

ELECTRICAL ENGINEERING

UNIVERSITY *of* WASHINGTON

Lecture 15: Multipliers

Acknowledgements

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UW (2013-2022)
GaTech (2022-present)

Roadmap for today

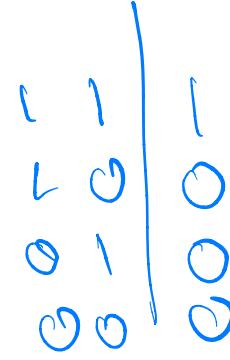
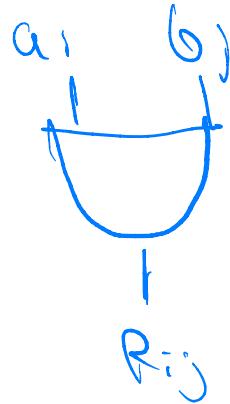
- Unsigned multiplication
- Signed multiplication (Baugh-Wooley)
- Booth encoding to improve performance
- Prefix computation.... **again?!!**

Multipliers

- How do you multiply binary numbers?

$$\begin{array}{r} 1 \ 0 \ 1 \ 1 \\ 1 \ 0 \ 1 \\ \hline 1 \ 0 \ 1 \ 1 \\ 0 \ 0 \ 0 \ 0 \\ 1 \ 0 \ 1 \ 1 \\ \hline 1 \ 1 \ 0 \ 1 \ 1 \ 1 \end{array} \quad \begin{array}{r} 11 \\ \times 5 \\ \hline \end{array}$$

a_i b_j
 \downarrow
 R_{ij}



- Multiplication
 - Generate partial products
 - Sum partial products together
 - $M \times N$ multiplication $\rightarrow M+N$ bit result

Multiplication

- Given multiplicand $Y = y_{n-1} y_{n-2} y_{n-3} \dots y_1 y_0$
- Given multiplicand $X = x_{m-1} x_{m-2} x_{m-3} \dots x_1 x_0$
- $$X \times Y = (\sum_{i=0}^{m-1} x_i 2^i) (\sum_{j=0}^{n-1} y_j 2^j) = \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} x_i y_j 2^{i+j}$$
- In the case $m=5, n=5$

	y_5	y_4	y_3	y_2	y_1	y_0	
	x_5	x_4	x_3	x_2	x_1	x_0	
	$x_0 y_5$	$x_0 y_4$	$x_0 y_3$	$x_0 y_2$	$x_0 y_1$	$x_0 y_0$	
	$x_1 y_5$	$x_1 y_4$	$x_1 y_3$	$x_1 y_2$	$x_1 y_1$	$x_1 y_0$	
	$x_2 y_5$	$x_2 y_4$	$x_2 y_3$	$x_2 y_2$	$x_2 y_1$	$x_2 y_0$	
	$x_3 y_5$	$x_3 y_4$	$x_3 y_3$	$x_3 y_2$	$x_3 y_1$	$x_3 y_0$	
	$x_4 y_5$	$x_4 y_4$	$x_4 y_3$	$x_4 y_2$	$x_4 y_1$	$x_4 y_0$	
	$x_5 y_5$	$x_5 y_4$	$x_5 y_3$	$x_5 y_2$	$x_5 y_1$	$x_5 y_0$	
	p_{11}	p_{10}	p_9	p_8	p_7	p_6	p_5
						p_4	p_3
						p_2	p_1
						p_0	

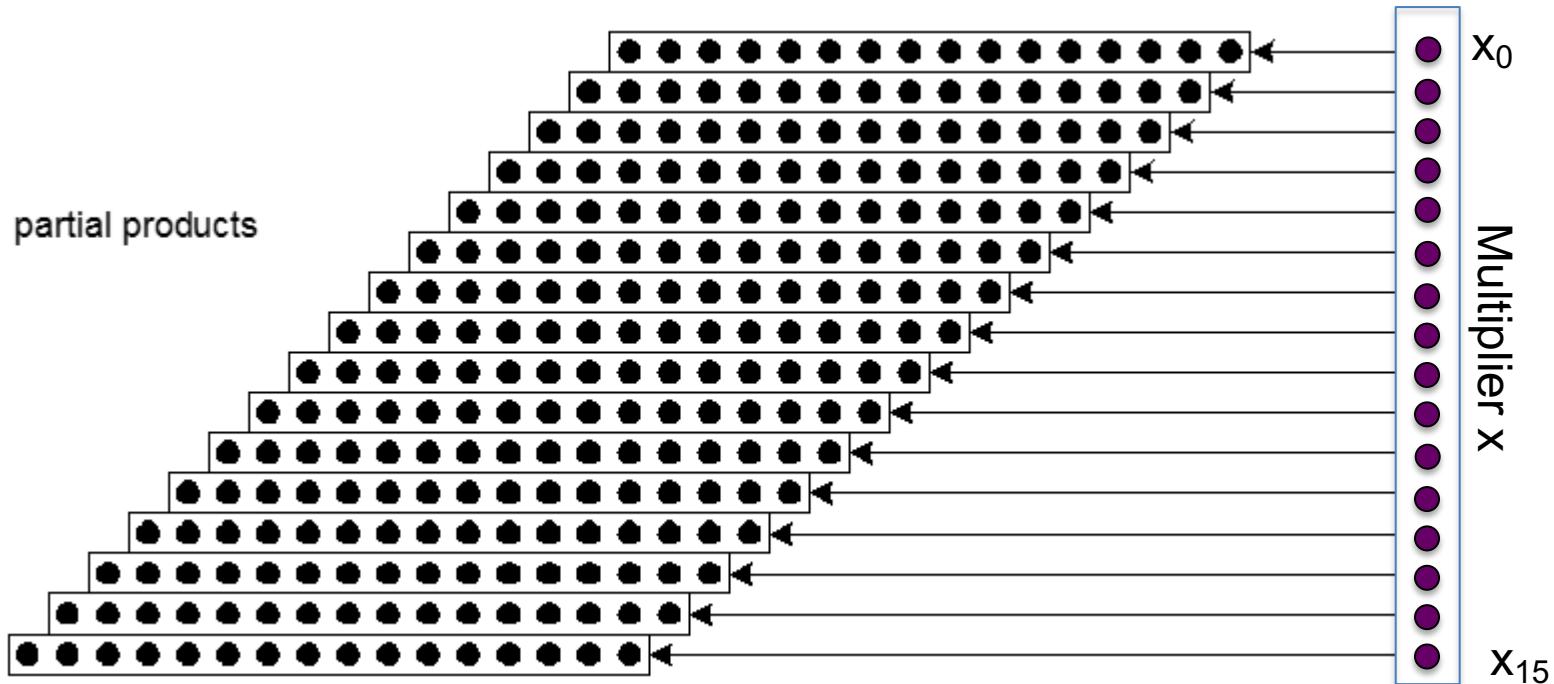
Annotations:

- A blue arrow labeled "Multiplicand" points to the column of y values ($y_5, y_4, y_3, y_2, y_1, y_0$).
- A blue arrow labeled "Multiplier" points to the row of x values ($x_5, x_4, x_3, x_2, x_1, x_0$).
- A large blue bracket on the right side of the table is labeled "Partial Products".
- A blue arrow labeled "Products" points to the final row of the table.

Source: Weste-Harris

Dot Diagram

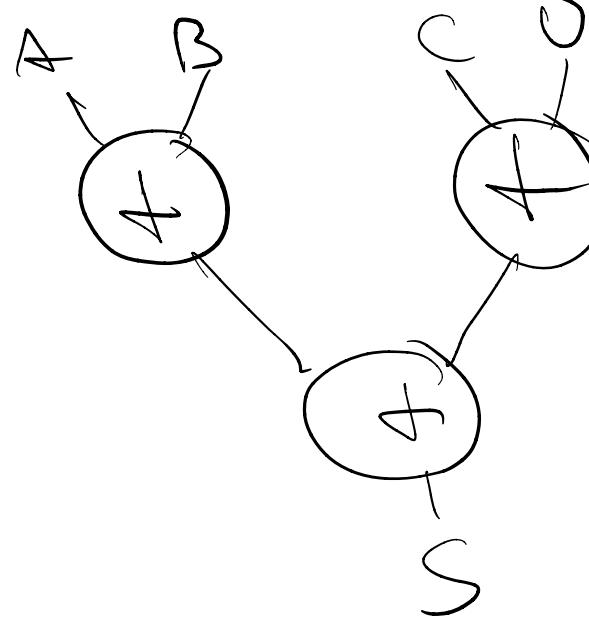
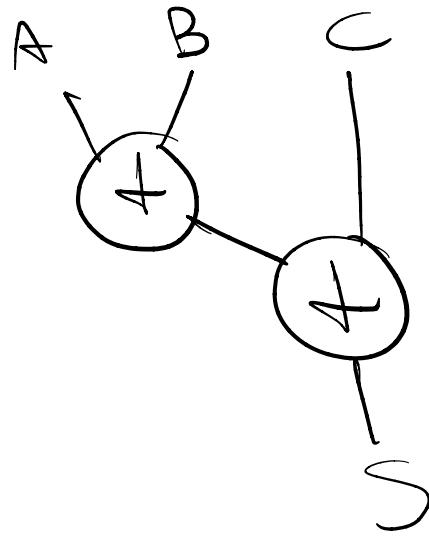
y
 x
 $y_2y_1y_0$
 $x_2x_1x_0$



Source: Weste-Harris

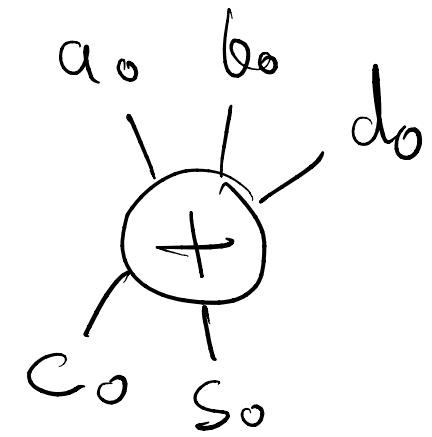
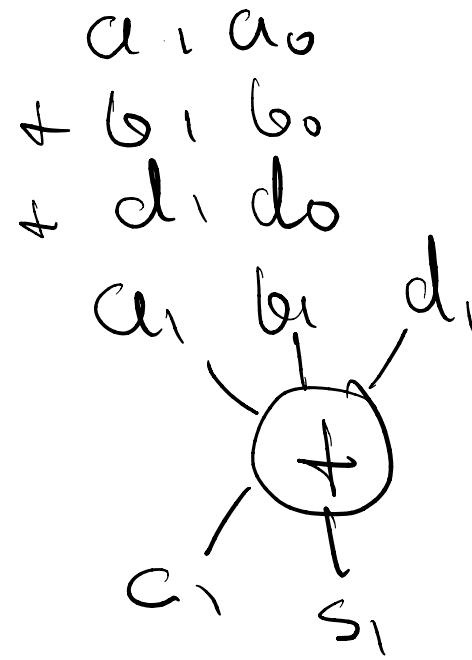
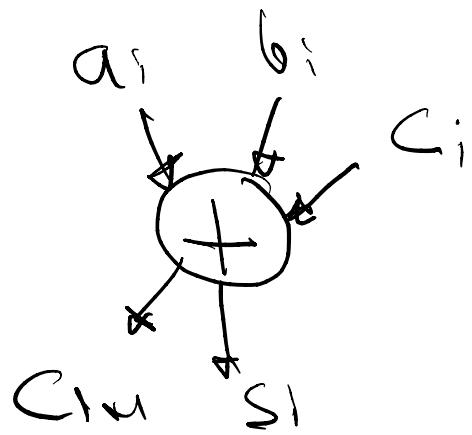
- Each “●” represents a bit-product

How to sum more than 2 numbers?



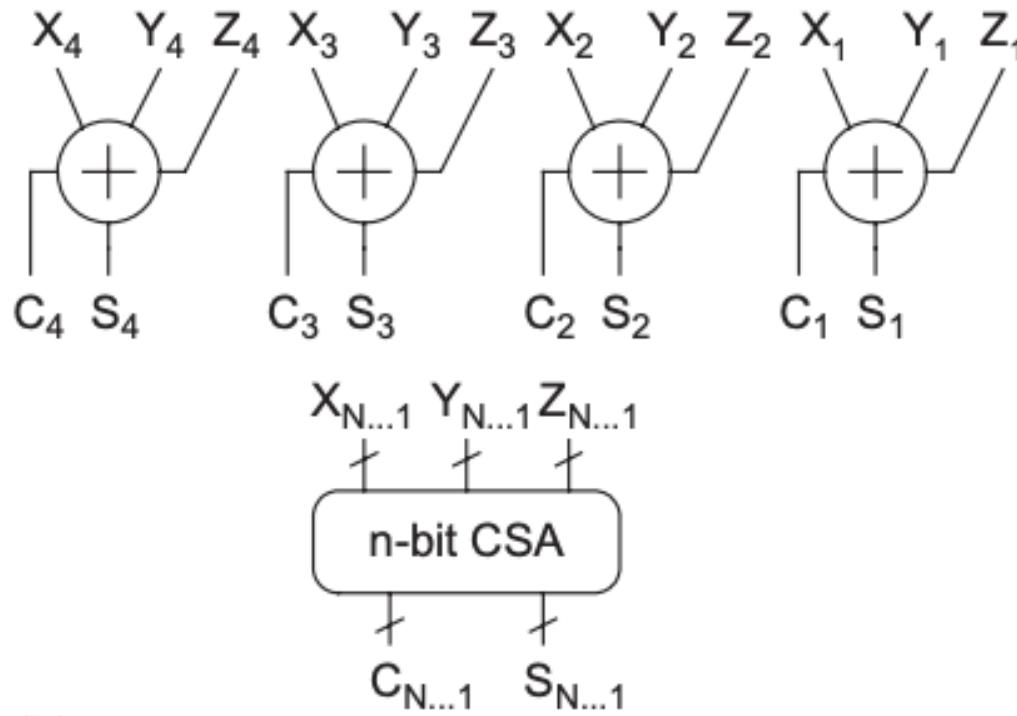
How to sum more than 2 numbers?

- Remember full-adders?



$$\begin{aligned} A + B + C &= s_1 s_0 + 2 \cdot c_1 c_0 \\ &= \underline{s_1 s_0} \\ &\quad + \underline{c_1 c_0 0} \end{aligned}$$

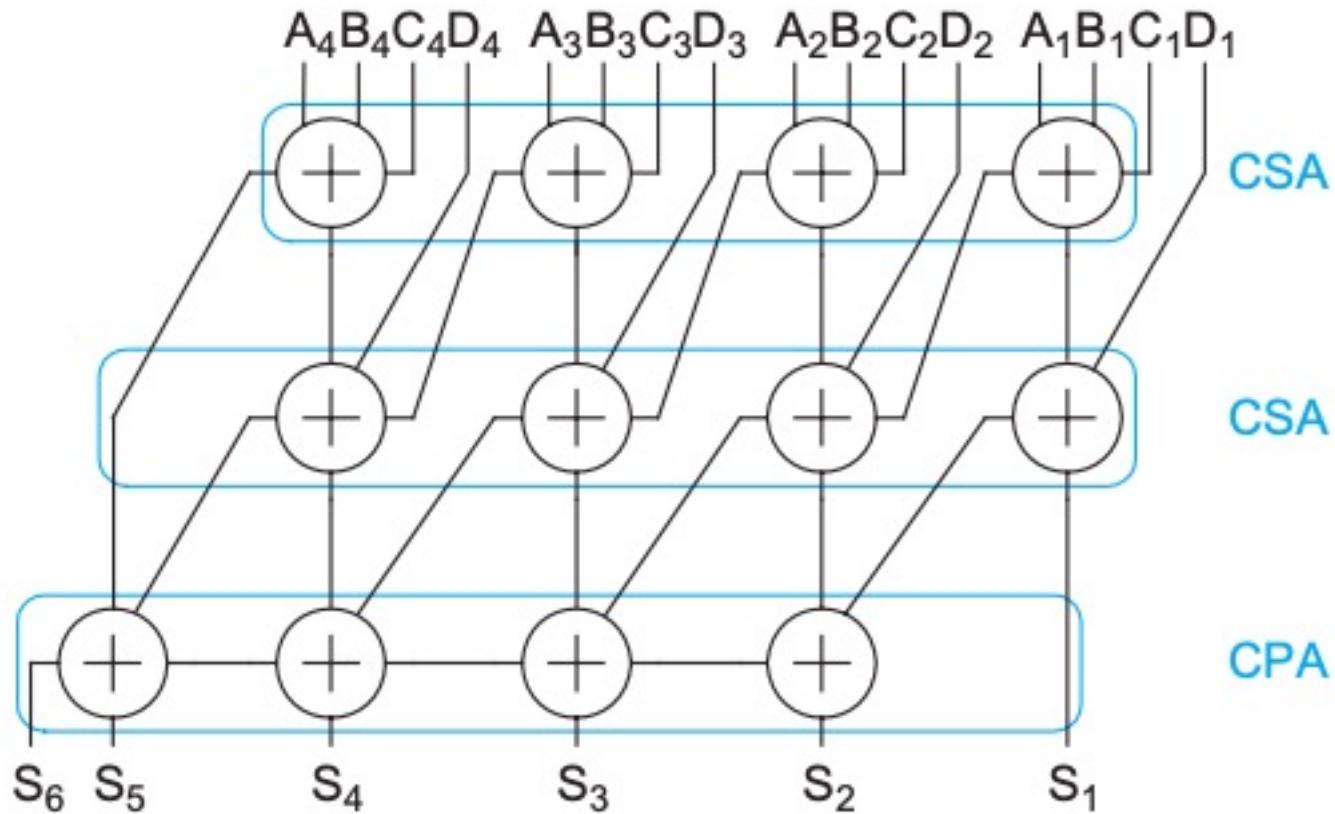
Carry Save Adder (CSA)



$$X + Y + Z = S + 2C$$

Source: Weste-Harris

CSAs to add four numbers



A₄ A₃ A₂ A₁
B₄ B₃ B₂ B₁
C₄ C₃ C₂ C₁
D₄ D₃ D₂ D₁

Source: Weste-Harris

Array Multiplier

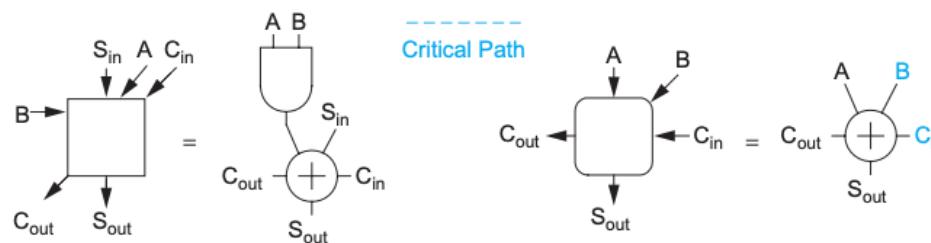
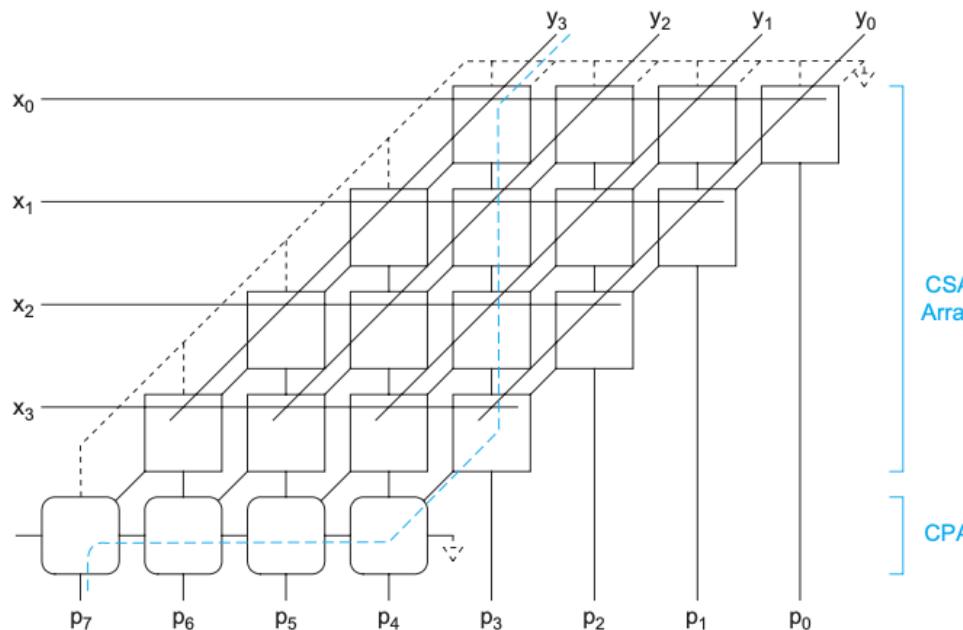
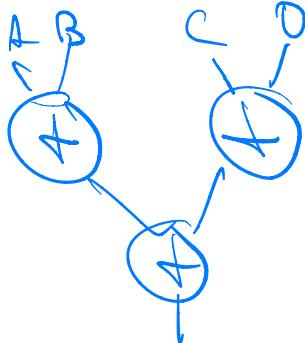


FIGURE 11.74 Array multiplier

Source: Weste-Harris

- Sequential addition of partial products (Analogous to Ripple-Carry Addition)
- Final addition of partial products is an Adder operation
 - All inputs do not arrive at the same time

Array Multiplier

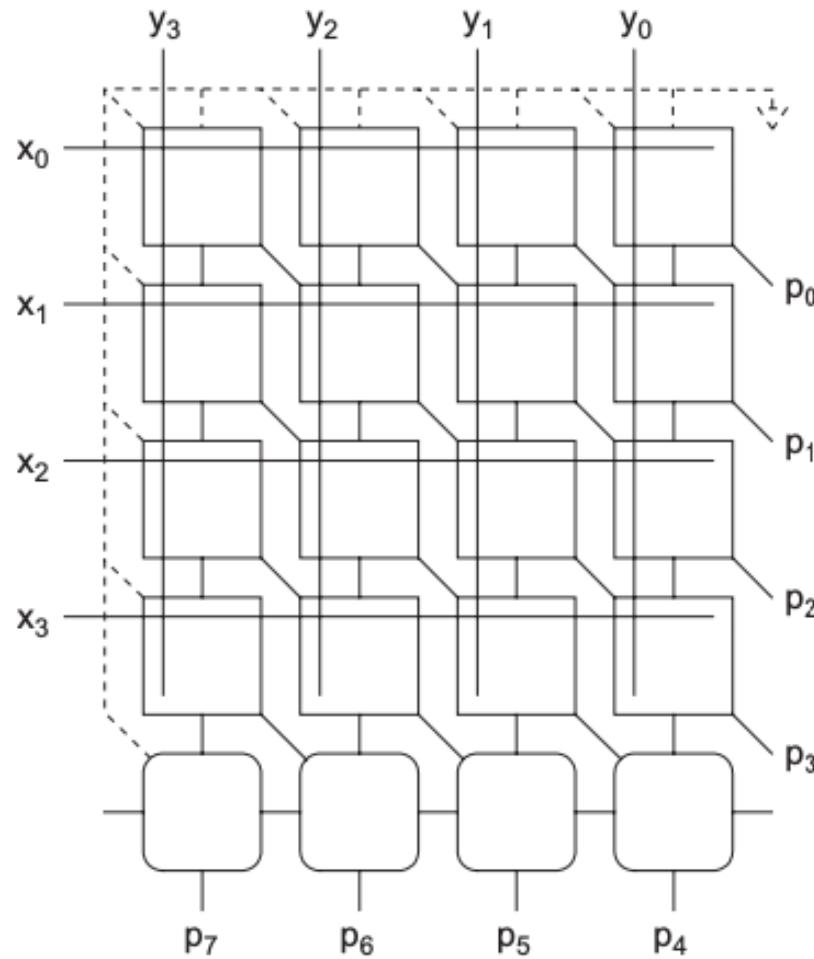


FIGURE 11.75 Rectangular array multiplier

Source: Weste-Harris

- “Straightening it” to be rectangular

Do you remember how to compute $-x$?

Two's complement

x . How do you compute $\neg x$?

$$\neg x = \overline{x} + 1$$

Signed Multiplication (Baugh-Wooley)

- Multiplier topology thus far has been signed multiplication
- $P_{signed} = X \times Y$

$$\begin{aligned} & \left(-x_{m-1} 2^{m-1} + \sum_{i=0}^{m-2} x_i 2^i \right) \left(-y_{n-1} 2^{n-1} + \sum_{j=0}^{n-2} y_j 2^j \right) \\ &= y_{n-1} x_{m-1} 2^{m+n-2} \\ &\quad - 2^{n-1} \sum_{i=0}^{m-2} y_{n-1} x_i 2^i - 2^{m-1} \sum_{j=0}^{n-1} x_{m-1} y_j 2^j \\ &\quad + \sum_{i=0}^{m-2} \sum_{j=0}^{n-2} x_i y_j 2^{i+j} \end{aligned}$$

$$X = x_m x_{m-2} \dots x_1 x_0$$

$$Y = y_n y_{n-2} \dots y_1 y_0$$

Signed Array Multiplier (Baugh-Wooley)

$$\sum_{i=0}^{m-2} \sum_{j=0}^{n-2} x_i y_j 2^{i+j}$$

$$\begin{array}{cccccc} & & & x_0y_4 & x_0y_3 & x_0y_2 & x_0y_1 & x_0y_0 \\ & & & x_1y_4 & x_1y_3 & x_1y_2 & x_1y_1 & x_1y_0 \\ & & & x_2y_4 & x_2y_3 & x_2y_2 & x_2y_1 & x_2y_0 \\ & & & x_3y_4 & x_3y_3 & x_3y_2 & x_3y_1 & x_3y_0 \\ & & & x_4y_4 & x_4y_3 & x_4y_2 & x_4y_1 & x_4y_0 \end{array}$$

Signed Array Multiplier (Baugh-Wooley)

$$\sum_{i=0}^{m-2} \sum_{j=0}^{n-2} x_i y_j 2^{i+j}$$

$$+ y_{n-1} x_{m-1} 2^{m+n-2}$$

x_5y_5

			x_0y_4	x_0y_3	x_0y_2	x_0y_1	x_0y_0
			x_1y_4	x_1y_3	x_1y_2	x_1y_1	x_1y_0
			x_2y_4	x_2y_3	x_2y_2	x_2y_1	x_2y_0
			x_3y_4	x_3y_3	x_3y_2	x_3y_1	x_3y_0
			x_4y_4	x_4y_3	x_4y_2	x_4y_1	x_4y_0

Signed Array Multiplier (Baugh-Wooley)

$$\begin{array}{r} \sum_{i=0}^{m-2} \sum_{j=0}^{n-2} x_i y_j 2^{i+j} \\ + y_{n-1} x_{m-1} 2^{m+n-2} \\ - 2^{m-1} \sum_{j=0}^{n-1} x_{m-1} y_i 2^i \end{array} \quad \begin{array}{ccccccccc} & & & & x_0 y_4 & x_0 y_3 & x_0 y_2 & x_0 y_1 & x_0 y_0 \\ & & & & x_1 y_4 & x_1 y_3 & x_1 y_2 & x_1 y_1 & x_1 y_0 \\ & & & & x_2 y_4 & x_2 y_3 & x_2 y_2 & x_2 y_1 & x_2 y_0 \\ & & & & x_3 y_4 & x_3 y_3 & x_3 y_2 & x_3 y_1 & x_3 y_0 \\ & & & & x_4 y_4 & x_4 y_3 & x_4 y_2 & x_4 y_1 & x_4 y_0 \\ x_5 y_5 & & & & & & & & \\ \overline{0} & \overline{0} & \overline{x_5 y_4} & \overline{x_5 y_3} & \overline{x_5 y_2} & \overline{x_5 y_1} & \overline{x_5 y_0} & \overline{0} & \overline{0} & \overline{0} & \overline{0} & \overline{1} \end{array}$$

Signed Array Multiplier (Baugh-Wooley)

$$\begin{array}{l} \sum_{i=0}^{m-2} \sum_{j=0}^{n-2} x_i y_j 2^{i+j} \\ + y_{n-1} x_{m-1} 2^{m+n-2} \\ - 2^{m-1} \sum_{j=0}^{n-1} x_{m-1} y_i 2^i \\ - 2^{n-1} \sum_{i=0}^{m-2} y_{n-1} x_i 2^i \end{array} \quad \begin{array}{ccccccccccccc} & & & & & x_0 y_4 & x_0 y_3 & x_0 y_2 & x_0 y_1 & x_0 y_0 \\ & & & & & x_1 y_4 & x_1 y_3 & x_1 y_2 & x_1 y_1 & x_1 y_0 \\ & & & & & x_2 y_4 & x_2 y_3 & x_2 y_2 & x_2 y_1 & x_2 y_0 \\ & & & & & x_3 y_4 & x_3 y_3 & x_3 y_2 & x_3 y_1 & x_3 y_0 \\ & & & & & x_4 y_4 & x_4 y_3 & x_4 y_2 & x_4 y_1 & x_4 y_0 \\ & & & & & x_5 y_5 & & & & & & & & & & & \\ \overline{0} & \overline{0} & \overline{x_5 y_4} & \overline{x_5 y_3} & \overline{x_5 y_2} & \overline{x_5 y_1} & \overline{x_5 y_0} & \overline{0} & \overline{1} \\ \overline{0} & \overline{0} & \overline{x_4 y_5} & \overline{x_3 y_5} & \overline{x_2 y_5} & \overline{x_1 y_5} & \overline{x_0 y_5} & \overline{0} & \overline{1} \end{array}$$

Signed Array Multiplier (Baugh-Wooley)

$$\begin{array}{c}
 \sum_{i=0}^{m-2} \sum_{j=0}^{n-2} x_i y_j 2^{i+j} \\
 + y_{n-1} x_{m-1} 2^{m+n-2} \\
 - 2^{m-1} \sum_{j=0}^{n-1} x_{m-1} y_i 2^i \\
 - 2^{n-1} \sum_{i=0}^{m-2} y_{n-1} x_i 2^i
 \end{array}
 \quad
 \begin{array}{ccccccccc}
 & & & & x_0 y_4 & x_0 y_3 & x_0 y_2 & x_0 y_1 & x_0 y_0 \\
 & & & & x_1 y_4 & x_1 y_3 & x_1 y_2 & x_1 y_1 & x_1 y_0 \\
 & & & & x_2 y_4 & x_2 y_3 & x_2 y_2 & x_2 y_1 & x_2 y_0 \\
 & & & & x_3 y_4 & x_3 y_3 & x_3 y_2 & x_3 y_1 & x_3 y_0 \\
 & & & & x_4 y_4 & x_4 y_3 & x_4 y_2 & x_4 y_1 & x_4 y_0
 \end{array}
 \quad
 \begin{array}{ccccccccc}
 & & & & & & & & 1 \\
 1 & 1 & \overline{x_5} y_4 & \overline{x_5} y_3 & \overline{x_5} y_2 & \overline{x_5} y_1 & \overline{x_5} y_0 & 1 & 1 \\
 1 & 1 & \overline{x_4} y_5 & \overline{x_3} y_5 & \overline{x_2} y_5 & \overline{x_1} y_5 & \overline{x_0} y_5 & 1 & 1 \\
 & & & & & & & & 1
 \end{array}$$

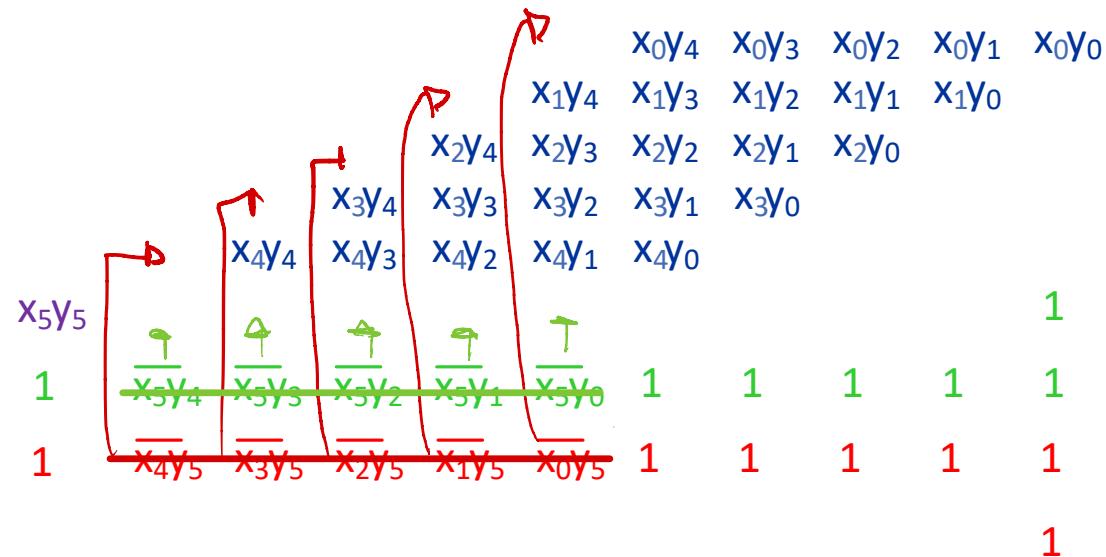
Signed Array Multiplier (Baugh-Wooley)

$$\sum_{i=0}^{m-2} \sum_{j=0}^{n-2} x_i y_j 2^{i+j}$$

$$+ y_{n-1} x_{m-1} 2^{m+n-2}$$

$$- 2^{m-1} \sum_{j=0}^{n-1} x_{m-1} y_i 2^i$$

$$- 2^{n-1} \sum_{i=0}^{m-2} y_{n-1} x_i 2^i$$



$$\begin{array}{r}
 & \overline{x_0y_5} \\
 & \overline{x_1y_4} \\
 & \overline{x_2y_3} \\
 & \overline{x_3y_2} \\
 & \overline{x_4y_1} \\
 & \overline{x_5y_0} \\
 \hline
 & \overline{x_0y_4} & \overline{x_0y_3} & \overline{x_0y_2} & \overline{x_0y_1} & \overline{x_0y_0} \\
 & \overline{x_1y_4} & \overline{x_1y_3} & \overline{x_1y_2} & \overline{x_1y_1} & \overline{x_1y_0} \\
 & \overline{x_2y_4} & \overline{x_2y_3} & \overline{x_2y_2} & \overline{x_2y_1} & \overline{x_2y_0} \\
 & \overline{x_3y_3} & \overline{x_3y_2} & \overline{x_3y_1} & \overline{x_3y_0} & \\
 & \overline{x_4y_2} & \overline{x_4y_1} & \overline{x_4y_0} & & \\
 & \overline{x_5y_1} & \overline{x_5y_0} & & & \\
 & \overline{x_0y_5} & & & & \\
 \hline
 & 1 & 1 & 1 & 1 & 1
 \end{array}$$

| | $\overline{x_5y_5}$ $\overline{x_5y_4}$ $\overline{x_5y_3}$ $\overline{x_5y_2}$ $\overline{x_5y_1}$ $\overline{x_5y_0}$

Removing leading 1's from signed numbers?

s bit number

$$2^5 \text{ (5 bits)} \quad 1 \ 1 \ 0 \ 1 \ 0 = -16 + 8 + 2 = -6$$

$$2^4 \text{ (4 bits)} \quad 1 \ 0 \ 1 \ 0 = -8 + 2 = -6$$

1 1 a₂ a₁ a₀

$$= -2^4 + 2^3 + a_2 2^2 + a_1 2^1 + a_0$$

$$= -2^3 + a_2 2^2 + a_1 2^1 + a_0$$

$$2^3 + 2^3 = 2 \cdot 2^3 = 2^4$$

$$-2^4 + 2^3 = -2^3$$

1 a₂ a₁ a₀

Signed Array Multiplier (Baugh-Wooley)

$$\sum_{i=0}^{m-2} \sum_{j=0}^{n-2} x_i y_j 2^{i+j}$$

$$+ y_{n-1} x_{m-1} 2^{m+n-2}$$

$$- 2^{m-1} \sum_{j=0}^{n-1} x_{m-1} y_i 2^i$$

$$- 2^{n-1} \sum_{i=0}^{m-2} y_{n-1} x_i 2^i$$

1	$\overline{x_0y_5}$	x_0y_4	x_0y_3	x_0y_2	x_0y_1	x_0y_0
$\overline{x_1y_5}$	x_1y_4	x_1y_3	x_1y_2	x_1y_1	x_1y_0	
$\overline{x_2y_5}$	x_2y_4	x_2y_3	x_2y_2	x_2y_1	x_2y_0	
$\overline{x_3y_5}$	x_3y_4	x_3y_3	x_3y_2	x_3y_1	x_3y_0	
$\overline{x_4y_5}$	x_4y_4	x_4y_3	x_4y_2	x_4y_1	x_4y_0	
1	x_5y_5	$\overline{x_5y_4}$	$\overline{x_5y_3}$	$\overline{x_5y_2}$	$\overline{x_5y_1}$	$\overline{x_5y_0}$

unsigned

A handwritten grid of circles on a white background. The grid consists of four rows and five columns. The first three rows contain green circles, while the fourth row contains black circles. The circles are roughly circular and vary slightly in size and position.

signed?

$$S_c \Sigma^{-k}$$

A handwriting practice sheet featuring the letter 'S' in red and green ink. The page is divided into four quadrants by a grid. The top-left quadrant contains five rows of the red 'S' in cursive script. The bottom-left quadrant contains three rows of the red 'S' in a bold, blocky font. The top-right quadrant contains three rows of the black 'S' in a cursive script. The bottom-right quadrant contains three rows of the black 'S' in a bold, blocky font. Each row of letters is accompanied by a row of small circles for tracing practice.

Signed Array Multiplier (Baugh-Wooley)

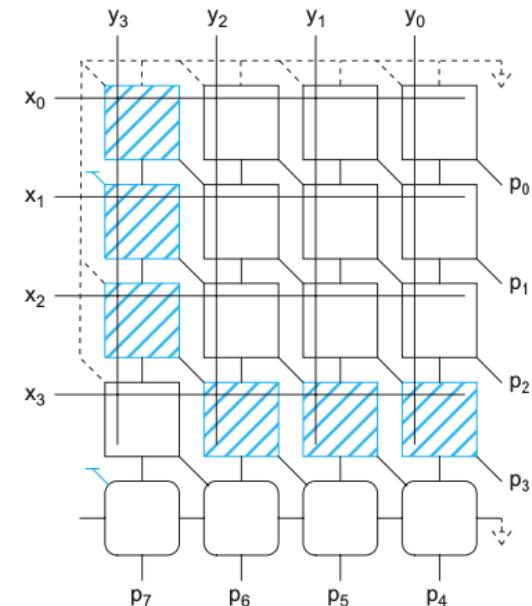
$$\sum_{i=0}^{m-2} \sum_{j=0}^{n-2} x_i y_j 2^{i+j}$$

$$+ y_{n-1} x_{m-1} 2^{m+n-2}$$

$$- 2^{m-1} \sum_{j=0}^{n-1} x_{m-1} y_i 2^i$$

$$- 2^{n-1} \sum_{i=0}^{m-2} y_{n-1} x_i 2^i$$

		$\frac{1}{x_1y_5}$	$\overline{x_0y_5}$	x_0y_4	x_0y_3	x_0y_2	x_0y_1	x_0y_0
			$\overline{x_2y_5}$	x_2y_4	x_2y_3	x_2y_2	x_2y_1	x_2y_0
			$\overline{x_3y_5}$	x_3y_4	x_3y_3	x_3y_2	x_3y_1	x_3y_0
			$\overline{x_4y_5}$	x_4y_4	x_4y_3	x_4y_2	x_4y_1	x_4y_0
1	x_5y_5	$\overline{x_5y_4}$	$\overline{x_5y_3}$	$\overline{x_5y_2}$	$\overline{x_5y_1}$	$\overline{x_5y_0}$		



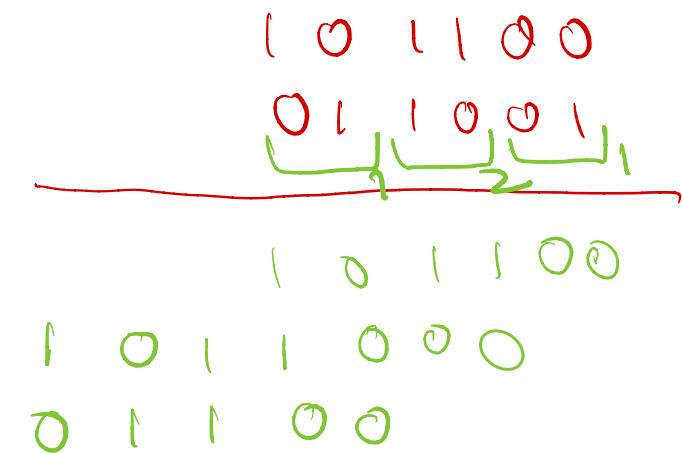
Source: Weste-Harris

FIGURE 11.78 Modified Baugh-Wooley two's complement multiplier

Reducing the Number of Partial Products

- Array multipliers require merging M partial products
 - We're adding too many partial products!
- Choose r for “radix- 2^r ” Booth Encoding
- → reduction to $\lfloor M/r \rfloor$ partial products

$$r=2$$



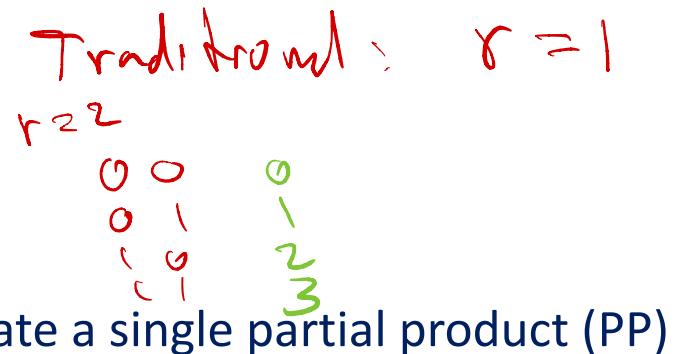
$$M \approx 5$$

$$3$$

$$S/2 = 2.5$$

Reducing the Number of Partial Products

- Array multipliers require merging M partial products
 - We're adding too many partial products!
- Choose r for “radix- 2^r ” Booth Encoding
- → reduction to M/r partial products
 - Groups of r multiplier bits looked at to generate a single partial product (PP)
 - Referred to as radix- 2^r encoding
 - Traditional radix-2 encoding: PP is selected as either 0 or Y (AND Array)
 - Radix-4 encoding: PP is selected as either 0, Y, 2Y or 3Y (Selection logic)



Similar ideas are relatively common in Digital VLSI. See for example:

Jeong, J., Collins, N., & Flynn, M. P. (2016). A 260 MHz IF Sampling Bit-Stream Processing Digital Beamformer With an Integrated Array of Continuous-Time Band-Pass $\Delta\Sigma$ Modulators. IEEE Journal of Solid-State Circuits, 51(5), 1168-1176.

Reducing the Number of Partial Products

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 - Traditional radix-2 encoding: PP is selected as either 0 or Y (AND Array)
 - Radix-4 encoding: PP is selected as either 0, Y, 2Y or 3Y (Selection logic)
 - Radix-4 and radix-8 encoding used more often
 - Increased complexity in generating partial products with $\uparrow r$
 - Use of even higher radix justified in rare cases
- $r \geq 2$ $r \geq 3$

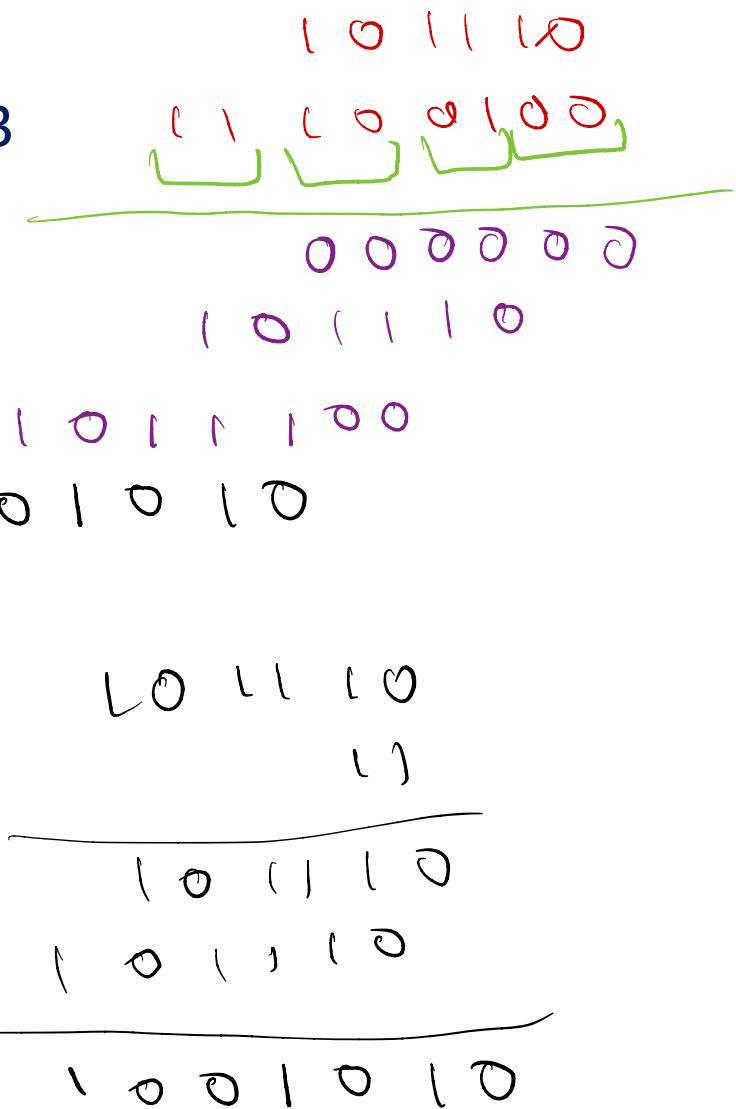
Booth Encoding (Radix-4)

- Idea: Scale partial product by 0, 1, 2 or 3
 - 2Y generated from a shift <<1
 - 3Y is a problem.

0	easy
1	easy
2	easy
3	hard
4	easy

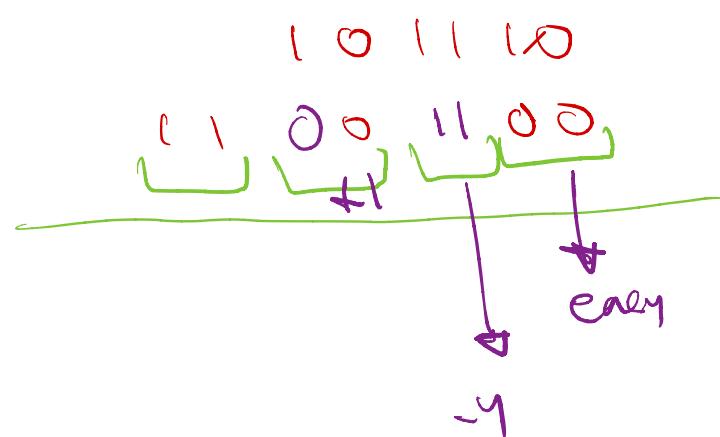
$$4 - 1 = 3 \Downarrow$$

$$3Y = 4Y - Y$$



Booth Encoding (Radix-4)

- Idea: Scale partial product by 0, 1, 2 or 3
 - 2Y generated from a shift <<1
 - 3Y is a problem.



0	easy
1	easy
2	easy
3	hard
4	easy

$$4 - 1 = 3 \Downarrow$$

$$3Y = 4Y - Y$$

Booth Encoding (Radix-4)

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0 1 0 0 0 1 1
+1 +1 -1

X 0 1 1 0 1 1 0
+1 +3 -1 +1 +2 +2
+2 -1 -1

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$$\begin{array}{r} 0 \ 1 \ 10 \ 10 \ 11 \\ \text{---} \\ \times \quad +1 \quad +2 \quad +3 \quad -1 \\ \times \quad +1 \quad +3 \quad -1 \quad -1 \\ \quad \quad +2 \quad -1 \quad -1 \quad -1 \end{array}$$

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$$2 = 4 - 2$$

$$\begin{array}{r} \times 0 \\ \times 1 \\ \times 2 \\ \times 3 \end{array} \xrightarrow{\quad} \begin{array}{r} \times 0 \\ \times 1 \\ \times 2 \\ \times (4-1) \end{array}$$

$$\begin{array}{r} \times 0 \\ \times 1 \\ \times 2 \\ \times 3 \end{array} \xrightarrow{\quad} \begin{array}{r} \times 0 \\ \times 1 \\ \times 2 \text{ or } \times (4-2) \\ \times (4-1) \end{array}$$

Booth Encoding (Radix-4)

$$2 = 4 - 2$$

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 - sometimes 2Y
 - sometimes 4Y-2Y

Booth Encoding (Radix-4)

$$2 = 4 - 2$$

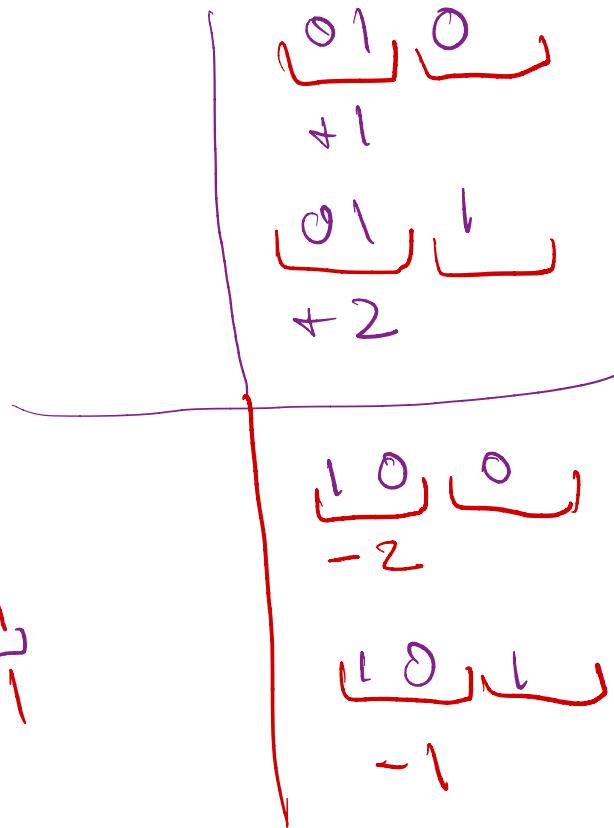
- Idea: Scale partial product by 0, 1, 2 or 3

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- 2Y implemented as
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- Why is this a good idea?

- We only need to look at the MSB bit of the previous 2-bit group! (No prefix 😊)



Booth-Encoding: rules and how it works

- Add extra group with leading 0s if needed
- Form partial products according to the following rules:
 - Add 4Y to next group based on MSB of current group

Inputs (numbers to multiply)			Outputs
Current Group		MSB from previous group	Partial product
x_{2i+1}	x_{2i}	x_{2i-1}	PP_i
0	0	0	0
0	1	0	Y
1	0	0	$-2Y$
1	1	0	$-Y$
0	0	1	Y
0	1	1	$2Y$
1	0	1	$-Y$
1	1	1	0

$$RP = \begin{cases} 0 \\ Y \\ -2Y \\ -Y \\ -Y \end{cases}$$

Tree Multipliers

- Array multipliers inherently process PPs serially
- This addition can also be performed in parallel

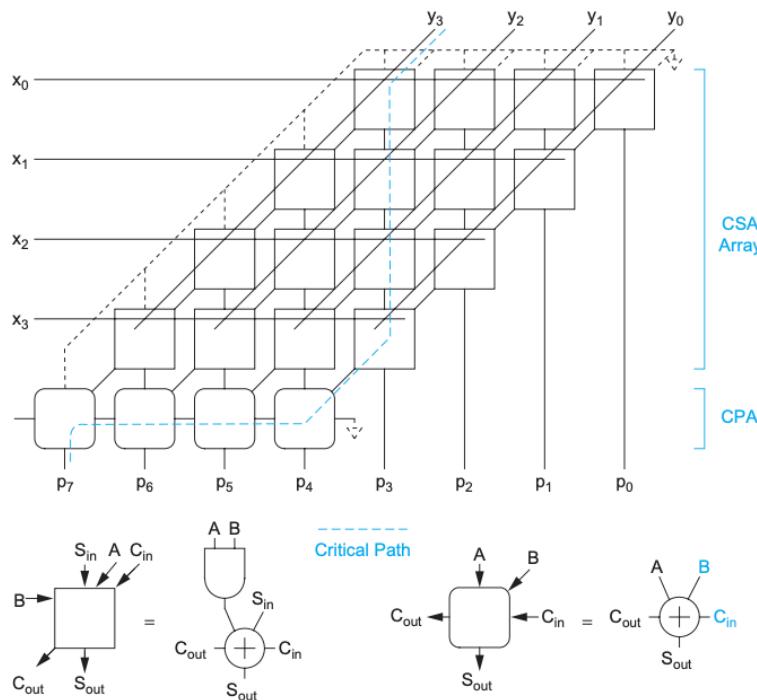


FIGURE 11.74 Array multiplier

Source: Weste-Harris

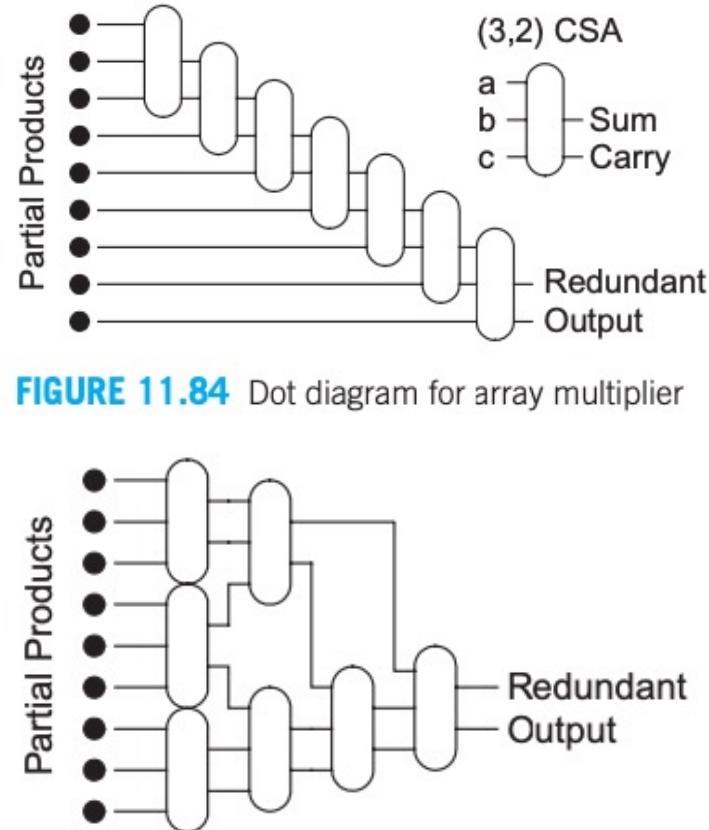


FIGURE 11.84 Dot diagram for array multiplier

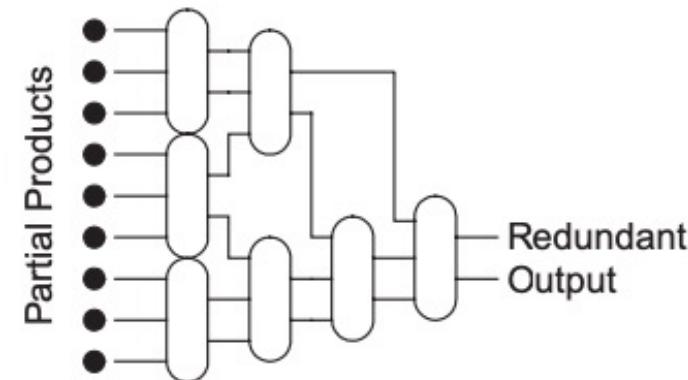


FIGURE 11.85 Dot diagram for Wallace tree multiplier

Tree Multipliers

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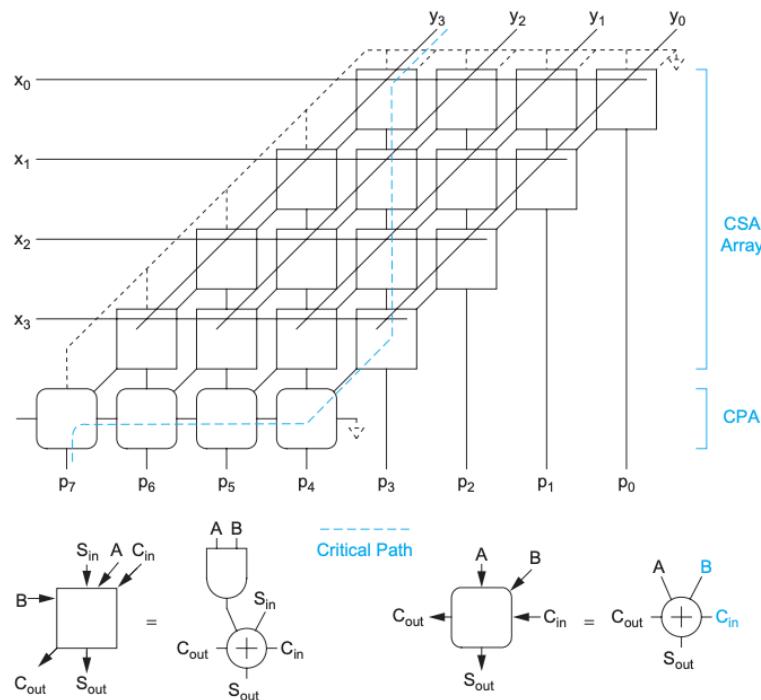


FIGURE 11.74 Array multiplier

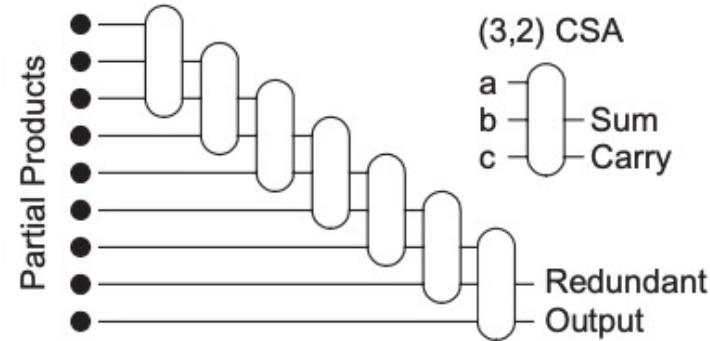


FIGURE 11.84 Dot diagram for array multiplier

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Questions/Assignments

- Signed booth multiplier?
- Non-equal width of the carry-save segment