EE 452 – Power Electronics Design, Fall 2021

Homework 3

Due Date: Thursday October 28th 2021, 11:59pm Pacific Time

Instructions. You must scan your completed homework assignment into a pdf file, and upload your file to the Canvas Assignment page by the due date/time above. All pages must be gathered into a single file of moderate size, with the pages in the correct order. Set your phone or scanner for basic black and white scanning. You should obtain a file size of hundreds of kB, rather than tens of MB. I recommend using the "Tiny Scanner" app. Please note that the grader will not be obligated to grade your assignment if the file is unreadable or very large.

Description In Problems 1–3 on the next page, the input voltage V_g is dc and positive with the polarity shown. Specify how to implement the switches using a minimal number of diodes and transistors, such that the converter operates over the entire range of duty cycles $0 \leq D \leq 1$. The switch states should vary as shown in Fig. 1. You may assume that the inductor current ripples and capacitor voltage ripples are small. For each problem, do the following:

(a) Realize the switches using SPST ideal switches, and explicitly define the voltage and current of each switch.

(b) Express the on-state current and off-state voltage of each SPST switch in terms of the converter inductor currents, capacitor voltages, and/or input source voltage.

(c) Solve the converter to determine the inductor currents and capacitor voltages, as in Chapter 2.

(d) Determine the polarities of the switch on-state currents and off-state voltages. Do the polarities vary with duty cycle?

(e) State how each switch can be realized using transistors and/or diodes, and whether the realization requires single-quadrant, current-bidirectional two-quadrant, voltage-bidirectional two-quadrant, or four-quadrant switches.

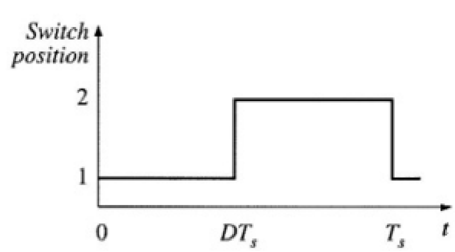
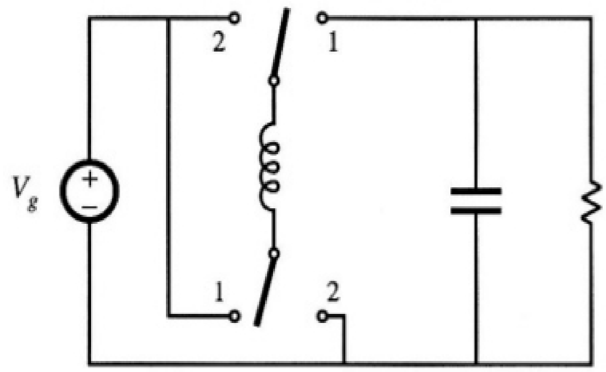
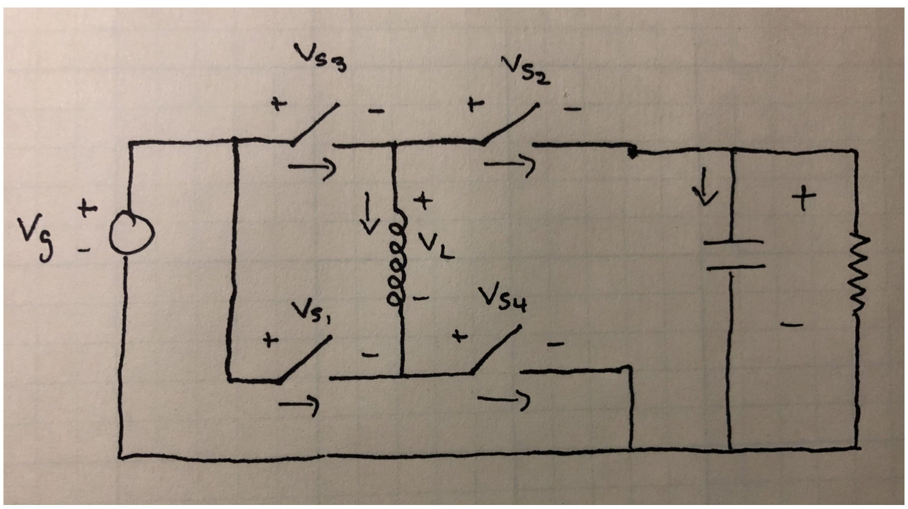


Figure 1: Switch control method for Problems 1–3.

a.)



Circuit for Problem 1.



b.)

Switch Position	Switch	Voltage	Current
1	1	$V_{S1} = 0$	$I_{S1} = i_L$
1	2	$V_{S2} = 0$	$I_{S2} = i_L$
1	3	$V_{S3} = V_g - v$	$I_{S3} = 0$
1	4	$V_{S4} = V_g$	$I_{S4} = 0$
2	1	$V_{S1} = V_g$	$I_{S1} = 0$
2	2	$V_{S2} = V_g - v$	$I_{S2} = 0$
2	3	$V_{S3} = 0$	$I_{S3} = -i_L$
2	4	$V_{S4} = 0$	$I_{S4} = -i_L$

c.)

Switch $[0 < t < DT_s]$

$$0 = V_g - v_L - v$$

$$v_L = V_g - v$$

$$\frac{di_L}{dt} = \frac{V_g - v}{L} \qquad [i_L \text{ slope}]$$

Switch $[DT_s < t < T_s]$

$$0 = V_g + v_L$$

$$v_L = -V_g$$

$$\frac{di_L}{dt} = \frac{-V_g}{L} \qquad [i_L \text{ slope}]$$

Total Volt Seconds over 1 period for Inductor Voltage (small ripple approx)

$$< v_L > = \frac{1}{T_S} \int_o^{T_S} v_L(t)dt$$

$$0 = \left[D \cdot (V_g - V)\right] + \left[D' \cdot (-V_g)\right]$$

$$0 = (D - D')V_g - DV$$

$$V = V_g \frac{(D - D')}{D} = V_g \frac{(2D - 1)}{D} \qquad [\text{Output Voltage } V]$$

Switch $[0 < t < DT_s]$

$$i_C = i_L - \frac{v}{R}$$

Switch $[DT_s < t < T_s]$

$$i_C = -\frac{v}{R}$$

Total Volt Seconds over 1 period for Capacitor Current (small ripple approx)

$$< i_C > = \frac{1}{T_S} \int_o^{T_S} i_C(t)dt$$

$$0 = \left[D \cdot (I - \frac{V}{R})\right] + \left[(D') \cdot (-\frac{V}{R})\right]$$

$$0 = DI - (D + D')(\frac{V}{R})$$

$$0 = DI - (\frac{V}{R})$$

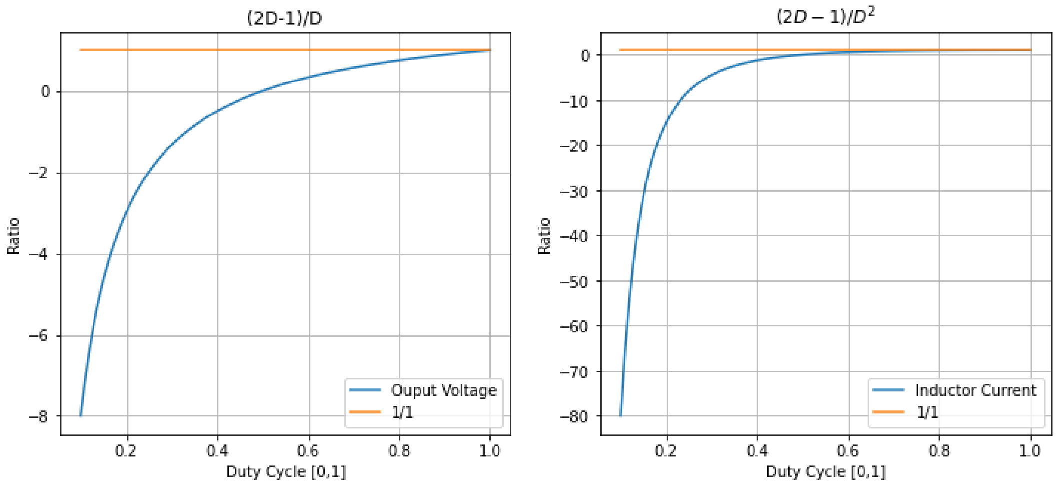
$$I = \frac{V}{RD}$$

$$I = V_g \frac{(2D - 1)}{D} \frac{1}{RD}$$

$$I = \frac{V_g}{R} \frac{(2D - 1)}{D^2} \qquad [\text{Inductor Current } I]$$

d.)

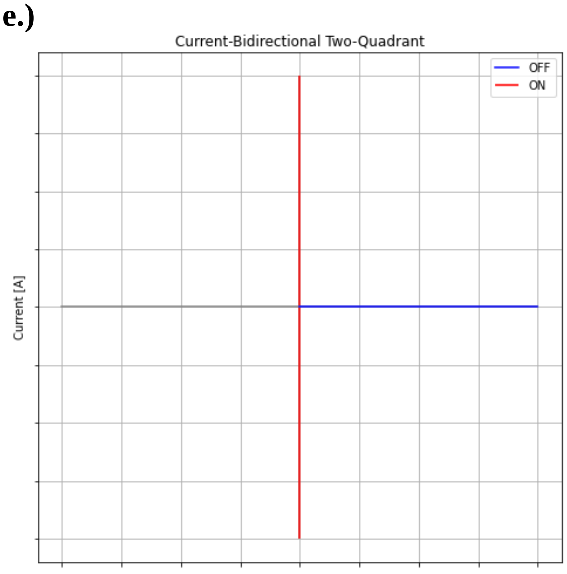
Switch Position	Switch	Voltage	Current
1	1	0	+/-
1	2	0	+/-
1	3	+	0
1	4	+	0
2	1	+	0
2	2	+	0
2	3	0	+/-
2	4	0	+/-



Output voltage ratio ranges from $(-\infty, 1]$, thus V can only be as large as V_g.
Switch voltage equals $V_g - V$ so it is always greater than or equal to 0.
Polarity **doesn't** vary with duty cycle.

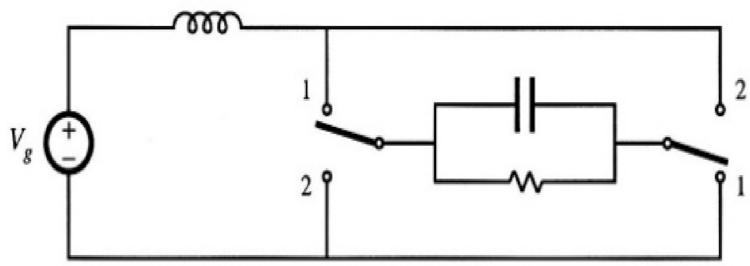
Inductor current ratio ranges from $(-\infty, 1]$.
Switch current is a function of inductor current only so polarity **varies** with the duty cycle.

e.)

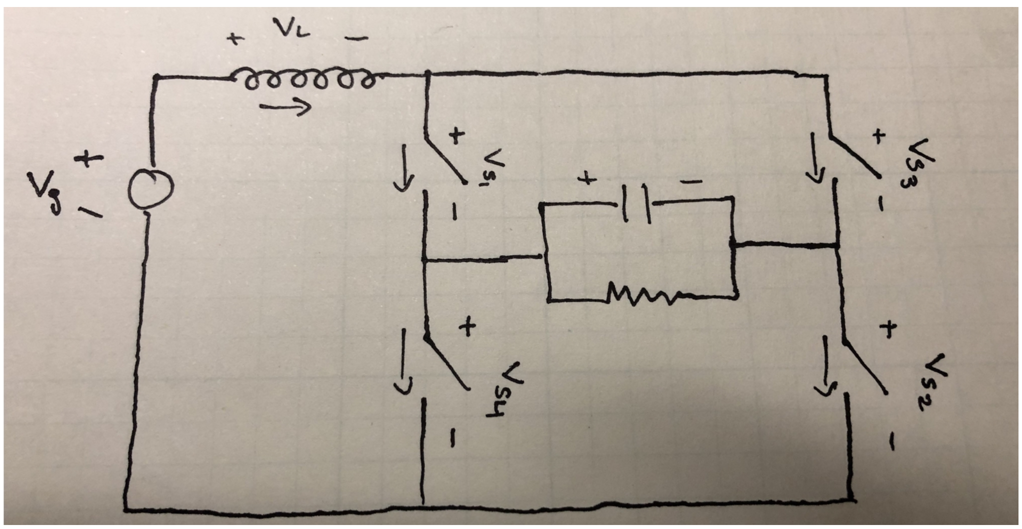


All switches are current-bidirectional two-quadrant. They can be realized with a power MOSFET.

a.)



Circuit for Problem 2.



b.)

Switch Position	Switch	Voltage	Current
1	1	$V_{S1} = 0$	$I_{S1} = i_L$
1	2	$V_{S2} = 0$	$I_{S2} = i_L$
1	3	$V_{S3} = v$	$I_{S3} = 0$
1	4	$V_{S4} = v$	$I_{S4} = 0$
2	1	$V_{S1} = -v$	$I_{S1} = 0$
2	2	$V_{S2} = -v$	$I_{S2} = 0$
2	3	$V_{S3} = 0$	$I_{S3} = i_L$
2	4	$V_{S4} = 0$	$I_{S4} = i_L$

c.) Switch $[0 < t < DT_s]$

$$0 = V_g - v_L - v$$

$$v_L = V_g - v$$

$$\frac{di_L}{dt} = \frac{V_g - v}{L} \qquad [i_L \text{ slope}]$$

Switch $[DT_s < t < T_s]$

$$0 = V_g - v_L + v$$

$$v_L = V_g + v$$

$$\frac{di_L}{dt} = \frac{V_g + v}{L} \qquad [i_L \text{ slope}]$$

Total Volt Seconds over 1 period for Inductor Voltage (small ripple approx)

$$< v_L > = \frac{1}{T_S} \int_o^{T_S} v_L(t)dt$$

$$0 = \left[D \cdot (V_g - V)\right] + \left[D' \cdot (V_g + V)\right]$$

$$0 = (D + D')V_g + (D' - D)V$$

$$0 = V_g + (1 - 2D)V$$

$$V = -V_g \frac{1}{1 - 2D} = V_g \frac{1}{2D - 1} \qquad [\text{Output Voltage } V]$$

Switch $[0 < t < DT_s]$

$$i_C = i_L - \frac{v}{R}$$

Switch $[DT_s < t < T_s]$

$$-i_L = i_C + \frac{v}{R}$$

$$i_C = -(i_L + \frac{v}{R})$$

Total Volt Seconds over 1 period for Capacitor Current (small ripple approx)

$$< i_C > = \frac{1}{T_S} \int_o^{T_S} i_C(t)dt$$

$$0 = \left[D \cdot (I - \frac{V}{R})\right] + \left[(D') \cdot (-I - \frac{V}{R})\right]$$

$$0 = (D - D')I + (D + D')(\frac{-V}{R})$$

$$0 = (D - D')I - (\frac{V}{R})$$

$$I = (\frac{V}{R})\frac{1}{D - D'}$$

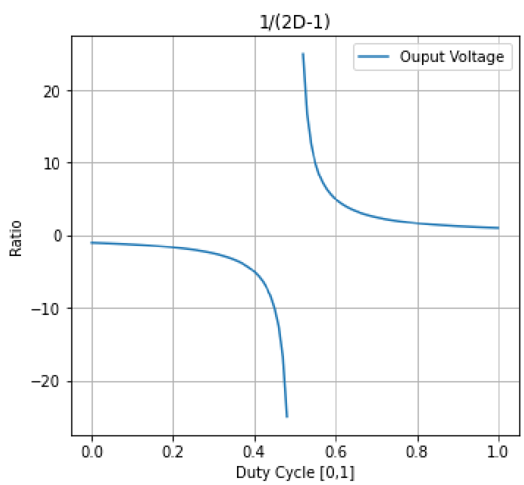
$$I = (\frac{V}{R})\frac{1}{2D - 1}$$

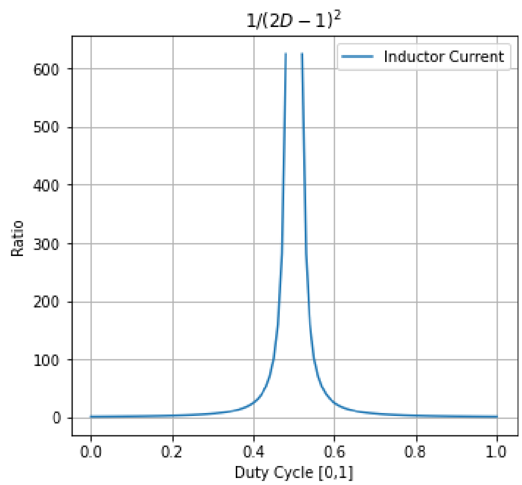
$$I = \frac{V_g}{R} \cdot \frac{1}{2D - 1} \cdot \frac{1}{2D - 1}$$

$$I = \frac{V_g}{R}\frac{1}{(2D - 1)^2} = \frac{V_g}{R}\frac{1}{(D - D')^2} \qquad [\text{Inductor Current } I]$$

d.)

Switch Position	Switch	Voltage	Current
1	1	0	+
1	2	0	+
1	3	+/-	0
1	4	+/-	0
2	1	+/-	0
2	2	+/-	0
2	3	0	+
2	4	0	+

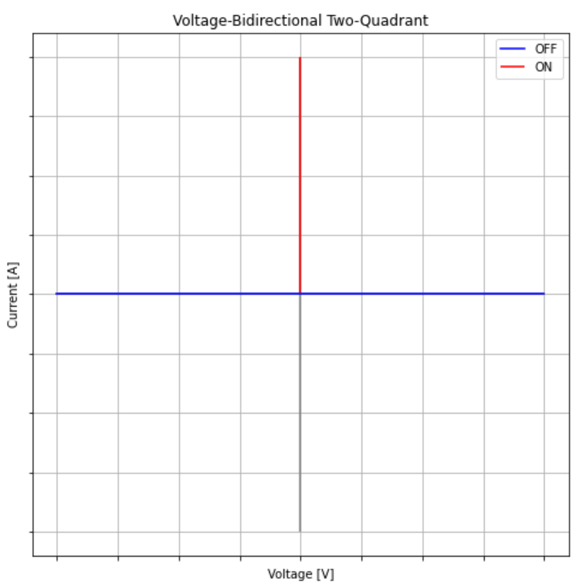




Output voltage ratio ranges from $(-\infty, -1)$ and $(1, \infty)$.
Switch voltage is a function of output voltage only so polarity **varies** with the duty cycle.

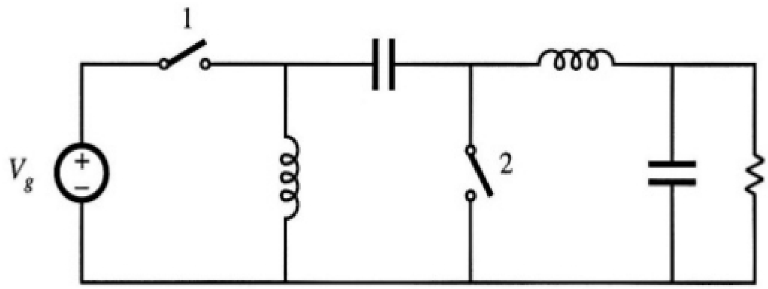
Inductor current ratio ranges from $(1, \infty)$.
Switch current is a function of inductor current so polarity **doesn't** vary with the duty cycle.

e.)

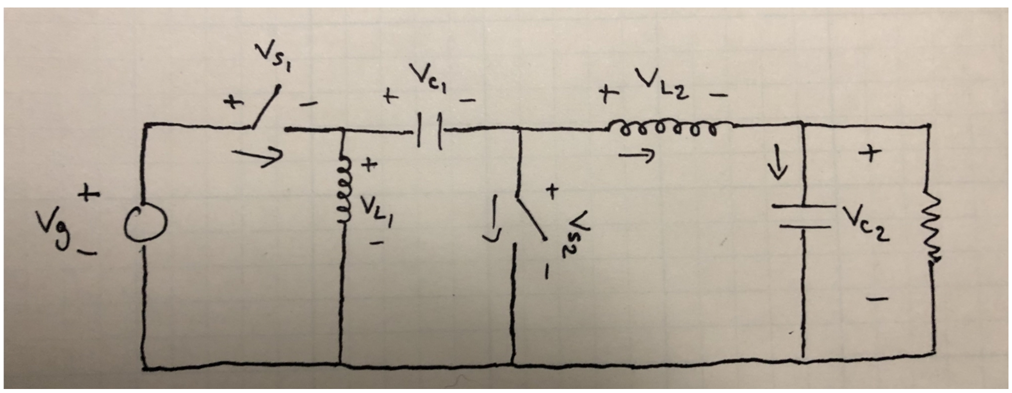


All switches are voltage-bidirectional two-quadrant. They can be realized with a series BJT transistor and diode.

a.)



Circuit for Problem 3.



b.)

Switch Position	Switch	Voltage	Current
1	1	$V_{S1} = 0$	$I_{S1} = i_{L1} + i_{L2}$
1	2	$V_{S2} = V_g - v_{C1}$	$I_{S2} = 0$
2	1	$V_{S1} = V_g - v_{C1}$	$I_{S1} = 0$
2	2	$V_{S2} = 0$	$I_{S2} = -(i_{L1} + i_{L2})$

c.)

Switch $[0 < t < DT_s]$

$$0 = V_g - v_{L_1}$$
$$v_{L_1} = V_g$$

Switch $[DT_s < t < T_s]$

$$0 = v_{L_1} - v_{C_1}$$
$$v_{L_1} = v_{C_1}$$

Switch $[0 < t < DT_s]$

$$0 = V_g - v_{C_1} - v_{L_2} - v$$
$$v_{L_2} = V_g - v_{C_1} - v$$

Switch $[DT_s < t < T_s]$

$$0 = -v_{L_2} - v$$
$$v_{L_2} = -v$$

Total Volt Seconds over 1 period for V_{L_1} (small ripple approx)

$$< v_{L_1} > = \frac{1}{T_S}\int_o^{T_S} v_{L_1}(t)dt$$
$$0 = \left[D \cdot (V_g)\right] + \left[D' \cdot (V_{C_1})\right]$$
$$V_{C_1} = -V_g\frac{D}{D'} \qquad \text{[Capacitor\_1 Voltage]}$$

Total Volt Seconds over 1 period for V_{L_2} (small ripple approx)

$$< v_{L_2} > = \frac{1}{T_S}\int_o^{T_S} v_{L_2}(t)dt$$
$$0 = \left[D \cdot (V_g - V_{C_1} - V)\right] + \left[D' \cdot (-V)\right]$$
$$0 = DV_g - DV_{C_1} - (D + D')'V$$
$$0 = DV_g - DV_{C_1} - V$$
$$V = DV_g - (V_g\frac{D}{D'})D$$
$$V = V_g \cdot D(1 + \frac{D}{D'}) \qquad \text{[Output Voltage } V\text{]}$$

Switch $[0 < t < DT_s]$

$$i_{C_1} = i_{L_2}$$

Switch $[DT_s < t < T_s]$

$$i_{C_1} = -i_{L_1}$$

Switch $[0 < t < DT_s]$

$$i_{C_2} = i_{L_2} - \frac{v}{R}$$

Switch $[DT_s < t < T_s]$

$$i_{C_2} = i_{L_2} - \frac{v}{R}$$

Total Volt Seconds over 1 period for i_{C_2} (small ripple approx)

$$< i_{C_2} > = \frac{1}{T_S}\int_o^{T_S} i_{C_2}(t)dt$$

$$0 = \left[D \cdot (I_{L_2} - \frac{V}{R})\right] + \left[D' \cdot (I_{L_2} - \frac{V}{R})\right]$$

$$0 = I_{L_2} - \frac{V}{R}$$

$$I_{L_2} = \frac{V}{R}$$

$$I_{L_2} = \frac{V_g}{R} \cdot D(1 + \frac{D}{D'}) \qquad \text{[Inductor Current } I_{L_2}\text{]}$$

Total Volt Seconds over 1 period for i_{C_1} (small ripple approx)

$$< i_{C_1} > = \frac{1}{T_S}\int_o^{T_S} i_{C_1}(t)dt$$

$$0 = \left[D \cdot (I_{L_2})\right] + \left[(D') \cdot (-I_{L_1})\right]$$

$$0 = DI_{L_2} - D' I_{L_1}$$

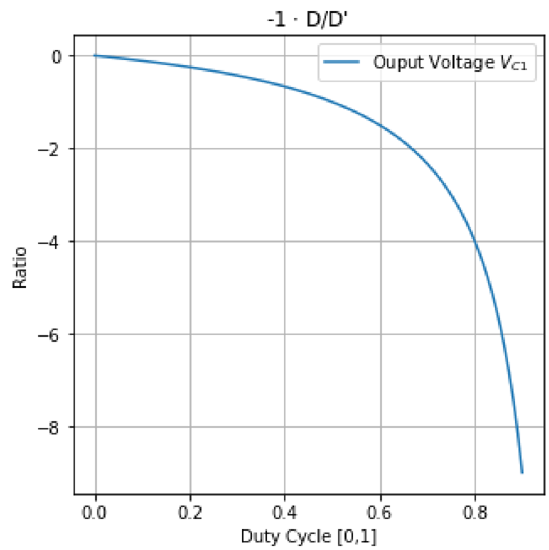
$$I_{L_1} = \frac{D}{D'} I_{L_2}$$

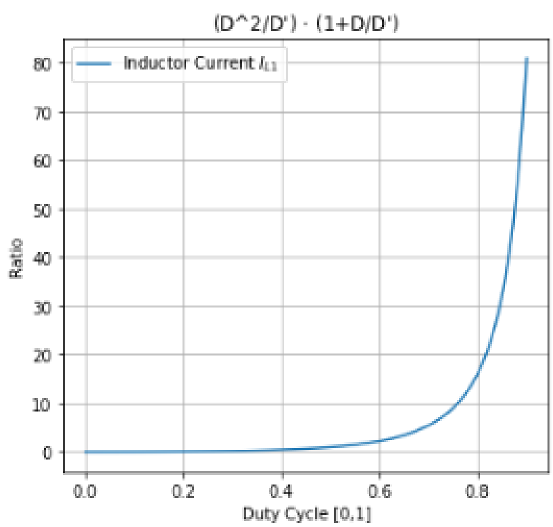
$$I_{L_1} = \frac{D}{D'} \frac{V}{R}$$

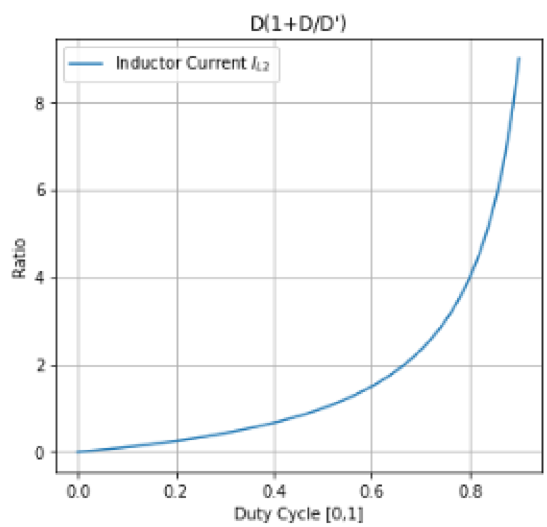
$$I_{L_1} = \frac{V_g}{R} \cdot \frac{D^2}{D'}(1 + \frac{D}{D'}) \qquad \text{[Inductor Current } I_{L_1}\text{]}$$

d.)

Switch Position	Switch	Voltage	Current
1	1	0	+
1	2	+	0
2	1	+	0
2	2	0	-



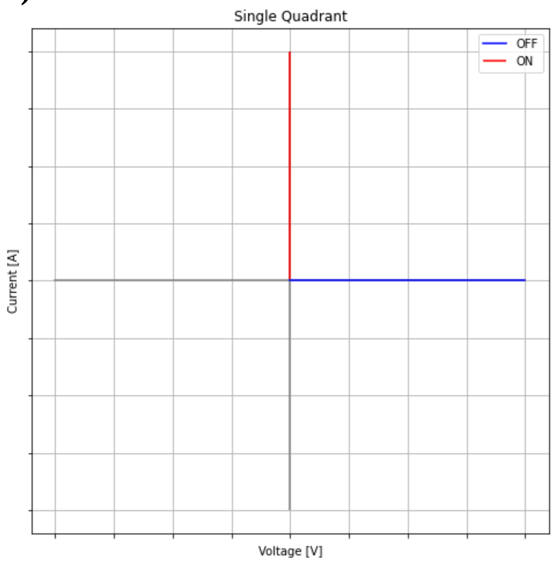




Output voltage ratio ranges from $(0, -\infty)$.
Switch voltage is a function of $V_g - V$ so polarity **doesn't** vary with the duty cycle.

Inductor current ratio ranges from $(0, \infty)$.
Switch current is sum of inductor currents so polarity **doesn't** vary with the duty cycle.

e.)



All switches are single quadrant. They can be realized by a BJT transistor.