EE 332 **Circuits and Devices II** **Autumn 2020**

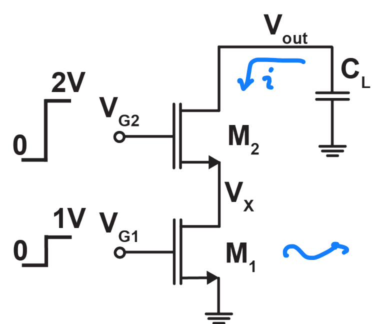
Midterm Exam 400 **Exam Date: Nov. 13 (5-7pm)**

Assume: VDD=1.5V, $Vth_{n,p}$=0.3V, $\mu_n C_{ox} = \frac{\cancel{200}\mu A}{V^2}$, $\mu_n = 2\mu_p, \lambda_p = 0.2V^{-1}$, $\lambda_n = 0.1V^{-1}, \gamma = 0$ for both NMOS and PMOS devices.

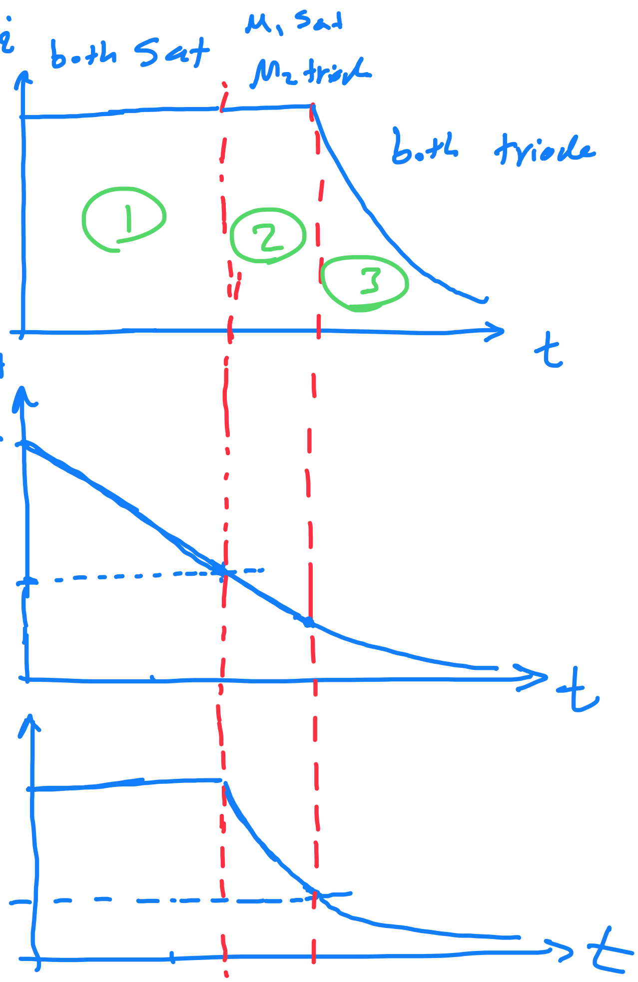
1. (15 pts) Suppose that while V_{out} at the beginning is at 5V, V_{G1} and V_{G2} will rise to 1V and 2V respectively and the capacitance starts discharging through $M_{1,2}$. Answer the following questions:

a) Find the regions of operation for M1 and M2 as C_L discharges. Also levels!!!

b) Plot V_x and V_{out} over time, mark the voltages where M1 or M2 transit into another operation region & I_D (a qualitative plot is enough, no need to solve the equations!).



$\rightsquigarrow V_{od_{1,2}} = 1 - 0.3 = 0.7V$



1 Both M_1 & M_2 are in Sat.

i is set by V_{gs_1} & since $I_1 = I_2$

$V_{gs_1} = V_{gs_2} \longrightarrow V_X = 2 - 1 = 1V$

V_X stays at 1V to keep $I_2 = I_1$, while V_{out}

V_{out} voltage drops linearly.

2 Once V_{out} reaches $V_X + V_{od_2} = 1.7V$

M_2 goes into triode, this causes

V_X to start dropping, but the current is

still fixed by M_1 since $V_X > V_{od}$.

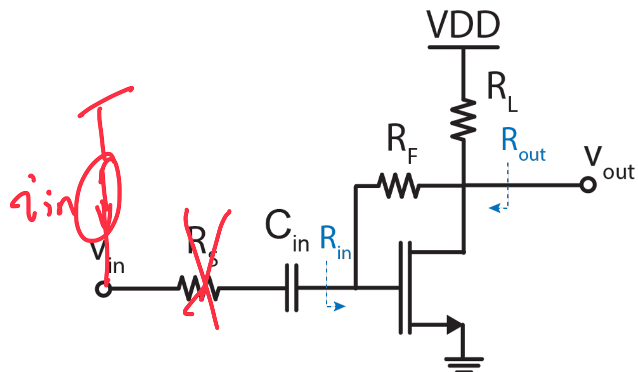
3 when V_X gets smaller than 0.7

M_1 also goes into triode.

Circuit behaves like an RC

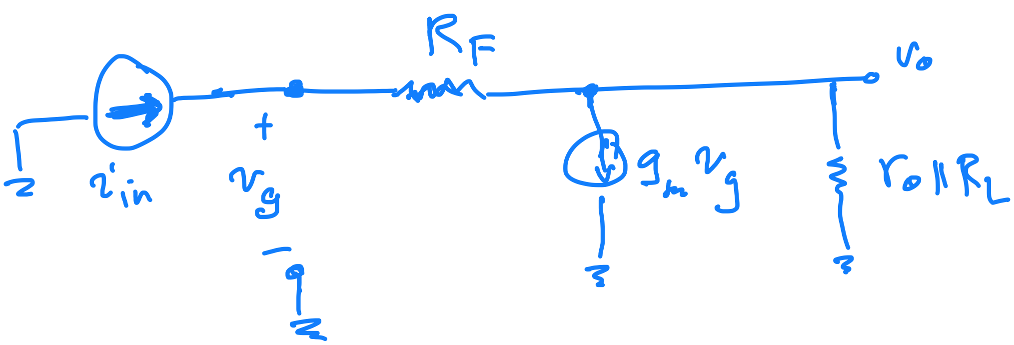
circuit until both V_X & V_{out} go to 0V.

2. (25 pts) For the single-stage amplifier given below, answer to the following questions. Assume Cin is very large and will be short in small-signal models. Your solution should only depend on gm, r_o, Rs, R_F, and R_L.



a) Draw the small-signal model.
b) Find the small-signal gain (Vout/~~Vin~~) (v_{out}/i_{in})
c) Find R_{out} and R_{in}. trm.

a)



b)

$$\begin{cases} i_{in} = g_m v_g + \dfrac{v_o}{r_o \| R_L} \\ v_g = v_o + i_{in} R_F \end{cases} \Rightarrow i_{in} = g_m v_o + i_{in} g_m R_F + \frac{v_o}{r_o \| R_L}$$

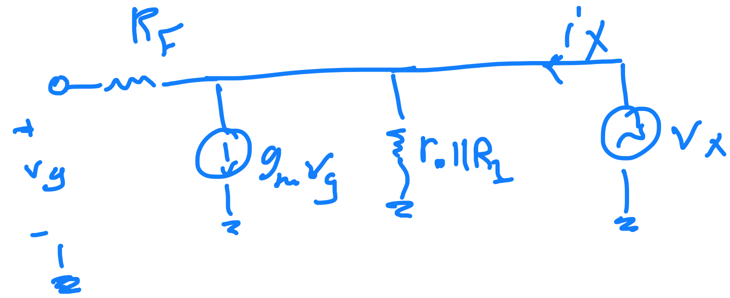
$$\Rightarrow \boxed{\frac{v_o}{i_{in}} = \frac{1 - g_m R_F}{g_m + \dfrac{1}{r_o \| R_L}}}$$

c)

$$R_{in} = \frac{V_g}{i_{in}} = \frac{v_o}{i_{in}} + R_F = \frac{1 - g_m R_F}{g_m + (r_o \| R_L)^{-1}} + R_F$$

$$= \frac{1 + R_F (r_o \| R_L)^{-1}}{g_m + (r_o \| R_L)^{-1}}$$

d) R_{out}:



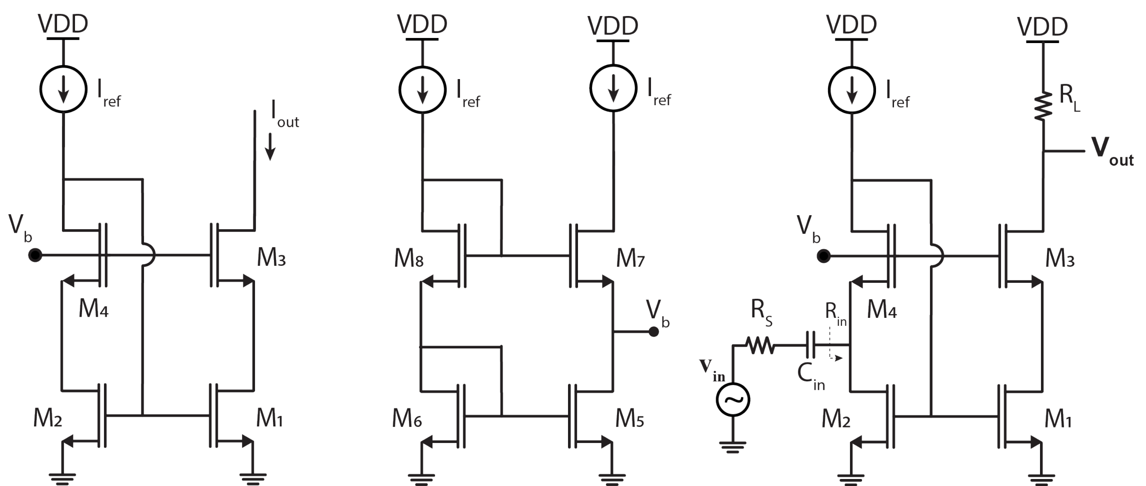
$$v_g = v_x$$

$$i_x = \frac{v_x}{r_o \| R_L} + g_m \cancel{v_g}^{\,v_x}$$

$$\Rightarrow \quad R_{out} = \frac{v_x}{i_x} = \frac{1}{g_m + r_o \| R_L} = \boxed{\frac{1}{g_m} \, \| \, r_o \, \| \, R_L}$$

3. (30 pts) For the current-mirror circuit shown in Figure a,

a) If $(W/L)_1=(W/L)_3=k(W/L)_2= k(W/L)_4$, what is the ratio of I_{out}/I_{ref}?
b) What is the minimum working voltage at the output as a function of V_{od} and V_b? What should be V_b value to minimize this voltage limit?
c) Suppose we want to use the circuit shown in Figure b to generate this V_b. Assuming W/L of $M_{5\text{-}7}$ is the same as W/L of M_2, what should be the ratio of $(W/L)_8/(W/L)_1$?



(a) (b) (c)

Now we decided to turn this current mirror into an amplifier shown in Figure c. Assume that C_{in} is a very large decoupling cap (short in small-signal) and the input voltage source has a series resistance of Rs:

d) Find the gain (Vout/Vin) and input and output impedances (assume $\boldsymbol{\lambda} = \mathbf{0}$). Write down your answers in terms of V_{od} ~~voltages of~~ $M_{1\text{-}4}$, Rs, R_L, and ~~I_{ref}~~. $g_{m_{1-4}}$, R_{ref}.

a) $\boxed{I_{out}/I_{ref} = k}$

b) $\boxed{V_{o,min} = V_b - V_{th}}$ → $\boxed{V_{b_{min}} = V_{OD_1} + V_{gs_3}}$

$V_{o,min} = 2V_{OD}$

Since $M_{1\text{-}4}$ have the same V_{od} → $V_{b_{min}} = V_{OD} + V_{gs}$

c) We need $V_b = V_{OD} + V_{gs}$

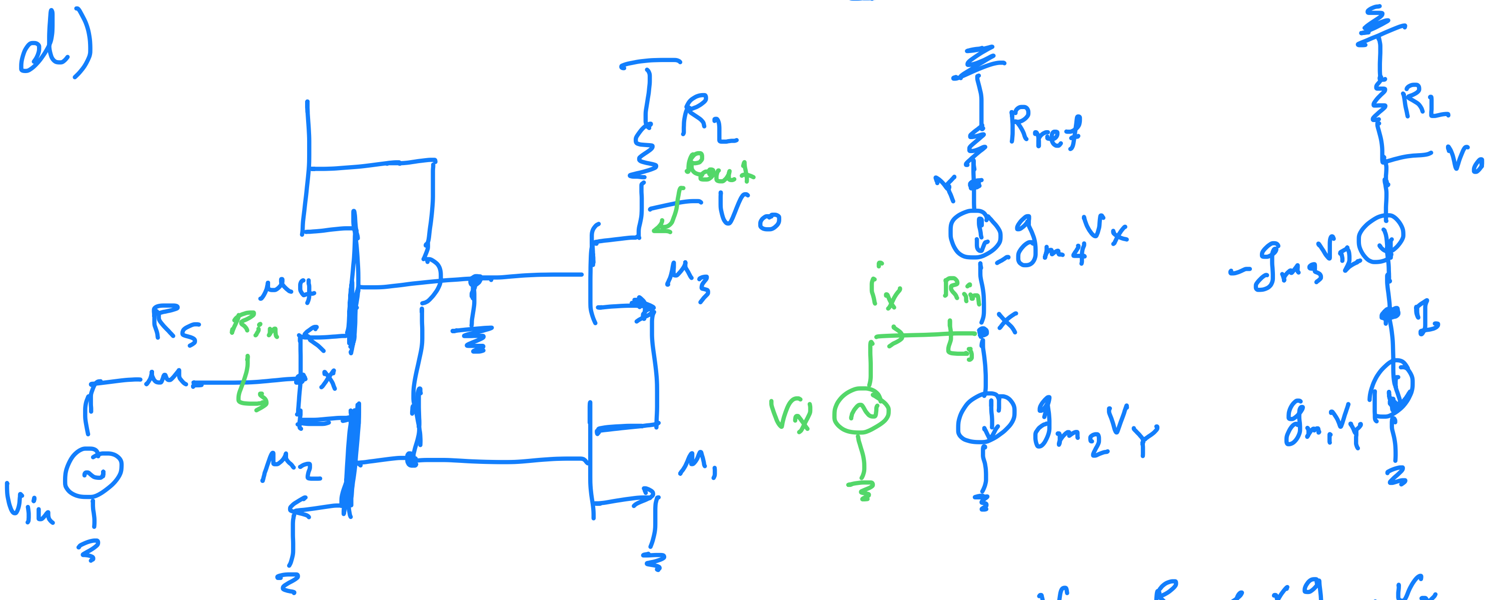
from circuit b: $V_b = \cancel{V_{gs}} + V_{gs_8} - \cancel{V_{gs}}$ ⇒ $V_{gs_8} = V_{gs} + V_{OD}$

⇒ $V_{OD_8} = V_{gs_8} - V_{th} = V_{gs} - V_{th} + V_{OD} = 2V_{OD}$

$\Rightarrow$ for the same current of I_{ref} to have

$V_{OD_8} = 2V_{OD}$, we need $\boxed{\left(\frac{W}{L}\right)_8 = \frac{1}{4}\left(\frac{W}{L}\right)}$

d)



$V_y = R_{ref} \times g_{m4} V_x$

R_{in}: $\quad i_x + (-g_{m4}V_x) = g_{m2} V_y$

$\Rightarrow \quad i_x = g_{m4} V_x + g_{m4} g_{m2} R_{ref} V_x$

$$\Rightarrow \boxed{R_{in} = \frac{V_x}{i_x} = \frac{1}{g_{m4}(1 + g_{m2} R_{ref})}}$$

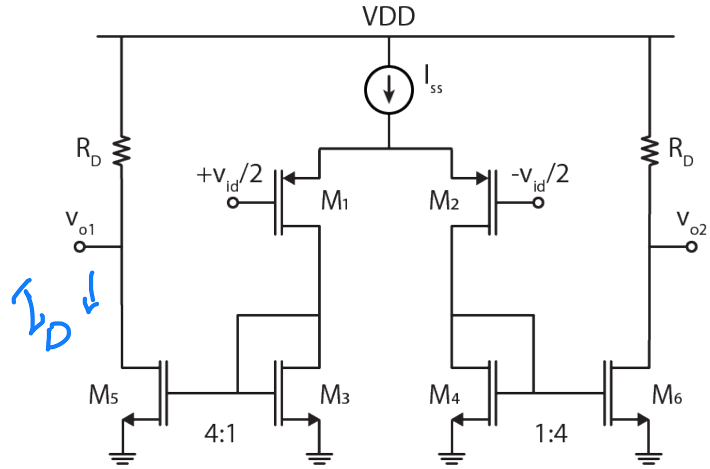
R_{out}: $\quad \boxed{R_{out} = R_L \parallel \infty = R_L}$

G_m: $\quad i_{out} = g_{m1} V_y = g_{m1} g_{m4} R_{ref} V_x$

$$V_x = \frac{R_{in}}{R_{in} + R_S} V_{in} \Rightarrow \boxed{G_m = \frac{g_{m1} g_{m4} R_{ref} R_{in}}{R_{in} + R_S}}$$

$\underline{A_v = -G_m R_{out}}$

4. (30 pts) For the differential amplifier shown below:



$$VDD = 1.2V, R_D = 1.5k\Omega,\ R_{SS} = 10k\Omega$$

$$\left(\frac{W}{L}\right)_{3,4} = 2\left(\frac{W}{L}\right)_{1,2} = 200,\ \left(\frac{W}{L}\right)_5 = 4\left(\frac{W}{L}\right)_3,\ \left(\frac{W}{L}\right)_6 = 4\left(\frac{W}{L}\right)_4$$

a) What should be the I_{SS} value in order to bias output nodes ($V_{o1/2}$) at VDD/2=0.6V?

b) Based on I_{SS} value from part a, what is the g_m and V_{od} of $M_{1,2}$?

c) Draw the differential-mode half circuit and calculate the differential gain ($\frac{v_{o2}-v_{o1}}{v_{id}}$)?

d) Draw the common-mode half circuit and calculate single ended common-mode gain ($\frac{v_{o1/2}}{v_{cm}}$)?

e) Find differential and single-ended peak-to-peak output swing.

f) If we implement the tail current with a PMOS with W/L=~~400~~ 200, what would be the input common-mode range?

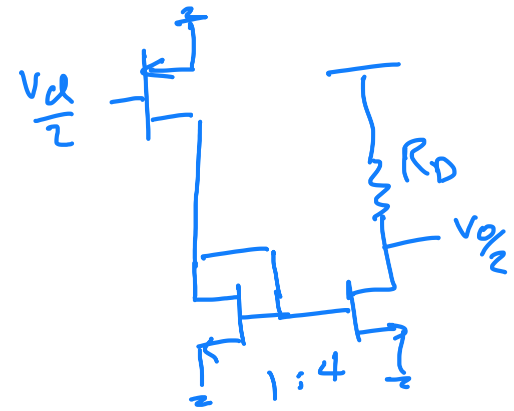
a) $V_o = 0.6V \rightarrow I_D = \frac{\cancel{V_{DD}-0.6}\ 0.6V}{1.5k\Omega} = 0.4mA$

$I_D = 4 \times \frac{I_{SS}}{2} = 2 I_{SS} \Rightarrow \boxed{I_{SS} = 0.2mA}$

b) $0.1mA = \frac{1}{2} \times 100 \times \frac{0.2mA}{V^2} \times V_{od}^2 \Rightarrow \boxed{V_{od_{1,2}} = 0.1V}$

$g_m = \frac{2I}{V_{od}} = \frac{0.2mA}{0.1} = \boxed{2mS}$

c)

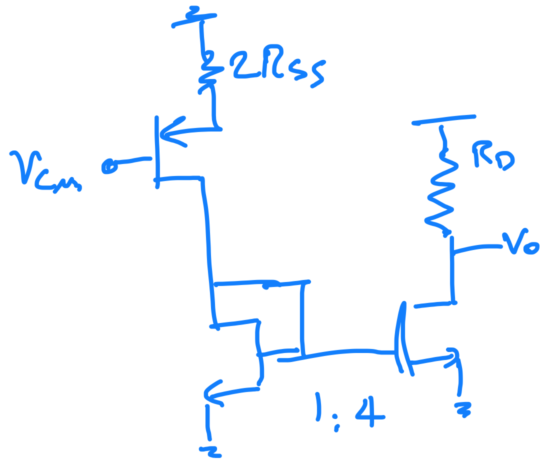


$$A_d = g_{m_1} \times 4 \times (R_D \| r_{o6})$$

$$r_{o6} = \frac{1}{\lambda_n I_D} = \frac{1}{0.1 \times 0.4mA} = 25k\Omega >> R_D$$

$$\Rightarrow A_d = 2mS \times 4 \times 1.5k = \boxed{12}$$

d)



$$A_{cm} = \frac{v_o}{v_{cm}} = \frac{g_{m_1}}{1+2R_{ss}\times g_{m}} \times 4 \times (R_D \cancel{\| r_{o6}}) \; R_D$$

$$= \frac{A_d}{1+g_{m_1}2R_{ss}} = \frac{12}{1+\underbrace{2mS\times 2\times 10k}_{40}} = \frac{12}{40}$$

$$\Rightarrow \boxed{A_{cm} \approx 0.3}$$

e)

$$\begin{cases} V_{out,max} = V_{DD} = 1.2V \\ V_{out,min} = V_{OD_{5,6}} = 0.1V \end{cases} \rightarrow \boxed{V_{swing,pp} = 1.1V}$$

single ended.

$$V_{OD_5}: \; 0.4\,mA = \frac{1}{2} \times \frac{0.4mA}{V^2} \times 200 \times V_{od}^2 \rightarrow \boxed{V_{od} = 0.1V}$$

Diff. peak-to-peak swing will be $2 \times 1.1 = \boxed{2.2V}$

f)

$$0.2\,mA = \frac{1}{2} \times \frac{0.2mA}{V^2} \times 200 \times V_{od_7}^2$$

$$\Rightarrow V_{od_7} = 0.1V$$

$$V_{in,cm} \leq V_{DD} - (V_{od_7} + V_{gs_{1,2}})$$

$$\Rightarrow \boxed{V_{in,cm} \leq \cancel{1.2 - (0.1 + 0.1 + 0.3)} \; 0.7V}$$

Additionally for M_1 & M_2 to stay in Sat:

$$\underbrace{V_{D_{1,2}}} - |V_{th}| \leq V_{in,CM}$$

$V_{gs_3} = V_{OD_3} + V_{th}$

$$\Rightarrow \quad V_{OD_3} \leq V_{in,CM}$$

$$\Rightarrow \quad \boxed{0.1V \leq V_{in,CM}}$$