EE 473/533
Homework 3
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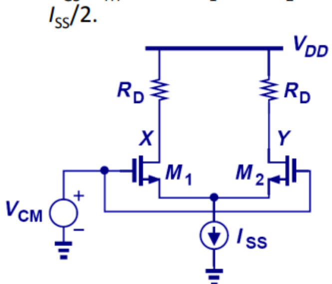
9.4. In the op amp of Fig. 9.21(b), $(W/L)_{1-8} = 100/0.5$, $I_{SS} = 1$ mA, and $V_{b1} = 1.7$ V. Assume that $\gamma = 0$.

(a) What is the maximum allowable input CM level?
(b) What is V_X?
(c) What is the maximum allowable output swing if the gate of M_2 is connected to the output?
(d) What is the acceptable range of V_{b2}?

Reference: https://inst.eecs.berkeley.edu/~ee105/sp08/lectures/lecture24.pdf

Equilibrium Overdrive Voltage

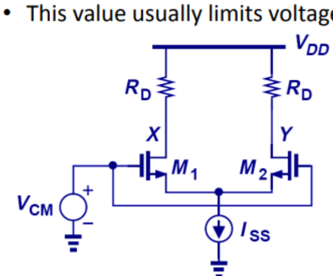
- The ***equilibrium overdrive voltage*** is defined as V_{GS}-V_{TH} when M_1 and M_2 each carry a current of $I_{SS}/2$.



$$(V_{GS} - V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

Minimum CM Output Voltage

- In order to maintain M_1 and M_2 in saturation, the common-mode output voltage cannot fall below V_{CM}-V_{TH}.
- This value usually limits voltage gain.



$$V_{DD} - R_D \frac{I_{SS}}{2} > V_{CM} - V_{TH}$$

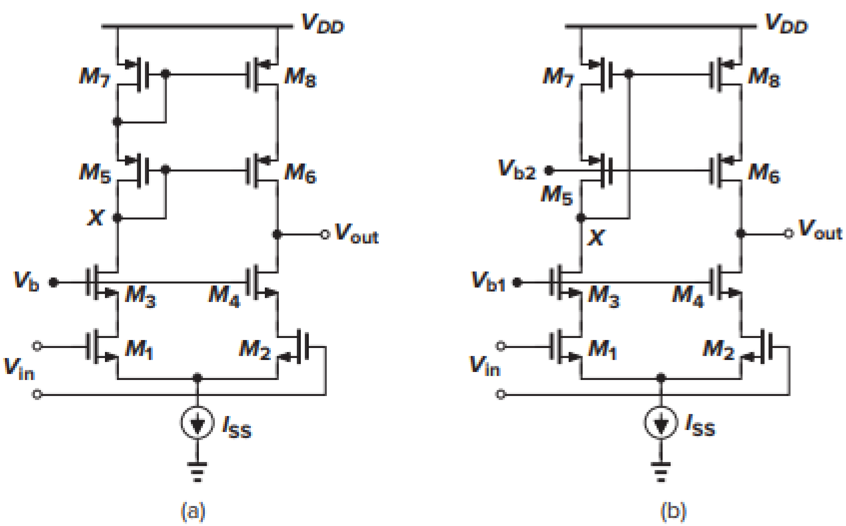


Figure 9.21 Cascode op amps with single-ended output.

9.4

Known

$\mu_n C_{ox} = 1.34E-4$
$\mu_p C_{ox} = 3.83E-5$ } 3.5x

$\frac{W}{L} = \frac{100}{0.5} = 200$

$\lambda = 0 \rightarrow V_{th} = V_{th0}$

$V_{thn} = 0.7V$

$V_{thp} = 0.8V$

a.) Max CM?

$V_{CM,max} - V_{th_2} < V_{b_1} - V_{gs_4}$

$V_{CM,max} < V_{b_1} - V_{gs_4} + V_{th_2}$

$V_{gs_4} = \sqrt{\frac{2I_D}{\mu_n C_{ox}(\frac{W}{L})}} + V_{thn} \Big|_{I_D = \frac{I_{SS}}{2} = 0.5mA}$

$V_{gs_4} = \sqrt{\frac{2(0.5mA)}{1.34E-4(200)}} + 0.7$

$V_{gs_4} = 0.193 + 0.7 = 0.893V$

$V_{CM,max} < V_{b_1} - V_{gs_4} + V_{th_2}$

$V_{CM,max} < \overset{1.7}{\cancel{0.893}} - 0.893 + 0.7$

$V_{CM,max} < 1.507V$

b.) Figure A

$V_x = V_{DD} - |V_{GS_5}| - |V_{GS_7}|$

$V_{GS_7} = \sqrt{\frac{2I_D}{\mu_p C_{ox}(\frac{W}{L})}} + V_{thp} = \sqrt{\frac{2(0.5mA)}{3.83E-5(200)}} + 0.8$

$V_{GS_7} = 0.361 + 0.8 =$

$V_{GS_7} = 1.161$

$V_x = V_{DD} - |V_{GS_7}|$

$V_x = 3 - 1.161$

$V_x = 1.839V$

Figure B

$V_x = V_{DD} - |V_{GS_7}|$

~~$V_x = 3$~~

c.) Max Swing?

$V_{out,max} = V_{CM,max} < 1.507V$

$V_{CM,min} - V_{th_2} > \sqrt{\frac{2I_D}{\mu_n C_{ox}(\frac{W}{L})}}$

$V_{CM,min} > \sqrt{\frac{2I_D}{\mu_n C_{ox}(\frac{W}{L})}} + V_{thn}$

$V_{CM,min} > \sqrt{\frac{2(0.5mA)}{1.34E-4(200)}} + 0.7$

$V_{CM,min} > 0.139 + 0.7$

$V_{CM,min} > 0.893$

$V_{out,swing} = V_{CM,max} - V_{CM,min}$

$V_{out,swing} = 1.507 - 0.893$

$V_{out,swing} = 0.614V$

d.) $V_{b_2,max} - V_{th_5} < |V_{DS_5}|$

$V_{b_2,max} < |V_{DS_5}| + V_{th_5}$

$< |V_x - (V_{DD} - V_{DS_7})| + V_{th_5}$

$< |V_x - V_{DD} + V_{DS_7}| + V_{th_5}$

$< |(V_{DD} - V_{GS_7}) - V_{DD} + (V_{GS_7} - V_{th_7})| + V_{th_5}$

$< |\cancel{V_{DD}} - \cancel{V_{GS_7}} - \cancel{V_{DD}} + \cancel{V_{GS_7}} - V_{th_7}| + V_{th_5}$

$< |-V_{th_7}| + V_{th_5}$

$< 2V_{thp}$

$< 2(0.8)$

$V_{b_2,max} < 1.6\text{ V}$

$V_{b_2,min} - V_{th_5} > \sqrt{\frac{2I_D}{\mu_p C_{ox}(\frac{W}{L})}} \quad \Big| \; I_D = \frac{I_{SS}}{2} = 0.5\text{ mA}$

$V_{b_2,min} > \sqrt{\frac{2I_D}{\mu_p C_{ox}(\frac{W}{L})}} + V_{thp}$

$V_{b_2,min} > \sqrt{\frac{2(0.5\text{ mA})}{3.83\text{E-}5\,(200)}} + 0.8$

$V_{b_2,min} > \overset{0.361}{\cancel{0.193}} + 0.8$

$V_{b_2,min} > 1.161\text{ V}$

~~$V_{out,swing} = V_{CM}$~~

$\boxed{1.161\text{ V} < V_{b_2} < 1.6\text{ V}}$

9.6. If in Fig. 9.23, $(W/L)_{1-8} = 100/0.5$ and $I_{SS} = 1$ mA,

(a) What CM level must be established at the drains of M_3 and M_4 so that $I_{D5} = I_{D6} = 1$ mA? How does this constrain the maximum input CM level?

(b) With the choice made in part (a), calculate the overall voltage gain and the maximum output swing.

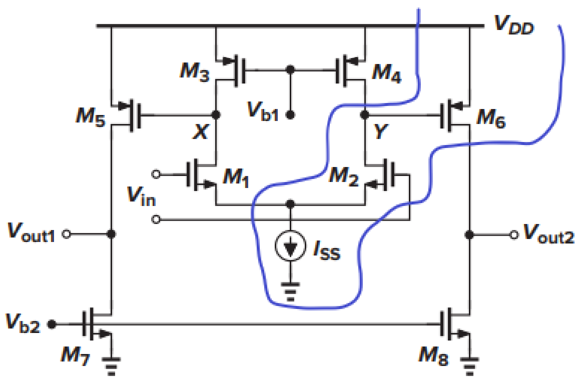
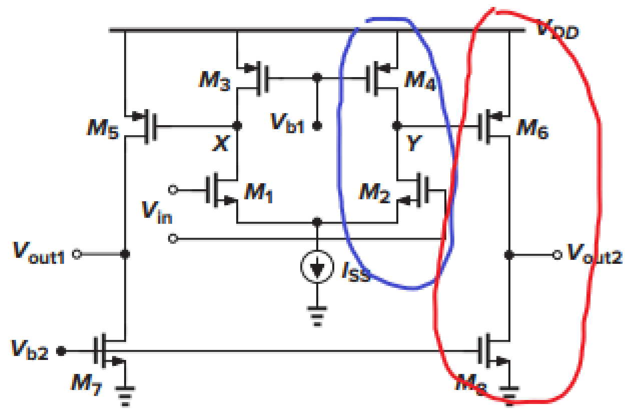


Figure 9.23 Simple implementation of a two-stage op amp.



9.6

a) $V_{CM,max} - V_{th_2} < V_{DD} - \cancel{V_{DS_4}}$ V_{GS_6}

$V_{CM,max} < V_{DD} - V_{GS_6} + V_{th_2}$

$V_{GS_6} = \sqrt{\dfrac{2I_D}{\mu_p C_{ox}\left(\frac{W}{L}\right)}} + V_{thp} \Big|_{I_D = 1mA}$

$V_{GS_6} = \sqrt{\dfrac{2(1mA)}{3.83E\text{-}5(200)}} + 0.8$

$V_{GS_6} = 0.511 + 0.8$

$V_{GS_6} = 1.311$ V

$V_{CM,max} < V_{DD} - V_{GS_6} + V_{th\,n}$

$V_{CM,max} < 3 - 1.311 + 0.7$

$V_{CM,max} < 2.39$ V

$V_{CM,min} - V_{th\,n} > \sqrt{\dfrac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} \Big|\ I_D = 0.5mA$

$V_{CM,min} > \sqrt{\dfrac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} + V_{th\,n}$

$V_{CM,min} > \sqrt{\dfrac{2(0.5mA)}{1.34E\text{-}4(200)}} + 0.7$

$V_{CM,min} > 0.193 + 0.7$

$V_{CM,min} > 0.893$ V

Known

$\mu_n C_{ox} = 1.34E\text{-}4$

$\mu_p C_{ox} = 3.83E\text{-}5$

$V_{th\,n} = 0.7$

$V_{th\,p} = 0.8$

$\frac{W}{L} = \frac{100}{0.5} = 200$

b.) $V_{out,\ swing} = V_{DD} - 2V_{DSat}$

$V_{out,\ swing} = V_{DD} - |V_{DS_6}| - |V_{DS_8}|$

$V_{DS_6} = \sqrt{\dfrac{2I_D}{\mu_p C_{ox}\left(\frac{W}{L}\right)}} \Big|_{I_D = 1mA} = \sqrt{\dfrac{2(1mA)}{3.83E\text{-}5\,(200)}} = 0.511\ V$

$V_{DS_8} = \sqrt{\dfrac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} \Big|_{I_D = 1mA} = \sqrt{\dfrac{2(1mA)}{1.34E\text{-}4\,(200)}} = 0.273\ V$

$V_{out,\ swing} = 3 - 0.511 - 0.273$

$V_{out,\ swing} = 2.215\ V$

Known

$\frac{W}{L} = \frac{100}{0.5} = 200$

$\lambda_n = 0.1$

$\lambda_p = 0.2$

$A_V = A_{V_1} \cdot A_{V_2}$

$A_V = g_{m_2}(r_{o2} // r_{o4}) \cdot g_{m_6}(r_{o6} // r_{o8})$

$g_{m_2} \cancel{A_{V_1}} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D} \quad \Big|_{I_D = \frac{I_{SS}}{2} = 0.5mA}$

$g_{m_2} = \sqrt{2(1.34E\text{-}4)(0.5mA)(200)} = 0.00518$

$g_{m_6} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right) I_D} \quad \Big|_{I_D = 1mA}$

$g_{m_6} = \sqrt{2(3.83E\text{-}5)(200)(1mA)} = 0.00391$

$r_{o2} = \dfrac{1}{\lambda_n I_D} \Big|_{I_D = 0.5mA} = \dfrac{1}{(0.1)(0.5mA)} = 20{,}000$

$r_{o8} = \dfrac{1}{\lambda_n I_D} \Big|_{I_D = 1mA} = \dfrac{1}{(0.1)(1mA)} = 10{,}000$

$r_{o4} = \dfrac{1}{\lambda_p I_D} \Big|_{I_D = 0.5mA} = \dfrac{1}{(0.2)(0.5mA)} = 10{,}000$

$r_{o6} = \dfrac{1}{\lambda_p I_D} \Big|_{I_D = 1mA} = \dfrac{1}{(0.2)(1mA)} = 5{,}000$

$r_{o2} // r_{o4} = 20k // 10k = 6.67k$

$r_{o6} // r_{o8} = 5k // 10k = 3.33k$

$A_V = (0.00518)(6.67k) \cdot (0.00391)(3.33k)$

$A_V = 34.54 \cdot 13.03$

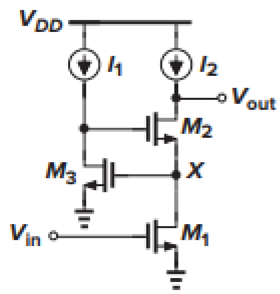
$\boxed{A_V = 450\ V/V}$

9.10. Suppose that in Fig. 9.88, $I_1 = 100\ \mu\text{A}$, $I_2 = 0.5$ mA, and $(W/L)_{1-3} = 100/0.5$. Assuming that I_1 and I_2 are implemented with PMOS devices having $(W/L)_P = 50/0.5$,

(a) Calculate the gate bias voltages of M_2 and M_3.

(b) Determine the maximum allowable output voltage swing.

(c) Calculate the overall voltage gain and the input-referred thermal noise voltage.



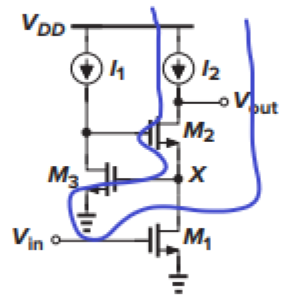
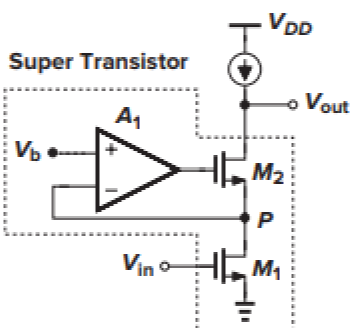
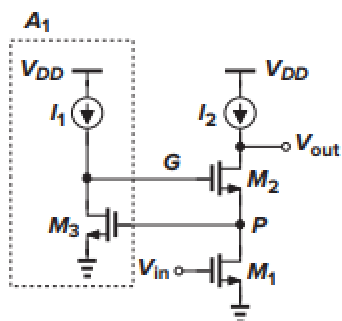


Figure 9.88





$$|A_v| \approx g_{m1}[r_{O2} + (A_1 + 1)g_{m2}r_{O2}r_{O1} + r_{O1}]$$
$$\approx g_{m1}g_{m2}r_{O1}r_{O2}(A_1 + 1)$$

9.10

Known

$\frac{W}{L} = \frac{100}{0.5} = 200$ [nmos]

$\frac{W}{L} = \frac{50}{0.5} = 100$ [pmos]

$V_{th_n} = 0.7$

$V_{th_p} = 0.8$

$\mu_n C_{ox} = 1.34E-4$

$\mu_p C_{ox} = 3.83E-5$

a.) Find ~~V_g~~ V_{GS_3}:

$$V_{GS_3} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} + V_{th_n} \Bigg|_{I_D = I_1 = 0.1\,mA}$$

$$V_{GS_3} = \sqrt{\frac{2(0.1\,mA)}{1.34E-4\,(200)}} + 0.7$$

$$V_{GS_3} = 0.086 + 0.7 =$$

$$\underline{V_{GS_3} = 0.786\ V}$$

Find V_{GS_2}:

$$V_{GS_2} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} + V_{th_n} \Bigg|_{I_D = I_2 = 0.5\,mA}$$

$$V_{GS_2} = 0.193 + 0.7$$

$$\underline{V_{GS_2} = 0.893\ V}$$

b.) $V_{out,\,swing} = V_{DD} - V_{DS_{I_2}} - V_{DSat_2} - V_{GS_3}$

$$V_{DSat_{I_2}} = \sqrt{\frac{2I_D}{\mu_p C_{ox}\left(\frac{W}{L}\right)_p}} \Bigg|_{I_D = 0.5\,mA} = \sqrt{\frac{2(0.5\,mA)}{3.83\,E-5\,(100)}} = 0.51\ V$$

$$V_{DSat_2} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} \Bigg|_{I_D = 0.5\,mA} = \sqrt{\frac{2(0.5\,mA)}{1.34E-4\,(200)}} = 0.193\ V$$

$$V_{out,\,swing} = 3 - 0.51 - 0.193 - 0.786$$

$$\underline{V_{out,\,swing} = 1.51\ V}$$

Known

$\mu_n C_{ox} = 1.34E-4$

$\mu_p C_{ox} = 3.83E-5$

$\frac{W}{L} = \frac{100}{0.5} = 200$ [nmos]

$\frac{W}{L} = \frac{50}{0.5} = 100$ [pmos]

$\lambda_n = 0.1$

$\lambda_p = 0.2$

C.) Given perfect current source:

$A_v = g_{m_1} g_{m_2} r_{o1} r_{o2} (A_1 + 1)$

A_1 is replaced with NMOS CS stage

$|A_1| = g_{m_3}(r_{o3} // r_{oI_1})$

$g_{m_3} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D} \Big|_{I_D = I_1 = 0.1\,mA}$

$g_{m_3} = \sqrt{1.34E-4\,(200)(0.1mA)\,2}$

$g_{m_3} = \cancel{0.00164} = 0.00232$

$g_{m_1} = g_{m_2} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D} \Big|_{I_D = I_2 = 0.5mA}$

$= \sqrt{1.34E-4\,(200)(0.5\,mA)\,2}$

$g_{m_1} = g_{m_2} = \cancel{0.00366}\ 0.00518$

$r_{o1} = r_{o2} = \frac{1}{\lambda_n I_D} = \frac{1}{0.1(0.5mA)} = 20{,}000$

$r_{o3} = \frac{1}{\lambda_n I_D} = \frac{1}{0.1\,(0.1mA)} = 100{,}000$

$r_{oI_1} = \frac{1}{\lambda_p I_D} = \frac{1}{0.2\,(0.1mA)} = 50{,}000$

$|A_1| = (\cancel{0.00164}\ 0.00232)(100k // 50k)$

$A_1 = \cancel{54.57}\ 77.17$

$A_v = (\cancel{0.00366})^2 (0.00518)^2 (20{,}000)^2 (\cancel{54.57}\ 77.17 + 1)$

$A_v = 297{,}847 \approx 300k\ V/V$ ← not correct

~~A~~ Vout is in parallel with I_2 [not perfect current source]

$A_v = g_{m_1}[\, r_{oI_2} // \underbrace{g_{m_2} r_{o1} r_{o2}(A_1+1)}\,]$ ← this is massive magnitude E7

$r_{oI_2} = \frac{1}{\lambda_p I_D} = \frac{1}{0.2\,(0.5\,mA)} = 10{,}000$

~~AA~~ $r_{oI_2} // g_{m_2} r_{o1} r_{o2}(A_1+1) \approx r_{oI_2}$

$A_v = g_{m_1} r_{oI_2}$

$A_v = (\cancel{0.00366}\ 0.00518)(10{,}000)$

$A_v \approx \cancel{36.6\ V/V}$

$A_v \approx 51.8\ V/V$

9.18. In this problem, we design a two-stage op amp based on the topology shown in Fig. 9.90. Assume a power budget of 6 mW, a required output swing of 2.5 V, and $L_{eff} = 0.5\ \mu m$ for all devices.

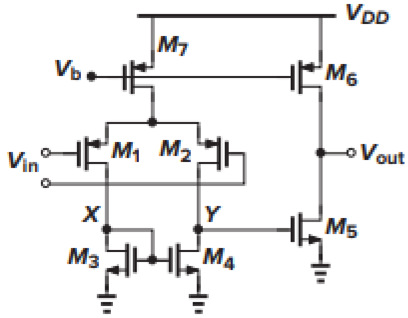


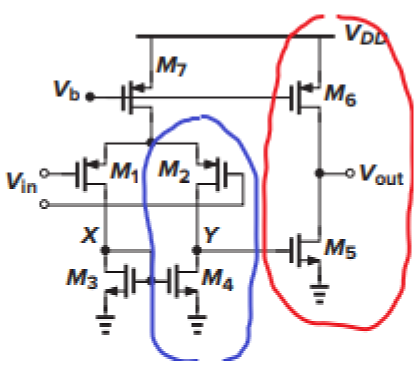
Figure 9.90

(a) Allocating a current of 1 mA to the output stage and roughly equal overdrive voltages to M_5 and M_6, determine $(W/L)_5$ and $(W/L)_6$. Note that the gate-source capacitance of M_5 is in the signal path, whereas that of M_6 is not. Thus, M_6 can be quite a lot larger than M_5.

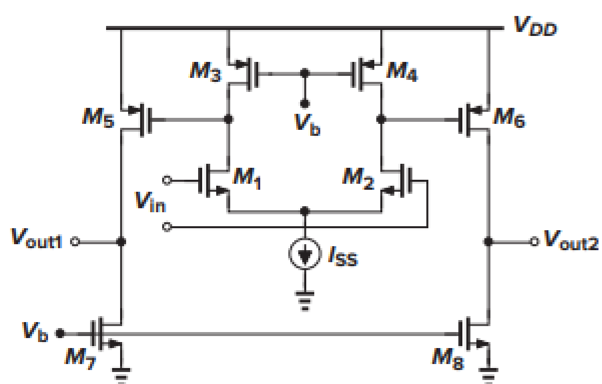
(b) Calculate the small-signal gain of the output stage.

(c) With the remaining 1 mA flowing through M_7, determine the aspect ratio of M_3 (and M_4) such that $V_{GS3} = V_{GS5}$. This is to guarantee that if $V_{in} = 0$ and hence $V_X = V_Y$, then M_5 carries the expected current.

(d) Calculate the aspect ratios of M_1 and M_2 such that the overall voltage gain of the op amp is equal to 500.



Stage 1: CS (pmos)
Stage 2: CS (nmos)



Compare to Differential Two-Stage Op Amp

9.18

Known

$L_{eff} = 0.5\text{E-}6$

$V_{out,swing} = 2.5\text{ V}$

$V_{DD} = 3\text{V}$

a.) Find $\left(\frac{W}{L}\right)_5$ and $\left(\frac{W}{L}\right)_6$

$V_{DS_5} + V_{DS_6} < V_{DD} - V_{out,swing}$

$V_{DS_5} + V_{DS_6} < 3 - 2.5$

$V_{DS_5} + V_{DS_6} < 0.5\text{ V}$ and $V_{DS_5} \approx V_{DS_6}$ so each get 0.25 V

$[V_{DS_5}]$ $0.25\text{ V} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}}\Bigg|_{I_D = 1mA}$

$0.25^2 = \frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}\Bigg|_{L = L_{eff} = 0.5\text{E-}6}$

$\frac{W}{L} = \frac{2I_D}{\mu_n C_{ox}(0.25)^2}$

$W = \frac{2I_D \cdot L}{\mu_n C_{ox}(0.25)^2} = \frac{2(1mA)(0.5\text{E-}6)}{1.34\text{E-}4\,(0.25)^2}$

$W_5 = 119\text{ E-}6\text{ m}$

$W_6 = \frac{2I_D \cdot L}{\mu_p C_{ox}(0.25)^2} = \frac{2(1mA)(0.5\text{E-}6)}{3.83\text{E-}5\,(0.25)^2}$

$W_6 = 418\text{E-}6\text{ m}$

b.) $A_{v_2} = g_{m_5}(r_{o5} \parallel r_{o6})$

$\left(\frac{W}{L}\right)_5 = \frac{119}{0.5} = 238.8$

$g_{m_5} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)I_D}\Big|_{I_D = 1mA}$

$g_{m_5} = \sqrt{2(1.34\text{E-}4)(238.8)(1mA)}$

$g_{m_5} = 0.008$

$r_{o5} = \frac{1}{\lambda_n I_D} = \frac{1}{0.1(1mA)} = 10{,}000$

$r_{o6} = \frac{1}{\lambda_p I_D} = \frac{1}{0.2(1mA)} = 5{,}000$

$r_{o5} \parallel r_{o6} = 3.33\text{ k}$

$A_{v_2} = (0.008)(3.33\text{k})$

$A_{v_2} = 26.66\text{ V/V}$

c.) $I_{D4} = \frac{I_{D7}}{2} = \frac{1mA}{2} = 0.5mA$

Find $\left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_3$

$V_{GS_3} = V_{GS_5}$

$\sqrt{\frac{2I_D'}{\mu_n C_{ox}\left(\frac{W}{L}\right)_3}} + \cancel{V_{thn}}\Bigg|_{I_D' = 0.5mA} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_5}} + \cancel{V_{thn}}\Bigg|_{I_D = 1mA}$

$\frac{2I_D'}{\cancel{\mu_n C_{ox}}\left(\frac{W}{L}\right)_3} = \frac{2I_D}{\cancel{\mu_n C_{ox}}\left(\frac{W}{L}\right)_5}$ $I_D' = \frac{I_D}{2}$

$\frac{\cancel{I_D}}{\left(\frac{W}{L}\right)_3} = \frac{2\cancel{I_D}}{\left(\frac{W}{L}\right)_5} \rightarrow \left(\frac{W}{L}\right)_3 = \frac{\left(\frac{W}{L}\right)_5}{2}$

Known:

$L_{eff} = 0.5E\text{-}6$

d.) $A_V = A_{V_1} \cdot A_{V_2} = 500$

$A_V = g_{m_2}(r_{o2} \parallel r_{o4}) \cdot 26.66 = 500$

$g_{m_2}(r_{o2} \parallel r_{o4}) = \frac{500}{26.66} = 18.75$

$g_{m_2} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_2 I_D} \quad / \; I_D = 0.5\text{ mA}$

$g_{m_2} = \sqrt{2(3.83E\text{-}5)(0.5\text{mA})\left(\frac{W}{L}\right)_2}$

$g_{m_2} = \sqrt{3.83E\text{-}8\left(\frac{W}{L}\right)_2}$

$r_{o2} = \frac{1}{\lambda_p I_D} = \frac{1}{(0.2)(0.5\text{mA})} = 10{,}000$

$r_{o4} = \frac{1}{\lambda_n I_D} = \frac{1}{0.1(0.5\text{mA})} = 20{,}000$

$r_{o2} \parallel r_{o4} = 6.67\text{k}$

$A_{V_1} = \left(\sqrt{3.83E\text{-}8\left(\frac{W}{L}\right)_2}\right)(6.67\text{k}) = 18.75 \text{ V/V}$

$3.83E\text{-}8\left(\frac{W}{L}\right)_2 = \left(\frac{18.75}{6.67\text{k}}\right)^2$

$\left(\frac{W}{L}\right)_2 = \frac{(18.75/6.67\text{k})^2}{3.83E\text{-}8}$

$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \underline{206.56} \longrightarrow W = 206.56\, L_{eff} = 103E\text{-}6 \text{ m}$