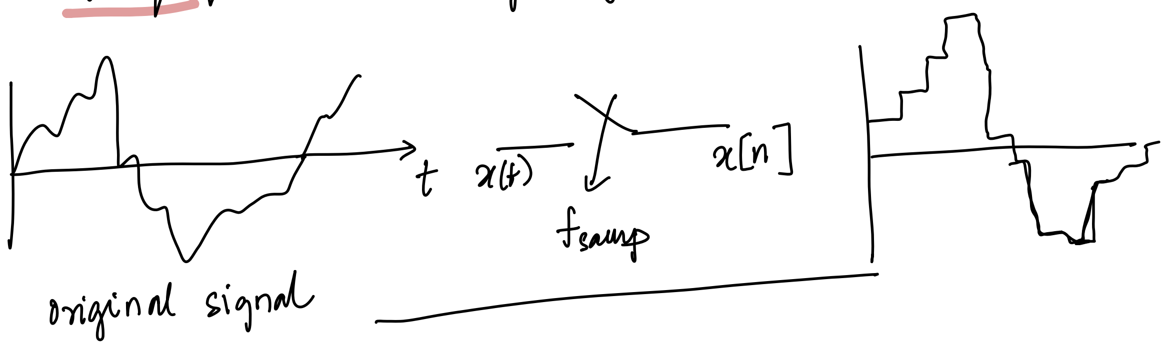
Lecture 7 (January 25, 2022)

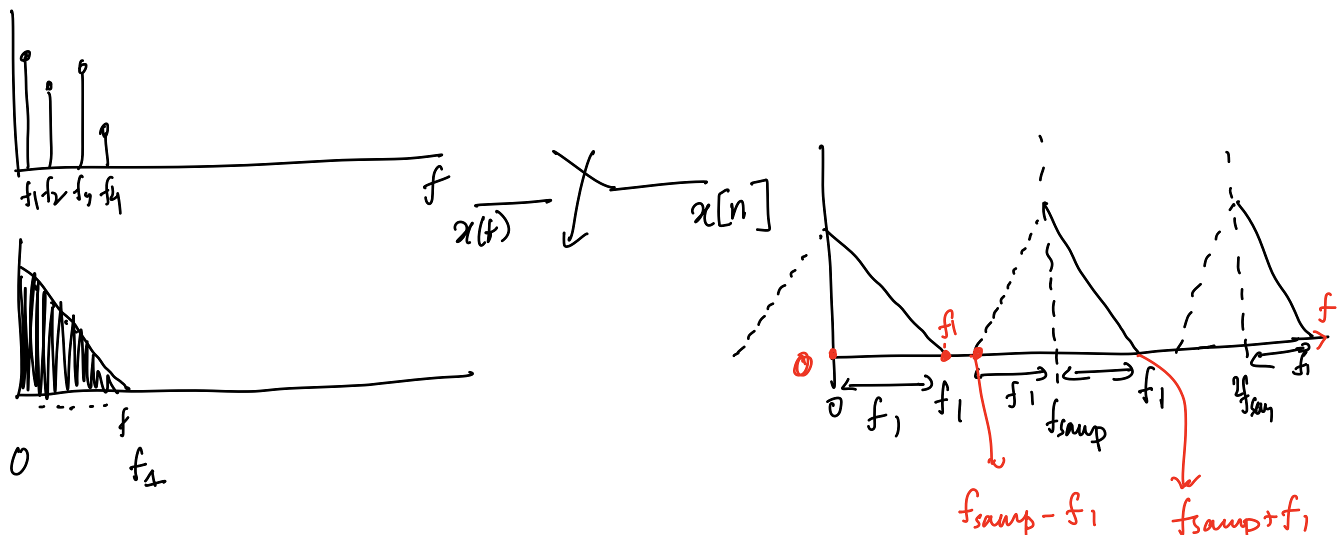
HW2 due on thursday (1/27, 4:30pm).

Lab: Nyquist frequency.

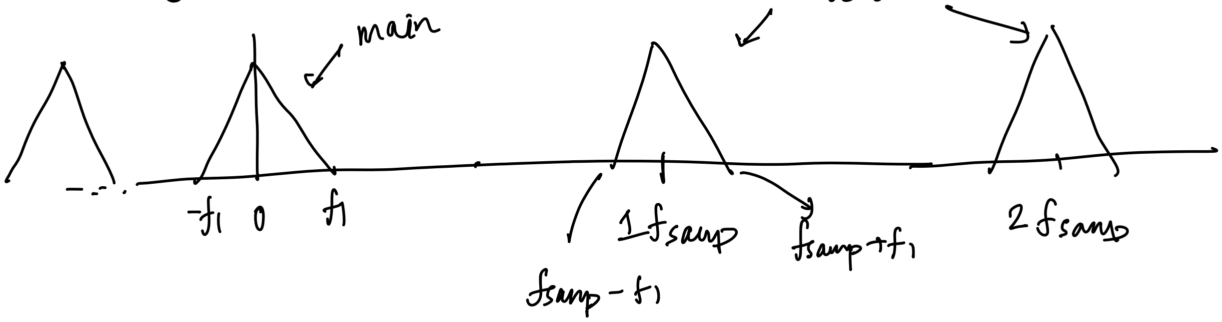
Min freq at which you get the original signal



original signal



@ Increase my sampling freq

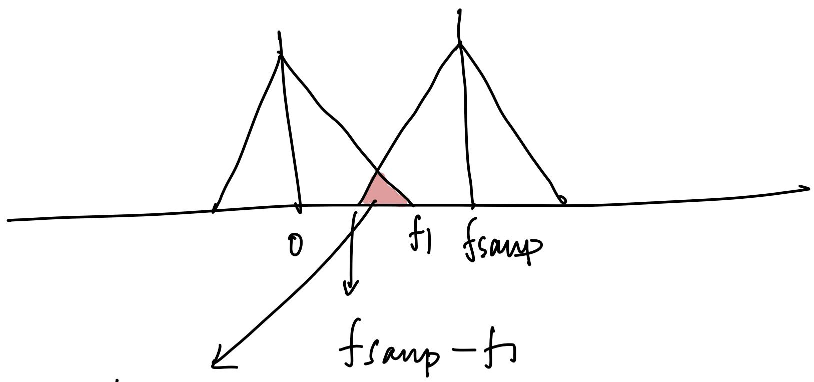


Reconstructing

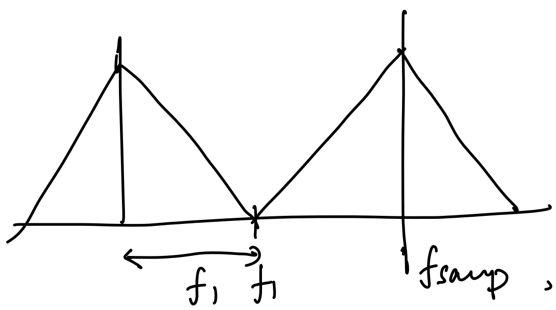
Anti aliasing filter. (low pass filter)

you get back the original signal.

(b) Reduce sampling freq

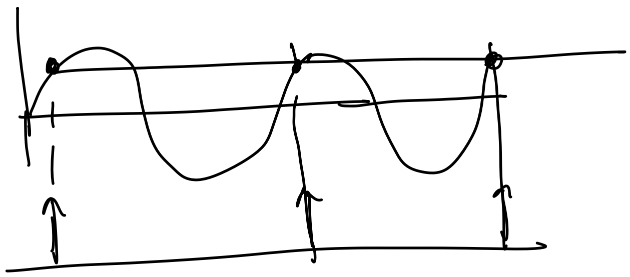


signal gets corrupted.



Boundary condition

$$f_{samp} \geq 2 f_1$$ (Nyquist criterion)



* Let the sampling freq be = f_{samp}.
* Let the input freq be = f_1

ⓐ If $f_1 < f_{samp}/2$

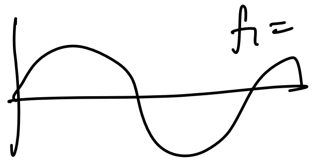
perfect output, content of freq $\Rightarrow f_1 = f_{output}$

ⓑ If not,

$$f_{output} = f_1 - k \cdot \frac{f_{samp}}{2} \quad (k > 0)$$

till
$$\boxed{0 < f_1^{eq} < \frac{f_{samp}}{2}}$$

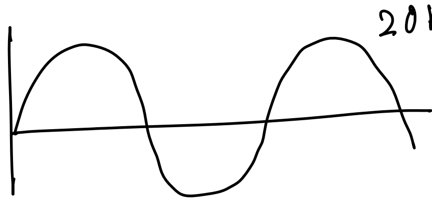
eg ⓐ



$f_1 = 1\text{kHz}$. $f_{samp} = 10\text{kHz}$

$f_{output} = f_1 = 1\text{kHz}$.

ⓑ

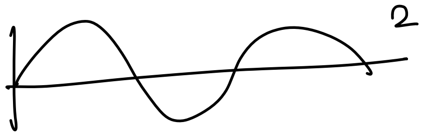


20kHz

$$\underbrace{f_{samp} = 10\text{kHz}}_{\frac{f_{samp}}{2} = 5\text{kHz}}$$

$f_{output} = 0\text{ Hz}$. $k = 4$

(c)



$f_{samp} = 10\text{kHz}$

$f_{output} = 21 - 5 = 16\text{ kHz}$

$\vdots$

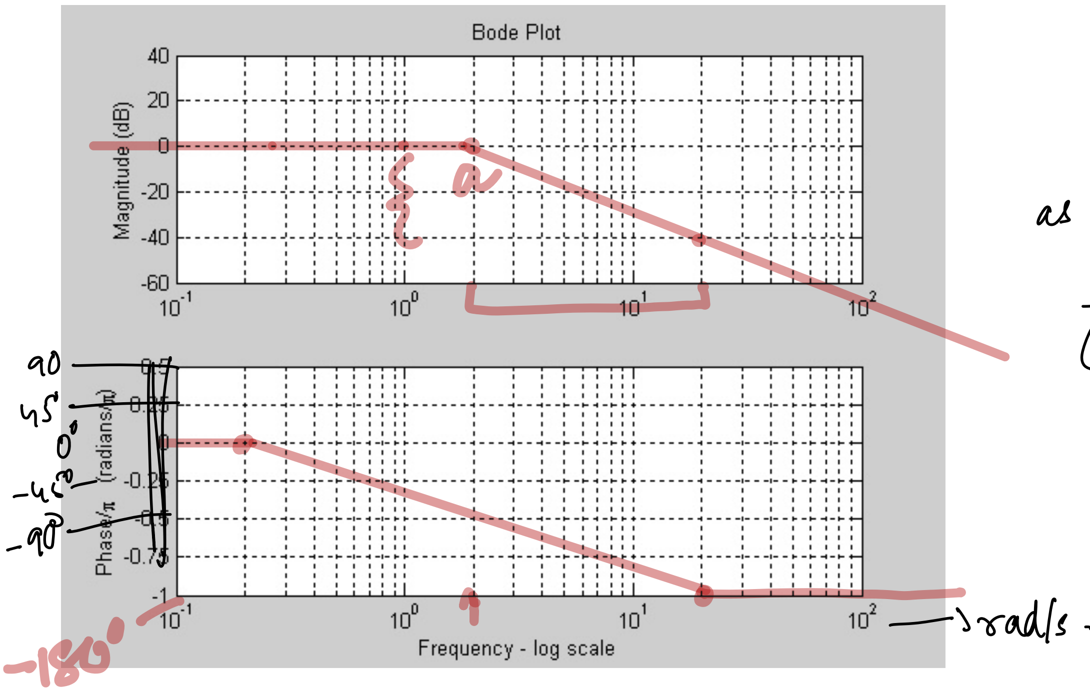
$= 1\text{ kHz}.$

Bode Plots

① Double pole.

$$F(s) = \frac{1}{(s/a+1)^2}$$

eg $a = 2$ rad/s



as $s \to \infty$

$\frac{s}{a} \gg 1$

$$\frac{1}{(\frac{s}{a})^2}$$

$$\frac{1}{(\frac{j\omega}{a})^2}$$

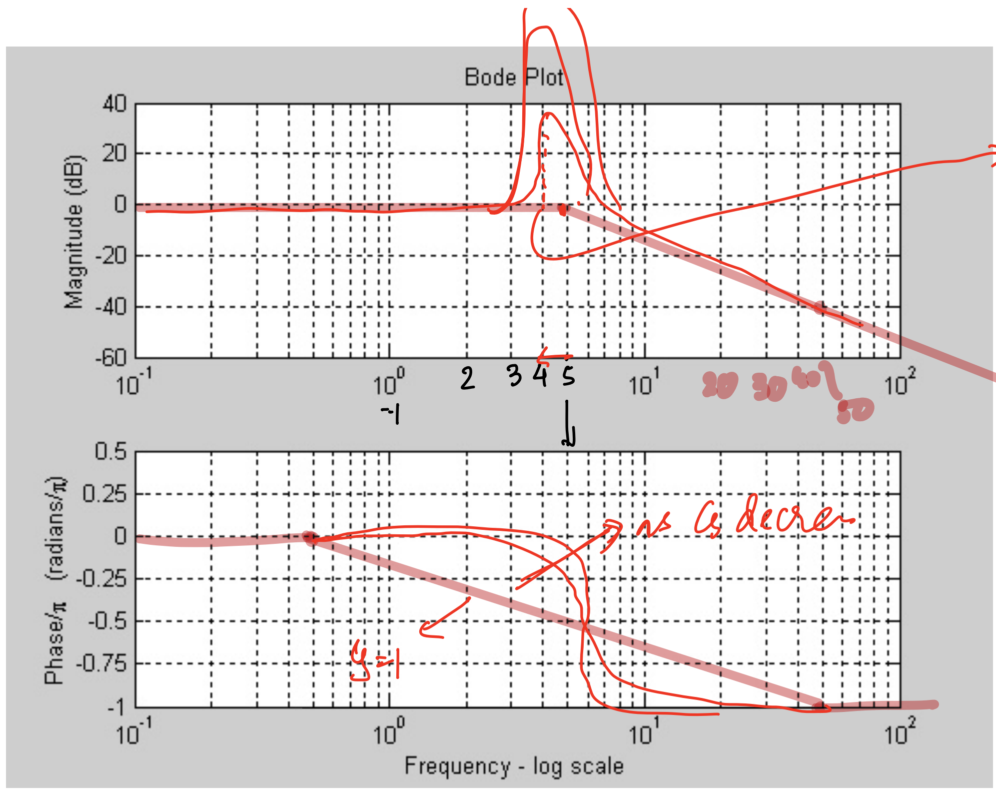
$$= \ominus \frac{a^2}{\omega^2}$$

$$T(s) = \left(\frac{s}{b}+1\right)^2$$

$$F(s) = \frac{1}{(s/\omega_0)^2 + 2\zeta\, s/\omega_0 + 1}$$

$$\frac{1}{(s/a+1)^2}$$

$$= \frac{1}{s^2/a^2 + 2s/a + 1}$$



$\omega_r = \omega_0\sqrt{1-\zeta^2}$

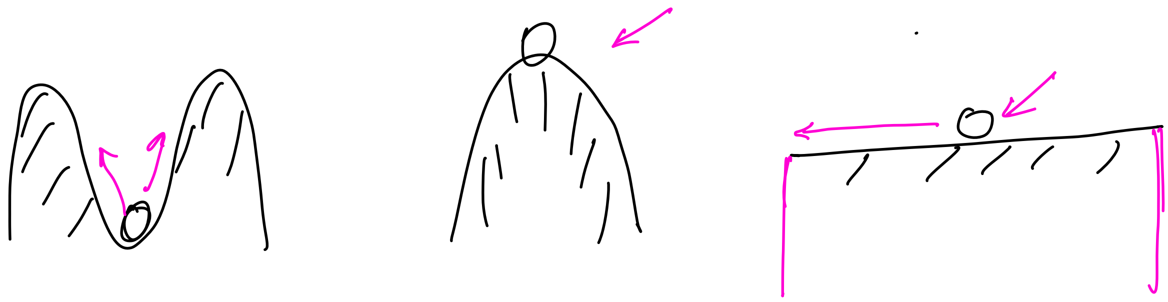
$\to \dfrac{1}{(s/\omega_0+1)^2}$

$$\frac{1}{(s/\omega_0)^2 + 2\zeta\, s/\omega_0 + 1}$$

as $\zeta \to 1$

$\omega_0 = 5$ rad/s

Stability (Control system).

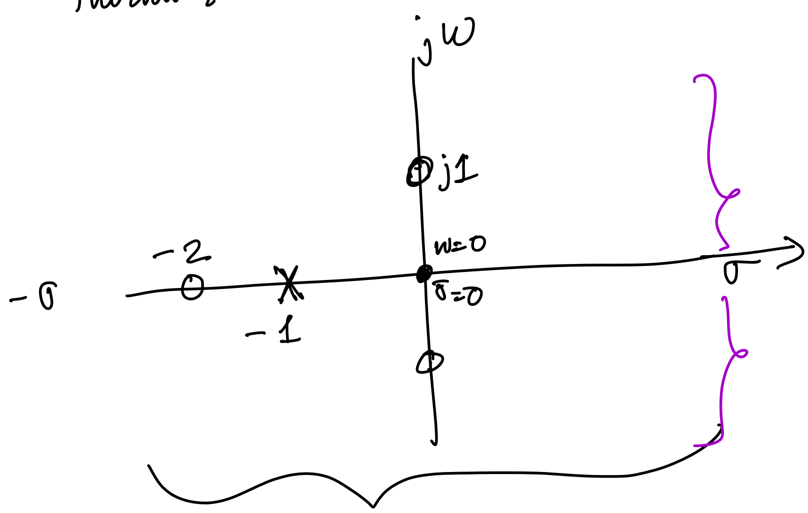


II

How to determine stability

Routh Array (Routh-Hurwitz)

Hurwitz



S plane.

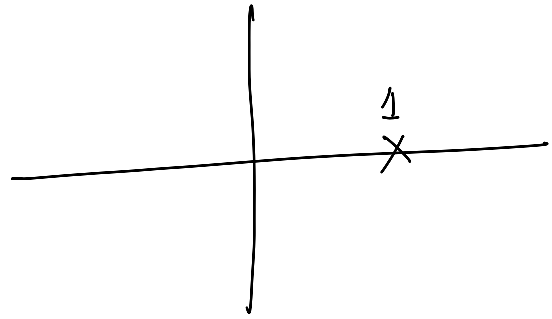
$$\frac{1}{s+1}$$

$$\frac{s+2}{s+1}$$

$$\frac{\boxed{(s-j1)}(s+2)}{(s+1)}$$

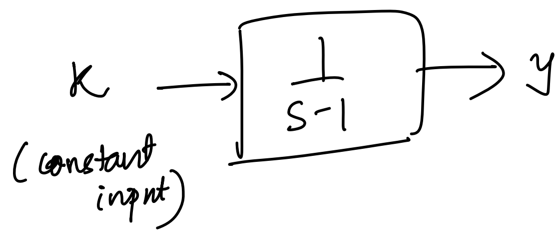
$$\frac{(s+j)(s-j)(s+2)}{(s+1)}$$

$$\frac{(s^2+1)(s+2)}{(s+1)}$$



$\longrightarrow \dfrac{1}{(s-1)}$

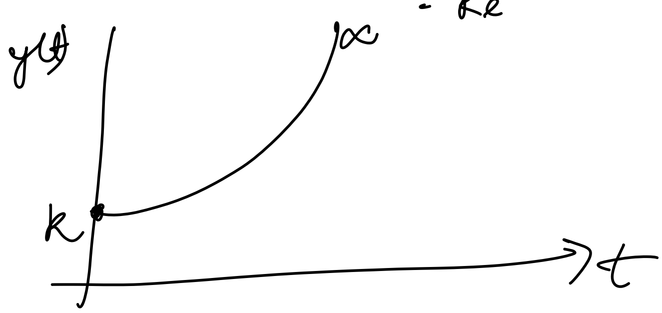
why unstable?



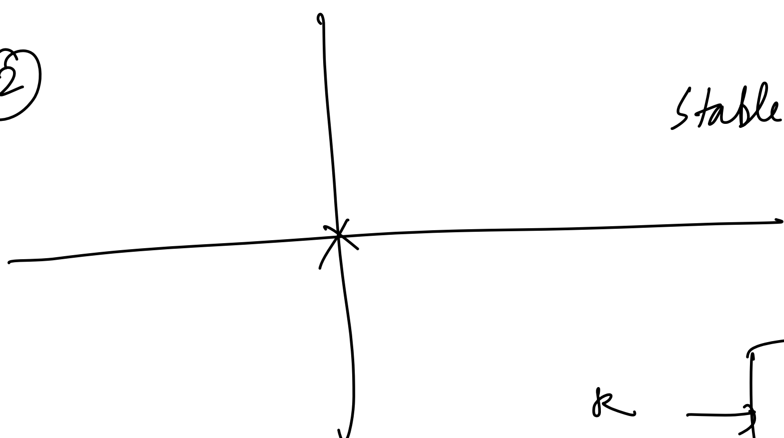
$$y(s) = \frac{k}{(s-1)}$$

$$y(t) = \mathcal{L}^{-1}\left[y(s)\right] = \mathcal{L}^{-1}\left(\frac{k}{s-1}\right)$$

$$= ke^{t}$$

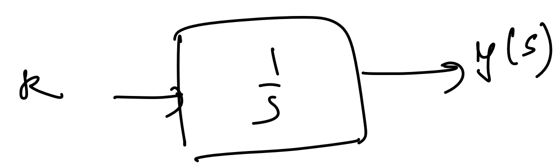


(2)

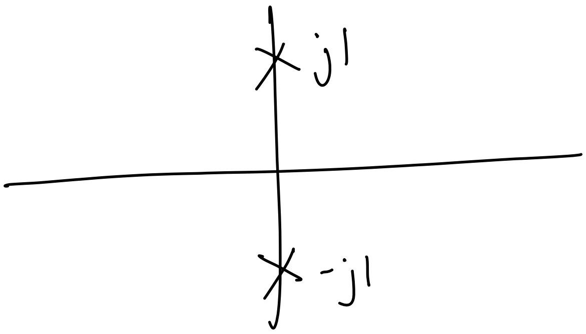


Stable?

$$T(s) = \frac{1}{s}$$



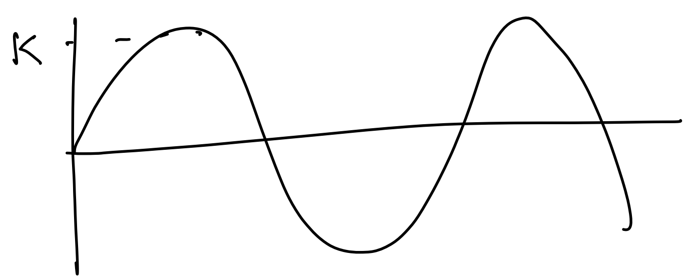
$$y(s) = \frac{k}{s} \qquad y(t) = \mathcal{L}^{-1}\left(\frac{k}{s}\right) = k$$

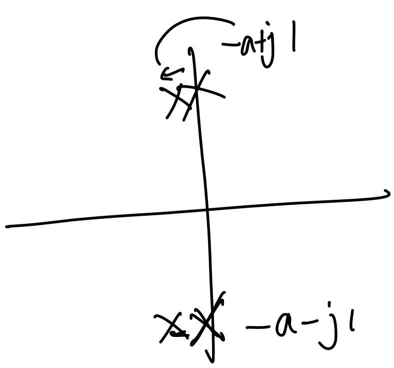


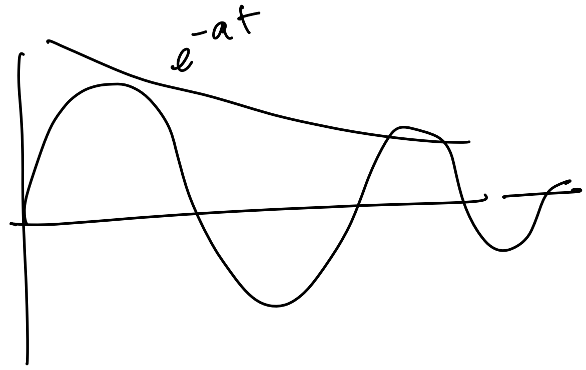
$$\frac{1}{s^2+1}$$

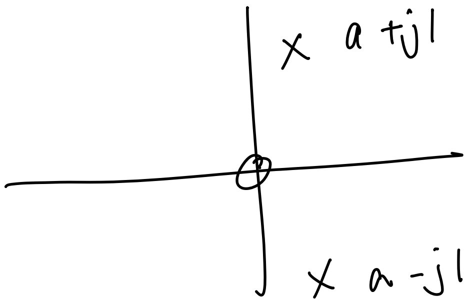
$$K \rightarrow \boxed{\frac{1}{s^2+1}} \rightarrow$$

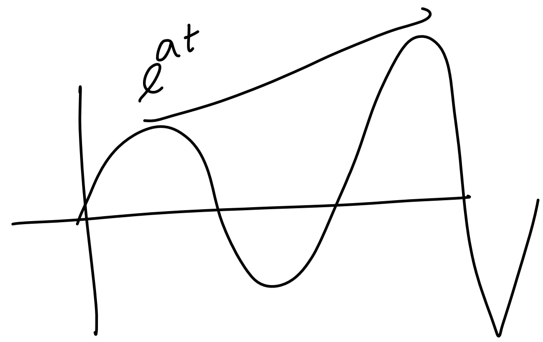
$$y(s) = \frac{K}{s^2+1} \quad \Big| \quad y(t) = \mathcal{L}^{-1} y(s) = K \cdot \sin(1 \cdot t)$$



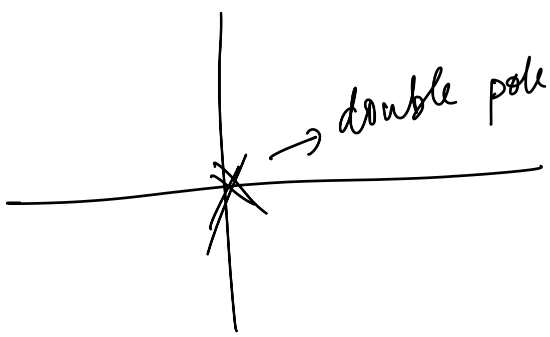








$$\frac{s}{(s+a)^2+1^2}$$

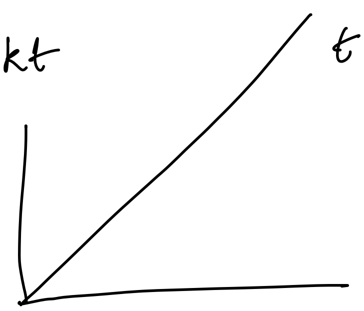


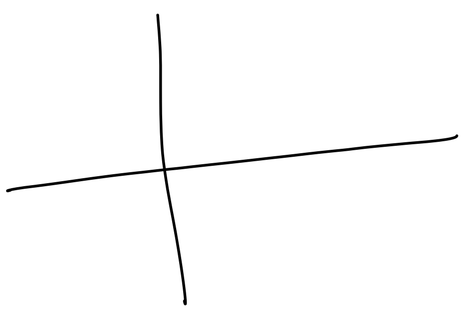
→ double pole at origin

$$F(s) = \frac{1}{s^2}$$

$$y(s) = \frac{k}{s^2}$$

$$y(t) = kt$$



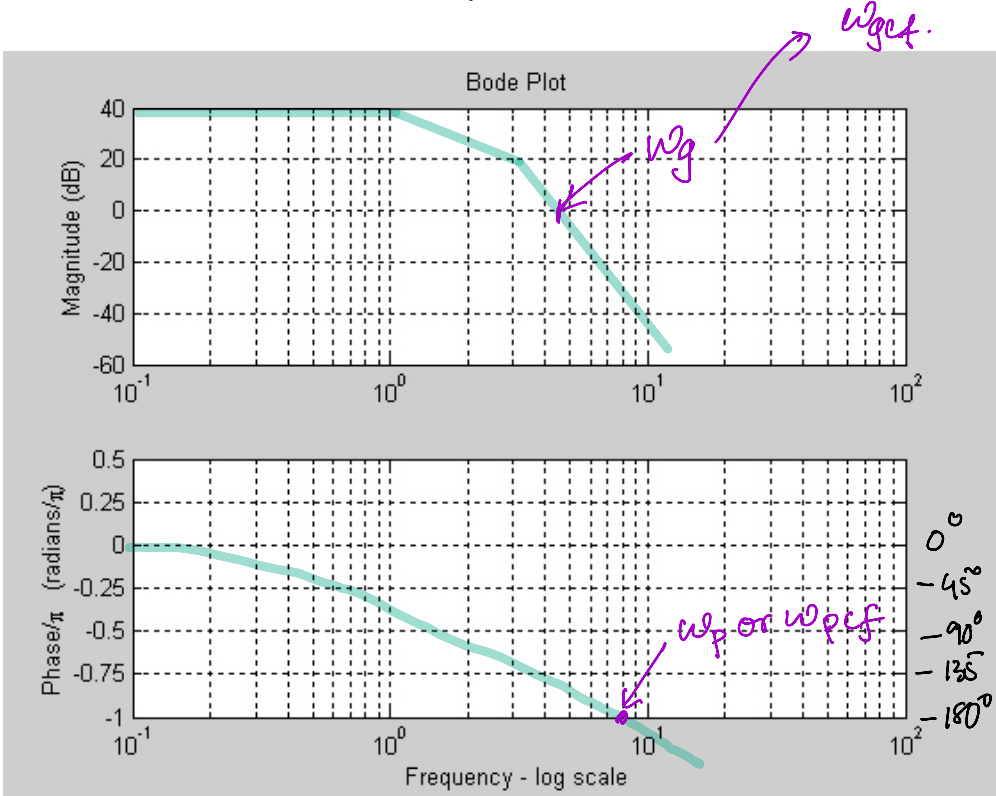


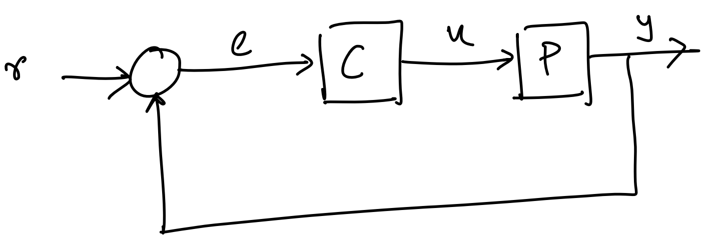
Absolute stability = Routh, Hurwitz

Stable / Not Stable.

Relative Stability.

Phase margin, Gain margin.





$$y = \frac{CP}{1+CP} \cdot r$$

$$\approx \frac{\cancel{CP}}{\cancel{CP}} \approx 1 \quad \therefore \; y = r$$

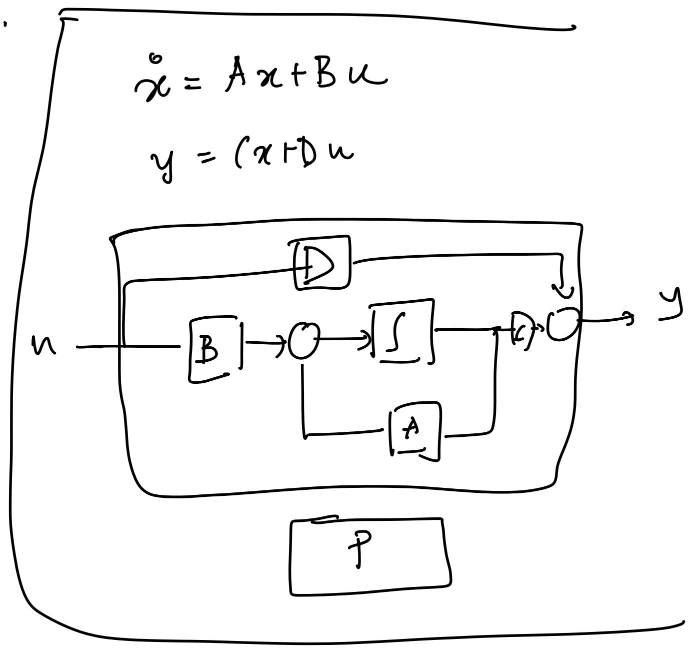
C: Controller transfer function

P: Plant

l = loop gain $:=$ $C \cdot P$.

also called open loop gain

closed loop gain $= \dfrac{l}{1+l}$.



$$y = \frac{l}{1+l} \cdot r$$

$$\boxed{l = -1}$$

$$y = \frac{l}{0} r = \infty$$

O/P goes to infinity.

$$l = C \cdot P$$

$$l(j\omega) = C(j\omega) P(j\omega) = -1.$$

$$l(j\omega) = -1.$$

$$\Rightarrow \boxed{|l(j\omega)| = 1 \quad \& \quad \measuredangle l(j\omega) = -180^\circ}$$

If you design a controller $C(s)$ [or $C(j\omega)$] such that $C(s) \cdot P(s)$ [or $C(j\omega) P(j\omega)$] is equal to -1, then your control design has resulted in an unstable system.

* Moral of control design

⇒ Design $C(s)$ (or $C(j\omega)$) such that, $C(s)P(s)$ [or $C(j\omega)P(j\omega)$] remains "as far" as possible from -1.

⇓

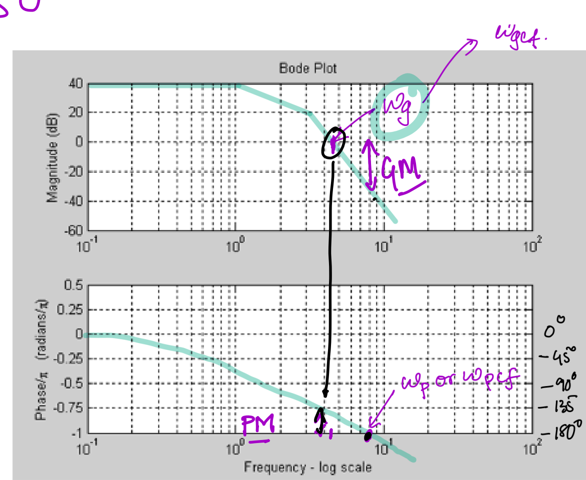
Concept of relative stability in terms of gain & phase margin.

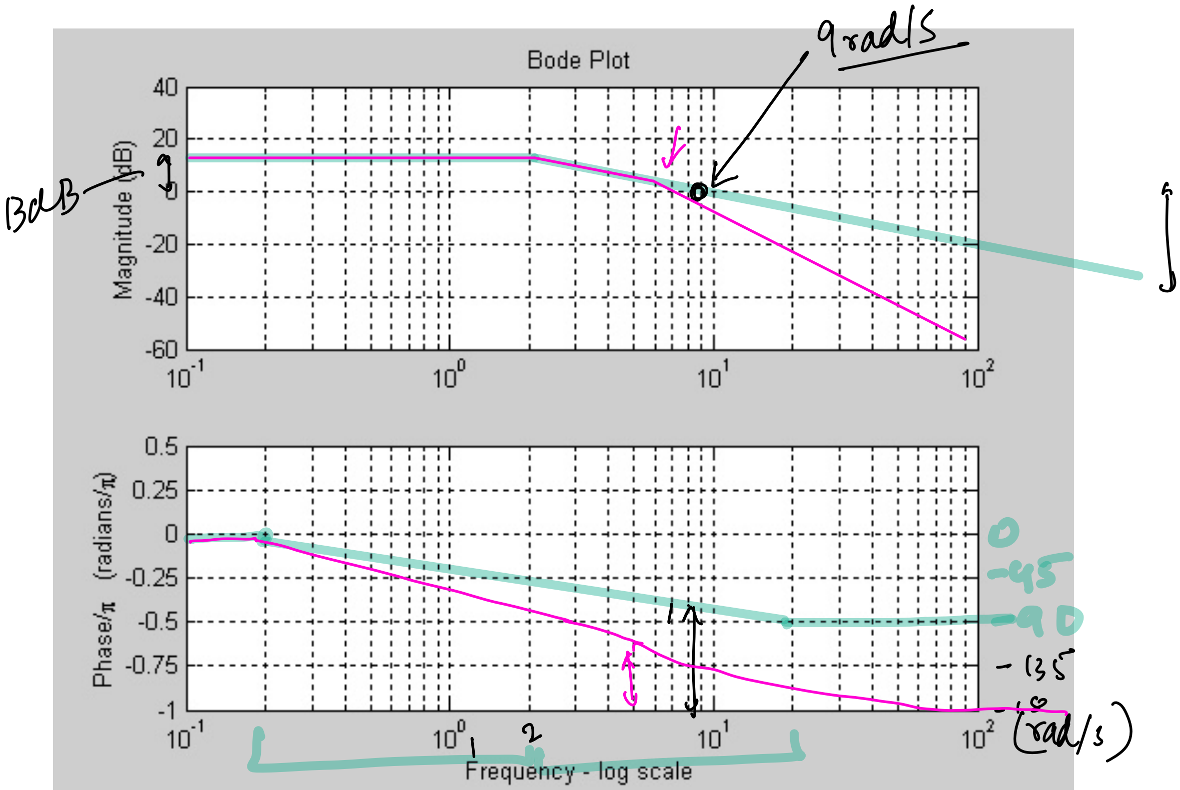
→ If $\boxed{|\ell(j\omega)| = 1}$, be as far away as possible from $\angle \ell(j\omega) = -180^\circ$

→ If $\underline{\angle \ell(j\omega) = -180^\circ}$, be as far away as possible from $|\ell(j\omega)| = 1$.

$$|\ell(j\omega)|_{\omega=\omega_g} = 1 = 0\,dB$$

$$\angle \ell(j\omega)|_{\omega=\omega_p} = -180^\circ$$





Ex $\boxed{\dfrac{10}{s+2}} \longrightarrow$ Mag, phase plot

$$\frac{10/2}{s/2+1} = \frac{5}{(s/2+1)} = (5)\left(\frac{1}{s/2+1}\right)$$

g_1 g_2

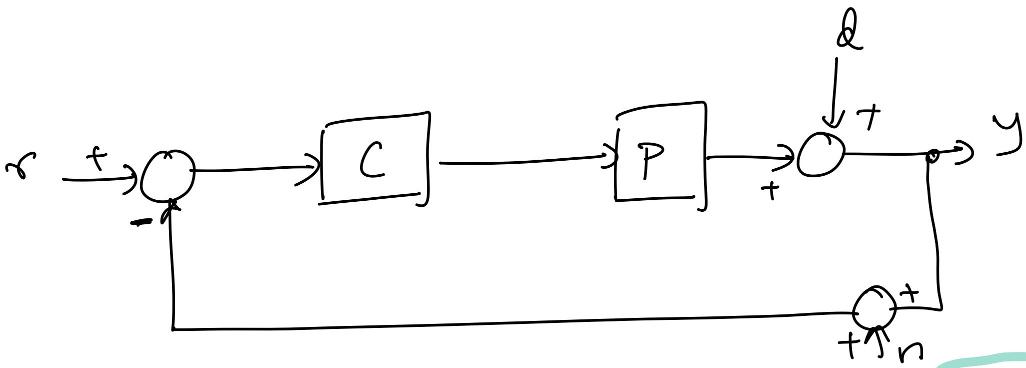
$$20\log_{10}(5) = 13\,dB$$

PM $\simeq 90+20$
$\simeq 110^0$

GM = infinite

$\left(\frac{s+2}{s+1}\right)$ → non minimum phase systems.

$\left(\frac{s-2}{s+1}\right)$ → minimum phase systems



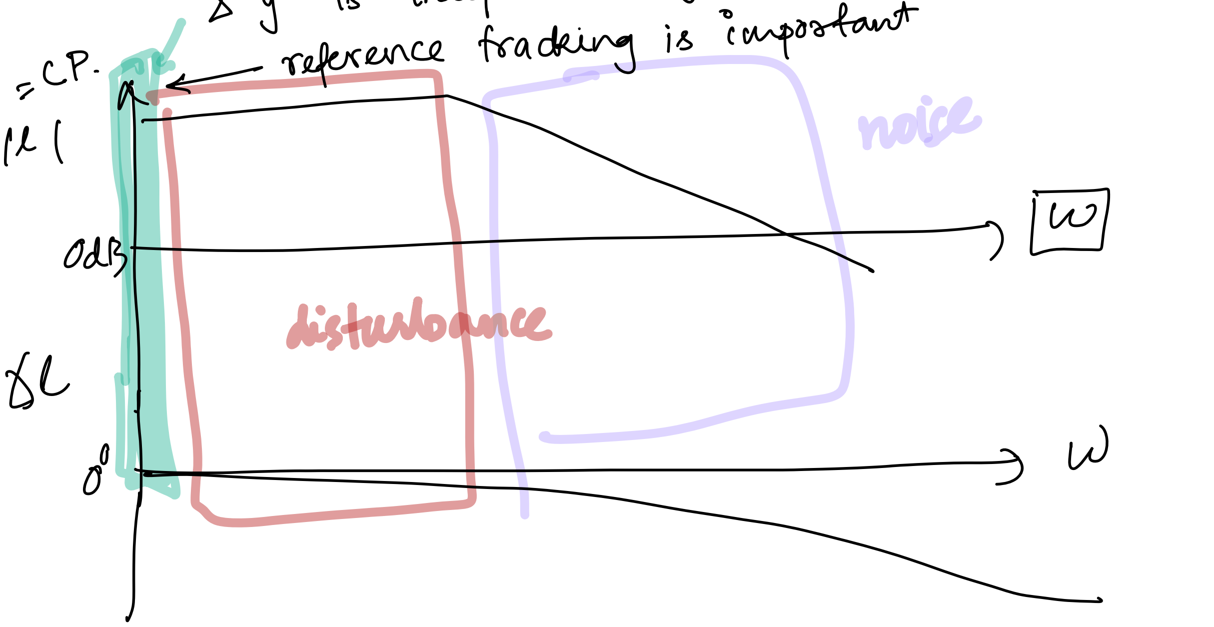
$$y = \boxed{\frac{CP}{1+CP}\cdot r + \frac{1}{1+CP}\cdot d - \frac{CP}{1+CP}\, n}$$

we wanted C to be very high

$\frac{1}{1+CP} \sim 0$

∴ $y \approx r$ (if $n=0$)

& y is independent of d.



$$y = \frac{CP}{1+CP} \cdot r$$

$$= \frac{l}{1+l} \, r$$

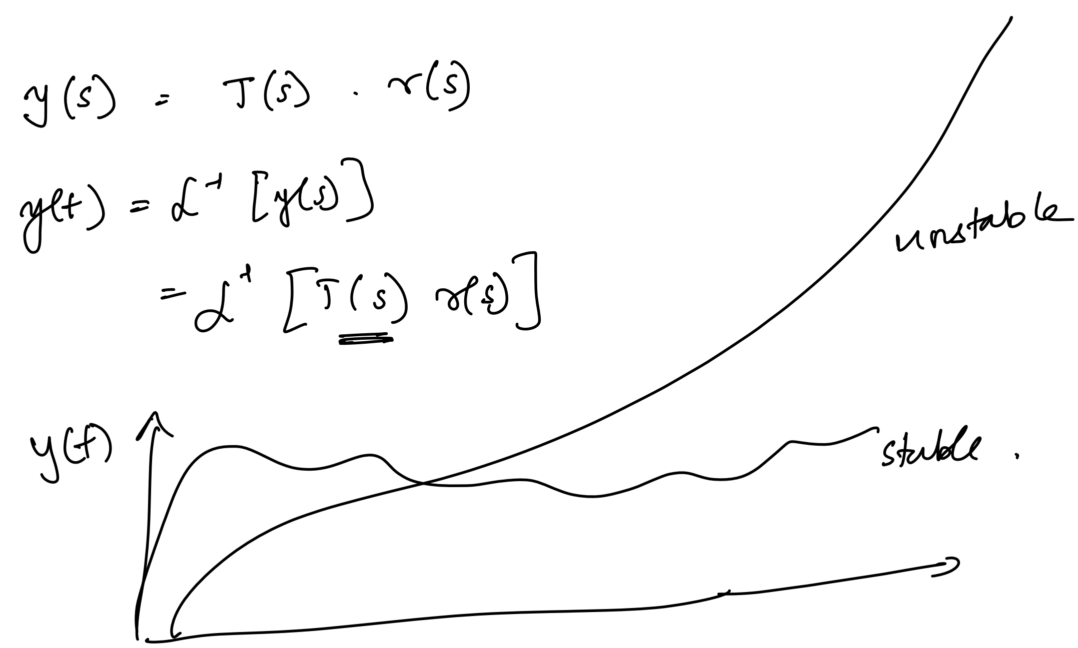
$$= T \, r$$

T: closed loop transfer function.
Complementary sensitivity transfer function.

$$y(s) = T(s) \cdot r(s)$$

$$y(t) = \mathcal{L}^{-1}[y(s)]$$

$$= \mathcal{L}^{-1}[\underline{\underline{T(s)}} \, r(s)]$$



$$T(s) = \frac{\boxed{l(s)}}{\boxed{1 + l(s)}}$$

$T(s) = -1$

You do the bode plot of $l(s) = C(s) \cdot P(s)$.
P(s) is given to you.
You have design freedom over C(s).

That helps you to give a particular shape to $l(s)$

↓

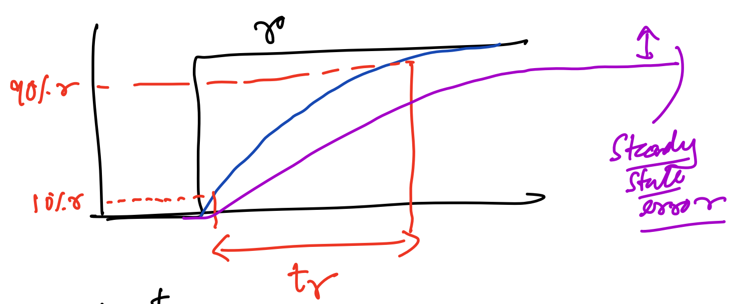
That will influence your output

$$\underbrace{y(t)} = \mathcal{L}^{-1}\left[\frac{l(s)}{1+l(s)} \cdot r(s)\right]$$

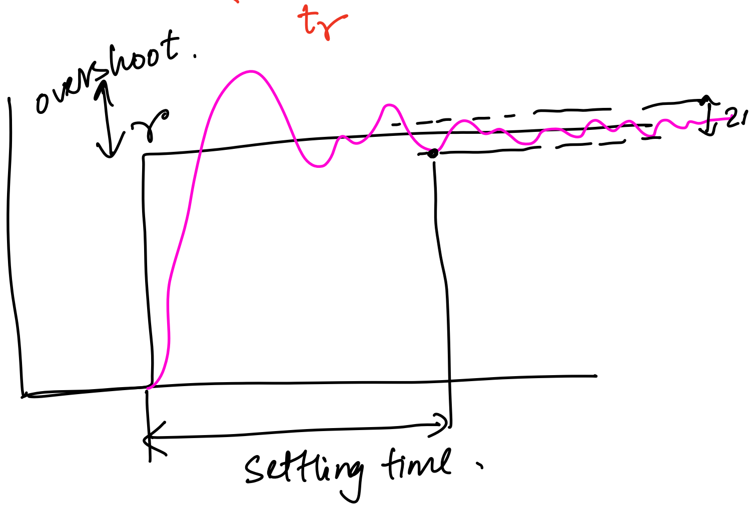
Client gives you information/design guideline in time domain

1) Steady state error :- difference b/w 'r' (reference) & 'y' (output).

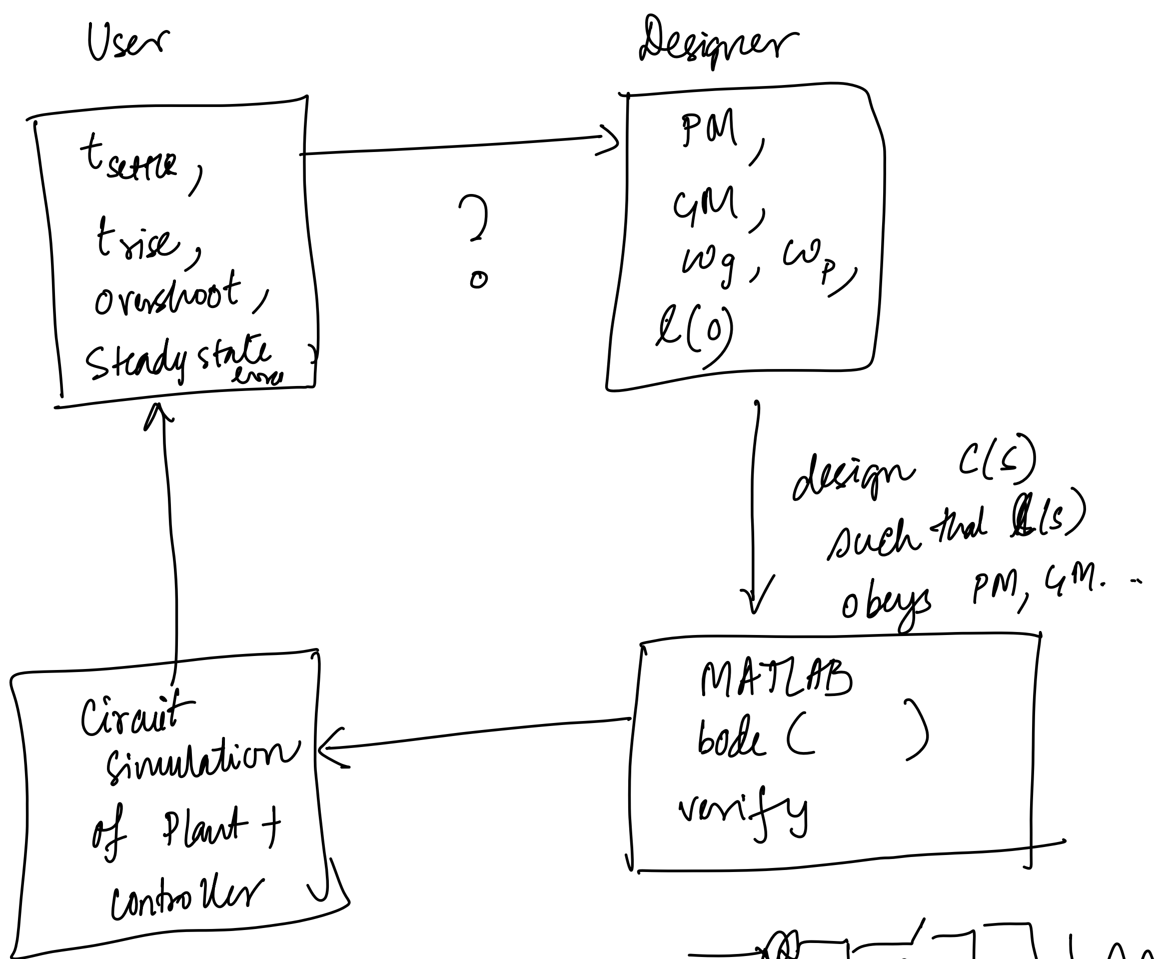
2) Rise time.

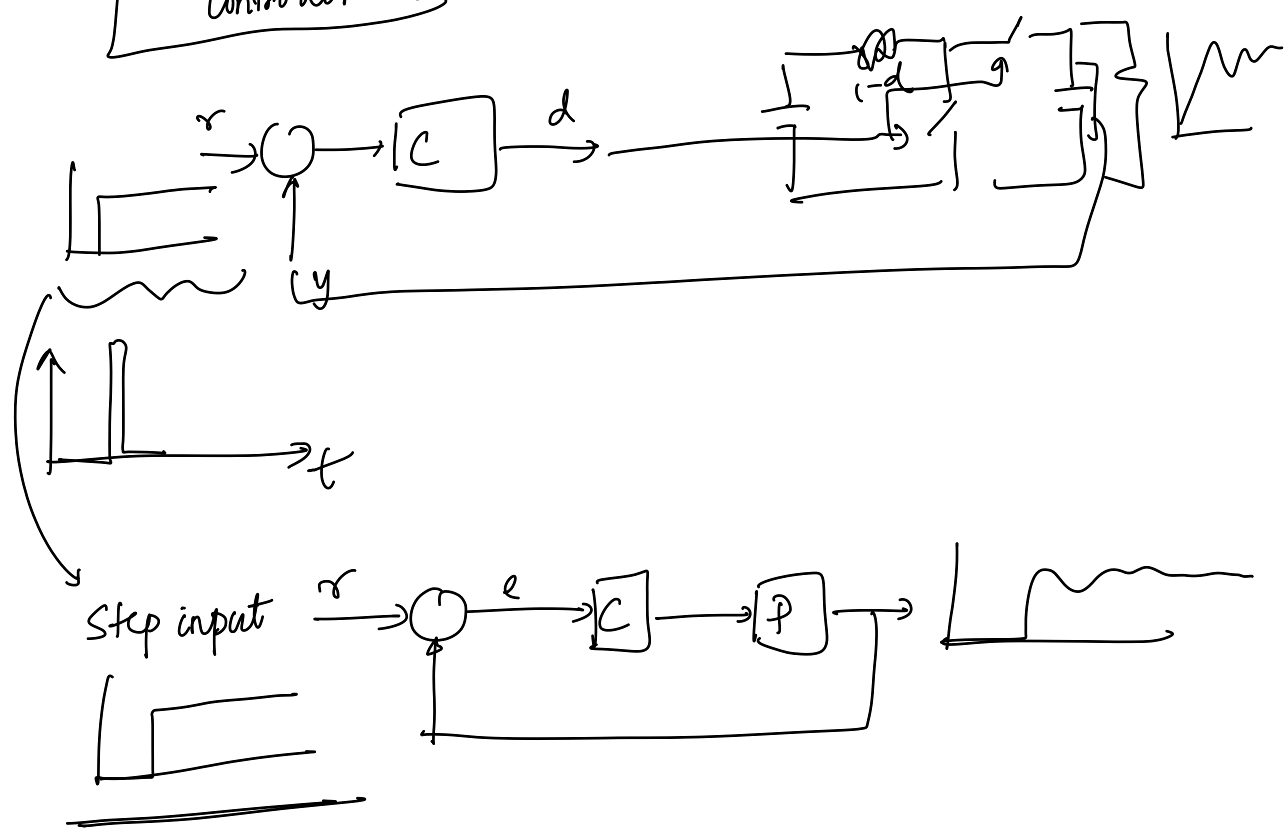


3) Settling time.



4) Overshoot.


User
Designer
t_{settle},
t_{rise},
overshoot,
Steady state error
?
PM,
GM,
ω_g, ω_p,
$e(0)$
design C(s)
such that L(s)
obeys PM, GM. ..
MATLAB
bode ( )
verify
Circuit simulation of Plant + controller


r
C
d
y
t
Step input
r
e
C
P