Problem 1 (David P Babin)

$$\tilde{x}(s) = (s\mathbf{I} - A)^{-1} B\, \tilde{u}(s)$$

$$\begin{bmatrix} \tilde{\imath}(s) \\ \tilde{v}(s) \end{bmatrix} = \left( \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -\frac{R_L}{L} & -\frac{D'}{L} \\ \frac{D'}{C} & -\frac{1}{RC} \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} \frac{V}{L} & \frac{1}{L} \\ -\frac{I}{C} & 0 \end{bmatrix} \cdot \begin{bmatrix} \tilde{d}(s) \\ \tilde{v}_{in}(s) \end{bmatrix}$$

$$\begin{bmatrix} \tilde{\imath}(s) \\ \tilde{v}(s) \end{bmatrix} = \frac{1}{CLRs^2 + (L + CRR_L)s + R_L + (D')^2 R} \begin{bmatrix} V + D'IR + CRVs & CRs + 1 \\ -R \cdot (IR_L - D'V + ILs) & D'R \end{bmatrix} \cdot \begin{bmatrix} \tilde{d}(s) \\ \tilde{v}_{in}(s) \end{bmatrix}$$

The transfer function relating the small signal voltage to the small signal duty ratio:

$$G_{vd}(s) = \frac{\tilde{v}(s)}{\tilde{d}(s)} = \frac{-R \cdot (IR_L - D'V + ILs)}{CLRs^2 + (L + CRR_L)s + R_L + (D')^2 R}$$

$\Downarrow$ Assume $R_L$=0

$$G_{vd}(s) = \frac{RD'V - RILs}{CLRs^2 + Ls + (D')^2 R} = \frac{RD'V}{(D')^2 R} \cdot \frac{1 - \frac{RIL}{RD'V} s}{1 + \frac{L}{(D')^2 R} s + \frac{CLR}{(D')^2 R} s^2}$$

$$G_{vd}(s) = \frac{V}{D'} \cdot \frac{1 - \frac{IL}{D'V} s}{1 + \frac{L}{(D')^2 R} s + \frac{CL}{(D')^2} s^2} = G_o \frac{1 - \frac{s}{\omega_z}}{1 + \frac{s}{Q\omega_o} + \left(\frac{s}{\omega_o}\right)^2}$$

So we have:

- $\boxed{G_o = G_{d0} = \frac{V}{D'}}$
- $\frac{1}{\omega_z} = \frac{IL}{D'V} \rightarrow \omega_z = \frac{D'V}{IL} = \frac{D'}{L} \cdot \frac{V}{I} = \frac{D'}{L} \cdot \frac{D'R \cdot \frac{V_{in}}{R_L + (D')^2 R}}{\frac{V_{in}}{R_L + (D')^2 R}} = \frac{D'}{L} \cdot D'R = \boxed{\frac{(D')^2 R}{L} = \omega_z}$
- $\frac{1}{\omega_o^2} = \frac{CL}{(D')^2} \rightarrow \omega_o^2 = \frac{(D')^2}{CL} \rightarrow \boxed{\omega_o = \frac{D'}{\sqrt{LC}}}$
- $\frac{1}{Q\omega_o} = \frac{L}{(D')^2 R} \rightarrow Q\omega_o = \frac{(D')^2 R}{L} \rightarrow Q = \frac{1}{\omega_o} \cdot \frac{(D')^2 R}{L} = \frac{\sqrt{LC}}{D'} \cdot \frac{(D')^2 R}{L} = \boxed{D'R\sqrt{\frac{C}{L}} = Q}$

Problem 2 Calculation

(a)

$\Delta i_L = \frac{V_g D T_s}{L}$ (increase L to reduce current ripple)

(b)

$P = 250 = \frac{{V_{out}}^2}{R} = \frac{1}{R}\left(\frac{V_g}{D'}\right)^2 \Longrightarrow D' = \frac{V_g}{\sqrt{RP}}$

To keep V_out constant, adjust R instead of D:

$$D' = 0.5 = \frac{24}{\sqrt{R * 250}} = \frac{24}{\sqrt{R_{new} * 100}} \Longrightarrow R = 9.216\Omega, R_{new,b} = 23.04\Omega$$

(c)

$f_{sw}$ doesn't affect $G_{vd}(s)$

(d)

To keep P_out constant, adjust both R and D

$$R_{new,d} = \frac{{V_{out}}^2}{P} = \frac{30^2}{250} = 3.6\Omega$$

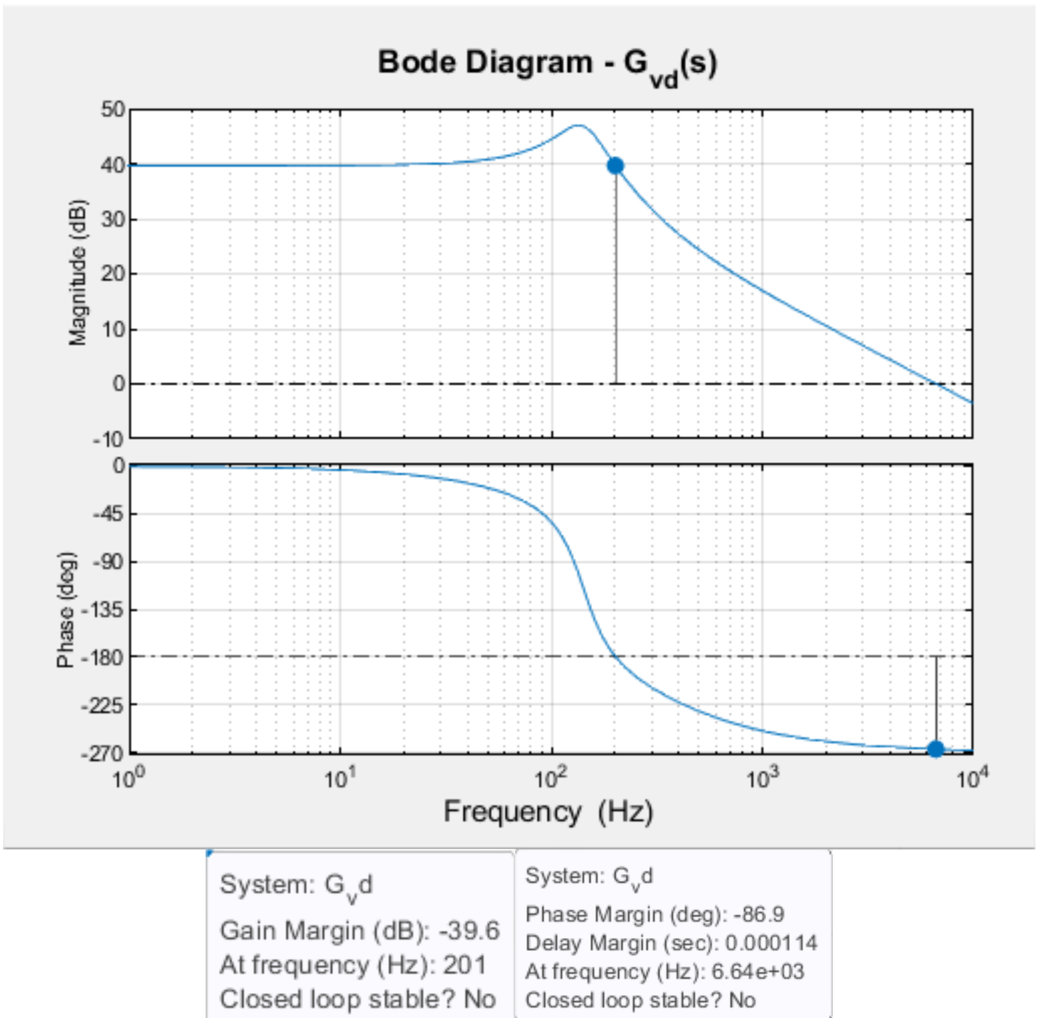
$$D_{new,d}{}' = \frac{V_g}{V_{out}} = \frac{24}{30} = 0.8$$

Problem 2 Bode Plots (Sergio Alexander Sunley Pocasangre)

**Problem 2:** We operate the boost converter of Problem 1 at the following nominal conditions

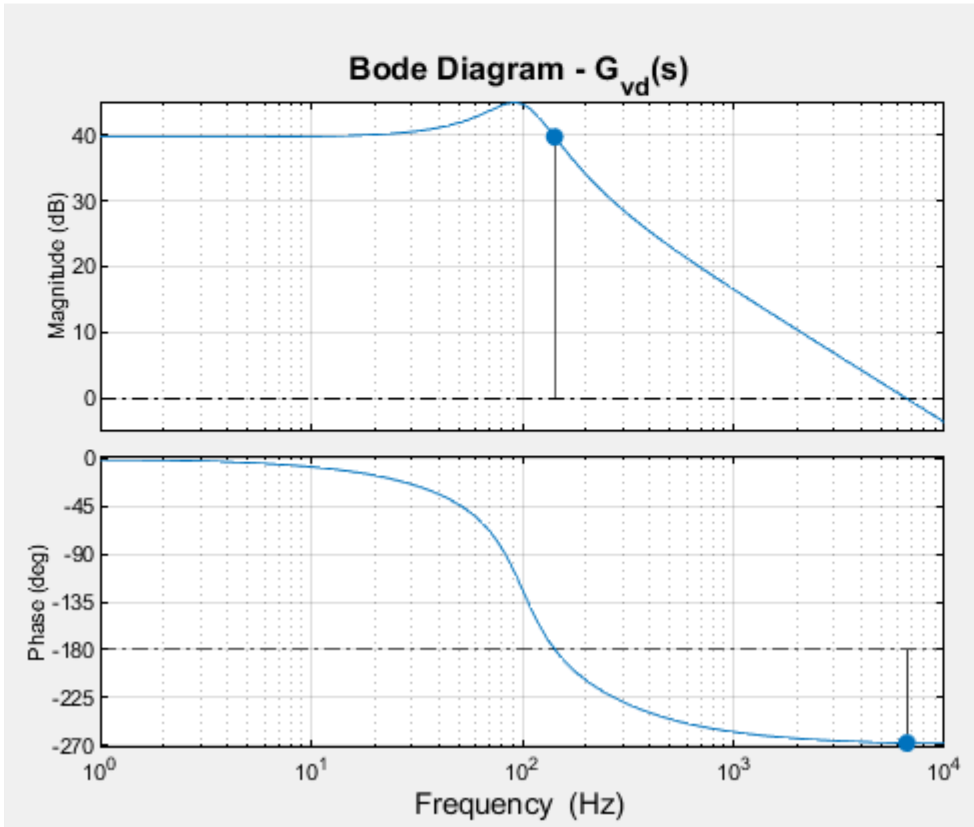
$V_{in}$ = 24V,
output voltage = 48V,
output power = 250 W,
$f_{sw}$ = 100 kHz,
L = 1.25 mH,
C = 250 µF.

**Bode plot with initial values**



Does the stability of the system $G_{vd}(s)$ with no compensation (controller, $C(s) = 1$) increases, decreases, or remains unaffected when

- **You redesign inductance, L to reduce the current ripple of i(t) shown in Figure 1.**

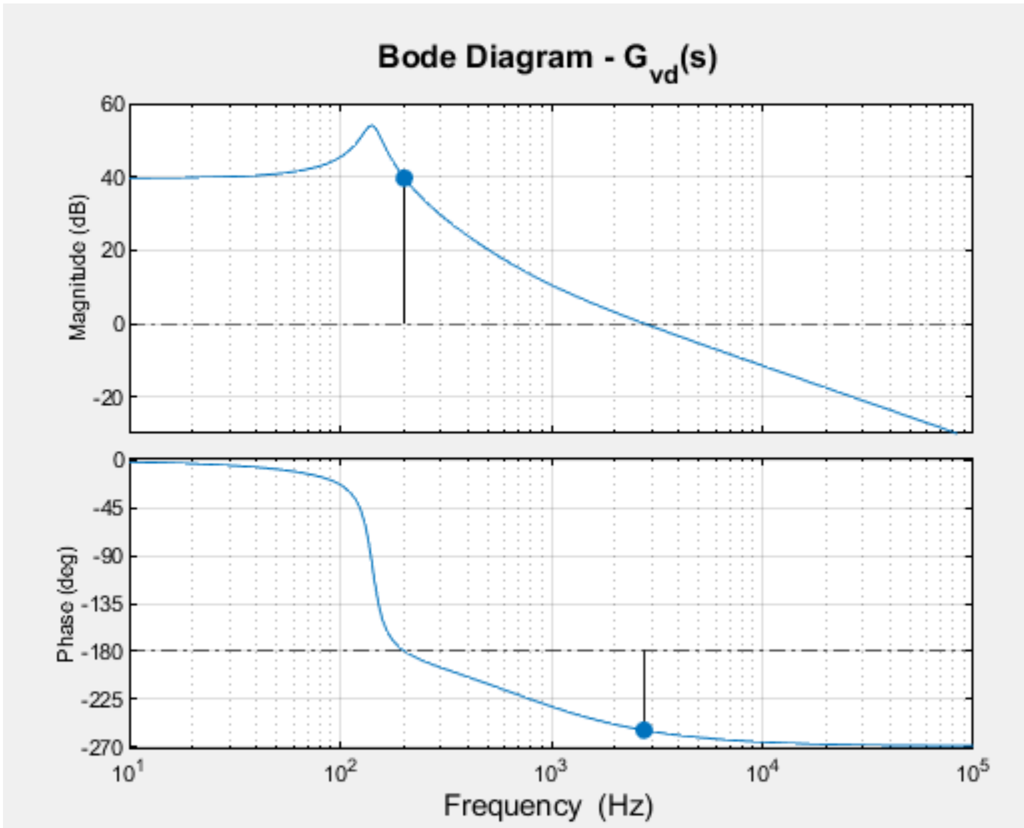



System: $G_v d$
Gain Margin (dB): -39.6
At frequency (Hz): 142
Closed loop stable? No

System: $G_v d$
Phase Margin (deg): -88.1
Delay Margin (sec): 0.000114
At frequency (Hz): 6.63e+03
Closed loop stable? No

Inductance value used: L = 2.50 mH

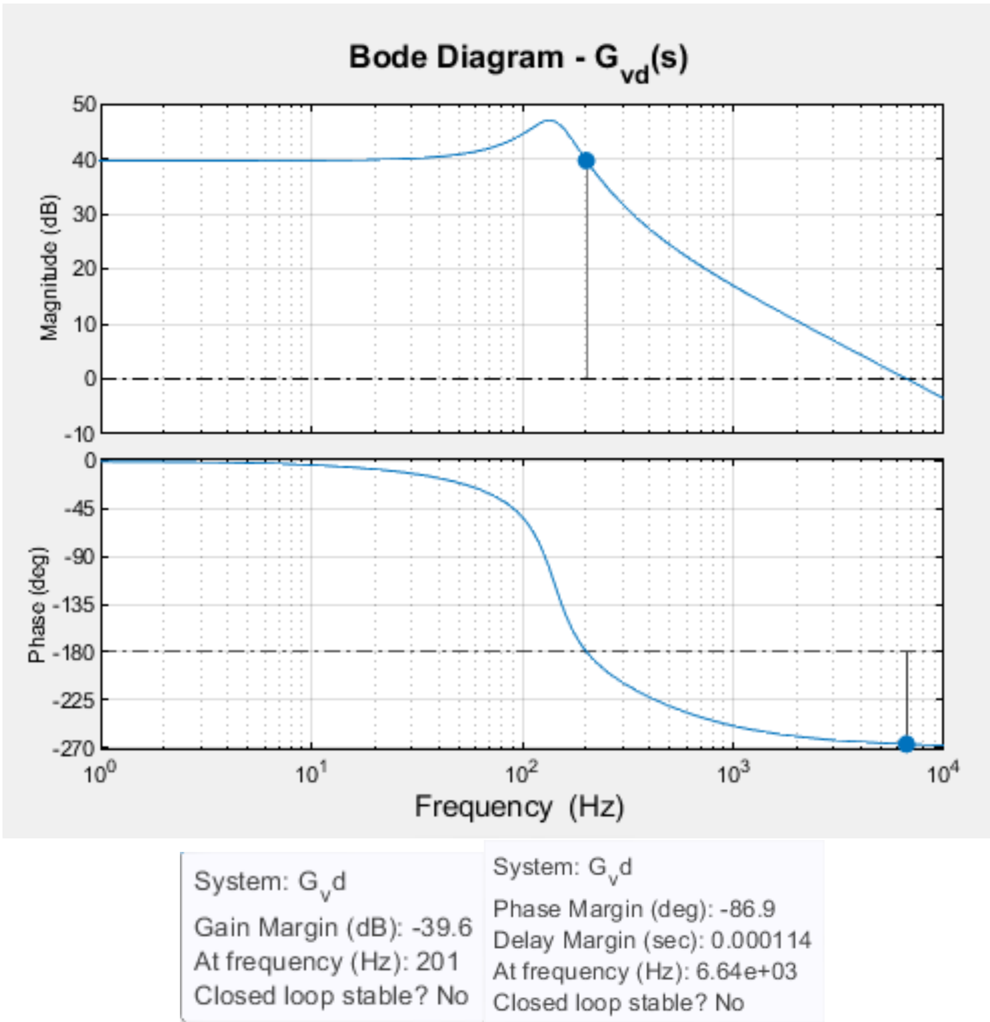
- **You change the output power to 100 W.**



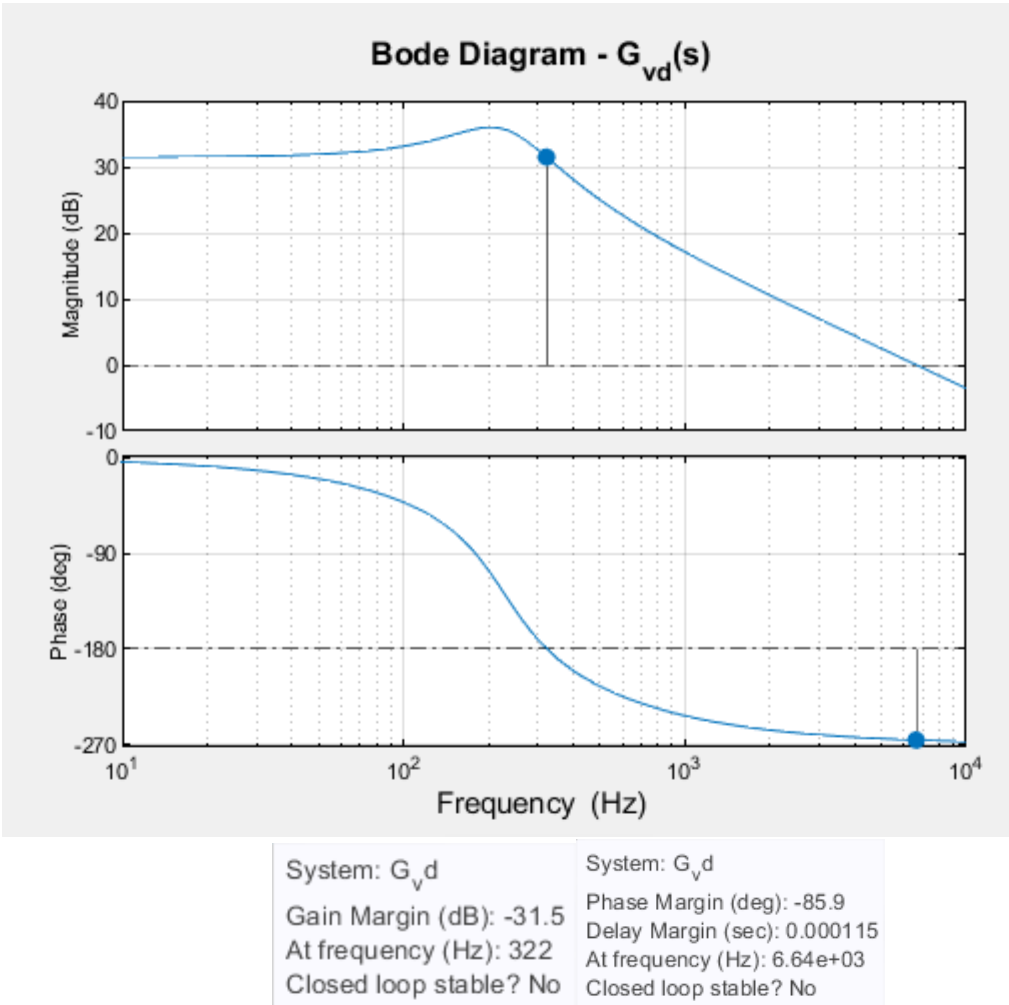

System: G_vd
Gain Margin (dB): -39.6
At frequency (Hz): 201
Closed loop stable? No

System: G_vd
Phase Margin (deg): -74.5
Delay Margin (sec): 0.000288
At frequency (Hz): 2.75e+03
Closed loop stable? No

- **You change the switching frequency to 10 kHz.**



- **You change the output voltage to be at 30 V.**




**Summary:**

For the **original system** (system with given values), the **gain margin (GM) is -39.6 dB** and **phase margin (PM) is -86.9°**. Since we have a negative phase margin, system is not stable.

- By increasing L (to reduce current ripple) PM increases to -88.1°. Gain Margin is not altered. Hence, **the stability of the system decreases with an increase of L (reducing current ripple)**

- By decreasing power to 100 W, PM decreases to -74.5°. GM is not altered. Hence, **the stability of the system increases when reducing output power.**

- **Changing switching frequency does not cause modification on stability (stability remains unaffected)**. Transfer function does not depend on switching frequency.

- Modifying the output voltage to 30 V (duty = 0.2) causes GM to decrease to -31.5 dB and PM to decrease to - 85.9°. **The stability of the system increases with the reduction of the output voltage**

Problem 3 (Anisha Chutani)

3. $G_1(s) = G_0\left(\dfrac{1}{1+\frac{s}{\omega_1}}\right)$ $\quad G_2(s) = \dfrac{1}{1+\frac{s}{\omega_2}}$

$G_1(s) = 100\left(\dfrac{1}{1+\frac{s}{2\pi 1000}}\right)$ $\quad G_2(s) = \dfrac{1}{1+\frac{s}{2\pi 10{,}000}}$

~~as s →~~

$100 \cdot \dfrac{1}{1+\frac{j\omega}{2\pi 100}}$

as $\omega \to 0$:

$|G_1(s)| \to 100 = 20\log 100$
$= 40\,dB$

$\angle G \to 0$

as $\omega \to \infty$

$|G| \to 0$ ??

$\angle G \to -90^\circ$

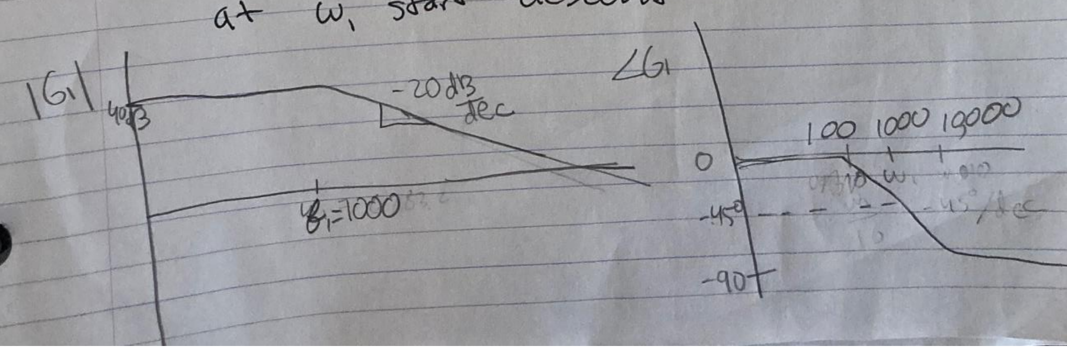
pole: $1 + \dfrac{s}{2\pi 1000} = 0$

$s = -2000\pi = -6283.2\,Hz$

mag. decreases at 20 dB/dec

& phase decreases one dec before to after
45°/dec

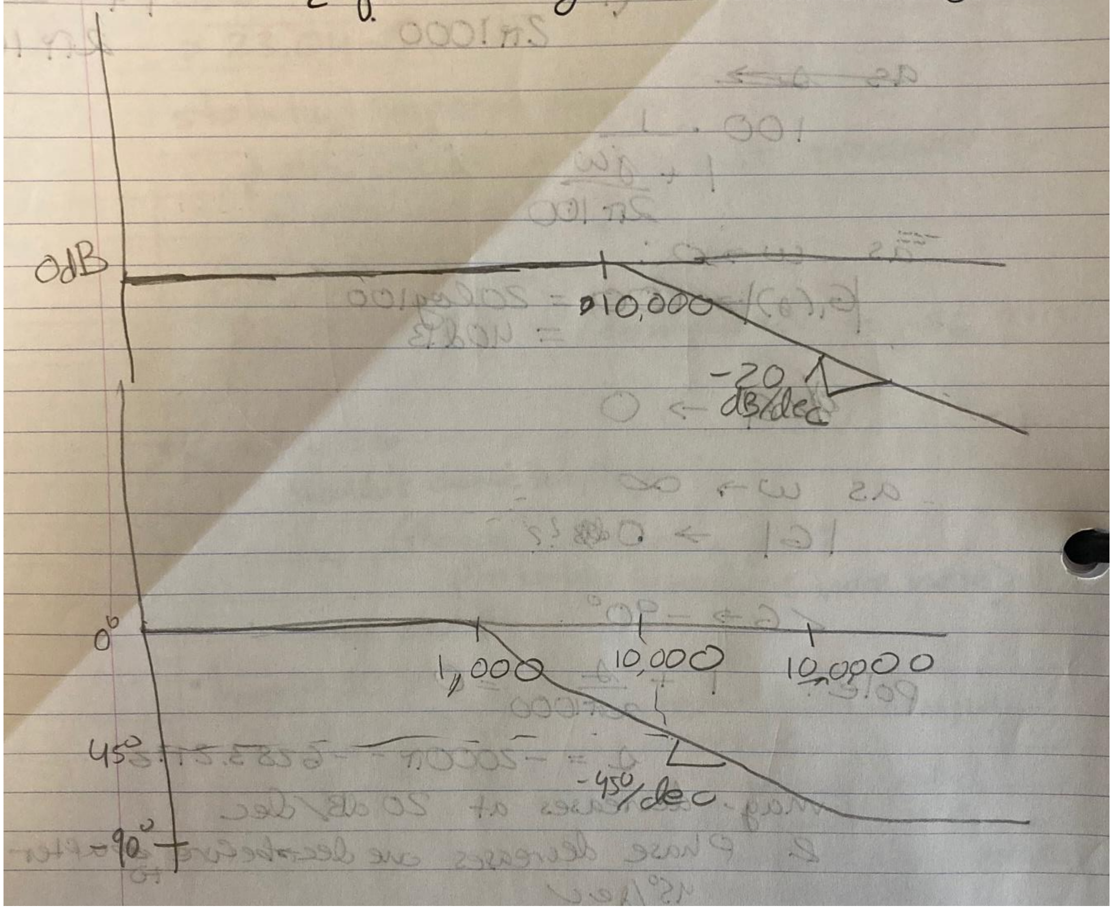
at $\omega_1$ start descent



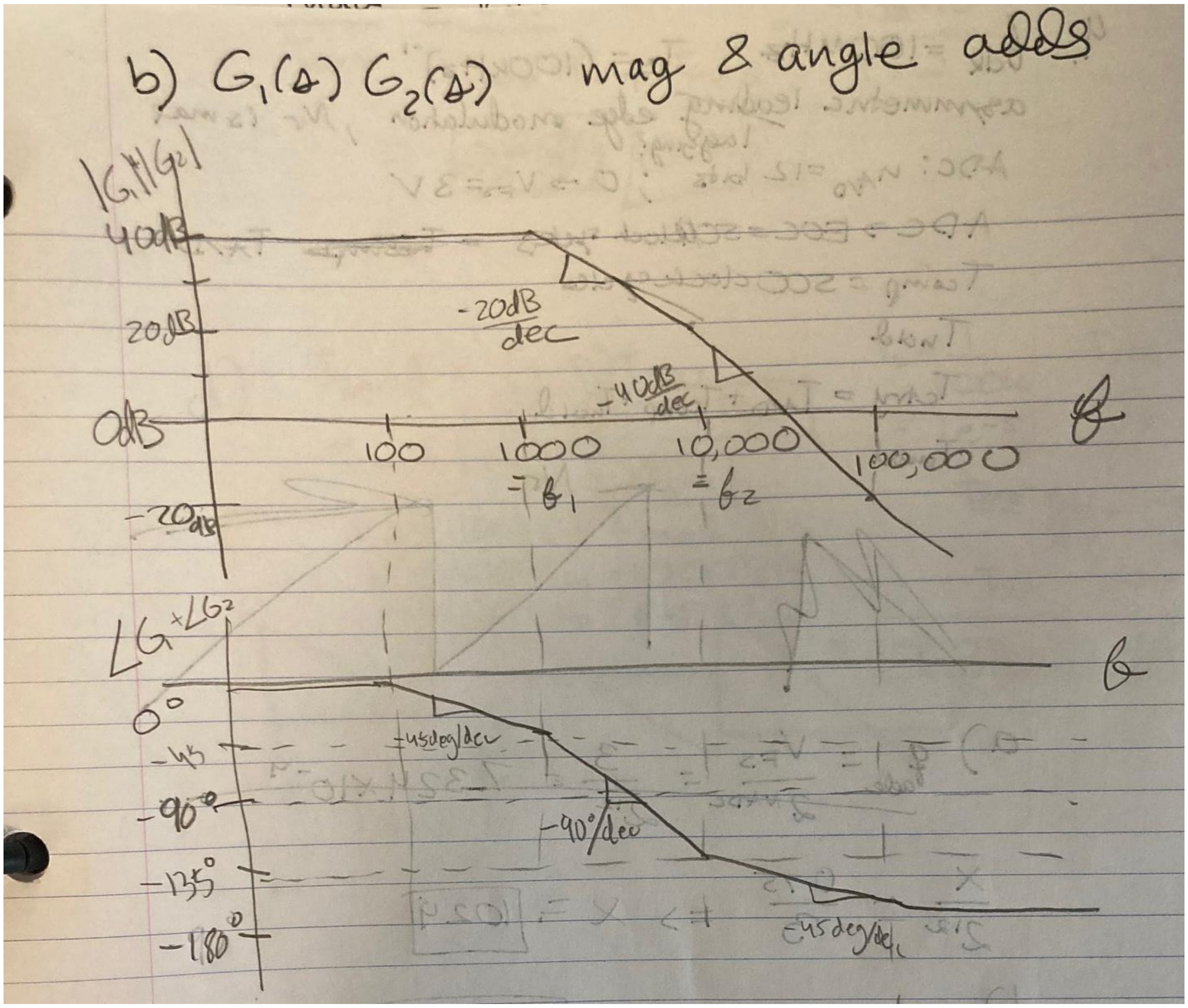
$$G_2 = \frac{1}{1 + \frac{s}{20{,}000\pi}}$$

same except $|G_2(0)| = 1$ (no magnitude scaling

& $\omega_2$ further right




**b)**




Verify results: