

EE 332 Circuits and Devices II Autumn 2022
Midterm Exam Exam Date: Nov. 14 (5-7pm)

Full Name: **Grade:** /100

SID :

Assume for all the problems: $V_{DD}=1.2V$, $|V_{th_{n,p}}|=0.3V$, $\mu_n C_{ox} = \frac{0.5mA}{V^2}$, $\mu_n = 2\mu_p$, $\lambda_p = \lambda_n = 0.1V^{-1}$, $\gamma = 0$ for both NMOS and PMOS devices.

Useful Equations:

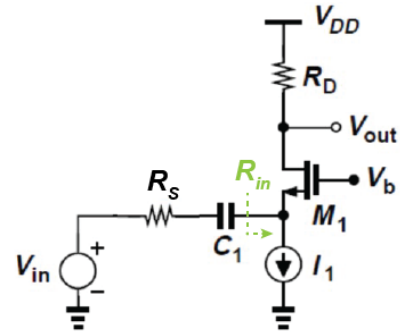
$$I_D = \frac{1}{2} \mu_{n,p} C_{ox} \left(\frac{W}{L} \right) (V_{GS} - V_{TH})^2$$

$$g_m = \mu_{n,p} C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) = \sqrt{2 \mu_{n,p} C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{V_{GS} - V_{TH}}$$

$$r_o = \frac{1}{\lambda I_D}$$

1. (40 pts) If $I_1 = 1\text{mA}$ (ideal current source), and C_1 is an AC-coupling capacitor (will be shorted in small-signal models and open for DC/bias analysis) answer following questions assuming $\lambda = 0$ for this problem. Notice that input voltage source has an impedance of $R_S = 50\Omega$.

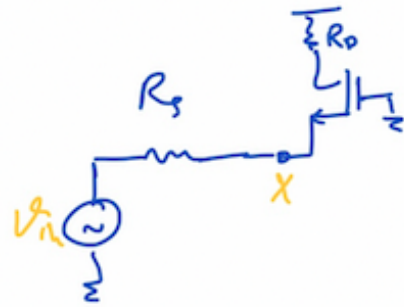
- Find $(W/L)_1$ such that the input impedance (R_{in}) will be 50Ω .
- Write down the small-signal gain equation in terms of small-signal parameters and R_D .
- Find R_D such that V_{out} bias point will be $V_{DD}/2$.
- Find small-signal gain using R_D from part c.
- If we realize I_1 current-source with a single NMOS device at a fixed gate bias of 0.5V , what would be the W/L for that device?
- What can be minimum V_b to make sure all transistors are in saturation?
- Assuming $V_{in} = 0.5 + 0.01\sin(\omega t)\text{V}$, what is the overall (bias and small-signal) V_{out} ? (C_1 will be shorted at all frequencies)
- What's the output voltage swing range (peak-to-peak)?



$$\begin{aligned}
 a) \quad R_{in} &= 1/g_m \Rightarrow g_m = \frac{1}{50} = 20 \text{ mS} \\
 \Rightarrow \frac{2 \cancel{I_D}}{V_{ov}} &= 20 \text{ mS} \Rightarrow \boxed{V_{ov} = 0.1 \text{ V}} \\
 I_D &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{ov}^2 \Rightarrow \boxed{\frac{W}{L} = 400}
 \end{aligned}$$

$$b) \frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_x} \times \frac{v_x}{v_{in}}$$

$$\begin{matrix} \swarrow & \searrow \\ g_m R_D & \frac{R_{in}}{R_{in} + R_S} = \frac{1}{2} \end{matrix}$$



$$\Rightarrow \boxed{A_v = \frac{1}{2} g_m R_D}$$

$$c) v_{out,DC} = V_{DD} - I_D R_D = V_{DD}/2 \Rightarrow \boxed{R_D = 600 \Omega}$$

$$d) A_v \approx \frac{1}{2} g_m R_D \Rightarrow \boxed{A_v = 6}$$

$$e) I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_{ov}^2 \Rightarrow \boxed{\frac{W}{L} = 100}$$

$0.5V - 0.3V = 0.2V$

$$f) v_{b,min} = v_{ov,tail} + v_{GS1} \quad \text{part a}$$

$$= 0.2V + 0.1V + 0.3V$$

$$\Rightarrow \boxed{v_{b,min} = 0.6V}$$

$$g) v_{out} = v_{out,DC} + v_{out,AC/SS}$$

$$= 0.6V + A_v \times v_{in}$$

$$\boxed{v_{out} = 0.6V + 0.06 \sin(\omega t)}$$

$$h) v_{out,max} = V_{DD}$$

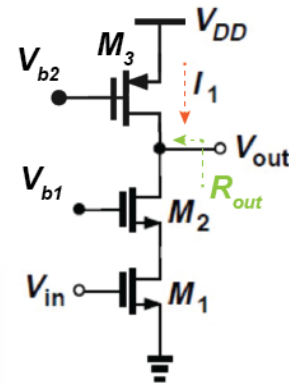
$$v_{out,min} = v_b - v_{th}$$

$$\Rightarrow v_{sw,pp} = 1.2 - (0.3)$$

$$\boxed{v_{sw,pp} = 0.9V}$$

2. (25 pts) For the Cascode amplifier shown in figure below all transistors are biased to have a V_{OD} ($V_{GS} - V_{th}$) of 0.2V with the drain current of current $I_1 = 1mA$ and $g_m r_o \gg 1$.

- Find R_{out} and R_{in} values for this amplifier.
- What is the gain of this amplifier?
- Calculate optimal V_{b1} to maximize the output swing.
- What's the output swing range (peak-to-peak)?



a) $R_{in} = \infty$

$$R_{out} = [r_{o1} + r_{o2} + \underbrace{g_{m2} r_{o2} r_{o1}}_{\sim g_{m2} r_{o2} r_{o1}}] \parallel r_{o3}$$

$$= \underbrace{\frac{2I_1}{V_{DD}} \times \frac{1}{\lambda I_1} \times \frac{1}{\lambda I_1}}_{100} \parallel \frac{1}{\lambda I_1}$$

$$\Rightarrow R_{out} \approx r_{o3} = \frac{1}{\lambda I_1} = \frac{1}{0.1 \times 1mA} = 10k\Omega$$

b) $A_v = -g_{m1} \times R_{out}$

$$= - \underbrace{\frac{2I_1}{V_{DD}}}_{10mS} \times \underbrace{R_{out}}_{10k\Omega} = -100$$

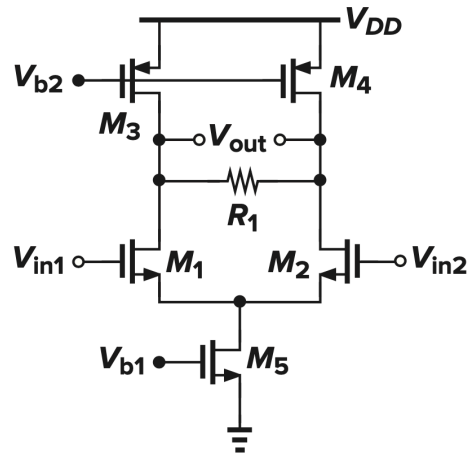
c) $V_{b1} = V_{OD1} + V_{DS2} = V_{OD1} + V_{OD2} + V_{th} =$
 $0.2V + 0.2V + 0.3V = 0.7V$

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$$d) \begin{cases} V_{out, max} = V_{pp} - V_{op2} = 1.2 - 0.2 = 1V \\ V_{out, min} = V_{op1} + V_{op2} = 0.2 + 0.2V = 0.4V \end{cases}$$

$$V_{sw, pp} = 1 - 0.4 = 0.6V$$

3. (35 pts) For the differential amplifier shown below:



- Assuming the target over-drive voltage (V_{od}) for all the transistors is 0.2V, find the V_{b1} , V_{b2} values.
- Find $(W/L)_{1,2}$, $(W/L)_{3,4}$, & $(W/L)_5$ such that current tail will be **8mA**.
- Draw the differential-mode half circuit and calculate the differential gain ($\frac{v_{out}}{v_{in1}-v_{in2}}$) in terms of small-signal parameters (g_m , r_o , and R_1)?
- Draw the common-mode half circuit and calculate single ended common-mode gain ($\frac{v_{out1/2}}{v_{in,cm}}$) in terms of small-signal parameters (g_m , r_o , and R_1)?
- For a differential gain of -10, what should be the value of R_1 ?
- Find differential output peak-to-peak swing.
- Find minimum input common-mode ($V_{in,cm}$) for this amplifier.

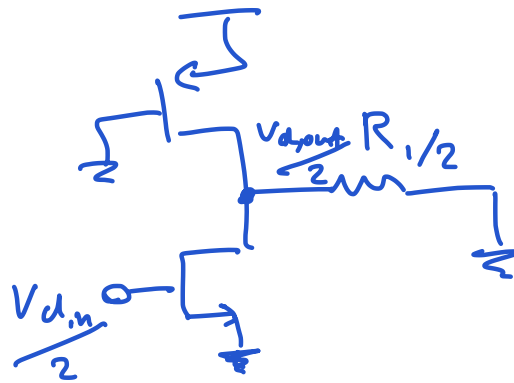
$$a) V_{b2} = V_{DD} - V_{GS3,4} = 1.2 - (0.2 + 0.3) = \boxed{0.7V}$$

$$V_{b1} = V_{GS5} + V_{th} = 0.2 + 0.3 = \boxed{0.5V}$$

b) using I_D eqn:

$$\boxed{\left(\frac{W}{L}\right)_{1,2} = 400} \quad \boxed{\left(\frac{W}{L}\right)_{3,4} = 800} \quad \boxed{\left(\frac{W}{L}\right)_5 = 800}$$

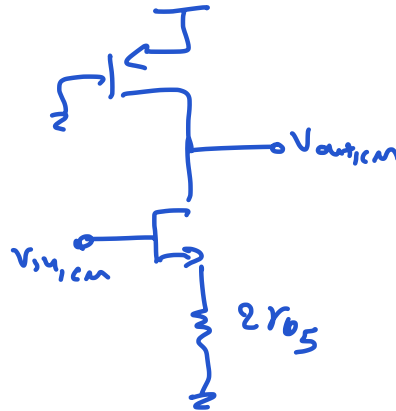
c) D.M



$$A_v = -g_{m1} (R_{i/2} \parallel r_{o1} \parallel r_{o2})$$

$$\approx -g_{m1} \frac{R_i}{2}$$

d) C.M



$$A_{v2} = \frac{-g_{m1} r_{o3}}{1 + g_{m1} \times 2r_{o5}}$$

e) $A_v \approx -\frac{2I_1}{V_{od}} \frac{R_i}{2} = -10$

$$= -\frac{2 \times 4 \text{ mA}}{0.2 \text{ V}} \times \frac{R_i}{2} = -10$$

$$\Rightarrow R_i = 500 \Omega$$

f) Single-ended Swing: $\begin{cases} V_{max} = 1.2 - 0.2 = 1 \text{ V} \\ V_{min} = 0.2 + 0.2 \text{ V} = 0.4 \text{ V} \end{cases}$

$$\text{Diff pp Swing} = 2 \times (1 - 0.4) = 1.2 \text{ V}$$

g) $V_{i, in, cm} \geq V_{GS1} + V_{ov1} \Rightarrow V_{i, in, cm} \geq 0.7 \text{ V}$