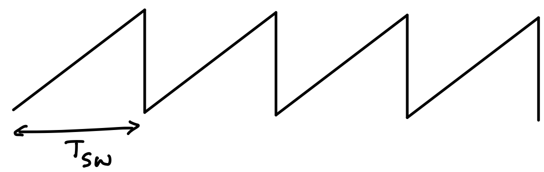
Lecture 10 (Feb 2, 2022)

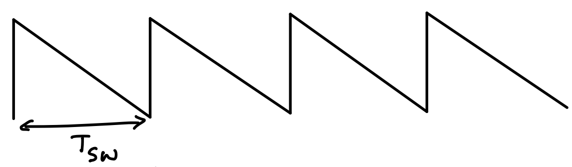
- HW 3 due now
- Midterm will be posted tomorrow.

Some clarification

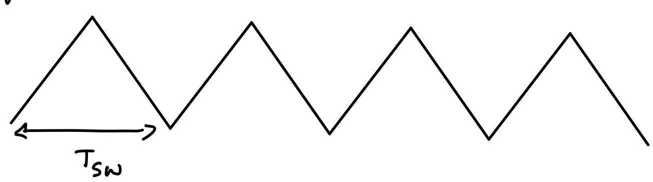
1. Lagging edge / Trailing edge



2. Leading edge.



3. Symmetrical



<u>ADC quantization</u>

v_{FS} = Full scale ADC voltage

N_{adc} = no of ADC bits = no of bits in digital output.

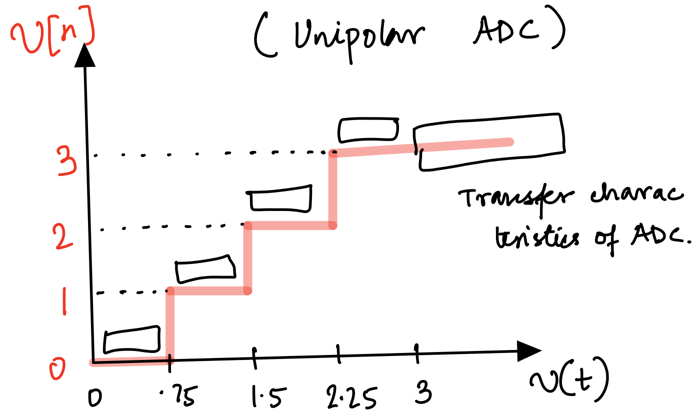
$2^{N_{adc}}$ = no of states / possible digital output

ΔV = Resolution

Eg. $v_{FS} = 3V$; $N_{adc} = 2$, $2^{N_{adc}} = 2^2 = 4$

$\Delta V = \frac{3}{4} = 0.75$

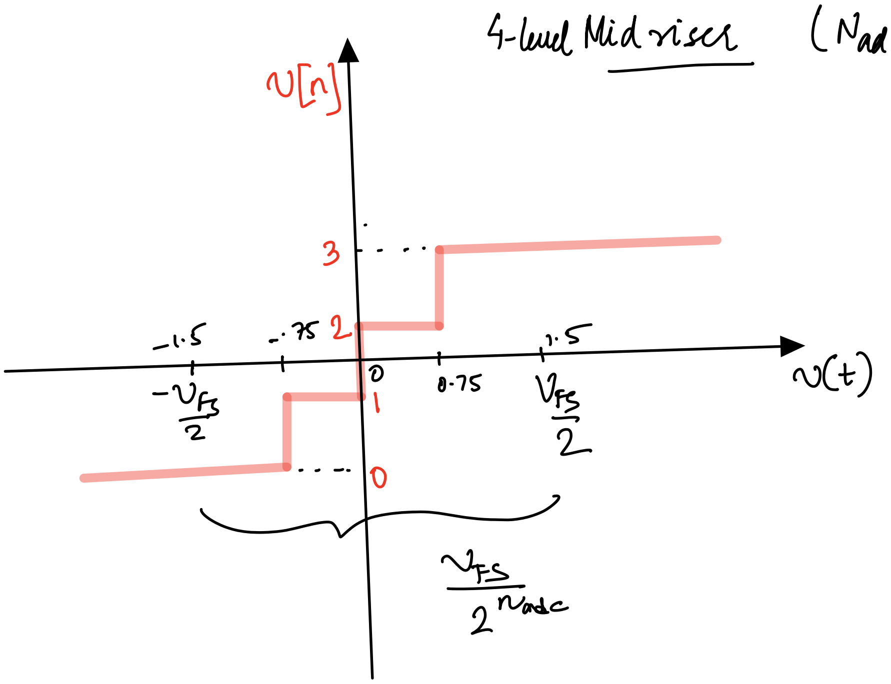
(Unipolar ADC)



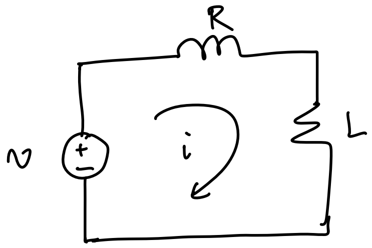
$v(t) \rightarrow \boxed{T(s)} \rightarrow v[n]$

(Bipolar ADC)

4-level Mid riser ($N_{adc} = 2$)



First Order System



$$\frac{1}{s+a} \quad \frac{1}{a} \qquad \frac{1}{s/\tau + 1}$$

$$v(t) = L\frac{di(t)}{dt} + R\,i(t)$$

$$\therefore \frac{L}{R}\frac{di(t)}{dt} = -\,i(t) + \frac{1}{R}v(t)$$

$$\frac{L}{R} = \tau$$

$$\boxed{\tau\frac{di(t)}{dt} = -\,i(t) + \frac{1}{R}v(t)}$$

First order system.

$i(t)$

How to solve? take Laplace.

$$s\tau\, i(s) - \tau\, i(0) = -i(s) + \frac{v(s)}{R}$$

$$\text{or,} \quad i(s)\,[s\tau + 1] = \tau\, i(0) + v(s)/R.$$

$$\therefore i(s) = \frac{\tau\, i(0)}{s\tau + 1} + \frac{v(s)}{R(s\tau+1)}$$

take Laplace inverse.

$$\lim_{t\to\infty} i(t)$$

$$\mathcal{L}^{-1}\left[i(s) = \frac{\tau\, i(0)}{s\tau+1} + \frac{v(s)}{R(s\tau+1)} \right]$$

$$i(t) = \frac{1}{\tau}\tau\, i(0)\, e^{-t/\tau} + \boxed{\mathcal{L}^{-1}\left[\frac{v(s)}{R(s\tau+1)}\right]}$$

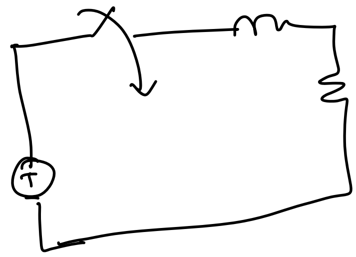
Assume i/p voltage $v(t)$ = step i/p :—

$$v(t) = \begin{cases} V & t > 0 \\ 0 & t < 0 \end{cases}$$

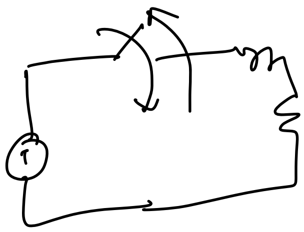
constant

$$\therefore v(s) = \mathcal{L}(v(t)) = \frac{V}{s}$$

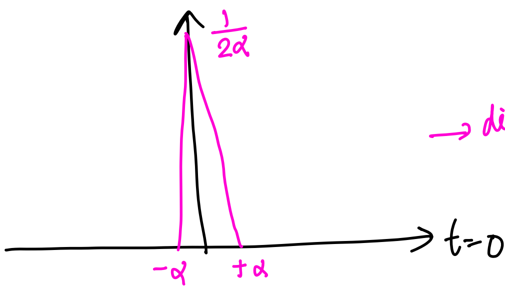
$v(t)$



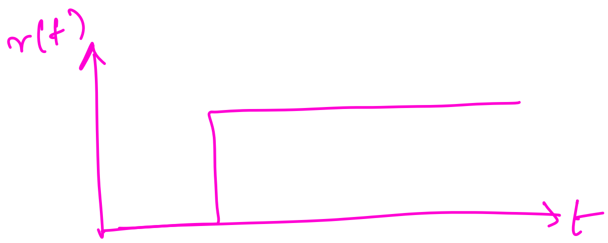
step function

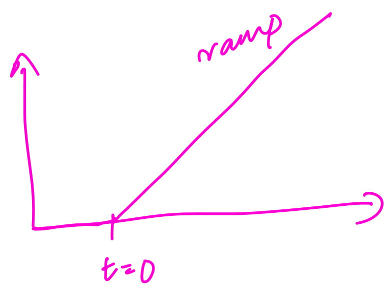


impulse.



→ dirac delta impulse





$$y(s) = \underline{\underline{P(s)}} \cdot r(s)$$

$$\boxed{P(s) = \frac{1}{s}}$$

① $r(t) = \delta(t) \rightarrow y(s) = \frac{1}{s} \cdot \mathcal{L}[\delta(t)] = \frac{1}{s}$

$y(t) = u(t)$

② $r(t) = u(t) \rightarrow y(s) = \frac{1}{s} \cdot \frac{1}{s} = \frac{1}{s^2} \quad \therefore y(t) = t\,u(t)$

$$\therefore\ i(t) = i(0)e^{-t/\tau} + \mathcal{L}^{-1}\frac{V}{R.\ s(s\tau+1)}$$

$$= i(0)e^{-t/\tau} + \frac{V}{R}\tau\left[\mathcal{L}^{-1}\left\{\frac{1}{\tau s(s\tau+1)}\right\}\right]$$

$$= i(0)e^{-t/\tau} + \frac{V}{R}\tau\left[\mathcal{L}^{-1}\frac{s\tau+1-s\tau}{\tau s(s\tau+1)}\right]$$

$$= i(0)e^{-t/\tau} + \frac{V}{R}\tau\ \mathcal{L}^{-1}\left(\frac{1}{s\tau} - \frac{1}{s\tau+1}\right)$$

$$= i(0)e^{-t/\tau} + \frac{V}{R}\frac{\tau}{\tau}\left[u(t) - e^{-t/\tau}u(t)\right]$$

$$i(t) = \underbrace{i(0)e^{-t/\tau}}_{\text{response due to initial state}} + \underbrace{\frac{V}{R}\left[1-e^{-t/\tau}\right]u(t)}_{\text{response due to input}}$$

forced response

At $t \to \infty$

$$i(t)\Big|_{t\to\infty} = i(\infty) = 0 + \frac{V}{R}\left[1-0\right] = \frac{V}{R}.$$

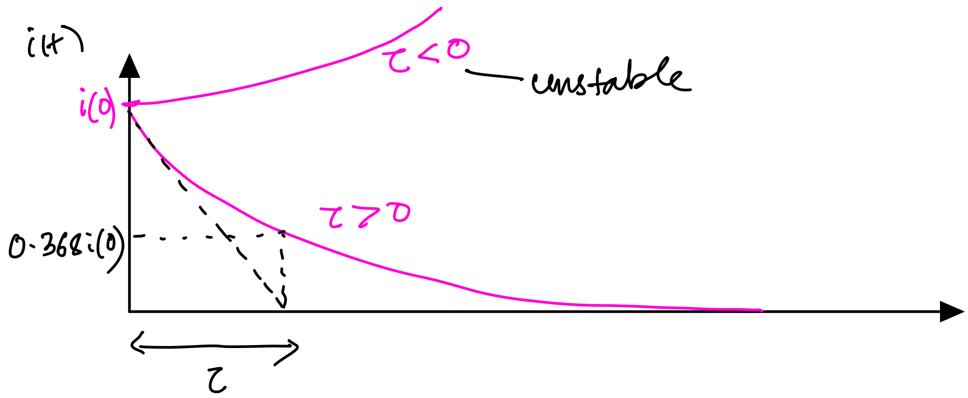
Find value theorem, $\quad i(t=\infty) = \lim_{s\to 0} s\, i(s)$

$$\lim_{s\to 0}\ s.\frac{i(0)}{s\tau+1} + \cancel{s}.\frac{V}{\cancel{s}R(s\tau+1)}$$

$$= 0 + \frac{V}{R}$$

$$i(t) = \underbrace{i(0)e^{-t/\tau}} + \underbrace{\frac{V}{R}\left[1-e^{-t/\tau}\right]u(t)}$$

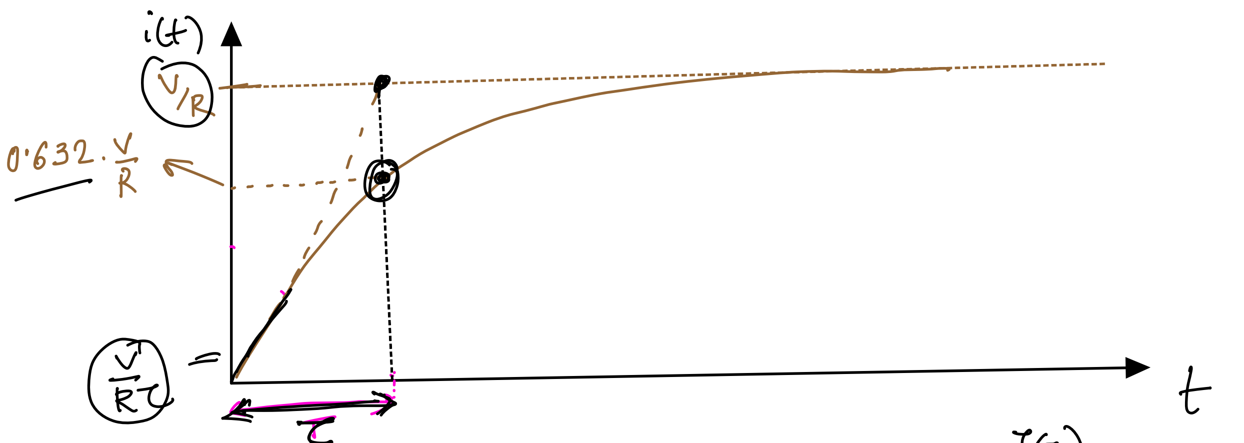
① Assume $V=0$ (no input) $\Rightarrow$ zero input response.



$$\frac{1}{s\tau+1}$$

$$s = -\frac{1}{\tau}$$

② First Order response ($i(0)=0$) $\Rightarrow$ zero i.c. response



$$i(t) = \frac{V}{R}\left(1-e^{-t/\tau}\right) \longrightarrow \frac{V}{R}\left(1-e^{-\tau/\tau}\right) = \frac{0.63V}{R}$$

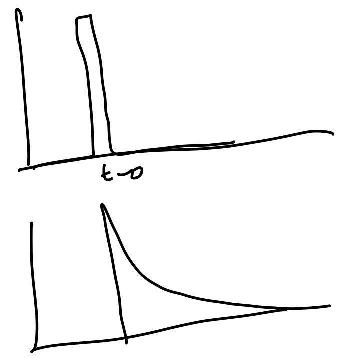
$$\frac{di(t)}{dt} = \frac{V}{R}\left(-\frac{1}{\tau}\right)e^{-t/\tau} = \frac{V}{R\tau}e^{-t/\tau}$$

$$\left.\frac{di(t)}{dt}\right|_{t=0} = \frac{V}{R\tau}$$

What about other inputs?

eg. (a) Impulse response ($i(0)=0$)

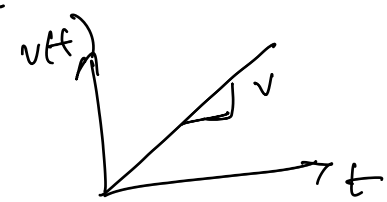
$$v(t) = V.\delta(t) \quad \therefore \quad v(s) = V$$



$$i(s) = \frac{v(s)}{R(s\tau+1)}$$

$$\therefore i(s) = \frac{V}{R(s\tau+1)}$$

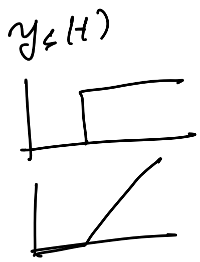
$$\therefore i(t) = \frac{V}{R\tau} e^{-t/\tau} \qquad t \geq 0.$$



Ramp Response

$$v(t) = V.t \;, t \geq 0 \qquad \therefore \quad v(s) = \frac{V}{s^2}$$

$$i(s) = \frac{v(s)}{R(s\tau+1)} = \frac{V}{s^2 R(s\tau+1)} \rightarrow i(t) = \mathcal{L}^{-1}[i(s)]$$



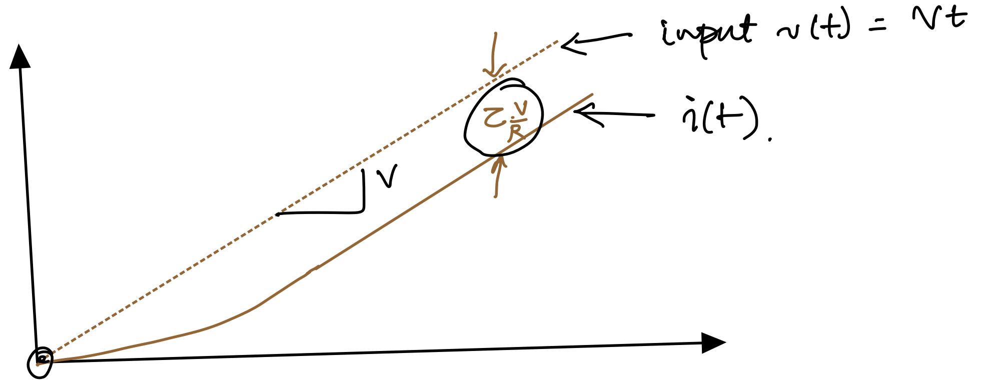
$$i_r(t) = \int_0^t i_s(t)\, dt = \underbrace{\int_0^t \frac{V}{R}\left[1 - e^{-t'/\tau}\right] u(t')}$$

$$= \frac{V}{R} t - \frac{V}{R}\tau\left[e^{-t'/\tau}\right]_0^t$$

$$= \frac{Vt}{R} + \frac{V}{R}\tau.\left[e^{-t/\tau} - 1\right]$$

$$= \frac{V}{R}\left(t - \tau\left(1 - e^{-t/\tau}\right)\right)$$

As $t \to \infty$, $\frac{Vt}{R}\Big|_{t\to\infty} - \frac{\tau.V}{R}.0 \rightsquigarrow$ steady error.



Final value :-

$$\lim_{s \to 0} s \cdot i(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{V}{s^{\cancel{2}} R (s\tau + 1)}$$

$$\lim_{s \to 0} \frac{V}{s \cdot R (s\tau + 1)} = \infty .$$

input → $v(t) = Vt$

response → $i(t) = \frac{V}{R}\left(t - \underbrace{\tau\left(1 - e^{-t/\tau}\right)}\right)$ $\quad \tau = \frac{L}{R}$

$i^{\text{expected}}(t) = \frac{Vt}{R}$

error in response $= i^{\text{expected}}(t) - i(t)$

$$= \frac{Vt}{R} - \frac{V}{R}\left(t - \tau\left(1 - e^{-t/\tau}\right)\right)$$

$$= \frac{V}{R}\tau\left(1 - e^{-t/\tau}\right)$$

As $t \to \infty$, $\frac{V\tau}{R}$

Second Order System.

$$L\frac{di}{dt} + i\cdot R + v = v_{in} \rightarrow \text{KVL}$$

$$i = C\frac{dv}{dt}$$

$$\left\{ LC\frac{d^2v}{dt^2} + CR\frac{dv}{dt} + v = v_{in} \right.$$

$$\Rightarrow \quad s^2 + \frac{R}{L}s + \frac{1}{LC}$$

$$\downarrow \mathcal{L} \downarrow$$

$$\left\{ \left(s^2LC + sRC + 1\right)v(s) = v_{in}(s) \right.$$

$$\left\{ v(s) = \frac{v_{in}(s)}{s^2LC + sRC + 1.} \right.$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad ; \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q\cdot\omega_0 = \frac{1}{RC}$$

$$\underbrace{Q = \frac{\sqrt{LC}}{RC} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Quality factor

$$v(s) = \frac{v_{in}(s)}{\left(\frac{s^2}{\omega_0}\right) + \frac{s}{Q\omega_0} + 1}$$

$$\boxed{Q = \frac{1}{2\zeta}}$$

Same as before, but we directly go to step response.

$$v_{in}(t) = \begin{cases} V_{in} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$\therefore v_{in}(s) = \frac{V_{in}}{s}$$

$$v(s) = \frac{v_{in}(s)}{s^2 LC + sRC + 1}$$

$$= \frac{v_{in}(s)/LC}{s^2 + \frac{sR\cancel{C}}{L\cancel{C}} + \frac{1}{LC}}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \qquad = \frac{v_{in}(s)\cdot \omega_0^2}{s^2 + s\cdot\frac{R}{L} + \omega_0^2}$$

$$2\zeta\omega_0 = \frac{R}{L}$$

$$\boxed{\therefore \zeta = \frac{1}{2} R \sqrt{\frac{L}{C}}} \qquad \boxed{v(s) = \frac{v_{in}(s)\ \omega_0^2}{s^2 + 2\cdot s\cdot\zeta\omega_0 + \omega_0^2}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$v_{in}(s) \rightarrow \boxed{\frac{\omega_0^2}{s^2 + 2s\zeta\omega_0 + \omega_0^2}} \rightarrow v(s)$$

step, impulse, ramp

$= \frac{V_{in}}{s}$ constant

$$s^2 + 2s\zeta\omega_0 + \omega_0^2 = 0$$

$$s = \frac{-2\zeta\omega_0 \pm \sqrt{(2\zeta\omega_0)^2 - 4\omega_0^2}}{2}$$

$$= -\zeta\omega_0 \pm \sqrt{(\zeta\omega_0)^2 - \omega_0^2}$$

$$= -\zeta\omega_0 \pm \omega_0 \underbrace{\sqrt{\zeta^2 - 1}}$$

$$\boxed{-\zeta\omega_0 \pm j\omega_0 \sqrt{1-\zeta^2}}$$

$$\sqrt{-1} = j$$

$$s_1 = -\zeta\omega_0 + \omega_0\sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta\omega_0 - \omega_0\sqrt{\zeta^2 - 1}$$

① $\zeta = 0$

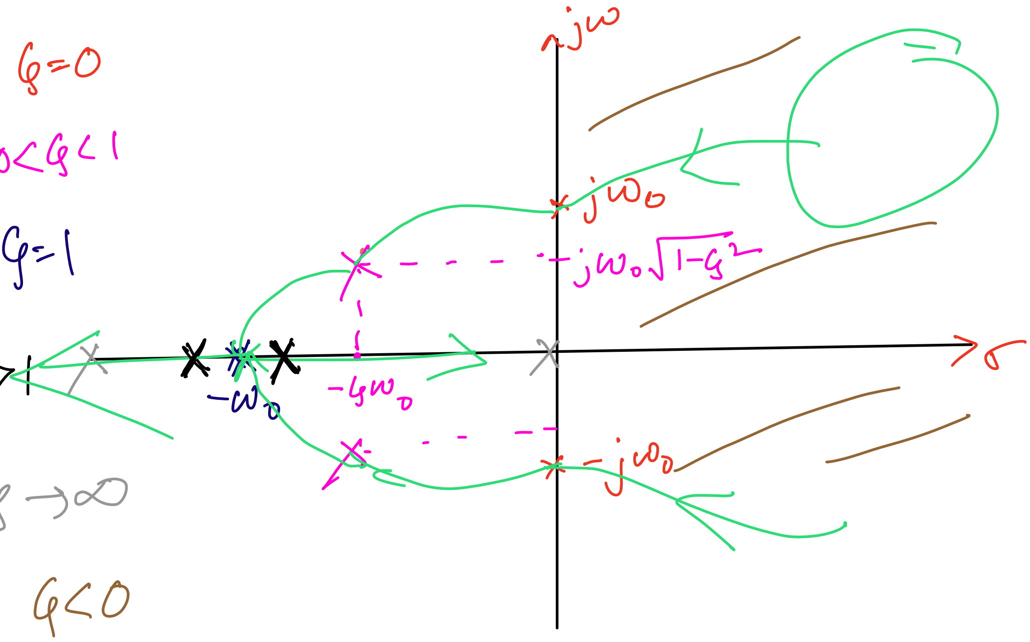
② $0 < \zeta < 1$

③ $\zeta = 1$

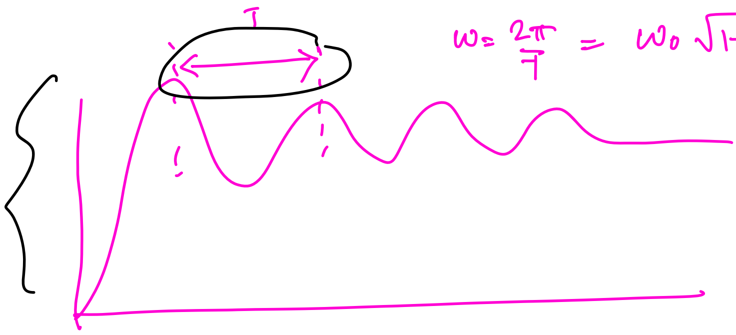
④ $\zeta > 1$

⑤ $\zeta \to \infty$

⑥ $\zeta < 0$

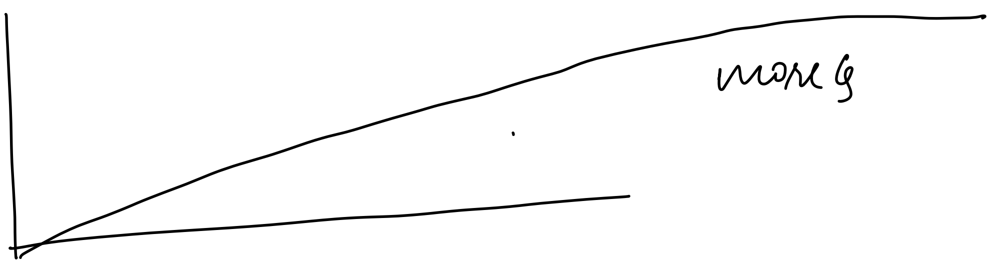


$\omega = \frac{2\pi}{T} = \omega_0\sqrt{1-\zeta^2}$



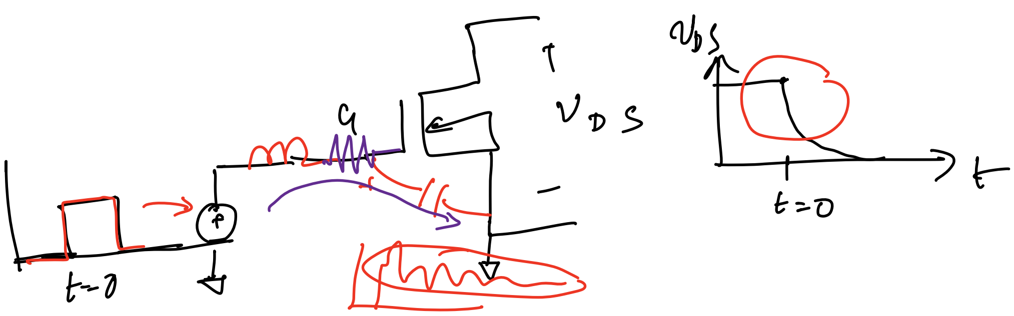
less ζ

$\zeta > 1$



more ζ

$PM = \frac{\zeta}{100}$; $PM\uparrow$, $\zeta\uparrow$.



$$v(s) = \frac{\omega_0^2 \, v_{in}(s)}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$\zeta = \frac{R}{2}\sqrt{\frac{L}{C}}$ damping factor

$$v(s) = \frac{v_{in}(s)}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1}$$

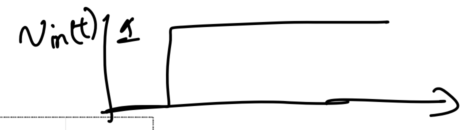
$$v(s) = \frac{\omega_0^2 \, V_{in}}{s(s^2 + 2\zeta\omega_0 s + \omega_0^2)}$$

$$\therefore \; v(t) = \mathcal{L}^{-1}[v(s)]$$

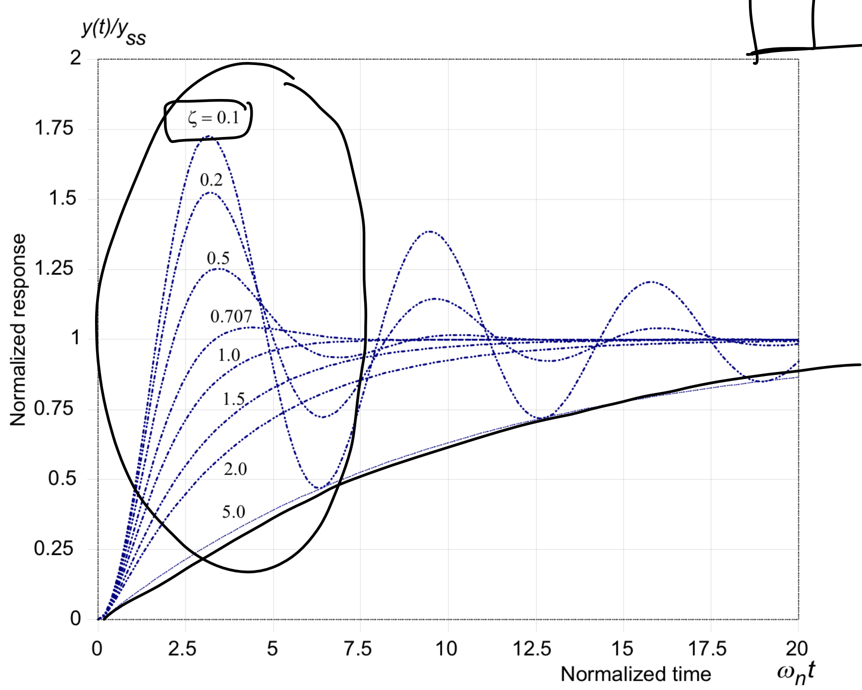
$$= V_{in}\left[1 - e^{-\zeta\omega_0 t}\cos(\omega_d t) - \frac{\zeta}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_0 t}\sin(\omega_d t)\right]$$

$$\omega_d = \omega_0\sqrt{1-\zeta^2} \qquad \varphi = \tan^{-1}\frac{\zeta}{\sqrt{1-\zeta^2}}$$

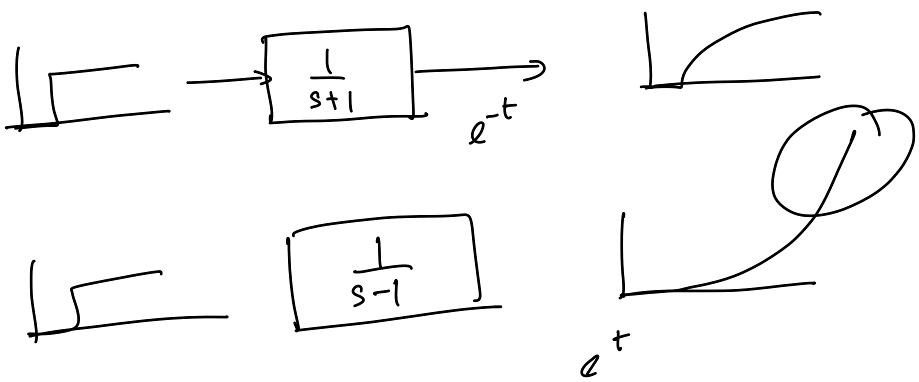
$$\therefore \; v(t) = V_{in}\left[1 - \frac{e^{-\zeta\omega_0 t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \varphi)\right]$$

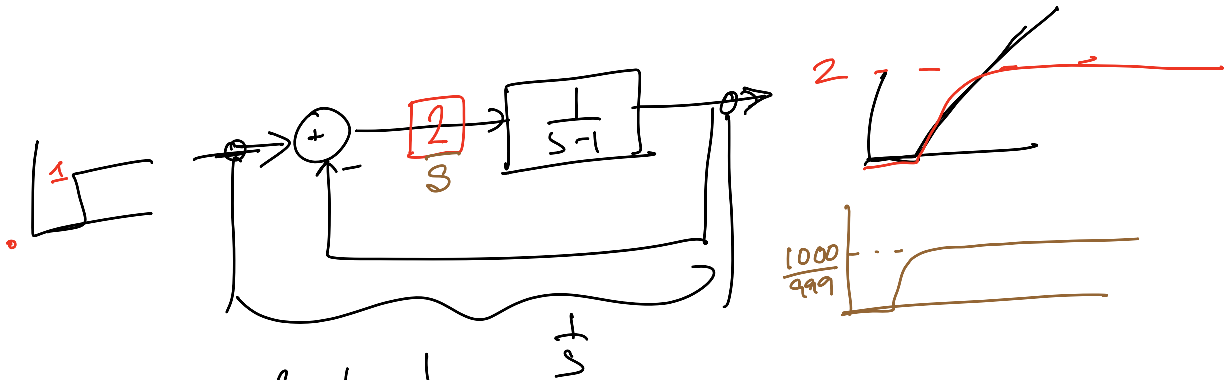


$v(t)$ = cap voltage



Open loop & Closed loop.



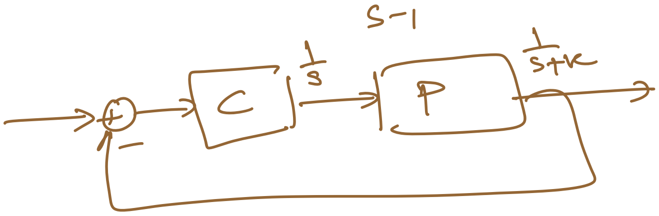


$$l = 1 \cdot \frac{1}{s-1}$$

$$\frac{l}{1+l} = \frac{\frac{1}{s-1}}{1+\frac{1}{s-1}} = \frac{1}{s\cancel{-1}+\cancel{1}} = \frac{1}{s}$$

$$\frac{l}{1+l} = \frac{\frac{2}{s-1}}{1+\frac{2}{s-1}} = \frac{2}{s+1} = \frac{2}{s+1}$$

$$\frac{l}{1+l} = \frac{\frac{1000}{s-1}}{1+\frac{1000}{s-1}} = \frac{1000}{s+999}$$



$$\frac{\frac{1}{s}\frac{1}{(s+k)}}{1+\frac{1}{s(s+k)}}$$

$$\frac{1}{s^2+ks+1}$$

Damping ratio	Input $f(t)$	Characteristic Response $y(t)$
$\boxed{0 \le \zeta < 1}$	$f(t) = u_r(t)$	$y_r(t) = \dfrac{1}{\omega_n^2}\left[t + \dfrac{e^{-\zeta\omega_n t}}{\omega_n}\left(2\zeta\cos\omega_d t + \dfrac{2\zeta^2-1}{\sqrt{1-\zeta^2}}\sin\omega_d t\right) - \dfrac{2\zeta}{\omega_n}\right]$
	$f(t) = u_s(t)$	$y_s(t) = \dfrac{1}{\omega_n^2}\left[1 - \dfrac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \psi)\right]$
	$f(t) = \delta(t)$	$y_\delta(t) = \dfrac{e^{-\zeta\omega_n t}}{\omega_n\sqrt{1-\zeta^2}}\sin(\omega_d t)$
$\zeta = 1$	$f(t) = u_r(t)$	$y_r(t) = \dfrac{1}{\omega_n^2}\left[t + \dfrac{2}{\omega_n}e^{-\omega_n t} + te^{-\omega_n t} - \dfrac{2}{\omega_n}\right]$
	$f(t) = u_s(t)$	$y_s(t) = \dfrac{1}{\omega_n^2}\left[1 - e^{-\omega_n t} - \omega_n te^{-\omega_n t}\right]$
	$f(t) = \delta(t)$	$y_\delta(t) = te^{-\omega_n t}$
$\zeta > 1$	$f(t) = u_r(t)$	$y_r(t) = \dfrac{1}{\omega_n^2}\left[t + \dfrac{\omega_n}{2\sqrt{1-\zeta^2}}\left(\tau_1^2 e^{-t/\tau_1} - \tau_2^2 e^{-t/\tau_2}\right) - \dfrac{2\zeta}{\omega_n}\right]$
	$f(t) = u_s(t)$	$y_s(t) = \dfrac{1}{\omega_n^2}\left[1 - \dfrac{\omega_n}{2\sqrt{\zeta^2-1}}\left(\tau_1 e^{-t/\tau_1} - \tau_2 e^{-t/\tau_2}\right)\right]$
	$f(t) = \delta(t)$	$y_\delta(t) = \dfrac{1}{2\omega_n\sqrt{\zeta^2-1}}\left(e^{-t/\tau_1} - e^{-t/\tau_2}\right)$

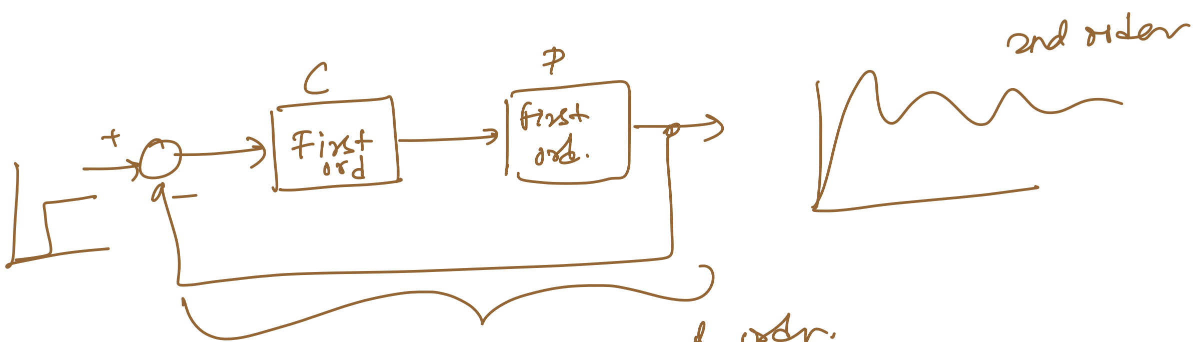
$\dfrac{PM}{100} = \zeta$

$30^\circ - 60^\circ$

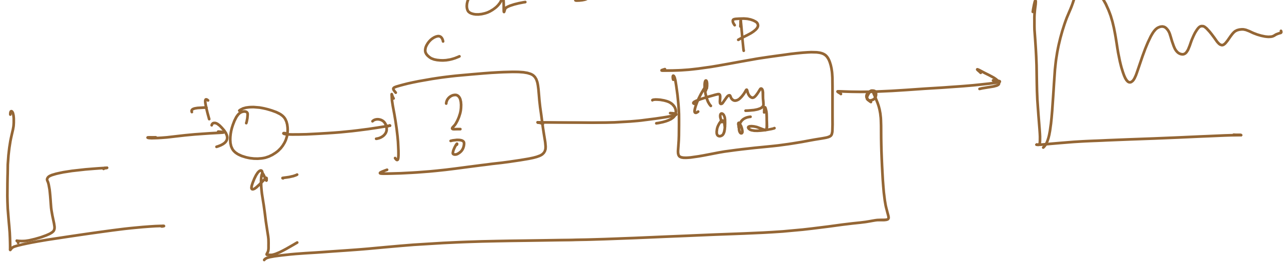
$\zeta = 0.3$ to 0.6

Notes:

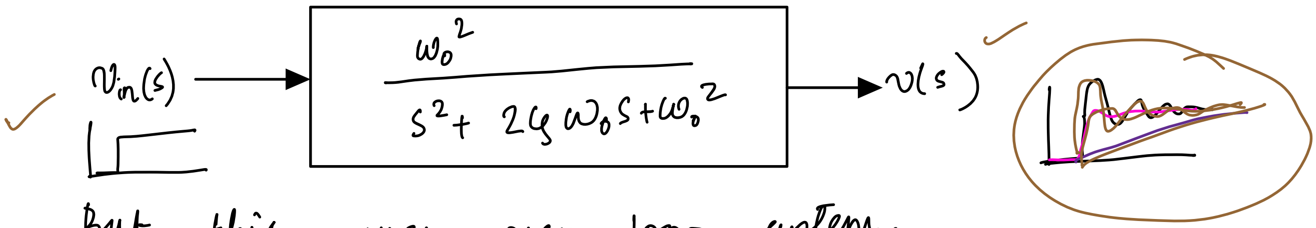
1. The damped natural frequency $\omega_d = \sqrt{1-\zeta^2}\omega_n$ for $0 \le \zeta < 1$.
2. The phase angle $\psi = \tan^{-1}\left(\zeta/\sqrt{1-\zeta^2}\right)$ for $0 \le \zeta < 1$.
3. For over-damped systems ($\zeta > 1$) the time constants are $\tau_1 = 1/\left(\zeta\omega_n - \sqrt{\zeta^2-1}\omega_n\right)$, and $\tau_2 = 1/\left(\zeta\omega_n + \sqrt{\zeta^2-1}\omega_n\right)$.



CL second ordr.

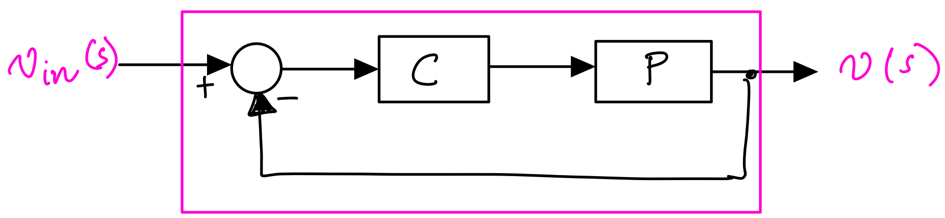


We did this:

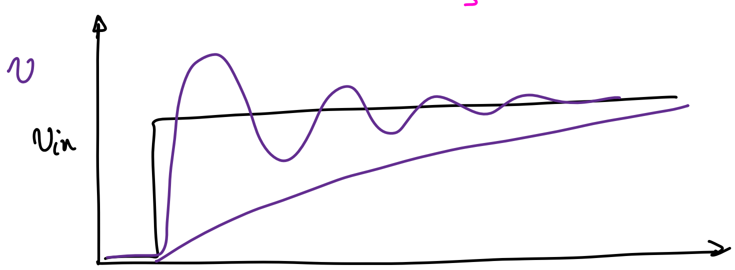


But this was open loop system.

How does this fit into our closed loop?

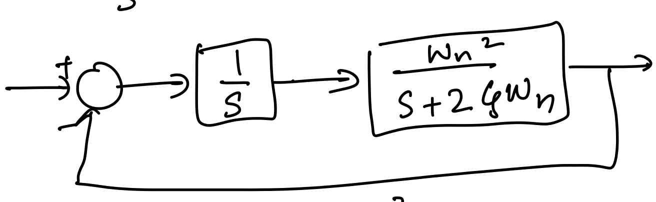


$$:= \frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$$



Eg. $\quad P = \frac{1}{s+a} \equiv \frac{\omega_n^2}{s+2\zeta\omega_n}$

$C = \frac{1}{s}$



$$\frac{CP}{1+CP} = \frac{\frac{\omega_n^2}{s(s+2\zeta\omega_n)}}{1+\frac{\omega_n^2}{s(s+2\zeta\omega_n)}} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

Then we can use all things we learnt....

Q: Where does stability come from?

$$\frac{1}{s^2+2\zeta\omega_0 s+\omega_0^2}$$

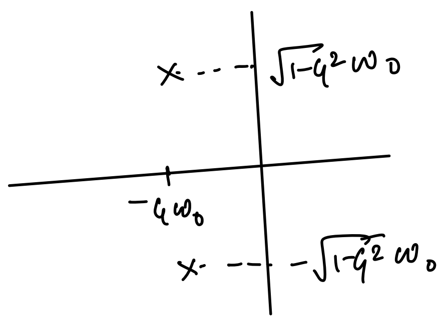
⓪ $\zeta=0 \Rightarrow s=\pm j\omega_0$.

① $0<\zeta<1$ (Underdamped)

$$s^2+2\zeta\omega_0 s+\omega_0^2 \Rightarrow s = -\frac{2\zeta\omega_0 \pm \sqrt{(4\zeta\omega_0)^2-4\omega_0^2}}{2}$$

$$= -\zeta\omega_0 \pm \sqrt{\zeta^2-1}\,\omega_0$$

$$= -\zeta\omega_0 \pm j\sqrt{1-\zeta^2}\,\omega_0$$



② $\zeta=1$ (Critically damped)

$s+2s\omega_0+\omega_0^2 \Rightarrow s=-\omega_0$ (double pole)

③ $\zeta>1$ (Over damped)

$-\zeta\omega_0+\sqrt{\zeta^2-1}\,\omega_0$

$-\zeta\omega_0-\sqrt{\zeta^2-1}\,\omega_0$

$\zeta\gg 1$ $\quad 0, \quad -2\zeta\omega_0$

Q. What happens if $P(s)$ & $C(s)$ are not as simple?

$$P = \frac{1}{s+a} \equiv \frac{1}{s+2\zeta\omega_n}$$

$$C = \frac{1}{s}$$

$$\frac{1}{s^2+2s+5}$$

$$\boxed{\text{Let, } \quad P(s) = \frac{n_p(s)}{d_p(s)} \quad \& \quad C(s) = \frac{n_c(s)}{d_c(s)}}$$

$$\therefore \; CLTF = \frac{PC}{1+PC} = \frac{\dfrac{n_p(s)n_c(s)}{d_p(s)d_c(s)}}{1 + \dfrac{n_p(s)n_c(s)}{d_p(s)d_c(s)}}$$

$$\therefore \; \frac{n_p(s)\, n_c(s)}{\boxed{d_p(s)\, d_c(s) + n_p(s)\, n_c(s)}}$$

poles of open loop plant : $d_p(s) = 0 \rightarrow$.

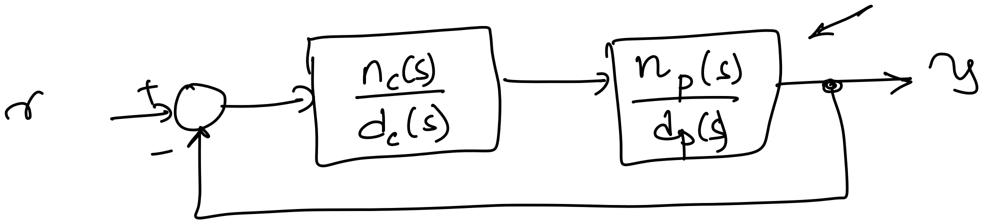
poles of closed loop : $d_p(s)d_c(s) + n_p(s)\,n_c(s) = 0. \rightarrow$

- We <u>ASSUME</u> $\rightarrow$ there will be '2' dominant pole.

when we solve

$$d_p(s)d_c(s) + n_p(s)\,n_c(s) = 0$$

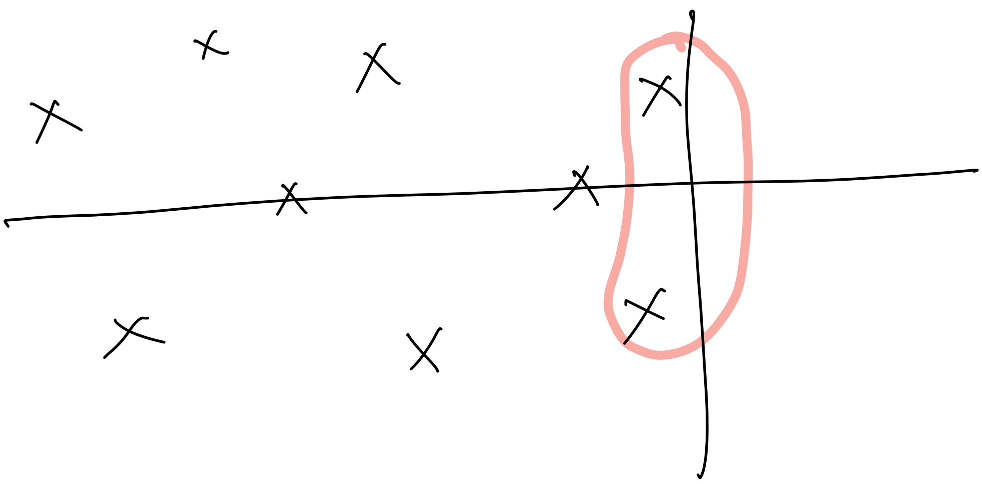
& hence, this will always have a response "like" a second order s/s.

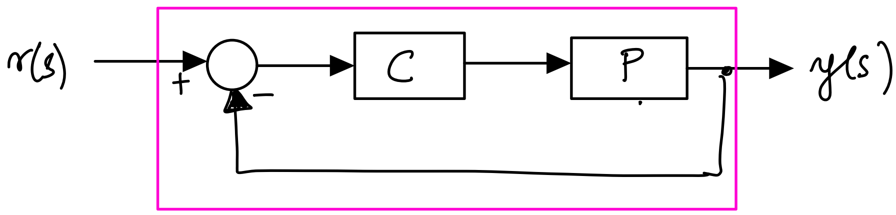


$$r \longrightarrow \boxed{\frac{n_p(s)\, n_c(s)}{d_p(s)\, d_c(s) + n_p(s)\, n_c(s)}} \longrightarrow y$$

$$d_p(s)\, d_c(s) + n_p(s)\, n_c(s) = 0$$

$$\max \operatorname{ord} \{ d_p d_c ,\ n_p n_c \}$$





$$\equiv$$

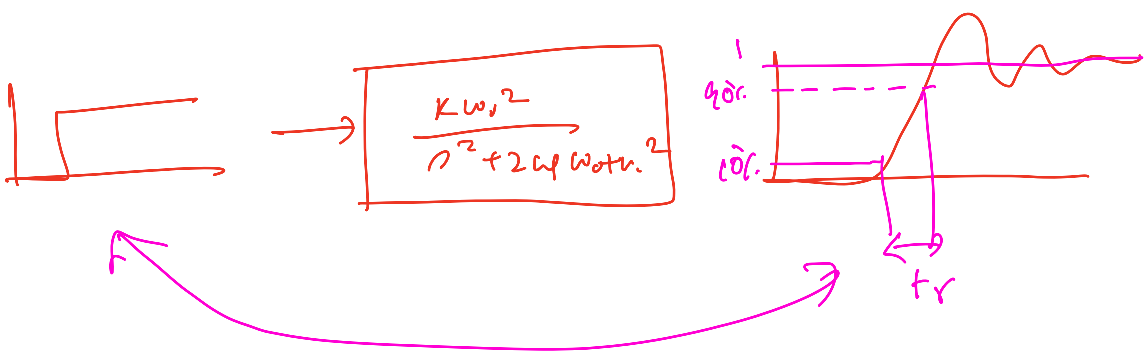
$$r(s) \longrightarrow \boxed{\frac{n_p(s)\, n_c(s)}{d_p(s)\, d_c(s) + n_p(s)\, n_c(s)}} \longrightarrow y(s)$$

$$\cong$$

$$r(s) \longrightarrow \boxed{\frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}} \longrightarrow y(s)$$

And then, $$\frac{C \cdot P}{1 + CP} = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$\boxed{\therefore\ C \cdot P = \boxed{\frac{\omega_0^2}{s(s + 2\zeta\omega_0)}}}$$



Bandwidth & Rise time

$$CLTF = \frac{K\,\omega_0^2}{s^2+2\zeta\omega_0^2+\omega_0^2}$$

Closed loop transfer function

as $s \to 0$, $\to K$

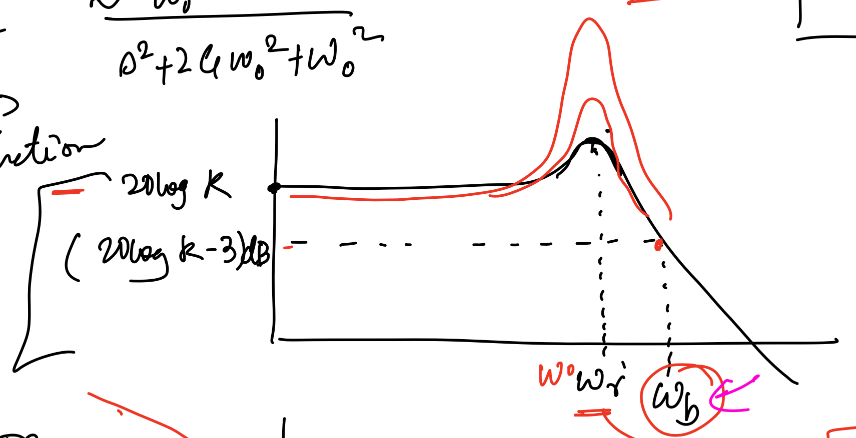
$\zeta = 0$

$$\boxed{\frac{K\,\omega_0^2}{s^2+\omega_0^2}}$$

$s = j\omega_0$

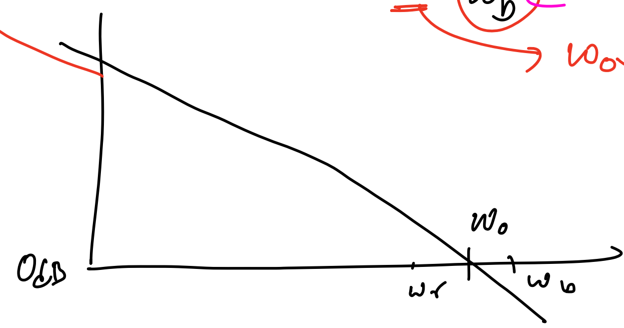
∞

$$\frac{K\omega_0^2}{j^2\omega_0^2+\omega_0^2} = \frac{K\omega_0^2}{0}$$



OLTF

open loop transfer function



$$\omega_r = \omega_0\sqrt{1-2\zeta^2} \sim \frac{\omega_b}{1.5}$$

$$\omega_b = 1.5\,.\,\omega_r \qquad \therefore \boxed{t_r = \frac{2.5}{\omega_b}}$$

Phase Margin and Damping Ratio (hence, overshoot)

Gain cross over frequency:

$|C.P| = 1 \quad \therefore \quad \left| \dfrac{\omega_0^2}{j\omega_c (j\omega_c + 2\zeta\omega_0)} \right| = 1$

$\therefore \boxed{\omega_c = \omega_0 \sqrt{1-2\zeta^2}}$

Phase margin $= 180^\circ + \angle C.P(j\omega)\big|_{\omega=\omega_c}$

$= 180^\circ + \left(-90^\circ - \tan^{-1}\dfrac{\omega_c}{2\zeta\omega_0}\right)$

$= 180^\circ + \left(-180^\circ + \tan^{-1}\dfrac{2\zeta\omega_0}{\omega_c}\right) = \tan^{-1}\dfrac{2\zeta\omega_0}{\omega_c}$

$PM = \tan^{-1}\dfrac{2\zeta\omega_0}{\omega_0\sqrt{1-2\zeta^2}} = \tan^{-1}\dfrac{2\zeta}{\sqrt{1-2\zeta^2}}$ rad

Phase margin $= \left[\tan^{-1}\dfrac{2\zeta}{\sqrt{1-2\zeta^2}}\right]$ rad

$= \left(\tan^{-1}\dfrac{2\zeta}{\sqrt{1-2\zeta^2}}\right)\cdot\dfrac{180}{\pi}$ degree

$\boxed{0 < \zeta < 0.6}$

$(\zeta < 0.6) \quad \simeq \quad 2\cdot\zeta\cdot\dfrac{180}{\pi} \quad \approx \quad \zeta\,(100)$ degree

$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$

$\tan^{-1} 2\zeta.$

$\tan^{-1}\theta \sim \theta.$

$\boxed{\dfrac{PM}{100} = \zeta \; (PM < 60^\circ)}$

Controller Design Guidelines

Step 1. Take input from user

- Maximum rise time (t_r)
- Maximum settling time (t_s)
- Maximum peak time (t_p)
- Maximum overshoot

Step 2. Use the following formulaes to convert time domain design requirements

↓

frequency domain design requirements.

$$t_r = \frac{\pi - \tan^{-1}\sqrt{\frac{1-\zeta^2}{\zeta}}}{\omega_0\sqrt{1-\zeta^2}} \sim \frac{2.5}{1.5\ \omega_0\sqrt{1-\zeta^2}} = \frac{5}{3\omega_0\sqrt{1-\zeta^2}}$$

$$t_p = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$$

M_p given

$$\boxed{M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}}$$ → given as per unit.

(eg. $V_{ref} = 100\,V$, max overshoot $= 2\,V$

$\therefore M_p = \frac{2}{100} = 0.02$)

$t_s = \frac{4}{\zeta\omega_n}$ (settles within 2% of reference)

$\frac{3}{\zeta\omega_n}$ (—— 5% ——)

Step 3 Once you have obtained ζ, ω_n

→ C(s) should have a pole at origin

$$C(s) = \frac{1}{s} \cdot \underbrace{q(s)}_{\text{rest of controller}}$$ → to ensure dc tracking

→ C(s) should have high gain such that |C·P| has high gain at frequencies where disturbance is present

→ C(s) should have low gain such that |C·P| has low gain at frequencies where noise is present

The open loop bode plot (C·P) should have the following (2) properties:—

① $$\boxed{PM = \left(\tan^{-1} \frac{2\zeta}{\sqrt{1-2\zeta^2}}\right) \cdot \frac{180}{\pi} \text{ degrees}}$$

② Gain cross over frequency,
$$\omega_c = \omega_0 \sqrt{1-2\zeta^2}$$

Design C(s) such that C(s) . P(s) has ω_c as the gain cross over freq & the corresponding phase margin = PM.

$$MP = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

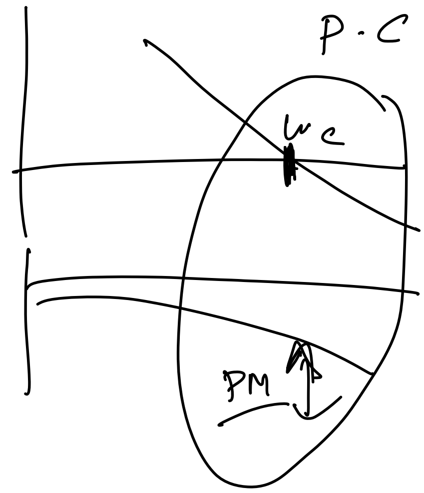
$$k := \ln(MP) = -\frac{\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\sqrt{1-\zeta^2} = -\frac{\pi\zeta}{k}$$

$$1-\zeta^2 = \frac{\pi^2\zeta^2}{k^2}$$

$$\therefore \zeta^2\left(\frac{\pi^2}{k^2}+1\right) = 1$$

$$\boxed{\therefore \zeta = \frac{1}{\sqrt{1+\frac{\pi^2}{\{\ln(MP)\}^2}}}}$$



Assume 2nd order s/s resp.

?

ω_0; ζ

Assume 2nd ord resp

t_r, t_s, MP

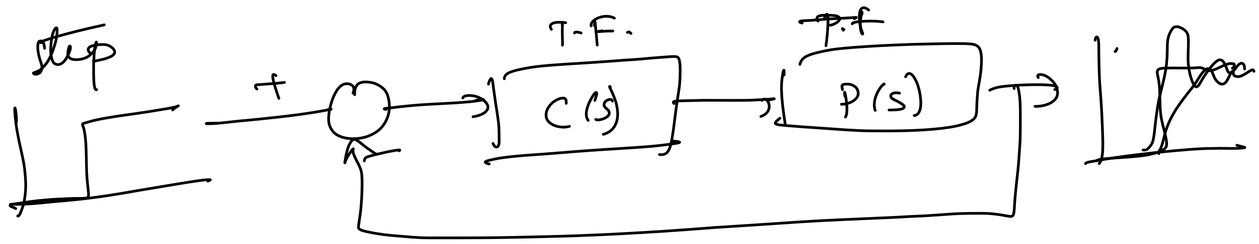
$\boxed{PM, \omega_c}$

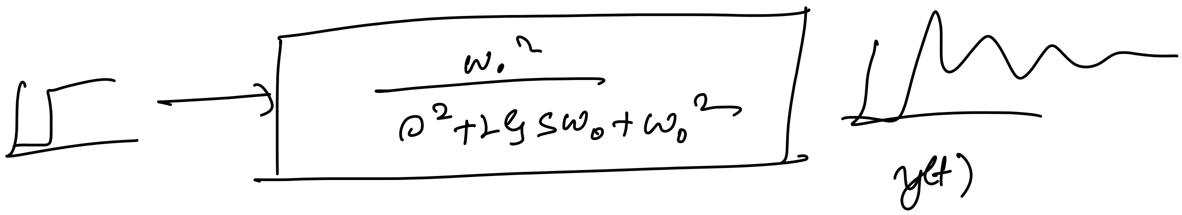
w/ the 2nd ord sys

$P(s) = \underline{\underline{\text{given.}}}$ $\quad \dfrac{K}{(s+1)(s+2)}$ $\downarrow$ $\boxed{PM,\ \omega_g}$

$\left[\, C(s) = \dfrac{\quad}{\quad} \right.$

C.P. = has PM at ω_g;
it has a gain crossover freq of ω_g

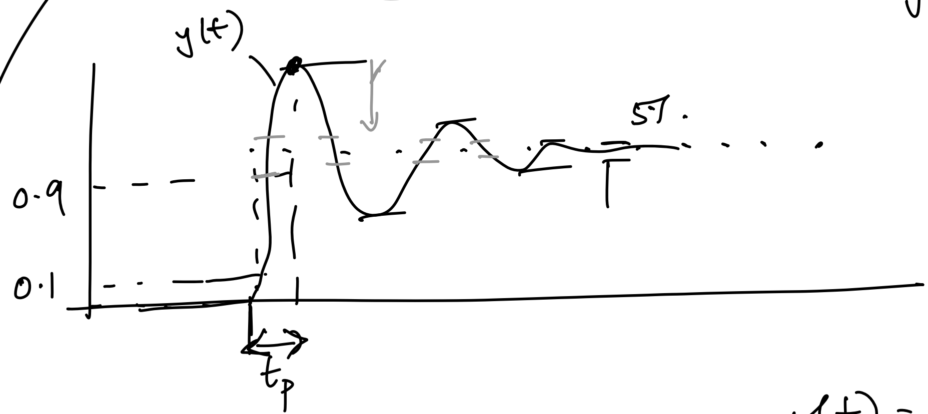




$$y(t)=\mathcal{L}^{-1}\left[\frac{1}{s}\cdot\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}\right]$$

$$y(t)=1-\frac{e^{-\zeta\omega_d t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t+\varphi) \qquad \boxed{0<\zeta<1}$$

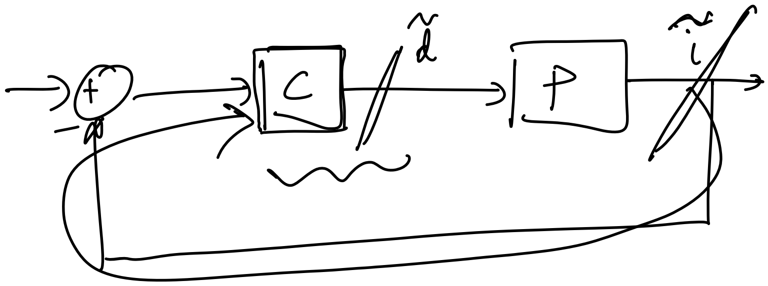
$$\varphi=\tan^{-1}\frac{\zeta}{\sqrt{1-\zeta^2}}$$

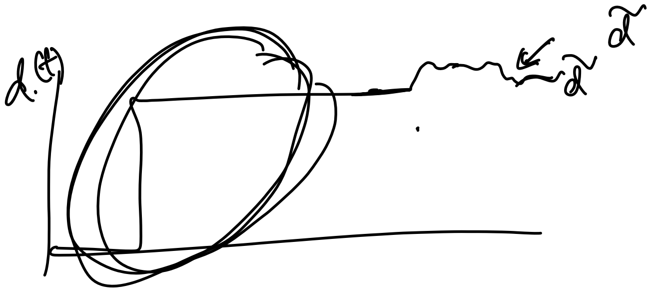


$=0.1$ $\qquad$ $=0.9$ $\qquad$ $y(t)=0.95$

$$\boxed{\frac{dy(t)}{dt}=0 \qquad t=?}\qquad t_p$$

$$\boxed{1-\frac{e^{-\zeta\omega_d t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t+\varphi)}=0.9$$





D.