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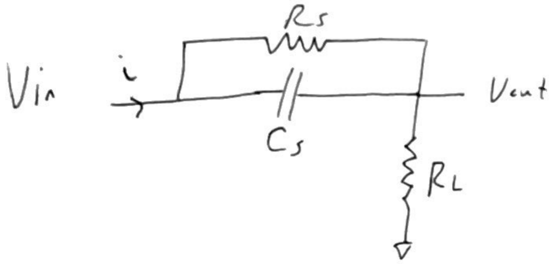


(a) $\dfrac{V_{out}(s)}{V_{in}(s)} = i \cdot R_L \quad , \quad i = \dfrac{V_{in}}{R_s \| \frac{1}{C_s s} + R_L}$

$$= \left(\frac{V_{in}}{\frac{R_s / C_s s}{R_s + \frac{1}{C_s s}} + R_L} \right) \left(\frac{R_s + \frac{1}{C_s s}}{R_s + \frac{1}{C_s s}} \right)$$

$$= \frac{V_{in}\left(R_s + \frac{1}{C_s s}\right)}{R_s / C_s s + R_L\left(R_s + \frac{1}{C_s s}\right)} \cdot \frac{C_s s}{C_s s}$$

$$= \frac{V_{in}(R_s C_s s + 1)}{R_s + R_L(R_s C_s s + 1)}$$

$$= V_{in}\left(\frac{R_s C_s s + 1}{R_s + R_L + R_s R_L C_s s} \right)$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{R_s C_s s + 1}{R_s + R_L + R_s R_L C_s s} \cdot R_L$$

$$\boxed{\frac{V_{out}(s)}{V_{in}(s)} = \frac{(R_s \| R_L) C_s s + \frac{R_L}{R_s + R_L}}{(R_s \| R_L) C_s s + 1}}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\left(\frac{(R_s \| R_L) C_s (R_s + R_L)}{R_L} s + 1 \right)\left(\frac{R_L}{R_s + R_L} \right)}{(R_s \| R_L) C_s s + 1}$$

$$= \frac{\left(\frac{(R_s \cdot \cancel{R_L}) \cancel{(R_s + R_L)} C_s}{\cancel{(R_s + R_L)} \cancel{R_L}} s + 1 \right)\left(\frac{R_L}{R_s + R_L} \right)}{(R_s \| R_L) C_s s + 1} = \boxed{\frac{R_s C_s s + 1}{(R_s \| R_L) C_s s + 1} \cdot \frac{R_L}{R_s + R_L}}$$

Plug in numbers:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{9k\Omega \cdot 1nF\, s + 1}{(9k\Omega \| 1k\Omega) \cdot 1nF \cdot s + 1} \cdot \frac{1k\Omega}{1k\Omega + 9k\Omega}$$

$$= \frac{9\mu s \cdot s + 1}{900ns \cdot s + 1} \cdot \frac{1}{10}$$

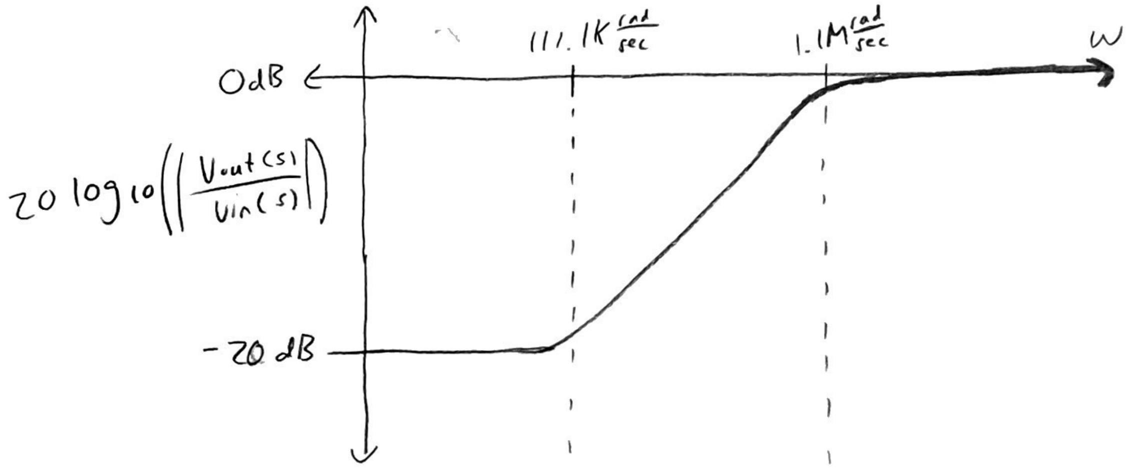
$$\omega_{pole} = \frac{1}{900ns} = 1.1\, M \frac{rad}{sec}$$

$$\omega_{zero} = \frac{1}{9\mu s} = 111.1\, K \frac{rad}{sec}$$

(b) Magnitude

At DC, $s=0$, and $\frac{V_{out}}{V_{in}} = \frac{1}{10} = 20\log_{10}\left(\frac{1}{10}\right) = -20\text{ dB}$

At very high frequency, $\frac{V_{out}}{V_{in}} = 1 \rightarrow = 0\text{dB}$



phase

At DC, $\frac{V_{out}}{V_{in}}$ has phase of 0

After the zero "turns on", we will have: $\frac{V_{out}(s)}{V_{in}(s)} = \frac{9\text{ ns}\cdot s+1}{1}\cdot\frac{1}{10}$

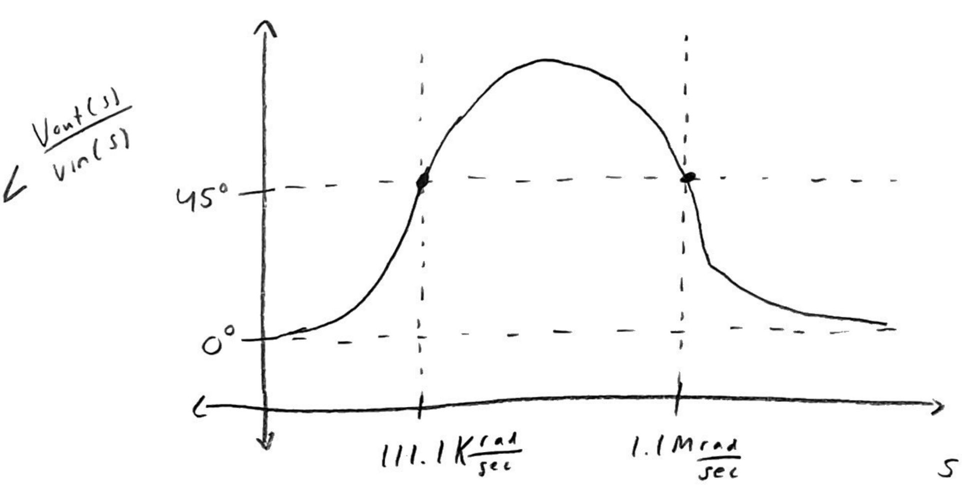
$= \frac{1}{10}\frac{j+1}{1} \longrightarrow$ phase of 45°

After the pole "turns on", we will have:

$\frac{V_{out}(s)}{V_{in}(s)} = \frac{9\text{ns}\cdot j\omega}{j+1}\cdot\frac{1}{10} \longrightarrow$ phase of 45°

At a high frequency:

$\frac{V_{out}(s)}{V_{in}(s)} = \frac{9\mu s\cdot j\omega}{900\text{ns}\cdot j\omega}\cdot\frac{1}{10} \longrightarrow$ phase of 0



c) $V_{in}(t) = 0.2 + \sin(\omega_p t)$

To solve for V_{out} we can consider the DC and non-DC parts of the input separately

$V_{in,DC}(t) = 0.2$

$V_{out,DC}(t) = 0.2 \cdot$ DC gain

$= 0.2 \cdot \frac{1}{10}$

$= 0.02$

$V_{in,AC}(t) = \sin(\omega_p t)$

solve for $\frac{V_{out}(s)}{V_{in}(s)}$ at $s = \omega_p = 1.1\,M \frac{rad}{sec}$

$$\frac{V_{out}}{V_{in}}\left(s = 1.1\,M\frac{rad}{sec}\right) = \frac{j\,9 \cdot 10^{-6} \cdot 1.1 \cdot 10^{6} + 1}{j\,900 \cdot 10^{-9} \cdot 1.1 \cdot 10^{6} + 1} \cdot \frac{1}{10}$$

$$= \frac{10j + 1}{j + 1} \cdot \frac{1}{10}$$

$$\left|\frac{V_{out}}{V_{in}}(\omega_p)\right| = \frac{1}{10} \cdot \frac{\sqrt{10^2+1}}{\sqrt{1+1}} = \frac{1}{10}\sqrt{\frac{101}{2}} = 0.710$$

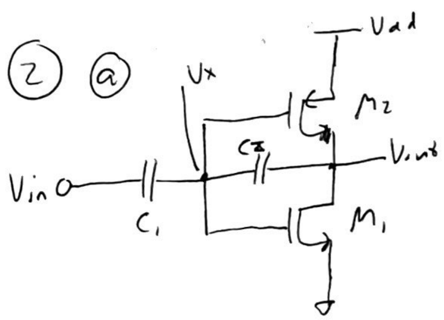
$$\angle \frac{V_{out}}{V_{in}}(\omega_p) = \frac{\tan^{-1}(10)}{\tan^{-1}(1)} = \frac{\angle 84.29}{\angle 45} \longrightarrow 39.29°$$

$V_{out,AC}(t) = V_{in,AC}(t) \cdot$ phasor

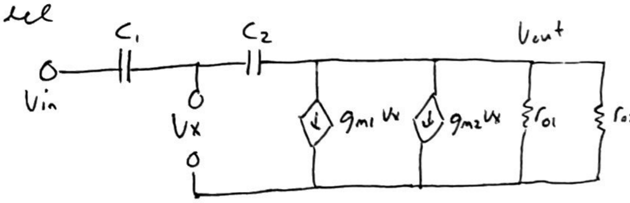
$V_{out,AC}(t) = 0.710 \sin(1.1 \cdot 10^{6} \cdot t + 39.29°)$

$$\boxed{V_{out}(t) = 0.02 + 0.710 \sin(1.1 \cdot 10^{6} t + 39.29°)}$$

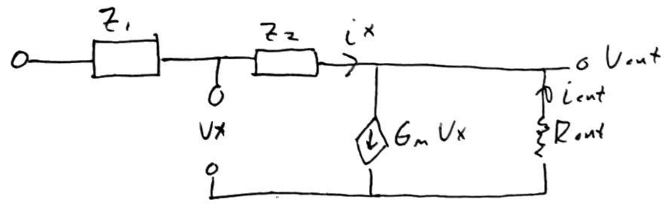
2) a)



Draw small-signal model →



I will solve this first by generalizing the capacitors to impedances - can plug in impedance of a capacitor later



$G_m = g_{m1} + g_{m2}$

$R_{out} = r_{o1} \,\|\, r_{o2}$

$$\frac{V_{in} - V_{out}}{Z_1 + Z_2} - \frac{V_{out}}{R_{out}} = G_m v_x \qquad V_x = V_{in} - \left(\frac{V_{in} - V_{out}}{Z_1 + Z_2}\right) Z_1$$

$$\frac{V_{in} - V_{out}}{Z_1 + Z_2} - \frac{V_{out}}{R_{out}} = G_m \left[V_{in} - \frac{V_{in} - V_{out}}{Z_1 + Z_2} Z_1 \right]$$

$$\frac{V_{in}}{Z_1 + Z_2} - \frac{V_{out}}{Z_1 + Z_2} - \frac{V_{out}}{R_{out}} = G_m V_{in} - \frac{G_m Z_1 V_{in}}{Z_1 + Z_2} + \frac{G_m V_{out} Z_1}{Z_1 + Z_2}$$

$$V_{in}\left[\frac{1 + G_m Z_1}{Z_1 + Z_2} - G_m\right] = V_{out}\left[\frac{1}{R_{out}} + \frac{1 + G_m Z_1}{Z_1 + Z_2}\right] \longrightarrow \times \frac{Z_1 + Z_2}{Z_1 + Z_2}$$

$$V_{in}\left[1 + G_m Z_1 - G_m Z_1 - G_m Z_2\right] = V_{out}\left[\frac{Z_1 + Z_2}{R_{out}} + 1 + G_m Z_1\right]$$

$$V_{in}\left[1 - G_m Z_2\right] = V_{out}\left[\frac{Z_1 + Z_2}{R_{out}} + 1 + G_m Z_1\right]$$

$$\frac{V_{out}}{V_{in}} = \frac{1 - G_m Z_2}{\frac{Z_1 + Z_2}{R_{out}} + 1 + G_m Z_1} \longrightarrow \text{now let } Z_1 = \frac{1}{C_1 s},\ Z_2 = \frac{1}{C_2 s}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1 - \frac{G_m}{C_2 s}}{\frac{\frac{1}{C_1 s} + \frac{1}{C_2 s}}{R_{out}} + 1 + \frac{G_m}{C_1 s}} \cdot \frac{s}{s} = \frac{s - \frac{G_m}{C_2}}{s + \left(\frac{1}{R_{out} C_1} + \frac{1}{R_{out} C_2} + \frac{G_m}{C_1}\right)} \cdot \frac{R_{out} C_1 C_2}{R_{out} C_1 C_2}$$

$$\boxed{\frac{V_{out}}{V_{in}}(s) = \frac{R_{out} C_1 (C_2 s - G_m)}{R_{out} C_1 C_2 s + C_2 + C_1 + G_m R_{out} C_2}}$$

Alternatively we can do:

$$\boxed{\frac{V_{out}}{V_{in}}(s) = \frac{s - \frac{G_m}{C_2}}{s + \left(\frac{1}{R_{out} C_1} + \frac{1}{R_{out} C_2} + \frac{G_m}{C_1}\right)}}$$

So we have:

$$\boxed{\begin{aligned} &\text{Zero:} \quad \omega_z = + \frac{G_m}{C_2} \\ &\text{pole:} \quad \omega_p = \frac{1}{R_{out} C_1} + \frac{1}{R_{out} C_2} + \frac{G_m}{C_1} \end{aligned}}$$

~~Let us assume~~ $\cancel{\omega_z < \omega_p}$

$\cancel{\frac{V_{out}(s)}{V_{in}(s)}}$

~~To make the Bode Plot~~

~~Let's find DC conditions and high freq conditions:~~

$\cancel{\frac{V_{out}}{V_{in}}(0) = \frac{+ G_m R_{out} C_1}{C_2 + C_1 + G_m R_{out} C_2}}$

$\cancel{\frac{V_{out}}{V_{in}}(s = \infty) = 1}$

~~The presence of a negative zero can be a little confusing. What this means is that the zero will already be "on" at DC so the gain will already be rising 20 dB/decade~~

To make the Bode Plot

- Let's assume that $\omega_s < \omega_p$
- Let's find key points & values of $\frac{V_{out}}{V_{in}}(s)$

- $s = 0$

$$\frac{V_{out}}{V_{in}}(s=0) = \frac{-G_m R_{out} C_1}{C_2 + C_1 + G_m R_{out} C_2} \longrightarrow \text{mag} = \left| \frac{+G_m R_{out} C_1}{C_2 + C_1 + G_m R_{out} C_2} \right| , \text{ phase} = +180^\circ$$

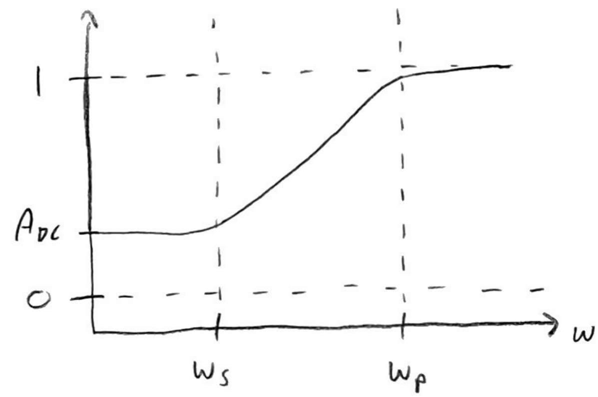
will assume this to be < 1

$s = \infty$

$$\frac{V_{out}}{V_{in}}(s=\infty) = 1 \longrightarrow \text{mag} = 1 , \text{ phase} = 0$$

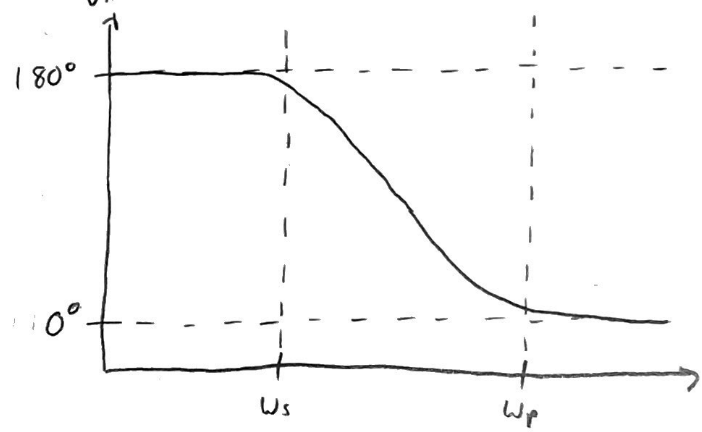
Mag

$\left| \frac{V_{out}}{V_{in}}(s) \right|$



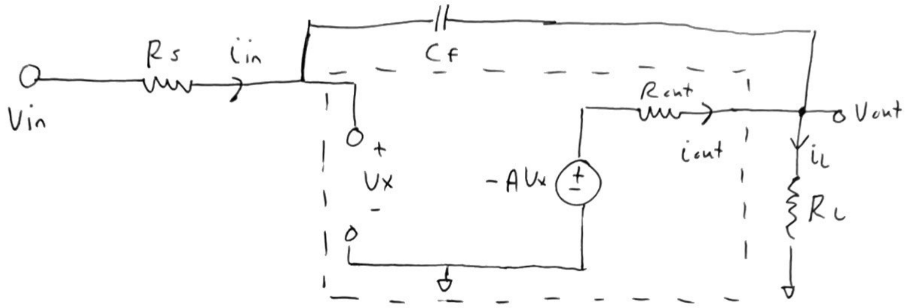
Phase

$\angle \frac{V_{out}(s)}{V_{in}(s)}$



$$A_{DC} = \left| \frac{G_m R_{out} C_1}{C_2 + C_1 + G_m R_{out} C_2} \right|$$

3 a



For solving this I will abstract C_f to a generalized impedance Z_f and will convert to $\frac{1}{C_f s}$ at the end

$$i_{in} + i_{out} = i_L \qquad i_{in} = \frac{V_{in} - V_{out}}{R_s + Z_f}, \quad i_L = \frac{V_{out}}{R_L}, \quad i_{out} = \frac{-A V_x - V_{out}}{R_{out}} \longrightarrow V_x = V_{in} - \overbrace{\left(\frac{V_{in} - V_{out}}{R_s + Z_f}\right)}^{i_{in}} R_s$$

$$i_{out} = \frac{-A\left(V_{in} - \frac{V_{in} - V_{out}}{R_s + Z_f} R_s\right) - V_{out}}{R_{out}}$$

$$\downarrow$$

$$\frac{V_{in} - V_{out}}{R_s + Z_f} + \frac{-A\left(V_{in} - \frac{V_{in} - V_{out}}{R_s + Z_f} R_s\right) - V_{out}}{R_{out}} = \frac{V_{out}}{R_L}$$

$$\left[\frac{V_{in} - V_{out}}{R_s + Z_f} - \frac{A V_{in}}{R_{out}} + \frac{A(V_{in} - V_{out})}{R_{out}(R_s + Z_f)} R_s - \frac{V_{out}}{R_{out}} = \frac{V_{out}}{R_L}\right](R_s + Z_f) R_{out}$$

$$R_{out} V_{in} - R_{out} V_{out} - A V_{in}(R_s + Z_f) + A V_{in} R_s - A V_{out} R_s - V_{out}(R_s + Z_f) = V_{out} X, \quad X = \frac{(R_s + Z_f) R_{out}}{R_L}$$

$$\cancel{R_{out} V_{in}} \; \cancel{A V_{in} R_s} \; V_{in}(R_{out} - A Z_f) = V_{out}\left[X + (R_s + Z_f) + A R_s + R_{out}\right]$$

$$\frac{V_{out}}{V_{in}} = \frac{R_{out} - A Z_f}{X + (R_s + Z_f) + A R_s + R_{out}} \longrightarrow \text{now substitute } Z_f = \frac{1}{C_f s}$$

$$\frac{V_{out}}{V_{in}} = \frac{R_{out} - \frac{A}{C_f s}}{X + \cancel{(R_s + Z_f)} + A R_s + R_{out} + \left(R_s + \frac{1}{C_f s}\right)} \cdot \frac{C_f s}{C_f s}$$

$$\boxed{\frac{V_{out}}{V_{in}}(s) = \frac{R_{out} C_f s - A}{C_f s\left(X + R_s(A+1) + R_{out}\right) + 1}, \quad X = \frac{R_s + Z_f}{R_L} R_{out}}$$

Continued on next page

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{R_{out} C_f s - A}{C_f s \left(\frac{R_s + \frac{1}{C_f s}}{R_L} R_{out} + R_s(A+1) + R_{out} \right) + 1}$$

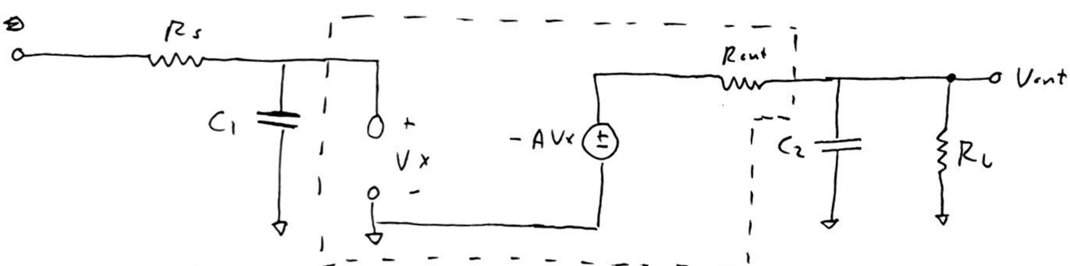
$$= \frac{R_{out} C_f s - A}{C_f s \left(\frac{R_s}{R_L} R_{out} + R_s(A+1) + R_{out} \right) + 1 + \frac{R_{out}}{R_L}}$$

$$= \frac{R_{out} C_f s - A}{C_f s \left[R_s \left(\frac{R_{out}}{R_L} + A + 1 \right) + R_{out} \right] + 1 + \frac{R_{out}}{R_L}}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{R_L (R_{out} C_f s - A)}{C_f s \left[R_s (R_{out} + R_L A + R_L) + R_{out} R_L \right] + R_L + R_{out}}$$

$$= \frac{\left(s - \frac{A}{R_{out} C_f} \right)(R_L)(R_{out} C_f)}{\left(s + \frac{R_L + R_{out}}{C_f \left[R_s (R_{out} + R_L A + R_L) + R_{out} + R_L \right]} \right) \left(C_f \left[R_s (R_{out} + R_L A + R_L) + R_{out} + R_L \right) \right)}$$

(b)



$C_1 = (1+B) C_f$, $C_2 = \frac{1+B}{B} C_f$

B = open loop gain

[redacted] $= \frac{A \cdot R_L}{R_{out} + R_L}$

(c) $$V_x = \frac{V_{in}}{R_s + \frac{1}{C_1 s}} \cdot \frac{1}{C_1 s} \quad \text{(voltage divider)}$$

$$\frac{V_x}{V_{in}} = \frac{1}{R_s C_1 s + 1}$$

$$V_{out} = \frac{-A V_x}{R_{out} + R_L \| \frac{1}{C_2 s}} \cdot \left(R_L \| \frac{1}{C_2 s}\right) \quad \text{(voltage divider)}$$

$$R_L \| \frac{1}{C_2 s} = \frac{R_L \frac{1}{C_2 s}}{R_L + \frac{1}{C_2 s}} \cdot \frac{C_2 s}{C_2 s} = \frac{R_L}{R_L C_2 s + 1}$$

$$V_{out} = \frac{-A V_x}{R_{out} + \frac{R_L}{R_L C_2 s + 1}} \cdot \frac{R_L}{R_L C_2 s + 1}$$

$$\frac{V_{out}}{V_x} = \frac{-A R_L}{R_{out} R_L C_2 s + R_{out} + R_L}$$

$$\boxed{\frac{V_{out}}{V_{in}}(s) = \frac{-A R_L}{(R_s C_1 s + 1)\left[R_{out} R_L C_2 s + R_{out} + R_L\right]}}$$

We can also write:

$$\frac{V_{out}}{V_{in}}(s) = \frac{-A R_L}{(R_s C_1 s + 1)(R_{out} R_L C_2 s + R_{out} + R_L)} \cdot \frac{\left(\frac{1}{R_{out} R_L C_2}\right)\left(\frac{1}{C_1 R_s}\right)}{\left(\frac{1}{R_{out} R_L C_2}\right)\left(\frac{1}{R_s C_1}\right)}$$

$$\boxed{\frac{V_{out}}{V_{in}}(s) = \left(\frac{-A R_L}{\left(s + \frac{1}{R_s C_1}\right)\left(s + \frac{R_{out} + R_L}{R_{out} R_L C_2}\right)}\right)\left(\frac{1}{R_{out} R_L C_2} \cdot \frac{1}{C_1 R_s}\right)}$$

where $C_1 = (1+B) C_f$, $C_2 = \left(\frac{1+B}{B}\right) C_f$, $B = \frac{A \cdot R_L}{R_{out} + R_L}$

(d) True Transfer Function

- One pole and one zero

$$\omega_z = \frac{A}{R_{out} C_f}, \quad \omega_p = \frac{R_L + R_{out}}{C_f\left[R_s(R_{out} + R_L A + R_L)\right] + R_{out} + R_L}$$

Miller Approx. Transfer function

- Two poles

$$\omega_{p1} = \frac{1}{R_s C_1}, \quad \omega_{p2} = \frac{R_{out} + R_L}{R_{out} R_L C_2} = \frac{R_{out} + R_L}{R_{out} R_L \left[\frac{1 + \frac{A \cdot R_L}{R_{out} + R_L}}{\frac{A \cdot R_L}{R_{out} + R_L}}\right] C_F} = \frac{R_{out} + R_L}{R_{out} R_L \left[\frac{R_{out} + R_L + A \cdot R_L}{A \cdot R_L}\right] C_F}$$

$$\omega_{p2} = \frac{R_{out} + R_L}{(R_{out} \cdot R_L)\left(\frac{R_{out} + R_L}{A \cdot R_L} + 1\right) C_f}$$

The true transfer function shows one zero and one pole whereas the Miller approximation leads us to two poles.