

EE 332: Devices and Circuits II

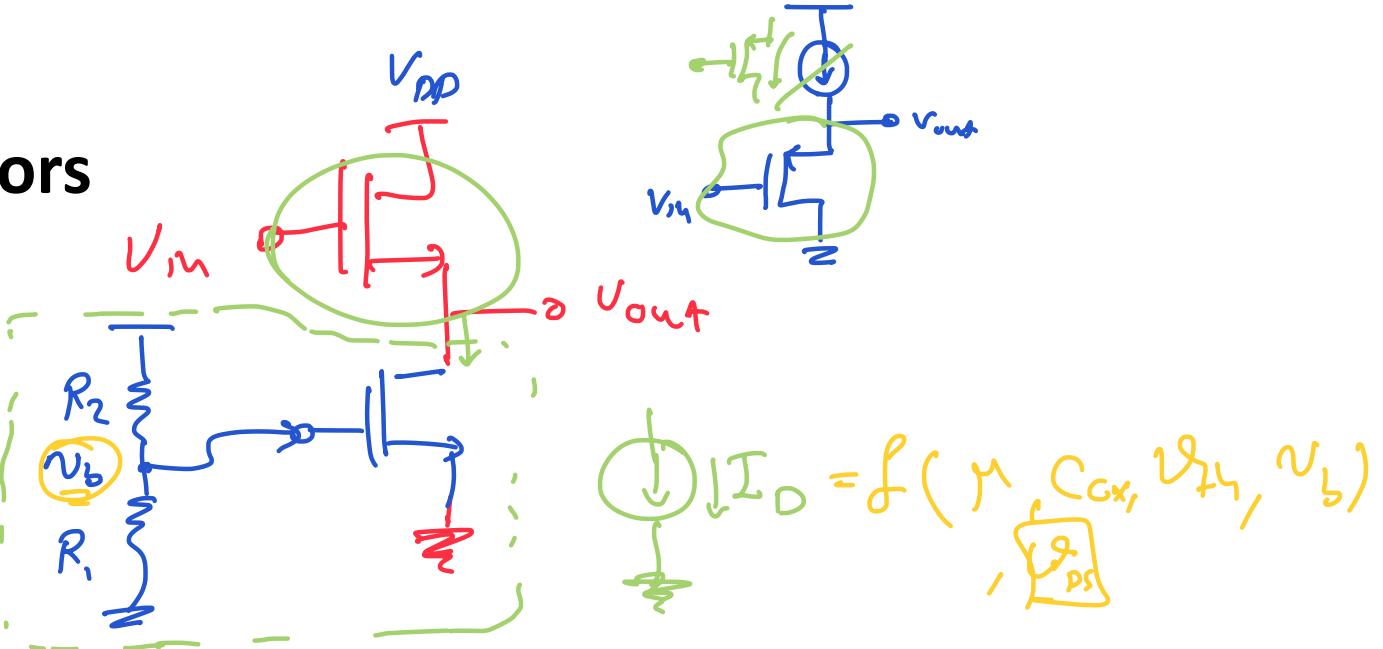
Lecture 5: Current Mirrors

Prof. Sajjad Moazeni

smoazen@uw.edu

Autumn 2022

$$V_{DD} \times \frac{R_1}{R_1 + R_2}$$



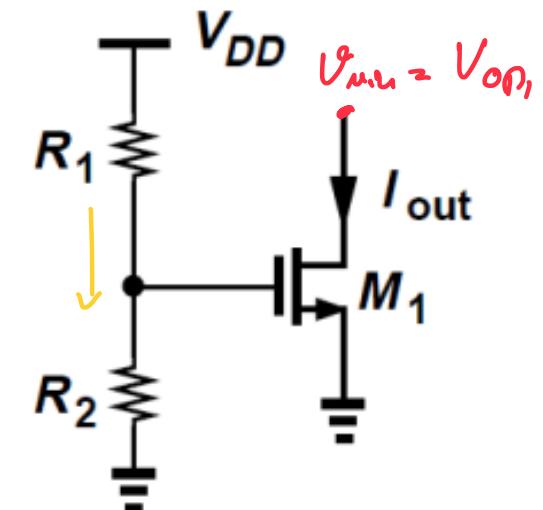
Basic Current Source

Ideal Current Source: ?

(large $R_s \Rightarrow$ big area X)

X Noise

$\frac{V_{DD}}{R_1 + R_2}$

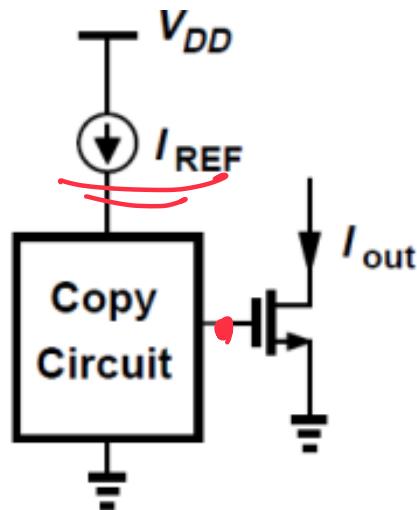
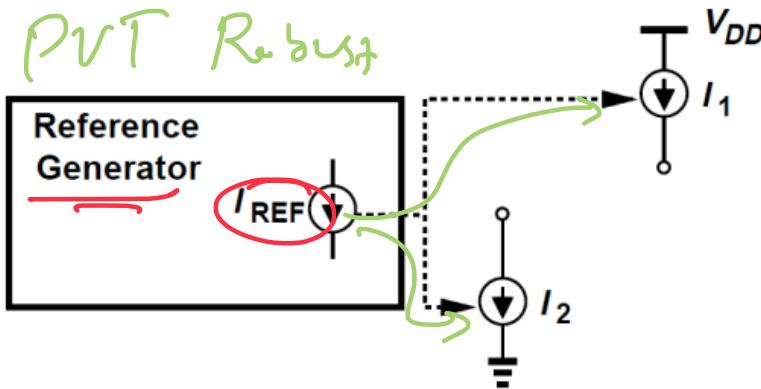


- Assuming M1 is in saturation, we can write

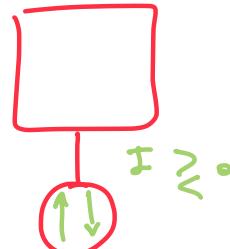
$$I_{out} \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(\frac{R_2}{R_1 + R_2} V_{DD} - V_{TH} \right)^2$$

- The threshold voltage may vary by 50 to 100 mV from wafer to wafer
- Both μ_n and V_{TH} exhibit temperature dependence
- We must seek other methods of biasing MOS current sources.

Copying currents (Current Mirroring)



- Use of a reference to generate various currents.
- Two identical MOS devices that have equal V_{GS} and operate in saturation carry equal currents
 - Can change the ratio with W/L
- Distributed “Ref” can be eventually in either Voltage or Current domain.
 - What are the Pros/Cons of each?



Basic Current Mirror

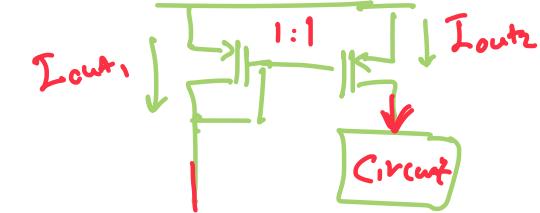
- Neglecting channel-length modulation (CLM), we can write

$$I_{REF} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right)_1 (V_{GS} - V_{TH})^2 \quad (1 + \lambda V_{DS})$$

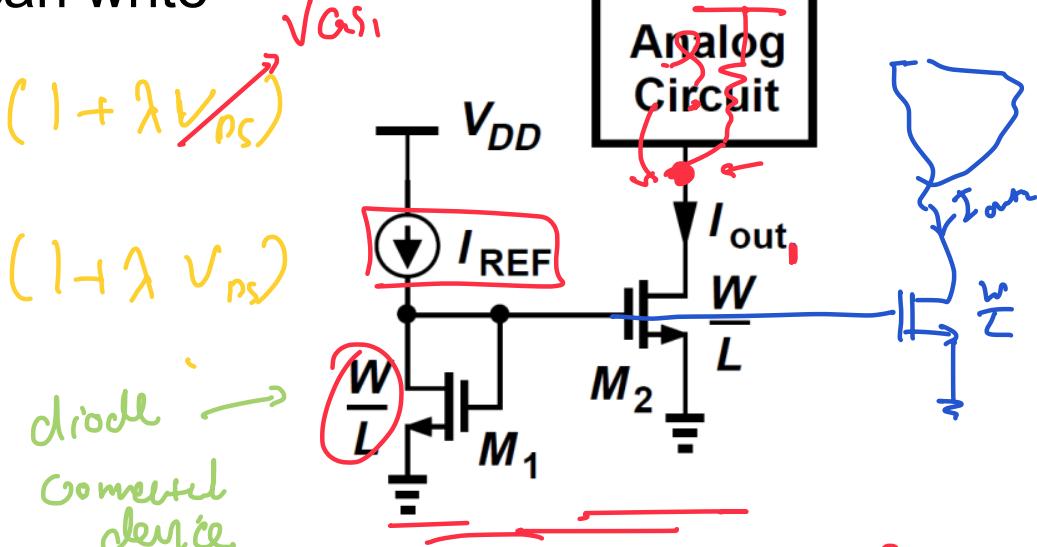
$$I_{out} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right)_2 (V_{GS} - V_{TH})^2 \quad (1 + \lambda V_{DS})$$

$$I_{out} = \frac{(W/L)_2}{(W/L)_1} I_{REF}$$

$$V_{GS1} = V_{GS2}$$



- Allows precise copying of the current with no dependence on process and temperature
- How would Channel-Length Modulation will impact this result?



$$V_{GS1} = V_{GS2}$$

$$g_{ds} = \frac{\partial I}{\partial V_{DS}} \quad \frac{1}{r_o} = g_{ds} = \frac{\partial I}{\partial V_{DS}}$$

Example

- Calculate the small-signal voltage gain of the circuit shown in Figure.

$$I_{D2} = I_{D1}$$

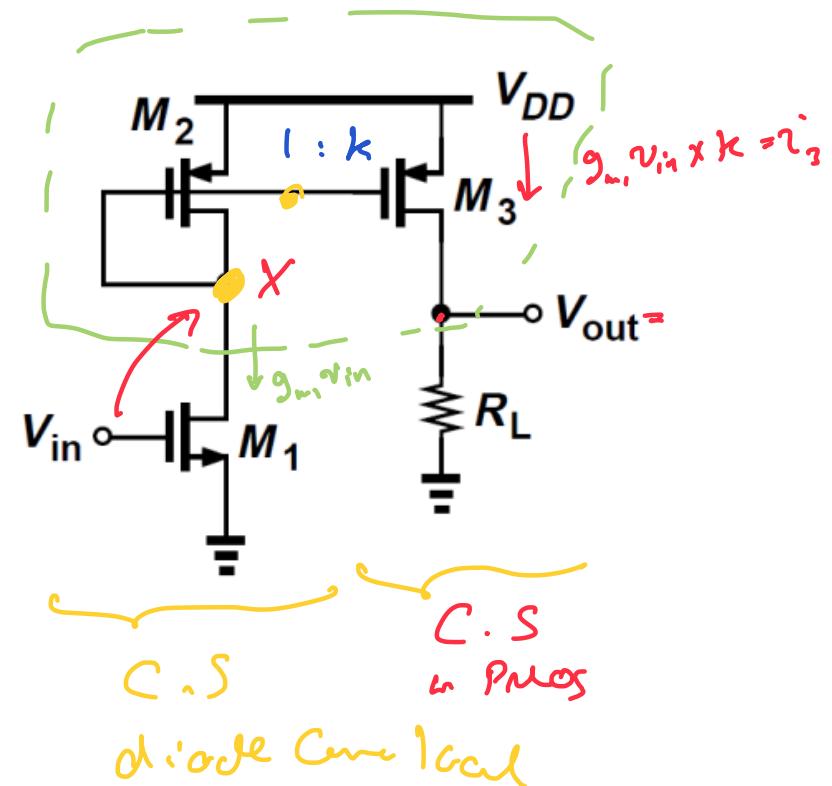
$$I_{D3} = I_{D2}(W/L)_3/(W/L)_2$$

$$\cdot A_v = ? \quad \frac{V_{out}}{V_{in}} = \frac{V_x}{V_{in}} \times \frac{V_{out}}{V_x}$$

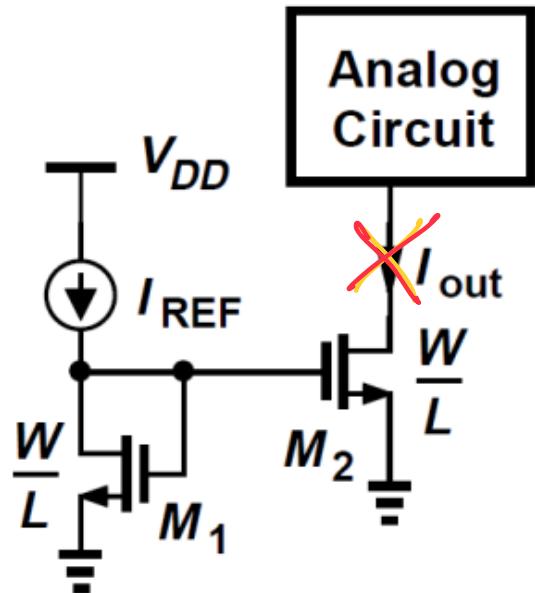
$$= (-g_{m_1} \times \frac{1}{g_{m_2}}) \times (-g_{m_3} \times R_L)$$

$$= g_{m_1} R_L \times \left(\frac{g_{m_3}}{g_{m_2}} \right)^K$$

$$\frac{(W/L)_3}{(W/L)_2} \times K \Rightarrow \frac{g_{m_3}}{g_{m_2}} = ? \quad K$$



Cascode Current Mirrors



$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right)_1 (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS1})$$

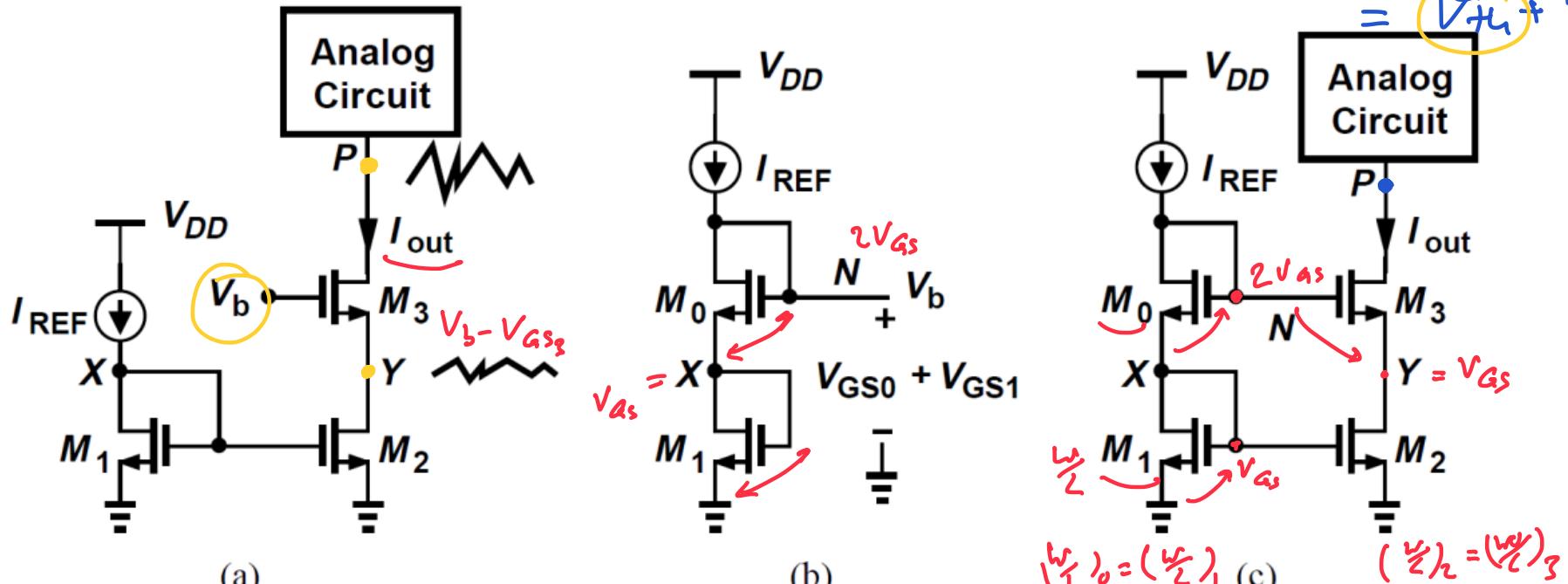
$$I_{D2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right)_2 (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS2})$$

$$\frac{I_{D2}}{I_{D1}} = \frac{(W/L)_2}{(W/L)_1} \cdot \underbrace{\frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}}}_{\text{depends on "Box" circuit.}}$$

- While $V_{DS1} = V_{GS1} = V_{GS2}$, V_{DS2} may not equal V_{GS2}
 - We can (a) force V_{DS2} to be equal to V_{DS1} , or (b) force V_{DS1} to be equal to V_{DS2} .
- $V_{PS_2} = V_{GS_1} = V_{PS_1}$

First Approach

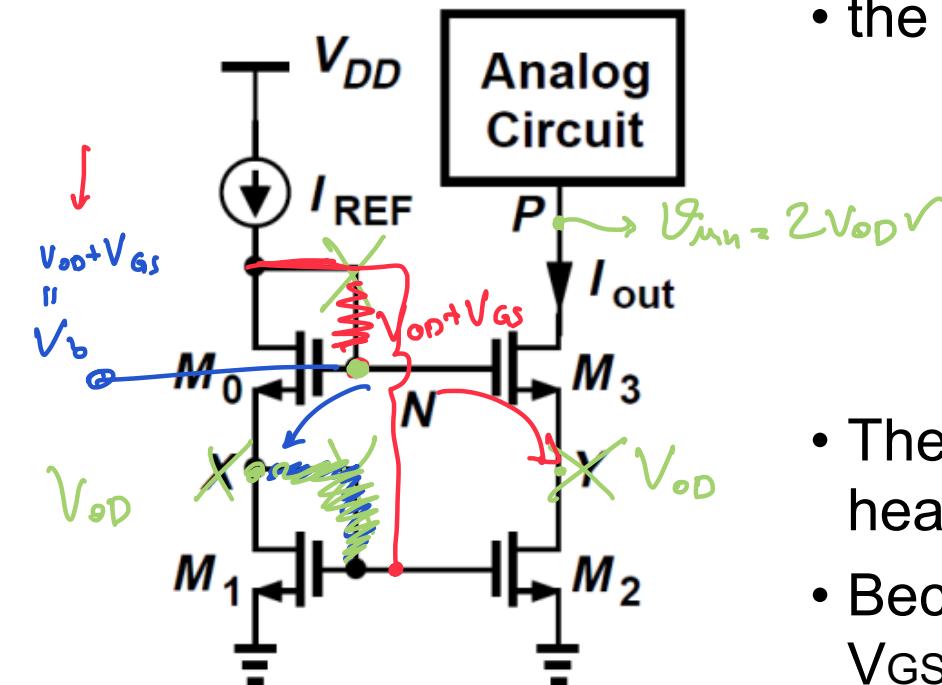
$$\cancel{V_{DS\min} @ P = V_{GS} + V_{OD}} \\ = \cancel{V_{Th}} + 2V_{OD}$$



$$V_b = 2V_{GS} \implies V_y = V_x = V_{GS}$$

- A cascode device can shield a current source, thereby reducing the voltage variations across it.
- But, how do we ensure that $V_{DS2} = V_{DS1}$?
- We must generate V_b such that $V_b - V_{GS3} = V_{DS1} (= V_{GS1})$

Example



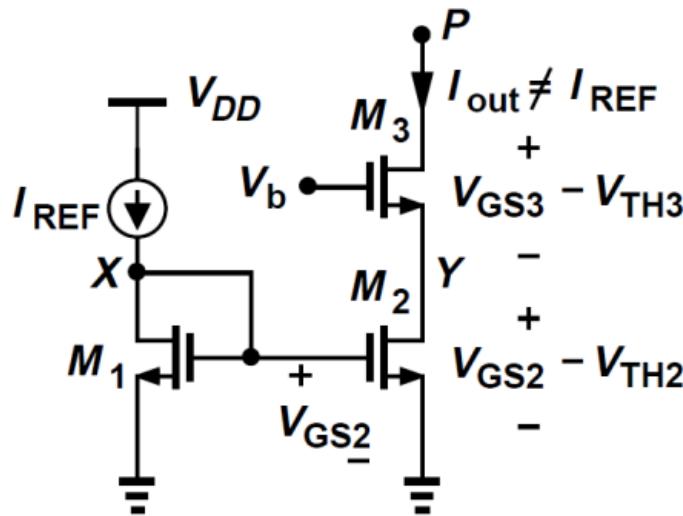
- the minimum allowable voltage at node P is equal to

$$V_N - V_{TH} = V_{GS0} + V_{GS1} - V_{TH}$$

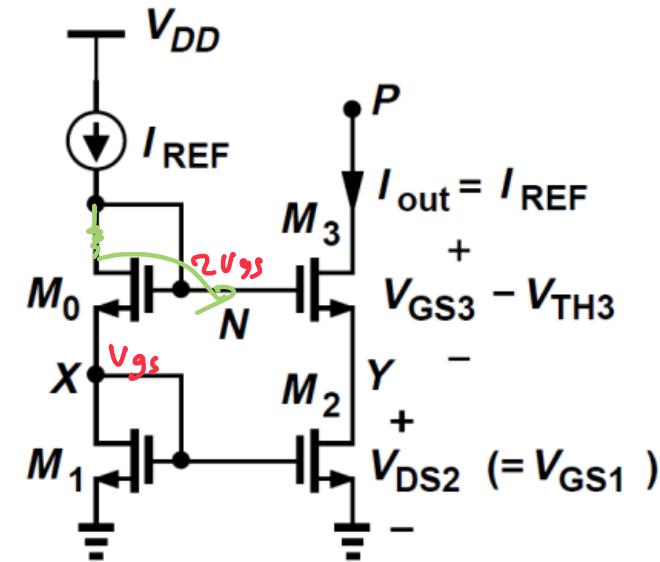
$$= (V_{GS0} - V_{TH}) + (V_{GS1} - V_{TH}) + V_{TH}$$

- The cascode mirror “wastes” one threshold voltage in the headroom.
 - Because $V_{DS2} = V_{GS2}$, whereas V_{DS2} could be as low as $V_{GS2}-V_{TH}$ while maintaining M2 in saturation.

Approach summary



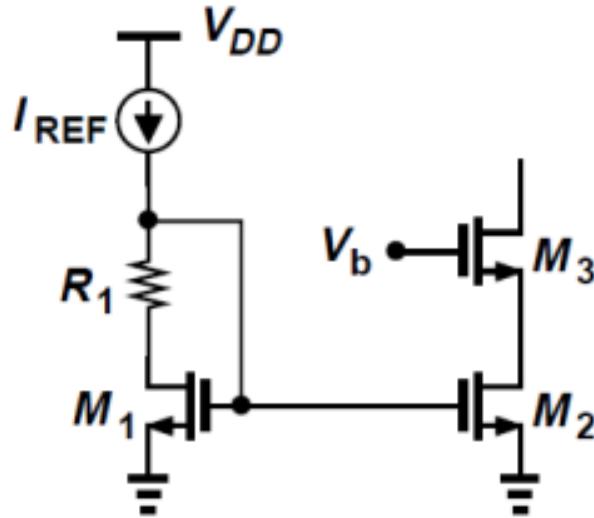
(a)



(b)

- In Fig(a), V_b is chosen to allow the lowest possible value of V_P but the output current does not accurately track I_{REF} .
- In Fig(b), a higher accuracy is achieved, but the minimum level at P is higher by one threshold voltage.

Second Approach

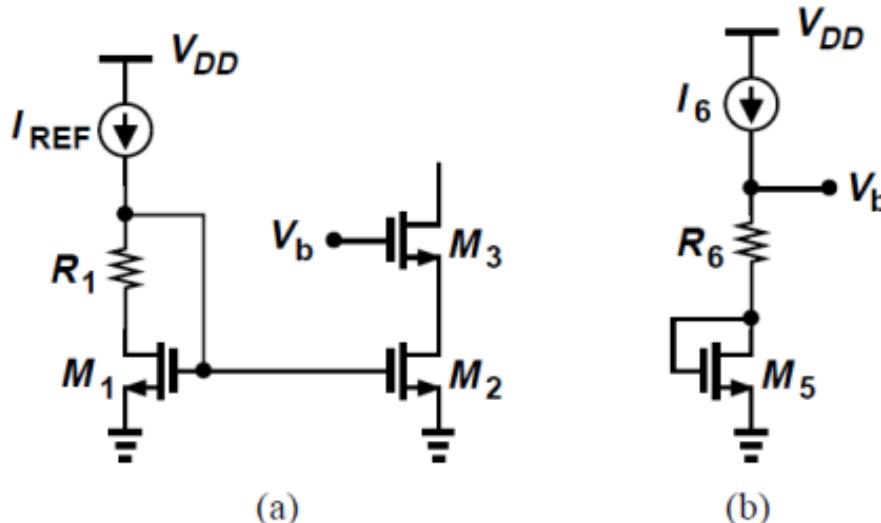


- Consider the branch shown in Fig. 5.16(b)
- As a candidate and write $V_b = V_{GS3} + R_6 I_6$.

$$R_1 I_{REF} \approx V_{TH1}$$
$$V_b = V_{GS3} + (V_{GS1} - V_{TH1})$$

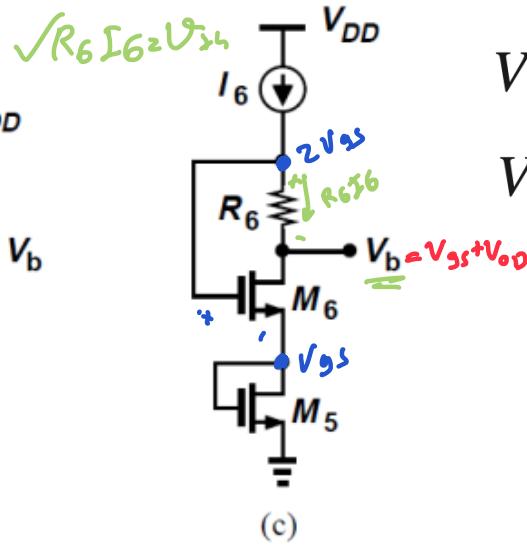
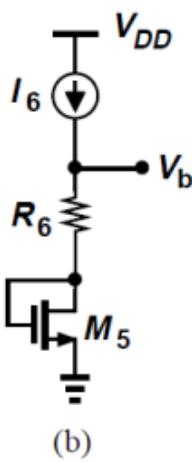
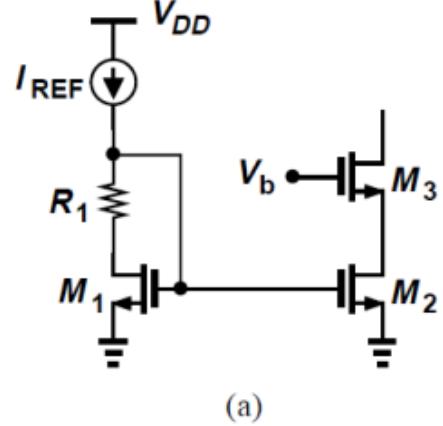
- Thus, from a small-signal point of view, the combination is close to a diode-connected device.
- But ...
 - (1) It may be difficult to guarantee that $R_1 I_{REF} \approx V_{TH1}$
 - (2) The generation of V_b is not straightforward.

Generate V_b



- Consider the branch shown in Fig(b) as a candidate and write $V_b = V_{GS5} + R_6 I_6$.
- Choose I_6 and $(W/L)_5$ such that: $V_{GS5} = V_{GS3}$
- However, the condition $R_6 I_6 = V_{GS1} - V_{TH1} = V_{GS1} - R_1 I_{REF}$ is hard to meet.
 - This condition translates to: $R_6 I_6 + R_1 I_{REF} = V_{GS1}$

Generate V_b



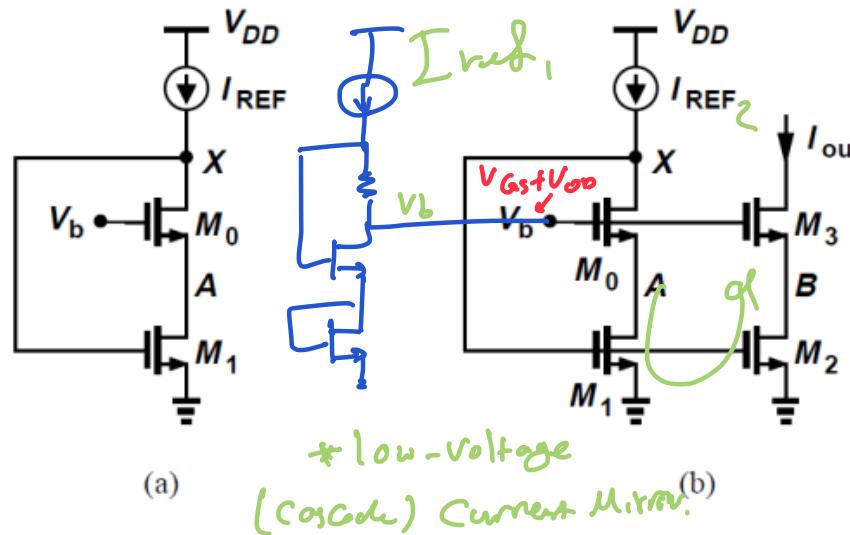
$$V_{GS5} = V_{GS3}$$

$$V_{GS6} - R_6 I_6 = V_{GS1} - V_{TH1}$$

$$= V_{GS1} - R_1 I_{REF}$$

- It is now possible to ensure that V_{GS6} and V_{GS1} track each other.
- For example, we may simply choose $I_6 = I_{REF}$, $R_6 = R_1$, and $(W/L)_6 = (W/L)_1$

Another circuit topology



- In this case

$$V_{DS1} = V_b - V_{GS0}$$

- Must have $V_b - V_{TH0} \leq V_X (= V_{GS1})$ for M_0 to be saturated and $V_{GS1} - V_{TH1} \leq V_A (= V_b - V_{GS0})$ for M_1 to be saturated.
- A solution exists if $V_{GS0} + (V_{GS1} - V_{TH1}) \leq V_b \leq V_{GS1} + V_{TH0}$
- We must therefore size M_0 to ensure its overdrive is well below V_{TH1} .

$\cancel{W_L}$ large $\Rightarrow V_{GS} \approx V_{TH}$

How to generate V_b

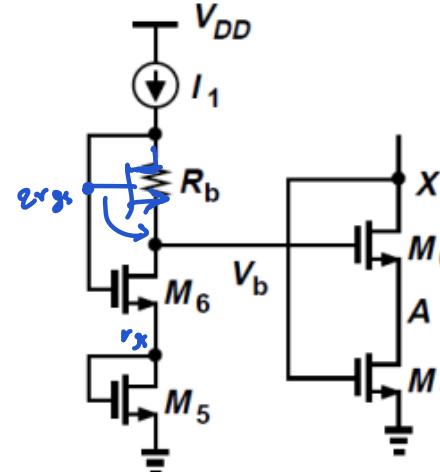
- In figure (a):
 - Need $V_b = V_{GS0} + (V_{GS1} - V_{TH1})$

- So we need to ...

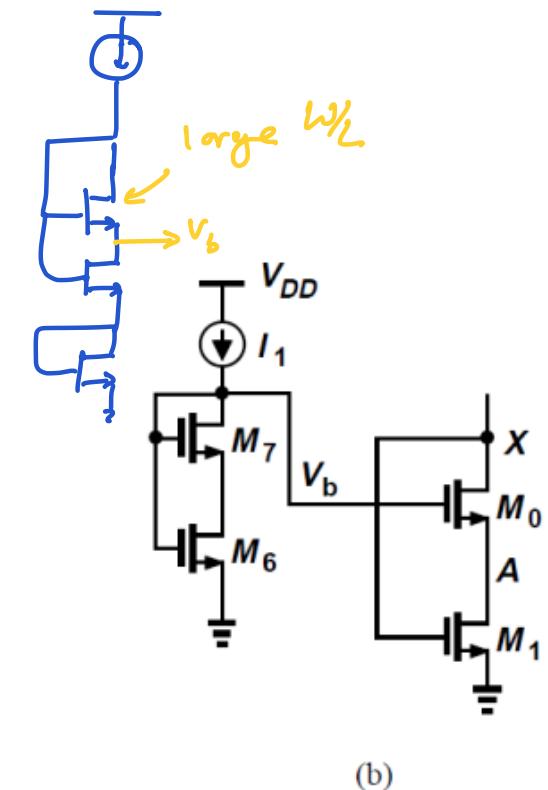
$$V_{GS5} \approx V_{GS0}$$

$$V_{DS6} = V_{GS6} - R_b I_1 \approx V_{GS1} - V_{TH1}$$

V_{TH}



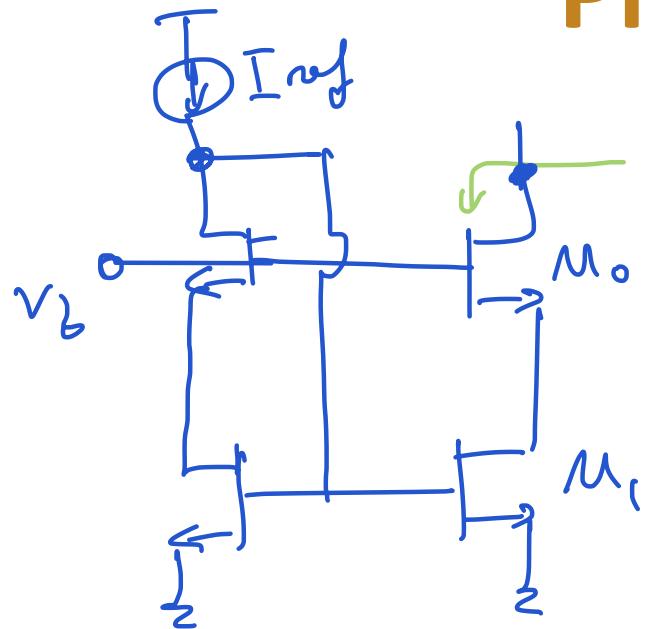
(a)



(b)

- Some inaccuracy nevertheless arises because M_5 does not suffer from body effect whereas M_0 does.
- Also, the magnitude of $R_6 \times I_1$ is not well-controlled.
- A simpler alternative is shown in Fig (b)

Practical Current Distribution



$$R_{out} \approx \frac{r_o + r_o b_1 g_m}{g_m} + r_o \approx r_o (g_m r_o)$$

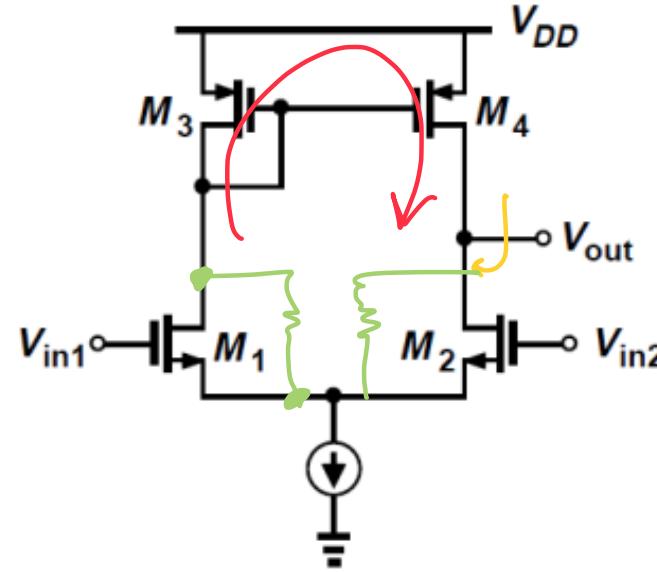
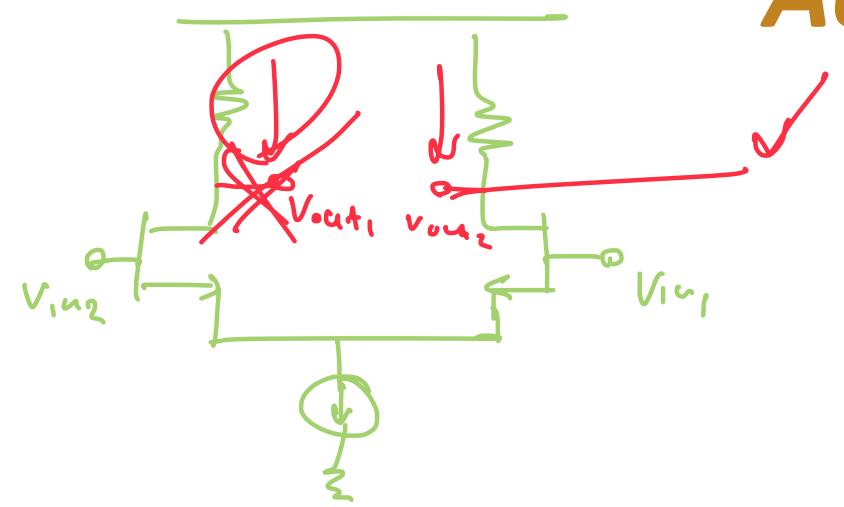
10-100

$$R_{out} \approx r_o = \frac{1}{2 I_D}$$

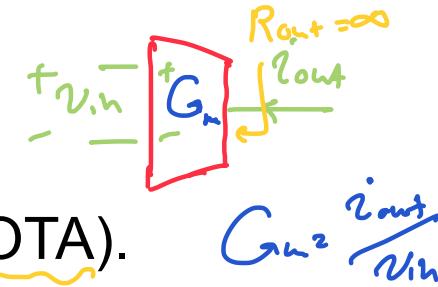
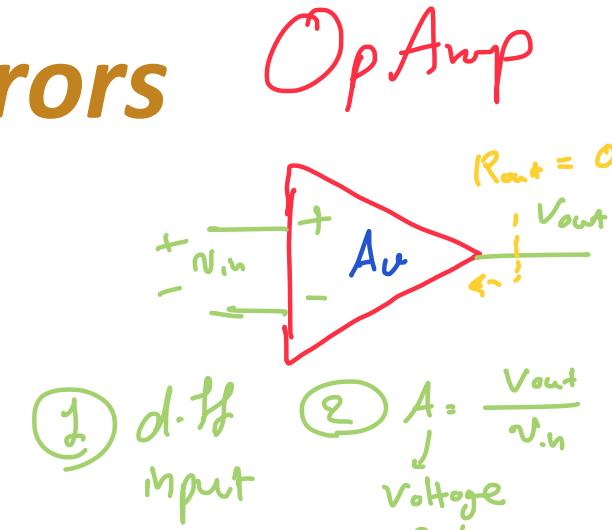
$\lambda \propto \frac{1}{L}$

Large L
↓
✓ Increases the r_o

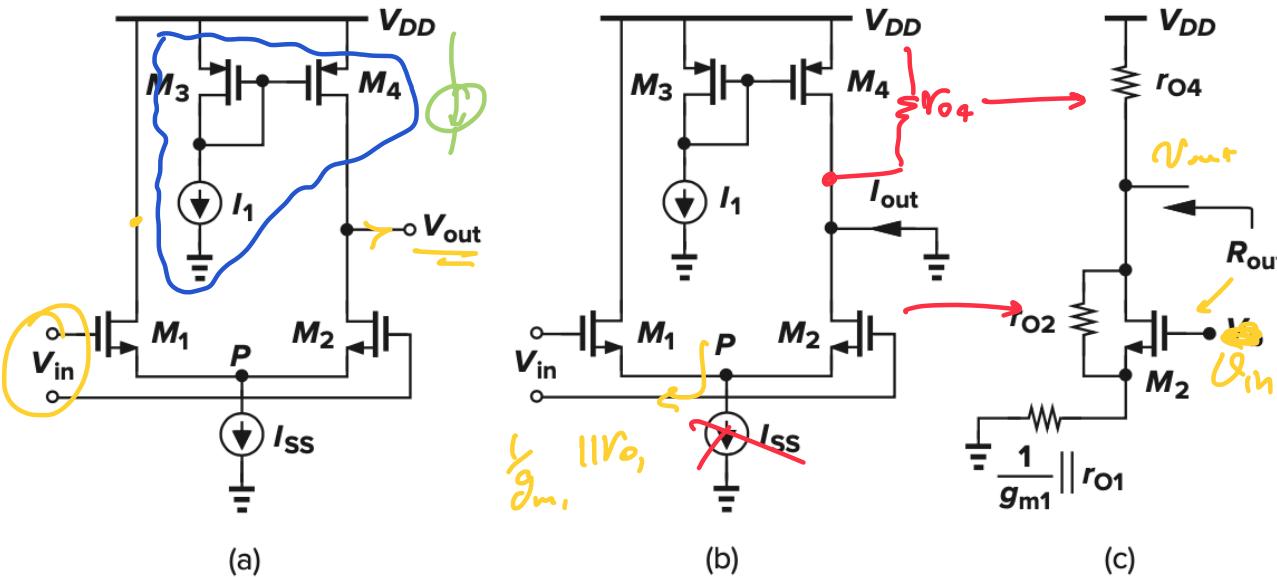
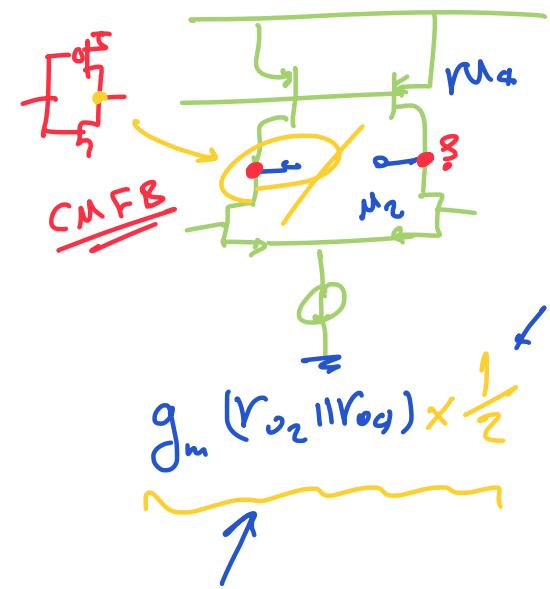
Active Current Mirrors



- A five-transistor “operational transconductance amplifier” (OTA).
- Note that the output is single-ended, hence the circuit is sometimes used to convert differential signals to a single-ended output.



Diff Pair + Active Mirror Load



- We may simply discard one output of a differential pair as shown above
- What is the small-signal gain?

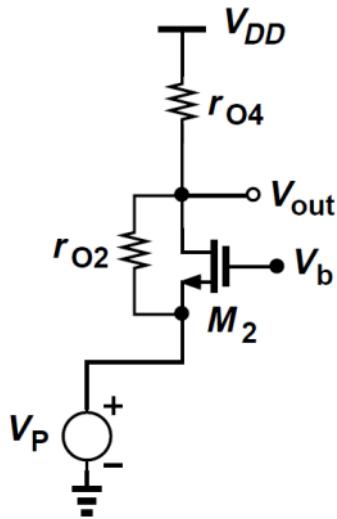
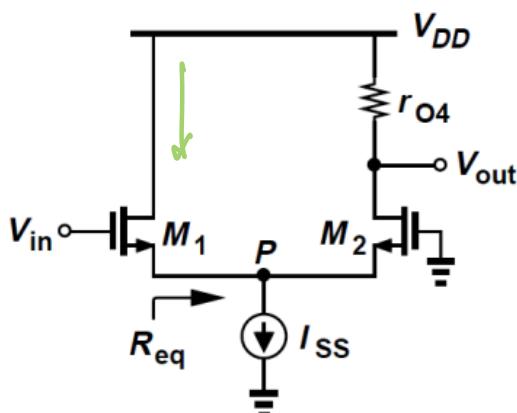
$$A_{v2} = \frac{V_{out}}{V_{in}}$$

no symmetry

$$|A_v| = \frac{g_{m1}}{2} [(2r_{o2}) \parallel r_{o4}]$$

Second Approach

- We calculate V_p / V_{in} and V_{out} / V_p



- Calculate V_{out} / V_p

$$\frac{V_{out}}{V_p} = \frac{(1 + g_m r_{O2}) r_{O4}}{r_{O2} + r_{O4}}$$

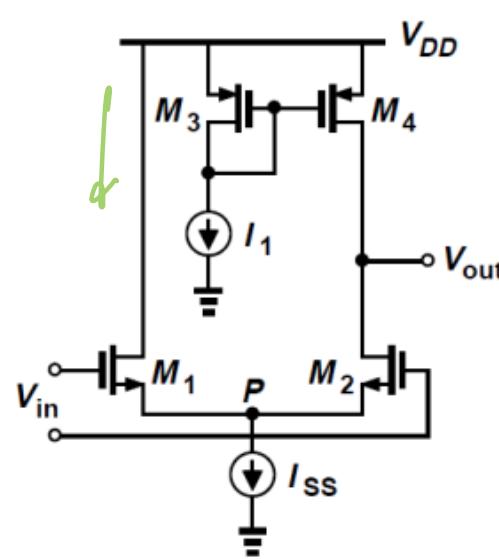
$$\frac{V_p}{V_{in}} = \frac{R_{eq} || r_{O1}}{R_{eq} || r_{O1} + \frac{1}{g_{m1}}} \quad R_{eq} = \frac{r_{O2} + r_{O4}}{1 + g_{m2} r_{O2}}$$

$$\frac{V_p}{V_{in}} = \frac{g_{m1} r_{O1} (r_{O2} + r_{O4})}{(1 + g_{m1} r_{O1})(r_{O2} + r_{O4}) + (1 + g_{m2} r_{O2}) r_{O1}}$$

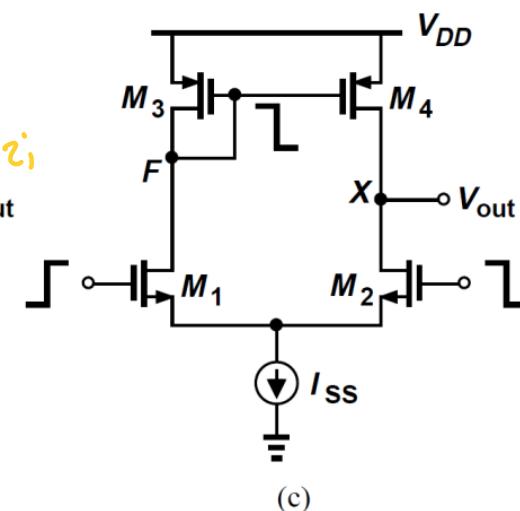
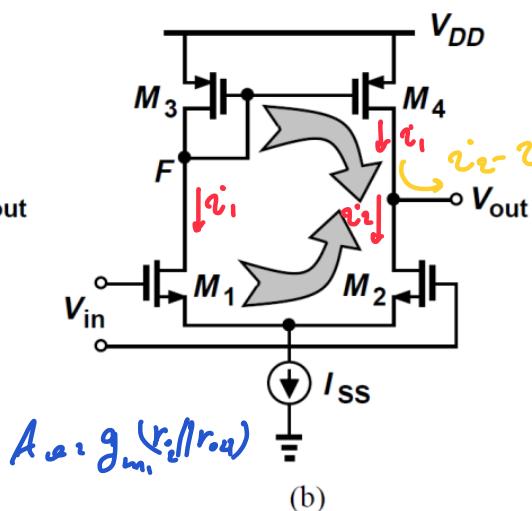
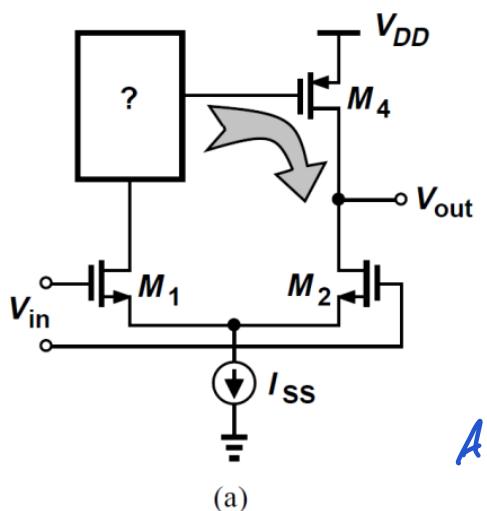
$$\begin{aligned} \frac{V_{out}}{V_{in}} &= \frac{g_{m2} r_{O2} r_{O4}}{2 r_{O2} + r_{O4}} \\ &= \frac{g_{m2}}{2} [(2 r_{O2}) || r_{O4}] \end{aligned}$$

R_{out}

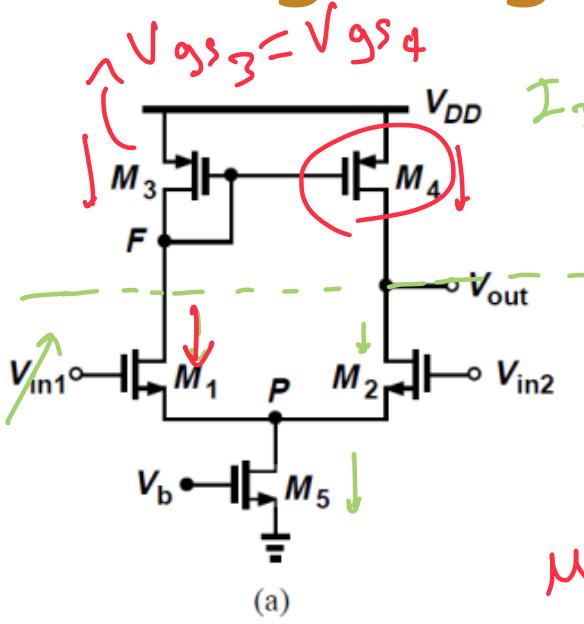
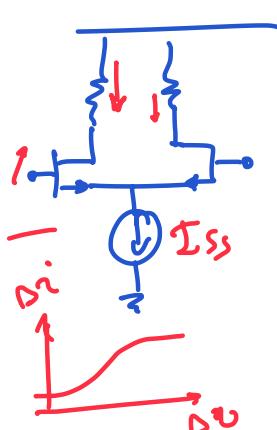
Differential Pair with Active Load



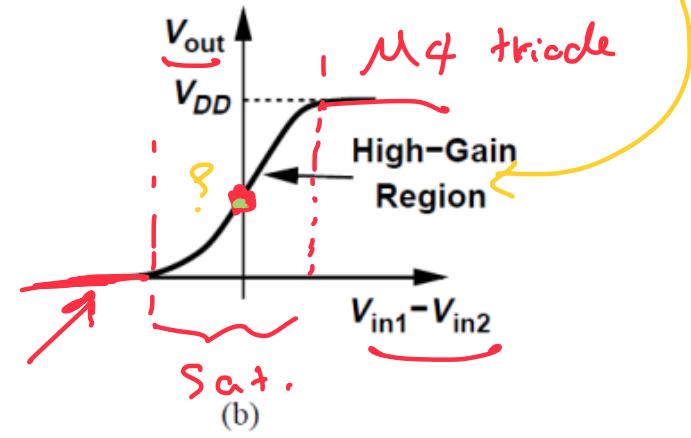
- The small-signal drain current of M_1 is “wasted.”
- It is desirable to utilize this current with proper polarity at the output.
- This can be accomplished by the five-transistor OTA, M_3 enhances the gain.
- The five-transistor OTA is also called a differential pair with active load.



Large-Signal Analysis

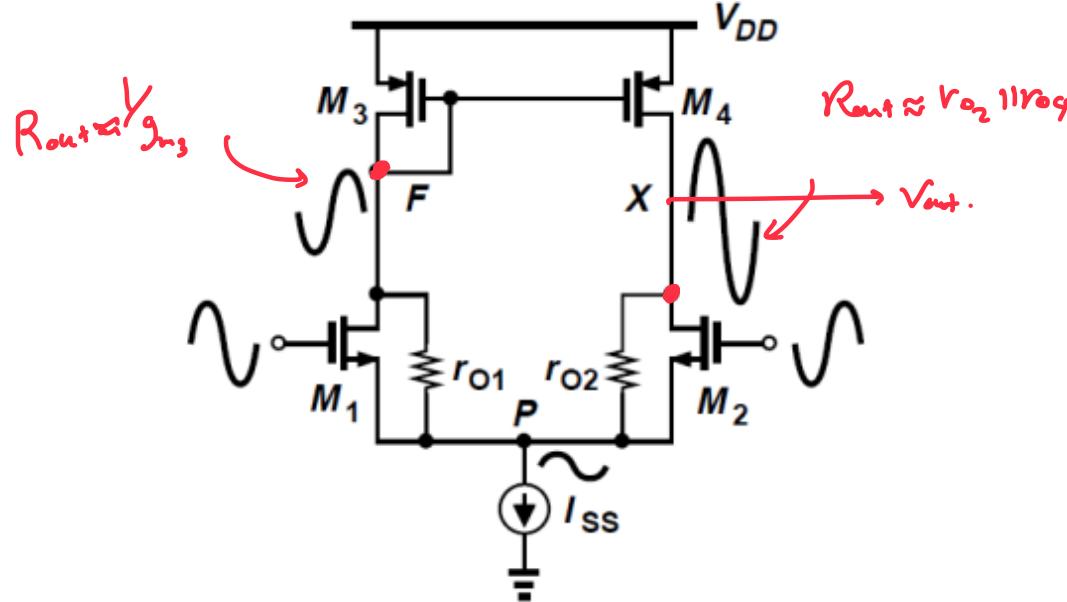


$$\sqrt{g_{s3}} = \sqrt{g_{s4}} \quad I_g = I_s \Rightarrow V_{DS3} = \sqrt{V_{DD}I_s} \Rightarrow V_{out+} = V_F = V_{DD} - V_{GS3}$$



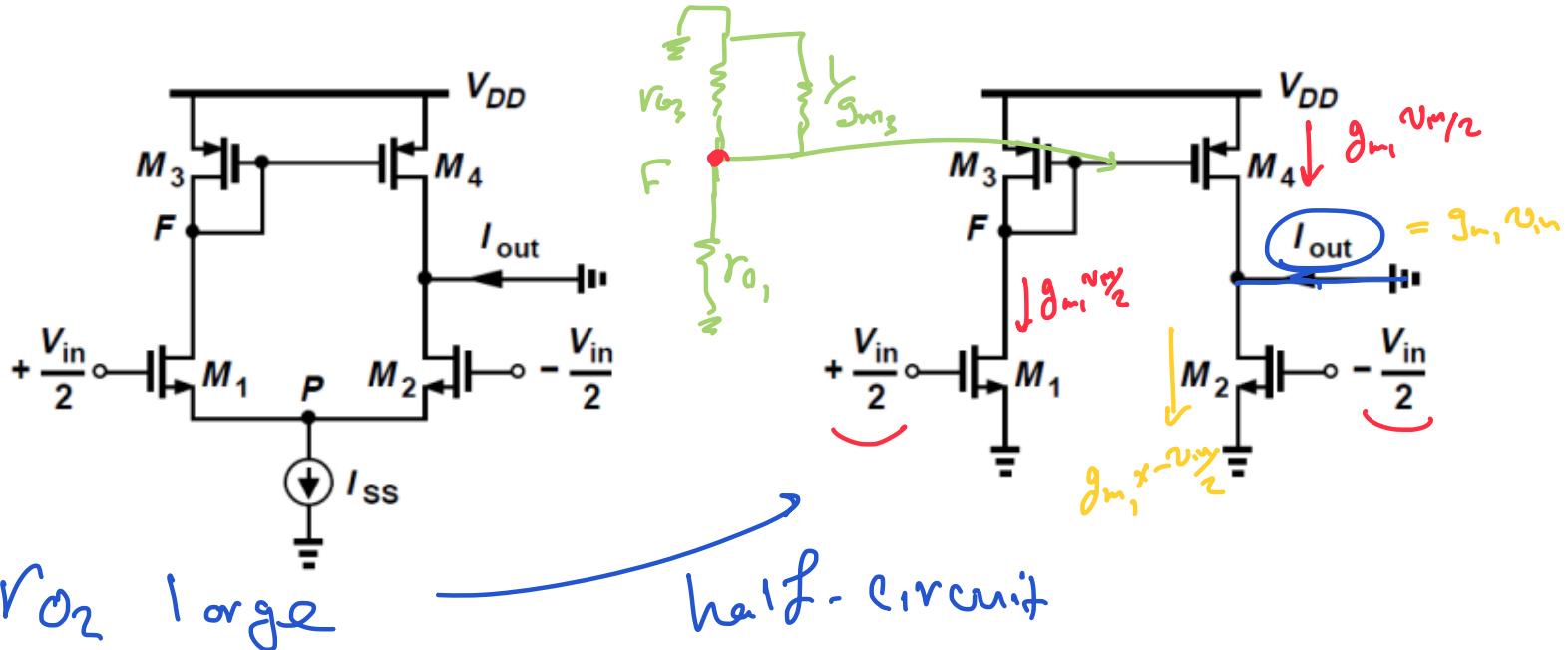
- If V_{in1} is much more negative than V_{in2} , $V_{out} = 0$.
- As V_{in1} approaches V_{in2} , the output voltage then depends on the difference between I_{d4} and I_{d2} . For a small difference between V_{in1} and V_{in2} , both M_2 and M_4 are saturated, providing a high gain.
- As V_{in1} becomes more positive than V_{in2} , allowing V_{out} to rise and eventually driving M_4 into the triode region .

Small-Signal Analysis



- With small differential inputs, the voltage swings at nodes F and X are vastly different.
- The effects of V_F and V_X at node P (through r_{o1} and r_{o2} , respectively) do not cancel each other and this node cannot be considered a virtual ground.

Approximate Analysis



- Node P can be approximated by a virtual ground.

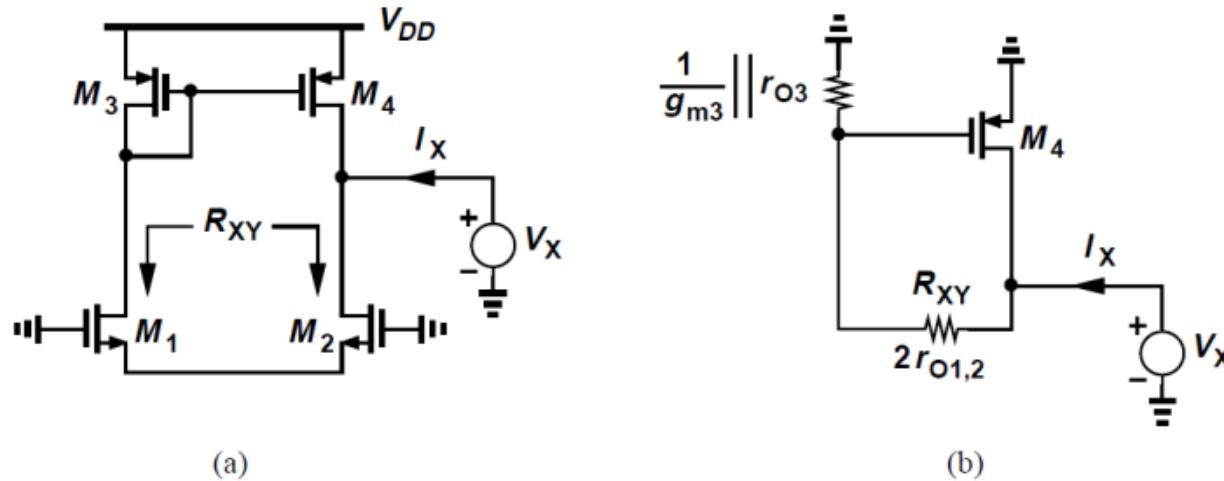
$$I_{D1} = |I_{D3}| = |I_{D4}| = g_{m1,2} V_{in} / 2$$

$$I_{D2} = -g_{m1,2} V_{in} / 2$$

$$I_{out} = -g_{m1,2} V_{in}$$

$$\underline{|G_m|} = g_{m1,2}$$

Calculation of R_{out}



- Any current flowing into M₁ must flow out of M₂, and the role of the two transistors can be represented by a resistor $R_{XY} = 2r_{O1,2}$
- The current drawn from V_X by R_{XY} is mirrored by M₃ onto M₄ with unity gain.

$$I_X = \frac{V_X}{2r_{O1,2} + \frac{1}{g_{m3}} \parallel r_{O3}} \left[1 + \left(\frac{1}{g_{m3}} \parallel r_{O3} \right) g_{m4} \right] + \frac{V_X}{r_{O4}}$$

- For $2r_{O1,2} \gg (1/g_{m3}) \parallel r_{O3}$

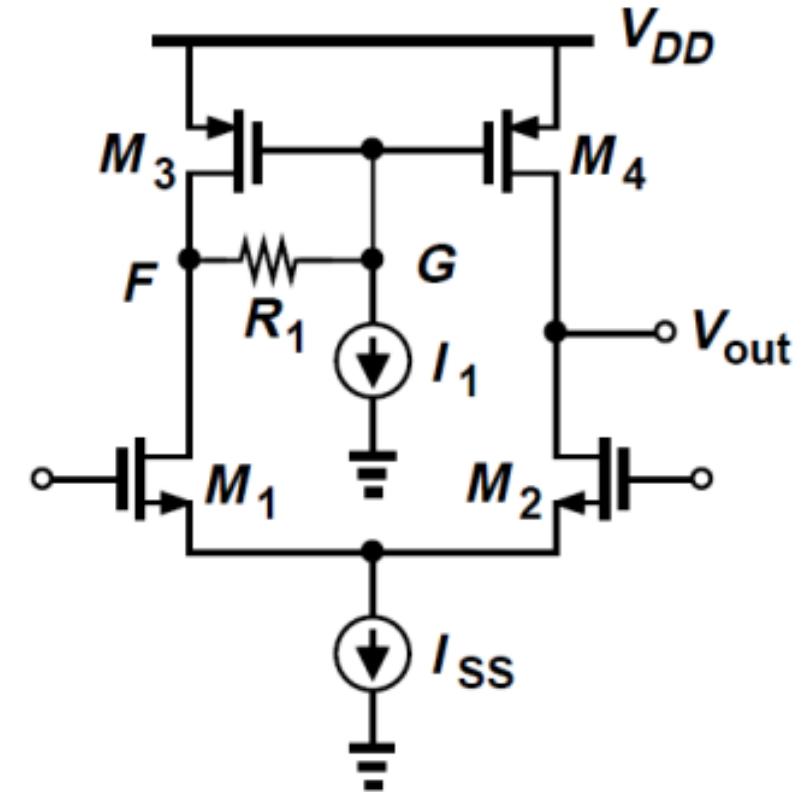
$$R_{out} \approx r_{O2} \parallel r_{O4}$$

Headroom Issues

- The five-transistor OTA does not easily lend itself to low-voltage operation.
- The value of I_1 must be much less than $I_{ss}/2$ and
- Insert a resistor in series with the gate and draw a constant current from it.

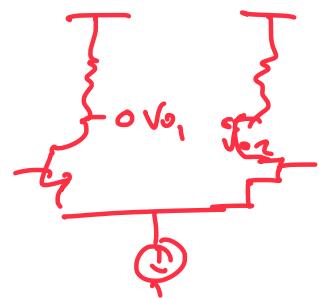
$$R_1 I_1 \leq V_{TH3}$$

- Now V_F can be larger than V_G



$$V_{out} < V_{DD} - V_{GS,3,4}$$

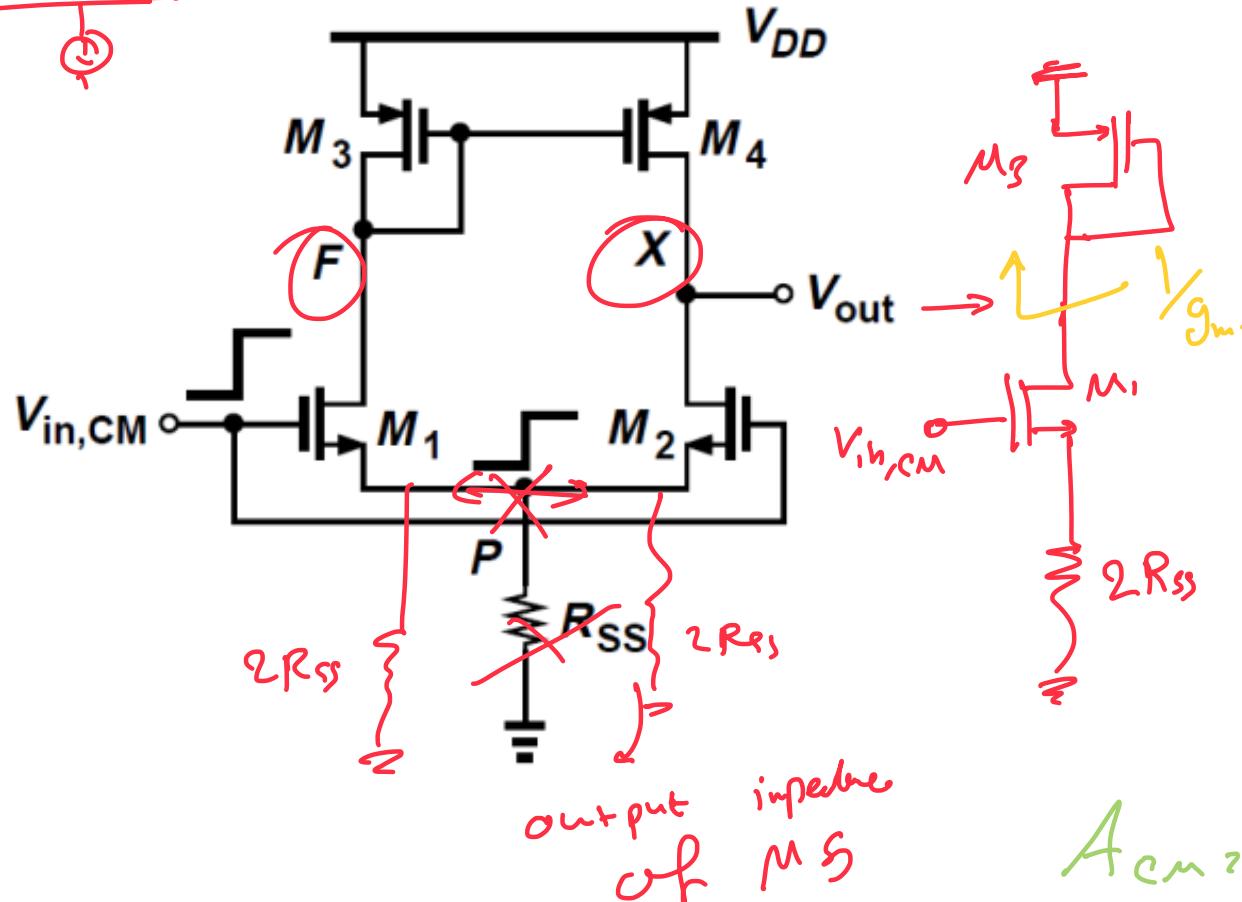
Common-Mode Properties



$$V_{o_1} - V_{o_2} = V_{d,out}$$

$$\frac{V_{d,out}}{V_{in,CM}} = 0$$

$$\frac{V_{o_1}}{V_{in,CM}} = \frac{-g_m R_D}{1 + g_m \times 2R_S}$$



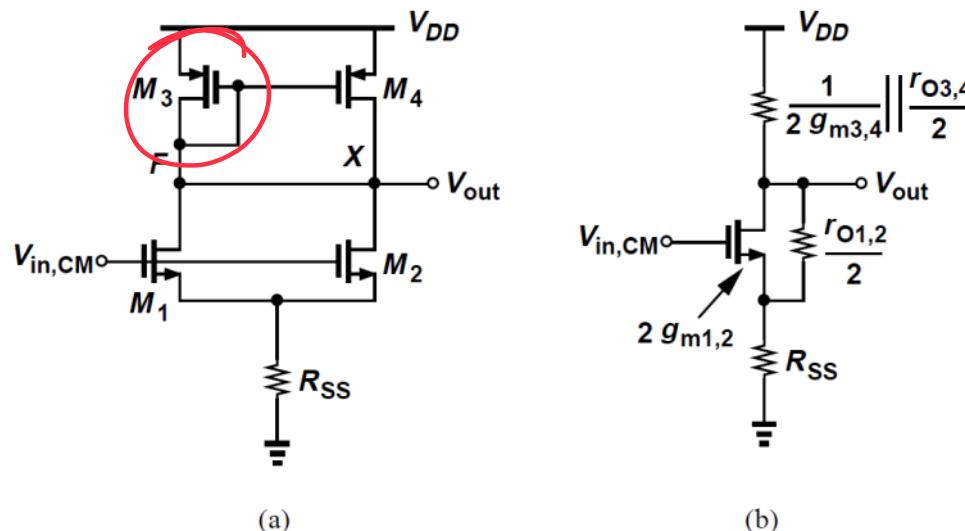
$$A_{CM} = \frac{\Delta V_{out}}{\Delta V_{in,CM}}$$

$$A_{CM} \approx \frac{\frac{1}{2g_{m3,4}} \parallel \frac{r_{O3,4}}{2}}{\frac{1}{2g_{m1,2}} + R_{SS}}$$

$$= \frac{-1}{1 + 2g_{m1,2}R_{SS}} \frac{g_{m1,2}}{g_{m3,4}}$$

$$A_{CM} = \frac{-g_{m1} \times \frac{1}{g_{m3}}}{1 + g_{m1} \times 2R_{SS}}$$

CMRR

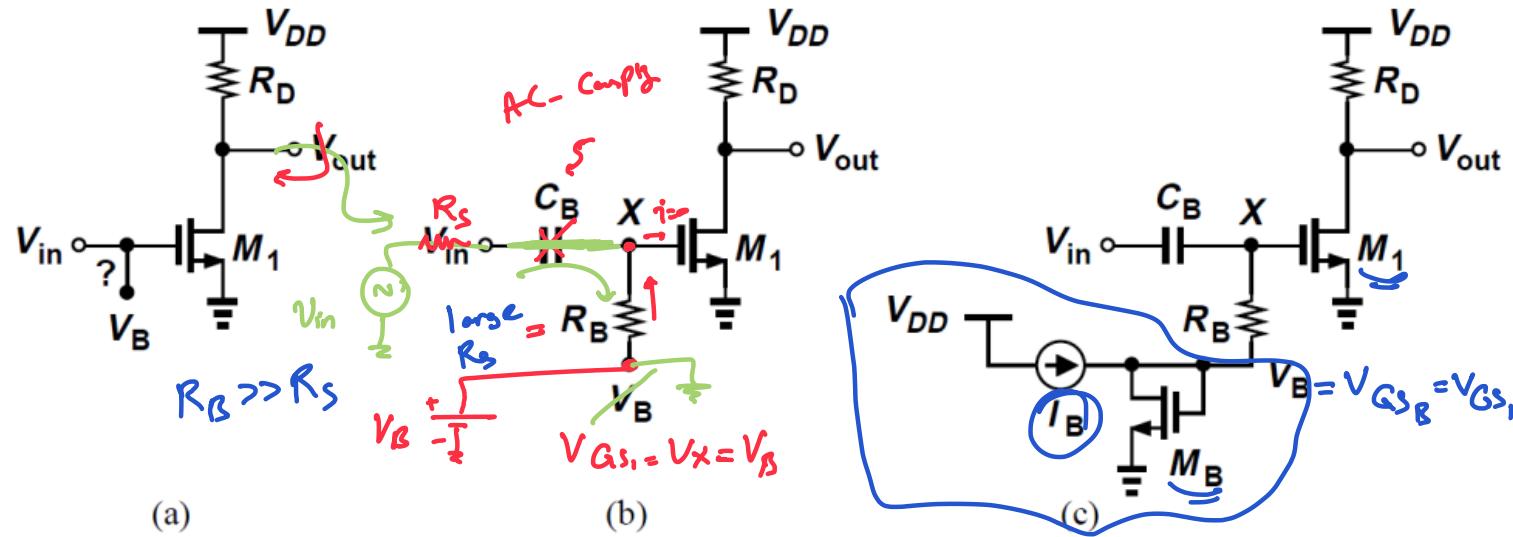


(a) Simplified circuit of Fig. 5.37, (b) equivalent circuit of (a).

$$\begin{aligned} \text{CMRR} &= \left| \frac{A_{DM}}{A_{CM}} \right| \\ &= g_{m1,2}(r_{O1,2} \parallel r_{O3,4}) \frac{g_{m3,4}(1 + 2g_{m1,2}R_{SS})}{g_{m1,2}} \\ &= (1 + 2g_{m1,2}R_{SS}) \underline{\underline{g_{m3,4}(r_{O1,2} \parallel r_{O3,4})}} \end{aligned}$$

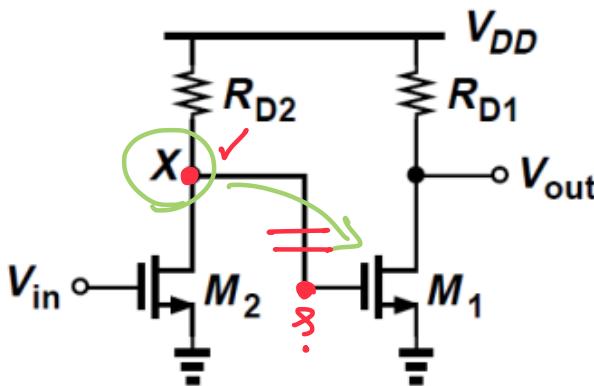
- Even with perfect symmetry, the output signal is corrupted by input CM variations.

Biasing Techniques



- Simple CS Stage
 - How do we ensure that V_B does not “fight” V_{in} ?
- Couple V_{in} capacitively and establish a high impedance for V_B .
- Node X in Fig (b) must have a dc path to a voltage.
- The bias voltage must be generated by a diode-connected device
- Typically select I_B about one-tenth to one-fifth of I_{D1} so as to minimize the power.

Direct Coupling



Direct coupling between two stages.

- Possible to remove the input coupling capacitor and provide the bias voltage from the preceding stage?
- The bias conditions of M_1 are influenced by those of M_2 .
- The PVT variations are amplified.
- One can employ direct coupling between two stages if each has a low gain.