Lab #2 - A

1. Load and Sensing Circuitry

Design calculations for voltage sensing divide for the load voltage measurement. Clearly show how 60 V upper limit was translated into the parameters for your divider. (10 pts)

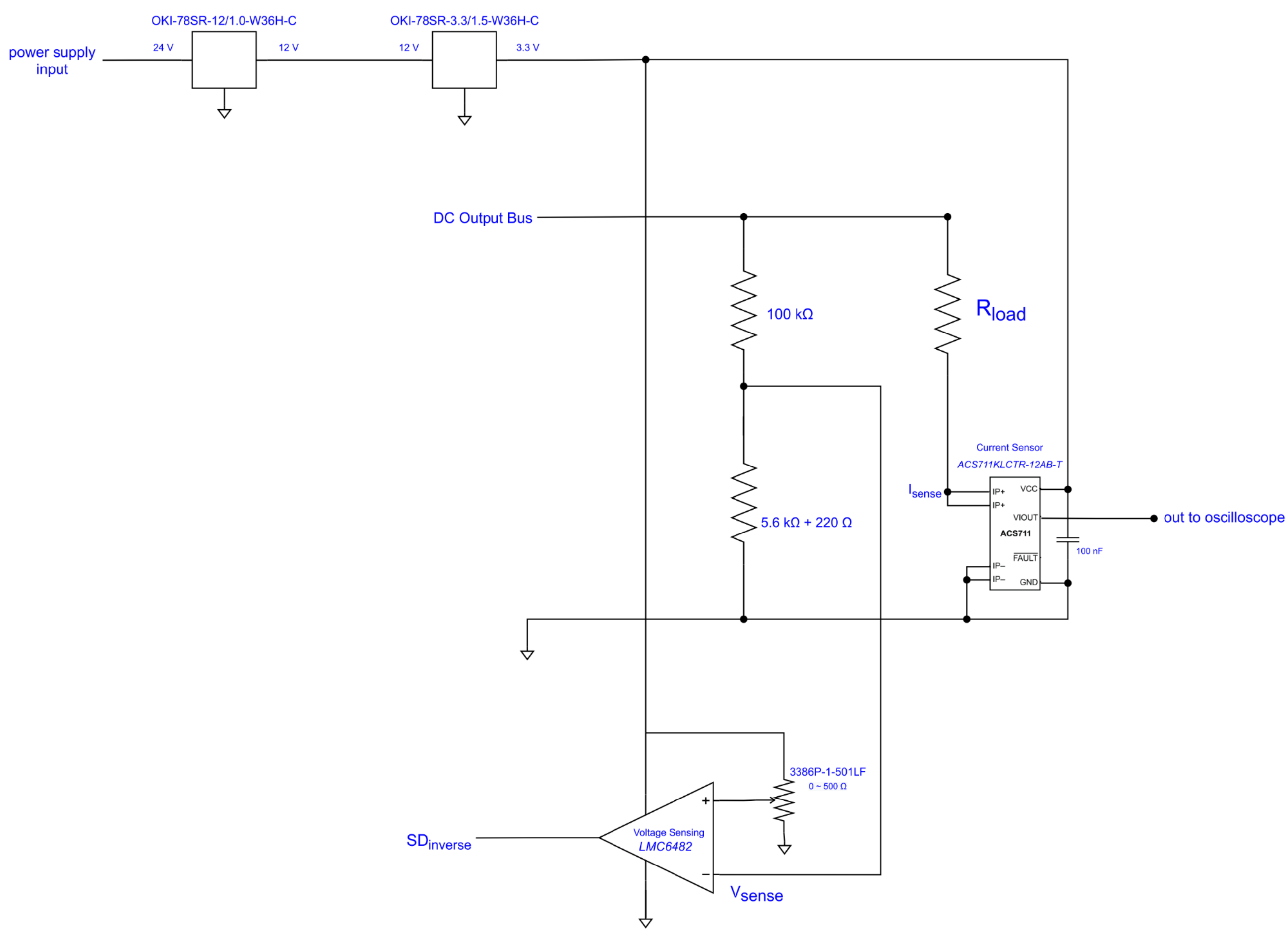
$R_{divider}$ = 100 kΩ + 5.6 kΩ + 220 Ω = 105.82 kΩ

V_{sense} = V_{in} (5.6k + 220) / (5.6k + 220 + 100k) = V_{in} (0.055)

For V_{in} = 24 V → V_{sense} = (24 V)(0.055) = 1.32 V

For V_{in} = 60 V → V_{sense} = (60 V)(0.055) = 3.3 V

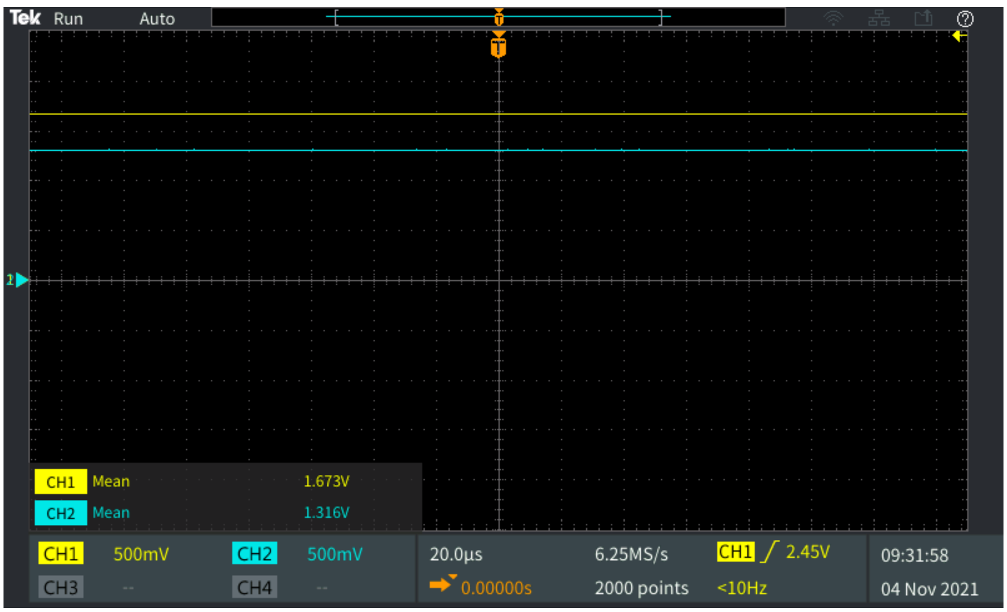
A schematic showing the details of your load and sensing circuitry. All parts and values should be clearly labeled. (10 pts)



2. Protection Logic

Capture 1: The current sensor voltage output and the scaled, sensed load voltage using the voltage divider circuit. Also record the used load resistance (20 pts) Calculate the expected output voltage of the current sensor and show your computations. Based on the expected current through the load, compare the expected and measured output voltages and comment on any differences. See the datasheet for sensitivity and details on voltage output. (10 pts)

CH1: Current Sense Out
CH2: V_{Sense}
R_{load} = 100 Ω
V_{in} = 24 V



For 0 Amps through the current sensor, the expected output is:
$V(0\ \text{Amps}) = \frac{1}{2} V_{cc} = \frac{1}{2}\ 3.3\ \text{V} = 1.65\ \text{V}$

The sensitivity of the device is 110 mV/A, thus the expected output at 0.24 Amps is:
V(0.24 Amps) = (110 mV / A) (0.24 A) + V(0 Amps) = 0.0264 V + 1.65 V = 1.6764

Current Sense Out: Expected - Measured = 1.6764 V - 1.673 V = 3.4 mV
V_{Sense}: Expected - Measured = 1.32 V - 1.316 V = 4 mV

Considering the accuracy of the oscilloscope DCV measurements, a difference of a few millivolts can be considered negligible. Thus, the current sensor and voltage divider are determined to be behaving as expected.

Capture 2/3: Capture the output voltage of the comparator and the reference voltage input in two instances one when the reference is greater than input and the other when it is lesser than input. Each capture should show the reference voltage, the input voltage, and the comparator output voltage. (20 pts)

CH1: V_{ref}
CH2: Comparator Output
CH3: V_{sense}

Image 1:

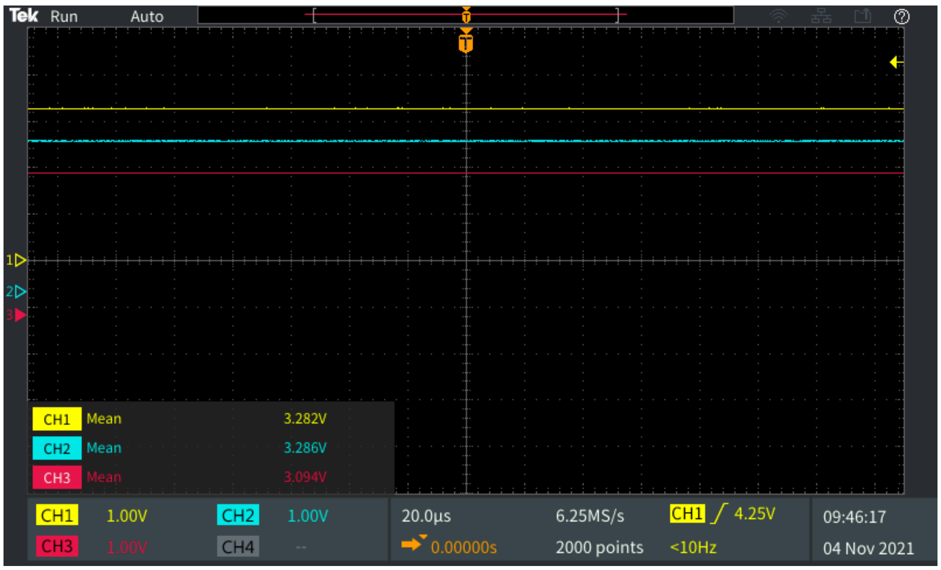
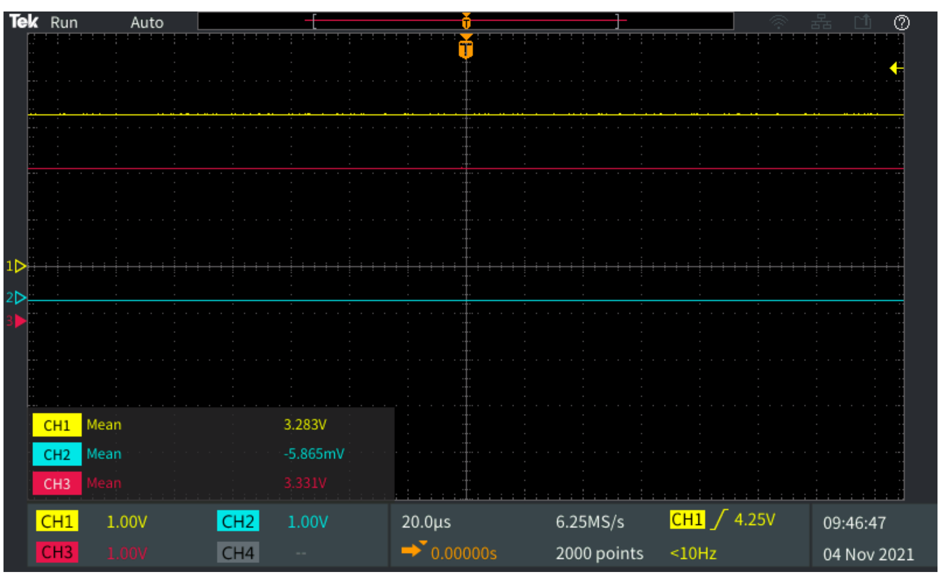


Image 2:



These captures were taken while the comparator was being tested on a breadboard. After the desired behavior was obtained, the comparator was installed alongside a threshold varying potentiometer to our circuit board.
To test the Comparator functionality prior to installation, a separate fixed voltage divider was assembled such that 3.3 V would be the output of V_{sense} given a power supply input of 24 V. Thus the power supply output could be slewed to increase or decrease V_{sense}. V_{ref} was fixed at ~3.3 V (the output of the 3.3 V regulator).

When $V_{sense} < V_{ref}$ (Image 1), the output of the comparator is HIGH.
When $V_{sense} > V_{ref}$ (Image 2 - note channel offsets), the output of the comparator is LOW.

Thus our comparator displayed the desired functionality and behavior.

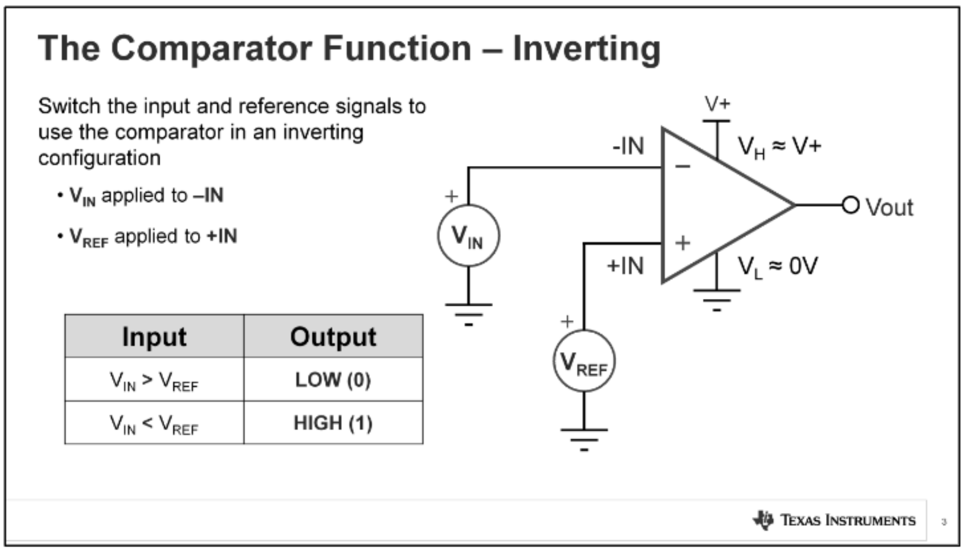
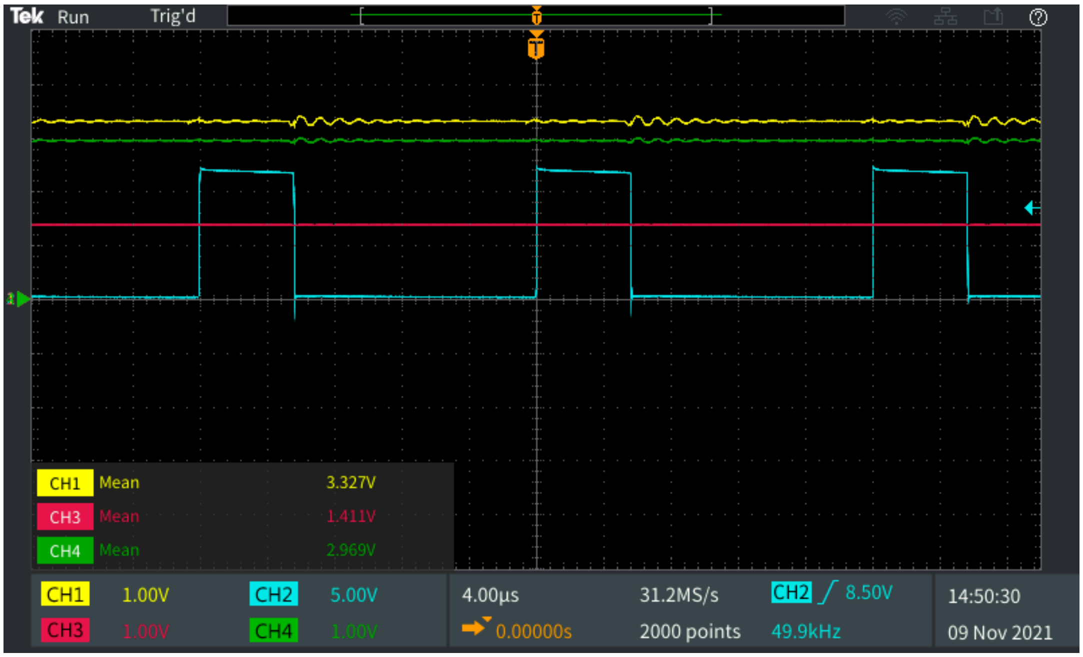
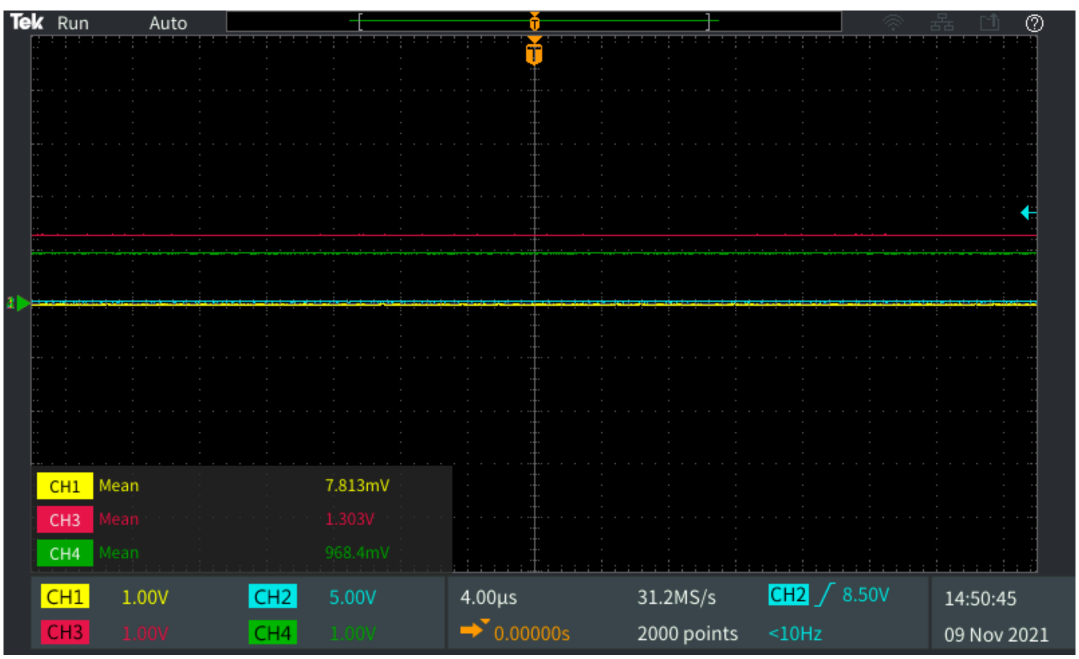


Figure: Comparator Function Reference

Capture 4: Capture the low-side gate driver output and show how it is being driven low when protection is active. Show functional protection at one threshold value (15-24V) by varying the potentiometer connected to your non-inverting input to show that it works over an adjustable range. (20 pts)

CH1: Comparator Output
CH2: Gate Driver LO
CH3: V_{sense}
CH4: V_{ref}





After obtaining the desired comparator functionality, the comparator was installed to our circuit board alongside a threshold varying potentiometer. Thus, by twisting the potentiometer, V_{ref} could be increased or decreased. Then, the pullup resistor tied to $SD_{inverse}$ on the gate driver was replaced with the output of the comparator, such that when the comparator is LOW, the Gate Driver is shut off.

When $V_{sense} < V_{ref}$ (Image 1), the output of the comparator is HIGH
and the Gate Driver is on

When $V_{sense} > V_{ref}$ (Image 2), the output of the comparator is LOW
and the Gate Driver is shut off

Note: With an input voltage of 24 V, V_{sense} was expected to be a steady 1.32 V, independent of the value of V_{ref}; however, due to an unfortuitous choice of potentiometer size, the value of V_{sense} did change slightly as V_{ref} was slewed, likely due to effective parallel resistances in the circuit. Although it likely would have been a better choice to have used a larger valued potentiometer, our choice does not significantly affect the functionality of the variable protection.

Optional for EE452 Take current sensor output voltage measurements for both 0 A and a different current. You should see a noticeable increase in voltage between the two. What is the range of possible voltage outputs from the current sensor? Is the 0A measurement in the middle? If not, what is the datasheet parameter that gives the value you found? (10pts)

Image 1:

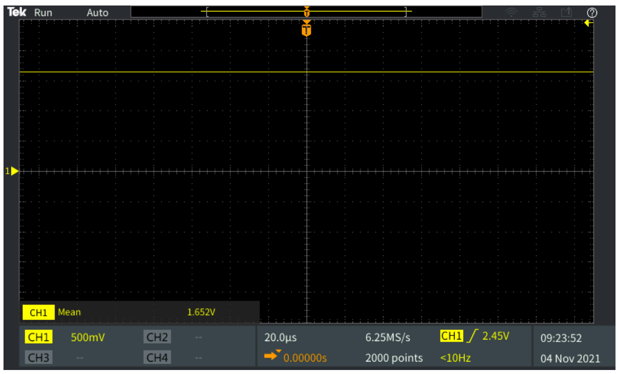
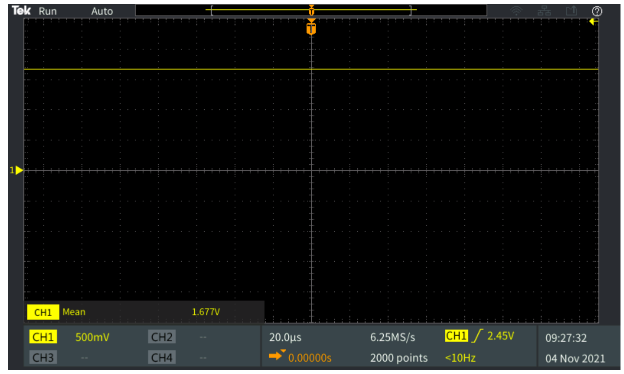


Image 2:



CH1: Current Sense Out
V_{in} = 24 V
R_{load} = 100Ω

For 0 Amps through the current sensor (image 1), the measured output is:
V(0 Amps) = 1.652 V

For 0.24 Amps through the current sensor (image 1), the measured output is:
V(0.24 Amps) = 1.677 V

The difference of 0.025 V corresponds to a measured current of (0.025 V) / (110 mV/A) = 0.23 A
This is very close to the anticipated 0.24 A.

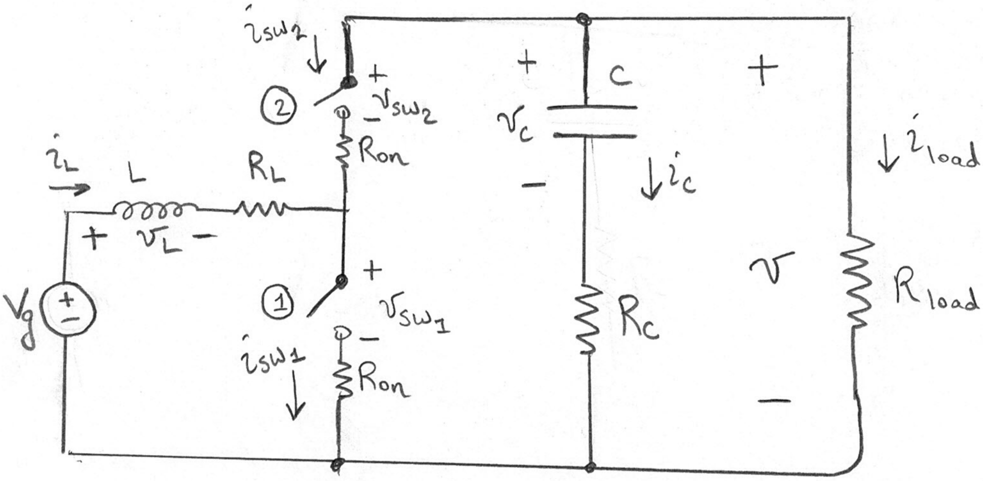
Note that the V(0 Amps) is within 2 mV of the expected midpoint. There exists an electrical offset voltage, $V_{OE(TA)}$ of ± 5 mV, which likely explains the offset.

The rated range of the device is -12.5 A to 12.5 A. For a 3.3V input, the rated output of the device is:

1.65 V ± (0.110 V/A * 12.5 A) = 1.65 V ± 1.375 V → [0.275V, 3.025V]

Characteristic	Symbol	Test Conditions	Min.	Typ.	Max.	Units
Optimized Accuracy Range	I_P		–12.5	–	12.5	A

Part (a) - Analytical Model



@ state 1

* $v_L = V_g - i_L R_L - i_L R_{on} \overset{SRA}{\approx} V_g - I_L R_L - I_L R_{on}$
* $i_c = -i_{load} = -\frac{v}{R_{load}} \overset{SRA}{\approx} -\frac{V}{R_{load}}$
* $i_{sw_1} = i_L \overset{SRA}{\approx} I_L$
* $v_{sw_2} = v - i_{sw_1} R_{on} = v - i_L R_{on} \overset{SRA}{\approx} V - I_L R_{on}$

@ state 2

* $v_L = V_g - i_L R_L - i_L R_{on} - v \overset{SRA}{\approx} V_g - I_L R_L - I_L R_{on} - V$
* $i_c = i_c = -i_{sw_2} - i_{Load} = i_L - i_{Load} = i_L - \frac{v}{R_{load}} \overset{SRA}{\approx} I_L - \frac{V}{R_{load}}$
* $i_{sw_2} = -i_L \overset{SRA}{\approx} -I_L$
* $v_{sw_1} = v - i_{sw_2} R_{on} = v + i_L R_{on} \overset{SRA}{\approx} V + I_L R_{on}$

- Using volt-second/charge-balance equations, derive an analytical model of the converter voltage conversion ratio V/V_g and plot the variation of the conversion ratio with variation of duty ratio D for $R_L = 40m\Omega, R_{on} = 10m\Omega, R_{load} = 50\Omega$. Grads use $R_{c,esr} = 0.1\Omega$ **(15 pts)**

→ by C a-s balance

$\langle i_c \rangle = \emptyset$ @ s.s.

$$D\left(-\frac{V}{R_{load}}\right) + D'\left(I_L - \frac{V}{R_{load}}\right) = \emptyset$$

$$D' I_L - \frac{V}{R_{load}} = \emptyset \Longrightarrow \underline{\underline{I_L = \frac{V}{D' R_{load}} = \frac{V_g}{R_L + R_{on} + (D')^2 R_{load}}}}$$

→ by L v-s balance

$\langle v_L \rangle = \emptyset$ @ s.s.

$$D(V_g - I_L R_L - I_L R_{on}) + D'(V_g - I_L R_L - I_L R_{on} - V) = \emptyset$$

$$V_g - I_L R_L - I_L R_{on} - D' V = \emptyset$$

substitute $I_L = \frac{V}{D' R_{load}}$

$$V_g - \frac{V}{D' R_{load}} R_L - \frac{V}{D' R_{load}} R_{on} - D' V = \emptyset$$

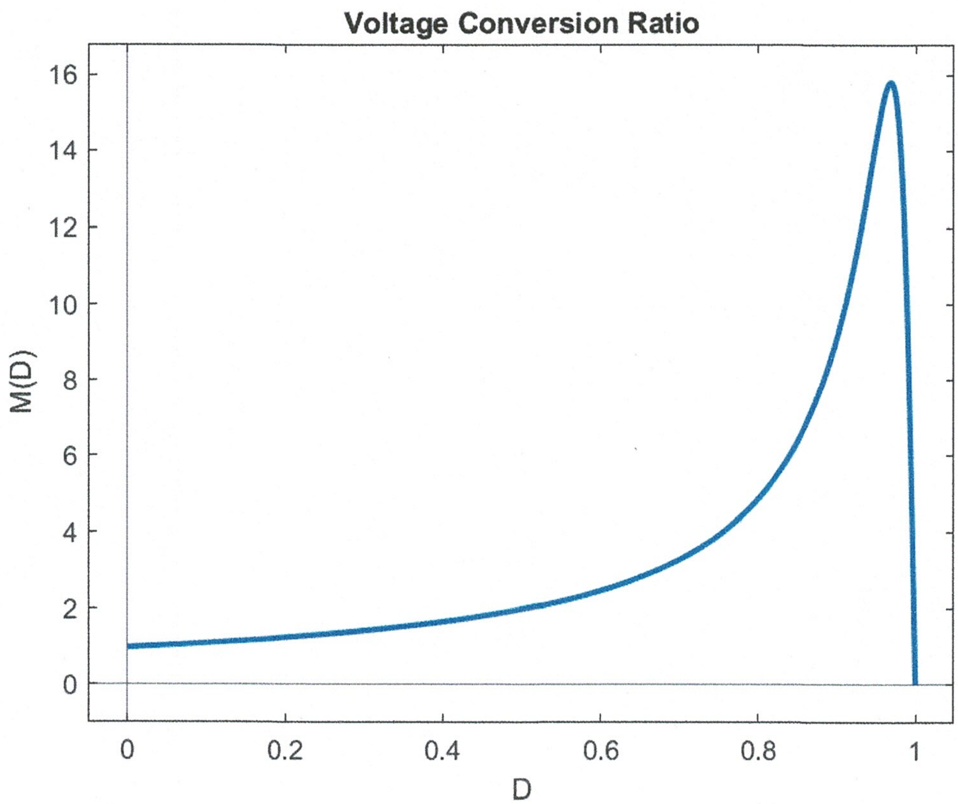
$$(V)\left(\frac{R_L + R_{on}}{D' R_{load}} + D'\right) = V_g \Longrightarrow V \, \frac{R_L + R_{on} + (D')^2 R_{load}}{D' R_{load}} = V_g$$

$$\Longrightarrow \underline{\underline{V = V_g \frac{D' R_{load}}{R_L + R_{on} + (D')^2 R_{load}}}}$$

$$M(D)_{\text{non-ideal}} = \frac{V_{out}}{V_{in}} = \frac{V}{V_g} = \frac{1}{V_g} \cdot V_g \frac{D' R_{load}}{R_L + R_{on} + (D')^2 R_{load}}$$

$$\boxed{M(D) = \frac{D' R_{load}}{R_L + R_{on} + (D')^2 R_{load}}}$$

figure shown on next page →


Voltage Conversion Ratio
M(D)
D
16
14
12
10
8
6
4
2
0
0
0.2
0.4
0.6
0.8
1

- Using the inductor current ripple equations, derive an analytical expression for the value of the inductance that will be included in your converter. **(5 pts)**

$$v_L = L\frac{di_L}{dt} \approx L\frac{2\Delta i_L}{\Delta t}$$

by SRA

$$V_L = L\frac{2\Delta I_L}{\Delta T} \Rightarrow L = \frac{V_L \Delta T}{2\Delta I_L} = \frac{(V_g - I_L R_L - I_L R_{on})(DT_s)}{2\Delta I_L} \text{ for } DT_s$$

$$\Rightarrow L = \frac{DT_s}{2\Delta I_L}\left(V_g - I_L(R_L + R_{on})\right)$$

where $I_L = \dfrac{V}{D' R_{load}}$

$$\boxed{L = \frac{DT_s}{2\Delta I_L}\left[V_g - \frac{V}{D' R_{load}}(R_L + R_{on})\right]}$$

where $T_s = \dfrac{1}{f_s}$

substituting $V = V_g \dfrac{D' R_{load}}{R_L + R_{on} + (D')^2 R_{load}}$

$$L = \frac{V_g D (D')^2 R_{Load} T_s}{(2\Delta I_L)(R_L + R_{on} + (D')^2 R_{load})}$$

- Give expressions for the average conduction loss and switching loss. Assume switching transition includes delay time and rise/fall time (details in lab lecture). (10 pts)

Conduction Loss

$$P_{cond,loss} = P_{cond,loss,1} + P_{cond,loss,2}$$

$$\boldsymbol{P_{cond,loss,1}} = I_{sw1,rms}^2 R_{on} = \boldsymbol{D\left(\frac{V}{D'R_{load}}\right)^2 R_{on}}$$

where:

$$I_{sw1,rms}^2 = \left(\sqrt{\frac{1}{T_s}\int_{T_s} i_{sw1}^2(t)\,dt}\right)^2 = \frac{1}{T_s}\int_{T_s} i_{sw1}^2(t)\,dt$$

$$= \frac{1}{T_s}[I_L^2 \cdot DT_S + 0 \cdot D'T_S] = DI_L^2 = D\left(\frac{V}{D'R_{load}}\right)^2$$

$$\boldsymbol{P_{cond,loss,2}} = I_{sw2,rms}^2 R_{on} = \boldsymbol{D'\left(\frac{V}{D'R_{load}}\right)^2 R_{on}}$$

where:

$$I_{sw2,rms}^2 = \left(\sqrt{\frac{1}{T_s}\int_{T_s} i_{sw2}^2(t)\,dt}\right)^2 = \frac{1}{T_s}\int_{T_s} i_{sw2}^2(t)\,dt$$

$$= \frac{1}{T_s}[0 \cdot DT_S + (-I_L)^2 \cdot D'T_S] = D'I_L^2 = D'\left(\frac{V}{D'R_{load}}\right)^2$$

So we have:

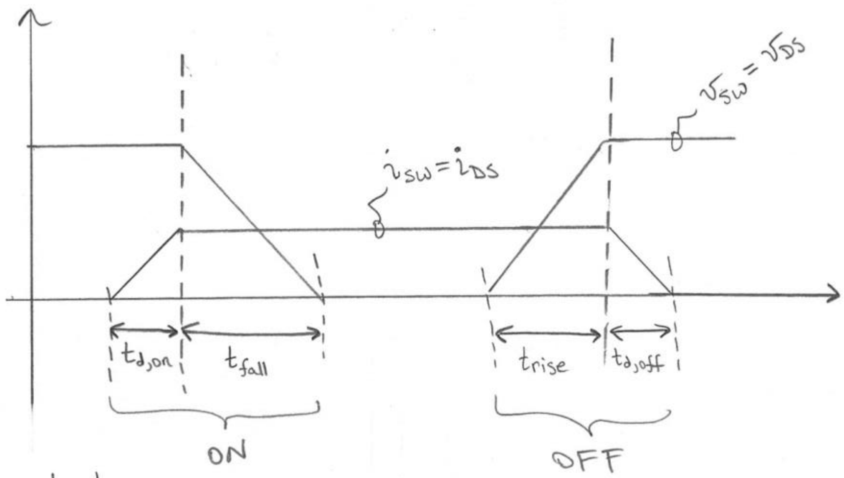
$$P_{cond,loss} = P_{cond,loss,1} + P_{cond,loss,2}$$

$$P_{cond,loss} = D\left(\frac{V}{D'R_{load}}\right)^2 R_{on} + D'\left(\frac{V}{D'R_{load}}\right)^2 R_{on}$$

$$P_{cond,loss} = R_{on}\left(\frac{V}{D'R_{load}}\right)^2 [D + D']$$

$$\boxed{\boldsymbol{P_{cond,loss} = R_{on}\left(\frac{V}{D'R_{load}}\right)^2}}$$

Switching Loss



for switch 1

$$\rightsquigarrow P_{sw_1,loss} = W_{sw_1,loss} f_s$$

$$W_{sw_1,loss} = W_{\substack{sw_1,loss \\ ON}} + W_{\substack{sw_1,loss \\ OFF}} = \frac{1}{2} v_{sw_1} i_{sw_1} \left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off} \right]$$

$$* W_{\substack{sw_1,loss \\ ON}} = \frac{1}{2}(t_{d,on})(i_{sw_1})(v_{sw_1}) + \frac{1}{2}(t_{fall})(v_{sw_1})(i_{sw_1})$$

$$= \frac{1}{2} v_{sw_1} i_{sw_1} \left[t_{d,on} + t_{fall} \right]$$

$$* W_{\substack{sw_1,loss \\ OFF}} = \frac{1}{2}(t_{rise})(v_{sw_1})(i_{sw_1}) + \frac{1}{2}(t_{d,off})(i_{sw_1})(v_{sw_1})$$

$$= \frac{1}{2} v_{sw_1} i_{sw_1} \left[t_{rise} + t_{d,off} \right]$$

$$W_{sw_1,loss} = \frac{1}{2} v_{sw_1} i_{sw_1} \left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off} \right]$$

$* v_{sw_1} = V + I_L R_{on} \Rightarrow v_{sw_1} = V$ (disregard R_{on} b.c. it is already accounted for in conduction loss)

$* i_{sw_1} = I_L$

$$W_{sw_1,loss} = \frac{1}{2} V I_L \left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off} \right]$$

$$W_{sw1,loss} = \frac{1}{2} V \left(\frac{V}{D' R_{load}} \right) \left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off} \right]$$

$$W_{sw1,loss} = \frac{1}{2} \cdot \frac{V^2}{D' R_{load}} \left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off} \right]$$

so,

$$\rightarrow P_{sw1,loss} = W_{sw1,loss} f_s = \frac{1}{2} \cdot \frac{V^2}{D' R_{load}} \left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off} \right] f_s$$

for switch #2 → since both MOSFETs will be of the same type: $t_{d,on}$, t_{fall}, t_{rise}, & $t_{d,off}$ will be the same for both as well.

$$P_{sw2,loss} = \frac{1}{2} v_{sw2} \hat{i}_{sw2} \left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off} \right] f_s$$

* $v_{sw_2} = V - I_L R_{on} \Rightarrow v_{sw_2} = V$ (by same reasoning as before)

* $i_{sw_2} = -I_L \Rightarrow i_{sw_2} = I_L$ (disregard (-) as P is a magnitude)

by similar derivation as for $P_{sw_2,loss}$

$$P_{sw2,loss} = \frac{1}{2} \cdot \frac{V^2}{D' R_{load}} \left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off} \right] f_s$$

Combining the losses of the 2 switches

$P_{sw,loss} = P_{sw1,loss} + P_{sw2,loss}$ $\Rightarrow$ $$\boxed{P_{sw,loss} = \frac{V^2 f_s}{D' R_{load}} \left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off} \right]}$$

substituting for V

$$P_{sw,loss} = \frac{V_g^2 D' R_{load} f_s}{\left[R_L + R_{on} + (D')^2 R_{load} \right]^2} \left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off} \right]$$

- Derive an expression for the efficiency of the boost converter. Plot the variation of the efficiency with duty cycle variation. (15 pts)

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{sw,loss} + P_{cond,loss} + P_{L,esr} + P_{C,esr}}$$

$$\boldsymbol{P_{out}} = V_{out} \cdot I_{out} = \frac{\boldsymbol{V^2}}{\boldsymbol{R_{load}}}$$

$$\boldsymbol{P_{L,esr}} = I_L^2 R_L = \left(\frac{V}{D' R_{load}}\right)^2 R_L = \left(\frac{\boldsymbol{V}}{\boldsymbol{R_{load}}}\right)^{\boldsymbol{2}} \frac{\boldsymbol{R_L}}{\boldsymbol{(D')^2}}$$

$$\boldsymbol{P_{C,esr}} = I_{C,rms}^2 \cdot R_{C,esr} = \frac{\boldsymbol{D}}{\boldsymbol{D'}} \cdot \left(\frac{\boldsymbol{V}}{\boldsymbol{R_{load}}}\right)^{\boldsymbol{2}} \boldsymbol{R_{C,esr}}$$

where:

$$I_{C,rms}^2 = \left(\sqrt{\frac{1}{T_s}\int_{T_s} i_c^2(t)\,dt}\right)^2 = \frac{1}{T_s}\int_{T_s} i_c^2(t)\,dt$$

$$= \frac{1}{T_s}\left[\int_0^{DT_s}\left(-\frac{V}{R_{load}}\right)^2 dt + \int_{DT_s}^{T_s}\left(I_L - \frac{V}{R_{load}}\right)^2 dt\right]$$

$$= \frac{1}{T_s}\left[\left(\frac{V}{R_{load}}\right)^2 \cdot DT_s + \left(I_L - \frac{V}{R_{load}}\right)^2 \cdot (T_s - DT_s)\right]$$

$$= \left(\frac{V}{R_{load}}\right)^2 \cdot D + \left(I_L - \frac{V}{R_{load}}\right)^2 \cdot D'$$

$$= \left(\frac{V}{R_{load}}\right)^2 D + \left[I_L^2 - 2I_L\frac{V}{R_{load}} + \left(\frac{V}{R_{load}}\right)^2\right] D'$$

$$= \left(\frac{V}{R_{load}}\right)^2 + D'\left[I_L^2 - 2I_L\frac{V}{R_{load}}\right]$$

$$= \left(\frac{V}{R_{load}}\right)^2 + D'\left[\left(\frac{V}{D' R_{load}}\right)^2 - 2\left(\frac{V}{D' R_{load}}\right)\frac{V}{R_{load}}\right]$$

$$= \left(\frac{V}{R_{load}}\right)^2 + \left(\frac{V}{R_{load}}\right)^2\left[\frac{1}{D'} - 2\right]$$

$$= \left(\frac{V}{R_{load}}\right)^2\left[1 + \frac{1}{D'} - 2\right] = \left(\frac{V}{R_{load}}\right)^2\left[\frac{1 - D'}{D'}\right]$$

$$= \frac{D}{D'} \cdot \left(\frac{V}{R_{load}}\right)^2$$

$$\boldsymbol{P_{cond,loss} = R_{on}\left(\frac{V}{D' R_{load}}\right)^2}$$

$$\boldsymbol{P_{sw,loss} = \frac{V^2 f_s}{D'\, R_{load}}\left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off}\right]}$$

$$\eta = \frac{P_{out}}{P_{out} + P_{sw,loss} + P_{cond,loss} + P_{L,esr} + P_{C,esr}}$$

$$\eta = \frac{\frac{V^2}{R_{load}}}{\frac{V^2}{R_{load}} + \frac{V^2 f_s}{D'\, R_{load}}\left[t_{d,on}+t_{fall}+t_{rise}+t_{d,off}\right] + \frac{V^2 R_{on}}{\left(D'\, R_{load}\right)^2} + \left(\frac{V}{R_{load}}\right)^2 \frac{R_L}{(D')^2} + \left(\frac{V}{R_{load}}\right)^2 \left[\frac{D}{D'}\right] R_C}$$

$$\eta = \frac{1}{1 + \frac{f_s}{D'}\left[t_{d,on}+t_{fall}+t_{rise}+t_{d,off}\right] + \frac{R_{on}}{(D')^2\, R_{load}} + \frac{R_L}{(D')^2 R_{load}} + \left[\frac{D}{D'}\right]\frac{R_C}{R_{load}}}$$

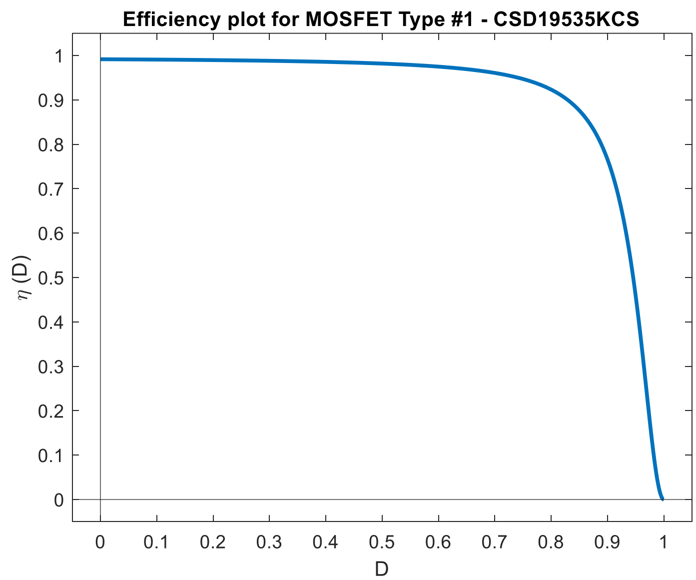
$$\boxed{\boldsymbol{\eta = \frac{1}{1 + \frac{f_s}{D'}\left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off}\right] + \frac{1}{(D')^2 R_{load}}\left[\, R_{on} + R_L + (D')\, D\, R_C \,\right]}}}$$

General Parameters

- (Internal) Inductor Resistance: $R_L = 40\ m\Omega$
- (Internal) MOSFET On-State Resistance: $R_{on} = 10\ m\Omega$
- Load Resistance: $R_{load} = 50\ \Omega$
- (Internal) Capacitor Resistance: $R_C = R_{c,esr} = 0.1\ \Omega$

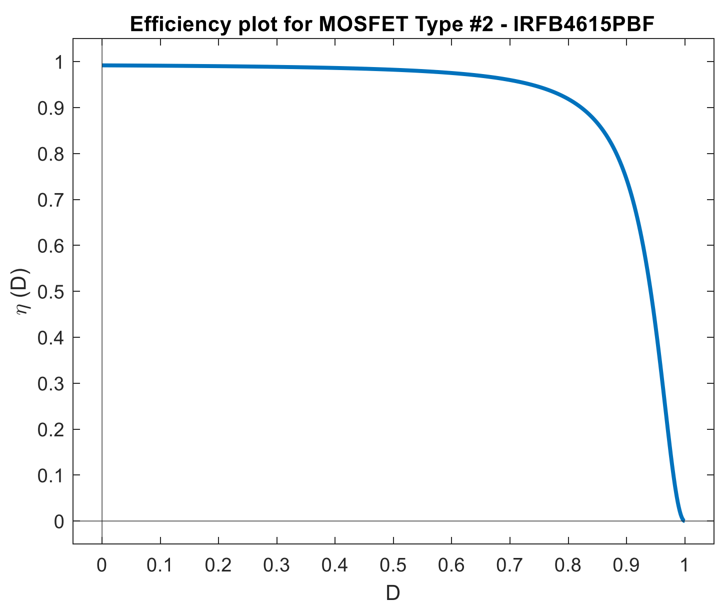
MOSFET Type #1 - CSD19535KCS

- Drain-to-Source On Resistance: $R_{on} = R_{DS(on)} = 3.4\ m\Omega$
- Turn On Delay Time: $t_{d(on)} = 32\ ns$
- Rise Time: $t_{rise} = t_r = 15\ ns$
- Turn Off Delay Time: $t_{d(off)} = 60\ ns$
- Fall Time: $t_{fall} = t_f = 5\ ns$



MOSFET Type #2 - IRFB4615PBF

- Drain-to-Source On Resistance: $R_{on} = R_{DS(on)} = 32\ m\Omega$
- Turn On Delay Time: $t_{d(on)} = 15\ ns$
- Rise Time: $t_{rise} = t_r = 35\ ns$
- Turn Off Delay Time: $t_{d(off)} = 25\ ns$
- Fall Time: $t_{fall} = t_f = 20\ ns$



For the following, use a switching frequency of 50kHz and an inductor current ripple of 15%.

- Given $V_{in} = 24V$, $V_{out} = 48V$, $R_L = 40m\Omega$,$R_{load} = 50\Omega$ and R_{on} values from both sets of MOSFETS, compute an inductance value from question 1. Compute the value of the duty cycle needed in both cases. **(20 pts)**
- Given $V_{in} = 24V$, $V_{out} = 48V$, $R_L = 40m\Omega$,$R_{load} = 50\Omega$ and R_{on} values from both sets of MOSFETS, compute the average conduction and switching loss in both cases.**(10 pts)**
- Given $V_{in} = 24V$, $V_{out} = 48V$, $R_L = 40m\Omega$,$R_{load} = 50\Omega$ and R_{on} values from both sets of MOSFETS, compute converter efficiencies.**(5 pts)**

Assume a capacitance C of 200 uF in all cases.

Based on the efficiencies calculated for each MOSFET, select one of the MOSFETS for your converter. Explain why **10 pts**.

General Data:

- Switching Frequency: $f_s = \frac{1}{T_s} = 50\ kHz$
- Inductor Current Ripple: $\Delta I_L = 15\%\ of\ I_L \ \rightarrow\ \Delta I_L = 0.15 \cdot I_L$
- Source Voltage: $V_g = V_{in} = 24\ V$
- Load Voltage: $V = V_{out} = 48\ V$
- (Internal) Inductor Resistance: $R_L = 40\ m\Omega$
- (Internal) Capacitor Resistance: $R_C = R_{c,esr} = 0.1\ \Omega$
- Load Resistance: $R_{load} = 50\ \Omega$

Duty Cycle:

$$V = V_g \frac{D'\ R_{load}}{R_L + R_{on} + (D')^2\ R_{load}}$$

$$\Rightarrow VR_L + VR_{on} + (D')^2 VR_{load} - V_g D'\ R_{load} = 0$$

$$\Rightarrow (D')^2\ \{VR_{load}\} - D'\{V_g R_{load}\} + \{VR_L + VR_{on}\} = 0$$

$$\Rightarrow D' = \frac{-(-V_g\ R_{load}) \pm \sqrt{(-V_g\ R_{load})^2 - 4(VR_{load})(VR_L + VR_{on})}}{2(VR_{load})}$$

$$\boldsymbol{D = 1 - \frac{V_g\, R_{load} \pm \sqrt{\left(V_g\, R_{load}\right)^2 - 4V^2 R_{load}(R_L + R_{on})}}{2\, V\, R_{load}}}$$

Inductor Value:

$$L = \frac{DT_s}{2 \cdot \Delta I_L}\left[V_g - \frac{V}{D'\, R_{load}}(R_L + R_{on})\right] = \frac{D}{2f_s \cdot \left(0.15 \cdot \left(\frac{V}{D'\, R_{load}}\right)\right)}\left[V_g - \frac{V}{D'\, R_{load}}(R_L + R_{on})\right]$$

$$\boldsymbol{L = \frac{D\, D'\, R_{load}}{0.3\, f_s\, V}\left[V_g - \frac{V}{D'\, R_{load}}(R_L + R_{on})\right]}$$

Conduction Loss:

$$\boldsymbol{P_{cond,loss} = \frac{V^2 R_{on}}{(D'\, R_{load})^2}}$$

Switching Loss:

$$\boldsymbol{P_{sw,loss} = \frac{V^2 f_s}{D'\, R_{load}}\left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off}\right]}$$

Efficiency:

$$\boldsymbol{\eta = \frac{1}{1 + \frac{f_s}{D'}\left[t_{d,on} + t_{fall} + t_{rise} + t_{d,off}\right] + \frac{1}{(D')^2 R_{load}}\left[\, R_{on} + R_L + (D')\, D\, R_C\,\right]}}$$

MOSFET Type #1 - CSD19535KCS

- ***Data***
  - Drain-to-Source On Resistance: $R_{on} = R_{DS(on)} = 3.4\ m\Omega$
  - Turn On Delay Time: $t_{d(on)} = 32\ ns$
  - Rise Time: $t_{rise} = t_r = 15\ ns$
  - Turn Off Delay Time: $t_{d(off)} = 60\ ns$
  - Fall Time: $t_{fall} = t_f = 5\ ns$

- ***Duty Ratio***

$$D = 1 - \frac{(24)(50) \pm \sqrt{[(24)(50)]^2 - 4(48)^2(50)[(40 \times 10^{-3}) + (3.4 \times 10^{-3})]}}{2(48)(50)} = \{0.5017, 0.9983\}$$

Note, choosing smaller D value as the larger one is not realizable in the physical circuit

$$\boxed{\boldsymbol{D = 0.5017}}$$

- ***Inductor Value***

$$L = \frac{(0.5017)(1-0.5017)(50)}{0.3(50\times10^3)(48)}\left[24 - \frac{48}{(1-0.5017)(50)}[(40\times10^{-3}) + (3.4\times10^{-3})]\right] = 4.1521\times10^{-4}$$

$$\boxed{\boldsymbol{L = 415.21\ \mu H}}$$

- ***Conduction Loss***

$$P_{cond,loss} = \frac{(48)^2(3.4\times10^{-3})}{[(1-0.5017)(50)]^2} = 0.012619$$

$$\boxed{\boldsymbol{P_{cond,loss} = 12.619\ mW}}$$

- ***Switching Loss***

$$P_{sw,loss} = \frac{(48)^2(50\times10^3)}{(1-0.5017)(50)}([32+5+15+60]\times10^{-9}) = 0.5179$$

$$\boxed{\boldsymbol{P_{sw,loss} = 517.9\ mW}}$$

- ***Efficiency***

$$\eta = \frac{1}{1+\frac{50\times10^3}{1-0.5017}([32+5+15+60]\times10^{-9})+\frac{1}{(1-0.5017)^2(50)}[\,(3.4\times10^{-3})+(40\times10^{-3})+(1-0.5017)(0.5017)(0.1)\,]} = 0.9835$$

$$\boxed{\boldsymbol{\eta = 0.9835}}$$

MOSFET Type #2 - IRFB4615PBF

- ***Data***
  - Drain-to-Source On Resistance: $R_{on} = R_{DS(on)} = 32\ m\Omega$
  - Turn On Delay Time: $t_{d(on)} = 15\ ns$
  - Rise Time: $t_{rise} = t_r = 35\ ns$
  - Turn Off Delay Time: $t_{d(off)} = 25\ ns$
  - Fall Time: $t_{fall} = t_f = 20\ ns$

- ***Duty Ratio***

$$D = 1 - \frac{(24)(50) \pm \sqrt{[(24)(50)]^2 - 4(48)^2(50)[(40 \times 10^{-3}) + (32 \times 10^{-3})]}}{2(48)(50)} = \{0.5029, 0.9971\}$$

Note, choosing smaller D value as the larger one is not realizable in the physical circuit

$$\boxed{\boldsymbol{D = 0.5029}}$$

- ***Inductor Value***

$$L = \frac{(0.5029)(1 - 0.5029)(50)}{0.3(50 \times 10^3)(48)}\left[24 - \frac{48}{(1 - 0.5029)(50)}[(40 \times 10^{-3}) + (32 \times 10^{-3})]\right] = 4.1424 \times 10^{-4}$$

$$\boxed{\boldsymbol{L = 414.24\ \mu H}}$$

- ***Conduction Loss***

$$P_{cond,loss} = \frac{(48)^2(32 \times 10^{-3})}{[(1 - 0.5029)(50)]^2} = 0.119345$$

$$\boxed{\boldsymbol{P_{cond,loss} = 119.3\ mW}}$$

- ***Switching Loss***

$$P_{sw,loss} = \frac{(48)^2(50 \times 10^3)}{(1 - 0.5029)(50)}([15 + 20 + 35 + 25] \times 10^{-9}) = 0.4403$$

$$\boxed{\boldsymbol{P_{sw,loss} = 440.3\ mW}}$$

- ***Efficiency***

$$\eta = \frac{1}{1 + \frac{50 \times 10^3}{1 - 0.5029}([32 + 5 + 15 + 60] \times 10^{-9}) + \frac{1}{(1 - 0.5029)^2(50)}[\,(32 \times 10^{-3}) + (40 \times 10^{-3}) + (1 - 0.5029)(0.5029)(0.1)\,]} = 0.9829$$

$$\boxed{\boldsymbol{\eta = 0.9829}}$$

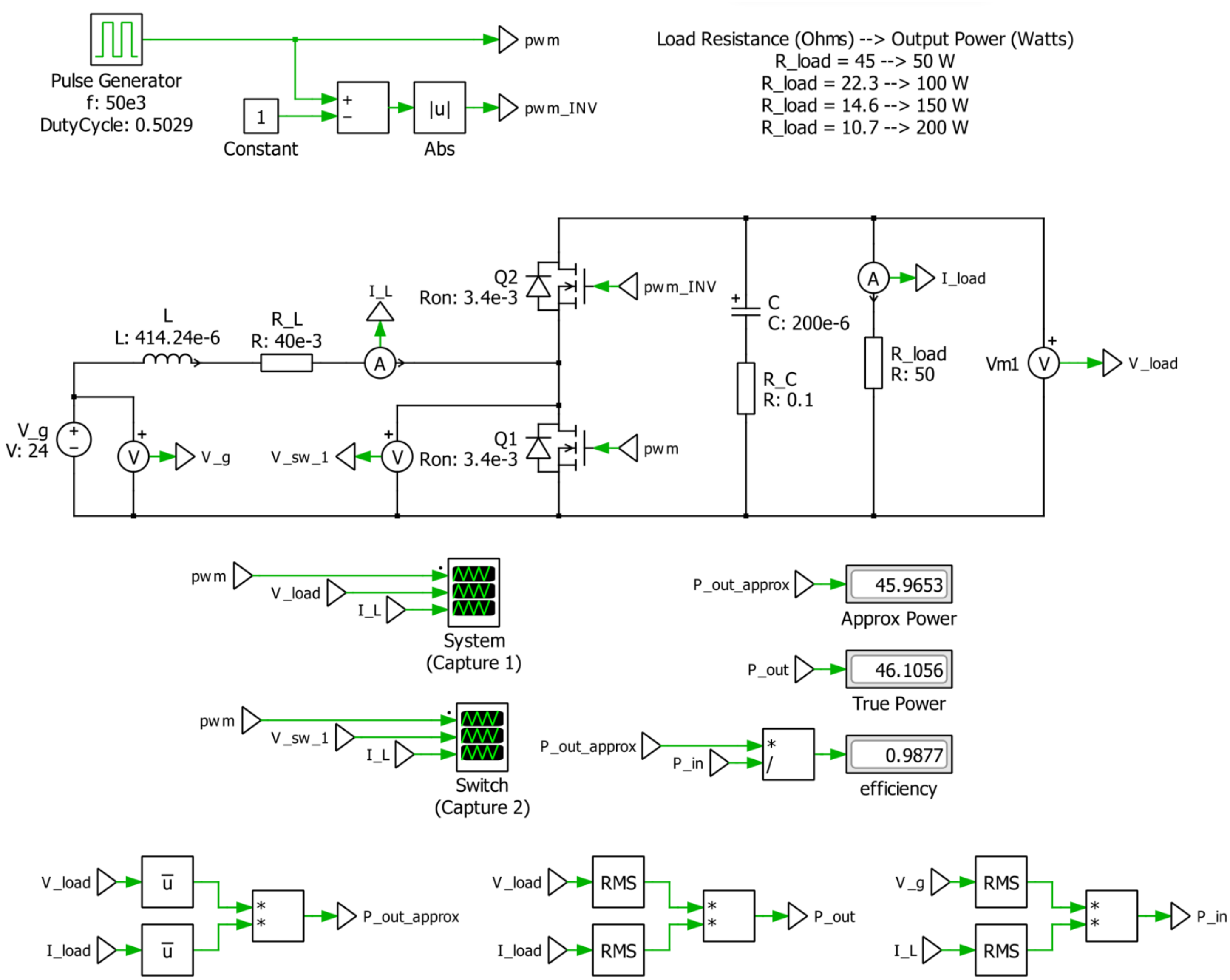
Selecting MOSFETs for converter:

Given the data, $\boxed{\textbf{we will use } \boldsymbol{MOSFET\ Type\ \#1 - CSD19535KCS}}$ for the following reasons:

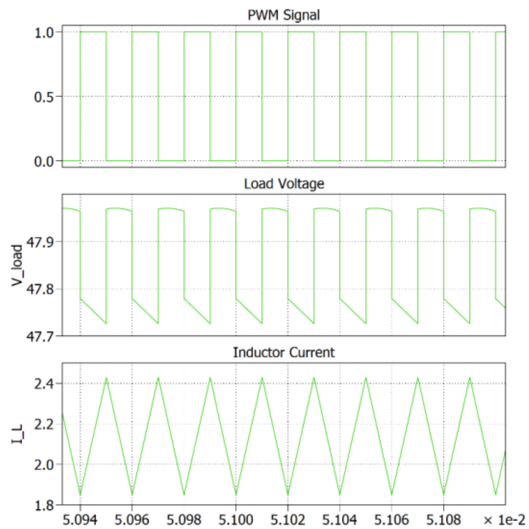
- Superior overall efficiency of $\eta = 0.9835$. In other words, there are fewer overall losses in the selected MOSFET at the given switching frequency $f_s = 50\ kHz$.
    - Note, if we were to hold the switching frequency constant at $f_s = 50\ kHz$ while using the selected MOSFET in higher power rated applications (by decreasing the value of R_{load} such that the load current increases), then the selected MOSFET (*Type #1 - CSD19535KCS*) would still be superior in terms of overall efficiency.
- Lower conduction losses for any given duty ratio.

Part (b) – Simulation

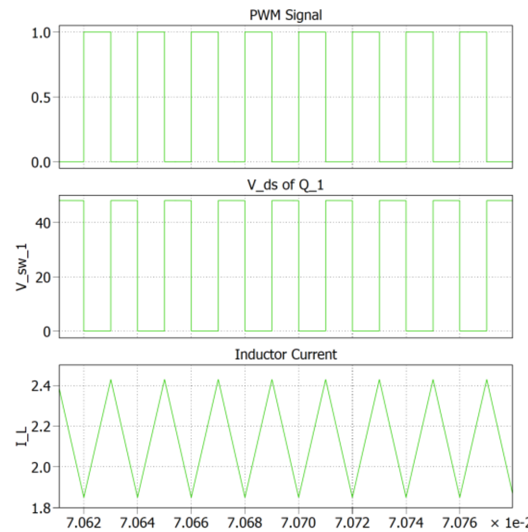
Simulate the efficiency at (i.e., when the power pulled from the converter is) 50 W, 100 W, 150 W, and 200 W.



Power pulled from converter: $P_{out} = 50\ W \ \rightarrow R_{load} = 45\ \Omega$

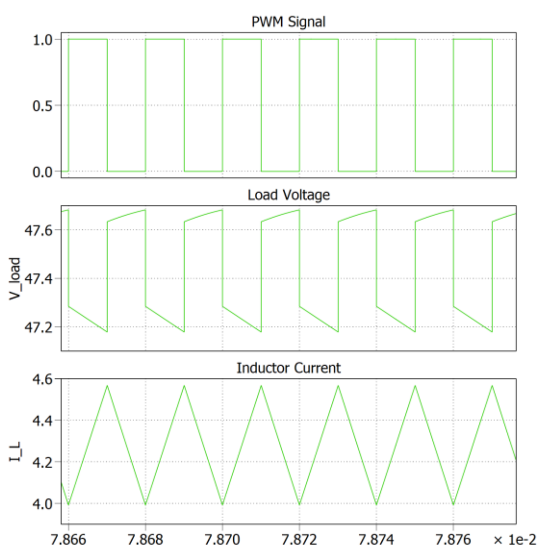


Capture 1: steady state I_L (inductor current) and V (load voltage) for $P_{out} = 50\ W \ \rightarrow R_{load} = 45\ \Omega$

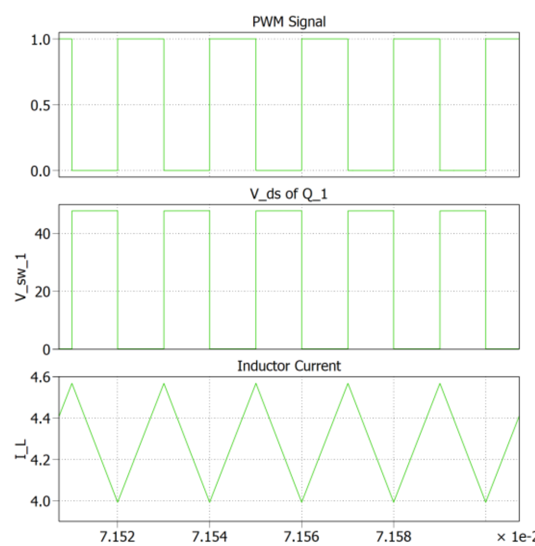


Capture 2: steady state I_L (inductor current) and $v_{DS,1}$ (drain-to-source voltage of MOSFET 1) *for* $P_{out} = 50\ W \ \rightarrow R_{load} = 45\ \Omega$

Power pulled from converter: $P_{out} = 100\ W \ \rightarrow R_{load} = 22.5\ \Omega$

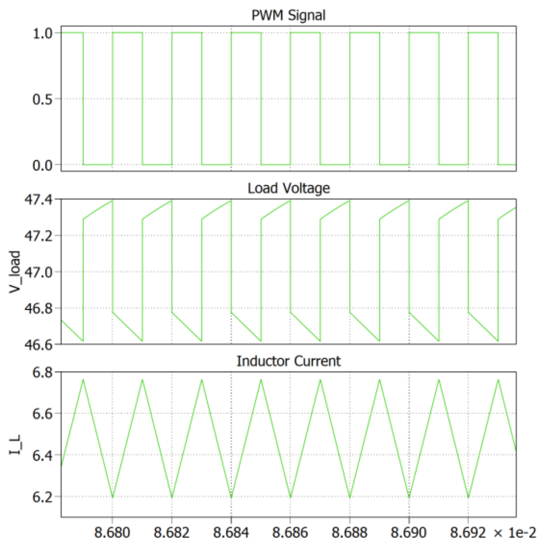


Capture 1: steady state I_L (inductor current) and V (load voltage) for $P_{out} = 100\ W \ \rightarrow R_{load} = 22.3\ \Omega$

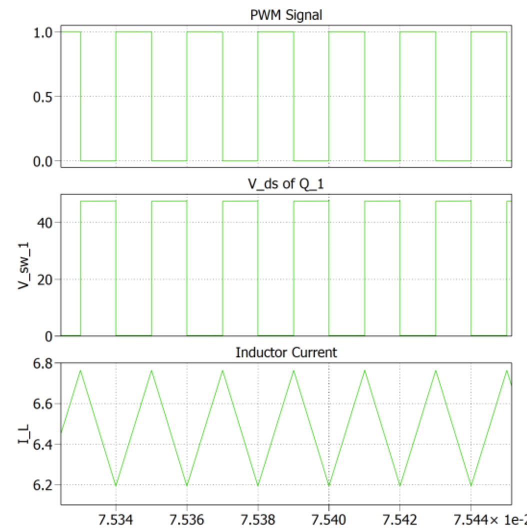


Capture 2: steady state I_L (inductor current) and $v_{DS,1}$ (drain-to-source voltage of MOSFET 1) *for* $P_{out} = 100\ W \ \rightarrow R_{load} = 22.3\ \Omega$

Power pulled from converter: $P_{out} = 150\ W \ \rightarrow R_{load} = 14.8\ \Omega$

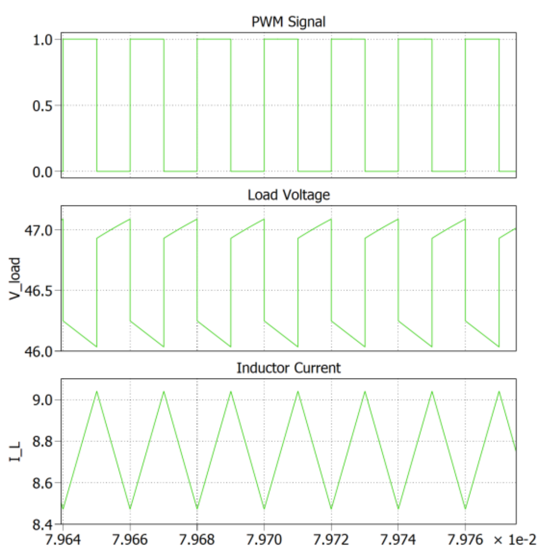


Capture 1: steady state I_L (inductor current) and V (load voltage) for $P_{out} = 150\ W \ \rightarrow R_{load} = 14.6\ \Omega$

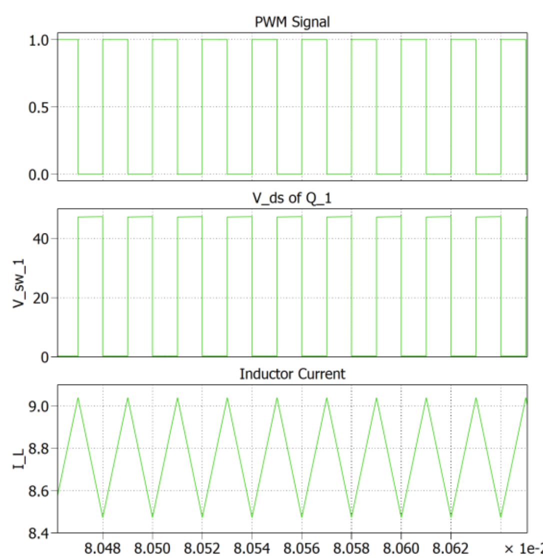


Capture 2: steady state I_L (inductor current) and $v_{DS,1}$ (drain-to-source voltage of MOSFET 1) *for* $P_{out} = 150\ W \ \rightarrow R_{load} = 14.6\ \Omega$

Power pulled from converter: $P_{out} = 200\ W \ \rightarrow R_{load} = 10.9\ \Omega$



Capture 1: steady state I_L (inductor current) and V (load voltage) for $P_{out} = 200\ W \ \rightarrow R_{load} = 10.7\ \Omega$



Capture 2: steady state I_L (inductor current) and $v_{DS,1}$ (drain-to-source voltage of MOSFET 1) *for* $P_{out} = 200\ W \ \rightarrow R_{load} = 10.7\ \Omega$

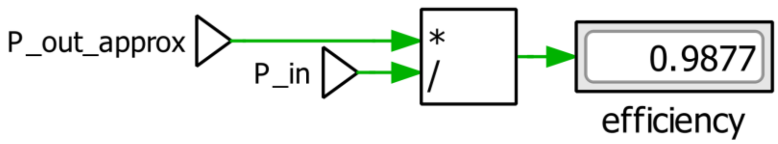
$\boldsymbol{I_L}$ *and* $\boldsymbol{v_{DS}}$ *relationship*:

The drain-to-source voltage v_{DS} is directly proportional to the inductor current I_L by the on-state resistance of the MOSFET R_{on}. Specifically, to satisfy Ohm's law: $v_{ds} = I_L R_{on}$.

Efficiency : *Analytical vs simulation result for* $\boldsymbol{R_{load} = 50\ \Omega}$

For *MOSFET Type #1 - CSD19535KCS*

- Analytical result: $\boldsymbol{\eta_{analytical} = 0.9835}$
- Simulation result: $\boldsymbol{\eta_{simulation} = 0.9877}$



We observe that $\{\boldsymbol{\eta_{analytical} = 0.9835}\} < \{\boldsymbol{\eta_{simulation} = 0.9884}\}$. The reason for slightly differing results may be attributed to multiple factors. The most important factor is the fact that the simulation model does not account for switching losses represented by the terms $t_{d(on)}, t_{rise}, t_{d(off)}\ \&\ t_{fall}$. In addition, the simulation employs true power (i.e., uses the RMS of current and voltage for power computations) while the analytical result employs an approximation of the true power (i.e., uses the average values of current and voltage for power computations).

Lab #2 - C

Inductor Design

	Inductance L	Resistance R
Expected	0.20 mH	40 mΩ
Measured	0.27 mH	35 mΩ

The measured inductor had $\frac{0.27mH-0.20mH}{0.20mH} = 35\%$ larger inductance than expected. The increased inductance results from some human error in winding the inductor. Approximately 1-2 windings were added, however the modifications were kept due to improved performance of the booster circuit. The increased inductance will help reduce the output current ripple.

Design Procedure

The objective is to solve for the inductor properties given ETD39 core geometry and Ferroxcube 3C97 material.

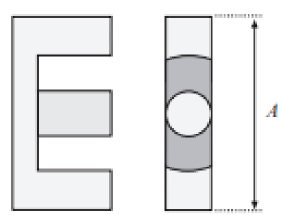
Using the material and inductor datasheets, the following properties are known:

$$I_{max},\ B_{max},\ \mu\ ,\ K_u,\ R,\ \rho$$

With the given parameters, the following is solved:

$$n,\ l_g,\ and\ A_w.$$

A2.4 ETD core data



Core type (A) (mm)	Geometrical constant K_g cm^5	Geometrical constant K_{gfe} cm^x	Cross-sectional area A_c (cm^2)	Bobbin winding area W_A (cm^2)	Mean length per turn MLT (cm)	Magnetic path length l_m (cm)	Thermal resistance R_{th} (°C/W)	Core weight (g)
ETD29	0.0978	$8.5\cdot10^{-3}$	0.76	0.903	5.33	7.20		30
ETD34	0.193	$13.1\cdot10^{-3}$	0.97	1.23	6.00	7.86	19	40
ETD39	0.397	$19.8\cdot10^{-3}$	1.25	1.74	6.86	9.21	15	60
ETD44	0.846	$30.4\cdot10^{-3}$	1.74	2.13	7.62	10.3	12	94
ETD49	1.42	$41.0\cdot10^{-3}$	2.11	2.71	8.51	11.4	11	124

Figure: ETD39 Core Geometry Parameters

1. Determine Core Size

$$K_g \geq \left(\frac{\rho L^2 I^2_{max}}{B^2_{max} R K_u}\right) 10^8\ (cm^5) \qquad \rho = 1.724 \cdot 10^{-6}|_{T=23C}\ or\ 2.3 \cdot 10^{-6}|_{T=100C}$$

Boost converter requirements are provided in table:

P_{out}	$100W$
V_{out}	$48V$
V_{in}	$24V$

Assuming high efficiency, then $I_{out} \approx 2A$ and $I_{in} \approx 4A$.
The max current is calculated by summing the inductor current and ripple current (15%). An additional safety factor of 15% is included.

$$I_{max} = 1.15(4 + 0.15(4)) \approx 5.3A$$

Using the v_L slope equation for the boost converter, the inductance can be solved.
Given: $D = 0.5$, $T_S = \frac{1}{f_{SW}} = \frac{1}{50kHz} = 20\mu s$, $i_L = 0.15(4) = 0.6A$ and $V_L \approx 24V$

$$L = \frac{V_L DT_S}{2\Delta i_L} = \frac{24 \cdot 0.5 \cdot 20 \cdot 10^{-6}}{2 \cdot 0.6} = 0.2mH$$

B_{sat} is provided using the core geometry ETD39 datasheet: $0.33\ T$. The material Ferroxcube 3C97 datasheet provides greater values, however these are not achievable due to the geometry. B_{max} is chosen to be 0.25 to satisfy the condition $B_{sat} \geq B_{max}$.

Determining *R*, depends on the power loss of the copper. Design requires the copper loss be less than 1W, such that $P_{cu} = I^2_{rms} R \leq 1W$. The I_{rms} value is the average inductor current (4A), then *R* is calculated to be

$$R \leq \frac{P_{cu}}{I^2_{rms}} = \frac{1W}{(4A)^2} = 60m\Omega$$

Lastly, the winding fill factor K_u is given as $0.5\ cm^2/cm^2$. Putting everything together results in core size

$$K_g \geq \left(\frac{(1.72 \cdot 10^6)\,(0.2 \cdot 10^{-3})^2\,(5)^2}{(0.25)^2\,(60 \cdot 10^{-3})\,(0.5)}\right) 10^8 = 0.13\ (cm^5)\,.$$

2. Determine the Air Gap Length

Core geometry ETD39 has given $A_c = 1.25\ cm^2$, $w_A = 1.74\ cm^2$, $MLT = 6.86\ cm$. The air gap can be solved by substituting the given parameter A_c as well as L, I_{max}, and B_{max} from the previous steps.

$$l_g = \left(\frac{\mu_0 L I^2_{max}}{B^2_{max} A_c}\right)10^4 = \left(\frac{(4\pi \cdot 10^{-6})(0.2 \cdot 10^{-3})\,(5)^2}{(0.25)^2\,(1.25)}\right)10^4 = 5.58 \cdot 10^{-4}\,(m) = 0.0558\,(cm)$$

3. Determine the Number of Turns

The number of turns is determined by substituting A_c, L, I_{max}, and B_{max} from the previous steps.

$$n = \left(\frac{L\, I_{max}}{B_{max} A_C}\right)10^4 = \left(\frac{(0.2 \cdot 10^{-3})(5)}{(0.25)\ (1.25)}\right)10^4 = 27 \text{ turns.}$$

4. Evaluate Wire Size

In most designs, the maximum wire size should be chosen (accounting for insulation).

$$A_W \leq \frac{K_u W_A}{n}\ (cm^2)$$
$$A_W \leq \frac{0.5 \cdot 1.74}{27} = 0.032\ (cm^2) = 32 \cdot 10^{-3}(cm^2)$$

Standard	Size $10^{-3}(cm^2)$
13 AWG	26.26
16 AWG	13.07

The area can have a maximum of 13 AWG wire, however 16 AWG was chosen for this design. The inductor resistance can be verified by

$$R = \frac{\rho \cdot n \cdot MLT}{A_w} = \frac{(2.3 \cdot 10^{-3})(27)(6.86)}{0.013} = 28.5\Omega$$

The wire size chosen satisfies the resistance requirements of the project however, a larger wire will help with carrying maximum current from the inductor.

5. Inductance per Turn

$$L = A_L(l_g)\, n^2$$

The inductance per turn (A_L), can be determined as a function of the air gap l_g.

	Inductance *L*	Turns *n*	$A_L(l_g)$
Expected	0.20 mH	27	274 nH
Measured	0.27 mH	28	344 nH

American wire gauge data

AWG#	Bare area, 10^{-3} cm^2	Resistance, 10^{-6} Ω/cm	Diameter, cm
0000	1072.3	1.608	1.168
000	850.3	2.027	1.040
00	674.2	2.557	0.927
0	534.8	3.224	0.825
1	424.1	4.065	0.735
2	336.3	5.128	0.654
3	266.7	6.463	0.583
4	211.5	8.153	0.519
5	167.7	10.28	0.462
6	133.0	13.0	0.411
7	105.5	16.3	0.366
8	83.67	20.6	0.326
9	66.32	26.0	0.291
10	52.41	32.9	0.267
11	41.60	41.37	0.238
12	33.08	52.09	0.213
13	26.26	69.64	0.190
14	20.02	82.80	0.171
15	16.51	104.3	0.153
16	13.07	131.8	0.137
17	10.39	165.8	0.122
18	8.228	209.5	0.109
19	6.531	263.9	0.0948
20	5.188	332.3	0.0874

Table: AWG Sizes

Recap

Table 1: Variables and their meanings.

Physical meaning	Variable	Units
wire resistivity	ρ	Ω·cm
peak winding current	I_{max}	A
inductance	L	H
winding resistance	R	Ω
winding fill factor	K_u	cm^2/cm^2
core maximum flux density	B_{max}	T
core cross-sectional area	A_c	cm^2
core window area	W_A	cm^2
mean length per turn	MLT	cm

1. **Determine Core Size**

$$K_g \ge \left(\frac{\rho L^2 I^2_{max}}{B^2_{max} R K_u}\right) 10^8\ (cm^5)$$

2. **Determine the Air Gap Length**

$$l_g = \left(\frac{\mu_0 L I^2_{max}}{B^2_{max} A_c}\right) 10^4\ (m)$$

3. **Determine the Number of Turns**

$$n = \left(\frac{L I_{max}}{B_{max} A_C}\right) 10^4$$

4. **Evaluate Wire Size**

$$A_W \le \frac{K_u W_A}{n}\ (cm^2)$$