EE 458/533 – Power Electronics Controls, Winter 2022

Final Exam

Duration: March 12 (00:01 am)- March 14th, 2022 (11:59 pm)

Name: ______________________________

Instructions. You must scan your completed final into a pdf file, and upload your file to the Canvas page by the due date - Monday 14th March, 11:59 pm. All pages must be gathered into a single file of moderate size, and your work (calculations, diagrams, and simulation results) should be presented in the same order as the questions. Please note that the grader will not be obligated to grade your assignment if the file is unreadable or excessively messy.
Be mindful of the following

- Use your lecture, homework simulations and solutions for HW 5 and 6. You can directly use those files.
- Make sure to include the simulation waveforms and zoom in appropriately to show tracking.
- Upload your simulation files and describe your working for partial marking.

Problem 1: Conceptual Questions [10 Points Total]

1. For a minimum phase system (system with no right half pole, zero or delay)would you agree that the phase plot is redundant? That is, if someone gives you only the magnitude plot, you can correctly derive the phase plot from it. Explain logically. If you want to explain mathematically, prove that there exists a one-to-one mapping between the gain and phase plots for a minimum phase system.

2. A 24 V battery is connected to the boost converter, whose output is then connected to a BLDC motor through a three phase inverter as shown in Figure 2. Can you model how would oscillations (disturbances) in the 24 V input supply appear in the dq -based current control loop for the motor? How would you modify your existing controller to actively cancel those disturbances. You may use additional voltage and current sensors.

3. For the three phase inverter, you have used the following modulation scheme in open loop:

$$
\begin{aligned}
d_a &= 0.5 + 0.5m\sin(\omega t) \\
d_b &= 0.5 + 0.5m\sin(\omega t - 2\pi/3) \\
d_c &= 0.5 + 0.5m\sin(\omega t + 2\pi/3)
\end{aligned}
$$

   For a input dc voltage $V_{\text{dc}} = 48V$ and $m = 0.9$, what is the maximum peak of ac voltage across three resistors at the output of the inverter connected in (a) star (b)delta connection?

4. For Fig.1 write the state equations with state $x = [i, v]^\top$ and and input $u = [d, V_{\text{in}}]^\top$ for the interval when MOSFET 1 is on and MOSFET 2 is off.

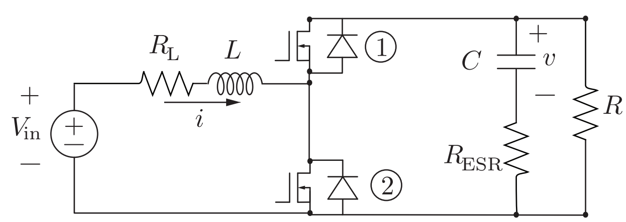


Figure 1: Boost Converter with ESR.

Exam Description In this assignment, we will finalize the design and simulation of the system below. The next two problems will allow you to focus on the boost simulation and motor drive simulation separately. In the final problem, both simulations will be integrated into a cohesive system as in Figure 2.

The boost input and output voltages are v_{g} and v, respectively, where v is across the output branch. The boost RL branch has inductance and resistance, L and R_{L}, and the boost output has capacitance C. The stator inductance and resistance are L_{s} and R_{s}. The boost and drive both have the same switching frequency of $f_{\mathrm{sw}} = 1/T_{\mathrm{sw}}$. Also, for both the boost converter and drive the sampling frequency should be the same as switching frequency (valley sampling). The values of $L, R_{\mathrm{L}}, C, L_{\mathrm{s}}$ and R_{s} from Homework 5 and 6 will be reused.

From here forward, we will also reuse the boost current controller and motor drive current controllers designed in Homeworks and class lectures. To distinguish between each control loop, we will denote the boost current control PI gains as $k_{\mathrm{p,i}}$ and $k_{\mathrm{i,i}}$, and the motor current control gains as $k_{\mathrm{p,m}}$ and $k_{\mathrm{i,m}}$. Following are the events you need to simulate.

- At $t = 0.5\,\mathrm{s}$, $i_{\mathrm{q,ref}}$ changes from $0\,\mathrm{A} \rightarrow -30\,\mathrm{A}$.
- At $t = 1.0\,\mathrm{s}$, $i_{\mathrm{d,ref}}$ changes from $0\,\mathrm{A} \rightarrow 10\,\mathrm{A}$.
- At $t = 1.5\,\mathrm{s}$, $i_{\mathrm{q,ref}}$ changes from $-30\,\mathrm{A} \rightarrow -20\,\mathrm{A}$.
- At $t = 2.0\,\mathrm{s}$, $i_{\mathrm{q,ref}}$ changes from $-20\,\mathrm{A} \rightarrow +20\,\mathrm{A}$.

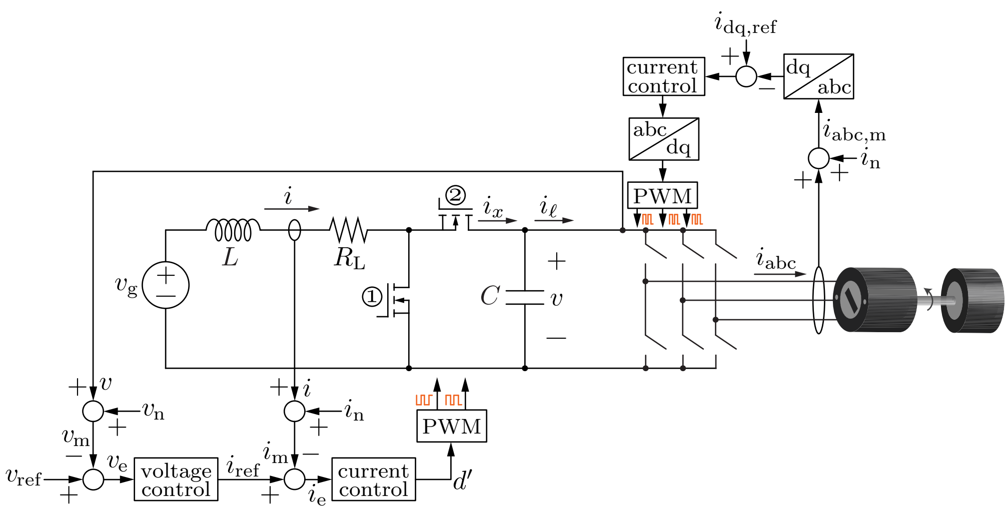
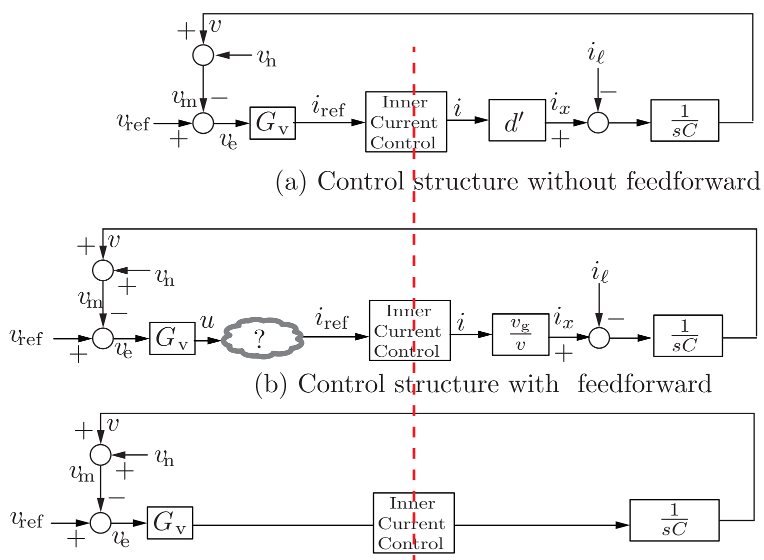


Figure 2: The integrated boost and motor drive system under consideration.

Problem 2, Boost voltage control, [30 Points Total]: Your objective is to design the voltage control loop and analyze the boost system. The new voltage controller will have PI gains denoted as $k_{\mathrm{p,v}}$ and $k_{\mathrm{i,v}}$. As mentioned above, we will reuse your inner current control loop design (along with the feed-forward in Homework 5 and 6) where the inner loop has a first order response $\ell_{\mathrm{i}}/(1+\ell_{\mathrm{i}}) = 1/(\tau_{\mathrm{i}}s+1)$ and $\omega_{\mathrm{i}} = \tau_{\mathrm{i}}^{-1} = \omega_{\mathrm{sw}}/10$. Note that:

- The simulation duration is 30 ms and v_{ref} is fixed at 48 V, and $V_{\mathrm{g}} = 24$ V.
- The load current is initialized at $i_\ell(0) = 2$ A. At $t = 15$ ms, i_ℓ changes from 2 A $\rightarrow$ 4 A.



(a) Control structure without feedforward

(b) Control structure with feedforward

(c) Simplified plant with feedforward

Figure 3: The feedforward control of the outer voltage controller of the boost converter under consideration. On the right side of the red-dotted line is your plant and on the left is the voltage controller, $G_v(s)$.

(a) **[10 Points]**: Implement a feed-forward in the voltage controller to remove the dependence on the load current (i_ℓ) and scalar gain (d') by following Figure 3. Since $v = v_{\mathrm{g}}/d'$, we can use this substitution, $d' = v_{\mathrm{g}}/v$ for feed-forward to avoid any potential divide-by-zero error in the simulation. Using Fig.3 (c), we deduce the open loop-gain to be

$$\ell_{\mathrm{v}}(s) = G_v(s)\frac{1}{1+s\tau_{\mathrm{i}}}\frac{1}{sC}. \tag{1}$$

You may use any method to choose your voltage controller gain. Choose a bandwidth (should be less than 1/10-th of the current controller bandwidth) and phase margin of your liking and tailor your design so that $k_{\mathrm{p,v}}, k_{\mathrm{i,v}}$ satisfy that. The only constraint we impose is that the controller should be of a proportional + integral type. With your PI controller, the closed-loop voltage response takes the following form

$$\frac{\ell_{\mathrm{v}}}{1+\ell_{\mathrm{v}}} = \frac{k_{\mathrm{p,v}}s + k_{\mathrm{i,v}}}{C\tau_{\mathrm{i}}s^3 + Cs^2 + k_{\mathrm{p,v}}s + k_{\mathrm{i,v}}}. \tag{2}$$

Specify units of $k_{\mathrm{p,v}}$ and $k_{\mathrm{i,v}}$ (hint: use circuit intuition).

Create a single Bode plot overlaid with $\ell_{\mathrm{i}}/(1+\ell_{\mathrm{i}})$ and $\ell_{\mathrm{v}}/(1+\ell_{\mathrm{v}})$. Comment on the frequency ranges where $\ell_{\mathrm{i}}/(1+\ell_{\mathrm{i}}) \approx 1$ and $\ell_{\mathrm{v}}/(1+\ell_{\mathrm{v}}) \approx 1$ and how that relates to the speed of the current and voltage loops.

(b) [**3 Points**]: Solve the inductor volt-second and capacitor charge balance equations to obtain the steady-state initial values of D' and I. Provide analytical expressions. (Hint: When solving for I, you will need to solve a quadratic equation. Check the values of both solutions to obtain the correct value of I.)

(c) [**7 Points**]: Simulate the dual-loop system in the ideal continuous domain including all the feed-forward controllers that you have designed. Correctly initialize capacitor voltage and inductor current. Provide a plot with v and v_{ref} overlaid, and another with i and i_{ref}, overlaid. Format your plots to provide a legend, and label axes appropriately.

(d) [**10 Points**]: Simulate the dual-loop system with a discretized controller. Provide a plot with v and v_{ref} overlaid, and another with i and i_{ref}, overlaid. Build on top of the Homework 6 solution file and place your voltage controller in the triggered subsystem. Attach screenshots of your simulation diagram.

Problem 3, Motor control and sensor path design, [35 Points Total]: Reuse models provided in class , verify that the model comprises a switched simulation of a bike motor drive system with current control in the synchronous dq frame. Feedforward should be included to reject cross-coupling and back EMF disturbances. For your simulation, assume you have an ideal electrical angle and frequency measurement, θ_e and ω_e, respectively, and measurements of the currents i_a, i_b, and i_c. The dc bus voltage is $V = 48\,\mathrm{V}$, the switching frequency is 10 kHz, and the machine parameters are:

- Pole pairs: $p/2 = 24$
- Stator inductance and resistance: $L = 240\,\mu\mathrm{H}$ and $R_\mathrm{L} = 125\,\mathrm{m}\Omega$
- Back EMF constant & peak flux linkage: $\lambda_\mathrm{m} = 0.024\,\mathrm{Vs/rad}$
- Wheel radius $r = 0.35\,\mathrm{m}$, and rider mass $m_\mathrm{r} = 75\,\mathrm{kg}$
- Wheel inertia: $J_\mathrm{w} = 0.12\,\mathrm{kg\,m}^2$
- Inertia from rider: $J_\mathrm{r} = m_\mathrm{r} r^2$
- Torque from friction $= \omega_\mathrm{m}\beta$ where $\beta = 1\,\mathrm{Nms/rad}$

Design the d and q axis current control loops to exhibit the first order response $T/(1+T) = 1/(\tau_\mathrm{i} s + 1)$ where $\omega_\mathrm{i} = \tau_\mathrm{i}^{-1} = \omega_\mathrm{sw}/10$. At $t = 0$, the stator currents, current commands, and motor speed are all at zero. Simulate the dq current commands given in the Exam Description and end the simulation after 5 seconds elapses. You should be able to use the values used in class for the current controller.

We will model the LAH25-NP hall-effect sensor. This sensor outputs a current $G_\mathrm{hall} i(t)$ where $G_\mathrm{hall} = 1/1000\,\mathrm{A/A}$ and i is the current flowing on one of the three phases (see Figure 4). The sensor current flows in a burden resistor $R_\mathrm{b} = 75\,\Omega$. The effective gain from the current i to op-amp input signal v_s is $G_\mathrm{s} = G_\mathrm{hall} R_\mathrm{b}$. This sensor gives a zero dc offset at v_s (in handout $v_\mathrm{dc} = 0$). The ADC pin voltage is $0 \le v_\mathrm{A/D} \le V_\mathrm{FS}$ where $V_\mathrm{FS} = 3\,\mathrm{V}$, the ADC has $n_\mathrm{A/D} = 12$ bits, and $x_\mathrm{A/D}$ is the integer produced by the ADC. Denote the maximum phase current as i_pk where $i_\mathrm{pk} = 40\,\mathrm{A}$. Only one sensing path is shown for clarity. All other measurements are ideal.

(a) **[10 Points]**: Assume we only sample once in a switching cycle, discretize your PI controller and place your controller inside of a triggered subsystem that is triggered

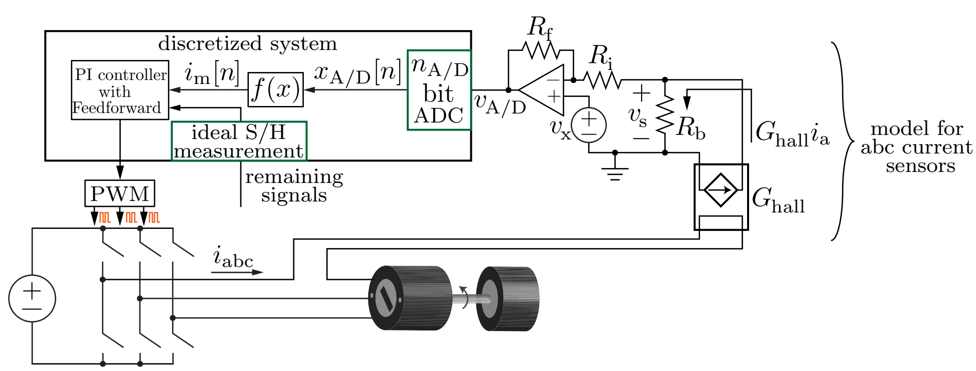


Figure 4: Motor drive with discretize controller along with sensing path scaling and circuitry.

each time the carrier starts ramping up (we will do only valley sampling for this). Once this first step is done and the discretized system works with ideal sensing, begin modifying your model with the current sensor model in Figure 4. The objective of the sensing path design is to ensure:

- When $i = 0$, the ADC pin voltage should be at the midpoint $v_{\mathrm{A/D}} = V_{\mathrm{FS}}/2$.
- When the current swings to the positive and negative peaks (i.e., $|i| = i_{\mathrm{pk}}$), the ADC pin voltage equals

$$\begin{aligned} v_{\mathrm{A/D}} &= V_{\mathrm{FS}} - \epsilon, \qquad \text{for } i = -i_{\mathrm{pk}} \\ v_{\mathrm{A/D}} &= \epsilon, \qquad \text{for } i = i_{\mathrm{pk}} \end{aligned}$$

where $\epsilon = 0.1$ V. Using the method taught in class, compute the ratio $R_{\mathrm{f}}/R_{\mathrm{i}}$ and voltage v_{x}. Assume the input resistor is $R_{\mathrm{i}} = 100\,\mathrm{k\Omega}$.

(b) **[10 Points]**: Given that the ADC has 12 bits, use your solution in part (a) to compute the factors m and b in the measurement recovery formula $i(t) \approx i_{\mathrm{m}}[n] = m x_{\mathrm{A/D}}[n] + b$. Add this to your simulation (see Figure 4) for each of the three current sensing paths.

(c) **[25 Points]**: Simulate the motor with the discretized controller and detailed sensor path models for the three-phase currents. Zoom in on one ac-cycle when non-zero torque is being applied, and plot the following: i) the actual current i_{a} and digitally reconstructed estimate $i_{\mathrm{m,a}}$ overlaid, and ii) the integer readout $x_{\mathrm{A/D,a}}$.

Simulate the sequence above with and without the rider (i.e., once when $J = J_{\mathrm{w}}$ with wheel only, and $J = J_{\mathrm{w}} + J_{\mathrm{r}}$ with wheel and rider). For each, provide the following plots, i) i_{d} and $i_{\mathrm{d,ref}}$ overlaid, ii) i_{q} and $i_{\mathrm{q,ref}}$, iii) bike speed in mph, iv) the three-phase sinusoidal stator currents, v) the three-phase sinusoidal back EMF waveforms, and vi) the electrical power absorbed by the back EMFs. Your plots must be formatted with appropriate labels using. Attach screenshots of your simulations and show all subsystems. Comment on how the two sets of simulations (with and without rider) for the sequence of current references in Exam description differ and why.

Problem 4, System integration, [25 Points Total]: Combine the boost controller with discretized dual-loop controller from Problem 2 and motor system from Problem 3 into a single functional system.

(a) [**18 Points**]: Carry out the dq current command events listed in the Exam Description where the boost converter is operated consistently with a 48 V reference for the output voltage of boots converter. Recompute the initial values for your boost converter since the motor currents are zero at startup. Your model should start at steady-state. Provide a plot with v and v_{ref} overlaid, and another with i and i_{ref}, overlaid. Also give $i_{\mathrm{d}}, i_{\mathrm{d,ref}}, i_{\mathrm{q}}$ and $i_{\mathrm{q,ref}}$. Provide plots as well as screenshots of your model.

(b) [**4 Points**]: For your boost converter voltage control, you used a feedforward of i_{ℓ}, which, as shown in Fig. 2 is the dc link current of the three phase inverter. Remove this current sensor from the simulation and instead obtain this feed-forward in terms of the three measured currents :i_a, i_b and i_c and the three duty ratios that you generate for each of the top switches of each leg, d_a, d_b and d_c.

(c) [**3 Points**]: For your boost converter voltage control, you had used another strategy to cancel the gain of d' by using v/v_{g}. For this feedforward, you could use the sensed values of the input voltage, v_{g} and output voltage, v or use their nominal value of 24 V and 48 V respectively. Explain the difference in the d and q axis motor current due to these two approaches.