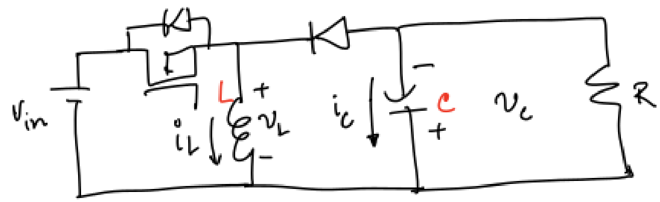
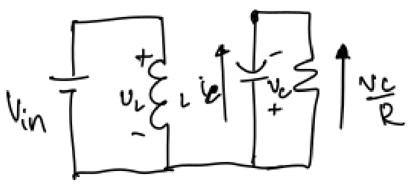
Lecture 3
January 11, 2022



Buck Boost converter

mode 1



Mode 1

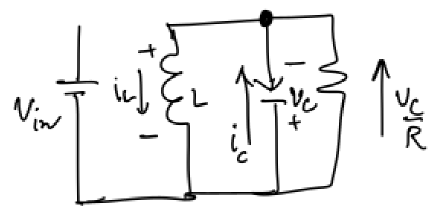
$$v_L = V_{in} = L\frac{di_L}{dt}$$

$$0 = i_C + \frac{v_C}{R}$$

$$0 = C\frac{dv_C}{dt} + \frac{v_C}{R}$$

$$\therefore \quad C\frac{dv_C}{dt} = -\frac{v_C}{R}$$

Mode 2



Mode 2

$$v_L = L\frac{di_L}{dt} = -v_C$$

$$i_L = i_C + \frac{v_C}{R}$$

$$\therefore \quad C\frac{dv_C}{dt} = -\frac{v_C}{R} + i_L$$

$$\langle x(t)\rangle_{Ts} = \int_t^{t+T_s} x(\tau)\,d\tau$$

$$0 = (V_{in})D + (-V_c)D'$$

$$0 = (-\frac{v_C}{R})D + (-\frac{v_C}{R} + I_L)D'$$

Balance Equations

$$\langle v_L(t)\rangle_{Ts} = L\frac{d}{dt}\langle i_L(t)\rangle_{Ts} = [v_L \text{ during mode 1}]\ d(t) + [v_L \text{ during mode 2}]\ d'(t)$$

$$= \langle v_{in}(t)\rangle_{Ts}\ d(t) + \langle -v_C(t)\rangle_{Ts}\ d'(t)$$

$$= \langle v_{in}(t)\rangle_{Ts}\ d(t) - \langle v_C(t)\rangle_{Ts}\ d'(t)$$

$$\langle i_C(t)\rangle_{Ts} = C\frac{d}{dt}\langle v_C(t)\rangle_{Ts} = [i_C \text{ during mode 1}]\ d(t) + [i_C \text{ during mode 2}]\ d'(t)$$

$$= \left(-\frac{\langle v_C(t)\rangle_{Ts}}{R}\right) d(t) + \left(-\frac{\langle v_C(t)\rangle_{Ts}}{R} + \langle i_L(t)\rangle_{Ts}\right) d'(t)$$

Notation

$\langle x(t)\rangle_{T_s}$ denotes the average of $x(t)$ over an interval of length T_s

Introduce Perturbation

$$\langle i_L(t)\rangle_{Ts} = I_L + \tilde{i}_L(t)$$

$$\langle v_C(t)\rangle_{Ts} = V_C + \tilde{v}_C(t)$$

$$\langle v_{in}(t)\rangle_{Ts} = V_{in} + \tilde{v}_{in}(t)$$

$$d(t) = D + \tilde{d}(t)$$

$$\begin{aligned} d'(t) &= 1 - d(t) \\ &= 1 - D - \tilde{d}(t) \\ &= D' - \tilde{d}(t) \end{aligned}$$

Rewrite Balance Equations ***$v_L(t)$***

$$\langle v_L(t)\rangle_{Ts} = L\frac{d}{dt}\langle i_L(t)\rangle_{Ts} = \langle v_{in}(t)\rangle_{Ts}\ d(t) + \langle -v_C(t)\rangle_{Ts}\ d'(t)$$

$$L\frac{d}{dt}\left[I_L + \tilde{i}_L(t)\right] = \left[V_{in} + \tilde{v}_{in}(t)\right]\left[D + \tilde{d}(t)\right] - \left[V_C + \tilde{v}_C(t)\right]\left[D' - \tilde{d}(t)\right]$$

$$L\frac{d}{dt}I_L + L\frac{d}{dt}\tilde{i}_L(t) = V_{in}D + V_{in}\tilde{d}(t) + \tilde{v}_{in}(t)D + \tilde{v}_{in}(t)\tilde{d}(t) - V_C D' + V_C\tilde{d}(t) - \tilde{v}_C(t)D' + \tilde{v}_C(t)\tilde{d}(t)$$

Note $\tilde{v}_C(t)\tilde{d}(t)$ and $\tilde{v}_{in}(t)\tilde{d}(t)$ are very small and can be ignored

Collect similar terms

Large signal terms (DC terms)

$$L\frac{d}{dt}I_L = V_{in}D + -V_C D' = 0$$

$$V_{in}D = V_C D' \text{ where } V_C = V_0$$

$$\therefore V_0 = \frac{V_{in}D}{D'} \qquad \textbf{1.}$$

Linear small signal

$$L\frac{d}{dt}\tilde{i}_L(t) = V_{in}\tilde{d}(t) + \tilde{v}_{in}(t)D + V_C\tilde{d}(t) - \tilde{v}_C(t)D' \qquad \textbf{2.}$$

Rewrite Balance Equations $i_C(t)$

$$\langle i_C(t)\rangle_{T_s} = C\frac{d}{dt}\langle v_C(t)\rangle_{T_s} = \left(\frac{\langle v_C(t)\rangle_{T_s}}{R}\right) d(t) + \left(-\frac{\langle v_C(t)\rangle_{T_s}}{R} + \langle i_L(t)\rangle_{T_s}\right) d'(t)$$

$$C\frac{d}{dt}\langle v_C(t)\rangle_{T_s} = \frac{-\langle v_C(t)\rangle_{T_s}}{R}\left(d(t) + d'(t)\right) + \langle i_L(t)\rangle_{T_s}\, d'(t)$$

$$C\frac{d}{dt}\langle v_C(t)\rangle_{T_s} = \frac{-\langle v_C(t)\rangle_{T_s}}{R} + \langle i_L(t)\rangle_{T_s}\, d'(t)$$

$$\therefore \quad C\frac{d}{dt}\left[V_C + \tilde{v}_C(t)\right] = \frac{-\left[V_C + \tilde{v}_C(t)\right]}{R} + \left[I_L + \tilde{i}_L(t)\right]\left[D' - \tilde{d}(t)\right]$$

Collect similar terms

1. Large signal terms (DC terms)

$$C\frac{d}{dt}V_C = \frac{V_C}{R} + I_L D' = 0$$

$$I_L = \frac{V_C}{RD'} \text{ where } V_C = V_0$$

$$\therefore I_L = \frac{V_0}{RD'} = \frac{I_0}{D'}$$

3.

2. Linear small signal

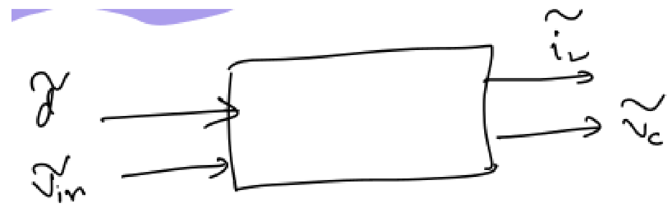
$$C\frac{d}{dt}\tilde{v}_C(t) = \frac{-\tilde{v}_C(t)}{R} - I_L\tilde{d}(t) + \tilde{i}_L(t)D'$$

4.

Linear small system

$$L\frac{d}{dt}\tilde{i_L}(t) = V_{in}\tilde{d}(t) + \tilde{v_{in}}(t)D + V_C\tilde{d}(t) - \tilde{v_C}(t)D'$$

$$C\frac{d}{dt}\tilde{v_C}(t) = \frac{-\tilde{v_C}(t)}{R} - I_L\tilde{d}(t) + \tilde{i_L}(t)D'$$



Want to find $\frac{\tilde{i}_L}{\tilde{d}}, \frac{\tilde{v}_c}{\tilde{d}}$

$$L\frac{d}{dt}\tilde{i_L}(t) = V_{in}\tilde{d}(t) + \tilde{v_{in}}(t)D + V_C\tilde{d}(t) - \tilde{v_C}(t)D'$$

$$L\left[s\tilde{I_L}(s) - \tilde{i_L}(0)\right] = V_{in}\tilde{d}(s) + \tilde{v_{in}}(s)D + V_C\tilde{d}(s) - \tilde{v_C}(s)D'$$

$$\text{control:} \quad \text{apply } \tilde{i_L}(0) = 0 \text{ and } \tilde{I_L}(s) = \langle i_L \rangle_{Ts}$$

$$sL\,\tilde{i_L}(s) = V_{in}\tilde{d}(s) + \tilde{v_{in}}(s)D + V_C\tilde{d}(s) - \tilde{v_C}(s)D' \qquad \textbf{5.}$$

$$C\frac{d}{dt}\tilde{v_C}(t) = \frac{-\tilde{v_C}(t)}{R} - I_L\tilde{d}(t) + \tilde{i_L}(t)D'$$

$$sC\,\tilde{v_C}(s) = \frac{-\tilde{v_C}(s)}{R} - I_L\tilde{d}(s) + \tilde{i_L}(s)D' \qquad \textbf{6.}$$

Solve System

Set $\tilde{v_{in}} = 0$

$$sL\,\tilde{i_L}(s) = V_{in}\tilde{d}(s) + \tilde{v_{in}}(s)D + V_C\tilde{d}(s) - \tilde{v_C}(s)D'$$

$$\left(sC + \frac{1}{R}\right)\,\tilde{v_C}(s) = -I_L\tilde{d}(s) + \tilde{i_L}(s)D'$$

Find $\frac{\tilde{i_L}(s)}{\tilde{d}(s)}$

$$\tilde{v_C}(s) = \frac{\tilde{i_L}(s)D' - I_L\tilde{d}(s)}{\left(sC + \frac{1}{R}\right)}$$

$$sL\,\tilde{i_L}(s) = (V_{in} + V_C)\,\tilde{d}(s) - \tilde{v_C}(s)D'$$

$$sL\,\tilde{i_L}(s) = (V_{in} + V_C)\,\tilde{d}(s) - D'\,\frac{\tilde{i_L}(s)D' - I_L\tilde{d}(s)}{sC + \frac{1}{R}}$$

$$sL\,\tilde{i_L}(s) = (V_{in} + V_C)\,\tilde{d}(s) - \frac{\tilde{i_L}(s)D'^2}{sC + \frac{1}{R}} + \frac{D'\,I_L\tilde{d}(s)}{sC + \frac{1}{R}}$$

$$\left(sL + \frac{\tilde{i_L}(s)D'^2}{sC + \frac{1}{R}}\right)\tilde{i_L}(s) = (V_{in} + V_C)\,\tilde{d}(s) + \frac{D'\,I_L}{sC + \frac{1}{R}}\tilde{d}(s)$$

$$\therefore \frac{\tilde{i_L}(s)}{\tilde{d}(s)} = \frac{(V_{in} + V_C)(sC + \frac{1}{R}) + D'\,I_L}{s^2\,LC + \frac{sL}{R} + D'^2}$$

Similarly find $\frac{\tilde{v_C}(s)}{\tilde{d}(s)}$ ...