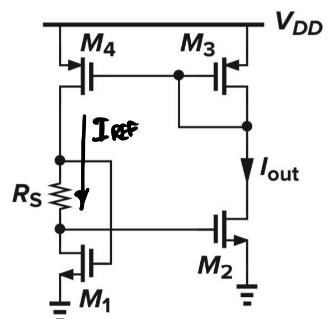
12.1. Derive an expression for I_{out} in Fig. 12.42.



THE PROBLEM STATEMENT IS NOT COMPLETE, MORE INFORMATION IS NEEDED.

ASSUME

$$(W/L)_4 = (W/L)_3 \ \&\ (W/L)_1 \neq (W/L)_2$$

FURTHER ASSUME

$\lambda = 0$ FOR ALL DEVICES.

Figure 12.42

$$\underbrace{I_{REF} = I_{OUT}}$$

$$V_{GS1} = V_{GS2} + I_{REF} R_S$$

USE SQUARE LAW EQNS. & SOLVE FOR V_{GS}

$$\sqrt{\frac{2I_{OUT}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} + \cancel{V_{TH1}} = \sqrt{\frac{2I_{OUT}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} + \cancel{V_{TH2}} + I_{OUT} R_S$$

ASSUME $V_{TH1} = V_{TH2}$ & SOLVE FOR I_{OUT}

$$\sqrt{\frac{2I_{OUT}}{\mu_n C_{ox}}}\left(\frac{1}{\sqrt{\frac{W}{L}_1}} - \frac{1}{\sqrt{\frac{W}{L}_2}}\right) = I_{OUT} R_S$$

$$\sqrt{I_{OUT}} = \frac{1}{R_S}\sqrt{\frac{2}{\mu_n C_{ox}}}\left(\frac{1}{\sqrt{W/L_1}} - \frac{1}{\sqrt{\frac{W}{L}_2}}\right)$$

$$\boxed{I_{OUT} = \frac{1}{R_S^2}\frac{2}{\mu_n C_{ox}}\left(\sqrt{\frac{L}{W_1}} - \sqrt{\frac{L}{W_2}}\right)^2}$$

12.2. Explain how the start-up circuit shown in Fig. 12.43 operates. Derive a relationship that guarantees that $V_X < V_{TH}$ after the circuit turns on.

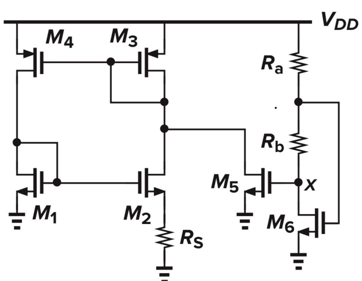
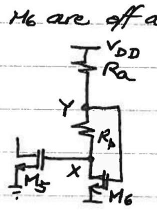


Figure 12.43

12.2 When the circuit turns on, initially both M_5 and M_6 are off and V_X and V_Y rise together, i.e., $V_X = V_Y$. When V_Y reaches V_{TH6}, V_X is also near V_{TH5}. Thus, M_6 and M_5 turn on almost simultaneously.



The surge in the drain current of M_5 turns the rest of the circuit on. As V_Y increases further, V_X begins to drop if M_6 is turned on sufficiently because the voltage gain of M_6 and R_b exceeds unity. For high values of V_Y, V_X can be lower than V_{TH5}.

Since $(V_{DD} - I_{D6} \cdot R_a - V_{TH})^2 \cdot \mu_n C_{ox}\left(\frac{W}{L}\right)_6 = I_{D6}$, we solve the quadratic equation:

$$R_a^2 I_{D6}^2 - I_{D6}\left(2R_a(V_{DD}-V_{TH}) + \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_6}\right) + (V_{DD}-V_{TH})^2 = 0$$

$$\Rightarrow I_{D6} = \frac{2R_a(V_{DD}-V_{TH}) + \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_6} + \sqrt{\left[2R_a(V_{DD}-V_{TH}) + \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_6}\right]^2 - 4R_a^2(V_{DD}-V_{TH})^2}}{2R_a^2}$$

This value is substituted in the other condition:

$$V_{DD} - I_{D6}(R_a + R_b) \leq V_{TH5}$$

to give the condition for turning off M_5.

12.5. In the circuit of Fig. 12.15, assume that Q_2 and Q_4 have a finite current gain β. Calculate the error in the output voltage.

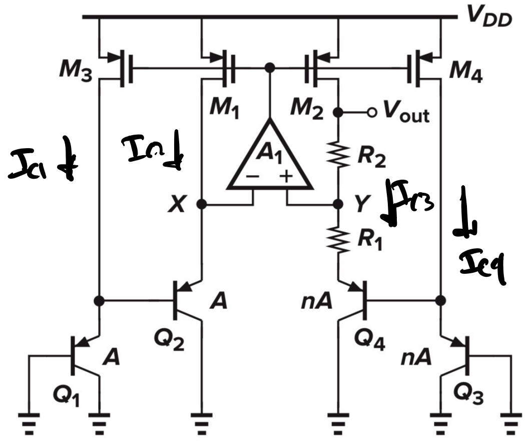


FIG 12.15

$I_c = \beta I_B$

FIRST FIND V_{out} WHEN $\beta = 0$.

ASSUME $\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_4$

& ASSUME IDEAL OP AMP ∴

$$V_X = V_Y \text{ \& } I_{S_{1,2,3,4}} = I_S$$

BECAUSE ALL PMOS THE SAME SIZE.

$$I_{c1} = I_{c2} = I_{c3} = I_{c4} = I_c$$

$$V_X = V_{BE1} + V_{BE2} = \ln\left(\frac{I_c}{I_S}\right) + \ln\left(\frac{I_c}{I_S}\right)$$

$$V_Y = V_{BE3} + V_{BE4} + I_c \cdot R_1$$

$$= \ln\left(\frac{I_c}{nI_S}\right) + \ln\left(\frac{I_c}{nI_S}\right) + I_c R_1$$

$$V_X = V_Y$$

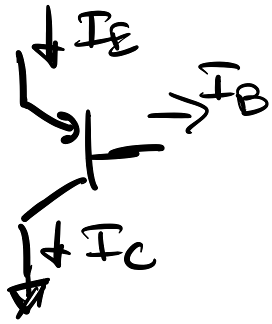
$$\ln\left(\frac{I_c}{I_S}\right) + \ln\left(\frac{I_c}{I_S}\right) = \ln\left(\frac{I_c}{nI_S}\right) + \ln\left(\frac{I_c}{nI_S}\right) + I_c R_1$$

$$2\ln(n) = I_c R_1$$

$$I_c = \frac{2\ln(n)}{R_1}$$

$$V_{out} = \frac{R_1}{R_2} 2\ln(n) + V_{BE3} + V_{BE4}$$

WITH β ERROR



$$I_E = I_c - I_B$$

$$I_E = I_c - \frac{I_c}{\beta}$$

$$= I_c\left(1 - \frac{1}{\beta}\right)$$

$$I_C = I_E \frac{1}{(1 - 1/\beta)} = I_E \frac{(1 + 1/\beta)}{1 - (1/\beta)^2} \qquad 1/\beta \ll 1$$

$$\approx I_E \left(\beta + 1/\beta\right)$$

$\therefore$ THE ERROR IN I_E IS

$$I_E = I_C \frac{\beta}{\beta + 1}$$

& THE ERROR VOLTAGE IS:

$$\Delta V_{OUT} = \frac{R_2}{R_1} \alpha \ln \left(\frac{\beta}{\beta + 1} \right)$$