EE 452 – Power Electronics Design, Fall 2021

Homework 6

Due Date: Tuesday November 23rd 2021, 11:59 pm Pacific Time

Instructions. You must scan your completed homework assignment into a pdf file, and upload your file to the Canvas Assignment page by the due date/time above. All pages must be gathered into a single file of moderate size, with the pages in the correct order. Set your phone or scanner for basic black and white scanning. You should obtain a file size of hundreds of kB, rather than tens of MB. I recommend using the "Tiny Scanner" app. Please note that the grader will not be obligated to grade your assignment if the file is unreadable or very large.

Description The core illustrated in Fig. 1(a) is 1 cm thick. All legs are 1 cm wide, except for the right-hand side vertical leg, which is 0.5 cm wide. You may neglect nonuniformities in the flux distribution caused by turning corners.

(a) Determine the magnetic circuit model of this device, and label the values of all reluctances in your model.

(b) Determine the inductance of the winding.

(c) A second winding is added to the same core, as shown in Fig. 1(b). Modify your model of part (a) to include this winding.

(d) The electrical equations for this circuit may be written in the form

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix},$$

Use superposition to determine analytical expressions and numerical values for L_{11}, L_{12}, and L_{22}.

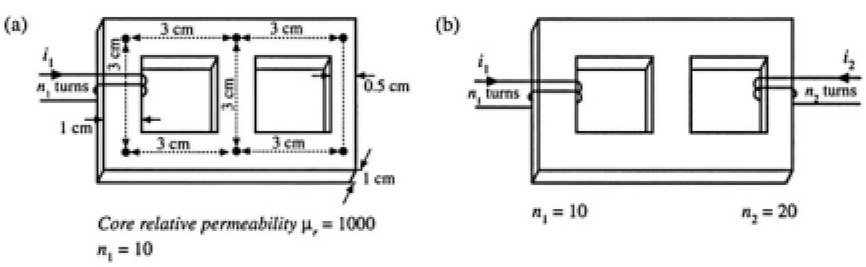


Figure 1: Magnetic core.

a.)

$$\mathcal{R}_1 = \frac{\ell_c}{\mu_0 \mu_r A_{c_1}}$$

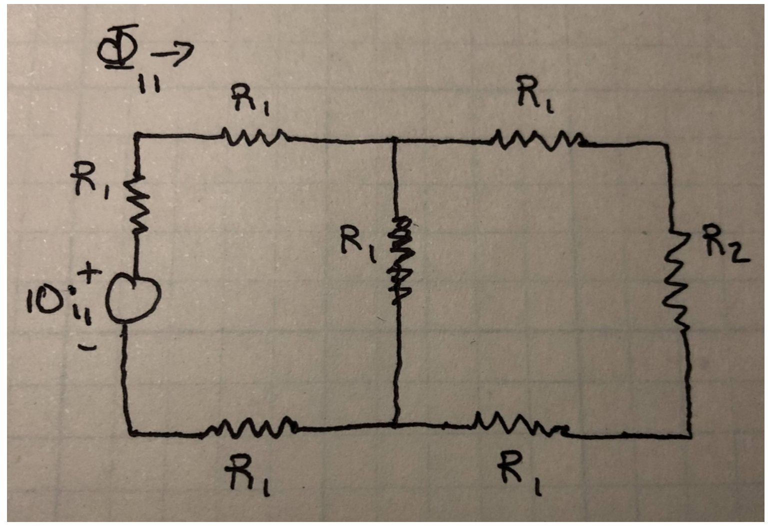
$$= \frac{3 \cdot 10^{-2}\ [m]}{(4\pi \cdot 10^{-7})\left[\frac{H}{m}\right](1000)(10^{-2} \cdot 10^{-2})\left[m^2\right]}$$

$$= 2.39 \cdot 10^5 H^{-1}$$

$$\mathcal{R}_2 = \frac{\ell_c}{\mu_0 \mu_r A_{c_2}}$$

$$= \frac{3 \cdot 10^{-2}\ [m]}{(4\pi \cdot 10^{-7})\left[\frac{H}{m}\right](1000)(10^{-2} \cdot 0.5 \cdot 10^{-2})\left[m^2\right]}$$

$$= 4.77 \cdot 10^5 H^{-1}$$



b.)

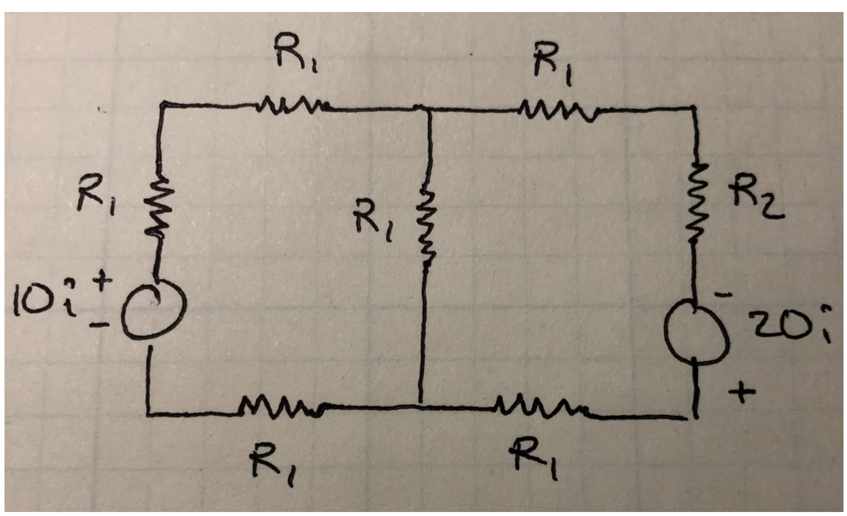
$$L = \frac{n^2}{\mathcal{R}_c + \mathcal{R}_g} = \frac{n^2}{\mathcal{R}_{eq}}$$

$$\mathcal{R}_{eq} = 3R_2 + [R_1 \parallel (2R_1 + R_2)] = 907183\ H^{-1}$$

$$= \frac{10^2}{90.7 \cdot 10^5\ [H^{-1}]}$$

$$= 110\mu H$$

c.)



d.)

$$\Phi = \Phi_1 + \Phi_2$$

$$\Phi_1 = \Phi_{11}[\text{flux generated from source 1}]$$
$$+ \; \Phi_{12}[\text{flux in winding 1 due to current from winding 2}]$$
$$\Phi_2 = \Phi_{22}[\text{flux generated from source 2}]$$
$$+ \; \Phi_{21}[\text{flux in winding 2 due to current from winding 1}]$$
$$\mathcal{R}_{eq_1} = 3R_2 + [R_1 \parallel (2R_1 + R_2)]$$
$$= 907183 \; H^{-1} \; [\text{source 1 only}]$$
$$\mathcal{R}_{eq_2} = 2R_1 + R_2 + [3R_1 \parallel R_1]$$
$$= 1133979 \; H^{-1} \; [\text{source 2 only}]$$

Turn off source 2

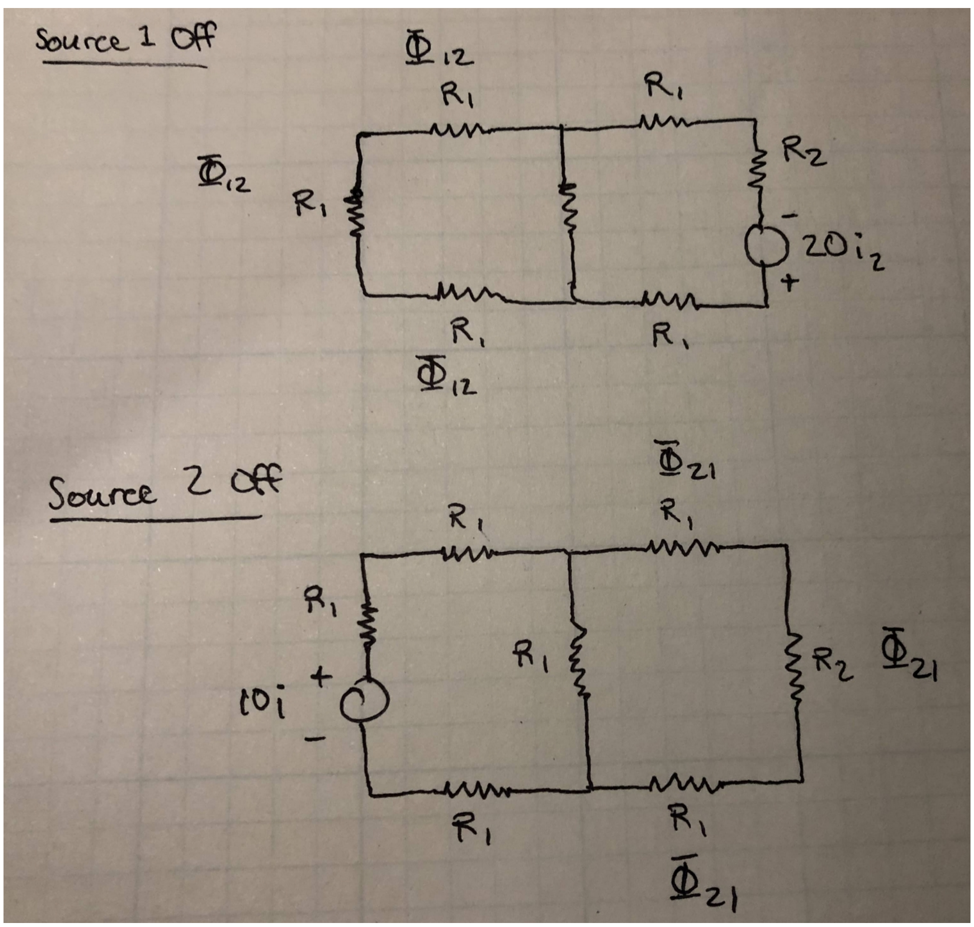
$$\Phi_{11} = \frac{n_1 i_1}{\mathcal{R}_{eq_1}}$$

$$\Phi_{21} = \Phi_{11} \cdot \frac{R_1}{R_1 + (2R_1 + R_2)} = 0.20 \; \Phi_{11}$$

Turn off source 1

$$\Phi_{22} = \frac{n_2 i_2}{\mathcal{R}_{eq_2}}$$

$$\Phi_{12} = \Phi_{22} \cdot \frac{R_1}{R_1 + 3R_1} = 0.25 \; \Phi_{22}$$



$$v_1 = n_1 \frac{d}{dt}(\Phi_1) = n_1 \frac{d}{dt}(\Phi_{11} + \Phi_{12})$$

$$v_1 = n_1 \frac{d}{dt}(\Phi_{11}) + n_1 \frac{d}{dt}(\Phi_{12})$$

$$v_1 = n_1 \frac{d}{dt}\left(\frac{n_1 i_1}{\mathcal{R}_{eq_1}}\right) + n_1 \frac{d}{dt}\left(0.25\frac{n_2 i_2}{\mathcal{R}_{eq_2}}\right)$$

$$v_1 = \frac{n_1^2}{\mathcal{R}_{eq_1}} \frac{d}{dt}(i_1) + 0.25\frac{n_1 n_2}{\mathcal{R}_{eq_2}} \frac{d}{dt}(i_2)$$

$$v_1 = L_{11}\frac{d}{dt}(i_1) + L_{12}\frac{d}{dt}(i_2)$$

$$L_{11} = \frac{n_1^2}{\mathcal{R}_{eq_1}}$$

$$L_{11} = \frac{10^2}{907183} = 110\mu H$$

$$L_{12} = 0.25\frac{n_1 n_2}{\mathcal{R}_{eq_2}}$$

$$L_{12} = 0.25\left(\frac{10 \cdot 20}{1133979}\right) = 44\mu H$$

$$v_2 = n_2 \frac{d}{dt}(\Phi_2) = n_2 \frac{d}{dt}(\Phi_{21} + \Phi_{22})$$

$$v_2 = n_2 \frac{d}{dt}(\Phi_{21}) + n_2 \frac{d}{dt}(\Phi_{22})$$

$$v_2 = n_2 \frac{d}{dt}\left(0.2\frac{n_1 i_1}{\mathcal{R}_{eq_1}}\right) + n_2 \frac{d}{dt}\left(\frac{n_2 i_2}{\mathcal{R}_{eq_2}}\right)$$

$$v_2 = 0.2\frac{n_1 n_2}{\mathcal{R}_{eq_1}} \frac{d}{dt}(i_1) + \frac{n_2^2}{\mathcal{R}_{eq_2}} \frac{d}{dt}(i_2)$$

$$v_2 = L_{12}\frac{d}{dt}(i_1) + L_{22}\frac{d}{dt}(i_2)$$

$$L_{12} = 0.2\frac{n_1 n_2}{\mathcal{R}_{eq_1}}$$

$$L_{12} = 0.2\left(\frac{10 \cdot 20}{907183}\right) = 44\mu H$$

$$L_{22} = \frac{n_2^2}{\mathcal{R}_{eq_2}}$$

$$L_{22} = \frac{20^2}{1133979} = 352\mu H$$