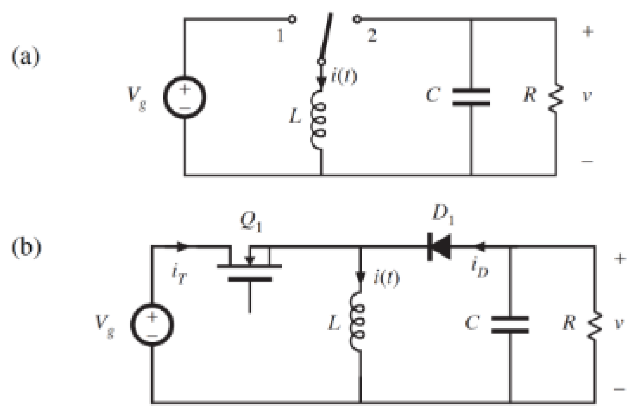
EE 452 – Power Electronics Design

Homework 1

Due Date: Monday October 11th 2021, 11:59pm

Instructions. You must scan your completed homework assignment into a pdf file, and upload your file to the Canvas Assignment HW#1 page by the due date/time above. All pages must be gathered into a single file of moderate size, with the pages in the correct order. Set your phone or scanner for basic black and white scanning. You should obtain a file size of hundreds of kB, rather than tens of MB. I recommend using the "Tiny Scanner" app. Please note that the grader will not be obligated to grade your assignment if the file is unreadable or very large.

Problem 1. Analysis and design of a buck-boost converter: A buck-boost converter is illustrated in Fig. 1(a), and a practical implementation using a transistor and diode is shown in Fig. 1(b).



(a) Find the dependence of the equilibrium output voltage V and inductor current I on the duty ratio D, input voltage V_g, and load resistance R. You may assume that the inductor current ripple and capacitor voltage ripple are small.

*Small-ripple approximation, $v \approx V$ and $i_L \approx I$

Switch closed (at position 1)

$$0 = V_g - v_L$$
$$v_L = V_g$$
$$\frac{di_L}{dt} = \frac{V_g}{L} \qquad [i_L \text{ slope}]$$

Switch open (at position 2)

$$0 = v_L - v$$
$$v_L = v$$
$$\frac{di_L}{dt} = \frac{v}{L} \qquad [i_L \text{ slope}]$$

Total Volt Seconds over 1 period for Inductor Voltage

$$\int_o^{T_S} v_L(t)dt$$
$$0 = DT_S \cdot V_g + (1-D)T_S \cdot V$$
$$0 = DV_g + (1-D)V$$
$$V = -V_g \frac{D}{(1-D)} = -V_g \frac{D}{D'} \qquad [\text{Output Voltage } V]$$

Switch closed (at position 1)

$$i_C = -\frac{v}{R} \approx -\frac{V}{R}$$

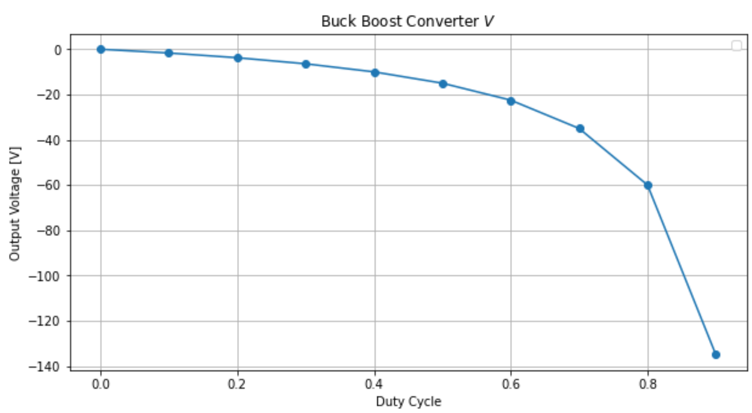
Switch open (at position 2)

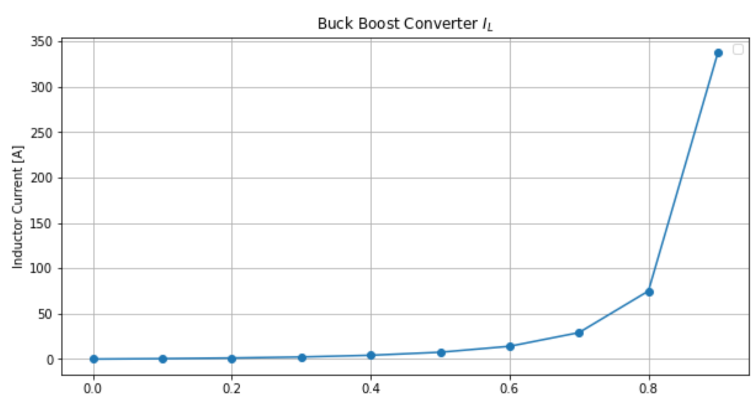
$$i_C = -i_L - \frac{v}{R}$$
$$i_C \approx -I - \frac{V}{R}$$

Total Volt Seconds over 1 period for Capacitor Current I

$$\int_o^{T_S} i_C(t)dt$$
$$0 = DT_S \cdot (\frac{-V}{R}) + (1-D)T_S \cdot (-I - \frac{V}{R})$$
$$0 = D(\frac{-V}{R}) + (1-D)(-I - \frac{V}{R})$$
$$0 = D(\frac{-V}{R}) + D'(\frac{-V}{R}) - D'I$$
$$0 = (D + D')(\frac{-V}{R}) - D'I$$
$$D'I = \frac{-V}{R}$$
$$I = \frac{-V}{D'R}$$
$$I = V_g \frac{D}{D'D'R} \qquad [\text{Inductor Current } I]$$

(b) Plot your results of part (a) over the range 0 ≤ D ≤ 1.





(c) DC design: For the specifications below:

- V_g = 15 V, V = −12 V, R = 4 Ω, f_s = 200 kHz

(i) Find D and I

Solving for D:

$$V = -V_g \frac{D}{D'}$$

$$\frac{D}{D'} = \frac{-V}{V_g}$$

$$D = \frac{-V}{V_g}(1 - D)$$

$$D - \frac{V}{V_g}D = \frac{-V}{V_g}$$

$$D(1 - \frac{V}{V_g}) = \frac{-V}{V_g}$$

$$D = \frac{-V/V_g}{(1 - V/V_g)}$$

$$D = 0.44$$

Solving for I

$$I = V_g \frac{D}{D' D' R}$$

$$I = 5.4 \text{ Amps}$$

(ii) Calculate the value of L that will make the peak inductor current ripple Δi equal to ten percent of the average inductor current I.

10% of I = 0.54A

$$2\Delta i = \frac{di_L}{dt} DT_S$$

$$\Delta i = \frac{V_g}{L} \frac{DT_S}{2}$$

$$0.54 = \frac{V_g}{L} \frac{DT_S}{2}$$

$$L = \frac{V_g}{0.54} \frac{DT_S}{2}$$

$$L = 30.9 \mu H$$

(iii) Choose C such that the peak output voltage ripple Δv is 0.1 V.

$$2\Delta v = \frac{dv_C}{dt} DT_S$$

$$\Delta v = \frac{i_C}{2C} DT_S$$

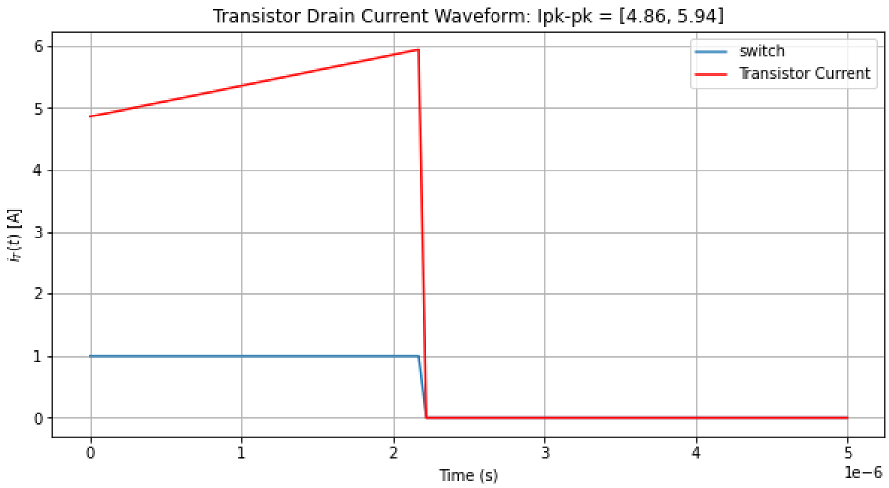
$$0.1 = \frac{-V}{2RC} DT_S$$

$$C = \frac{-V}{2R \cdot 0.1} DT_S$$

$$C = 33.3 \mu F$$

(d) Sketch the transistor drain current waveform $i_T(t)$ for your design of part (c). Include the effects of inductor current ripple, and label numerical values and axes. What is the peak value of $i_T(t)$? Also sketch $i_T(t)$ for the case when L is decreased such that Δi is 50% of I. What happens to the peak value of i_T in this case?

Part c design:



50% of I = 2.7A

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L = (Vg/2.7)*D*Ts/2
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L = 6.2e-06 uH
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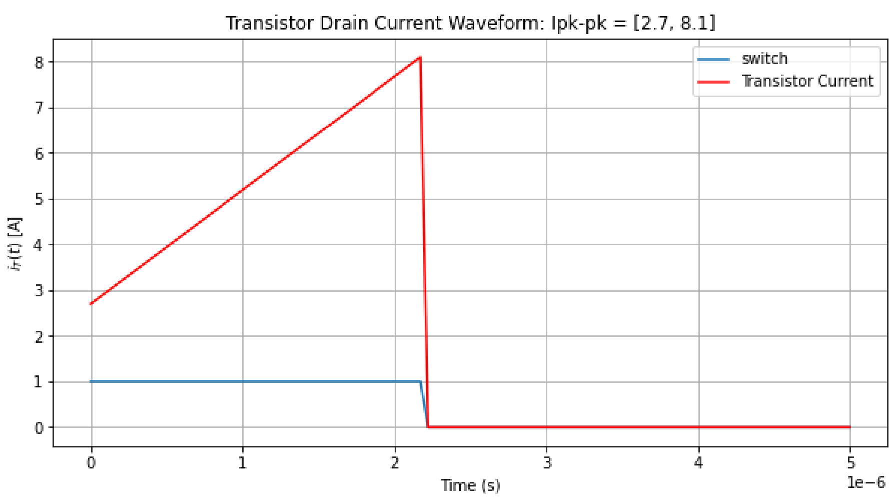
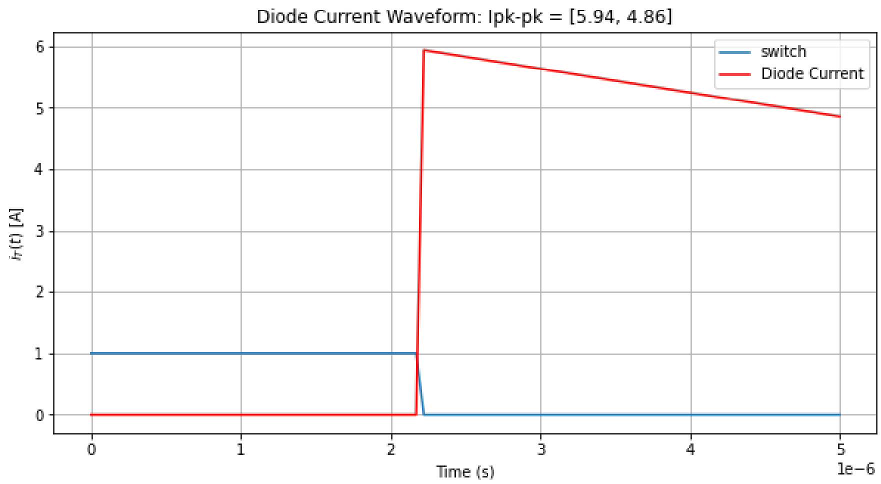
As L decreases, $i_T(t)$ increases. The positive slope is dependant on $\frac{V_g}{L}$.

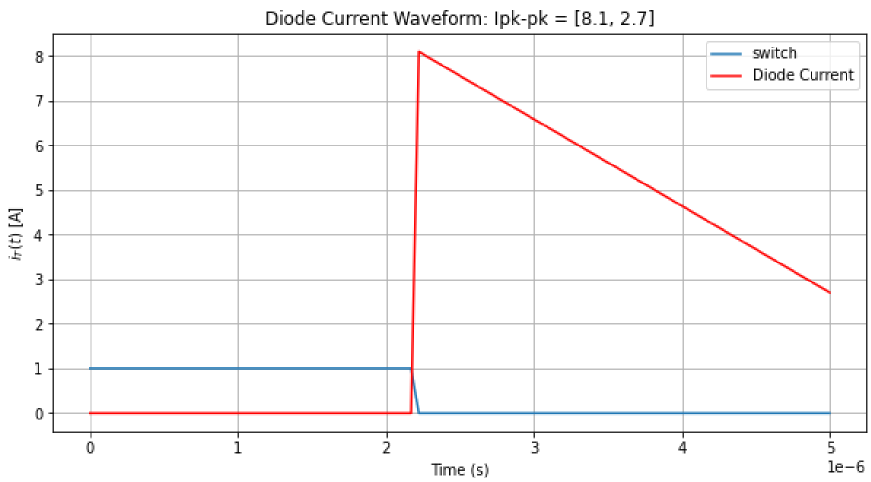


(e) Sketch the diode current waveform $i_D(t)$ for the two cases of part d.

Part c design:



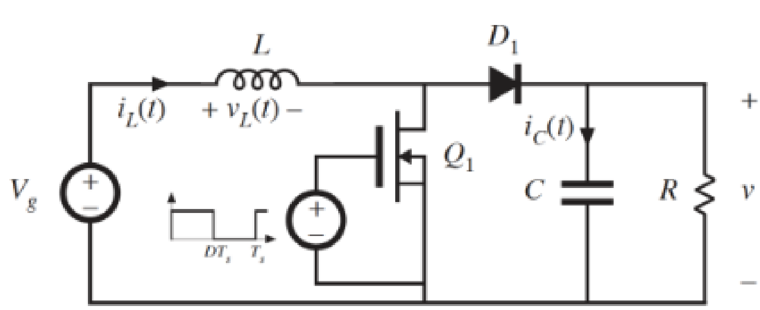
50% of I = 2.7A



Problem 2. The boost converter illustrated in Fig. 2 operates with the following conditions:

- V_g = 3.3 V, V = 5 V, f_s = 500 kHz

All elements are ideal, and the converter operates in steady state with small inductor current ripple and small capacitor voltage ripple.



(a) What is the duty cycle?

Switch closed (at position 1)

$$0 = V_g - v_L$$
$$v_L = V_g$$

Switch open (at position 2)

$$0 = V_g - v_L - v$$
$$v_L = V_g - v$$

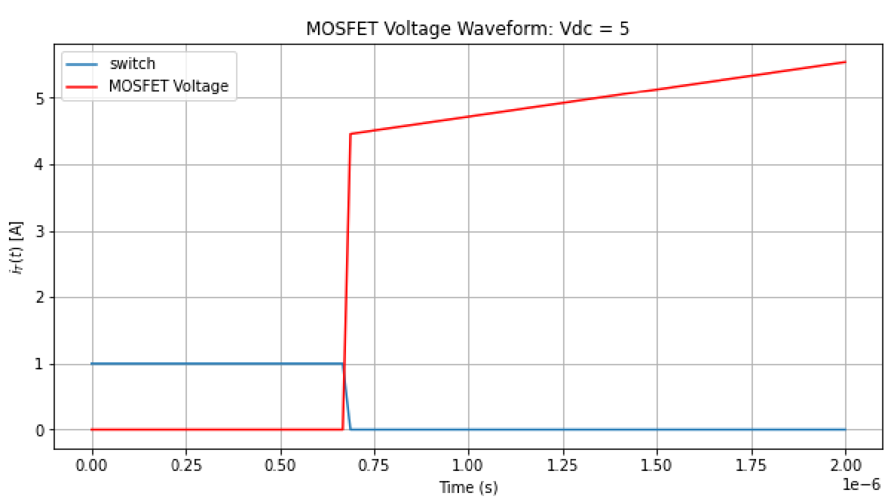
Total Volt Seconds over 1 period for Inductor Voltage

$$\int_o^{T_S} v_L(t)dt$$
$$0 = DT_S \cdot V_g + (1 - D)T_S \cdot (V_g - V)$$
$$0 = DV_g + D'(V_g - V)$$
$$0 = V_g(D + D') - VD'$$
$$0 = V_g - VD'$$
$$V = \frac{V_g}{D'} = \frac{V_g}{1 - D}$$

Solving for D:

$$1 - D = \frac{V_g}{V}$$
$$D = 1 - \frac{V_g}{V}$$
$$D = 0.34$$

(b) Sketch the waveform of the MOSFET drain-to-source voltage, v_{DS}.



(c) Find the dc component of the voltage waveform in Part b.

$$\int_{o}^{T_S} v_{FET}(t)dt = (1 - D)V = 3.3V$$