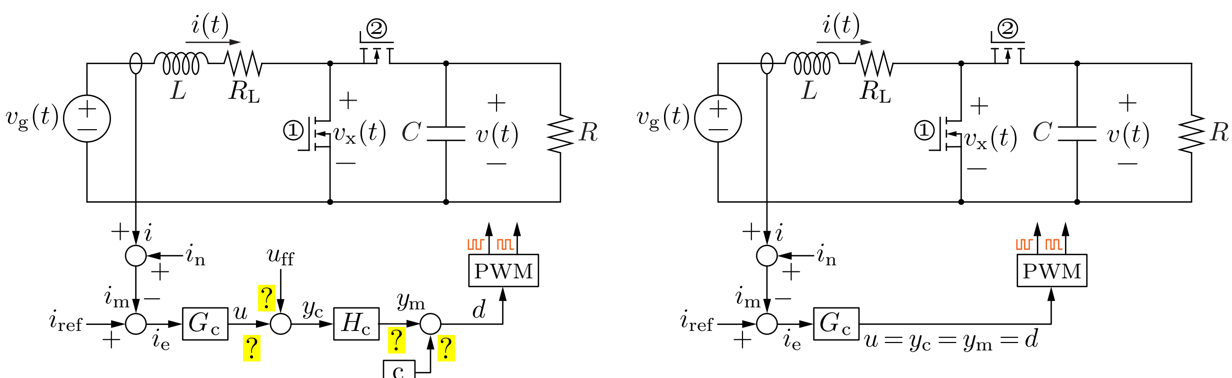
EE 458/533 – Power Electronics Controls, Winter 2022

Homework 5

Due Date: Friday March 3rd 2022, 4:30pm PT

Problem 1: Consider a boost converter where the winding and MOSFET on resistances are lumped into R_L. The diagrams in Figs. 1(a) represents the version used in lab and 1(b) represent the improved control structure that implements feed-forward. Below, you will model the improved controller and then compare it to the lab implementation. Assume all sensing path gains are equal to 1 and that a PI controller is used.



(a) Improved current controller

(b) Existing current controller in lab

Figure 1: Boost converter with current control.

Perform the following:

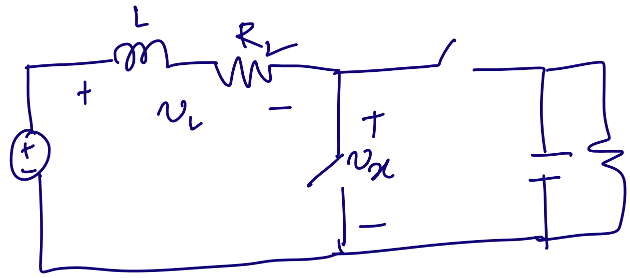
(a) Derive the voltage $\langle v_\mathrm{x}(t)\rangle$(average value of $v_\mathrm{x}(t)$ over one switching period) in terms of duty, d and output voltage, $\langle v(t)\rangle$

(b) Derive the KVL in the input loop to obtain a relationship between $v_\mathrm{g}(t)(= \langle v_\mathrm{g}(t)\rangle)$, L, R_L,$\langle v_\mathrm{x}(t)\rangle$ and $\langle i(t)\rangle$.

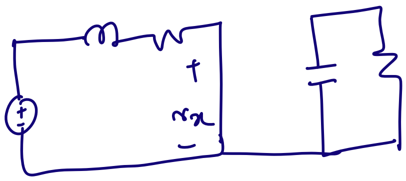
(c) Assume the controller, G_c produces an output, d, verify that Fig. 2 (a) shows the correct implementation of your KVL derived in the preceding question.

(d) We want to implement feed-forward to simplify our plant transfer function to be only $1/(sL + R_L)$ so that the controller design becomes simplified as in Fig. 2 (c). To achieve this, sketch or provide a numerical relationship between the controller output,u, and the actual duty ratio, d, that goes into the PWM block. (Note under all circumstances, $0 < d < 1$). Compare the relationship you have derived with Fig. 1 (a) to identify H_c, c, u_ff and the signs of the sum blocks shown with ?.

(e) Design your controller, G_c by plant-inversion formulae to achieve a bandwidth of 1/10-th of your switching frequency.

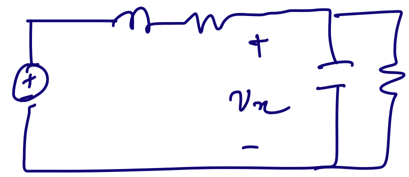


$0 < t < T_{ON}$



$v_x = 0$

$\underbrace{T_{ON} < t < T_{sw}}_{T_{OFF}}$



$v_x = v$

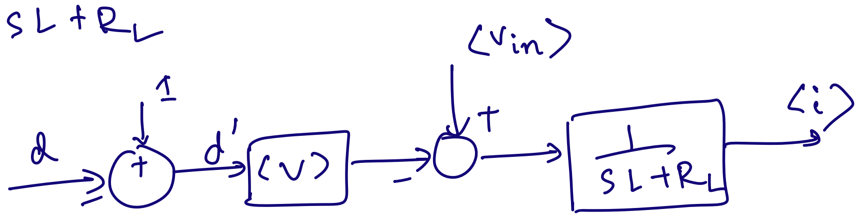
$$\therefore \quad \langle v_x \rangle = 0 \cdot d + \langle v \rangle \cdot d'$$

$$\langle v_x \rangle = \langle v \rangle \cdot d'$$

$$\langle V_{in} \rangle = L \frac{d\langle i \rangle}{dt} + \langle i \rangle R_L + \langle v_x \rangle$$

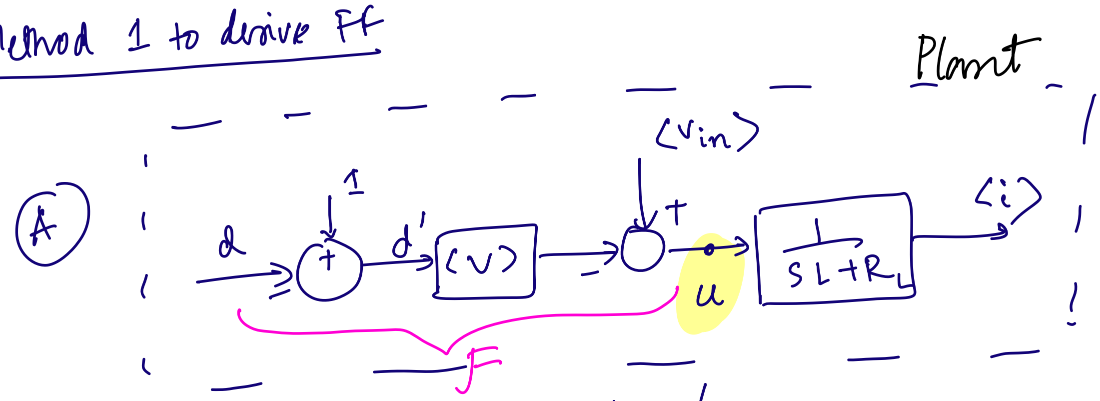
or, $$\langle V_{in} \rangle - \langle v \rangle d' = L \frac{d\langle i \rangle}{dt} + \langle i \rangle R_L$$

or, $$\frac{\langle V_{in} \rangle - \langle v \rangle d'}{sL + R_L} = \langle i \rangle$$



plant eq

Method 1 to derive FF



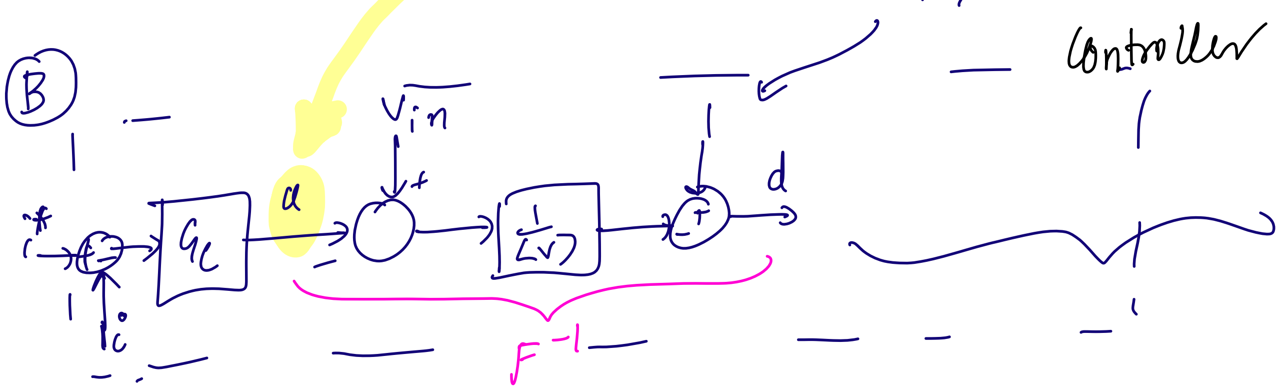
$$u = \langle V_{in}\rangle - \langle v\rangle d'$$

$$= \langle V_{in}\rangle - \langle v\rangle(1-d)$$

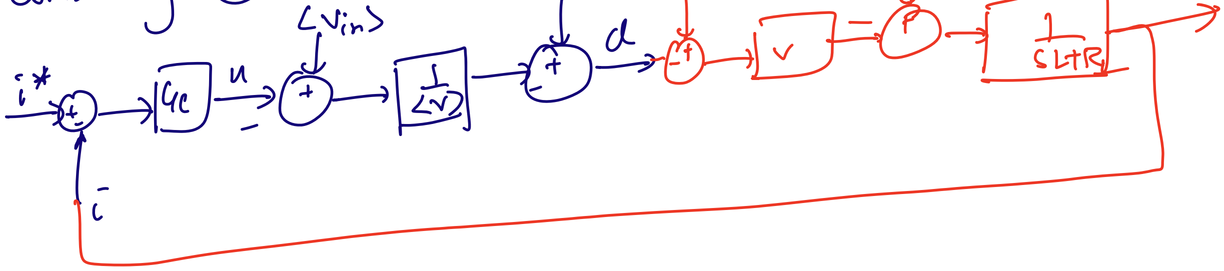
$$\therefore \quad (1-d)\langle v\rangle = \langle V_{in}\rangle - u$$

$$\Rightarrow \quad 1-d = \frac{\langle V_{in}\rangle - u}{\langle v\rangle}$$

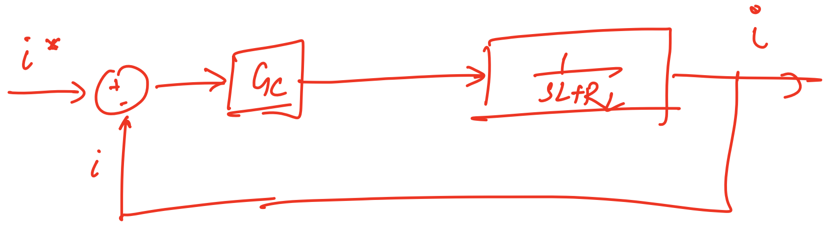
$$\therefore \quad d = 1 - \frac{\langle V_{in}\rangle - u}{\langle v\rangle}$$



Combining (A) & (B),



$\Rightarrow$



$$H = \frac{1}{\langle V \rangle}$$

$$u_{ff} = V_{in}$$

$$C = 1$$

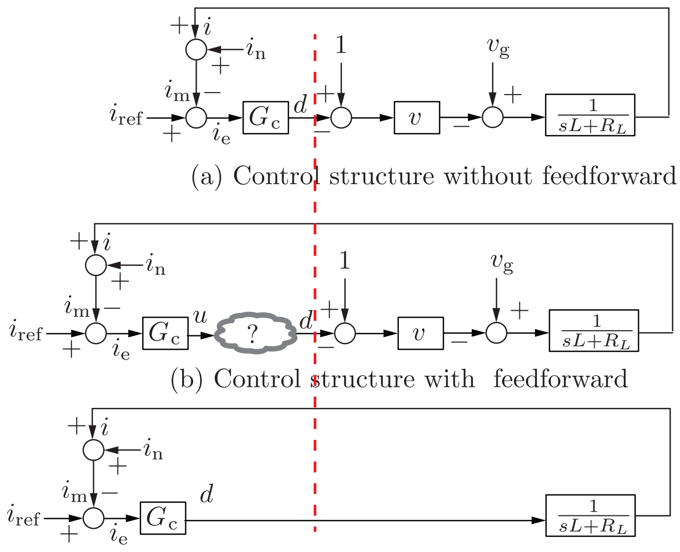


Figure 2: Control simplification with feed-forward. On the right side of the red-dashed line is your plant and on the left is your controller.

Problem 2: Simulate the improved implementation in Problem 1 where $V_{\mathrm{g}} = 24\,\mathrm{V}$, $f_s = 10\,\mathrm{kHz}$, $L = 1.3\,\mathrm{mH}$, $R_{\mathrm{L}} = 60\,\mathrm{m}\Omega$, $C = 250\,\mu\mathrm{F}$, and $R = 10\,\Omega$. Use the white noise block[1] to simulate the current sensor noise. Initialize your simulation with a reference of $i_{\mathrm{ref}} = 3\,\mathrm{A}$ such that the states are all in steady-state and the error, i_{e}, is zero. Perform the following:

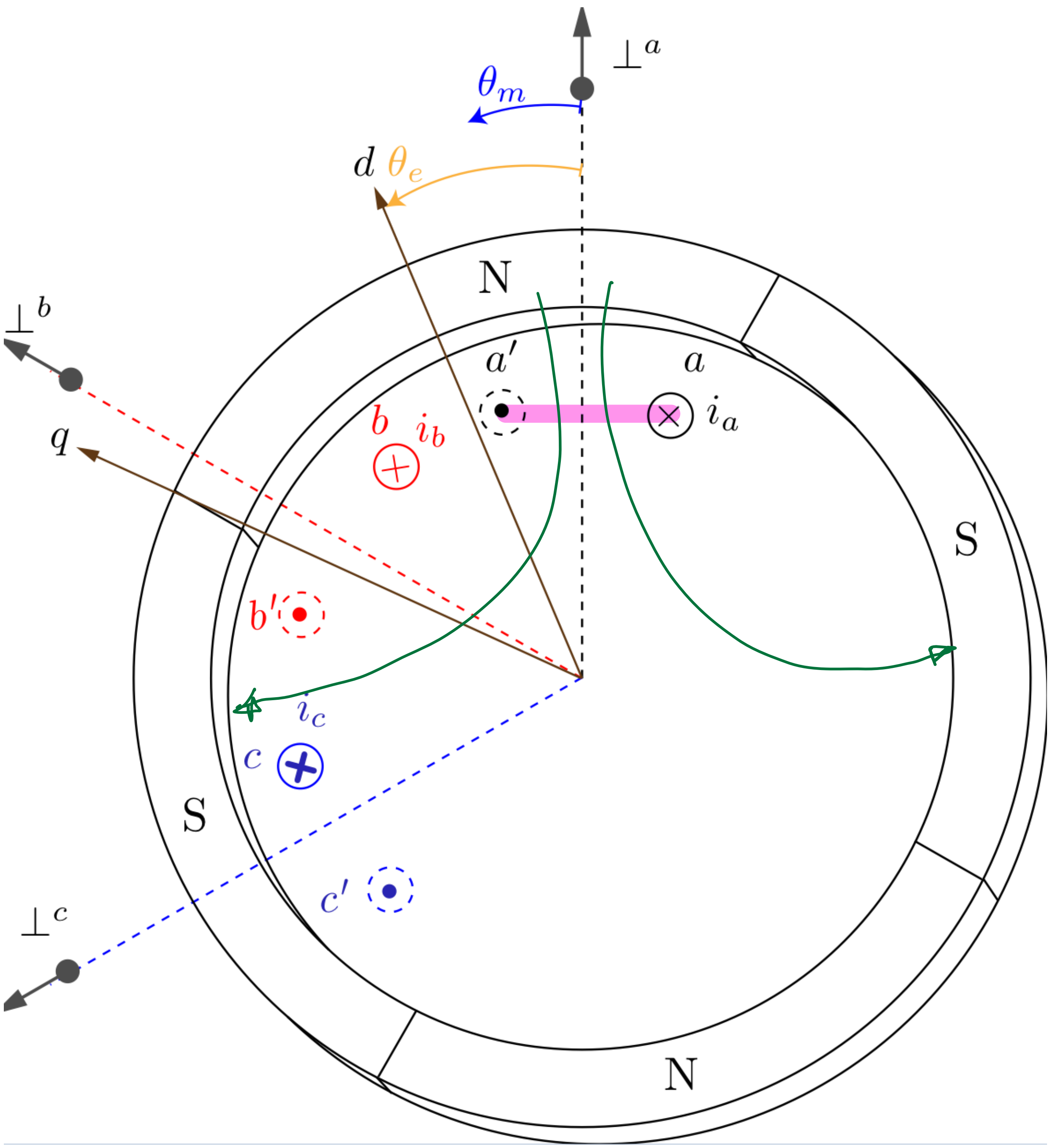
(a) Create a switched simulation for improved implementation in Fig. 1 with and without feedforward where the following events below occur. Use your control gains from 2(a). Overlay the $i(t)$ waveforms for with and without feedforward (for without feedforward, use the controller you developed in lab) on the same plot to facilitate comparison. Comment on how they differ.

- At $t = 5\,\mathrm{ms}$, i_{ref} changes from $3\,\mathrm{A} \rightarrow 5\,\mathrm{A}$.
- At $t = 10\,\mathrm{ms}$, v_{g} changes from $24\,\mathrm{V} \rightarrow 28\,\mathrm{V}$.
- At $t = 15\,\mathrm{ms}$, R changes from $10\,\Omega \rightarrow 15\,\Omega$.

[1] Configure the white noise block with zero mean, 50 mA standard deviation, and sample time T_{s}.

Problem 3: We will build on the results from Homework 4, Using Faraday's Law, give the expressions for the induced back EMF voltages, e_{a}, e_{b}, e_{c}, within the a, b, and c phase windings, respectively. In addition, draw the equivalent circuits looking into the a, b, and c windings where the back EMFs, e_{a}, e_{b}, e_{c}, are clearly labeled along with the speed-dependent voltages $\mathcal{E}_{\mathrm{a}}$, $\mathcal{E}_{\mathrm{b}}$, $\mathcal{E}_{\mathrm{c}}$ and inductive voltage drops. Be careful with all circuit element polarities.

– Class Lecture



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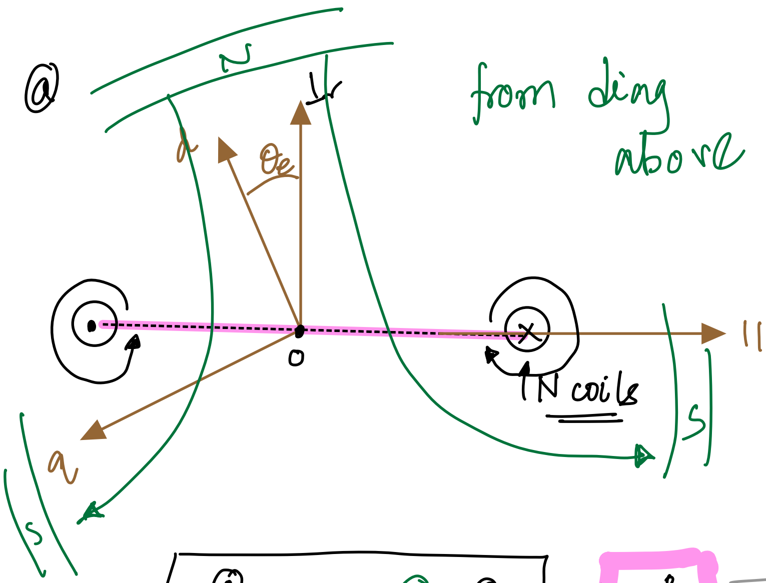
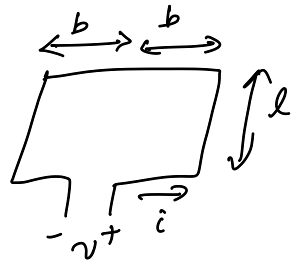
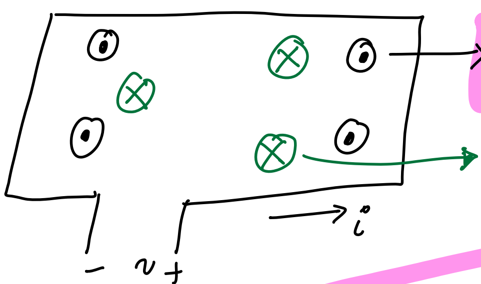


from diag above





$L.i$ → #1 flux

$\Phi = \int_S \bar{B}\cdot d\bar{l}$ → #2 flux

$\phi = (B_{pk}\cdot A)\cos(\theta_e)$ for one coil

λ = Flux linkage = $N(B_{pk}A)\cos(\theta_e)$ for N coil

$N\Phi_{pk}\cos(\theta_e)$

$\lambda_{pk}\cos(\theta_e)$

* Remember, sign convention
CCW +ve for arrow coming out of paper.

So, net flux linkage coming out of paper is a function of angle θ_e

$\therefore \quad \lambda(\theta_e) = -N.\,\Phi_{pk}\cos(\theta_e) + Li$

(sign convention to accomodate the $\odot$ rotation)

Induced emf :-

$$-\frac{d\lambda(\theta_e)}{dt} = -\frac{d}{dt}\left[-\lambda_{pk}\cos\left(\theta_m \frac{P}{2}\right) + Li\right]$$

$$= \lambda_{pk} \cdot \frac{d}{dt}\left\{\cos\left(\theta_m \frac{P}{2}\right)\right\} - L\frac{di}{dt}$$

$$= -\lambda_{pk}\sin\left(\theta_m \frac{P}{2}\right)\frac{P}{2}\frac{\partial}{\partial t}(\theta_m) - L\frac{di}{dt}$$

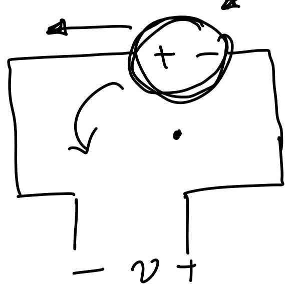
$$= \underbrace{-\lambda_{pk}\sin(\theta_e)\frac{P}{2}\omega_m - L\frac{di}{dt}}$$

$\lambda_{pk}\sin(\theta_e)\omega_e$

Direction

$\lambda_{pk}\sin(\theta_e)\frac{P}{2}\omega_m =: \mathcal{E}$

back emf



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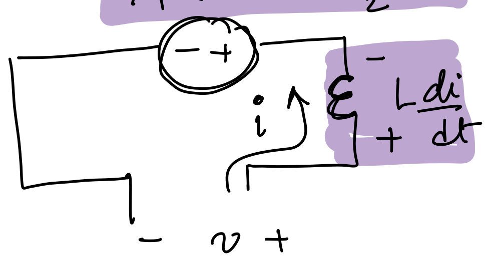
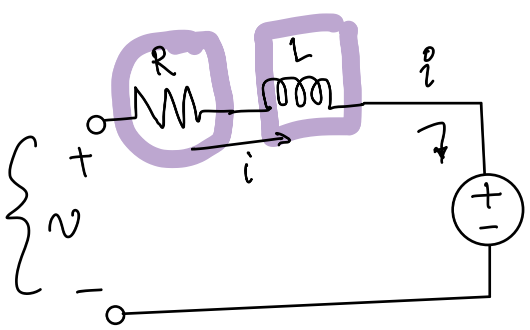
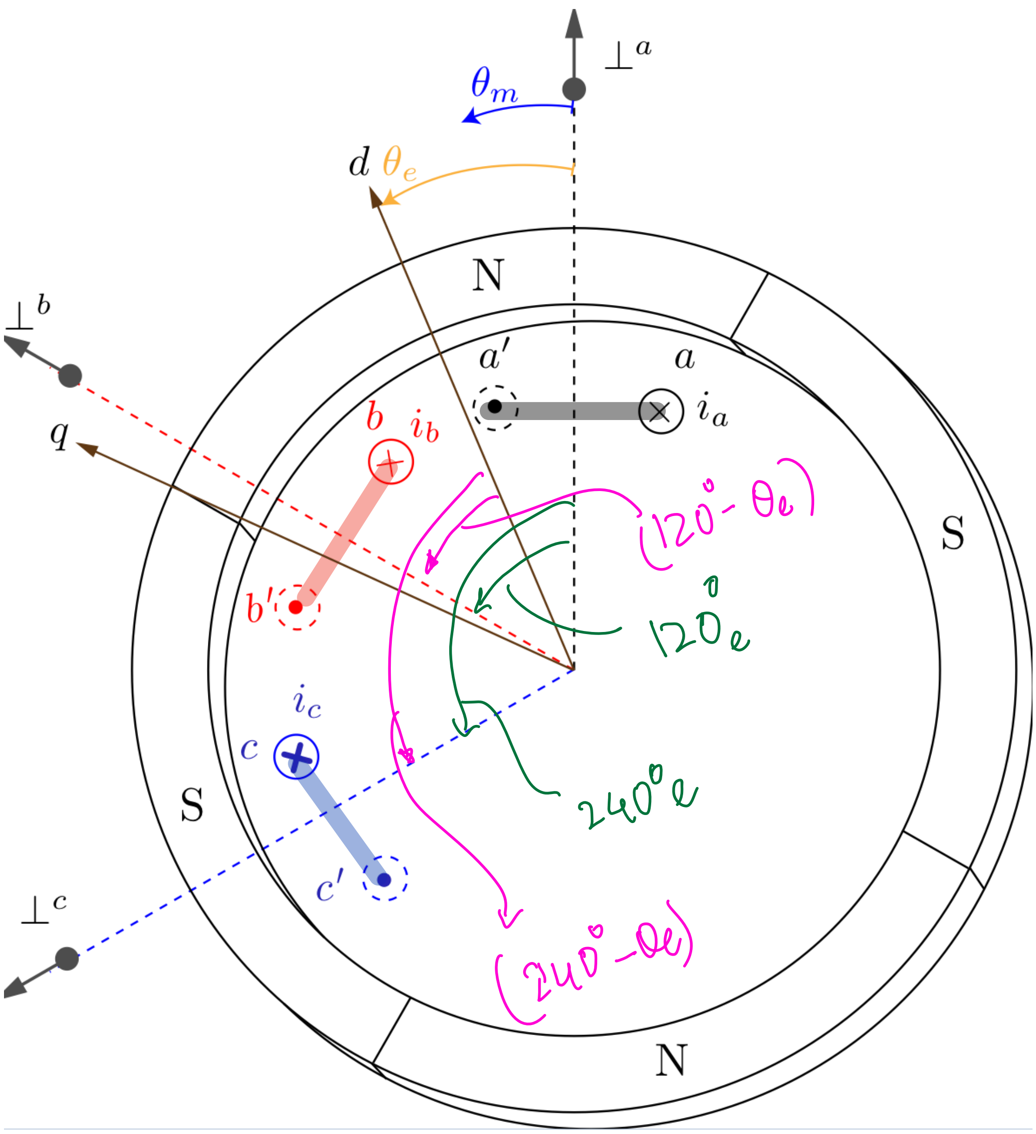


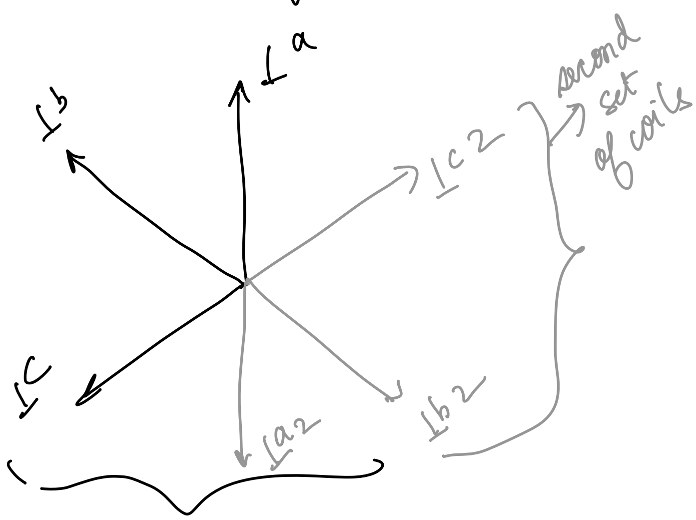


$\mathcal{E} = \lambda_{pk}\,\omega_e \sin(\theta_e)$

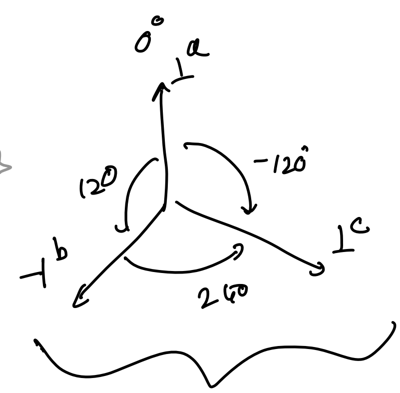


$\perp^a$
θ_m
d θ_e
N
$\perp^b$
a'
a
i_a
q
b i_b
$(120^\circ - \theta_e)$
S
b'
120°_e
i_c
c
S
$240^\circ e$
$\perp^c$
c'
$(240^\circ - \theta_e)$
N

(c) For three sets of coil.



mechanical 2π



electrical 2π

$$\lambda_{a_1a_2} = \underline{-\lambda_{pk}\cos(\theta_e) + Li_a}$$

$$\lambda_{b_1b_2} = \underline{-\lambda_{pk}\cos\left(\frac{2\pi}{3} - \theta_e\right)} + Li_b = -\lambda_{pk}\cos\left(\theta_e - \frac{2\pi}{3}\right) + Li_b$$

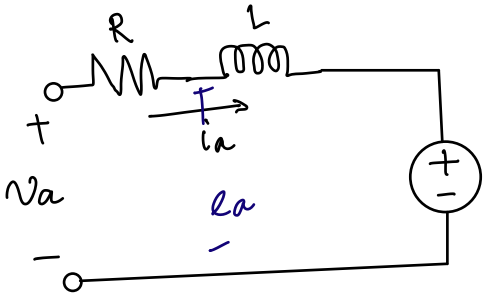
$$\lambda_{c_1c_2} = \underline{-\lambda_{pk}\cos\left(\frac{4\pi}{3} - \theta_e\right) + Li_c} = -\lambda_{pk}\cos\left(\theta_e - \frac{4\pi}{3}\right) + Li_c$$

<u>induced emf:-</u>

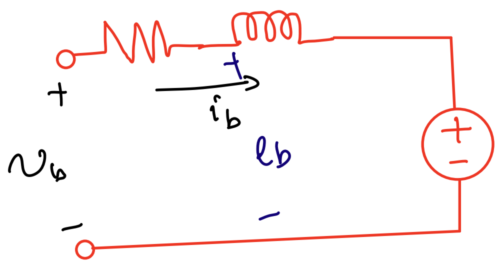
Coil a_1a_2: $\left(-\frac{d\lambda_{a_1a_2}}{dt}\right) = -\lambda_{pk}\omega_e\sin(\theta_e) - L\frac{di_a}{dt}$

Coil b_1b_2: $\left(-\frac{d\lambda_{b_1b_2}}{dt}\right) = -\lambda_{pk}\omega_e\sin\left(\theta_e - \frac{2\pi}{3}\right) - L\frac{di_b}{dt}$

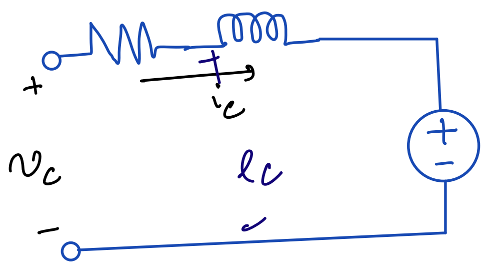
Coil c_1c_2: $\left(-\frac{d\lambda_{c_1c_2}}{dt}\right) = \underbrace{\underbrace{-\lambda_{pk}\omega_e\sin\left(\theta_e - \frac{4\pi}{3}\right)}_{\text{speed dependent emfs}} - L\frac{di_c}{dt}}_{\text{net back emfs.}}$



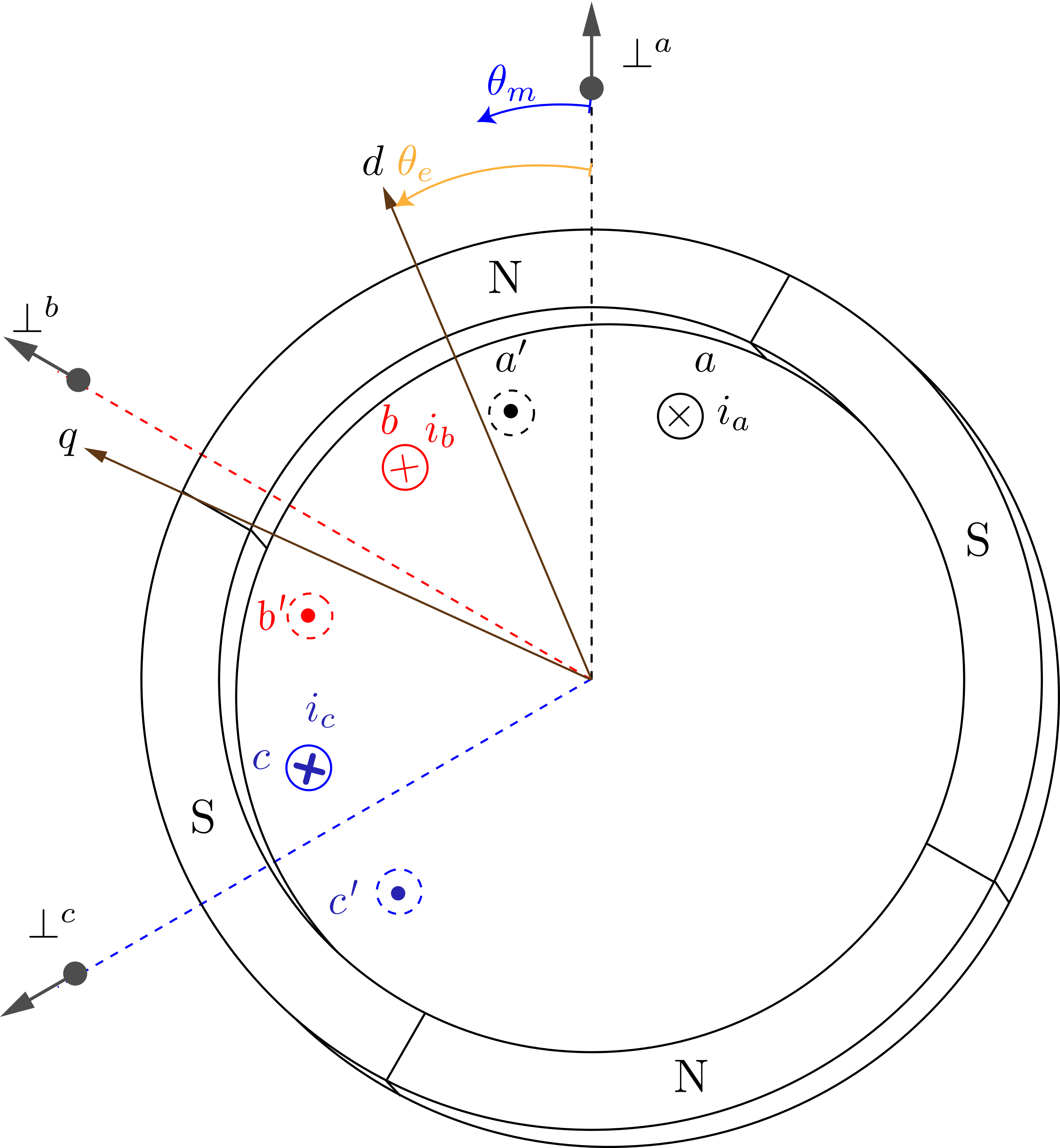
$$\mathcal{E}_a = \lambda_{pk}\,\omega_e \sin(\theta_e)$$



$$\mathcal{E}_b = \lambda_{pk}\,\omega_e \sin\left(\theta_e - \frac{2\pi}{3}\right)$$



$$\mathcal{E}_c = \lambda_{pk}\,\omega_e \sin\left(\theta_e - \frac{4\pi}{3}\right)$$



$\perp^a$
θ_m
d
θ_e
$\perp^b$
N
a'
a
i_a
b
i_b
q
S
b'
i_c
c
S
c'
$\perp^c$
N