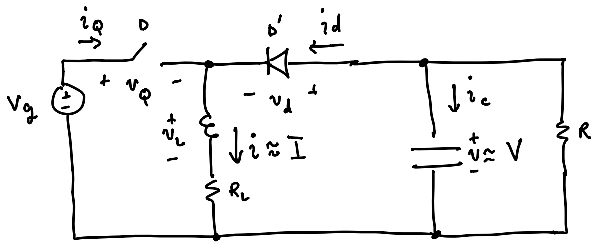
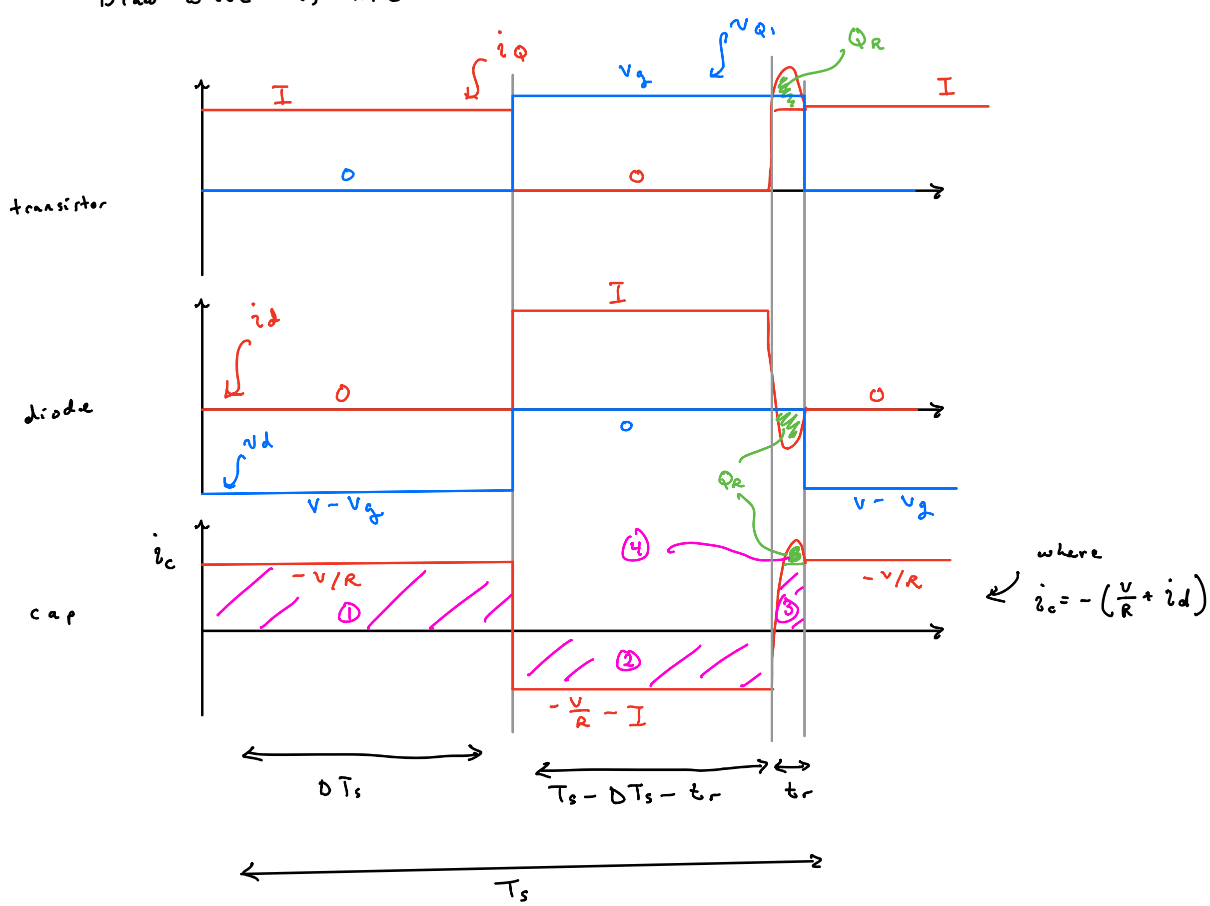
Homework # 5

Problem 1



Draw waveforms like in lecture



- Volt - sec balance

$$\langle v_L \rangle = 0 = D(V_g - I R_L) + D'(V - I R_L)$$

$$\boxed{= DV_g - I R_L + D'V = 0}$$ ← KVL loop eqn.

- Charge balance

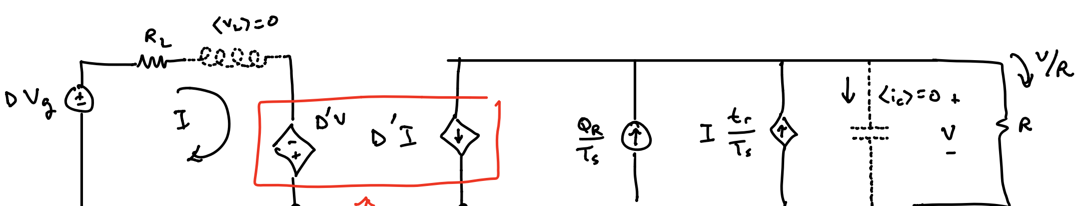
$$\langle i_c \rangle = \frac{\text{sum of areas}}{T_s}$$

$$= \frac{\overset{①}{DT_s(-V/R)} + \overset{②}{\overbrace{(T_s - DT_s - t_r)}}(-V/R - I) + \overset{③}{t_r(-V/R)} + \overset{④}{Q_R}}{T_s}$$

$$= \frac{-DT_s \frac{V}{R} + \left(-T_s V/R - I T_s + DT_s V/R + I D T_s + t_r V/R + I t_r\right) - t_r V/R + Q_R}{T_s}$$

$$= \boxed{-V/R - D'I + I\frac{t_r}{T_s} + \frac{Q_R}{T_s} = 0}$$ ← KCL eqn

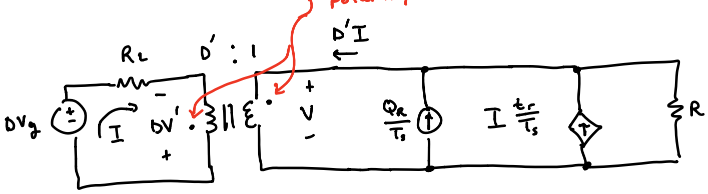
Draw equiv ckt



draw as xfmr.

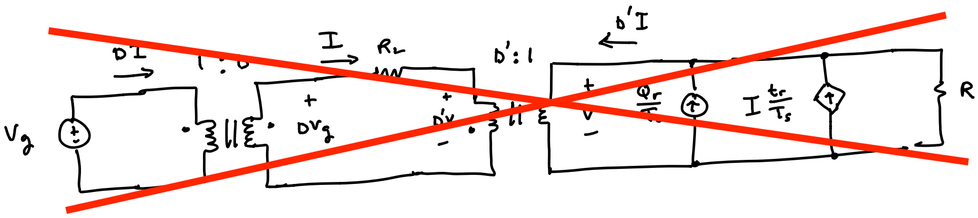
⇓

dots placed to respect polarity



could add another xfmr here too

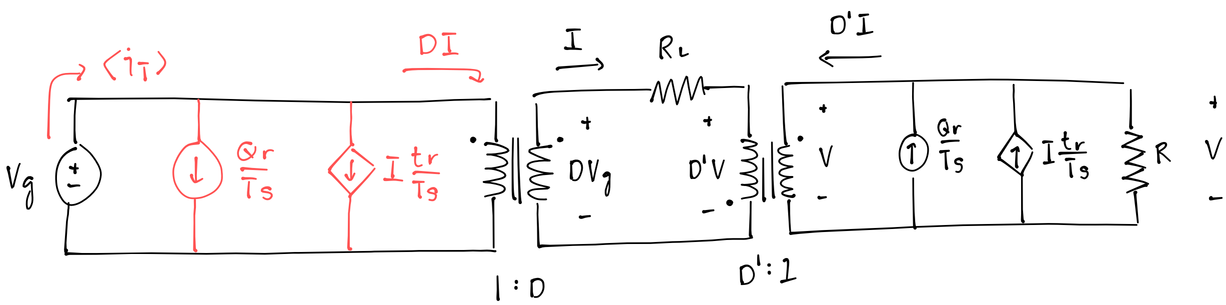
Final Form with ideal xfmrs:



* correction

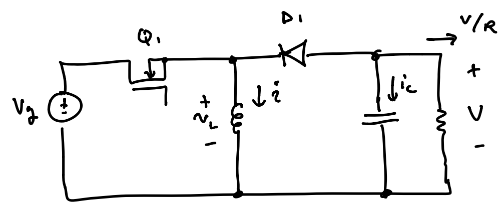
$$\langle P_{in}\rangle = V_g \langle i_T\rangle \neq V_g DI$$

$$\langle i_T\rangle = DI + I t_r/T_s + Q_r/T_s$$

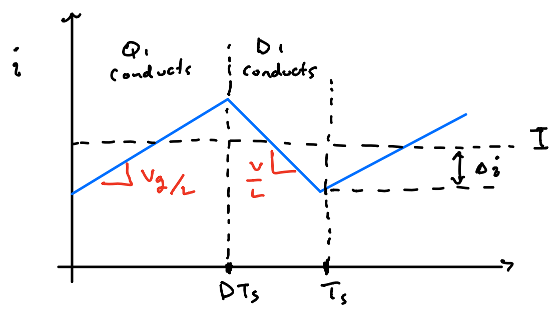


Problem # 2

a) Show DCM happens when $K < K_{crit.}$



• Look @ ripple amplitude in CCM



$$v_L = L \frac{2\Delta i}{\Delta t}$$

↳ look @ config when Q_1 conducts

$$\Rightarrow \quad V_g = L \frac{2\Delta i}{DT_s}$$

$$\Rightarrow \quad \Delta i = \frac{DT_s}{2L} V_g \qquad (1)$$

In steady-state, it is well known that

$$M = \frac{V}{V_g} = \frac{-D}{1-D} \qquad (2)$$

- Now look @ dc component of i using charge balance

$$\langle i_c \rangle = 0 = D\left(-\frac{v}{R}\right) + D'\left(-I - v/R\right)$$

solve for I

$$\Rightarrow \quad 0 = -\frac{v}{R} + D'(-I)$$

$$I = -\frac{V}{R D'}$$

$*$ use (2)

$$= \cancel{-}\frac{1}{R\underbrace{(1-D)}_{D'}}\left(\underbrace{\cancel{-} V_g \frac{D}{1-D}}_{V}\right)$$

$$= \frac{V_g D}{R(1-D)^2} \qquad (3)$$

$$= I$$

Using (1) & (3), CCM occurs if

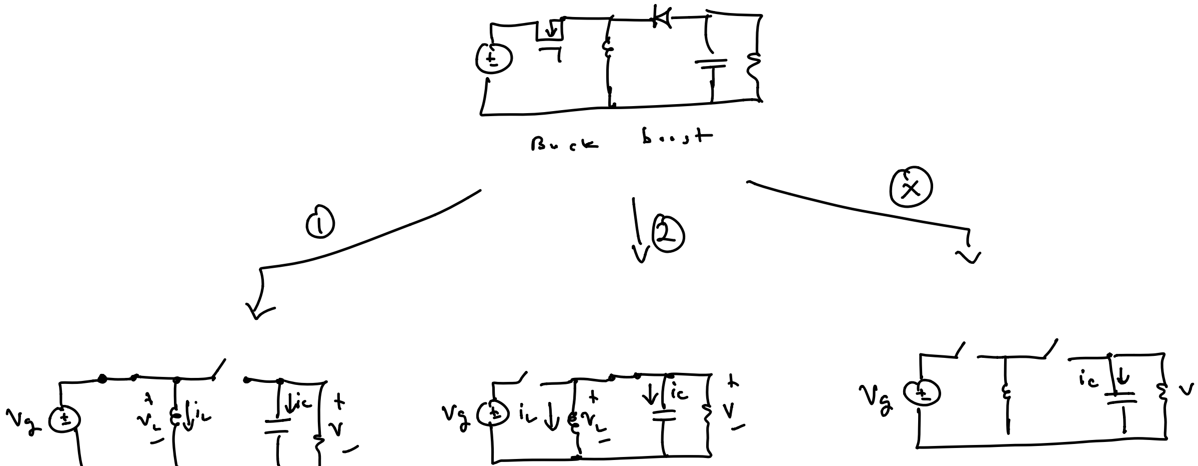
$$\underbrace{\frac{\cancel{D} T_s}{2L} \cancel{V_g}}_{\Delta i} > \underbrace{\frac{\cancel{V_g} \cancel{D}}{R(1-D)^2}}_{I} \qquad (4)$$

Put (4) into $\underbrace{K}_{\text{physics}} > \underbrace{K_{crit}}_{\text{control}}$ form

$$\rightarrow \boxed{\underbrace{(1-D)^2}_{K_{crit}} > \underbrace{\frac{2L}{R T_s}}_{K}} \quad \boxed{2a}$$

2.b) Compute v/v_g.

First need to analyze 3 ckt configurations:



duration DT_s | duration $D_2 T_s$ | Duration $D_3 T_s$

duty for uncontrolled diode ontime.

Need three golden equations to relate V, I, D_2

- Volt-sec balance gives:

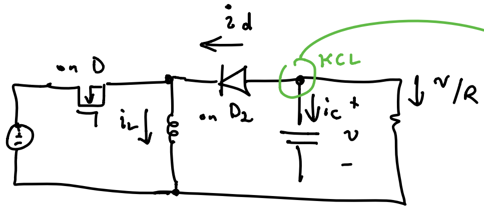
SRA okay for cap voltage.

$$\langle v_L \rangle = 0 = D(V_g) + D_2(V) + D_3(0)$$

$$\boxed{\begin{aligned} &= DV_g + D_2 V \\ &= 0 \end{aligned}} \quad (5)$$

key eqn for later

- Look @ KCL on load side



KCL gives

pulsed diode current

$$i_c = -v/R - i_d \quad (6)$$

Take avg of both sides of (6)

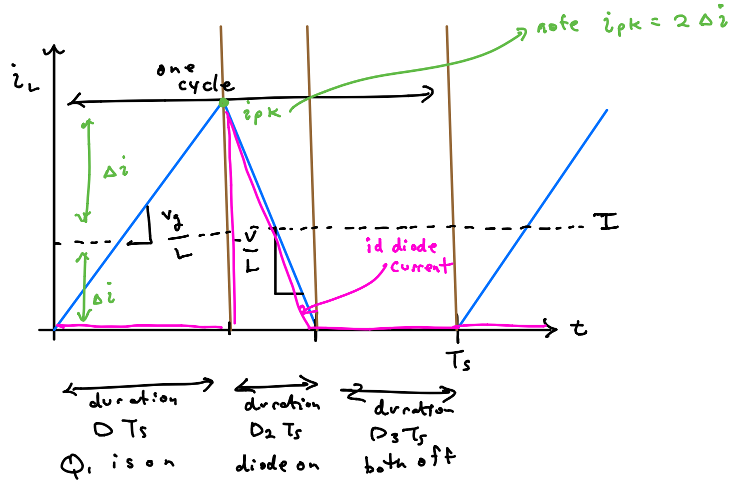
$$\Rightarrow \cancel{\langle i_c \rangle}^{\,0} = -\frac{V}{R} - \langle i_d \rangle \quad (7)$$

must be zero in s.s.

(7) implies

$$\boxed{\langle i_d \rangle = -\frac{V}{R}} \qquad (8)$$

- And now analyze i_L and i_d waveforms to get I and $\langle i_d \rangle$



Before computing $\langle i_L \rangle$ & $\langle i_d \rangle$, first compute i_{pk}. When Q_1 is on for duration DT_s in config ①, we know $2\Delta i = i_{pk}$. Hence,

$$v_L = L\frac{2\Delta i}{\Delta t}$$

$$\underset{①}{\text{config}} \Rightarrow \quad V_g = L\frac{2\Delta i}{DT_s} = L\frac{i_{pk}}{DT_s}$$

$$\Rightarrow \quad i_{pk} = V_g\frac{DT_s}{L} \qquad (9)$$

Integrate / average over 1 cycle to get $\langle i_L \rangle = I$ value:

$$\langle i_L \rangle = I = \frac{1}{T_s}\underbrace{\int_0^{T_s} i_L(t)\,dt}_{\text{area}}$$

$$= \frac{1}{\cancel{T_s}}\,\frac{1}{2}\,i_{pk}\,(D + D_2)\cancel{T_s}$$

$$= \frac{i_{pk}(D + D_2)}{2}$$

* use (9) for i_{pk}

$$\boxed{\begin{aligned} &= \frac{V_g D T_s}{L} \frac{(D + D_2)}{2} \\ &= I \end{aligned}} \quad \Bigg\} \ (10)$$

And diode avg current is

$$\langle i_d \rangle = \frac{1}{T_s} \int_0^{T_s} i_d(t)\, dt$$

$$= \frac{1}{\cancel{T_s}} \frac{1}{2} i_{pk} D_2 \cancel{T_s}$$

$$= \frac{i_{pk} D_2}{2}$$

* use (9) again

$$\boxed{\begin{aligned} &= \frac{V_g D T_s}{L} \frac{D_2}{2} \\ &= \langle i_d \rangle \end{aligned}} \quad \Bigg\} \ (11)$$

- 4 green boxes above are our key eqns. Rest is algebra to isolate V/V_g.

Use (5) to get D_2

$$D_2 = -D \frac{V_g}{V} \qquad (12)$$

Now equate (8) & (11) for $\langle i_d \rangle$

$$\underbrace{-\frac{V}{R}}_{(8)} = \underbrace{\frac{V_g D T_s}{L} \frac{D_2}{2}}_{(11)}$$

* plug in (11) for D_2

$$= \frac{V_g D T_s}{2L} \overbrace{\left(-D \frac{V_g}{V}\right)}^{D_2}$$

$$= -\frac{V_g^2 D^2 T_s}{2VL}$$

$$\Rightarrow \quad \frac{v^2}{v_g^2} = \left(\frac{v}{v_g}\right)^2 = \underbrace{\frac{RT_s}{2L}}_{K^{-1}} D^2$$

recall

$$K = \frac{2L}{RT_s}$$

$$= \frac{D^2}{K}$$

$$\Rightarrow \quad M = \left(\frac{v}{v_g}\right) = \sqrt{\frac{D^2}{K}}$$

2b

$$\boxed{= \frac{D}{\sqrt{K}} = \frac{v}{v_g}}$$

3c) Plot v/v_g for $K = 0.1$ for $0 \le D \le 1$.

$$\frac{v}{v_g} = \frac{D}{\sqrt{0.1}} \approx 3.16\,D$$