EE 458/533 – Power Electronics Controls
Homework 2
Due Date: Thursday January 20th 2021

1.

A) $\dot{x} = \begin{bmatrix} f_1(x,u) \\ f_2(x,u) \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \langle i(t)\rangle \\ \langle v(t)\rangle \end{bmatrix}$ where $u = \begin{bmatrix} d(t) \\ \langle v_g(t)\rangle \end{bmatrix}$

$$\frac{d}{dt}\begin{bmatrix} \langle i(t)\rangle \\ \langle v(t)\rangle \end{bmatrix} = \begin{bmatrix} \frac{1}{L}\left[\langle v_g(t)\rangle - 2R_{on}\langle i(t)\rangle - \langle v(t)\rangle\right] d(t) + \frac{1}{L}\left[-\langle v_g(t)\rangle - 2R_{on}\langle i(t)\rangle - \langle v(t)\rangle\right] d'(t) \\ \frac{1}{C}\left[\langle i(t)\rangle - \frac{\langle v(t)\rangle}{R}\right] d(t) + \frac{1}{C}\left[\langle i(t)\rangle - \frac{\langle v(t)\rangle}{R}\right] d'(t) \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} \langle i(t)\rangle \\ \langle v(t)\rangle \end{bmatrix} = \begin{bmatrix} \frac{1}{L}\left[(2d(t)-1)\langle v_g(t)\rangle - 2R_{on}\langle i(t)\rangle - \langle v(t)\rangle\right] \\ \frac{1}{C}\left[\langle i(t)\rangle - \frac{\langle v(t)\rangle}{R}\right] \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} \langle i(t)\rangle \\ \langle v(t)\rangle \end{bmatrix} = \begin{bmatrix} \frac{1}{L}\left[2d(t)\langle v_g(t)\rangle - \langle v_g(t)\rangle - 2R_{on}\langle i(t)\rangle - \langle v(t)\rangle\right] \\ \frac{1}{C}\left[\langle i(t)\rangle - \frac{\langle v(t)\rangle}{R}\right] \end{bmatrix}$$

$$\langle v_L(t)\rangle = L\frac{d}{dt}\left[I + \langle \hat{i}(t)\rangle\right] = \left[2[D+\hat{d}(t)][V_g + \langle \hat{v}_g(t)\rangle] - [V_g + \langle \hat{v}_g(t)\rangle] - 2R_{on}[I + \langle \hat{i}(t)\rangle] - [V + \langle \hat{v}(t)\rangle]\right]$$

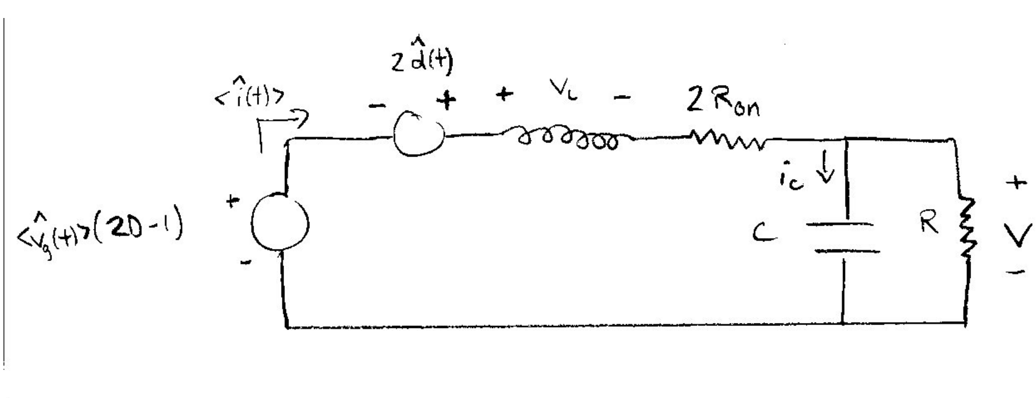
$$\langle v_L(t)\rangle = L\frac{d}{dt}\left[I + \langle \hat{i}(t)\rangle\right] = \left[2DV_g + 2D\langle \hat{v}_g(t)\rangle + 2\hat{d}(t)V_g + 2\hat{d}(t)\langle \hat{v}_g(t)\rangle - V_g - \langle \hat{v}_g(t)\rangle - 2R_{on}I - 2R_{on}\langle \hat{i}(t)\rangle - V - \langle \hat{v}(t)\rangle\right]$$

$$\langle \hat{v}_L(t)\rangle = L\frac{d}{dt}\langle \hat{i}(t)\rangle = \left[2D\langle \hat{v}_g(t)\rangle + 2\hat{d}(t)V_g - \langle \hat{v}_g(t)\rangle - 2R_{on}\langle \hat{i}(t)\rangle - \langle \hat{v}(t)\rangle\right]$$

$$\langle \hat{v}_L(t)\rangle = L\frac{d}{dt}\langle \hat{i}(t)\rangle = \left[(2D-1)\langle \hat{v}_g(t)\rangle + 2\hat{d}(t)V_g - 2R_{on}\langle \hat{i}(t)\rangle - \langle \hat{v}(t)\rangle\right]$$

$$\langle i_C(t)\rangle = C\frac{d}{dt}\left[V + \langle \hat{v}(t)\rangle\right] = \left[[I + \langle \hat{i}(t)\rangle] - \frac{1}{R}[V + \langle v(t)\rangle]\right]$$

$$\langle \hat{i}_C(t)\rangle = C\frac{d}{dt}\langle \hat{v}(t)\rangle = \left[\langle \hat{i}(t)\rangle - \frac{\langle v(t)\rangle}{R}\right]$$



B) Taking the derivative with respect to x_1 and x_2, the same values for A and B are found such that $\dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t)$.

$$\frac{d}{dt}\langle \hat{i}(t)\rangle = \frac{1}{L}\left[(2D-1)\langle \hat{v}_g(t)\rangle + 2\hat{d}(t)V_g - 2R_{on}\langle \hat{i}(t)\rangle - \langle \hat{v}(t)\rangle\right]$$

$$\frac{d}{dt}\langle \hat{v}(t)\rangle = \frac{1}{C}\left[\langle \hat{i}(t)\rangle - \frac{\langle v(t)\rangle}{R}\right]$$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix} \frac{\partial f_1}{\partial \langle \hat{i}(t)\rangle} & \frac{\partial f_1}{\partial \langle \hat{v}(t)\rangle} \\ \frac{\partial f_2}{\partial \langle \hat{i}(t)\rangle} & \frac{\partial f_2}{\partial \langle \hat{v}(t)\rangle} \end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix} \frac{-2R_{on}}{L} & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix} \frac{\partial f_1}{\partial \hat{d}(t)} & \frac{\partial f_1}{\partial \langle \hat{v}_g(t)\rangle} \\ \frac{\partial f_2}{\partial \hat{d}(t)} & \frac{\partial f_2}{\partial \langle \hat{v}_g(t)\rangle} \end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix} \frac{-2V_g}{L} & \frac{2D-1}{L} \\ 0 & 0 \end{bmatrix}$$

C)

$$\hat{y}(t) = \hat{x}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \langle \hat{i}(t)\rangle \\ \langle \hat{v}(t)\rangle \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \hat{d}(t) \\ \langle \hat{v}_g(t)\rangle \end{bmatrix}$$

$$\hat{y}(s) = \left(C(sI-A)^{-1}B + E\right)\hat{u}(s)$$

$$G(s) = \begin{bmatrix} G_{id}(s) & G_{ig}(s) \\ G_{vd}(s) & G_{vg}(s) \end{bmatrix} =$$

$$\begin{bmatrix} \frac{2V_g(CRs+1)}{CLRs^2 + (2CRR_{on}+L)s + 2R_{on} + R} & \frac{(2D-1)(CRs+1)}{CLRs^2 + (2CRR_{on}+L)s + 2R_{on} + R} \\ \frac{2RV_g}{CLRs^2 + (2CRR_{on}+L)s + 2R_{on} + R} & \frac{R(2D-1)}{CLRs^2 + (2CRR_{on}+L)s + 2R_{on} + R} \end{bmatrix}$$

D) Given d=0.5, $G_{ig}(s)$ and $G_{vg}(s)$ will have no response since they have terms of (2D-1).
Half the switching frequency is 25kHz/2 = 12.5kHz, however the cross over frequencies occur at frequencies much greater than this.

The system is stable due to large positive phase margin in $G_{id}(s)$. Phase margin shows how much the system gain can be increased before it is unstable. There is also small positive phase margin in $G_{vd}(s)$.

TF	GM (dB)	PM (deg)	WG (Hz)	WP (Hz)	Stability
$G_{id}(s)$	inf	90.01	nan	106,182	yes
$G_{ig}(s)$	inf	inf	nan	nan	yes
$G_{vd}(s)$	inf	2.25	nan	41,180	yes
$G_{vg}(s)$	inf	inf	nan	nan	yes