EE 452 – Power Electronics Design, Fall 2021

Homework 7

Due Date: Friday December 3rd 2021, 11:59 pm Pacific Time

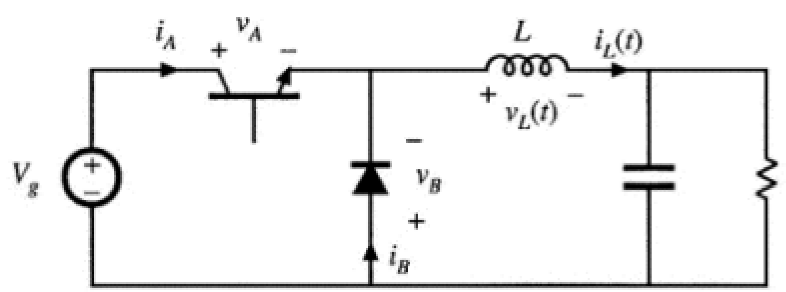
Instructions. You must scan your completed homework assignment into a pdf file, and upload your file to the Canvas Assignment page by the due date/time above. All pages must be gathered into a single file of moderate size, with the pages in the correct order. Set your phone or scanner for basic black and white scanning. You should obtain a file size of hundreds of kB, rather than tens of MB. I recommend using the "Tiny Scanner" app. Please note that the grader will not be obligated to grade your assignment if the file is unreadable or very large.

Description A simple buck converter operates with a 50 kHz switching frequency and a dc input voltage of $V_g = 40\,\text{V}$. The output voltage is $V = 20\,\text{V}$. The load resistance is $R \geq 4\,\Omega$.

(a) Determine the value of the output filter inductance L such that the peak-to-average inductor current ripple is 10% of the dc component I.

(b) Determine the peak steady-state inductor current $I_{\max}$.

(c) Design an inductor which has the values of L and $I_{\max}$ from parts (a) and (b). Use a ferrite EE core, with $B_{\max} = 0.25\,\text{T}$. Choose a value of winding resistance such that the inductor copper loss is less than or equal to 1 W at room temperature. Assume $K_u = 0.5$. Specify: core size (core type and geometrical constant K_g), gap length, wire size (AWG), and number of turns. Use tables in Appendix D of book as necessary.



Constants	Value
μ_0	$4\pi \cdot 10^{-7}$ H/m
$\rho\ [23C]$	$1.724 \cdot 10^{-6}\Omega$
$\rho\ [100C]$	$2.3 \cdot 10^{-6}\Omega$

Buck Converter	Requirement
f_s	50 kHz
V_g	40 V
V	20 V
R	$\geq 4\Omega$
Δi_L	10% I_{DC}

a.)

Volt Seconds

Switch $[0 < t < DT_s]$

$$0 = V_g - v_L - v$$

$$v_L = V_g - v$$

$$\frac{di_L}{dt} = \frac{V_g - v}{L} \qquad [i_L \text{ slope}]$$

Switch $[DT_s < t < T_s]$

$$0 = -v_L - v$$

$$v_L = -v$$

$$\frac{di_L}{dt} = \frac{-v}{L} \qquad [i_L \text{ slope}]$$

$$< v_L > = \int_o^{T_S} i_C(t)dt$$

$$0 = DT_S \cdot (V_g - V) + D'T_S \cdot (-V)$$

$$0 = DV_g - V$$

$$V = DV_g$$

$$\frac{V}{V_g} = D$$

$$D = \frac{V}{V_g} = \frac{20}{40} = 0.5$$

Charge Balance

Switch $[0 < t < DT_s]$

$$i_C = i_L - \frac{v}{R}$$

Switch $[DT_s < t < T_s]$

$$i_C = i_L - \frac{v}{R}$$

$$< i_C > = \int_o^{T_S} i_C(t)dt$$

$$0 = DT_S \cdot (I - \frac{V}{R}) + D'T_S \cdot (I - \frac{V}{R})$$

$$I = \frac{V}{R}$$

$$I \leq \frac{20}{4}$$

$$I \leq 5A$$

Output filter inductance L

$$\Delta i_L = 0.1\ (5) = 0.5A$$

$$v_L = L\frac{2\Delta i_L}{\Delta t}$$

$$L = \frac{v_L \Delta t}{2\Delta i_L}$$

$$= \frac{VD'T_s}{2\Delta i_L}$$

$$= \frac{VD'}{2\Delta i_L \cdot f_s}$$

$$= \frac{24 \cdot 0.5}{2 \cdot 0.5 \cdot 50 \cdot 10^3}$$

$$\boxed{= 0.2mH}$$

b.) I_{max} is the sum of the inductor current *I*, the ripple current (10%), and a safety factor (15%).

Imax

$$i_{safety} = 0.15\,(I + \Delta i_L)$$

$$I_{max} = I + \Delta i_L + i_{safety}$$

$$I_{max} = 1.15\,(I + \Delta i_L)$$

$$I_{max} = 1.15\,(5 + 0.5)$$

$$I_{max} = 6.325\,A$$

c.)

Recap

1. **Determine Core Size**

$$K_g \geq \left(\frac{\rho L^2 I^2_{max}}{B^2_{max} R K_u}\right) 10^8\ (cm^5)$$

2. **Determine the Air Gap Length**

$$l_g = \left(\frac{\mu_0 L I^2_{max}}{B^2_{max} A_c}\right) 10^4\ (m)$$

3. **Determine the Number of Turns**

$$n = \left(\frac{L I_{max}}{B_{max} A_c}\right) 10^4$$

4. **Evaluate Wire Size**

$$A_W \leq \frac{K_u W_A}{n}\ (cm^2)$$

Table 1: Variables and their meanings.

Physical meaning	Variable	Units
wire resistivity	ρ	Ω·cm
peak winding current	$I_{\max}$	A
inductance	L	H
winding resistance	R	Ω
winding fill factor	K_u	cm^2/cm^2
core maximum flux density	$B_{\max}$	T
core cross-sectional area	A_c	cm^2
core window area	W_A	cm^2
mean length per turn	MLT	cm

Inductor	Requirement
B_{max}	0.25 T
K_u	0.5
P_{cu}	1W

1.

Determine Inductor Resistance based on the copper power loss requirements

$$I_L = 5A$$

$$P_{cu} = I_{rms}^2 R \leq 1W$$

$$R \leq \frac{P_{cu}}{I_{AVG}^2}$$

$$R \leq \frac{1}{5^2}$$

$$R \leq 0.04\Omega = 40m\Omega$$

Use ρ at 100C

$$K_g \geq \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u} 10^8$$

$$K_g \geq \frac{(2.3 \cdot 10^{-6})(0.2 \cdot 10^{-3})^2 (6.325^2)}{(0.25^2)(0.04)(0.5)} 10^8$$

$$K_g \geq 0.29\ (cm^5)$$

$$\rho = 2.3 \cdot 10^{-6}$$

$$L = 0.2 \cdot 10^{-3}$$

$$I_{max} = 6.325$$

$$B_{max} = 0.25$$

$$R = 0.04$$

$$K_u = 0.5$$

Core size must be greater than 0.29 (cm^5).

2. Air gap is dependent upon the core cross-sectional area A_c. Due to the core size constraint, the possible cores are EE30, EE50, EE60, and EE70/68/19.

For this design, core EE30 is initially chosen with cross section area 1.09 cm^2. However, the calculated wire resistance is greater than the acceptable resistance due to the copper power loss. Therefore the EE50 core was ultimately chosen.

	Geometric Constant (Kg)	Cross-sectional Area (Ac)	Core Window Area W_A	MLT
EE30	0.857 cm^5	1.09 cm^2	0.476 cm^2	6.6 cm
EE50	0.909 cm^5	2.26 cm^2	1.78 cm^2	10.0 cm

Air Gap

$$l_g = \left(\frac{u_0\ L\ I_{max}^2}{B_{max}^2\ A_c}\right) 10^4$$

$$l_g = \left(\frac{(4\pi \cdot 10^{-7})(0.2 \cdot 10^{-3})(6.325^2)}{(0.25^2)(2.26)}\right) 10^4$$

$$\boxed{l_g = 0.00071\ m = 0.071\ cm}$$

Resistance

$$R = \rho \frac{n(MLT)}{A_W}$$

$$R = (0.2 \cdot 10^{-3}) \frac{23 \cdot 10}{0.039}$$

$$R = 0.014 < 0.04\Omega$$

3. Plug in values from previous steps.

Number of Turns

$$n = \left(\frac{L\ I_{max}}{B_{max}\ A_c} \right) 10^4$$

$$n = \left(\frac{(0.2 \cdot 10^{-3})\,(6.325)}{(0.25)\,(2.26)} \right) 10^4$$

$$n = 23 \text{ turns}$$

4. Design uses the largest wire size given constraints. The wire size is calculated to be less than or equal to 0.039 cm^2, so a **12 AWG** wire is chosen.

$$A_W \leq \frac{K_u W_A}{n}$$

$$A_W \leq \frac{0.5 \cdot 1.78}{23}$$

$$A_W \leq 0.039\ cm^2 = 39 \cdot 10^{-3}\ cm^2$$

D.2 EE CORE DATA

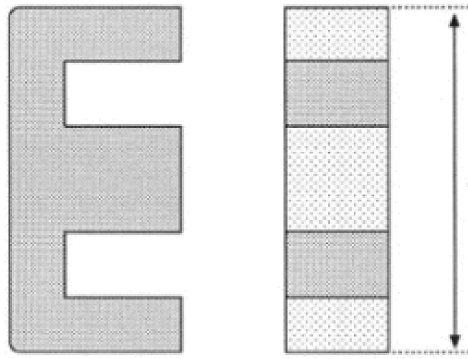


Fig. D.2

Core type (A) (mm)	Geometrical constant K_g (cm^5)	Geometrical constant K_{gfe} (cm^x)	Cross-sectional area A_c (cm^2)	Bobbin winding area W_A (cm^2)	Mean length per turn MLT (cm)	Magnetic path length ℓ_m (cm)	Core weight (g)
EE12	$0.731 \cdot 10^{-3}$	$0.458 \cdot 10^{-3}$	0.14	0.085	2.28	2.7	2.34
EE16	$2.02 \cdot 10^{-3}$	$0.842 \cdot 10^{-3}$	0.19	0.190	3.40	3.45	3.29
EE19	$4.07 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	0.23	0.284	3.69	3.94	4.83
EE22	$8.26 \cdot 10^{-3}$	$1.8 \cdot 10^{-3}$	0.41	0.196	3.99	3.96	8.81
EE30	$85.7 \cdot 10^{-3}$	$6.7 \cdot 10^{-3}$	1.09	0.476	6.60	5.77	32.4
EE40	0.209	$11.8 \cdot 10^{-3}$	1.27	1.10	8.50	7.70	50.3
EE50	0.909	$28.4 \cdot 10^{-3}$	2.26	1.78	10.0	9.58	116
EE60	1.38	$36.4 \cdot 10^{-3}$	2.47	2.89	12.8	11.0	135
EE70/68/19	5.06	$75.9 \cdot 10^{-3}$	3.24	6.75	14.0	18.0	280

American wire gauge data

AWG#	Bare area, 10^{-3} cm^2	Resistance, 10^{-6} Ω/cm	Diameter, cm
0000	1072.3	1.608	1.168
000	850.3	2.027	1.040
00	674.2	2.557	0.927
0	534.8	3.224	0.825
1	424.1	4.065	0.735
2	336.3	5.128	0.654
3	266.7	6.463	0.583
4	211.5	8.153	0.519
5	167.7	10.28	0.462
6	133.0	13.0	0.411
7	105.5	16.3	0.366
8	83.67	20.6	0.326
9	66.32	26.0	0.291
10	52.41	32.9	0.267
11	41.60	41.37	0.238
12	33.08	52.09	0.213
13	26.26	69.64	0.190
14	20.02	82.80	0.171
15	16.51	104.3	0.153
16	13.07	131.8	0.137
17	10.39	165.8	0.122
18	8.228	209.5	0.109
19	6.531	263.9	0.0948
20	5.188	332.3	0.0874