EE 332: Devices and Circuits II

Lecture 6: Frequency Response

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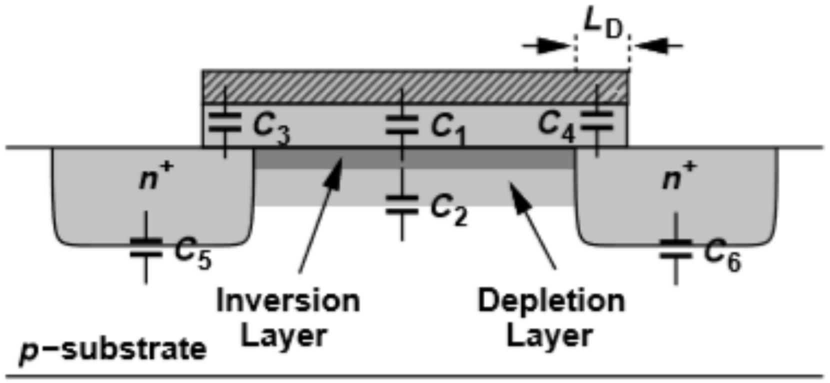
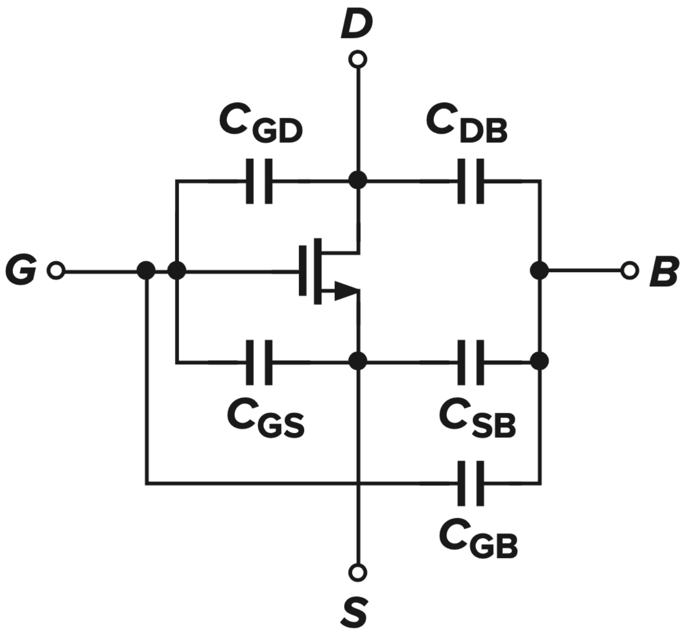
ELECTRICAL & COMPUTER ENGINEERING

MOS Device Capacitances





- Device capacitances cause frequency dependent Av, Z_{in}, Z_{out}, etc.
- Capacitance exists almost between every two of the four terminals
- Larger device -> Larger caps!

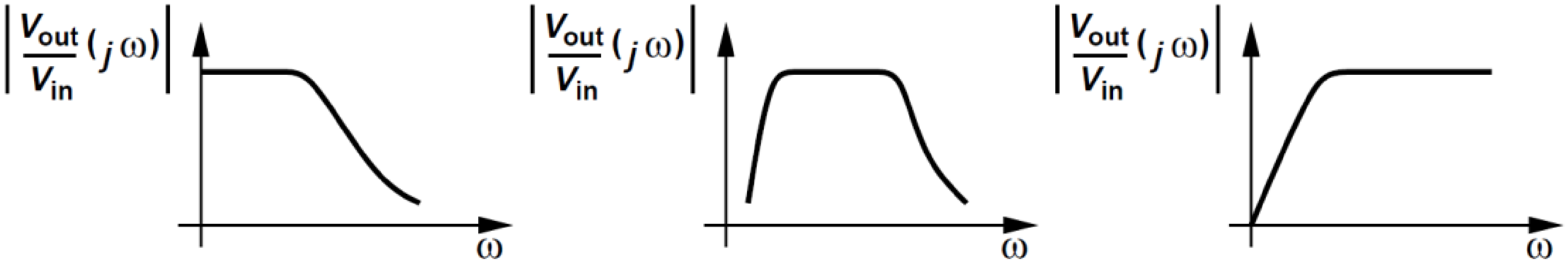
Frequency Response

$$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_0}{s^n + b_{n-1} s^{n-1} + \ldots + b_0}$$

$$T(s) = a_m \frac{(s - Z_1)(s - Z_2)\ldots(s - Z_m)}{(s - P_1)(s - P_2)\ldots(s - P_n)}$$

$$H(s) = \frac{b_n}{a_m} \cdot \frac{\left(1 + \frac{s}{\omega_{z1}}\right)\left(1 + \frac{s}{\omega_{z2}}\right)\cdots\left(1 + \frac{s}{\omega_{zn}}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)\cdots\left(1 + \frac{s}{\omega_{pm}}\right)}$$

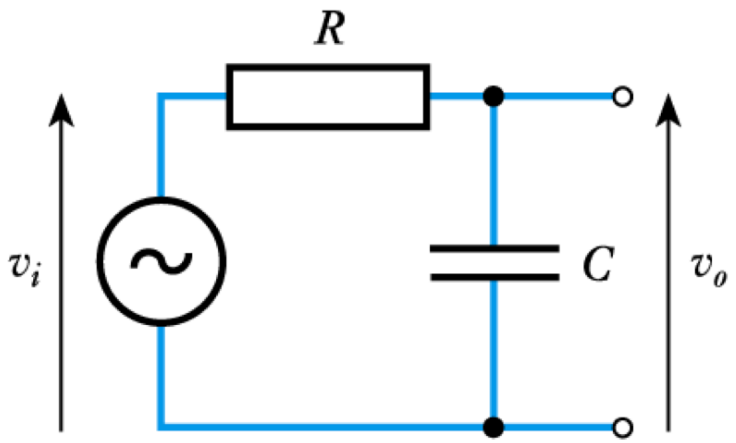
General Considerations

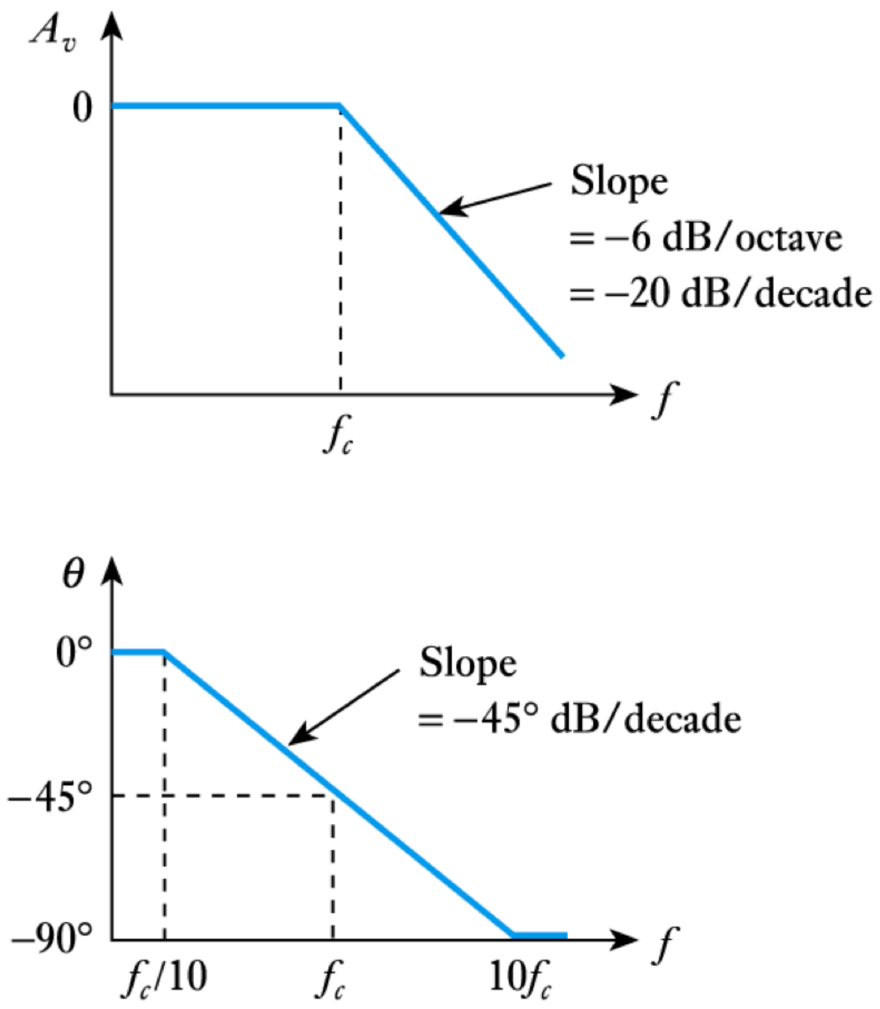


(a) (b) (c)

- Zeros and poles are respectively defined as the roots of the numerator and denominator of the transfer function.
- In this chapter, we are primarily interested in the magnitude of the transfer function.
- The magnitude of a complex number a + jb is given by $\sqrt{a^2 + b^2}$

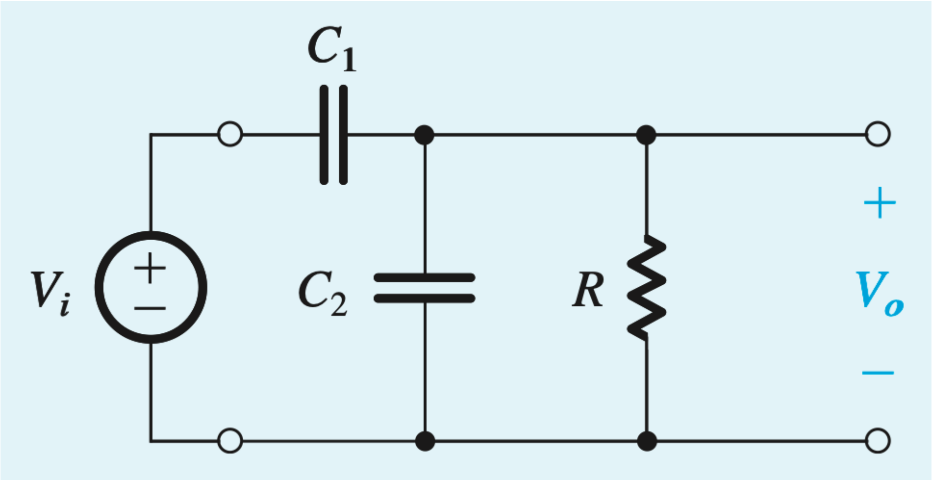
Bode Plot



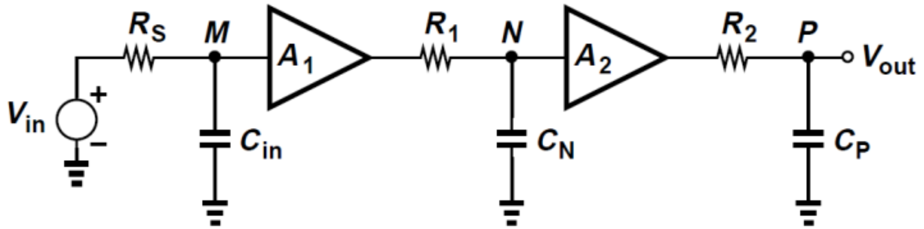


(b) Low-pass circuit

Example 1



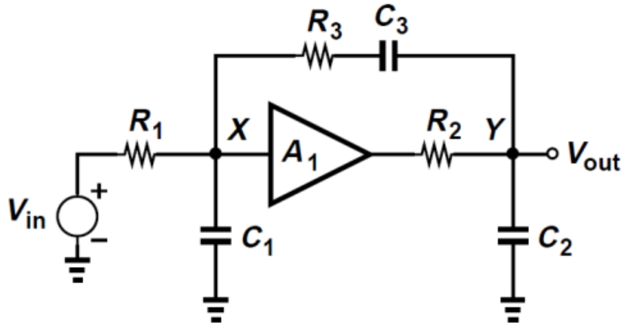
Association of Poles with Nodes



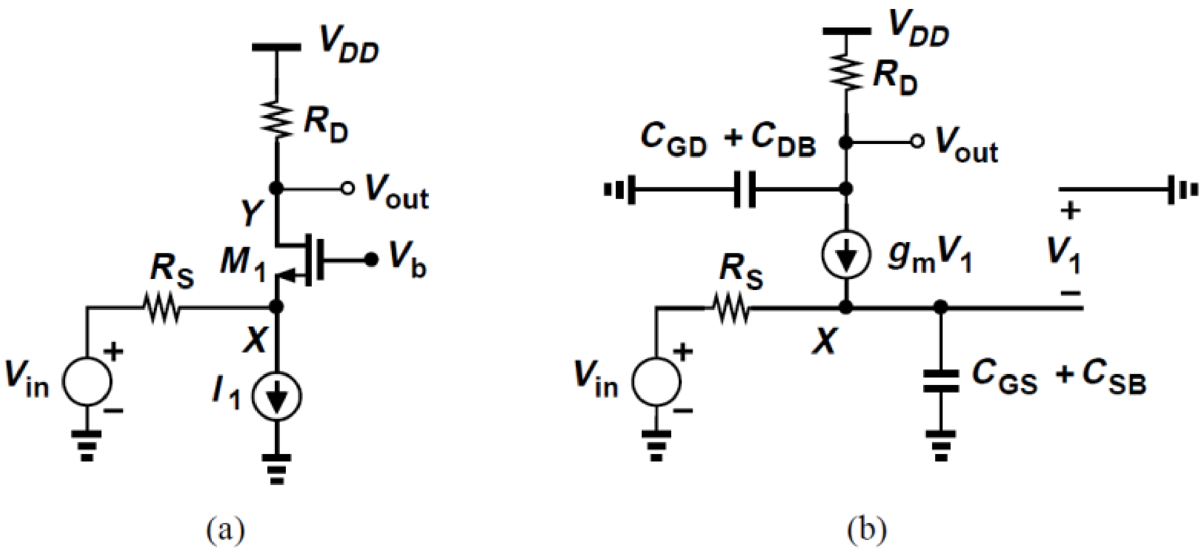
- The overall transfer function can be written as

$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + R_S C_{in} s} \cdot \frac{A_2}{1 + R_1 C_N s} \cdot \frac{1}{1 + R_2 C_P s}$$

- Each node in the circuit contributes one pole to the transfer function.
- Not valid in general. Example:



Example: Common-Gate Stage



- At node X:

$$\omega_{in} = \left[(C_{GS} + C_{SB}) \left(R_S \Bigg\| \frac{1}{g_m + g_{mb}} \right) \right]^{-1}$$

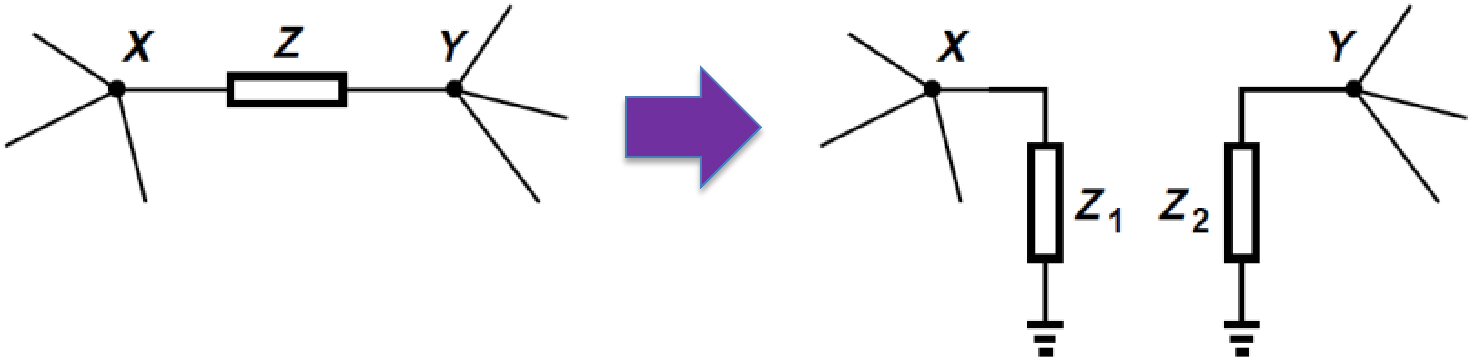
- At node Y:

$$\omega_{out} = [(C_{DG} + C_{DB}) R_D]^{-1}$$

- The overall transfer function:

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb}) R_D}{1 + (g_m + g_{mb}) R_S} \cdot \frac{1}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

Miller effect

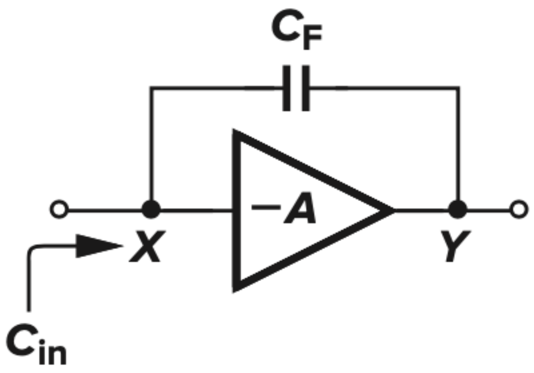


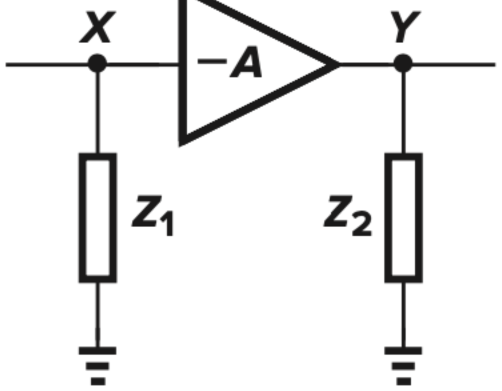
(a) (b)

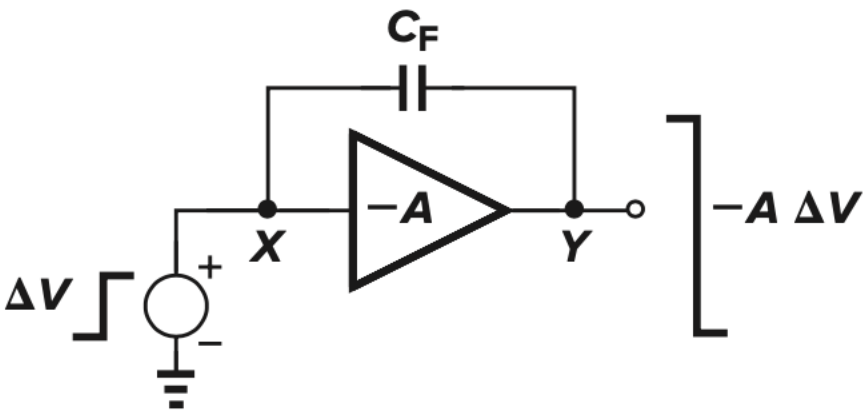
$$\frac{V_X - V_Y}{Z} = \frac{V_X}{Z_1}$$

$$Z_1 = \frac{Z}{1 - \frac{V_Y}{V_X}} \qquad Z_2 = \frac{Z}{1 - \frac{V_X}{V_Y}}$$

Example







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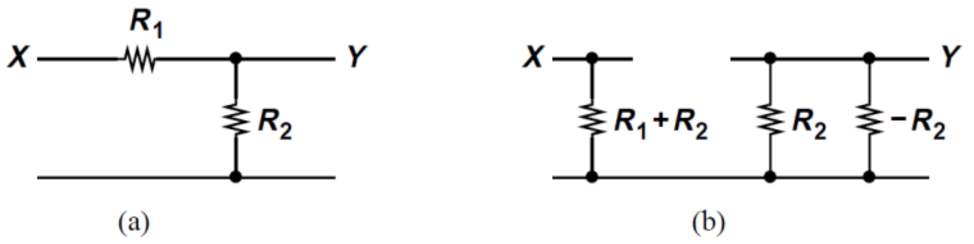
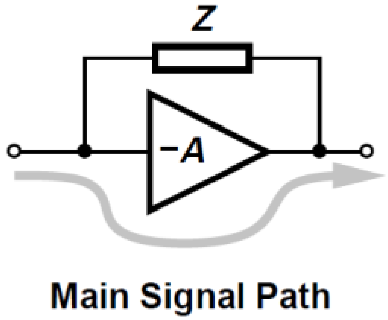
Validity of Miller's Theorem



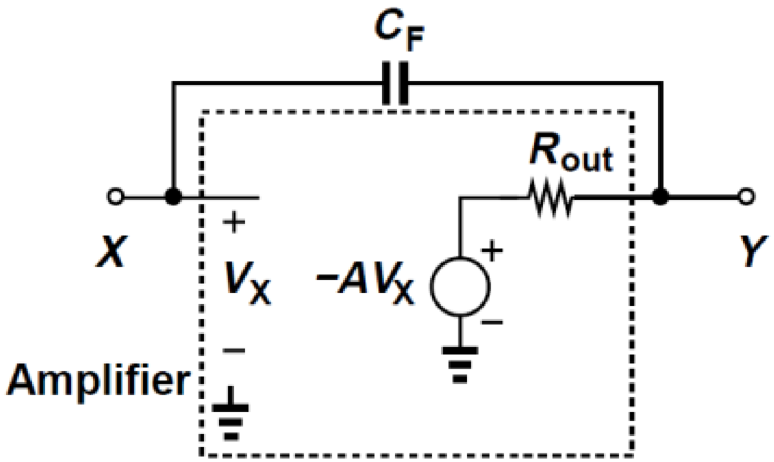
Improper application of Miller's theorem



Typical case for valid application of Miller's theorem.

- Miller's theorem does not stipulate the conditions under which this conversion is valid.
- If the impedance Z forms the **only signal path between X and Y** , then the conversion is often invalid.

Example

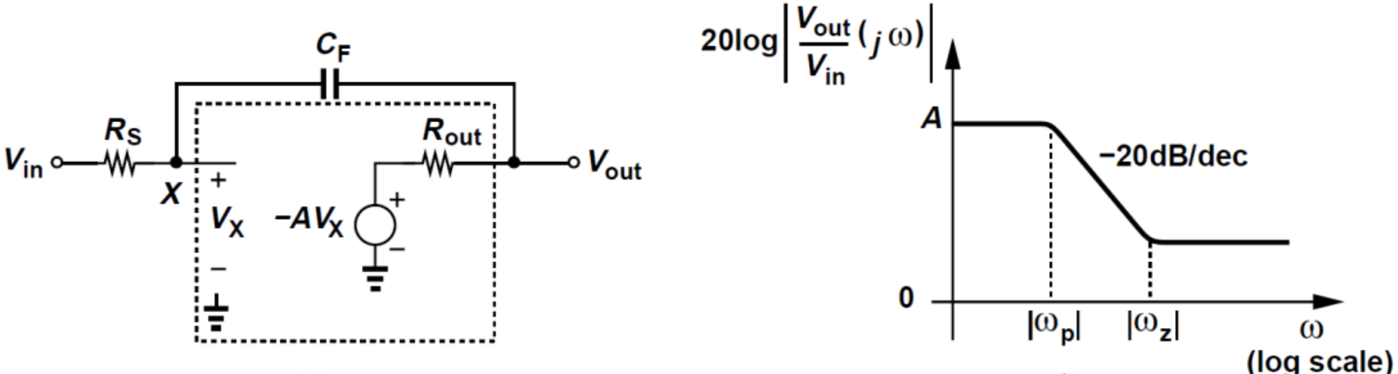


- The value of Av = V_Y / V_X must be calculated at the frequency of interest.
- In the figure, the equivalent circuit reveals that $V_Y \neq -AV_X$ at high frequencies.
- In many cases we use the low-frequency value of V_Y / V_X to gain insight.
- We call this approach "Miller's approximation."

Example

- Direct Calculation:

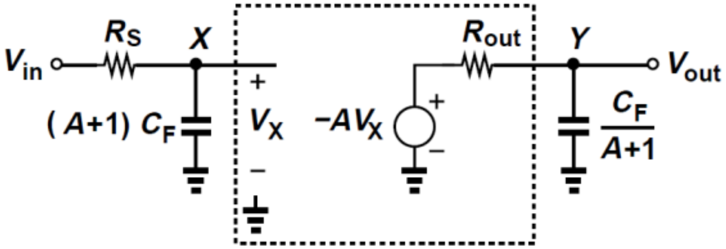
$$\frac{V_{out}}{V_{in}}(s) = \frac{R_{out} C_F s - A}{[(A+1)R_S + R_{out}]C_F s + 1}$$



(a) (b)

- Miller Approximation:

$$\frac{V_{out}}{V_{in}}(s) = \frac{-A}{[(1+A)R_S C_F s + 1]\left(\frac{1}{1+A^{-1}} C_F R_{out} s + 1\right)}$$



(c)

•Miller's approximation has eliminated the zero and predicted two poles for the circuit!

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Miller's approximation

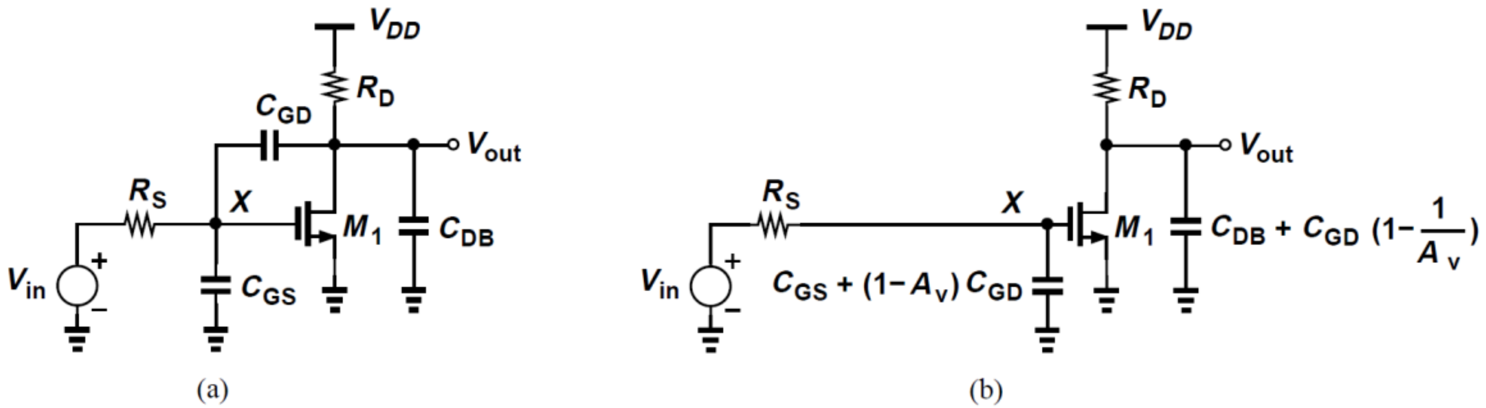
- *Miller's approximation*:

  (1) it may eliminate zeros

  (2) it may predict additional poles

  (3) it does not correctly compute the "output" impedance

Common-Source Stage



- The magnitude of the "input" pole (using Miller approximation):

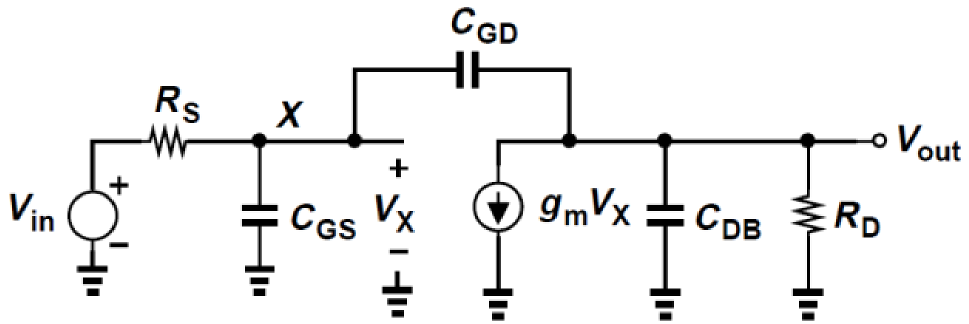
$$\omega_{in} = \frac{1}{R_S[C_{GS} + (1 + g_m R_D)C_{GD}]}$$

- At the output node

$$\omega_{out} = \frac{1}{R_D(C_{DB} + C_{GD})}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{-g_m R_D}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

Direct Analysis



$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1+g_mR_D)C_{GD} + R_SC_{GS} + R_D(C_{GD}+C_{DB})]s + 1}$$

- While the denominator appears rather complicated, it can yield intuitive expressions for the two poles. $|\omega_{p1}| \ll |\omega_{p2}|$
- "Dominant pole" approximation.

$$\omega_{p1} = \frac{1}{R_S(1+g_mR_D)C_{GD} + R_SC_{GS} + R_D(C_{GD}+C_{DB})}$$

- The intuitive approach provides a rough estimate with much less effort.

"Dominant Pole" Approximation

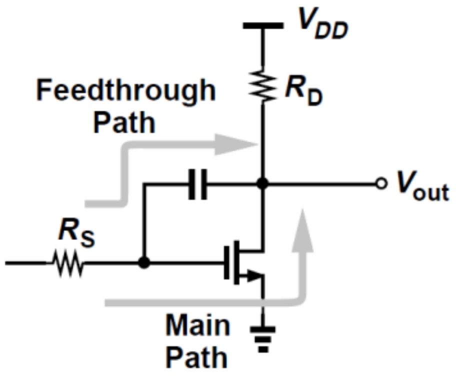
- Assume a two pole transfer function:

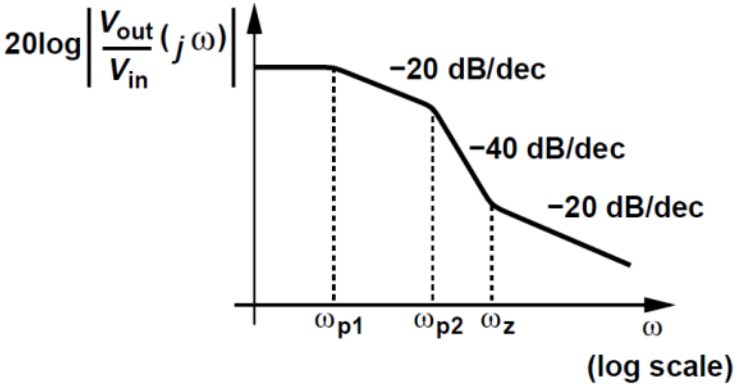
$$D = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right)$$

$$= \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$$

- If one pole will be dominant (: $|\omega_{p1}| \ll |\omega_{p2}|$), then:

Zero in Transfer Function





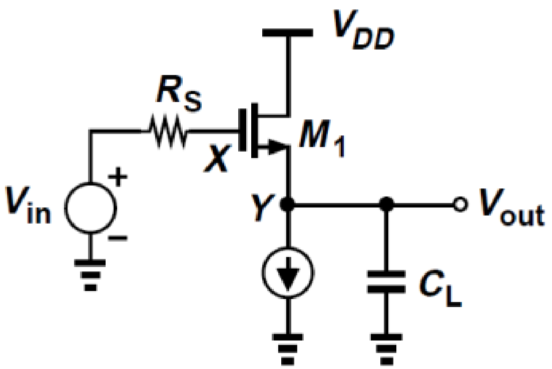
$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1}$$

- The transfer function of exhibits a zero given by

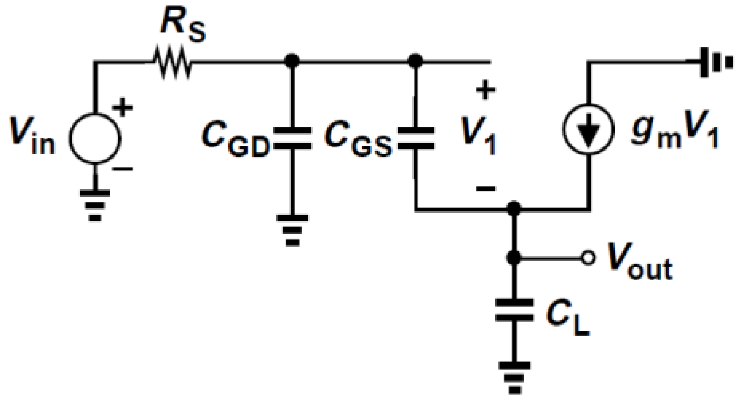
$$\omega_z = +g_m/C_{GD}$$

- CGD provides a feedthrough path that conducts the input signal to the output at very high frequencies.

Source Followers



(a)



(b)

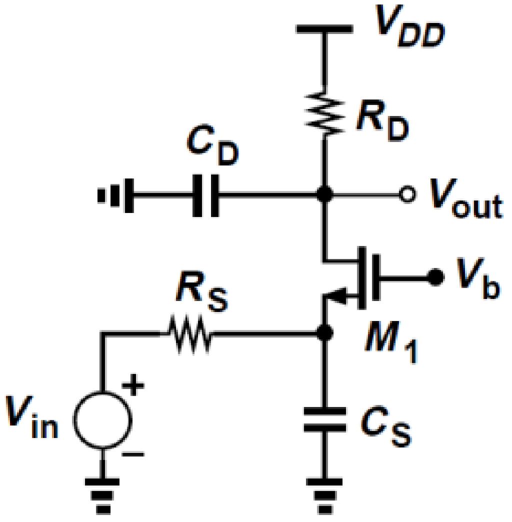
- The strong interaction between nodes X and Y through C_{GS} in makes it difficult to associate a pole with each node.

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$

- Contains a zero in the left half plane. Why?

Common-Gate Stage

- A transfer function (w/o channel length modulation):



$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \frac{1}{\left(1 + \frac{C_S}{g_m + g_{mb} + R_S^{-1}} s\right)(1 + R_D C_D s)}$$

- No Miller multiplication of capacitances.
- R_D is typically maximized, so the dc level of the input signal must be quite low.
- As an amplifier in cases where a low input impedance is required in cascode stages.

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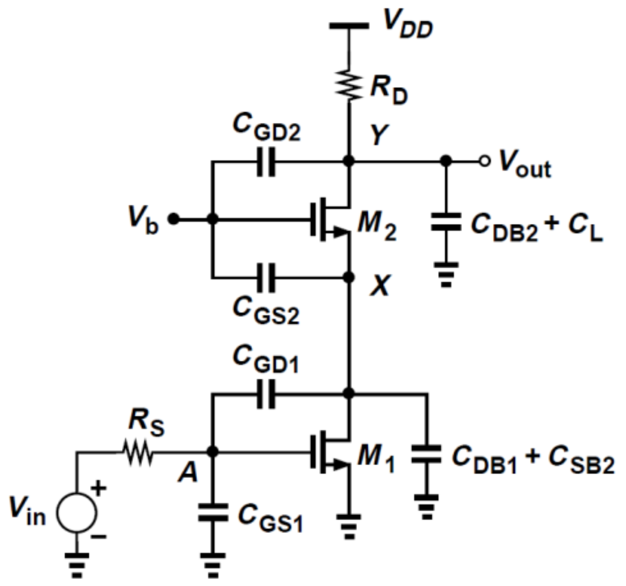
Cascode Stage



High-frequency model of a cascode stage.

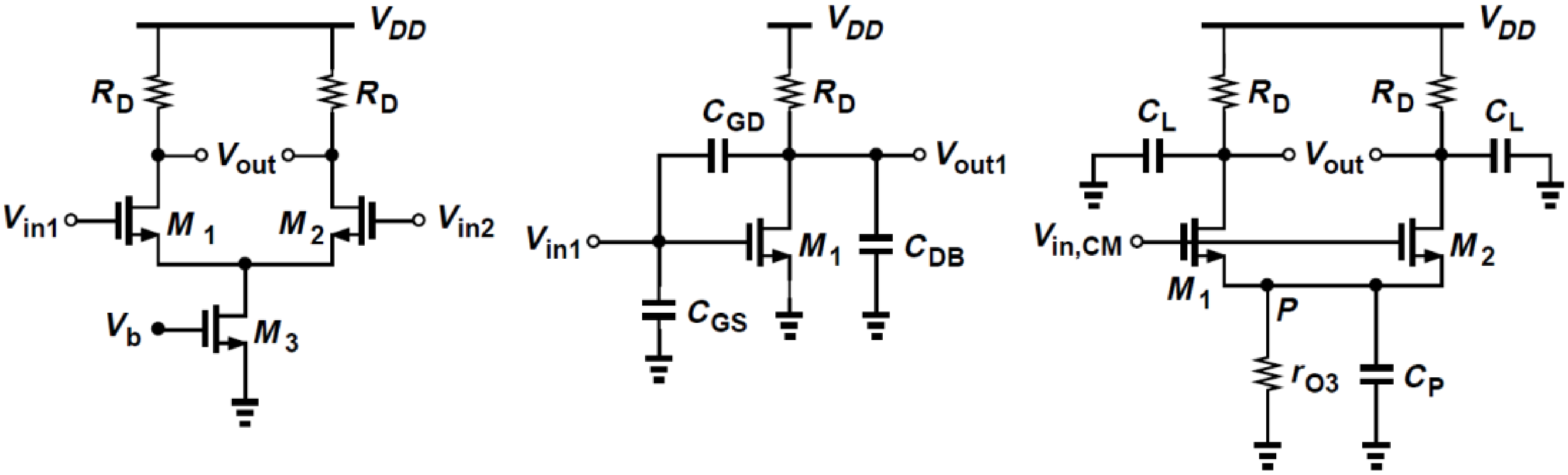
$$\omega_{p,A} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2} + g_{mb2}}\right)C_{GD1}\right]}$$

$$\omega_{p,X} = \frac{g_{m2} + g_{mb2}}{2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}}$$

$$\omega_{p,Y} = \frac{1}{R_D(C_{DB2} + C_L + C_{GD2})}$$

- Miller effect is less significant in cascode amplifiers than in common-source stages.
- But $\omega_{p,X}$ is typically quite higher than the other two.
- What if R_D is replaced by a current source?
  - Pole at node X may be quite lower, but transfer function will not affect much by this. See example.

Differential Pair

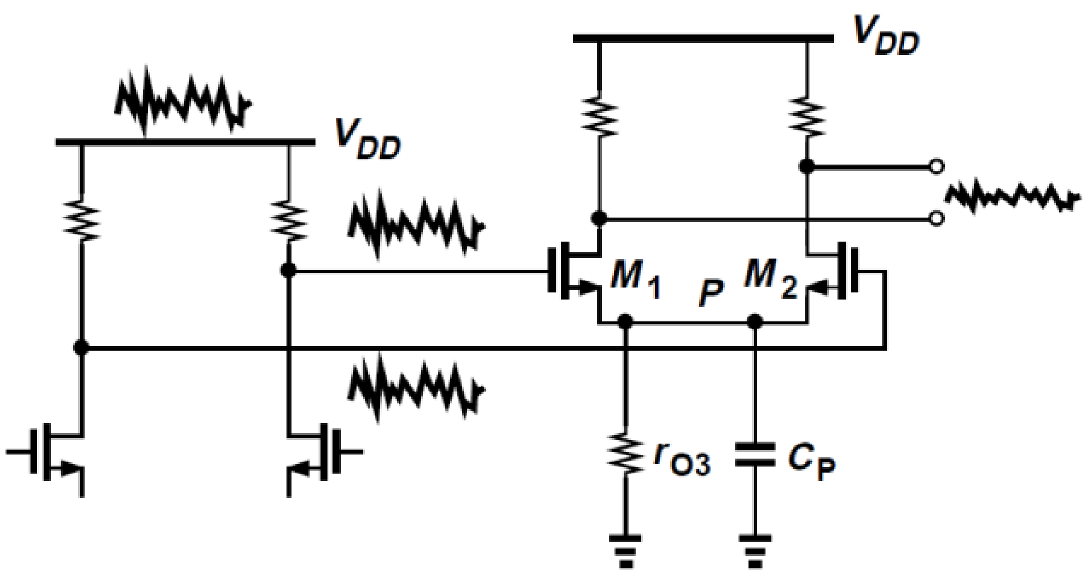


- For differential signals, the response is identical to that of a common-source stage.
- the common-mode rejection of the circuit degrades considerably at high frequencies.

$$A_{v,CM} = -\frac{\Delta g_m \left[R_D \middle\| \left(\frac{1}{C_L s} \right) \right]}{(g_{m1} + g_{m2}) \left[r_{O3} \middle\| \left(\frac{1}{C_P s} \right) \right] + 1}$$

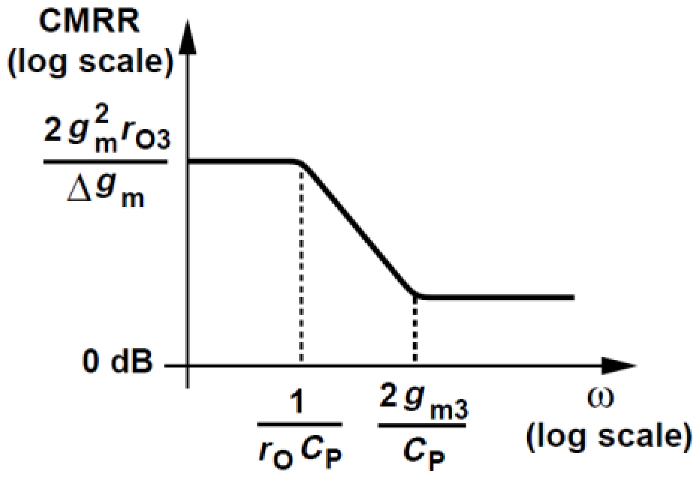
- Channel-length modulation, body effect, and other capacitances are neglected.

Differential Pair



$$\text{CMRR} \approx \frac{g_m}{\Delta g_m}\left[1 + 2g_m\left(r_{O3}||\frac{1}{C_P s}\right)\right]$$

$$\approx \frac{g_m}{\Delta g_m}\,\frac{r_{O3}C_P s + 1 + 2g_m r_{O3}}{r_{O3}C_P s + 1}$$



- This transfer function contains a zero and a pole.
- The magnitude of the zero is much greater than the pole.

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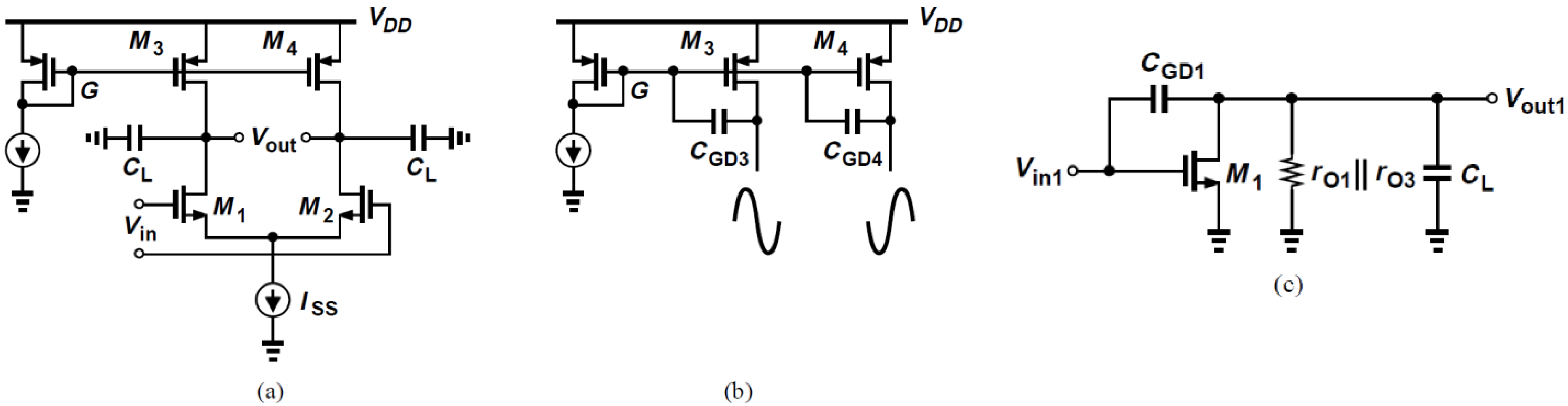
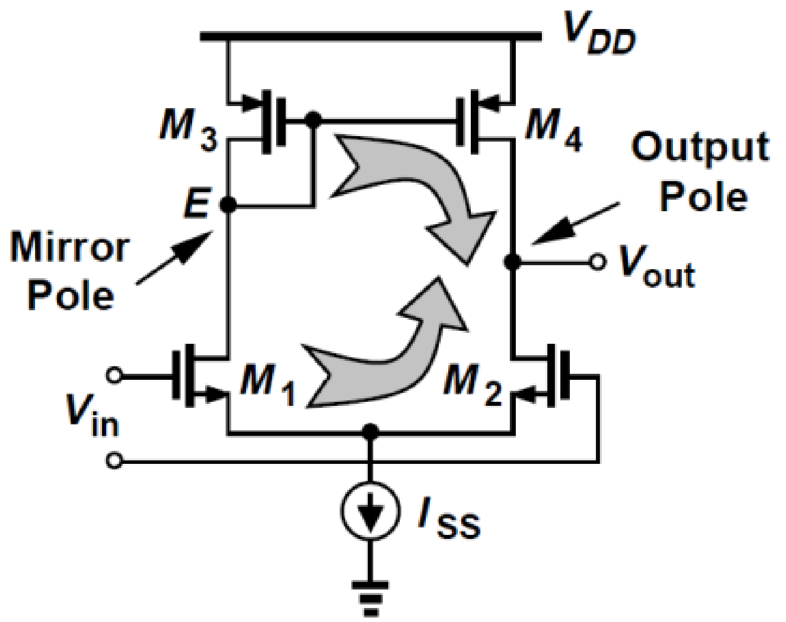
Differential Pair



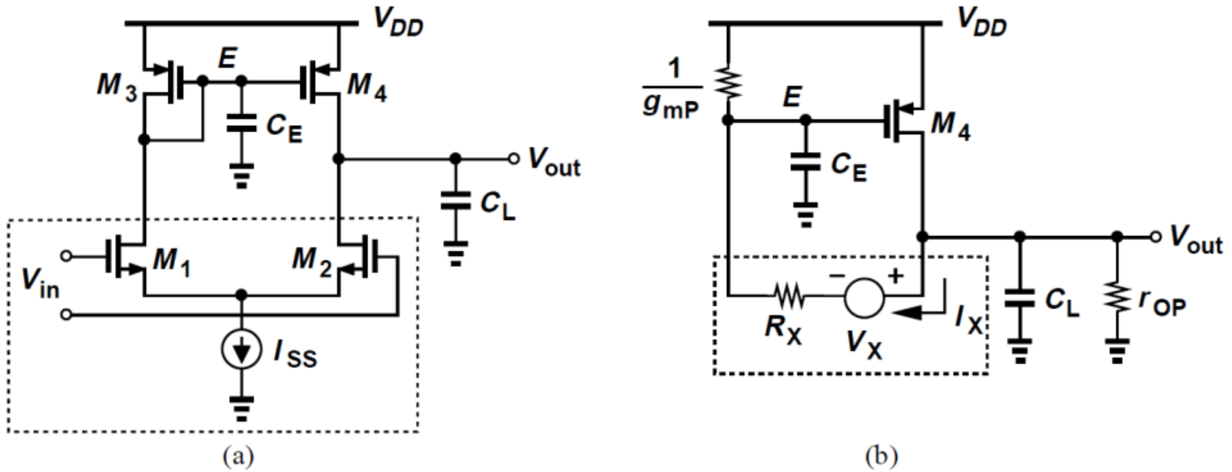
- Frequency response of differential pairs with high-impedance loads.
- Fig (b) C_{GD3} and C_{GD4} conduct equal and opposite currents to node G, making this node an ac ground.
- The differential half circuit is depicted in Fig. (c).
- More in chapter 10 …

Differential Pair with Active Load



- How many poles does this circuit have?
- The severe trade-off between g_m and C_{GS} of PMOS devices results in a pole that impacts the performance of the circuit.
- The pole associated with node E is called a “mirror pole.”

Active Load



- Replacing Vin, M1, and M2 by a Thevenin equivalent (Direct calculation method)

$$V_X = g_{mN} r_{ON} V_{in} \qquad R_X = 2r_{ON}$$

$$\frac{V_{out}}{V_{in}} = \frac{g_{mN} r_{ON}(2g_{mP} + C_E s) r_{OP}}{2r_{OP} r_{ON} C_E C_L s^2 + [(2r_{ON} + r_{OP})C_E + r_{OP}(1 + 2g_{mP} r_{ON})C_L]s + 2g_{mP}(r_{ON} + r_{OP})}$$

$$\omega_{p1} \approx \frac{1}{(r_{ON} \| r_{OP})C_L} \qquad \omega_{p2} \approx \frac{g_{mP}}{C_E}$$

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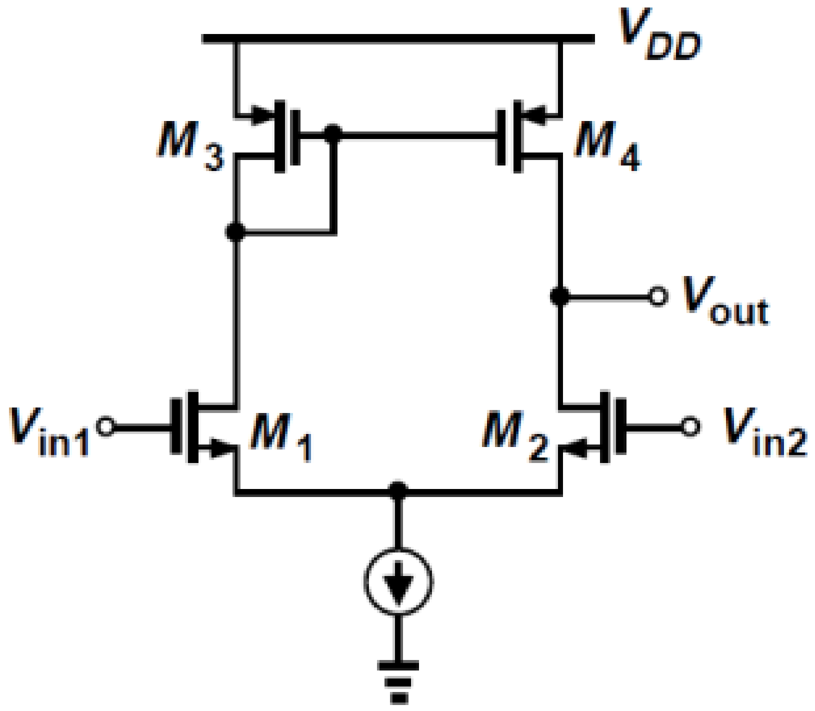
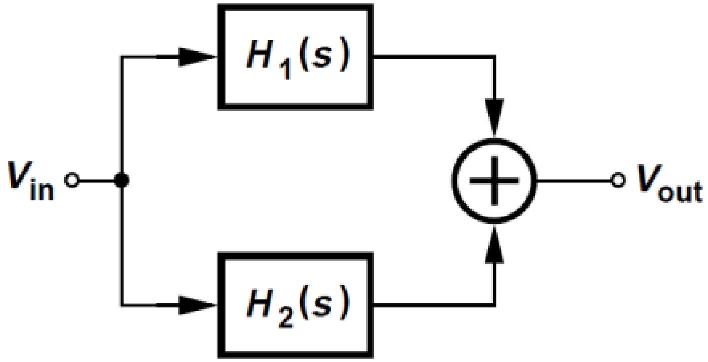


Example



- Can this (the zero) occur if H1(s) and H2(s) are first-order low-pass circuits?
- $H_1(s)$ = $A_1/(1+s/\omega_{p1})$ and $H_2(s)$ = $A_2/(1+s/\omega_{p2})$

$$\frac{V_{out}}{V_{in}}(s) = \frac{\left(\frac{A_1}{\omega_{p2}} + \frac{A_2}{\omega_{p1}}\right)s + A_1 + A_2}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

- The overall transfer function contains a zero.

Active Load

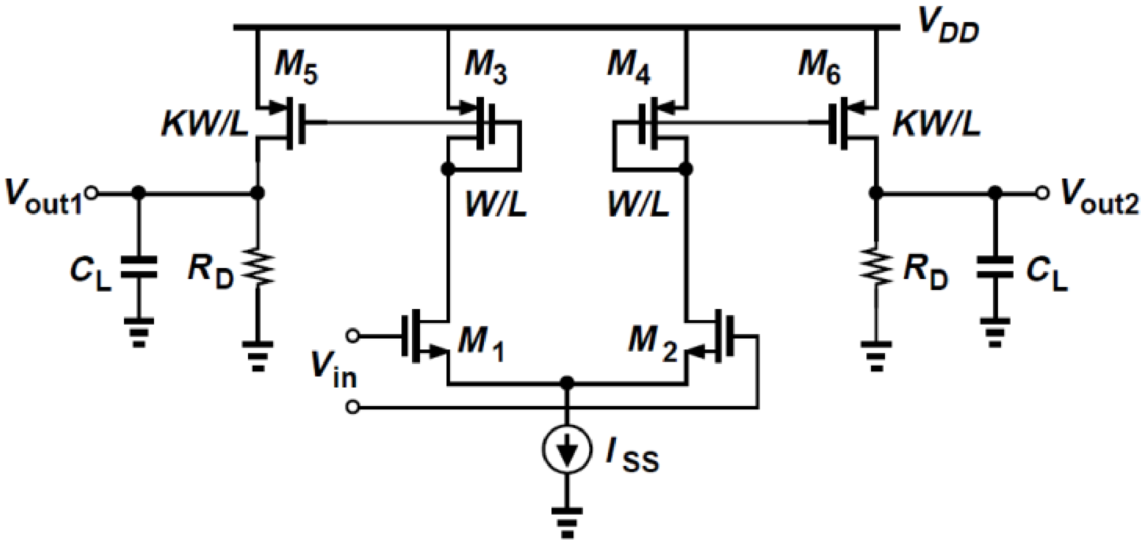
$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + s/\omega_{p1}} \left(\frac{1}{1 + s/\omega_{p2}} + 1 \right)$$

$$= \frac{A_0(2 + s/\omega_{p2})}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}$$

- A zero with a magnitude of $2g_{mP}/C_E$ in the left half plane.
- The appearance of this zero can be understood by noting that the circuit consists of:
- a “slow path” (M1,M3 and M4) $A_0/[(1 + s/\omega_{p1})(1 + s/\omega_{p2})]$ in parallel with
- a “fast path” (M1 and M2) by $A_0/(1 + s/\omega_{p1})$

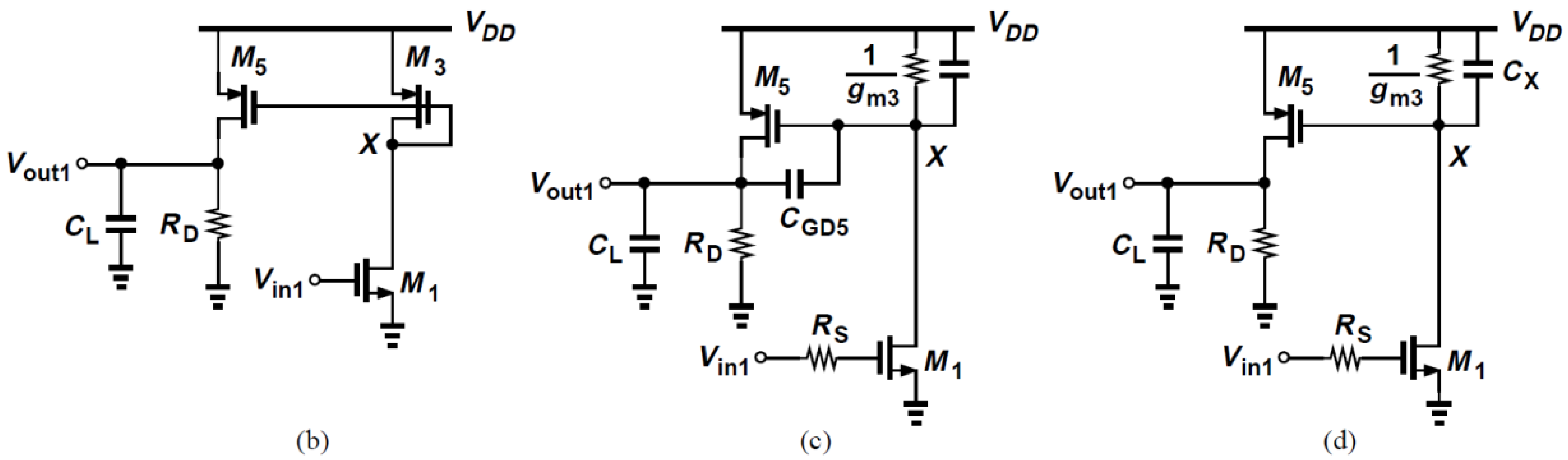
Example

- Estimate the low-frequency gain and the transfer function of this circuit.



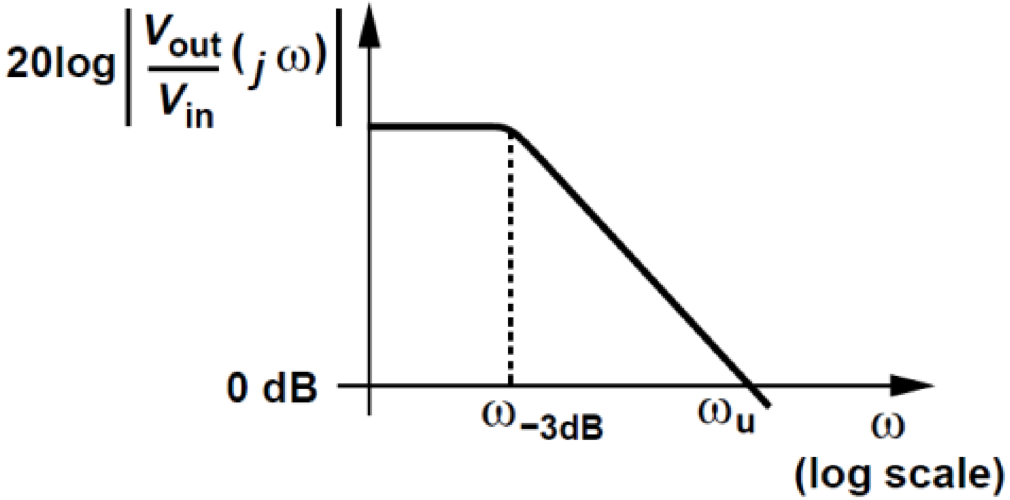
(a)

Example



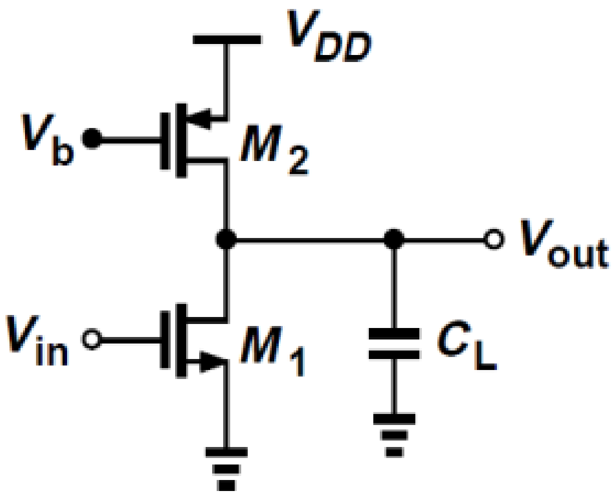
V_{DD}
M_5
M_3
X
V_{out1}
C_L
R_D
V_{in1}
M_1
(b)
V_{DD}
M_5
$\frac{1}{g_{m3}}$
X
V_{out1}
C_{GD5}
C_L
R_D
R_S
V_{in1}
M_1
(c)
V_{DD}
M_5
$\frac{1}{g_{m3}}$
C_X
X
V_{out1}
C_L
R_D
R_S
V_{in1}
M_1
(d)

Gain-Bandwidth Trade-Offs



- We wish to maximize both the gain and the bandwidth of amplifiers.
- We are interested in both the 3-dB bandwidth, ω_{-3dB} , and the “unity-gain” bandwidth, ω_u

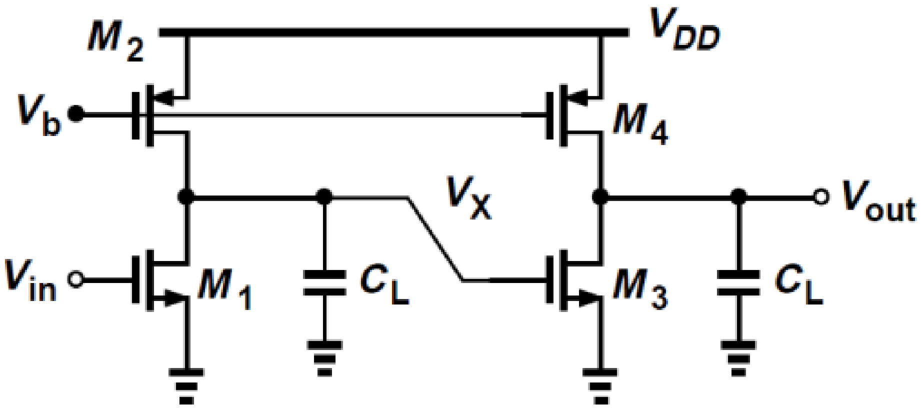
One pole circuit



$$\begin{aligned}\mathrm{GBW} &= A_0\omega_p \\ &= g_{m1}(r_{O1}||r_{O2})\frac{1}{2\pi(r_{O1}||r_{O2})C_L} \\ &= \frac{g_{m1}}{2\pi C_L}\end{aligned}$$

$$\begin{aligned}\omega_u &= \sqrt{A_0^2-1}\omega_p \\ &\approx A_0\omega_p\end{aligned}$$

Multi-Pole Circuits



- It is possible to increase the GBW product by cascading two or more gain stages.
- Assume the two stages are identical and neglect other capacitances.

$$\frac{V_{out}}{V_{in}} = \frac{A_0^2}{(1+\frac{s}{\omega_p})^2}$$

$$\omega_{-3dB} = \sqrt{\sqrt{2}-1}\omega_p \approx 0.64\omega_p$$

$$\mathrm{GBW} = \sqrt{\sqrt{2}-1}A_0^2\omega_p$$

- While raising the GBW product, cascading reduces the bandwidth.