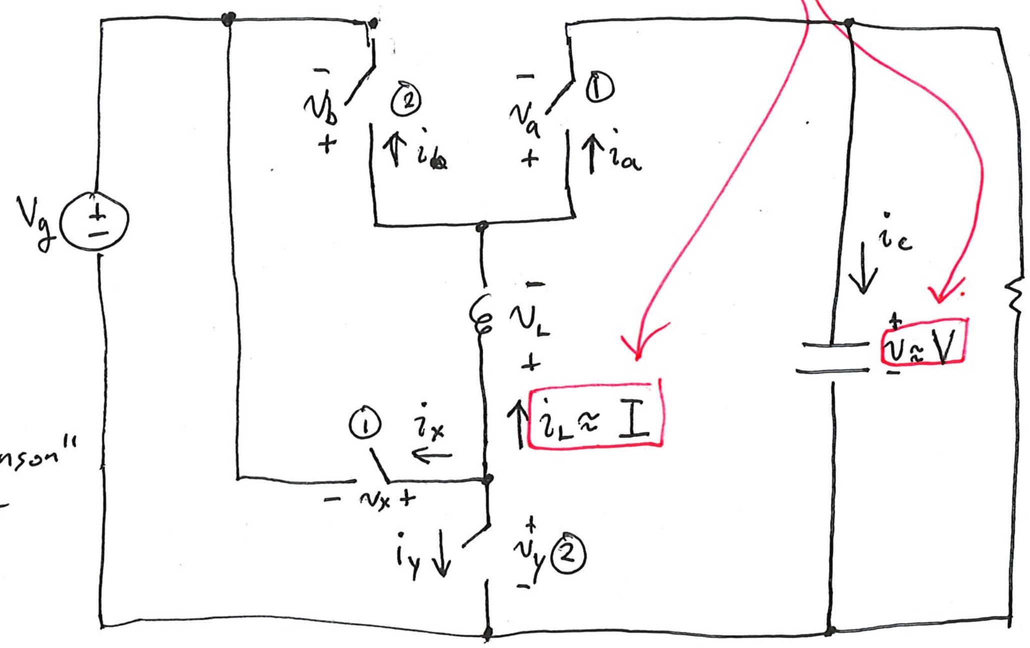
Homework #3 Solutions

Problem #1

[A] Redraw w/ SPST's & define polarities of voltages & currents.

"Small ripple approximation" (SRA) applied here



Called a "Watkins-Johnson" Converter

[B] Express ~~polarities of~~ $v_a, v_b, v_x, v_y, i_a, i_b, i_x, i_y$ in terms of V_g, I, V.

off state sw voltages	on-state sw currents
$v_a = V_g - V$	$i_a = I$
$v_b = V_g - V$	$i_b = I$
$v_x = V_g$	$i_x = -I$
$v_y = V_g$	$i_y = -I$

$\boxed{C}$ • Solve ckt using Ch 2 approach. I suggest you draw ckt for configurations ① & ②, then use KVL to find v_L & KCL to find i_c in both settings.

• Apply volt-second & charge balance equations:

$$\langle v_L \rangle = 0 = D(\overbrace{V_g - V}^{\text{KVL in ①}}) + (1-D)(\overbrace{-V_g}^{\text{KVL in ②}})$$

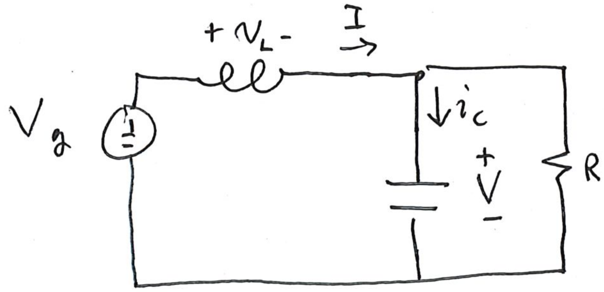
$$= V_g(2D-1) - DV \qquad (1)$$

$$\langle i_c \rangle = 0 = D(\overbrace{I - \frac{V}{R}}^{\text{KCL in ①}}) + (1-D)(\overbrace{-\frac{V}{R}}^{\text{KCL in ②}})$$

$$= -\frac{V}{R} + DI \qquad (2)$$

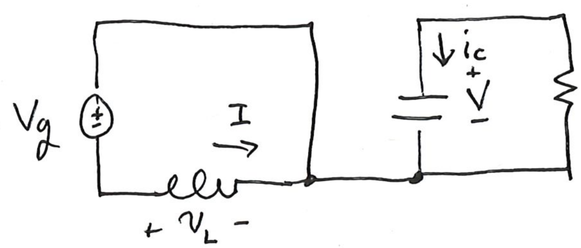
①

• circuit in config ①



②

• circuit in config ②



• (1)–(2) are 2 equations with 2 unknowns (V, I). Solve for V, I:

▫ Get V from (1)

$$\boxed{V = V_g \frac{2D-1}{D}} \qquad (3) \qquad \boxed{C}$$

▫ (3) → (2) to get I

$$I = \frac{V}{DR} = \boxed{\frac{V_g}{R}\,\frac{(2D-1)}{D^2} = I} \quad (4)$$

$\boxed{D}$ Determine polarities of off sw voltages & on currents using solutions from $\boxed{c}$.

- sw a

sw a

- $v_a = V_g - V = V_g - V_g\left(\frac{2D-1}{D}\right)$

$= V_g\left(1 - \frac{2D-1}{D}\right) = V_g\left(\frac{D-2D+1}{D}\right)$

$= \underbrace{V_g}_{\oplus \text{ only}} \underbrace{\left(\frac{1-D}{D}\right)}_{\oplus \text{ only for } 0<D<1} \Rightarrow \boxed{v_a \text{ can be } \oplus \text{ only}}$

product of two ⊕ #'s is also positive

sw a

- $i_a = I = \underbrace{\frac{V_g}{R}}_{\oplus} \underbrace{\frac{1}{D^2}}_{\oplus} \underbrace{(2D-1)}_{\substack{\oplus \text{ for } D>1/2 \\ \ominus \text{ for } 0<D<1/2}} \Rightarrow \boxed{i_a \text{ can be } \oplus \text{ or } \ominus}$, depending on D

$\oplus \times \oplus \times (\oplus \text{ or } \ominus)$

↳ i_a ⊕ for $D>1/2$

i_a ⊖ for $0<D<1/2$

- sw b is same as sw a

sw b

$\Rightarrow \boxed{v_b \oplus \text{ only}}, \boxed{i_b \oplus \text{ or } \ominus}$

- sw x

$v_x = V_g \Rightarrow \boxed{v_x \oplus \text{ only}}$

sw x

$i_x = -I$

$= -\frac{V_g}{R}\frac{1}{D^2}(2D-1) \Rightarrow \boxed{i_x \oplus \text{ or } \ominus}$

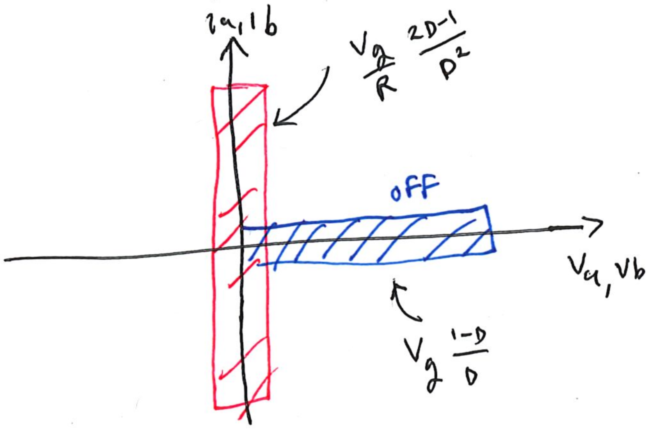
- sw Y same as sw X

sw y

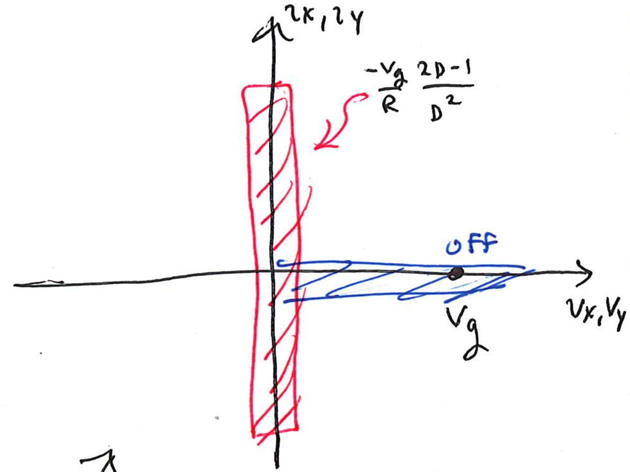
$\Rightarrow \boxed{v_y \oplus \text{ only}, \quad i_y \oplus \text{ or } \ominus}$

look @ quadrants

sw a & sw b



sw x & sw Y



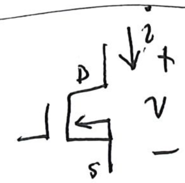
↑ Two quadrant, current bidirectional

↳ sw a & b are MOSFETs

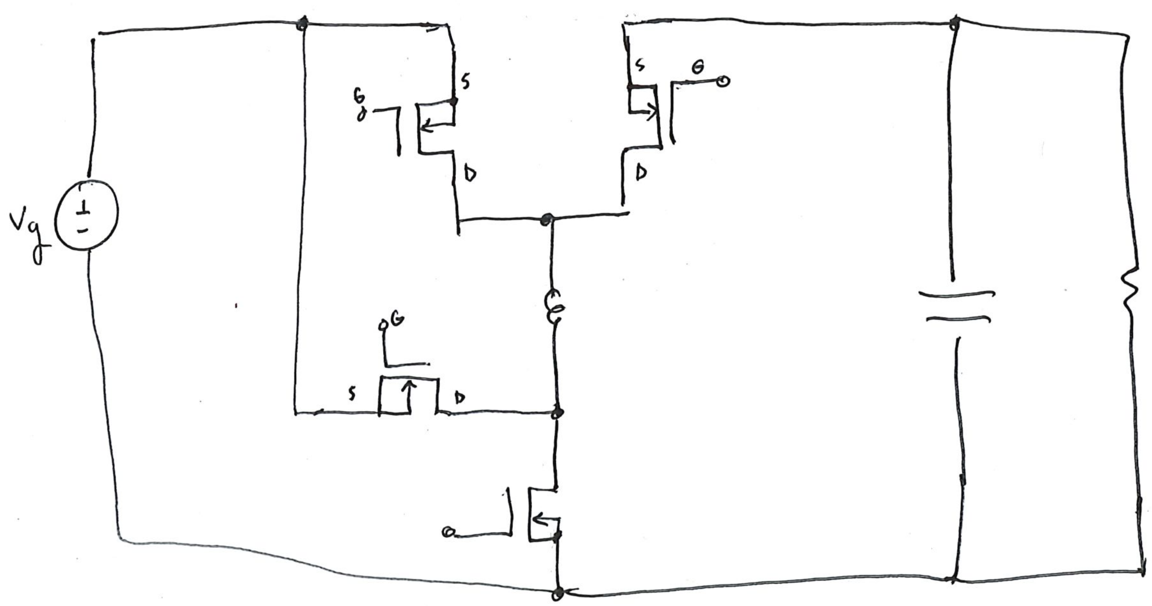
↑ two quadrant, current bidirection

↳ sw x, sw y also MOSFETs

MOSFET = sign convention

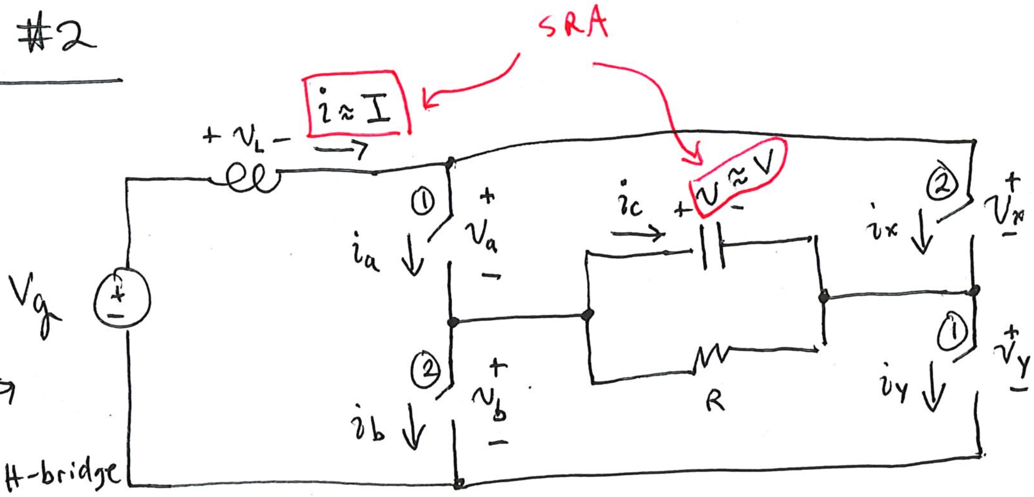


Realization:



Problem #2

A



called a "current source H-bridge Inverter"

B Write sw v's & i's in terms of V_g, I, V.

off state sw voltages	on state sw currents
$v_a = -V$	$i_a = I$
$v_b = +V$	$i_b = I$
$v_x = +V$	$i_x = I$
$v_y = -V$	$i_y = I$

C Apply voltsecond & charge balance ... similar to ~~chee~~ HW2 problem #1.

$$\langle v_L \rangle = 0 = D(V_g - V) + (1-D)(V_g + V)$$

$$= V_g + V(1-2D) \qquad (1)$$

$$\langle i_c \rangle = 0 = D\left(I - \frac{V}{R}\right) + (1-D)\left(-I - \frac{V}{R}\right)$$

$$= -\frac{V}{R} + I(2D-1) \qquad (2)$$

- Solve for V, I.
- Rearrange (2)

$$\rightarrow \quad I = \frac{V}{R}\,\frac{1}{2D-1} \qquad (3)$$

- Put (3) → (1) to get

$$V = \frac{-V_g}{1-2D} = \boxed{V_g \frac{1}{2D-1} = V} \qquad (4)$$

- (4) → (3) gives

$$\boxed{I = \frac{V_g}{R}\,\frac{1}{(2D-1)^2}} \qquad (5)$$

C

$\boxed{D}$: Look at sw v's & i's in $\boxed{B}$ using solution in $\boxed{C}$

sw a

$$v_a = -V = \underbrace{-V_g}_{\ominus}\,\underbrace{\frac{1}{2D-1}}_{\substack{\oplus \text{ for } D > 1/2 \\ \ominus \text{ for } 0 < D < 1/2}} \Rightarrow v_a \text{ can be } \oplus \text{ or } \ominus$$

$$i_a = I = \underbrace{\frac{V_g}{R}}_{\oplus}\,\underbrace{\frac{1}{(2D-1)^2}}_{\substack{\oplus \text{ only b/c} \\ \text{denominator} \\ \text{is squared}}} \Rightarrow i_a \oplus \text{ only}$$

sw Y

- sw Y is same as sw a → v_Y ⊕ or ⊖, i_Y ⊕ only

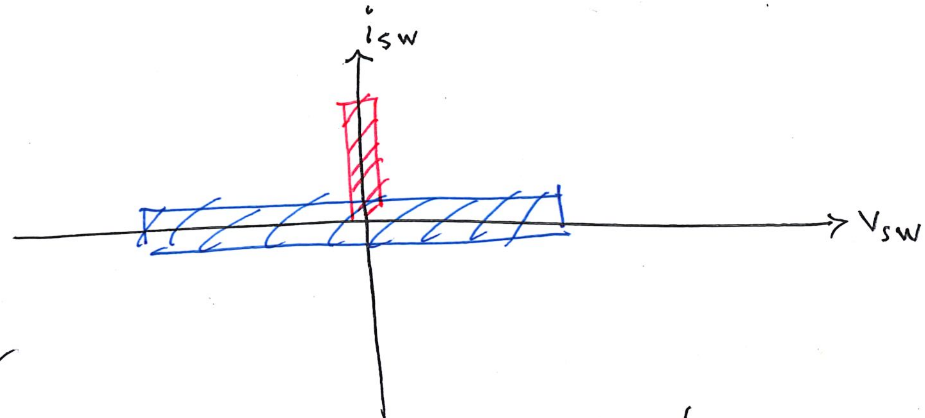
• sw b

sw b:

$$v_b = +V = \underbrace{V_g}_{+} \; \underbrace{\frac{1}{2D-1}}_{+ \text{ or } -} \Rightarrow v_b \; + \text{ or } -$$

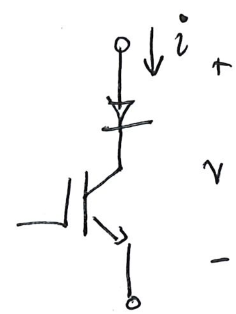
$$i_b = \text{same} \ldots \Rightarrow i_a \; + \text{ only}$$

↳ all switches experience same polarities

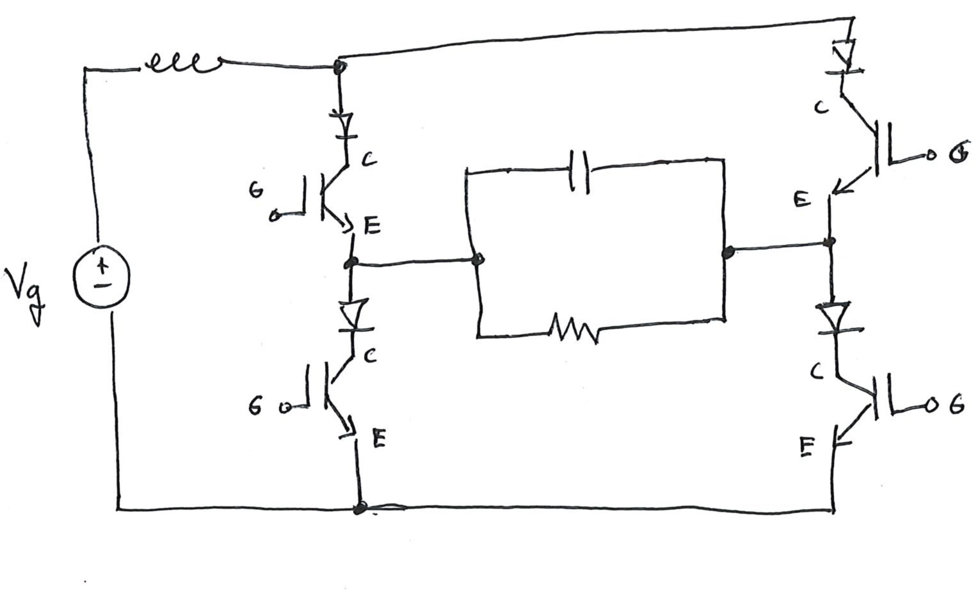


↳ 2 quadrant, voltage bidirectional

↳ all are series diode & IGBT (or MOSFET)

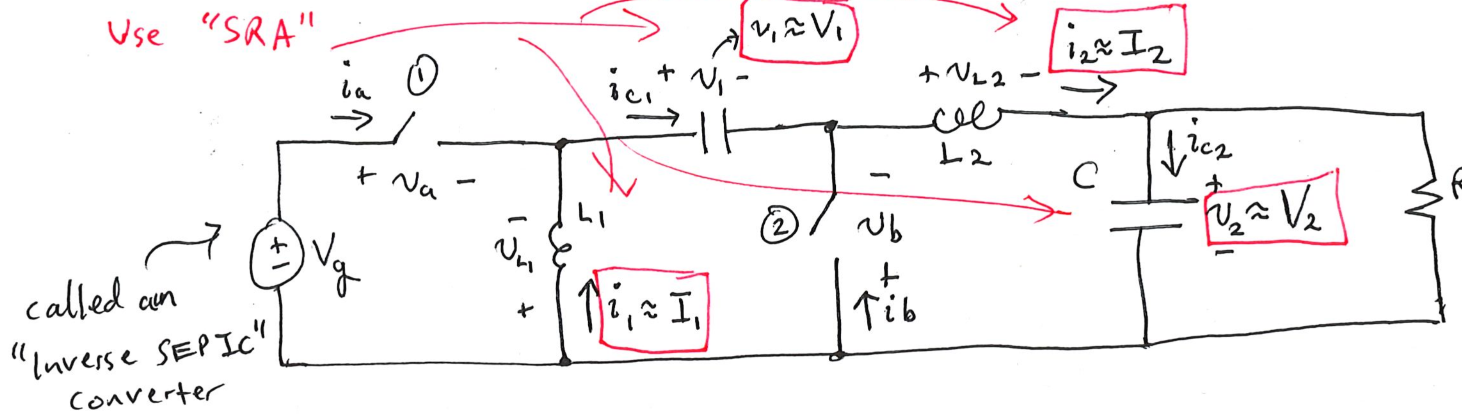


Realization:



Problem # 3

$\boxed{A}$ already done in prompt... just need to define v's & i's in circuit & switches.



$\boxed{B}$ Express v_a, i_a, v_b, i_b in terms of I_1, I_2, V_1, V_2

off-state sw voltages	on-state sw currents
$v_a = V_g - V_1$	$i_a = I_2 - I_1$
$v_b = -(V_g - V_1) = V_1 - V_g$	$i_b = I_2 - I_1$

$\boxed{C}$ Solve for I_1, I_2, V_1, V_2

$$\langle v_{L_1} \rangle = 0 = D(\underbrace{-V_g}_{*\text{ KVL ①}}) + (1-D)(\underbrace{-V_1}_{*\text{ KVL ②}})$$

$$= -D V_g - V_1 D' \quad \text{let } D' := 1-D \qquad (1)$$

$$\langle v_{L_2} \rangle = 0 = D(\underbrace{V_g - V_1 - V_2}_{*\text{ KVL ①}}) + (1-D)(\underbrace{-V_2}_{*\text{ KVL ②}})$$

$$= D(V_g - V_1) - V_2 \qquad (2)$$

$$\langle i_{c_1} \rangle = 0 = D(\underbrace{I_2}_{*\text{KCL ①}}) + (1-D)(\underbrace{I_1}_{*\text{KCL ②}}) \qquad (3)$$

$$\langle i_{c_2} \rangle = 0 = \underbrace{I_2 - \frac{V_2}{R}}_{*\text{ KCL same for ① \& ②}} \qquad (4)$$

Helpful Note: To get quantities marked with (*), redraw ckt for both switch ~~[illegible]~~ configurations. Then carefully look @ KVL & KCL for configurations ① & ②. I skipped this step since I am more experienced. I suggest you not skip this step.

- We now have 4 equations ((1)–(4)) & 4 unknowns (I_1, I_2, V_1, V_2). Next solve system of equations w/ algebra

- (1) gives

$$\boxed{V_1 = \frac{-D}{D'} V_g} \checkmark \qquad (5)$$

- (5) → (2) gives

*from (2) directly

substitute (5)

$$V_2 = D(V_g - V_1)$$

$$= D\left(V_g + \frac{D}{D'} V_g\right)$$

$$= D V_g \left(1 + \frac{D}{D'}\right) = D V_g \left(\frac{D' + D}{D'}\right)$$

$1 - \cancel{D} + \cancel{D}$

$$= \boxed{\frac{D}{D'} V_g = V_2} \checkmark \qquad (6)$$

- Rearrange (4) then substitute (6)

$$I_2 = \frac{V_2}{R} = \boxed{\frac{D V_g}{D' R} = I_2} \checkmark \qquad (7)$$

- Rearrange (3) then substitute (7)

$$I_1 = \frac{-D}{D'} I_2 = \boxed{\frac{-D^2}{D'^2} \frac{V_g}{R} = I_1} \checkmark \qquad (8)$$

$\boxed{c}$

$\boxed{D}$ Look @ polarities for v_a, i_a, v_b, i_b.

Re-examine sw v's & currents we started with and substitute solutions in (5)-(8).

sw a

- $v_a = V_g - V_1 = V_g + \frac{D}{D'} V_g$

$= V_g\left(1 + \frac{D}{D'}\right) = V_g\left(\frac{\overbrace{D' + D}^{1-D+D}}{D'}\right)$

$= \frac{V_g}{D'} \rightarrow$ $\boxed{v_a \oplus \text{ only}}$ since V_g & D' both $\oplus$ only

- $i_a = I_2 - I_1 = I_2 + \frac{D}{D'} I_2$

$= I_2\left(1 + \frac{D}{D'}\right) = \frac{I_2}{D'}$

$= \frac{D}{D'^2}\frac{V_g}{R} \rightarrow$ $\boxed{i_a \oplus \text{ only}}$ since V_g, R, D, D' all $\oplus$ only

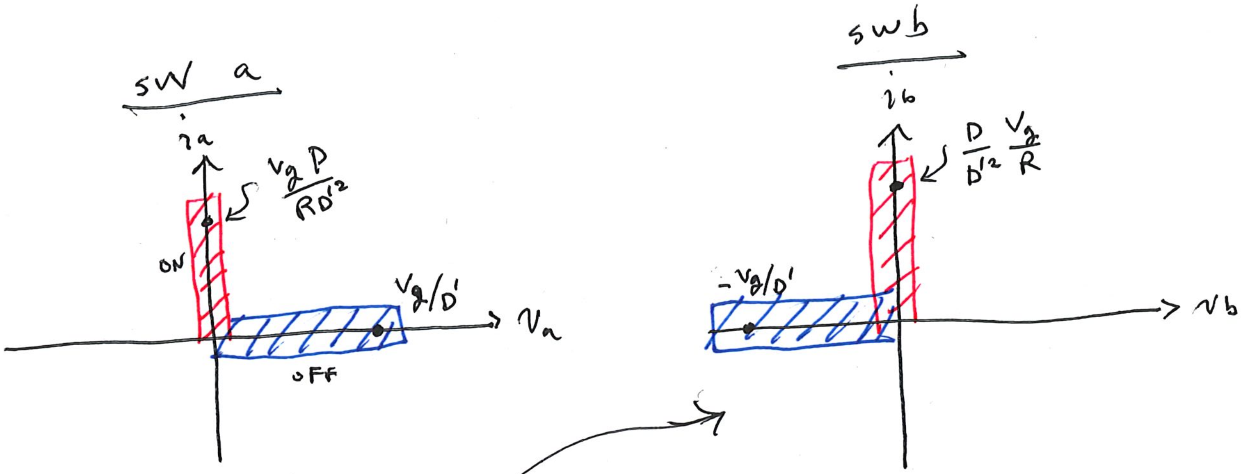
sw b

- $v_b = V_1 - V_g = -v_a = -\frac{V_g}{D'} \rightarrow$ $\boxed{v_b \ominus \text{ only}}$

- $i_b = \ldots$ same as $i_a = \frac{D}{D'^2}\frac{V_g}{R} \rightarrow$ $\boxed{i_b \oplus \text{ only}}$

D

$\boxed{E}$ Look @ quadrants & pick devices



Both are "single quadrant" devices

- sw a = MOSFET or IGBT
- sw b = Diode

Realization below ↓