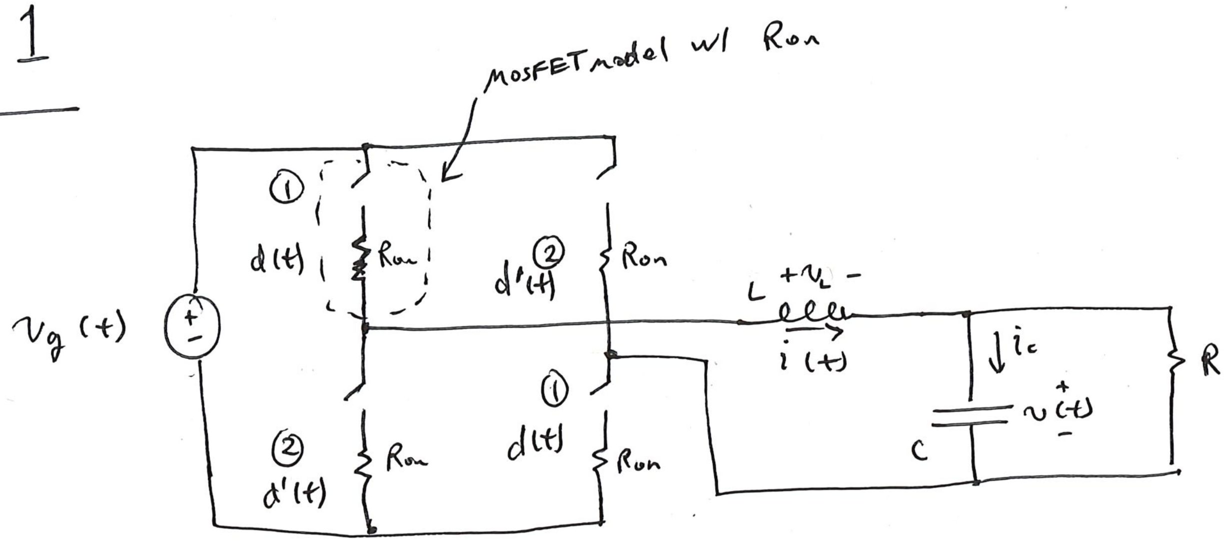
Homework # 1 solution

Problem 1



(a) Get avg equiv circuit

$$L\frac{d\langle i_L(t)\rangle}{dt} = d(t)\overbrace{\left(\langle v_g(t)\rangle - 2R_{on}\langle i(t)\rangle - \langle v(t)\rangle\right)}^{①\ v_L}$$
$$+\, d'(t)\underbrace{\left(-\langle v_g(t)\rangle - 2R_{on}\langle i(t)\rangle - \langle v(t)\rangle\right)}_{②}$$

$$= (d(t) - (\overbrace{1-d(t)}^{d'(t)}))\langle v_g(t)\rangle - 2R_{on}\langle i(t)\rangle - \langle v(t)\rangle$$

$$= (2d(t) - 1)\langle v_g(t)\rangle - 2R_{on}\langle i(t)\rangle - \langle v(t)\rangle$$

← eqn (1.1)

$$C\frac{d\langle v(t)\rangle}{dt} = d(t)\overbrace{\left(\langle i(t)\rangle - \frac{\langle v(t)\rangle}{R}\right)}^{①} + d'(t)\overbrace{\left(\langle i(t)\rangle - \frac{\langle v(t)\rangle}{R}\right)}^{②}$$

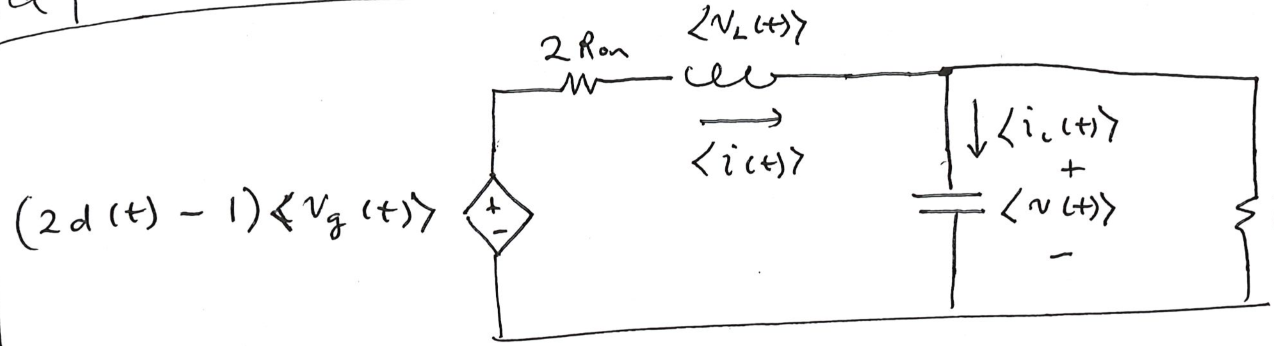
$$= \langle i(t)\rangle - \frac{\langle v(t)\rangle}{R} \quad (1.2)$$

Draw as equiv ckt

(1) → KVL for inductor

(2) → KCL for cap

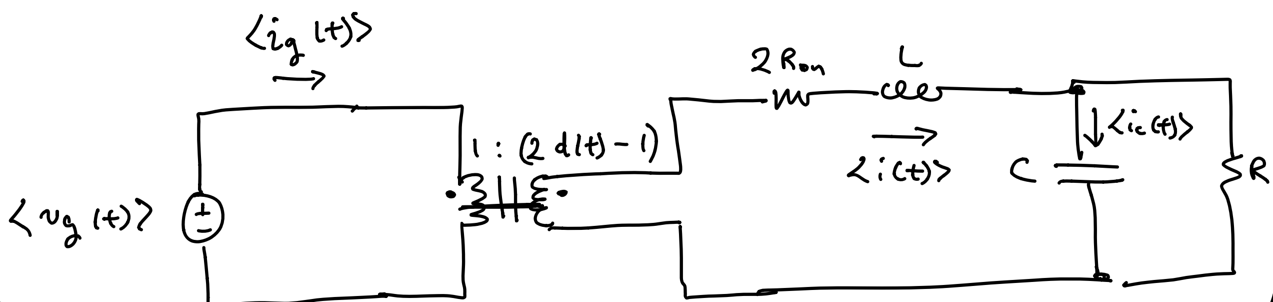
1a



OR

which can also be drawn as

1a



This representation is more complete since it shows the input source side current too.

(b) Put into Form $\dot{x} = f(x, u)$

$\div$ (1.1) by L $\&$ $\div$ (1.2) by C to get

$$\boxed{1b}$$

$$\dot{x} = \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \langle i(t) \rangle \\ \langle v(t) \rangle \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \frac{1}{L}\left[(2d(t)-1)\langle v_g(t)\rangle - 2R_{on}\langle i(t)\rangle - \langle v(t)\rangle\right] \\ \frac{1}{C}\left[\langle i(t)\rangle - \frac{\langle v(t)\rangle}{R}\right] \end{bmatrix}}_{f(x,u)}$$

$$= \begin{bmatrix} f_1(x,u) \\ f_2(x,u) \end{bmatrix}$$

where $u = \begin{bmatrix} d(t) \\ \langle v_g(t) \rangle \end{bmatrix}$

(c) Derive S.S. X given U

- Look at (1.1) - (1.2) where $d(t) = D$, $\langle v_g(t) \rangle = V_g$ and set derivatives to zero, AKA "balance equations"
- we get:

$$0 = (2D-1)V_g - 2R_{on}I - V \qquad (1.3)$$

&

$$0 = I - \frac{V}{R} \qquad (1.3)$$

From (1.3) $\rightarrow$ $V = IR$ (1.4)

(1.4) $\rightarrow$ (1.3)

$$0 = (2D-1)V_g - 2R_{on}I - IR$$

$$= (2D-1)V_g - I(2R_{on} + R)$$

$$\rightarrow \boxed{I = \frac{(2D-1)V_g}{(2R_{on}+R)}} \qquad (1.5)$$

(1.5) $\rightarrow$ (1.4)

$$\boxed{V = \frac{(2D-1)V_g R}{(2R_{on}+R)}} \qquad (1.6)$$

$$\rightarrow X = \begin{bmatrix} I \\ V \end{bmatrix} = \frac{(2D-1)V_g}{2R_{on}+R}\begin{bmatrix} R \\ 1 \end{bmatrix} = X \qquad (1.7)$$

(d) Get linearized model. $\dot{x} \approx A\,(x(t) - X) + B\,(u(t) - U)$ (4)

- Compute A

$$A = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}\right|_{X,U} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial \langle i \rangle} & \frac{\partial f_1}{\partial \langle v \rangle} \\ \frac{\partial f_2}{\partial \langle i \rangle} & \frac{\partial f_2}{\partial \langle v \rangle} \end{bmatrix}\right|_{X,U}$$

$$= \boxed{\begin{bmatrix} -\frac{2R_{on}}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} = A} \qquad (1.8)$$

- Compute B

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial d} & \frac{\partial f_1}{\partial \langle v_g \rangle} \\ \frac{\partial f_2}{\partial d} & \frac{\partial f_2}{\partial \langle v_g \rangle} \end{bmatrix}$$

$$= \left.\begin{bmatrix} \frac{2}{L}\langle v_g(t)\rangle & \frac{2}{L}d(t) \\ 0 & 0 \end{bmatrix}\right|_{X,U} = \boxed{\begin{bmatrix} \frac{2}{L}V_g & \frac{2D-1}{L} \\ 0 & 0 \end{bmatrix} = B} \qquad (1.9)$$

- Combine result in (C) w/ A & B:

$\boxed{1d}$

$$\dot{x} = \underbrace{\begin{bmatrix} -\frac{2R_{on}}{L} & -\frac{1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix}}_{A} \left(\underbrace{\begin{bmatrix} \langle i(t) \rangle \\ \langle v(t) \rangle \end{bmatrix}}_{x(t)} - \underbrace{\frac{(2D-1)V_g}{2R_{on}+R} \begin{bmatrix} R \\ 1 \end{bmatrix}}_{X} \right) + \ldots$$

...

$$
+ \underbrace{\begin{bmatrix} \frac{2}{L} V_g & \frac{2D-1}{L} \\ 0 & 0 \end{bmatrix}}_{B} \left(\underbrace{\begin{bmatrix} d(t) \\ \langle v_g(t) \rangle \end{bmatrix}}_{u(t)} - \underbrace{\begin{bmatrix} D \\ V_g \end{bmatrix}}_{U} \right) \qquad (1.10)
$$

1d

e) See plots in solution simulations.

As mentioned in pdf, let $D = d(0)$ & $V_g = v_g(0)$

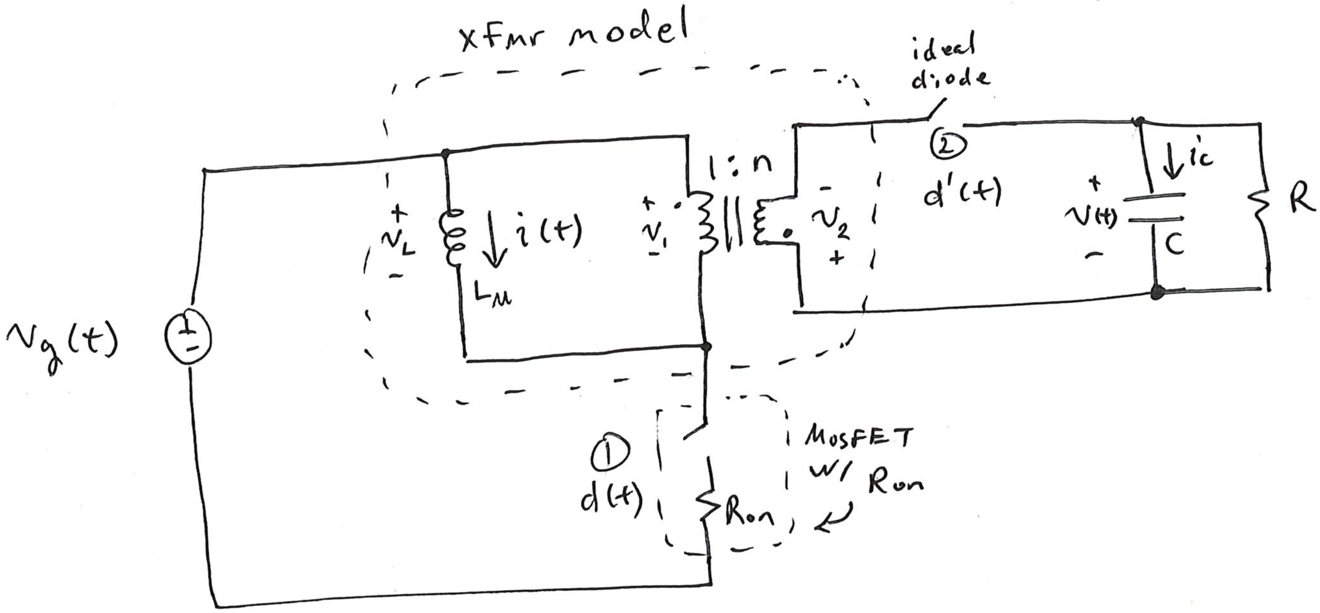
$$
\Rightarrow \quad D = \tfrac{1}{2} \;\; \& \;\; V_g = 100V \qquad (1.11)
$$

Plugging in (1.11) → (1.7) we get

$$
X = \cancel{\frac{(\overbrace{2\tfrac{1}{2} - 1}^{0})100V}{2R_{on} + R}}^{\,0} \begin{bmatrix} R \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
$$

Problem #2

Flyback w/ transformer model:



(a) Get avg ckt.

- Follow details from HW5 & Flyback to get RHS of eqns below:

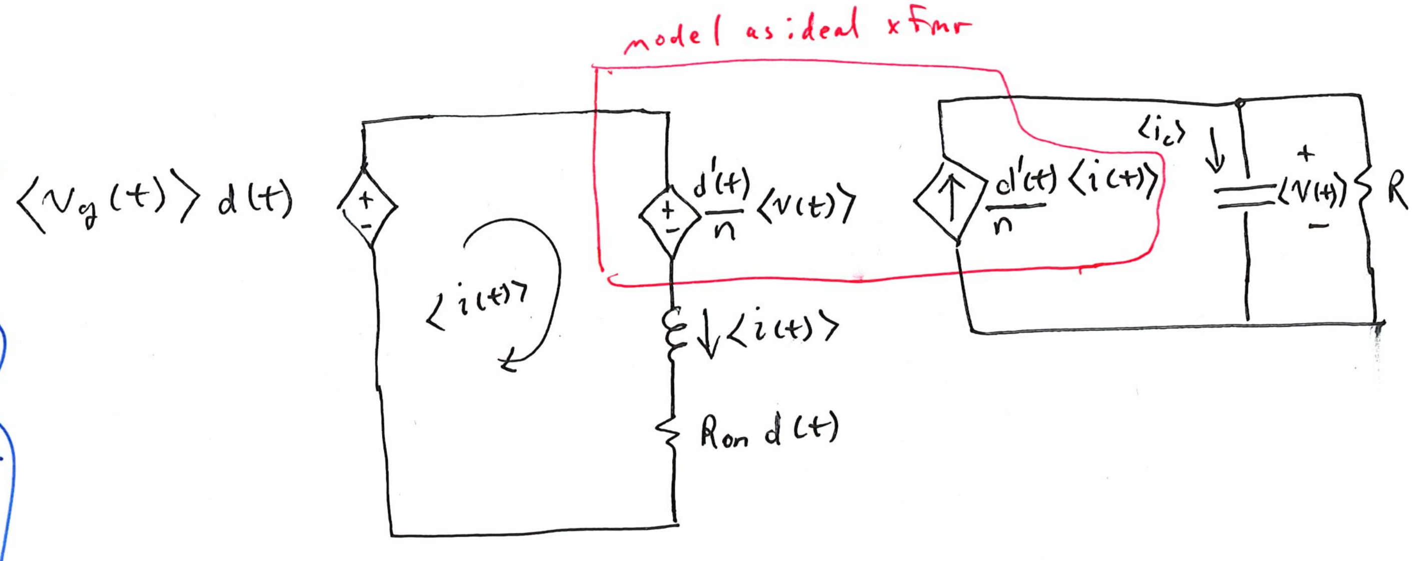
$$L\,\frac{d\langle i_L(t)\rangle}{dt} = d(t)\overbrace{\Big(\langle v_g(t)\rangle - \langle i(t)\rangle R_{on}\Big)}^{①\ v_L} - d'(t)\underbrace{\Big(\big(\langle v(t)\rangle\big)\frac{1}{n}}_{②\ v_L} \qquad (2.1)$$

and

$$C\,\frac{d\langle v(t)\rangle}{dt} = d(t)\left(\frac{\overbrace{-\langle v(t)\rangle}^{i_c\ ①}}{R}\right) + d'(t)\overbrace{\left(\frac{\langle i(t)\rangle}{n} - \frac{\langle v(t)\rangle}{R}\right)}^{②\ i_c}$$

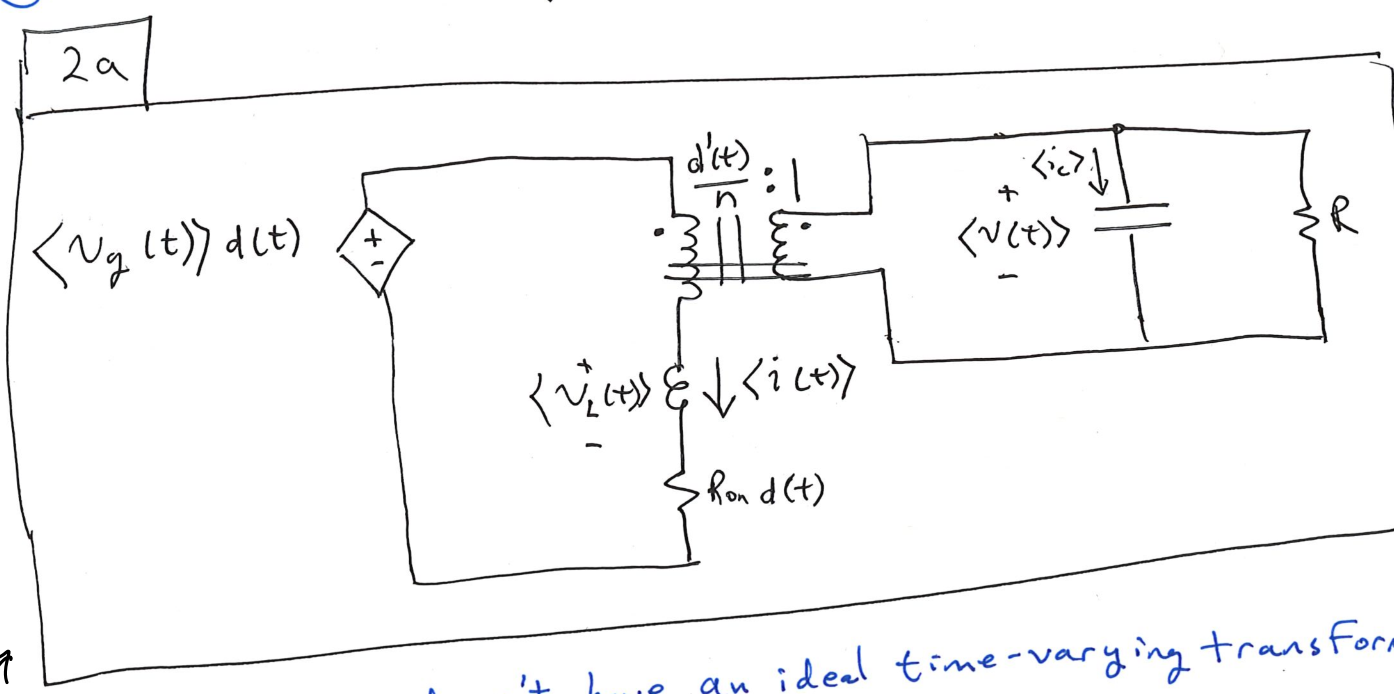
$$= -\frac{\langle v(t)\rangle}{R} + d'(t)\,\frac{\langle i(t)\rangle}{n} \qquad (2.2)$$

Draw as



for PLECS simulation

$\Downarrow$

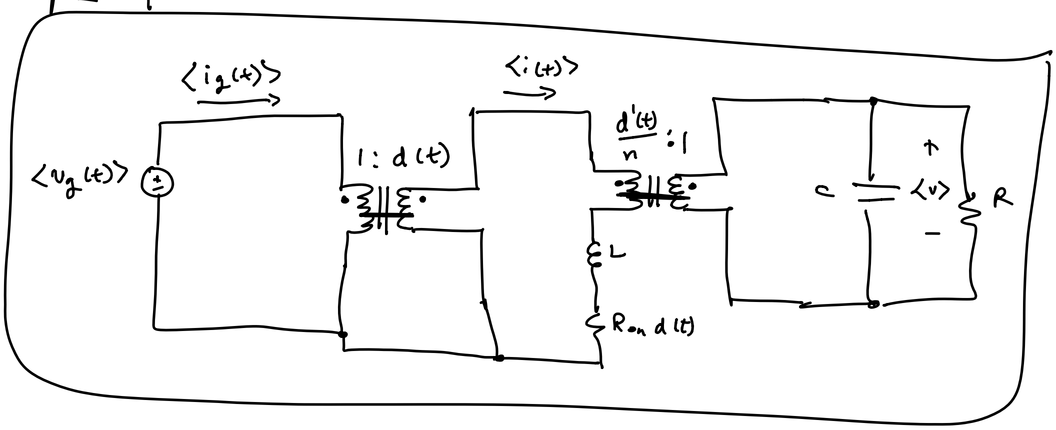


OR

* Since PLECS doesn't have an ideal time-varying transformer, you have to build your averaged simulation using the circuit above

Can also draw equivalently as

2a



This diagram is more complete & shows the input source.

(b) Put into form $\dot{x} = f(x, u)$

from (2.1) – (2.2) we get

$$\dot{x} = \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \langle i(t) \rangle \\ \langle v(t) \rangle \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{L}\left(d(t)\left(\langle v_g(t) \rangle - \langle i(t) \rangle R_{on}\right) - \frac{(1-d(t))}{n}\langle v(t) \rangle \right) \\ \frac{1}{C}\left(-\frac{\langle v(t) \rangle}{R} + (1-d(t))\frac{\langle i(t) \rangle}{n} \right) \end{bmatrix}$$

$$= f(x, u)$$

again $u = \begin{bmatrix} d(t) \\ \langle v_g(t) \rangle \end{bmatrix}$

(c) Derive S.S. X.

Solve balance equations ... obtain V, I in terms of independent variables (V_g, D). Set (2.1) – (2.2) to 0.

$$0 = D\left(V_g - I R_{on}\right) - \frac{(1-D)}{n} V \qquad (2.3)$$

$$0 = -\frac{V}{R} + (1-D)\frac{I}{n} \qquad (2.4)$$

Get I from (2.4),

$$I = \frac{Vn}{R(1-D)} \qquad (2.5)$$

(2.5) → (2.3) to get

$$0 = D\left(V_g - \frac{V n R_{on}}{R(1-D)}\right) - \frac{(1-D)}{n}V$$

$$= DV_g - V\left(\frac{D n R_{on}}{R(1-D)} + \frac{(1-D)}{n}\right)$$

solve for V

$$\rightarrow \quad V = \frac{DV_g}{\left(\frac{D n R_{on}}{R(1-D)} + \frac{1-D}{n}\right)}$$

$$\boxed{= \frac{D(1-D)RV_g n}{D n^2 R_{on} + (1-D)^2 R} = V} \qquad (2,6)$$

Put $(2.6) \longrightarrow (2.5)$

$$I = \frac{n}{\cancel{R(1-D)}} \frac{D\cancel{(1-D)}\cancel{R}V_g n}{(Dn^2 R_{on} + 1-D)}$$

$$\rightarrow \boxed{\frac{n^2 V_g D}{Dn^2 R_{on} + (1-D)^2 R} = I} \qquad (2,7)$$

now put into vector form

$$X = \begin{bmatrix} I \\ V \end{bmatrix} = \frac{n D V_g}{(D n^2 R_{on} + (1-D)^2 R)} \begin{bmatrix} n \\ (1-D) R \end{bmatrix} \qquad (2.8)$$

2(d): Get linearized model: $\dot{x} \approx A(x(t) - X) + B(u(t) - U)$

$$A = \left. \begin{bmatrix} \frac{\partial f_1}{\partial \langle i \rangle} & \frac{\partial f_1}{\partial \langle v \rangle} \\ \frac{\partial f_2}{\partial \langle i \rangle} & \frac{\partial f_2}{\partial \langle v \rangle} \end{bmatrix} \right|_{X,U}$$

$$= \boxed{\begin{bmatrix} \frac{-1}{L} D R_{on} & \frac{-(1-D)}{L\, n} \\ \frac{(1-D)}{C\, n} & \frac{-1}{RC} \end{bmatrix} = A} \qquad (2.8)$$

and,

$$B = \left. \begin{bmatrix} \frac{\partial f_1}{\partial d} & \frac{\partial f_1}{\partial \langle v_g \rangle} \\ \frac{\partial f_2}{\partial d} & \frac{\partial f_2}{\partial \langle v_g \rangle} \end{bmatrix} \right|_{X,U} = \boxed{\begin{bmatrix} \frac{1}{L}\left(V_g - I R_{on} + \frac{V}{n}\right) & \frac{1}{L} D \\ \frac{-1}{C}\left(\frac{I}{n}\right) & 0 \end{bmatrix} = B} \qquad (2.9)$$

where the result in 2.8 gives us I.

Putting it all together

$$\dot{x} \approx \underbrace{\begin{bmatrix} \frac{-DR_{on}}{L} & \frac{-(1-D)}{nL} \\ \frac{1-D}{nC} & \frac{-1}{RC} \end{bmatrix}}_{A} \left(\underbrace{\begin{bmatrix} \langle i(t) \rangle \\ \langle v(t) \rangle \end{bmatrix}}_{x(t)} - \underbrace{\frac{nDV_g}{(n^2 R_{on} + 1 - D)} \begin{bmatrix} n \\ (1-D)R \end{bmatrix}}_{X} \right)$$

$$+ \underbrace{\begin{bmatrix} \frac{1}{L}\left(V_g - I R_{on} + \frac{V}{n}\right) & \frac{1}{L} D \\ -\frac{I}{nC} & 0 \end{bmatrix}}_{B} \left(\underbrace{\begin{bmatrix} d(t) \\ \langle v_g(t) \rangle \end{bmatrix}}_{u(t)} - \underbrace{\begin{bmatrix} D \\ V_g \end{bmatrix}}_{U} \right)$$

$\boxed{2(d)}$

2(e) See simulation plots.

Code calculates I s.s. variable first... all else carries forward logically.

Contents


```
clear all
clc
```

Problem 1 set up

```
% Parameters
fs = 25e3;
T = 1/fs;
L = 300e-6;
C = 10e-6;
R = 10;
Ron = 15e-3;

% Input values before and after step changes
vg_before = 100;
vg_after = 75;
tStepVg = 20e-3;

% duty cycle
A_before =.7;
A_after = .9;
freq=2*pi*50;
tStepDuty = 40e-3;

% In part (c) we get the ss values below
Vg = vg_before;
D = .5*(1+.7*sin(freq*0));
I = -(1-2*D)*Vg/(2*Ron+R);
V = -R*(1-2*D)*Vg/(2*Ron+R);

% State Space Set up
A=[-2*Ron/L -1/L; 1/C -1/(R*C)];
B=[2*Vg/L -(1-2*D)/L; 0 0];
X=[I; V];
U=[D; Vg];
```

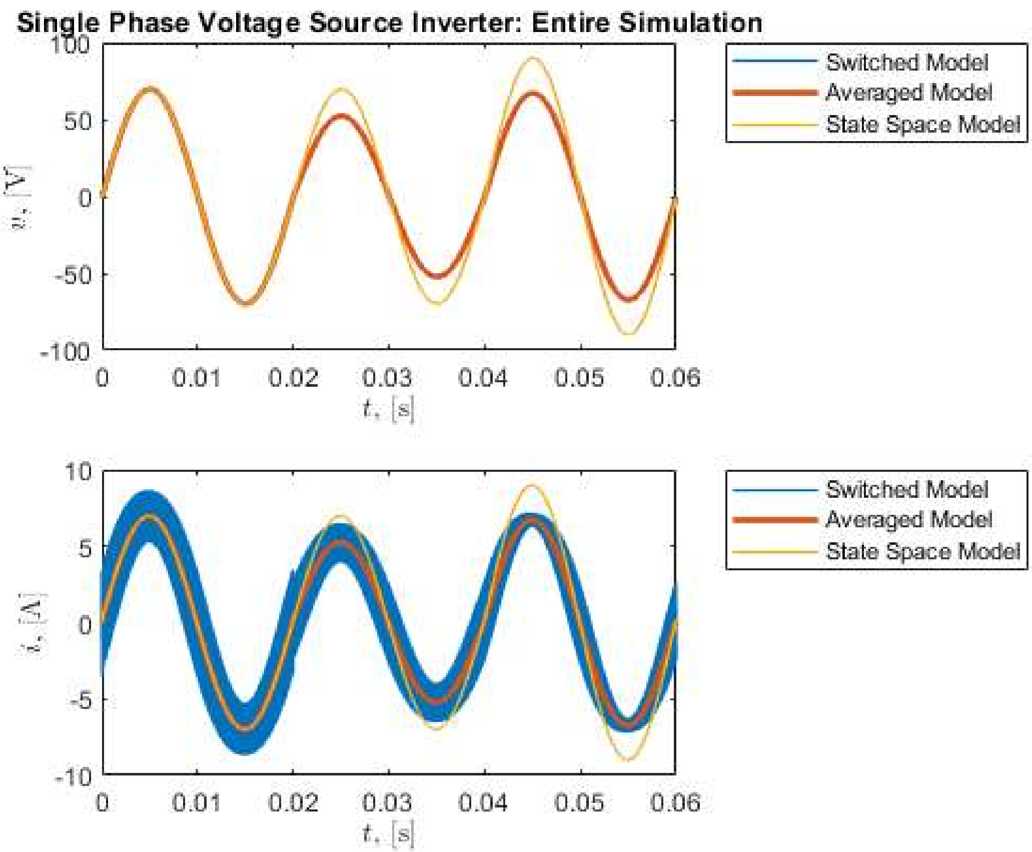
Problem 1 Run Simulation

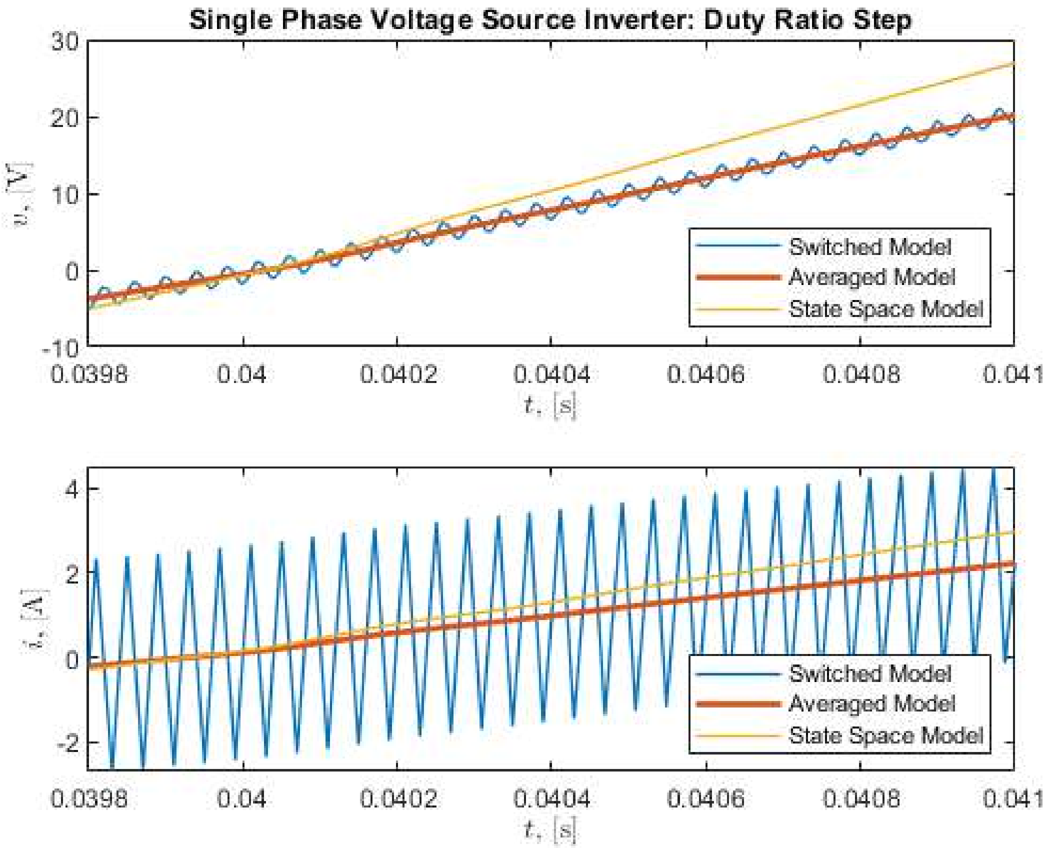
```
tStop = 60e-3;

[t_sw, x_sw, y_sw] = sim('Sim_EE452_HW6_Prob1_Switched', tStop);

i_sw = x_sw(:,1);
v_sw = x_sw(:,2);

[t_avg, x_avg, y_avg] = sim('Sim_EE452_HW6_Prob1_Averaged', tStop);

i_avg = x_avg(:,1);
v_avg = x_avg(:,2);

[t_ss,x_ss]=sim('Sim_EE452_HW6_Prob1_StateSpace_Simulink', tStop);

i_ss=x_ss(:,1);
v_ss=x_ss(:,2);
```

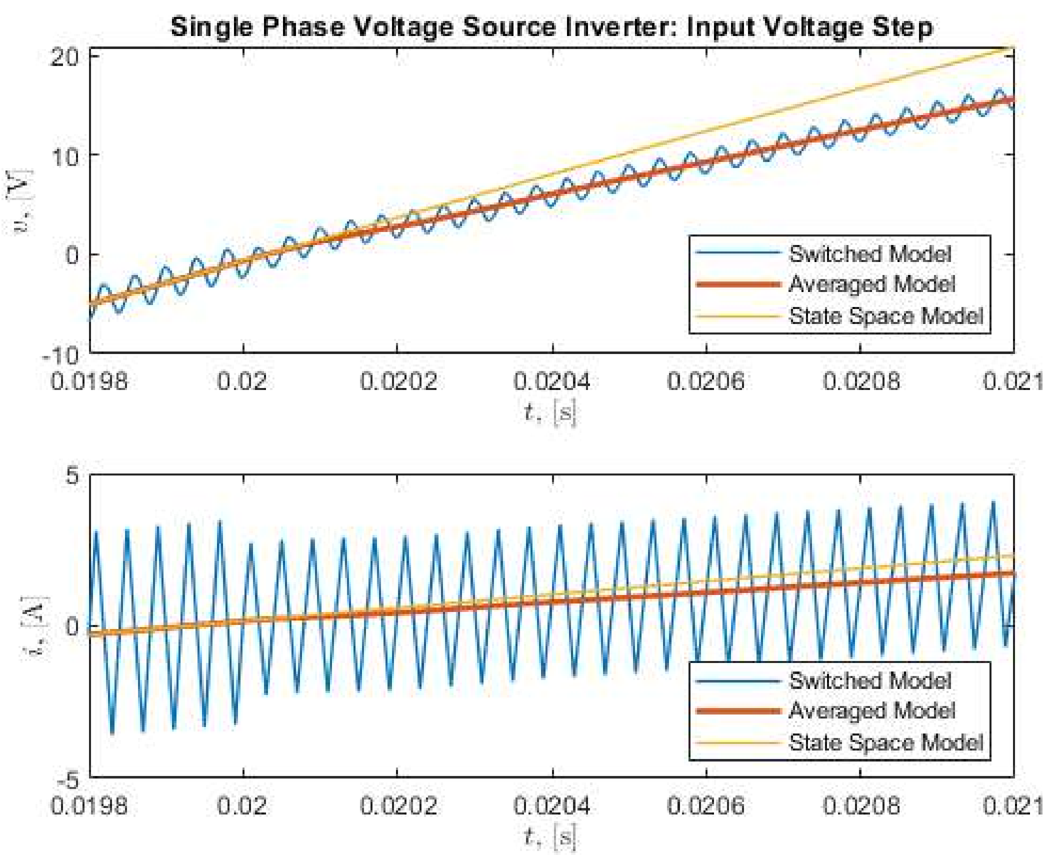
Problem 1 Plot Results

```
close all

for i = 1:3

figure (i)

a1 = subplot(2,1,1);
plot(t_sw, v_sw, 'LineWidth', 1)
hold on
plot(t_avg, v_avg, 'LineWidth',2)
plot(t_ss,v_ss,'LineWidth',1)
xlabel('$t$, [s]','Interpreter','latex');
ylabel('$v$, [V]','Interpreter','latex');
legend('Switched Model', 'Averaged Model','State Space Model','Location','SouthEast')

a2 = subplot(2,1,2);
plot(t_sw, i_sw, 'LineWidth', 1)
hold on
plot(t_avg, i_avg, 'LineWidth',2)
plot(t_ss,i_ss,'LineWidth',1)
xlabel('$t$, [s]','Interpreter','latex');
ylabel('$i$, [A]','Interpreter','latex');
legend('Switched Model', 'Averaged Model','State Space Model','Location','SouthEast')

if i == 1
    linkaxes([a1 a2],'x')
    xlim([0, tStop])
    subplot(2,1,1);
    legend('Switched Model', 'Averaged Model','State Space Model','Location','NorthEastOut
side')
    title('Single Phase Voltage Source Inverter: Entire Simulation')
    subplot(2,1,2);
    legend('Switched Model', 'Averaged Model','State Space Model','Location','NorthEastOut
side')
elseif i == 2
    linkaxes([a1 a2],'x')
```

```
    xlim([tStepDuty - 5*T, tStepDuty + 25*T])
    subplot(2,1,1);
    title('Single Phase Voltage Source Inverter: Duty Ratio Step')
else
    linkaxes([a1 a2],'x')
    xlim([tStepVg - 5*T, tStepVg + 25*T])
    subplot(2,1,1);
    title('Single Phase Voltage Source Inverter: Input Voltage Step')
end

end
```







Problem 2 Set up

```
% Parameters
n=2;
Lm=300*10^-6;
tStop = 60e-3;

% duty cycle
d_before=0.5;
d_after=0.75;

% In part (c) we get the ss values below
Vg = vg_before;
D = d_before;
I = n^2*D*Vg/(D*Ron*n^2+(1-D)^2*R);
V = D*(1-D)*R*n*Vg/(D*Ron*n^2+(1-D)^2*R);

A=[-D^2*Ron/Lm -(1-D)/(Lm*n); (1-D)/(C*n) -1/(R*C)];
B=[-Ron*I/Lm+V/(Lm*n)+Vg/L D/L; -I/(C*n) 0];
X=[I; V];
U=[D; Vg];
```

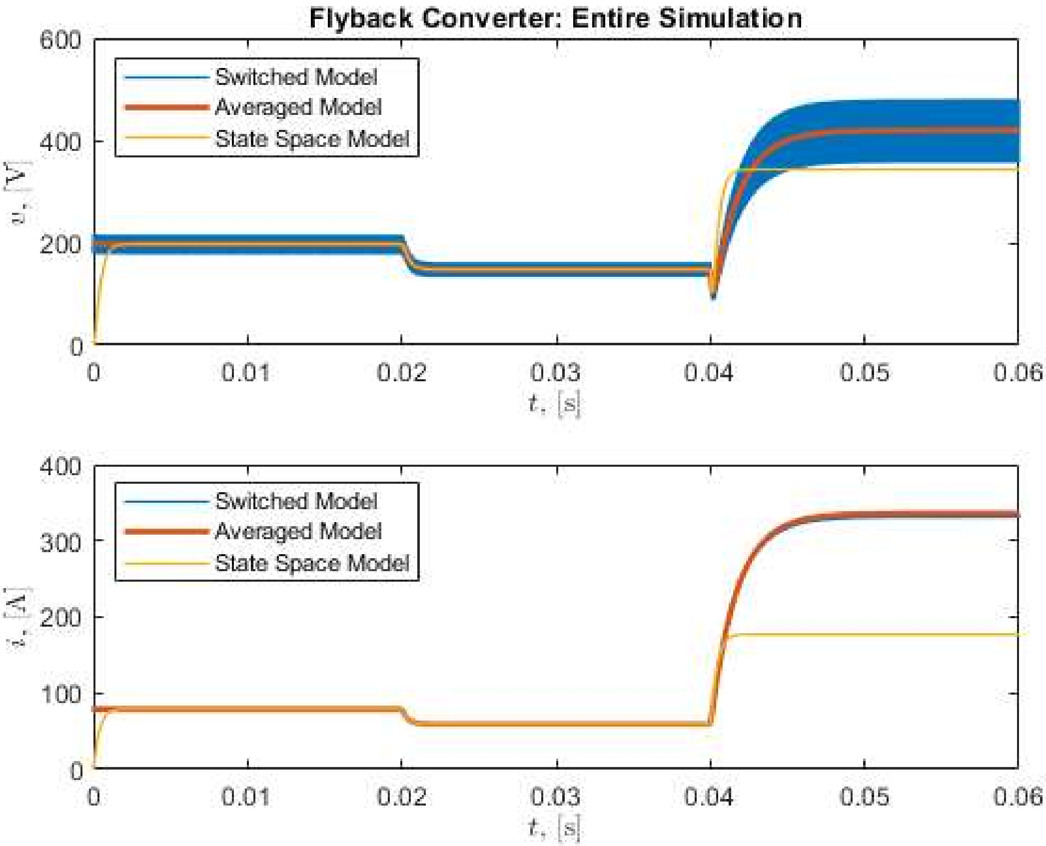
Problem 2 Run Simulation

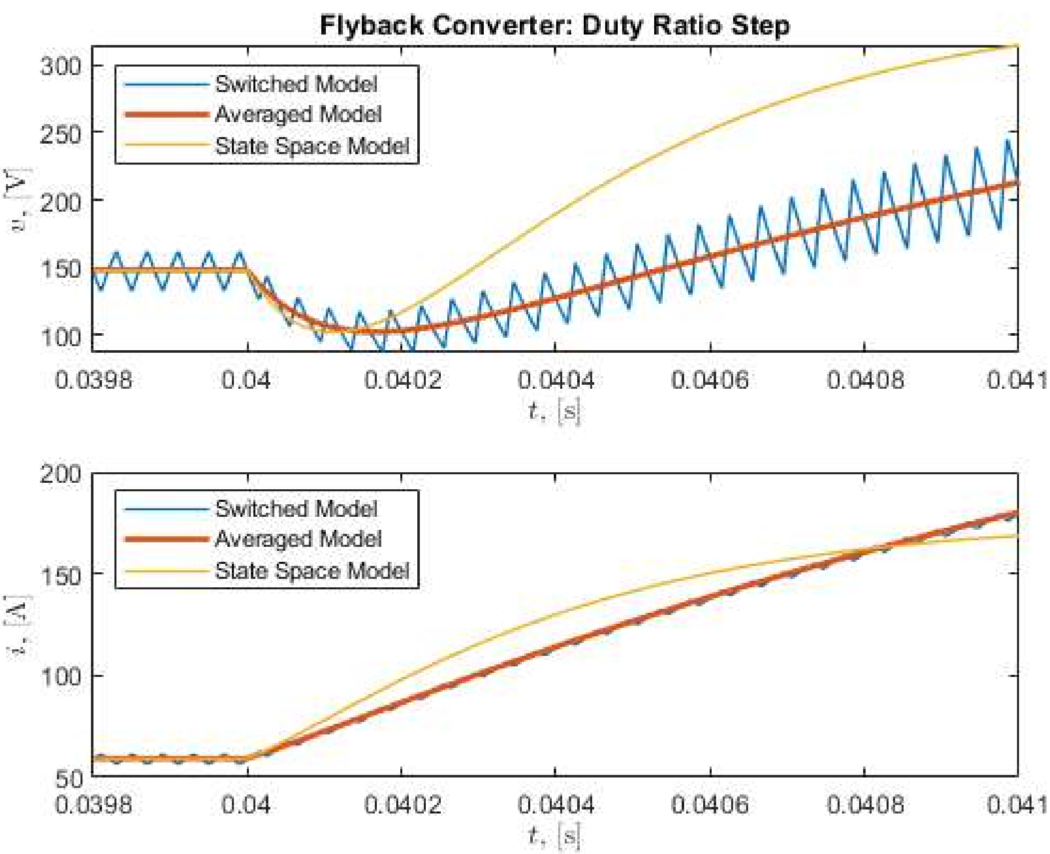
```
tStop = 60e-3;

[t_sw2, x_sw2, y_sw2] = sim('Sim_EE452_HW6_Prob2_Switched', tStop);

i_sw2 = y_sw2(:,1);
v_sw2 = y_sw2(:,2);

[t_avg2, x_avg2, y_avg2] = sim('Sim_EE452_HW6_Prob2_Averaged', tStop);

i_avg2 = y_avg2(:,1);
v_avg2 = y_avg2(:,2);

[t_ss,x_ss]=sim('Sim_EE452_HW6_Prob2_StateSpace_Simulink', tStop);

i_ss=x_ss(:,1);
v_ss=x_ss(:,2);
```

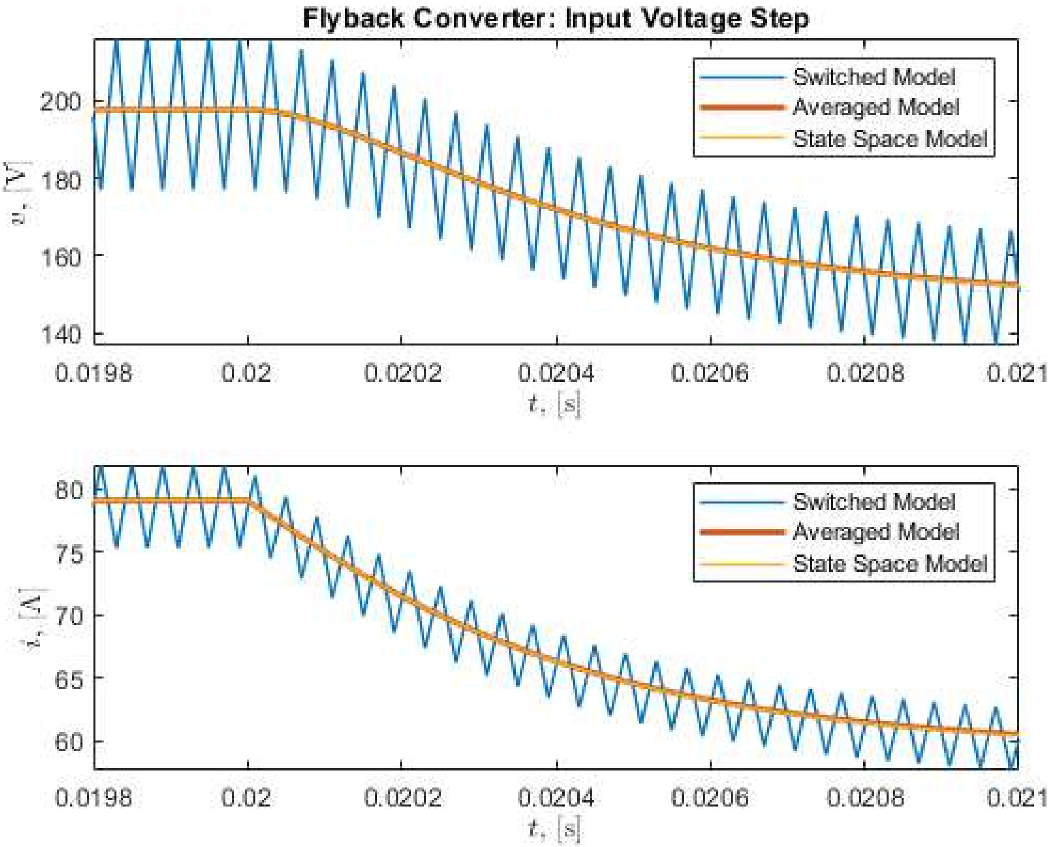
Problem 2 Plot Results

```
for i = 4:6

    figure (i)

a1 = subplot(2,1,1);
plot(t_sw2, v_sw2, 'LineWidth', 1)
hold on
plot(t_avg2, v_avg2, 'LineWidth',2)
plot(t_ss,v_ss,'LineWidth',1)
xlabel('$t$, [s]','Interpreter','latex');
ylabel('$v$, [V]','Interpreter','latex');
legend('Switched Model', 'Averaged Model','State Space Model')
```

```
a2 = subplot(2,1,2);
plot(t_sw2, i_sw2, 'LineWidth', 1)
hold on
plot(t_avg2, i_avg2, 'LineWidth',2)
plot(t_ss,i_ss,'LineWidth',1)
xlabel('$t$, [s]','Interpreter','latex');
ylabel('$i$, [A]','Interpreter','latex');
legend('Switched Model', 'Averaged Model','State Space Model')

if i == 4
    linkaxes([a1 a2],'x')
    xlim([0, tStop])
    subplot(2,1,1)
    legend('Switched Model', 'Averaged Model','State Space Model','Location','NorthWest')
    title('Flyback Converter: Entire Simulation')
    subplot(2,1,2)
    legend('Switched Model', 'Averaged Model','State Space Model','Location','NorthWest')
elseif i == 5
    linkaxes([a1 a2],'x')
    xlim([tStepDuty - 5*T, tStepDuty + 25*T])
    subplot(2,1,1)
    legend('Switched Model', 'Averaged Model','State Space Model','Location','NorthWest')
    title('Flyback Converter: Duty Ratio Step')
    subplot(2,1,2)
    legend('Switched Model', 'Averaged Model','State Space Model','Location','NorthWest')
elseif i == 6
    linkaxes([a1 a2],'x')
    xlim([tStepVg - 5*T, tStepVg + 25*T])
    subplot(2,1,1);
    title('Flyback Converter: Input Voltage Step')
end

end
```


Flyback Converter: Entire Simulation
Switched Model
Averaged Model
State Space Model
v, [V]
t, [s]
Switched Model
Averaged Model
State Space Model
i, [A]
t, [s]


Flyback Converter: Duty Ratio Step
Switched Model
Averaged Model
State Space Model
v, [V]
t, [s]
Switched Model
Averaged Model
State Space Model
i, [A]
t, [s]



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Hence

$$\boxed{f_{sw} \leq f_{sw_{max}} = 1\,MHZ}$$ 2a

2b) find $F_{sw_{min}}$ for a 2^{16} bit counter.

- The slowest switching occurs when Nr is maximized.
- Since we have 16 bits, then

$$N_r \leq N_{r_{max}} = 2^{16} = 65,536$$

- If we set $N_r = N_{r_{max}}$ & plug into equation (2.1),

we get

$$N_{r_{max}} = \frac{f_{clk}}{2 f_{sw}}$$

$$\Rightarrow F_{sw} = \frac{f_{clk}}{2 N_{r_{max}}} = \frac{100\,\mu HZ}{2 \cdot 2^{16}} = \frac{100\,\mu HZ}{2^{17}}$$

$$\boxed{\approx 762.9\,HZ = f_{sw_{min}}}$$ 2b

2c) Resolution with $N_r = 2^{16}$? 2c

$$\rightarrow \quad q_D = \frac{1}{N_{r_{max}}} = \frac{1}{2^{16}} \boxed{\approx 1,52 \times 10^{-5} = q_D}$$

$\boxed{\Rightarrow \text{Resolution of } 0,00152\ \%}$