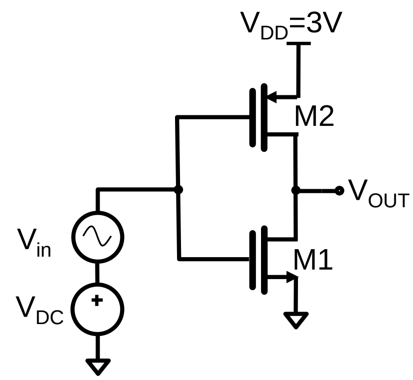
Name: SOLUTIONS

50 points total

EE 473 — Quiz 1 — Winter 2022

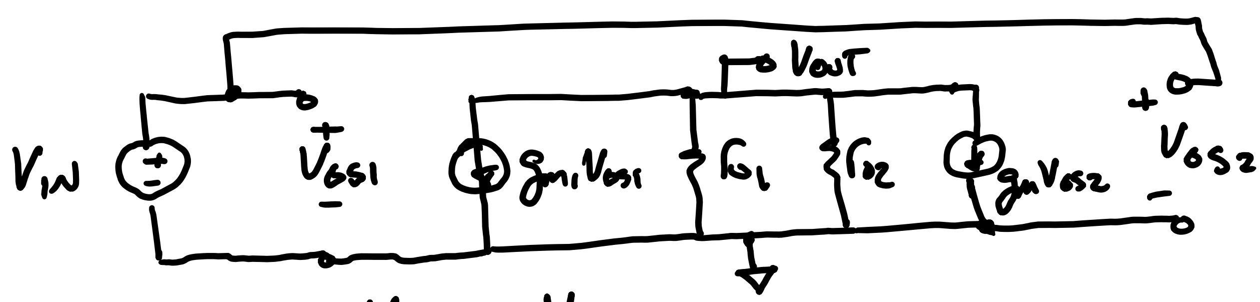
1) Assume that M1 and M2 are in the saturation region and that V_{DC} is set to produce a DC V_{OUT} which is optimal for maximum headroom and output voltage swing. Also, assume that the drain currents of M1 and M2 are $I_D = 0.5mA$ (Total: 25 points)
Device characteristics: $\mu_n C_{ox} = 4mA/V^2$, $\mu_p C_{ox} = 2mA/V^2$
$(W/L)_{M1} = (10\mu m/0.5\mu m)$, and $(W/L)_{M2} = (20\mu m/0.5\mu m)$, $\lambda_p = \lambda_n = 0.1V^{-1}$ and $V_{THn} = 0.7V$ and $V_{THp} = 0.8V$.



a) What is the small-signal gain $A_v = \frac{V_{OUT}}{V_{in}}$? (15 points)

b) What is the optimal DC value of V_{OUT} to produce a maximum peak-to-peak output swing? This question is ***not*** asking you to compute V_{OUT} DC using the drain current equations for M1 and M2, but rather find the optimal VOUT DC given the $(V_{GS}-V_{TH})$ of M1 and M2. (5 points) What is the corresponding peak output swing? (5 points)

PART a) DRAW SMALL-SIGNAL AC CIRCUIT.



$$V_{GS1} = V_{GS2} = V_{IN}$$

$$G_M = g_{m1} + g_{m2}$$

$$A_V = -(g_{m1}+g_{m2}) \cdot (r_{o1} \,//\, r_{o2})$$

$$= -(g_{m1}+g_{m2}) \left(\frac{r_{o1} \cdot r_{o2}}{r_{o1}+r_{o2}}\right)$$

$$= -\left(\frac{2I_{D1}}{(V_{GS}-V_{TH})_1} + \frac{2I_{D2}}{(V_{GS}-V_{TH})_2}\right) \cdot \frac{\frac{1}{\lambda I_1} \cdot \frac{1}{\lambda I_{D2}}}{\frac{1}{\lambda I_{D1}} + \frac{1}{\lambda I_{D2}}}$$

$I_{D1} = I_{D2} = 0.5\,mA \qquad \lambda_n = \lambda_p = \lambda = 0.1$

$$A_V = -2I_D\left(\frac{1}{(V_{GS}-V_{TH})_1} + \frac{1}{(V_{GS}-V_{TH})_2}\right) \cdot \frac{\frac{1}{\lambda \cancel{I_D}}}{\cancel{1+1}}$$

$$= -\left(\frac{1}{(V_{GS}-V_{TH})_1} + \frac{1}{(V_{GS}-V_{TH})_2}\right) \cdot \frac{1}{\lambda}$$

NEED TO FIND $(V_{GS}-V_{TH})_1$ & $(V_{GS}-V_{TH})_2$. BOTH ALSO NEEDED FOR PART "B".

$$(V_{GS1}-V_{TH})_1 = \sqrt{\frac{2I_D}{\mu_n C_{ox} W/L}} = \sqrt{\frac{2 \cdot 0.5\,mA}{4\,mA/V^2\,(10/0.5)}}$$

$$= 111\,mV$$

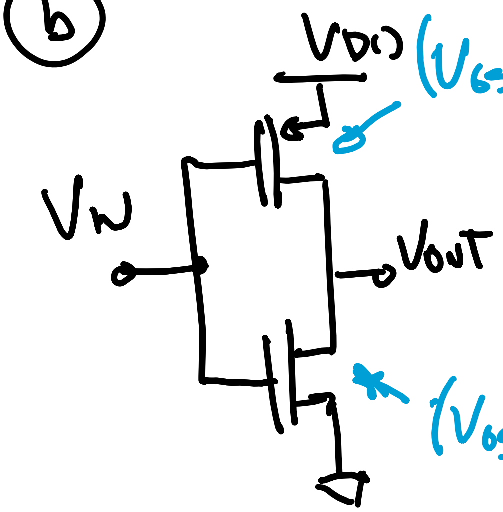
$$(V_{GS}-V_{TH})_2 = \sqrt{\frac{2 \cdot 0.5\,mA}{2\,mA/V^2\,\left(\frac{20}{0.5}\right)}}$$

$$= 111\,mV$$

$$\therefore A_V = -\left(\frac{2}{111\,mV}\right)\frac{1}{0.1\,V^{-1}} = -180\,V/V$$

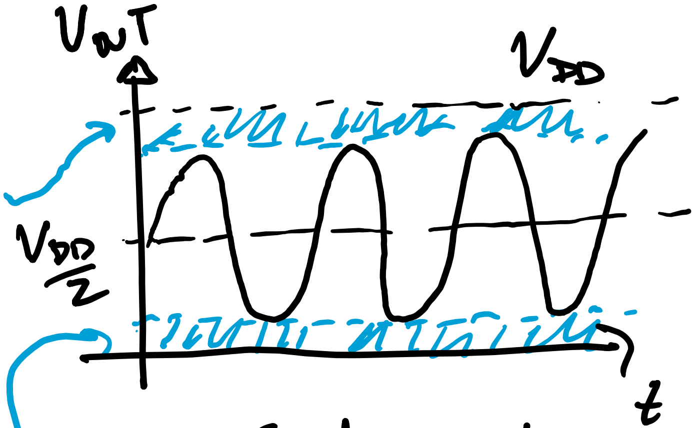
$$\boxed{A_V = -180\,V/V}$$

(b)



$(V_{GS}-V_{TH})_2 = 0.111V$

$(V_{GS}-V_{TH})_1 = 0.11V$

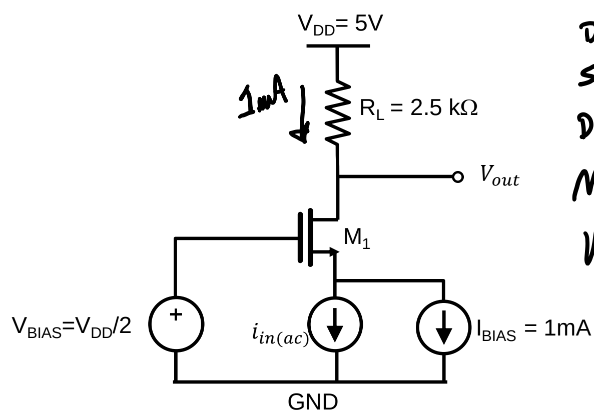


- BECAUSE $V_{DSAT1} = V_{DSAT2}$, OPTIMAL $\boxed{V_{OUT\,DC} = V_{DD}/2}$
- $V_{PK\text{-}PK\,MAX} = V_{DD} - 2V_{DSAT}$
  $= 3V - 2(0.111V)$

$\boxed{V_{PK\text{-}PK\,MAX} = 2.778V}$

2) For the below single-transistor amplifier, both an ideal DC current source and ideal AC current are applied to the input. It is fair to assume M_1 is in the saturation region and behaves like a "square law" device. Find the DC value of V_{OUT} (5 points) – note: ignore the body affect (λ=0) to make the calculation of the DC bias easier. Next, draw the small signal circuit for this amplifier and derive an expression for the small-signal AC transresistance (v_{out}/i_{in}), state any assumptions made to get your answer. Lastly, compute the value of the transresistance. (20 points) Note: for the AC-SS analysis you cannot ignore the body effect – e.g. $\lambda=0.01V^{-1}$
Device characteristics: $\mu_n C_{ox} = 5mA/V^2$, $(W/L)_{M1} = (10\mu m/1\mu m)$, $\lambda=0.01V^{-1}$ and $V_t=0.7V$.

DC VALUE V_{OUT}

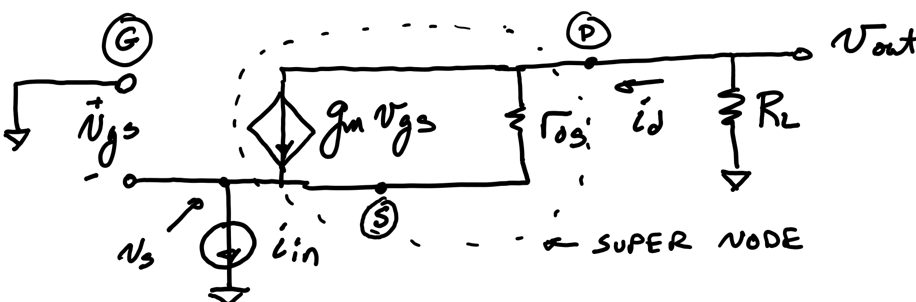


DC CURRENT AT THE SOURCE IS 1mA ∴ DC DRAIN CURRENT OF M1 IS 1mA.

$V_{OUT} = 5V - 1mA \cdot 2.5k\Omega$

$= 2.5V$

TRANSRESISTANCE v_{out}/i_{in}



$r_{ds} = \frac{1}{\lambda I_D}$

$= \frac{1}{0.01 \cdot 1\times10^{-3}}$

$= \frac{1}{1\times10^{-5}} = 100k\Omega$

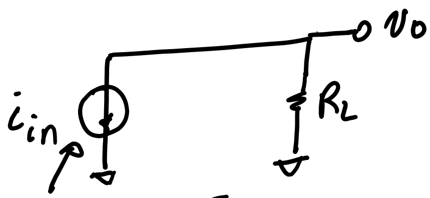
VERY LARGE IGNORE

THIS PROBLEM CAN BE SOLVED A NUMBER OF WAYS. HOWEVER, YOU COULD REALIZE THAT WHAT CURRENT GOES IN THE SOURCE COMES OUT THE DRAIN. OR, THE g_m & r_{ds} ELEMENTS ARE IN A SUPER NODE ∴

$i_d = i_{in}$

$v_o = -(-i_{in}) \cdot R_L = i_{in} R_L$

$$\boxed{\frac{v_o}{i_{in}} = R_L = 2.5k\Omega}$$



DIRECTION OF INPUT SOURCE NEGATIVE

IF YOU DID NOT SEE THE SUPER NODE, A HARDER WAY TO DO THIS IS WRITE TWO KCL EQNS @ v_s & v_o

KCL @ (SOURCE)

$$v_{gs} = -v_s$$

$$i_{in} - g_m v_s + \frac{v_s - v_o}{r_{ds}} = 0$$

SOLVE FOR v_s

$$\frac{v_s}{r_{ds}} - g_m v_s = \frac{v_o}{r_{ds}} - i_{in}$$

$$v_s\left(\frac{1}{r_{ds}} - g_m\right) = \frac{v_o}{r_{ds}} - i_{in}$$

$$v_s = \frac{\frac{v_o}{r_{ds}} - i_{in}}{\left(\frac{1}{r_{ds}} - g_m\right)}$$

$$\frac{1}{r_{ds}} << g_m$$

$$v_s = \frac{\frac{v_o}{r_{ds}} - i_{in}}{-g_m}$$

$$v_s = \frac{i_{in} - \frac{v_o}{r_{ds}}}{g_m} \quad (1)$$

KCL @ v_o (DRAIN)

$$g_m \cdot (-v_s) + \frac{v_o - v_s}{r_{ds}} + \frac{v_o}{R_L} = 0$$

$$-g_m v_s + \frac{v_o}{r_{ds}} - \frac{v_s}{r_{ds}} + \frac{v_o}{R_L} = 0$$

SUB (1)

$$-\cancel{g_m}\left(\frac{i_{in} - \frac{v_o}{r_{ds}}}{\cancel{g_m}}\right) + \frac{v_o}{r_{ds}} - \frac{\left(i_{in} - \frac{v_o}{r_{ds}}\right)}{g_m r_{ds}} + \frac{v_o}{R_L} = 0$$

IGNORE

$$i_{in} - \cancel{\frac{v_o}{r_{ds}}} + \cancel{\frac{v_o}{r_{ds}}} - \frac{i_{in}}{g_m r_{ds}} - \cancel{\frac{v_o}{g_m r_{ds} \cdot r_{ds}}} + \frac{v_o}{R_L} = 0$$

$$\frac{i_{in}}{g_m r_{ds}} << i_{in} \qquad \frac{v_o}{g_m r_{ds}^2} << \frac{v_o}{R_L}$$

$$i_{in} = \frac{-v_o}{R_L}$$

DIRECTION OF i_{in} IS NEGATIVE ∴

$$\boxed{\frac{v_o}{i_{in}} = R_L}$$

← SAME ANS AS BEFORE.