Homework2 Solution

October 28, 2022

1 Problem 1

1.1 a

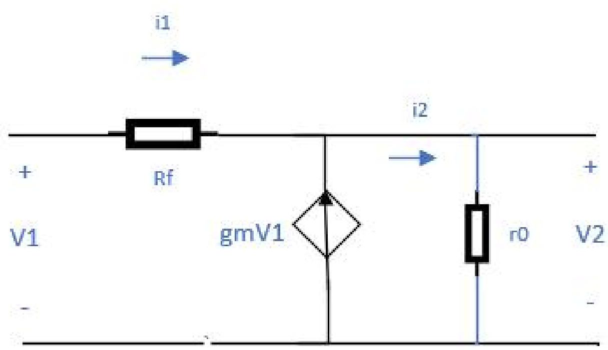

Figure 1: Problem 1a

$$\frac{V1 - V2}{R_f} = i_1, i_1 = i_2 - g_m V_1$$

$$\frac{V1 - V2}{R_f} = i_2 - g_m V_1, i_2 = \frac{V_2}{r_0}$$

$\Rightarrow$

$$\frac{V_1 - V_2}{Rf} = \frac{V_2}{r_0} - g_m V_1 \Rightarrow V_1(\frac{1}{R_f} + gm) = V_2(\frac{1}{r_0} + \frac{1}{R_f})$$

$\Rightarrow$

$$V_1(r_0 + g_m r_0 R_f) = V_2(R_f + r_0)$$

$\Rightarrow$

$$\frac{V_2}{V_1} = \frac{r_0 + g_m r_0 R_f}{R_f + r_0}$$

1.2 b

$R_{in} = \frac{V_1}{i_1}$, and $i_1 = \frac{V_2}{r_0} - g_m V_1$

also,

$$i_1 = \frac{V_1}{r_0}(\frac{r_0 + g_m r_0 R_f}{R_f + r_0})$$

Hence,

$$\frac{V_1}{i_1} = \frac{1}{(\frac{1+g_m R_f}{R_f+r_0}) - g_m} = \frac{R_f + r_0}{1 - g_m r_0}$$

$\Rightarrow R_{in} = \frac{R_f+r_0}{1-g_m r_0}$

1.3 c

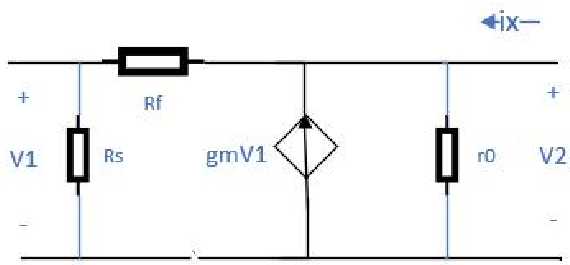

Figure 2: Problem 1C

$$R_{out} = \frac{V_2}{i_x}, i_x = \frac{V_2}{r_0} - g_m V_1 + \frac{V_2}{R_f + R_s}$$

also,

$$\frac{V_1}{V_2} = \frac{R_s}{R_f + R_s}$$

$$i_x = \frac{V_2}{r_0} - \frac{g_m V_2 R_s}{R_f + R_s} + \frac{V_2}{R_f + R_s}$$

$$i_x = V_2 \left(\frac{1}{r_0} + \frac{1}{R_f + R_s} (-g_m R_s + 1) \right)$$

$$= V_2(\frac{R_f + R_s + r_0(-g_m R_s + 1)}{r_0(R_f + R_s)})$$

$\Rightarrow$

$$R_{out} = \frac{V_2}{i_x} = \frac{r_0(R_f + R_s)}{R_f + R_s + r_0(1 - g_m R_s)}$$

2 Problem 2

2.1 a

First assume the NMOS is in saturation region

$$I_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{gs} - V_{TH}\right)^2\left(1 + \lambda V_{DS}\right)$$

Which is

$$I_{DS} = (800uA)(1 + 0.1(1 - 500I_{DS}))$$

Hence, $I_{DS} = 846uA$, let's verify is it in saturation region?
If I_{DS} = 846mA, V_{DS} = 1 - 500 $\times$ I_{DS} = 0.577V, so $V_{DS} > V_{GS} - V_{TH}$, it is in saturation region.

2.2 b

$$g_m = \frac{2I_D}{V_{GS} - U_{TH}} = \frac{2 \times 846\mu A}{0.8 - 0.4} = \frac{4.23\mu A}{V}$$

$$r_0 = \frac{1}{\lambda I_{DS}} = \frac{1}{0.1V^{-1} \cdot 846\mu A} = 11.82k\Omega$$

2.3 c

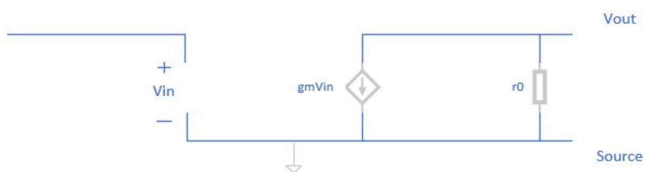

Figure 3: Problem 2C

2.4 d

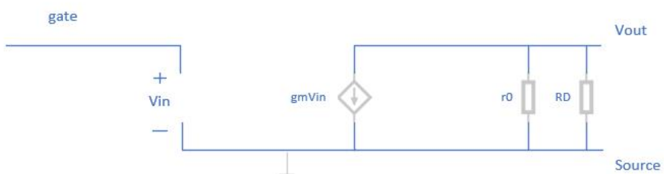

Figure 4: Problem 2D

2.5 e

$$\frac{V_{out}}{r_0} + \frac{V_{out}}{R_d} + g_m V_{in} = 0$$
$$V_{out}(\frac{1}{r_o} + \frac{1}{R_d}) = -g_m V_{in}$$
$$\frac{V_{out}}{V_{in}} = -gm(r_o//R_d)$$

2.6 f

$$\frac{V_{out}}{V_{in}} = -gm(r_o//R_d)$$
$$\frac{V_{out}}{V_{in}} = \frac{4.23mA}{V} \frac{11.82k\Omega \times 500\Omega}{11.82k\Omega + 500\Omega}$$
$$\frac{V_{out}}{V_{in}} = 2.03$$

3 Problem 3

3.1 a

The small signal diagram shown below:

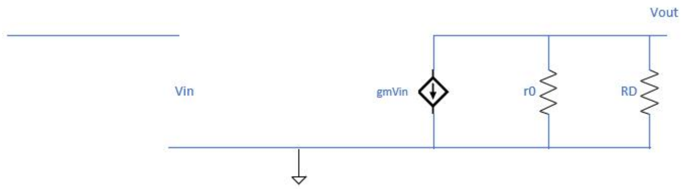

Figure 5: Problem 3A

$$A_v = \frac{v_{\text{out}}}{v_{\text{in}}} = -g_m\left(R_D//r_o\right)$$

where

$$r_o = \frac{1}{I_D\lambda}$$
$$g_m = \frac{2I_D}{V_{GS}-V_{TH}}$$
$$R_D//r_o = \frac{R_D r_o}{R_D + r_o}$$

Input impedance:

$$Z_{in} = \infty$$

Output impedance:

$$Z_{\text{out}} = R_D//r_o$$

3.2 b

The smallest $V_{out} = 0$,
The $V_{outmax} = V_{DD} - |V_{SD}| = V_{DD} - |V_{gs} - V_{TH}| = 1 - 0.1 = 0.9V$

3.3 c

$$A_v = g_m\left(r_o//R_D\right) \approx g_m R_D = 20$$
$$g_m = \mu_p C_{ox}\left(\frac{W}{L}\right)\left(V_{GS} - V_{TH}\right)$$

From the equations above, where we have assumed $r_o >> R_D$, one obtains $\frac{W}{L} = 200, I_D = 50\mu A$, we then determine $r_o = \frac{1}{\lambda I_D} = 200k\Omega >> R_D$, the original assumption holds.