Lecture # 4, 10/6/2021

Last time:

- 1st 1/2 of motor modeling doc.

  → Finish 2nd 1/2 on Friday.

Today

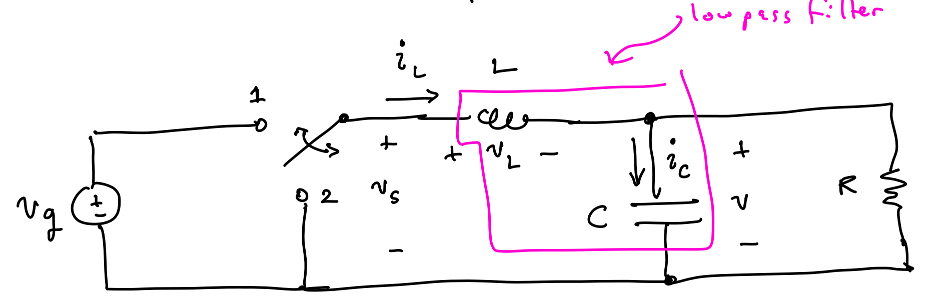
- Ch2, "balance equations"
- HW1 due next Monday 11:59 pm PT.

Chapter 2 → Steady-state Analysis

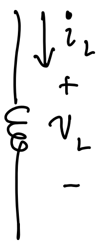
periodic steady state due to switching

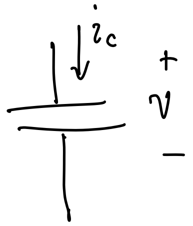
Consider the "Buck" converter

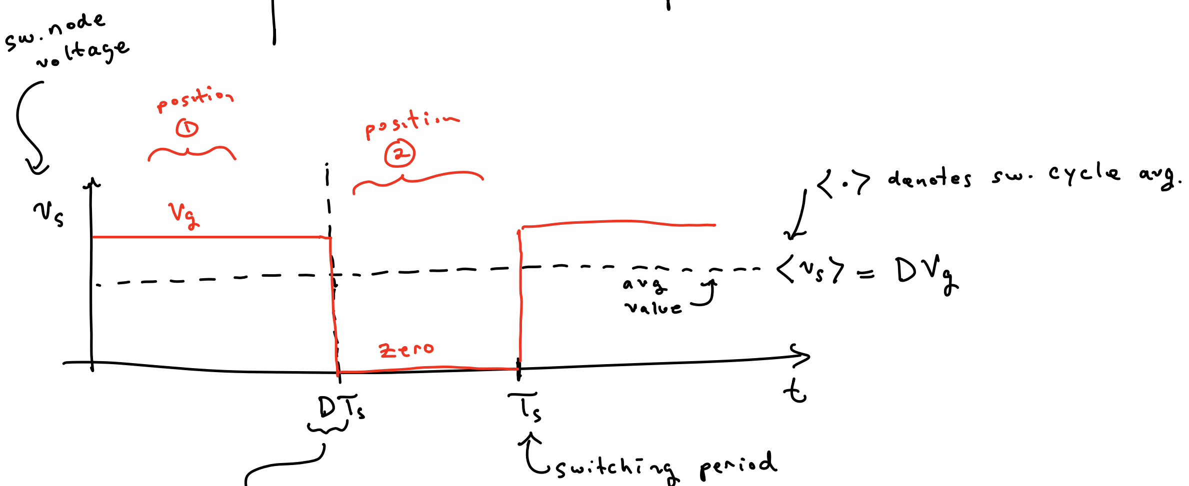
↳ step-down



A comment on polarity / sign conventions for analysis







D is "duty ratio"

$0 < D < 1$

$$D = \frac{\text{time on}}{T_s}$$

$f_s = T_s^{-1}$, typically kHZ to MHZ range

Avg value = dc component

$$\langle v_s \rangle = \frac{1}{T_s}\int_0^{T_s} v_s(t)\,dt$$

$$= \frac{1}{T_s}\left(\int_0^{DT_s} V_g\,dt + \cancelto{0}{\int_{DT_s}^{T_s} \text{zero}\,dt}\right)$$

$$= \frac{1}{T_s} V_g \underbrace{\int_0^{DT_s} 1\,dt}_{DT_s}$$

$$= \frac{D\cancel{T_s}}{\cancel{T_s}} V_g$$

$$= DV_g$$

Q: What about the output voltage?

A: LC is a lowpass filter. Dc component of sw. terminal voltage appears @ output.

$$\Rightarrow v = DV_g$$

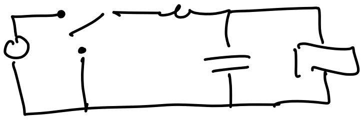
$$\rightarrow \boxed{\underbrace{\frac{v}{V_g} = \frac{\text{output}}{\text{input}}}_{\text{ratio called}} = D} \leftarrow \text{specific to buck}$$

"M" in book → s.s. voltage conversion ratio

* Need a more generalized method to derive $\frac{v}{v_g} = M$ for any converter

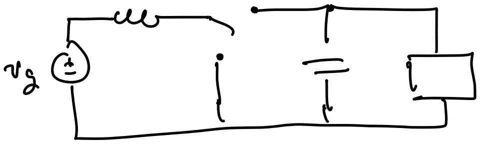
Overview of 3 common converters

Buck :



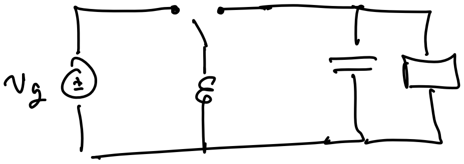
$M = D$

Boost :



$M = \frac{1}{1-D} > 1$

Buck Boost :



$M = \frac{-D}{1-D}$

General method to derive M hinges on

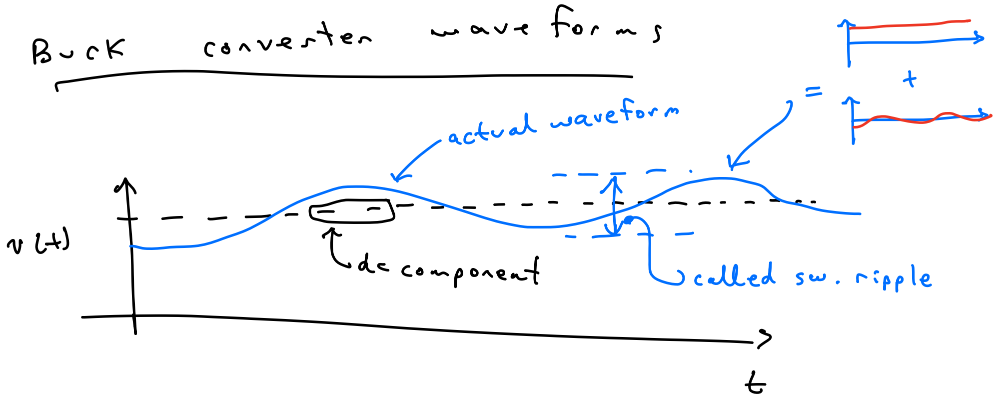
→ volt-second balance for L's

→ charge (AKA amps-seconds) balance for C's

Intuition:

Energy doesn't build up in L's & C's of the converter. Input energy goes to output.

- Buck converter waveforms



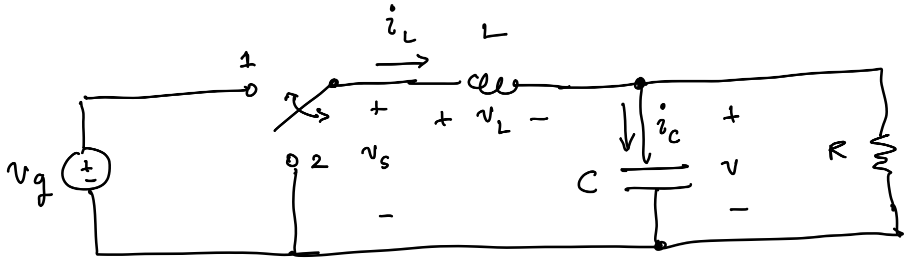
$$v(t) = \underset{\substack{\uparrow \\ \text{dc comp.}}}{V} + v_{ripple}(t)$$

In most designs $\| v_{ripple} \| < V$

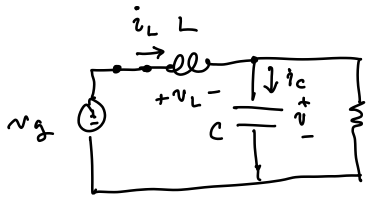
and $v(t) \approx V$

- General Method to Derive S.S. Input-Output Relations

- Start w/ a given converter ckt.
- Draw diagram for each sw. config. & compute ckt voltages & currents



config ①



- Look @ L voltage

$$v_L = v_g - V \qquad (1)$$

b/c small ripple @ output

$v \approx V$ = dc component

Hence, (1) becomes

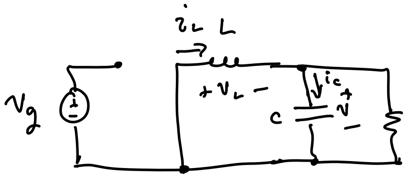
$$v_L = V_g - V \qquad (2a)$$

$$L\frac{di_L}{dt} = v_L \qquad (3a)$$

$$= \underbrace{V_g - V}_{\text{constant}}$$

$$\Rightarrow \underset{\text{slope}}{i_L} = \underbrace{\frac{di_L}{dt} = \frac{V_g - V}{L}}_{\oplus\ \text{slope}}$$

config ②



- Inductor voltage

$$v_L = -v$$

∴ small ripple ...

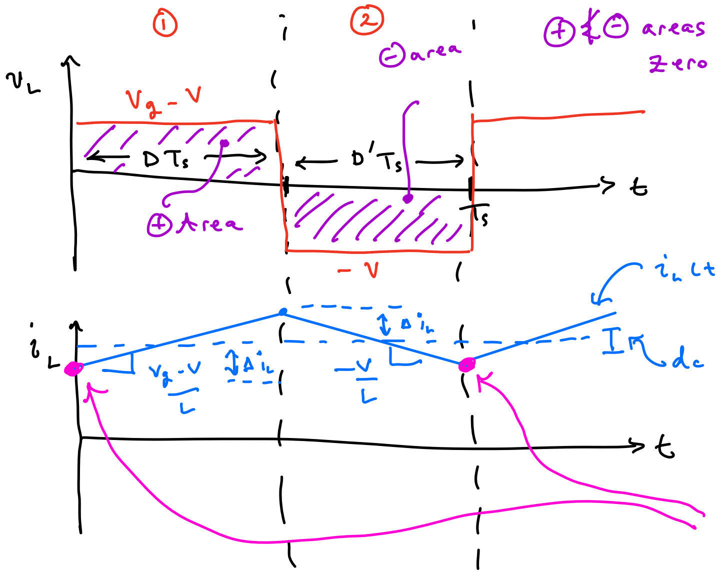
$$v_L = -V \qquad (2b)$$

$$L\frac{di_L}{dt} = v_L \qquad (3b)$$

$$= -V$$

$$\underbrace{\frac{di_L}{dt} = \frac{-V}{L}}_{\ominus\ \text{slope}}$$

Look @ L waveforms



⊕ & ⊝ areas are equal & must avg to zero over 1 cycle.

where

$D' = 1 - D$

$i_L(t)$ looks like a piecewise linear waveform

dc component

same value of i_L. Since $E_L = \frac{1}{2} L i_L^2$, energy stored at these 2 points across a cycle is the same.

Principle of volt-second balance

know $v_L = L \frac{di_L}{dt} \rightarrow \frac{di_L}{dt} = \frac{v_L}{L}$

in integral form $\rightarrow$ $\overbrace{\underbrace{i_L(T_s)}_{\text{init.}} - \underbrace{i_L(0)}_{\text{final}}}^{\text{zero}} = \frac{1}{L} \int_0^{T_s} v_L(t)\, dt$

b/c final & initial values same in steady state

$$\Rightarrow \qquad 0 = \frac{1}{L} \int_0^{T_s} v_L(t)\, dt$$

÷ by L & × by $\frac{1}{T_s}$

rewrite as $0 = \frac{1}{T_s} \underbrace{\int_0^{T_s} v_L(t)\, dt}_{\text{area under curve}} = \langle v_L \rangle$

Area above & below in rectangles must be equal

$$\underset{\substack{\uparrow\\ \text{zero in}\\ \text{s.s.}}}{\lambda} = \underbrace{\int_0^{T_s} v_L(t)\,dt = (V_g - V)(D\cancel{T_s}) + (-V)(D'\cancel{T_s})}_{\text{volt-sec balance eqn.}} = 0$$

Solve for V

$0 =$

$=$

$\Rightarrow V =$

$\rightarrow M =$

- What about Cap? ↓
- Principle of cap charge balance

$$i_c(t) = \qquad\qquad \rightarrow \quad \frac{dv_c}{dt} =$$

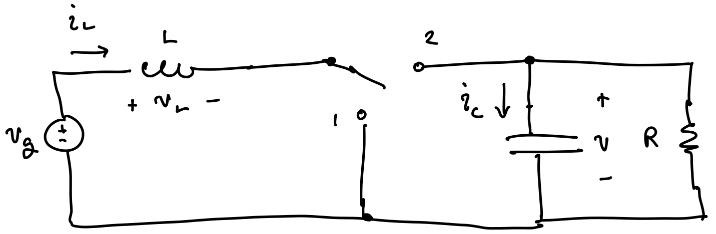
integral form $\quad v_c(T_s) - v_c(0) =$

$0 =$

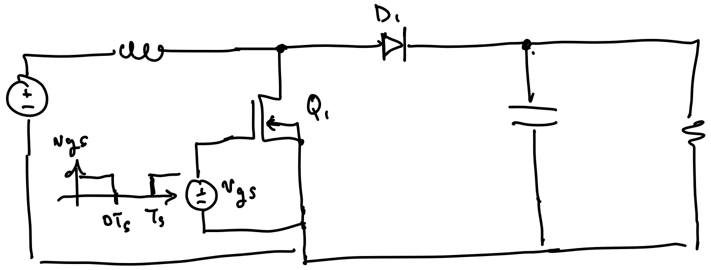
$0 =$

charge $= \int_0^{T_s} i_c(t)\,dt$

- **Boost** Example to see both balance equations



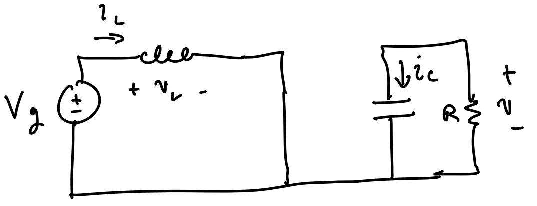
↓ realization

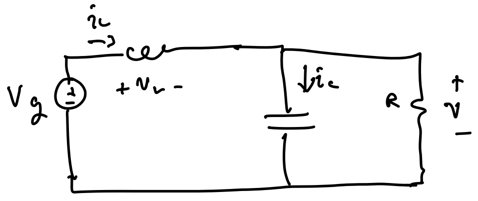


Back to analysis

config ①

config ②





inductor v_L & cap i_c

v_L =

i_c =

apply small ripple approx

v_L =

i_c =

$v \approx$, i_L =

inductor v_L & cap i_c

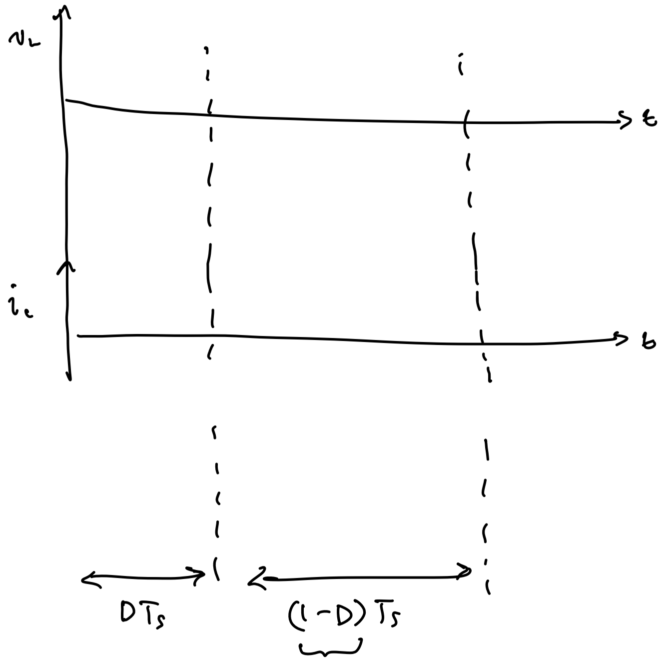
v_L =

i_c =

v_L =

i_c =

Look @ waveforms



Write balance equations

volt second:

$0 =$

solve for V

$0 =$

$\rightarrow \quad V = \qquad\qquad (*)$

$$\longrightarrow \boxed{M(D) = \frac{V}{V_g} =}$$

Boost conversion

Charge balance ... needed to set I

$0 =$

solve for I

$0 =$

$I =$

substitue expression of V ... use $(*)$

$$\Longrightarrow \boxed{I =}$$