Homework #2

#1) Look @ switched ckt first

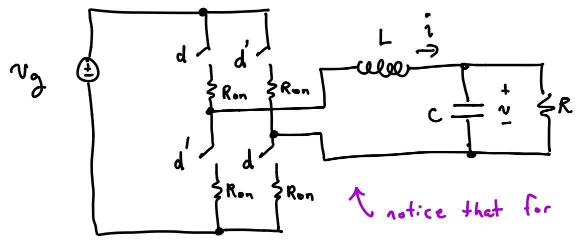


Fig 1

↑ notice that for config ① w/ d or ② w d', i will flow across 2 Ron elements. Hence, 2Ron can be modeled in series w/ L.

a) Average the sw. ports

Following p.9-10 of lecture #4, draw the H-bridge as two sets of sw. ports

$\Rightarrow$

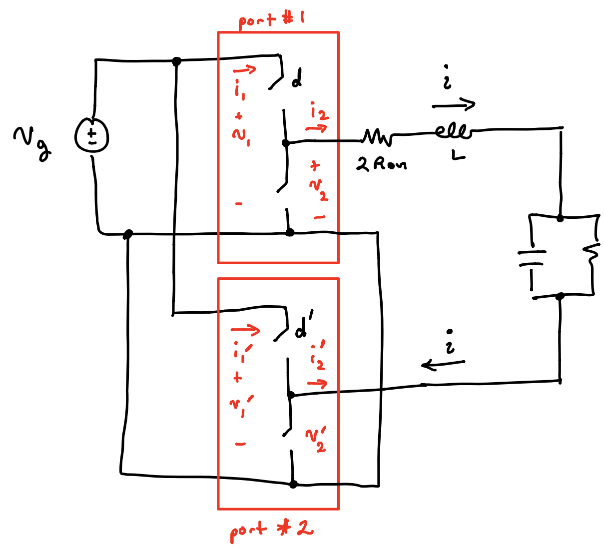


Fig 2

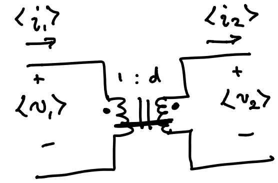
- Avg port #1

$$\langle v_2\rangle = d\langle v_1\rangle + d'\cancel{(0)}^{\,0} \qquad (i)$$

$$\langle i_1\rangle = d\langle i_2\rangle + d'\cancel{(0)}^{\,0} \qquad (ii)$$

put in xfmr form

$$\frac{\langle v_1\rangle}{1} = \frac{\langle v_2\rangle}{d} \Rightarrow$$



$$\underbrace{\langle i_1\rangle - d\langle i_2\rangle = 0}_{\text{of form}}$$

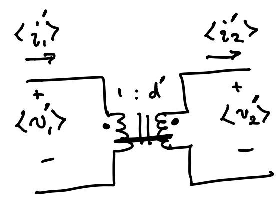
windings coil 1 × current into dot 1 + # windings coil 2 × current into dot 2 + . . . = 0

• Avg port # 2

$$\langle v_2'\rangle = d'\langle v_1'\rangle + d\,\cancelto{0}{(0)}$$
$$\langle i_1'\rangle = d'\langle i_2'\rangle + d\,\cancelto{0}{(0)}$$

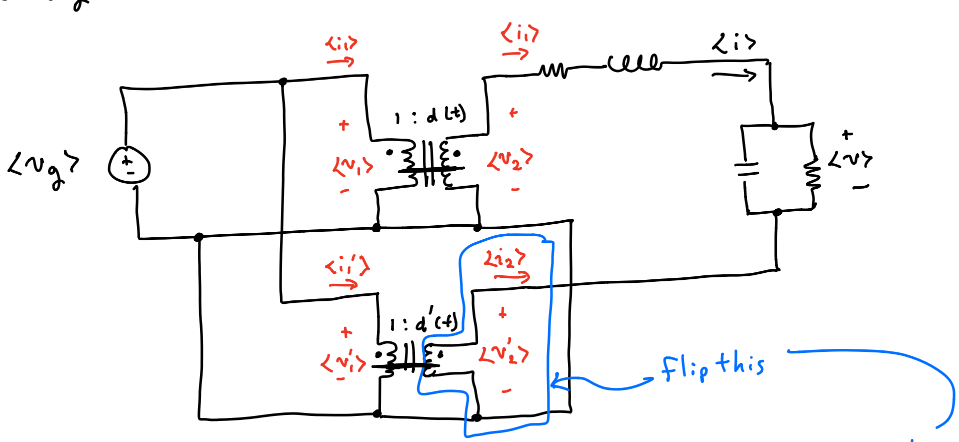
put in xfmr form

$$\frac{\langle v_1'\rangle}{1} = \frac{\langle v_2'\rangle}{d'} \Rightarrow$$

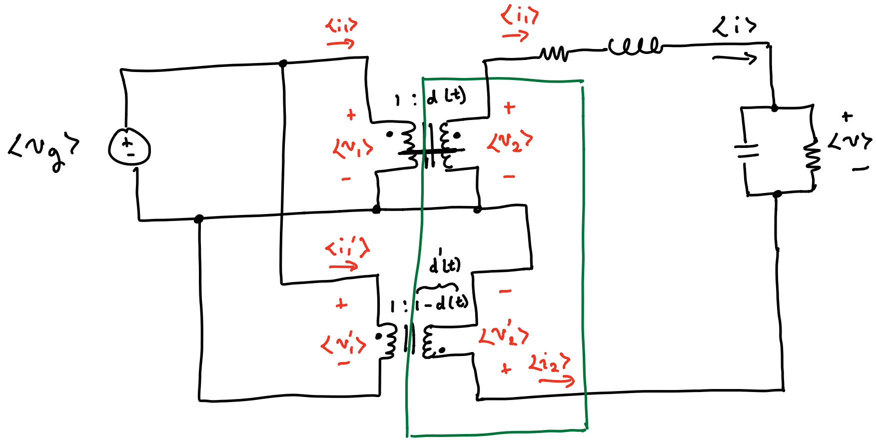


$$\langle i_1'\rangle - d'\langle i_2'\rangle = 0$$

• Draw avg version of Fig 2 :



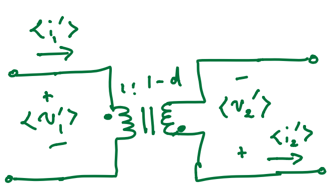
Let's manipulate this setup to simplify. First flip secondary coil on port #2. Note dot on that coil flips.



↑ easy to see secondary coils in series

Next step is to combine both xfmrs into one. 2 series coils w/ n_1 & n_2 turns are equivalent of one coil w/ $n_1 + n_2$ turns.

To make easier to see method of combining 2 xfmrs, focus on bottom xfmr for port #2

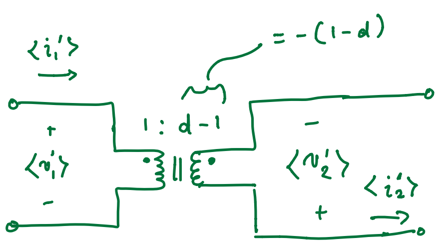


$\iff$

$$\frac{\langle v_1' \rangle}{1} = \frac{\langle v_2' \rangle}{1-d} \qquad (1)$$

$$\langle i_1' \rangle (1) + (-\langle i_2' \rangle)(1-d) = 0 \qquad (2)$$

This ckt is a pain b/c dot is on bottom of secondary. Can get equiv xfmr w/ dot on top by multiplying $(1-d)$ turns by (-1). We get

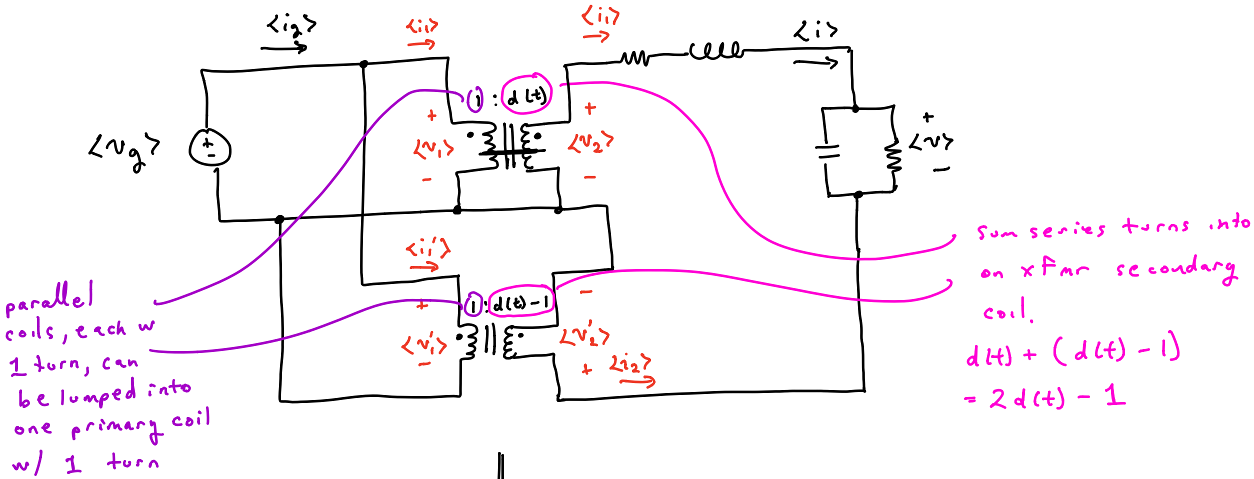


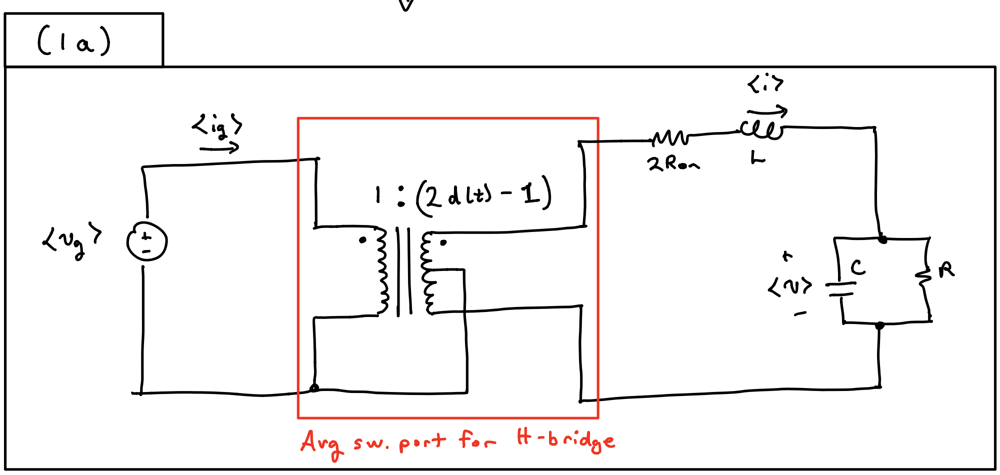
$\iff$

$$\frac{\langle v_1' \rangle}{1} = \overbrace{\frac{-\langle v_2' \rangle}{d-1}}^{= \frac{\langle v_2' \rangle}{1-d}} \qquad (3)$$

$$\langle i_1' \rangle (1) + \underbrace{\langle i_2' \rangle (d-1)}_{= (-\langle i_2' \rangle)(1-d)} = 0 \qquad (4)$$

Since (1)-(2) & (3)-(4) are equiv, we get





This is the same as in HW 1. But this solution is most complete since is shows that secondary midpoint is with respect to ⊖ rail of $\langle v_g \rangle$ supply.

b) Derive small-signal equiv ckt

Focus on Avg sw. port xfmr equiv in (1a)

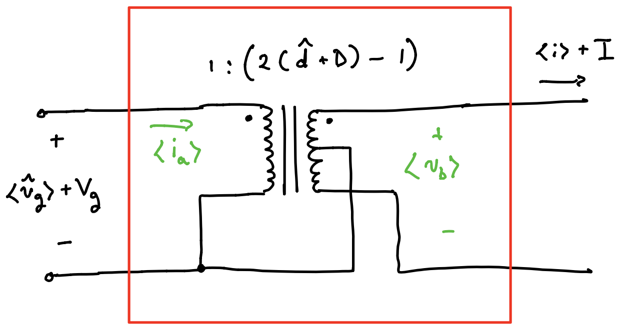


Fig 3

- Let $\langle i_a \rangle$ & $\langle v_b \rangle$ denote input/output of new equiv ckt from (1a).

- $\langle v_b \rangle$ depends on $\langle v_g \rangle$ input
- $\langle i_a \rangle$ depends on $\langle i \rangle$ state
- Compute $\langle v_b \rangle$

$$\langle v_b \rangle = \left(V_g + \langle \hat{v}_g \rangle\right)\left(2(\hat{d} + D) - 1\right)$$

$$= \left(V_g + \langle \hat{v}_g \rangle\right)\left((2D-1) + 2\hat{d}\right)$$

$$= \underbrace{V_g(2D-1)}_{\substack{\text{dc} \\ \text{comp.}}} + \underbrace{\langle \hat{v}_g \rangle (2D-1) + 2V_g\hat{d}}_{\substack{\text{1st order linear} \\ \text{terms}}} + \underbrace{\cancel{2\hat{d}\langle \hat{v}_g \rangle}}_{\text{2nd order}}^{\approx 0} \quad (5) \leftarrow \text{KVL for secondary}$$

- Compute $\langle i_a \rangle$

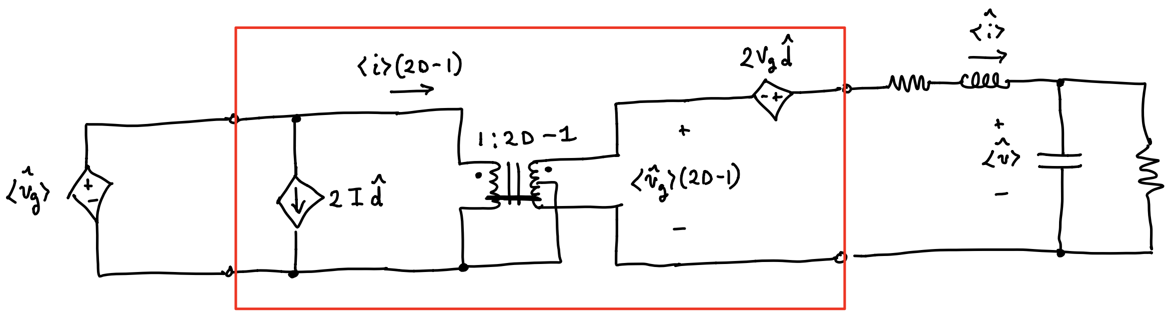
$$\langle i_a \rangle = \left(I + \langle \hat{i} \rangle\right)\left(2(\hat{d} + D) - 1\right)$$

$$= \left(I + \langle \hat{i} \rangle\right)\left((2D-1) + 2\hat{d}\right)$$

$$= \underbrace{I(2D-1)}_{\substack{\text{dc} \\ \text{comp.}}} + \underbrace{\langle \hat{i} \rangle (2D-1) + 2I\hat{d}}_{\substack{\text{1st order linear} \\ \text{terms}}} + \underbrace{\cancel{2\hat{d}\langle \hat{i} \rangle}}_{\text{2nd order}}^{\approx 0} \quad (6) \leftarrow \text{KCL for primary}$$

- Next, discard dc terms in (5) – (6) and also discard V_g & I in Fig 3. Use KVL / KCL relations implied in (5) – (6) to get

(1 b)



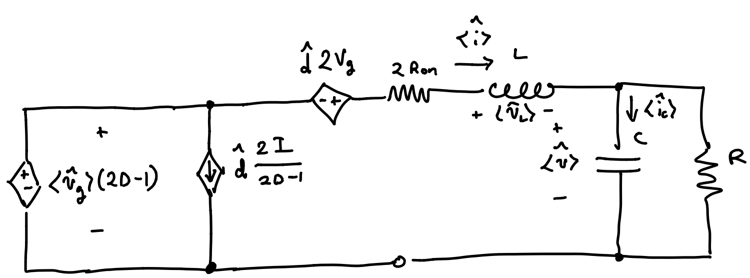
Linearized sw. port

Fig 4

1c) Derive $\frac{d}{dt}\langle \hat{i} \rangle$ & $\frac{d}{dt}\langle \hat{v} \rangle$.

Push all ckt elements to right hand side of xfmr.

$\Downarrow$



$$\frac{d}{dt}\langle \hat{i} \rangle = \frac{1}{L}\langle \hat{v}_L \rangle$$

$$= \frac{1}{L}\left(\langle \hat{v}_g \rangle (2D-1) + \hat{d}\,2V_g - 2R_{on}\langle \hat{i} \rangle - \langle \hat{v} \rangle\right)$$

&

$$\frac{d}{dt}\langle \hat{v} \rangle = \frac{1}{C}\langle \hat{i}_c \rangle$$

$$= \frac{1}{C}\left(\langle \hat{i} \rangle - \langle \hat{v} \rangle / R\right)$$

$$\Downarrow \text{ put in matrix form}$$

(1 c)

$$\frac{d}{dt}\begin{bmatrix} \langle \hat{i} \rangle \\ \langle \hat{v} \rangle \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{2R_{on}}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}}_{A} \begin{bmatrix} \langle \hat{i} \rangle \\ \langle \hat{v} \rangle \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{2V_g}{L} & \frac{2D-1}{L} \\ 0 & 0 \end{bmatrix}}_{B} \begin{bmatrix} \hat{d} \\ \langle \hat{v}_g \rangle \end{bmatrix}$$

matches solution in HW 1 !

1 d) Given $C = I_{2\times 2}$ & $E = 0_{2\times 2}$, compute freq domain model.

- From Lecture #7, we know:

$$\hat{y}(s) = \left(C(sI - A)^{-1}B + E \right) \hat{u}(s)$$

$$* \; C = I, \quad E = 0_{2\times 2}$$

$$= (sI - A)^{-1} B \,\hat{u}(s)$$

$$= \left(s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -\frac{2R_{on}}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \right)^{-1} B \,\hat{u}(s)$$

$$= \begin{bmatrix} s + \frac{2R_{on}}{L} & \frac{1}{L} \\ -\frac{1}{C} & s + \frac{1}{RC} \end{bmatrix}^{-1} B \,\hat{u}(s)$$

$$* \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \overbrace{\frac{1}{\left(s + \frac{2R_{on}}{L}\right)\left(s + \frac{1}{RC}\right) - \left(\frac{1}{L}\right)\left(-\frac{1}{C}\right)} \underbrace{\begin{bmatrix} s + \frac{1}{RC} & -\frac{1}{L} \\ \frac{1}{C} & s + \frac{2R_{on}}{L} \end{bmatrix} \overbrace{\begin{bmatrix} \frac{2V_g}{L} & \frac{2D-1}{L} \\ 0 & 0 \end{bmatrix}}^{B}}_{*}}^{(sI-A)^{-1}} \hat{u}(s)$$

$$= \frac{1}{s^2 + s\left(\frac{1}{RC} + \frac{2R_{on}}{L}\right) + \frac{2R_{on}}{RLC} + \frac{1}{LC}} \cdot \frac{RLC}{RLC} \quad (*)$$

$$= \frac{RLC}{s^2 RLC + s\left(L + 2R_{on}RC\right) + 2R_{on} + R} \left[\begin{array}{c|c} \left(s + \frac{1}{RC}\right)\left(\frac{2V_g}{L}\right) + \cancelto{0}{\left(-\frac{1}{L}\right)(0)} & \left(s + \frac{1}{RC}\right)\left(\frac{2D-1}{L}\right) + \cancelto{0}{\left(-\frac{1}{L}\right)(0)} \\ \hline \left(\frac{1}{C}\right)\left(\frac{2V_g}{L}\right) + \cancelto{0}{\left(s + \frac{2R_{on}}{L}\right)(0)} & \left(\frac{1}{C}\right)\left(\frac{2D-1}{L}\right) + \cancelto{0}{\left(s + \frac{2R_{on}}{L}\right)(0)} \end{array}\right] \hat{u}(s)$$

$$= \frac{1}{s^2 RLC + s\left(L + 2R_{on}RC\right) + 2R_{on} + R} \left[\begin{array}{c|c} R\cancel{L}C\left(s + \frac{1}{RC}\right)\left(\frac{2V_g}{\cancel{L}}\right) & R\cancel{L}C\left(s + \frac{1}{RC}\right)\left(\frac{2D-1}{\cancel{L}}\right) \\ R\cancel{L}\cancel{C}\left(\frac{1}{\cancel{C}}\right)\left(\frac{2V_g}{\cancel{L}}\right) & R\cancel{L}\cancel{C}\left(\frac{1}{\cancel{C}}\right)\left(\frac{2D-1}{\cancel{L}}\right) \end{array}\right] \hat{u}(s)$$

$$= \underbrace{\frac{1}{s^2 RLC + s\left(L + 2R_{on}RC\right) + 2R_{on} + R} \left[\begin{array}{c|c} 2V_g\left(sRC + 1\right) & (2D-1)\left(sRC+1\right) \\ \hline 2V_g R & (2D-1)R \end{array}\right]}_{G(s) \in \mathbb{C}^{2\times 2}} \begin{bmatrix} \hat{d}(s) \\ \langle \hat{v}_g \rangle(s) \end{bmatrix}$$

$$= \begin{bmatrix} \langle \hat{i} \rangle(s) \\ \langle \hat{v} \rangle(s) \end{bmatrix}$$

where
$$G(s) = \begin{bmatrix} G_{id}(s) & G_{ig}(s) \\ G_{vd}(s) & G_{vg}(s) \end{bmatrix}$$

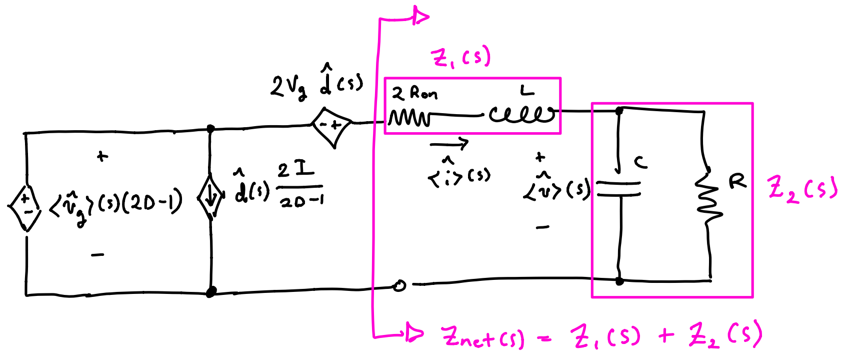
and

(1d)

$$\boxed{\begin{aligned} G_{id}(s) &= \frac{2V_g\left(sRC + 1\right)}{s^2 RLC + s\left(L + 2R_{on}RC\right) + 2R_{on} + R} \\ G_{ig}(s) &= \frac{(2D-1)\left(sRC+1\right)}{s^2 RLC + s\left(L + 2R_{on}RC\right) + 2R_{on} + R} \\ G_{vd}(s) &= \frac{2V_g R}{s^2 RLC + s\left(L + 2R_{on}RC\right) + 2R_{on} + R} \\ G_{vg}(s) &= \frac{(2D-1)R}{s^2 RLC + s\left(L + 2R_{on}RC\right) + 2R_{on} + R} \end{aligned}}$$

1e) Re-derive result in (1c) using ckt analysis in s-domain.

Look @ ckt:



Follow method in Lecture #6:

- First calculate a few things we need later:

□ $Z_{net}(s) = Z_1(s) + Z_2(s)$

$$= (2R_{on} + sL) + \left(\frac{1}{sC} \parallel R\right)$$

$$= (2R_{on} + sL) + \frac{\frac{1}{sC}R}{R + \frac{1}{sC}} \cdot \frac{sC}{sC} = 2R_{on} + sL + \frac{R}{sRC+1}$$

$$= \frac{(2R_{on} + sL)(sRC+1) + R}{sRC+1}$$

$$= \frac{s^2RLC + s(2R_{on}RC + L) + 2R_{on} + R}{sRC+1}$$

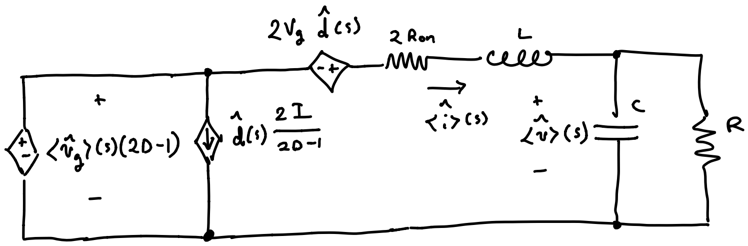
□ Voltage Divider

$$\frac{R \parallel \frac{1}{sC}}{R \parallel \frac{1}{sC} + 2R_{on} + sL} = \frac{\frac{R\frac{1}{sC}}{R+\frac{1}{sC}} \cdot \frac{sC}{sC}}{\frac{R\frac{1}{sC}}{R+\frac{1}{sC}} \cdot \frac{sC}{sC} + 2R_{on} + sL}$$

$$= \frac{\frac{R}{\cancel{sRC+1}}}{\frac{s^2RLC + s(2R_{on}RC + L) + 2R_{on} + R}{sRC+1}}$$

$$= \frac{R}{s^2 RLC + s(2R_{on}RC + L) + 2R_{on} + R}$$

- Use Superposition



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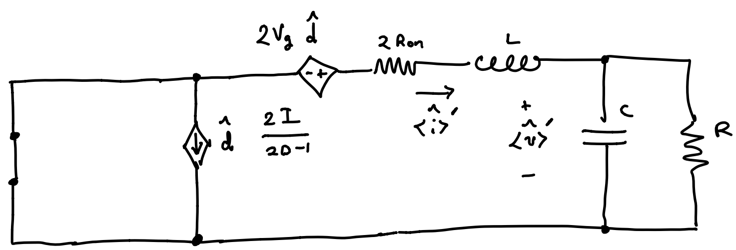
ckt (a) : set $\langle\hat{v}_g\rangle = 0$



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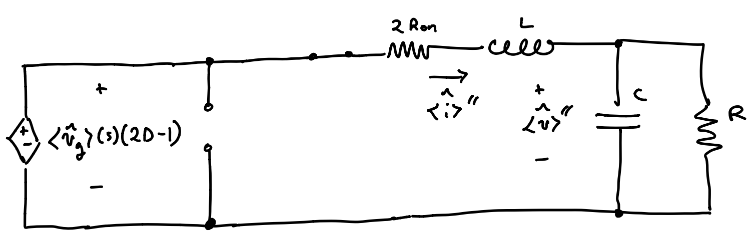
ckt (b) : set $\hat{d} = 0$



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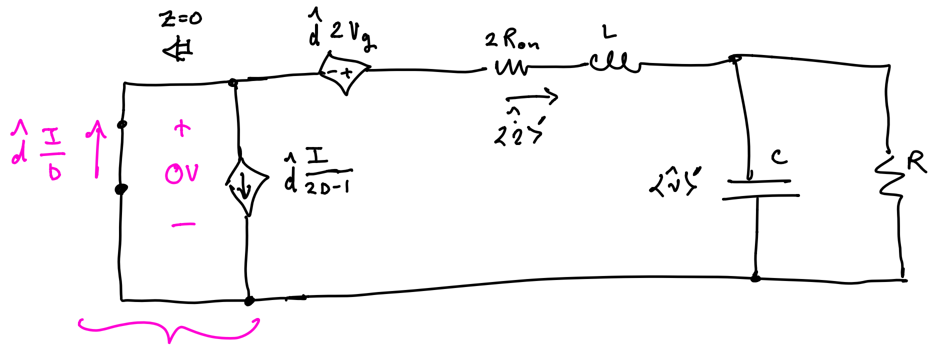
Solution

$$\langle\hat{i}\rangle(s) = \langle\hat{i}\rangle'(s) + \langle\hat{i}\rangle''(s)$$

$$\langle\hat{v}\rangle(s) = \langle\hat{v}\rangle'(s) + \langle\hat{v}\rangle''(s)$$

• Solve ckt ⓐ for $\langle \hat{i} \rangle'(s)$, $\langle \hat{v} \rangle'(s)$



doesn't effect
rest of ckt

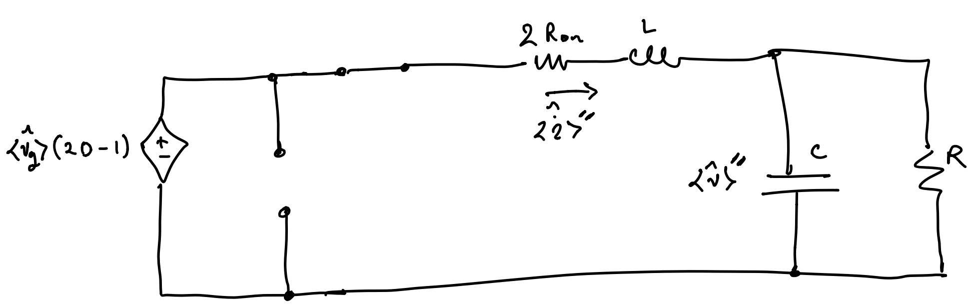
$$\langle \hat{i} \rangle'(s) = \hat{d}(s)\, 2V_g \frac{1}{Z_{net}}$$

$$= \hat{d}(s)\, 2V_g \frac{sRC + 1}{s^2 RLC + s(2R_{on}RC + L) + 2R_{on} + R}$$

Get $\langle \hat{v} \rangle'$

$$\langle \hat{v} \rangle'(s) = \hat{d}(s)\, 2V_g \underbrace{\frac{R}{s^2 RLC + s(2R_{on}RC + L) + 2R_{on} + R}}_{\text{voltage div.}}$$

• Solve ckt ⓑ for $\langle i \rangle''(s)$, $\langle v \rangle''(s)$



Get $\langle \hat{i} \rangle''$

$$\langle \hat{i} \rangle''(s) = \langle \hat{v}_g \rangle(s)\,(2D-1)\,\frac{1}{Z_{net}(s)}$$

$$= \langle \hat{v}_g \rangle(s)\,(2D-1)\,\frac{sRC+1}{s^2RLC + s(2R_{on}RC + L) + 2R_{on} + R}$$

Get $\langle \hat{v} \rangle''$

$$\langle \hat{v} \rangle''(s) = \langle \hat{v}_g \rangle(s)\,(2D-1)\,\underbrace{\frac{R}{s^2RLC + s(2R_{on}RC + L) + 2R_{on} + R}}_{\text{voltage div.}}$$

- Put together to get soln.

(1e)

$$\boxed{\begin{aligned}
\langle \hat{i} \rangle(s) &= \langle \hat{i} \rangle'(s) + \langle \hat{i} \rangle''(s) \\
&= \hat{d}(s)\,\underbrace{2V_g\,\frac{sRC+1}{s^2RLC + s(2R_{on}RC + L) + 2R_{on} + R}}_{G_{id}(s)} \\
&\quad + \langle \hat{v}_g \rangle(s)\,\underbrace{(2D-1)\,\frac{sRC+1}{s^2RLC + s(2R_{on}RC + L) + 2R_{on} + R}}_{G_{ig}(s)} \\
\langle \hat{v} \rangle(s) &= \langle \hat{v} \rangle'(s) + \langle \hat{v} \rangle''(s) \\
&= \hat{d}(s)\,\underbrace{2V_g\,\frac{R}{s^2RLC + s(2R_{on}RC + L) + 2R_{on} + R}}_{G_{vd}(s)} \\
&\quad + \langle \hat{v}_g \rangle(s)\,\underbrace{(2D-1)\,\frac{R}{s^2RLC + s(2R_{on}RC + L) + 2R_{on} + R}}_{G_{vg}(s)}
\end{aligned}}$$

same result as part (1d)!