Homework1 Solution

October 19, 2022

1 Problem 1

(a).

$$S^2 = \frac{1}{2} \Rightarrow S = \sqrt{2}/2$$

(b).
Voltage should scale as S, because

$$E_{ox} = \left(V_{DD} - V_{\text{th}}\right)/t_{ox}$$

where the t_{ox} is gate oxide thickness
To keep E_{ox} constant, V scales as S, V_{th} scales as S.
(c).
Density of transistor $\sim 1/s^2$
Drain source current in saturation region $\sim$ S

$$I_{ds} = \frac{1}{2}\mu_n \text{ Cox } \frac{W}{L}\left(V_{GS} - V_{TH}\right)^2$$

C_{ox} scales as 1/S, as voltages goes as S, $I_{ds} \sim S$
g$_m$ remain constant

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{VDS\text{const}} = \mu_n C_{ax} \frac{W}{L}\left(V_{GS} - V_{TH}\right)$$

C_{ox} scales as 1/S, Hence g_m remain constant
junction capacitance $\sim \frac{S^2}{S} = S$, C $\sim \frac{Area}{Distance}$
$r_o \sim 1$ $r_o = \frac{1}{\lambda \times I_{ds}}$, λ scales with 1/s while I_{ds} scales as S, hence no change in r_o
(d).

$$\begin{gathered}\text{Idt} = \text{dVC}\\ \text{dV} \sim \text{S}\\ \text{C} \sim \text{S}\\ \text{I} \sim \text{S} \Rightarrow \text{dt} \sim \text{S}\end{gathered}$$

1/dt $\sim$ 1/S, the speed f scales as 1/S
(e).
$P = C_{GS} V_{DD}^2 f$, single device scales as $S \times S_2 \times 1/S = S^2$
Total power consumption: $S^2 \times \frac{1}{S^2} = 1$ remains unchanged.
(f).
Could be fabrication technique, could be quantum effects influence device performance.

2 Problem 2

a.
Before switch turn on, $V_{gs} = 0 > V_{TH} = -0.4$, there is no current in PMOS
After switch turns on, $V_{SD} = 1V > V_{SG} + V_{TP} = 0.6V$, PMOS is in saturation regime.

$$I_{SD} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(|V_{GS} - V_{TH}|\right)^2 = 277\mu A$$

For the Cap, $i = C\frac{dV}{dt} \Rightarrow \frac{dV}{dt} = 2.77 \times 10^9 \frac{V}{S}$ Capacitor will charge linearly because of the constant current until V_x = 0.4V, then V_{GD} = -0.4 = V_{TH}

$$V_x(t) = V_x(t=0) + \int \frac{dV_x}{dt} = 0.4 \text{ V}$$

$\Rightarrow t = 1.44 \times 10^{-10}s$
b.
At first, PMOS is in saturation region, after 1.44×10^{10} s capacitor charge, PMOS now is in triode region.

$$I_{SD} = \mu_p C_{ox}\frac{W}{L}\left[(V_{SG} - |V_{TH}|) \cdot V_{SD} - \frac{1}{2}V_{SD}^2\right]$$

$$\frac{\partial I_{SD}}{\partial V_{SV}} = \mu_p C_{OX}\frac{W}{\tau}\left(V_{SG} - |V_{TH,p}| - V_{SD}\right)$$

v_{SD} will goes from 0.6 to 0, we will use an average value by assuming $V_{SD} = 0.3$, so we can approximate the transistor as resistor.
$\frac{\partial I_{SD}}{\partial V_{SG}} = 4.62 \times 10^{-4}\Omega^{-1} \Rightarrow \frac{\partial V_{SG}}{\partial I_{SD}} = 2167\Omega$
Now we have this circuit:

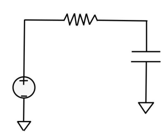

Figure 1: Problem2-b

$V_x(t) = V_{fin} - (V_{fin} - V_{init})\, e^{\frac{-(t-t_1)}{RC}} = 1 - (1.0 - 0.4)e^{\frac{-\left(t-1.44\times10^{-10}\right)}{2.167\times10^{-10}}}$
After $\Delta t = t - t1 = RC$, V_x will be $V_f - (V_f - V_i) \times e^{-1} = 0.78V$ and $t = 3.6 \times 10^{-10}$s
$\frac{\partial Vx(t)}{\partial t} = \frac{V_f - V_i}{RC} = \frac{0.6V}{2.167\times10^{-10}s} = 2.77 \times 10^9 \frac{V}{S}$

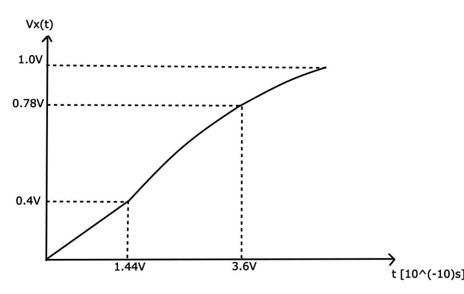

Figure 2: Problem2-b-2

c.
In this situation, the circuit won't work very well because the PMOS will off when $V_{SG} = 0.3$ because $V_{TH} = 0.4V$. If V_x starts at a high voltage before V_{SW} goes 1 to 0, then it will drop to 0.4, and the current will stay flowing, so Vx will never read Vin.
We can use the following circuit:

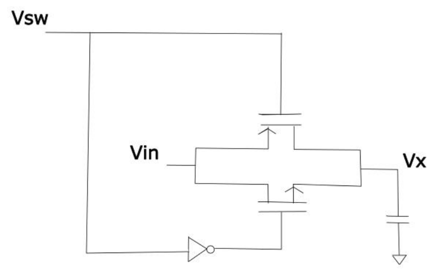

Figure 3: Problem2c

The NMOS can solve this issue when Vin is low and PMOS can solve this issue when Vin is high.

3 Problem 3

a.

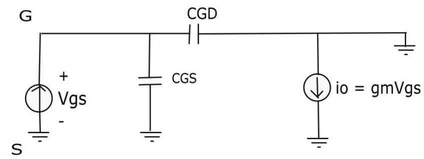

Figure 4: Problem3A

Note that V_{gs} is a small signal.

$$i = j\omega(CGD + CGS)Vgs \text{ and } i_o = gmV_{gs}$$

so,

$$|\beta| = \frac{i_0}{i_i} = 1 \Rightarrow \frac{g_m}{\omega_T\left(C_{GD} + C_{GS}\right)} = 1$$

$$\omega_T = \frac{g_m}{C_{GD} + c_{GS}}$$

$$f_t = \frac{w_t}{2 \times \pi} = \frac{g_m}{2 \times \pi \times (C_{gd} + C_{gs})}$$

c.
$g_m = \mu_n C_{0x} \frac{W}{L}\left(V_{GS} - V_{TH}\right)$
$C_{GD} + C_{GS} \approx C_{0\times} WL.$
Plug into the original f_t from part a can prove the equation

4 Problem 4

NMOS:
From book, we know

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{8.85 \times 10^{-14} \times 3.9}{9 \times 10^{-7}} = 3.83fF/(um)^2$$

also, $\mu_n = UO$, Hence,

$$g_m = \sqrt{2u_n C_{ox} \frac{w}{L} I_{DS}} = 3.66 \frac{\text{mA}}{\text{V}}$$

$$r_o = \frac{1}{\lambda I_{DS}} = 20k\Omega$$

$$g_m r_o = 73.3$$

PMOS:

$$g_m = \sqrt{2N_p C_{ox} \frac{W}{L} I_{DS}} = 1.96 \frac{mA}{V}$$

$$r_o = \frac{1}{\lambda I_{DS}} = 10k\Omega$$

$$g_m r_o = 19.6$$

5 Problem 5

NMOS as a example, we have, in saturation:

$$I_D \approx \frac{1}{2}\mu_n C_{ax}\frac{W}{L}\left(V_{GS}-V_{TH}\right)^2\left(1+\lambda V_{DS}\right)$$

$$\begin{aligned} g_m &= \mu_n C_{ox}\frac{W}{L}\left(V_{GS}-V_{TH}\right)\left(1+\lambda V_{DS}\right) \\ &= \sqrt{2\mu_n C_{ox}(W/L) I_D\left(1+\lambda V_{DS}\right)} \end{aligned}$$

λV_{DS} is small,

$$gm = \sqrt{2\mu_n C_{ox}(W/L) I_D}$$

$$\begin{aligned} r_O &= \frac{\partial V_{DS}}{\partial I_D} \\ &= \frac{1}{\partial I_D/\partial V_{DS}} \\ &= \frac{1}{\frac{1}{2}\mu_n C_{ax}\frac{W}{L}\left(V_{GS}-V_{TH}\right)^2\cdot\lambda} \\ &\approx \frac{1}{\lambda I_D} \end{aligned}$$

so,

$$g_m r_0 = \frac{\sqrt{2\mu_n C_{ox}\frac{W}{L} I_{DS}}}{\lambda I_{DS}}$$

Since $\lambda \propto \frac{1}{L}$, we can rewrite

$$g_m r_o = C\cdot\sqrt{\frac{L}{I_{DS}}}$$

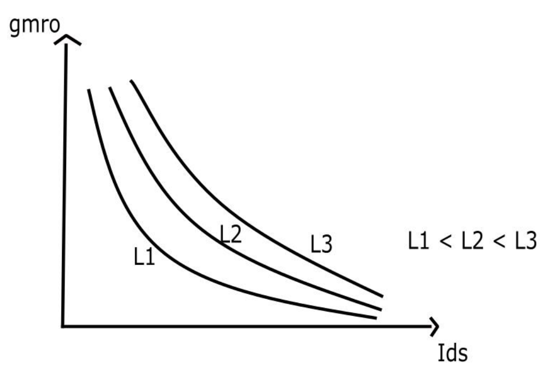

Figure 5: Problem5