Lecture # 4, 10/6/2021

Last time:

- 1st 1/2 of motor modeling doc.

  → Finish 2nd 1/2 on Friday.

Today

- Ch 2, "balance equations"

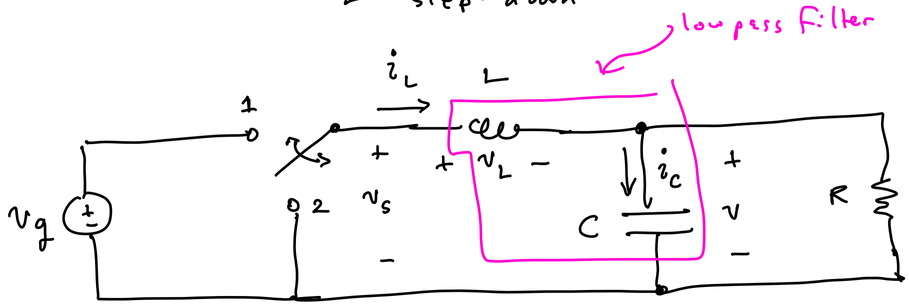
- HW1 due next Monday 11:59 pm PT.

Chapter 2 → Steady-state Analysis

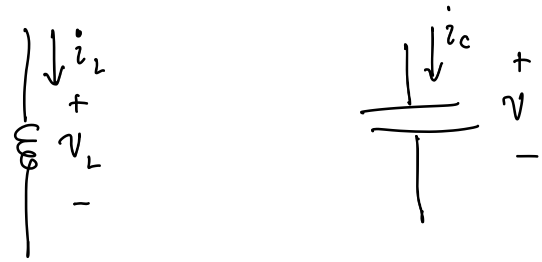
periodic steady state due to switching

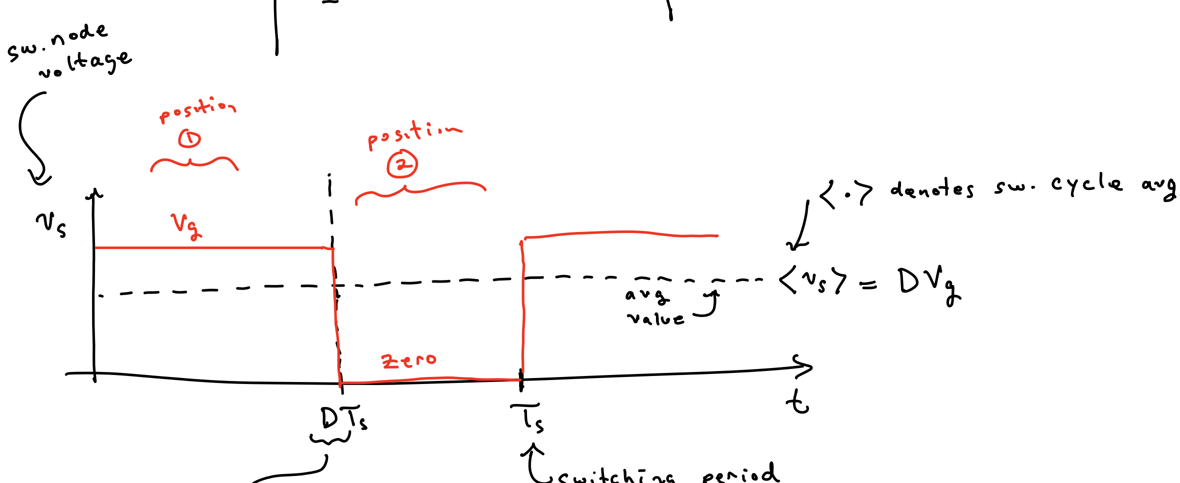
Consider the "Buck" converter

step-down



A comment on polarity / sign conventions for analysis





D is "duty ratio"

$0 < D < 1$

$$D = \frac{\text{time on}}{T_s}$$

$f_s = T_s^{-1}$, typically kHz to MHz range

Avg value = dc component

$$\langle v_s \rangle = \frac{1}{T_s} \int_0^{T_s} v_s(t)\, dt$$

$$= \frac{1}{T_s} \left(\int_0^{DT_s} V_g \, dt + \cancel{\int_{DT_s}^{T_s} \text{zero} \, dt}^{\,0} \right)$$

$$= \frac{1}{T_s} V_g \underbrace{\int_0^{DT_s} 1 \, dt}_{DT_s}$$

$$= \frac{D\cancel{T_s}}{\cancel{T_s}} V_g$$

$$= DV_g$$

Q: What about the output voltage?

A: LC is a lowpass filter. Dc component of sw. terminal voltage appears @ output.

$$\Rightarrow v = DV_g$$

$$\rightarrow \underbrace{\boxed{\frac{v}{V_g} = \frac{\text{output}}{\text{input}} = D}}_{\text{ratio called}}$$

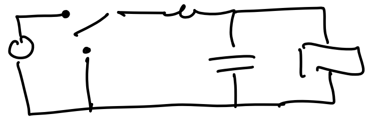
specific to buck

"M" in book $\rightarrow$ s.s. voltage conversion ratio

* Need a more generalized method to derive $\frac{v}{v_g} = M$ for <u>any</u> converter

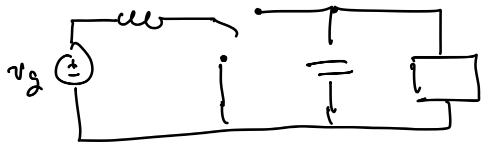
Overview of 3 common converters

Buck :



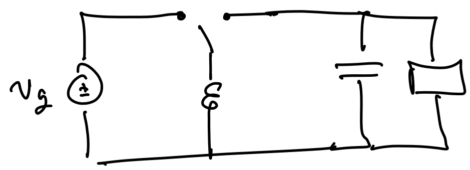
$M = D$

Boost :



$M = \frac{1}{1-D} > 1$

Buck Boost :



$M = \frac{-D}{1-D}$

General method to derive M hinges on

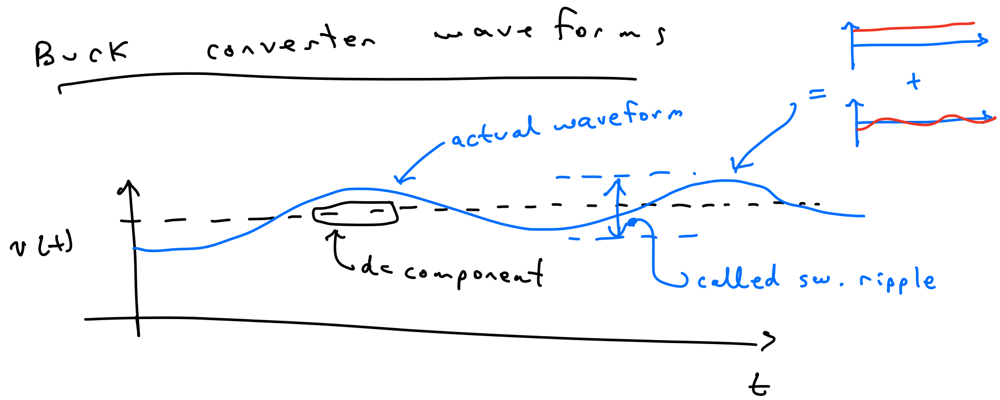
→ volt-second balance for L's

→ charge (AKA amps-seconds) balance for C's

Intuition:

Energy doesn't build up in L's & C's of the converter. Input energy goes to output.

– Buck converter waveforms



$$v(t) = \underset{\substack{\uparrow \\ \text{dc comp.}}}{V} + v_{ripple}(t)$$

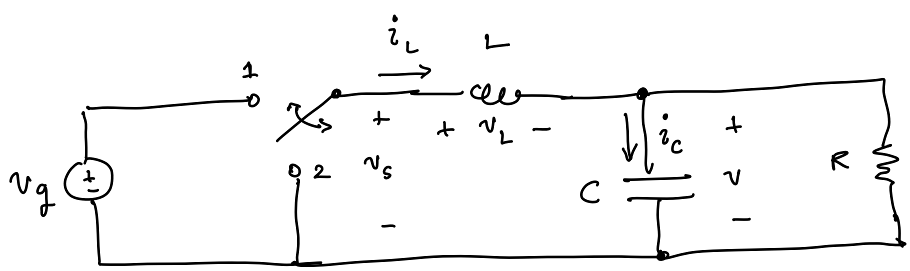
In most designs $\| v_{ripple} \| < V$

and $v(t) \approx V$

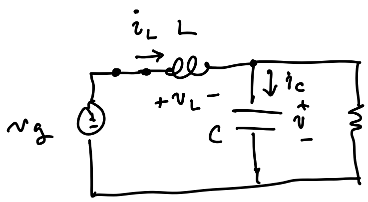
– General Method to Derive S.S. $\underbrace{\text{Input-Output}}$ Relations

and get all S.S. I's & V's in ckt

- Start w/ a given converter ckt.
- Draw diagram for each sw. config. & compute ckt voltages & currents



config ①



- Look @ L voltage

$$v_L = v_g - v \qquad (1)$$

b/c small ripple @ output

$$v \approx V = \text{dc component}$$

Hence, (1) becomes

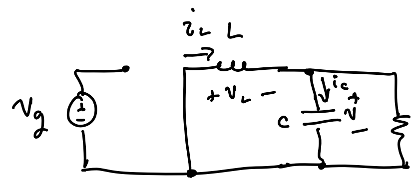
$$v_L = V_g - V \qquad (2a)$$

$$L \frac{di_L}{dt} = v_L \qquad (3a)$$

$$= \underbrace{V_g - V}_{\text{constant}}$$

$$\Rightarrow \underset{\text{slope}}{i_L} = \frac{di_L}{dt} = \underbrace{\frac{V_g - V}{L}}_{\oplus \text{ slope}}$$

config ②



- Inductor voltage

$$v_L = -v$$

∴∴ small ripple ...

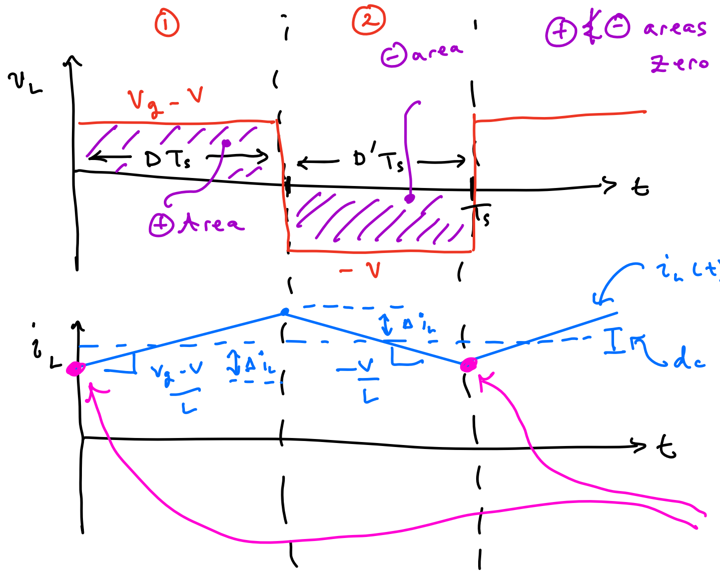
$$v_L = -V \qquad (2b)$$

$$L \frac{di_L}{dt} = v_L \qquad (3b)$$

$$= -V$$

$$\underbrace{\frac{di_L}{dt} = \frac{-V}{L}}_{\ominus \text{ slope}}$$

Look @ L waveforms



⊕ & ⊖ areas are equal & must avg to zero over 1 cycle.

where

$D' = 1 - D$

$i_L(t)$ looks like a piecewise linear waveform

same value of i_L. Since $E_L = \frac{1}{2} L i_L^2$, energy stored at these 2 points across a cycle is the same.

Principle of volt-second balance

know $v_L = L \frac{di_L}{dt} \rightarrow \frac{di_L}{dt} = \frac{v_L}{L}$

in integral form $\rightarrow$ $\overbrace{\underbrace{i_L(T_s)}_{\text{init.}} - \underbrace{i_L(0)}_{\text{final}}}^{\text{zero}} = \frac{1}{L} \int_0^{T_s} v_L(t)\,dt$

b/c final & initial values same in steady state

$$\Rightarrow \quad 0 = \frac{1}{L} \int_0^{T_s} v_L(t)\,dt$$

$\div$ by L & $\times$ by $\frac{1}{T_s}$

rewrite as $0 = \frac{1}{T_s} \underbrace{\int_0^{T_s} v_L(t)\,dt}_{\text{area under curve}} = \langle v_L \rangle$

Area above & below in rectangles must be equal

$$\lambda = \int_0^{T_s} v_L(t)\,dt = \underbrace{(V_g - V)(D\cancel{T_s}) + (-V)(D'\cancel{T_s}) = 0}_{\text{volt-sec balance eqn.}}$$

area under v_L curve

zero in s.s.

Solve for V

$$0 = (V_g - V)D - V\underbrace{(1-D)}_{D'}$$

$$= DV_g - V$$

$$\Rightarrow V = DV_g$$

$$\rightarrow M = \frac{V}{V_g} = \frac{D\cancel{V_g}}{\cancel{V_g}} = D$$

- What about Cap?

- Principle of cap charge balance

$$i_c(t) = C\frac{dv(t)}{dt} \rightarrow \frac{dv}{dt} = \frac{i_c}{C}$$

integral Form

$$\underbrace{v(T_s) - v(0)}_{\text{zero}} = \frac{1}{C}\int_0^{T_s} i_c(t)\,dt$$

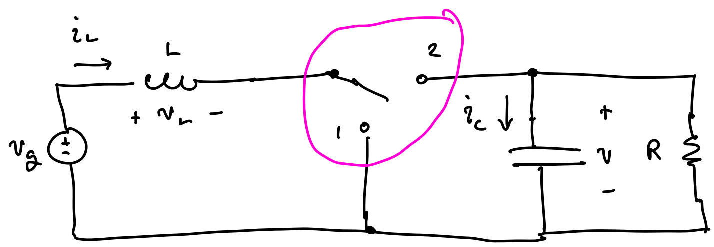
same

$$0 = \frac{1}{C}\int_0^{T_s} i_c(t)\,dt \qquad \times \text{ by } C, \div \text{ by } T_s$$

$$0 = \frac{1}{T_s}\underbrace{\int_0^{T_s} i_c(t)\,dt}_{\text{charge}} = \langle i_c \rangle \text{ Avg value}$$

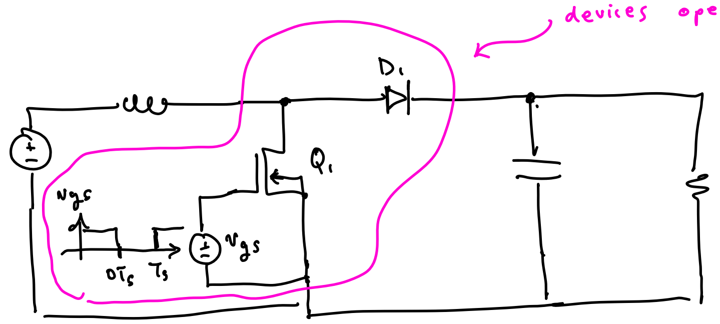
$$\overbrace{\text{charge}}^{Q} = \int_0^{T_s} i_c(t)\, dt$$

- Boost Example to see both balance equations



realization

devices operate complementarily

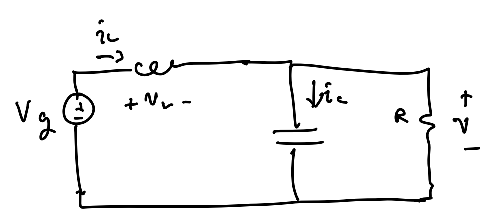


Back to analysis

config ①

config ②

Duration DT_s



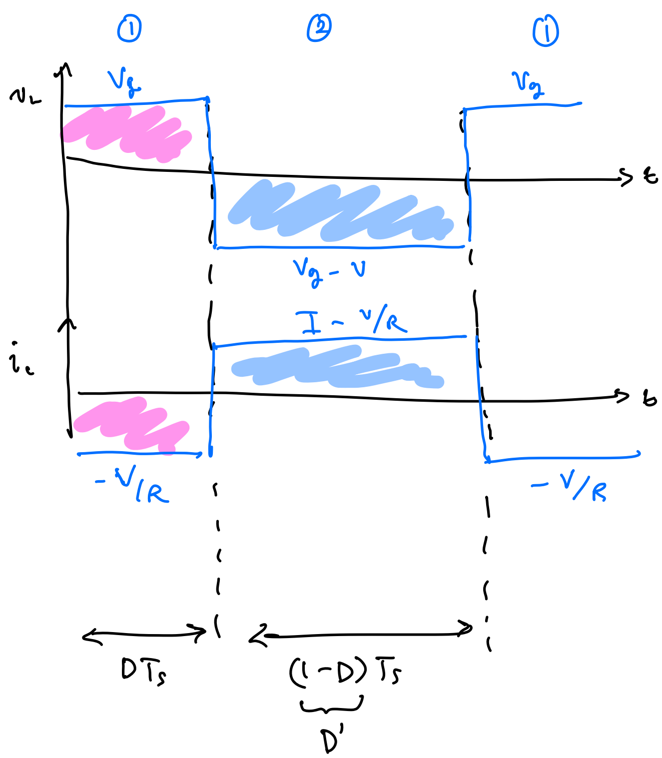
Duration $D'T_s$

inductor v_L & cap i_c	inductor v_L & cap i_c
$v_L = V_g$	$v_L = V_g - v$
$i_c = -v/R$	$i_c = i_L - v/R$
$v_L = V_g$	$v_L = V_g - V$
$i_c = -V/R$	$i_c = I - \frac{V}{R}$

apply small ripple approx

$\hookrightarrow$ $v \approx V$, $i_L = I$

Look @ waveforms



Write balance equations

volt second:

$$0 = \underbrace{(V_g)(D\cancel{T_s})} + \underbrace{(V_g - V)(1-D)\cancel{T_s}}$$

solve for V

$$0 = V_g(\cancel{D} + 1 - \cancel{D}) - V(1-D)$$

$$\rightarrow \quad V = \frac{V_g}{1-D} \quad (*)$$

$$\rightarrow \boxed{M(D) = \frac{V}{V_g} = \frac{1}{1-D}}$$

Boost conversion

Charge balance ... needed to set I

$$0 = \underbrace{(-V/R)(D\cancel{T_s})} + \underbrace{(I - V/R)(1-D)\cancel{T_s}}$$

solve for I

$$0 = -\frac{V}{R}(\cancel{D} + 1 - \cancel{D}) + I(1-D)$$

$$I = \frac{V}{(1-D)R}$$

substitute expression of V ... use (*)

$$\Longrightarrow \boxed{I = \frac{V_g}{(1-D)^2 R}}$$