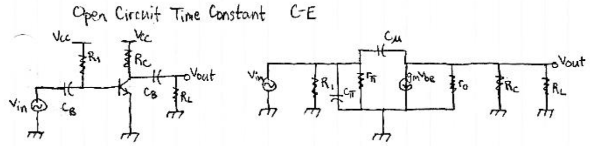
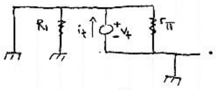
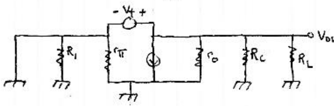
Open Circuit Time Constant C-E



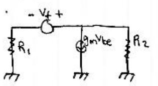
- 

C_π: $\quad V_t = 0 \quad$ (short C_μ)

$R = \frac{V_t}{i_t} = 0 \Rightarrow R_{C_\pi} = 0$

- 

C_μ: (short C_π)

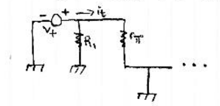


General form: $\frac{V_t}{i_t} = R_1 + R_2 + g_m R_1 R_2$

$R_{C\mu} = R_1 // r_\pi + r_o // R_c // R_L + g_m (R_1 // r_\pi)(r_o // R_c // R_L)$

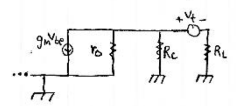
$2\pi f_H = \omega_H = \frac{1}{0 \cdot C_\pi + R_{C\mu} C_\mu}$

Short Circuit Time Constant C-E

- 

$V_t = (R_1 // r_\pi) i_t \quad \leftarrow C_{B_1}$

$R_{B_1} = \frac{V_t}{i_t} = R_1 // r_\pi$



C_{B_2}: $i_t = g_m V_{be}$

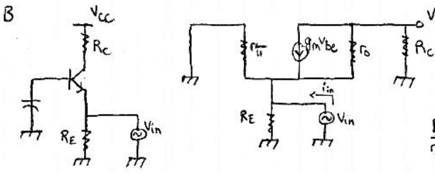
$V_t = g_m V_{be} (r_o // R_c)$

$R_{B2} = \frac{V_t}{i_t} = (r_o // R_c) + R_L$

$\therefore 2\pi f_L = \omega_L = \sum \frac{1}{R_{B_i} C_{B_i}}$

$\omega_L = \frac{1}{R_{B_1} C_1} + \frac{1}{R_{B_2} C_2}$

C-B



$A_v \approx g_m R_c$

$V_{out} = -R_c \left(g_m V_{be} + \frac{V_{out} - V_{in}}{r_o} \right)$

$\frac{R_c}{r_o} V_{out} + V_{out} = R_c g_m V_{in} + \frac{R_c}{r_o} V_{in}$

$V_{out}\left[1 + \frac{R_c}{r_o}\right] = V_{in}\left[g_m R_c + \frac{R_c}{r_o}\right]$

$A_v = \frac{V_{out}}{V_{in}} = \frac{r_o g_m R_c + R_c}{r_o + R_c} = \frac{R_c[1 + g_m r_o]}{R_c + r_o}$

Input Resistance:

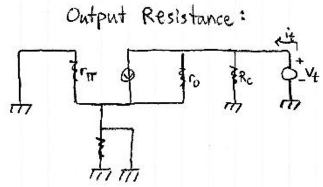
$i_{in} = \frac{V_{in}}{R_E} + \frac{V_{in}}{r_\pi} + \frac{V_{in} - V_{out}}{r_o} - g_m V_{be}$

$i_{in} = V_{in}\left[\frac{1}{R_E} + \frac{1}{r_\pi} + \frac{1}{r_o} + g_m\right] + \underbrace{\frac{V_{out}}{r_o}}_{0}, \quad V_{out} = g_m R_c V_{in}$

$i_{in} = V_{in}\left[\frac{1}{R_E} + \frac{1}{r_\pi} + \frac{1}{r_o} + \frac{1}{1/g_m}\right]$

$R_{in} = \frac{V_{in}}{i_{in}} = R_E // \underbrace{r_\pi // \frac{1}{g_m}}_{1/g_m} // r_o$

Output Resistance:



$i_t = g_m V_{be}$

$V_t = g_m V_{be} (r_o // R_c)$

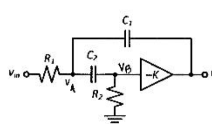
$R_{out} = \frac{V_t}{i_t} = r_o // R_c$

Part 2:
Find the transfer function of the circuit below in terms of k, C_1, C_2, R_1, and R_2 and answer questions (a) through (c)

a) Find the transfer function of the circuit below.
b) What type of filter is this?
c) What is the DC gain of the step response for this circuit? $\quad$ step $= \frac{1}{s} \quad$ gain $= \frac{1}{s} \cdot \frac{V_{out}}{V_{in}}\Big|_{s=0}$

$-V_B K = V_o$

$V_B = \frac{-V_o}{K}$



a.)

- $\frac{V_A - V_{in}}{R_1} + \frac{V_A - V_o}{\frac{1}{sC_1}} + \frac{V_A + V_o/K}{\frac{1}{sC_2}} \;\; V_A\left[\frac{1}{R_1} + sC_1 + sC_2\right] = \frac{V_{in}}{R_1} + V_o\left[sC_1 + \frac{sC_2}{K}\right]$

- $\frac{-V_o/K}{R_2} + \frac{V_o/K - V_A}{1/sC_2} = 0 \quad = \quad \frac{V_o}{K}\left[\frac{1}{R_2} + sC_2\right] = V_A[sC_2]$

$V_A = \frac{V_o}{K}\left[\frac{1 + sC_2R_2}{sR_2C_2}\right]$

$V_A[1 + sC_1R_1 + sC_2R_1] - V_o\left[sC_1R_1 + \frac{sC_2R_1}{K}\right] = V_{in}$

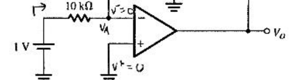
$\frac{V_o}{K}\left[\frac{1 + sC_2R_2}{sR_2C_2}\right][1 + sC_1R_1 + sC_2R_1] - \frac{V_o}{K}[sC_1R_1K + sC_2R_1] = V_{in}$

$\frac{V_o}{K}\left[\frac{1 + sC_1R_1 + sC_2R_1 + sC_2R_2 + s^2C_1C_2R_1R_2 + s^2C_2^2R_1R_2 - s^2C_1C_2R_1R_2K - s^2C_2^2R_1R_2}{sR_2C_2}\right] = V_{in}$

$V_o = V_{in} \frac{sKR_2C_2}{1 + s(C_1R_1 + C_2R_1 + C_2R_2) + s^2 C_1C_2R_1R_2(1-K)}$

$\frac{V_o}{V_{in}} = \frac{sKR_2C_2}{1 + s(C_1R_1 + C_2R_1 + C_2R_2) + s^2C_1C_2R_1R_2(1-K)}$ ✓

(a) Find expressions for the voltage v_o in terms of R_L. Assume the op amp is ideal.
(b) What should be the value of R_L so that v_o = -16 V?
(c) Now assume that the op-amp has an input offset voltage of v_{os}. Find the expressions for v_o in terms of v_{os}, and R_L. Using R_L from part (b), what is the output offset voltage if v_{os} is ±4 mV?



a.)

$\frac{V_A - V_B}{10k} = \frac{1}{10k} \Rightarrow \frac{-V_B}{10k} = \frac{1}{10k} \Rightarrow V_B = -1\,V$

$\frac{V_B}{50} + \frac{V_B - V_o}{R_L} = 0$

$V_B\left(\frac{1}{50} + \frac{1}{R_L}\right) = \frac{V_o}{R_L}$

$V_B\left(\frac{R_L + 50}{50\cancel{R_L}}\right) = \frac{V_o}{\cancel{R_L}}$

$V_o = -\frac{R_L}{50} - 1$

b.) $-16 = \frac{-R_L}{50} - 1$

$R_L = 750\,\Omega$

c.) $V_A = \pm 4\,mV = V_{os}$

$\frac{V_{os} - 1}{10k} + \frac{V_{os} - V_B}{10k} = 0 \Rightarrow V_{os}\left(\frac{1}{10k} + \frac{1}{10k}\right) = \frac{V_B + 1}{10k}$

$2V_{os} = V_B + 1$

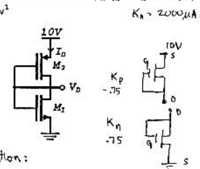
$\frac{V_B}{50} + \frac{V_B - V_o}{R_L} = 0 \Rightarrow V_B\left(\frac{R_L + 50}{50\cancel{R_L}}\right) = \frac{V_o}{\cancel{R_L}}$

$V_o = \left[\frac{R_L + 50}{50}\right](2V_{os} - 1)$

$V_o = -16 \pm .128\,V$ ✓

Find the current I_D and voltage V_O if W/L of both transistors is 20/1. Assume $V_{TP} = -0.75\,V$, $\lambda = 0$, and $\mu_p C_{ox} = K_p' = 40\,\mu A/V^2$, $V_{TN} = +0.75\,V$, and $\mu_n C_{ox} = K_n' = 100\,\mu A/V^2$.

$K_p = 800\mu A/V^2 \qquad K_n = 2000\mu A/V^2$



Assume in saturation:

$I_D = \frac{1}{2}(800 \cdot 10^{-6})(V_{SG_p} - 0.75)^2 = \frac{1}{2}(2000 \cdot 10^{-6})(V_{GS_n} - 0.75)^2$

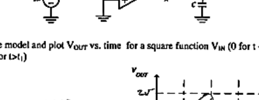
$V_{SG} = V_S - V_g = 10V - V_D$

$V_{GS} = V_g - V_S = V_D$

$\boxed{V_D = 4.04\,V \quad I_D = 10.8\,mA}$ ✓

NMOS	PMOS
$V_{ov} = 3.29\,V$	$V_{ov} = 5.21\,V$
$V_{DS} = V_D = 4.04$	$V_{DS} = 10 - 4.04 = 5.96$
✓ $V_{DS} > V_{ov}$	$V_{DS} > V_{ov}$ ✓

Problem 3 (20 Points): Use the circuit below to answer the following questions. The op-amp is *ideal*. $2R_1 = R_2$.

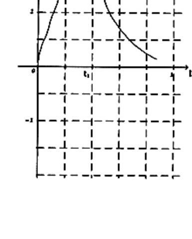


a) Use ideal diode model and plot V_{OUT} vs. time for a square function V_{IN} (0 for t < 0, -1 V for t_1 > t > 0, 0 V for t > t_1)

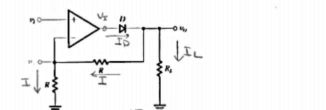
$t < 0$
$V_{in} = 0$
all currents are zero
$V_{out} = 0$

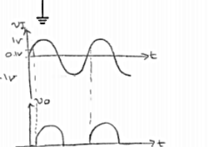
$0 < t < t_1$
$V_{in} = -1$
D_2: ON D_1: Off
$I = \frac{1}{R_1} = I_{D2}$
$V_{out} = \frac{R_2}{R_1} \times 1V = 2V$
depending on t_1 & time constant may or maynot reach 2V

$t_1 < t$
$V_{in} = 0$

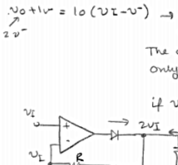


Problem 3 (25 Points): For the circuit should below, draw the output waveform for the input sinusoidal waveform with peak-peak voltage of 2 V and DC of 0 V. Assume the op-amp has a voltage gain of 10 V/V and its output saturates at ±15 V. Use constant voltage drop model for diode D with V_{ON} of 1 V.





v_I : positive
assume diode ON
$I_D = I + I_L$
$v_X = 10(v_I - v^-)$
$v_O = v_X - 1V \Rightarrow v_X = v_O + 1V$
$v_O = 2R \times I = 2R \times \frac{v^-}{R} = 2v^-$
$v_O + 1V = 10(v_I - v^-) \rightarrow 2v^- + 1 = 10(v_I - v^-)$
$2v^-$
$12v^- + 1 = 10v_I \Rightarrow v^- = \frac{10v_I - 1}{12}$
The current flow is positive only if v^- is positive or $10v_I > 1$ or $v_I > 0.1V$
if v^- is negative: the current flow direction is shown on the left.
It can be seen that this violates KCL at node v_O → diode turns off & v_X saturates to $-15V$
all currents become zero & $v_O = 0$



This is a rectifier

Bode and Phase $\tan^{-1}(\omega/\omega_o)$

constant: K	$20\log_{10}(\lvert K\rvert)$	$K>0$ $0°$; $K<0$ $\pm180°$	
S	Δ 20 dB/dec, $\omega_o=1$	$+90°$	
$\frac{1}{S}$	∇ -20 dB/dec, $\omega_o=1$	$-90°$	
$\frac{1+\frac{j\omega}{\omega_o}}{1}$	Δ 20 dB/dec, ω_o	$0°$ … $90°$; $0.1\omega_o$ ω_o $10\omega_o$; $\Delta 45°$/dec.	
$\frac{1}{1+\frac{j\omega}{\omega_o}}$	∇ 20 dB/dec, ω_o	$0°$ … $-90°$; $0.1\omega_o$ ω_o $10\omega_o$	

Constant Voltage Drop
$V_D = V_{ON}$, $I_D > 0$

$V_D < V_{ON}$ $I_D = 0$

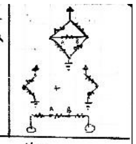
Ideal Mode
$V_D = 0$ $I_D > 0$

$V_D < 0$ $I_D = 0$

$\omega_{3dB} \to \omega_b = 1/\tau$ $\tau = CR$ or L/R

Low Pass $\frac{K}{1+(s/\omega_o)}$

High Pass $\frac{Ks}{s+\omega_o}$



$I_1 = I_s e^{V_1/V_T}$, $I_2 = I_s e^{V_2/V_T}$, $V_T = \frac{kT}{q}$

$\frac{I_2}{I_1} = e^{(V_2-V_1)/V_T}$ $V_2 - V_1 = V_T \ln\left(\frac{I_2}{I_1}\right)$

$V_T = 0.0862T$ mV (25.8 mV @ 20°C)

Normalize: $(d = 1/Q)$

$\left(1 + \frac{ds}{\omega_n} + \frac{s^2}{\omega_n^2}\right)\Big|_{\omega_n=1} = 1 + ds + s^2$

- Reverse Bias: current doubles every 10°C rise in temp
- Forward Bias: current doubles every 5°C rise in temp. [const. volt]
- Const. Current: $\downarrow$ 2mV / $\uparrow$ 1°C

- $V_{Id} = V^+ - V^-$
- $V_{CM} = \frac{1}{2}(V^+ + V^-)$
- $V^- = V_{Icm} - V_{Id}/2$
- $V^+ = V_{Icm} + V_{Id}/2$
- $V_o = A_d V_{Id} + A_{CM} V_{CM}$
- $A_d V_{Id} = V_{out}$ amplifier differential gain
- $A_{CM} = \frac{V_o}{V_{CM}}$ common mode gain
- CMRR $= \frac{A_d}{A_{CM}}$ common mode rejection ratio

- $n_i = 1.5 \cdot 10^{10} / cm^3$ silicone
- $\frac{x_n}{x_p} = \frac{N_A}{N_D}$ $\epsilon_S = 11.7\epsilon_o$, $\epsilon_o = 8.85 \cdot 10^{-14}$ F/cm
- $W = x_n + x_p$
- $W = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_o + V_R)}$ width of depletion; not always present
- $Q_j = q\frac{N_A N_D}{N_A + N_D} AW$ $q = 1.6 \times 10^{-19}$
- $= A\sqrt{2\epsilon_s q\left(\frac{N_A N_D}{N_A + N_D}\right)(V_o + V_R)}$

- $V_o = A_d\left[V_{Id} + \frac{V_{CM}}{CMRR}\right]$ (equiv. statement as above)
- $V_o = A_d\left[V_{Id} + V_{os} + \frac{V_{CM}}{CMRR}\right]$ (input offset voltage)
- $R_{in} = \frac{v_i}{i_i}$ (input resistance)
- $\frac{d}{dt} V_{out}\big|_{max}$ (slew rate)
- $A_o\omega_n = \omega_t$ $\omega_n = 3dB$ ω_t: $\lvert A(s)\rvert = 1$

Gain Bandwidth Product

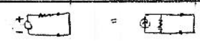
Large Signal Mode

$I_D = I_S\left(e^{-V/V_T} - 1\right)$

$\frac{V_{out}}{V_{in}} = \frac{A_1 A_2 \dots A_n}{\left(1+\frac{s}{\omega_{p_1}}\right)\left(1+\frac{s}{\omega_{p_2}}\right)\dots\left(1+\frac{s}{\omega_{p_n}}\right)} = \frac{A_1 A_2 A_n}{\sqrt{2}}$

$\rho = \frac{R_1}{R_2}$ $\gamma = \frac{C_2}{C_1}$ $T_1 = R_1' C_1' \omega_n$, $T_2 = R_2' C_2' \omega_n$

$A_o(v^+ - v^-) - R_o i = V_o$ (output resistance)

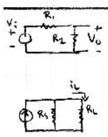


$=$

Open Circuit / Short Circuit:
- Break circuit at terminals (create o.c.)
- Find V_{oc}. $V_{oc} = V_{th}$ (nodal analysis)
- Create short circuit at terminals
- Find voltage at node closest to terminals
- Solve for current I_{sc}
- Solve for $R_{Th} = \frac{V_{Th}}{I_{sc}}$. ✓

Open Circuit / External Source
- Break circuit at terminals
- Find V_{oc}. $V_{oc} = V_{th}$
- Deactivate all independent sources (voltage or current). Leave depend src!
- Solve using mesh currents (add voltages) Have current in terms of V_{ex}
- Solve for $R_{Th} = \frac{V_{ex}}{i_{ex}}$.

(V_{ex} should cancel out!)



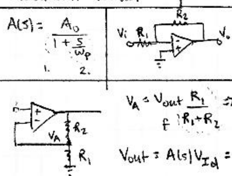
$V_o = \frac{V_i R_2}{R_1 + R_2}$

$i_L = i_o\frac{R_S}{R_S + R_L}$

$20\log\left(\frac{V_1}{V_2}\right) \longleftrightarrow 10^{dB/20}$

$f(t)$	$F(s)$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$

$A(s) = \frac{A_o}{1+\frac{s}{\omega_p}}$ 1. 2.



$\frac{V_o}{V_i} = \frac{-R_2}{R_1}$

$A_v V_{Id}$, R_o

$V_A = V_{out}\frac{R_1}{R_1+R_2} \Rightarrow \frac{V_A}{V_{out}} = \frac{R_1}{R_1+R_2}$

$V_{out} = A(s) V_{Id} = A(s)(V_{in} - V_A)$

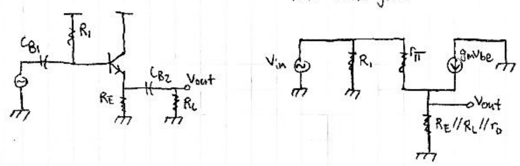
R_{in}

$V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$

Short Circuit Time Constant (SCTC) C-C:

$2\pi f_L = \omega_L = \sum \frac{1}{R_{B_i} C_{B_i}}$

Mid-band gain:



Let $R_E' = R_E \,//\, R_L \,//\, r_o$

$V_{in} = V_{out} + V_{be}$

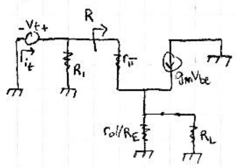
$\beta = g_m r_\pi$ $V_{out} = \left(g_m V_{be} + \frac{V_{be}}{r_\pi}\right) R_E' = \left(\frac{\beta+1}{r_\pi}\right) V_{be} \cdot R_E'$

[substitute] $V_{out} \approx g_m R_E' V_{be} \Rightarrow V_{be} = \frac{V_{out}}{g_m R_E'}$

$V_{in} = V_{out} + \frac{V_{out}}{g_m R_E'}$

$\frac{V_{out}}{V_{in}} = \frac{g_m R_E'}{1 + g_m R_E'}$

C_{B_1}: Short input and other large capacitors; test voltage at specific capacitor



$V_t = V_{be} + \left[r_o // R_E // R_L\right]\left(\frac{V_{be}}{r_\pi} + g_m V_{be}\right)$

$i_t = \frac{V_{be}}{r_\pi}$

$R = \frac{V_t}{i_t} = r_\pi + (r_o // R_E // R_L)\left(\frac{V_{be}}{r_\pi} + g_m V_{be}\right)\frac{r_\pi}{V_{be}}$

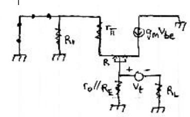
$\beta = g_m r_\pi$

Find equivalent resistance

$R = r_\pi + (r_o // R_E // R_L)(1 + \beta)$

$R_{C_1} = R_1 // R$

C_{B_2}:



KCL:

$i_t + \frac{V_{be}}{r_\pi} + g_m V_{be} = 0$

$i_t = \frac{-V_{be}}{r_\pi} - g_m V_{be}$

$V_t = -V_{be}$

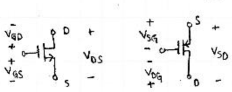
$R = \frac{V_t}{i_t} = \frac{V_{be}}{\frac{V_{be}}{r_\pi} + g_m V_{be}} = \frac{1}{\frac{1}{r_\pi} + g_m} = \frac{1}{\frac{1}{r_\pi} + \frac{1}{1/g_m}}$

$= r_\pi // 1/g_m$

$R_{C_2} = R_L + r_o // R_E // R$

lower freq cutoff $\omega_L = \frac{1}{R_{C_1} C_1} + \frac{1}{R_{C_2} C_2}$

NMOS PMOS



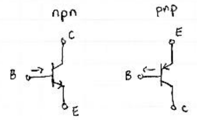
$V_{ov} = V_{GS} - V_t$ or $V_{SG} - \lvert V_t\rvert$

NMOS	PMOS	
$V_{GS} < V_t$	$V_{SG} < \lvert V_t\rvert$	Cutoff Condition ($i_D = 0$)
$V_{GD} > V_t$, $V_{DS} < V_{ov}$	$V_{DG} > \lvert V_t\rvert$, $V_{SD} < \lvert V_{ov}\rvert$	Triode Condition
$i_D = k_n'\left(\frac{W}{L}\right)\left[V_{ov} - \frac{1}{2}V_{DS}\right]V_{DS}$	$i_D = k_p'\left(\frac{W}{L}\right)\left[\lvert V_{ov}\rvert - \frac{1}{2}V_{SD}\right]V_{SD}$	Triode Region
$V_{GD} \le V_t$, $V_{DS} \ge V_{ov}$	$V_{DG} \le \lvert V_t\rvert$, $V_{SD} \ge \lvert V_{ov}\rvert$	Saturation Condition
$i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{ov}^2$	$i_D = \frac{1}{2}k_p'\left(\frac{W}{L}\right)V_{ov}^2$	Saturation Region

- Large Signal
  $r_o = \left[\frac{\partial I_D}{\partial v_{DS}}\right]^{-1}$ (saturation)
  $r_o = \frac{1}{\lambda I_D} = \frac{V_A}{I_D}$
- $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$
- $k_n' = \mu_n C_{ox}$
- $k_n = \mu_n C_{ox}\left(\frac{W}{L}\right)$
- @ v_{DS} small, MOSFET behaves as linear resistance
  $r_{DS} = \frac{1}{g_{DS}} = \frac{1}{k_n V_{ov}}$

Active Mode BJT
- $i_C = I_S e^{V_{BE}/V_T}$ $i_C = \alpha i_E$ $i_B = (1-\alpha) i_E = \frac{i_E}{\beta+1}$
- $i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{V_{BE}/V_T} \Rightarrow i_C = \beta i_B$ $i_E = (\beta+1) i_B$
- $i_E = \frac{i_C}{\alpha} = \left(\frac{I_S}{\alpha}\right) e^{V_{BE}/V_T}$ $\beta = \frac{\alpha}{1-\alpha}$ $\alpha = \frac{\beta}{\beta+1}$

npn pnp



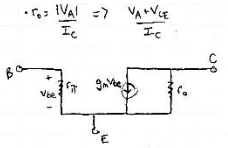
(for pnp transistor, replace V_{BE} with V_{EB})

$\beta_{forced} = \frac{i_C}{i_B}\Big|_{saturation}$

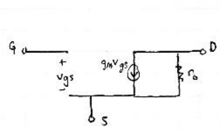
- $g_m = \frac{I_C}{V_T}\Big|_{V_T = 25\,mV}$
- $r_\pi = \frac{V_T}{I_B} = \beta\frac{V_T}{I_C} = \frac{\beta}{g_m}$
- $r_o = \frac{\lvert V_A\rvert}{I_C} \Rightarrow \frac{V_A + V_{CE}}{I_C}$

	npn	pnp
cutoff: EBJ reverse, CBJ reverse	$I_B = 0$, $I_C = 0$	$I_B = 0$, $I_C = 0$
active: EBJ forward, CBJ reverse	$V_{BE} = 0.7$ V; @ threshold of sat. $\hookrightarrow V_{CE} = 0.3$ V	$V_{EB} = 0.7$ V; $V_{EC} = 0.3$ V
saturation: EBJ forward, CBJ forward	$V_{BE} = 0.7$ V; $V_{CE_{sat}} = 0.2$ V	$V_{EB} = 0.7$ V; $V_{CE_{sat}} = 0.2$ V

$\beta_f = \beta_{fo}\left(1 + \frac{V_{CE}}{V_A}\right)$



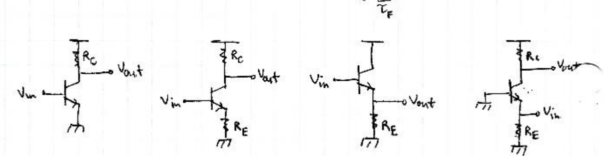
Small signal BJT

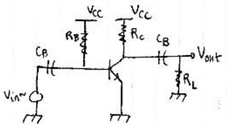


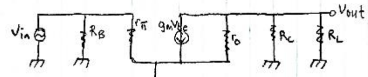
Small signal MOSFET

capacitors $\to$ short
current source $\to$ open circuit
(all DC components are deactivated)

	C-E	C-E w/ deg	C-C	C-B
R_{in}	r_π moderate	$r_\pi(1+g_m R_E) = r_\pi + \beta R_E$ high	$r_\pi(1+g_m R_E)$ high	$\frac{1}{g_m} // R_E$ low
R_{out}	$r_o // R_C \approx R_C$ moderate	$R_C // r_o(1+g_m R_E) \approx R_C$ moderate	$\frac{1}{g_m} // R_E$ low	$R_C // r_o \approx R_C$ moderate
A_v	$-g_m R_C$ high	$\frac{-g_m R_C}{1+g_m R_E}$ moderate	$\frac{g_m R_E}{1+g_m R_E} \approx 1$ low	$g_m R_C$ high
ω_H	$\frac{1}{C_\mu R_C}$ moderate; ignores R_S	$\frac{1}{C_\pi(R_E // 1/g_m) + C_\mu R_{out}}$ low	$\frac{1}{C_\pi(1/g_m // R_E)}$ high; $\approx \frac{1}{\tau_F}$	$\frac{1}{C_\mu R_{out}}$ moderate





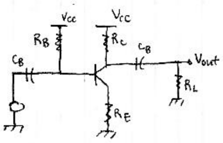


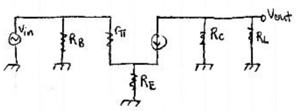
C-E

Midband gain:
(all caps are shorted)

$v_{in} = v_{be}$

$$\frac{V_{out}}{V_{in}} = -g_m(r_o // R_C // R_L) \approx -g_m(R_C // R_L)$$





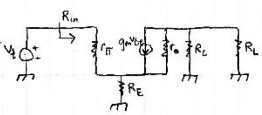
C-E with degeneration

Midband Gain

$$V_{in} = V_{be} + R_E\left(g_m V_{be} + \frac{V_{be}}{r_\pi}\right)$$

$$V_{out} = -g_m(R_C // R_L)$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_m(R_C // R_L)}{1 + g_m R_E}$$



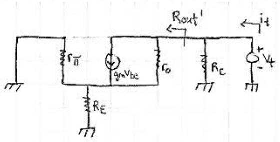
R_{in}: replace signal with test voltage
ignore any capacitors

$$i_t = \frac{V_{be}}{r_\pi}$$

C-E with degeneration

$$V_t = V_{be} + R_E\left(\frac{V_{be}}{r_\pi} + g_m V_{be}\right) = V_{be} + R_E \frac{V_{be}}{r_\pi} + R_E g_m V_{be}$$

$$R_{in} = \frac{V_t}{i_t} = r_\pi + R_E + R_E g_m r_\pi \approx r_\pi(1 + g_m R_E)$$



$R_{out} = R_C // R_{out}'$

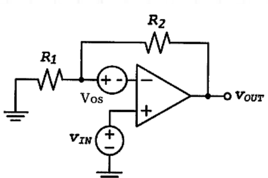
$$i_t = g_m v_{be} + \frac{V_t - (-V_{be})}{r_o}$$

C-E with degeneration

$$V_{be} = -(r_\pi // R_E)\, i_t \quad \text{[substitute]}$$

When finding Rout, ignore all input signals
ignore any capacitors

$$i_t = -g_m(r_\pi // R_E)\, i_t + \frac{V_t - (r_\pi // R_E)\, i_t}{r_o}$$



$$V_t = i_t r_o + g_m(r_\pi // R_E)\, i_t r_o + (r_\pi // R_E)\, i_t$$

$$V_t = i_t\left[r_o + g_m(r_\pi // R_E) r_o + r_\pi // R_E\right]$$

$$R_{out} = \frac{V_t}{i_t} = r_o + g_m(r_\pi // R_E) r_o + r_\pi // R_E$$

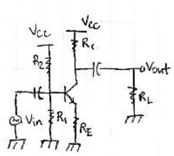
$$V_{out} = \frac{V_{in}}{R_1} \times R_2 + V_{in} + \frac{V_{OS}}{R_1} \times R_2 + V_{OS}$$

if $R_E \ll r_\pi \Rightarrow R_{out} = r_o + g_m r_o R_E + R_E$

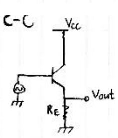
$$\approx r_o(1 + g_m R_E)$$

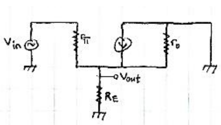
- Summary

	C-E	C-E w/ degen.
Gain	$-g_m(R_C // R_L)$	$\frac{-g_m(R_C // R_L)}{1 + g_m R_E}$
R_{in}	$r_\pi // R_1 // R_2$	$r_\pi(1 + g_m R_E) // (R_1 // R_2)$
R_{out}	$R_C // r_o$	$R_C // r_o(1 + g_m R_E)$



C-C





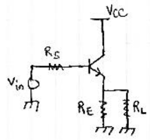
$V_{be} = V_{in} - V_{out}$

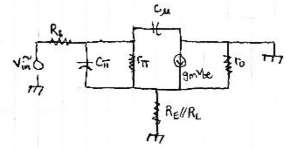
KCL: $\frac{V_{in} - V_{out}}{r_\pi} + g_m(V_{in} - V_{out}) + \frac{0 - V_{out}}{r_o // R_E} = 0$

$$V_{in}\left(\frac{1}{r_\pi} + g_m\right) = V_{out}\left(\frac{1}{r_\pi} + g_m + \frac{1}{R_E}\right)$$

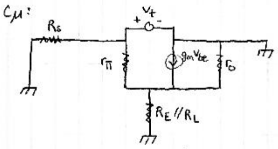
$$\frac{V_{out}}{R_{in}} = \frac{g_m R_E}{1 + g_m R_E}$$

Open Circuit Time Constant (OCTC) C-C:





C_μ:



$$i_t = \frac{V_{be}}{r_\pi}$$

$$V_t = V_{be} + R_E'\left(\frac{V_{be}}{r_\pi} + g_m V_{be}\right)$$

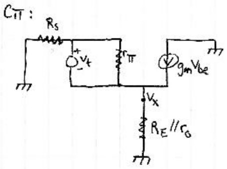
$R_E' = R_E // R_L // r_o$

$$R = \frac{V_t}{i_t} = r_\pi + R_E' + R_E' g_m r_\pi \approx r_\pi(1 + g_m R_E')$$

$R_{C\mu} = R_S // R$

$R_{C\mu} = R_S // (r_\pi(1 + g_m R_E')) \approx R_S$

C_π:



$V_{be} = V_t$

$$i_t = \frac{V_t}{r_\pi} + \frac{V_x + V_t}{R_S}$$

$$V_x = (R_E // r_o)\left(g_m V_t - \frac{V_x + V_t}{R_S}\right)$$

$$i_t = V_t\left[\frac{1}{r_\pi} + \frac{1}{R_S}\right] + V_x\left[\frac{1}{R_S}\right] \qquad V_x\left[1 + \frac{R_E // r_o}{R_S}\right] = \left[R_E // r_o\right]\left[g_m - \frac{1}{R_S}\right] V_t$$

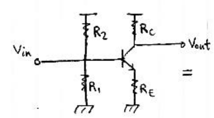
$$i_t = \frac{V_t}{r_\pi // R_S} + \frac{\left[R_E // r_o\right]\left[g_m - 1/R_S\right]}{R_S + R_E // r_o}$$

$$i_t = V_t \underbrace{\left[\frac{1}{r_\pi // R_S} + \frac{\left[R_E // r_o\right]\left[g_m - 1/R_S\right]}{R_S + R_E // r_o}\right]}_{A} \qquad R_{O\pi} = \frac{1}{A}$$

$$\omega_H = \frac{1}{R_{O\mu} C_\mu + R_{O\pi} C_\pi}$$

Multi Stage Amplifiers

1. Simplifying Stages



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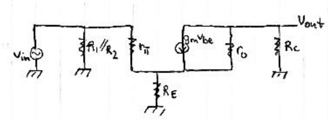

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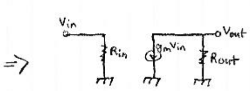
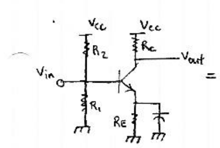


$R_{in} = R_1 // R_2 // [r_\pi(1 + g_m R_E)]$

$R_{out} = R_C$

$$G_M = \frac{i_{out}}{V_{in}}\bigg|_{V_{out}=0} = \frac{g_m V_{be}}{V_{be} + R_E\left(\frac{V_{be}}{r_\pi} + g_m V_{be}\right)}$$



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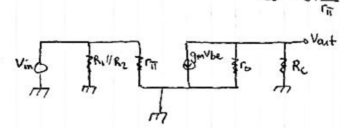

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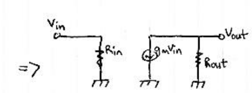
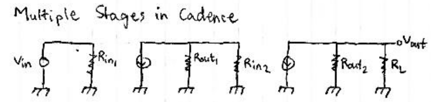


$R_{in} = R_1 // R_2 // r_\pi$

$R_{out} = R_C$

$$G_M = \frac{i_{out}}{V_{in}}\bigg|_{V_{out}=0} = \frac{g_m V_{be}}{V_{be}} = g_m$$

2. Multiple Stages in Cadence



$$\frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_{in_2}} \cdot \frac{V_{in_2}}{V_{in_1}} = -G_M(R_{out_2} // R_L) + G_{M_1}(R_{in_2} // R_{out_1})$$