Homework5 Solution

November 13, 2022

1 Problem 1

1.1 a

To find the max output swing, we have
M2:

$$V_b - V_{TH} < V_{OUT}$$
$$V_b > V_x + V_{TH}$$

M1:

$$V_{in} - V_{TH} < V_x$$
$$V_{in} > V_{TH}$$

$\Rightarrow V_{max} = V_{DD}$ AND $V_{min} = V_{OD1} + V_{OD2}$
Assume the V_{OUT} is in the middle of the swing.

$$V_{OUT} = \frac{V_{DD}}{2} + \frac{V_{OD1}}{2} + \frac{V_{OD2}}{2}$$

That means to maximize the output swing, we should find $\frac{V_{OD1}}{2} + \frac{V_{OD2}}{2} \rightarrow \min$

$$A_v = g_{m1}g_{m2}(r_o)^2 = (r_o)^2 \frac{(I_D)^2 * 4}{V_{OD1}V_{OD2}}$$

And

$$r_0 = \frac{1}{\lambda I_D} = \frac{1}{0.2 * 1 \times 10^{-3}} = 5000\Omega$$

We can get the product of V_{OD1} and V_{OD2}, which is 0.0625, also, $x + y \geq 2\sqrt{xy} = 0.5$
Hence, we can choose the $V_{GS} - V_{TH} = 0.25V$ for each transistor.

1.2 b

If $V_{OD1} = 0.25V$ and $V_{OD2} = 0.25V$

$$g_m = \frac{2I_d}{V_{OD}} = \frac{2 * 1 * 10^{-3}}{0.25V} = 0.008$$

Also

$$g_m = \mu_n C_{ox}(W/L)(V_{GS} - V_{TH})(1 + \lambda V_{DS})$$

and as section A shows, $V_x = 0.375$
We can get $\frac{W}{L} \approx 306$

1.3 c

For M1:

$$V_{OD1} = V_{GS} - V_{TH}$$

$\Rightarrow V_{GS} = V_{IN} = 0.55V$
For M2:

$$V_{OD2} = V_{GS} - V_{TH}$$
$$V_{OD2} = V_b - V_x - V_{TH}$$

and as section A shows, $V_x = 0.375$, we can get $V_b = 0.925V$ And the maximum swing is from 0.5V to 1V

1.4 d

$$V_{\text{out, min}} = 1 - 0.3 - 0.25 + 0.25 = 0.7 \text{ V}$$
$$\text{Swing: } \tfrac{1-0.7}{2} = 0.15 \text{ V}$$

1.5 e

The bias current remains at 1mA, for this PMOS $gm_p = \frac{2I_D}{V_{OD}} = 0.02$

$$g_m = \mu_p C_{ox} \frac{W}{L} V_{OD}$$

We can know $\frac{W}{L} = 4000$

For NMOS:

$$gm_n = \frac{2I_D}{V_{OD}} = 0.008$$

Also,

$$|A_v| = gm_n[gm_n(r_o)^2||r_o] = 39$$

1.6 f

Because $\lambda \propto 1/L$, length of PMOS double, λ is half $\Rightarrow \lambda = 0.1$
Hence,

$$r_p = \frac{1}{\lambda I_D} = 10000\Omega$$

$\Rightarrow R_{out}$ does not change much, Hence A_v almost unchanged.

Also, $\frac{W}{L} = 2000$

1.7 e

It will increase parasitic capacitance, and large width gives large capacitance, which limit the speed.

Problem 1

(a) Consider left half

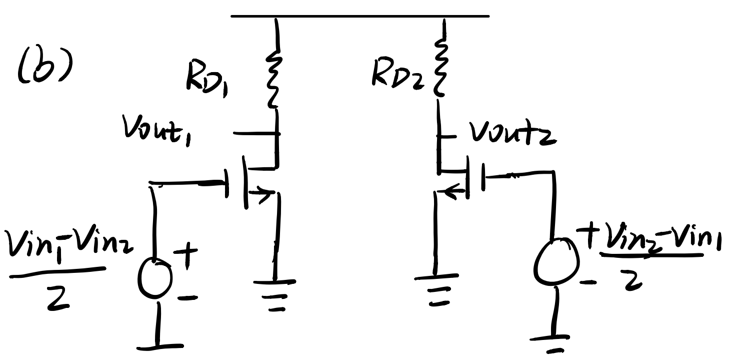
$$P = I_{SS} V_{DD} = 2mW \Rightarrow I_{SS} = 2mA$$

The differential peak-to-peak swing is

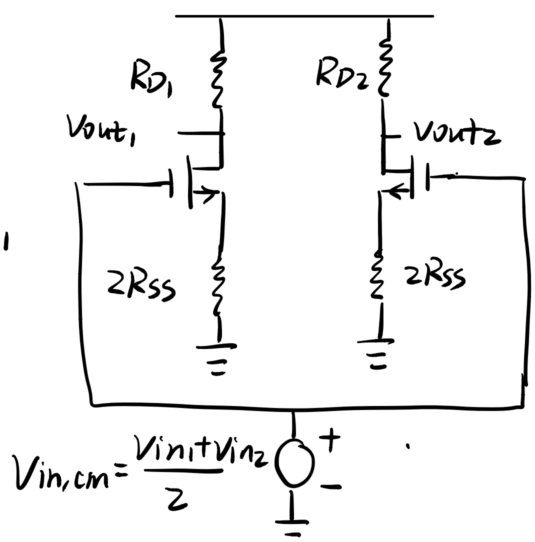
$$2 I_{SS} R_D = 2 \times 2 \times 10^{-3} A \cdot R_D = 1V$$

$$\Rightarrow R_D = 0.25 k\Omega$$

(b)



differential half-circuit



common half-circuit.

(c) $A_v = -g_m R_D = -5$

$$g_m = \frac{5}{R_D} = \frac{5}{250} = 0.02\ \Omega^{-1}$$

$$g_m = \frac{2I_D}{V_{OD}} \Rightarrow V_{OD} = \frac{2I_D}{g_m} = \frac{2 \times 10^{-3}}{0.02} = 0.1V$$

$$\frac{W}{L} = \frac{g_m^2}{2\, I_D\, \mu_n C_{ox}} = \frac{4\times10^{-4}}{2\times10^{-3}\times \cancel{200\times10^{-6}}} = \frac{4}{2\times10^{-3}}$$

$$= 1000$$

(d) According to Eq. 4.14

$$\Delta V_{in1} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} = \sqrt{\frac{2\times2\times10^{-3}}{200\times10^{-6}\times2000}}$$

$$= \sqrt{\frac{\cancel{4}\times10^{-3}}{\cancel{4}\times\cancel{10^{5}}\times\cancel{10^{-6}}\ 10^{-1}}}$$

$$= \sqrt{10^{-2}} = 0.1\,V$$

(e)

$$A_v = \frac{R_D}{\frac{1}{g_m}+2R_{SS}} = \frac{250}{50+2R_{SS}} = 0.05$$

$$50\times0.05+0.05\times2R_{SS} = 250$$

$$R_{SS} = 4.95\,k\Omega \approx 5\,k\Omega$$

(f)

$$r_o = \frac{1}{\lambda I_D} = 5\,k\Omega$$

$$\lambda = \frac{1}{5k\Omega\times2mA} = 0.1\,V^{-1} \Rightarrow L = 2L_0 = 130\,nm.$$

(g)

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{OD}^2$$

$$W/L = 2I_D/(\mu_n C_{ox} V_{OD}^2)$$

$$= 2\times2\times10^{-3}/(2\times10^{-4}\times0.2^2)$$

$$= 4\times10^{-3}/(0.08\times10^{-4}) = 500.$$

$\Rightarrow W = 500 \times 130nm = 65\mu m.$

(h) $V_{in,cm} \geq V_{OD\,current\,source} + V_{GS_1} = 0.2 + 0.2 + 0.3 = 0.7V$

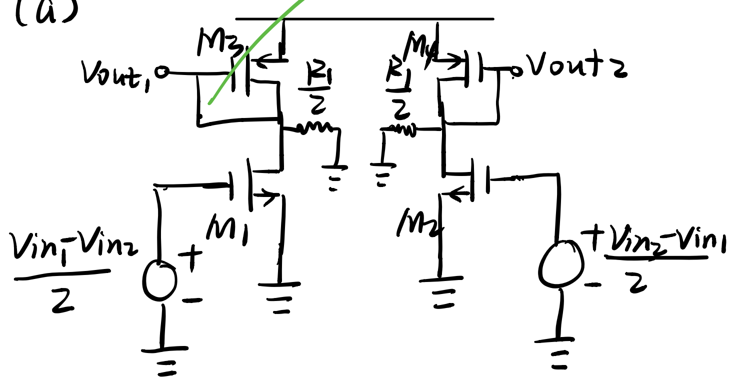
$$V_{in,cm} \leq \min[\underbrace{V_{DD} - R_D\frac{I_{SS}}{2} + V_{Th}}_{0.8V},\ V_{DD}]$$

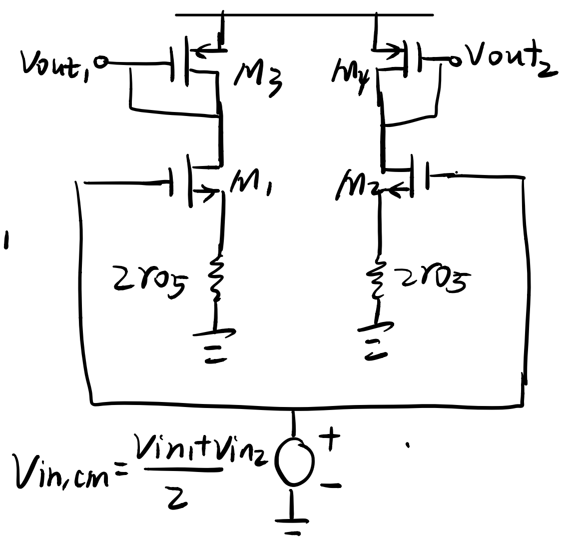
$\therefore \quad 0.7V \leq V_{in,cm} \leq 0.8V$

Problem 2.

always in saturation

(a)

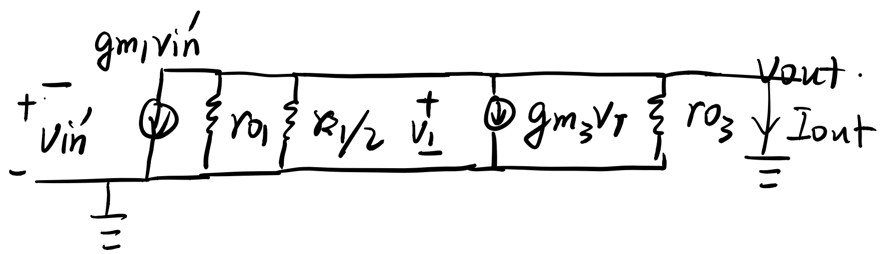




differential half-circuit common half-circuit.

(b) Only consider the left side now and denote $\frac{V_{in_1} - V_{in_2}}{2}$ as V_{in}'

small signal model.



Determine Gm: $G_m = \dfrac{I_{out}}{v_{in}'}$

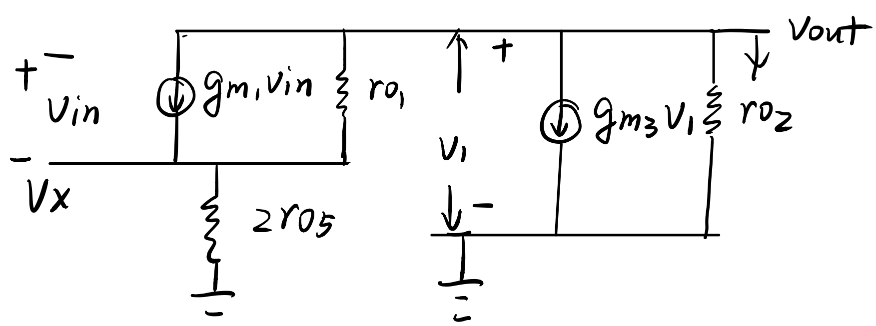
$I_{out} = -g_{m1}v_{in}'$ $\therefore G_m = -g_{m1}$

Determine Rout:

$$v_{out}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}} + \frac{2}{R_1} + g_{m3}\right) = I_{out}$$

$$R_{out} = \frac{1}{1/r_{o1} + 1/r_{o3} + 2/R_1 + g_{m3}}$$

$$\therefore A_{v,DM} = \frac{g_{m1}}{1/r_{o1} + 1/r_{o3} + 2/R_1 + g_{m3}}$$



Determine Gm: $-\dfrac{v_{in}}{(2r_{o5})//r_{o1}} - g_{m1}v_{in} = I_{out}$

$$\Rightarrow G_m = -\frac{1}{(2r_{o5})//r_{o1}} - g_{m1}$$

Determine Rout

$$R_{out} = (r_{o1} + 2r_{o5}) \, // \, r_{o2} \, // \, (1/g_{m3})$$

$$A_{v,cm} = -\left(g_{m1} + \frac{1}{(2r_{o5})//r_{o1}}\right)(r_{o1} + 2r_{o5}) \, // \, r_{o2} \, // \, (1/g_{m3})$$

(c) $V_{OD1} + V_{OD5} \leq V_{out,single} \leq V_{DD} - |V_{Th3}|$

Note the common mode level will be $V_{DD} - V_{SG3}$ where V_{SG3} is set by current source M5

Single-end Swing: $V_{DD} - |V_{Th3}| - (V_{DD} - V_{SG3}) = |V_{OD3}|$

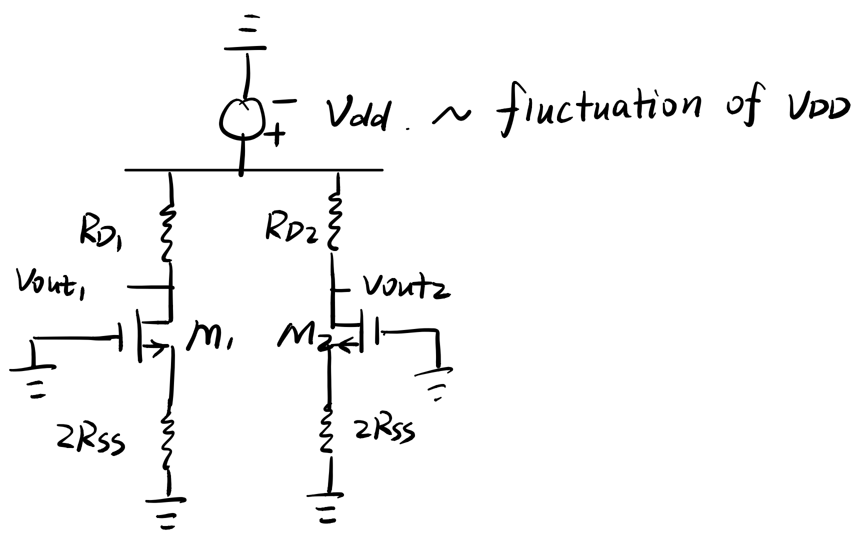
Differential swing: $2|V_{OD3}|$

(d) $V_{OD5} + V_{GS1} \leq V_{in,common} \leq V_{DD} - V_{OD3} + V_{Th1}$

Problem 3.

(a) No change in differential mode,
have change in common mode.

(b)



common mode half-circuit.

(c) Note although the gate is grounded in the small signal model, there is current I_D flow through $M_{1,2}$. (I_D is DC operatin point)

Look at the left branch,

According to Fig. 3.56

The output impedence seeing through the drain of M_1 is

$$(1+g_{m1}r_o)\cdot 2R_{SS} + r_o \approx 2R_{SS}\, g_{m1} r_o + r_o$$

The it's in parallel with R_{D1}

So the total output impedance is

$$(2R_{SS}\, g_{m1} r_o + r_o) \,//\, R_{D1}$$

Then calculate G_m: Once vout is grounded,

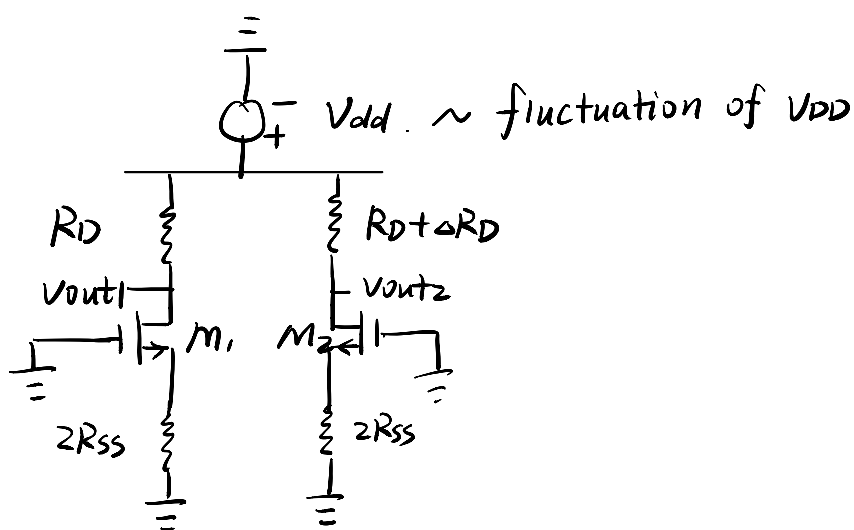
$$G_m = -1/R_{D1}$$

$$\therefore \quad A_v = -G_m R_{out} = \frac{1}{R_{D1}} \cdot [(2R_{SS} g_m r_o + r_o) // R_{D1}]$$

$$\approx 1$$

So the resultant vout fluctuation is the same as the fluctuation of VDD, i.e., vdd.

(d) Here we use a small signal method to solve it. Though the circuit is not fully symmetric, it's still valid to use the Lemma on page.113, thus the half-circuit:



common mode half-circuit.

From (3c)

$$|A_{v_i}| = \frac{R_{Di} // [(2R_{SS} g_{mi} r_{oi} + r_{oi})]}{R_{Di}}$$

$$\Delta V_{out} = (A_{v1} - A_{v2}) v_{dd} = \left\{ \frac{R_D // [2R_{SS} g_m r_o + r_o]}{R_D} - \frac{(R_D + \Delta R_D) // [2R_{SS} g_m r_o + r_o]}{R_D + \Delta R_D} \right\} v_{dd}$$

when R_{Di} is small compared to $r_o + 2R_{SS} g_m r_o$, $\Delta V_{out} = 0$

problem 4

(a) The differential output swing is

$$I_{SS} \cdot R_D$$

Single ended swing is

$$I_{SS} R_D / 2$$

Also note: the output swing is also related to the DC $V_{in,cm}$. $-V_{in} - |V_{ThP}| < -V_{out}$, thus $V_{out} < V_{in} + |V_{ThP}|$

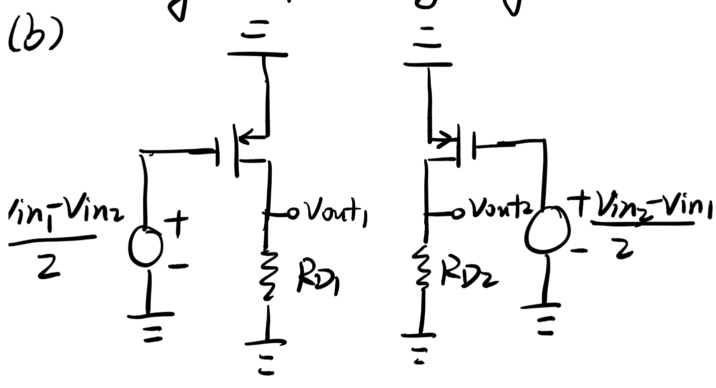
As we'll discuss in part d,

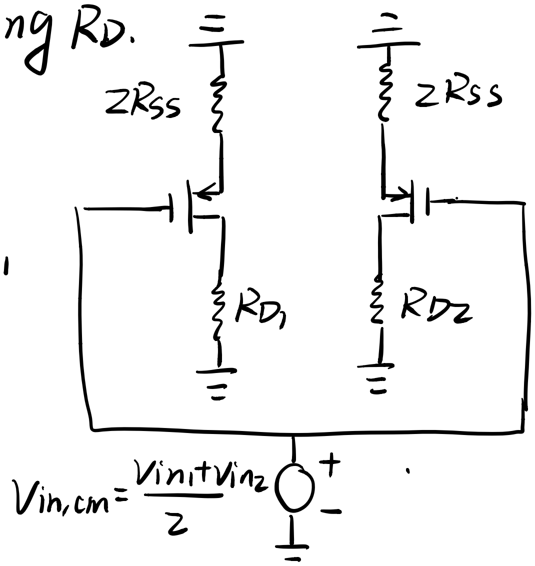
$$V_{in,CM} \leq V_{DD} - V_{OD,C} - V_{SG1}$$

$$\therefore \quad V_{out} < V_{DD} - V_{OD,C} - V_{OD1}$$

Thus the maximum single end p-p swing is $V_{DD} - V_{OD,C} - V_{OD1}$, this swing is limited by V_{OD}. This means one can't increase voltage swing infinitely by increasing R_D.

(b)





differential half-circuit

common half-circuit.

(c) DM: single end common source amplifier.

$$A_{v,DM} = -g_m R_D$$

CM: degenerated common source amplifier.

$$A_{v,CM} = -\frac{R_D}{1/g_m + 2R_{SS}}$$

(d) When $V_{in} = V_{DD}$, $V_{SG} < V_{Th}$.

As V_{in} continues to decrease, when

$V_{SG} - |V_{ThP}| = 0$ The transistors are on.

If the current source requires $V_{OD,c}$ overdrive voltage,

$$V_{DD} - V_{OD,c} - V_{in,cm} \geq |V_{ThP}|$$

$$V_{in,CM} \leq V_{DD} - V_{OD,c} - |V_{ThP}|$$

However, the current flows through the transistors requires certain overdrive voltage $V_{OD} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$

So $V_{DD} - V_{OD,C} - V_{in,cm} \geq \underbrace{|V_{ThP}| + V_{OD}}_{V_{SG}}$

$\Rightarrow$ $V_{in,cm} \leq V_{DD} - V_{OD,c} - V_{SG}$.

As $V_{in,CM}$ continue to decrease, when

$$V_{SG} - |V_{ThP}| \geq V_{SD}$$

i.e., $V_{DD} - V_{OD,c} - V_{in,CM} - |V_{ThP}| \geq V_{DD} - V_{OD,c} - V_{out}$

The transistors are in triode region.

$\Rightarrow \quad V_{in,cm} \leq V_{out} - |V_{ThP}|$

Note V_{out} is $\frac{1}{2} I_{SS} R_D$ in common mode,

to maintain in saturation region

$$V_{in,cm} > \frac{1}{2} I_{SS} R_D - |V_{ThP}|$$

if $\frac{1}{2} I_{SS} R_D - |V_{ThP}| < 0$, then

(this is easy to satisfy by design)

For $V_{in,cm} > 0$, conditionis always satisfied.

In that case $\quad 0 \leq V_{in,cm} \leq V_{DD} - V_{OD,C} - V_{SG}$.

(e) $\quad V_{OD,C} + V_{GS} \leq V_{in,NMOS} \leq \min\left[V_{DD} - R_D \frac{I_{SS}}{2} + V_{THN},\ V_{DD}\right]$

Note that when $V_{DD} - R_D \frac{I_{SS}}{2} + V_{ThN} > V_{DD}$

the upper limit of V_{DD} is always satisfied, so

$$V_{OD,C} + V_{GS} \leq V_{in,NMOS} \leq V_{DD}$$

For NMOS we only have a lower bound

, while for PMOS we only have a upper bound.