EE 452 – Power Electronics Design, Fall 2021

Homework 5

Due Date: Monday November 15th 2021, 11:59 pm Pacific Time

Instructions. You must scan your completed homework assignment into a pdf file, and upload your file to the Canvas Assignment page by the due date/time above. All pages must be gathered into a single file of moderate size, with the pages in the correct order. Set your phone or scanner for basic black and white scanning. You should obtain a file size of hundreds of kB, rather than tens of MB. I recommend using the "Tiny Scanner" app. Please note that the grader will not be obligated to grade your assignment if the file is unreadable or very large.

Problem 1: The buck-boost converter of Fig. 1 is implemented with a MOSFET and a p-n diode. The MOSFET can be modeled as ideal, but the diode exhibits a substantial reverse-recovery process, with reverse recovery time t_r and recovered charge Q_r. In addition, the inductor has winding resistance R_L. The converter operates in the continuous conduction mode (the small ripple approximation can be used). Derive an equivalent circuit that models the dc components of the converter waveforms and that accounts for the loss elements described above.

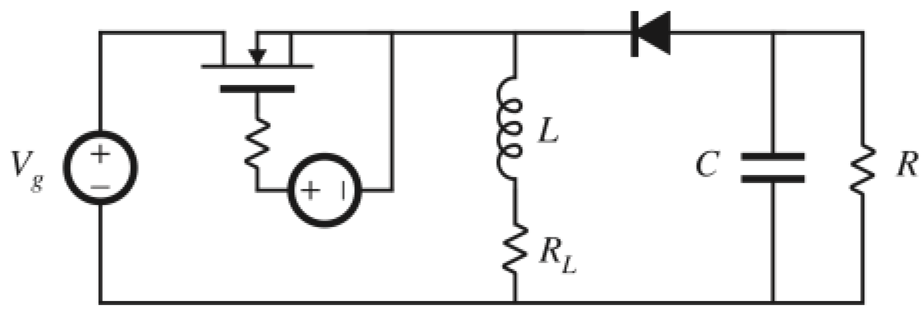
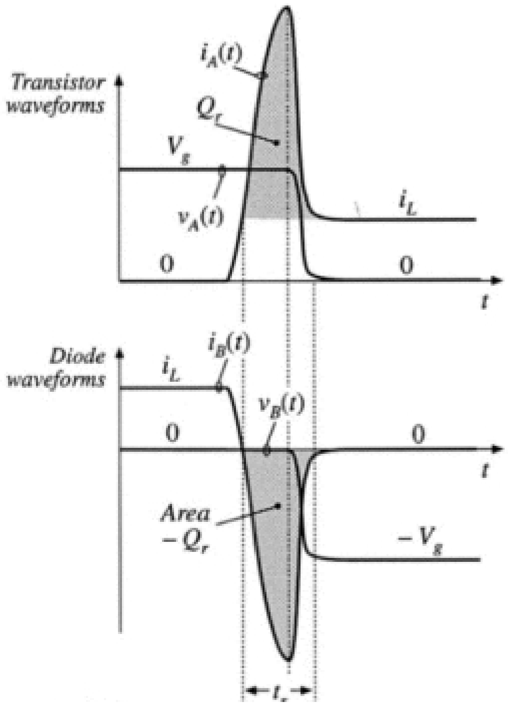


Figure 1: Buck-boost converter of Problem 1.



Transistor Current

$$
\begin{aligned}
< i_t > &= \int_o^{T_S} i_t(t)dt \\
&= D(I_L) + \text{Reverse Recovery} + D'(0) \\
&= D(I_L) + \frac{Q_R}{T_s} + \frac{t_r I_L}{T_s} + D'(0) \\
&= DI_L + \frac{Q_R}{T_s} + \frac{t_r I_L}{T_s}
\end{aligned}
$$

Switch $[0 < t < DT_s]$

$$
\begin{aligned}
0 &= V_g - v_L - IR_L \\
v_L &= V_g - IR_L
\end{aligned}
$$

Switch $[DT_s < t < T_s]$

$$
\begin{aligned}
0 &= IR_L + v_L - v \\
v_L &= v - IR_L
\end{aligned}
$$

Total Volt Seconds over 1 period for Inductor Voltage

$$\int_o^{T_S} v_L(t)dt$$

$$0 = D(V_g - IR_L) + D'(V - IR_L)$$

$$0 = DV_g - IR_L + D'V \qquad \text{[Equivalent Voltage Circuit]}$$

Capacitor Current

Switch $[0 < t < DT_s]$

$$i_C = -i_D - \frac{v}{R}$$

Switch $[DT_s < t < T_s]$

$$i_C = -i_D - \frac{v}{R}$$

Diode Reverse Recovery $[i_D]$

$$< i_D > = \int_o^{T_S} i_D(t)dt$$

$$i_D = D(0) + D'(I_L) - \text{Reverse Recovery}$$

$$i_D = D' I_L - \frac{Q_R}{T_s} - \frac{t_r I_L}{T_s}$$

Total Charge Balance over 1 period for Capacitor Current I

$$< i_C > = \int_o^{T_S} i_C(t)dt$$

$$0 = D(-i_D - \frac{-V}{R}) + D'(-i_D - \frac{V}{R})$$

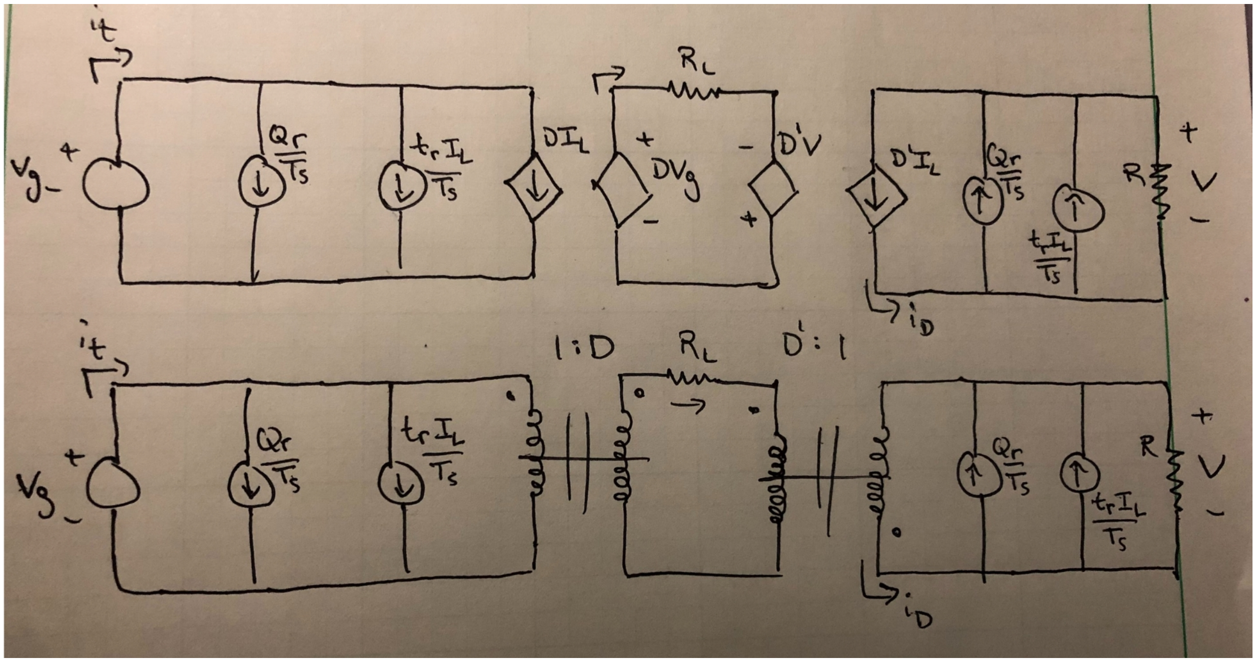
$$0 = \frac{-V}{R} - i_D$$

$$0 = \frac{-V}{R} - D' I_L - \left(-\frac{Q_R}{T_s} - \frac{t_r I_L}{T_s}\right)$$

$$0 = -D' I_L + \frac{Q_R}{T_s} + \frac{t_r I_L}{T_s} - \frac{V}{R}$$

$$0 = D' I_L - \frac{Q_R}{T_s} - \frac{t_r I_L}{T_s} + \frac{V}{R} \qquad \text{[Equivalent Current Circuit]}$$

Equivalent circuit: models DC components of converter waveforms and accounts for loss elements



Problem 2: The elements of the buck-boost converter of Fig. 2 are ideal: all losses may be ignored. Your results for parts (a) and (b) should agree with Table 5.2 in the book.

1. Show that the converter operates in discontinuous conduction mode when $K < K_{\text{crit}}$ and derive expressions for K and K_{crit}.
2. Derive an expression for the dc conversion ratio V/V_g of the buck-boost converter operating in discontinuous conduction mode.
3. For $K = 0.1$, plot V/V_g over the entire range $0 \leq D \leq 1$.
4. ~~Sketch the inductor voltage and current waveforms for $K = 0.1$ and $D = 0.3$. Label salient features.~~
5. ~~What happens to V at no load ($R \to \infty$)? Explain why, physically.~~

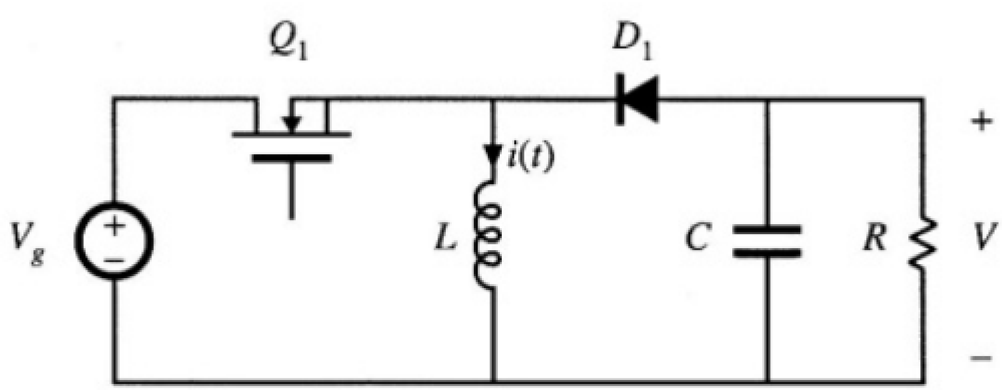


Figure 2: Buck-boost converter of Problem 2.

1.

If $I < \Delta i$, then the slope will reach the y=0 axis and have moments where it is discontinuous and doesn't conduct current.

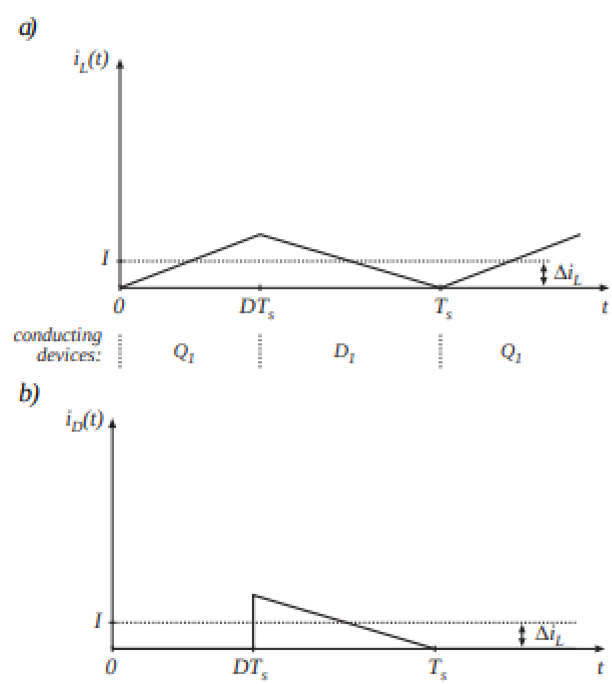


Fig. 5.3. Buck converter waveforms at the boundary between the continuous and discontinuous conduction modes: (a) inductor current $i_L(t)$, (b) diode current $i_D(t)$.

Switch $[0 < t < DT_s]$

$$\frac{di_L}{dt} = \frac{V_g}{L} \quad [i_L \text{ slope}]$$

Switch $[DT_s < t < T_s]$

$$\frac{di_L}{dt} = \frac{V}{L} \quad [i_L \text{ slope}]$$

Switch $[0 < t < DT_s]$

$$i_C = -\frac{V}{R}$$

Switch $[DT_s < t < T_s]$

$$i_C = -I - \frac{V}{R}$$

Total Charge Balance over 1 period for Capacitor Current I

$$I = V_g \frac{D}{D' D' R} \quad [\text{Inductor Current } I]$$

Ripple

$$2\Delta i = (i_L \text{ slope}) \cdot DT_s$$

$$\Delta i = \frac{V_g}{2L} \cdot DT_s$$

DCM

$$I_L < \Delta i$$

$$V_g \frac{D}{D' D' R} < \frac{V_g}{2L} \cdot DT_s$$

$$\frac{1}{D' D' R} < \frac{1}{2L} \cdot T_s$$

$$\frac{2L}{RT_s} < D' D'$$

$$K = \frac{2L}{RT_s}$$

$$K_{crit} = D' D'$$

2. **Output Voltage**

Switch $[0 < t < D_1 T_s]$

$$0 = V_g - v_L$$
$$v_L = V_g$$

Switch $[D_1 T_s < t < (D_1 + D_2)T_s]$

$$0 = v_L - v$$
$$v_L = v$$

Switch $[(D_1 + D_2)T_s < t < T_s]$

$$v_L = 0$$

Total Volt Seconds over 1 period for Inductor Voltage

$$< v_L > = \int_o^{T_S} v_L(t)dt$$
$$0 = D_1(V_g) + D_2(V) + D_3(0)$$
$$V = \frac{-D_1 V_g}{D_2} \qquad \text{[Output Voltage]}$$
$$D_2 = \frac{-D_1 V_g}{V} \qquad \text{[Solve in terms of } D_1\text{]}$$

Thus

$$< i_D > = \frac{V_g}{2L} D_1 D_2 T_s$$
$$\frac{V}{R} = \frac{V_g}{2L} D_1 \left(\frac{-D_1 V_g}{V} \right) T_s$$
$$\frac{V}{R} = -\frac{D_1^2 V_g^2}{2L \cdot V} T_s$$
$$\left[\frac{V}{R}\right] \frac{2L \cdot V}{T_s} = \left[-\frac{D_1^2 V_g^2}{2L \cdot V} T_s \right] \frac{2L \cdot V}{T_s}$$
$$V^2 \frac{2L}{RT_s} = -D_1^2 V_g^2$$
$$V^2 K = -D_1^2 V_g^2$$
$$\frac{V^2}{V_g^2} = -\frac{D_1^2}{K}$$
$$\frac{V}{V_g} = -\frac{D_1}{\sqrt{K}}$$
$$\frac{V}{V_g} = -D_1 \cdot \sqrt{\frac{RT_s}{2L}}$$

Capacitor Charge Balance

$$i_D = i_C + \frac{v}{R} \Big|_{i_C = 0}$$
$$i_D = \frac{V}{R}$$

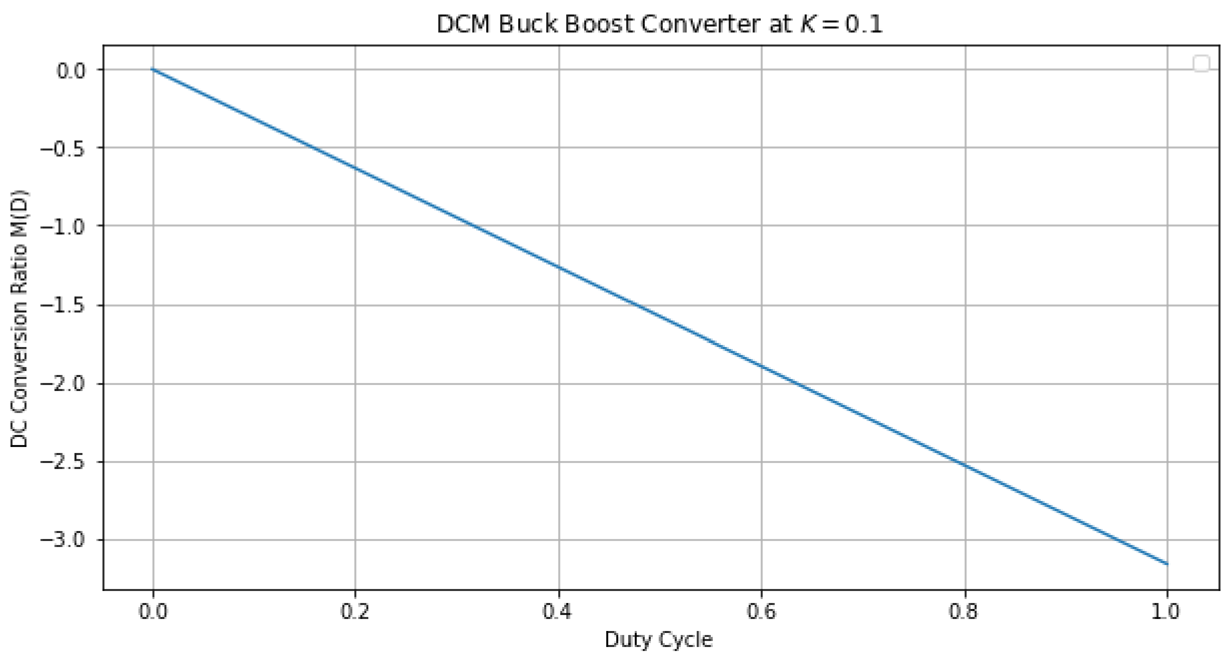
Peak Current:

$$i_D(D_1 T_s) = i_{pk} = \frac{V_g}{L} D_1 T_s$$

Average Current:

$$< i_D > = \frac{1}{T_s} \int_o^{T_S} i_D(t)dt$$
$$= \frac{1}{T_s} \cdot \frac{1}{2} i_{pk} D_2 T_s$$
$$= \frac{1}{T_s} \frac{1}{2} \left(\frac{V_g}{L} D_1 T_s \right) D_2 T_s$$
$$= \frac{V_g}{2L} D_1 D_2 T_s$$

3.



DCM Buck Boost Converter at $K = 0.1$
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
DC Conversion Ratio M(D)
0.0
0.2
0.4
0.6
0.8
1.0
Duty Cycle