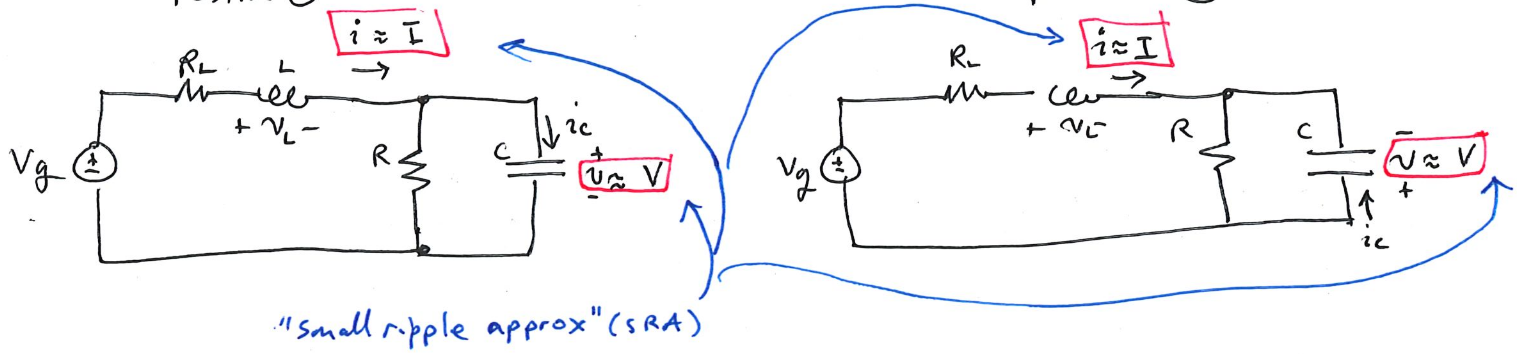
HW #2 Solution

Problem #1

1(a) Draw as equiv ckt.

- Look @ both configurations

Postion 1 — postion 2



- Volt sec balance w/ SRA

$$\langle v_L \rangle = 0 = (V_g - IR_L - V)D + (V_g - IR_L + V)(1-D) \qquad (1)$$

- Charge balance w/ SRA

$$\langle i_c \rangle = 0 = \left(I - \frac{V}{R}\right)D + \left(-I - \frac{V}{R}\right)(1-D) \qquad (2)$$

- Simplify (1)

$$0 = V_g(D+1-D) - IR_L(D+1-D) - VD + V - VD$$

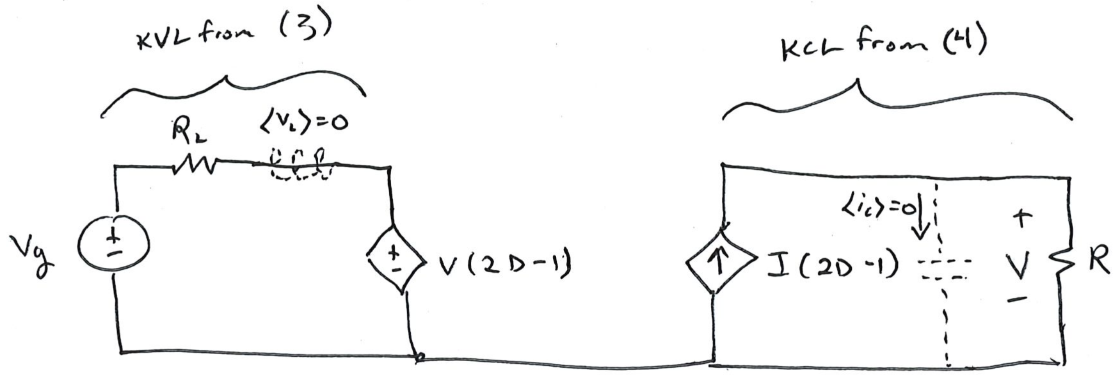
$$= \boxed{V_g - IR_L - V(2D-1) = 0 \qquad (3) \longrightarrow \text{KVL Loop eqn}}$$

- Simplify (2)

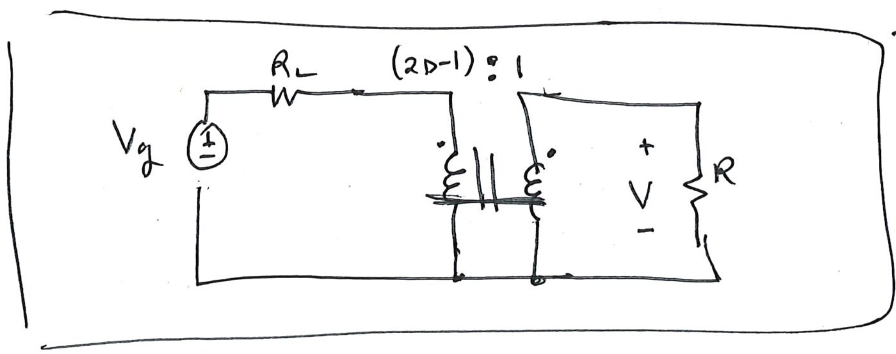
$$0 = ID - I + ID - \frac{V}{R}D - \frac{V}{R} + \frac{V}{R}D$$

$$= \boxed{I(2D-1) - \frac{V}{R} = 0 \qquad (4) \longrightarrow \text{KCL node eqn}}$$

$$\hookrightarrow I = \frac{V}{R}\frac{1}{2D-1} \qquad (5) \text{ save for later}$$



$\Downarrow$ equiv.



$\boxed{1(a)}$

Equivalent model

$-1(b)$ Solve for $\frac{V}{V_g}$.

(5) → (3) gives

$$0 = V_g - \frac{V R_L}{R}\frac{1}{(2D-1)} - V(2D-1) = V_g - V\left(\frac{R_L}{R(2D-1)} + \frac{(2D-1)^2 R}{R(2D-1)}\right)$$

$$= V_g - V\,\frac{R_L + R(2D-1)^2}{R(2D-1)} = 0$$

$$\rightarrow \frac{V}{V_g} = \frac{R(2D-1)}{R_L + R(2D-1)^2}$$

$$\boxed{= \frac{2D-1}{\frac{R_L}{R} + (2D-1)^2} = \frac{V}{V_g}, \quad (6)} \quad \boxed{1(b)}$$

note

$$\lim_{R_L \to 0} \frac{V}{V_g} = \frac{1}{2D-1}$$ → same as ideal transformer above!

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1 (c) Plot V/Vg

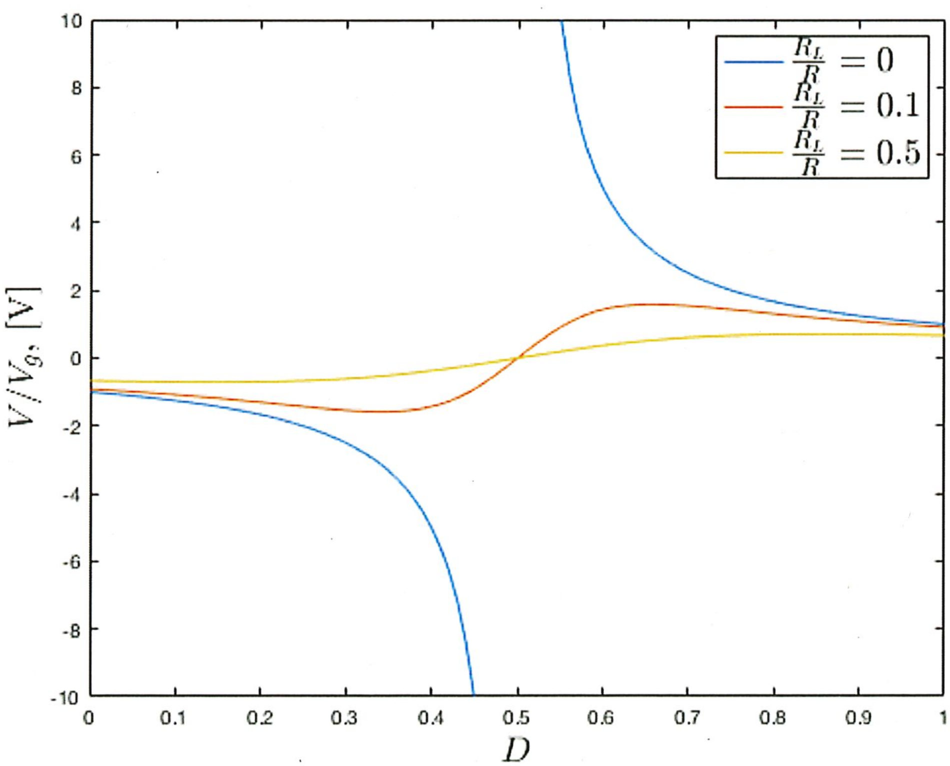
```
clear all
clc

Rratio = [0 0.1 0.5];
D = 0:0.01:1;

VoverVg = zeros(length(Rratio), length(D));

for i = 1:3
    VoverVg(i,:) = (2*D-1)./(Rratio(i) + (2*D-1).^2);
end
```

```
close all
plot(D,VoverVg(1,:), D,VoverVg(2,:), D,VoverVg(3,:))
xlabel('$D$','Interpreter','latex','fontsize',18);
ylabel('$V/V_g$, [V]','Interpreter','latex','fontsize',18);
ylim(10*[-1 1])
legend({'$\frac{R_L}{R} = 0$', '$\frac{R_L}{R} = 0.1$',...
    '$\frac{R_L}{R} = 0.5$'}, 'Interpreter','latex', 'fontsize',18)
```



1 (d) Compute η

$$P_{out} = \frac{V^2}{R}$$

& $P_{in} = V_g I$

* use (4)

$$= \frac{V_g V}{R} \frac{1}{2D-1}$$

$$\Rightarrow \eta = \frac{P_{out}}{P_{in}} = \frac{V^2}{\cancel{R}} \frac{\cancel{R}(2D-1)}{V_g V} = \frac{V}{V_g}(2D-1)$$

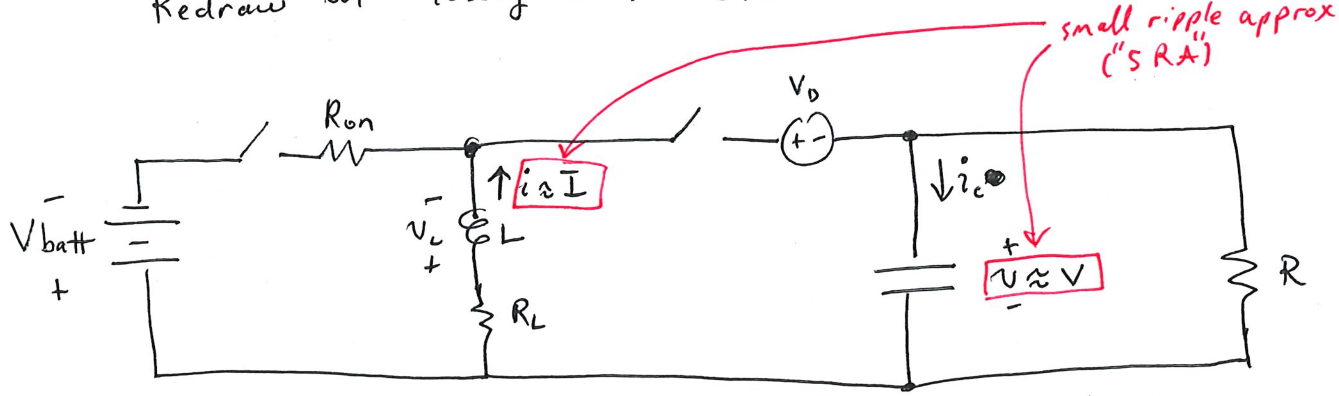
* use (6)

$$= \boxed{\frac{(2D-1)^2}{\left(\frac{R_L}{R} + (2D-1)^2\right)} = \eta} \quad \boxed{1(d)}$$

5 4

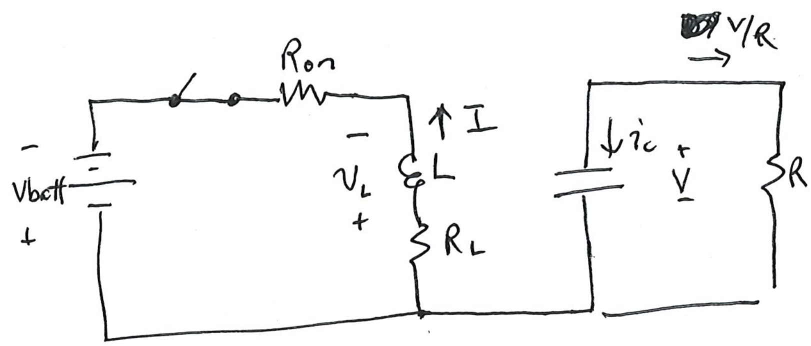
Problem #2

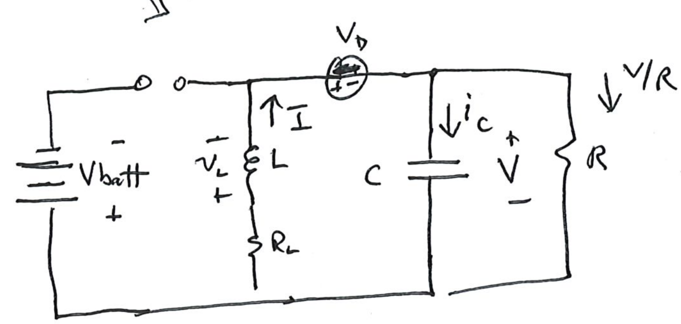
Redraw w/ lossy ckt elements



configuration ① duration DT

Conf. ②, duration $(1-D)T = D'T$





- Volt sec balance w/ "SRA"

$$0 = \langle v_L \rangle = (V_{batt} - IR_L - IR_{on})D + (-V - IR_L - V_D)(1-D)$$

$$V_{batt}D - IR_L - IR_{on}D - V\underbrace{(1-D)}_{D'} - V_D\underbrace{(1-D)}_{D'}$$

KVL loop

$$\boxed{= V_{batt}D - IR_L - IR_{on}D - VD' - V_D D' = 0} \quad (1)$$

- Charge balance w/ SRA

$$0 = \langle i_c \rangle = \left(-\frac{V}{R}\right)D + \left(I - \frac{V}{R}\right)(1-D)$$

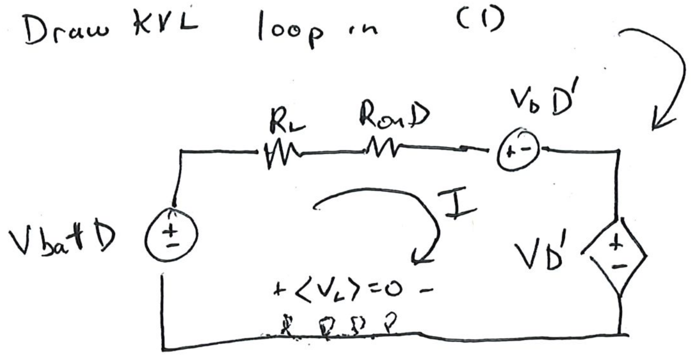
$$= \boxed{-\frac{V}{R} + I(1-D) = 0} \quad (2)$$

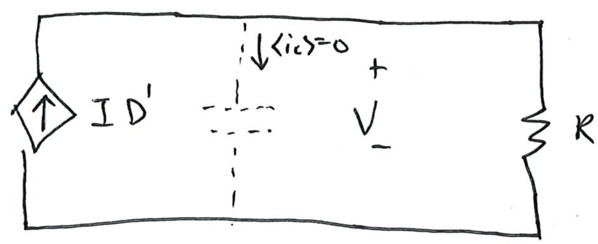
KCL node eqn.

2(a) Make equiv ckt

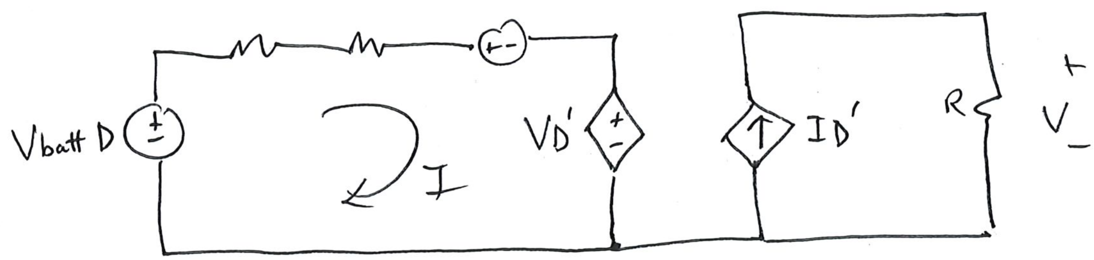
Draw KVL loop in (1)

Draw KCL node eqn in (3)

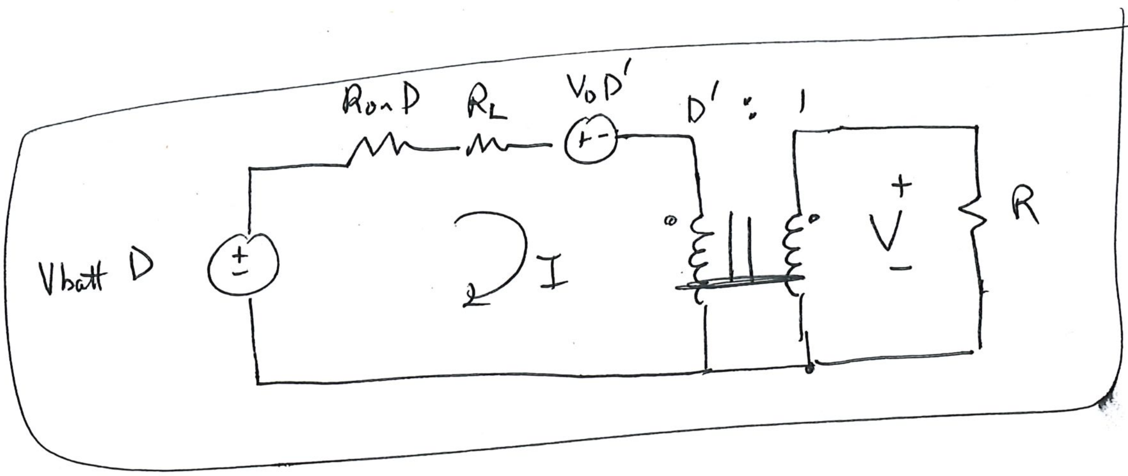




combine



⇓ equiv ckt w/ transformer



2 (d)

2 (b) Solve for V & η

Rearrange (2) to get I

$$I = \frac{V}{R}\frac{1}{1-D}$$

$$= \frac{V}{RD'} \qquad (3)$$

Put (3) → (1)

$$0 = V_{batt}D - \frac{V}{RD'}(R_L + R_{on}D) - VD' - V_D D'$$

$$= V_{batt}D - V\underbrace{\left(\frac{R_L}{RD'} + \frac{R_{on}}{R}\frac{D}{D'} + D'\right)}_{= \frac{R_L + R_{on}D + RD'^2}{RD'}} - V_D D'$$

$$\Rightarrow \boxed{V = \frac{RD'}{R_L + R_{on}D + RD'^2}\left(V_{batt}D - V_D D'\right)} \quad \boxed{2b} \qquad (4)$$

Now get η. First get P_{in} & P_{out} from equiv ckt.

$$P_{in} = I\,V_{batt}D \quad \& \quad P_{out} = V^2/R$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{V^2}{R}\frac{1}{I\,V_{batt}D}$$

* use (3) for I

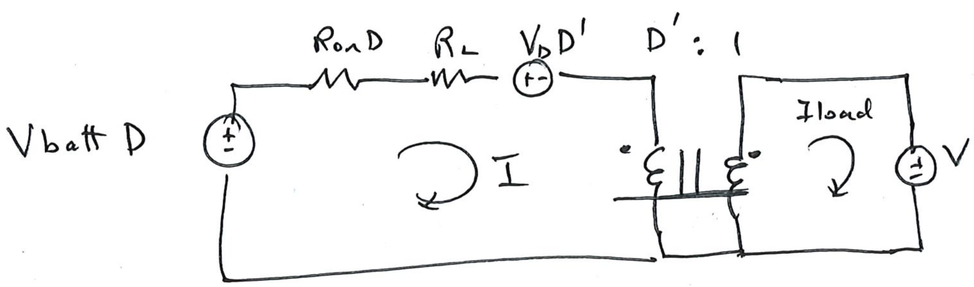
$$= \frac{V^{\cancel{2}}}{\cancel{R}}\frac{1}{V_{batt}D}\frac{\cancel{R}D'}{\cancel{V}} = \frac{D'}{V_{batt}D}V$$

use last result in (4)

$$\boxed{= \frac{D'}{V_{batt}D}\frac{RD'}{R_L + R_{on}D + RD'^2}\left(V_{batt}D - V_D D'\right) = \eta} \quad \boxed{2c} \qquad (5)$$

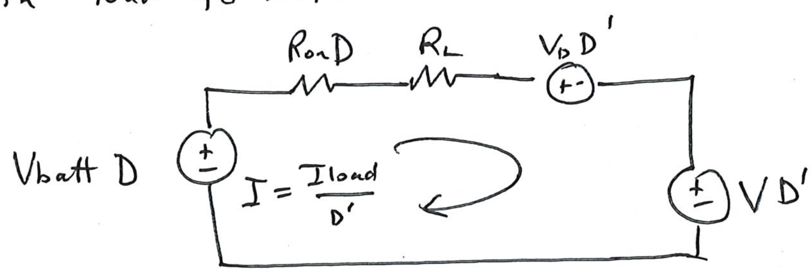
2 (c) Solve for $R_{L\,max}$ s.t $\eta > 0.8$

Equiv ckt becomes :



we know $\frac{I_{load}}{I} = D' \rightarrow I = \frac{I_{load}}{D'}$

• push load to LHS



now

$$P_{out} = \frac{I_{load}}{D'} V D' = I_{load} V \qquad (6)$$

& $$P_{in} = P_{out} + P_{loss} = I_{load} V + \left(\frac{I_{load}}{D'}\right)^2 R_{on} D + \left(\frac{I_{load}}{D'}\right)^2 R_L + \frac{I_{load}}{D'} V_D D' \qquad (7)$$

$$\Rightarrow \eta = \frac{P_{out}}{P_{in}} = \frac{I_{load} V}{I_{load} V + \left(\frac{I_{load}}{D'}\right)^2 R_{on} D + \left(\frac{I_{load}}{D'}\right)^2 R_L + \frac{I_{load}}{\cancel{D'}} V_D \cancel{D'}}$$

$$= \frac{I_{load} V D'^2}{I_{load} V D'^2 + I_{load}^2 R_{on} D + I_{load}^2 R_L + I_{load} V_D D'^2}$$

$$= \eta > \underbrace{\eta_{min} = 0.8}_{\text{min } \eta \text{ allowed}} \qquad (8)$$

Rearrange inequality in (8) to solve for R_L

$$\frac{I_{load} V D'^2}{\eta_{min}} < I_{load} V D'^2 + I_{load}^2 R_{on} D + I_{load}^2 R_L + I_{load} V_D D'^2$$

$$\rightarrow \quad R_L < \left(\frac{I_{load} V D'^2}{\eta_{min}} - I_{load} D'^2 (V + V_D) - I_{load}^2 R_{on} D \right) \frac{1}{I_{load}^2}$$

$$= \left(\frac{V D'^2}{I_{load} \eta_{min}} - \frac{(V+V_D) D'^2}{I_{load}} - R_{on} D \right)$$

$$= 0.0531 \, \Omega$$

$$\rightarrow \quad \boxed{\text{need } \quad R_L < 53.1 \, m\Omega} \quad \boxed{2\,(c)}$$