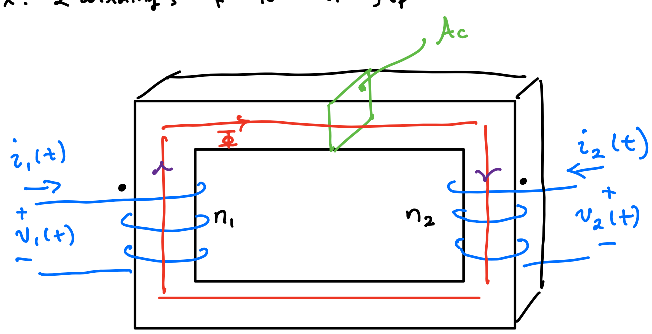
Lecture # 25 , 12/1/21

- HW 7 due Friday

- Today
  - ↳ Finish magnetics chapter (finish eddy currents, transformer)
  - ↳ Next time: chapter on "Converter Structures" — Ch 6 in 2nd Ed, Ch ? in 3rd Ed.

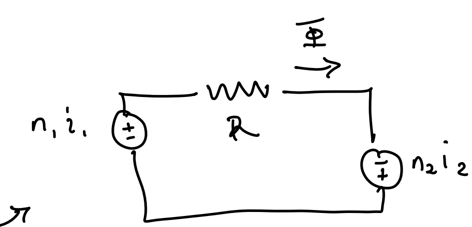
- Transformers

ex: 2 windings & no air gap



Core reluctance

$$\mathcal{R} = \frac{l_m}{\mu A_c}$$



& $\underbrace{\mathcal{F}}_{\text{MMF}} = n_1 i_1 + n_2 i_2$

$$\longrightarrow \quad \underbrace{n_1 i_1 + n_2 i_2}_{\text{sources}} = \underbrace{\Phi \mathcal{R}}_{\text{drop}} \qquad (1)$$

- If $\mathcal{R} = 0 \longrightarrow$ Recover ideal xfmr eqns.

  Look @ (1)

$$\boxed{n_1 i_1 + n_2 i_2 = 0} \qquad (1') \qquad \text{Ampere's Law}$$

  And Faraday's Law gives

$$v_1 = n_1 \frac{d\Phi}{dt} \quad \& \quad v_2 = n_2 \frac{d\Phi}{dt}$$

$$\Rightarrow \frac{d\Phi}{dt} = \boxed{\frac{v_1}{n_1} = \frac{v_2}{n_2}} \quad (2)$$

look familiar?

  (2) & (1') imply perfect $\eta = 100\%$ power transfer

- Look at realistic case where $\mathcal{R} \neq 0$

  Apply Faraday again

$$v_1 = n_1 \frac{d\Phi}{dt} \quad \& \quad v_2 = \ldots$$

  need to solve for Φ from (1)

  (1) gives

$$\Phi = \frac{n_1 i_1 + n_2 i_2}{\mathcal{R}} \qquad (3)$$

  Substitute & get

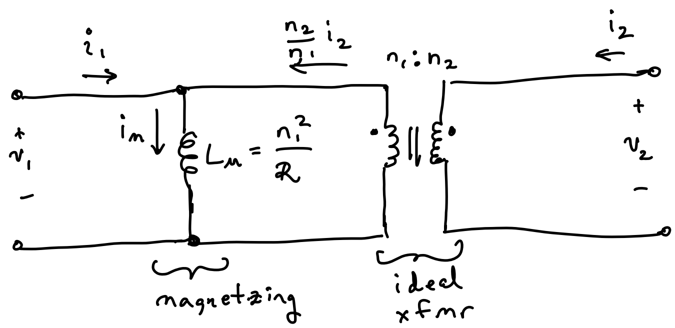
$$v_1 = n_1 \frac{d}{dt}\left(\frac{n_1 i_1 + n_2 i_2}{\mathcal{R}}\right)$$

$$= \underbrace{\frac{n_1^2}{\mathcal{R}}}_{L_\mu = \text{"magnetizing inductance"}} \frac{d}{dt}\left(i_1 + \frac{n_2}{n_1} i_2\right) \qquad (4)$$

$$v_2 = n_2 \frac{d}{dt}\left(\frac{n_1 i_1 + n_2 i_2}{\mathcal{R}}\right)$$

⋮
similar story.

Eq (4) implies the ckt:



apply KCL to get i_μ.

$$i_\mu = i_1 + \frac{n_2}{n_1} i_2$$

Observations:

- Remove coil #2 → recover "regular ol'" inductor w/ value L_μ.
- L_m causes departure from idealized xfmr eqns.

$$\cancel{n_1 i_1 + n_2 i_2 = 0} \quad \text{not true}$$

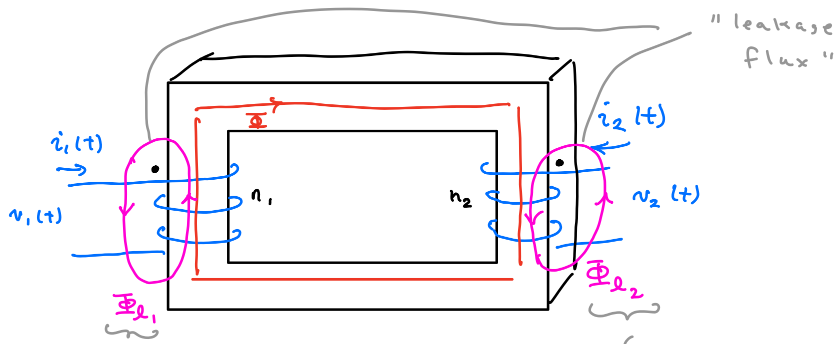
- What about saturation?

$$i_\mu(t) = \frac{1}{L_\mu} \int_0^t v_1(t)\, dt$$

$$\& \quad v_1 = n_1 \frac{d\Phi}{dt} = n A_c \frac{dB(t)}{dt}$$

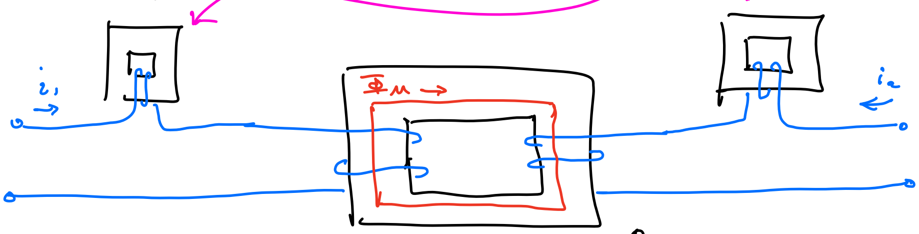
$$\rightarrow \quad B(t) = \frac{1}{n_1 A_c} \underbrace{\int_0^t v_1(t)\, dt}_{}$$

B reaching Bsat is caused by too many "volt-seconds"

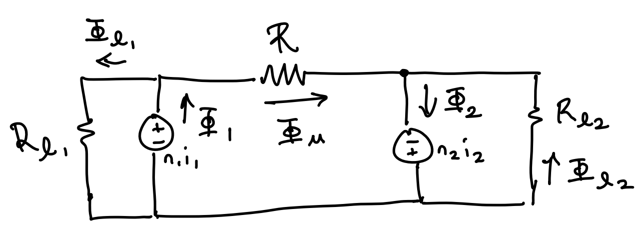


equiv to

leakage causes series inductance



magnetic equiv ckt:



Solve for Φ_1 & Φ_2 & Apply Faraday's Law

$$\underbrace{v_1 = n_1 \frac{d\Phi_1}{dt} \quad \& \quad v_2 = n_2 \frac{d\Phi_2}{dt}}$$

after some algebra

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \underbrace{\begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix}}_{\text{relate to } L_M \text{ \& leakages ?}} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$L_{12} = \frac{n_1 n_2}{\mathcal{R}} = \frac{n_2}{n_1} \frac{1}{L_M}$$

$$L_{11} = L_{\ell_1} + \frac{n_1}{n_2} L_{12}$$

$$L_{22} = L_{\ell_2} + \frac{n_2}{n_1} L_{12}$$

n_e = "effective turns ratio" $= \sqrt{\frac{L_{22}}{L_{11}}}$

↳ in ideal case $= \frac{n_2}{n_1}$

K = "coupling coefficient" $= \frac{L_{11}}{\sqrt{L_{11} L_{12}}}$

$$0 \leq K \leq 1$$

as $k \to 1$, approach ideal xfmr where $n_e \to \frac{n_2}{n_1}$

looks like