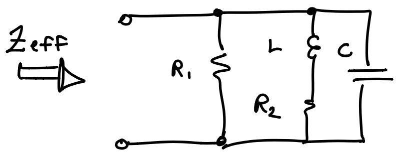
Homework #3

- Problem #1

Calculate $Z_{eff}(s)$ of ckt below & put into standard form:



$$Z_{eff}(s) = \frac{1}{\frac{1}{R_1} + \frac{1}{sL+R_2} + \frac{1}{1/sC}} \cdot \frac{R_1(sL+R_2)\left(\frac{1}{sC}\right)}{R_1(sL+R_2)\left(\frac{1}{sC}\right)}$$

$$= \frac{R_1(sL+R_2)\left(\frac{1}{sC}\right)}{(sL+R_2)\left(\frac{1}{sC}\right) + R_1\left(\frac{1}{sC}\right) + R_1(sL+R_2)} \cdot \frac{sC}{sC}$$

$$= \frac{R_1(sL+R_2)}{(sL+R_2) + R_1 + R_1(s^2LC + sR_2C)}$$

* collect terms by order

$$= \frac{sR_1L + R_1R_2}{s^2R_1LC + s(L+R_1R_2C) + \underline{R_2 + R_1}} \cdot \frac{\frac{1}{R_2+R_1}}{\frac{1}{R_2+R_1}}$$

* massage into form $G = G_0 \dfrac{\left(1-\frac{s}{\omega_z}\right)}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$

Observation:

* - to get 1 term in denom, must ÷ by (R_2+R_1)

$$= \frac{\frac{1}{R_1+R_2}\left(R_1R_2 + sR_1L\right)}{1 + s\frac{(L+R_1R_2C)}{R_1+R_2} + s^2\boxed{\frac{R_1LC}{R_1+R_2}}}$$

reshuffled order of terms slightly to match targeted form above

this equals $\frac{1}{\omega_o^2}$

* Next steps

- Look @ s^2 coefficient in denom & try to deduce $\frac{1}{\omega_o^2}$ factor

$$\Rightarrow \boxed{\frac{1}{\omega_0} = \sqrt{\frac{R_1LC}{R_1+R_2}}} \quad (1)$$

- Now look at s coeff. in denom. It follows that

$$s \quad \frac{L+R_1R_2C}{R_1+R_2} = \frac{1}{Q\omega_0}$$

* use (1)

$$= \frac{1}{Q}\sqrt{\frac{R_1LC}{R_1+R_2}}$$

Now solve for Q

$$\frac{(\quad)}{\sqrt{(\quad)}} = \frac{(\quad)^1}{(\quad)^{1/2}} = (\quad)^{1/2} = \sqrt{(\quad)}$$

$$Q = \sqrt{\frac{R_1LC}{R_1+R_2}}\ \frac{R_1+R_2}{L+R_1R_2C}$$

$$\boxed{\begin{aligned} &= \frac{\sqrt{(R_1LC)(R_1+R_2)}}{L+R_1R_2C} \\ &= Q \end{aligned}} \quad (2)$$

- denom is basically fully defined in desired form now. Focus on num next!

$$= \frac{\frac{1}{R_1+R_2}\left(R_1R_2 + sR_1L\right)\cdot\frac{R_1R_2}{R_1R_2}}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_o}\right)^2}$$

where Q & ω_o are in (1)–(2)

* Next
- want num in $G_o\left(1+\frac{s}{\omega_z}\right)$ form. Factor out R_1R_2 to get "1" term.

$$= \frac{\overbrace{\frac{R_1 R_2}{R_1+R_2}}^{G_0}\left(1 + s\,\overbrace{\frac{\cancel{R_1} L}{\cancel{R_1} R_2}}^{\frac{-1}{\omega_z} = \frac{-L}{R_2}}\right)}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$\boxed{= \frac{G_0\left(1 - \frac{s}{\omega_z}\right)}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2} = Z_{ef}(s)}$$

done !

$\boxed{\# 1}$

where

$$\boxed{\begin{aligned} G_0 &= \frac{R_1 R_2}{R_1+R_2} \\ \omega_z &= -\frac{R_2}{L} \\ Q &= \frac{\sqrt{(R_1 L C)(R_1+R_2)}}{L + R_1 R_2 C} \\ \omega_0 &= \sqrt{\frac{R_1+R_2}{R_1 L C}} \end{aligned}}$$

— Problem #2

a) Draw $G_1(j\omega)$ & $G_2(j\omega)$ asymptote plots

• Look @ $G_1(j\omega)$. Use material in lecture # 8.

$f=\frac{\omega}{2\pi}$	$\|G_1(j\omega)\|$	$\|G_1(j\omega)\|_{dB}$	$\angle G_1(j\omega)$
0	100	40 dB	0°
$f_0 = 1KHz$	$\frac{100}{\sqrt{2}}$	$\approx (40-3)$ dB	-45°
∞	drops as $\left(\frac{f}{f_0}\right)^{-1}$	$\rightarrow -20$dB/decade slope	-90°

$$G_1 = \frac{100}{1+\frac{s}{\omega_0}}$$

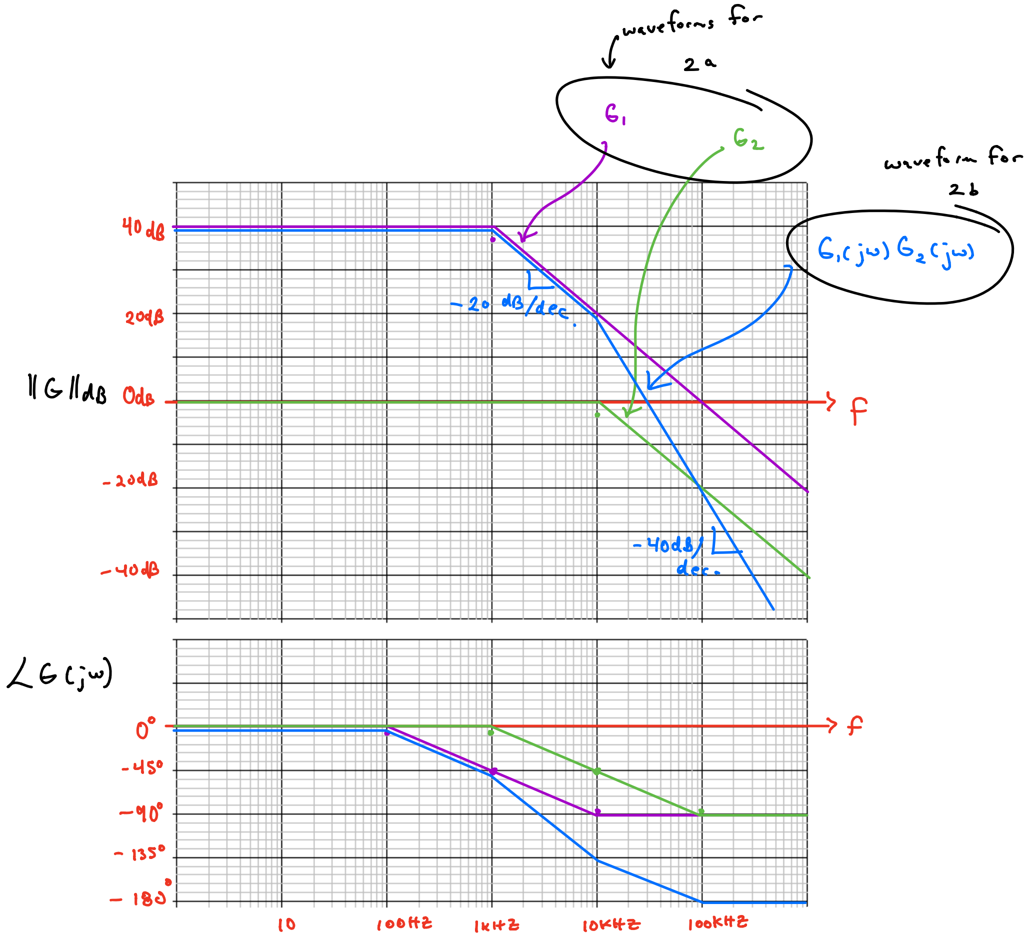
$\omega_0 = 2\pi\, 1KHz$

• Look @ $G_2(j\omega)$:

$f=\frac{\omega}{2\pi}$	$\|G_2(j\omega)\|$	$\|G_2(j\omega)\|_{dB}$	$\angle G_2(j\omega)$
0	1	0 dB	0°
$f_0 = 10KHz$	$\frac{1}{\sqrt{2}}$	≈ -3 dB	-45°
∞	drops as $\left(\frac{f}{f_0}\right)^{-1}$	$\rightarrow -20$dB/decade slope	-90°

$$G_2 = \frac{1}{1+\frac{s}{2\pi\, 10KHz}}$$

b) Add breakpoints of asymptotes to approximate product of G_1 & G_2.



- Problem #3

a) Find $x_{A/D}$ given $v_{A/D} = 0.75\text{ V}$:

$$
\begin{aligned}
x_{A/D} &= \text{round}\left((2^{n_{A/D}} - 1)\frac{v_{A/D}}{V_{FS}}\right) \\
&= \text{round}\left((2^{12}-1)\left(\frac{0.75\text{ V}}{3\text{ V}}\right)\right) \\
&= \text{round}(1{,}023.75) \\
&= \boxed{1{,}024 \quad \text{when } v_{A/D} = 0.75\text{V}}
\end{aligned}
$$

3a

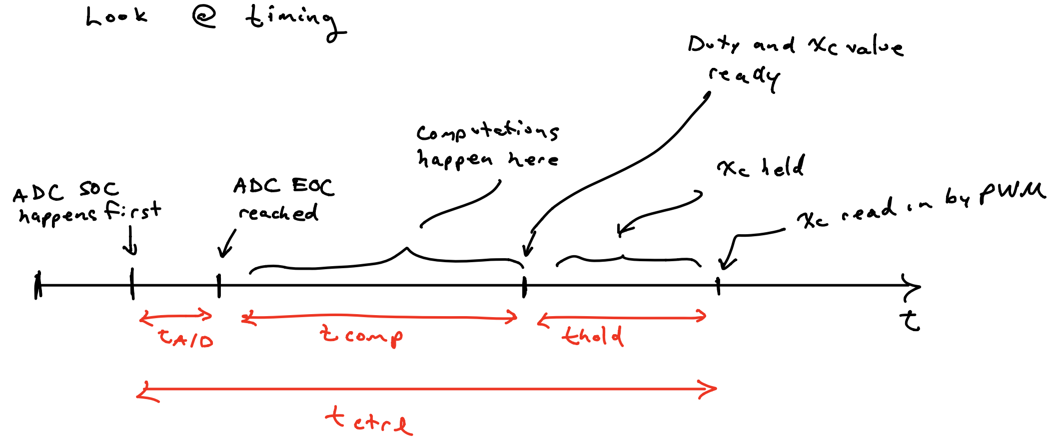
b) ADC voltage resolution

$$q_{A/D} = \frac{V_{FS}}{2^{n_{A/D}}} = \frac{3\text{ V}}{2^{12}} \approx \boxed{732\ \mu\text{V} \approx q_{A/D}}$$

3b

c) Compute Thold & Thold / Tctrl.

Look @ timing



Waveform diagram below gives more expletion of timing illustrated above. From above, it is clear that

$$t_{ctrl} = t_{A/D} + t_{comp} + t_{hold}$$

Since the PWM is configured to read x_c only @ counter reset, it follows that t_{ctrl} is fixed at

$$\underbrace{t_{ctrl} = T_s}$$

↳ PWM reads x_c once per sw. cycle.

Calculations below are easier if t_{ctrl} written in terms of T_{clk}.

$$\Rightarrow \quad t_{ctrl} = T_s = (100kHz)^{-1} = \left(\frac{100MHz}{10^3}\right)^{-1}$$

$$= \left(\frac{f_{clk}}{10}\right)^{-1} = \frac{10^3}{f_{clk}} = 10^3 T_{clk}$$

Now solve for t_{hold}:

$$\Rightarrow \quad t_{hold} = t_{ctrl} - t_{A/D} - t_{comp}$$

$*$ where $t_{A/D} = 50\,T_{clk}$

$t_{comp} = 500\,T_{clk}$

$t_{ctrl} = 1000\,T_{clk}$

$$= 1000\,T_{clk} - 50\,T_{clk} - 500\,T_{clk}$$

$$\boxed{\begin{aligned} &= 450\,T_{clk} = 4\mu s \\ &= t_{hold} \end{aligned}} \quad \boxed{3c}$$

and

$$\boxed{\frac{t_{hold}}{t_{ctrl}} = \frac{450\,T_{clk}}{1000\,T_{clk}} = 0.45 = 45\%} \quad \boxed{3c}$$

↳ This tells us that 45% of the time interval between ADC SOC & when x_c is read by PWM is unused. Hence, we have a 45% margin to do extra computations if necessary.

d) Get t_{ctrl}, t_{mod}, & t_d.

- From part c, we know $\boxed{t_{ctrl} = T_s = 1000\, T_{clk}.}$
- For asymmetric leading edge carrier,

$$\boxed{t_{mod} = D' T_s = 1000\, D' T_{clk}}$$

- Sum up

$$t_d = t_{ctrl} + t_{mod} = 1000\, T_{clk} + 1000\, D' T_{clk}$$

$$= 1000\, T_{clk} (\overbrace{1 + D'}^{1+1-D})$$

$$\boxed{t_d = 1000\, T_{clk} (2 - D)} \quad \boxed{3d}$$

(e) & (f): See next page.

Note
$$N_r = \frac{T_{clk}}{T_s} - 1 = \frac{100\,MHz}{100\,kHz} - 1$$

$$= \frac{\cancel{100} \times 10^{\cancel{6}\,3}}{\cancel{100} \times \cancel{10^3}} - 1 = 1000 - 1$$

$$= 999$$

Answer the questions below:

(a) If the voltage on the ADC pin, $v_{A/D}$, is 0.75 V, what digital integer value, $x_{A/D}$, will the ADC produce?

(b) What is the voltage resolution, $q_{A/D}$, of the ADC analog input?

(c) Compute T_{hold} and the ratio T_{hold}/T_{ctrl}. What does the value of T_{hold}/T_{ctrl} tell us about the computational burden on the processor? What happens to the ratio T_{hold}/T_{ctrl} as the controller requires more computations?

(d) Compute T_{ctrl}, T_{mod}, and T_d. Express your results in terms of T_{clk} and D.

(e) Sketch the binary PWM output $c(t)$ and sampled ADC value $x_{A/D}$ waveforms onto Figure 2. Overlay $x_{A/D}$ on the same plot as $v_{A/D}$.

(f) Add the following items onto Figure 2: Label the maximum feasible integer value of $x_{A/D}$ vertical axis, specify the DPWM counter maximum N_r, label the duty D, and the delays T_{ctrl} and T_{mod}. Also draw the delays $T_{A/D}, T_{comp}$, and T_{hold} within T_{ctrl} and try to draw them to scale.

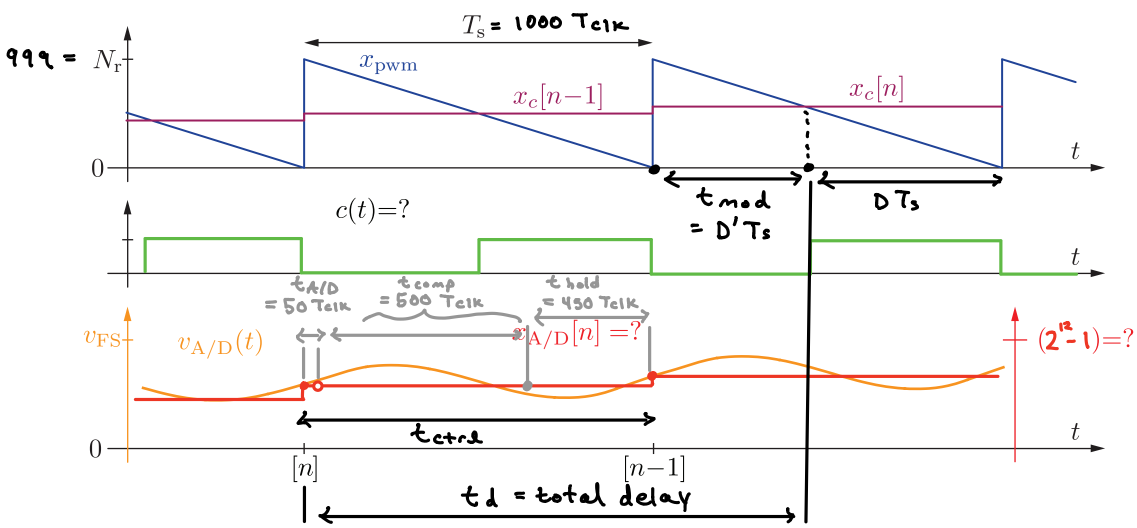


Figure 2: Digital system waveforms for Problem 3.