Lecture 6 (Jan 20, 2022)

Reminder: HW1 due tomorrow 4:30pm

HW2 has been posted on CANVAS. (due next thursday).

Linear Averaged Model

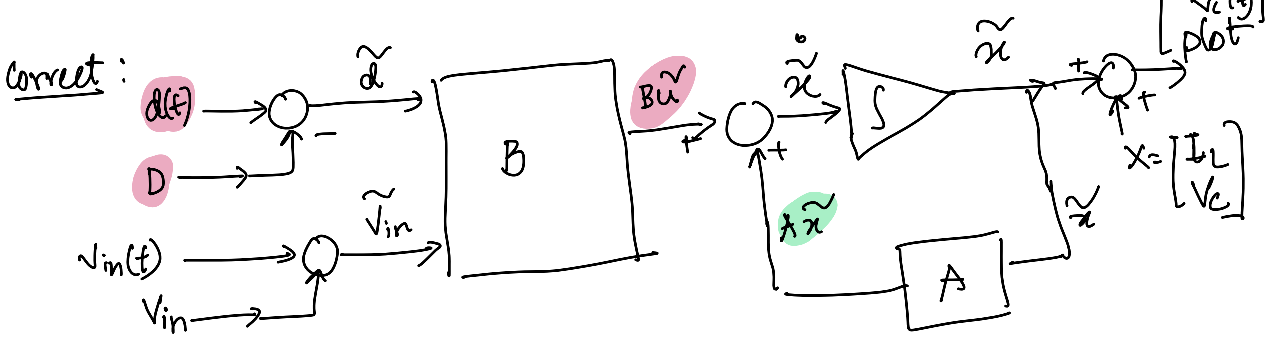
$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$$

$\rightarrow$ $\underbrace{x(t)}_{\text{actual variable}} = X + \tilde{x}$ (or $\bar{x} + \tilde{x}$)

$\tilde{x}$: small signal ac/perturbation.

X (or $\bar{x}$): large signal / dc / steady state value

Correct:

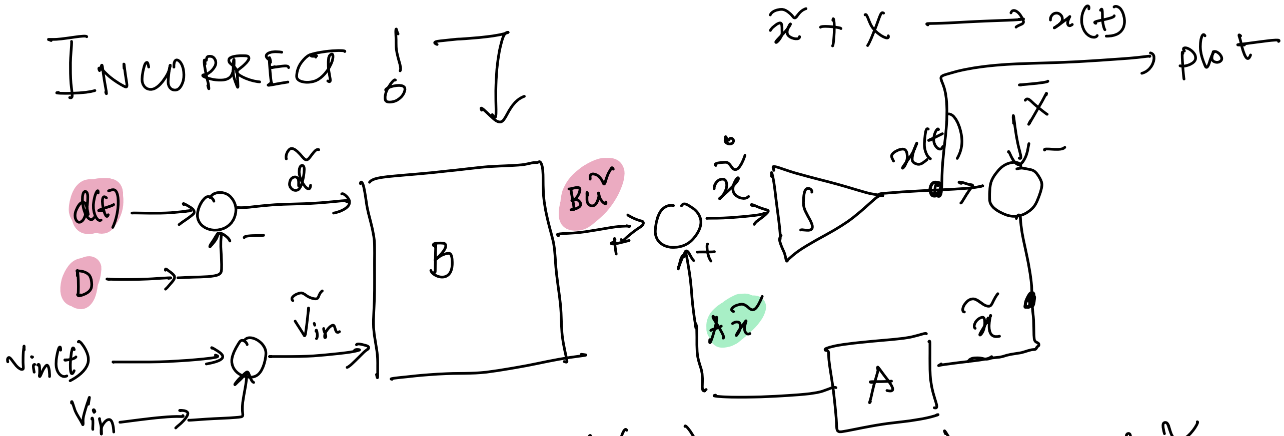


$$\int \dot{\tilde{x}} = \tilde{x}$$

$\tilde{x} \xrightarrow{?} x(t)$

$\tilde{x} + X \longrightarrow x(t)$

INCORRECT!



$$\frac{d}{dt}(x(t)) = \frac{d}{dt}(X + \tilde{x}) = 0 + \frac{d}{dt}\tilde{x}$$

Recapitulation

Buck Boost converter

↓

Identify the modes 1, 2

↓

$$u = \begin{bmatrix} d \\ V_{in} \end{bmatrix} \longrightarrow \boxed{\dot{x} = f(x, u)} \longrightarrow x = \begin{bmatrix} i_L \\ v_c \end{bmatrix}$$

Non linear

↓ Linearized the model.

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$$
$$y = C\tilde{x} + D\tilde{u}$$

↓ Laplace transform

$$\tilde{x}(s) = (sI-A)^{-1} B \tilde{u}(s)$$
$$y(s) = \left[C(sI-A)^{-1}B + D\right] \tilde{u}(s)$$

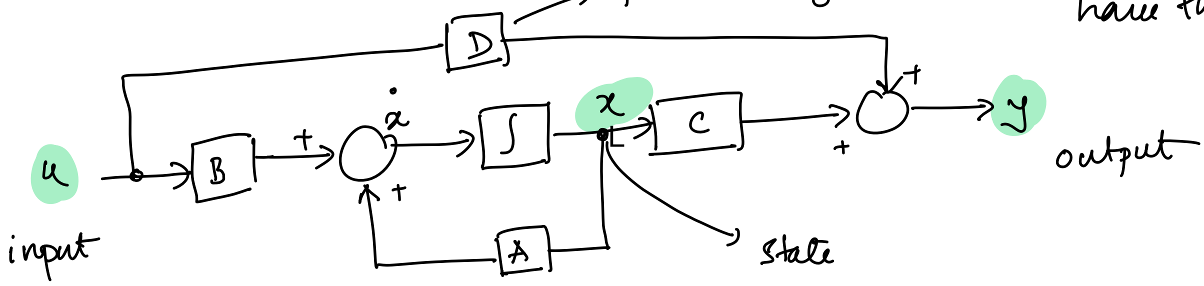
$$\dot{x} = Ax + Bu \longrightarrow \text{input}$$

state $\swarrow$ ($\dot{x}$)

$$y = Cx + Du$$

$y \longrightarrow$ output

$\rightarrow$ feedthrough ~ s/s rarely have this



$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

$$sx(s) = Ax(s) + Bu(s) \longrightarrow sIx(s) - Ax(s) = Bu(s)$$

$$\Rightarrow x(s) = (sI - A)^{-1} Bu(s)$$

$I \rightarrow$ identity matrix

$$y(s) = Cx(s) + Du(s)$$

$$\therefore y(s) = C(sI-A)^{-1} B u(s) + D u(s)$$

$$\boxed{\therefore y(s) = \left[C(sI-A)^{-1} B + D\right] u(s)}$$

input output relationship

$$\boxed{\begin{gathered} y = Cx \\ \underbrace{\begin{bmatrix} \tilde{i}_L \\ \tilde{v}_c \end{bmatrix}}_{2\times 1} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} \tilde{i}_L \\ \tilde{v}_c \end{bmatrix}}_{2\times 1} \end{gathered}} \quad \checkmark \qquad D = 0$$

$$\underbrace{(\tilde{i}_L + \tilde{v}_c)}_{1} = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_{1\times 2} \underbrace{\begin{bmatrix} \tilde{i}_L \\ \tilde{v}_c \end{bmatrix}}_{2\times 1}$$

$$y(s) = \left[C(sI-A)^{-1}B + D\right] u(s)$$

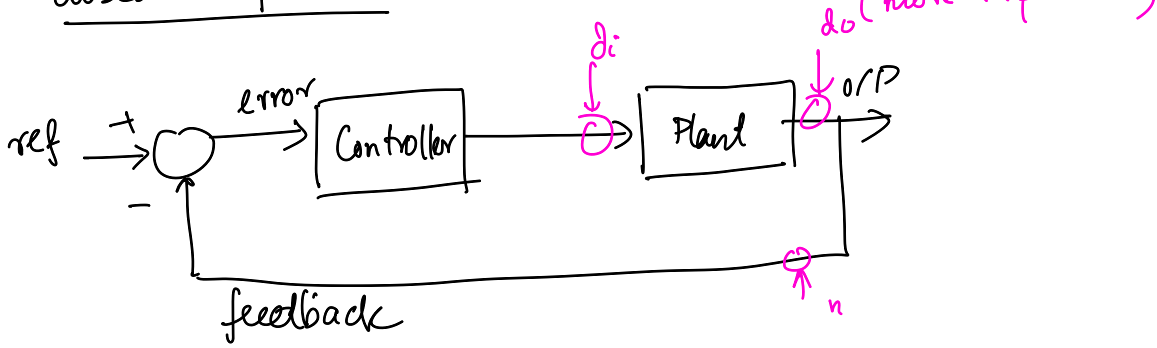
if $C = I$, $D = 0$

$$y(s) = (sI-A)^{-1}B\, u(s) = \tilde{x}(s)$$

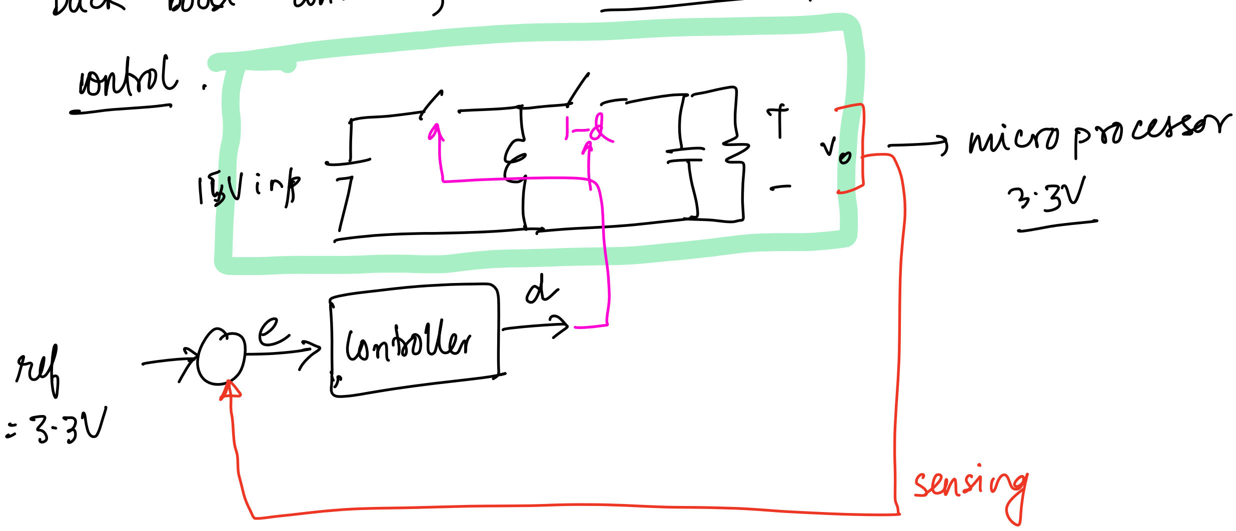
$$\frac{\tilde{i}_L}{\tilde{d}}, \quad \frac{\tilde{v}_c}{\tilde{d}}, \quad \frac{\tilde{i}_L}{\tilde{v}_{in}}, \quad \frac{\tilde{v}_c}{\tilde{v}_{in}}$$

Q: why do we need $\dfrac{\tilde{i}_L}{\tilde{d}}$?

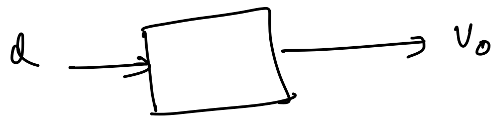
closed loop s/s.



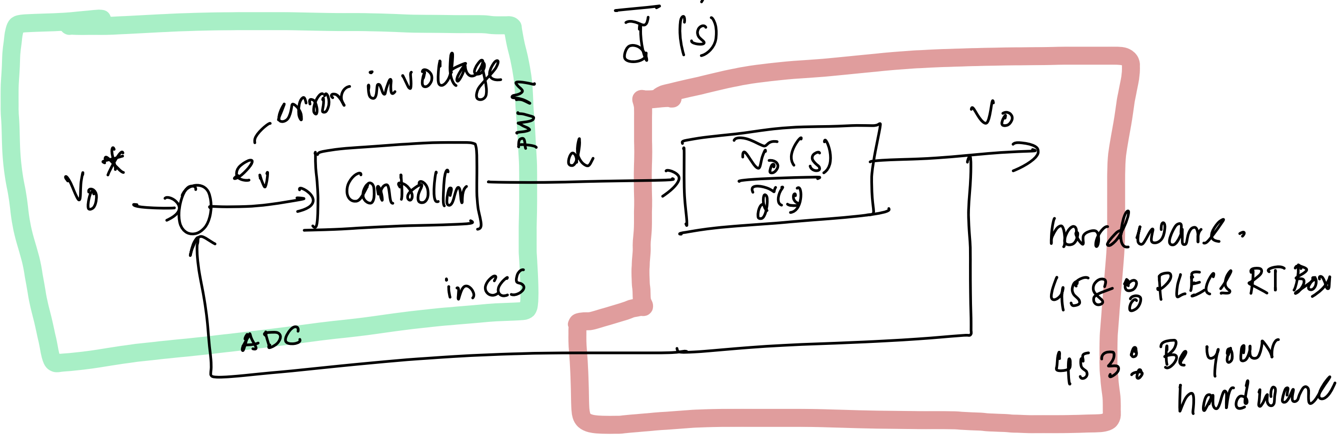
Buck-boost converter, someone asks you to do a voltage control.



The green box "effectively" becomes a black box

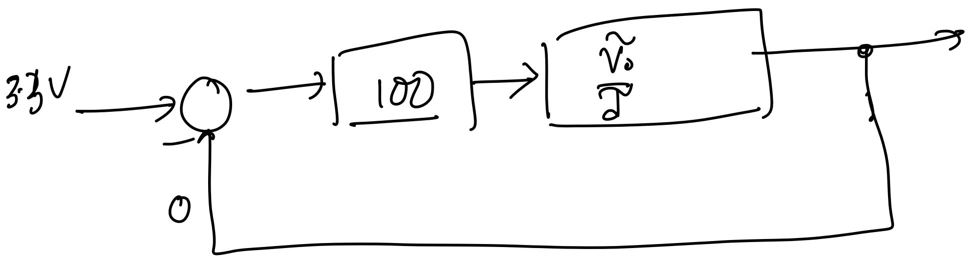


$\frac{\tilde{v}_o}{\tilde{d}}(s)$ = transfer function.



Question: How do we design a controller?

Ans : Maybe have a very high gain = 100



$$u(s) \longrightarrow \boxed{G(s) = \frac{\tilde{i}_L(s)}{\tilde{d}(s)}} \longrightarrow y(s)$$

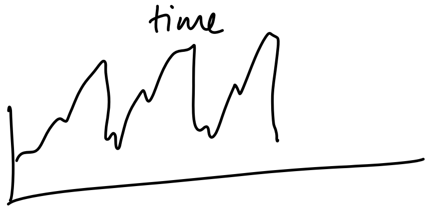
$$\boxed{s = \sigma + j\omega}$$

real

imag

$$g(t) \longrightarrow \boxed{\mathcal{F}} \longrightarrow G(\omega)$$

time



$\longrightarrow$ information content in freq domain

$$\mathcal{F}(g(t)) = G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt .$$

$$\mathcal{F}^{-1}(G(\omega)) = g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)\, e^{j\omega t}\, d\omega .$$

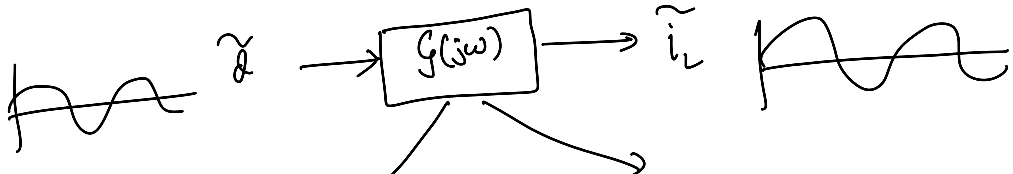
Why do we need laplace tranform.

$g(t) = e^{at}$; $a > 0$

$$\int_{-\infty}^{\infty} e^{at}\, e^{-j\omega t}\, dt$$

$$\int_{-\infty}^{\infty} e^{-\sigma t}\, e^{at}\, e^{-j\omega t}$$

$\underline{s = \sigma + j\omega}$



magnitude   phase

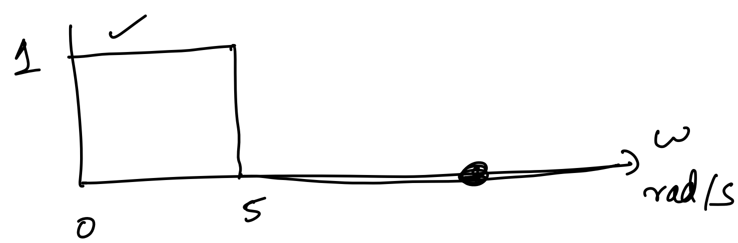
$|G(j\omega)|$ $\measuredangle G(j\omega)$

at $\omega = 1$ rad/s, $\quad G(j\cdot 1) = 1 \quad \measuredangle G(j1) = 90°$

$\tilde{d} = \sin(1t)$

$i_L(\ ) = |G(j\omega)| \cdot |\tilde{d}(j\omega)| \ \measuredangle G(j\omega) + \tilde{d}(j\omega)$

$G(j\omega) =$



$\tilde{d} = 2\sin(\omega t) \qquad \omega = 2$ rad/s.

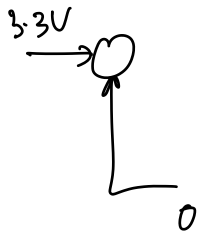
$|\tilde{i}_L| = 2 \times 1 = 2 \cdot \sin(\omega t)$

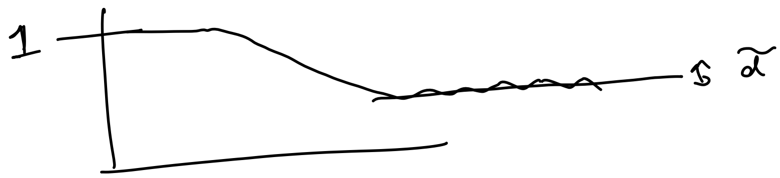
$\tilde{d} = 2\sin(10 \cdot t)$

$|\tilde{i}_L| = 0$

$\tilde{i}_L(j\omega) = G(j\omega) \cdot d(j\omega)$

$= |G(j\omega)|\,|d(j\omega)| \Big[\underbrace{\measuredangle G(j\omega) + \measuredangle d(j\omega)}_{\text{net angle.}} \Big]$





$\underline{g(j\omega)}$

sin	all
tan	cos

$|a+jb| = \sqrt{a^2+b^2}$

$\angle(a+jb) = \tan^{-1}\frac{b}{a}$.

atan$\left(\frac{b}{a}\right)$

atan2(b,a)

Bode Plots

$g(j\omega) \longrightarrow \underbrace{|G(j\omega)|}_{\text{magnitude}}$; $\angle G(j\omega)$ → phase

eg. $G(j\omega) = \frac{1}{j\omega}$

$G(s) = \frac{1}{s}$

$\left|\frac{1}{j\omega}\right| = \frac{1}{\omega}$

$\left|\frac{1}{j\omega}\right| = \frac{|1|}{|j\omega|} = \frac{1}{\sqrt{\omega^2}} = \frac{1}{\omega}$

$\angle g(j\omega) = ?$ $\quad \angle \frac{1}{j\omega} = \angle \frac{j}{j^2\omega}$

$= \angle \frac{j}{-\omega}$

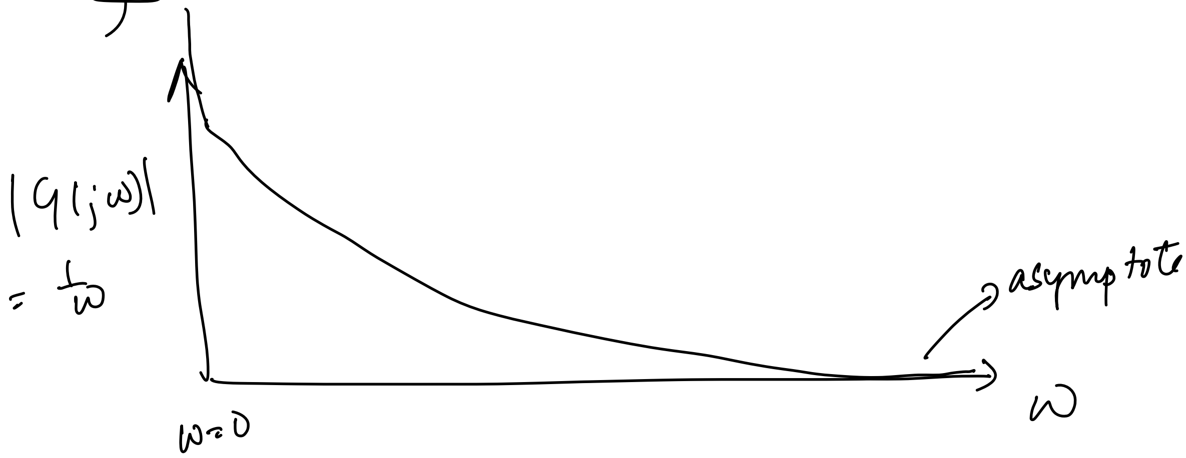
$\angle 1 = 0$

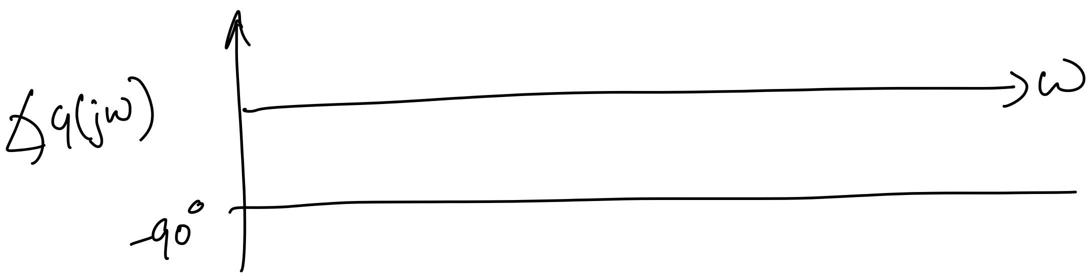
$= \angle -\frac{j}{\omega}$

$\angle a+jb = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{-1/\omega}{0}$ $\quad = \angle\, 0 - \frac{j}{\omega}$

$= \tan^{-1} -\infty = -\frac{\pi}{2}$.

$\angle j\omega = 90^\circ$





Deci Bel

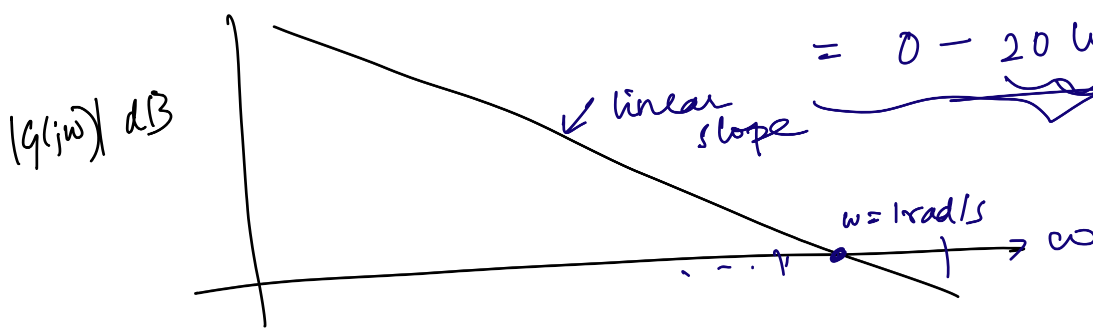
$$|G(j\omega)| \text{ in deciBel} = 20 \log_{10} |G(j\omega)|$$

$$20 \log_{10} \left| \frac{1}{j\omega} \right| = 20 \log_{10} \frac{1}{\omega}$$

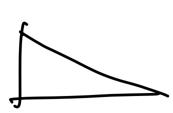
$$= 20 \log_{10} 1 - 20 \log_{10} \omega$$

$$= 0 - 20 \log_{10} \omega$$

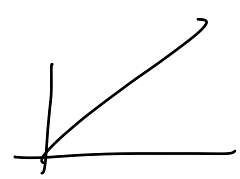
$= 1$

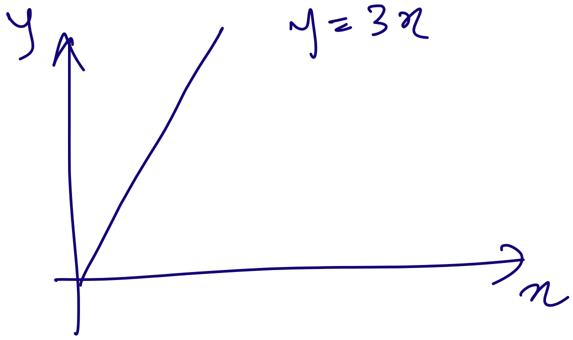


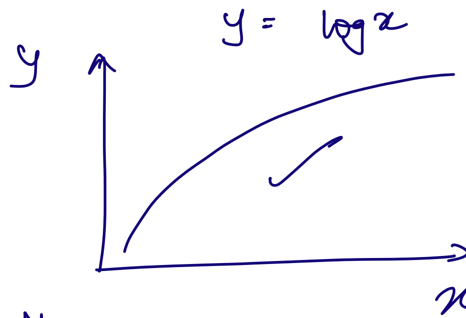
$y = -kx$

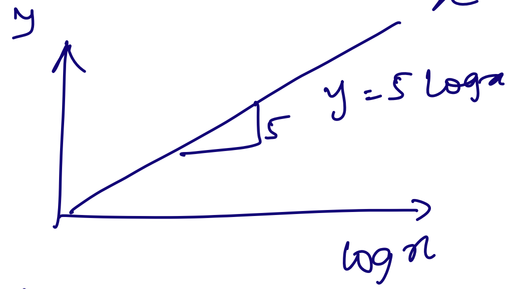


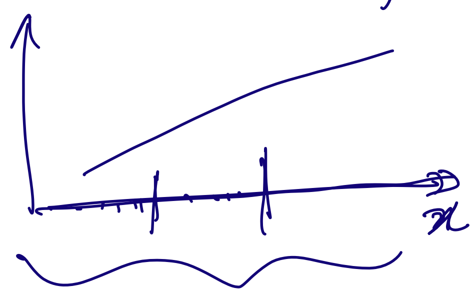
$y = mx$











Bode Plots.

① Constant Gain $G(s) = K_0$ $(K_0 \in \mathbb{R}^1)$

eg $K_0 = 5, \ldots$

② Real Pole (Simple) $G(s) = \dfrac{1}{s/a + 1}$ $(a \in \mathbb{R}^1)$

③ Real zero $G(s) = \dfrac{s/b + 1}{1}$ $(b \in \mathbb{R}^1)$

④ ③ & ② Combined $G(s) = \dfrac{s/b + 1}{s/a + 1}$

(i) $a \ll b$

(ii) $a \gg b$.

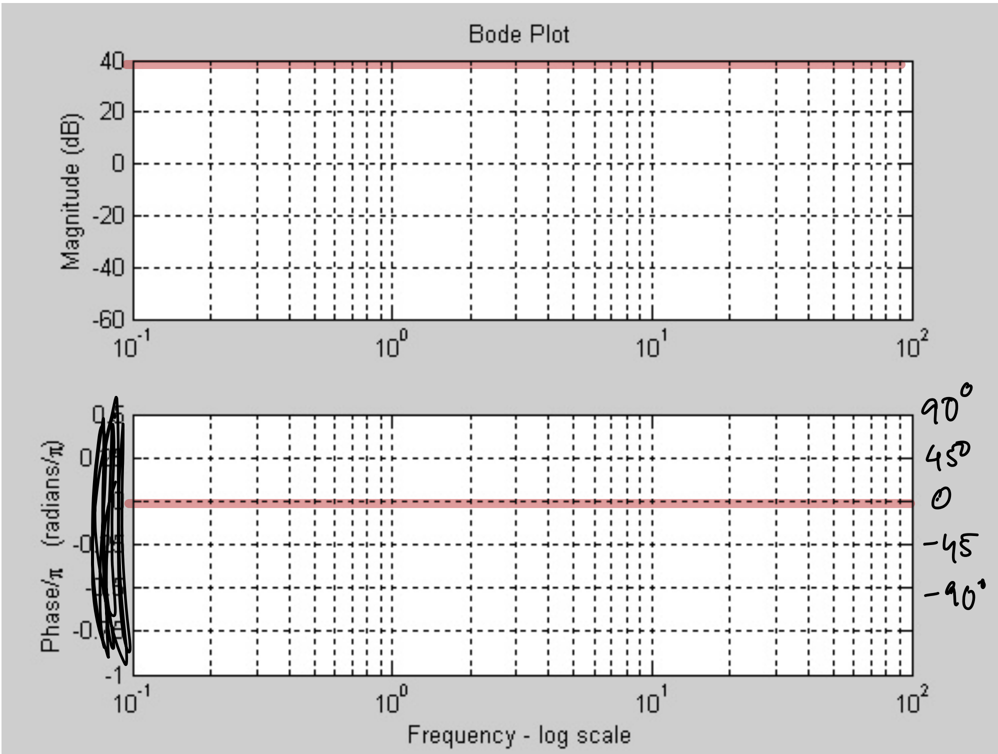
5:37

① $g(s) = K_0 \quad ; \quad K_0 = 100$

$\measuredangle a+jb = \tan^{-1}\frac{b}{a}$

$\therefore g(j\omega) = K_0 = 100$ $\qquad \tan^{-1}\frac{0}{100} =$

$|g(j\omega)| = 100 = 40\,dB$

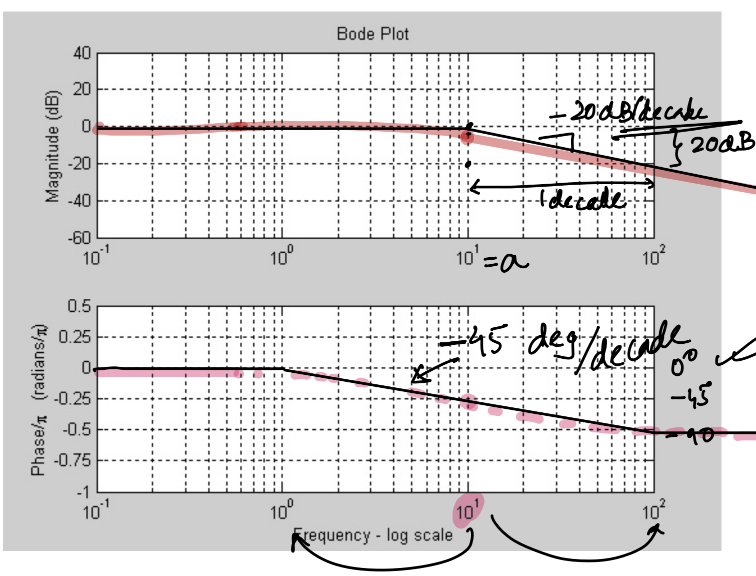


$\measuredangle g(j\omega) = 0$

② $g(s) = \frac{1}{s/a+1} \Rightarrow g(j\omega) = \frac{1}{\frac{j\omega}{a}+1}$

$a = 10$ rad/s.

$|G(j\omega)| = \frac{1}{\sqrt{1^2 + \frac{\omega^2}{a^2}}}$



very small ω

$\omega << a$

$|G(j\omega)| = 1$

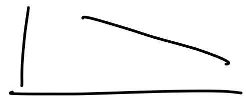
$0\,dB$

$\omega >> a$

$|G(j\omega)| = \frac{a}{\omega}$

$20\log\frac{a}{\omega} = 20\log a - 20\log\omega$

if $\omega \gg a$, $\quad |G(j\omega)| \simeq \underbrace{-20\log\omega}$



$\omega = a$

$$G(j\omega) = \frac{1}{\sqrt{1+\frac{\omega^2}{a^2}}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

$$= 20\lg\frac{1}{\sqrt{2}} = -3\,dB$$

$G(j\omega) = \dfrac{1}{1+j\frac{\omega}{a}}$ $\quad \measuredangle G(j\omega) = 0^\circ \quad (\omega \ll a) \qquad a = 10\,rad/s.$

$\dfrac{1}{1+0}$ $\qquad\qquad = -90^\circ \quad (\omega \gg a)$

eg $\quad \dfrac{1}{1+j10^6} \approx \dfrac{1}{j10^6}$

$$\measuredangle G(j\omega) = \measuredangle \frac{1}{1+\frac{j\omega}{a}} = \angle \frac{1-j\frac{\omega}{a}}{\underbrace{\left(1+\frac{j\omega}{a}\right)\left(1-\frac{j\omega}{a}\right)}_{\underbrace{1-\left(-\frac{\omega^2}{a^2}\right)}}}$$

$(a+b)(a-b) = a^2 - b^2$

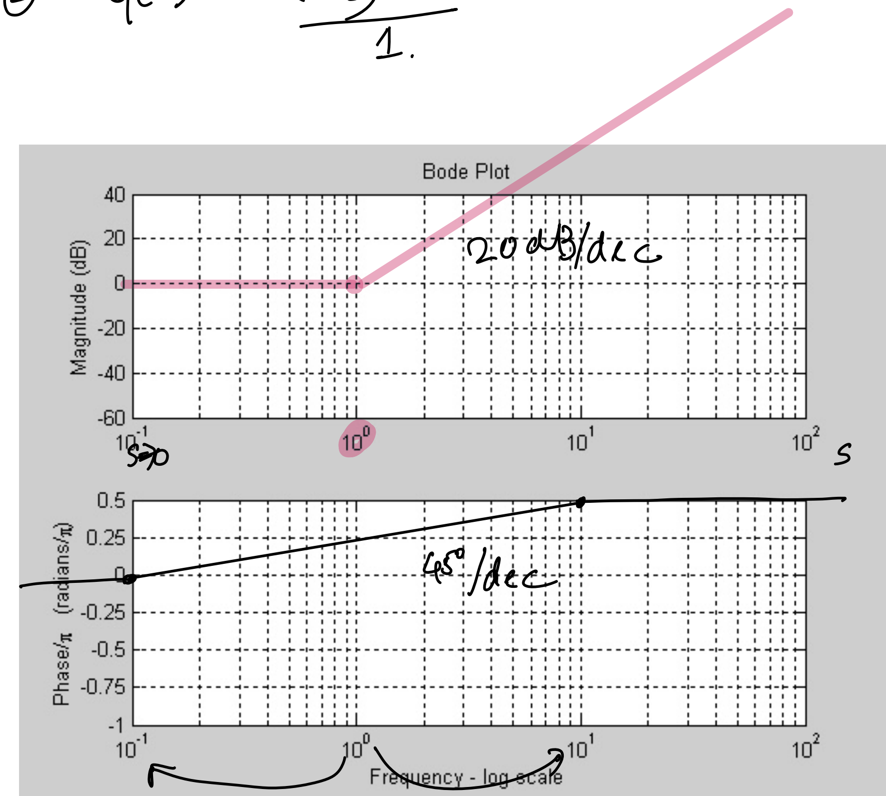
$$\angle \frac{1}{1+\frac{\omega^2}{a^2}} - \frac{j\omega/a}{1+\frac{\omega^2}{a^2}} = \tan^{-1}\frac{-\frac{\omega/a}{\cancel{(1+\omega^2/a^2)}}}{\frac{1}{\cancel{(1+\frac{\omega^2}{a^2})}}} = \tan^{-1} -\frac{\omega}{a}$$

$$= -\tan^{-1}\frac{\omega}{a}$$

at $\omega = a \Rightarrow -45^\circ$.

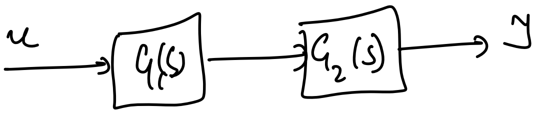
③ $G(s) = \dfrac{(s/b)+1}{1.}$

$\underline{b = 1 \text{ rad/s}}$



④ $\dfrac{(s/b+1)}{(s/a+1)} = ?$

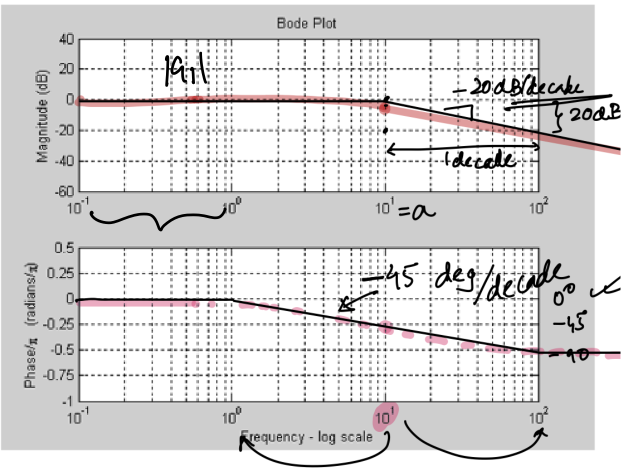
$G_1(s) \cdot \quad G_2(s)$

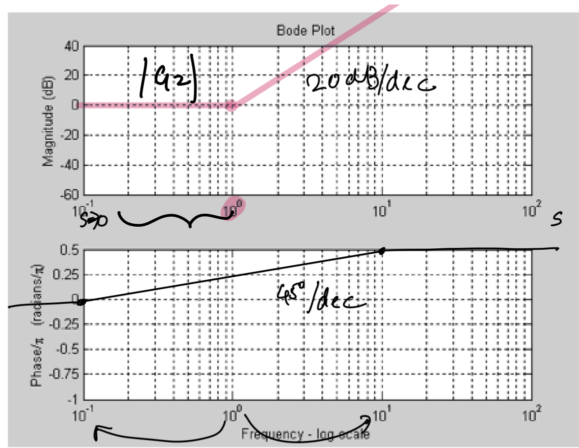


$\dfrac{y}{u} = G_1(s) \cdot G_2(s)$

$20 \log_{10} \left[|G_1(j\omega)| \cdot |G_2(j\omega)| \right]$

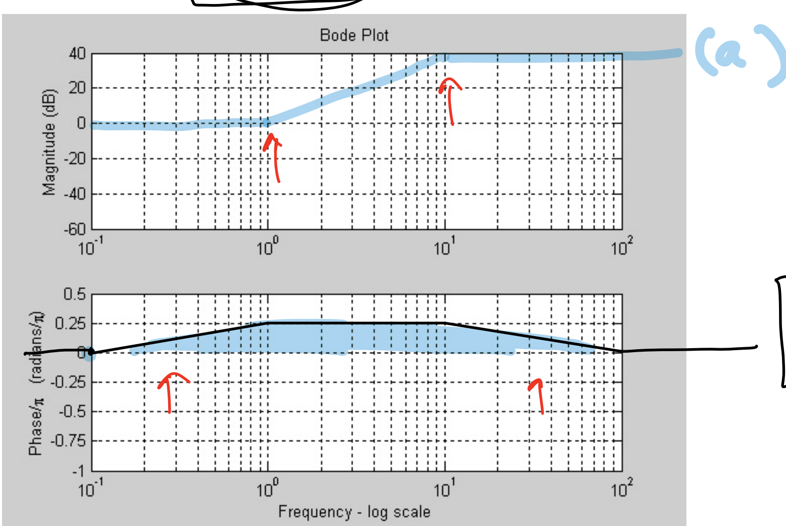
$= 20 \log_{10} |G_1(j\omega)| + 20 \log_{10} |G_2(j\omega)|$





$$G(s) = \boxed{\frac{s/b+1}{s/a+1}} = G_1 \cdot G_2 \qquad b = 1 \text{ rad/s}$$

$$a = 10 \text{ rad/s.}$$



-90°

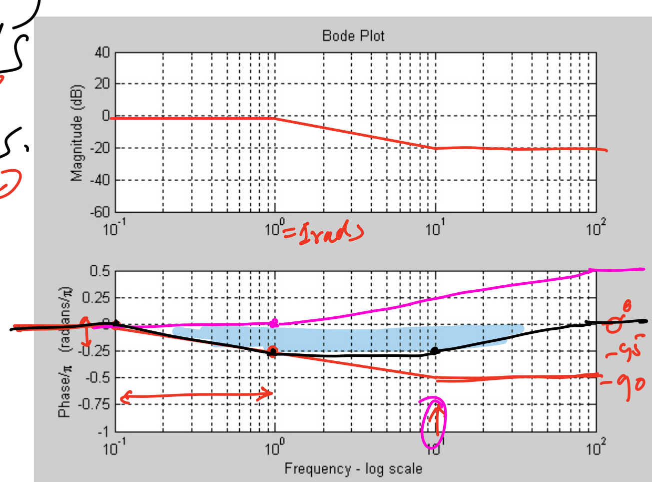
$+90^\circ$

$\boxed{\text{lead lag compensator}}$

$$\angle G(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

$$G(s) = \frac{s/b+1}{s/a+1} \qquad b = 10 \text{ rad/s}$$

$$a = 1 \text{ rad/s.}$$



lag lead compensator

$$\boxed{\frac{\tilde{i}_L(s)}{\tilde{d}(s)} = \frac{(V_{in}+V_c)\left(sC+\frac{1}{R}\right)+D'I_L}{s^2LC+\frac{sL}{R}+D'^2}}$$

$$G(s) = \frac{1}{s^2LC+\frac{sL}{R}+D'^2}$$

$$\boxed{\frac{1}{s+a}} = \underbrace{\boxed{\frac{1}{a\left(s/a+1\right)}}}$$ time constant form.

$s \to 0$ $\frac{1}{a}$

$$\boxed{G_1 \cdot G_2 = \underbrace{\frac{1}{(s/a+1)}}}$$

$" \frac{1}{a}$

0°

$$\frac{1}{D'^2\left[\frac{s^2LC}{D'^2}+\frac{sL}{RD'^2}+1\right]}$$

$$\Rightarrow \quad G_1 = \frac{1}{D'^2}$$

$$G_2 = \frac{1}{s^2\frac{LC}{D'^2}+\frac{sL}{RD'^2}+1}$$

$\left(\frac{1}{s/a+1}\right)$ First order transfer function

$$G_2(s) = \frac{1}{\underbrace{(s/\omega_0)^2+2\zeta\, s/\omega_0+1}}$$