Homework4 Solution

November 7, 2022

1 Problem 1

1.1 a

$$A_v = \frac{g_m R_S}{1 + g_m) R_S}$$

and

$$gm = \frac{2I_D}{V_{OD}} = 0.01$$

$\Rightarrow$

$$A_V = 0.91$$

$$R_{out} = r_o // R_S // \frac{1}{g_m} \approx \frac{1}{g_m}$$

Hence, $R_{out} = 90.1\Omega$ and $R_{in} = \infty\Omega$
Also,

$$\begin{aligned} V_{DD} - V_{OUT} > (V_{IN} - V_{OUT}) - V_{TH} = \quad & V_{GS} - V_{TH} \\ V_{GS} > V_{TH} & \end{aligned}$$

Vout max should be min($V_{DD} - V_{od}$,$V_{IN} - V_{th}$)
$\Rightarrow V_{OUT} < min(V_{DD} - V_{od}, V_{IN} - V_{th})V$
$\star$ The question did not give V_{IN}, so no points will be deducted for this voltage swing question

1.2 b

$$A_v = -\frac{g_{m1}}{g_{m2}}$$

because

$$\frac{1}{r_{o1}} = 1/\frac{1}{\lambda I_D} = \frac{1}{10k}\Omega.\text{too small, ignore it}$$

Hence, $A_V \approx -1V$
$R_{in} = \infty\Omega$ while

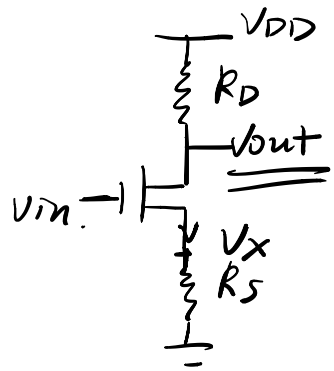
$$R_{out} = r_o // \frac{1}{g_{m2}} \approx \frac{1}{g_{m2}} = 100\Omega$$

Also,

$$\begin{aligned} V_{OUT} &> V_{GS} - V_{TH} = 0.2 \\ V_{DD} - V_{OUT} &> V_{TH} \end{aligned}$$

$\Rightarrow 2.5V > V_{OUT} > 0.2V$
Voltage swing is 0.2V-2.5V

1.
$R_S = 100\,\Omega$
$R_D = 10\,k\Omega$
$V_{DD} = 1V$
$V_{Th} = 0.3V$
$\mu_n C_{ox} = 100\,\mu A/V^2$



(a) $A_V = \dfrac{R_D}{1/g_m + R_S}$

$= 10.$

$10/g_m + 10 R_S = R_D$

$10/g_m = 9\,k\Omega$

$g_m = \dfrac{10}{9000} = \dfrac{1}{900}\ \Omega^{-1}$

$V_{out,max} = V_{DD}$
$V_{out,min}$
$V_{out} > V_{in} - V_{Th}.$

$\because\ V_{out,DC} = \dfrac{V_{DD}}{2} = 0.5V$

$I_D = \dfrac{V_{DD} - V_{out,DC}}{R_D} = \dfrac{0.5V}{10k\Omega} = 0.05\,mA.$

$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{Th})$

$\dfrac{W}{L} = \dfrac{g_m^2}{2\mu_n C_{ox} I_D} = \dfrac{(1/900)^2}{2 \times 100\times10^{-6} \times 0.05\times10^{-3}}$

$= 123.5.$

$V_{GS} = \dfrac{g_m}{\mu_n C_{ox} \frac{W}{L}} + V_{Th} = 0.39V.$

$V_X = I_D \cdot R_S = 0.05\times10^{-3} A \cdot 100\,\Omega = 0.005V$

$\therefore\ V_{in} = V_{GS} + V_x = 0.395\,V$

b) With $\lambda \neq 0$, we have r_o in the small signal model. And r_o is in parallel with R_D in the small signal model. in series with R_s (but R_s is very small, ignore it)

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.1 \times 0.05 \times 10^{-3}} = 200\,k\Omega$$

So, the new gain

$$r_o \parallel R_D = 9.5\,k\Omega$$

$$A_v' = \frac{R_D \parallel r_o}{1/g_m + R_S}$$

$$= \frac{9.5\,k\Omega}{0.9\,k\Omega + 0.1\,k\Omega} = 9.5$$, drop a little bit.

c) $V_{out,max} = V_{DD}$ when $I_D = 0$.

$$V_{out,min} = V_{GS} - V_{Th} + V_x$$
$$= 0.395 - 0.3$$
$$= 0.095\,V$$

So the voltage swing is $\frac{1 - 0.095}{2} \cong 0.45\,V$

(d) t_{ox} will affect g_m.

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{Th})$$

$t_{ox} \uparrow$ by 10% $\rightarrow$ $C_{ox}' = \frac{1}{1.1} C_{ox}$

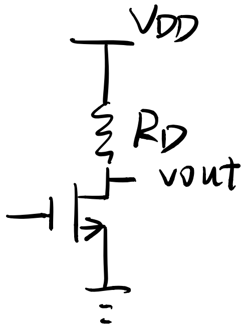
$\approx 0.91\,C_{ox}$

$g_m' = 0.91\,g_m$. $\quad A_v' = \dfrac{R_D}{1/0.91g_m + R_s} = \dfrac{10\times10^3}{989+100}$

$= 9.18.$

drop a bit. Compared to 9.5

(e) w/o source degeneration.



Assume that the V_{OD} is the same in this case.

Then the gain is

$$A_{v,w/o} = (R_D \parallel r_o)\,g_m$$
$$= 9.5\,k\Omega \times \frac{1}{900}\,\Omega^{-1}$$
$$= 10.6$$

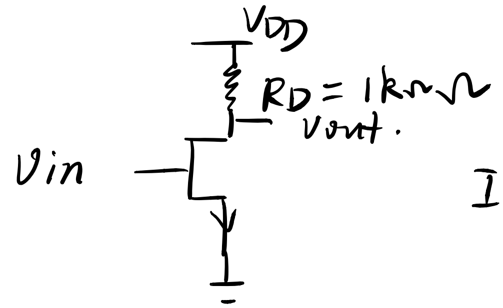
Then after the oxide gets 10% thicker, $g_m' = 0.91\,g_m$.

$$A_{v,w/o} = 9.5\,k\Omega \times 0.91 \times \frac{1}{900} = 9.6$$

With degenerate: $9.5 \rightarrow 9.18$ (3% change)

without degenerate: $10.6 \rightarrow 9.6$ (9% change)

2. (a)



$$I_D = \left(V_{DD} - \frac{V_{DD}}{2}\right) / R_D$$

$$= \frac{0.5}{1000} = 0.5mA$$

$$A_V = g_m R_D = 10.$$

$$g_m = \frac{10}{1 \times 10^3 \Omega} = 10^{-2} \Omega^{-1}$$

$$= \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\frac{W}{L} = g_m^2 / (2 \mu_n C_{ox} I_D)$$

$$= 10^{-4} / (\cancel{2} \times 10^{-4} \times \cancel{0.5} \times 10^{-3})$$

$$= 1000$$

$$(V_{GS} - V_{TH}) = \frac{g_m}{\mu_n C_{ox} \frac{W}{L}}$$

$$= \frac{10^{-2}}{10^{-4} \times 1000} = 0.1V$$

$V_{GS,bias} = 0.1 + 0.3 = 0.4V$

(b) in the small signal model, R_D is in parallel with R_L.

So the new $R_D' = R_L // R_D = \frac{1000 * 100}{1000 + 100}$

$= 91\Omega$

$A_v = R_D' g_m = 91 \times 10^{-2} = 0.91$, very small!

(c) I will need increase $\left(\frac{W}{L}\right)$ dramatically, but it's unrealistic

(d) $V_{DD} \cdot I_D = 1V \times 0.5mA = 0.5mW.$

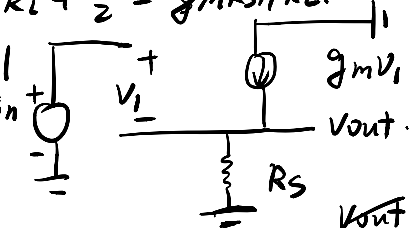
$100 // 1000 = \frac{10^5}{1000}$

(e) $R_S I_{D2} = \frac{V_{DD}}{2} \Rightarrow I_{D2} = \frac{V_{DD}}{2R_S} = \frac{1V}{2 \times 1k\Omega} = 0.5mA$

$A_v = \frac{g_m R_S // R_L}{g_m R_S // R_L + 1} = 0.5 \Rightarrow \frac{1}{2} g_m R_S // R_L + \frac{1}{2} = g_m R_S // R_L.$

$g_m R_S // R_L = 1$

$g_m = 0.01$



$\frac{W}{L} = \frac{g_m^2}{2 \mu_n C_{ox} I_{D2}}$

$= \frac{\cancel{10^{-4}}}{[illegible]} = 10^3$

$\cancel{2\times10^{-4}\times0.5\times10^{-3}}$

$$(V_{GS}-V_{Th}) = \frac{g_m}{\mu_n C_{ox}\times W/L} = \frac{10^{-2}}{\cancel{10^{-4}}_{0.1}\times\cancel{10^{3}}} = 0.1$$

$$V_{in,bias} = 0.1 + \frac{V_{DD}}{2} + 0.3 = 0.9\,V$$

$$R_{out} = \frac{\quad}{+\cancel{V_{out}}\cdot g_m + \frac{\cancel{V_{out}}}{R_S}} = \frac{R_S}{g_m R_S + 1}$$

$$G_m = g_m$$

(f) Gain of Common source stage should be 20

$$R_D g_{m1} = 20 \qquad g_{m1} = \frac{20\,V}{1k\Omega} = 2\times10^{-2}\,\Omega^{-1}$$

$$V_{out} = 0.5+0.4 = 0.9V \qquad I_{D1} = \frac{1-0.9}{1000} = 0.1mA.$$

$$W/L = \frac{g_{m1}^2}{2\mu_n C_{ox} I_D} = \frac{4\times\cancel{10^{-4}}}{2\times\cancel{10^{-4}}\times0.1\times10^{-3}} = 2\times10^4.$$

$$V_{GS1}-V_{Th} = \frac{g_{m1}}{\mu_n C_{ox}\, W/L} = \frac{2\times10^{-2}}{10^{-4}\times2\times10^4} = 10^{-2}\,V$$

$$V_{GS1} = 0.31V$$

(g)

$$P = 0.5mA\times1V + 0.1mA\times1 = 0.6mW > 0.5mW \text{ in (d)}$$

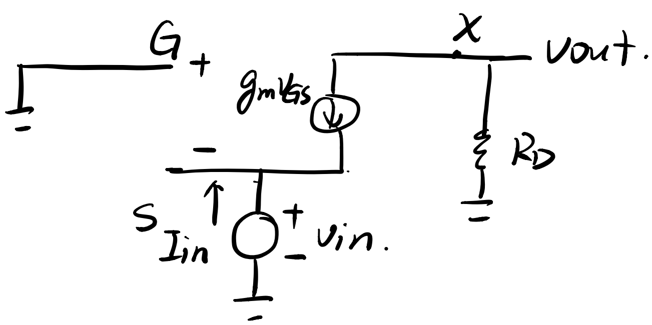
3.

(a) For gain.

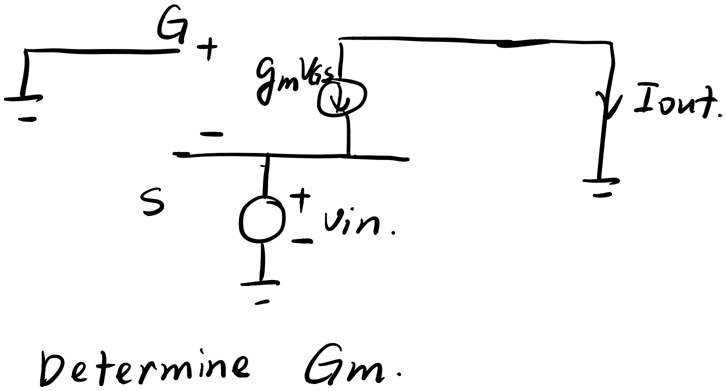
$$V_{GS} = -V_{in}$$

$$G_m = \frac{I_{out}}{V_{in}} = -g_m.$$

$$R = R_D$$



Small signal model.



Determine Gm.

$A_v = -G_m \cdot R = g_m R_D = 50$

input impedance $\quad Z_{in} = 1/g_m$

$= 50\,\Omega$

$\Rightarrow g_m = 0.02 \quad \Rightarrow R_D = 2500\,\Omega$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_1} \quad \Rightarrow \quad \frac{W}{L} = \frac{g_m^2}{2\mu_n C_{ox} I_1} = \frac{4\times 10^{-4}}{2\times\cancel{10^{-4}}\times\cancel{0.2}\,0.1\times 10^{-3}}$$

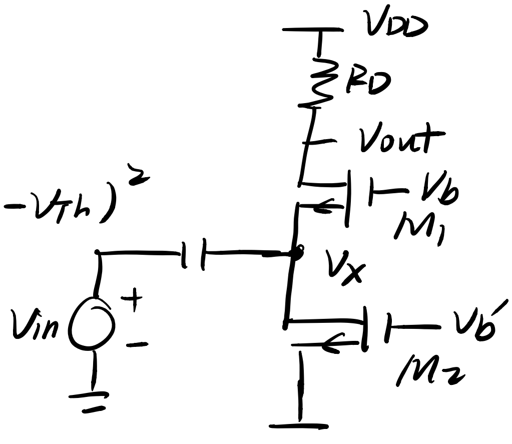
$$= 10^4$$

$$V_{GS} - V_{Th} = \frac{g_m}{\mu_n C_{ox}\frac{W}{L}} = \frac{0.02}{10^{-4}\times 10^4} = 0.02\,V$$

(b) $\quad V_{GS} - V_{Th} = 0.5 - 0.3 = 0.2\,V$

$I_1 = 0.2\,mA.$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{Th})^2$$



$$\Rightarrow \frac{W}{L} = \frac{2I_1}{\mu_n C_{ox}(V_{GS}-V_{Th})^2}$$

$$= 100$$

(c) $\quad V_x > V_b' - V_{Th}$

$V_{out} - V_x > V_b - V_{Th}$

$V_{out} > V_x + (V_{GS_1} - V_{Th}) > V_b' - V_{Th} + (V_{GS} - V_{Th})$

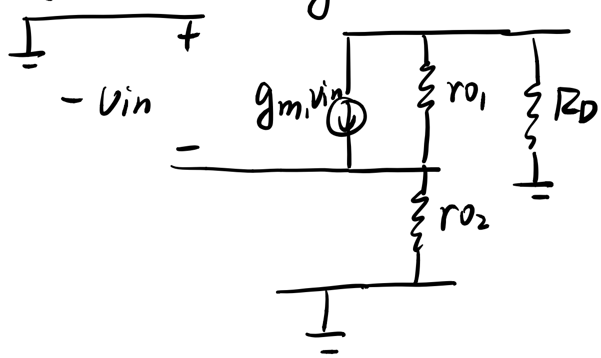
$\cong 0.2V$

Upper limit: $V_{DD} - I_1 R_D = 1 - 0.2\times10^{-3}\times 2500$

$= 0.5V.$

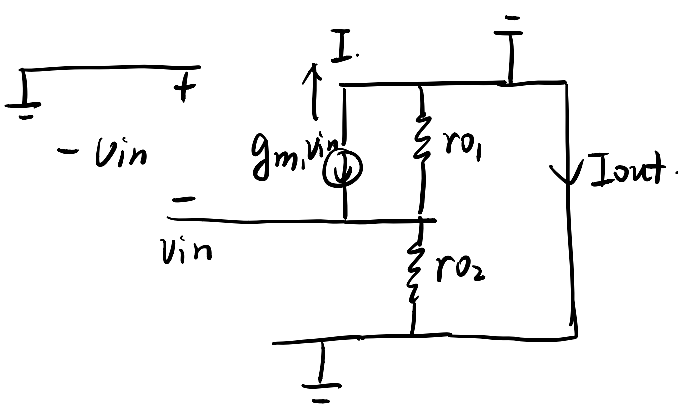
Voltage swing: $0.15V$

(d) The small signal model.



Use Lemma.

- Determine Gm.



$$I_{out} = g_m v_{in} + v_{in}/r_{o1}//r_{o2}$$

$$G_m = g_m + \frac{1}{r_{o1}//r_{o2}} \qquad r_o = \frac{1}{\lambda_1 I_D} = \frac{1}{0.1\times 0.2mA}$$

$$= 50k\Omega$$

$$R = (r_{o1} + r_{o2}) // R_D$$

$$\approx R_D$$

$$A_v' = G_m R = \left(g_{m1} + \frac{1}{r_{o1} // r_{o2}}\right) (r_{o1} + r_{o2}) // R_D$$

Note previously $A_v = \left(g_{m1} + \frac{1}{r_{o1}}\right) r_{o1} // R_D$

$$= \left(0.02 + \frac{1}{50000}\right) 50k // 2.5k$$

$$= 47.6$$

(Note in problem a we ignored r_o)

Now $A_v' = \left(0.02 + \frac{1}{25000}\right) 100k // 2.5k$

$= 48.9$, the total gain slightly increases.

(e) $f = \frac{1}{R_{in} C}$

$$= \frac{1}{50C} \lesssim 300$$

$$\Rightarrow \quad C \geq \frac{1}{6} F$$