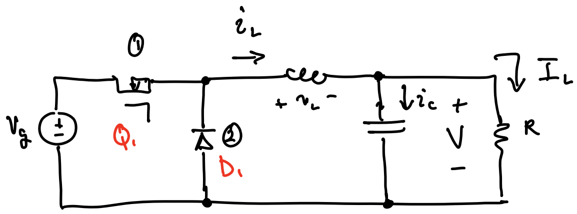
Lecture #17 11/10/21

Today : Ch 5 on DCM (today only) } read chapter efficiently over next few days

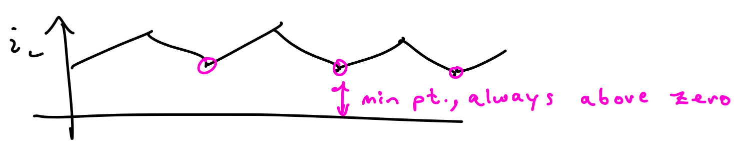
Look ahead: Inductor Design

- Discontinuous Conduction Mode (DCM) - Ch 5

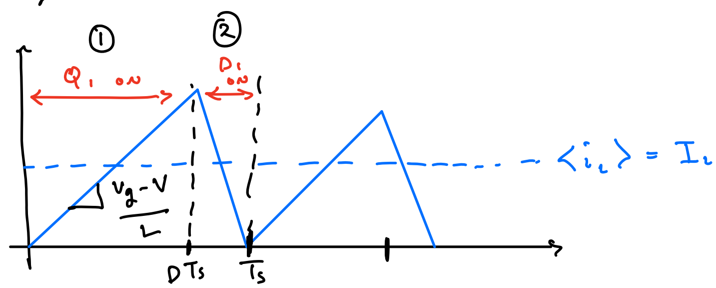
Buck example



All analysis so far is for "Continuous Conduction Mode" (CCM)



@ Boundary, we see



know $v_L \approx L \frac{2\Delta i}{\Delta t}$

↳ look @ config ①

$$\Rightarrow \quad V_g - V = L\,\frac{2\Delta i}{DT_s}$$

solve for Δi

$$\Delta i = \frac{(V_g - V)DT_s}{2L} \qquad (1)$$

Recall, can use V-sec balance to derive $\underbrace{M = \frac{V}{V_g} = D}_{\text{ideal buck}}$

$$\Rightarrow \quad V = DV_g \quad (2)$$

(2) → (1) gives

$$\Delta i = \frac{(V_g - DV_g)DT_s}{2L}$$

$$= \frac{V_g(1-D)DT_s}{2L} \qquad (3)$$

Look @ load

$$I_L = \frac{V}{R} \quad \leftarrow \text{due to charge balance}$$

$$= \frac{DV_g}{R} \qquad (4)$$

DCM occurs if

$$\underset{\text{use (4)}}{I_L} < \underset{\text{use (3)}}{\Delta i_L}$$

$$\Downarrow$$

$$\underbrace{\frac{\cancel{D}\cancel{V_g}}{R}}_{\text{load dc comp.}} < \underbrace{\frac{\cancel{V_g}(1-D)\cancel{D}T_s}{2L}}_{\substack{\Delta i_L \\ \text{ripple size}}}$$

① Obtain DCM boundary eqn:

$$\boxed{\text{DCM occurs if}: \quad \frac{1}{R} < \frac{(1-D)T_s}{2L}} \quad (6)$$

for buck

call R_{crit} when

$$\frac{1}{R_{crit}} = \frac{(1-D)T_s}{2L}$$

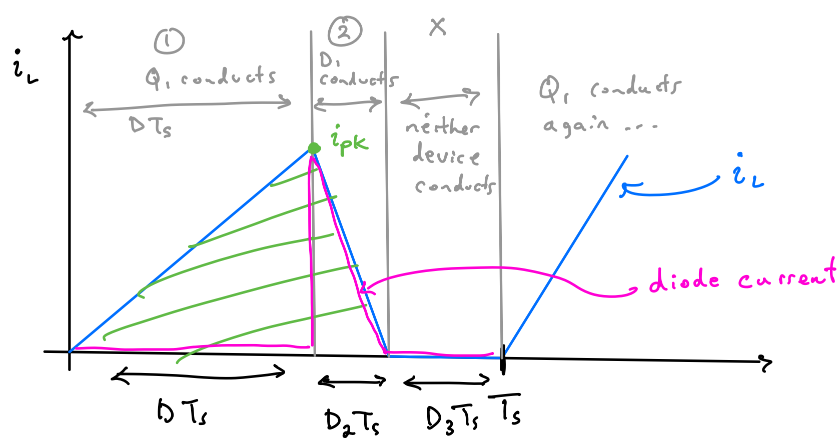
$$\hookrightarrow \boxed{R_{crit} = \frac{2L}{D'T_s}} \quad (7)$$

Look closer @ (6) & rearrange it to get

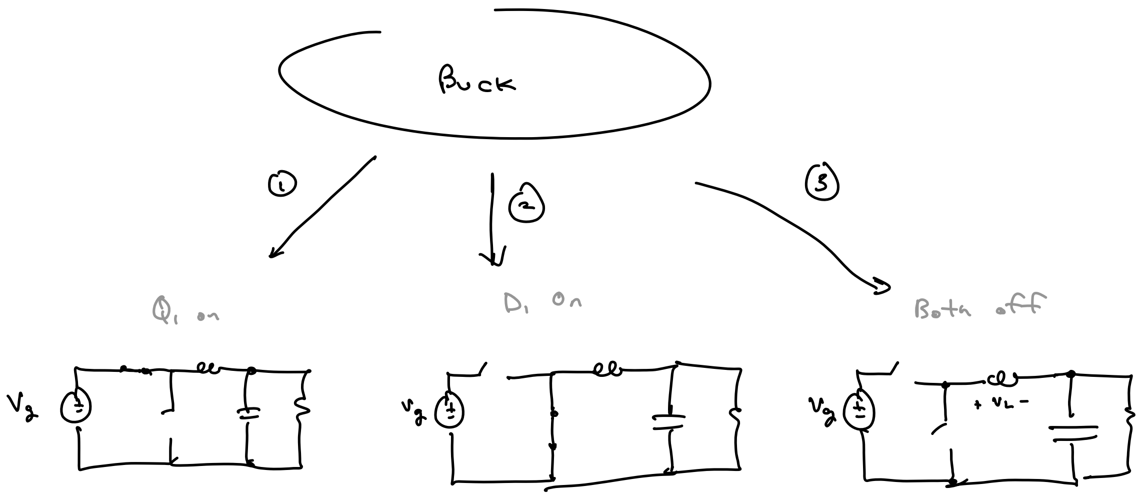
$$\underbrace{\frac{2L}{RT_s}}_{\substack{\text{physics} \\ \text{"K"}}} < \underbrace{D'}_{\substack{\text{control side} \\ K_{crit}(D)}} \quad (8)$$

↳ also know range D lies in (between 0 & 1 are possible)

– Crossing DCM boundary



Now Get 3 regions / ckts



Caution

→ $\cancel{i_L \approx I_L}$ SRA for i_L no longer is true

→ $v \approx V$ ✓ SRA still okay for cap.

When does DCM happen?

→ "light load" → large R, less power

→ inductor intentionally designed for large ripple

→ can get smaller/cheaper inductor

→ facilitate soft switches

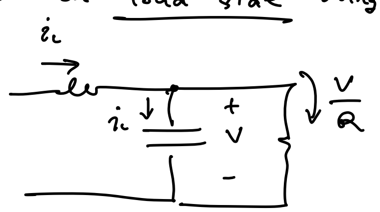
- Look @ v-sec balance

$$\boxed{\langle v_L \rangle = 0 = D\overbrace{(V_g - V)}^{1} + D_2\overbrace{(-V)}^{2} + D_3\overbrace{(0)}^{3} \qquad (9)}$$

→ Extra unknown... need 3rd eqn. Will have 2 charge balance eqns

* Note $D + D_2 + D_3 = 1$

- Charge balance on load side using KCL analysis



KCL gives

$$i_L(t) = i_c(t) + v/R$$

$\downarrow$ avg both sides

$$\underbrace{\langle i_L \rangle}_{I_L} = \cancel{\langle i_c \rangle}^{\,0} + \frac{V}{R}$$

must be zero b/c s.s.

$$\Rightarrow \boxed{I_L = \frac{V}{R}} \qquad (10)$$

equal

- Compute $\langle i_L \rangle$ using integral of i_L waveform

$$\underbrace{\langle i_L \rangle}_{I_L} = \frac{1}{T_s} \int_0^{T_s} \underbrace{i_L(t)\, dt}_{\text{area of } \triangle}$$

$$\boxed{\begin{aligned} &= \frac{1}{\cancel{T_s}} \frac{1}{2} i_{pk} (D + D_2) \cancel{T_s} \\ &= I_L \end{aligned}} \qquad (11)$$

Last step: Do algebra to eliminate D_2 ... then can get V's & I's

Solve it. (9) gives

$$0 = D(V_g - V) + D_2(-V)$$

$$\Rightarrow \boxed{D_2 = D \frac{V_g - V}{V}} \qquad (12)$$

Equate (10) & (11)

$$\frac{V}{R} = \frac{1}{2} \underbrace{\frac{DT_s}{L}(V_g - V)}_{i_{pk}\text{. From } \Delta i_L \text{ eqn when } Q_1 \text{ on}} (D + D_2)$$

use (12)

$$= \frac{1}{2}\frac{DT_s}{L}(V_g - V)\left(D + D\frac{V_g - V}{V}\right)$$

$$= \frac{1}{2}\frac{D^2 T_s}{L}(V_g - V)\left(\frac{\cancel{V} + V_g - \cancel{V}}{V}\right)$$

$$= \underbrace{\frac{1}{2}\frac{D^2 T_s}{L}(V_g - V)\left(\frac{V_g}{V}\right)}_{RHS}$$

$$= \underbrace{\frac{V}{R}}_{LHS}$$

$\times$ by $\frac{R}{V_g}$

$$\underbrace{\frac{V}{V_g}}_{LHS} = D^2\frac{RT_s}{2L}(V_g - V)\frac{1}{V} \quad = D^2 \overbrace{\frac{RT_s}{2L}}^{\text{call this } K^{-1}}\left(\frac{V_g}{V} - 1\right)$$

from earlier, see (8)

rewrite

$$\frac{V}{V_g} - \frac{D^2}{K}\frac{V_g}{V} + \frac{D^2}{K} = 0$$

want $M = \frac{V}{V_g}$

$\times$ by $\frac{V}{V_g}$

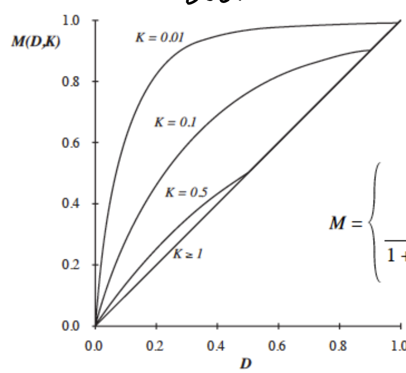
get quadratic form

$$\left(\frac{v}{v_g}\right)^2 + \frac{D^2}{k}\left(\frac{v}{v_g}\right) - \frac{D^2}{k} = 0$$

apply quadratic eqn to

set $\mathcal{M} = \frac{v}{v_g}$

Buck

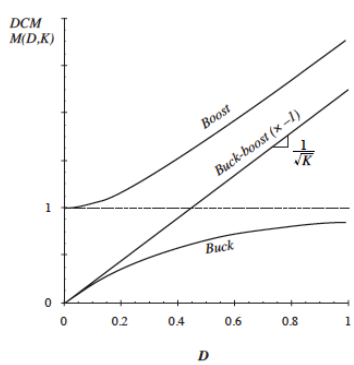


$$M = \begin{cases} D & \text{for } K > K_{crit} \\ \dfrac{2}{1+\sqrt{1+4K/D^2}} & \text{for } K < K_{crit} \end{cases}$$

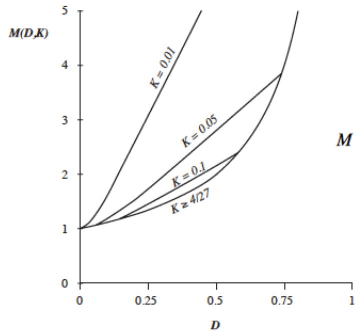
Table 5.2. Summary of CCM-DCM characteristics for the buck, boost, and buck-boost converters

Converter	$K_{crit}(D)$	*DCM* $M(D,K)$	*DCM* $D_2(D,K)$	*CCM* $M(D)$
Buck	$(1-D)$	$\dfrac{2}{1+\sqrt{1+4K/D^2}}$	$\dfrac{K}{D}M(D,K)$	D
Boost	$D(1-D)^2$	$\dfrac{1+\sqrt{1+4D^2/K}}{2}$	$\dfrac{K}{D}M(D,K)$	$\dfrac{1}{1-D}$
Buck-boost	$(1-D)^2$	$-\dfrac{D}{\sqrt{K}}$	$\sqrt{K}$	$-\dfrac{D}{1-D}$

with $K = 2L/RT_s$. *DCM occurs for* $K < K_{crit}$.



- DCM buck and boost characteristics are asymptotic to M = 1 and to the DCM buck-boost characteristic
- DCM buck-boost characteristic is linear
- CCM and DCM characteristics intersect at mode boundary. Actual M follows characteristic having larger magnitude
- DCM boost characteristic is nearly linear



$$M = \begin{cases} \dfrac{1}{1-D} & \text{for } K > K_{crit} \\ \dfrac{1+\sqrt{1+4D^2/K}}{2} & \text{for } K < K_{crit} \end{cases}$$

Approximate M in DCM:

$$M \approx \frac{1}{2} + \frac{D}{\sqrt{K}}$$