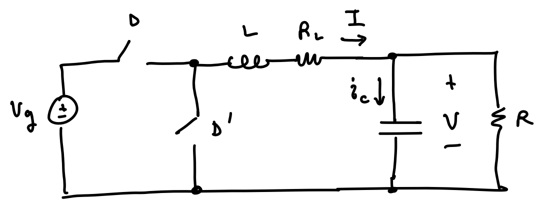
Solution



where

$V_g = 40V$

$V = 20V$

$f_s = 50KHZ$

$R \geq R_{min} = 4\,\Omega$

a) Find L such that $\Delta i / I = 0.1$

- Ignore R_L effect for now since it is very small and has minimal impact on result.

- Volt-second balance gives

$$0 = (V_g - V)D + (-V)D'$$

$$= DV_g - V$$

$$\Rightarrow \quad D = \frac{V}{V_g}$$

- Charge balance gives

$$0 = (I - V/R)D + (I - V/R)D'$$

$$= I - V/R$$

$$\Rightarrow \quad I = V/R$$

* look @ heaviest load when $R = R_{min} = 4\,\Omega$

$$\left.\begin{aligned} &= \frac{V}{R_{min}} \\ &= I \end{aligned}\right\} \quad (1)$$

- Look @ ripple:

$$V_L = L\,\frac{2\Delta i}{\Delta t}$$

* see config ①

$$\Rightarrow \quad (V_g - V) = L\,\frac{2\Delta i}{DT_s} \qquad (2)$$

$$\left.\begin{aligned}&\text{need } \frac{\Delta i}{I} = 0.1 \quad \Rightarrow \quad \Delta i = 0.1 I \\ &\text{\& } T_s = \frac{1}{f_s} \\ &\text{\& (1) gives us } I.\end{aligned}\right\} \text{ Plug into (2)}$$

Substituting into (2) gives

$$V_g - V = L \, \frac{\overbrace{2 \, \underbrace{\tfrac{1}{10}}_{0.1} \, I}^{\Delta i}}{D/f_s}$$

$$= L \, \frac{f_s \overbrace{I}^{V/R_{min}}}{5D}$$

$$= L \, \frac{V f_s}{5 R_{min} D}$$

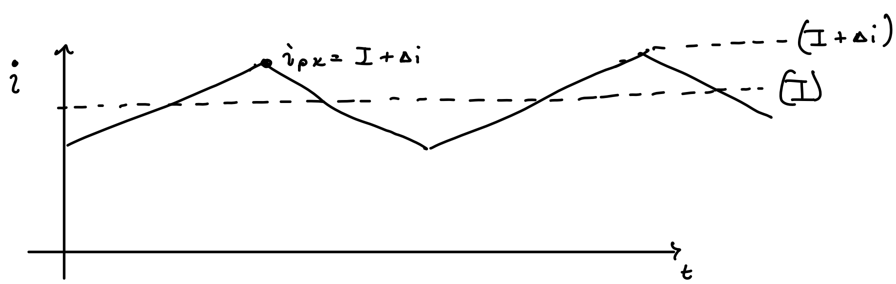
solve for L

$$L = \frac{(V_g - V)\, 5 R_{min} \overbrace{D}^{D = \frac{V}{V_g} = \frac{20V}{40V} = 0.5}}{V f_s}$$

$$= \frac{(40V - 20V)\, 5 \times 4\Omega \times 0.5}{20V \times 50kHz}$$

$$= \boxed{200\mu H = L}$$

b) Determine I_{max}



$$i_{pk} = I + \Delta i$$
$$= I + 0.1I$$
$$= 1.1I$$

* use (i)

$$\underbrace{= 1.1 \frac{V}{R_{min}}}= i_{pk}$$

- i_{pk} is the theoretical exact peak of the current.
- In real applications, need to deal with transients and other cases where i might exceed i_{pk} momentarily.
- A more practical choice is

→ Pick $I_{max} = \underbrace{1.25}_{\text{gives 25\% "safety" margin}}\, i_{pk}$

$$\boxed{\begin{aligned} &= 1.25 \times 1.1 \frac{V}{R_{min}} \\ &= I_{max} \end{aligned}}$$

this solution uses engineering judgement and is not unique.

c) Design inductor via K_g method.

Goal: Need $I^2 R_L < P_{loss,max} = 1\,W$

$$\Rightarrow R_L < \frac{P_{loss,max}}{I^2}$$

let $R := \dfrac{P_{loss,max}}{I^2}$ = max allowable resistance

Step 1) Pick core big enough

$$K_g \stackrel{\Delta}{=} \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u} 10^8$$

$$= \frac{\rho L^2 \left(1.25 \times 1.1 \times \frac{V}{R_{min}}\right)^2}{B_{max}^2 R K_u} 10^8$$

* evaluate at

$\rho = 2.3 \times 10^{-6} \Omega \cdot cm$ @ 100°C

$L = 200 \mu H$

$V = 20V$, $R_{min} = 4\Omega$

$B_{max} = 0.25T$

$K_u = 0.5$ ← realistic utilization factor

$$\approx 347.9 \times 10^{-3} \ cm^5$$

Look @ table. Smallest available core which exceeds $K_{g min}$ above is

→ $\boxed{K_g = 0.909 \ cm^5 \text{ for "EE50" geometry}}$

core size chosen

Step #2) Pick air gap

$$l_g = \frac{\mu_o L I_{max}^2 10^4}{B_{max}^2 A_c}$$

* where $A_c = 2.26 \ cm^2$ for EE50

$$\approx \boxed{841 \ \mu m = l_g}$$

Step #3) Pick Turns

$$n = \text{round}\left(\frac{L\, I_{max}}{B_{max} A_c} 10^4\right)$$

$* \text{ where } A_c = 2.26 \text{ cm}^2$ for EE50

$$= \boxed{25 \text{ turns} = n}$$

Step 4) Check wire size

$$A_w \leq \frac{K_u W_A}{n} \quad [\text{cm}^2]$$

$* \; W_A = 1.78 \text{ cm}^2$

$$\approx 35.6 \times 10^{-3} \text{ cm}^2$$

↳ Implies need AWG#12 or above (smaller wire)

- AWG #12 $A_w = 33.1 \times 10^{-3} \text{ cm}^2$.

Sanity check

Check $R_{actual} = \rho \dfrac{n\,(MLT)}{A_w}$

$* \; (MLT) = 10 \text{ cm}$ for EE50

$$= \frac{(2.3 \times 10^{-6}\ \Omega\text{-cm})(25)(10\text{cm})}{33.1 \times 10^{-3}\ \text{cm}^2} \approx \overbrace{17 m\Omega}^{\text{"actual" } R \text{ @ dc}} < \overbrace{R = 40 m\Omega}^{\text{max R value we targeted}}$$

↳ success! we hit our target.