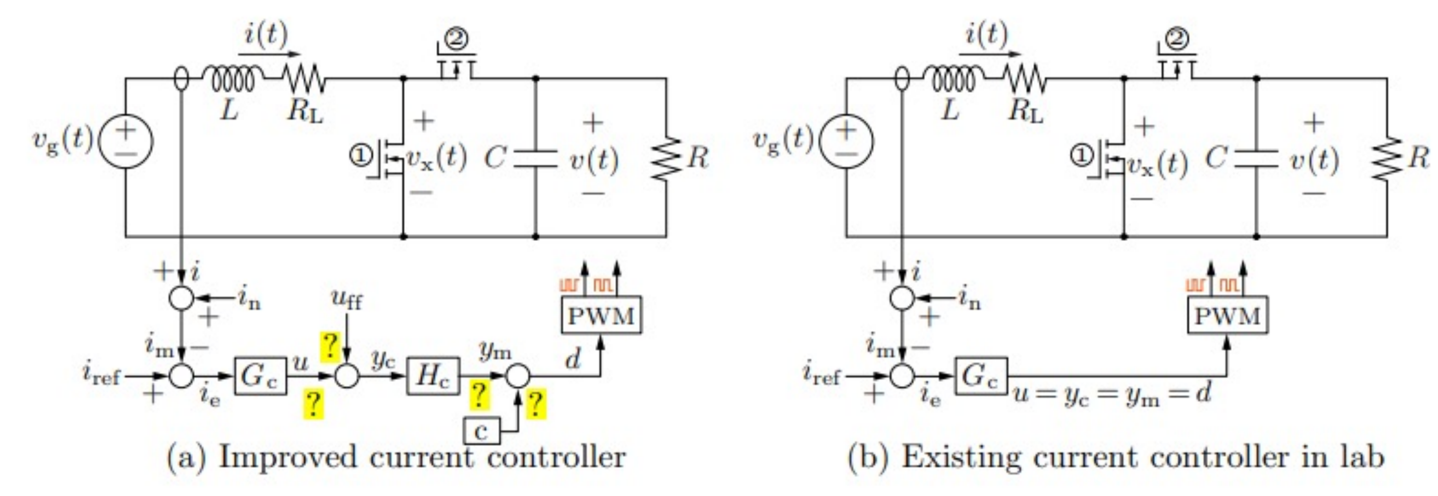
HW5 Problem 1 (Kaden William Fitzpatrick)

Wednesday, March 2, 2022 5:54 PM

→ Boost converter

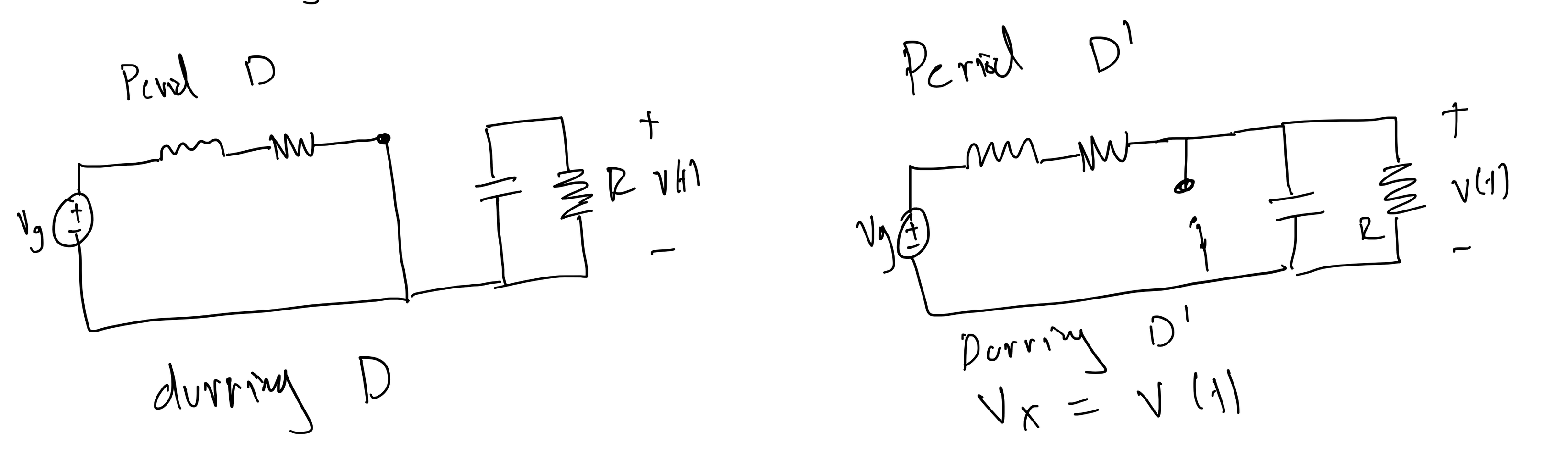
- winding + Mosfet on resistances lumped into R_L
- 1a → control structure
- 1b → feed forward structure



(a) Improved current controller (b) Existing current controller in lab

a.) Derive the voltage $\langle V_x(t)\rangle$ average value of $V_x(t)$ over one switching period in terms of duty d and output voltage

Period D Period D'



durring D

$V_x = 0$

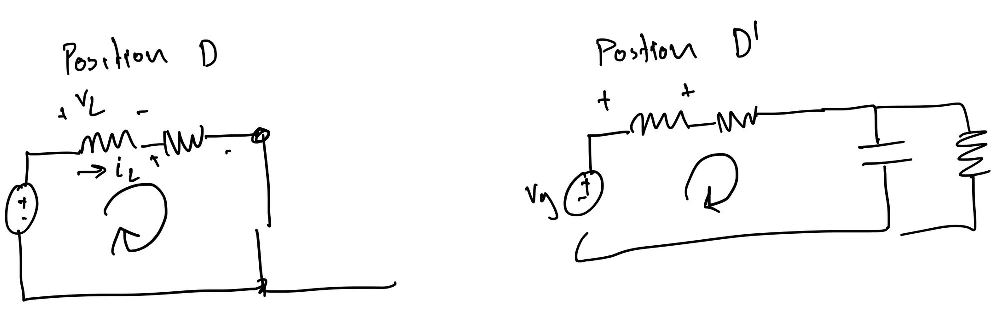
Durring D'

$V_x = V(t)$

$$\boxed{\langle V_x(t)\rangle = \langle V(t)\rangle[1-d(t)]}$$

b.) Derive KVL in the input loop to obtain a relationship between $v_g(t) (= \langle v_g(t)\rangle)$, L, R_L, $\langle V_x(t)\rangle$ & $\langle i(t)\rangle$

Position D Position D'



$v_g = i(t)R_L$ $\qquad$ $V_g = i(t)R_L + V_x$

$$\boxed{\langle v_g(t)\rangle = L\frac{d\langle i(t)\rangle}{dt} + R_L\langle i(t)\rangle + \langle V_x(t)\rangle}$$

c.) Assume the controller, G_c produces an output, d, verify that fig 2(a) shows the correct implentation of your KVL derived in proceding question

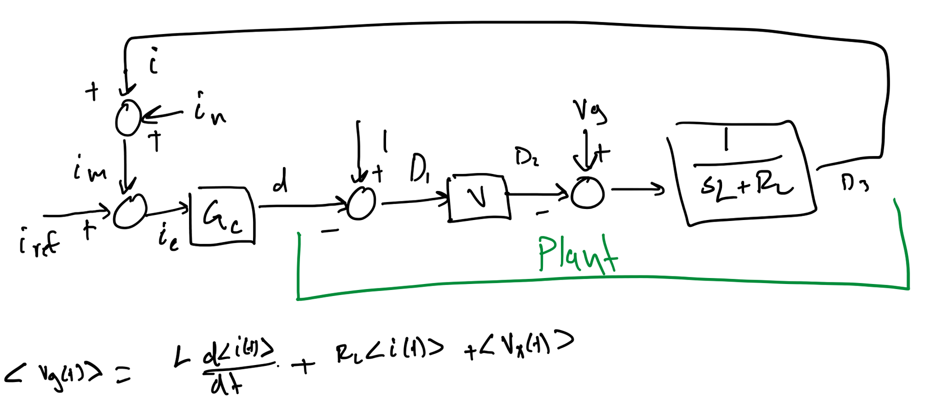


fig 2a

$\langle v_g(t)\rangle = L\frac{d\langle i(t)\rangle}{dt} + R_L\langle i(t)\rangle + \langle V_x(t)\rangle$

$D = [1-d]\cdot V$ $\qquad$ $D_2 = V_g - V[1-d]$ $\qquad$ $D_3 = \frac{[V_g - V[1-d]]}{sL + R_L} = i$

$i_m = i_n + i$ $\qquad$ $i_e = i_{ref} - i_m$

$d = G_c \cdot i_e \rightarrow d = G_c(i_{ref} - i_m)$ $\qquad$ $d = G_c(i_{ref} - i_n - i)$

$$d = G_c\left(i_{ref} - i_n - \frac{[V_g - V[1-d]]}{sL + R_L}\right)$$

$\langle v_g(t)\rangle = L\frac{d\langle i(t)\rangle}{dt} + R_L\langle i(t)\rangle + \langle V_x(t)\rangle \Rightarrow V_g = L\frac{di}{dt} + iR_L + V_x$

$$i = \frac{[V_g - V[1-d]]}{sL + R_L}$$

$L\frac{di}{dt}$

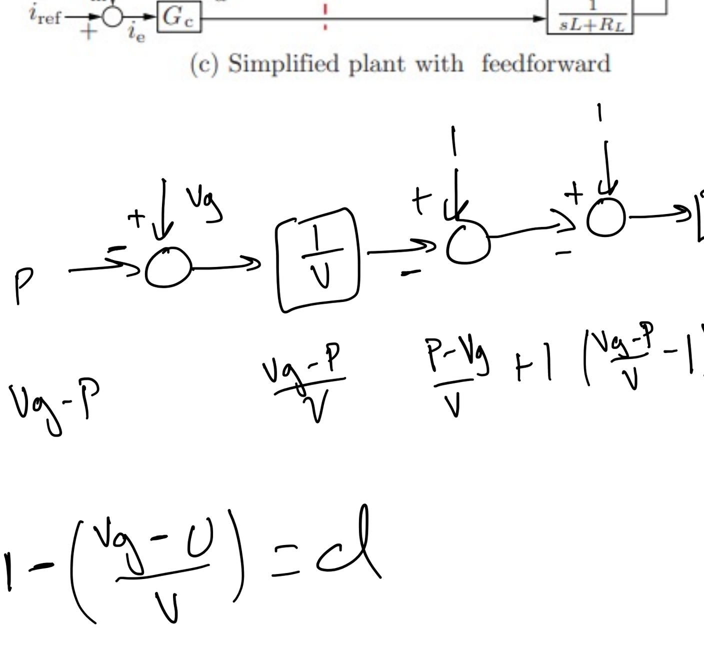
$$i(s) = \frac{V_g(s) - V(s)[1-d]}{(sL + R_L)}$$

$$\mathcal{L}^{-1}\left((sL+R_L)i(s) + V(s)[1-d(s)]\right) = \left(V_g(s)\right)\mathcal{L}^{-1}$$

$$\mathcal{L}^{-1}\left(sLi(s) + i(s)R_L + V(s)[1-d(s)]\right) = \mathcal{L}^{-1}[V_g(s)]$$

$$\rightarrow \boxed{L\frac{di(t)}{dt} + i(t)R_L + V_x(t) = V_g(t)} ✓$$

d.) We want to implement Feed-Forward to simplify transfer function $\rightarrow \frac{1}{sL+R_L}$ so that the controller design becoms simplifed as in Fig 2c



(a) Control structure without feedforward

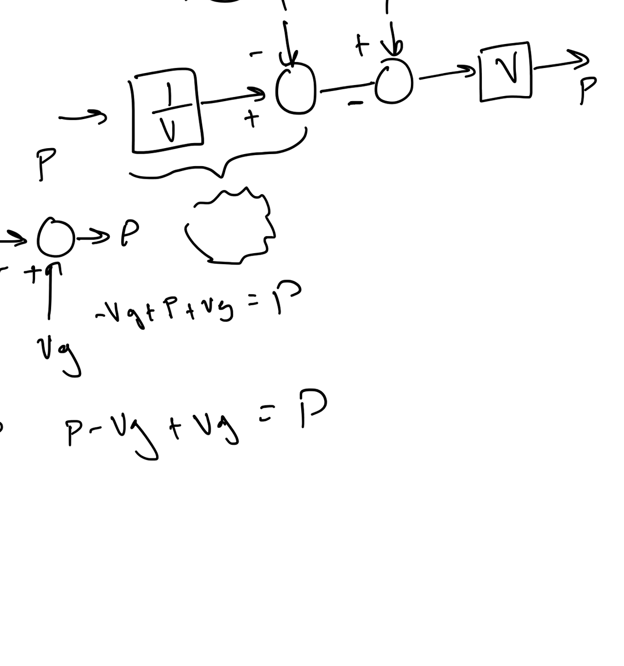
(b) Control structure with feedforward

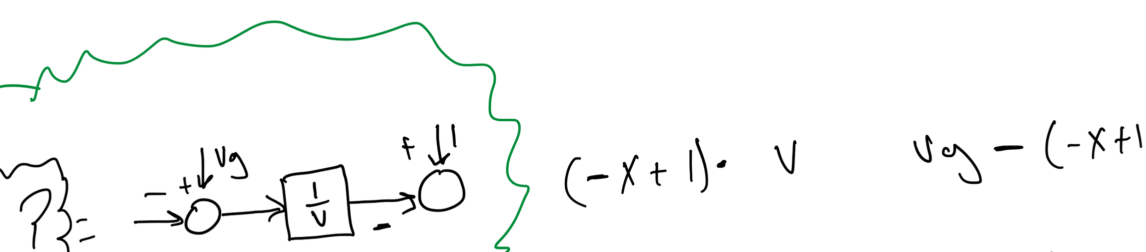
(c) Simplified plant with feedforward

$i(s) = \frac{V_g(s) - V(s)[1-d]}{sL + R_L}$

F

F^{-1}

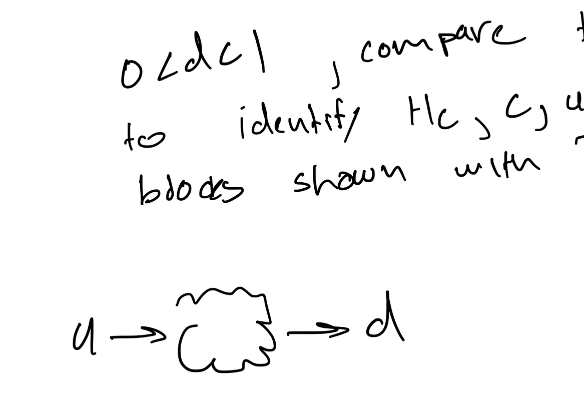




$-V_g + P + V_g = P$

$V_g - P$ $\qquad$ $\frac{V_g - P}{V}$ $\qquad$ $\frac{P - V_g}{V} + 1$ $\qquad$ $\left(\frac{V_g - P}{V} - 1\right) + 1$ $\qquad$ $V_g - P$ $\qquad$ $P - V_g + V_g = P$

$$1 - \left(\frac{V_g - U}{V}\right) = d$$



$(-X+1)\cdot V$ $\qquad$ $V_g - (-X+1)V$

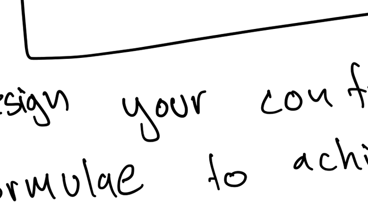
Provide a numerical relationship between controller output U and actual duty ratio, d, that goes into PWM block $0 < d < 1$, compare the relationship with fig 1a. to identify H_c, c, u_{ff} and the signs of the sum blocks shown with ?

$\underbrace{V_g - (-X+1)V}_{d}$ → before $\frac{1}{sL+R_L}$

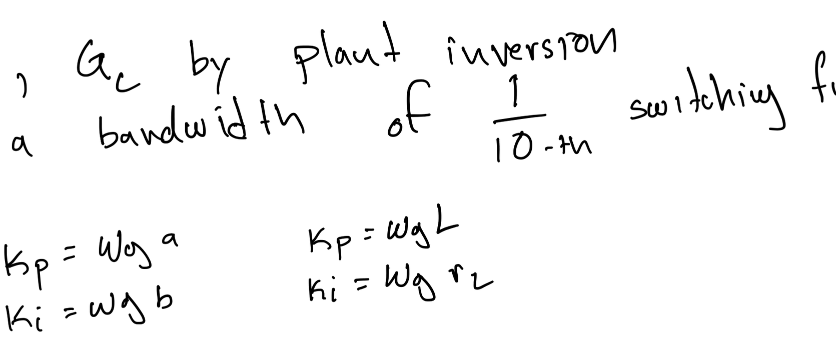
$$\boxed{d = 1 - \left(\frac{V_g - U}{V}\right)}$$

relationship

from my block diagram above the relationship is proved correct



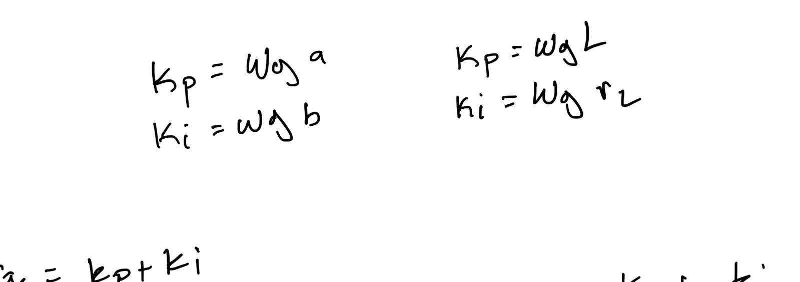
from figure 1(a)



$V_g = u_{ff}$

$c = 1$

$H = \frac{1}{V}$



e.) Design your controller, G_c by plant inversion formulae to achieve a bandwidth of $\frac{1}{10\text{-th}}$ switching freq

$f_{bw} = \frac{f_{sw}}{10}$ $\qquad$ $K_p = \omega_g a$, $K_i = \omega_g b$ $\qquad$ $K_p = \omega_g L$, $K_i = \omega_g r_L$

$\ell = \frac{\omega_g}{s}$ $\qquad$ $G_c = k_p + \frac{k_i}{s}$

↳ ① DC tracking
② 90° PM
③ ω_g = bandwidth

$\frac{K_p s + k_i}{s}$

$\ell = G_c \frac{1}{as+b}$

$a = L$

$b = R_L$

$$\frac{\omega_g}{s} = K_p + \frac{k_i}{s} \cdot \frac{1}{sL + R_L}$$

$$\frac{\omega_g}{s} = \frac{(K_p s + K_i)}{s} \cdot \frac{1}{(sL + R_L)}$$

$$\frac{\omega_g}{s} = \frac{\omega_g L s + \omega_g R_L}{s} \cdot \frac{1}{Ls + R_L}$$

$$\frac{\omega_g}{s} = \frac{\omega_g}{s}\frac{(Ls + R_L)}{1} \cdot \frac{1}{Ls + R_L}$$

$$\frac{\omega_g}{s} = \frac{\omega_g}{s} ✓$$

$$G_c = \text{controller} = \frac{k_p s + k_i}{s} = \frac{\omega_g L s + \omega_g R_L}{s} = \frac{\omega_g(Ls + R_L)}{s}$$

$\omega_g = \frac{\omega_{sw}}{10}$

$\omega = 2\pi f$

$\frac{\omega}{2\pi} = f$

$$\boxed{G_c = \frac{\omega_{sw}(Ls + R_L)}{10s} \qquad G_c = \frac{f_{sw}(Ls + R_L)}{20\pi s}}$$

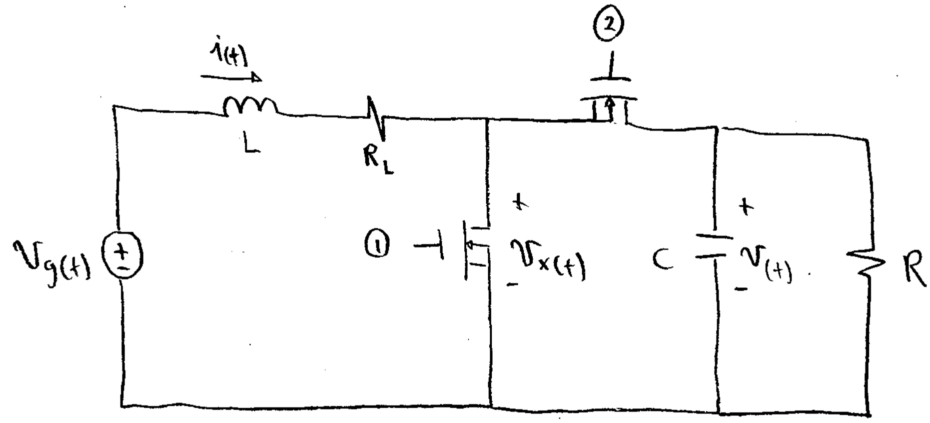
$K_p = \frac{10{,}000 \cdot 2\pi}{10} \cdot 0.0013$

Problem 1 (Sergio Alexander Sunley Pocasangre)

Student: Sergio Sunley

Power Electronics Controls

HW 5



a) $\langle v_{x(t)} \rangle = (0)\, d + \langle v_{(t)} \rangle (1-d)$

$$\boxed{\langle v_{x(t)} \rangle = (1-d) \langle v_{(t)} \rangle}$$

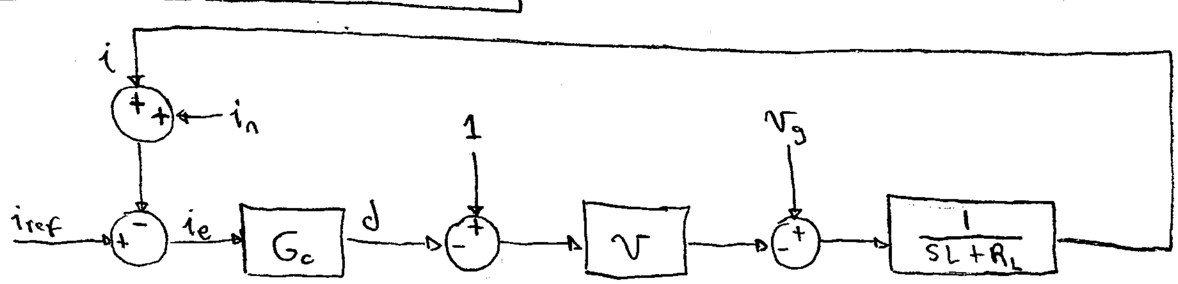
b)
$$\boxed{\langle v_{g(t)} \rangle = L \frac{d \langle i_{(t)} \rangle}{dt} + R_L \langle i_{(t)} \rangle + v_{x(t)}}$$

c) $v_{g(s)} = SL\, i_{(s)} + R_L\, i_{(s)} + v_{x(s)}$; $v_{x(s)} = (1-d)\, v_{(s)}$

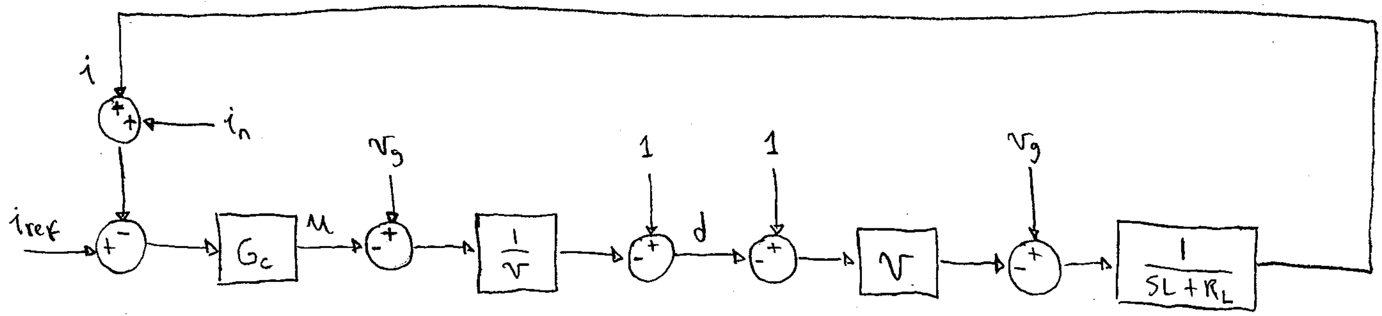
$v_{g(s)} = SL\, i_{(s)} + R_L\, i_{(s)} + (1-d)\, v_{(s)}$

$i_{(s)} (SL + R_L) = v_{g(s)} - (1-d)\, v_{(s)}$

$$\boxed{i_{(s)} = \frac{v_{g(s)} - (1-d)\, v_{(s)}}{SL + R_L}}$$



d)

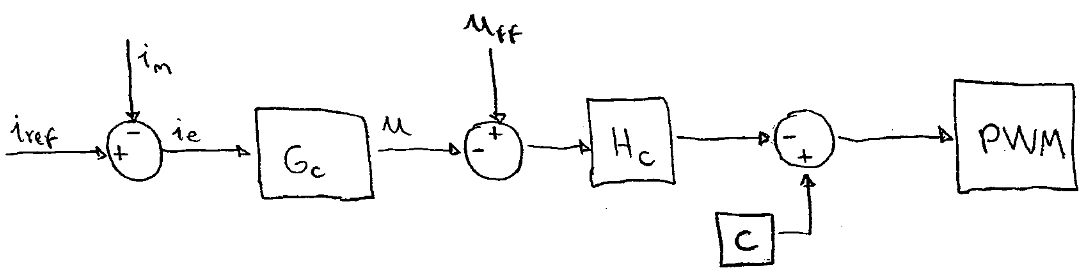


$$i(s) = \frac{v_g(s) - (1-d)\,v(s)}{sL+R_L} \leftarrow v_g - (1-d)\,v = u$$

$$\frac{v_g - u}{v} = (1-d)$$

$$\boxed{d = 1 - \frac{v_g - u}{v}}$$

For figure 1(a)



$$\boxed{\begin{aligned} H_c &= \frac{1}{v} \\ u_{FF} &= v_g \\ C &= 1 \end{aligned}}$$

e) $G_c P = \frac{\omega_g}{s}$

$G_c = \frac{\omega_g}{s} \frac{1}{P}$; $P = \frac{1}{sL + R_L}$

$G_c = \frac{\omega_g}{s} \frac{1}{\frac{1}{sL+R_L}}$

$G_c = \frac{\omega_g (sL + R_L)}{s} = \frac{\omega_g L s + \omega_g R_L}{s} = \omega_g L + \frac{\omega_g R_L}{s}$

$G_c = k_p + \frac{k_i}{s} \rightarrow k_p = \omega_g L$; $\omega_g = 0.1\, \omega_{sw} = 0.1(2\pi) f_{sw}$

$k_i = \omega_g R_L$ $\qquad \omega_g = 0.2\pi f_{sw}$

$$\boxed{\begin{aligned} k_p &= 0.2\pi f_{sw} L \\ k_i &= 0.2\pi f_{sw} R_L \end{aligned}}$$

$$G_{c(s)} = k_p + \frac{k_i}{s} = \frac{k_p s + k_i}{s} = \boxed{\frac{0.2\pi f_{sw} L s + 0.2\pi f_{sw} R_L}{s}}$$

Problem 2

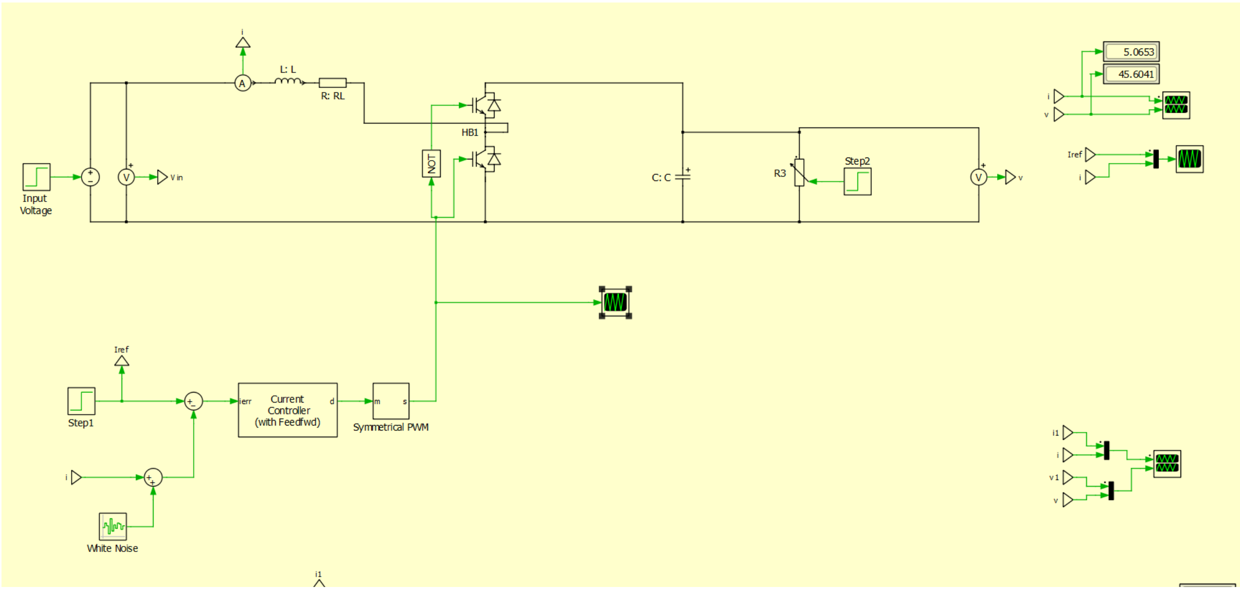
$k_p = 0.2\pi f_{sw} L = 0.2(\pi)(10\times10^3)(1.3\times10^{-3}) = 8.168$

$k_i = 0.2\pi f_{sw} R_L = 0.2(\pi)(10\times10^3)(60\times10^{-3}) = 376.991$

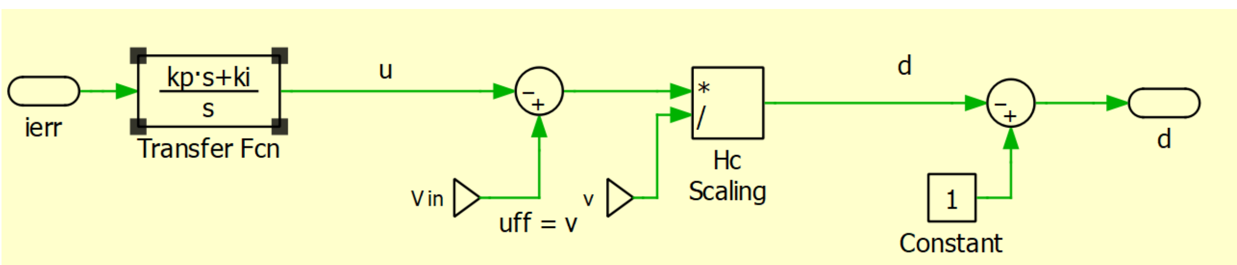
$$G_{c(s)} = \frac{8.168\, s + 376.991}{s}$$

Problem 2 (Kale Martin Strange)

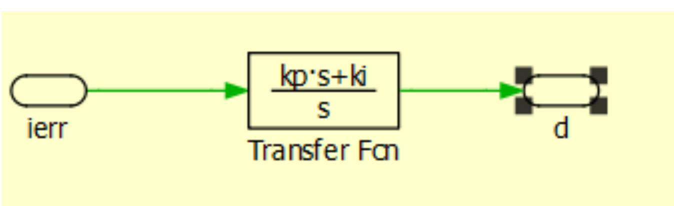
Problem #2:

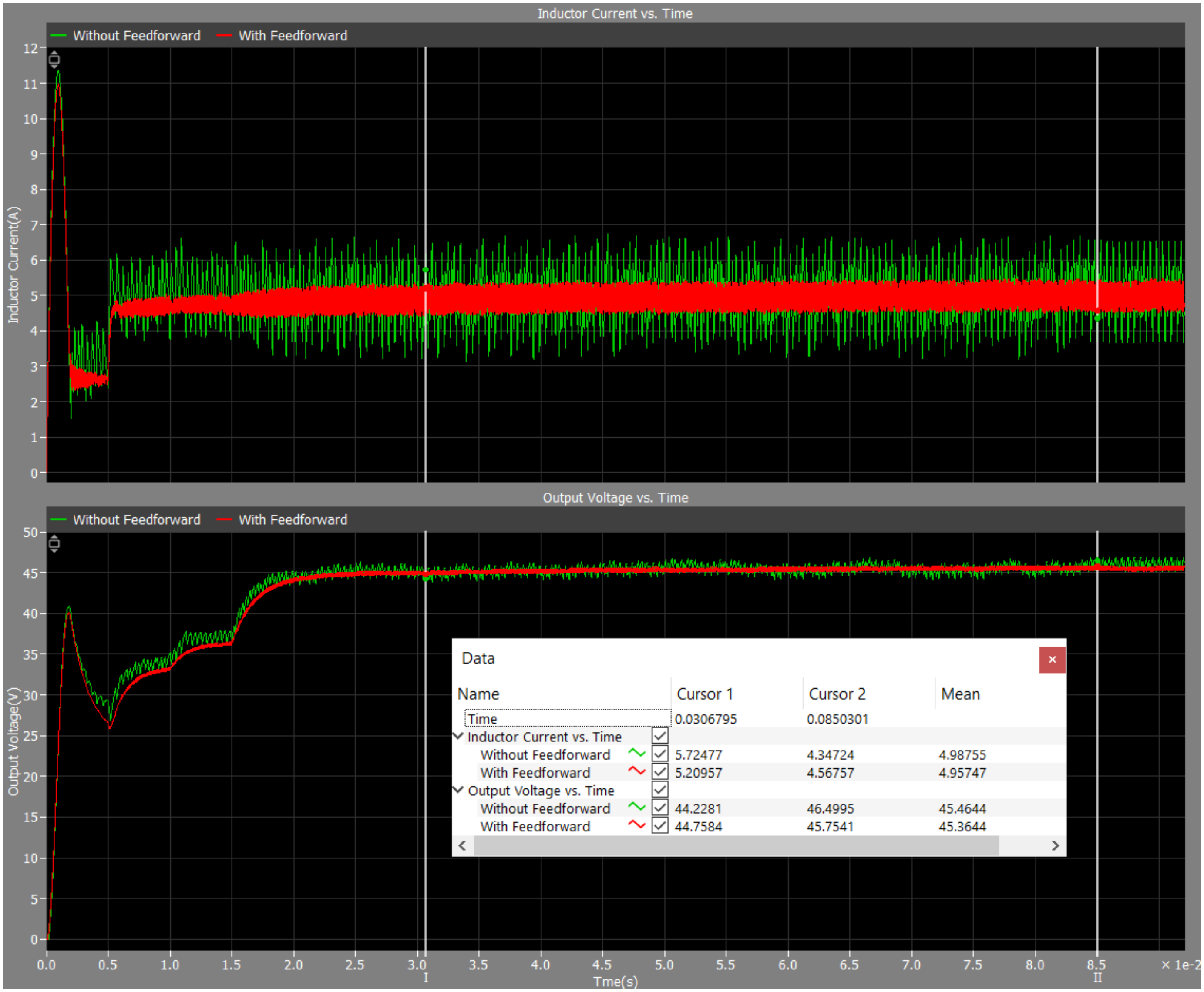


Controller With Feedforward:

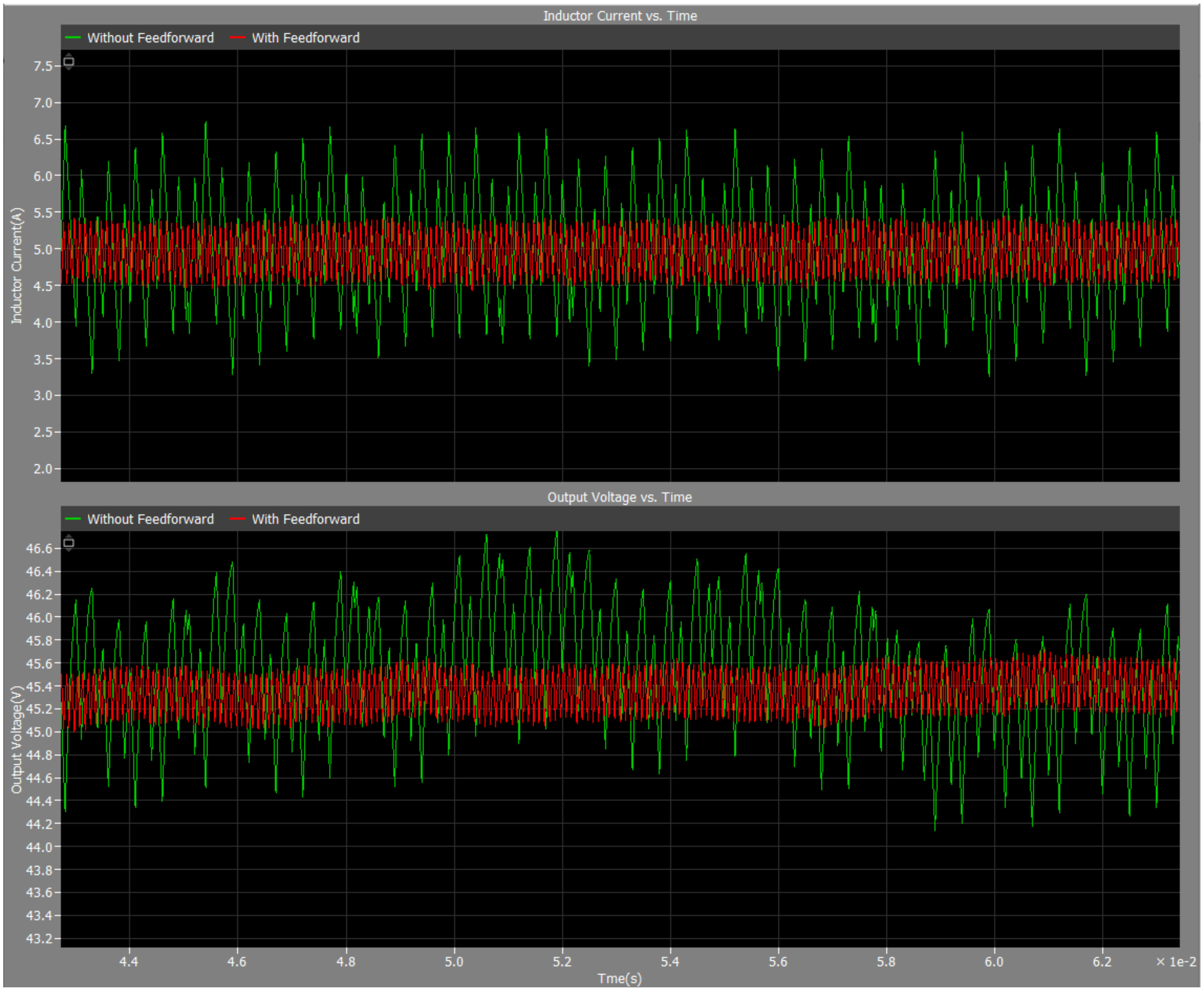


Controller Without Feedforward:

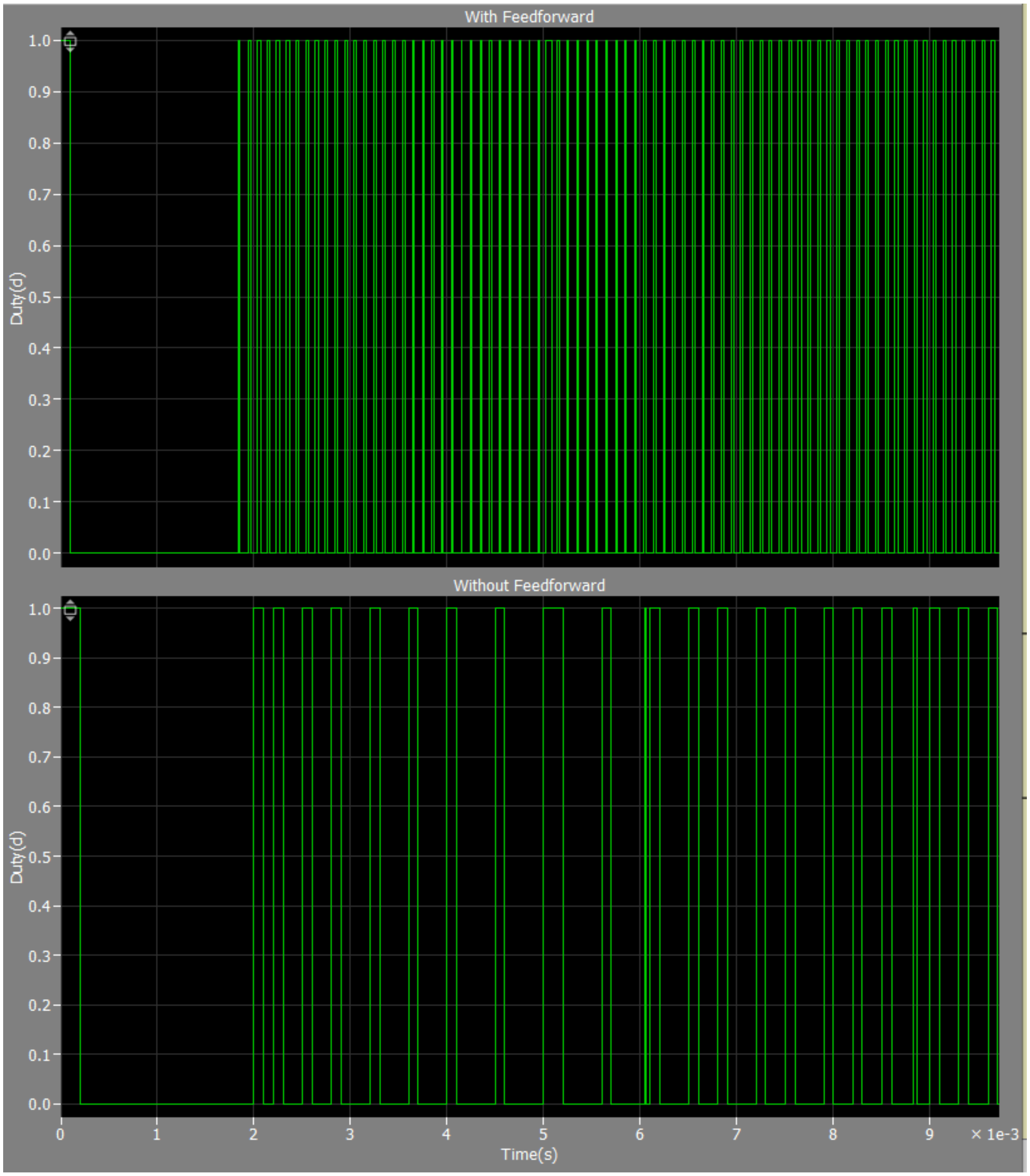




Inductor Current vs. Time
Without Feedforward
With Feedforward
Inductor Current(A)
Output Voltage vs. Time
Without Feedforward
With Feedforward
Output Voltage(V)
Tme(s)
× 1e-2
Data
Name
Cursor 1
Cursor 2
Mean
Time 0.0306795 0.0850301
Inductor Current vs. Time
Without Feedforward 5.72477 4.34724 4.98755
With Feedforward 5.20957 4.56757 4.95747
Output Voltage vs. Time
Without Feedforward 44.2281 46.4995 45.4644
With Feedforward 44.7584 45.7541 45.3644



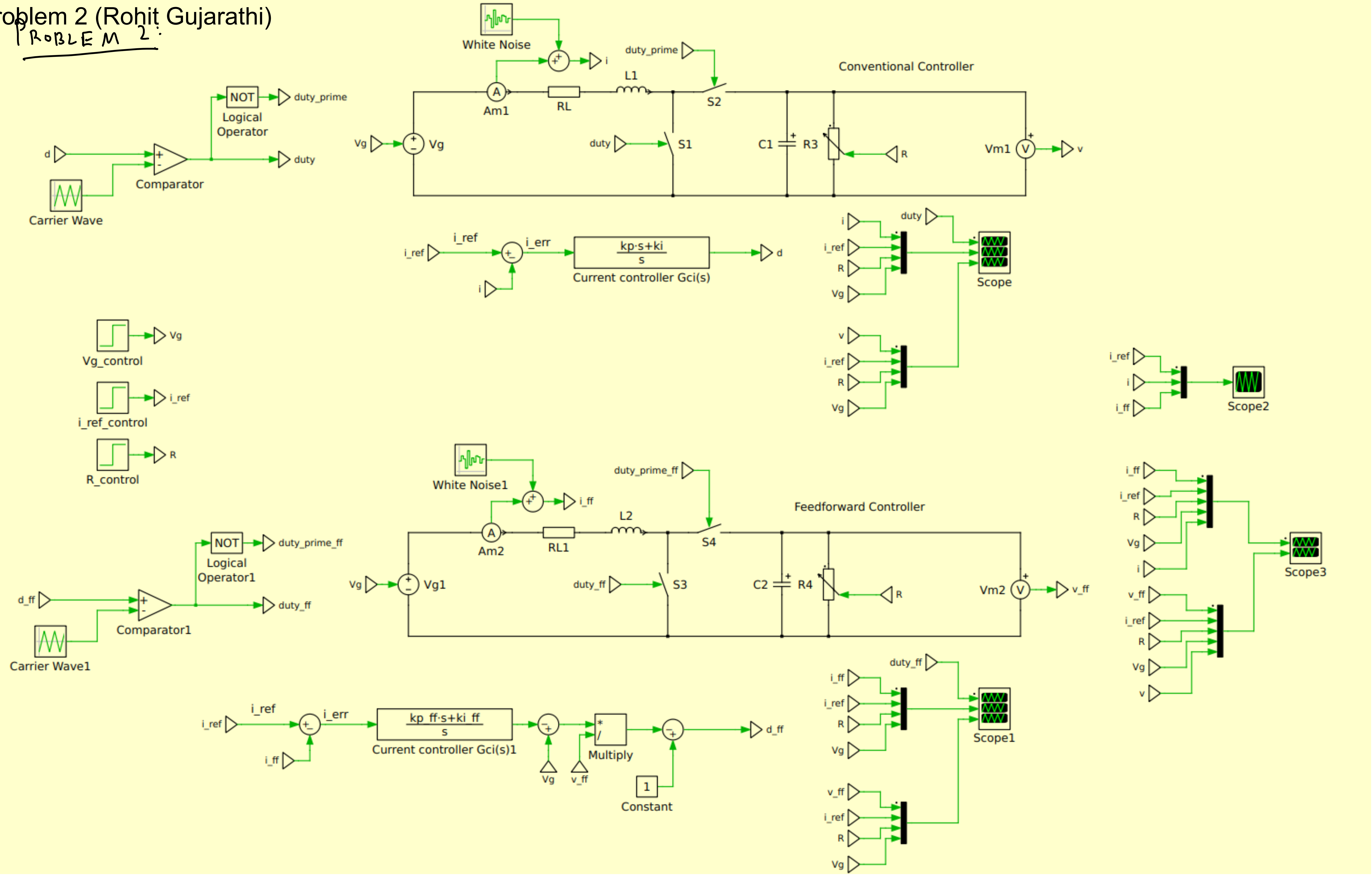
Inductor Current vs. Time
Without Feedforward
With Feedforward
Inductor Current(A)
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
Output Voltage vs. Time
Without Feedforward
With Feedforward
Output Voltage(V)
46.6
46.4
46.2
46.0
45.8
45.6
45.4
45.2
45.0
44.8
44.6
44.4
44.2
44.0
43.8
43.6
43.4
43.2
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
6.2
× 1e-2
Tme(s)

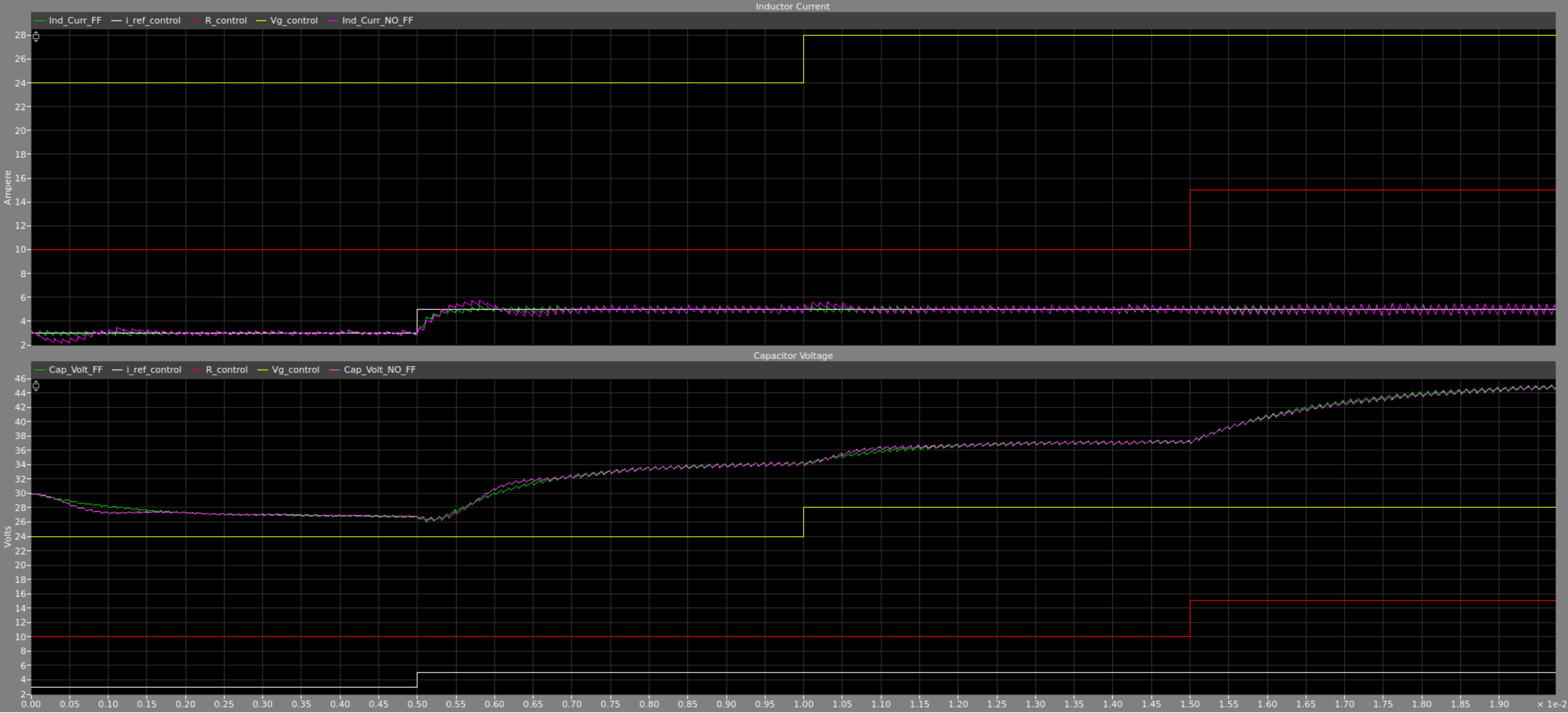


The controller with forward feedback has a much quicker response time and has less than half the current and voltage ripple. Additionally, the output signal varies less with input white noise.

Problem 2 (Rohit Gujarathi)

PROBLEM 2:





Inductor Current
Ind_Curr_FF
i_ref_control
R_control
Vg_control
Ind_Curr_NO_FF
Ampere
Capacitor Voltage
Cap_Volt_FF
i_ref_control
R_control
Vg_control
Cap_Volt_NO_FF
Volts
× 1e-2

Differences

→ The settling time is the same in both cases.

→ The overshoot has reduced in case of feedforward.

→ The risetime is the same in both cases.

→ Feedforward is better at rejecting changes in V_g as can be seen at 10ms. There is a bump in the control without feedforward.

→ Both are similar in rejecting changes in R.

→ The control without feedforward is quick to respond to changes making the response "jerky".

Problem 3: We will build on the results from Homework 4, Using Faraday's Law, give the expressions for the induced back EMF voltages, e_a, e_b, e_c, within the a, b, and c phase windings, respectively. In addition, draw the equivalent circuits looking into the a, b, and c windings where the back EMFs, e_a, e_b, e_c, are clearly labeled along with the speed-dependent voltages $\mathcal{E}_a$, $\mathcal{E}_b$, $\mathcal{E}_c$ and inductive voltage drops. Be careful with all circuit element polarities.

Assumptions:

- Equal number of turns for each of the coils:

$$N = N_a = N_b = N_c$$

- Maximum magnetic ring flux linking a coil is the same for each of the phase coils:

$$\Phi_{max} = \Phi_{max,a} = \Phi_{max,b} = \Phi_{max,c}$$

Flux Linkage:

- $\lambda_a = -N_a\Phi_{max,a}\cos(\theta_{e,a}) + Li_a = -N\Phi_{max}\cos(\theta_e) + Li_a$
- $\lambda_b = -N_b\Phi_{max,b}\cos(\theta_{e,b}) + Li_b = -N\Phi_{max}\cos\left(\theta_e - \frac{2\pi}{3}\right) + Li_b$
- $\lambda_c = -N_c\Phi_{max,c}\cos(\theta_{e,c}) + Li_c = -N\Phi_{max}\cos\left(\theta_e - \frac{4\pi}{3}\right) + Li_c$

Back EMF:

- $e_a = -\frac{d\lambda_a}{dt} = -\frac{d}{dt}\{-N\Phi_{max}\cos(\theta_e) + Li_a\} = N\Phi_{max}\frac{d}{dt}\{\cos(\theta_e)\} - L\frac{d}{dt}\{i_a\}$

$$= -N\Phi_{max}\sin(\theta_e)\cdot\frac{d}{dt}\{\theta_e\} - L\frac{di_a}{dt} = -N\Phi_{max}\sin(\theta_e)\cdot\omega_e - L\frac{di_a}{dt}$$

$$\boxed{\boldsymbol{e_a = -N\omega_e\Phi_{max}\sin(\theta_e) - L\frac{di_a}{dt}}}$$

- $e_b = -\frac{d\lambda_b}{dt} = -\frac{d}{dt}\left\{-N\Phi_{max}\cos\left(\theta_e - \frac{2\pi}{3}\right) + Li_b\right\} = N\Phi_{max}\frac{d}{dt}\left\{\cos\left(\theta_e - \frac{2\pi}{3}\right)\right\} - L\frac{d}{dt}\{i_b\}$

$$= -N\Phi_{max}\sin\left(\theta_e - \frac{2\pi}{3}\right)\cdot\frac{d}{dt}\left\{\theta_e - \frac{2\pi}{3}\right\} - L\frac{di_b}{dt} = -N\Phi_{max}\sin\left(\theta_e - \frac{2\pi}{3}\right)\cdot\omega_e - L\frac{di_b}{dt}$$

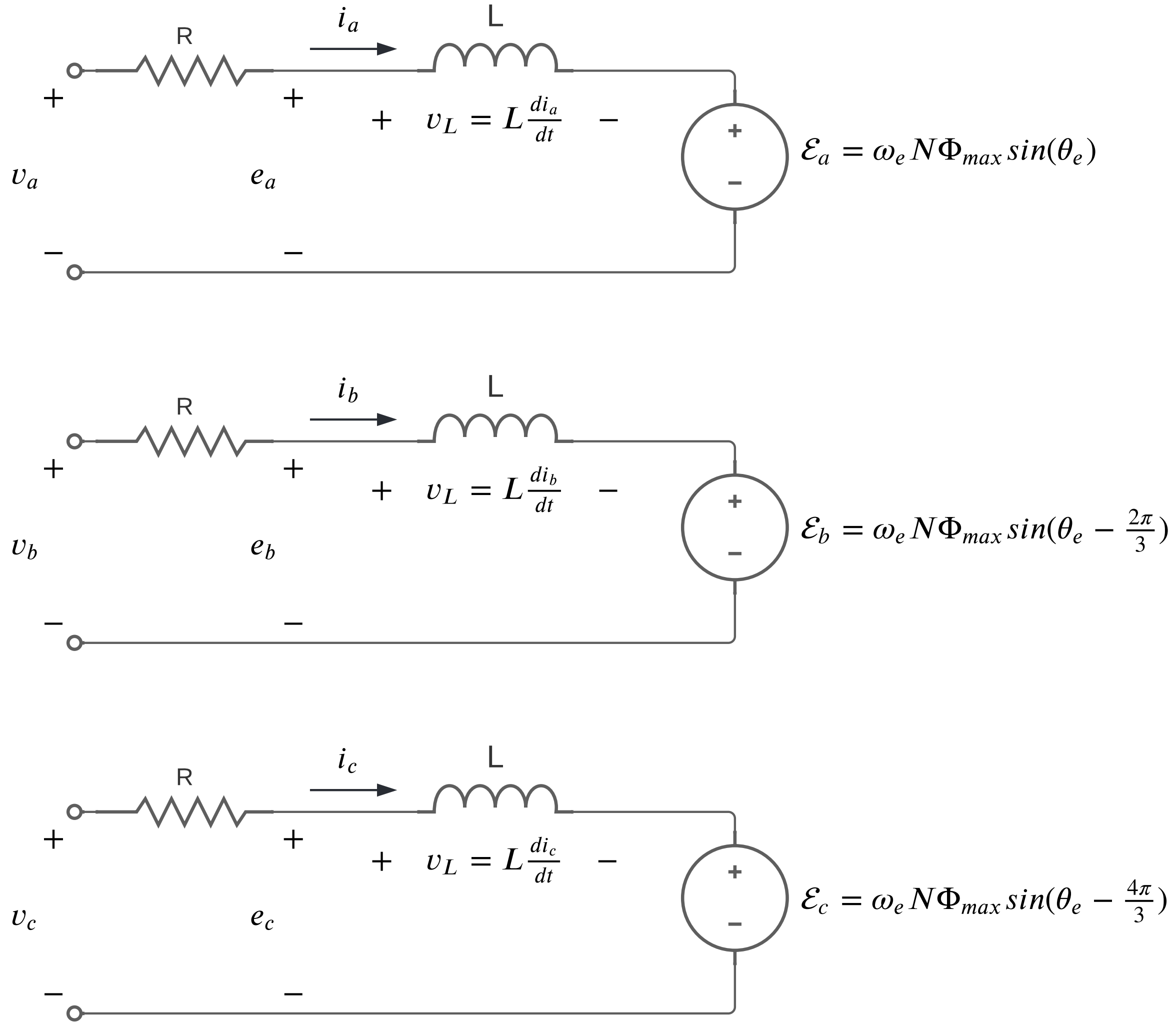
$$\boxed{\boldsymbol{e_b = -N\omega_e\Phi_{max}\sin\left(\theta_e - \frac{2\pi}{3}\right) - L\frac{di_b}{dt}}}$$

- $e_c = -\frac{d\lambda_c}{dt} = -\frac{d}{dt}\left\{-N\Phi_{max}\cos\left(\theta_e - \frac{4\pi}{3}\right) + Li_c\right\} = N\Phi_{max}\frac{d}{dt}\left\{\cos\left(\theta_e - \frac{4\pi}{3}\right)\right\} - L\frac{d}{dt}\{i_c\}$

$$= -N\Phi_{max}\sin\left(\theta_e - \frac{4\pi}{3}\right)\cdot\frac{d}{dt}\left\{\theta_e - \frac{4\pi}{3}\right\} - L\frac{di_c}{dt} = -N\Phi_{max}\sin\left(\theta_e - \frac{4\pi}{3}\right)\cdot\omega_e - L\frac{di_c}{dt}$$

$$\boxed{\boldsymbol{e_c = -N\omega_e\Phi_{max}\sin\left(\theta_e - \frac{4\pi}{3}\right) - L\frac{di_c}{dt}}}$$

Equivalent Circuits:



Note, the equivalent circuits also include the effects of winding resistance R, assuming that: $R = R_a = R_b = R_c$.

Notation:

- v → terminal voltage of the armature (i.e., stator) coils
- v_L → inductance of the armature coils
- e → back EMF
- Ɛ → speed-dependent back EMF

Problem 3 (Prajakta Popat Wable)

Que.3. for phase (a)

$$\lambda_a = -N\phi_{max}\cos\theta_e + L i_a$$

induced back EMF voltage,

$$e_a = -\frac{d\lambda_a}{dt}.$$

$$= -\frac{d}{dt}\left(-\cancel{N}\phi_{max}\cos\theta_e) + L i_a\right)$$

$$= -N\phi_{max}\omega_e\sin\theta_e - L\frac{di_a}{dt}.$$

$$e_a = N\phi_{max}\omega_e\sin\theta_e.$$

for phase (b)

$$\lambda_b = -N\phi_{max}\cos(\theta_e - 2\pi/3) + L i_b.$$

$$e_b = -\frac{d\lambda_b}{dt}$$

$$= -N\phi_{max}\omega_e\sin(\theta_e - 2\pi/3) + \left(-L\frac{di_b}{dt}\right)$$

$$= -N\phi_{max}\omega_e\sin(\theta_e - 2\pi/3) - L\frac{di_b}{dt}.$$

$$e_b = N\phi_{max}\omega_e\sin(\theta_e - 2\pi/3)$$

for phase ⓒ

$$\lambda_c = -N\phi_{max}\cos\left(\theta_e + 2\pi/3\right) + L i_c.$$

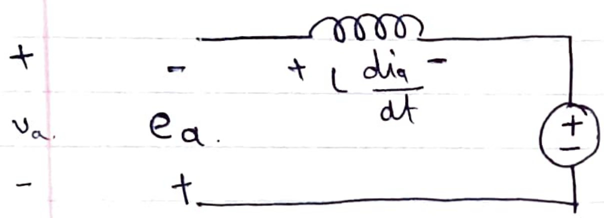
$$e_c = -\frac{d\lambda_c}{dt}.$$

$$= -\frac{d}{dt}\left(-N\phi_{max}\cos\left(\theta_e + 2\pi/3\right) + L i_c\right).$$

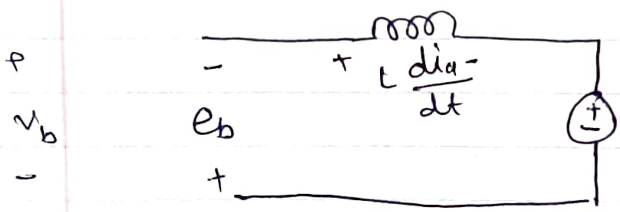
$$= -N\phi_{max}\,\omega_e \sin\left(\theta_e + 2\pi/3\right) - L\frac{di_c}{dt}.$$

$$\cancel{e_c = N\phi_{max}\,\omega_e\cos\left(\theta_e + 2\pi/3\right)}$$

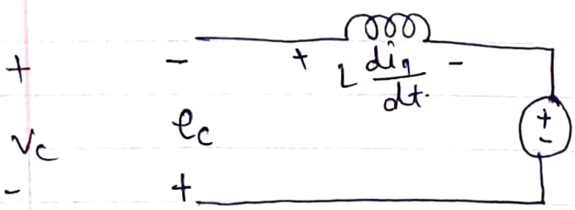
$$e_c = N\phi_{max}\,\omega_e\sin\left(\theta_e + 2\pi/3\right)$$



$E_a = N\phi_{max}\,\omega_e\sin\theta_e$



$E_b = N\phi_{max}\,\omega_e\sin\left(\theta_e - 2\pi/3\right)$



$E_c = N\phi_{max}\,\omega_e\sin\left(\theta_e + 2\pi/3\right)$