EE 458 – Power Electronics Controls
Homework 4

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1 Output Voltage Control of Buck-Boost Converter

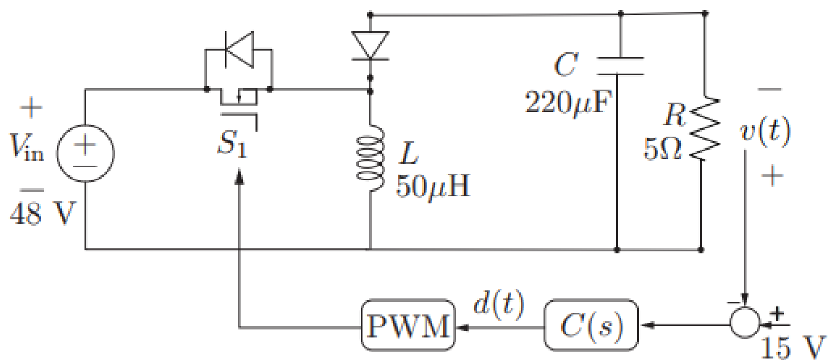


Figure 1: Buck-Boost Converter

Buck-Boost System

$$V = V_g \frac{D}{D'}$$

$$I = \frac{V}{RD'}$$

$$\hat{y}(s) = \Big(C(sI - A)^{-1}B + E\Big)\hat{u}(s)$$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix} \frac{\partial f_1}{\partial \langle \hat{i}(t)\rangle} & \frac{\partial f_1}{\partial \langle \hat{v}(t)\rangle} \\ \frac{\partial f_2}{\partial \langle \hat{i}(t)\rangle} & \frac{\partial f_2}{\partial \langle \hat{v}(t)\rangle} \end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix} 0 & \frac{-D'}{L} \\ \frac{D'}{C} & \frac{-1}{RC} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix} \frac{\partial f_1}{\partial \hat{d}(t)} & \frac{\partial f_1}{\partial \langle \hat{v}_g(t)\rangle} \\ \frac{\partial f_2}{\partial \hat{d}(t)} & \frac{\partial f_2}{\partial \langle \hat{v}_g(t)\rangle} \end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix} \frac{V_g + V}{L} & \frac{D}{L} \\ \frac{-I}{C} & 0 \end{bmatrix}$$

$$\hat{y}(s) = \Big(C(sI - A)^{-1}B + E\Big)\hat{u}(s)$$

$$\hat{y}(s) = G(s)u(s)$$

$$G(s) = \begin{bmatrix} G_{id}(s) & G_{ig}(s) \\ G_{vd}(s) & G_{vg}(s) \end{bmatrix} = \begin{bmatrix} \left(\frac{V_g}{D'}\right)\frac{RCs + (D+1)}{RLCs^2 + Ls + D'^2R} & \frac{D(RCs+1)}{RLCs^2 + Ls + D'^2R} \\ \left(\frac{V_g}{D'^2}\right)\frac{-DLs + D'^2R}{RLCs^2 + Ls + D'^2R} & \frac{DD'R}{RLCs^2 + Ls + D'^2R} \end{bmatrix}$$

1.1 Plant Transfer Function

$$G_{vd}(s) = \left(\frac{V_g}{D'^2}\right)\frac{-DLs + D'^2R}{RLCs^2 + Ls + D'^2R}$$

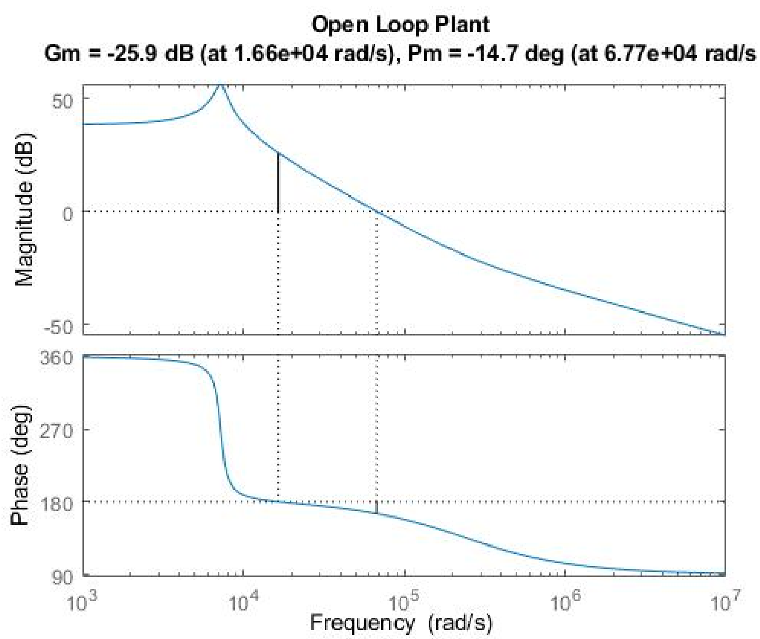


Figure 2: Open-Loop Plant: P(s)

1.2 Controller $C(s)$

Requirements:

- DC tracking
- High bandwidth but less than 10% f_{sw}
- 52°phase margin

```
s = tf('s');
% Plant Transfer Function (Open Loop)
    p = tf([0,-86821875,21168000000000.0],[4851,4410000,256000000000.0]);
    margin(p);
% C(s)P(s) where C(s) = Ki/s
% Target Bandwidth is 5000Hz
    Ki = 331.1;
    margin(Ki/s*p);
% C(s)P(s) where C(s) = Kp + Ki/s
% Add zero at crossover frequency
    fc = 5000;
    wc = 2*pi*fc;
    Kp = Ki/(wc);
    C = Kp + Ki/s;
    margin(C*p);
% C(s)L(s)P(s) where L(s) = (1+alpha*T*s)/(1+T*s)
    alpha = 5000;
    T = 1/(100*wc);
    lead = (1+alpha*T*s)/(1+T*s);
    bode(lead);
    margin(C*lead*p);
% Closed Loop Response C(s)L(s)P(s)
    li = C*lead*p;
    step(li/(li+1));
```

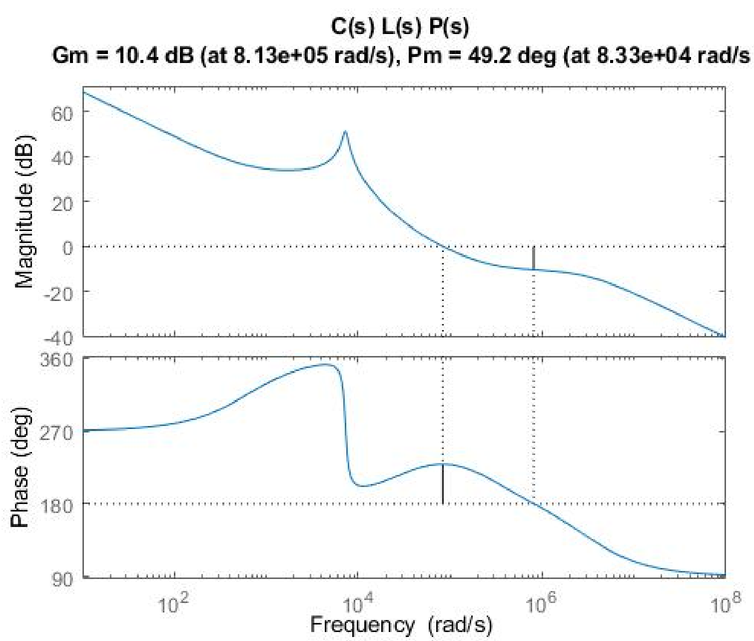


Figure 3: Phase Margin 49.2° at 13.25 kHz

1.3 PLECS

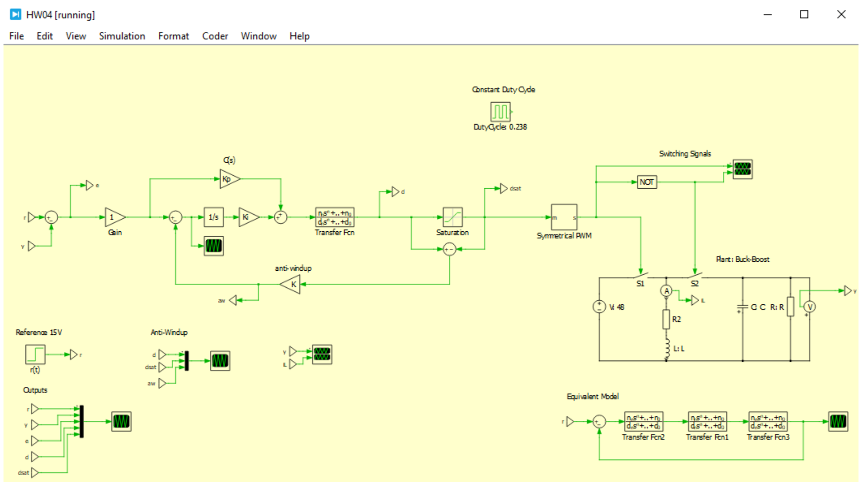


Figure 4: PLECS Implementation with Anti-Windup Logic

$$C(s) = \left(K_p + \frac{Ki}{s}\right)\left(\frac{1+\alpha Ts}{1+Ts}\right)$$

$$K_i = 331.1$$

$$K_p = 0.0105$$

$$\alpha = 5000$$

$$T = \frac{1}{100 \cdot (2\pi 5000)}$$

1.4 Increase Controller Bandwidth

I designed my controller to be approximately max bandwidth, given the controller requirements. (Phase margin is just under 52 degrees). In general, increasing bandwidth decreases the rise time but increases settling time. The controller will have more oscillations that eventually settle to the steady state value. If the controller bandwidth is increased beyond stability, the controller will always oscillate and never settle to the steady state value.

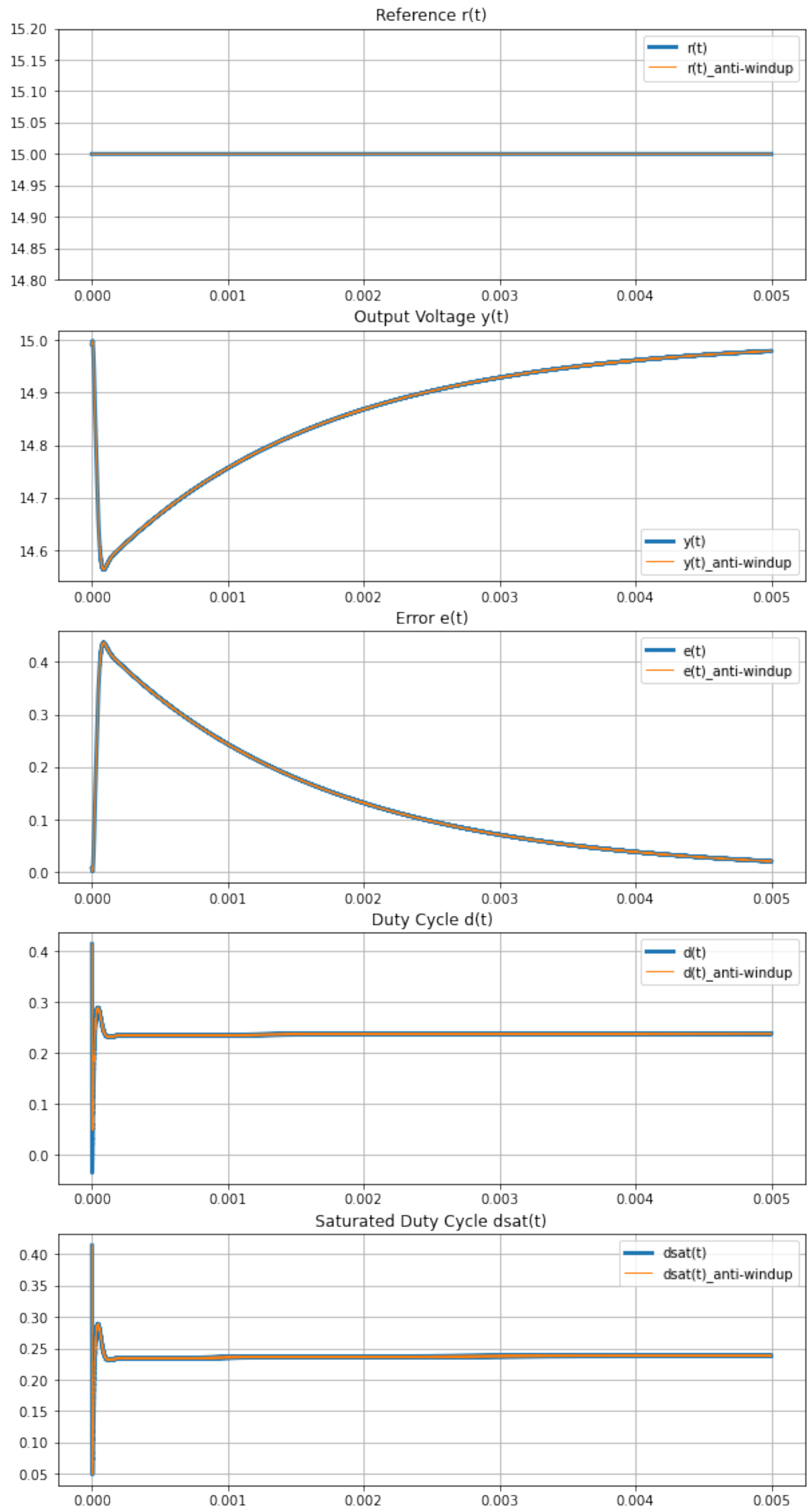


Figure 5: Closed Loop Response: With and Without Anti-Windup

1.5 Tustin Transformation

$$\frac{d(s)}{e(s)} = C(s) = \left(K_p + \frac{Ki}{s}\right)\left(\frac{1+\alpha T s}{1+Ts}\right)$$

$$\frac{d(s)}{e(s)} = \frac{(K_p s + K_i)(\alpha T s + 1)}{s(Ts+1)}$$

$$\frac{d(s)}{e(s)} = \frac{\left(\frac{K_i}{s} + K_p\right)(\alpha T s + 1)}{Ts+1}$$

$$\left.\frac{d(z)}{e(z)}\right|_{s=\frac{2}{T_{samp}}\frac{1-z^{-1}}{1+z^{-1}}} = \frac{\left(\frac{K_i T_{samp}}{2}\frac{1+z^{-1}}{1-z^{-1}} + K_p\right)\left(\frac{2\alpha T}{T_{samp}}\frac{1-z^{-1}}{1+z^{-1}} + 1\right)}{\frac{2T}{T_{samp}}\frac{1-z^{-1}}{1+z^{-1}} + 1}$$

$$= \frac{\left(K_i T_{samp}(1+z^{-1}) + 2K_p(1-z^{-1})\right)\left(2\alpha T(1-z^{-1}) + T_{samp}(1+z^{-1})\right)}{2(1-z^{-1})\left(2T(1-z^{-1}) + T_{samp}(1+z^{-1})\right)}$$

$$= \frac{-2\alpha K_i T_{samp} T(z^{-2}) + 2\alpha K_i T_{samp} T + 4\alpha K_p T(z^{-2}) - 8K_p T + T_{samp}(z^{-1}) + T_{samp}}{4T(z^{-2}) - 8T(z^{-1}) + 4T - T_{samp}(z^{-1}) - T_{samp}}$$

$$= \frac{\left(-(2\alpha K_i T_{samp} T) + (4\alpha K_p T)\right)z^{-2} + \left(T_{samp}\right)z^{-1} + \left(2\alpha K_i T_{samp} T + -8K_p T + T_{samp}\right)}{\left(4T\right)z^{-2} - \left(8T + T_{samp}\right)z^{-1} + \left(4T - T_{samp}\right)}$$

$$d(z)\left(4T - T_{samp}\right) - z^{-1}d(z)\left(8T + T_{samp}\right) + z^{-2}d(z)\left(4T\right) =$$

$$e(z)\left(2\alpha K_i T_{samp} T + -8K_p T + T_{samp}\right) + e(z)z^{-1}\left(T_{samp}\right) + e(z)z^{-2}\left(-(2\alpha K_i T_{samp} T) + (4\alpha K_p T)\right)$$

$$d(z) = z^{-1}d(z)\left(\frac{8T + T_{samp}}{4T - T_{samp}}\right) - z^{-2}d(z)\left(\frac{4T}{4T - T_{samp}}\right) +$$

$$e(z)\left(\frac{2\alpha K_i T_{samp} T + -8K_p T + T_{samp}}{4T - T_{samp}}\right) + e(z)z^{-1}\left(\frac{T_{samp}}{4T - T_{samp}}\right) + e(z)z^{-2}\left(\frac{-(2\alpha K_i T_{samp} T) + (4\alpha K_p T)}{4T - T_{samp}}\right)$$

Thus

$$d[k] = d[k-1]\left(\frac{8T + T_{samp}}{4T - T_{samp}}\right) - d[k-2]\left(\frac{4T}{4T - T_{samp}}\right) +$$

$$e[k]\left(\frac{2\alpha K_i T_{samp} T + -8K_p T + T_{samp}}{4T - T_{samp}}\right) + e[k-1]\left(\frac{T_{samp}}{4T - T_{samp}}\right) + e[k-2]\left(\frac{-(2\alpha K_i T_{samp} T) + (4\alpha K_p T)}{4T - T_{samp}}\right)$$

$$\alpha_1 = \frac{8T + T_{samp}}{4T - T_{samp}}$$

$$\alpha_2 = \frac{4T}{4T - T_{samp}}$$

$$\beta_0 = \frac{2\alpha K_i T_{samp} T + -8K_p T + T_{samp}}{4T - T_{samp}}$$

$$\beta_1 = \frac{T_{samp}}{4T - T_{samp}}$$

$$\beta_2 = \frac{-(2\alpha K_i T_{samp} T) + (4\alpha K_p T)}{4T - T_{samp}}$$

$$N = 2$$

```
c2d(C*lead,Tsamp,'tustin')
ans =

  11.12 z^2 - 21.39 z + 10.27
  ---------------------------
    z^2 - 0.4059 z - 0.5941
```

2 Three-Phase Permanent Magnet Motor

2.1 Induced Back Elecrtromotive Force Voltage (EMF)

$$\begin{aligned}
&\Phi\,(B_{pk}A)\,cos(\theta_e)\\
\lambda &= N(B_{pk}A)cos(\theta_e)\\
&= N\Phi_{pk}cos(\theta_e)\\
&= \lambda_{pk}cos(\theta_e)\\
\epsilon &= -\frac{d\lambda(\theta_e)}{dt} - \frac{d}{dt}[-\lambda_{pk}cos(\theta_m\frac{p}{2}) - Li]\\
&= \lambda_{pk}\frac{d}{dt}cos(\theta_m\frac{p}{2}) - L\frac{di}{dt}\\
&= -\lambda_{pk}sin(\theta_m\frac{p}{2})\frac{p}{2}\cdot\frac{d}{dt}(\theta_m) - L\frac{di}{dt}\\
&= -\lambda_{pk}sin(\theta_e)\frac{p}{2}\omega_m - L\frac{di}{dt}
\end{aligned}$$

Problem 2: Consider the three-phase permanent magnet motor where the outer ring of magnets can rotate relative to a set of stationary coils within its inner diameter. Note that this setup is very similar to the e-bike experiment. Only the a-phase winding has been drawn on Figure 2.

(a) Annotate Figure 2 with the items below:

- Draw the flux density B-field lines between all the magnetic poles. Also show how the field lines exist within the magnets.
- Define the d and q axes for the permanent magnets.
- Define the a-phase voltage polarity.
- Define the a-phase current positive direction such that its flux points toward the $\perp$ vector.
- Indicate the direction of the flux produced by the a-phase coil by adding arrows to the green flux circles.
- Label the mechanical angle, θ_{m}.
- Label one full electrical rotation of 360°. Write the equation of how θ_{e} and θ_{m} are related for this problem.
- Sketch the approximate placement of the b and c phase coils to form the full-three motor.

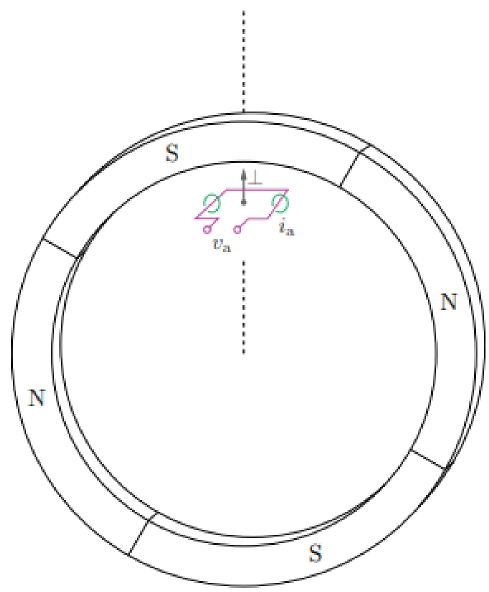


Figure 2: Permanent magnet motor setup.

(b) Denote the flux produced by the magnetic ring which cuts through the phase-a loop area as $\Phi(\theta_{\mathrm{e}})$. The flux linkage through the coil, which has N turns, is denoted as $\lambda(\theta_{\mathrm{e}})$. Consider Figure 3 where we are looking at a side view of the phase-a coil in Figure 2. Annotate the diagram in Figure 3 by showing the following for each angle configuration:

- Label the position of the poles above the coil.
- Draw the flux cutting through the coil.
- Fill in the two pink circles with crosses or dots to indicate the positive flow of coil current i.
- Add arrows to the green circles to show the flux density direction produced by i.
- Fill in the flux and flux linkage equations in terms of $N, i,$ and $\Phi_{\max}$, where $\Phi_{\max} > 0$ is the peak flux through the coil.

(c) Using the result from (b), give the form of the functions $\Phi(\theta_{\mathrm{e}})$ and $\lambda(\theta_{\mathrm{e}})$. Use Faraday's Law to compute the induced back electromotive force voltage $\mathcal{E}$.

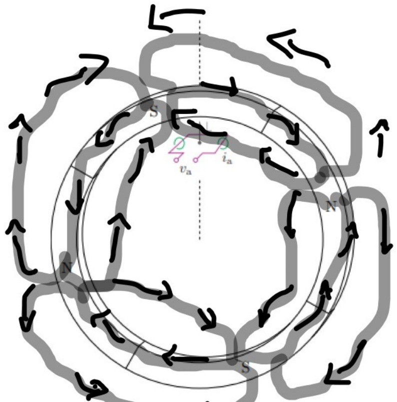


Figure 2: Permanent magnet motor setup.

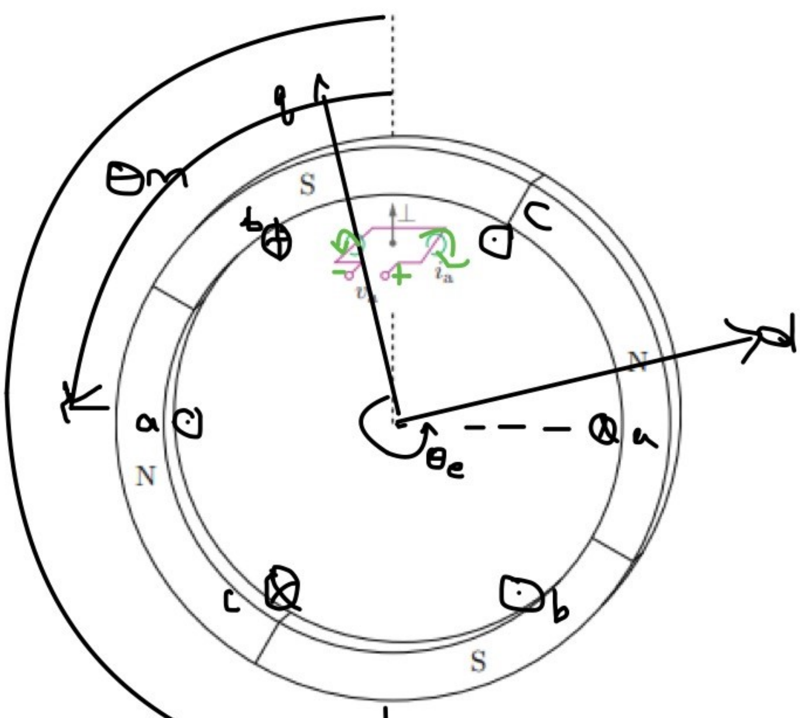
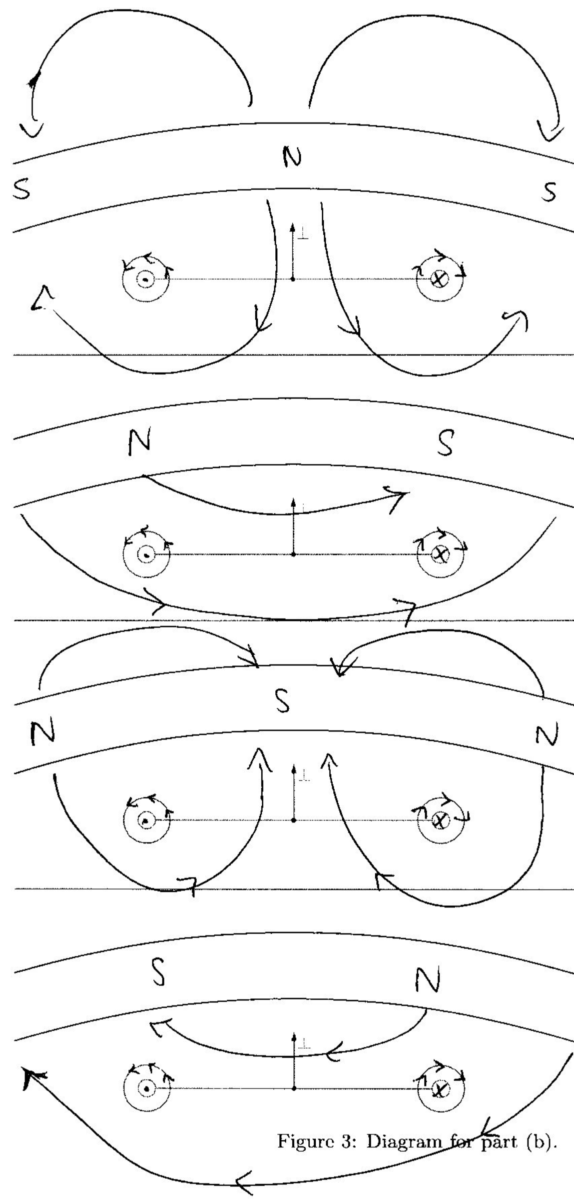


Figure 2: Permanent magnet motor setup.

$\theta_e = 360°$



when $\theta_e = 0$, give the following:

$\Phi(\theta_e) = -\Phi_{max}$

$\lambda(\theta_e) = -N\Phi_{max} + Li$

when $\theta_e = 90°$, give the following:

$\Phi(\theta_e) = 0$

$\lambda(\theta_e) = Li$

when $\theta_e = 180°$, give the following:

$\Phi(\theta_e) = \Phi_{max}$

$\lambda(\theta_e) = N\Phi_{max} + Li$

when $\theta_e = 270°$, give the following:

$\Phi(\theta_e) = 0$

$\lambda(\theta_e) = Li$

Figure 3: Diagram for part (b).