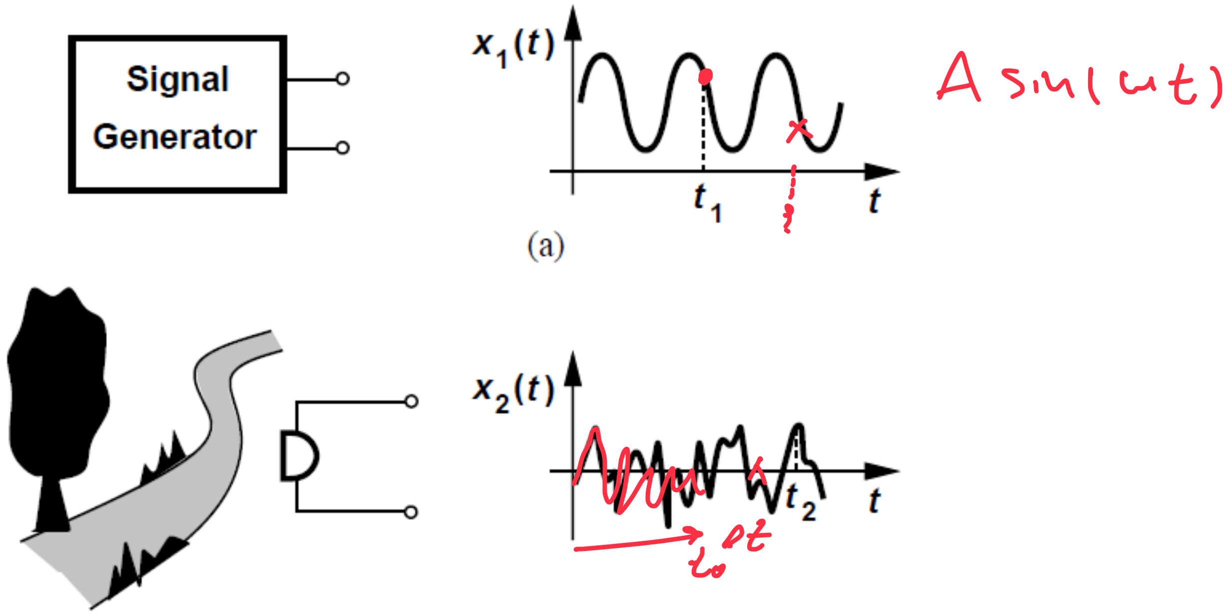
EE 332: Devices and Circuits II

Lecture 8: Noise

Prof. Sajjad Moazeni
smoazeni@uw.edu
Autumn 2022

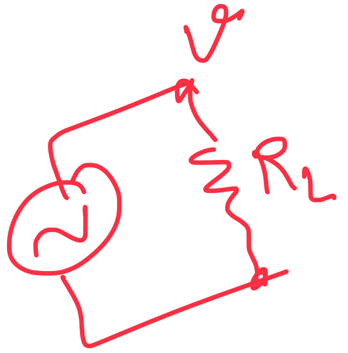
ELECTRICAL & COMPUTER ENGINEERING

Statistical Characteristics of Noise

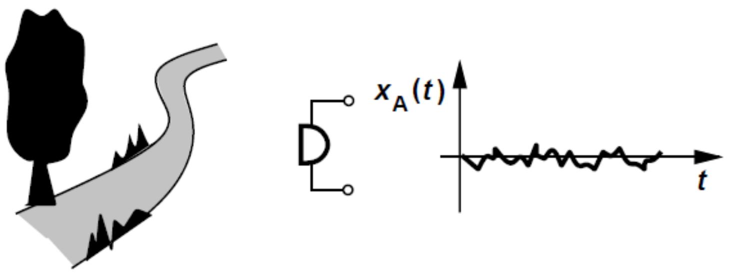


- Noise is a random process
- Value of $x_1(t_1)$ can be predicted from observed waveform, that of $x_2(t_2)$ cannot
  - Difference between deterministic and random phenomena
- Instantaneous value of noise in time domain is unpredictable

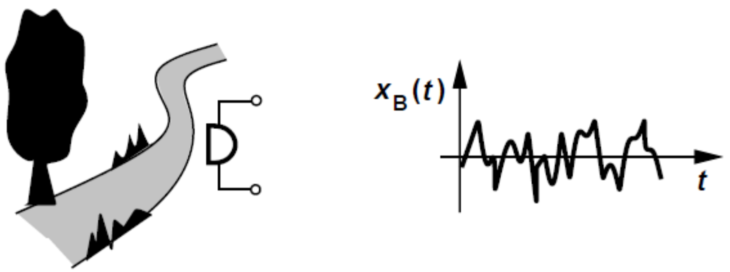
Statistical Characteristics of Noise



$P = \frac{v^2}{R_L}$



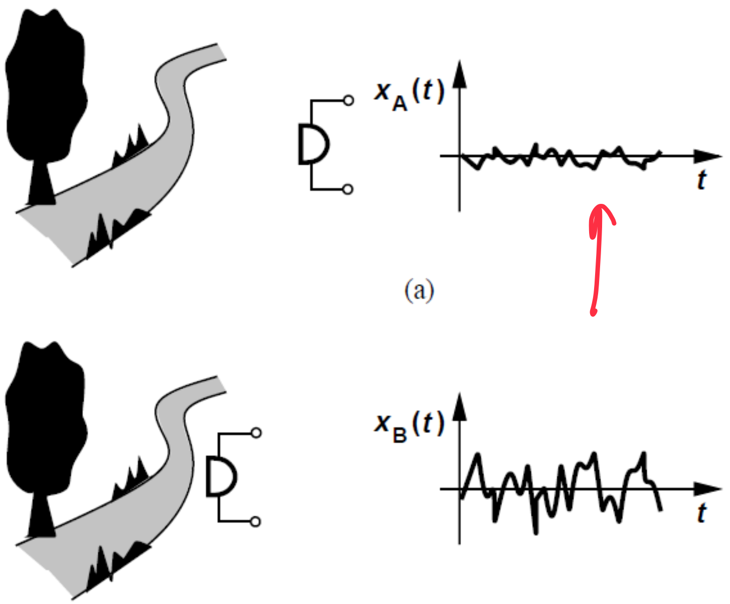
(a)



- Need for a "statistical model" for noise
- Average power of noise is predictable
  - Applicable to most sources of noise in circuits
- Average power delivered by a periodic voltage *v(t)* with period *T* to a load resistance R_L is defined as

$$P_{av} = \frac{1}{T}\int_{-T/2}^{+T/2} \frac{v^2(t)}{R_L} dt$$

Statistical Characteristics of Noise



- For a random signal (aperiodic), measurement must be carried out over a long time

Watts

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} \frac{x^2(t)}{R_L} dt$$

- $x(t)$ is a voltage quantity
- $x_A(t)$ delivers more power to a resistive load than $x_B(t)$

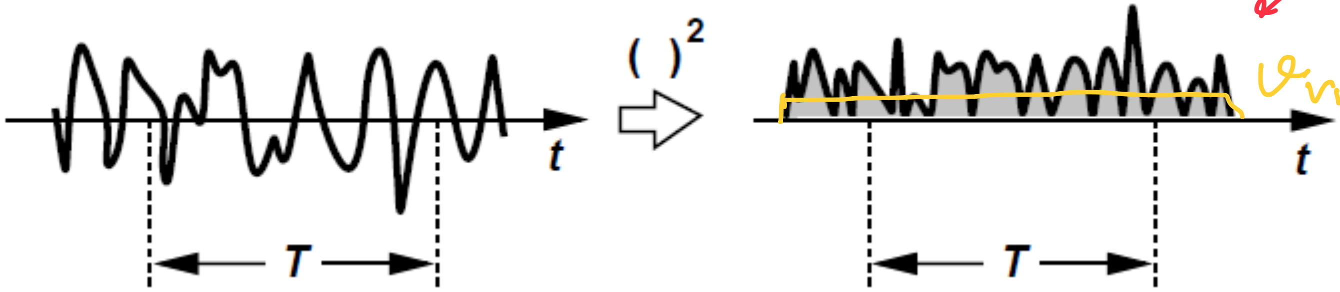
Statistical Characteristics of Noise

$A \sin(\omega t)$

$\downarrow$

$v_{rms} = A/\sqrt{2}$

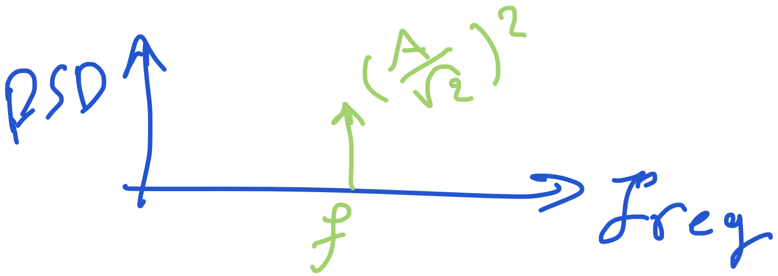
P_{out}



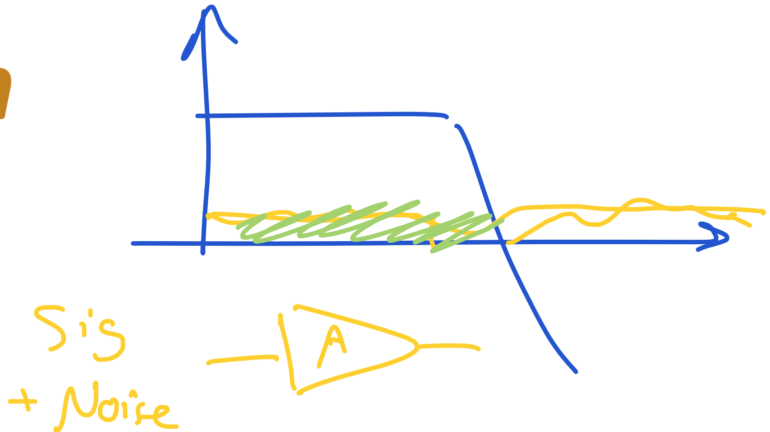
- To calculate average power of (noise) signal *x(t)*
  - Square the signal
  - Find area under resulting waveform for a long period *T*
  - Normalize area to *T*
- For simpler calculations, P_{av} is defined as

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} x^2(t)dt$$

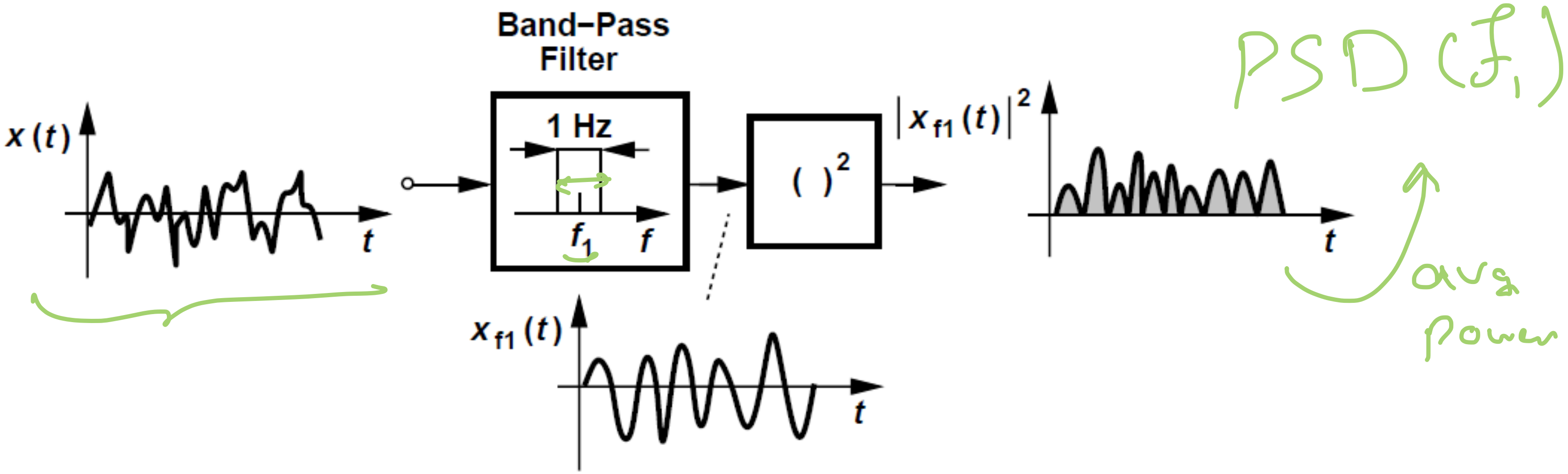
- P_{av} is expressed in V^2 rather than *W*
- RMS voltage for noise can be defined as $\sqrt{P_{av}} = v_{rms}$



Noise Spectrum



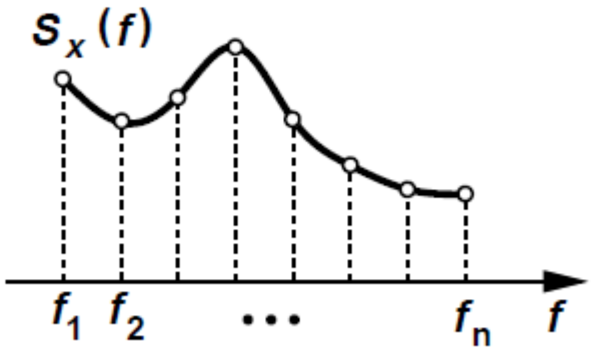
- Spectrum describes the *frequency content* of noise
- Also called ***Power Spectral Density (PSD)***
  - Shows how much power signal carries at each frequency
- PSD $S_x(f)$ of a noise waveform $x(t)$ is defined as the average power carried by $x(t)$ in a 1-Hz bandwidth around f
- Calculation of $S_x(f_1)$, i.e., power contained in a specific frequency f_1:



Noise Spectrum

- Repeating previous procedure with bandpass filters with different center frequencies, the overall PSD $S_x(f)$ can be constructed

PSD



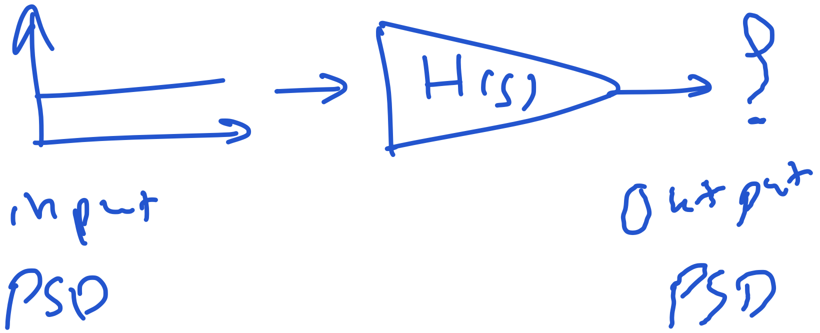
$$\int_{-\infty}^{+\infty} PSD(f)\,df = P_{av}$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{\infty} x^2(t)\,dt$$

- $S_x(f)$ Represents the power carried by signal (or noise) at all frequencies
  - Generally measured in Watts per Hertz

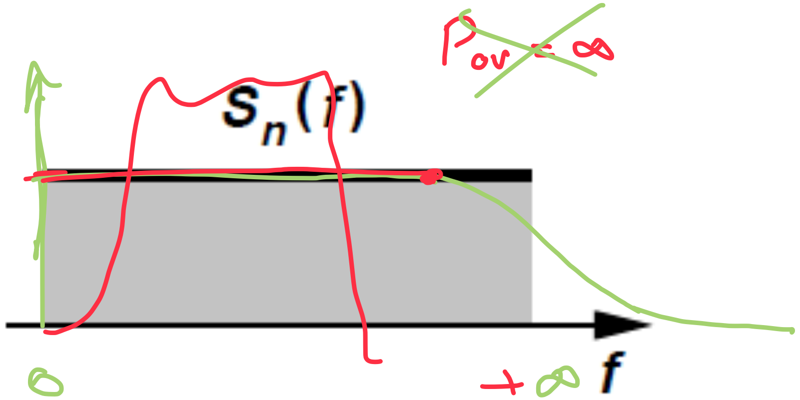
$$\frac{W}{Hz} \xrightarrow{\sqrt{\ }} \frac{V}{\sqrt{Hz}} \qquad \frac{V^2}{Hz}$$

Noise Spectrum



- As with P_{av}, it is customary to eliminate R_L from $S_x(f)$
- $S_x(f)$ is expressed in V^2/Hz rather than W/Hz
- Also common to take the square root of $S_x(f)$, expressing result in $V/\sqrt{Hz}$
- Common type of noise PSD is "white noise"
  - Displays same value at all frequencies
- White noise does not exist strictly speaking since total power carried by noise cannot be infinite
  - Noise spectrum that is flat *in the band of interest* is usually called white

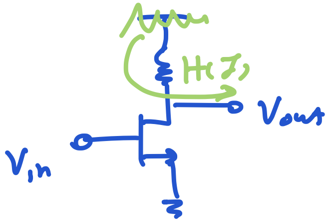
$PSD \propto \frac{1}{f}$



$x(t) \rightarrow \boxed{\text{LTI}} \rightarrow y(t)$

$H(s)$

Theorem



- If a signal with spectrum $S_x(f)$ is applied to a linear time-invariant (LTI) system with transfer function $H(s)$, then the output spectrum $S_Y(f)$ is given by

$\frac{v_y}{v_x} = H$

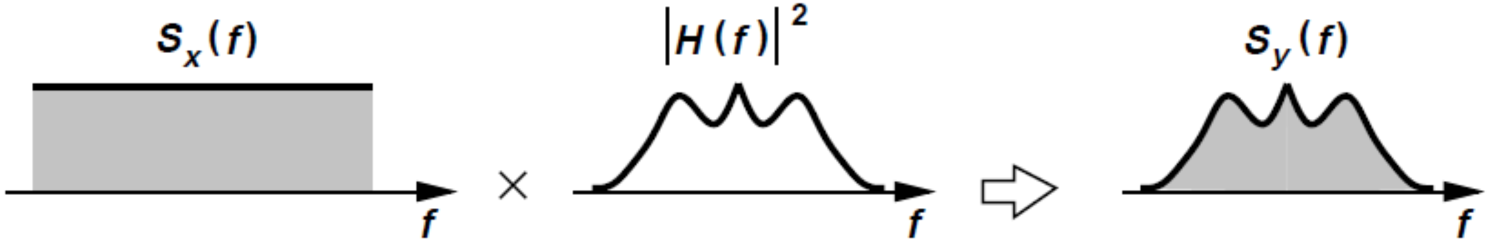
$$\boxed{S_Y(f) = S_x(f)|H(f)|^2}$$

where $H(f) = H(s = j2\pi f)$

PSD of noise @ output.

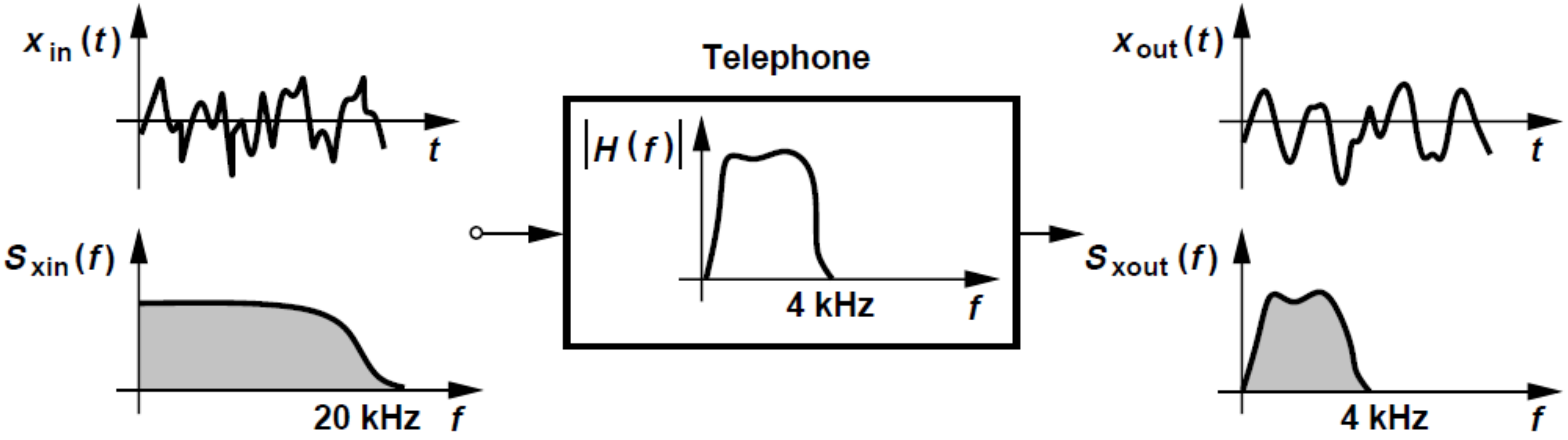
PSD of input noise

- Spectrum of signal is "shaped" by the transfer function of the system (see Fig. below)



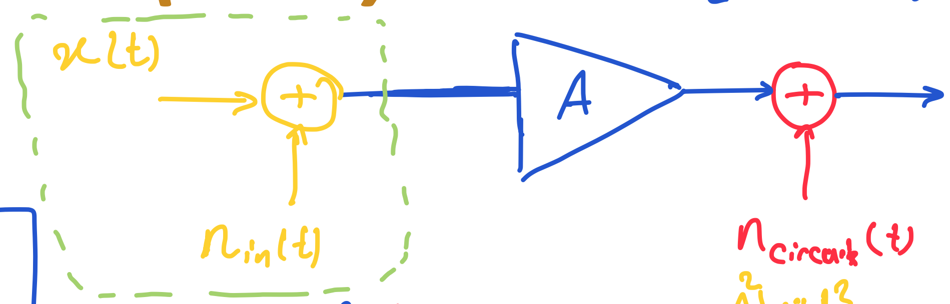
Theorem: Example

- Regular telephones have a bandwidth of approximately 4kHz and suppress higher frequency components in caller's voice
- Due to limited bandwidth, $x_{out}(t)$ exhibits slower changes than $x_{in}(t)$
  - Can be difficult to recognize the caller's voice



Signal-to-Noise Ratio (SNR)

$A[x(t) + n_{in}(t)] + n_{circui}$



- Signal-to-noise ratio (SNR) is defined as

dB

$$\boxed{\mathrm{SNR} = \frac{P_{sig}}{P_{noise}}}$$

$SNR_{in} = \frac{|x(t)|^2}{|n_{in}(t)|^2}$ $SNR_{out} = \frac{A^2|x(t)|^2}{A^2|n_{in}(t)|^2 + |n_{cir}|^2}$

- SNR of a noise-corrupted signal should be high for it to be intelligible
  - Audio signals require a minimum SNR of 20 dB

Noise Figure $= \frac{SNR_{in}}{SNR_{out}} > 1$

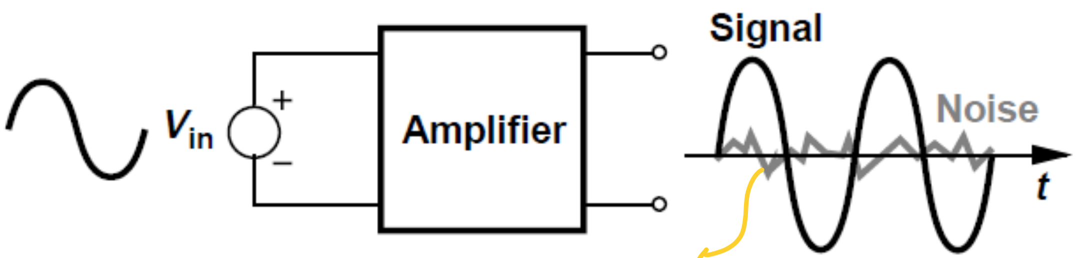
- For a sinusoid with peak amplitude A, $P_{sig} = A^2/2$ ← $A\sin(\omega t)$
- The total average power carried by noise is equal to the area under its spectrum

$$P_{noise} = \int_{-\infty}^{+\infty} S_{noise}(f)df$$

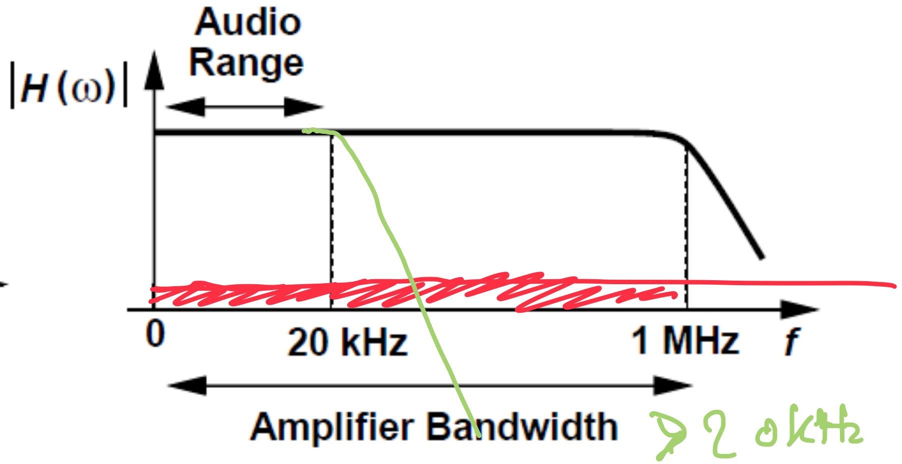
- P_{noise} can be very large if $S_{noise}(f)$ spans a wide frequency range

Signal-to-Noise Ratio (SNR)

- Below amplifier provides a bandwidth of 1 MHz while sensing an audio signal
  - What can be wrong?



added by the Amp.



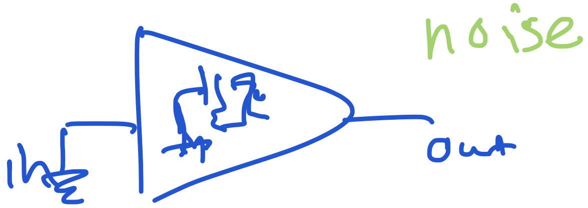
$\gg$ 20kHz

50x

$$SNR = \frac{P_{sig}}{P_{Noise}}$$

For white Noise

$\int PSD(f)\,df = PSD \times BW$



Noise Analysis Procedure

- Output signal of a given circuit is corrupted by noise sources within the circuit
  - Interested in noise observed at the output

thermal
P/N MOS
Res, Photodiode, channels PSD
Properties of noise source

- Four steps:
  1. Identify the sources of noise and note the spectrum of each
  2. Find the transfer function from each noise source to the output $H(f)$
  3. Use the theorem $S_Y(f) = S_x(f)|H(f)|^2$ to calculate output noise spectrum contributed by each noise source
  4. Add all the output spectra, accounting for correlated and uncorrelated sources

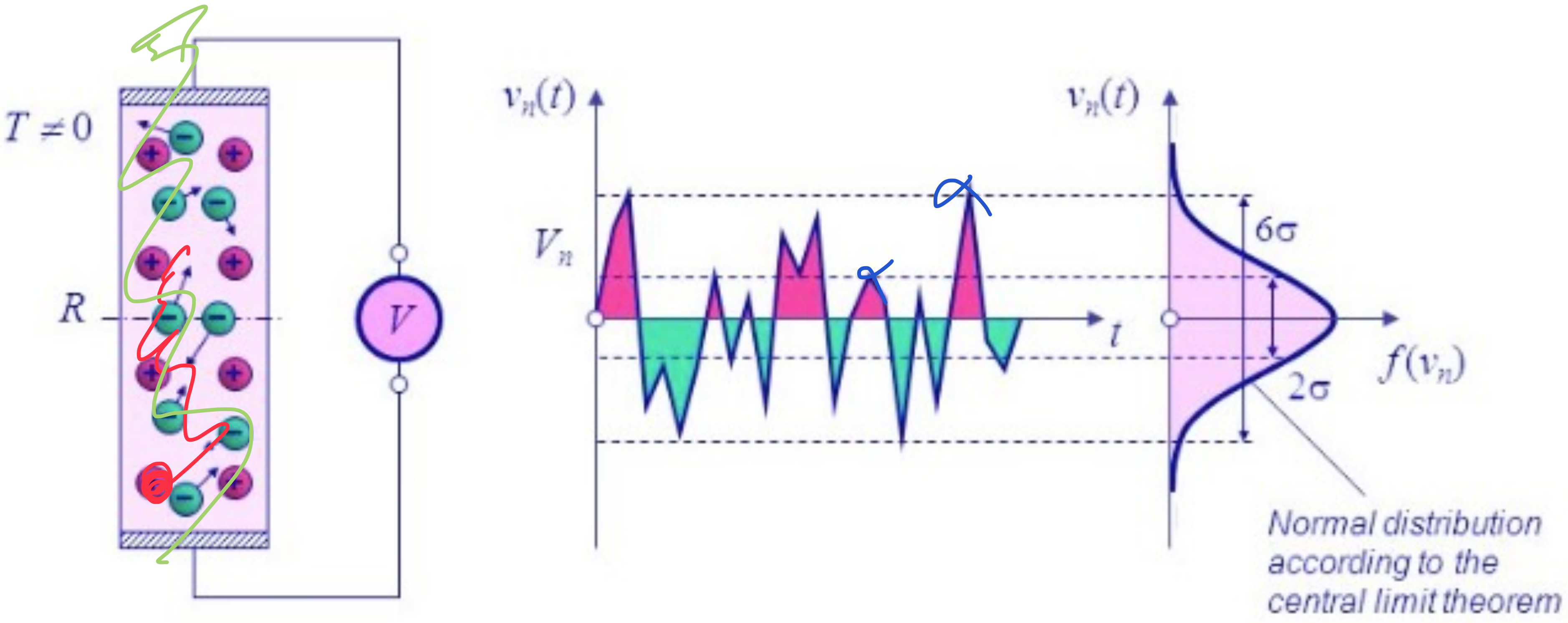
$$PSD_{out}(f) = \sum PSD_i(f)$$

- Integrate the output noise spectrum from -∞ to +∞ to get total output noise power

Resistor Thermal Noise

- Random motion of electrons in a conductor induces fluctuations in the voltage measured across it even though the average current is zero

T=temp



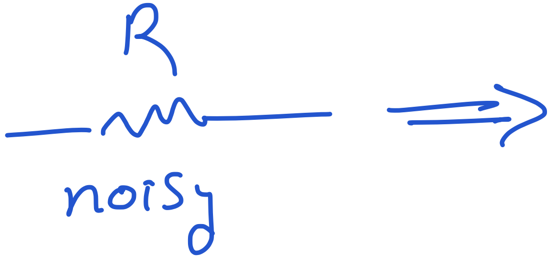
Resistor Thermal Noise

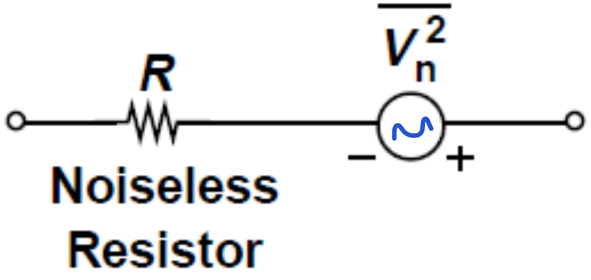
- Thermal noise of a resistor *R* can be modeled by a series voltage source, with one-sided spectral density

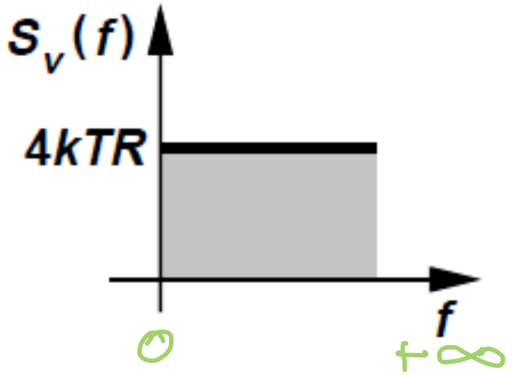
$$S_v(f) = 4kTR, \quad f \geq 0$$

$\frac{V^2}{Hz}$

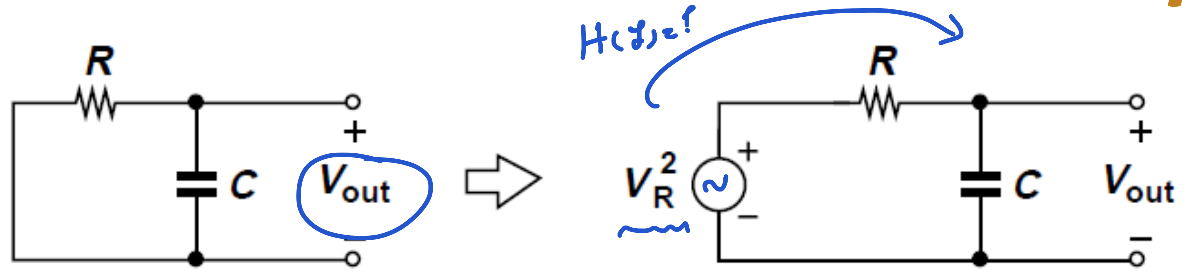
- Here, k = 1.38x10^{-23} J/K is the Boltzmann constant
- $S_v(f)$ is expressed in V^2/Hz, we also write $\overline{V_n^2} = 4kTR$
- For a 50-Ω resistor at T = 300K, thermal noise is 8.28 X 10^{-19} V^2/Hz, or 0.91 nV/√Hz
- $S_v(f)$ is flat up to 100THz, and is "white" for our purposes





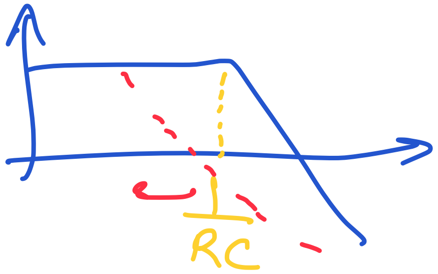


Resistor Thermal Noise: Example



- To find: Noise spectrum and total noise power in V_{out}
- Solution: Noise spectrum of R is given by $S_v(f) = 4kTR$ (V^2/Hz)
- Modeling noise by a series voltage source V_R, transfer function from V_R to V_{out} is

$$\boxed{\frac{V_{out}}{V_R}(s) = \frac{1}{RCs+1}}$$



- Using theorem, noise spectrum at the output $S_{out}(f)$ is

PSD

$$S_{out}(f) = S_v(f)\left|\frac{V_{out}}{V_R}(j\omega)\right|^2 \quad |H(f)|^2$$

$$= 4kTR\frac{1}{4\pi^2R^2C^2f^2+1} \rightarrow \int_0^{+\infty}$$

$\omega = 2\pi f$

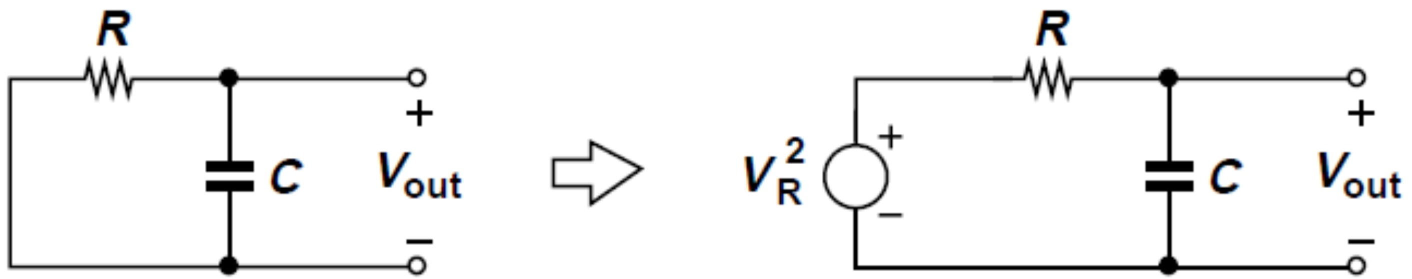
Resistor Thermal Noise: Example

2x R

4kTR noise power

BW

1/2 x



2x

- White noise spectrum of the resistor is shaped by a low-pass characteristic
- Total noise power at the output is

$$P_{n,out} = \int_0^{\infty} \frac{4kTR}{4\pi^2 R^2 C^2 f^2 + 1} df$$

- The integral reduces to

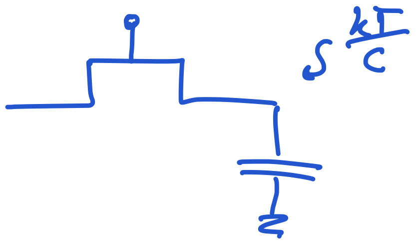
$$P_{n,out} = \frac{2kT}{\pi C} \tan^{-1} u \Big|_{u=0}^{u=\infty}$$

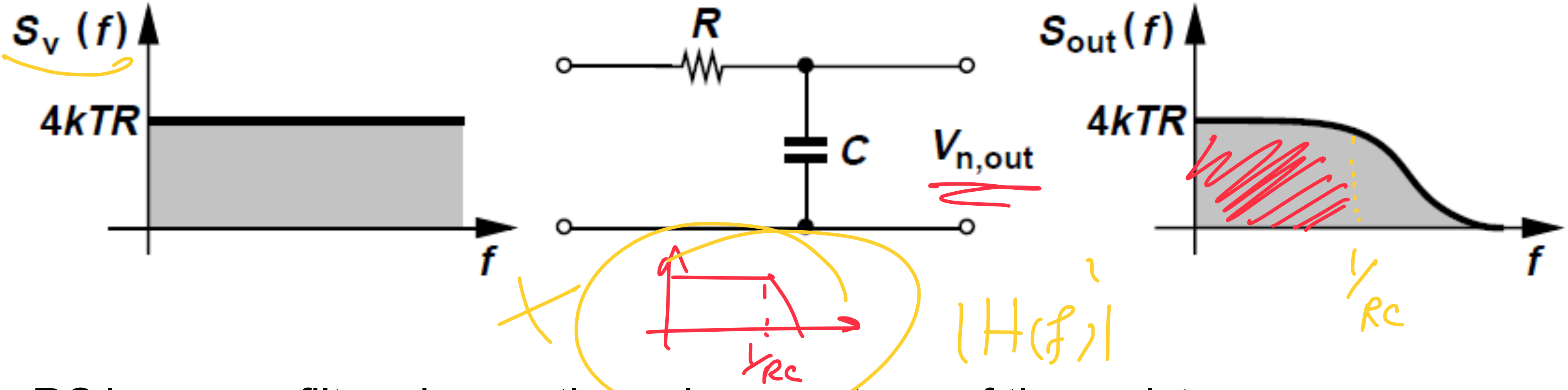
$$= \boxed{\frac{kT}{C}}$$

independent of R value!

- The unit of $P_{n,out}$ is $V^2/\cancel{Hz}$, $\sqrt{kT/C}$ may be considered as the total rms voltage measured at the output

Resistor Thermal Noise: Example

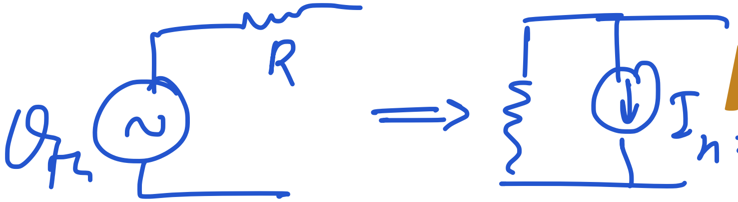




- The *RC* low-pass filter shapes the noise spectrum of the resistor
- Total noise at the output (area under $S_{out}(f)$) is independent of the resistance *R*
- Intuitively, this is because for larger values of R, noise per unit bandwidth increases but the overall bandwidth of the circuit decreases

PSD

- *kT/C* noise can only be decreased by increasing C (if T is fixed)

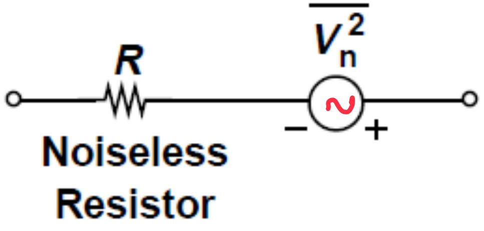


$I_n = v_{th}/R$

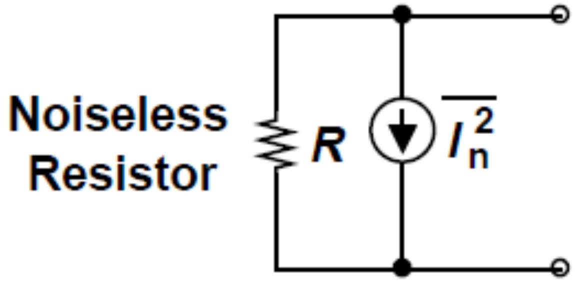
Resistor Thermal Noise

- Thermal noise of a resistor can be represented by a parallel current source too

Noisy Model for Res.



$\equiv$

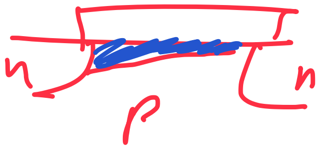


Norton Model

- This representation is equivalent to series voltage source representation with

$$\overline{I_n^2} = \frac{\overline{v_n^2}}{R^3} = \frac{4kTR}{R^2} \longrightarrow \overline{I_n^2} = 4kT/R$$

- $\overline{I_n^2}$ is expressed in A^2/Hz
- This notation assumes a 1-Hz bandwidth
- Depending on circuit topology, one model may simplify calculations than the other



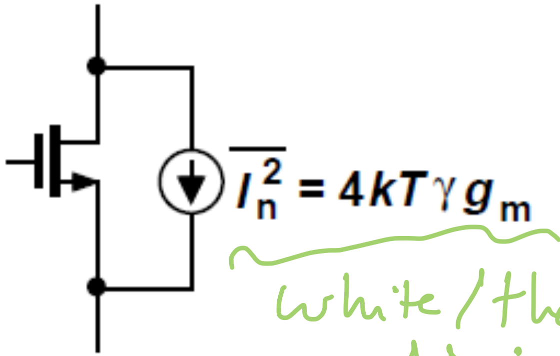
MOSFET Thermal Noise

- MOS transistors exhibit thermal noise with the most significant source being the noise generated in the channel
- For long-channel MOS devices operating in saturation, the channel noise can be modeled by a current source connected between the drain and source terminals with a spectral density

$$\overline{I_n^2} = 4kT\gamma g_m$$

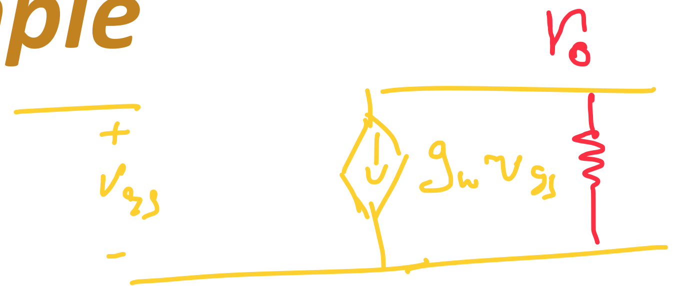
- The coefficient 'γ' (not the body effect coefficient) is derived to be 2/3 for long-channel transistors and is higher for submicron MOSFETs
- As a rule of thumb, assume $\gamma = 1$

technology node & region of op.



white/thermal Noise

MOSFET Thermal Noise: Example



- The maximum output noise occurs if the transistor sees only its own output impedance as the load, i.e., if the external load is an ideal current source
- Output noise voltage spectrum is given by

$$S_{out}(f) = S_{in}(f)|H(f)|^2$$

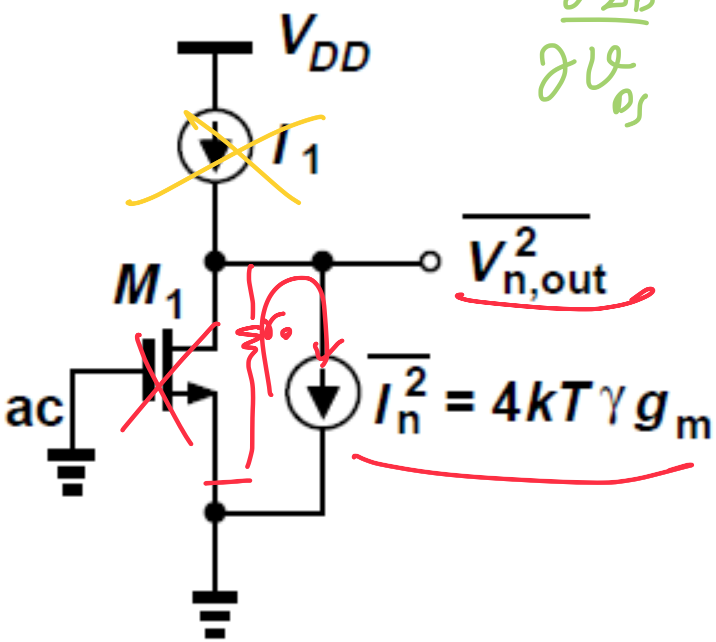
PSD of Noise @ output

$$\overline{V_n^2} = \overline{I_n^2} r_O^2$$

$$= (4kT\gamma g_m) r_O^2$$

$$H(f) = \frac{V_{n,out}}{I_n} =$$

$$\frac{\partial I_D}{\partial V_{DS}}$$

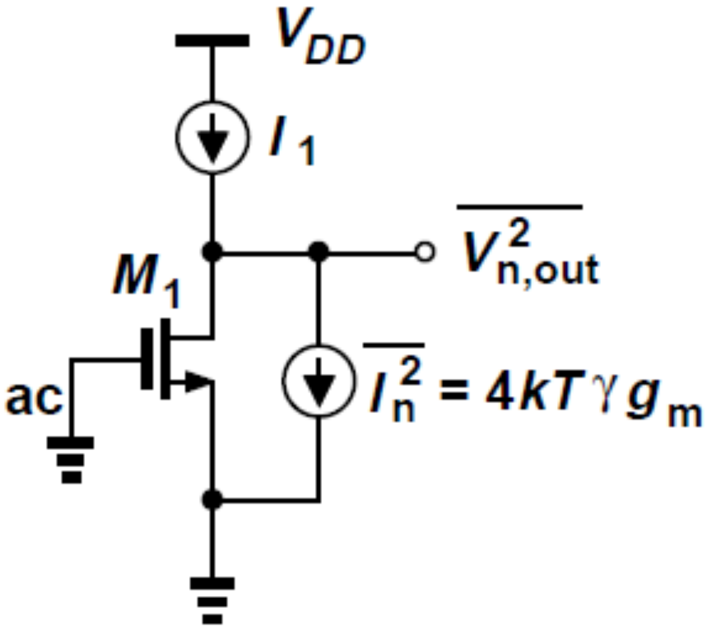


$$\overline{V_{n,out}^2} = (r_o)^2 \times \overline{I_n^2}$$

MOSFET Thermal Noise: Example

- Noise current of a MOS transistor decreases if g_m drops

- Noise measured at the output of the circuit does not depend on where the input terminal is because input is set to zero for noise calculation



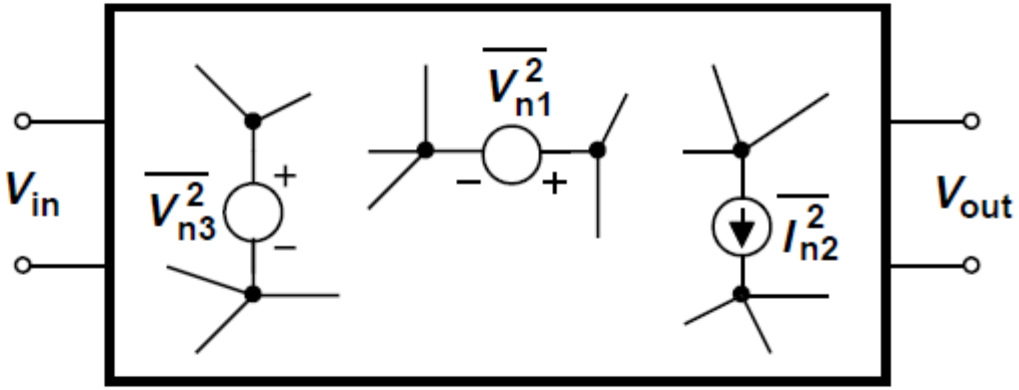
- The output resistance r_O does not produce noise because it is not a physical resistor! → It's only due to modeling of C.L.M

$1/f$ Noise.

- Another noise source is "Flicker Noise"
  - Due to some dangling bonds at the surface -> carriers can get randomly trapped!
  - Frequency dependent (S = 1/f)

$S(f) \propto 1/f$

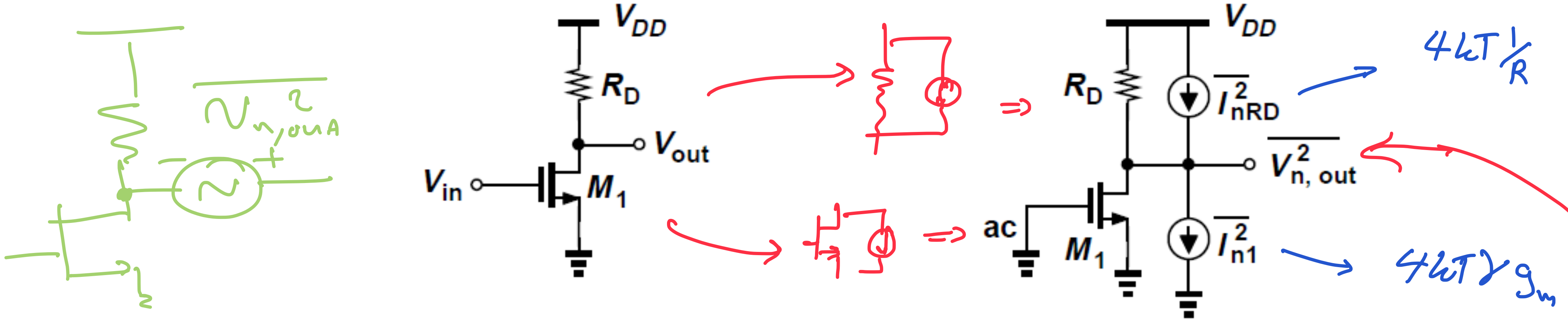
Representation of Noise in Circuits



- To find the output noise, the input is set to zero and total noise is calculated at the output due to all the noise sources in the circuit

- This is how noise is measured in laboratories and in simulations

Representation of Noise in Circuits: Example



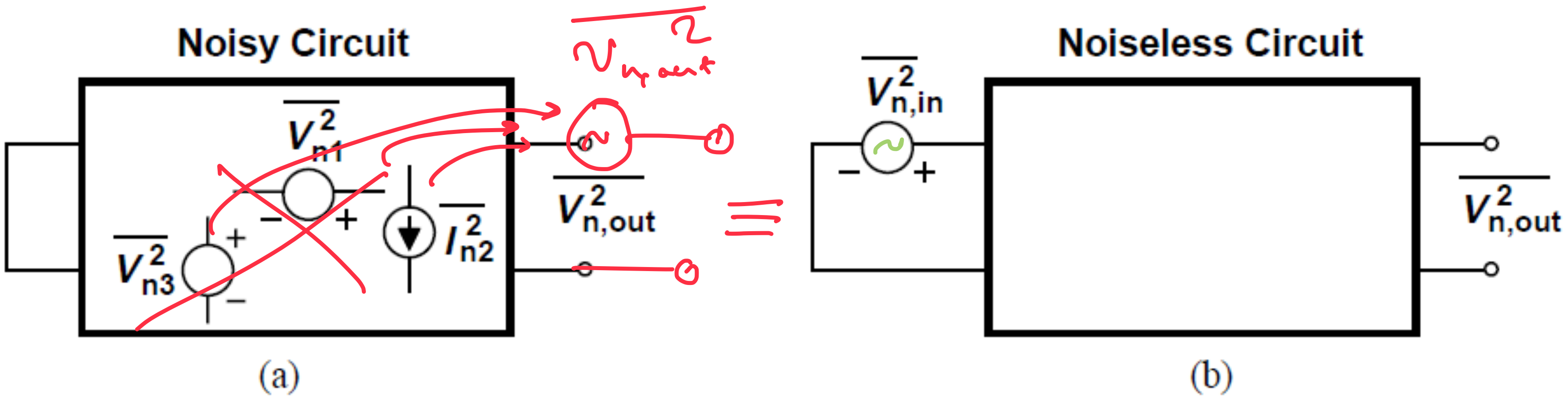
- Find total output noise voltage of the common-source stage

$$V_{out_n}/I_{n_1} = R_D \| r_o \longrightarrow PSD_{n_{RD}} = 4kT/R_D \times (R_D \| r_o)^2$$

$$V_{out_n}/I_{n_{RD}} = R_D \| r_o \longrightarrow PSD_{n_{M_1}} = 4kT\gamma g_m \times (R_D \| r_o)^2$$

$$PSD_{n,out} = PSD_{n_{RD}} + PSD_{n_{M_1}}$$

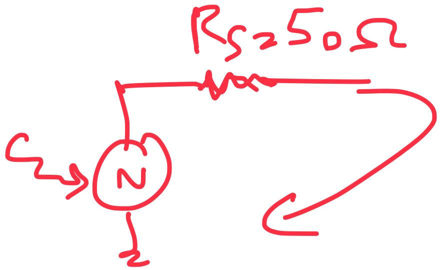
Input-Referred Noise

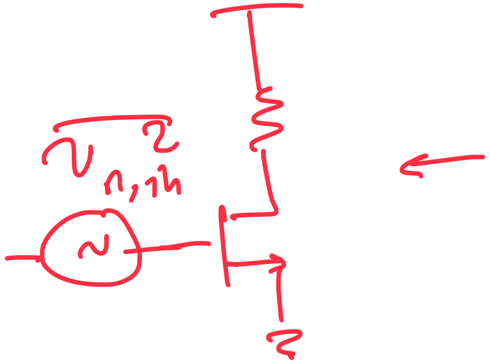


- Input-referred noise represents the effect of all noise sources in the circuit by a single source $\overline{V_{n,in}^2}$, at the input such that the output noise in Fig. b is equal to that in Fig. a
- If the voltage gain is A_v, then we must have

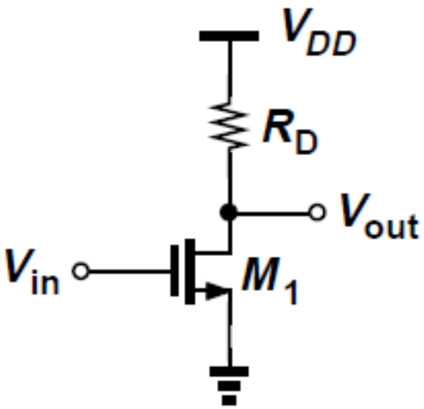
$$\overline{V_{n,out}^2} = A_v^2 \overline{V_{n,in}^2}$$

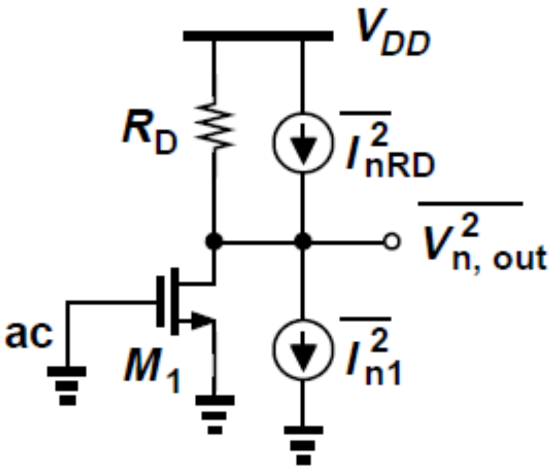
- The input-referred noise voltage in this simple case is simply the output noise divided by the gain squared.





Input-Referred Noise: Example





- For the simple CS stage, the input-referred noise voltage is given by …

$$\overline{v_{n,out}^2} = \left(4kT\gamma g_m + 4kT\frac{1}{R_D}\right)(r_o \| R_D)^2$$

$\div A^2$ where $A = -g_m(R_D \| r_o)$

$$\overline{v_{n,in}^2} = \left(4kT\gamma g_m + 4kT\frac{1}{R_D}\right) / g_m^2 = \frac{4kT\gamma}{g_m} + \frac{4kT g_m^2}{R_D}$$