EE 559/400 – Power Electronics Controls, Winter 2019

Homework 4

Due Date: Friday March 15th, 2019, 5pm

Instructions. You must scan your completed homework assignment into a pdf file, and upload your file to the Canvas Assignment page by 5 pm Pacific Time on Friday March 15th, 2019. All pages must be gathered into a single file of moderate size, with the pages in the correct order. Set your phone or scanner for basic black and white scanning. You should obtain a file size of hundreds of kB, rather than tens of MB. I recommend using the "Tiny Scanner" app. Please note that the grader will not be obligated to grade your assignment if the file is unreadable or very large.

Problem 1: See Figure 1 on the next page. Consider the signal chain from a current $i(t)$, through a hall-effect sensor, an op-amp circuit, an analog to digital converter (ADC), and a subsequent inversion calculation that is designed to recover the current as $i_{\mathrm{m}}[n] \approx i(t)$. Below are the system constraints, ratings, and notation to be mindful of:

- The converter is rated to a consume ac current swings between $-i_{\mathrm{pk}} \leq i \leq i_{\mathrm{pk}}$ when transfering full power.
- The current sensor output voltage, v_{s}, is

  $$v_{\mathrm{s}}(t) = G_{\mathrm{s}} i(t) + v_{\mathrm{dc}} \tag{1}$$

  where G_{s} is the sensor gain in units of V/A, and v_{dc} is the dc offset voltage at the sensor output.
- The op-amp has feedback and input resistors R_{f} and R_{i}, respectively. This circuit is fed by the sensor output voltage, v_{s}, and an adjustable dc-bias voltage v_{x} provided by an IC or a voltage divider. The op-amp output voltage, which is denoted as $v_{\mathrm{A/D}}$, is fed to the ADC pin.
- The ADC has an analog sensing range between 0 V and V_{FS}, and the digital output is an unsigned integer, x, with 2^n bits.

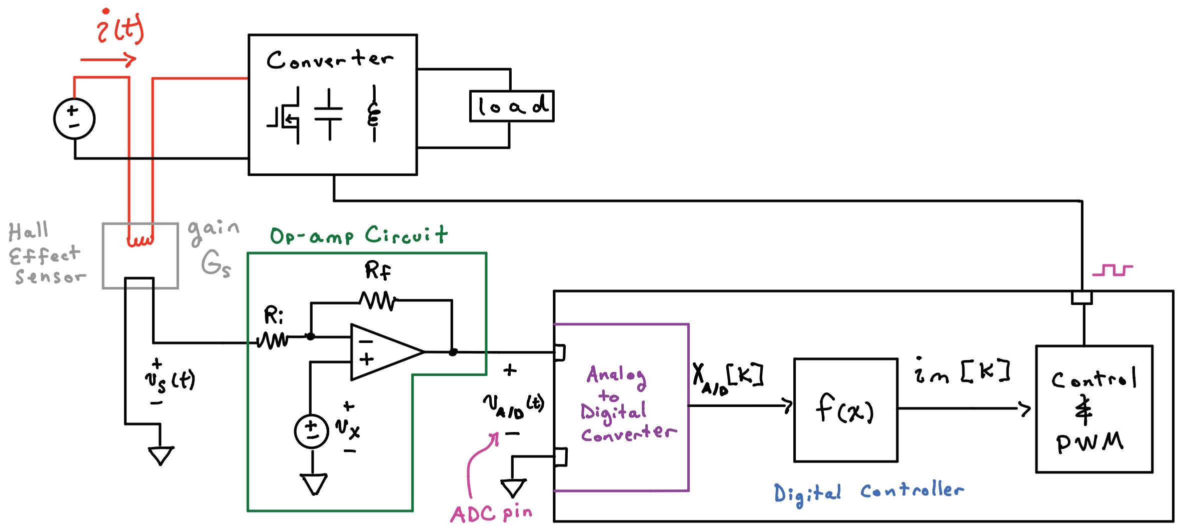


Figure 1: A physical current, $i(t)$, and its measurement signal-chain.

(a) Derive an analytic expression for the voltage that appears across the ADC pin, $v_{\mathrm{A/D}}$, in terms of $v_{\mathrm{x}}, R_{\mathrm{f}}/R_{\mathrm{i}}, G_{\mathrm{s}}, v_{\mathrm{dc}}$, and i. Hint: Use the provided handout on the op-amp "Golden Rules.".

(b) Consider the following design objectives:

- When $i = 0$, the ADC pin voltage should be at the midpoint $v_{\mathrm{A/D}} = V_{\mathrm{FS}}/2$.
- When the current swings to the positive and negative peaks (i.e., $|i| = i_{\mathrm{pk}}$), the ADC pin voltage equals

$$\begin{aligned} v_{\mathrm{A/D}} &= V_{\mathrm{FS}} - \epsilon, & \text{for } i = -i_{\mathrm{pk}} \\ v_{\mathrm{A/D}} &= \epsilon, & \text{for } i = i_{\mathrm{pk}} \end{aligned}$$

where $\epsilon \ll V_{\mathrm{FS}}/2$ is a small safety margin (typically a few percent of the ADC analog input voltage range) to prevent damage to the ADC.

Sketch the ADC pin voltage, $v_{\mathrm{A/D}}$, and full-rated converter current, $i = i_{\mathrm{pk}} \sin(2\pi t/T_{\mathrm{ac}})$, over one sinusoidal AC cycle of period T_{ac}. You can ignore the effect of ripple. Clearly label $\pm i_{\mathrm{pk}}$ points on the $i(t)$ plot. Clearly label V_{FS}, $V_{\mathrm{FS}}/2$, and ϵ on the $v_{\mathrm{A/D}}(t)$ plot.

(c) Design the parameters v_{x} and $R_{\mathrm{f}}/R_{\mathrm{i}}$ to achieve the objectives in (b). Express v_{x} and $R_{\mathrm{f}}/R_{\mathrm{i}}$ in terms of V_{FS} , ϵ, G_{s}, i_{pk} and v_{dc}.

(d) Substitute your result from (b) into (a), and show that the ADC pin voltage is

$$v_{\mathrm{A/D}}(t) = \frac{V_{\mathrm{FS}}}{2} \pm (?) i(t). \tag{2}$$

Fill in the blanks in red with the correct sign and specify the factor in terms of the sensor gain G_{s} and op-amp resistor ratio $R_{\mathrm{f}}/R_{\mathrm{i}}$.

(e) Derive an analytic expression for $x_{\mathrm{A/D}}[n]$ in terms of the number of bits $n_{\mathrm{A/D}}$ and the ADC analog voltage $v_{\mathrm{A/D}}$. To capture quantization from the ADC, use the "round$(\cdot)$" operator to denote the conversion to an integer value.

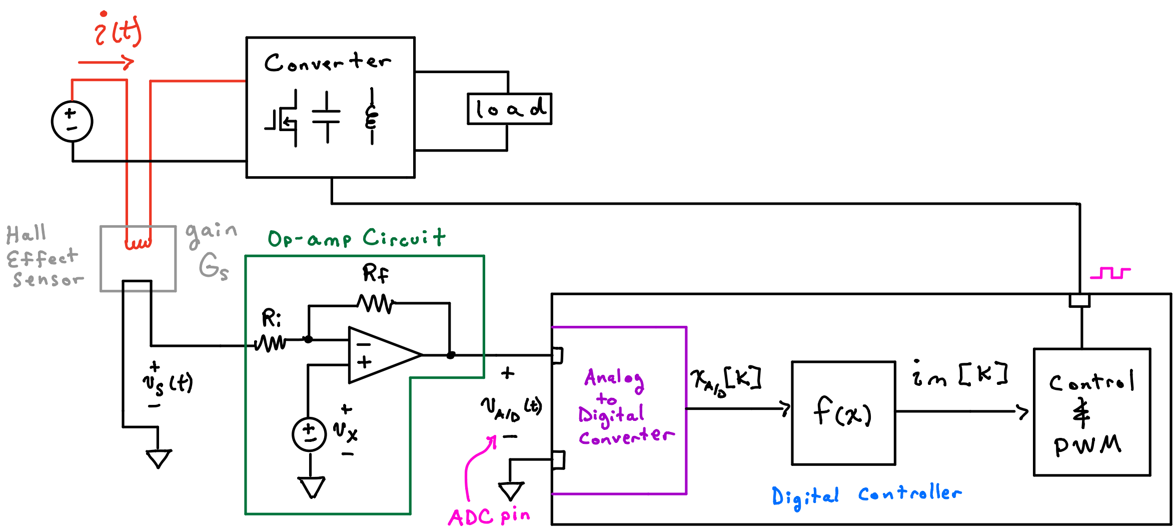
(f) Using your result in (d) and (e), derive the digitally implemented formula which recovers an estimate of the physical current, which we denote as $i_{\mathrm{m}}[n]$. Your result should be given in the form

$$i(t) \approx i_{\mathrm{m}}[n] = m x_{\mathrm{A/D}}[n] + b \tag{3}$$

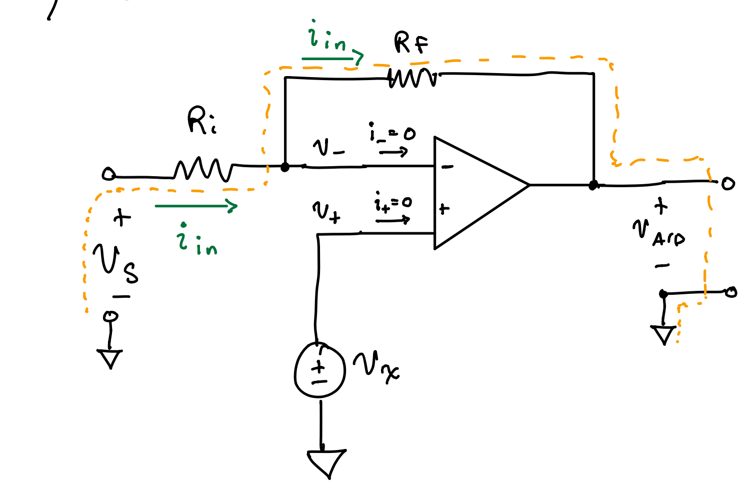
where both m and b are written in terms of $R_{\mathrm{f}}/R_{\mathrm{i}}$, V_{FS}, G_{s}, and the number of bits $n_{\mathrm{A/D}}$.

Sensing Path Design for AC Signals

Consider the signal chain:



a) Closer look @ op-amp



- Apply Golden Rule #1

Since $i_- = i_+ \approx 0$

then i_{in} flows through R_F & R_i.

Look @ KVL loop

$$0 = v_s - i_{in}(R_i + R_F) - v_{A/D} \qquad (1)$$

• Apply Golden Rule # 2

$$\underbrace{v_- = v_+}_{\text{Rule\#2}} = \underbrace{v_x}_{\oplus\text{terminal}} = \boxed{\begin{aligned} &\underbrace{v_s - i_{in} R_{in}}_{\text{from input side}} \quad (2) \\ = \; &\underbrace{v_{A/D} + i_{in} R_f}_{\text{from output side}} \quad (3) \end{aligned}}_{\ominus\text{ terminal}}$$

Solve for i_{in} using (2)

$$\Rightarrow \quad i_{in} = \frac{v_s - v_x}{R_{in}} \qquad (4)$$

Rearrange (3) to obtain V_{adc}

$$\Rightarrow \quad V_{A/D} = v_x - i_{in} R_f$$

* use (4)

$$= v_x - \left(\frac{v_s - v_x}{R_{in}}\right) R_f$$

$$= v_x - \frac{R_f}{R_i}\left(v_s - v_x\right) \qquad (5)$$

$$= v_{A/D}$$

- Use sensor equation. Recall

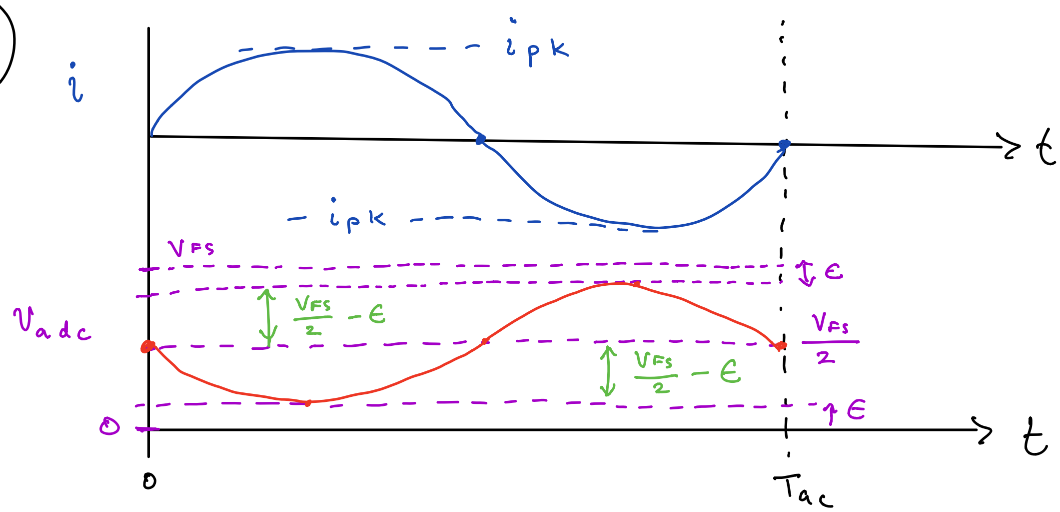
$$v_s = G_s\, i + V_{dc} \qquad (6)$$

(6) → (5) gives

$$\boxed{\begin{aligned} v_{A/D} &= v_x - \frac{R_f}{R_i}\left(G_s i + V_{dc} - v_x\right) \\ &= \underbrace{v_x\left(1 + \frac{R_f}{R_i}\right) - \frac{R_f}{R_i} V_{dc}}_{\text{dc offset terms}} - \underbrace{\frac{R_f}{R_i} G_s\, i}_{\text{sensed signal term}} \end{aligned}} \qquad (7)$$

(a)

b)



c) Look @ First objective in (b). Set $i = 0$ and $v_{A/D} = \frac{V_{FS}}{2}$, then apply eq (7) from part (a).

$$\underbrace{\frac{V_{FS}}{2}}_{V_{adc}} = v_x\left(1 + \frac{R_f}{R_i}\right) - \frac{R_f}{R_i} V_{dc} - \cancel{\frac{R_f}{R_i} G_s i} \rightarrow 0$$

$$\Rightarrow v_x = \frac{1}{1 + \frac{R_f}{R_i}}\left(\frac{\bar{V}_{adc}}{2} + \frac{R_f}{R_i} V_{dc}\right) \quad (8)$$

still need to solve for this...

Now look @ second objective in (b). See plot in (b) & notice that ac component of $v_{A/D}(t)$ has a peak amplitude $\frac{V_{FS}}{2} - \epsilon$ in both directions. Use this insight to make our lives easier. Set dc component in eq (7) to zero & $V_{A/D} = \frac{V_{FS}}{2} - \epsilon$ = ac component, then evaluate when $i = -i_{pk}$ to capture peak ⊕ swing.

$$\Rightarrow \frac{V_{FS}}{2} - \epsilon = -\frac{R_f}{R_i} G_s\left(-i_{pk}\right)$$

$$\Rightarrow \boxed{\frac{R_f}{R_i} = \frac{1}{G_s\, i_{pk}}\left(\frac{V_{FS}}{2} - \epsilon\right)}\ \boxed{(c)} \qquad (9)$$

(9) → (8) gives

$$\boxed{v_x = \frac{1}{1 + \frac{1}{G_s\, i_{pk}}\left(\frac{V_{FS}}{2} - \epsilon\right)}\left(\frac{V_{FS}}{2} + \frac{1}{G_s\, i_{pk}}\left(\frac{V_{FS}}{2} - \epsilon\right) v_{dc}\right)} \qquad (10)$$

$\boxed{(c)}$

(d) Put result from (c) into (a) as a sanity check.

Recall from (a), that

$$V_{A/D} = v_x\left(1 + \frac{R_f}{R_i}\right) - \frac{R_f}{R_i} v_{dc} - \frac{R_f}{R_i} G_s\, i$$

* use (8) for v_x to keep things simpler

$$= \underbrace{\frac{1}{\cancel{1 + \frac{R_f}{R_i}}}\left(\frac{V_{FS}}{2} + \frac{R_f}{R_i} v_{dc}\right)}_{v_x}\cancel{\left(1 + \frac{R_f}{R_i}\right)} - \frac{R_f}{R_i} v_{dc} - \frac{R_f}{R_i} G_s\, i$$

$$= \left(\frac{V_{FS}}{2} + \cancelto{0}{\frac{R_f}{R_i} v_{dc}} \right) - \cancelto{0}{\frac{R_f}{R_i} v_{dc}} - \frac{R_f}{R_i} G_s i$$

$$= \boxed{\frac{V_{FS}}{2} - \frac{R_f}{R_i} G_s \, i(t) = v_{A/D}(t)} \quad \text{d} \qquad (11)$$

... I will take a few extra steps to shed further insight.

* use (9) for $\frac{R_f}{R_i}$

$$= \frac{V_{FS}}{2} - \underbrace{\frac{1}{\cancel{G_s} i_{pk}} \left(\frac{V_{FS}}{2} - \epsilon \right)}_{\frac{R_f}{R_i}} \cancel{G_s} \, i$$

desired offset term. Good!

this factor normalizes & scales $i(t)$ as needed.

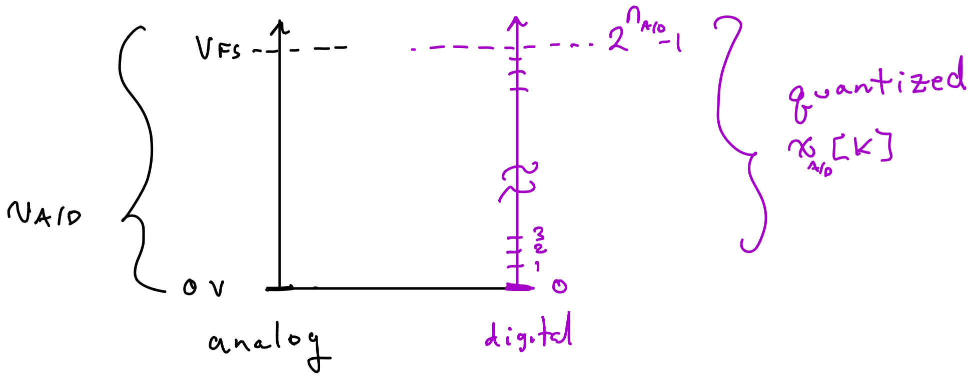
$$= \frac{V_{FS}}{2} - \boxed{\left(\frac{V_{FS}}{2} - \epsilon \right) \frac{i(t)}{i_{pk}}}$$

Success!

$$= v_{A/D}(t)$$

(e) Derive equation for digital value $x_{A/D}[n]$.

• Look @ analog & digital scales together



$x_{A/D}[n]$ is just a scaled version of $v_{A/D}$, of the form

$$\boxed{x_{A/D}[k] = \underbrace{\text{round}}_{\substack{\text{captures}\\ \text{quantization}\\ \text{of ADC.}}}\left(\underbrace{\frac{v_{A/D}}{V_{FS}}\cdot\left(2^{n_{A/D}}-1\right)}_{\substack{\text{normalization}\\ \text{\& scaling}}}\right)} \quad \text{(e)} \qquad (12)$$

(f) Recover i_m via a linear function.

• Ignoring quantization, (12) becomes

$$x_{A/D}[n] \approx \frac{\left(2^{n_{A/D}}-1\right)}{V_{FS}}\, v_{A/D}[n]$$

* substitute (11) and let $i_m[k] \approx i(t)$

$$= \frac{2^{n_{A/D}}-1}{V_{FS}}\left(\frac{V_{FS}}{2} - \frac{R_f}{R_i} G_s \, i_m[k]\right)$$

- Now solve for $i_m[k]$.

$$\Rightarrow \quad \frac{R_f}{R_i} G_s \, i_m[k] = \frac{V_{FS}}{2} - \frac{V_{FS}}{2^{n_{A/D}}-1} x_{A/D}[k]$$

$$\Rightarrow \quad \boxed{i_m[k] = -\frac{R_i V_{FS}}{R_f G_s \left(2^{n_{A/D}}-1\right)} x_{A/D}[n] + \frac{R_i}{R_f G_s}\,\frac{V_{FS}}{2}}$$

(f)