Problem 1: Conceptual Questions [20 Points Total]

Converters and devices.

(a) **[5 Points]** Why is the flyback converter useful in applications where large voltage conversion ratios are needed?

- The conversion ratio is $V/V_g = n\frac{D}{1-D}$, where n can be chosen to give a large (or very small) conversion ratio. This allows you to keep D in a range where efficiency is high.

(b) **[5 Points]** In converters with half-bridges, dead-time is necessary. What is dead-time and why is it needed?

- Deadtime is the interval where both devices have a low gate-source voltage & should not conduct.
- This aviods a potential short ckt which is caused by devices having a non-zero turn-on & turn-off transition times.

Magnetics.

(a) **[5 Points]** A ferromagnetic material is exposed to an external magnetic field B_{vac}. Why is the magnetic field inside the material larger than B_{vac} and how does this relate to the material permeability μ? Describe what is happening at the atomic level inside the material.

- Field in material is higher b/c dipoles in material align w/ $\vec{B}_{vac}$ & contribute additional flux density.
- μ is proportional to how easily dipoles in material align with an external field $\vec{B}_{vac}$.

(b) **[5 Points]** What is saturation and what is the effective permeability of a material during saturation? Describe what is happening at the atomic level inside the material.

- All dipoles are fully aligned w/ $\vec{B}_{vac}$ during saturation.
- effective permeability is μ_0 (free space permeability).

Problem 2: Magnetics [30 Points Total] Consider the magnetic core in Fig. 1. Assume that the core geometry was designed such that the reluctance through path a, path b, and path c are identical with value $\mathcal{R}$. Winding #1 has n_1 turns.

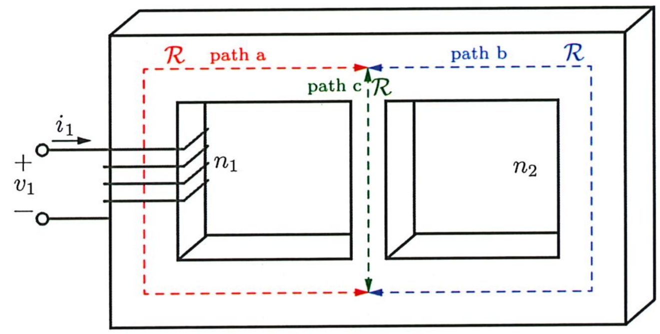
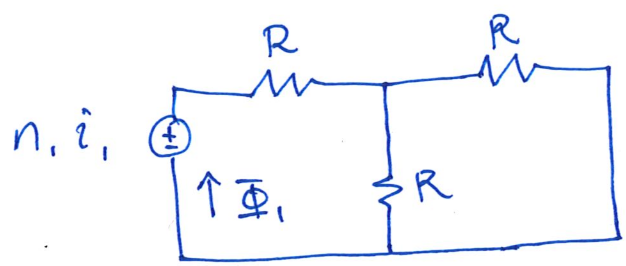


Figure 1: A magnetic core with one coil.

(a) **[5 Points]** Draw the magnetic circuit model of this device.



(b) **[5 Points]** Compute the inductance, L, for coil #1 in terms of $\mathcal{R}$ and n_1.

- Ampere's Law: $n_1 i_1 = \Phi_1 \left(R + \underbrace{R/2}_{R\|R}\right)$

$$= \frac{3}{2}\Phi_1 R$$

$$\Rightarrow \Phi_1 = \frac{2}{3} n_1 i_1 \frac{1}{R}$$

- Faraday's Law: $v_1(t) = n_1 \frac{d\Phi_1}{dt} = \underbrace{\frac{2n_1^2}{3R}}_{L} \frac{di_1}{dt}$

$$\Rightarrow \boxed{L = \frac{2n_1^2}{3R}}$$

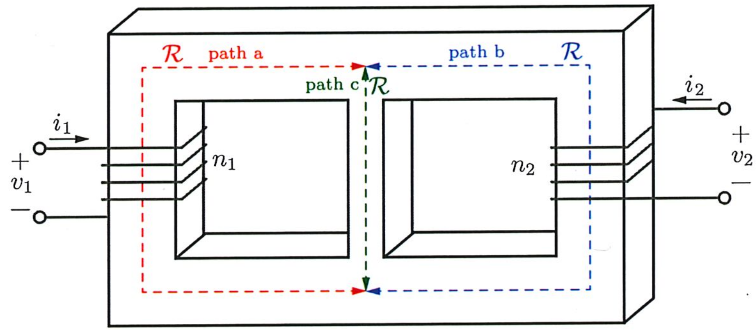
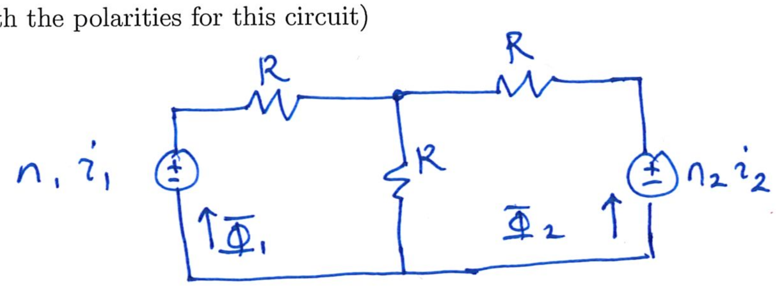


Figure 2: A magnetic core with two coils.

(c) **[5 Points]** A second winding is added to the same core, as shown in Fig. 2. Modify your magnetic circuit model in part (a) to include this winding. (Hint: Be careful with the polarities for this circuit)

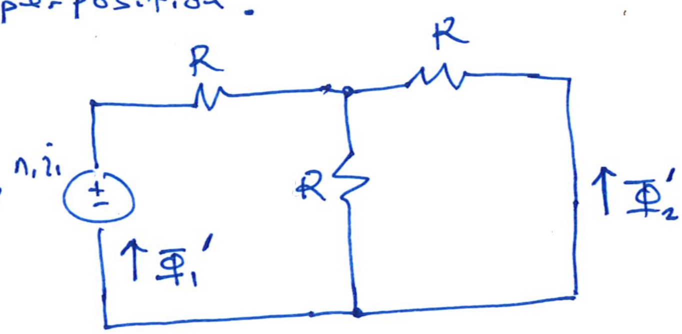


(d) **[15 Points]** Determine analytical expressions of L_{11}, L_{12}, and L_{22} in terms of $\mathcal{R}, n_1$ and n_2, where

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}.$$

- As done in HW7, apply superposition.

→ First set $n_2 i_2 = 0$



$$n_1 i_1 = \Phi_1'(R + \overbrace{R/2}^{R \| R})$$

$$= \frac{3\Phi_1'}{2} R$$

$$\Rightarrow \boxed{\Phi_1' = \frac{2 n_1 i_1}{3R}} \quad (1)$$

and outer KVL loop gives:

$$n_1 i_1 - \Phi_1' R + \Phi_2' R = 0$$

$$\Rightarrow \Phi_2' = \frac{1}{R}\left(\Phi_1' R - n_1 i_1\right)$$

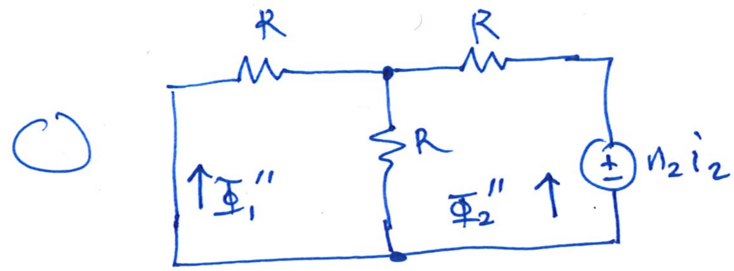
$*$ sub (1)

$$= \frac{1}{R}\left(\frac{2 n_1 i_1}{3\cancel{R}}\cancel{R} - n_1 i_1\right)$$

$$= \frac{1}{R} n_1 i_1 \left(\overbrace{\frac{2}{3} - 1}^{-1/3}\right)$$

$$= \boxed{-\frac{n_1 i_1}{3R} = \Phi_2' \qquad (2)}$$

$\Rightarrow$ Next set $n_1 i_1 = 0$ $\rightarrow$



$$n_2 i_2 = \Phi_2''\left(R + \frac{R}{2}\right) = \frac{3R}{2}\Phi_2''$$

$$\Rightarrow \boxed{\Phi_2'' = \frac{2 n_2 i_2}{3R} \qquad (3)}$$

and outer KVL loop again

$$0 = n_2 i_2 - \Phi_2'' R + \Phi_1'' R$$

$$\Longrightarrow \Phi_1'' = \frac{\Phi_2'' R - n_2 i_2}{R}$$

* sub (3)

$$= \frac{1}{R}\left(\frac{2 n_2 i_2}{3 \cancel{R}} \cancel{R} - n_2 i_2\right)$$

$$= \frac{1}{R} n_2 i_2 \left(\frac{2}{3} - 1\right)$$

$$= \boxed{-\frac{n_2 i_2}{3R} = \Phi_1'' \qquad (4)}$$

- Apply Superposition:

$$\begin{cases} \Phi_1 = \Phi_1' + \Phi_1'' = \dfrac{2}{3}\dfrac{n_1 i_1}{R} - \dfrac{n_2 i_2}{3R} \\[2ex] \Phi_2 = \Phi_2' + \Phi_2'' = -\dfrac{n_1 i_1}{3R} + \dfrac{2}{3}\dfrac{n_2 i_2}{R} \end{cases}$$

Apply Faraday's Law:

$$v_1 = n_1 \frac{d\Phi_1}{dt} = \frac{2 n_1^2}{3R}\frac{di_1}{dt} - \frac{n_1 n_2}{3R}\frac{di_2}{dt}$$

$$v_2 = n_2 \frac{d\Phi_2}{dt} = -\frac{n_1 n_2}{3R}\frac{di_1}{dt} + \frac{2 n_2^2}{3R}\frac{di_2}{dt}$$

$$\rightarrow \boxed{\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} \dfrac{2}{3}\dfrac{n_1^2}{R} & -\dfrac{n_1 n_2}{3R} \\[2ex] -\dfrac{n_1 n_2}{3R} & \dfrac{2}{3}\dfrac{n_2^2}{R} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}}$$

Problem 3: Inductor Design [25 Points Total] Consider the toroidal core with permeability μ, mean path radius r, air gap ℓ_g, and cross-sectional area A_c.

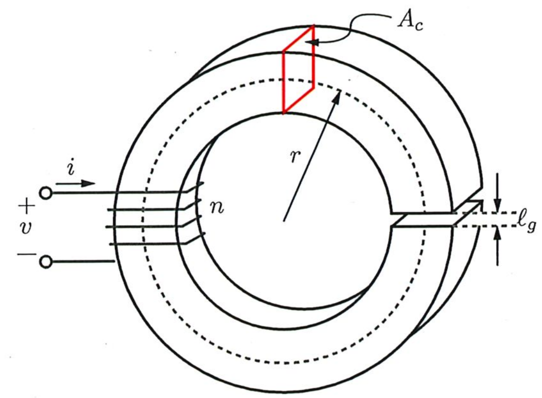
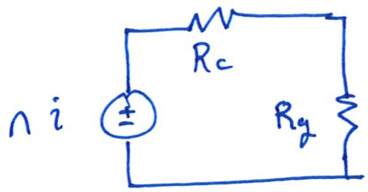


Figure 3: A gapped inductor.

(a) **[5 Points]** Draw the magnetic circuit model of this device and derive the total reluctance, $\mathcal{R}_{\text{net}}$, around the flux path in terms of μ, μ_o, r, ℓ_g, and A_c.



$$R_{net} = R_c + R_g$$

$$= \frac{2\pi r - l_g}{\mu A_c} + \frac{l_g}{\mu_o A_c}$$

$$\boxed{= \frac{2\pi r}{\mu A_c} + \frac{l_g}{A_c}\left(\frac{1}{\mu_o} - \frac{1}{\mu}\right) = R_{net}}$$

(b) **[5 Points]** Compute an analytical expression for the coil inductance in terms of μ, μ_o, n, r, ℓ_g, and A_c.

- Ampere's Law gives:

$$ni = \Phi R_{net}$$

$$\rightarrow \Phi = \frac{ni}{R_{net}} = \frac{ni}{\frac{2\pi r}{\mu A_c} + \frac{l_g}{A_c}\left(\frac{1}{\mu_o} - \frac{1}{\mu}\right)}$$

- And Faraday says

$$v = n\frac{d\Phi}{dt} = n\frac{d}{dt}\left(\frac{ni}{\frac{2\pi r}{\mu A_c} + \frac{l_g}{A_c}\left(\frac{1}{\mu_o} - \frac{1}{\mu}\right)}\right)$$

$$= \underbrace{\frac{n^2}{\frac{2\pi r}{\mu A_c} + \frac{l_g}{A_c}\left(\frac{1}{\mu_o} - \frac{1}{\mu}\right)}}_{L}\frac{di}{dt}$$

$$\Rightarrow \boxed{L = \frac{n^2 A_c}{\frac{2\pi r}{\mu} + l_g\left(\frac{1}{\mu_o} - \frac{1}{\mu}\right)}}$$

(c) **[5 Points]** Suppose you are given a maximum expected current I_{sat} and a material with saturation flux density of B_{sat}. Compute the air gap, ℓ_g, which gives saturation when $i \geq I_{\text{sat}}$. In other words, $B = B_{\text{sat}}$ when $i = I_{\text{sat}}$. Write your expression of ℓ_g in terms of $I_{\text{sat}}, B_{\text{sat}}, \mu, \mu_o, n,$ and r.

- In general, Ampere's Law says

$$n\, i = \Phi\, R_{net}$$

$$\& \quad \Phi = B A_c$$

$$= \underbrace{B \cancel{A_c}}_{\Phi} \left(\frac{2\pi r}{\mu \cancel{A_c}} + \frac{l_g}{\cancel{A_c}} \left(\frac{1}{\mu_o} - \frac{1}{\mu} \right) \right)$$

- Evaluate @ $i = I_{sat}$ & $B = B_{sat}$

$$n\, I_{sat} = B_{sat} \left(\frac{2\pi r}{\mu} + l_g \left(\frac{1}{\mu_o} - \frac{1}{\mu} \right) \right)$$

solve for l_g

$$\rightarrow \quad \frac{n\, I_{sat}}{B_{sat}} - \frac{2\pi r}{\mu} = l_g \left(\frac{1}{\mu_o} - \frac{1}{\mu} \right)$$

$$\rightarrow \quad \boxed{l_g = \frac{1}{\frac{1}{\mu_o} - \frac{1}{\mu}} \left(\frac{n\, I_{sat}}{B_{sat}} - \frac{2\pi r}{\mu} \right)}$$

(d) **[5 Points]** Substitute your expression for ℓ_g in part (c) into your inductance expression in part (b). Simplify your inductance expression such that the inductance is written purely in terms of $n, I_{\text{sat}}, B_{\text{sat}}$, and A_c.

Substitute results

$$L = \frac{n^2 A_c}{\frac{2\pi r}{\mu} + \underbrace{\frac{1}{\cancel{\left(\frac{1}{\mu_0} - \frac{1}{\mu}\right)}}\left(\frac{n I_{sat}}{B_{sat}} - \frac{2\pi r}{\mu}\right)}_{l_g}\cancel{\left(\frac{1}{\mu_0} - \frac{1}{\mu}\right)}}$$

$$= \frac{n^2 A_c}{\cancel{\frac{2\pi r}{\mu}} + \frac{n I_{sat}}{B_{sat}} - \cancel{\frac{2\pi r}{\mu}}}$$

$$= \frac{n^2 A_c}{\frac{n I_{sat}}{B_{sat}}} = \boxed{\frac{n A_c B_{sat}}{I_{sat}} = L}$$

(e) [**2.5 Points**] Fill in the blank: As the maximum expected current I_{sat} increases, the necessary air gap ℓ_g increases.

(f) [**2.5 Points**] Fill in the blank: Suppose you fix the number of turns n and design the air gap according to your solution in part d. As the saturation field B_{sat} increases, the inductance L increases.

Problem 4: Discontinuous Conduction Mode [25 Points Total] Consider the converter in Fig. 4. The input voltage is V_g, and output voltage is V. The MOSFET has duty ratio of D for configuration ①. All losses in the converter can be neglected and the switching frequency is denoted as $f_s = 1/T_s$. The inductor current and diode current are denote i and i_d.

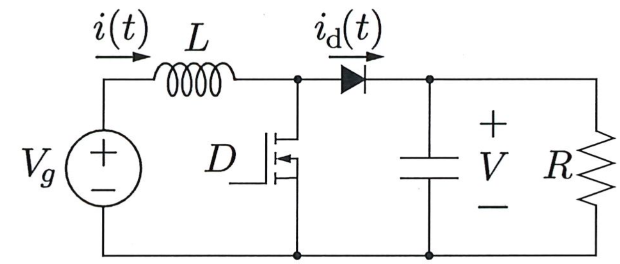
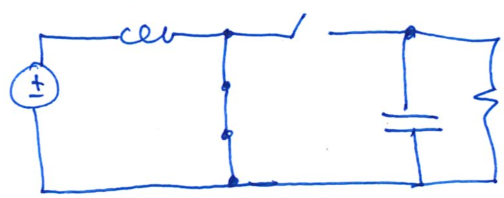


Figure 4: Ideal boost converter for DCM analysis.

(a) **[5 Points]** From here forward, consider the converter operating in discontinuous conduction mode (DCM). Draw the 3 possible circuit configurations and specify which devices are conducting. Denote configurations ①, ②, and ③ as having duty ratios D, D_2, and D_3, respectively.

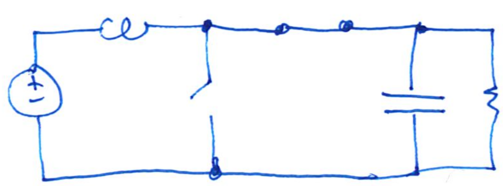
- Config ①



MOSFET conducts

DT_s duration

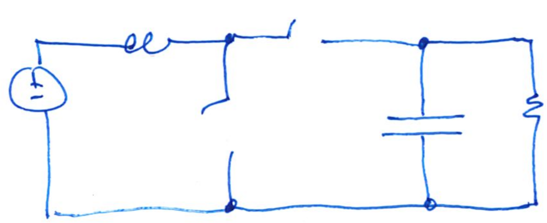
- Config ②



Diode conducts

$D_2 T_s$ duration

- Config ③



No devices conduct

$D_3 T_s$

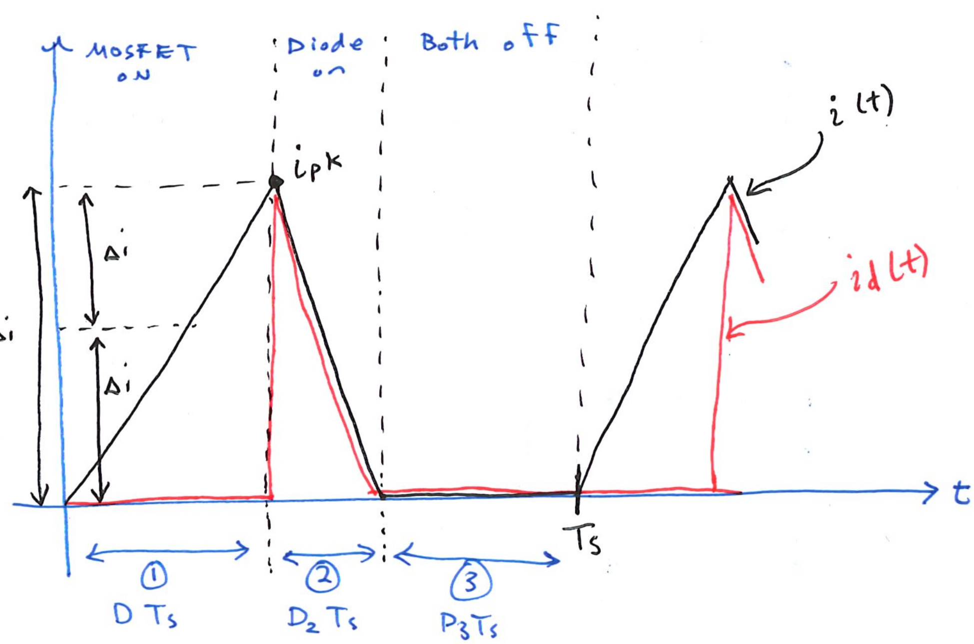
(b) **[5 Points]** Write down the volt-second balance equation and compute D_2 in terms of V, V_g, and D.

$$\langle v_L \rangle = 0 = D(V_g) + D_2(V_g - V) + \cancel{D_3(0)}^{\,0}$$

$$\Rightarrow D_2 = \frac{-D V_g}{V_g - V}$$

$$= \boxed{\frac{D V_g}{V - V_g} = D_2}$$

(c) **[5 Points]** Draw the DCM waveforms for the inductor current, i, and diode current, i_d, on the same plot. Clearly label the inductor current peak value as i_{pk} and label Δi in your plot.



(d) **[5 Points]** Obtain an expression for the averaged diode current, $\langle i_d \rangle$ in terms of V_g, V, D, T_s, and L. (Hint: First compute i_{pk} and also apply your result from part (b))

- Look @ $v_L = L \frac{i_{pk}}{DT_s}$ to get i_{pk}

$$\Rightarrow V_g = L \frac{i_{pk}}{DT_s} \Rightarrow i_{pk} = V_g \frac{DT_s}{L} \quad (1)$$

- Take avg of i_d waveform

$$\langle i_d \rangle = \frac{1}{T_s} \int_0^{T_s} i_d \, dt = \frac{1}{\cancel{T_s}} \left(\frac{1}{2} i_{pk} D_2 \cancel{T_s} \right)$$

* use (1)

$$= \frac{1}{2} \underbrace{V_g \frac{DT_s}{L}}_{i_{pk}} D_2$$

* use result for D_2 in (b)

$$= \frac{1}{2} V_g \frac{DT_s}{L} \underbrace{\frac{DV_g}{(V - V_g)}}_{D_2}$$

$$= \boxed{\frac{T_s}{2L} \frac{D^2 V_g^2}{(V - V_g)} = \langle i_d \rangle}$$

(e) **[5 Points]** Compute the charge-balance equation at the capacitor. Apply your result from part (d) solve for V/V_g in terms of D and K where $K := 2L/(RT_s)$. You will need so to solve a quadratic expression to obtain your final result.

KCL @ cap node gives

$$i_c = i_d - \frac{v}{R}$$

Average both sides

$$\Rightarrow \langle i_c \rangle = 0 = \langle i_d \rangle - \frac{\langle v \rangle}{R}$$

$$= \langle i_d \rangle - \frac{V}{R}$$

* use (3) from part (d)

$$= \frac{T_s}{2L} \frac{D^2 V_g^2}{V - V_g} - \frac{V}{R} = 0$$

multiply by R

$$\rightarrow 0 = \underbrace{\frac{R\,T_s}{2L}}_{1/K} \frac{D^2 V_g^2}{V - V_g} - V = \frac{D^2}{K} \frac{V_g^2}{(V - V_g)} - V$$

Multiply by $V - V_g$

$$\rightarrow 0 = \frac{D^2}{K} V_g^2 - V(V - V_g) = \frac{D^2}{K} V_g^2 - V^2 + V V_g \quad (4)$$

$\div$ by $-V_g^2$ and rearrange into quadratic form

$$\rightarrow 0 = +\left(\frac{V}{V_g}\right)^2 - \left(\frac{V}{V_g}\right) - \frac{D^2}{K}$$

$$= ax^2 + bx + c \quad \text{where} \quad x = \frac{V}{V_g}, \quad a = 1, \; b = -1, \; c = -\frac{D^2}{K}$$

Solve for $x = V/V_g$ & keep ⊕ solution

$$\rightarrow \frac{V}{V_g} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{1 + \sqrt{1 - 4(1)(-D^2/K)}}{2}$$

$$\boxed{= \frac{1 + \sqrt{1 + 4D^2/K}}{2} = \frac{V}{V_g}}$$