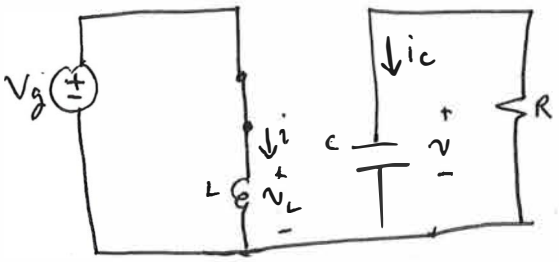
HW # 1 Solution

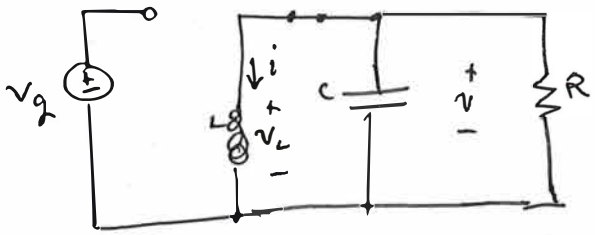
Problem # 1

1(a) Express V and I in terms of D, V_g, & R

Position ① Duration DT



Position ② Duration D'T = (1-D)T



Volt second balance

$$\langle v_L \rangle = D V_g + \overbrace{(1-D)}^{D'} V = 0 \qquad (1)$$

charge balance

$$\langle i_c \rangle = \left(-\frac{V}{R}\right) D + \left(-I - \frac{V}{R}\right)(1-D) = 0 \qquad (2)$$

Rearrange (1)

$$\boxed{V = -V_g \frac{D}{1-D}} \quad \boxed{1(a)} \qquad (3)$$

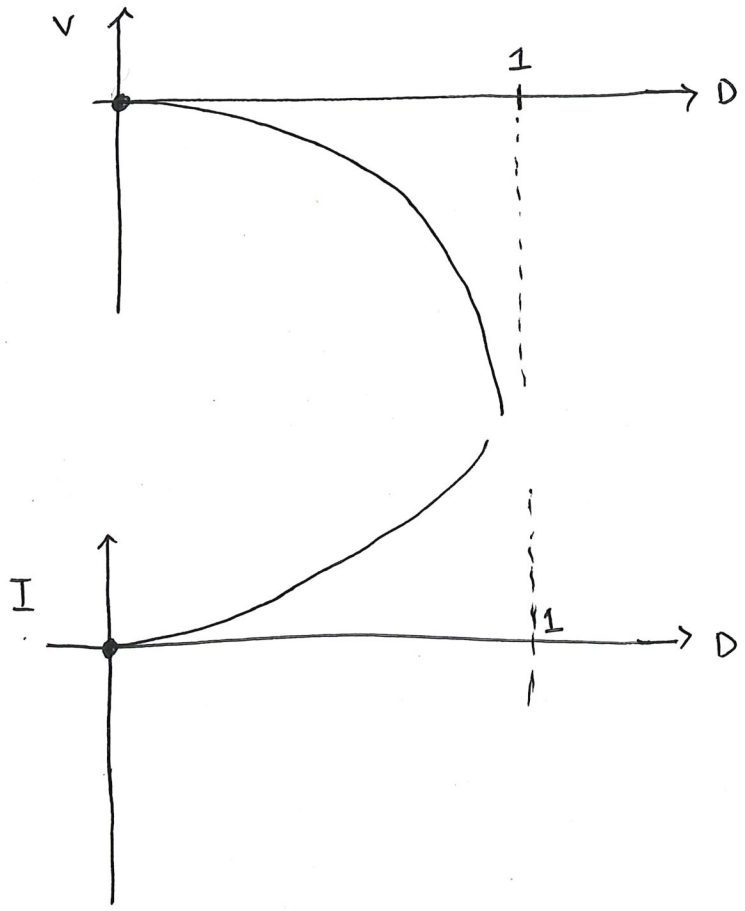
Rearrange (2)

$$-\frac{V}{R} D = (1-D)\left(I + \frac{V}{R}\right) \Rightarrow I = -\frac{V}{R}\frac{D}{1-D} - \frac{V}{R} \qquad (4)$$

*use (3) in (4)

$$\Rightarrow I = \frac{-V}{R}\left(\frac{D}{1-D} + 1\right) = \boxed{\frac{V_g}{R}\frac{D}{1-D}\left(\frac{D}{1-D} + 1\right) = I} \quad \boxed{1(a)} \qquad (5)$$

1(b) Plot V ∓ I versus D



only shape
∓ ~~data~~ zero values
@ D = 0 matter.

1 c) Design problem:

(1) Find D & I

Rearrange (1)

$$0 = DV_g - DV + V$$

$$= D(V_g - V) + V$$

$$\Rightarrow D = \frac{-V}{V_g - V}$$

$$= \frac{V}{V - V_g}$$

$$= \frac{(-12\,V)}{(-12\,V) - 15\,V}$$

$$= \frac{12}{27} = \frac{4 \cdot 3}{9 \cdot 3}$$

$$= \frac{4}{9} \approx 0{,}44 \approx D$$ 1 c) (i)

and (5) gives

$$I = \frac{15\,V}{4\,\Omega} \frac{\frac{4}{9}}{1 - \frac{4}{9}} \left(\frac{\frac{4}{9}}{1 - \frac{4}{9}} + 1 \right) \approx 5{,}4\,A$$ 1 c)(i)

- (ii) for 1C

  • Find L s.t. $\Delta i = 0.1\,I$

  • for position ① we get

$$L\,\frac{2\Delta i}{DT} = V_g \quad \Rightarrow \quad L = \frac{DTV_g}{2\Delta i}$$

  * where $\Delta i = 0.1\,I$ & I comes from before

$\Rightarrow$

$$L = \frac{DTV_g}{2\times 0.1\times I}$$

$$= \frac{DV_g}{2\times 0.1\,I f_s} \approx 30.9\,\mu H$$

$$= L$$

- (iii) Find C s.t. $\Delta v = 0.1\,V$

  • For position ①

$$C\,\frac{2\Delta v}{DT} = \frac{|V|}{R}$$

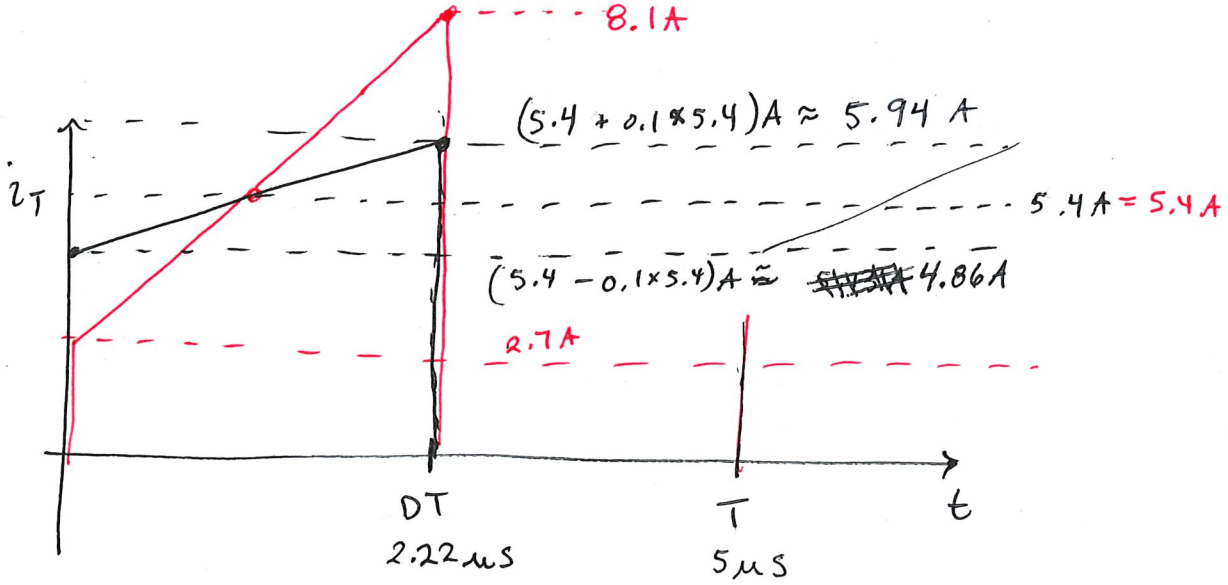
$$\rightarrow \quad C = \frac{DT|V|}{2R\Delta v}$$

$$= \frac{D|V|}{2R\times 0.1V} \approx 33.3\,\mu F$$

$$= C$$

(ii) 1C) (iii)

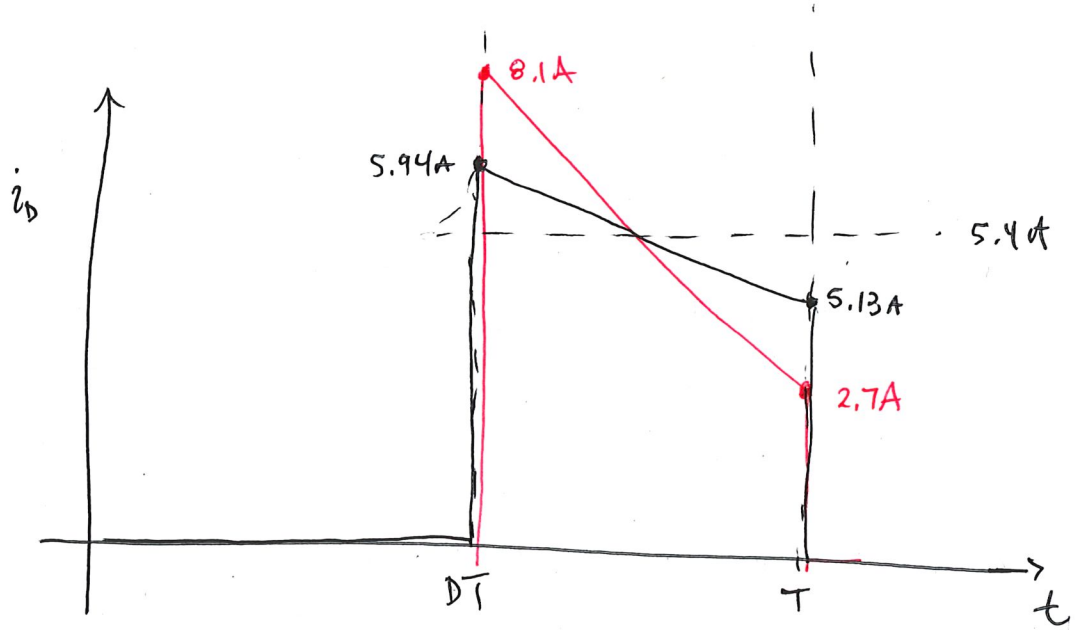
1 (d)



case for $\Delta i = 0.1\ I$

case for $\Delta i = 0.5\ I$

1 (e)



Problem #2

2(a) Volt second balance gives

$$\langle V_L \rangle = V_g D + (V_g - V)(1-D) = 0$$

$$\rightarrow \cancel{V_g D} + V_g - \cancel{V_g D} - V(1-D) = 0$$

$$\rightarrow \frac{V_g}{V} = 1 - D$$

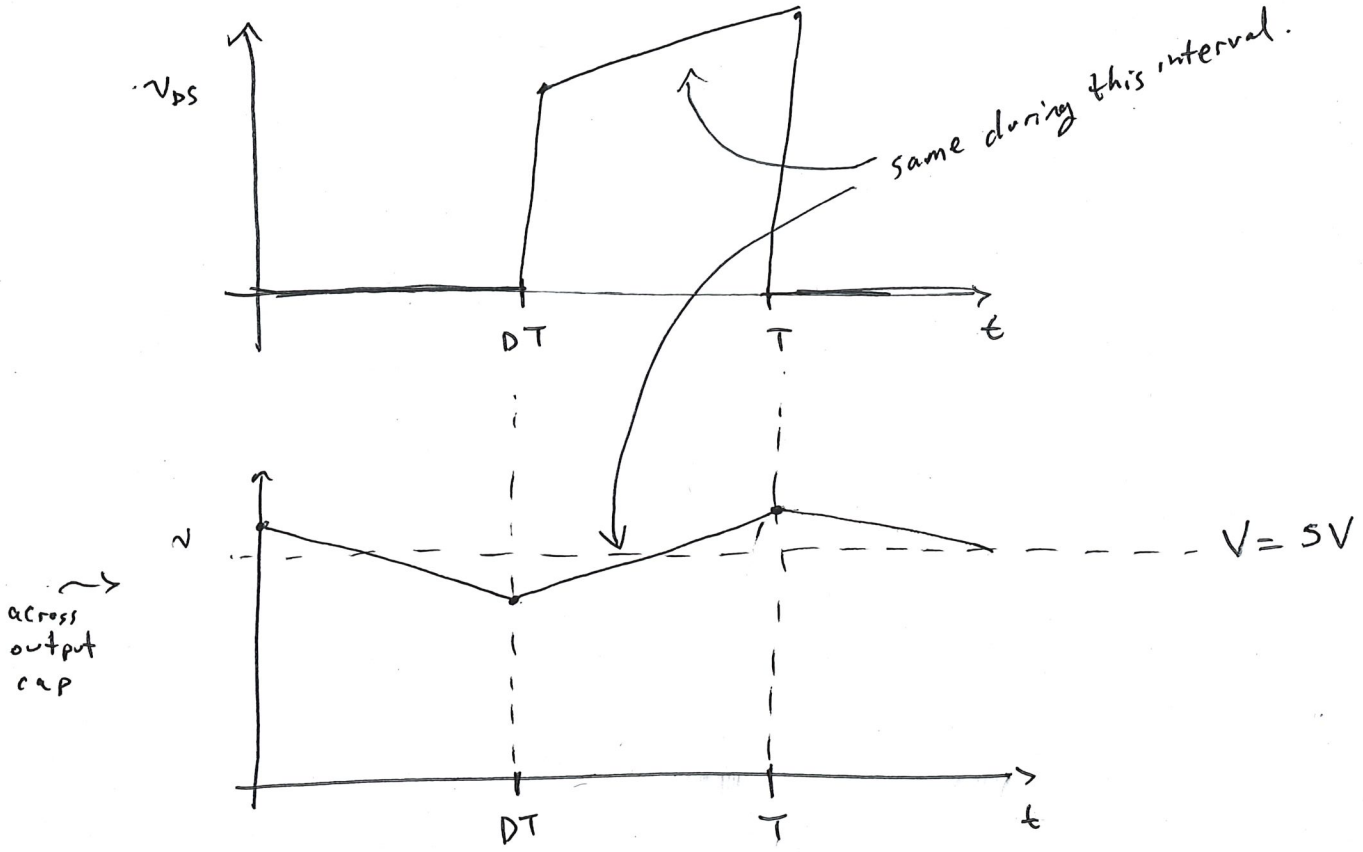
$$\rightarrow D = 1 - \frac{V_g}{V}$$

$$= 1 - \frac{3.3V}{5V} = D$$

$$\approx 0,34$$

2(a)

2(b) V_{DS} looks like



2(c) DC component of v_{ds} is same as average v_{ds} over 1 cycle

$$\langle v_{ds} \rangle = \frac{1}{T}\int_0^T v_{ds}(T) = \frac{1}{T}\left(\int_0^{DT} 0\, dT + \int_{DT}^{T} v(T)\, dT\right)$$

output cap voltage

Same as area of square equal to $V(1-D)T$

$$= \frac{1}{T} V(1-D)T$$

$$= V(1-D) = \langle v_{ds} \rangle \quad \text{2(c)}$$

$$\approx 5V(1-0.34) \approx 3.3\,V$$