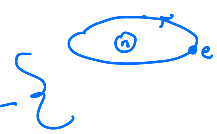
Lecture # 20, 11/17/2021

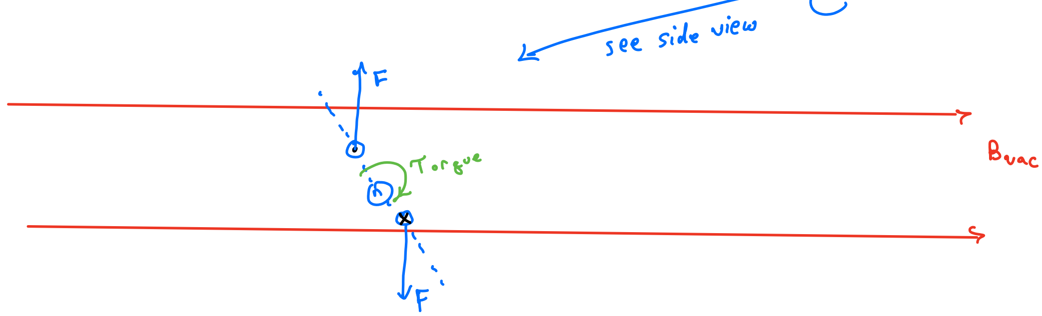
- Midterm
  - ↳ hope to have grades soon.

- Last Time
  - ↳ magnetics fundamentals

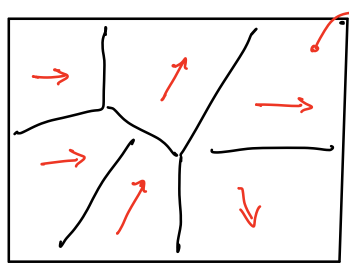
- Today
  - ↳ Magnetic circuit analogy
  - ↳ Gapped inductor

– Magnetic dipole torque action





– Ferromagnetic material domains

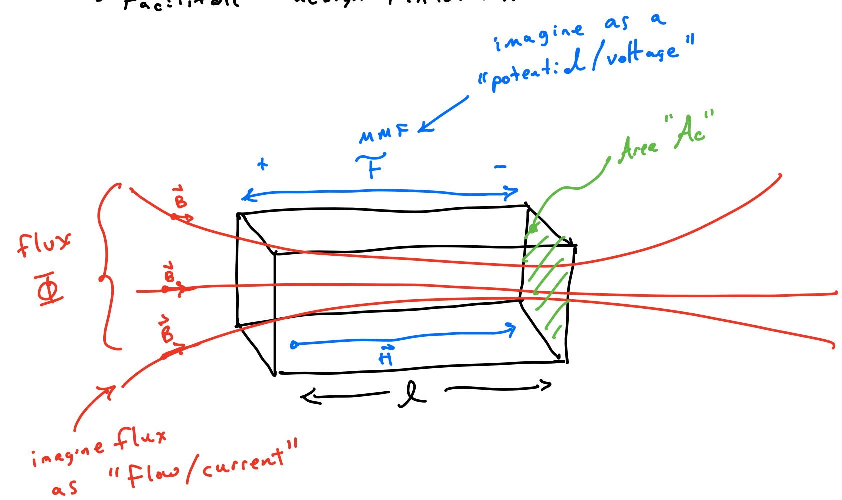


all atoms in a domain have $\vec{B}$ pointing in same direction

- why ?
  - ↳ Quantum mechanics

– Magnetic Circuit Analogy

- Facilitate design / intuition



$$\mathcal{F} = H\ell \quad \text{"across ends"} \qquad (1)$$

Recall

$$H = \frac{B}{\mu} \quad \& \quad B = \frac{\Phi}{A_c}$$

Rewrite (1)

$$\mathcal{F} = \underbrace{\frac{B}{\mu}}_{H}\ell = \overbrace{\frac{\Phi}{\mu A_c}}^{H}\ell$$

* reshuffle

$$= \underbrace{\frac{\ell}{\mu A_c}}_{=:\,\mathcal{R}}\Phi$$

"Reluctance"

Similar to $R = \dfrac{\text{length}}{\text{conductivity} \cdot \text{area}}$

$$= \boxed{\mathcal{R}\,\Phi = \mathcal{F}}$$

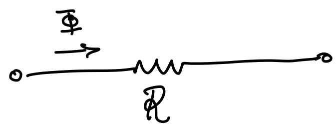
magnetic analogy

Like Ohm's Law

$$R\,I = V$$

electric

Redraw as a ckt:



$+ \quad \mathcal{F} = \Phi \mathcal{R} \quad -$

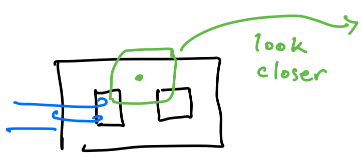
- Strategy for complex core structures
  - Break up core into "chunks" & compute $\mathcal{R}$ for each piece
  - windings are "sources" of MMF, draw as voltage sources
  - Solve ckt for flow(s) Φ in each piece

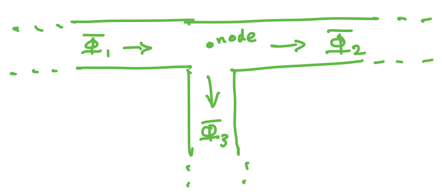
- Analogies to Kirchhoff's Laws:

  * KCL - like law

$$\sum \Phi_k = 0 \quad @ \text{ any node}$$

example:





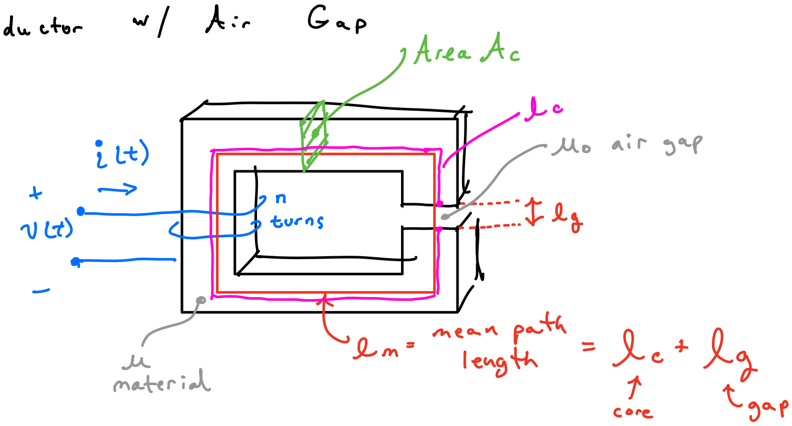
$$\Phi_1 = \Phi_2 + \Phi_3$$

* KVL - like law

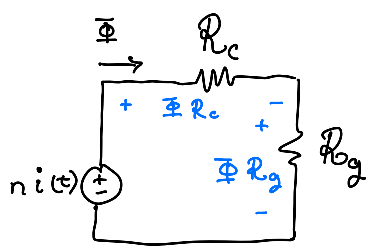
Ampere says

$$\underbrace{\oint \vec{H}\cdot d\vec{l}}_{\substack{\text{sum } \mathcal{R}_k \Phi_k \\ \text{"drops"}}} = \underbrace{i_{enclosed}}_{\substack{\text{sum up to } n_k i_k \\ \text{"sources" in loop}}}$$

— Inductor w/ Air Gap



Redraw as a ckt



Solve as a ckt

$$ni = \Phi(\mathcal{R}_c + \mathcal{R}_g)$$

where $\mathcal{R}_c = \dfrac{l_c}{\mu A_c}$

$$\mathcal{R}_g = \frac{l_g}{\mu_0 A_c}$$

Solve for Φ.

- Apply Faraday's Law

$$v(t) = n \frac{d\Phi}{dt}$$

$$* \text{ where } \quad \Phi = \frac{n\, i(t)}{R_c + R_g}$$

$$= n \frac{d}{dt}\left(\frac{n\, i(t)}{R_c + R_g}\right)$$

$$= \underbrace{\frac{n^2}{R_c + R_g}}_{L} \frac{d\, i(t)}{dt}$$

where

$$L = \frac{n^2}{R_c + R_g}$$

$$= \frac{n^2}{\frac{l_c}{\underset{\mu_r \mu_0}{\mu} A_c} + \frac{l_g}{\mu_0 A_c}}$$

$$\boxed{= \frac{n^2}{\frac{1}{\mu_0 A_c}\left(\frac{l_c}{\mu_r} + l_g\right)} = L}$$

observe

- As $l_g \uparrow$, then $L \downarrow$

HW preview (get started now please)