EE 458 – Power Electronics Controls
Experiment 2 Pre-Lab Assignment: Boost Converter

Kevin Egedy

1 Boost Converter Computations

1.1 Part A

Steady-state inductor current I_L

$$\langle v_L(t)\rangle = L\frac{d}{dt}\langle i(t)\rangle = \left[\Big(\langle v_g(t)\rangle - \langle i(t)\rangle R_L\Big)d(t) + \Big(\langle v_g(t)\rangle - \langle i(t)\rangle R_L - \langle v(t)\rangle\Big)d'(t)\right]$$

$$\langle v_L(t)\rangle = L\frac{d}{dt}\langle i(t)\rangle = \left[\langle v_g(t)\rangle - \langle i(t)\rangle R_L - \langle v(t)\rangle d'(t)\right]$$

$$V_L(t) = L\frac{d}{dt}I = V_g - IR_L - VD' = 0$$

$$VD' = V_g - IR_L$$

$$V = \frac{V_g - IR_L}{D'}\bigg|_{I=\frac{V}{D'R}}$$

$$V = \frac{V_g - \frac{V}{D'R}R_L}{D'}$$

$$V = \frac{V_g}{D'} - \frac{VR_L}{D'^2R}$$

$$V + \frac{VR_L}{D'^2R} = \frac{V_g}{D'}$$

$$V = \frac{V_g}{D'} \cdot \frac{1}{1 + \frac{R_L}{D'^2R}}$$

$$48 = \frac{24}{D'} \cdot \frac{1}{1 + \frac{60 \cdot 10^{-3}}{D'^2 100}} \rightarrow D' = 0.4988$$

$$V = \frac{24}{0.4988} \cdot \frac{1}{1 + \frac{60 \cdot 10^{-3}}{(0.4988)^2 100}} = 48V$$

Steady-state capacitor voltage V

$$\langle i_C(t)\rangle = C\frac{d}{dt}\langle v(t)\rangle = \left[\left(\frac{-\langle v(t)\rangle}{R}\right)d(t) + \left(\langle i(t)\rangle - \frac{\langle v(t)\rangle}{R}d'(t)\right)\right]$$

$$\langle i_C(t)\rangle = C\frac{d}{dt}\langle v(t)\rangle = \left[\frac{-\langle v(t)\rangle}{R} + \langle i(t)\rangle d'(t)\right]$$

$$I_C(t) = C\frac{d}{dt}V = \frac{-V}{R} + ID' = 0$$

$$ID' = \frac{V}{R}$$

$$I = \frac{V}{D'R}$$

$$I = \frac{1}{D'R}\cdot\frac{V_g}{D'}\cdot\frac{1}{1+\frac{R_L}{D'^2R}}$$

$$I = \frac{V_g}{D'^2R}\cdot\frac{1}{1+\frac{R_L}{D'^2R}}$$

$$I = \frac{24}{(0.4988)^2 100}\cdot\frac{1}{1+\frac{60\cdot 10^{-3}}{(0.4988)^2 100}} = 0.96A$$

1.2 Part B

Small signal $\frac{d}{dt}\langle\hat{i}(t)\rangle$

$$\langle v_L(t)\rangle = L\frac{d}{dt}\langle i(t)\rangle = \left[\Big(\langle v_g(t)\rangle - \langle i(t)\rangle R_L\Big)d(t) + \Big(\langle v_g(t)\rangle - \langle i(t)\rangle R_L - \langle v(t)\rangle\Big)d'(t)\right]$$

$$\langle v_L(t)\rangle = L\frac{d}{dt}\langle i(t)\rangle = \Big[\langle v_g(t)\rangle - \langle i(t)\rangle R_L - \langle v(t)\rangle d'(t)\Big]$$

$$\langle v_L(t)\rangle = L\frac{d}{dt}\left[I + \langle\hat{i}(t)\rangle\right] = \left[[V_g + \langle\hat{v_g}(t)\rangle] - \left[I + \langle\hat{i}(t)\rangle\right]R_L - [V + \langle\hat{v}(t)\rangle]\left[D' - \hat{d}(t)\right]\right]$$

$$\langle v_L(t)\rangle = L\frac{d}{dt}\left[I + \langle\hat{i}(t)\rangle\right] = \left[[V_g + \langle\hat{v_g}(t)\rangle] - \left[I + \langle\hat{i}(t)\rangle\right]R_L - D'V - D'\langle\hat{v}(t)\rangle + \hat{d}(t)V + \hat{d}(t)\langle\hat{v}(t)\rangle\right]$$

Remove large signals (DC terms)

$$\frac{d}{dt}\langle\hat{i}(t)\rangle = \frac{1}{L}\Big[\langle\hat{v_g}(t)\rangle - \langle\hat{i}(t)\rangle R_L - D'\langle\hat{v}(t)\rangle + \hat{d}(t)V\Big]$$

Small signal $\frac{d}{dt}\langle\hat{v}(t)\rangle$

$$\langle i_C(t)\rangle = C\frac{d}{dt}\langle v(t)\rangle = \left[\left(\frac{-\langle v(t)\rangle}{R}\right)d(t) + \left(\langle i(t)\rangle - \frac{\langle v(t)\rangle}{R}d'(t)\right)\right]$$

$$\langle i_C(t)\rangle = C\frac{d}{dt}\langle v(t)\rangle = \left[\frac{-\langle v(t)\rangle}{R} + \langle i(t)\rangle d'(t)\right]$$

$$\langle i_C(t)\rangle = C\frac{d}{dt}\left[V + \langle\hat{v}(t)\rangle\right] = \left[\frac{-\left[V + \langle\hat{v}(t)\rangle\right]}{R} + \left[I + \langle\hat{i}(t)\rangle\right]\left[D' - \hat{d}(t)\right]\right]$$

$$\langle i_C(t)\rangle = C\frac{d}{dt}\left[V + \langle\hat{v}(t)\rangle\right] = \left[\frac{-\left[V + \langle\hat{v}(t)\rangle\right]}{R} + ID' - I\hat{d}(t) + \langle\hat{i}(t)\rangle D' - \langle\hat{i}(t)\rangle\hat{d}(t)\right]$$

Remove large signals (DC terms)

$$\frac{d}{dt}\langle\hat{v}(t)\rangle = \frac{1}{C}\left[\frac{-\langle\hat{v}(t)\rangle}{R} - I\hat{d}(t) + \langle\hat{i}(t)\rangle D'\right]$$

System Inputs $\dot{x}$

$$\dot{x} = \frac{d}{dt}\begin{bmatrix}\langle\hat{i}(t)\rangle\\ \langle\hat{v}(t)\rangle\end{bmatrix} = \begin{bmatrix}\frac{1}{L}\left[\langle\hat{v}_g(t)\rangle - \langle\hat{i}(t)\rangle R_L - D'\langle\hat{v}(t)\rangle + \hat{d}(t)V\right]\\ \frac{1}{C}\left[\frac{-\langle\hat{v}(t)\rangle}{R} - I\hat{d}(t) + \langle\hat{i}(t)\rangle D'\right]\end{bmatrix}$$

System Output $\hat{y}(s)$

$$\hat{y}(s) = \left(C(sI - A)^{-1}B + E\right)\hat{u}(s)$$

$$A = \begin{bmatrix}\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2}\\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2}\end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix}\frac{\partial f_1}{\partial\langle\hat{i}(t)\rangle} & \frac{\partial f_1}{\partial\langle\hat{v}(t)\rangle}\\ \frac{\partial f_2}{\partial\langle\hat{i}(t)\rangle} & \frac{\partial f_2}{\partial\langle\hat{v}(t)\rangle}\end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix}\frac{-R_L}{L} & \frac{-D'}{L}\\ \frac{D'}{C} & \frac{-1}{RC}\end{bmatrix}$$

$$B = \begin{bmatrix}\frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2}\\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2}\end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix}\frac{\partial f_1}{\partial\hat{d}(t)} & \frac{\partial f_1}{\partial\langle\hat{v}_g(t)\rangle}\\ \frac{\partial f_2}{\partial\hat{d}(t)} & \frac{\partial f_2}{\partial\langle\hat{v}_g(t)\rangle}\end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix}\frac{V}{L} & \frac{1}{L}\\ \frac{-I}{C} & 0\end{bmatrix} = \begin{bmatrix}\frac{V_g}{D'L}\cdot\frac{1}{1+\frac{R_L}{D'^2R}} & \frac{1}{L}\\ \frac{-V_g}{D'^2RC}\cdot\frac{1}{1+\frac{R_L}{D'^2R}} & 0\end{bmatrix}$$

System Output expressed in equivalent $G(s)$

$$\hat{y}(s) = \Big(C(sI - A)^{-1}B + E\Big)\hat{u}(s)$$

$$\hat{y}(s) = G(s)u(s)$$

$$G(s) = \begin{bmatrix} G_{id}(s) & G_{ig}(s) \\ G_{vd}(s) & G_{vg}(s) \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{D'RV_g}{D'^2R + R_L}\right) \frac{RCs + 2}{RLCs^2 + (RR_LC + L)s + (D'^2R + R_L)} & \frac{RCs + 1}{RLCs^2 + (RR_LC + L)s + (D'^2R + R_L)} \\ \left(\frac{-RV_g}{D'^2R + R_L}\right) \frac{Ls + (R_L - D'^2R)}{RLCs^2 + (RR_LC + L)s + (D'^2R + R_L)} & \frac{D'R}{RLCs^2 + (RR_LC + L)s + (D'^2R + R_L)} \end{bmatrix}$$

1.3 Part B

Compute $G_{id}(s) = \frac{\hat{i_L}}{\hat{d}}$

$$G_{id}(s) = \left(\frac{D'RV_g}{D'^2R + R_L}\right) \frac{RCs + 2}{RLCs^2 + (RR_LC + L)s + (D'^2R + R_L)}$$

$$G_{id}(s) = \frac{10.56s + 96}{4.4 \cdot 10^{-5}s^2 + 0.0134s + 24.94}$$

1.4 Part C

Compute $G_{vd}(s) = \frac{\hat{v}}{\hat{d}}$

$$G_{vd}(s) = \left(\frac{-RV_g}{D'^2R + R_L}\right) \frac{Ls + (R_L - D'^2R)}{RLCs^2 + (RR_LC + L)s + (D'^2R + R_L)}$$

$$G_{vd}(s) = \frac{2388.45 - 0.02s}{4.4 \cdot 10^{-5}s^2 + 0.0134s + 24.94}$$

1.5 Part D

Compute $G_{vi}(s) = \dfrac{G_{vd}(s)}{G_{id}(s)}$

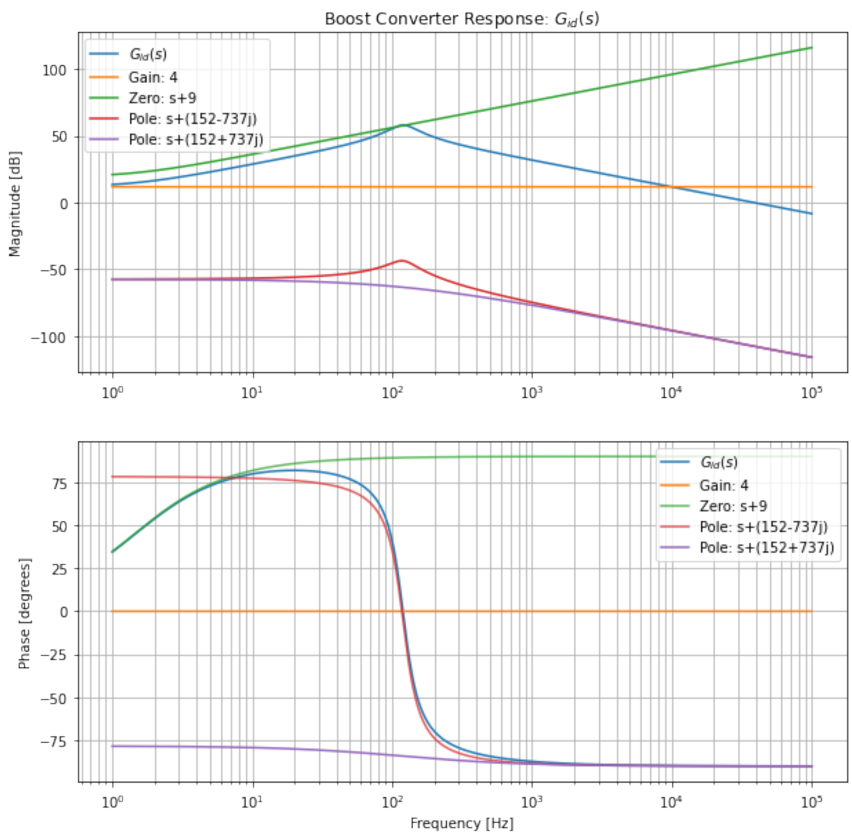
$$\frac{G_{vd}(s)}{G_{id}(s)} = \frac{\left(\dfrac{-RV_g}{D'^2R+R_L}\right)\dfrac{Ls+(R_L-D'^2R)}{RLCs^2+(RR_LC+L)s+(D'^2R+R_L)}}{\left(\dfrac{D'RV_g}{D'^2R+R_L}\right)\dfrac{RCs+2}{RLCs^2+(RR_LC+L)s+(D'^2R+R_L)}} = \left(\frac{-1}{D'}\right)\frac{Ls+(R_L-D'^2R)}{RCs+2}$$

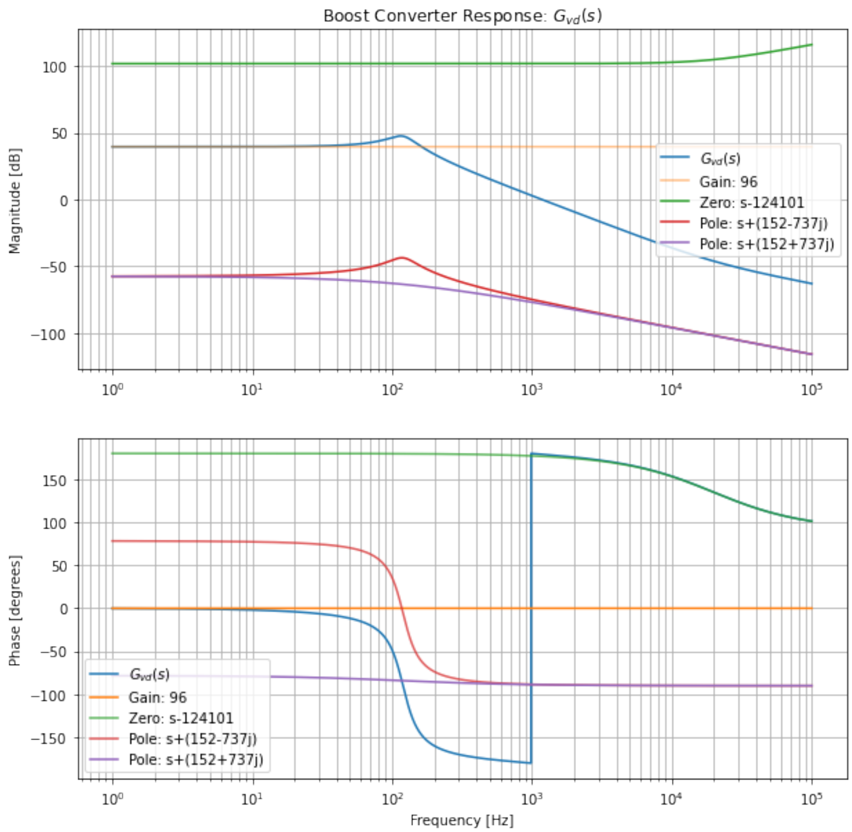
$$\frac{G_{vd}(s)}{G_{id}(s)} = \frac{2388.45-0.02s}{10.56s+96}$$

1.6 Part E

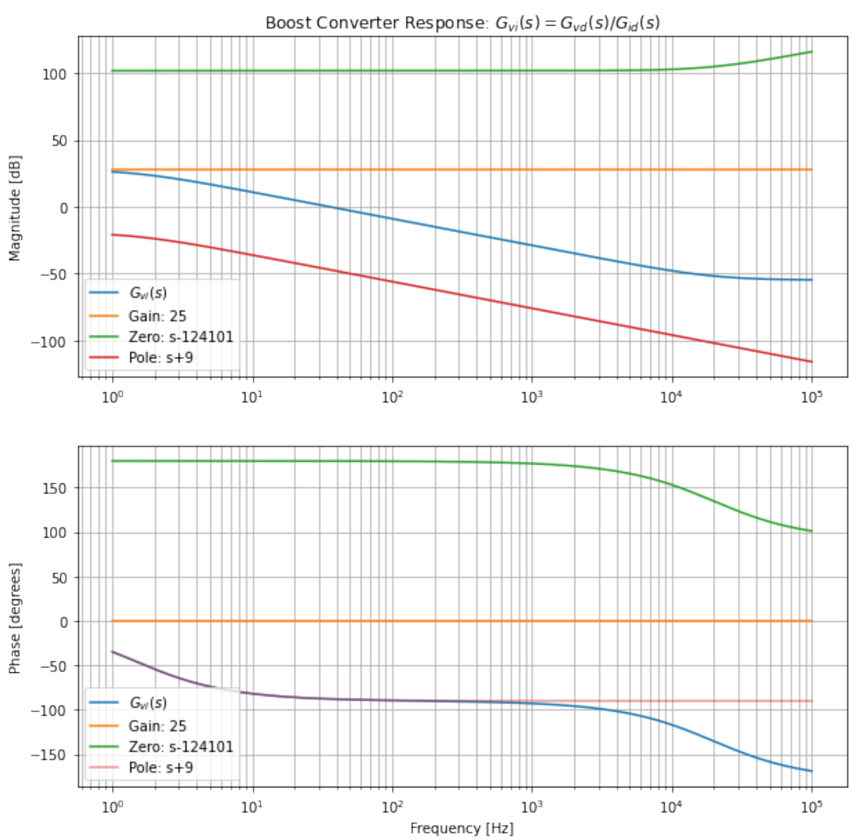
Extra Credit

TF	Gain	Zero	Pole
G_{id}	3.85	-9.09	[-152.27 + 737.32j, -152.27 - 737.32j]
G_{vd}	95.77	124,101.23	[-152.27 + 737.32j, -152.27 - 737.32j]
G_{vd}/G_{id}	24.88	124,101.23	-9.09





Boost Converter Response: $G_{vd}(s)$
Magnitude [dB]
$G_{vd}(s)$
Gain: 96
Zero: s-124101
Pole: s+(152-737j)
Pole: s+(152+737j)
Phase [degrees]
Frequency [Hz]



Boost Converter Response: $G_{vi}(s) = G_{vd}(s)/G_{id}(s)$
Magnitude [dB]
$G_{vi}(s)$
Gain: 25
Zero: s-124101
Pole: s+9
Phase [degrees]
Frequency [Hz]