Problem 1

(1a) $V_{P,min} = V_N - V_{Th} = V_{GS0} + V_{GS1} - V_{Th}$

$= (V_{GS0} - V_{Th}) + (V_{GS1} - V_{Th}) + V_{Th}$.

That is overdrive voltage of M_0 and M_1 plus additional one threshold voltage.

(1b) This is a cascode amplifier,

$R_{out} = (1 + g_{m3} r_{o3}) r_{o2} + r_{o3}$

$\approx g_{m3} r_{o3} r_{o2}$

(1c) For M_1

$I_{REF} = k_1 V_{OD,1}^2$ where $k = \frac{1}{2} \mu_n C_{ox} (\frac{W}{L})$

For M_2

$I_{out} = k_2 V_{OD,2}^2$

$= k_2 V_{OD,2}^2$

Note $\begin{cases} V_{GS1} = V_{GS2} \\ V_{Th1} = V_{Th2} \end{cases}$ $\therefore V_{OD,1} = V_{OD,2}$

$\therefore \frac{k_2}{k_1} = \frac{(W/L)_2}{(W/L)_1} = 10$, the ratio between $M_{0,3}$ is the same as $M_{1,2}$, so

$\frac{(W/L)_3}{(W/L)_0} = 10$.

(1d) $V_{p,min} = V_{OD_0} + V_{OD_1} + V_{Th} = 0.5V$

$V_{OD_0} + V_{OD_1} = 0.2V$

$\therefore V_{OD} = 0.1V$

For M_2 and M_3

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OD}^2 \Rightarrow \left(\frac{W}{L}\right)_{2,3} = 400$$

$$\text{So} \left(\frac{W}{L}\right)_{0,1} = \frac{400}{10} = 40.$$

(1e) $V_N = V_{GS_1} + V_{GS_0}$

$$I_{D_{0,1}} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{0,1} V_{OD_{0,1}}^2 \Rightarrow$$

$$V_{OD_{0,1}} = \sqrt{\frac{2I_{D_{0,1}}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{0,1}}} = \sqrt{\frac{2\times 0.1\times 10^{-3}}{5\times 10^{-4}\times 40}} = 0.1V$$

$\therefore V_{GS_{0,1}} = V_{OD_{0,1}} + V_{Th} = 0.4V$

$\therefore V_N = 0.8V$

$$\therefore R = \frac{V_{DD} - V_N}{I_{D_{0,1}}} = \frac{1.5 - 0.8}{0.1\times 10^{-3}} = 7k\Omega$$

Problem 2

(2a) $M_{3,4}$ are current source load of the common source half circuit.

M_7 is used to bias V_b.

(2b)

$$\frac{(W/L)_5}{(W/L)_8} = \frac{I_{ss}}{I_{ref}} = 5 \Rightarrow (W/L)_5 = 5\times 50 = 250.$$

Now, the current flowing through M6 is given by

$(W/L)_6 = I_{D6}/I_{ref} \cdot (W/L)_8$

Let $I_{D6} = 2.5 I_{ref} \Rightarrow (W/L)_6 = 2.5 \times 50 = 125$

As $(\frac{W}{L})_7 = 50$ and $I_{D3} = I_{D4} = \frac{1}{2} I_{SS} = 2.5 I_{ref} = I_{D6}$
$= I_{D7}$

So $(\frac{W}{L})_3 = (\frac{W}{L})_4 = (\frac{W}{L})_7 = 50$

(2C) $I_{ref} = 125\mu A \Rightarrow I_1 = I_2 = 2.5 \times 125 \mu A$
$= 312.5 \mu A$.

$I_{SS} = 125\mu A \times 5 = 625 \mu A$.

For M5 :

$I_{SS} = \frac{1}{2} \mu_n C_{ox} (\frac{W}{L})_5 V_{OD_5}^2$

$\Rightarrow V_{OD_5} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} (W/L)_5}}$

$= \sqrt{\frac{2 \times 625 \times 10^{-6}}{5 \times 10^{-4} \times 250}}$

$= 0.1 V$

$V_{out\,min} = V_{OD5} + V_{GS_1} - V_{Th}$

$= V_{GS_1} - 0.2V$ $(\frac{W}{L})_{1,2}$ is not given

Another limit posted by M3 in saturation

$$V_{out,max} = V_{DD} - V_{OD3}$$

$$V_{OD3} = \sqrt{\frac{2I_1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}} = \sqrt{\frac{I_{SS}}{2.5\times10^{-4}\times50}} = 0.22V$$

So the output range is $V_{GS1} - 0.2 \sim 1.28V$

(2d) $V_{in,CM} \geq 0.1V + V_{GS1}$

$$V_{in,CM} \leq V_{out} - V_{Th}$$
$$\leq 0.98V$$

$\therefore 0.1V + V_{GS1} \leq V_{in,cm} \leq 0.98V$

(2e) The output voltage is not well defined. It will depend on the specific value for the common mode input voltage

Problem 3.

(3a) $|A_v| = g_{m1,2}(r_{O2} \| r_{O4})$

At DC (or common mode), $V_{in1} = V_{in2}$.

So $I_{D1} = I_{D2} = \frac{1}{2} I_D = 0.5mA$

$$r_{O2} = \frac{1}{\lambda_n I_{D2}} = \frac{1}{0.1\times0.5\times10^{-3}} = 20k\Omega$$

$$r_{o4} = \frac{1}{\lambda_p I_{D4}} = \frac{1}{0.2 \times 0.5\times10^{-3}} = 10k\Omega$$

$$|A_v| = g_{m1,2}\,(r_{o2} \parallel r_{o4}) = 20$$

$$g_{m1,2} = \frac{\cancel{20}}{\cancel{20}/3\ k\Omega} = 3\times10^{-3}\ \Omega^{-1}$$

$$= \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_{1,2} I_{D1}} \qquad g_m = \frac{2I_D}{V_{OD}}$$

$$\left(\frac{W}{L}\right)_{1,2} = \frac{g_{m1,2}^2}{2\mu_n C_{ox} I_{D1}}$$

$$= \frac{9\times\cancel{10^{-6}}}{\cancel{2\times3}\times\cancel{10^{-4}}\times0.5\times\cancel{10^{-3}}} = 18.$$

(3b)

$$\left(\frac{W}{L}\right)_{3,4} = \frac{2I_{D3,4}}{\mu_p C_{ox} V_{OD}^2}$$

$$= \frac{1\times10^{-3}}{2.5\times10^{-4}\times0.2^2}$$

$$= 100$$

$$V_{OD1,2} = \frac{2I_{D1,2}}{g_{m1,2}} = \frac{2\times10^{-3}}{3\times10^{-3}} = 0.67V$$

$$V_{OD3,4} = 0.2V$$

$$g_{m3,4} = \frac{2I_{D3,4}}{V_{OD3,4}} = \frac{1\times10^{-3}}{0.2} = 5\times10^{-3}\ \Omega^{-1}$$

(3c) $V_{out,min} \geq V_{OD5} + V_{OD2}$

$V_{out,max} \leq V_{DD} - V_{OD4} = 1.3V$

(3d) The output DC bias is

$V_{out} = V_{DD} - V_{SG4}$

So the maximum swing is

$V_{DD} - V_{OD4} - (V_{DD} - V_{SG4}) = V_{SG4} - V_{OD4} = |V_{THP}| = 0.3V$

(3e)

$$r_{o_5} = \frac{1}{\lambda_n I_D} = \frac{1}{0.1 \times 1 \times 10^{-3}} = 10\,k\Omega$$

$$r_{o1,2} = \frac{1}{\lambda_n I_{D1,2}} = \frac{1}{0.1 \times 0.5 \times 10^{-3}} = 20\,k\Omega$$

$$r_{o3,4} = \frac{1}{\lambda_p I_{D3,4}} = \frac{1}{0.2 \times 0.5 \times 10^{-3}} = 10\,k\Omega$$

$$\text{CMRR} = (1 + 2 g_{m1,2} r_{o_5})\, g_{m3,4} (r_{o1,2} \,//\, r_{o3,4})$$

$$\approx 2 g_{m1,2} r_{o5}\, g_{m3,4} (r_{o1,2} \,//\, r_{o3,4})$$

$$= \cancel{2 \times 3} \times \cancel{10^{-3}} \times 10 \times \cancel{10^{3}} \times \cancel{5} \times \cancel{10^{-3}} \times \frac{200}{\cancel{30}} \times \cancel{10^{3}}$$

$$= 2000$$