EE 332: Devices and Circuits II

Lecture 6: Frequency Response

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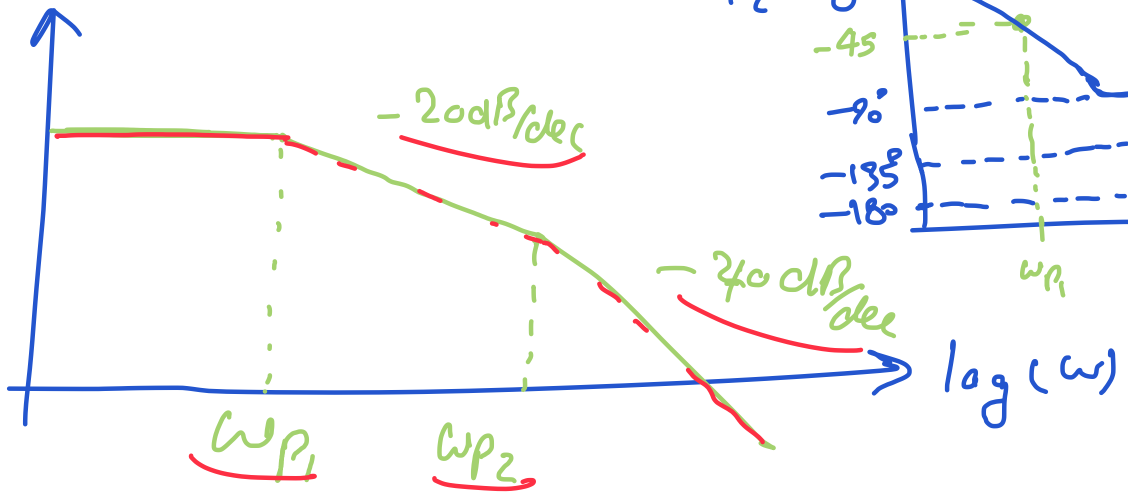
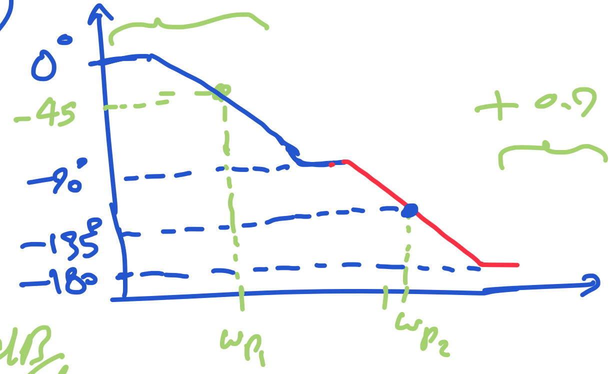
$$|H(s)| = \frac{1}{\left(1+\frac{s}{\omega_{p_1}}\right)\left(1+\frac{s}{\omega_{p_2}}\right)} \rightarrow \angle H(s) = -\tan^{-1}\left(\frac{\omega}{\omega_{p_1}}\right) - \tan^{-1}\left(\frac{\omega}{\omega_{p_2}}\right)$$

$|H(s)|$ in dB

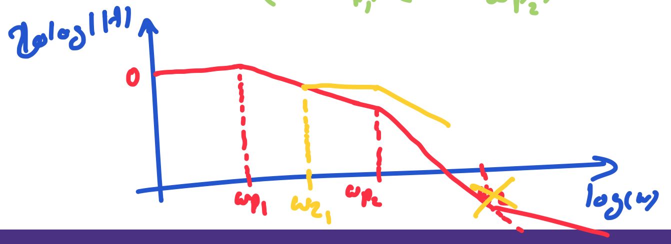


$A = -1$



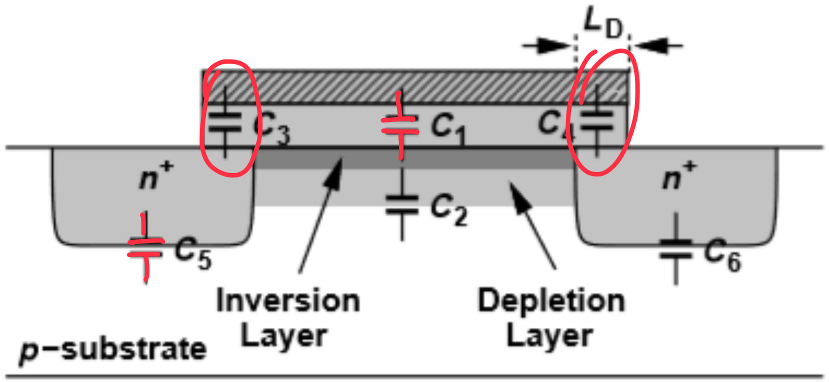
$+0.9$

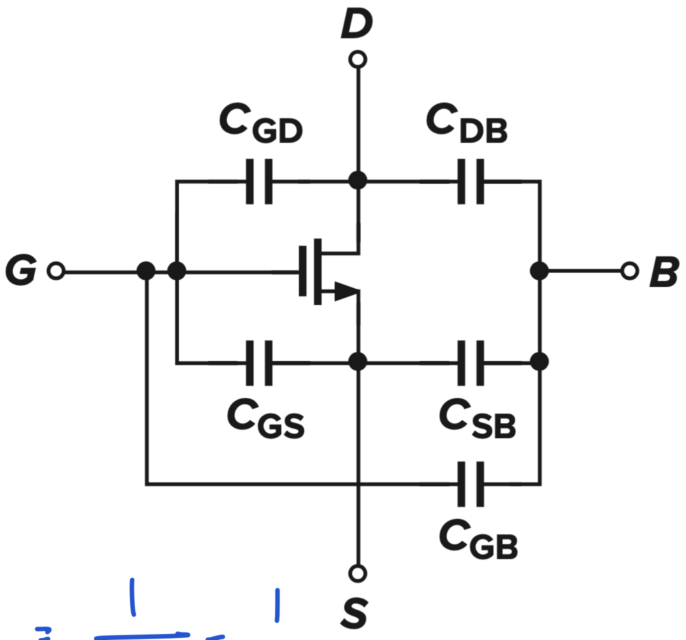
$$H(s) = \frac{\left(1+\frac{j\omega}{\omega_{z_1}}\right)}{\left(1+\frac{j\omega}{\omega_{p_1}}\right)\left(1+\frac{j\omega}{\omega_{p_2}}\right)}$$



MOS Device Capacitances

L → $Z_L = sL = j\omega L$





$C \propto WL$

$\propto W, L, E$

C → $Z_c = \frac{1}{sC} = \frac{1}{j\omega C}$

- Device capacitances cause frequency dependent Av, Z_{in}, Z_{out}, etc.
- Capacitance exists almost between every two of the four terminals
- Larger device -> Larger caps!

except D-S

Frequency Response

$v_{in} \sin(\omega t) \rightarrow \boxed{\text{LTI}} \rightarrow v_{out} = A_\omega \sin(\omega t + \varphi_\omega)$

$\boxed{s \longrightarrow j\omega}$ $\quad |T(j\omega)| \quad \measuredangle T(j\omega)$

$N(s_z) = 0 \longrightarrow s_z$ zero

$D(s_p) = 0 \longrightarrow s_p$ pole

$$T(s) = \frac{\overbrace{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_0}^{N(s)}}{\underbrace{s^n + b_{n-1} s^{n-1} + \ldots + b_0}_{D(s)}} \Rightarrow T(s) = a_m \frac{(s - Z_1)(s - Z_2)\ldots(s - Z_m)}{(s - P_1)(s - P_2)\ldots(s - P_n)}$$

m zeros

n poles

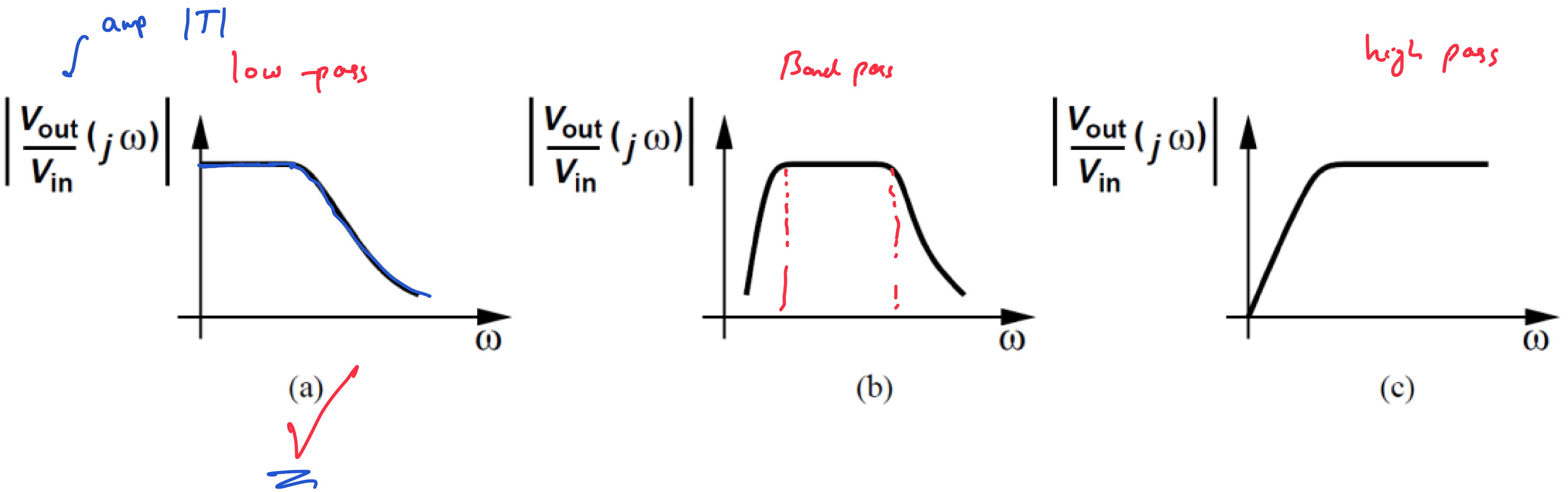
$Z = -\omega_{z_1}, -\omega_{z_2}, \ldots, -\omega_{z_n}$

$P = -\omega_{p_1}, -\omega_{p_2}, \ldots, -\omega_{p_m}$

$$H(s) = \frac{b_n}{a_m} \cdot \frac{\left(1 + \frac{s}{\omega_{z1}}\right)\left(1 + \frac{s}{\omega_{z2}}\right)\cdots\left(1 + \frac{s}{\omega_{zn}}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)\cdots\left(1 + \frac{s}{\omega_{pm}}\right)}$$

DC gain

General Considerations

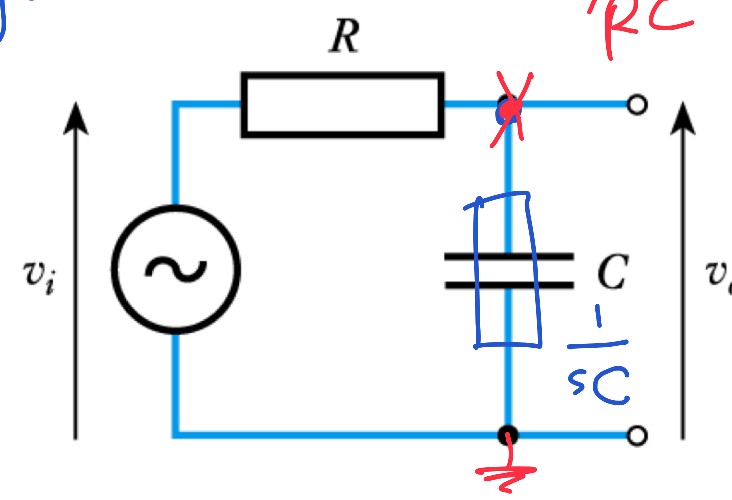


- Zeros and poles are respectively defined as the roots of the numerator and denominator of the transfer function.
- In this chapter, we are primarily interested in the magnitude of the transfer function.
- The magnitude of a complex number a + jb is given by $\sqrt{a^2 + b^2}$

Bode Plot

$$\frac{1}{sC} = \frac{-j}{\omega C}$$

$s = j\omega$



$\frac{1}{RC} \rightarrow A_v$

$$T = \frac{v_o}{v_{in}} = ?$$

$$\boxed{* \quad \omega = 2\pi f}$$

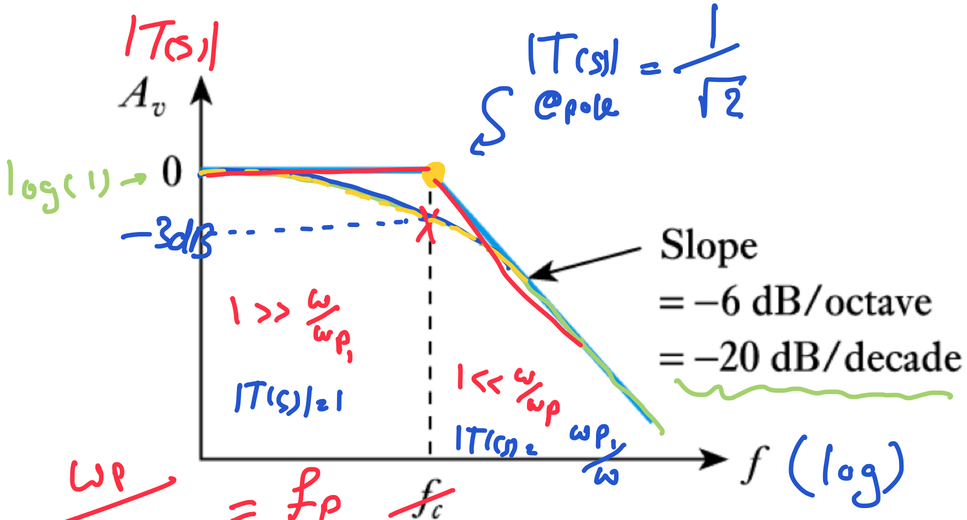
$$A_v = 20\log(|T(s)|)$$

$$T(s) = \frac{Z_C}{Z_C + Z_R} = \frac{1/sC}{1/sC + R} = \frac{1}{1 + sRC}$$

$$\left(\text{pole @ } -\frac{1}{RC}\right) = \frac{1}{1 + j\omega/\omega_{p1}} \rightarrow |T(s)|, \ |\angle T(s)|$$

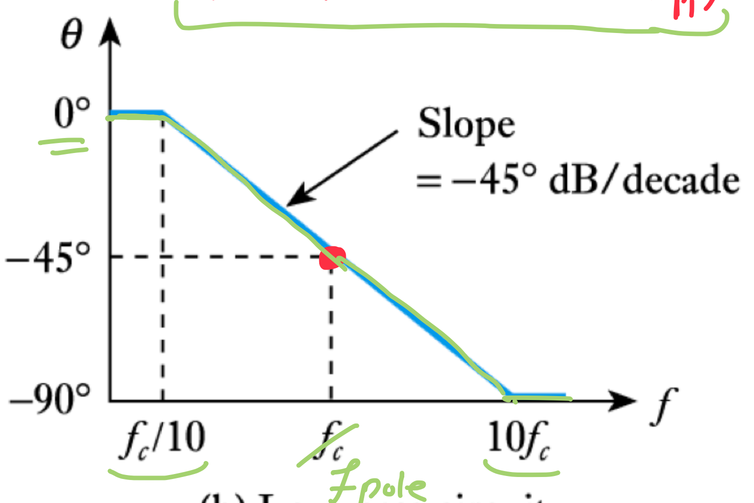
$$\omega_{p1} = 1/RC$$

$$|T(s)| = \frac{1}{\sqrt{1 + (\omega/\omega_{p1})^2}} \rightarrow 20\log|T(s)| = \dots$$



$$\frac{\omega_p}{2\pi} = f_p \quad \cancel{f_c}$$

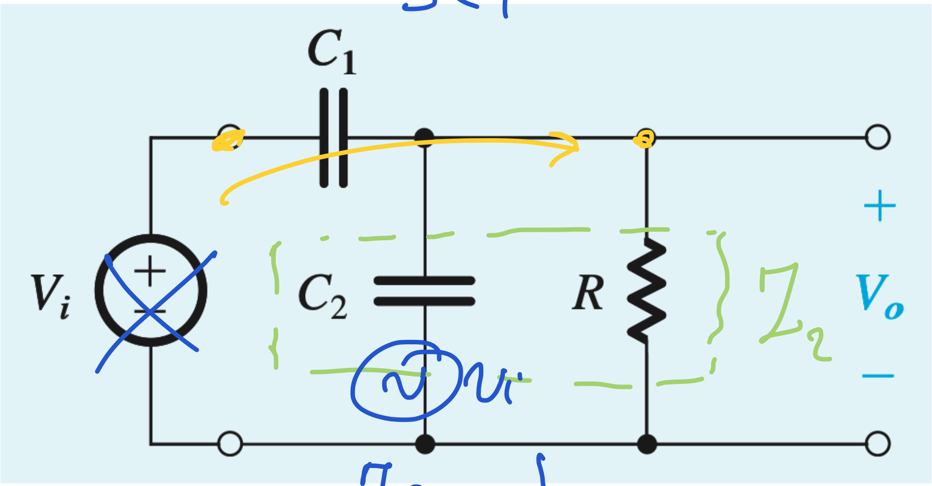
$$\angle T(s) = -\tan^{-1}(\omega/\omega_{p1})$$



(b) Low-pass circuit

Example 1

$Z_{C_1} = \frac{1}{sC_1}$



$Z_{C_2} = \frac{1}{sC_2}$

$$Z_2(s) = ? \quad \frac{1}{\frac{1}{R} + \cancel{\frac{1}{Z_{C_2}}}\, sC_2} = \frac{R}{1+sRC_2}$$

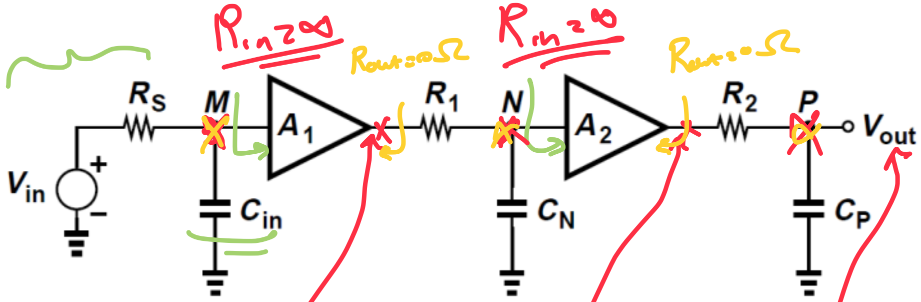
$$\frac{V_o}{V_i}(s) = \frac{Z_2}{Z_2 + Z_{C_1}} = \frac{\frac{RsC_1}{\cancel{1+sRC_2}}}{\frac{RsC_1}{\cancel{1+sC_2R}} + \frac{(1+sRC_2)}{\cancel{sC_1}}}$$

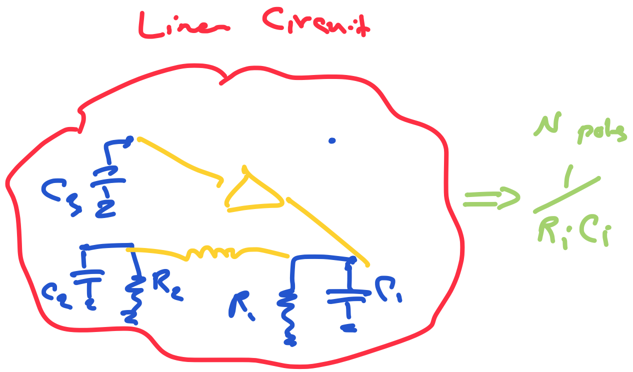
$$= \frac{sRC_1}{1+sR(C_1+C_2)}$$

→ zero @ $\omega_z = 0$

→ pole @ $\omega_p = -1/R(C_1+C_2)$

Association of Poles with Nodes

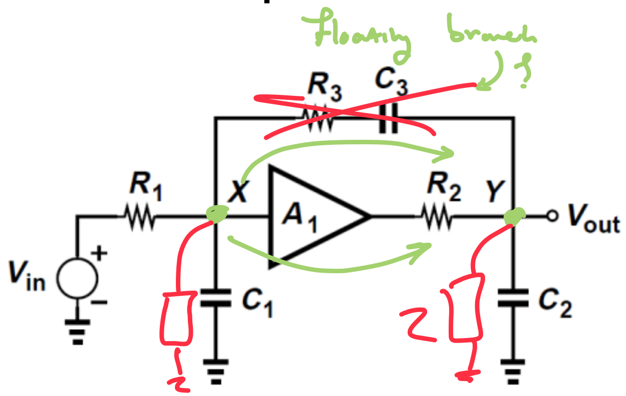




- The overall transfer function can be written as

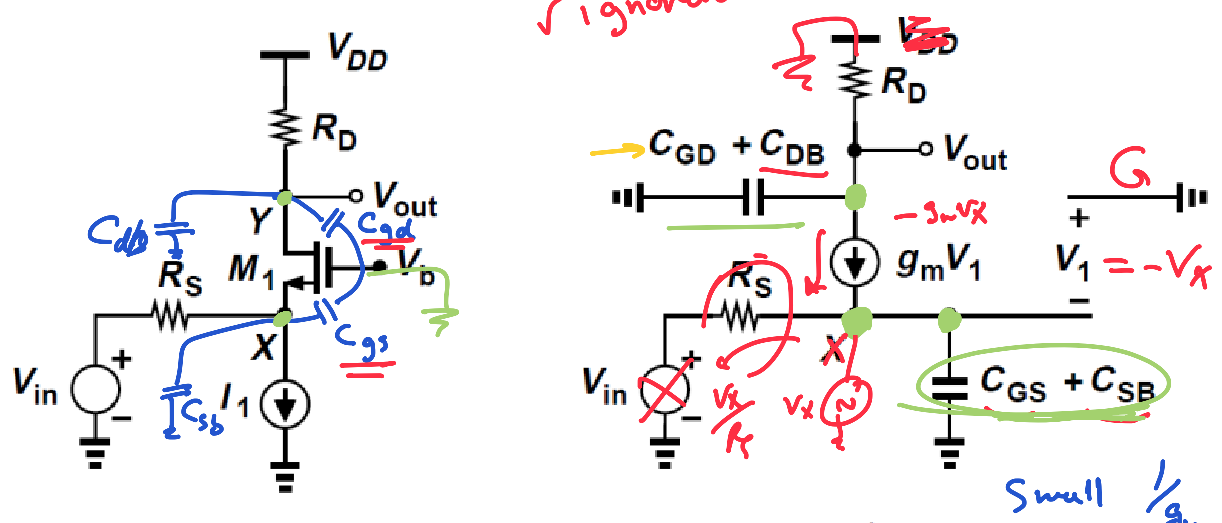
$$\frac{V_M}{V_{in}} \times \frac{V_N}{V_M} \times \frac{V_{out}}{V_N} = \frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1+R_SC_{in}s}\cdot\frac{A_2}{1+R_1C_Ns}\cdot\frac{1}{1+R_2C_Ps}$$

- Each node in the circuit contributes one pole to the transfer function.
- Not valid in general. Example:



Example: Common-Gate Stage ✓

✓ ignored r_o



(a) (b)

small $1/g_m$

- At node X:

$$\omega_{in} = \left[(C_{GS} + C_{SB}) \left(R_S \,\Big\|\, \frac{1}{g_m + \cancel{g_{mb}}} \right) \right]^{-1}$$

$\omega_{out} \ll \omega_{in}$

- At node Y:

$$\omega_{out} = [(C_{DG} + C_{DB}) R_D]^{-1}$$

$1/g_m \ll R_D$

- The overall transfer function:

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + \cancel{g_{mb}}) R_D}{1 + (g_m + \cancel{g_{mb}}) R_S} \cdot \frac{1}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

DC gain

2 pole

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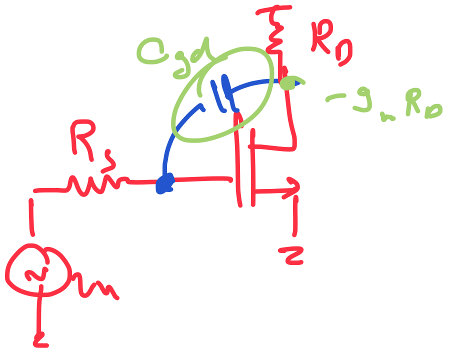
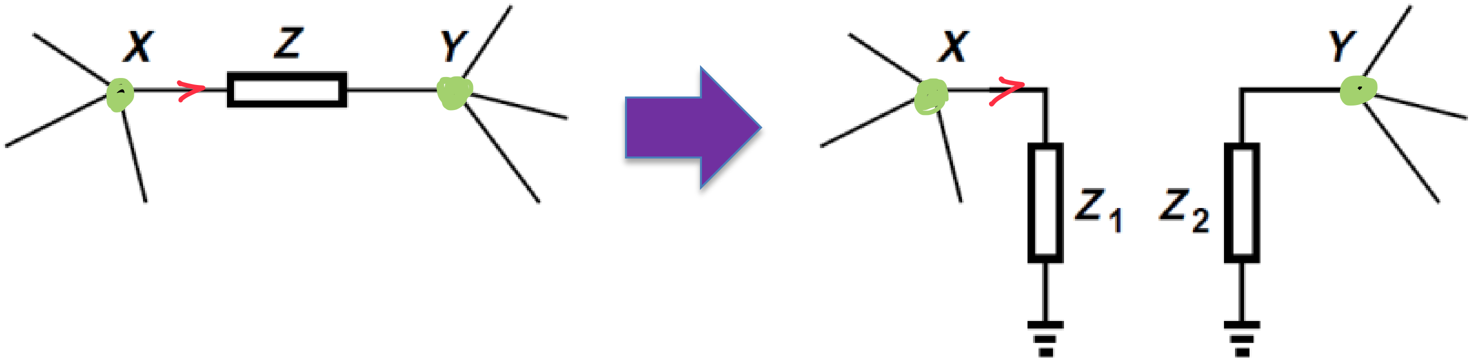


Miller effect



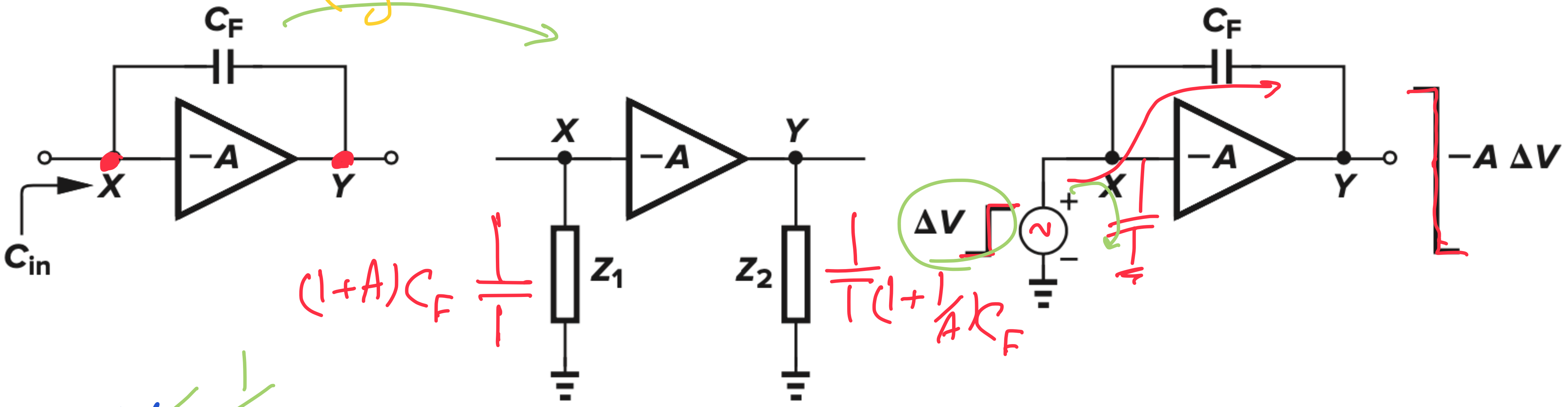
(a) (b)

$$I_Z = \frac{V_X - V_Y}{Z} = \frac{V_X}{Z_1}$$

$$\boxed{Z_1 = \frac{Z}{1 - \frac{V_Y}{V_X}}} \qquad \boxed{Z_2 = \frac{Z}{1 - \frac{V_X}{V_Y}}}$$

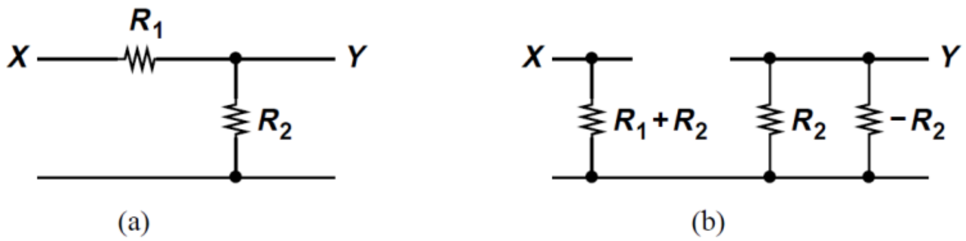
$$\frac{V_y}{V_x} = -A \rightarrow \text{low freq gain (ignore Caps ✓)}$$

Example

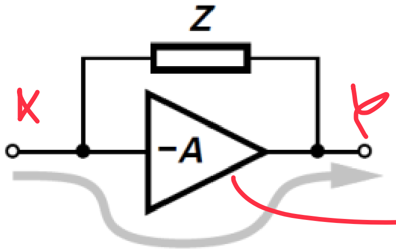


$$\left\{\begin{aligned} Z_1 &= \frac{\cancel{Z_F}\ \frac{1}{sC_F}}{1+A} \\ Z_2 &= \frac{Z_F}{1+\frac{1}{A}} \end{aligned}\right. \Longrightarrow \left\{\begin{aligned} Z_1 &= \frac{1}{s(1+A)C_F} \\ Z_2 &= \frac{1}{s(1+\frac{1}{A})C_F} \end{aligned}\right. \xrightarrow{A \gg 1} \boxed{\begin{aligned} C_1 &= (1+A)C_F \\ C_2 &\approx C_F \end{aligned}}$$

Validity of Miller's Theorem



Improper application of Miller's theorem

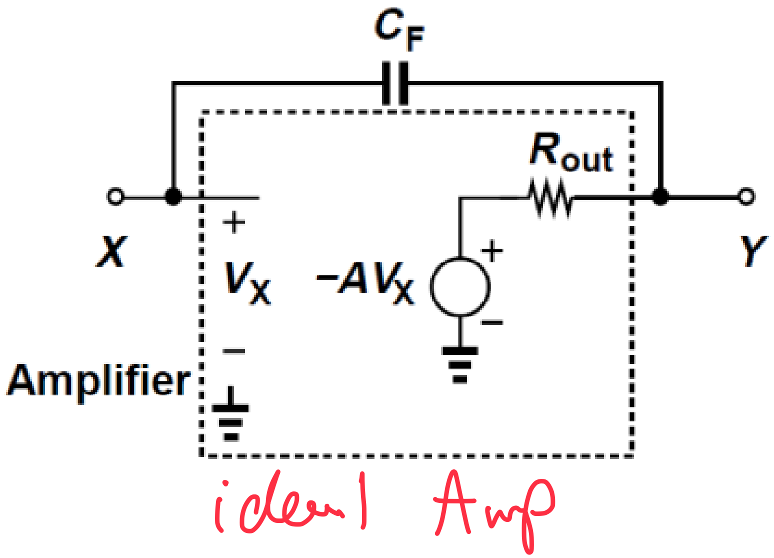


two paths betwen X & Y
A >> 1

Typical case for valid application of Miller's theorem.

- Miller's theorem does not stipulate the conditions under which this conversion is valid.
- If the impedance Z forms the **only signal path between X and Y** , then the conversion is often invalid.

Example



- The value of Av = V_Y / V_X must be calculated at the frequency of interest.
- In the figure, the equivalent circuit reveals that $V_Y \neq -AV_X$ at high frequencies.
- In many cases we use the low-frequency value of V_Y / V_X to gain insight.
- We call this approach "Miller's approximation."

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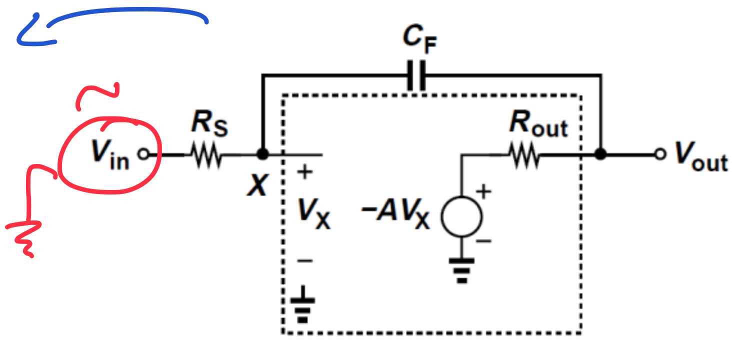
Example

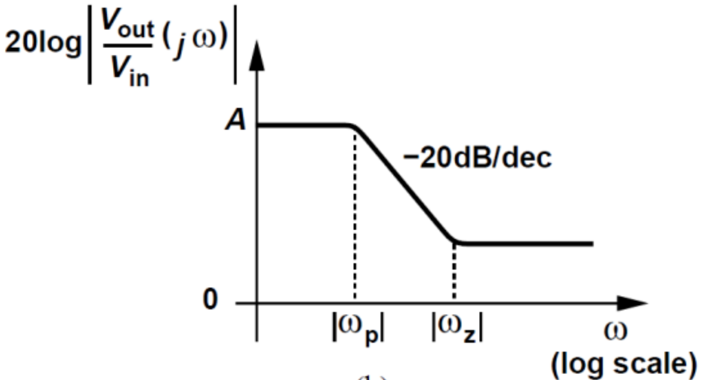
- Direct Calculation:

$$\frac{V_{out}}{V_{in}}(s) = \frac{R_{out}C_F s - A}{[(A+1)R_S + R_{out}]C_F s + 1}$$

1 Zero & 1 pole



(a)

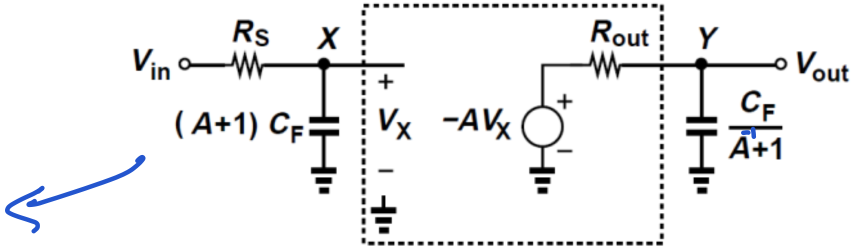


(b)

- Miller Approximation:

$$\frac{V_{out}}{V_{in}}(s) = \frac{-A}{[(1+A)R_S C_F s + 1]\left(\frac{1}{1+A^{-1}}C_F R_{out} s + 1\right)}$$

No zeros & 2 poles



(c)

Miller Est.

•Miller's approximation has eliminated the zero and predicted two poles for the circuit!

✓ Dominant pole

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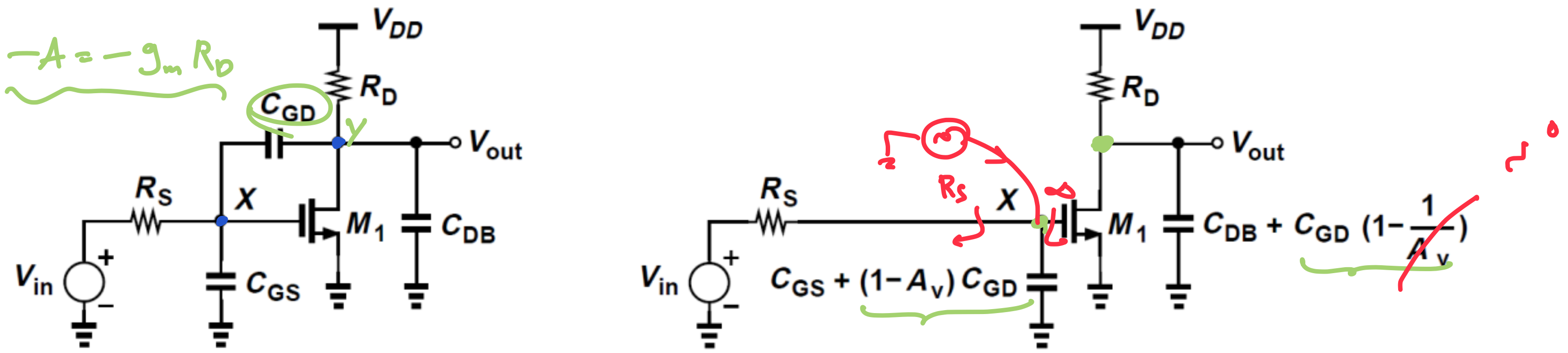
Miller's approximation

- *Miller's approximation*:
  (1) it may eliminate zeros
  (2) it may predict additional poles
  (3) it does not correctly compute the "output" impedance

✓ predicts the $\underbrace{\text{dominant pole}}_{\sim 3\text{dB BW of Amp.}}$

Miller effect

Common-Source Stage



(a) (b)

- The magnitude of the “input” pole (using Miller approximation):

$A_v = -g_m R_D \Rightarrow$

$$\omega_{in} = \frac{1}{R_S[C_{GS} + (1 + g_m R_D)C_{GD}]}$$

- At the output node

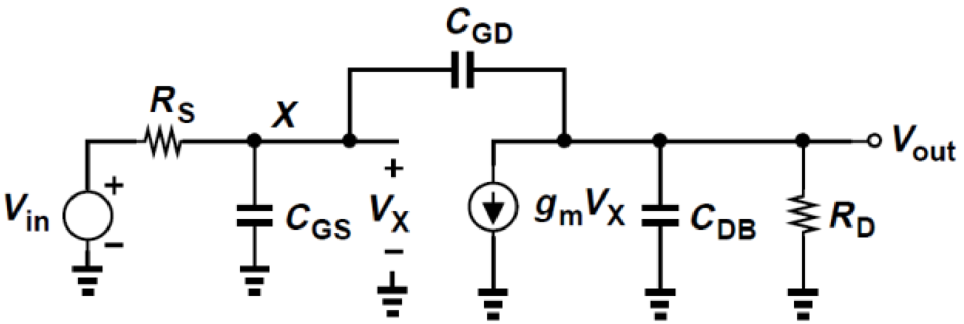
$$\omega_{out} = \frac{1}{R_D(C_{DB} + C_{GD})}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{-g_m R_D}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

$-\omega_{in}, -\omega_{out}$

Direct Analysis

1 zero
2 pole



$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1}$$

- While the denominator appears rather complicated, it can yield intuitive expressions for the two poles. $|\omega_{p1}| \ll |\omega_{p2}|$
- "Dominant pole" approximation.

$$\omega_{p1} = \frac{1}{R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})}$$

- The intuitive approach provides a rough estimate with much less effort.

"Dominant Pole" Approximation

- Assume a two pole transfer function:

$\omega_{p_1} << \omega_{p_2}$

$$D = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right)$$

$$= \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$$

$$\underbrace{A s^2 + B s + C}$$

$\frac{1}{\omega_{p_1}}$

- If one pole will be dominant ($|\omega_{p1}| \ll |\omega_{p2}|$), then:

$$\begin{cases} \omega_{p_1} \approx \frac{1}{B} \\ \omega_{p_2} \approx \frac{B}{A} \end{cases}$$

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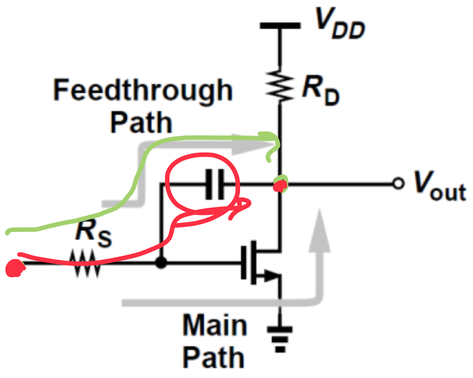
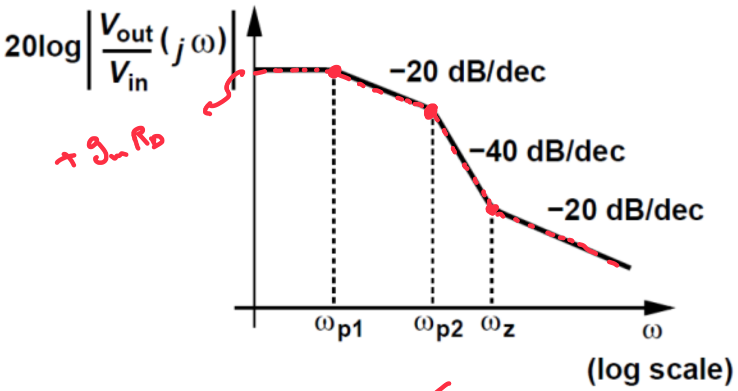
Zero in Transfer Function





$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1}$$

- The transfer function of exhibits a zero given by

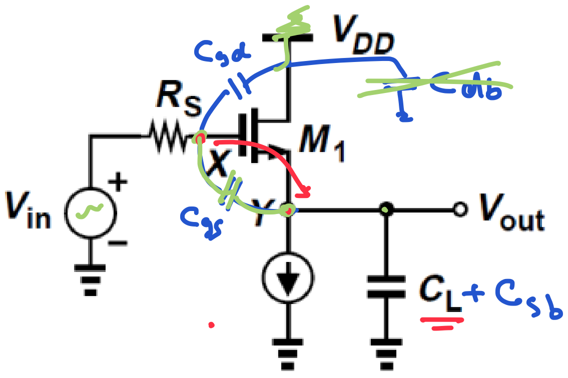
$$\omega_z = +g_m/C_{GD} \quad >> \omega_{p_1}, \omega_{p_2}$$

- CGD provides a feedthrough path that conducts the input signal to the output at very high frequencies.

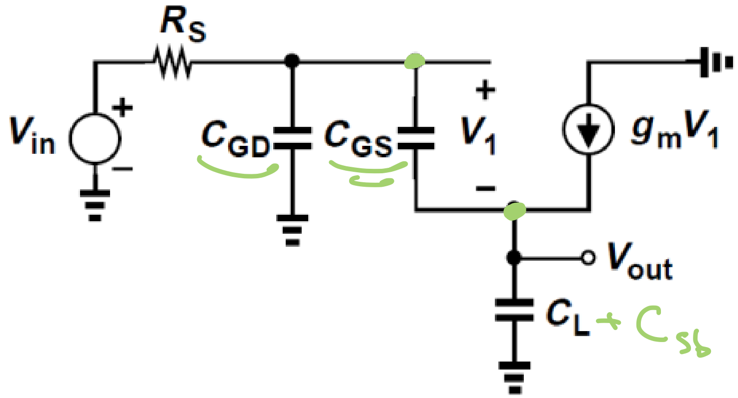
$A_v < 1$

don't use Miller ✗

Source Followers



(a)



(b)

- The strong interaction between nodes X and Y through C_{GS} in makes it difficult to associate a pole with each node.

$\omega_z = -\frac{C_{gs}}{g_m}$

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$

→ { 1 Zero, 2 poles

roots

- Contains a zero in the left half plane. Why?

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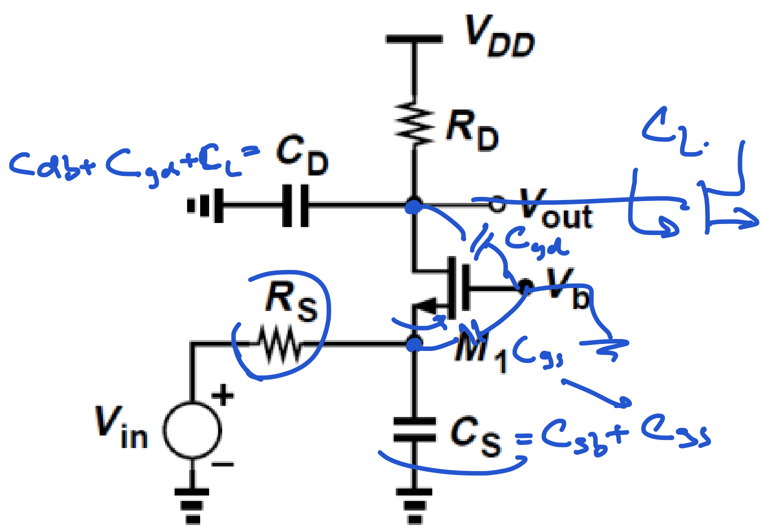
√ No need for Miller

Common-Gate Stage

- A transfer function (w/o channel length modulation):

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \frac{1}{\left(1 + \frac{C_S}{g_m + \cancel{g_{mb}} + R_S^{-1}} s\right)(1 + R_D C_D s)}$$

$R_S \parallel 1/g_m$

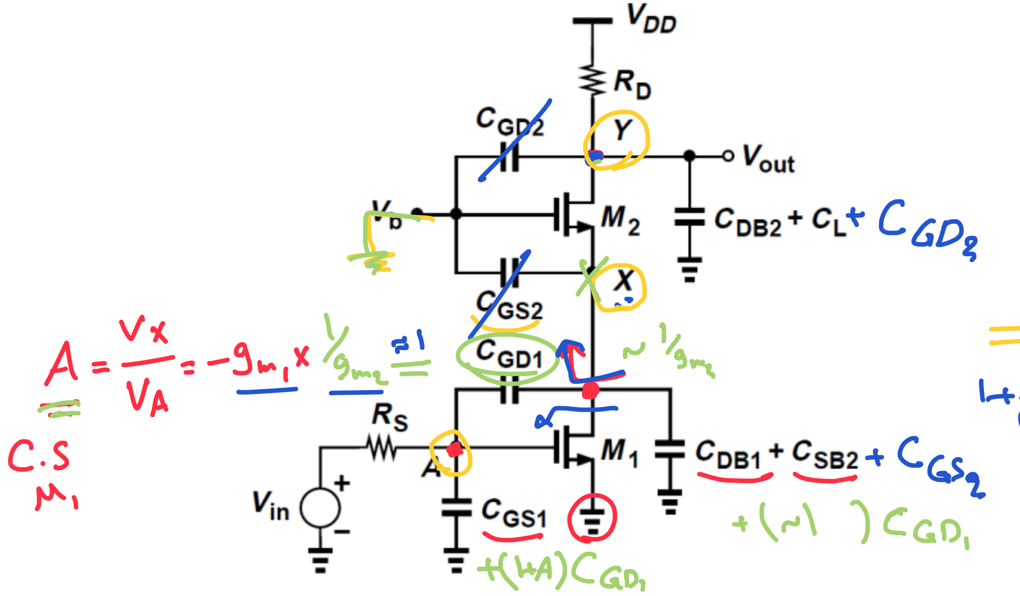


- No Miller multiplication of capacitances.
- RD is typically maximized, so the dc level of the input signal must be quite low.
- As an amplifier in cases where a low input impedance is required in cascode stages.

Cascode Stage

r_{o1} & r_{o2} >> $1/g_{m1,2}$

✓ Miller effect is acceptable to use



High-frequency model of a cascode stage.

input pole

$$\omega_{p,A} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2} + g_{mb2}}\right)C_{GD1}\right]}$$

Miller Cap of C_{GD1}

$R_X = r_{o1} \parallel 1/g_{m2} \approx 1/g_{m2}$

$1 + \frac{1}{A} \approx 2$

$$\omega_{p,X} = \frac{g_{m2} + g_{mb2}}{2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}}$$

$$\omega_{p,Y} = \frac{1}{R_D(C_{DB2} + C_L + C_{GD2})}$$

output pole

- Miller effect is less significant in cascode amplifiers than in common-source stages.
- But $\omega_{p,X}$ is typically quite higher than the other two. $\omega_{pX} \gg \omega_{pY}, \omega_{pA}$
- What if R_D is replaced by a current source?
    - Pole at node X may be quite lower, but transfer function will not affect much by this. See example.

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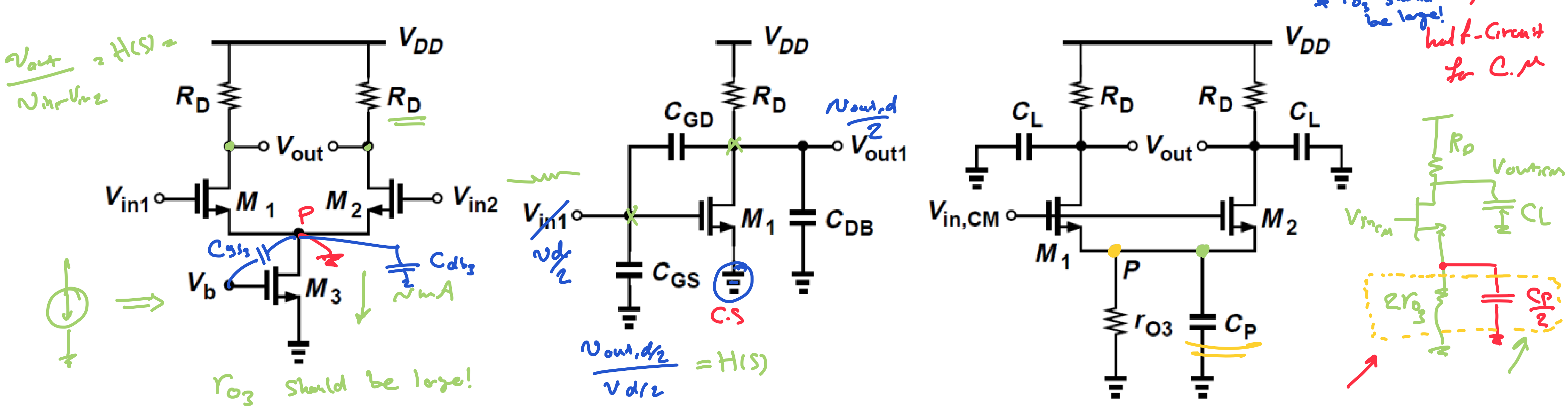
Differential Pair



- For differential signals, the response is identical to that of a common-source stage.
- the common-mode rejection of the circuit degrades considerably at high frequencies.

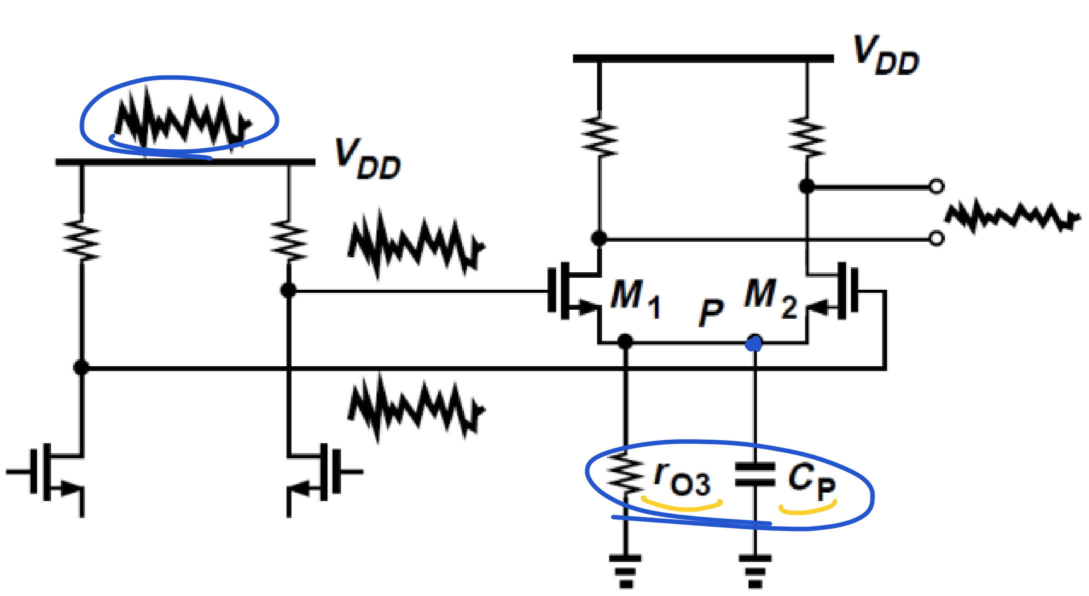
Mismatch M_1 & M_2

$$A_{v,CM} = -\frac{\Delta g_m \left[R_D \,\middle\|\, \left(\frac{1}{C_L s}\right)\right]}{(g_{m1}+g_{m2})\left[r_{O3} \,\middle\|\, \left(\frac{1}{C_P s}\right)\right]+1}$$

Z_{SS}

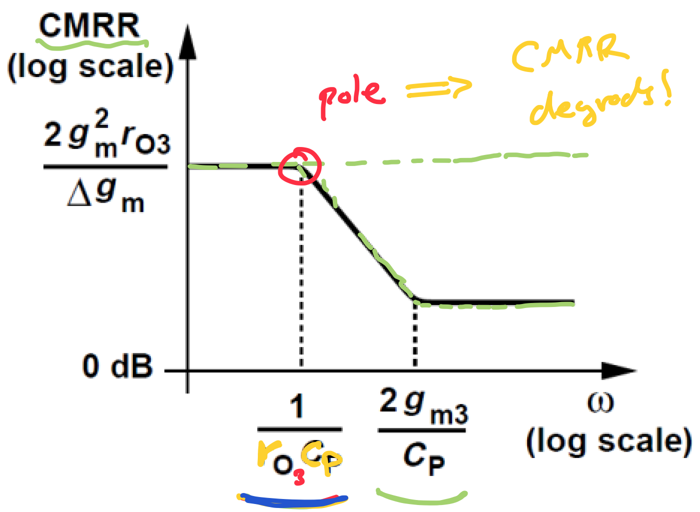
- Channel-length modulation, body effect, and other capacitances are neglected.

Differential Pair



$$\left|\frac{A_{DM}}{A_{CM}}\right| = \text{CMRR} \approx \frac{g_m}{\Delta g_m}\left[1 + 2g_m\left(r_{O3}\|\frac{1}{C_P s}\right)\right]$$

$$\approx \frac{g_m}{\Delta g_m}\,\frac{r_{O3}C_P s + 1 + 2g_m r_{O3}}{r_{O3}C_P s + 1}$$

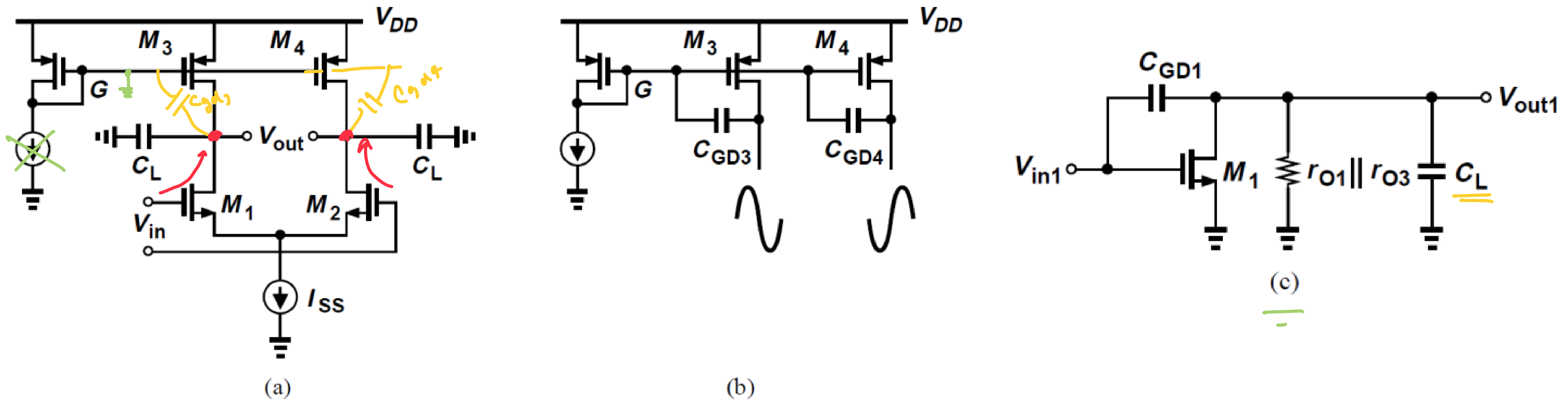


⇓

how large can be M_3

- This transfer function contains a zero and a pole.
- The magnitude of the zero is much greater than the pole.

Differential Pair



- Frequency response of differential pairs with high-impedance loads.
- Fig (b) C_{GD3} and C_{GD4} conduct equal and opposite currents to node G, making this node an ac ground.
- The differential half circuit is depicted in Fig. (c).
- More in chapter 10 …

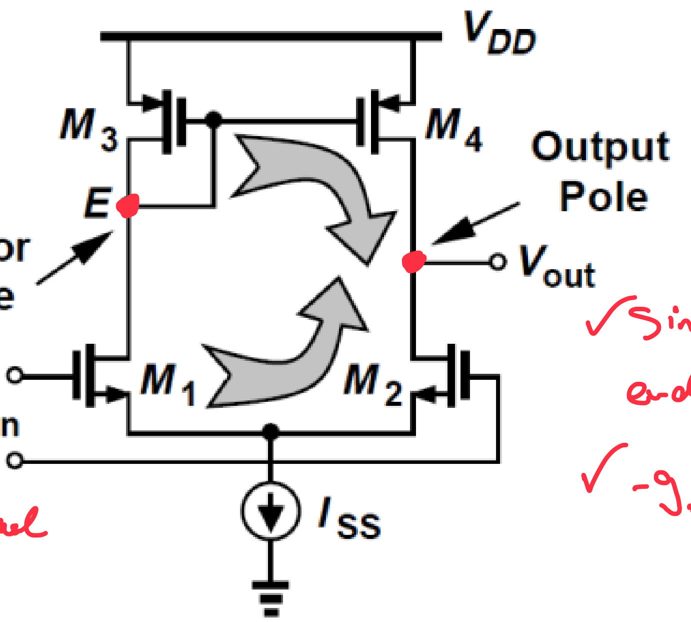
Differential Pair with Active Load

$CMRR = \frac{A_{DM}}{A_{CM-DM}}$ book

$CMRR = \frac{A_{DM}}{A_{CM}}$ Single ended output

* Current Mirror

Output pole ≠



differential

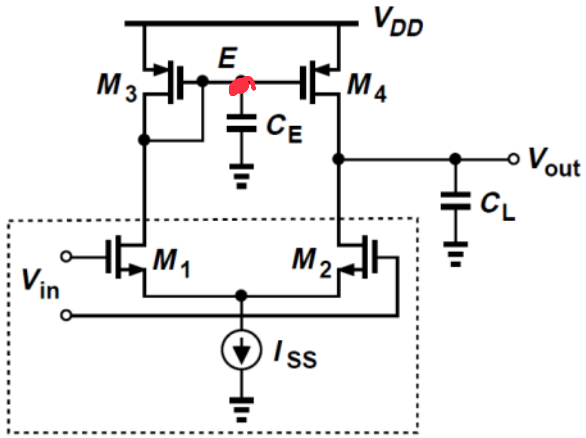
✓ Single ended Output

✓ $-g_m(r_{o2} \| r_{o4})$

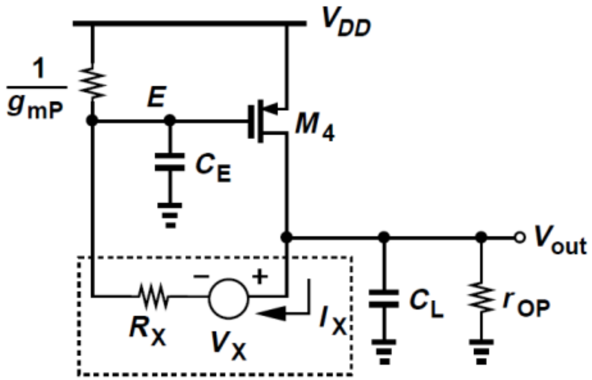
- How many poles does this circuit have?
- The severe trade-off between g_m and C_{GS} of PMOS devices results in a pole that impacts the performance of the circuit.
- The pole associated with node E is called a “mirror pole.”

ELECTRICAL & COMPUTER ENGINEERING

Active Load



(a)



(b)

- Replacing Vin, M1, and M2 by a Thevenin equivalent (Direct calculation method)

$$V_X = g_{mN} r_{ON} V_{in} \qquad R_X = 2r_{ON}$$

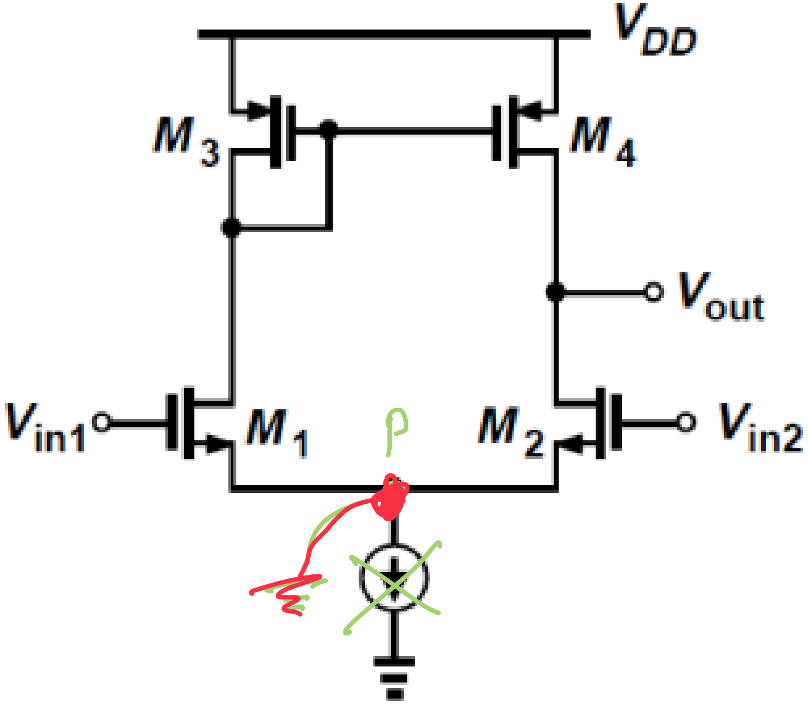
1 Zero

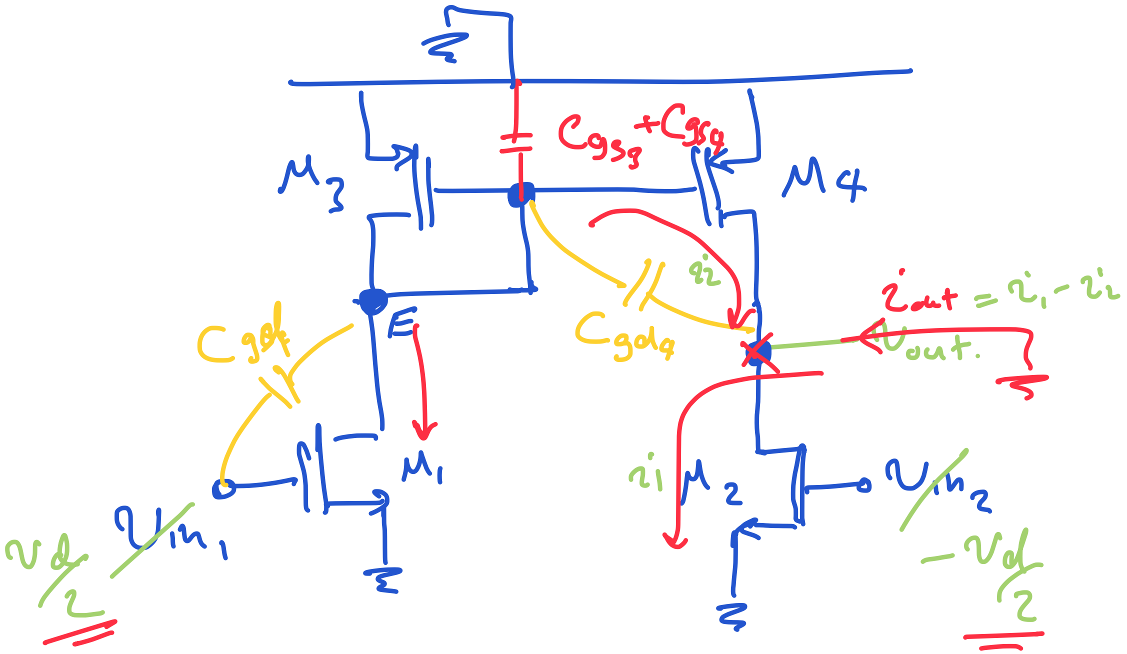
$$\frac{V_{out}}{V_{in}} = \frac{g_{mN} r_{ON}(2g_{mP} + C_E s) r_{OP}}{2r_{OP} r_{ON} C_E C_L s^2 + [(2r_{ON} + r_{OP})C_E + r_{OP}(1 + 2g_{mP} r_{ON})C_L]s + 2g_{mP}(r_{ON} + r_{OP})}$$

$$\omega_Z = \frac{-2g_{mP}}{C_E} = 2\omega_{p_2} \qquad \omega_{p1} \approx \frac{1}{(r_{ON} \| r_{OP})C_L} \qquad \omega_{p2} \approx \frac{g_{mP}}{C_E}$$

$R_E \approx 1/g_{m_3} \,\|\, r_{o_1} \approx 1/g_{m_3}$

5T-OTA





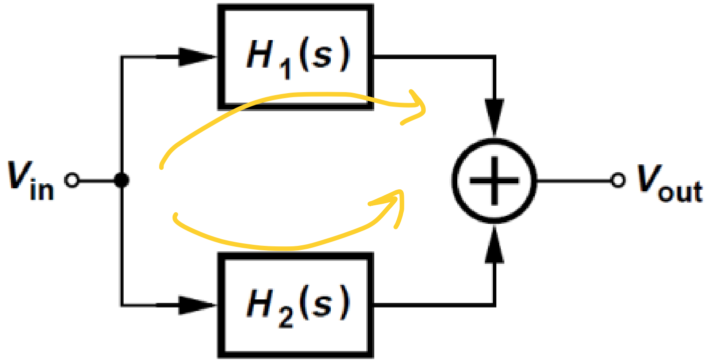
$$\frac{v_{out}}{v_E} = -g_{m4} \times (r_{o2} \| r_{o4})$$

$$G_m(s) = \frac{-g_{m2}}{(1+j\omega/\omega_{P_{out}})} + \frac{-g_{m1}}{(1+j\omega/\omega_{P_{out}})(1+j\omega/\omega_E)}$$

$$C_E = C_{GS_3} + C_{GS_4} + C_{db_1} + C_{db_3} + (1+\cancel{8}\,0)\,C_{gd4} + C_{gd_1}$$

$$C_{out} = C_{db_2} + C_{db_4} + (1+\sim 1)\,C_{gd4} + C_{gd_2}$$

Example



$H(s) = H_1(s) + H_2(s)$

- Can this (the zero) occur if H1(s) and H2(s) are first-order low-pass circuits?
- $H_1(s) = A_1/(1 + s/\omega_{p1})$ and $H_2(s) = A_2/(1 + s/\omega_{p2})$

$$\frac{V_{out}}{V_{in}}(s) = \frac{\left(\frac{A_1}{\omega_{p2}} + \frac{A_2}{\omega_{p1}}\right)s + A_1 + A_2}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

- The overall transfer function contains a zero.

Active Load

$$\frac{1}{\left(1+\frac{s}{\omega_{p_1}}\right)} + \frac{1}{\left(1+\frac{s}{\omega_{p_1}}\right)\left(1+\frac{s}{\omega_{p_2}}\right)}$$

$$= \frac{\cancel{2+\frac{s}{\omega_{p2}}}\ \cancel{+1}\quad 2\left(1+\frac{s}{2\omega_{p_2}}\right)}{\left(1+\frac{s}{\omega_{p_1}}\right)\left(1+\frac{s}{\omega_{p_2}}\right)}$$

Mirror pole

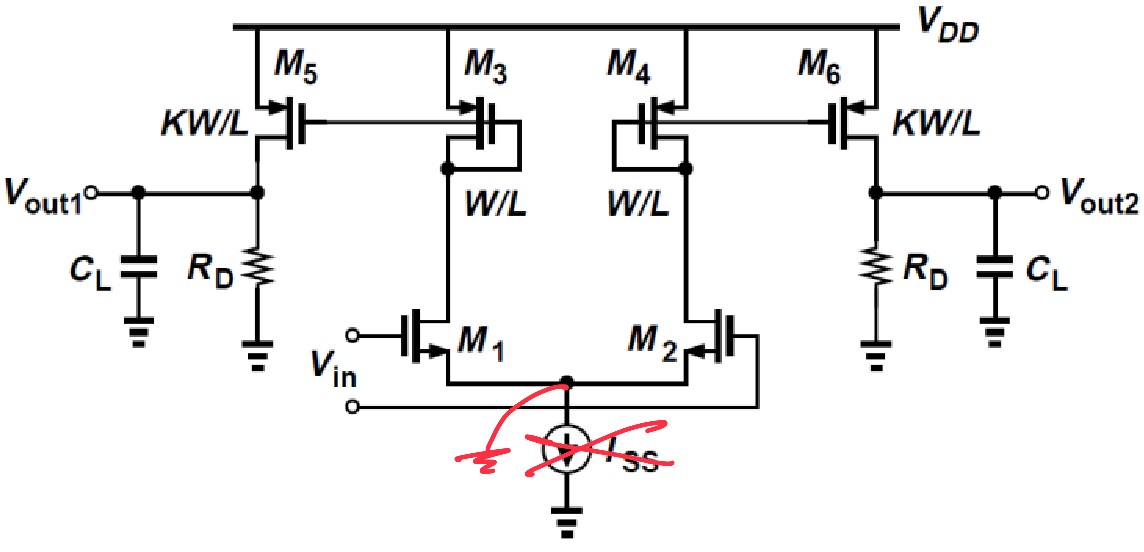
$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1+s/\omega_{p1}}\left(\frac{1}{1+s/\omega_{p2}}+1\right)$$

$$= \frac{A_0(2+s/\omega_{p2})}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$$

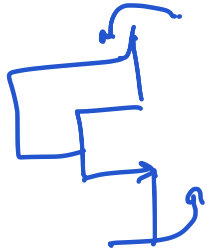
- A zero with a magnitude of $2g_{mP}/C_E$ in the left half plane.
- The appearance of this zero can be understood by noting that the circuit consists of:
- a “slow path” (M1,M3 and M4) $A_0/[(1+s/\omega_{p1})(1+s/\omega_{p2})]$ in parallel with
- a “fast path” (M1 and M2) by $A_0/(1+s/\omega_{p1})$

Example

- Estimate the low-frequency gain and the transfer function of this circuit.

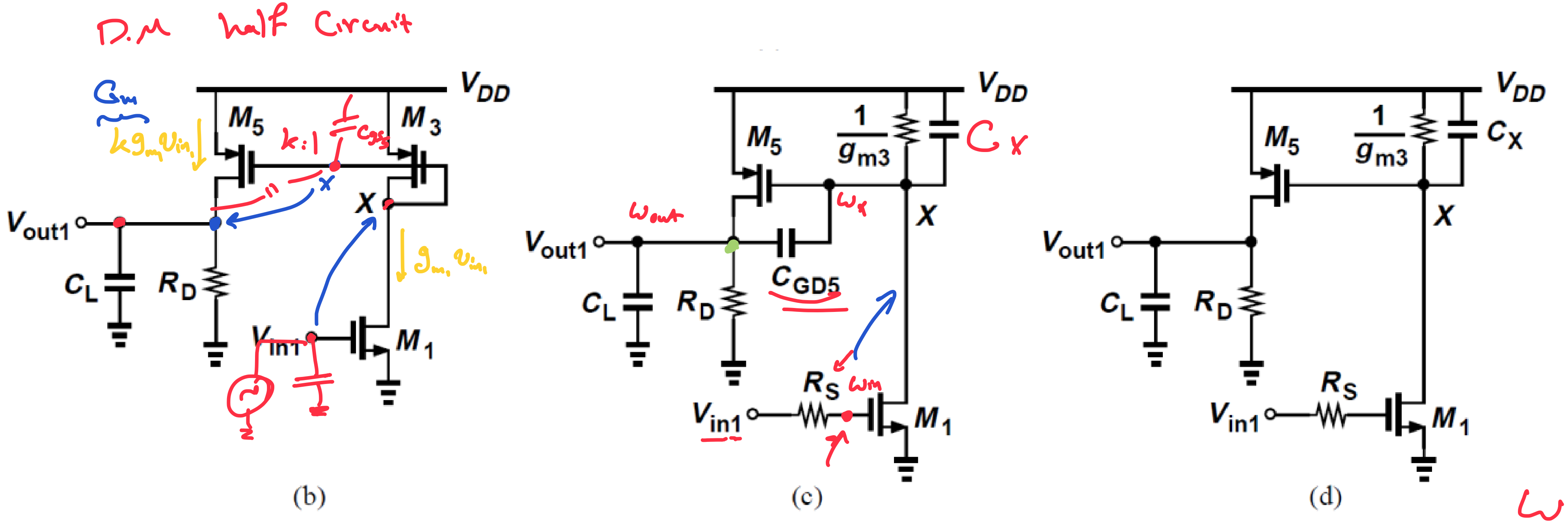


(a)



$1/g_m$

Example



$$A_v = \frac{v_{out1}}{v_{in1}} = k \cdot g_{m1} (R_D \| r_{o5})$$

$$A_v = \frac{V_x}{V_{in1}} \times \frac{V_{out}}{V_x} = \ldots$$

CS

poles	R_{eq}	C_{eq}
ω_{in}	R_S	$C_{gs1} + C_{gd1}\left(1 + \frac{g_{m1}}{g_{m3}}\right)$
ω_x	$1/g_{m3}$ $\cancel{\| r_{o1} \| r_{o3}}$	$C_{gs3} + C_{gs5} + C_{db1} + C_{db3} + C_{gd5}(1 + g_{m5} R_D)$
ω_{out}	$R_D \cancel{\| r_{o5}}$	$C_L + C_{db5} + C_{gd5}(1 + \sim 1)$

$\omega = \frac{1}{RC}$ ✓

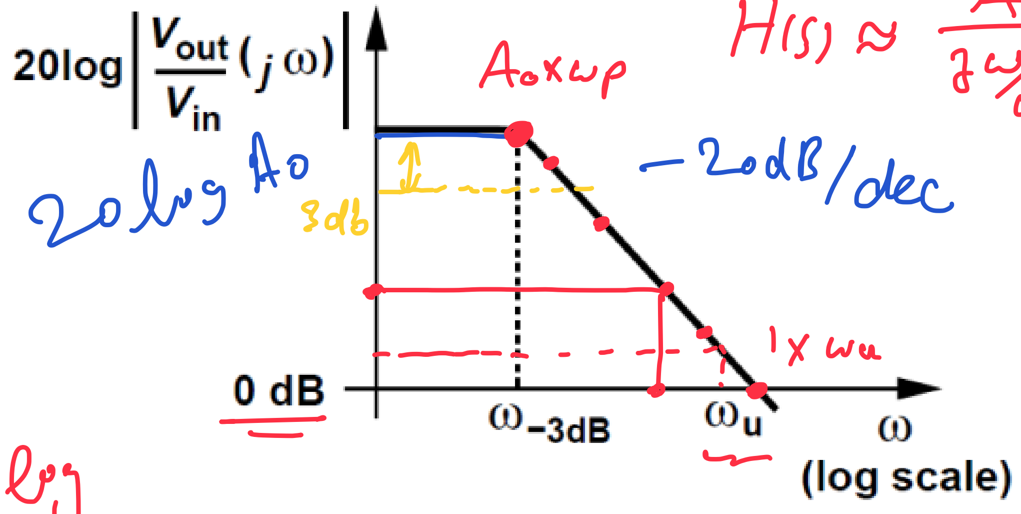
Gain-Bandwidth Trade-Offs

$\omega_{p_1} \ll \omega_{p_2}, \omega_{p_3}, \ldots$

dominant pole!

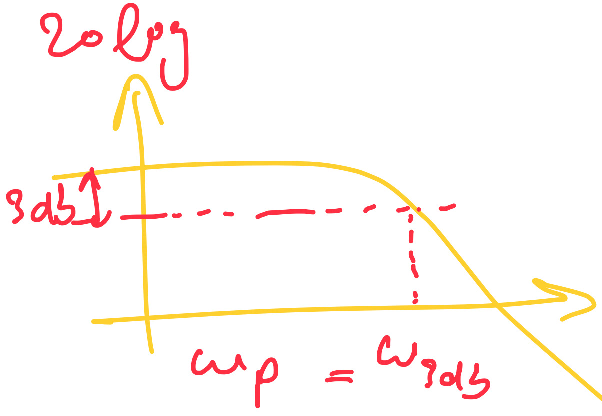
$H(s) \approx \dfrac{A_0}{j\omega/\omega_p} \Rightarrow |H(\omega)| \times \omega = A_0 \times \omega_p$

$$H(s) = \frac{A_0}{1 + s/\omega_p}$$



$$|H(s)| = \frac{A_0}{\sqrt{1+(\omega_u/\omega_p)^2}} = 1$$

$$\Rightarrow A_0^2 = 1 + (\omega_u/\omega_p)^2 \Rightarrow \boxed{\omega_u = \omega_p\sqrt{A_0^2-1}}$$



$A_0^2 \gg 1 \Rightarrow \boxed{\omega_u = A_0 \times \omega_p}$

freq @ $|H(s)| = 1$

0dB

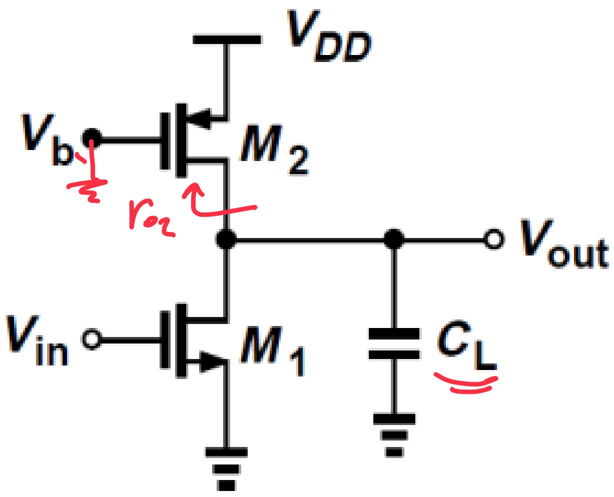
- We wish to maximize both the gain and the bandwidth of amplifiers.
- We are interested in both the 3-dB bandwidth, ω_{-3dB} , and the “unity-gain” bandwidth, ω_u

$A_v = -G_m R_{out}$

$\omega_p = \dfrac{1}{R_{out} C_L}$

$\boxed{\omega_u = \dfrac{G_m}{C_L}}$

One pole circuit



* ω_u is one of first metrics to find!

↳ $G_m \longrightarrow R_{out}, \ldots$

$G_m = \dfrac{2I}{V_{OD}}$

$$\text{GBW} = A_0\omega_p$$

$$= \overbrace{g_{m1}(r_{O1}\cancel{\|r_{O2}})}^{\text{gain}} \overbrace{\frac{1}{2\pi(r_{O1}\cancel{\|r_{O2}})C_L}}^{\omega_p}$$

$$G_m = \boxed{\frac{g_{m1}}{2\pi C_L}}$$

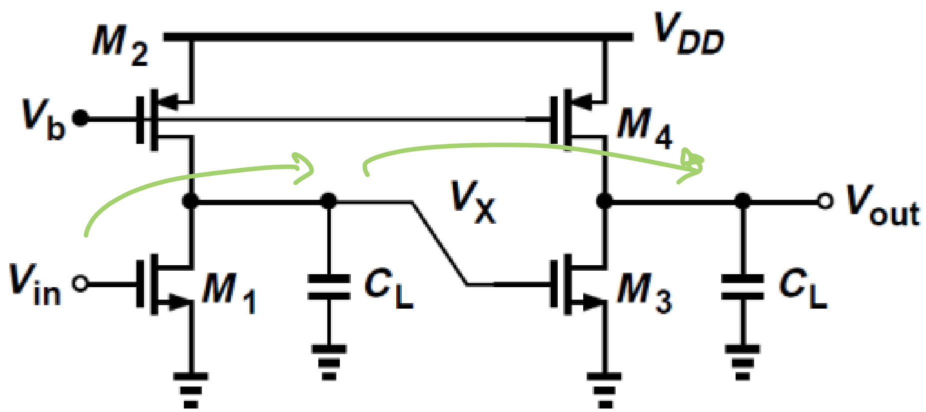
$$\omega_u = \sqrt{A_0^2 - 1}\,\omega_p$$

$$\approx A_0\omega_p = \text{GBW}$$

Multi-Pole Circuits

$H_1(s) = H_2(\,)$

$= \dfrac{A_o}{1+\frac{s}{\omega p}}$



- It is possible to increase the GBW product by cascading two or more gain stages.
- Assume the two stages are identical and neglect other capacitances.

$$\frac{V_{out}}{V_{in}} = \frac{A_0^2}{(1+\frac{s}{\omega_p})^2}$$

$H_1 \times H_2$

$$\omega_{-3dB} = \sqrt{\sqrt{2}-1}\omega_p$$
$$\approx 0.64\omega_p$$

$$\text{GBW} = \sqrt{\sqrt{2}-1}A_0^2\omega_p$$

- While raising the GBW product, cascading reduces the bandwidth.