Lecture 5 (18th Jan 2022)

Thursday before class

+ 24 hour extension. (Assignment 1).

HW 2 will be uploaded on Thursday.

Step 1

Write down the state eq in mode ①, ② & combine them to get the "average" (over a switching cycle) model.

Average Model:

$$L \frac{d\langle i_L(t)\rangle_{T_s}}{dt} = (v_L \text{ in mode ①})\, d(t) + (v_L \text{ in mode ②})\, d'(t).$$

$$C \frac{d\langle v_c(t)\rangle_{T_s}}{dt} = (i_c \text{ in mode ①})\, d(t) + (i_c \text{ in mode ②})\, d'(t).$$

Specific example of buck boost.

$$\langle v_L(t)\rangle_{T_s} = L \frac{d}{dt} \langle i_L(t)\rangle_{T_s} = \left(\langle v_{in}(t)\rangle_{T_s}\right) \cdot d(t) - \left(\langle v_c(t)\rangle_{T_s}\right) d'(t)$$

$$\langle i_c(t)\rangle_{T_s} = C \frac{d}{dt} \langle v_c(t)\rangle_{T_s} = \left(-\frac{\langle v_c(t)\rangle_{T_s}}{R}\right) d(t) + \left(-\frac{\langle v_c(t)\rangle_{T_s}}{R} + \langle i_L(t)\rangle_{T_s}\right) d'(t)$$

$$= -\frac{\langle v_c(t)\rangle_{T_s}}{R} + \langle i_L(t)\rangle_{T_s} \cdot d'(t).$$

How do we simulate this in PLECs?

$$L\frac{d\langle i_L\rangle}{dt} = \langle v_{in}\rangle d - \langle v_c\rangle d'$$

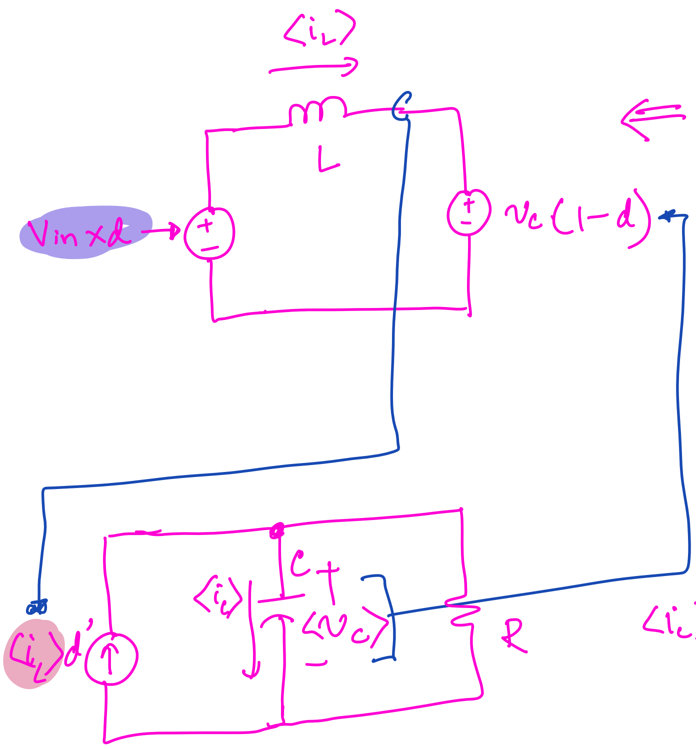
$$C\frac{d\langle v_c\rangle}{dt} = -\frac{\langle v_c\rangle}{R} + \langle i_L\rangle d'.$$

these are still variables of time, I am just using a shorthand.

$$\dot{x} = f(x) \qquad x = \begin{bmatrix} \langle i_L\rangle \\ \langle v_c\rangle \end{bmatrix}$$

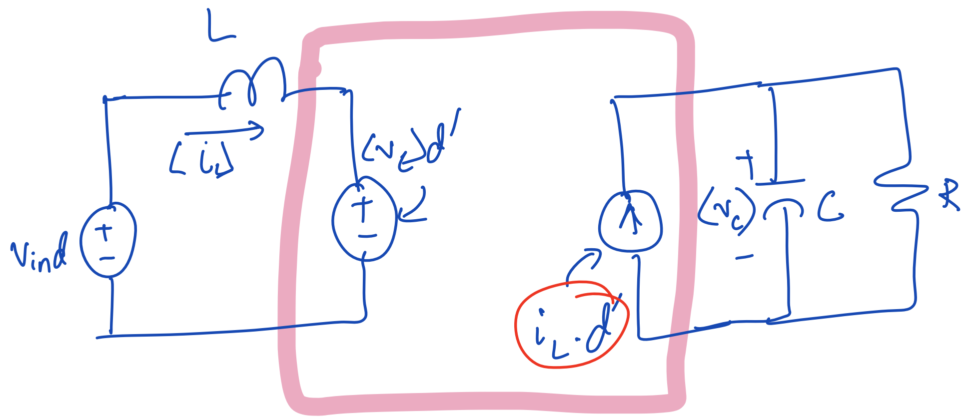
ⓐ Solve an ODE

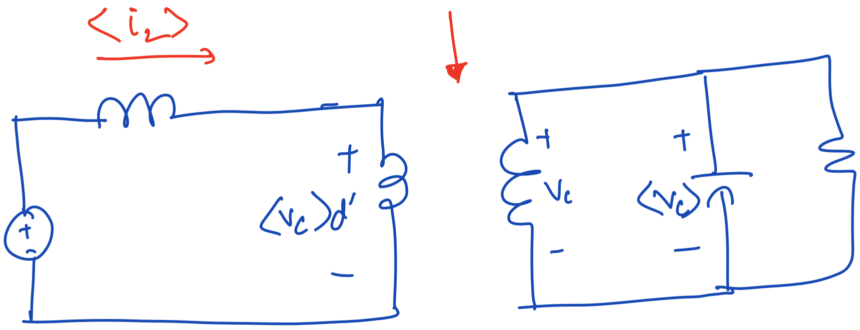
ⓑ Circuit representation.



$$\overbrace{L\frac{d\langle i_L\rangle}{dt}} = \langle v_{in}\rangle d - \langle v_c\rangle d'$$

$$\langle i_c\rangle = \overbrace{C\frac{d\langle v_c\rangle}{dt}} = -\frac{\langle v_c\rangle}{R} + \overbrace{\langle i_L\rangle d'}.$$





$$\underbrace{n_1 : n_2}_{d' : 1}$$

$n_1 = ?$ d'

$n_2 = ?$ 1

$$\boxed{\frac{V_1}{n_1} = \frac{V_2}{n_2}}$$

$$n_1 i_1 = n_2 i_2$$

$\underbrace{d' \langle i_L \rangle}$ $(1) \cdot (i_L \cdot d')$

Volt / turn

Step 2. Obtain the steady state value. $(\bar{x}, \bar{u})$
or, (X, U)

$$L \frac{d}{dt}\langle i_L \rangle = \langle v_{in} \rangle d - \langle v_c \rangle d'$$

$$C \frac{d}{dt}\langle v_c \rangle = -\frac{\langle v_c \rangle}{R} + \langle i_L \rangle d'$$

$v_{in} = V_{in} + \tilde{v}_{in}$

$$L \frac{d}{dt}(I_L + \tilde{i}_L) = (V_{in} + \tilde{v}_{in})(D + \tilde{d}) - (V_c + \tilde{v}_c)(D' - \tilde{d})$$

$$C \frac{d}{dt}(V_c + \tilde{v}_c) = -\frac{(V_c + \tilde{v}_c)}{R} + (I_L + \tilde{i}_L)(D' - \tilde{d})$$

Only solve for large signal. part

$$L \frac{dI_L}{dt} = V_{in} D - V_c D' = 0 \qquad \therefore \; V_c = V_{in} \cdot \frac{D}{D'}$$

$$C \frac{d}{dt} V_c = -\frac{V_c}{R} + I_L D' = 0 \qquad I_L = \frac{V_c}{R \cdot D'}$$

$$U = \begin{bmatrix} D \\ V_{in} \end{bmatrix} = \bar{u} \qquad X = \begin{bmatrix} I_L \\ V_c \end{bmatrix} = \bar{x} = \begin{bmatrix} V_c/(R \cdot D') \\ V_{in} \frac{D}{D'} \end{bmatrix} = \bar{x}$$

$$= \begin{bmatrix} \frac{V_{in} D}{R D'^2} \\ \frac{V_{in} D}{D'} \end{bmatrix}$$

Step 3 Obtain A & B to get the linearized average model.

$$\dot{\tilde{x}} = A(x-\bar{x}) + B(u-\bar{u})$$

$\tilde{x} = x - \bar{x}$
$\tilde{u} = u - \bar{u}$

$$A = \left.\frac{\partial f}{\partial \tilde{x}}\right|_{x=\bar{x},\, u=\bar{u}} \qquad B = \left.\frac{\partial f}{\partial \tilde{u}}\right|_{x=\bar{x},\, u=\bar{u}}$$

$$x = \begin{bmatrix} \langle i_L \rangle \\ \langle v_c \rangle \end{bmatrix} \qquad u = \begin{bmatrix} d \\ \langle v_{in} \rangle \end{bmatrix}$$

$$f = \begin{bmatrix} f_1(x,u) \\ f_2(x,u) \end{bmatrix}$$

$$L \frac{d\langle i_L \rangle}{dt} = \langle v_{in} \rangle d - \langle v_c \rangle d'$$

$$C \frac{d\langle v_c \rangle}{dt} = -\frac{\langle v_c \rangle}{R} + \langle i_L \rangle d'$$

$d' = 1-d \qquad \frac{\partial d'}{\partial d} = -1$

$$f_1 = \frac{\langle v_{in} \rangle d - \langle v_c \rangle d'}{L}$$

$\frac{V_{in}}{L} + \frac{V_c}{L}$

$$f_2 = -\frac{\langle v_c \rangle}{RC} + \frac{\langle i_L \rangle d'}{C}$$

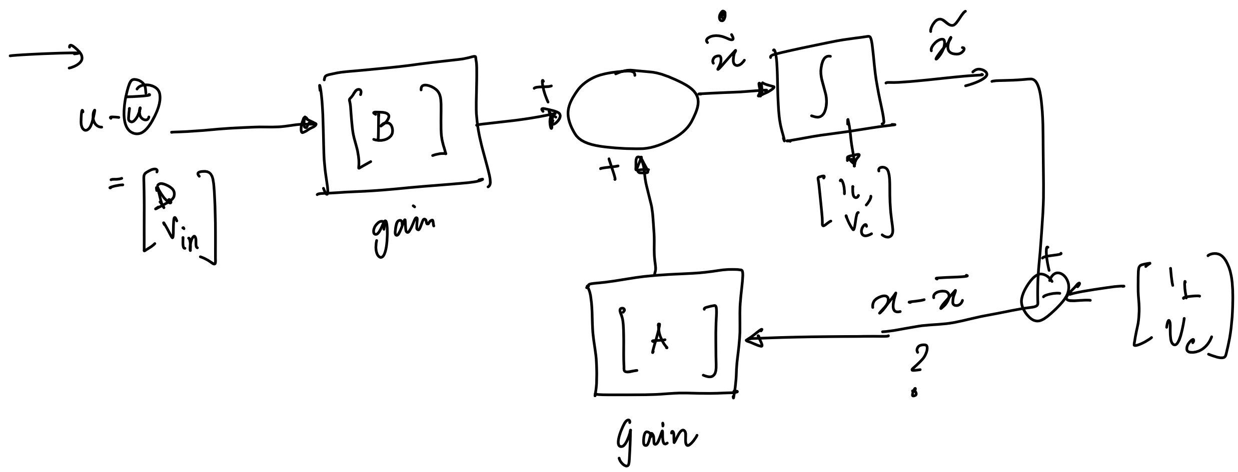
$$A = \left.\begin{bmatrix} \frac{\partial f_1}{\partial \tilde{i}_L} & \frac{\partial f_1}{\partial \tilde{v}_c} \\ \frac{\partial f_2}{\partial \tilde{i}_L} & \frac{\partial f_2}{\partial \tilde{v}_c} \end{bmatrix}\right|_{\substack{i_L = I_L,\, d = D \\ v_c = V_C,\, v_{in} = V_{in}}} = \begin{bmatrix} 0 & -\frac{D'}{L} \\ \frac{D'}{C} & -\frac{1}{RC} \end{bmatrix}$$

D, V_{in}

$$B = \left.\begin{bmatrix} \frac{\partial f_1}{\partial \tilde{D}} & \frac{\partial f_1}{\partial \tilde{v}_{in}} \\ \frac{\partial f_2}{\partial \tilde{D}} & \frac{\partial f_2}{\partial \tilde{v}_{in}} \end{bmatrix}\right|_{\substack{i_L = \\ \vdots}} = \begin{bmatrix} \frac{V_{in} + V_C}{L} & \frac{D}{L} \\ -\frac{i_L}{C} & 0 \end{bmatrix}$$

$$\boxed{\dot{\tilde{x}} = A\,(x - \bar{x}) + B\,(u - \bar{u})}$$

You know A, B for a particular D, Vin.



$$\dot{x} = f(x, u)$$

$$\dot{x} = \underbrace{\cancel{f(\bar{x}, \bar{u})}}_{0} + \underbrace{\frac{\partial f}{\partial x}}_{A} \cdot \underbrace{(x - \bar{x})}_{\tilde{x}} + \underbrace{\frac{\partial f}{\partial u}}_{B} \cdot \underbrace{(u - \bar{u})}_{\tilde{u}}$$

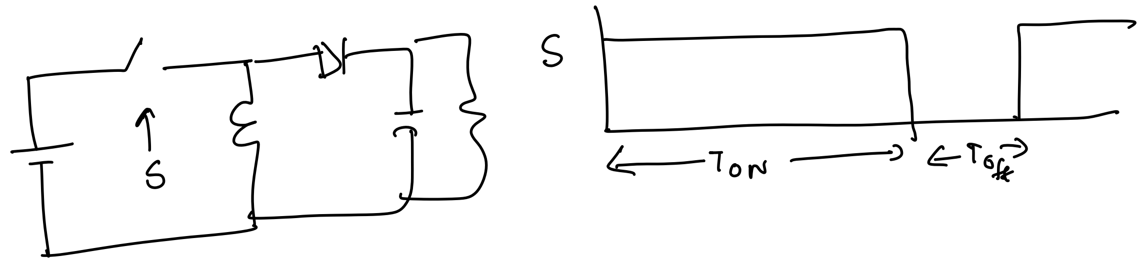
$$x - \bar{x} = \tilde{x}$$

$$x = \bar{x} + \tilde{x}$$

$$\dot{x} = \underbrace{\dot{\bar{x}}}_{=0} + \dot{\tilde{x}}$$

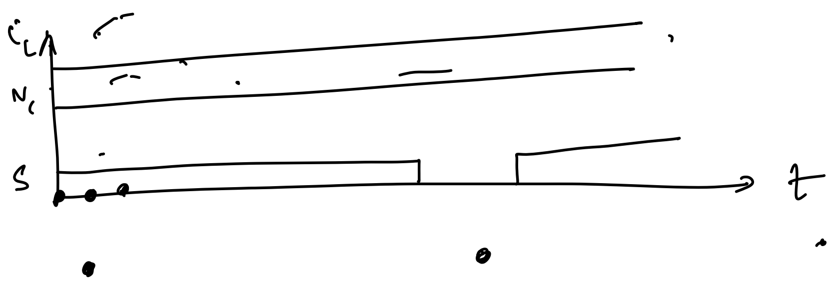
$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$$

PLECS simulation



Fixed simulation step size

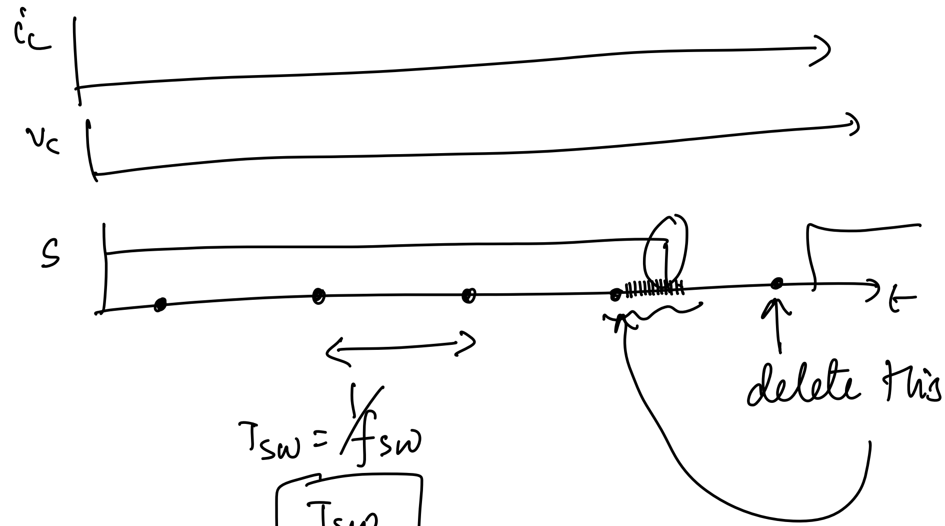
ⓐ



too large time steps - bad results

too small " " → good " unnecessarily long simulation

ⓑ Variable time step.

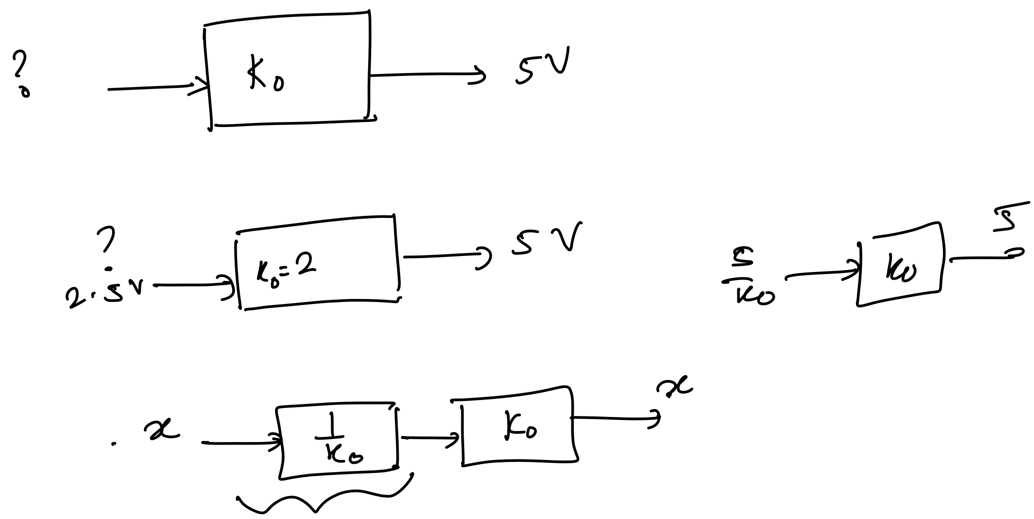


delete this point

$T_{sw} = 1/f_{sw}$

$$\boxed{\frac{T_{sw}}{10}}$$

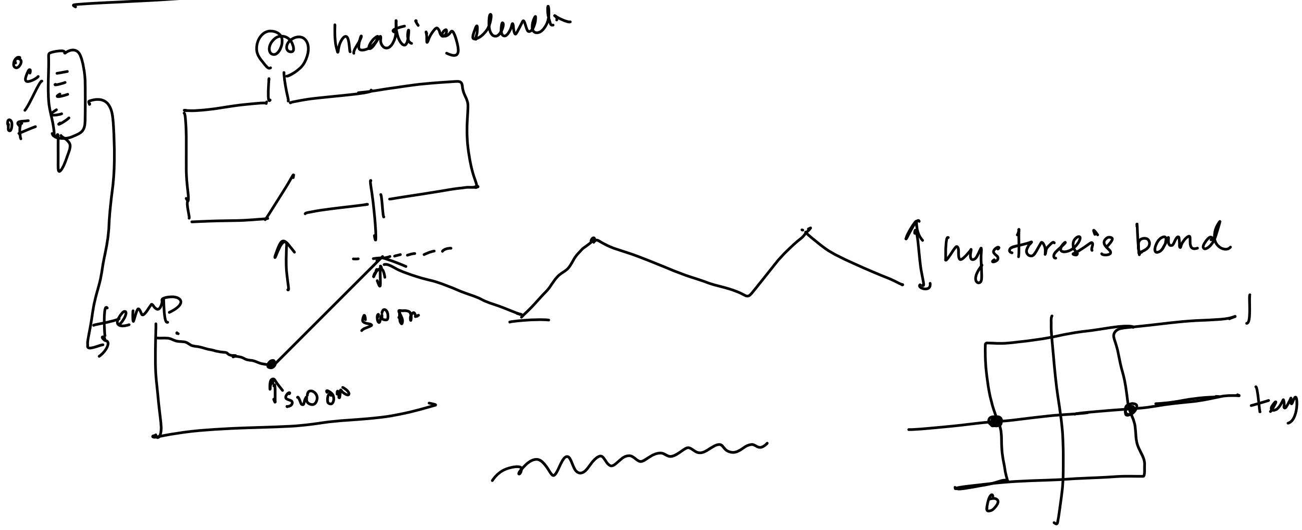
Control



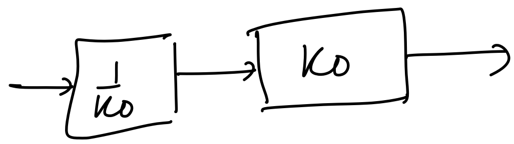
open loop control

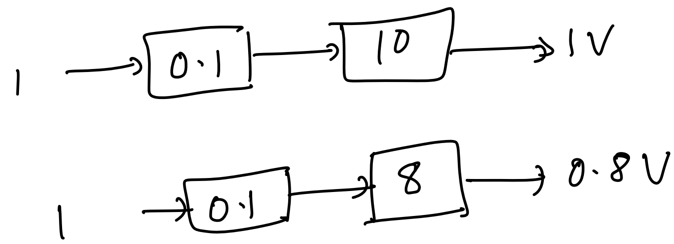
Buck converter - cheap.

on-off control / bang-bang control.

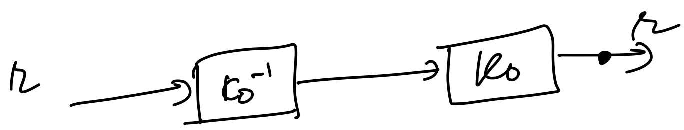


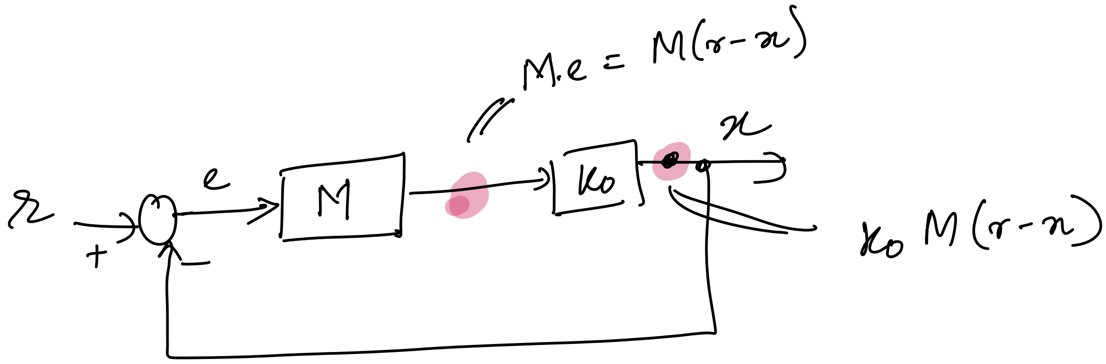
Closed Loop





$$10(1+T^{0.001})$$





Ideally $r = x$ is what I want.

$e = r - x$

$$K_0 M(r-x) = x$$
$$K_0 M r - K_0 M x = x$$

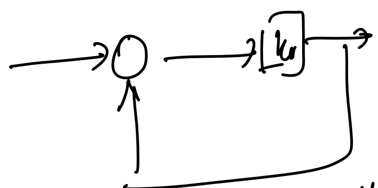
$$k_0 M r = (k_0 M + 1) x$$

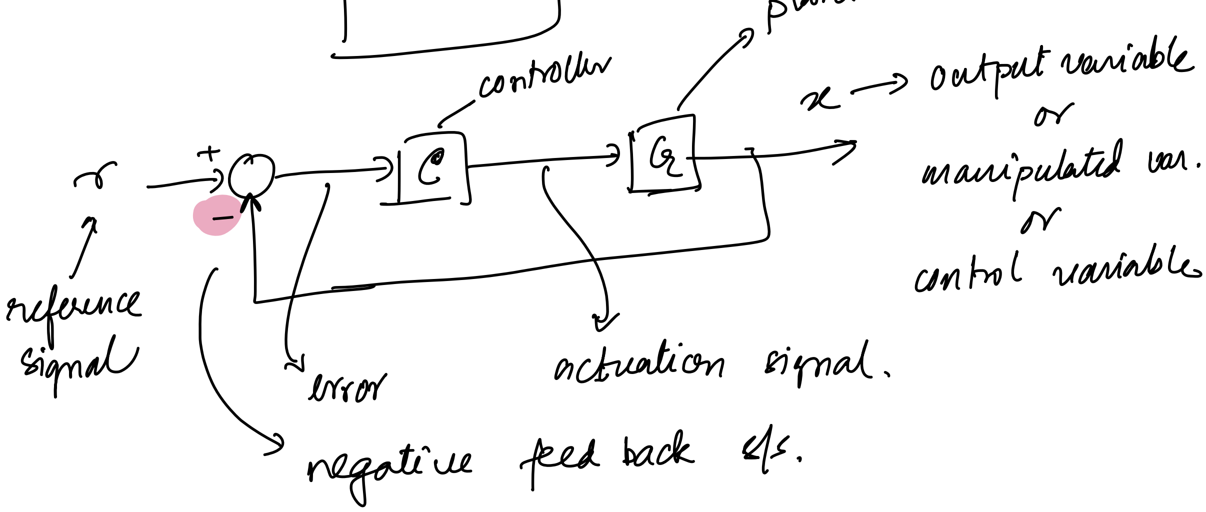
$$\therefore \quad x = \frac{k_0 M r}{1 + k_0 M}$$

$$x = r \cdot \frac{1}{1 + \frac{1}{k_0 M}}$$

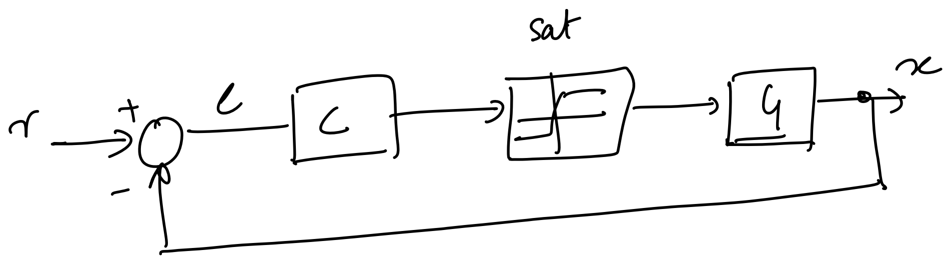
when $M \to \infty$, $x \simeq r$.

Basic of closed loop control s/s.

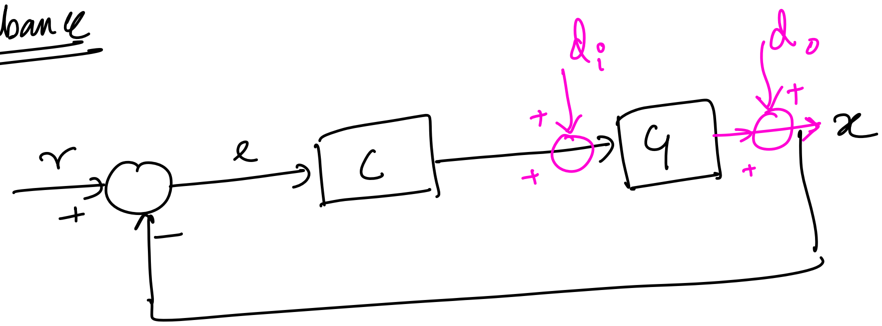




$$x = \frac{G \cdot C}{1 + G \cdot C} \cdot r.$$

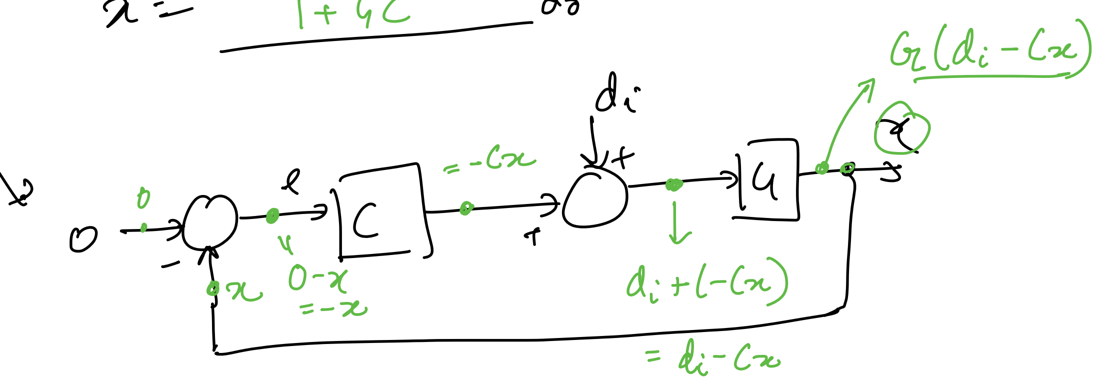


disturbance



$$x = \frac{G}{1+GC} d_i \qquad x = \underline{\qquad} r$$

$$x = \frac{1}{1+GC} d_o$$



$$G(d_i - Cx) = x$$

$$\Rightarrow G.d_i - GCx = x$$

$$G d_i = (GC+1)x$$

$$\therefore \quad x = \frac{G}{1+GC} d_i \quad \Bigg| \quad x = \frac{1}{1+GC} d_o$$

We do not want disturbance to impact the output variable (x)

So, I do not want d_i / d_o to impact x

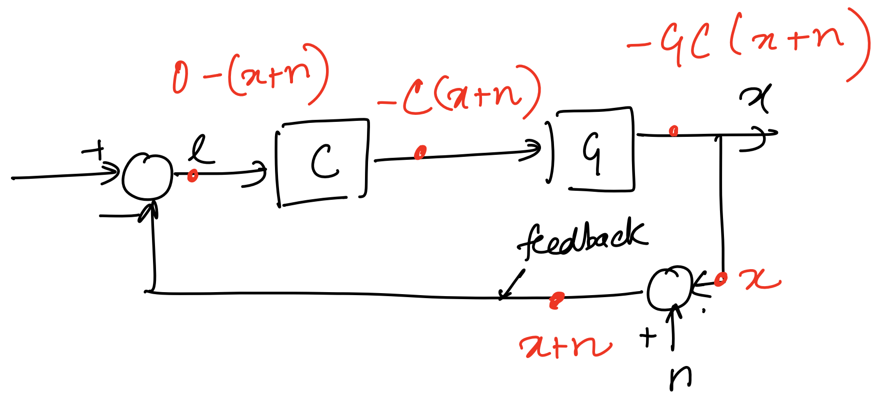
$x = \underbrace{\text{very small}} \; d_i$ or d_o.

$$x = \frac{1}{1+GC} d_o = \frac{1}{GC} d_o \text{ if } G \text{ is large}, \quad \underline{x = 0}$$

$$x = \frac{G}{1+GC} d_i = \frac{G \, d_i}{G C} = \frac{1}{C} d_i = 0.$$

G is large $\therefore$ $GC \gg 1$

<u>Noise</u>



$$-GC(x+n) = x$$

$$-GCx = (1+GC)x$$

$$\therefore \quad x = -\frac{GC}{1+GC} \cdot n.$$

C large, (controller) $\quad x = -\frac{G \cdot C}{G \cdot C} n = -n.$

Simultaneously large controller gain (C)

ensures

① reference tracking

② disturbance rejection

③ enhances noise.

$$x = \frac{GC}{1+GC} r + \frac{1}{1+GC} d_o + \frac{G}{1+GC} d_i - \frac{GC}{1+GC} n$$

rare

$$\boxed{x = T r + S d_o - T n}$$

Complementary sensitivity

Sensitivity