EE 437/538B: Integrated Systems Capstone/Design of Analog Integrated Circuits and Systems

Lecture 2: Channel Pulse Model

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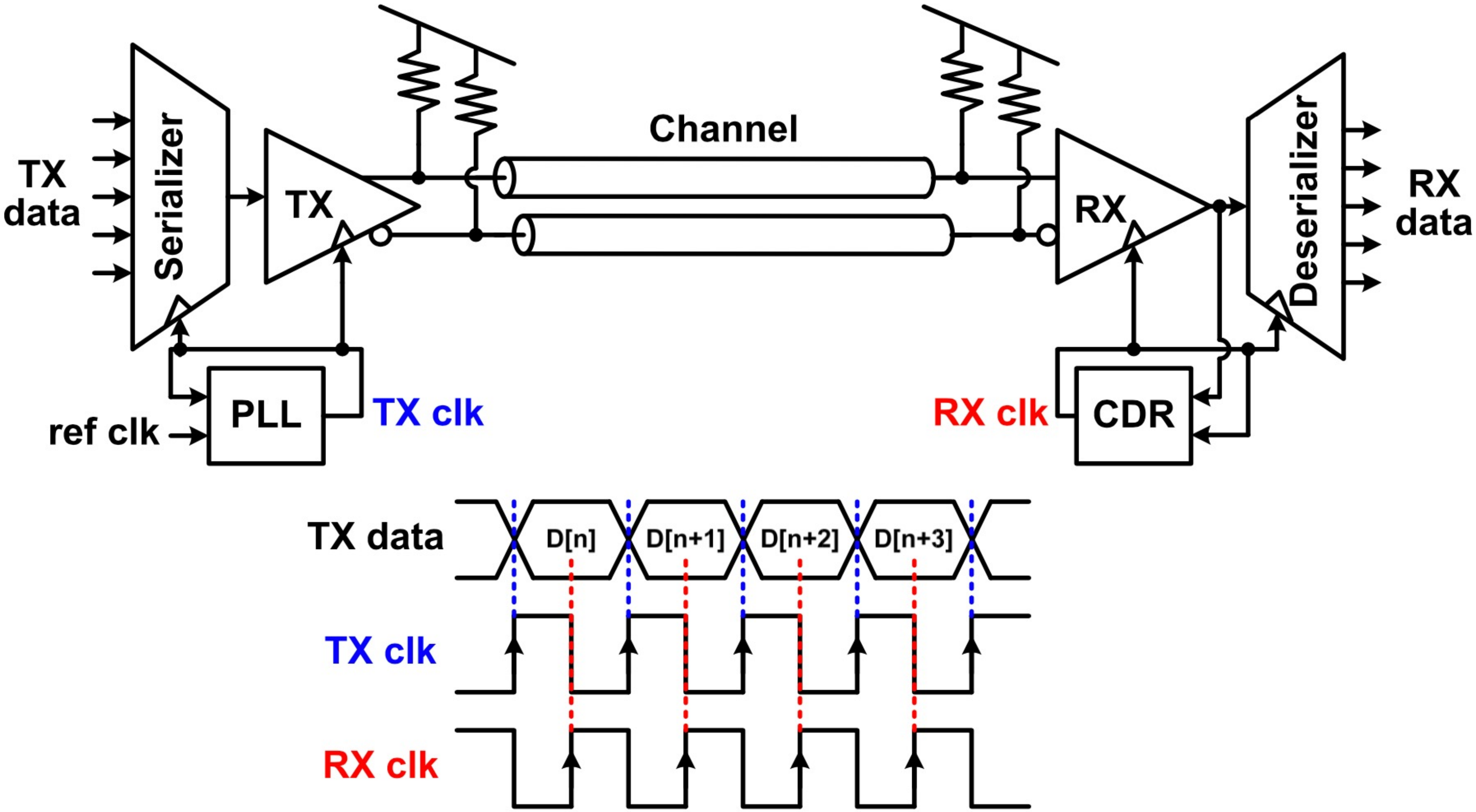
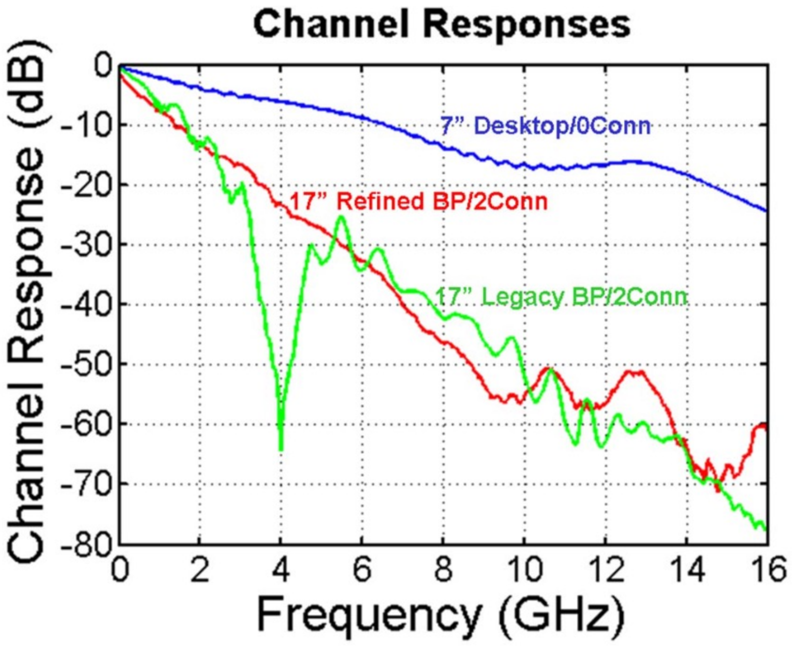
ELECTRICAL & COMPUTER ENGINEERING

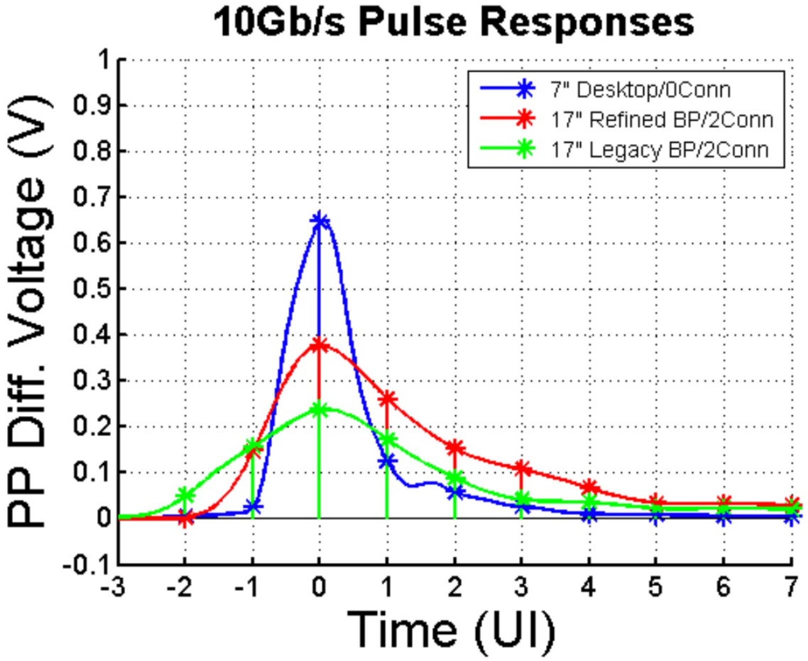
High-Speed Electrical Link System

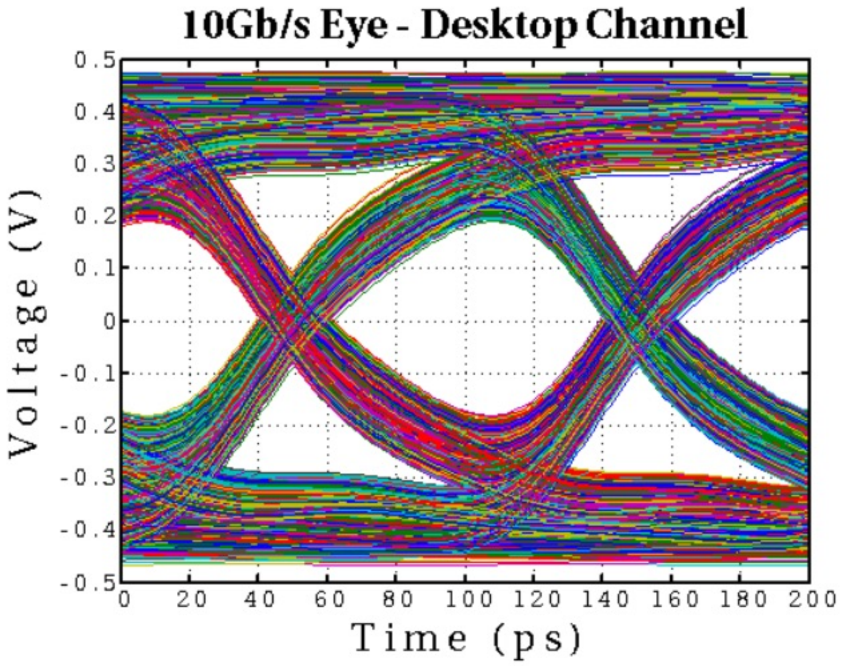


[Sam Palermo]

Channel Example







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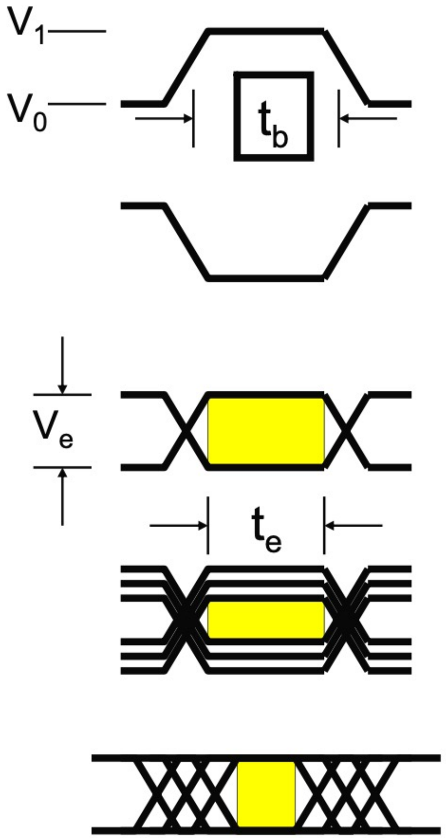
Eye Diagrams



This is a “1”

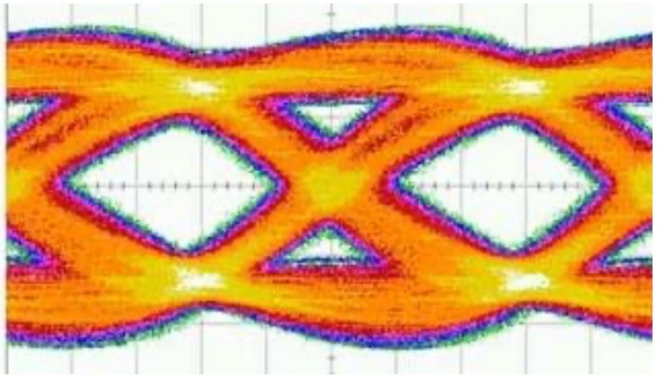
This is a “0”

Eye Opening - space between 1 and 0

With voltage noise

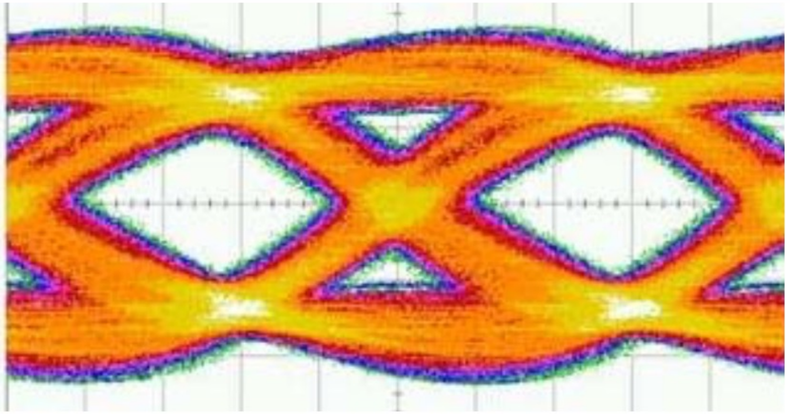
With timing noise

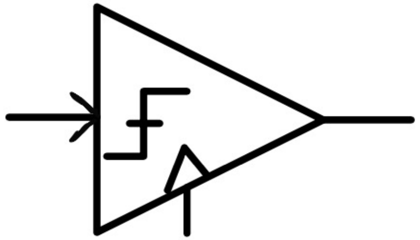
With Both!



[Elad Alon]

BER





- **BER = Bit Error Rate**
  - **Average # of wrong received bits / total transmitted bits**
- **Simplified example: (voltage only)**

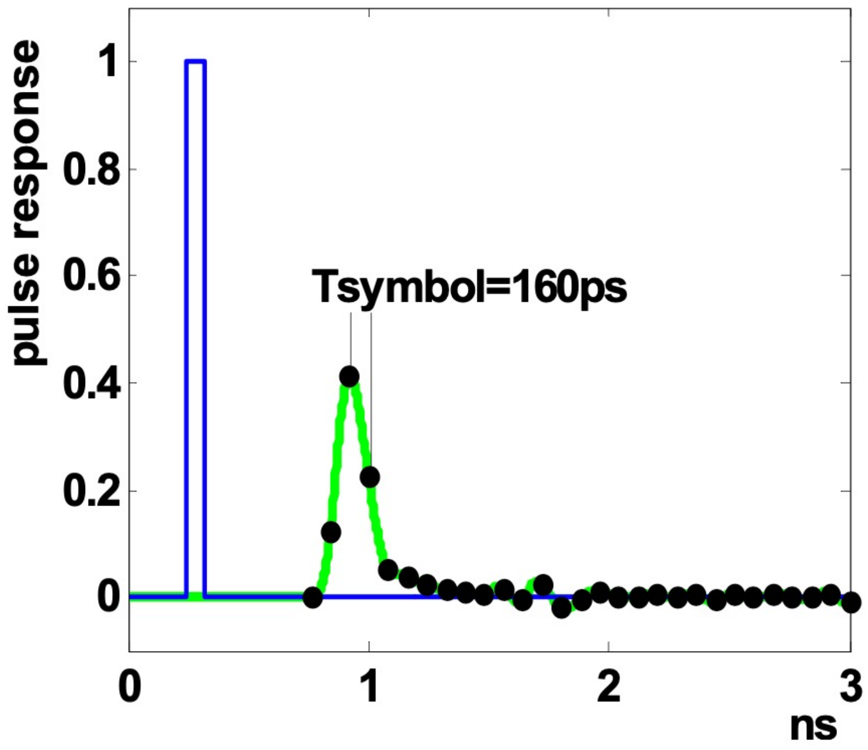
$$BER = \frac{1}{2} erfc\left(\frac{V_{in,ampl} - V_{off}}{\sqrt{2}\sigma_{noise}}\right)$$

- **BER = 10^{-12}: $(V_{in,ampl} - V_{off}) = 7\sigma_n$**
- **BER = 10^{-20}: $(V_{in,ampl} - V_{off}) = 9.25\sigma_n$**

[Elad Alon]

Inter-Symbol Interference

- Channel is band-limited
  - I.e., dispersive (low pass)
  - Short TX pulses get spread out
  - Low latency
- Also get reflections
  - Z mismatches, connectors, etc.
  - Longer latency



EE290C Lecture 5 3

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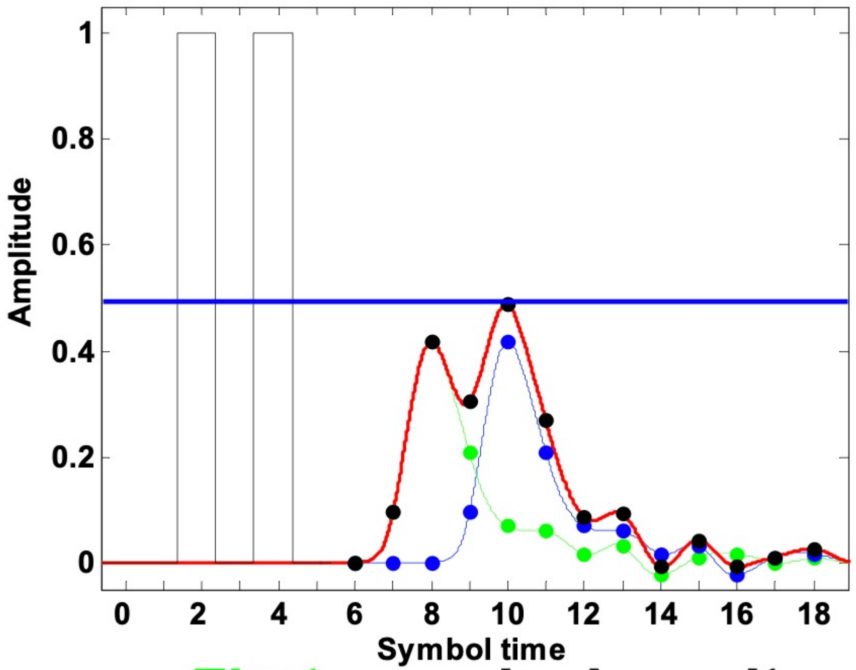
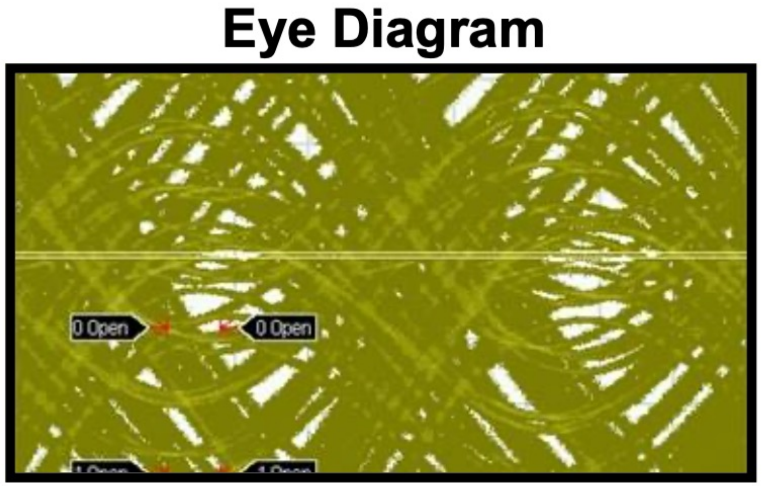
Why ISI Matters





- First sample doesn't even reach RX threshold
  - Suffers ISI from all previous zero bits
- Middle sample hardly different from first
  - 0.2 trailing ISI (from previous symbol) and 0.1 leading ISI (from next symbol)

[Elad Alon]

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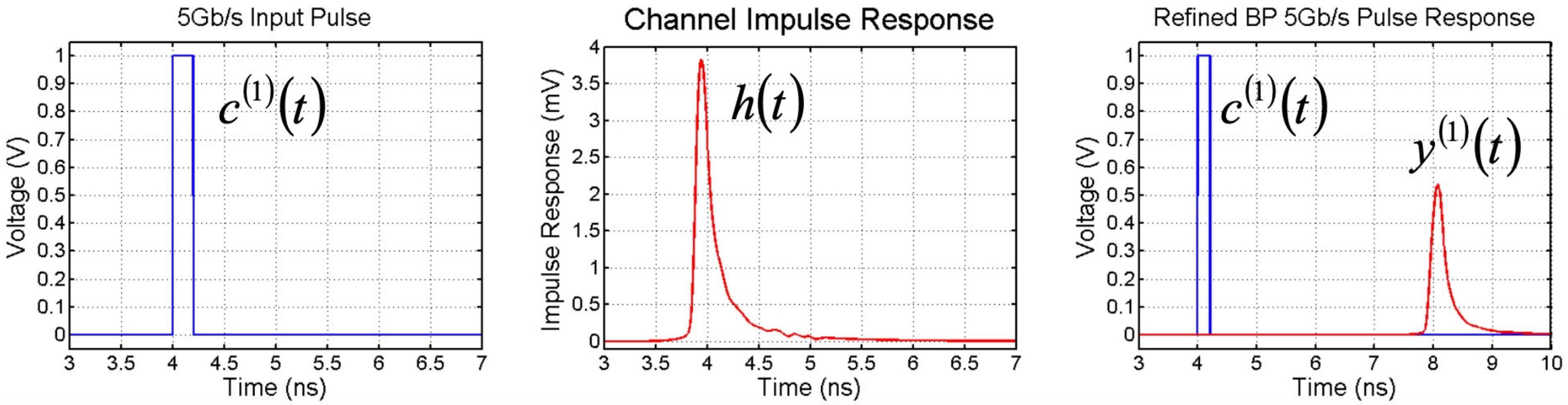
Inter-Symbol Interference (ISI)

- Previous bits residual state can distort the current bit, resulting in inter-symbol interference (ISI)
- ISI is caused by
  - Reflections, Channel resonances, Channel loss (dispersion)
- Pulse Response

$$y^{(1)}(t) = c^{(1)}(t) * h(t)$$

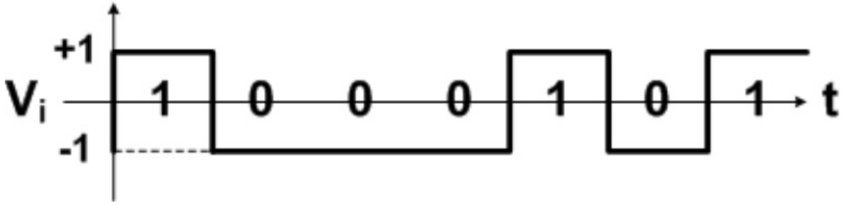


[Sam Palermo]

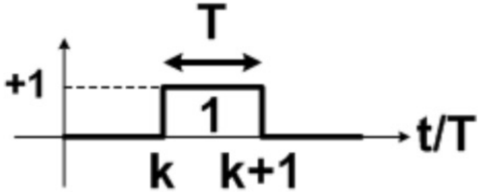
NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated "1"s and "0"s

Data = "1000101"

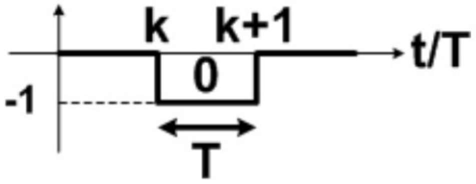


"1" Symbol



$$c_k^{(1)}(t) \equiv u(t - kT) - u(t - (k+1)T)$$

"0" Symbol



$$c_k^{(0)}(t) = -c_k^{(1)}(t)$$

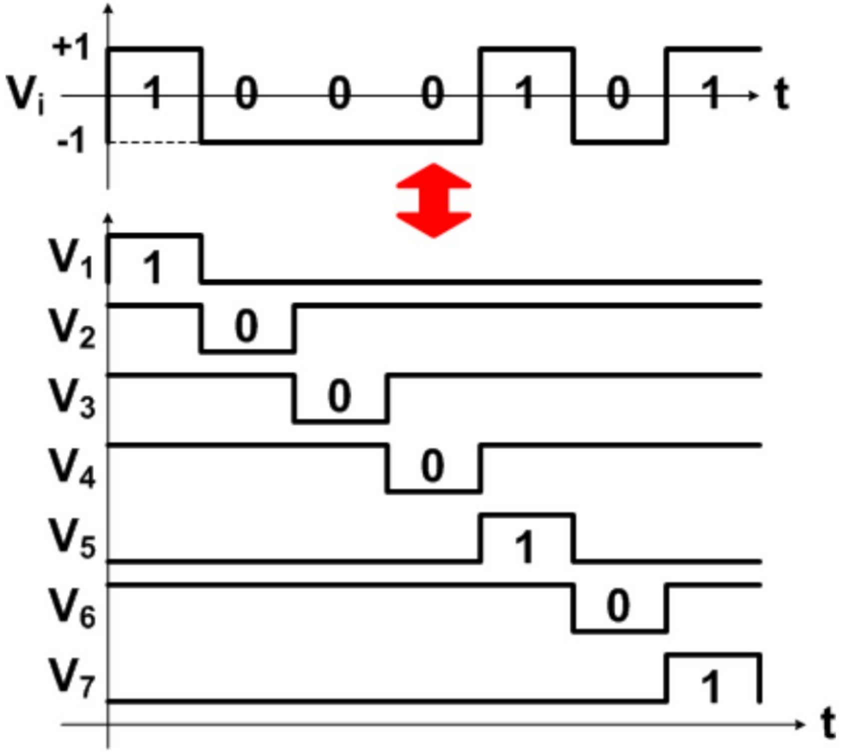
$$\text{where} \quad u(t) \equiv \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

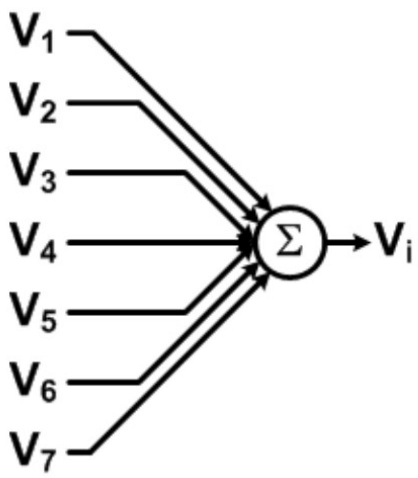
[Song]

[Sam Palermo]

NRZ Data Modeling

- An NRZ data stream can be modeled as a superposition of isolated "1"s and "0"s





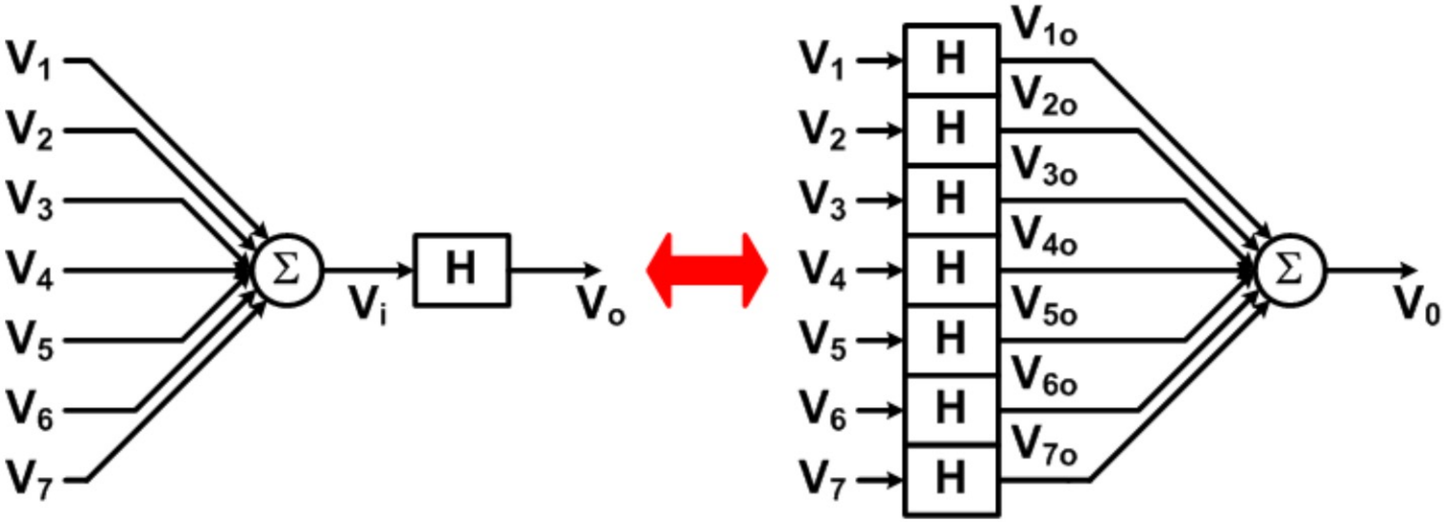
$$V_i(t) = \sum_{k=-\infty}^{\infty} c_k^{(d_k)}(t)$$

[Song]

6 [Sam Palermo]

Channel Response to NRZ Data

- Channel response to NRZ data stream is equivalent to superposition of isolated pulse responses



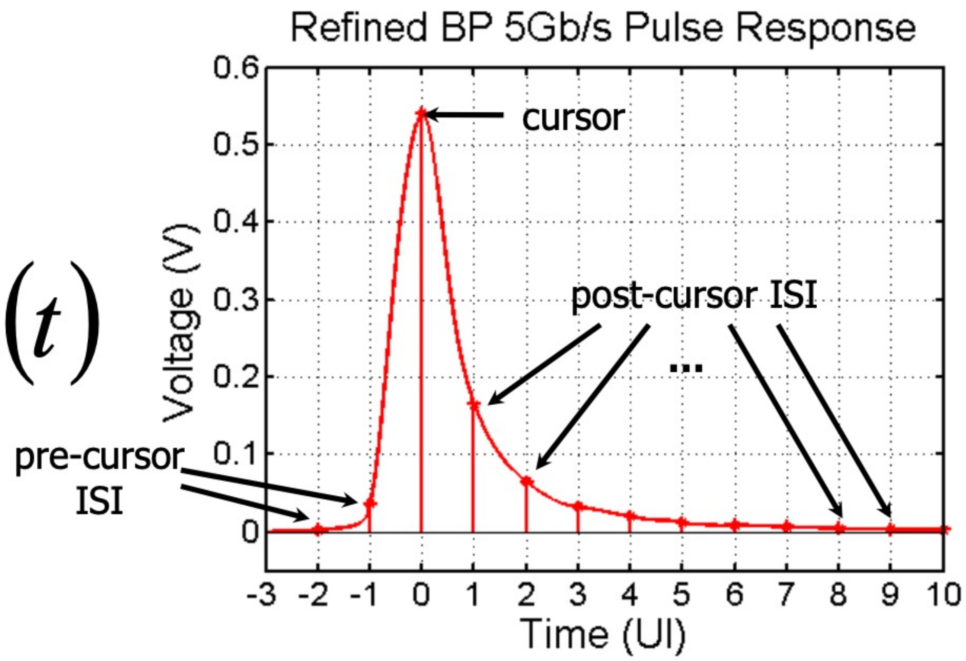
$$V_o(t) = H(V_i(t)) = \sum_{k=-\infty}^{\infty} H\left(c_k^{(d_k)}(t)\right) = \sum_{k=-\infty}^{\infty} y^{(d_k)}(t - kT)$$

[Song]

[Sam Palermo]

Channel Pulse Response

$$y^{(d_k)}(t) = c^{(d_k)}(t) * h(t)$$



$y^{(1)}(t)$ sampled relative to pulse peak:

[… 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 …]

k =[… -2 1 0 1 2 3 4 5 6 …]

By Linearity: $y^{(0)}(t) = -1 * y^{(1)}(t)$

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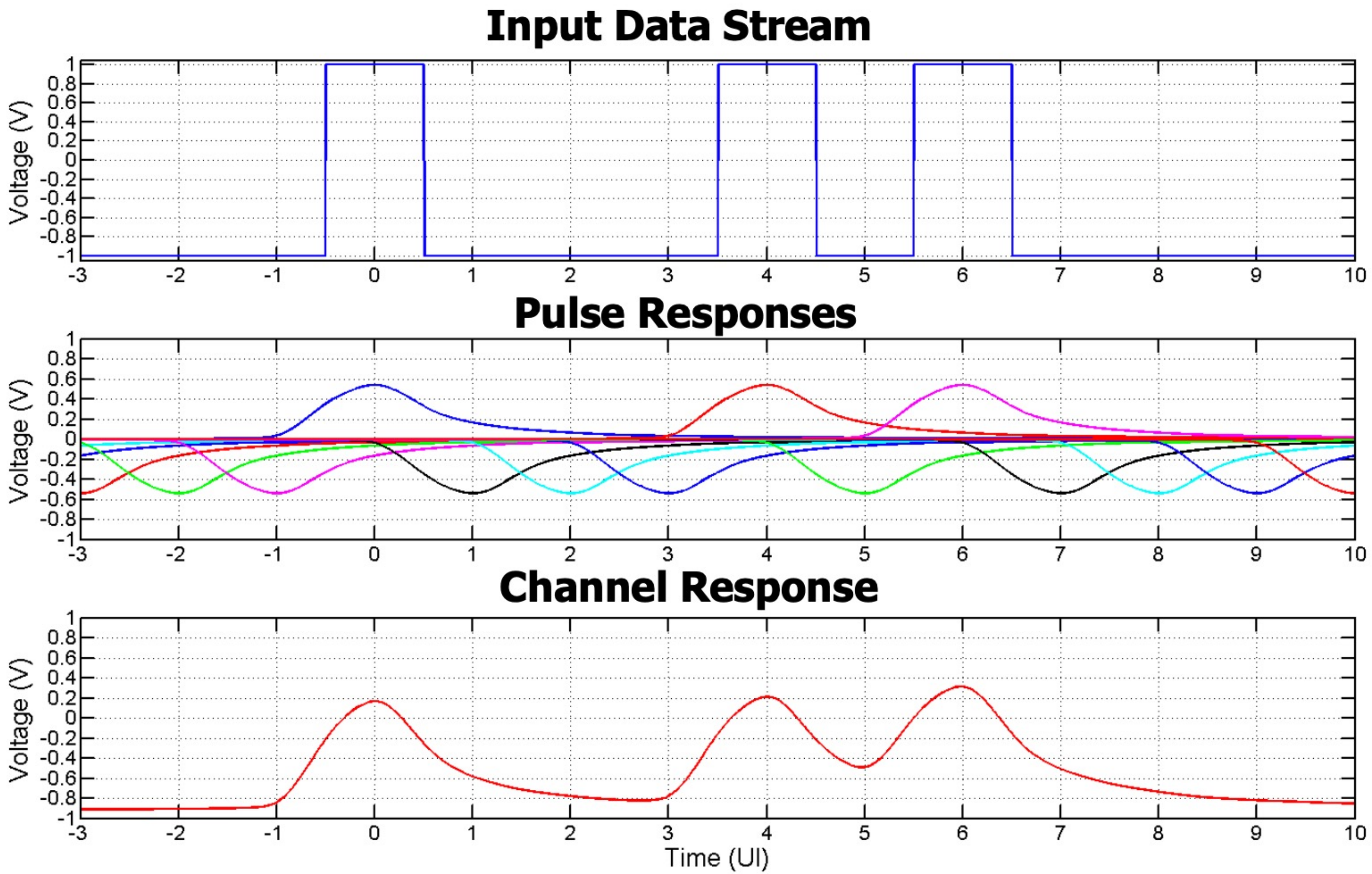
Channel Data Stream Response



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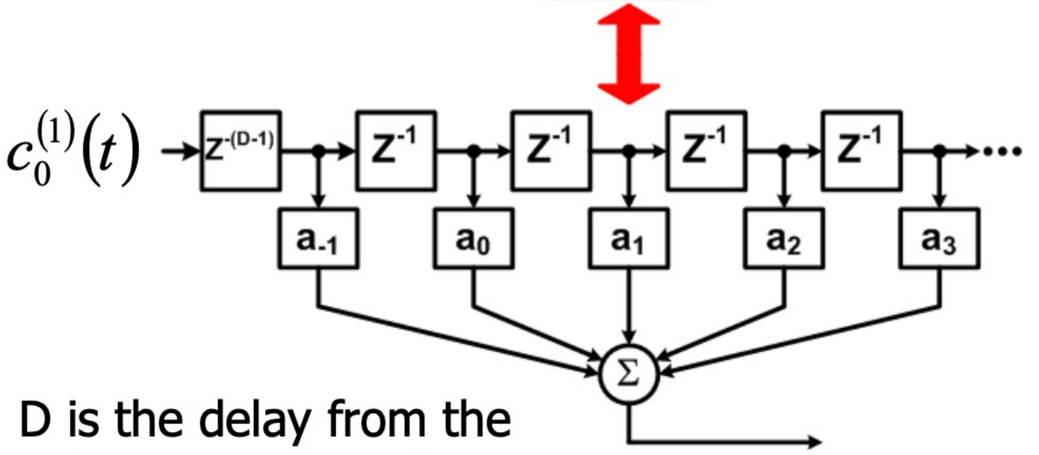
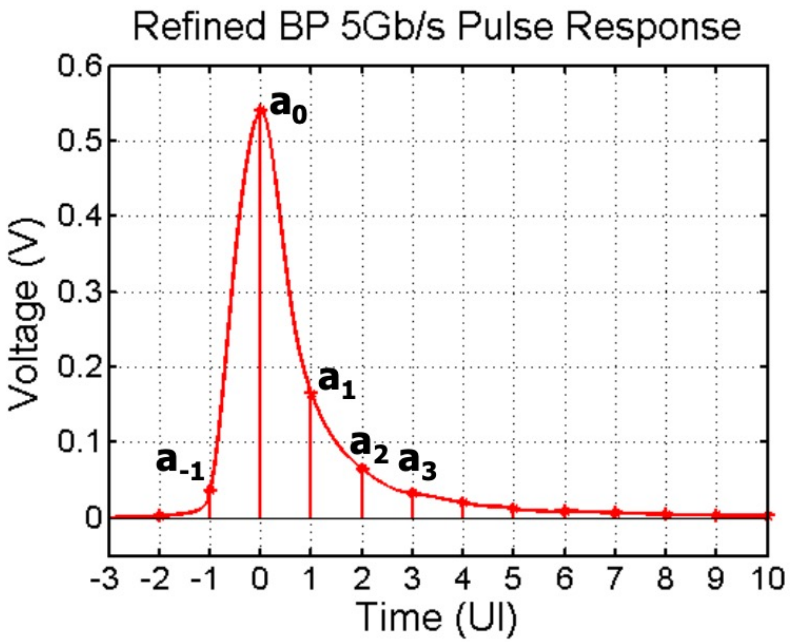
Channel “FIR” Model

$$c_0^{(1)}(t) \rightarrow \boxed{H} \rightarrow H\left(c_0^{(1)}(t)\right) = y_0^{(1)}(t)$$



D is the delay from the channel input to the output pulse peak

$$H\left(c_0^{(1)}(t)\right) = y_0^{(1)}(t)$$



$y^{(1)}(t)$ sampled relative to pulse peak:

[… 0.003 0.036 0.540 0.165 0.065 0.033 0.020 0.012 0.009 …]

$a = [\ \ldots\ a_{-2}\quad a_{-1}\quad a_0\quad a_1\quad a_2\quad a_3\quad a_4\quad a_5\quad a_6\quad \ldots]$

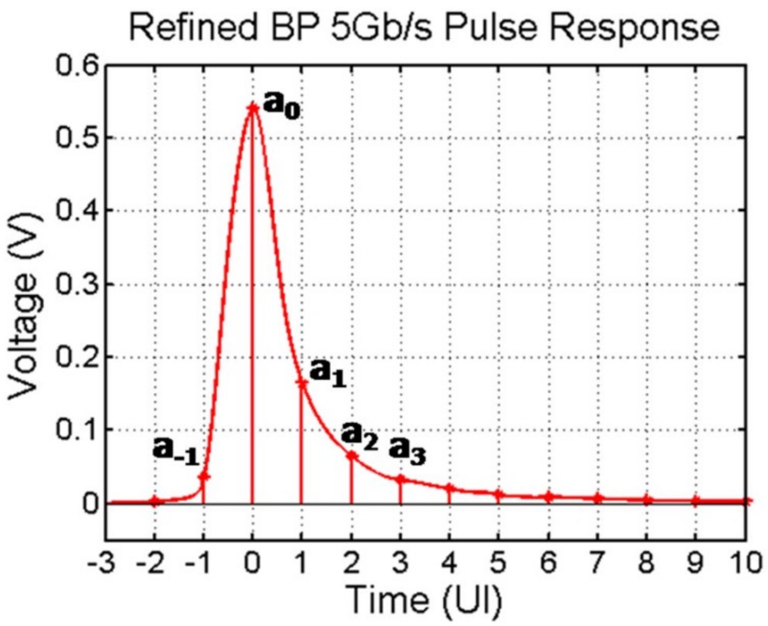
[Sam Palermo]

Peak Distortion Analysis

- Can estimate worst-case eye height and data pattern from pulse response
- Worst-case "1" is summation of a "1" pulse with all negative non k=0 pulse responses

$$\boxed{s_1(t) = y_0^{(1)}(t) + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(d_k)}(t-kT)\Big|_{y(t-kT)<0}}$$

- Worst-case "0" is summation of a "0" pulse with all positive non k=0 pulse responses



$$\boxed{s_0(t) = y_0^{(0)}(t) + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(d_k)}(t-kT)\Big|_{y(t-kT)>0}}$$

[Sam Palermo]

Peak Distortion Analysis

- Worst-case eye height is $s_1(t)$-$s_0(t)$

$$s(t) = s_1(t) - s_0(t) = \left(y_0^{(1)}(t) - y_0^{(0)}(t)\right) + \left(\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(d_k)}(t-kT)\Big|_{y(t-kT)<0} - \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(d_k)}(t-kT)\Big|_{y(t-kT)>0} \right)$$

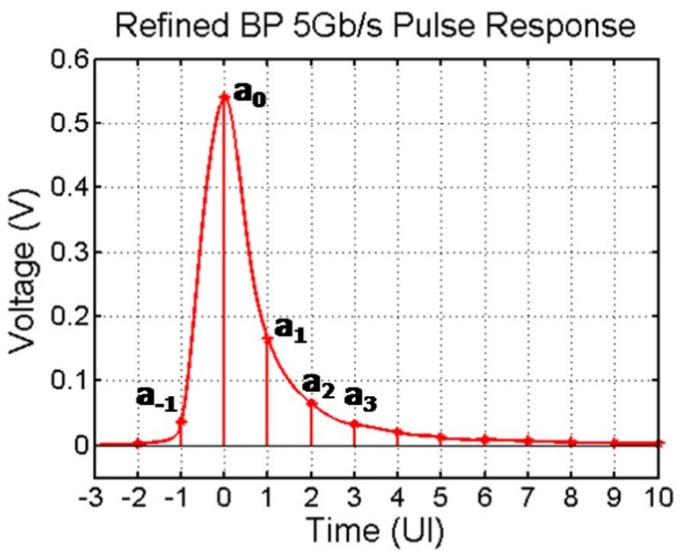
$$\text{Because } y_0^{(0)}(t) = -1\left(y_0^{(1)}(t)\right)$$

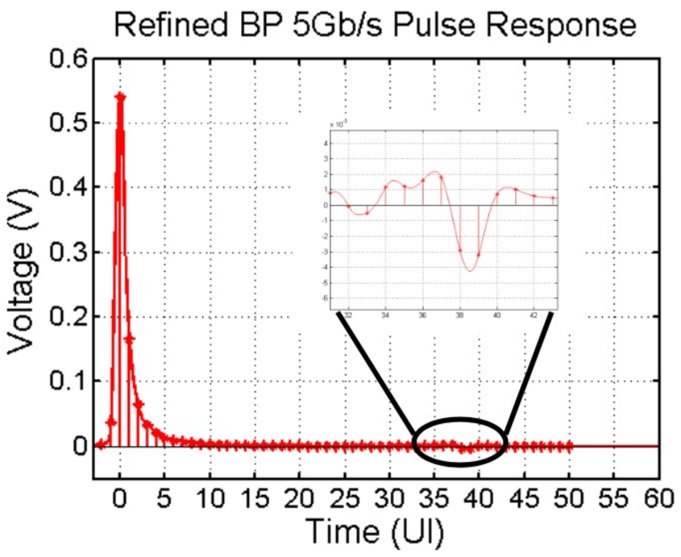
$$s(t) = 2\left(\underbrace{y_0^{(1)}(t) + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)<0}}_{\text{"1" pulse worst-case "1" edge}} - \underbrace{\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)>0}}_{\text{"1" pulse worst-case "0" edge}} \right)$$

- If symmetric "1" and "0" pulses (linearity), then only positive pulse response is needed

[Sam Palermo]

Peak Distortion Analysis Example 1



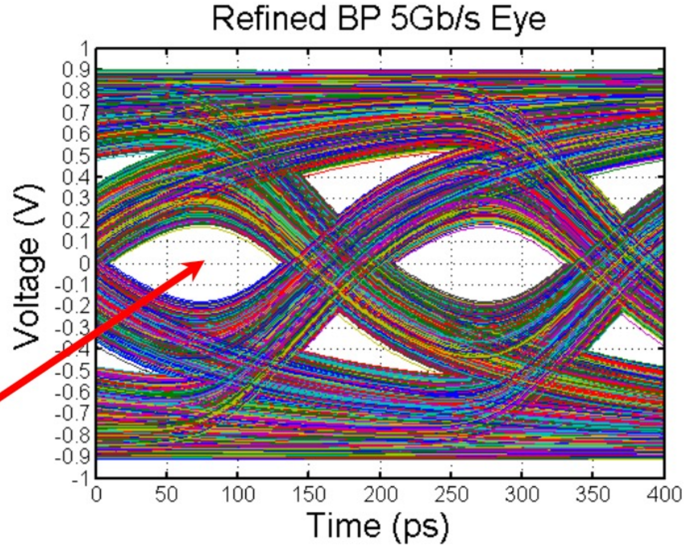


$$y_0^{(1)}(t) = 0.540$$

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)<0} = -0.007$$

$$\sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} y^{(1)}(t-kT)\Big|_{y(t-kT)>0} = 0.389$$

$$s(t) = 2(0.540 - 0.007 - 0.389) = 0.288$$



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Worst-Case Bit Pattern

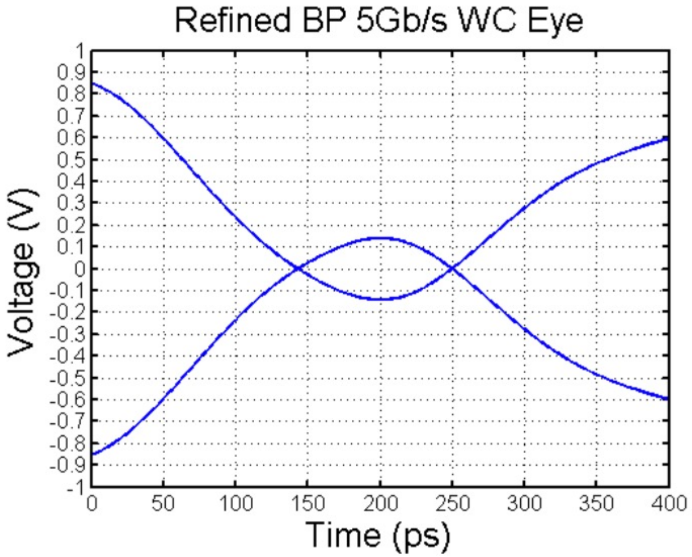
- Pulse response can be used to find the worst-case bit pattern

$$\text{Pulse} \quad \mathrm{a} = [\dots\ a_{-2} \quad a_{-1} \quad a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad \dots]$$

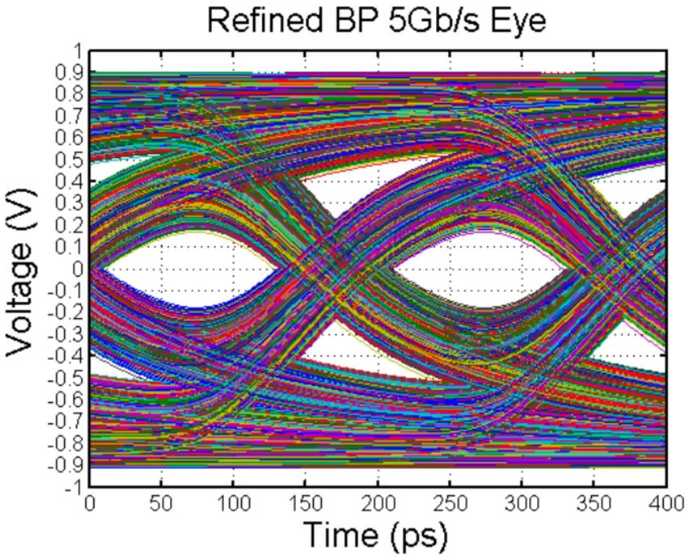
- Flip pulse matrix about cursor a_0 and the bits are the inverted sign of the pulse ISI

$$[\dots\ -sign(a_6) \quad -sign(a_5) \quad -sign(a_4) \quad -sign(a_3) \quad -sign(a_2) \quad -sign(a_1) \quad 1 \quad -sign(a_{-1}) \quad -sign(a_{-2}) \quad \dots]$$

Worst-Case Bit Pattern Eye



10kbits Eye



[Sam Palermo]

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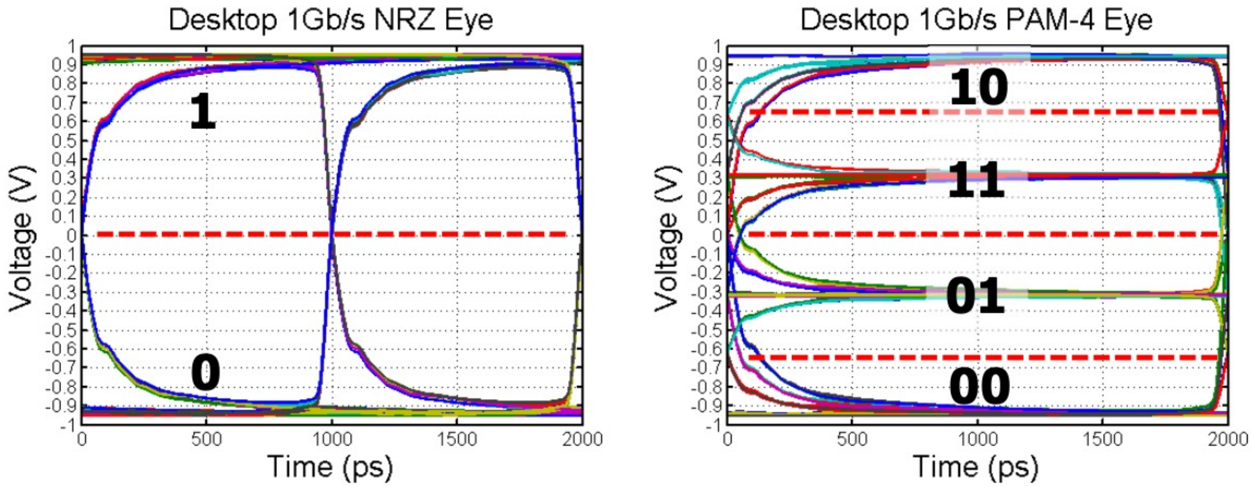
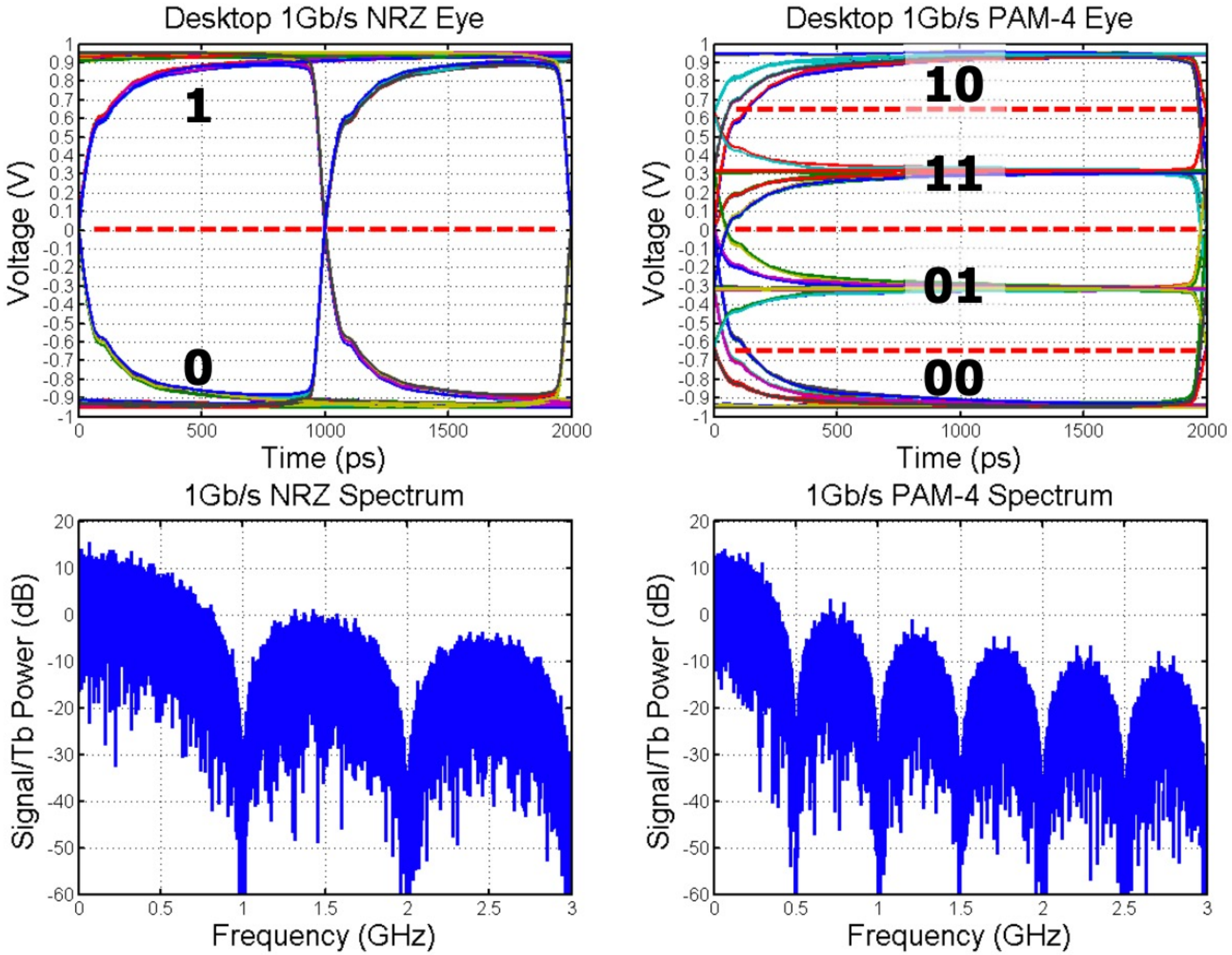
PAM-2 (NRZ) vs PAM-4 Modulation

- Binary, NRZ, PAM-2
  - Simplest, most common modulation format
- PAM-4
  - Transmit 2 bits/symbol
  - Less channel equalization and circuits run ½ speed



[Sam Palermo]

Modulation Frequency Spectrum



Majority of signal power in 1GHz bandwidth

Majority of signal power in 0.5GHz bandwidth

[Sam Palermo]

Nyquist Frequency

- Nyquist bandwidth constraint:
  - The theoretical minimum required system bandwidth to detect R_S (symbols/s) without ISI is $R_S/2$ (Hz)
  - Thus, a system with bandwidth $W=1/2T=R_S/2$ (Hz) can support a maximum transmission rate of $2W=1/T=R_S$ (symbols/s) without ISI

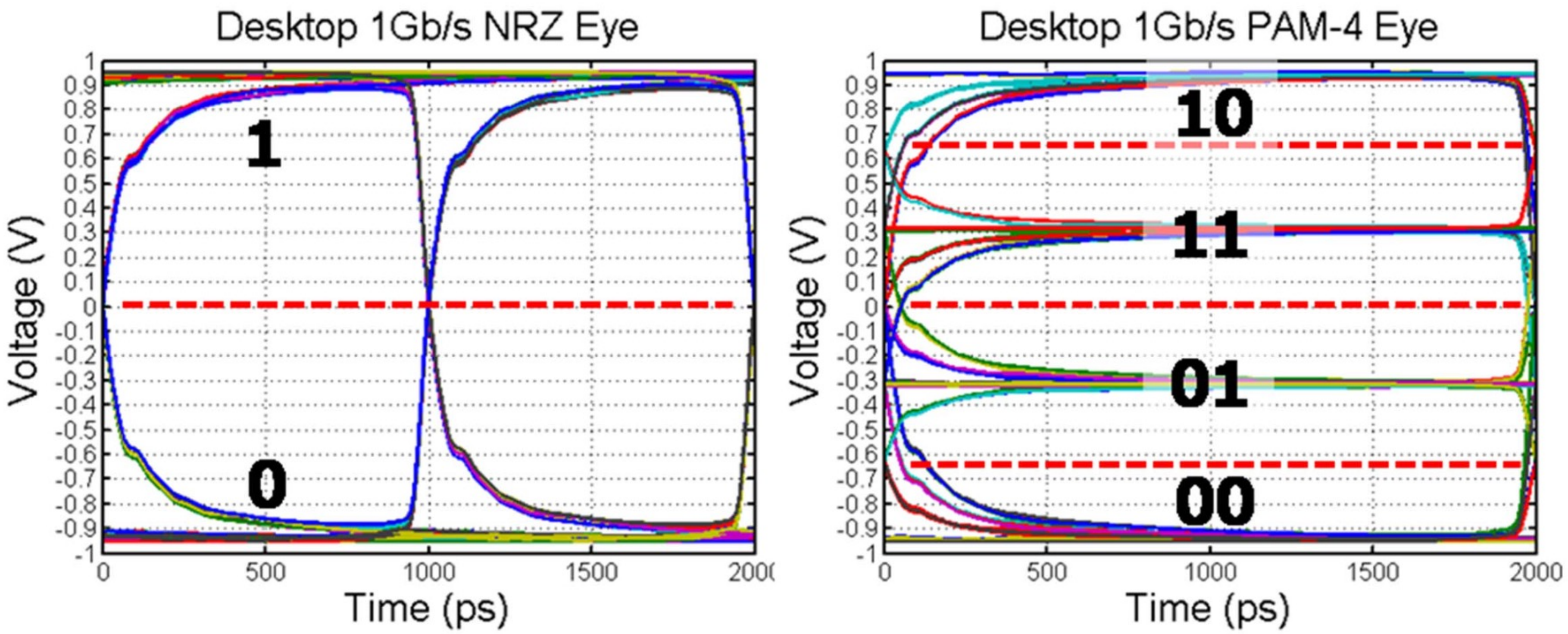
$$\frac{1}{2T} = \frac{R_S}{2} \le W \Rightarrow \frac{R_S}{W} \le 2 \quad \text{(symbols/s/Hz)}$$

- For ideal Nyquist pulses (sinc), the required bandwidth is only $R_S/2$ to support an R_S symbol rate

Modulation	Bits/Symbol	Nyquist Frequency
NRZ	1	$R_S/2=1/2T_b$
PAM-4	2	$R_S/2=1/4T_b$

[Sam Palermo]

NRZ vs PAM-4



- PAM-4 should be considered when
  - Slope of channel insertion loss (S_{21}) exceeds reduction in PAM-4 eye height
    - Insertion loss over an octave is greater than 20*log10(1/3)=-9.54dB
  - On-chip clock speed limitations

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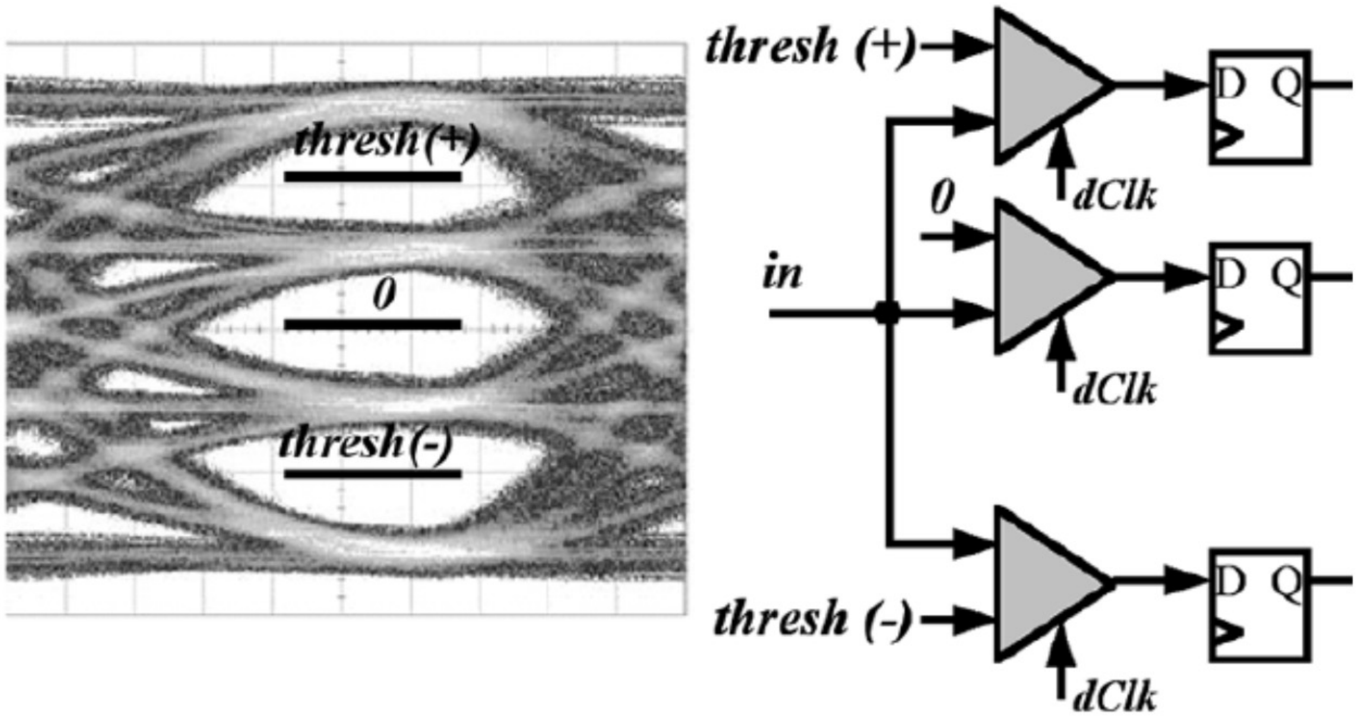
PAM-4 Receiver



[Stojanovic JSSC 2005]

- 3x the comparators of NRZ RX

[Sam Palermo]