EE 332: Devices and Circuits II

Lecture 7: Feedback

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ELECTRICAL & COMPUTER ENGINEERING

Feedback Systems

- Feedback Examples?

PID

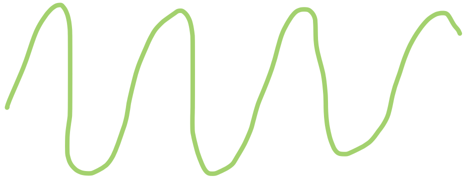
Oven heating → temp sensor

locks

biological → pace maker

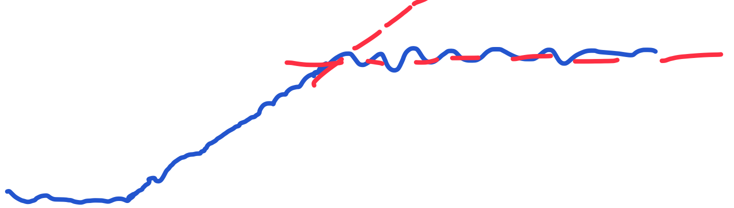
- Positive vs. Negative Feedback

Oscillator



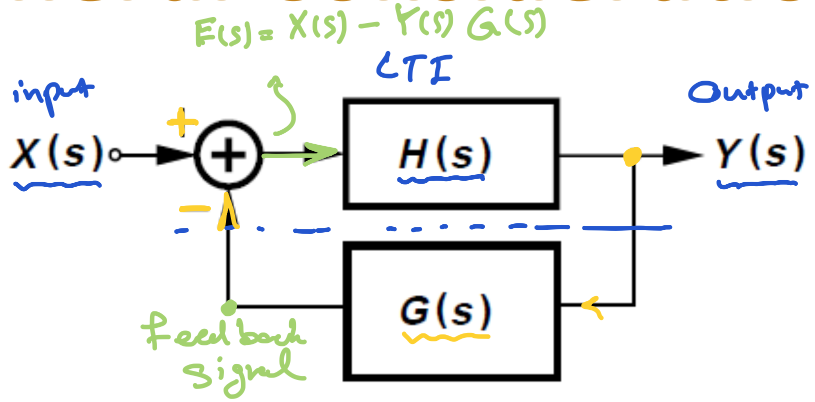
Stabalize

keep track of st



target

General Considerations



- Above figure shows a negative feedback system
- *H*: Feedforward network & *G: F*eedback network
- Feedback error is given by *X(s) – G(s)Y(s)*

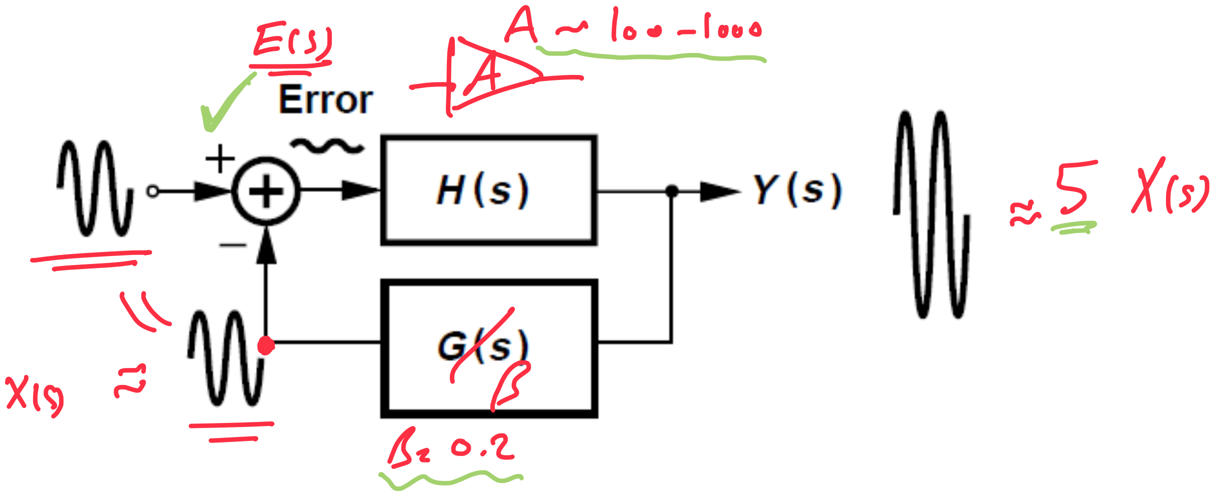
$$Y(s) = H(s)\overbrace{[X(s) - G(s)Y(s)]}^{E(s)}$$

- Thus

$$\underset{\text{closed loop gain}}{\frac{Y(s)}{X(s)}} = \frac{\overset{\text{open-loop gain}}{H(s)}}{1 + \underbrace{G(s)H(s)}_{\text{loop gain}}}$$

- *H(s)* is called the "open-loop" transfer function, *Y(s)/X(s)* is called the "closed-loop" transfer function, and G(s)H(s) is the "loop-gain"

General Considerations



- In most cases, *H(s)* represents an amplifier and *G(s)* is a frequency-independent quantity
- In a well-designed negative feedback system, the error term is minimized, making the output of *G(s)* an “accurate” copy of the input and hence the output of the system a faithful (scaled) replica of the input
- In subsequent developments, *G(s)* is replaced by a frequency-independent quantity *β* called the *feedback factor*

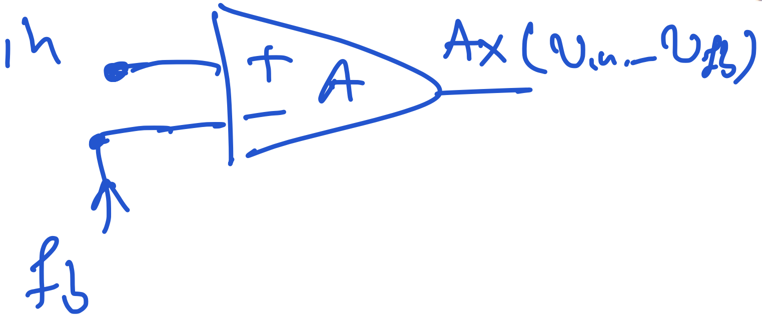
$\beta < 1$

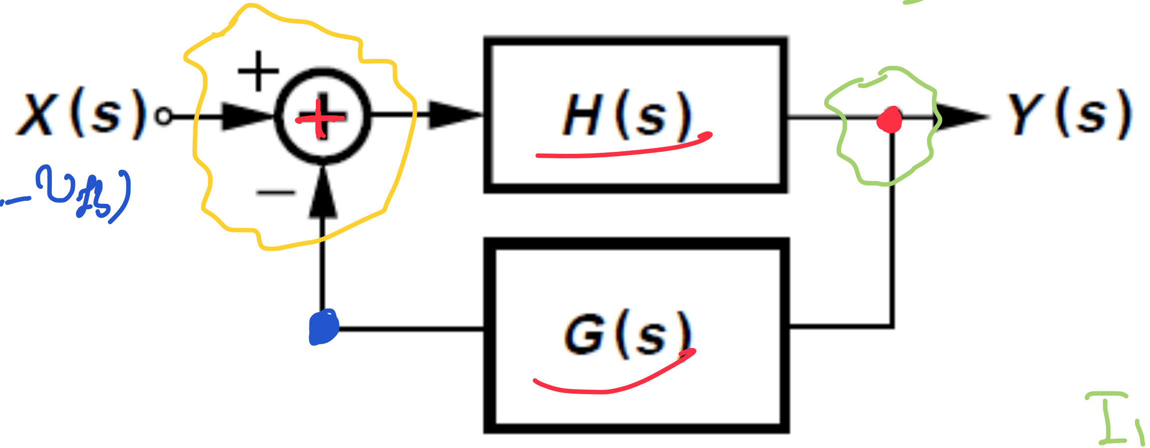
4 types of
* fb circuits

General Considerations

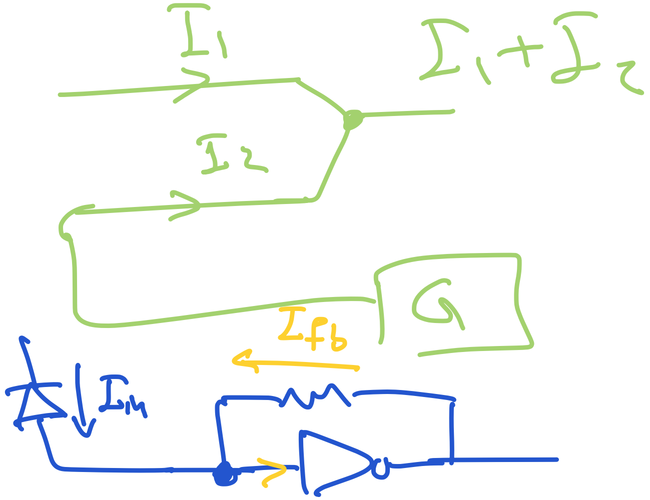
Volt or Current domain

Sense : Volt or Current.





KCL for Current Correction



- Four elements of a feedback system
    - The feedforward amplifier $H(s)$
    - A means of sensing the output
    - The feedback network $G(s)$
    - A means of generating the feedback error, i.e., a subtractor (or an adder)

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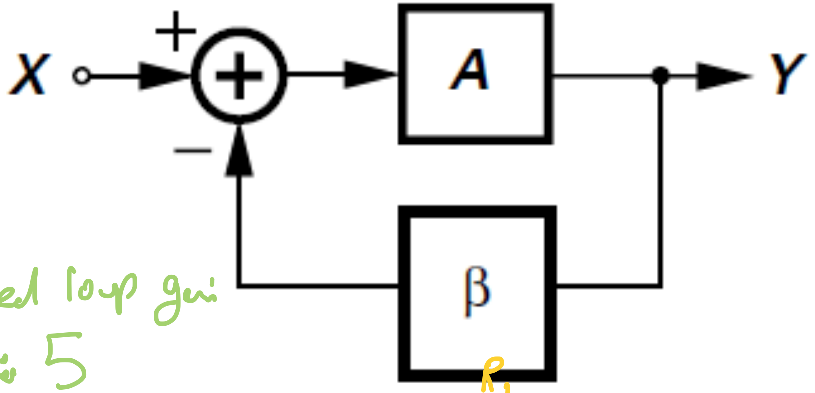
Properties of Feedback Circuits

βA = loop gain

$\beta < 1$

<u>ex.</u> $\beta = 0.2$
$A = 1000$



closed loop gain
≈ 5
% 0.5 error

R_1
R_2
$\beta = \frac{R_2}{R_1 + R_2}$

$\frac{Y}{X} - \frac{1}{\beta}$
actual closed loop gain
est.

$$\frac{\frac{Y}{X} - \frac{1}{\beta}}{1/\beta} = \frac{1}{\beta A}$$

- For a more general case, gain desensitization is quantified by writing

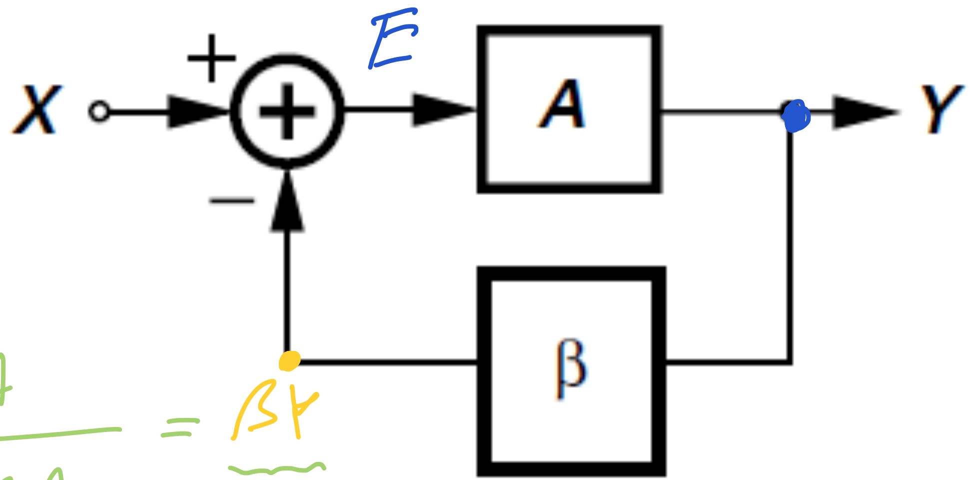
$$\begin{cases} H(s) = A \\ G(s) = \beta \end{cases} \rightarrow \frac{Y}{X} = \frac{A}{1 + \beta A}$$

$$\approx \frac{1}{\beta}\left(1 - \frac{1}{\beta A}\right)$$

$\frac{1}{\beta A} \approx 0$ $\longrightarrow$ $\boxed{\frac{Y}{X} \approx \frac{1}{\beta}}$

- It is assumed $\beta A >> 1$; even if open-loop gain A varies by a factor of 2, Y/X varies by a small percentage since $1/(\beta A) << 1$

Properties of Feedback Circuits



$$X \frac{\beta A}{1+\beta A} = \beta Y$$

$$\beta A \gg 1 \longrightarrow \beta Y \approx X$$

$$E = X - \beta Y$$

$$= X - \beta \overbrace{\left(\frac{A}{1+\beta A}\right) X}^{Y}$$

$$= X \left(\frac{1}{1+\beta A}\right)$$

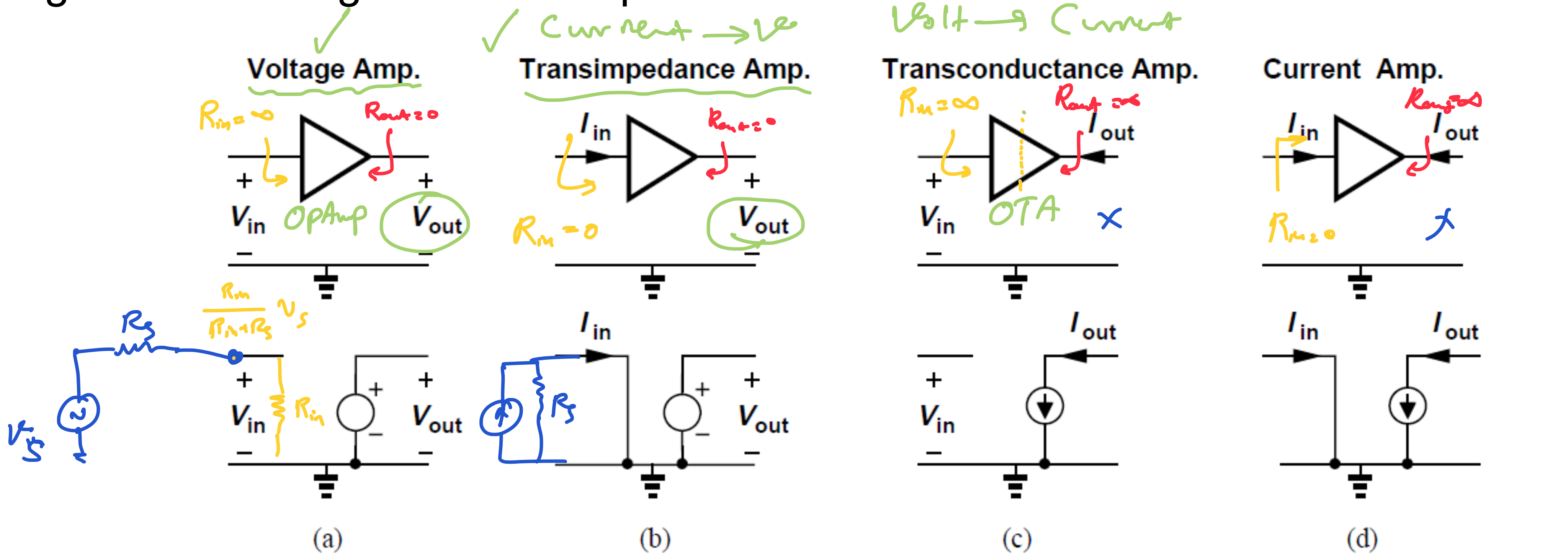
$$\text{if } \beta A \gg 1 \longrightarrow E(s) \approx 0$$

- The quantity βA is called the "loop gain"
- The output of the feedback network is equal to $\beta Y = X \cdot \beta A/(1+\beta A)$ approaching X as βA becomes much greater than unity

R_{in} & R_{out}

Types of Amplifiers

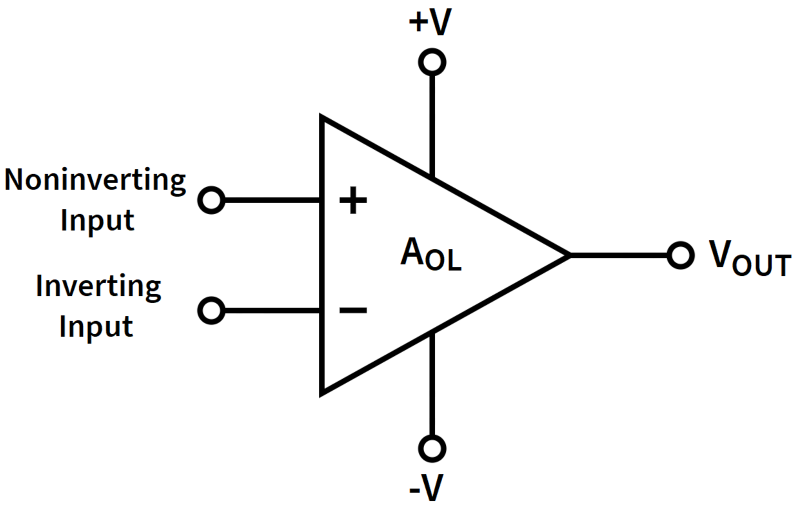
- Four possible amplifier configurations depending on whether the input and output signals are voltage or current quantities



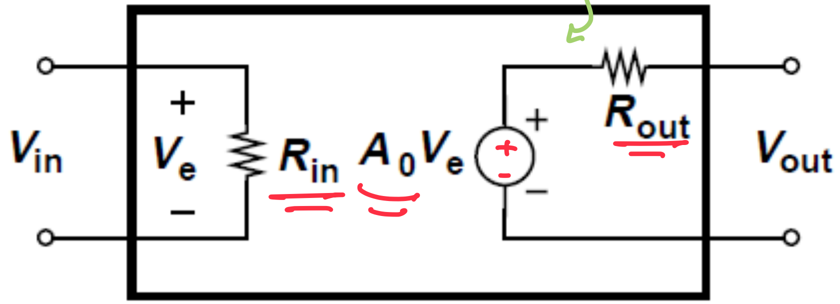
- Figs. (a) – (d) show the four amplifier types with the corresponding idealized models

OpAmp

Volt. Volt.



thevenin model



$$A_0(s) = \frac{A_0}{1+\frac{s}{\omega_{p1}}}$$

✓ Diff input

✓ large gain

V:V

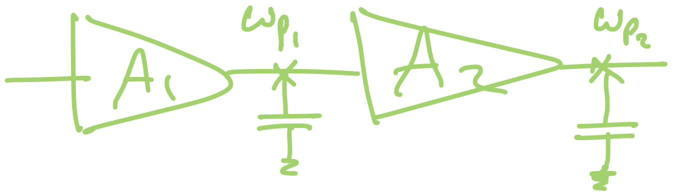
V:I

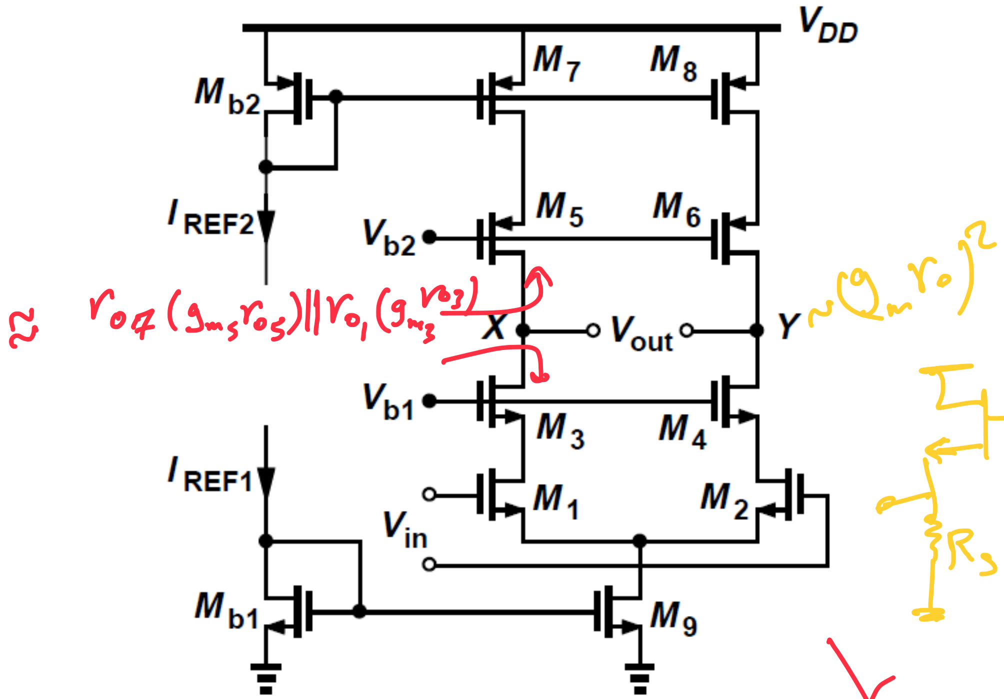
OpAmp vs. OTA

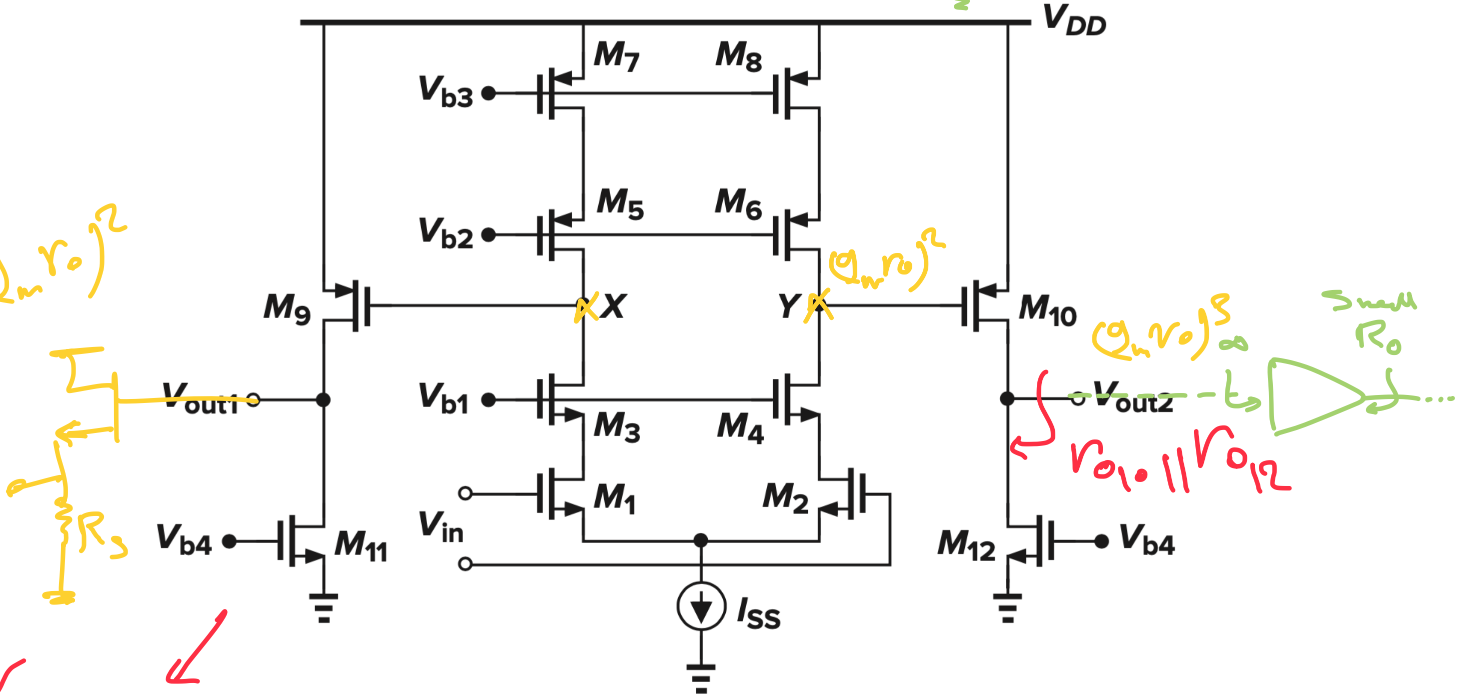
OpAmp

- ***Providing large-gain by:***
  - Cascoding technique $\times(g_m r_o)$
  - Cascading multiple stages
  - etc.

Stability & Compensation



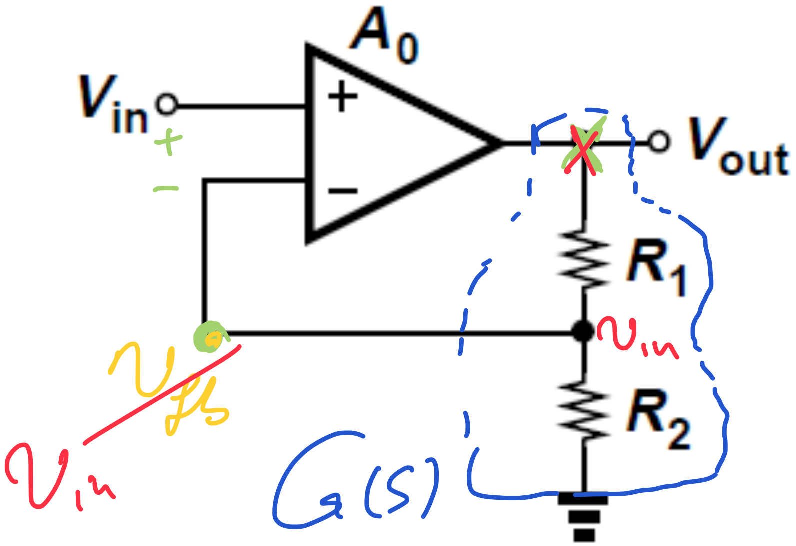




R_o = large $\Rightarrow$ OTA

Properties of Feedback Circuits

- **Gain Desensitization:** (Non-inverting feedback amplifier example)



Closed-loop gain $\frac{v_{out}}{v_{in}} = ?$

$$A = \frac{A_0}{1+\beta A_0} \xrightarrow{\beta A_0 \gg 1} \boxed{\frac{1}{\beta} = 1 + \frac{R_1}{R_2}}$$

$$\beta = \frac{R_2}{R_1+R_2}$$

$$\beta A \gg 1 \Longrightarrow E \approx 0 \Longrightarrow v_{in} = v_{fb}$$

$$v_{out} \times \frac{R_2}{R_1+R_2} = v_{in}$$

- Gain can be controlled with higher accuracy unaffected by PVT variations
- Closed-loop gain is less sensitive to device parameters than the open-loop gain, hence called “gain desensitization”

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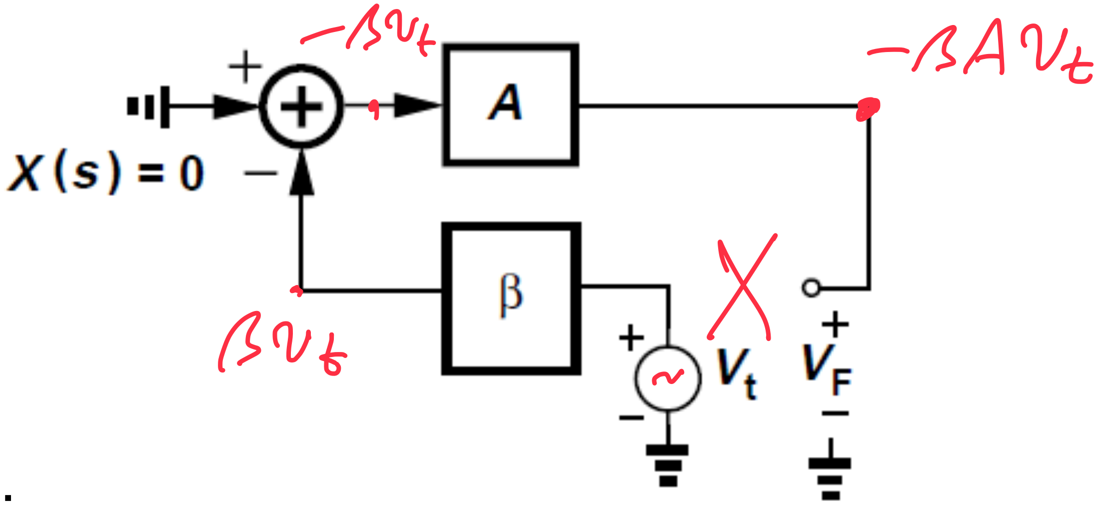
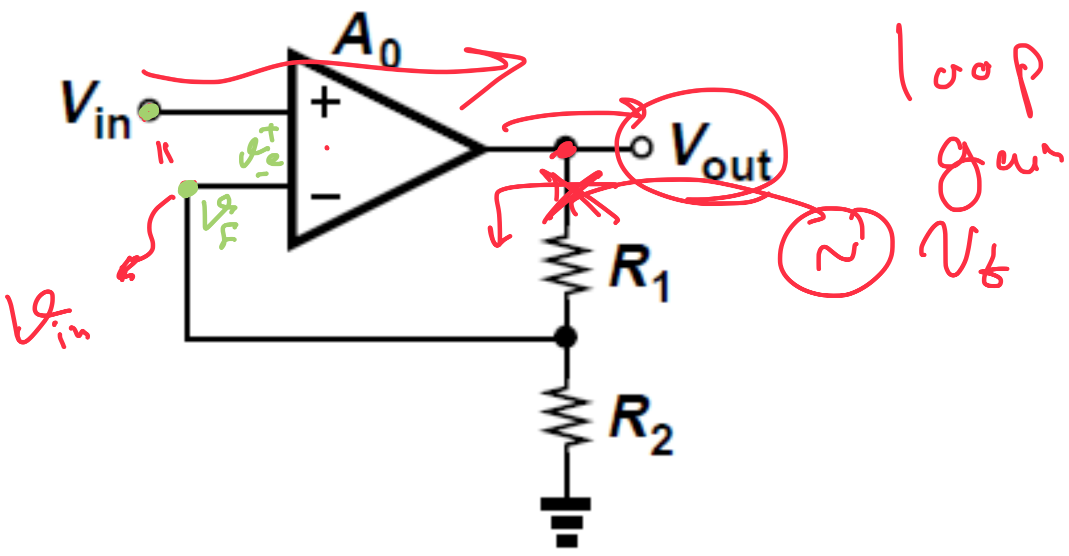
Closed loop gain = $\frac{\text{Open loop gain}}{1 + \text{loop gain}}$

Calculation of Loop Gain



- To calculate the loop gain:
  - Set the main input to (ac) zero
  - Inject a test signal in the “right” direction
  - Follow the signal around the loop and obtain the value that returns to the break point
  - Negative of the transfer function thus obtained is the loop gain
- Loop gain is a dimensionless quantity
- In above figure, $V_t\beta(-1)A = V_F$ and hence $V_F/V_t = -\beta A$

Calculation of Loop Gain: Example



Open loop = A_0

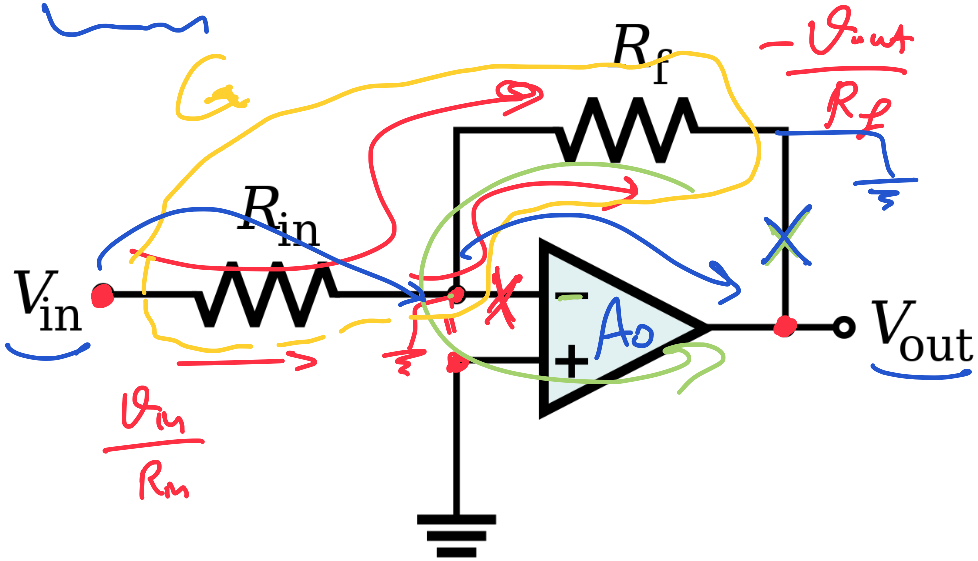
$$\text{loop gain} = -A_0 \times \frac{R_2}{R_1 + R_2}$$

OpAmp gain ≠

$$\text{Closed-loop gain} = \frac{\text{Open loop gain}}{1 + \text{loop gain}}$$

Calculation of Loop Gain: Example

(inverting feedback amplifier example)



$$\text{loop gain} = \frac{-R_{in}}{R_{in}+R_f} A_o \neq \text{open loop gain}$$

(A) $$\text{C.L gain} = \frac{A_o}{1+\underbrace{\beta A_o}_{\text{loop gain}}}$$

$$\Rightarrow \frac{\frac{-R_f}{R_{in}+R_f} A_o}{1+\frac{R_{in}}{R_{in}+R_f}\cancel{A_o}} \longrightarrow \frac{-\frac{R_f}{R_{in}}}{\cancel{1+\frac{R_f}{R_{in}}}}$$

input to Amp = 0 $\Rightarrow v_- = v_+$

$\Rightarrow v_- = 0\,V$

(B) $$\text{C.L gain} \Rightarrow \frac{v_{in}}{R_{in}} = \frac{-v_{out}}{R_f}$$

$$\Rightarrow \boxed{\text{C.L gain} = \frac{-R_f}{R_{in}}}$$

$$\text{Open loop gain} = \frac{-R_f}{R_{in}+R_f} \times A$$

Sense and Return Mechanisms

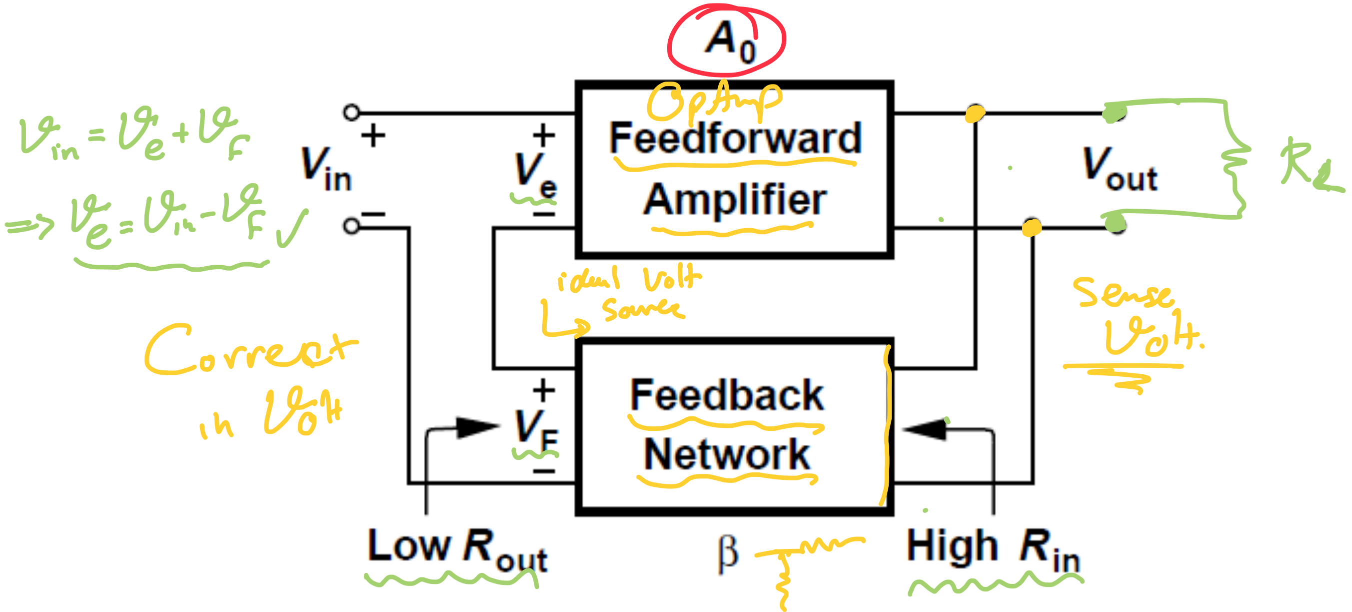
Correction

- Placing a circuit in a feedback loop requires sensing an output signal and returning a fraction of it to the summing node at the input

- Four types of feedback:
  - $\boxed{\text{Voltage-Voltage}}$ (*We only cover this type in our lecture, Sec. 8.2.1*)
  - Voltage-Current
  - Current-Current
  - Current-Voltage

- **First term is the quantity sensed at the output, and the second term is the type of signal returned to the input**

Voltage-Voltage Feedback



- Also called "series-shunt" feedback; first term refers to the *input* connection and second to the *output* connection
- We can write $V_F = \beta V_{out}$, $V_e = V_{in} - V_F$, $V_{out} = A_0(V_{in} - \beta V_{out})$, and hence

Closed-loop gain

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{A_0}{1+\beta A_0}} \longrightarrow 1/\beta$$

- βA_0 is the loop gain and the overall gain has dropped by $1+ \beta A_0$ >> 1

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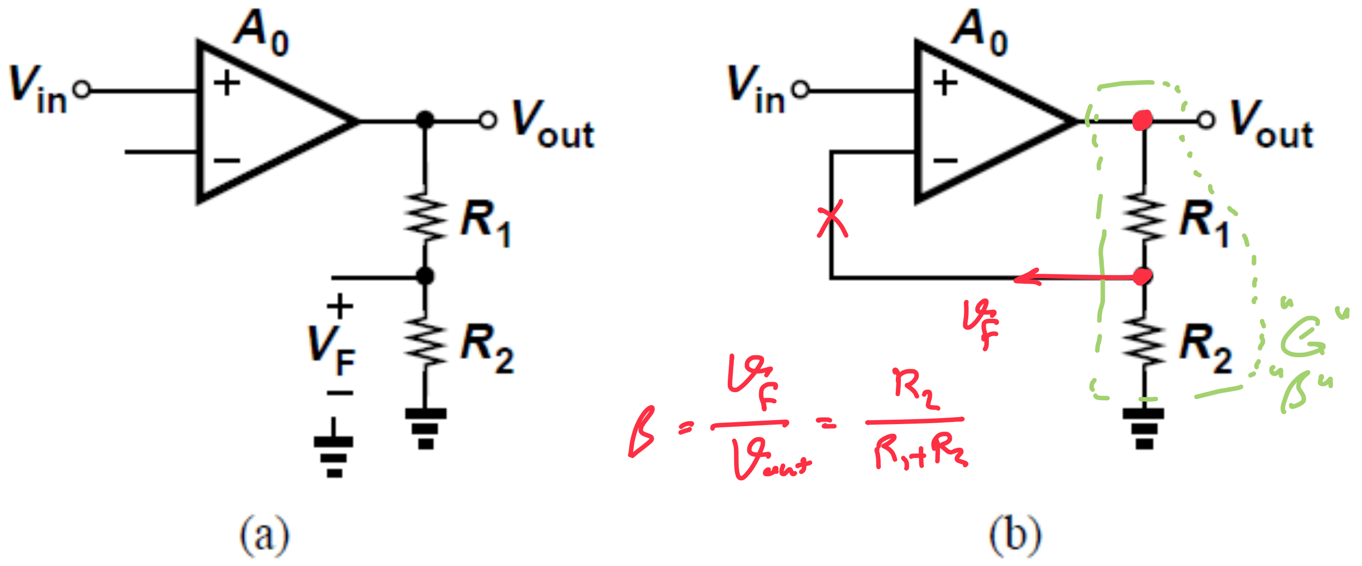
Voltage-Voltage Feedback

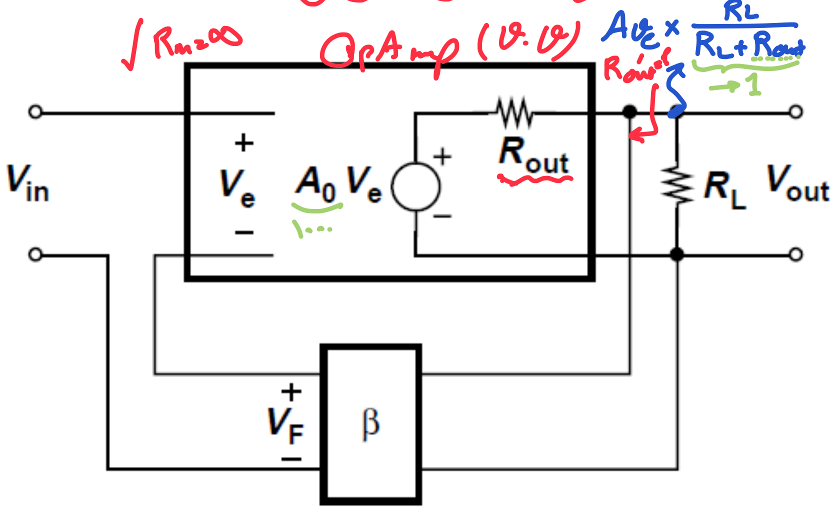


- As an example of voltage-voltage feedback, a differential voltage amplifier with single-ended output can be used as the forward amplifier and a resistive divider as the feedback network [Fig. (a)]
- The sensed voltage V_F is placed in series with the input to perform subtraction of voltages

Properties of Feedback Circuits

- **Terminal Impedance Modification (output impedance)**

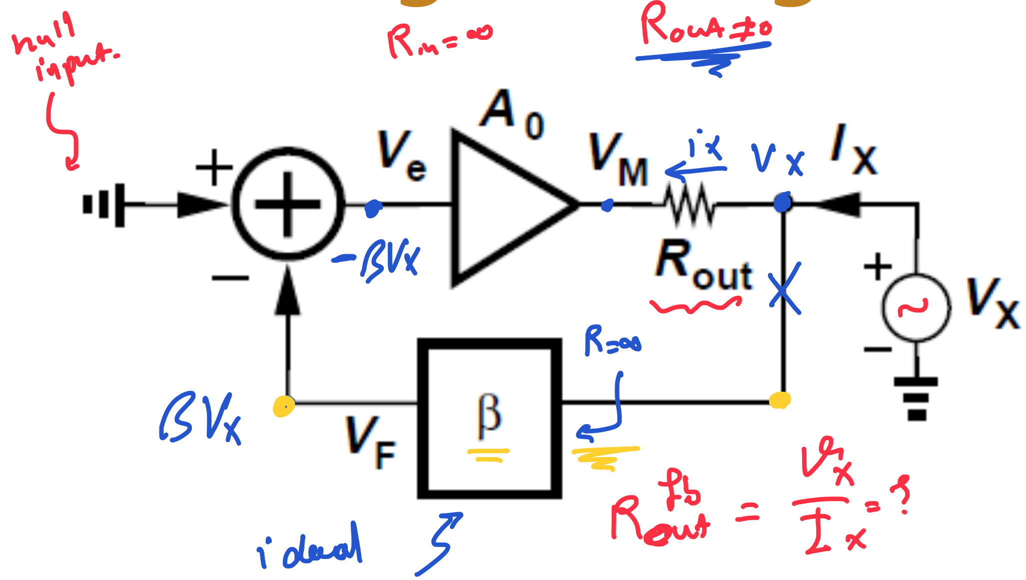
R_{out} w/o feedback is R_{out}!



- If output is loaded by resistor R_L, in open-loop configuration, output decreases in proportion to $R_L/(R_L+R_{out})$
- In closed-loop V_{out} is maintained as a constant replica of V_{in} regardless of R_L as long as loop gain is much greater than unity
- Circuit “stabilizes” output voltage despite load variations, behaves as a voltage source and exhibits low output impedance

* Voltage Output

Voltage-Voltage Feedback: Output Resistance



$V_M = A_0 x - \beta V_x$

$$i_x = \frac{V_x - V_M}{R_{out}} = \frac{V_x - (-A_0\beta)V_x}{R_{out}} = \frac{(1+\beta A_0)V_x}{R_{out}}$$

$$\Rightarrow \boxed{R_{out}^{w\,fb} = \frac{V_x}{i_x} = \frac{R_{out}}{1+\beta A_0}}$$

1 + loop gain >> 1

- In the above model, R_{out} represents the output impedance of the amplifier
- Setting input to zero and applying a voltage at the output, we write $V_F = \beta V_X$, $V_e = \beta V_X$, $V_M = \beta A_0 V_X$ and hence $I_X = [V_X - (-\beta A_0 V_X)]/R_{out}$ (*if current drawn by feedback network is neglected)

lowers the Rout

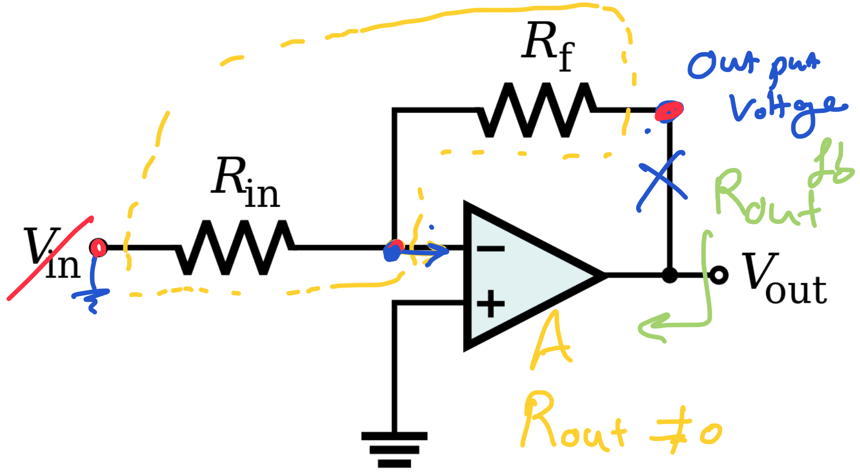
⇒ improving Rout

$$\boxed{\frac{V_X}{I_X} = \frac{R_{out}}{1 + \beta A_0}}$$

$1+\beta A$

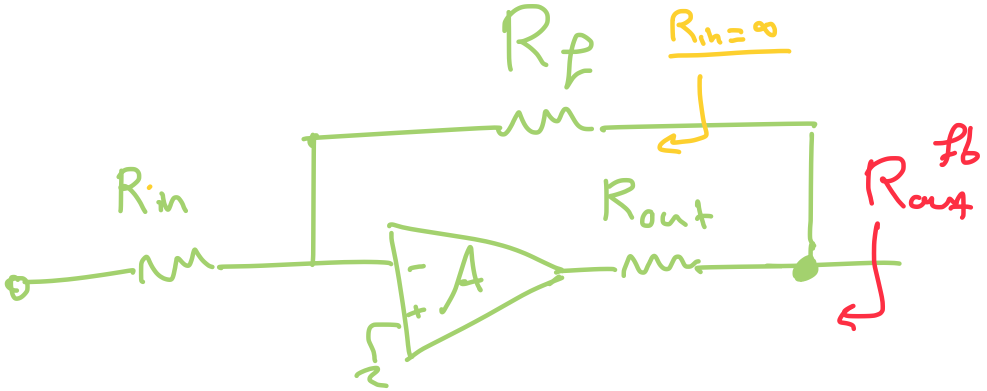
- Output impedance and gain are lowered by the same factor

Calculation of Output Impedance: Example



$R_{out} \neq 0$

$R_{in} = \infty$



$$R_{out}^{fb} = \frac{R_{out}}{1+\beta A}$$

$$\beta = \frac{R_{in}}{R_{in}+R_f}$$

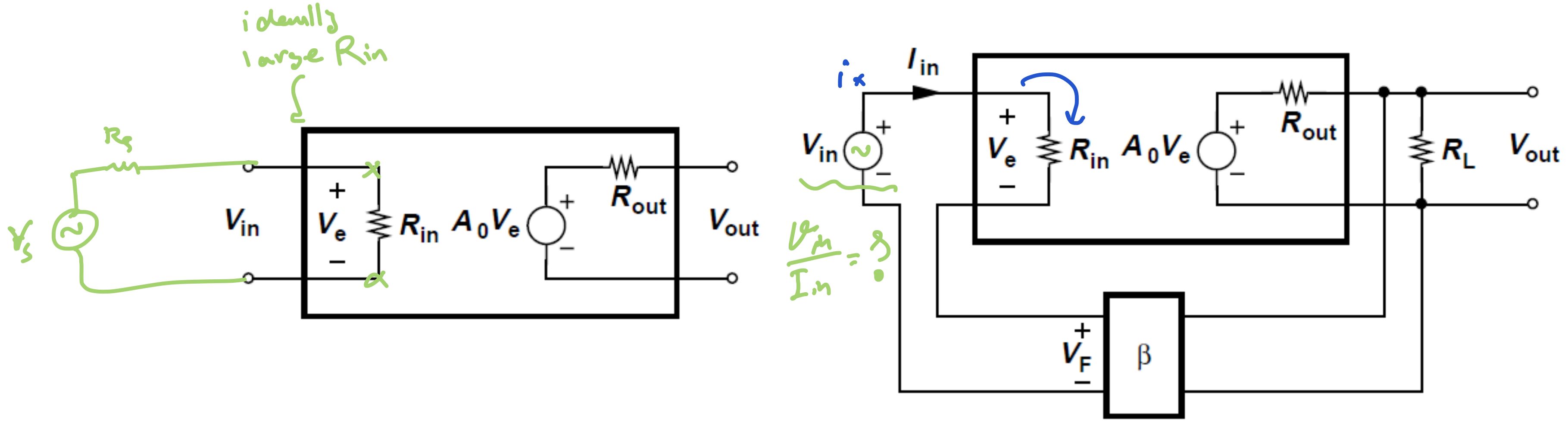
KVL & kcl

$$R_{out}^{fb} = (R_{in}+R_f) \parallel \frac{R_{out}}{1+\beta A}$$

$$(R_{in}+R_f) \gg \frac{R_{out}}{1+\beta A} \Rightarrow \text{large } R_{in} \,\&\, R_f$$

Properties of Feedback Circuits

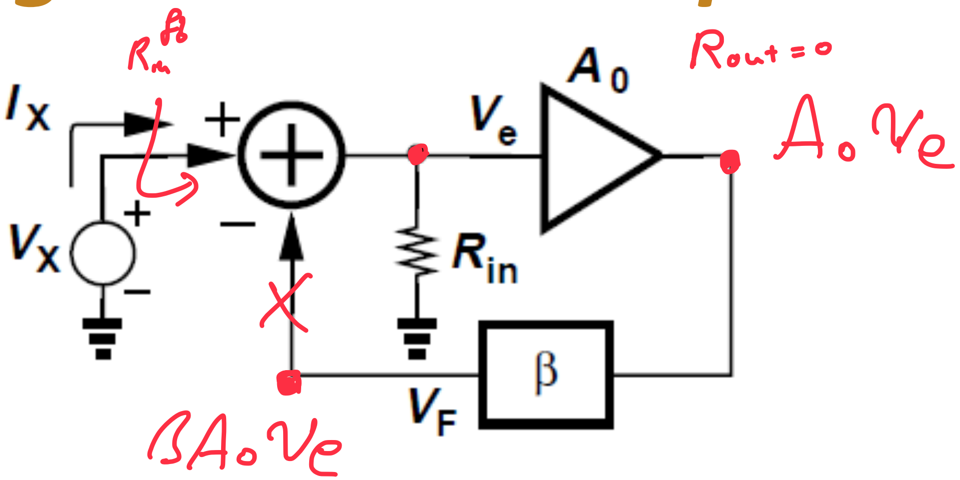
- **<u>Terminal Impedance Modification (input impedance)</u>**



(a) (b)

- Voltage-voltage feedback also modifies input impedance
- In Fig. (a) [open-loop], R_{in} of the forward amplifier sustains the entire V_{in}, whereas only a fraction in Fig. (b) [closed-loop]
- I_{in} is less in the feedback topology compared to open-loop system, suggesting increase in the input impedance

Voltage-Voltage Feedback: Input Resistance



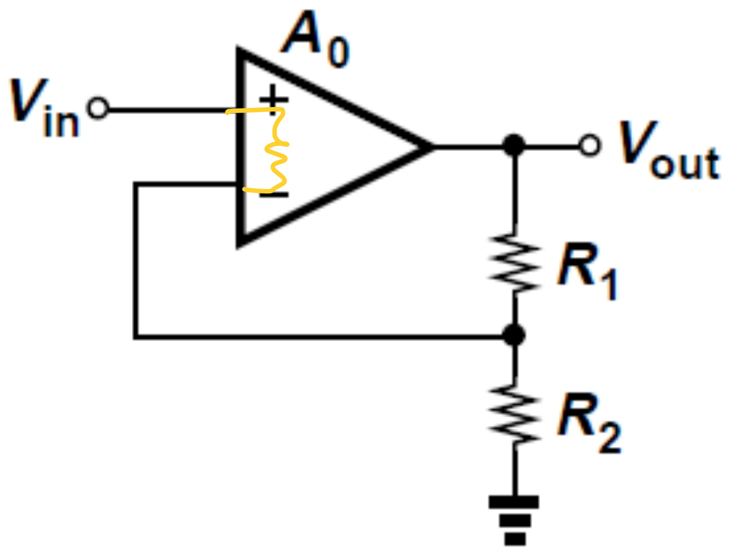
- $V_e = I_X R_{in}$ and $V_F = \beta A_0 I_X R_{in}$ => $V_e = V_X - V_F = V_X - \beta A_0 I_X R_{in}$
- Hence, $I_X R_{in} = V_X - \beta A_0 I_X R_{in}$ and

$$\boxed{\frac{V_X}{I_X} = R_{in}(1 + \beta A_0)}$$

- Input impedance increases by the factor $1+\beta A_0$, bringing the circuit closer to an ideal voltage amplifier
- Voltage-voltage feedback decreases output impedance and increases input impedance, useful as a buffer stage by a factor of 1+ loop. gain

Properties of Feedback Circuits

- **Terminal Impedance Modification**



$$R_{in}^{fb} = R_{in} \times (1 + \beta A_0)$$

$$\frac{R_2}{R_1 + R_2}$$

* large R_1 & R_2

Rules of feedback Amp

Feedback modifies input & output impedances (by a factor of 1+ *βA*)

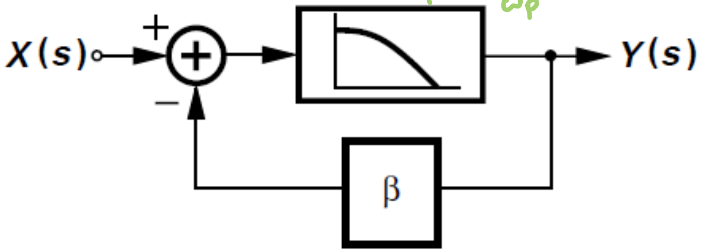
- increase or decrease of impedances depend on the feedback type
  - Feedback always improves the impedance …

Properties of Feedback Circuits

- **Bandwidth Modification:**

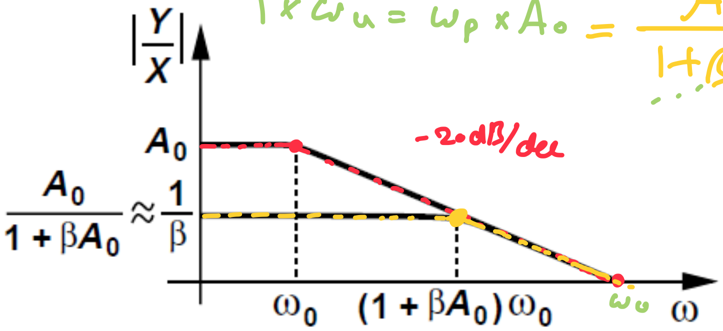
one pole Amp

$A(s) = \dfrac{A_0}{1+s/\omega_p}$



$A(s) = \dfrac{A_0}{1+s/\omega_0}$

$1 \times \omega_u = \omega_p \times A_0 = \dfrac{A_0}{1+\beta A_0} \times (1+\beta A_0)\omega_0$



- Suppose the feedforward amplifier above has a one-pole transfer function with A_0 as the low-frequency gain and ω_0 as the 3-dB bandwidth

$$A(s) = \frac{A_0}{1 + \dfrac{s}{\omega_0}}$$

pole of amp.

- Transfer function of the closed-loop system is

$A(s) / 1+\beta A(s)$

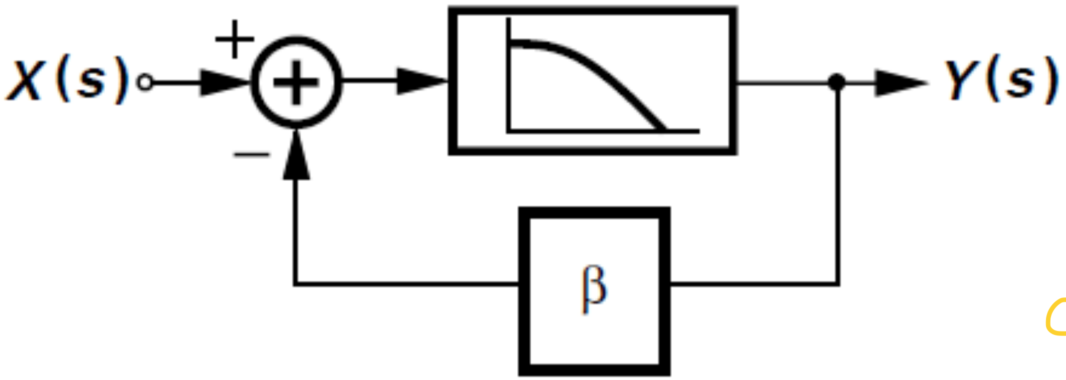
c.l gain @ low-freq

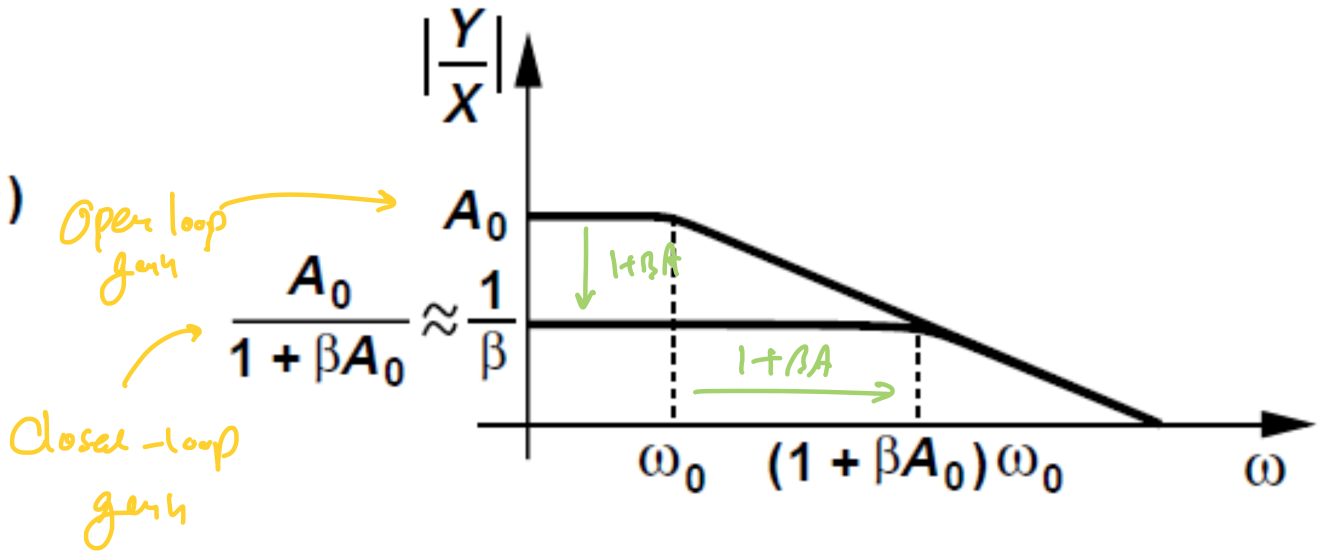
$$\frac{Y}{X}(s) = \frac{\dfrac{A_0}{1+\dfrac{s}{\omega_0}}}{1+\beta\dfrac{A_0}{1+\dfrac{s}{\omega_0}}} = \frac{A_0}{1+\beta A_0 + \dfrac{s}{\omega_0}} = \frac{\dfrac{A_0}{1+\beta A_0}}{1+\dfrac{s}{(1+\beta A_0)\omega_0}}$$

$\omega_p^{cl} = (1+\beta A_0)\omega_0$

Properties of Feedback Circuits

- **Bandwidth Modification:**

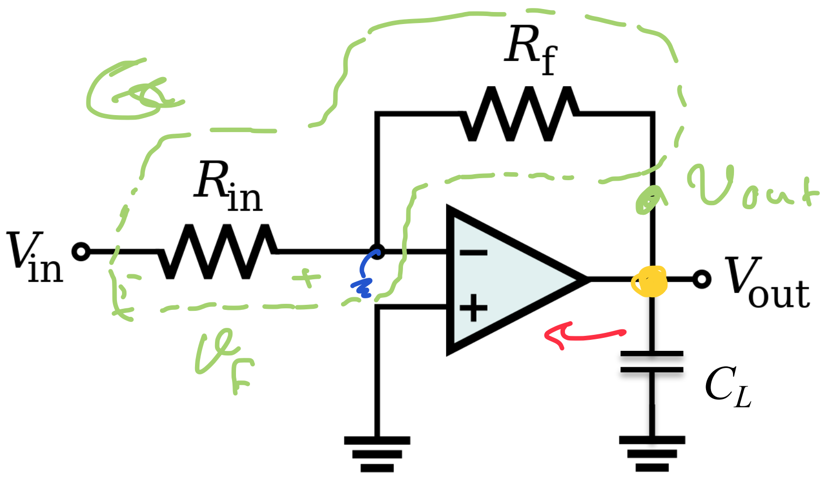




- The closed-loop gain at low frequencies is reduced by a factor of $1 + \beta A_0$, and the 3-dB bandwidth is increased by the same factor, revealing a pole at $(1 + \beta A_0)\omega_0$.
- If *A* is large enough, closed-loop gain remains approximately equal to 1/*β*
- At high frequencies, A drops so that *βA* is comparable to unity and closed-loop gain falls below 1/*β*

Properties of Feedback Circuits

- **Bandwidth Modification:**



$$A_o = -G_m R_{out} \qquad \omega_u = \frac{G_m}{C_L}$$

$$\omega_p = \frac{1}{R_{out} C_L}$$

$$\beta = \frac{v_F}{v_{out}} = \frac{R_{in}}{R_{in}+R_f}$$

$$\omega_p^{cl} = \omega_p (1+\beta A_o)$$

$$\omega_u^{cl} = \omega_u = \frac{G_m}{C_L}$$

$$A_{cl} = -\frac{R_f}{R_{in}}$$

$$A_{open\ loop} \neq A_o$$