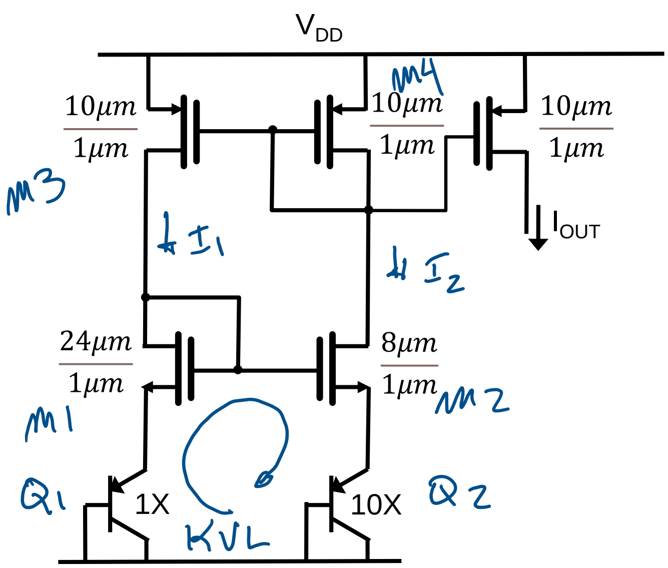
Name: SOLUTIONS

50 points total

EE 473 **Quiz 2** **Winter 2022**

1) a) For the below circuit, assume all MOS devices are in the saturation region. ***Derive*** a closed-form expression for the value of the output current, I_{OUT}, then compute the value. The following parameters may, or may not be needed, $\mu_n C_{ox} = 20mA/V^2, \mu_p C_{ox} = 10mA/V^2$, $V_{TH(NMOS)} = 0.7V$ and $V_{TH(PMOS)} = 0.8V$, $\lambda_n = 0$, and $\lambda_p = 0$. Ignore the body effect ($\gamma = 0$). (25 points)



$\lambda = 0 \Rightarrow r_o \to \infty$

$\Rightarrow I_1 = I_2 = I_{OUT}$ IF ALL DEVICES IN SATURATION.

KVL AROUND ABOVE LOOP:

$$V_{BE1} + V_{GS_{M1}} - V_{GS_{M2}} - V_{BE2} = 0$$

$$V_T \ln\left(\frac{I_1}{I_{S1}}\right) + (V_{DSAT} + V_{th})_{M1} - (V_{DSAT} + V_{th})_{M2} - V_T \ln\left(\frac{I_2}{n I_{S1}}\right) = 0$$

ASSUME $V_{th1} = V_{th2}$

$$V_T \ln\left(\frac{I_1}{I_2} \frac{n \cdot I_{S1}}{I_{S1}}\right) + (V_{DSAT M1} - V_{DSAT M2}) = 0$$

$$V_T \ln(n) = (V_{DSAT M2} - V_{DSAT M1})$$

$$V_T \ln(n) = \sqrt{\frac{2 \cdot I_{OUT}}{\mu_n C_{ox}}} \left(\sqrt{\left(\frac{L}{W}\right)_{M2}} - \sqrt{\left(\frac{L}{W}\right)_{M1}} \right)$$

SOLVE FOR I_{OUT}

$$\sqrt{\frac{2 \cdot I_{OUT}}{\mu_n C_{ox}}} = \frac{V_T \ln(n)}{\sqrt{\left(\frac{L}{W}\right)_{M2}} - \sqrt{\left(\frac{L}{W}\right)_{M1}}}$$

$$I_{OUT} = \frac{\mu_n C_{ox}}{2} \left(\frac{V_T \ln(n)}{\sqrt{\left(\frac{L}{W}\right)_{M2}} - \sqrt{\left(\frac{L}{W}\right)_{M1}}} \right)^2$$

$$= \frac{20\,mA/V^2}{2} \left(\frac{25\,mV \ln(10)}{\sqrt{\frac{1}{8}} - \sqrt{\frac{1}{24}}} \right)^2$$

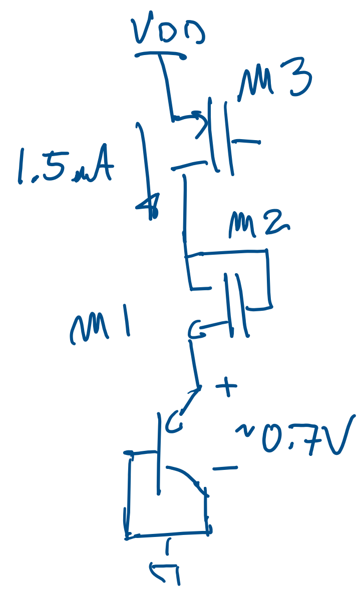
$$\boxed{\begin{aligned} I_{OUT} &= 1.484\,mA \\ &\cong 1.5\,mA \end{aligned}}$$

b) For the circuit in part a), what is the minimum supply voltage (V_{DD}) that can be used for the current source to operate properly (10 points)

LOWEST VOLTAGE ?

THE LOWEST VOLTAGE COULD BE DETERMINED BY THE LEFT LEG w/ I_1 OR THE RIGHT LEG (I_2).
SOLUTIONS WILL DO BOTH LEGS, BUT EITHER ANS WILL BE ACCEPTED.

ASSUME $V_{BEQ_1} \cong 0.7V$



$$V_{DD_{MIN}} = V_{BE1} + V_{GSM1} + V_{DSAT\,M3}$$

$$= V_{BE1} + V_{DSAT_{M1}} + V_{tn_N} + V_{DSAT\,M3}$$

$$V_{BE} \cong 0.7\,V$$

$$V_{DSAT\,M1} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{\frac{2 \cdot 1.5mA}{20\frac{mA}{V^2}\left(\frac{24}{1}\right)}} \approx 79mV \cong 80mV$$

$$V_{DSAT\,M4} = \sqrt{\frac{2I_D}{\mu_p C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{\frac{2 \cdot 1.5mA}{10\frac{mA}{V^2}\left(\frac{10}{1}\right)}} \approx 173mV \cong 175mV$$

$$V_{tn_N} = 0.7V \quad V_{tn\,p} = 0.8V$$

$$\therefore \quad V_{DD,MIN,LEFT} = 0.7V + 0.08V + 0.7 + 0.175 + 0.8V$$

$$= 2.455\,V$$

MIN VDD DUE TO THE RIGHT LEG.

$$V_{BE2} \cong 0.7V$$

$$V_{DD,MIN,RIGHT} = V_{BE2} + V_{DSAT\,M2} + V_{GS\,M4}$$

$$= V_{BE2} + V_{DSAT\,M2} + V_{DSAT_{M4}} + V_{tnp}$$

$$= 0.7V + 0.14 + 0.175 + 0.8$$

$$\boxed{= 1.815}$$

$$V_{DSAT\,M2} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} = \sqrt{\frac{2 \cdot 1.5mA}{20\frac{mA}{V^2}\left(8/1\right)}} \cong 140mV$$

$V_{DSAT\,M4}$: FROM ABOVE

$V_{tnn} = 0.7\,V$

$V_{tnp} = 0.8\,V$

· LEFT LEG LIMITS LOW VOLTAGE!

c) Describe the relationship between the output current and temperature. This can be described as a proportionality if you want – e.g. $I_{OUT} \alpha\ (T)$. (15 points)

$$I_{OUT} = \frac{\mu_n C_{ox}}{2} \left(\frac{V_T \ln(n)}{\sqrt{\left(\frac{L}{W}\right)_{M2}} - \sqrt{\left(\frac{L}{W}\right)_{M1}}} \right)^2$$

$$V_T = \frac{KT}{q} \quad \& \quad \mu_n \propto T^{-3/2}$$

ALL OTHER VARIABLES INDEPENDENT OF TEMP. $\therefore$

$$I_{OUT} \propto T^{-3/2} (T)^2$$

$$\boxed{I_{OUT} \propto \sqrt{T}}$$