EE 452 – Power Electronics Design, Fall 2021

Homework 2

Due Date: Wednesday October 20th 2021, 11:59 PM Pacific Time

Instructions. You must scan your completed homework assignment into a pdf file, and upload your file to the Canvas Assignment HW#1 page by the due date/time above. All pages must be gathered into a single file of moderate size, with the pages in the correct order. Set your phone or scanner for basic black and white scanning. You should obtain a file size of hundreds of kB, rather than tens of MB. I recommend using the "Tiny Scanner" app. Please note that the grader will not be obligated to grade your assignment if the file is unreadable or very large.

Problem 1. In the converter of Fig. 1, the inductor has winding resistance R_L. All other losses can be ignored. The switches operate synchronously: each is in position 1 for $0 < t < DT_s$, and in position 2 for $DT_s < t < T_s$.

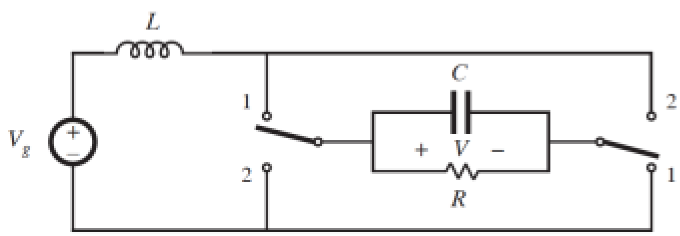
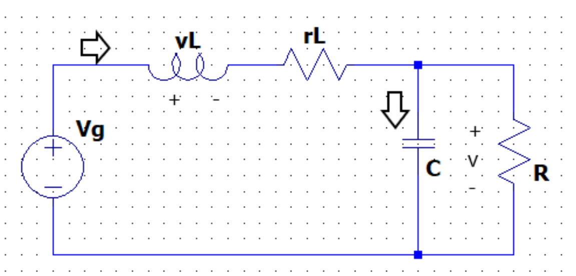
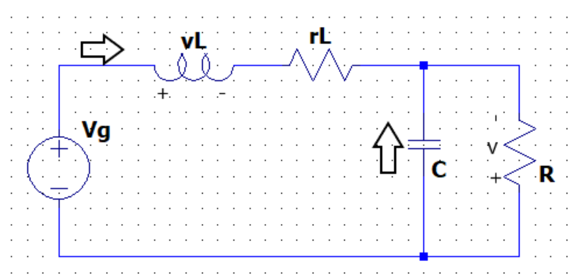


Figure 1: Nonideal current-fed bridge converter for Problem 1.

(a) Derive an equivalent circuit model for this converter, including the inductor winding resistance R_L.





Switch $[0 < t < DT_s]$

$$i_C = i_L - \frac{v}{R}$$

Switch $[DT_s < t < T_s]$

$$i_C = -i_L - \frac{v}{R} = -(i_L + \frac{V}{R})$$

Total Volt Seconds over 1 period for Capacitor Current (small ripple approx)

$$< i_C > = \frac{1}{T_S} \int_o^{T_S} i_C(t)dt$$

$$0 = \left[DT_S \cdot (I - \frac{V}{R})\right] + \left[(D')T_S \cdot (-I - \frac{V}{R})\right]$$

$$0 = DT_S I - D'T_S I - (DT_S \frac{V}{R} + D'T_S \frac{V}{R})$$

$$0 = (D - D')T_S I - (D + D')T_S(\frac{V}{R})$$

$$0 = (D - D')I - \frac{V}{R}$$

$$I = \frac{V/R}{D - D'} \qquad \text{[Inductor Current } I\text{]}$$

Switch $[0 < t < DT_s]$

$$0 = V_g - v_L - i_L R_L - v$$

$$v_L = V_g - i_L R_L - v$$

$$\frac{di_L}{dt} = \frac{V_g - i_L R_L - v}{L} \qquad [i_L \text{ slope}]$$

Switch $[DT_s < t < T_s]$

$$0 = V_g - v_L - i_L R_L + v$$

$$v_L = V_g - i_L R_L + v$$

$$\frac{di_L}{dt} = \frac{V_g - i_L R_L + v}{L} \qquad [i_L \text{ slope}]$$

Total Volt Seconds over 1 period for Inductor Voltage (small ripple approx)

$$< v_L > = \frac{1}{T_S} \int_o^{T_S} v_L(t)dt$$

$$0 = \left[DT_S \cdot (V_g - IR_L - V)\right] + \left[D'T_S \cdot (V_g - IR_L + V)\right]$$

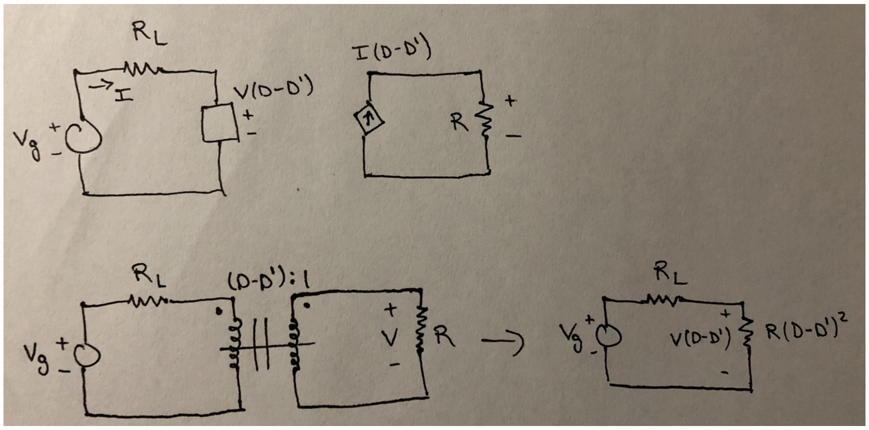
$$0 = DT_S V_g - DT_S IR_L - DT_S V + D'T_S V_g - D'T_S IR_L + D'T_S V$$

$$0 = (D + D')T_S V_g - (D + D')T_S IR_L - DT_S V + D'T_S V$$

$$0 = T_S V_g - T_S IR_L - (D - D')T_S V$$

$$V = \frac{T_S V_g - T_S IR_L}{(D - D')T_S}$$

$$V = \frac{V_g - IR_L}{D - D'} = \frac{V_g - IR_L}{2D - 1} \qquad \text{[Output Voltage } V\text{]}$$



(b) Derive an expression for the nonideal voltage conversion ratio $M = V/V_g$.

Nonideal Voltage Conversion Ratio $M = V/V_g$

$$V = \frac{V_g - IR_L}{D - D'} \qquad \text{and } I = \frac{V/R}{D - D'}$$

$$V = \frac{V_g - \left(\frac{V/R}{D-D'}\right) R_L}{D - D'}$$

$$V = \frac{V_g}{D - D'} - \frac{\left(\frac{V/R}{D-D'}\right) R_L}{D - D'}$$

$$V = \frac{V_g}{D - D'} - V\frac{R_L/R}{(D - D')^2}$$

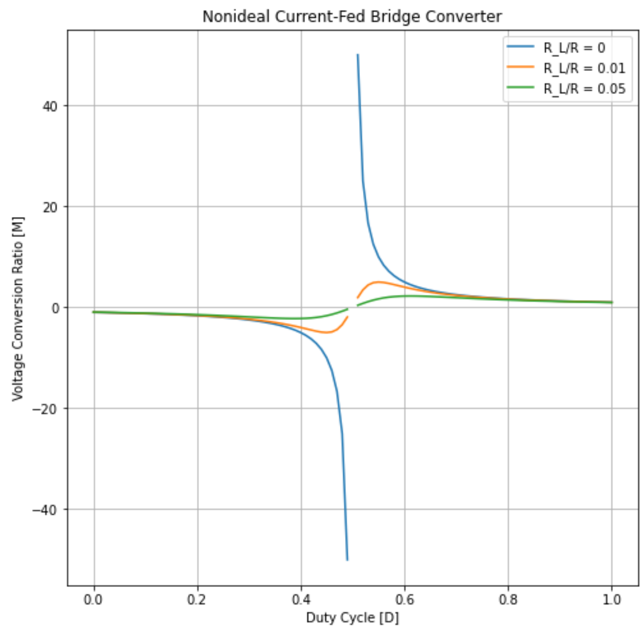
$$V + V\frac{R_L/R}{(D - D')^2} = \frac{V_g}{D - D'}$$

$$V = \frac{\frac{V_g}{D-D'}}{\left(1 + \frac{R_L/R}{(D-D')^2}\right)}$$

$$V = V_g \cdot \frac{1}{D - D'} \cdot \frac{1}{1 + \frac{R_L/R}{(D-D')^2}}$$

$$M = \frac{V}{V_g} = \frac{1}{D - D'} \cdot \frac{1}{1 + \frac{R_L/R}{(D-D')^2}}$$

(c) Plot the voltage conversion ratio M over the range $0 \le D \le 1$, for R_L/R =0, 0.01, and 0.05.



(d) Derive an expression for the efficiency η. Manipulate your expression into a form similar to the textbook Eq. (3.35).

Efficiency $\eta = \frac{P_{out}}{P_{in}}$

$$\frac{P_{out}}{P_{in}} = \frac{V_{out} I_{out}}{V_g I} = M(D) \cdot \frac{I_{out}}{I}$$

$$I = \frac{V/R}{D - D'}$$

$$I_{out} = \frac{V}{R}$$

$$\eta = \frac{1}{D - D'} \cdot \frac{1}{1 + \frac{R_L/R}{(D-D')^2}} \cdot V/R \cdot \left(\frac{V/R}{D - D'}\right)^{-1}$$

$$\eta = \frac{1}{D - D'} \cdot \frac{1}{1 + \frac{R_L/R}{(D-D')^2}} \cdot V/R \cdot \frac{D - D'}{V/R}$$

$$\eta = \frac{1}{1 + \frac{R_L/R}{(D-D')^2}}$$

Problem 2. A single-cell lithium-polymer battery is to be used to power a 3.3 V load. The battery voltage can vary over the usable range 3.0 V $\leq V_{batt} \leq$ 4.2 V. It has been decided to use a buck-boost converter for this application, as illustrated in Fig. 2 below. A suitable MOSFET transistor has been found for Q1, having an on resistance of $R_{on} = 50\,\text{m}\Omega$. A low-VF (low forward voltage) Schottky diode is employed for D1; this diode can be modeled as a fixed voltage drop of $V_D = 0.2$ V (with no series resistance). The inductor has winding resistance R_L. All other sources of loss can be neglected. In your analysis, denote the dc component of the inductor current as I and define it as pointing upwards in the diagram.

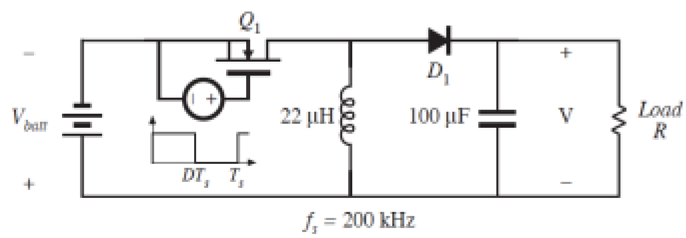
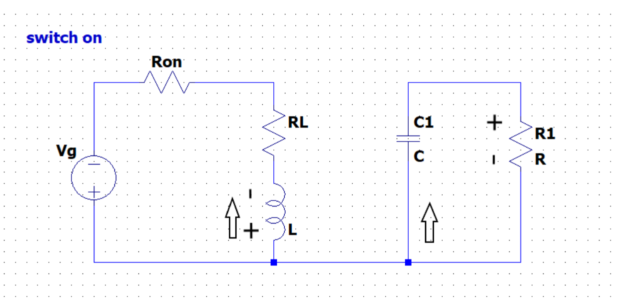
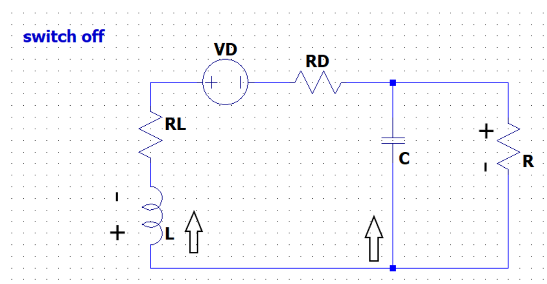


Figure 2: Circuit for Problem 2. Nonideal buck-boost converter powering a 3.3 V load from a lithium-polymer battery.

(a) Derive an equivalent circuit that models the dc properties of this converter and contains an ideal dc transformer. Include the transistor, diode, and inductor conduction losses as described above. Your equivalent circuit model should correctly describe the converter dc input port. Give analytical expressions for all elements in your model. "Analytical expressions" are equations or expressions that are written in terms of variable names such as D, R_{on}, V_D, etc., and that do not have numerical values substituted.





* Current through capacitor should be reversed.

Switch $[0 < t < DT_s]$

$$i_C = -\frac{v}{R}$$

Switch $[DT_s < t < T_s]$

$$i_C = i_L - \frac{v}{R}$$

Total Volt Seconds over 1 period for Capacitor Current (small ripple approx)

$$< i_C > = \frac{1}{T_S}\int_o^{T_S} i_C(t)dt$$

$$0 = \left[DT_S \cdot (-\frac{V}{R})\right] + \left[(D')T_S \cdot (I - \frac{V}{R})\right]$$

$$0 = -(D + D')T_S\frac{V}{R} + D'T_S I$$

$$0 = D'I - \frac{V}{R}$$

$$I = \frac{V/R}{D'} \qquad \text{[Inductor Current } I\text{]}$$

Switch $[0 < t < DT_s]$

$$0 = -V_{batt} + i_L R_{on} + i_L R_L + v_L$$

$$v_L = V_{batt} - i_L R_{on} - i_L R_L$$

$$\frac{di_L}{dt} = \frac{V_{batt} - i_L R_{on} - i_L R_L}{L} \qquad [i_L \text{ slope}]$$

Switch $[DT_s < t < T_s]$

$$0 = -v_L - i_L R_L - V_D - i_L R_D - v$$

$$v_L = -(i_L R_L + V_D + i_L R_D + v)$$

$$\frac{di_L}{dt} = \frac{-(i_L R_L + V_D + i_L R_D + v)}{L} \qquad [i_L \text{ slope}]$$

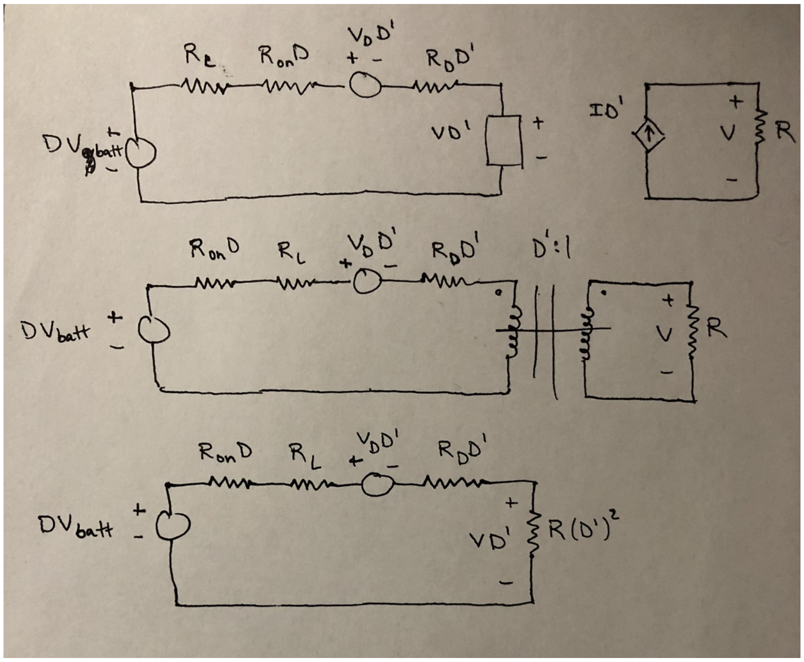
Total Volt Seconds over 1 period for Inductor Voltage (small ripple approx)

$$< v_L > = \frac{1}{T_S}\int_o^{T_S} v_L(t)dt$$

$$0 = [DT_S \cdot (V_{batt} - IR_{on} - IR_L)] + (-1)\left[D'T_S \cdot (IR_L + V_D + IR_D + V)\right]$$

$$0 = -(D + D')T_S IR_L + DT_S(V_{batt} - IR_{on}) - D'T_S(V_D + IR_D + V)$$

$$0 = -IR_L + D(V_{batt} - IR_{on}) - D'(V_D + IR_D + V)$$



(b) Solve your model to find analytical expressions for the converter output voltage V and efficiency η. Express V and η in terms of the following variables: R, R_L, R_{on}, D, D', V_D, and V_{batt}.

$$V = \frac{1}{D'} \cdot \left(DV_{batt} - D'V_D \right) \cdot \left(\frac{R(D')^2}{R(D')^2 + R_{on}D + R_L + R_D D'} \right)$$

$$V = \left(\frac{DV_{batt} - D'V_D}{D'} \right) \cdot \left(\frac{1}{1 + \frac{R_{on}D + R_L + R_D D'}{R(D')^2}} \right)$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{VD'I}{DV_{batt}I}$$

$$\eta = V \cdot \frac{D'}{DV_{batt}}$$

$$\eta = \left(\frac{DV_{batt} - D'V_D}{D'} \right) \cdot \left(\frac{1}{1 + \frac{R_{on}D + R_L + R_D D'}{R(D')^2}} \right) \cdot \frac{D'}{DV_{batt}}$$

$$\eta = \left(\frac{DV_{batt} - D'V_D}{DV_{batt}} \right) \cdot \left(\frac{1}{1 + \frac{R_{on}D + R_L + R_D D'}{R(D')^2}} \right)$$

$$\eta = \left(1 - \frac{D'V_D}{DV_{batt}} \right) \cdot \left(\frac{1}{1 + \frac{R_{on}D + R_L + R_D D'}{R(D')^2}} \right)$$

(c) (*Required for grads, optional 5 points bonus for undergrads*) It is decided that the converter must operate with an efficiency of at least 80% under the following operating condition:

- Input voltage $V_{batt} = 4.0$ V
- Output voltage $V = 3.3$ V
- Load current $I_{load} = 2$ A

Assume that a control system (not shown in Fig. 2) adjusts the duty cycle to 0.5 regulate the output voltage to be $V = 3.3$ V. To meet the above requirements, how large can the inductor winding resistance R_L be?

$$\eta = \left(1 - \frac{D' V_D}{D V_{batt}}\right) \cdot \left(\frac{1}{1 + \frac{R_{on} D + R_L + R_D D'}{R(D')^2}}\right)$$

$$1 + \frac{R_{on} D + R_L + R_D D'}{R(D')^2} = \left(1 - \frac{D' V_D}{D V_{batt}}\right) \cdot \left(\frac{1}{\eta}\right)$$

$$\frac{R_{on} D + R_L + R_D D'}{R(D')^2} = \left(\frac{1}{\eta}\right) \cdot \left(1 - \frac{D' V_D}{D V_{batt}}\right) - 1$$

$$R_{on} D + R_L + R_D D' = \left[\left(\frac{1}{\eta}\right) \cdot \left(1 - \frac{D' V_D}{D V_{batt}}\right) - 1\right] R(D')^2$$

$$R_L = \left[\left(\frac{1}{\eta}\right) \cdot \left(1 - \frac{D' V_D}{D V_{batt}}\right) - 1\right] R(D')^2 - R_{on} D - R_D D'$$

```
Vbatt = 4
eta = 0.8
Ron = 0.05
VD = 0.2
D = 0.5
D_p = 1-D
R = 3.3/2
R_L = ((1/eta)*(1-(D_p*VD)/(D*Vbatt))-1)*R*D_p**2 - Ron*D
print(f'R_L = {round(R_L,3)} Ohms')
```

```
R_L = 0.052 Ohms
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