EE 332: Devices and Circuits II

Lecture 4: Differential Amplifiers

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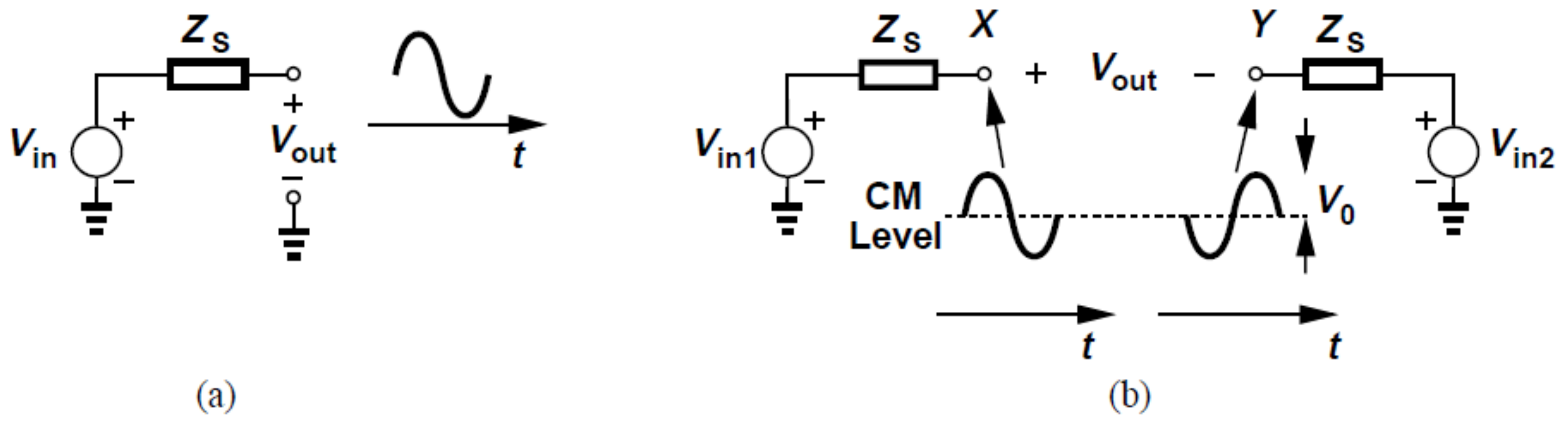
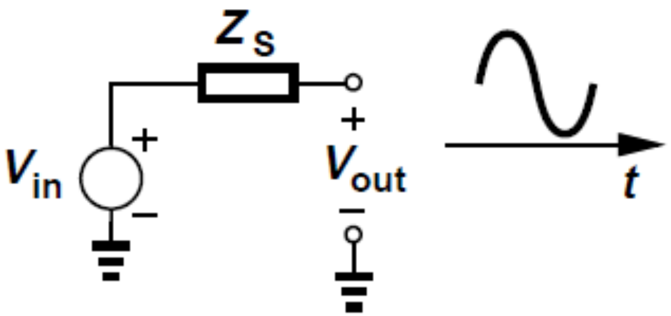
Single-Ended and Differential Operation

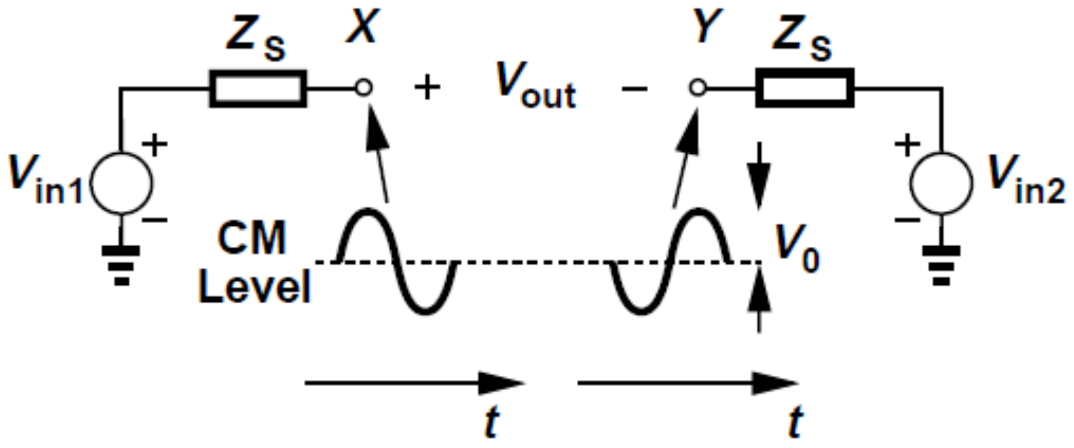
- A "single-ended" signal is one that is measured with respect to a fixed potential, usually the ground [Fig. (a)]
- A differential signal is one that is measured between two nodes that have equal and opposite signal excursions around a fixed potential [Fig. (b)]



- The "center" potential in differential signaling is called the "common-mode" (CM) level
  - bias value of the voltages, i.e., value in the absence of signals

Single-Ended (SE) vs. Differential



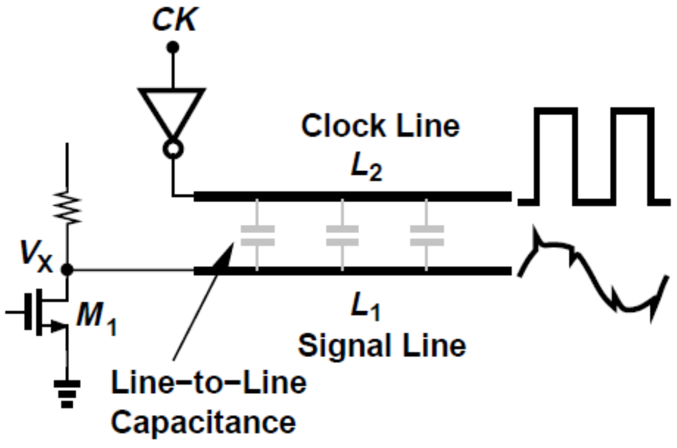


$V_{CM} = ?$

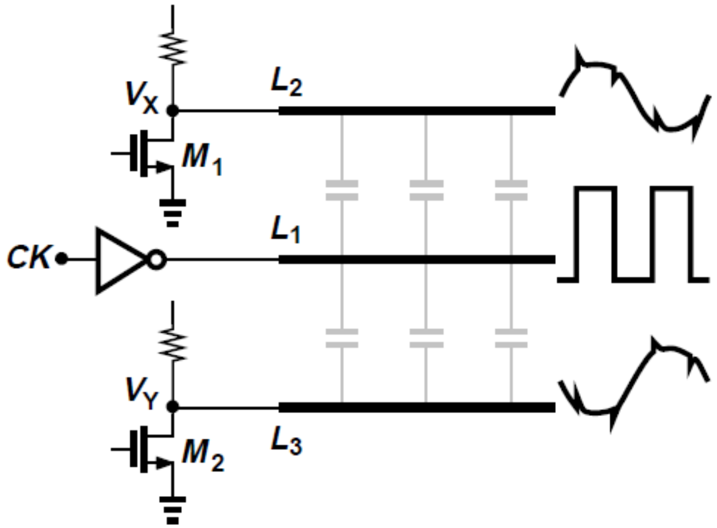
$V_d = ?$

Advantages of Differential Operation (1)

- Higher immunity to "environmental" noise in differential operation as compared to single-ended signaling



(a)

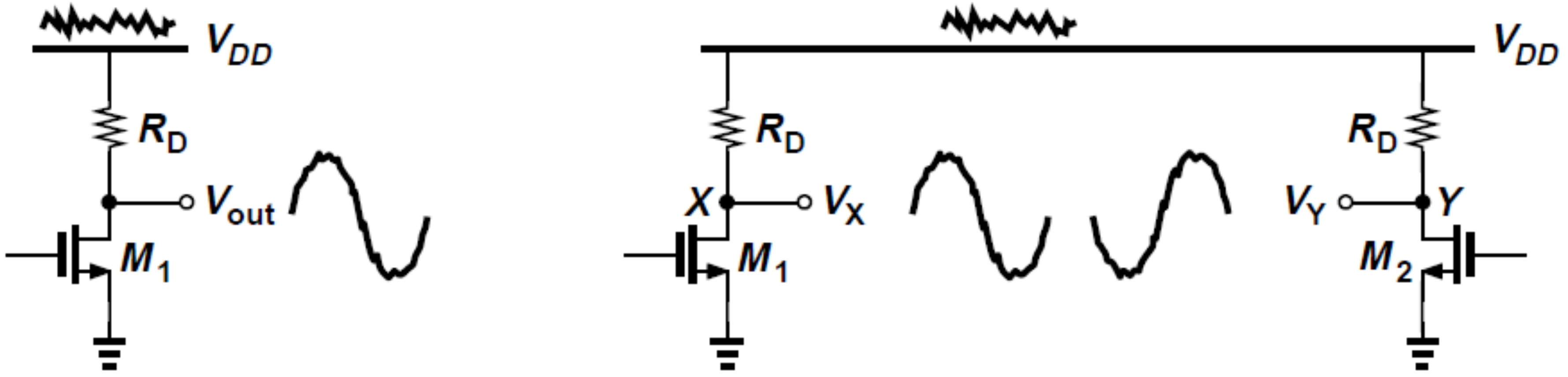


(b)

- In Fig. (a), transitions on the clock line L_2 corrupt the signal on sensitive signal line L_1 due to capacitive coupling between the lines
- If the sensitive signal is distributed as two equal and opposite phases as in Fig. (b), the clock line placed midway disturbs the differential phases equally and keeps the difference intact, called common-mode (CM) rejection

Advantages of Differential Operation

- CM rejection also occurs with noisy supply voltages

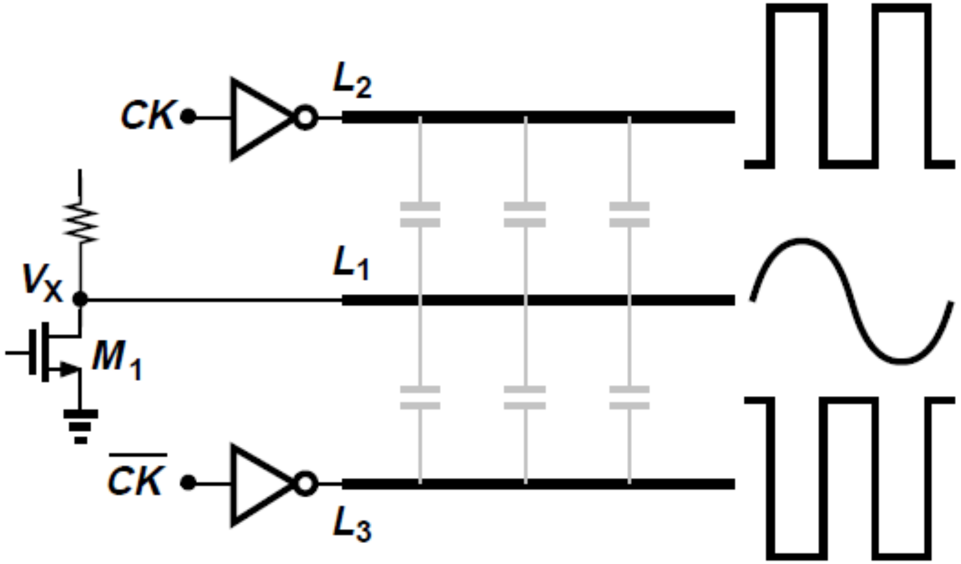


(a) (b)

- In the CS stage of Fig. (a), if V_{DD} varies by ΔV, then V_{out} changes by roughly the same amount, i.e., output is susceptible to noise on V_{DD}
- In the symmetric circuit of Fig. (b), noise on V_{DD} affects V_X and V_Y, but not $V_X - V_Y = V_{out}$
- The differential circuit is more robust to supply noise than its single-ended counterpart

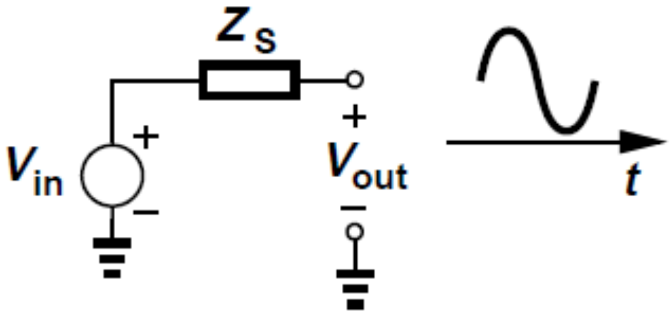
Advantages of Differential Operation

- Differential operation is as beneficial for sensitive signals ("victims") as for noisy lines ("aggressors")

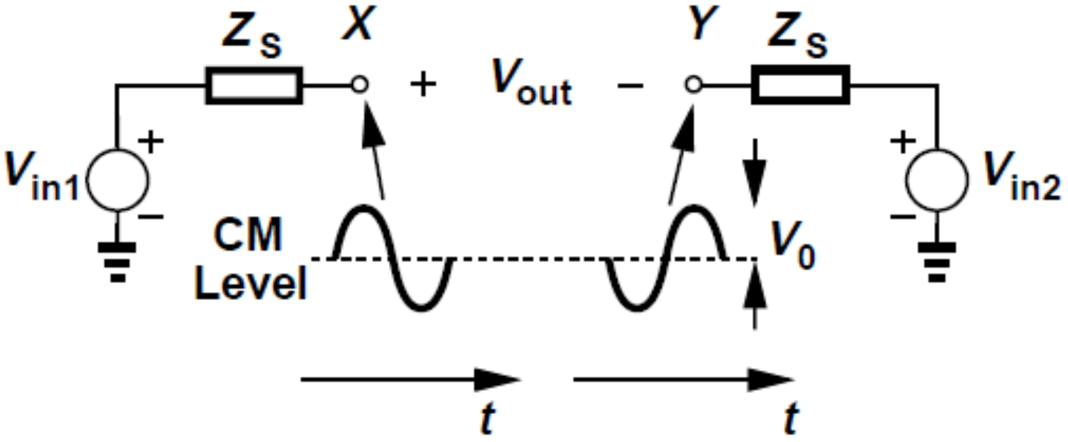


- Clock signal is distributed in differential form on two lines
- With perfect symmetry, the components coupled from *CK* and $\overline{CK}$ to the signal line cancel each other

SE vs. Differential (Voltage Swing)



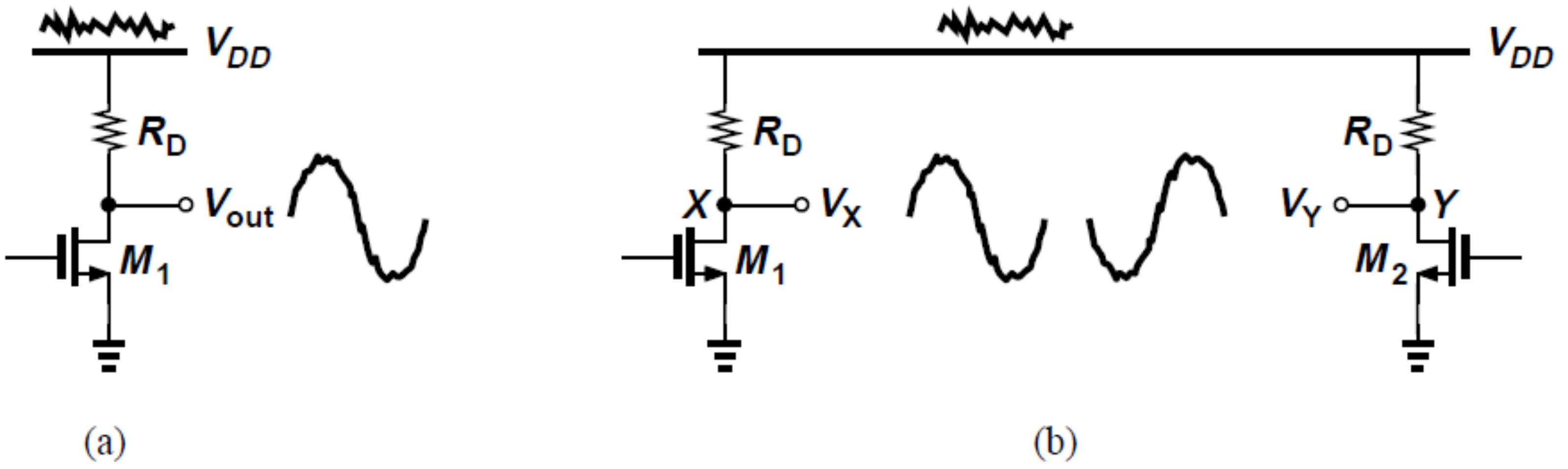
(a)



(b)

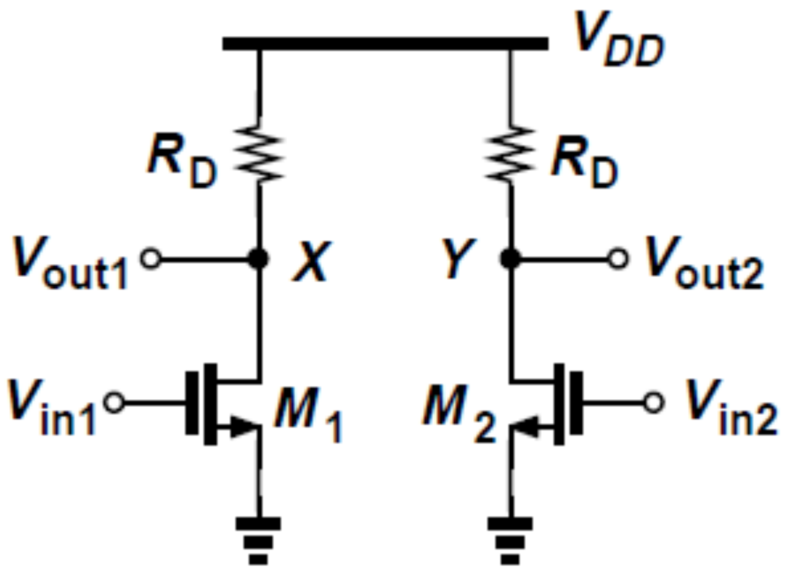
- Suppose each single-ended output in Fig. (b) has a peak amplitude of V_0
    - Then single-ended peak-to-peak swing is $2V_0$ and differential peak-to-peak swing is $4V_0$

Advantages of Differential Operation



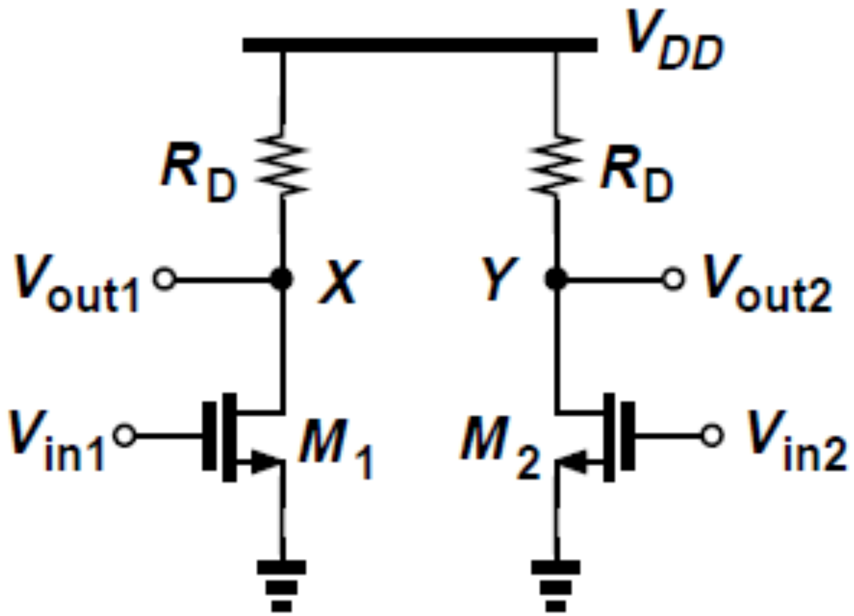
- Differential signaling increases maximum achievable voltage swings
- In the above differential circuit, the maximum output swing at *X* or *Y* is equal to $V_{DD} - (V_{GS} - V_{TH})$
- For $V_X - V_Y$, the maximum swing is $2[V_{DD} - (V_{GS} - V_{TH})]$
- Other advantages of differential circuits include simpler biasing and higher linearity
- Advantages of differential operation outweigh the possible increase in area

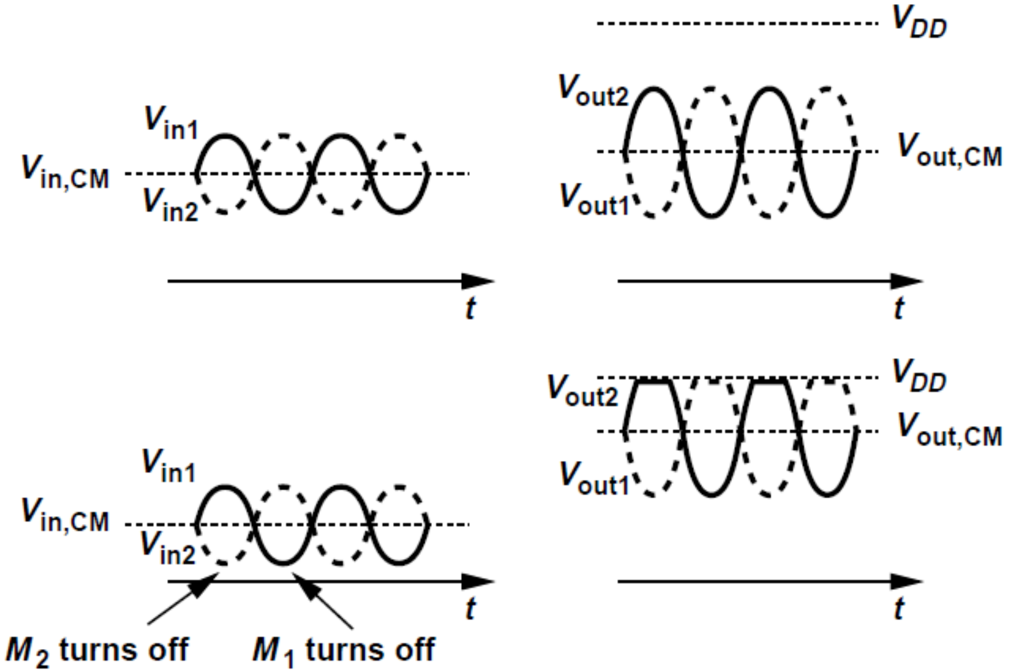
Basic Differential Pair: Introduction



- The simple differential circuit shown incorporates two identical single-ended paths to process the two phases
- Two differential inputs V_{in1} and V_{in2}, having a certain CM level $V_{in,CM}$ are applied to the gates
- The outputs are differential too and swing around the output CM level $V_{out,CM}$
- This circuit offers all advantages of differential signaling: supply noise rejection, higher output swings, etc.

Basic Differential Pair: Introduction





- As the input CM level, $V_{in,CM}$ changes, so do the bias currents of M_1 and M_2, thus varying both the transconductance of the devices and the output CM level
- Bias currents of the devices should have minimal dependence on the input CM level

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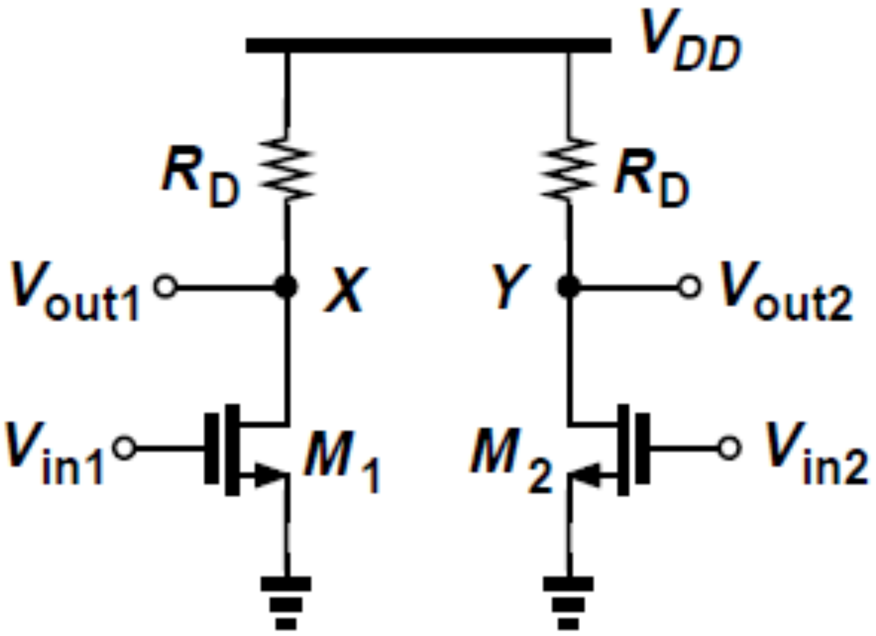
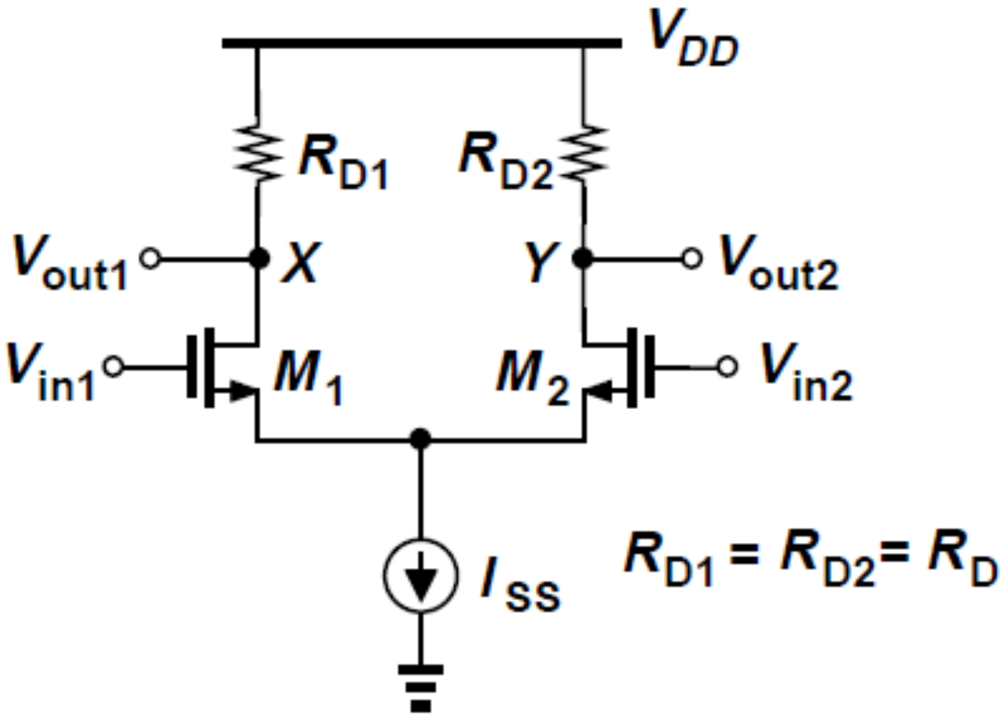
Pseudo-Differential Pair



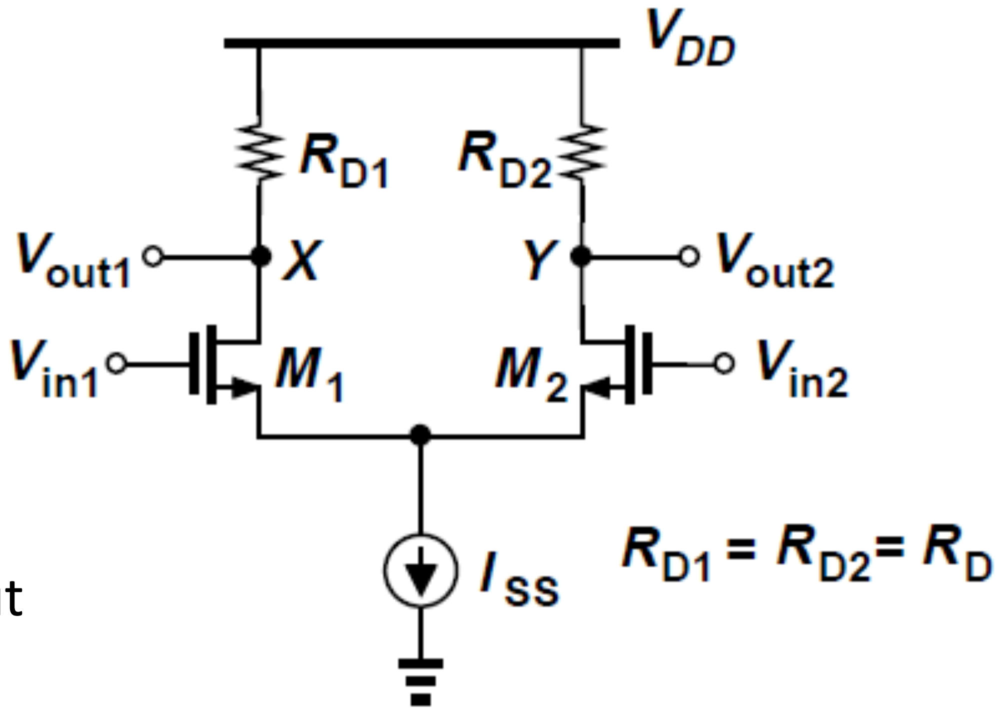
Basic Differential Pair



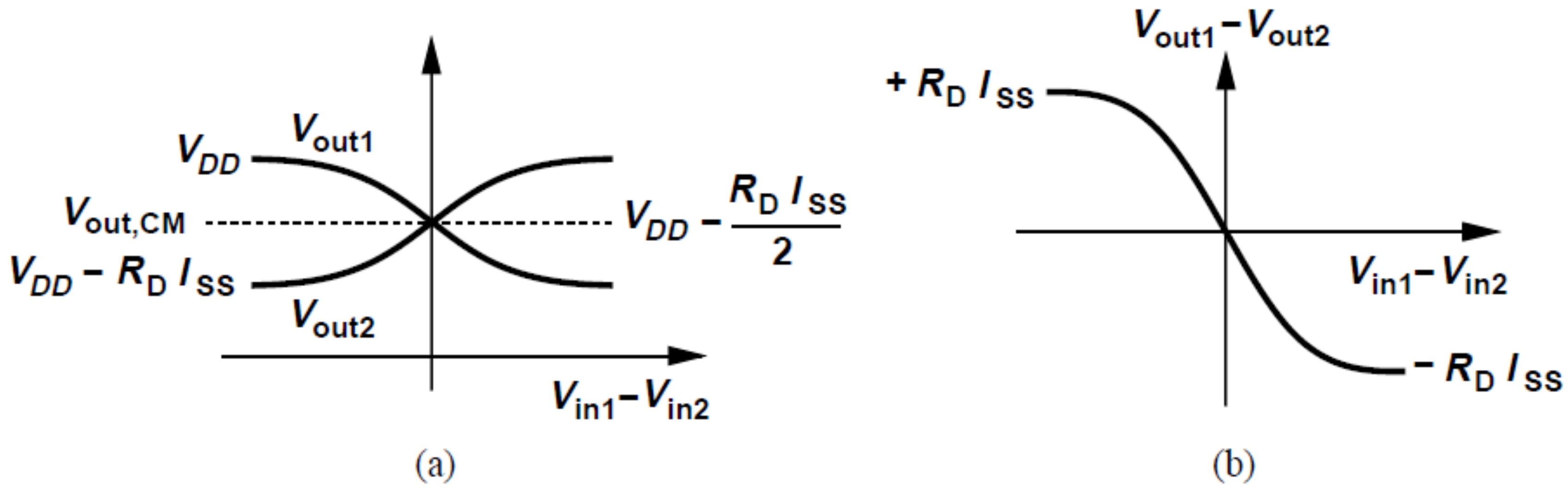
- A “differential pair” incorporates a current source I_{SS} to make $I_{D1} + I_{D2}$ independent of $V_{in,CM}$
- If $V_{in1} = V_{in2}$, the bias current of both M_1 & M_2 is $I_{SS}/2$ and the output CM level is $V_{DD} - R_D I_{SS}/2$

Basic Differential Pair: Qualitative Analysis

- When $V_{in1} << V_{in2}$,
  - M_1 is off, M_2 is on and $I_{D2} = I_{SS}$
  - $V_{out1} = V_{DD}$ and $V_{out2}=V_{DD} - R_D I_{SS}$
- As V_{in1} is brought closer to V_{in2}
  - M_1 gradually turns on, drawing a fraction of I_{SS} from R_{D1} and lowering V_{out1}
    => Since $I_{D1} + I_{D2} = I_{SS}$, I_{D2} falls and V_{out2} rises
- For $V_{in1}=V_{in2}$, $V_{out1}=V_{out2}=V_{DD} - R_D I_{SS}/2$, which is the output CM level
- When V_{in1} becomes larger than V_{in2}, I_{D1} becomes higher than I_{D2} and V_{out1} drops below V_{out2}
- For sufficiently large $V_{in1} - V_{in2}$, M_1 "hogs" all of I_{SS}, turning M2 off, therefore, $V_{out1} = V_{DD} - R_D I_{SS}$ and $V_{out2} = V_{DD}$



Basic Differential Pair: Qualitative Analysis



- The circuit contains three differential quantities: $V_{in1} - V_{in2}$, $V_{out1} - V_{out2}$, and $I_{D1} - I_{D2}$
- The maximum and minimum levels at the output are well-defined and independent of the input CM level
- The small-signal gain (slope of $V_{out1} - V_{out2}$ versus $V_{in1} - V_{in2}$) is maximum for $V_{in1} = V_{in2}$ and gradually falls to zero as $|V_{in1} - V_{in2}|$ increases
- Circuit becomes more nonlinear as input voltage swing increases

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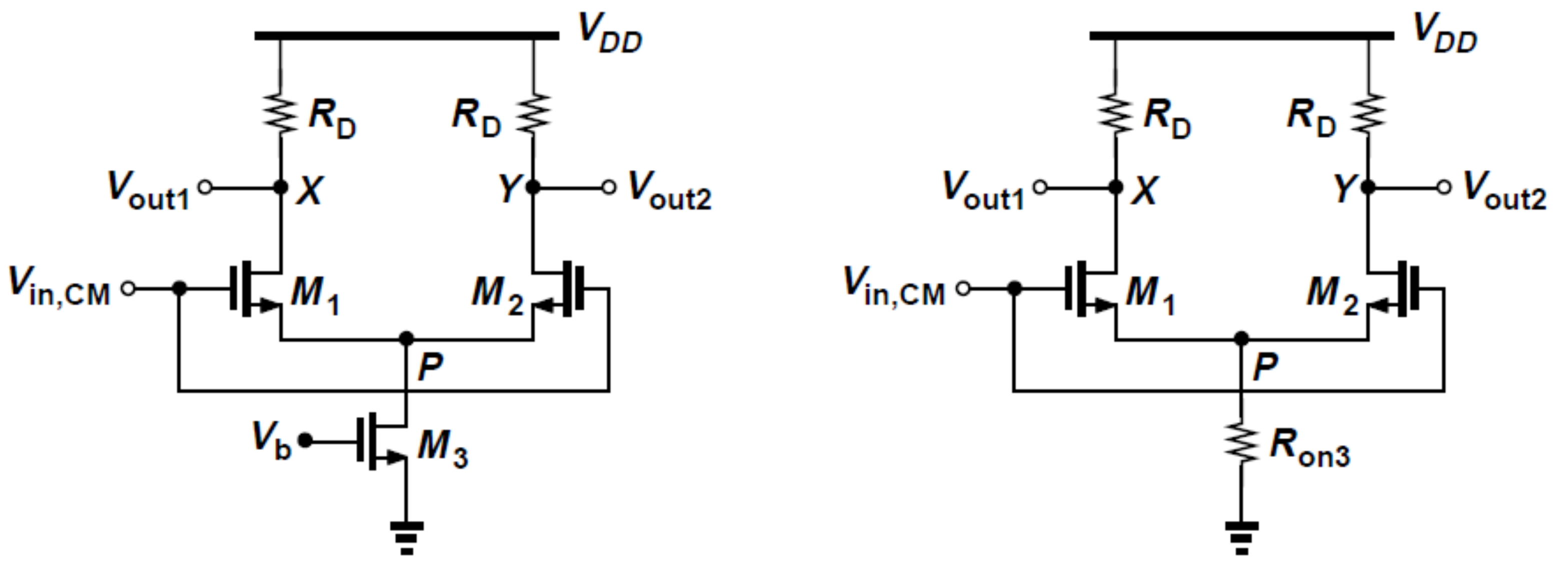
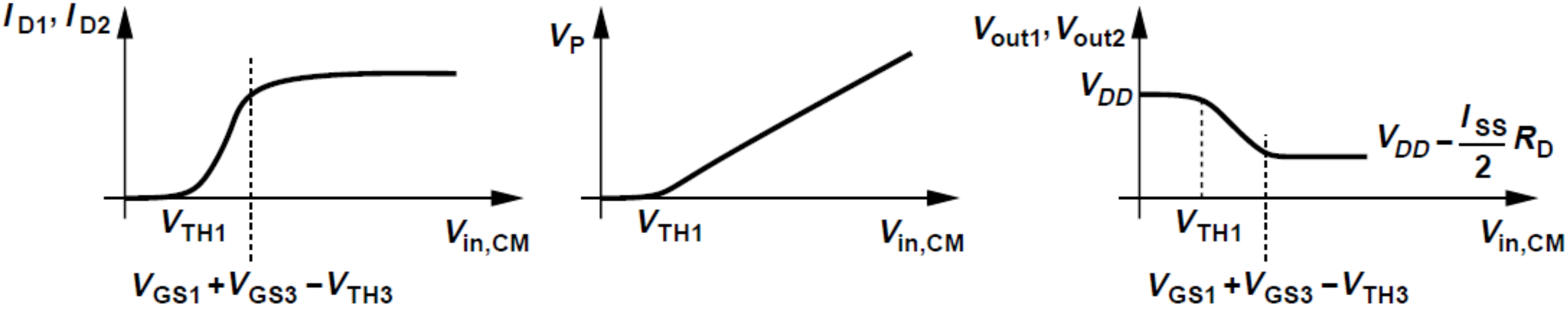
Basic Differential Pair: Common-mode behavior



(a) (b)

- Tail current source suppresses the effect of input CM level variations on the output level
- Set $V_{in1} = V_{in2} = V_{in,CM}$ and vary $V_{in,CM}$ from 0 to V_{DD} [Fig. (a)]
- Due to symmetry, $V_{out1} = \mathrm{V}_{\mathrm{out2}}$
- For $V_{in,CM} = 0$, M_1 and M_2 are off, $I_{D3} = 0$ and M_3 operates in the deep triode region [Fig. (b)]
- With $I_{D1} = I_{D2} = 0$, circuit is incapable of signal amplification; $V_{out1} = V_{out2} = V_{DD}$, and $V_P = 0$

Basic Differential Pair: Common-mode behavior



(c)

- M_1 and M_2 turn on if $V_{in,CM} \geq V_{TH}$
- Beyond this point, I_{D1} and I_{D2} continue to increase and V_P also rises [Fig. (c)]
- M_1 and M_2 act as a source follower, forcing V_P to follow $V_{in,CM}$
- When $V_{in,CM}$ is sufficiently high, V_{DS3} exceeds $V_{GS3} - V_{TH3}$, and M_3 operates in saturation so that $I_{D1}+I_{D2}$ is constant
- For proper operation, $V_{in,CM} \geq V_{GS1} + (V_{GS3} - V_{TH3})$ [why?]

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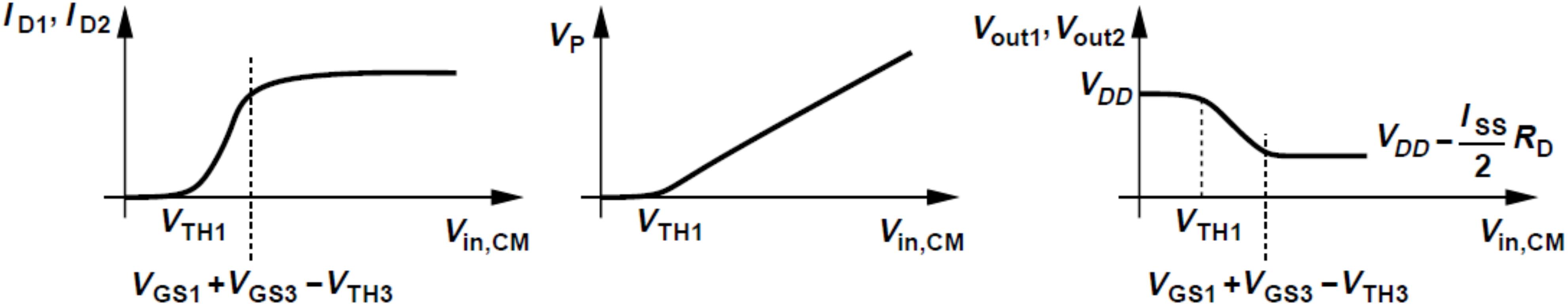
Basic Differential Pair: Common-mode behavior



(c)

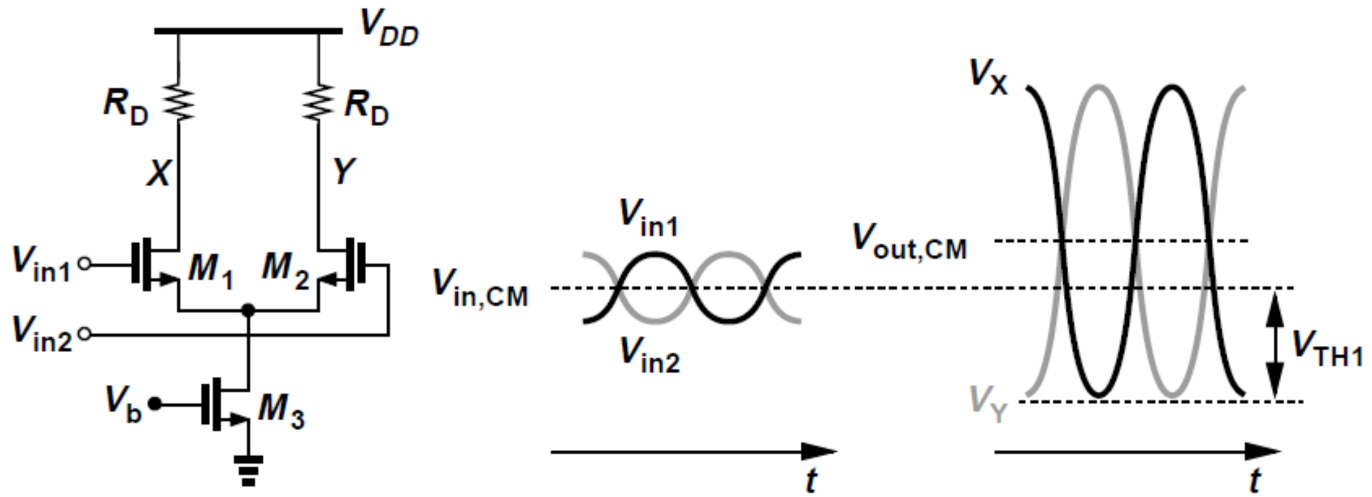
- As $V_{in,CM}$ rises further, V_{out1} and V_{out2} stay relatively constant, therefore, M_1 and M_2 enter the triode region if $V_{in,CM} > V_{out1} + V_{TH} = V_{DD} – R_D I_{SS}/2 + V_{TH}$

- The allowable value of $V_{in,CM}$ is bounded as follows:

$$V_{GS1} + (V_{GS3} - V_{TH3}) \leq V_{in,CM} \leq \min\left[V_{DD} - R_D \frac{I_{SS}}{2} + V_{TH},\ V_{DD}\right]$$

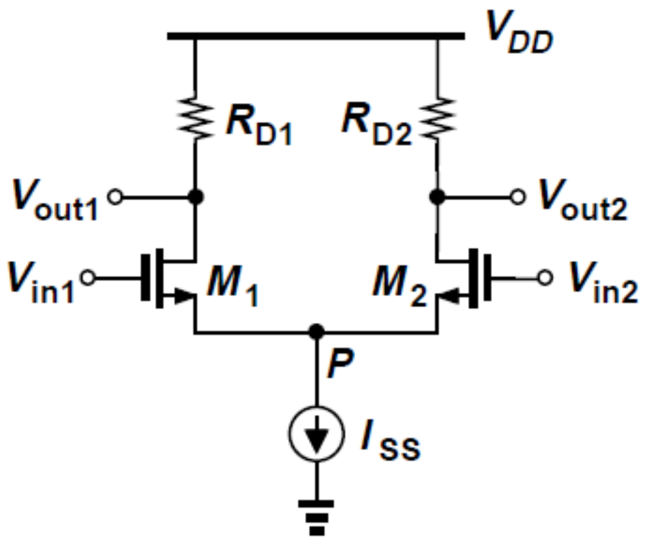
- Beyond the upper bound, the CM characteristics of Fig. (c) do not change, but the differential gain drops

Basic Differential Pair: Output Swings



- For M_1 & M_2 to remain saturated: each output can go as high as V_{DD} and as low as $V_{in,CM} - V_{TH}$
- $V_{in,CM} > V_{GS1} + (V_{GS3} - V_{TH3})$
- With this choice of $V_{in,CM}$, single-ended peak-to-peak swing is $V_{DD} - (V_{GS1} - V_{TH1}) - (V_{GS3} - V_{TH3})$
- Overall Diff. output swing = ?

*Basic Differential Pair: Large-signal Analysis



Objective: determine $V_{out1} - V_{out2}$ as a function of $V_{in1} - V_{in2}$

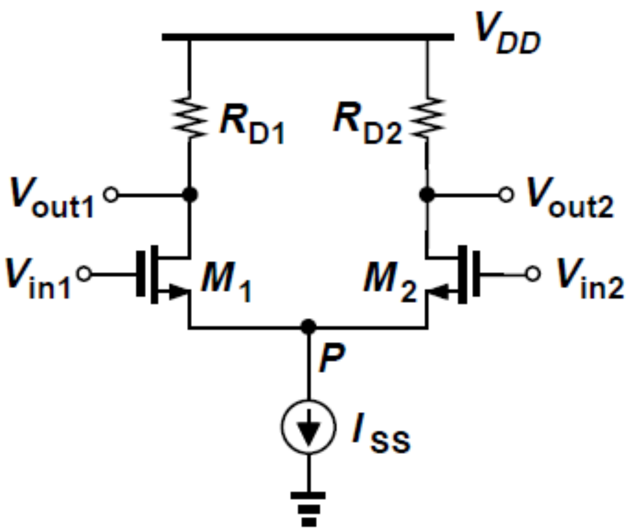
- If $R_{D1} = R_{D2} = R_D$, we have:

$$\begin{aligned} V_{out1} &= V_{DD} - R_{D1}I_{D1} \\ V_{out2} &= V_{DD} - R_{D2}I_{D2} \\ V_{out1} - V_{out2} &= R_{D2}I_{D2} - R_{D1}I_{D1} = R_D(I_{D2} - I_{D1}) \end{aligned}$$

- Assume the circuit is symmetric, M_1 and M_2 are saturated and $\lambda = 0$
- Since $V_P = V_{in1} - V_{GS1} = V_{in2} - V_{GS2}$, $V_{in1} - V_{in2} = V_{GS1} - V_{GS2}$
- For a square-law device: $V_{GS} = \sqrt{\dfrac{2I_D}{\mu_n C_{ox}\frac{W}{L}}} + V_{TH}$, Therefore: $(V_{GS} - V_{TH})^2 = \dfrac{I_D}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}$

***Extra Slides (not main course materials)**

*Basic Differential Pair: Large-signal Analysis



- It follows from previous derivation that

$$V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}}}$$

- To find $I_{D1} - I_{D2}$, square both sides of above eqn., and recognize that $I_{D1} + I_{D2} = I_{SS}$

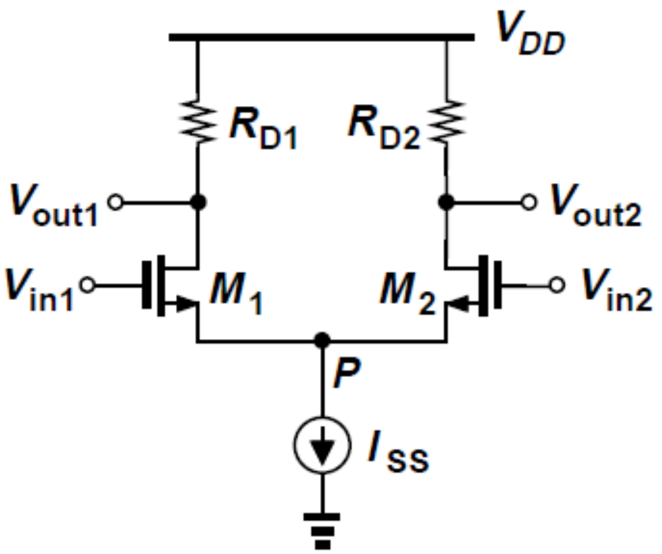
$$(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox}\frac{W}{L}}(I_{SS} - 2\sqrt{I_{D1}I_{D2}})$$

- Thus, $\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}$

- Squaring both sides and noting that $4I_{D1}I_{D2} = (I_{D1} + I_{D2})^2 - (I_{D1} - I_{D2})^2$, we arrive at

$$(I_{D1} - I_{D2})^2 = -\frac{1}{4}\left(\mu_n C_{ox}\frac{W}{L}\right)^2 (V_{in1} - V_{in2})^4 + I_{SS}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})^2$$

*Basic Differential Pair: Large-signal Analysis



- Thus

$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1} - V_{in2})^2}$$

$$= \sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}(V_{in1} - V_{in2})\sqrt{1 - \frac{\mu_n C_{ox}(W/L)}{4I_{SS}}(V_{in1} - V_{in2})^2}$$

- $I_{D1} - I_{D2}$ is an odd function of $V_{in1} - V_{in2}$, falling to zero for $V_{in1} = V_{in2}$

- As $|V_{in1} - V_{in2}|$ increases from zero, $|I_{D1} - I_{D2}|$ increases
- To find the equivalent G_m of M_1 and M_2, denote $I_{D1} - I_{D2}$ and $V_{in1} - V_{in2}$ as ΔI_D and ΔV_{in} respectively
- It can be shown that

$$\frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox}W/L} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}W/L} - \Delta V_{in}^2}}$$

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*Basic Differential Pair: Large-signal Analysis

- For $\Delta V_{in} = 0$, G_m is maximum and equal to $\sqrt{\mu_n C_{ox}(W/L) I_{SS}}$

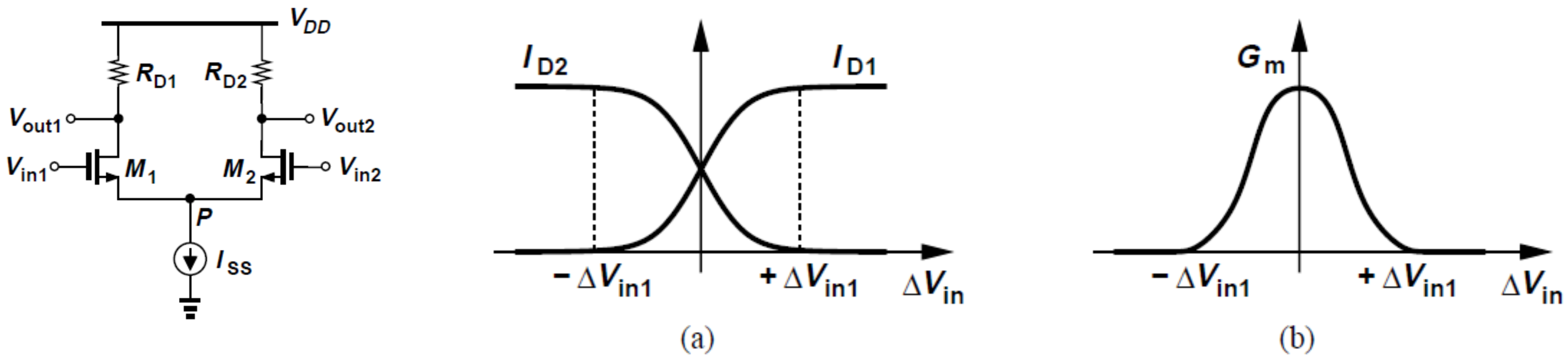
- Since $V_{out1} - V_{out2} = R_D \Delta I_D = -R_D G_m \Delta V_{in}$, small-signal differential voltage gain in equilibrium condition is

$$|A_v| = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} R_D$$

- Since each transistor carries $I_{SS}/2$ in equilibrium, the factor $\sqrt{\mu_n C_{ox}(W/L) I_{SS}}$ is the same as g_m, i.e., $|A_V| = g_m R_D$

- Previous result suggests that G_m falls to zero for $\Delta V_{in} = \sqrt{2 I_{SS}/(\mu_n C_{ox} W/L)}$.

*Basic Differential Pair: Large-signal Analysis

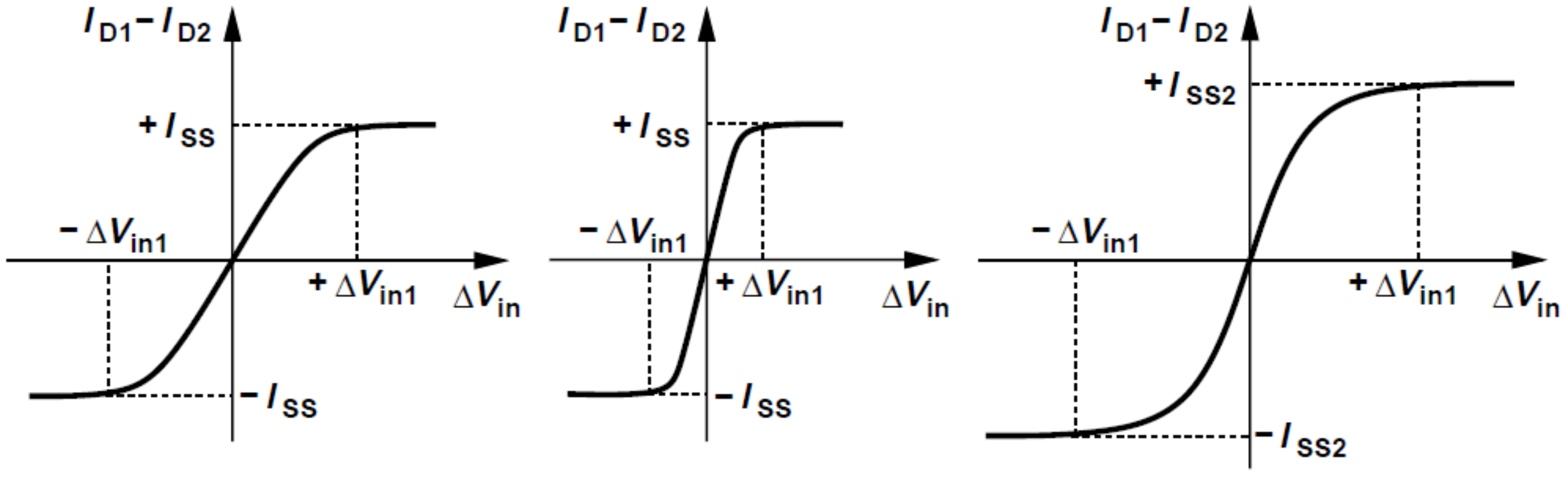


- As ΔV_{in} exceeds a limit ΔV_{in1}, one transistor carries the entire I_{SS}, turning off the other
- For this ΔV_{in}, $I_{D1} = I_{SS}$, and $\Delta V_{in1} = V_{GS1} - V_{TH}$ since M_2 is nearly off, thus

$$\Delta V_{in1} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$$

- For $\Delta V_{in} > \Delta V_{in1}$, M_2 is off and the equation derived for ΔI_D no longer holds [Fig. (a)]
- G_m is maximum for $\Delta V_{in} = 0$ and falls to zero for $\Delta V_{in} = \Delta V_{in1}$ [Fig. (b)]

*Basic Differential Pair: Large-signal Analysis



(a) (b) (c)

- As W/L increases, ΔV_{in1} decreases, narrowing the input range across which both devices are on [Fig. (b)]
- As I_{SS} increases, both the input range and the output current swing increase [Fig. (c)]
- Intuitively, circuit becomes more linear as I_{SS} increases or W/L decreases

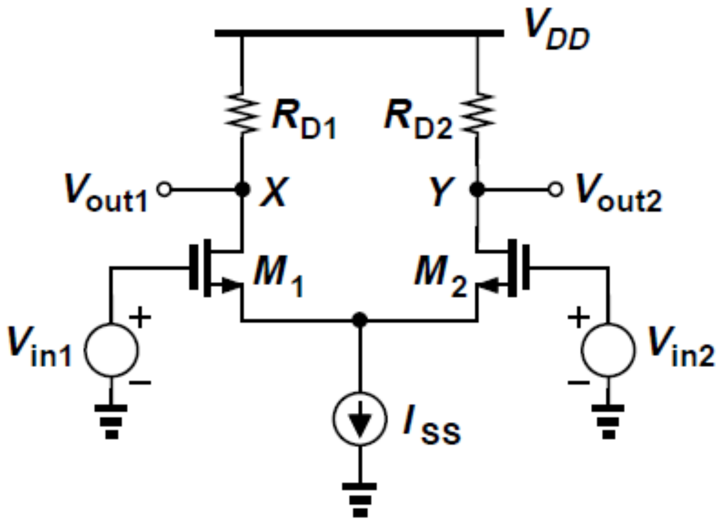
*Basic Differential Pair: Large-signal Analysis

- ΔV_{in1} represents the maximum differential input the circuit can "handle"
- ΔV_{in1} can be tied to the overdrive voltage of M_1 and M_2 in equilibrium
- For zero differential input, $I_{D1} = I_{D2} = I_{SS}/2$, yielding

$$(V_{GS} - V_{TH})_{1,2} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

- Thus, ΔV_{in1} is equal to √2 times the equilibrium overdrive
- Increasing ΔV_{in1} to improve linearity increases overdrive of M_1 and M_2, which for a given I_{SS} is achieved only by decreasing W/L and hence g_m, thereby reducing differential gain
- Alternatively, I_{SS} can be increases but with higher power consumption

Basic Differential Pair: Small-signal Analysis

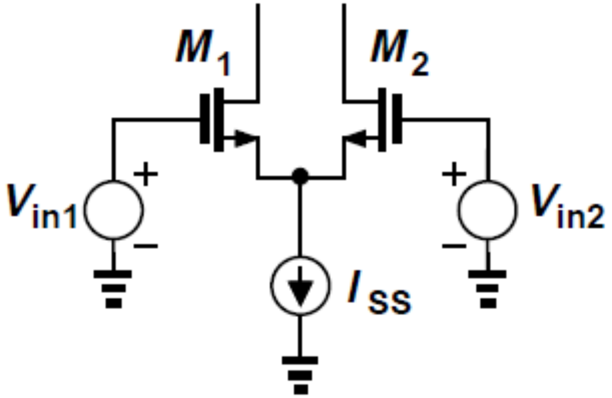


- Assume M_1 and M_2 are saturated and apply small-signal inputs V_{in1} and V_{in2}
- The differential gain $(V_{out1} - V_{out2})/(V_{in1} - V_{in2})$ was found to be $\sqrt{\mu_n C_{ox} I_{SS} W/L} R_D$ from large-signal analysis
- Since each transistor carries approximately $I_{SS}/2$ current in the vicinity of equilibrium, this expression reduces to $g_m R_D$
- Assume $R_{D1} = R_{D2} = R_D$, the small-signal analysis is carried out using two methods

  * ***Read Method 1 on your own! (Book pp. 110-111)***

Half-Circuit Technique

- The half-circuit technique can be applied even if the two inputs are not fully differential



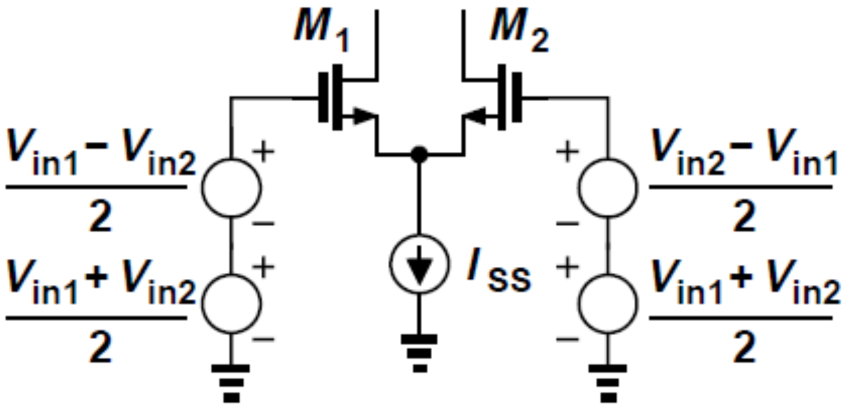
- The unsymmetrical inputs V_{in1} and V_{in2} each can be viewed as the sum of a differential component and a common-mode component, as

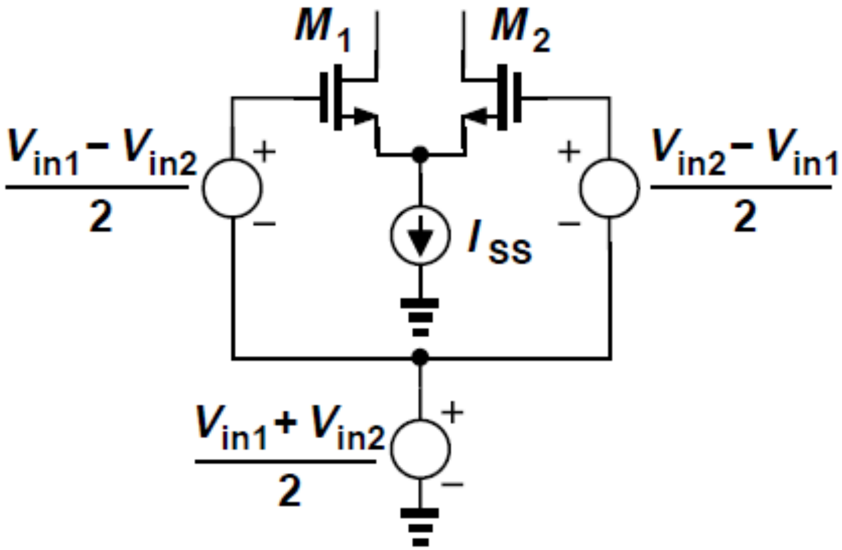
$$V_{in1} = \frac{V_{in1} - V_{in2}}{2} + \frac{V_{in1} + V_{in2}}{2}$$

$$V_{in2} = \frac{V_{in2} - V_{in1}}{2} + \frac{V_{in1} + V_{in2}}{2}$$

Half-Circuit Technique

- The circuit senses a combination of a differential input and a common-mode variation
- Effect of each type of input can be computed by superposition, with the half-circuit applied to the differential-mode operation

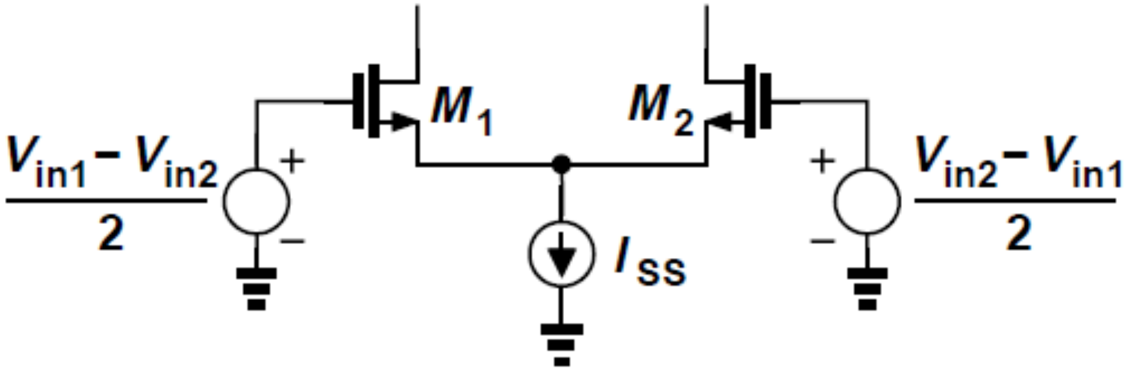




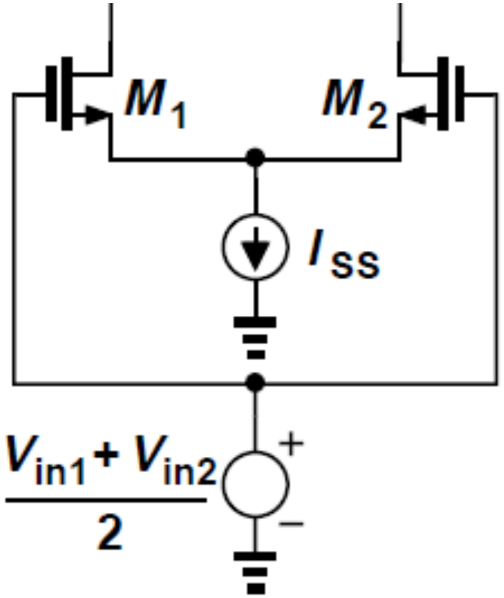
Half-Circuit Technique

- Unsymmetrical inputs V_{in1} and V_{in2} are superposed:

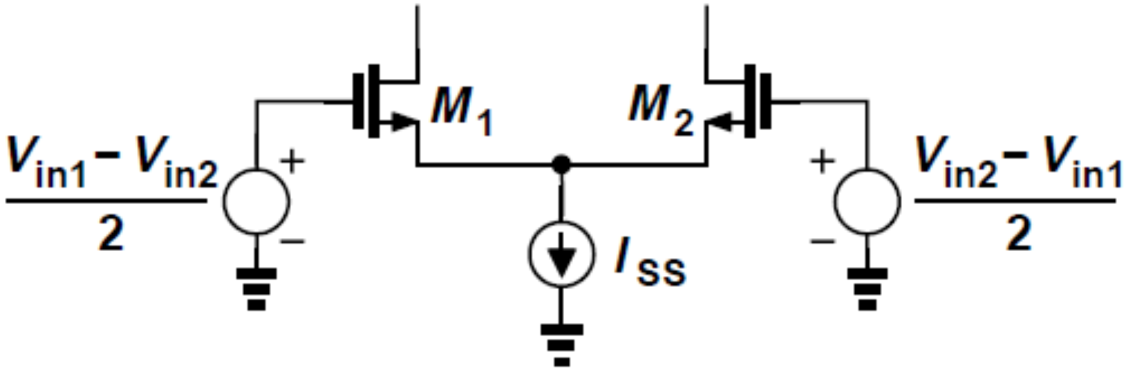
Differential-Mode



Common-Mode



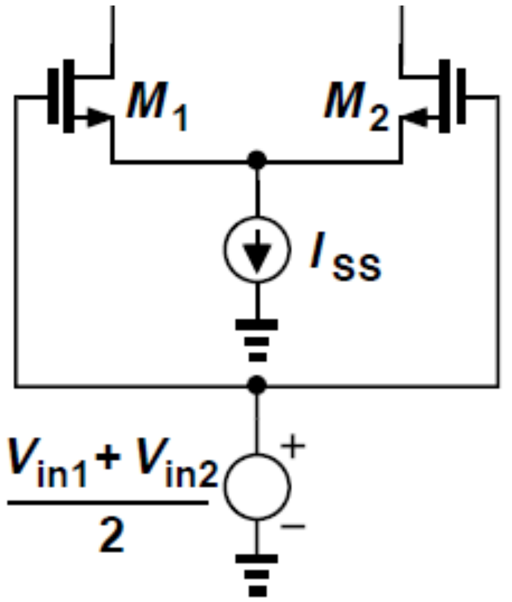
Half-Circuit Technique: Differential-Mode



For Differential-Mode:

Shared node between half-circuits is virtual ground!

Half-Circuit Technique: Common-Mode

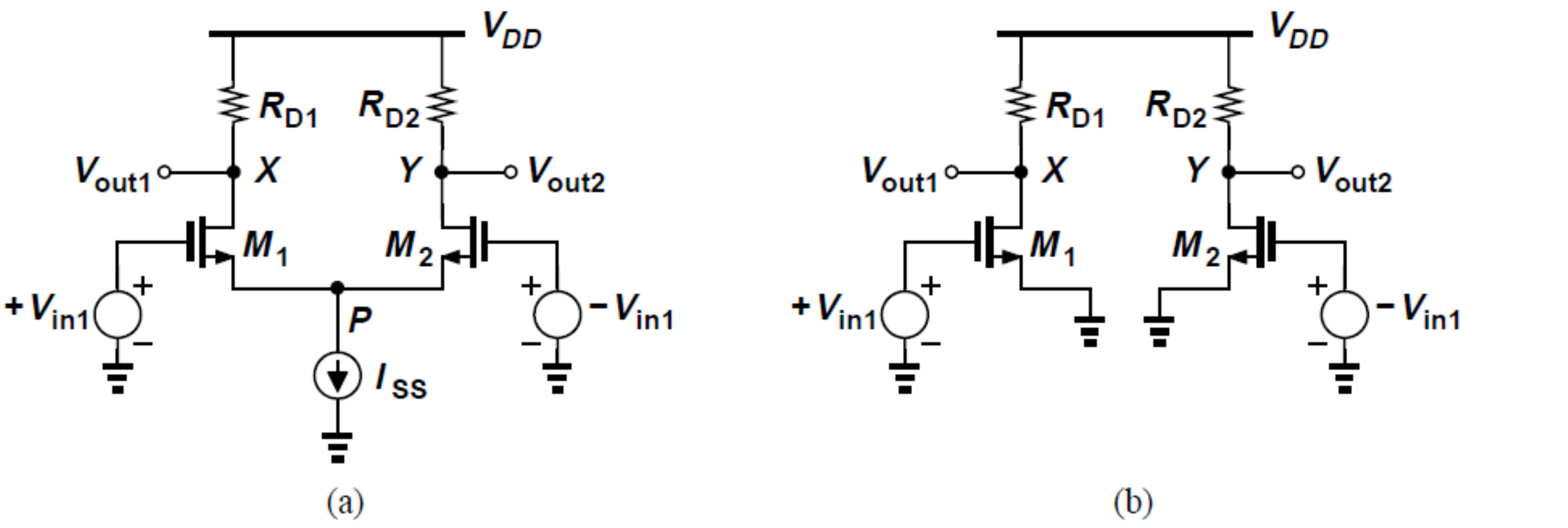


For Common-Mode:

Shared branch connecting half-circuits has zero current!

(or similar voltage nodes from each half can be shorted!)

Basic Differential Pair: DM



- Using the half-circuit concept, V_P experiences no change node *P* can be considered "ac ground" or virtual ground and the circuit can be decomposed into two separate halves
- We can write $V_X/V_{in1} = -g_m R_D$ and $V_Y/(-V_{in1}) = -g_m R_D$
- V_{in1} and $-V_{in1}$ represent the voltage *change* on each side
- Thus, $(V_X - V_Y)/(2V_{in1}) = -g_m R_D$, same result as in Method 1

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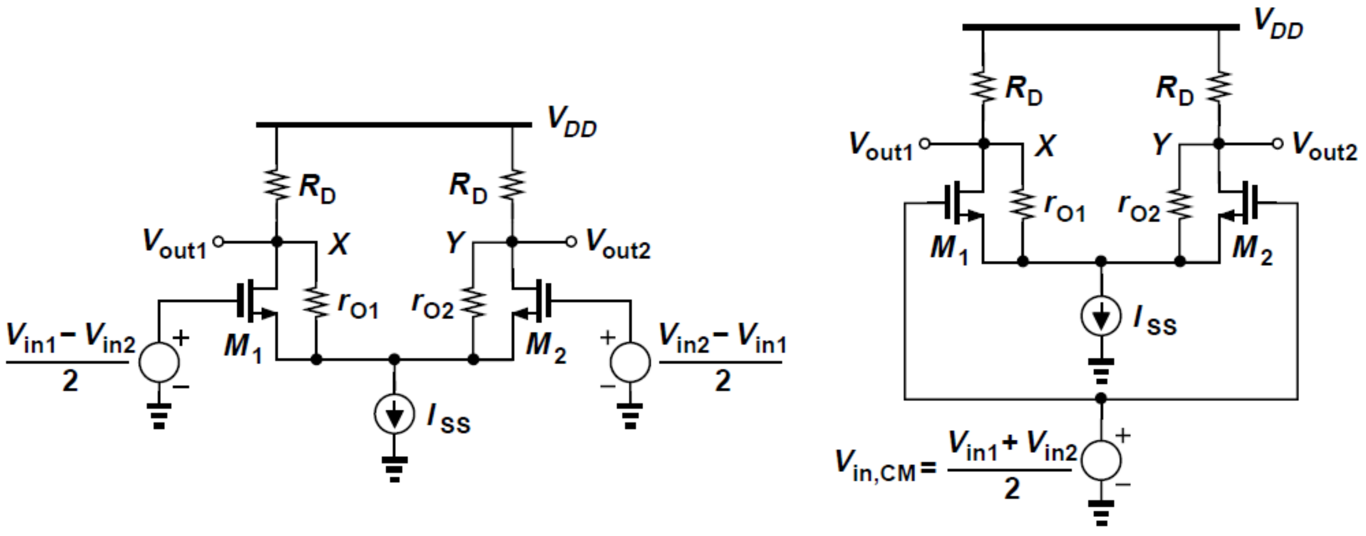
Half-Circuit Technique: Example

- For differential-mode operation, circuit reduces to Fig. (a)



(a) (b)

- Thus,

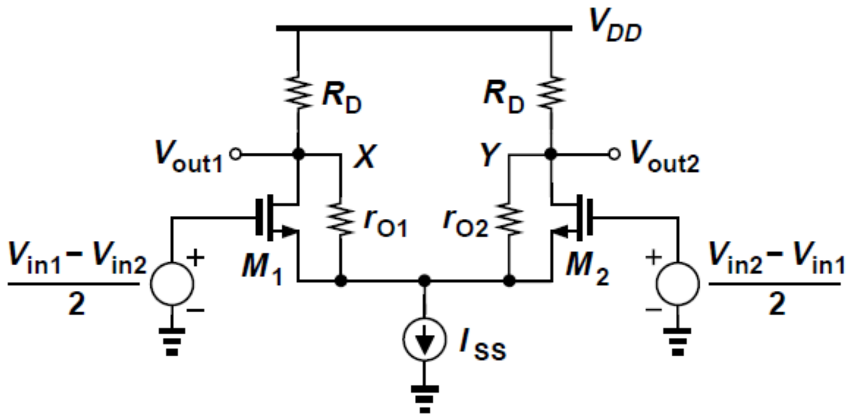
$$V_X = -g_m(R_D \| r_{O1})\frac{V_{in1} - V_{in2}}{2}$$

$$V_Y = -g_m(R_D \| r_{O2})\frac{V_{in2} - V_{in1}}{2}$$

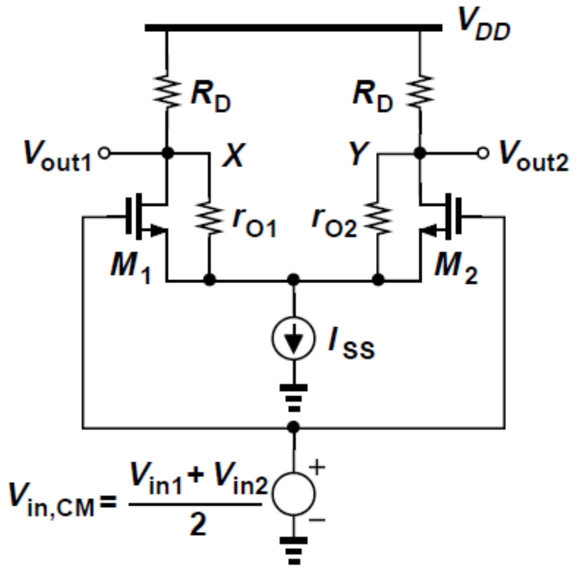
$$V_X - V_Y = -g_m(R_D \| r_O)(V_{in1} - V_{in2})$$

Half-Circuit Technique: Example

- For common-mode operation, circuit reduces to that in Fig. (b)



(a)

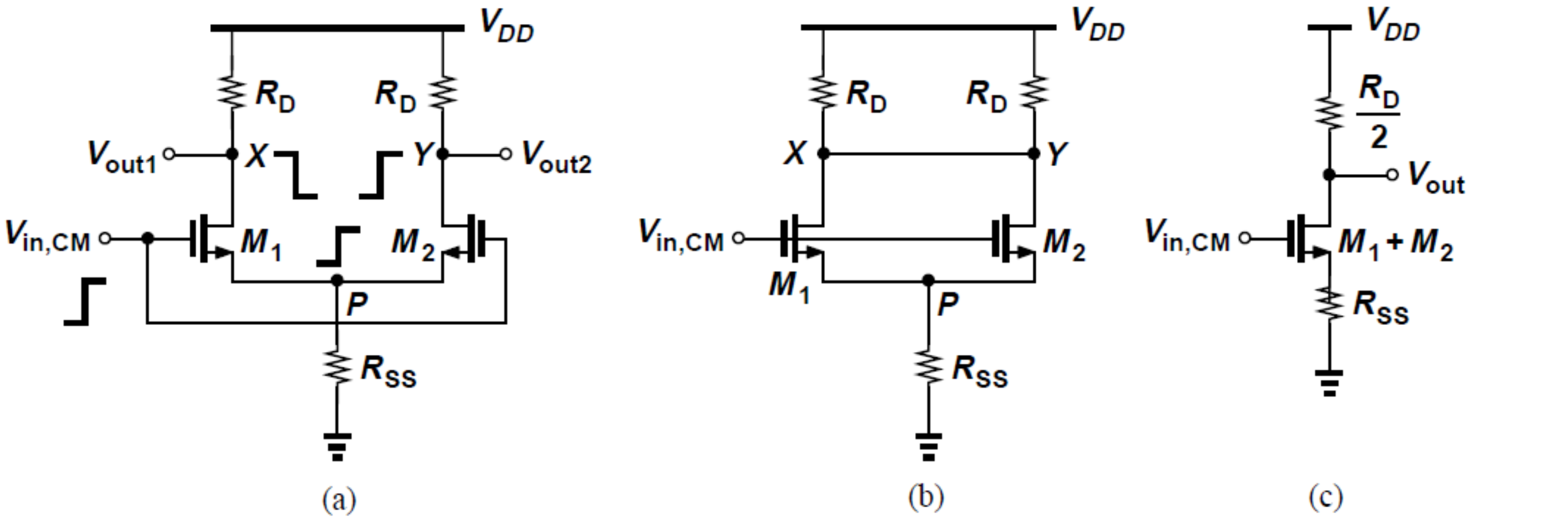


(b)

- If circuit is fully symmetric and I_{SS} is an ideal current source, the currents drawn by M_1 and M_2 from R_{D1} and R_{D2} are exactly equal to $I_{SS}/2$ and independent of $V_{in,CM}$
- V_X and V_Y remain equal to $V_{DD} – R_D(I_{SS}/2)$ and do not vary with $V_{in,CM}$, therefore, circuit simply amplifies $V_{in1} – V_{in2}$ while eliminating the effect $V_{in,CM}$

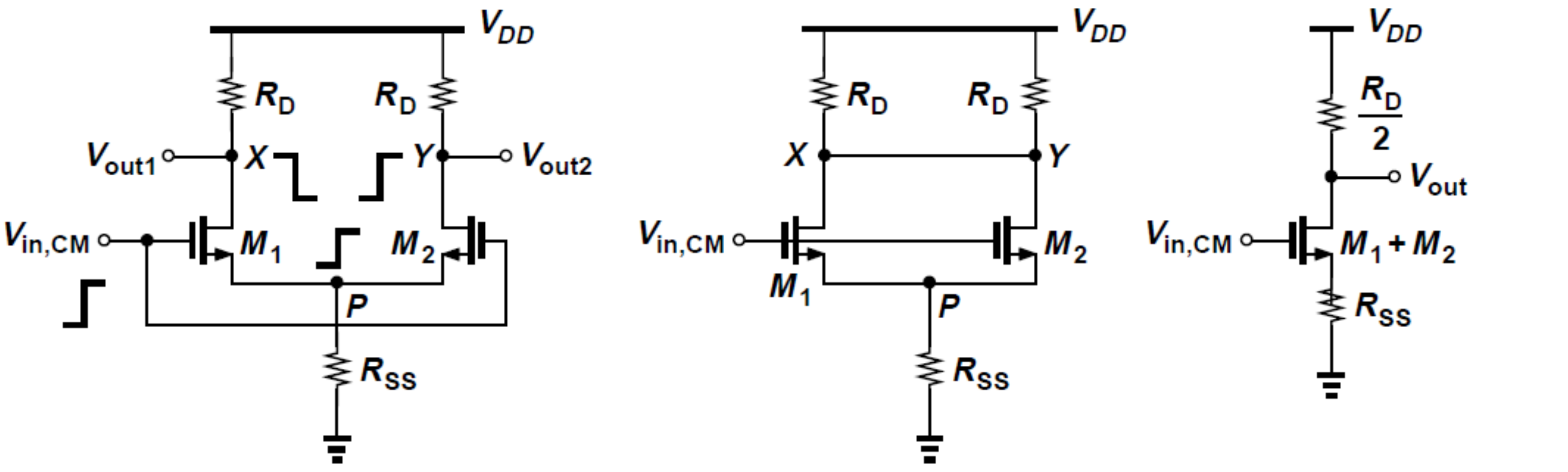
Basic Differential Pair: Common-Mode Response

- In reality, the differential pair is not fully symmetric and the tail current source exhibits a finite output impedance
- A fraction of the input CM variations appear at the output



- First assume that circuit is symmetric but tail current source has a finite output impedance R_{SS} [Fig. (a)]
- Increase in $V_{in,CM}$ causes V_P to increase and both V_X, V_Y to drop, which remain equal due to symmetry [Fig. (b)]

Basic Differential Pair: Common-Mode Response



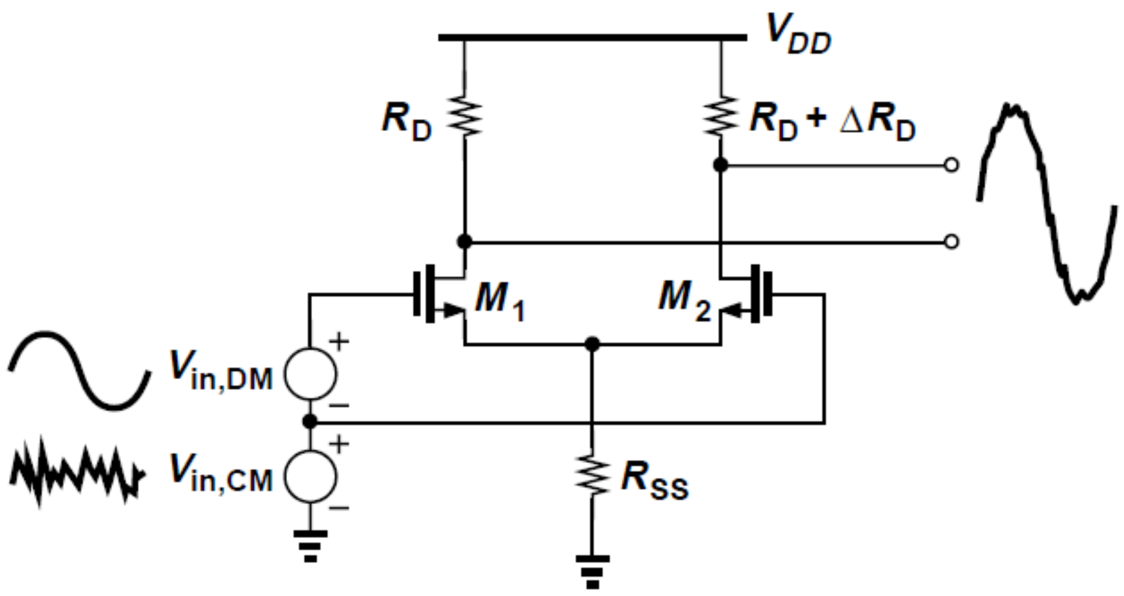
(a) (b) (c)

- M_1 and M_2 are "in parallel" and can be reduced to one composite device with twice the width, bias current and transconductance
- "Common-mode gain" of the circuit is ($\lambda = \gamma = 0$)

$$A_{v,CM} = \frac{V_{out}}{V_{in,CM}} = -\frac{R_D/2}{1/(2g_m) + R_{SS}}$$

- Differential output (V_x-V_y) will not be affected by $V_{in,cm}$ …

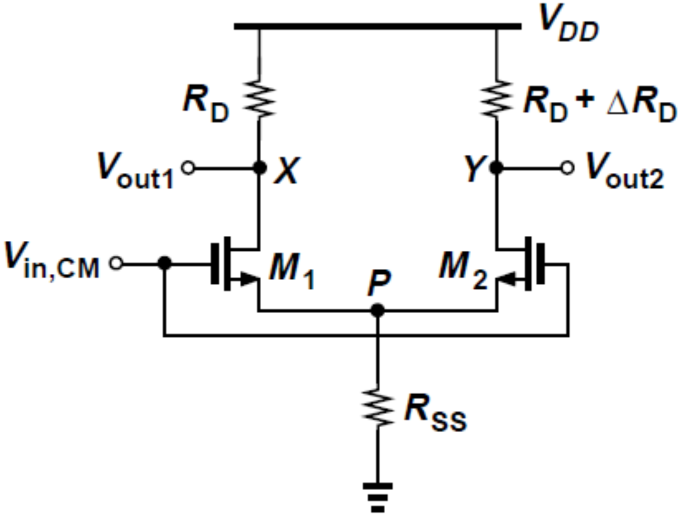
Basic Differential Pair: Common-Mode Response



- Common-mode response depends on output impedance of tail current source and asymmetries in the circuit
- Two effects:
  - Variation of output CM level (in the absence of mismatches)
  - Conversion of input CM variations to output differential components (more severe)

*Basic Differential Pair: Common-Mode Response

- There is variation in differential output due to change in $V_{in,CM}$ since the circuit is not fully symmetric, i.e., slight mismatches between the two sides



- $R_{D1} = R_D$, $R_{D2} = R_D + \Delta R_D$, where ΔR_D denotes a small mismatch and circuit is otherwise symmetric ($\lambda = \gamma = 0$ for M_1 and M_2)
- M_1 and M_2 operate as one source follower, raising V_P by

$$\Delta V_P = \frac{R_{SS}}{R_{SS} + \frac{1}{2g_m}} \Delta V_{in,CM}$$

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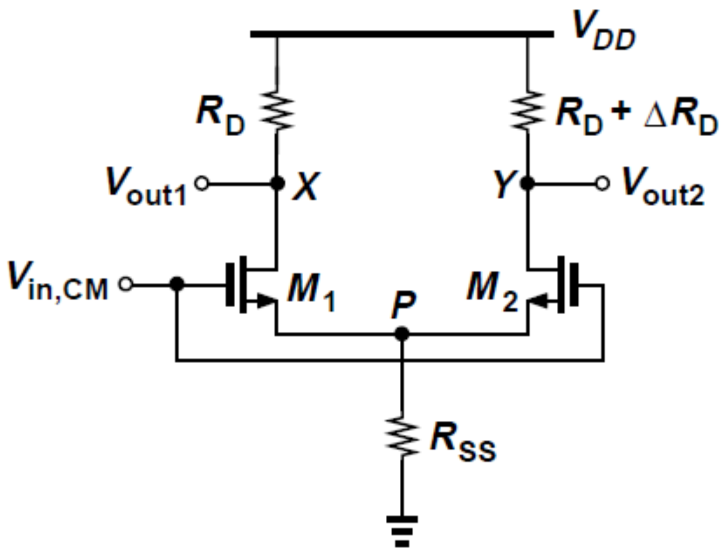
*Basic Differential Pair: Common-Mode Response



- Since M_1 and M_2 are identical, I_{D1} and I_{D2} increase by $[g_m/(1 + 2g_m R_{SS})]\Delta V_{in,CM}$
- V_X and V_Y change by different amounts

$$\Delta V_X = -\Delta V_{in,CM} \frac{g_m}{1 + 2g_m R_{SS}} R_D$$

$$\Delta V_Y = -\Delta V_{in,CM} \frac{g_m}{1 + 2g_m R_{SS}} (R_D + \Delta R_D)$$

- Common-mode change at the input introduces a differential component at the output – common-mode to differential conversion

*Common-Mode Response

- Common-mode rejection ratio (CMRR) is defined as the desired gain divided by undesired gain

$$\mathrm{CMRR} = \left| \frac{A_{DM}}{A_{CM-DM}} \right|$$

- If only g_m mismatch is considered, it can be shown that

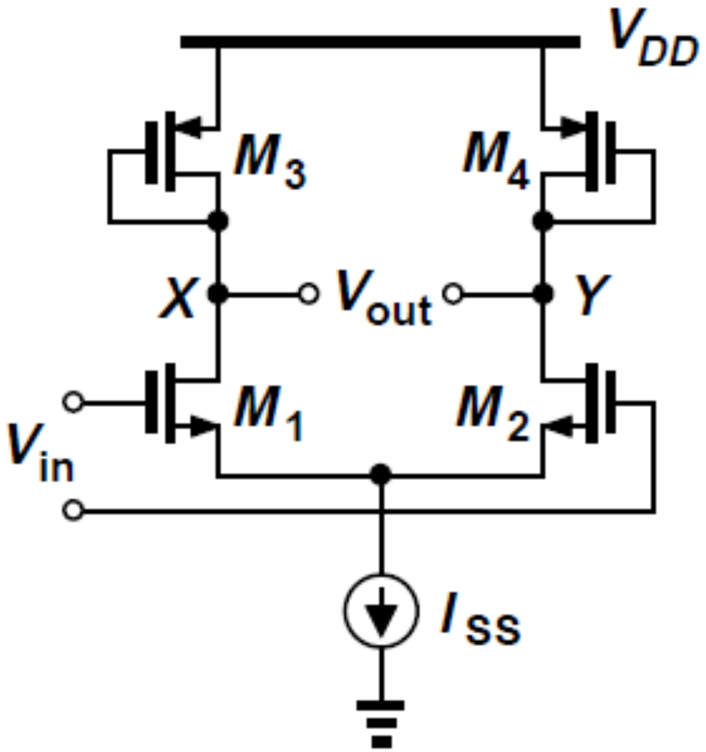
$$|A_{DM}| = \frac{R_D}{2} \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{1 + (g_{m1} + g_{m2})R_{SS}}$$

- Hence,

$$\begin{aligned}\mathrm{CMRR} &= \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{2\Delta g_m} \\ &\approx \frac{g_m}{\Delta g_m}(1 + 2g_m R_{SS})\end{aligned}$$

- g_m denotes the mean value, i.e., $g_m = (g_{m1} + g_{m2})/2$
- $2g_mR_{SS} >> 1$ and hence $\mathrm{CMRR} \approx 2g_m^2 R_{SS}/\Delta g_m$

Differential Pair with MOS Loads



Differential Pair with MOS Loads