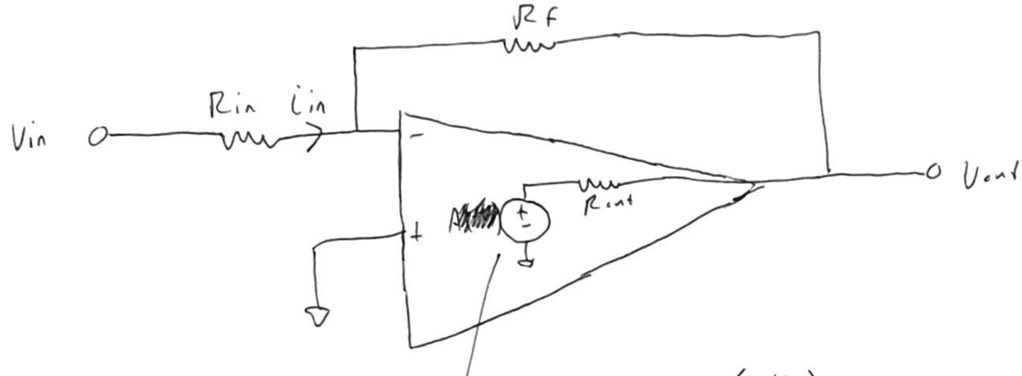
① · for this question parts a and b I will just solve for a generalized transfer function first and then I can adjust s and A as necessary to get results for ~~the~~ different parts

· for a generalized transfer function, we will describe the amp as follows:

$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_p}}, \quad A_0 = \text{DC gain}, \quad \omega_p = \text{pole frequency}$$



$$A(s) \cdot (v^+ - v^-) = A(s) \cdot (-v_x)$$

$$\frac{V_{in} - V_{out}}{R_{in} + R_f} = i_{in}, \quad V_{out} - i_{in}R_{out} = A(s)(-v_x), \quad v_x = V_{in} - i_{in}R_{in}$$

$$V_{out} - i_{in}R_{out} = A(s)\left(-V_{in} + i_{in}R_{in}\right)$$

$$V_{out}(s) - \frac{V_{in}(s) - V_{out}(s)}{R_f + R_{in}} R_{out} = A(s)\left[-V_{in}(s) + \frac{V_{in} - V_{out}}{R_f + R_{in}} R_{in}\right]$$

$$V_{out}(s) - \frac{V_{in}(s)R_{out}}{R_f + R_{in}} + \frac{V_{out}(s)}{R_f + R_{in}} R_{out} = -A(s)V_{in}(s) + \frac{A(s)R_{in}V_{in}(s)}{R_f + R_{in}} - \frac{A(s)R_{in}V_{out}(s)}{R_f + R_{in}}$$

$$R_f + R_{in}\left(V_{out}(s)\left[1 + \frac{R_{out}}{R_f + R_{in}} + \frac{A(s)R_{in}}{R_f + R_{in}}\right] = V_{in}(s)\left(-A(s) + \frac{A(s)R_{in}}{R_f + R_{in}} + \frac{R_{out}}{R_f + R_{in}}\right)\right)$$

$$V_{out}(s)\left[R_f + R_{in} + R_{out} + A(s)R_{in}\right] = V_{in}(s)\left[-A(s)(R_f + R_{in}) + A(s)R_{in} + R_{out}\right]$$

$$V_{out}(s)\left[R_f + R_{out} + R_{in}(1 + A(s))\right] = V_{in}(s)\left[R_{out} - A(s)R_f\right]$$

$$\boxed{\frac{V_{out}(s)}{V_{in}(s)} = \frac{R_{out} - A(s)R_f}{R_f + R_{out} + R_{in}(1 + A(s))}}$$

ⓐ set $s = 0$, $A_0 = \infty$, $R_{out} = 0$

$\hookrightarrow A(s) = A_0 = \infty$

$$\frac{V_{out}}{V_{in}} = \frac{R_{out} - A_0 R_f}{R_f + R_{in}(1 + A_0)}$$

as $A_0 \to \infty$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{-A_0 R_f}{R_{in} A_0} = \frac{-R_f}{R_{in}}}$$

$$\boxed{\frac{V_{out}}{V_{in}} = -5}$$

(b) Now, ~~Aless~~ $A_0 = 1000$, $\omega_p = 2\pi \cdot 10^7$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{R_{out} - A(s)R_f}{R_f + R_{out} + R_{in}(1 + A(s))}$$

$$= \frac{R_{out} - \left(\frac{A_0}{1+\frac{s}{\omega_p}}\right)R_f}{R_f + R_{out} + R_{in} + \frac{R_{in}A_0}{1+\frac{s}{\omega_p}}} \cdot \frac{1+\frac{s}{\omega_p}}{1+\frac{s}{\omega_p}}$$

$$= \frac{R_{out}\left(1+\frac{s}{\omega_p}\right) - A_0 R_f}{(R_f + R_{out} + R_{in})\left(1+\frac{s}{\omega_p}\right) + R_{in}A_0}$$

$$= \frac{R_{out} + \frac{R_{out}s}{\omega_p} - A_0 R_f}{(R_f + R_{out} + R_{in}) + \frac{(R_f + R_{out} + R_{in})s}{\omega_p} + A_0 R_{in}}$$

$$= \frac{\frac{R_{out}s}{\omega_p} + (R_{out} - A_0 R_f)}{\left(\frac{R_f + R_{out} + R_{in}}{\omega_p}\right)s + R_f + R_{out} + R_{in} + A_0 R_{in}}$$

$$= \frac{\left[\left(\frac{R_{out}}{\omega_p}\right)\left(\frac{1}{R_{out} - A_0 R_f}\right)s + 1\right](R_{out} - A_0 R_f)}{\left[\left(\frac{R_f + R_{out} + R_{in}}{\omega_p}\right)\left(\frac{1}{R_f + R_{out} + R_{in} + A_0 R_{in}}\right)s + 1\right](R_f + R_{out} + R_{in} + A_0 R_{in})}$$

note that this is the closed-loop gain

To find the closed loop gain we can set $s = 0$

$$\frac{V_{out}(s=0)}{V_{in}(s=0)} = \frac{100 - 1000 \cdot 5000}{1000 + 5000 + 100 + 1000 \cdot 1000}$$

$$\boxed{\frac{V_{out}(s=0)}{V_{in}(s=0)} = -4.97}$$

↳ closed loop gain

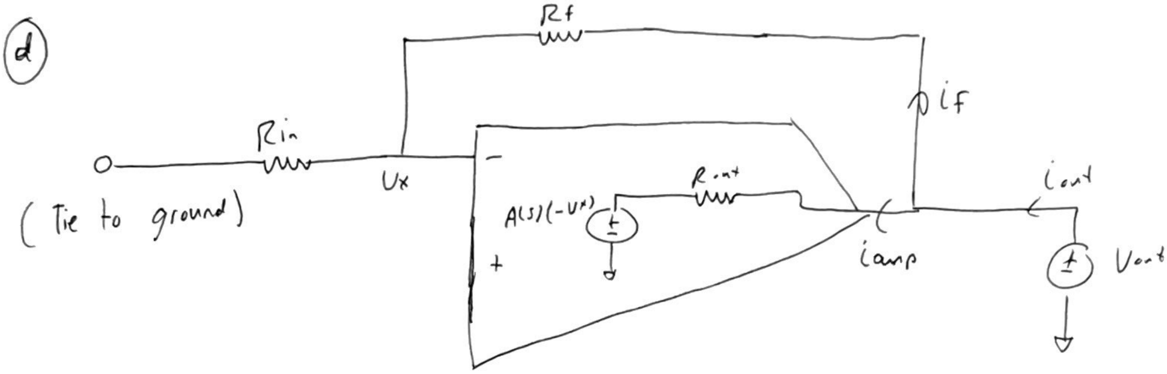
$$\omega_z' = \frac{(\omega_p)(R_{out} - A_0 R_f)}{R_{out}} = \frac{(2 \cdot \pi \cdot 10^7)(100 - 1000 \cdot 5000)}{100} = -3.14 \cdot 10^{12} \ \frac{\text{rad}}{\text{sec}}$$

$$\omega_p' = \frac{(\omega_p)(R_f + R_{out} + R_{in} + A_0 R_{in})}{R_f + R_{out} + R_{in}} = \frac{(2\pi \cdot 10^7)(1000 + 100 + 5000 + 1000 \cdot 1000)}{1000 + 100 + 5000} = 10.36 \cdot 10^9 \ \frac{\text{rad}}{\text{sec}}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\left(\frac{-s}{3.14 \cdot 10^{12}} + 1\right)}{\left(\frac{s}{10.36 \cdot 10^9} + 1\right)}(-4.97)$$

(c) fractional error $= \left|\frac{-5 + 4.97}{5}\right| = \frac{0.03}{5} = 0.006 = 0.6\%$

(d)



$$\frac{V_x(s)}{V_{out}(s)} = \frac{V_{out}(s)\cdot R_{in}}{R_{in}+R_F}$$

$$i_f^{(s)} = \frac{V_{out}(s)\cdot}{R_{in}+R_F}$$

$$V_{out}^{(s)} - i_{amp} R_{out} = A(s)(-V_x)$$

$$V_{out}(s) - i_{amp} R_{out} = A(s)\left(\frac{-V_{out}\cdot R_{in}}{R_{in}+R_f}\right)$$

$$V_{out}(s)\left[1 + A(s)\frac{R_{in}}{R_{in}+R_F}\right] = i_{amp} R_{out}$$

$$\frac{V_{out}(s)}{R_{out}}\left[1 + \frac{A(s) R_{in}}{R_{in}+R_f}\right] = i_{amp}(s)$$

$$R_{out}^* = \frac{V_{out}(s)}{i_{out}(s)}, \quad i_{out}(s) = i_f(s) + i_{amp}(s)$$

$$= \frac{\cancel{V_{out}(s)}}{\frac{\cancel{V_{out}(s)}}{R_{in}+R_f} + \frac{\cancel{V_{out}(s)}}{R_{out}}\left[1 + \frac{A(s)R_{in}}{R_{in}+R_f}\right]} \cdot \frac{R_{in}+R_f}{R_{in}+R_f}$$

$$= \frac{R_{in}+R_f}{\frac{1}{1} + \frac{R_{in}+R_f+A(s)R_{in}}{R_{out}}} \quad \frac{R_{out}}{R_{out}}$$

output impedance

$$\boxed{\begin{aligned} &= \frac{(R_{in}+R_f)(R_{out})}{R_{out} + R_{in} + R_f + A(s)R_{in}} \\ &= \frac{(1000+5000)(100)}{100+1000+5000+1000\cdot 1000} \\ &= 0.596\,\Omega \end{aligned}}$$

Problem 2.

(a) $$A(s) = \frac{A_0}{(1+\frac{s}{\omega_0})}$$

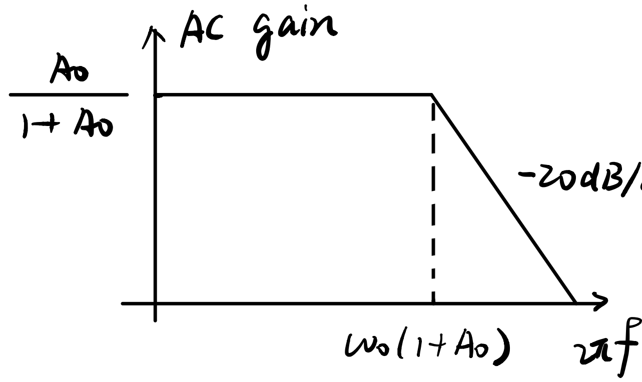
loop gain $= A(s)$ So $$V_{out}/V_{in} = \frac{A(s)}{1+A(s)}$$

or $\beta = 1$

$$= \frac{A_0}{(1+\frac{s}{\omega_0}) + A_0}$$

$$= \frac{A_0/(1+A_0)}{1 + \frac{s}{(1+A_0)\omega_0}}$$

Thus the pole is modified as $(1+A_0)\omega_0$, but the gain is also modified as $A_0/(1+A_0) \approx 1$



loop-gain $= \frac{1}{10} A(s)$

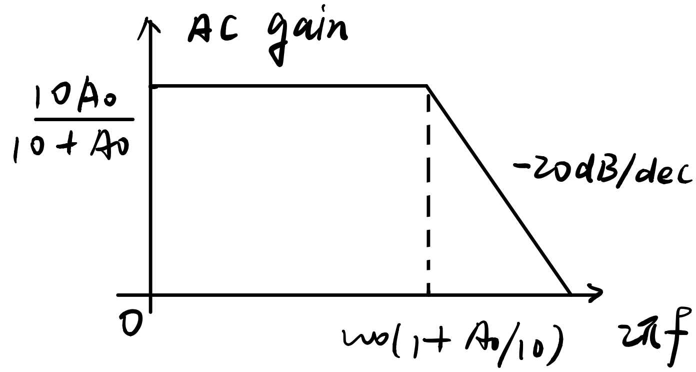
(b) $$\frac{V_{out}}{V_{in}} = \frac{A(s)}{1+\beta A(s)} = \frac{A_0/(1+\frac{s}{\omega_0})}{1 + 1/10 \cdot A_0/(1+s/\omega_0)}$$

$$= \frac{A_0}{(1+s/\omega_0) + 1/10 \cdot A_0}$$

$$= \frac{A_0}{A_0/10 + 1 + s/\omega_0}$$

$$= \frac{A_0/(1+A_0/10)}{1 + s/\omega_0(1+A_0/10)}$$

Bode plot see the next page.



(c) $$\text{loop gain} = \frac{1/Cs}{R+\frac{1}{Cs}} \cdot A(s)$$

$$= \underbrace{\frac{1}{RCs+1} A(s)}$$

$$\text{gain} = \frac{A(s)}{1+\frac{1}{RCs+1}A(s)}$$

$$= \frac{A_0/(1+\frac{s}{\omega_0})}{1+\frac{1}{RCs+1}A_0/(1+\frac{s}{\omega_0})}$$

$$= \frac{A_0(RCs+1)}{(RCs+1)(\frac{s}{\omega_0}+1)+A_0} \quad \text{denote } \frac{1}{RC}=\omega_1$$

$$= \frac{(\frac{s}{\omega_1}+1)}{\frac{1}{A_0}\frac{s}{\omega_1}\cdot\frac{s}{\omega_0}+\frac{1}{A_0}(\frac{s}{\omega_1}+\frac{s}{\omega_0})+\frac{A_0+1}{A_0}}$$

Let the denominator = 0.

$$\frac{1}{(A_0+1)\omega_0\omega_1}s^2+\frac{\omega_0+\omega_1}{(A_0+1)\omega_0\omega_1}s+1 = 0$$

One can use the dominant pole approximation, so that the coefficient of s is roughly $1/\omega_{p1}$

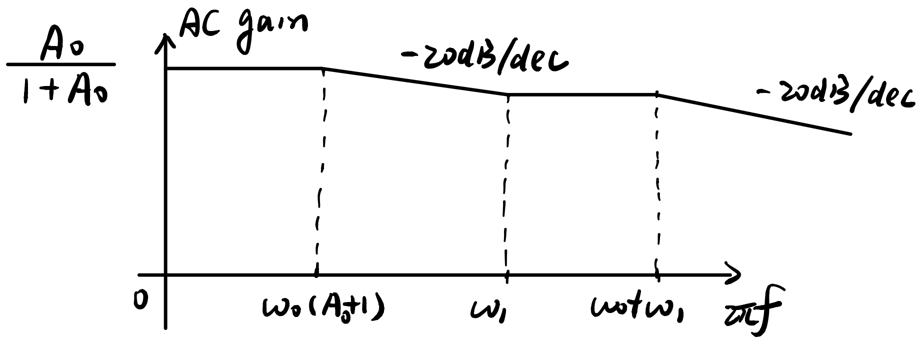
$$\therefore \begin{cases} \dfrac{1}{\omega_{p1}} = \dfrac{\omega_0+\omega_1}{(A_0+1)\omega_0\omega_1} \Rightarrow \omega_{p1} = \dfrac{(A_0+1)\omega_0\omega_1}{\omega_0+\omega_1} \\ \dfrac{1}{\omega_{p1}\omega_{p2}} = \dfrac{1}{(A_0+1)\omega_0\omega_1} \Rightarrow \omega_{p2} = \omega_0+\omega_1 \end{cases}$$

Assume $\omega_0 \ll \omega_1$

Note $\omega_{p1} \ll \omega_{p2}$

$$\begin{cases} \omega_{p1} = \dfrac{(A_0+1)\omega_0}{1+\dfrac{\omega_0}{\omega_1}} \approx (A_0+1)\omega_0 \\ \omega_{p2} \approx \omega_1+\omega_0 . \end{cases}$$

We need to make sure that $(A_0+1)\omega_0 \ll \omega_1+\omega_0$ so that our approximation is valid.



A more exact solution:

$$\Rightarrow \omega_{p1,2} = \frac{-\left(\frac{1}{\omega_0}+\frac{1}{\omega_1}\right) \pm \sqrt{\left(\frac{1}{\omega_0}+\frac{1}{\omega_1}\right)^2 - 4(A_0+1)\frac{1}{\omega_0\omega_1}}}{2\frac{1}{\omega_0\omega_1}}$$

Since we use the notation $\left(1+\frac{s}{\omega}\right)$,

$\omega_{p1,2} = -\omega_{1,2}$

$$\left(\frac{1}{\omega_0}+\frac{1}{\omega_1}\right) \pm \sqrt{\left(\frac{1}{\omega_0}+\frac{1}{\omega_1}\right)^2 - 4(A_0+1)\frac{1}{\omega_0\omega_1}}$$

$$= \frac{(\omega_0+\omega_1) \pm \sqrt{(\omega_0+\omega_1)^2 - 4(A_0+1)\omega_0\omega_1}}{2\,\frac{1}{\omega_0\omega_1}}$$

$$= \frac{(\omega_0+\omega_1) \pm \sqrt{(\omega_0+\omega_1)^2 - 4(A_0+1)\omega_0\omega_1}}{2}$$

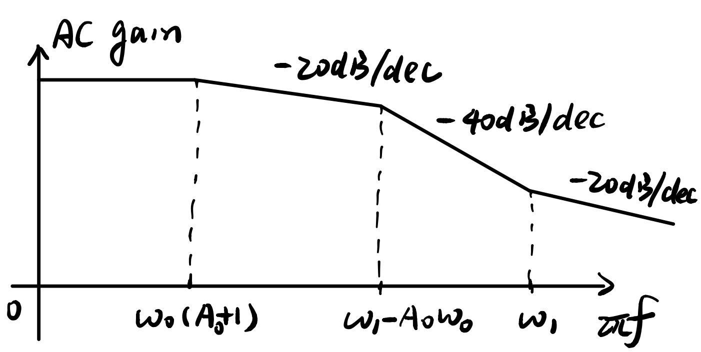
let's consider the case where $\omega_1 \gg \omega_0$.

$$\approx \frac{(\omega_0+\omega_1) \pm \omega_1\sqrt{(\omega_0/\omega_1+1)^2 - 4(A_0+1)\omega_0/\omega_1}}{2}$$

$$\approx \frac{(\omega_0+\omega_1) \pm \omega_1\sqrt{2(\omega_0/\omega_1)+1 - 4(A_0+1)\omega_0/\omega_1}}{2}$$

$$\approx \frac{(\omega_0+\omega_1) \pm [\omega_1 - (2A_0+1)\omega_0]}{2}$$

$$= \omega_1 - A_0\omega_0 \quad \text{or} \quad (A_0+1)\omega_0$$



Note also that this topology adds a zero at $\omega_1 = \frac{1}{RC}$