Lecture 15 (22nd February 2022)

HW 4 due on 24th feb.

Lab 2 should be completed by this week

- today's class delves into topics of Lab 3.

ABC to DQ transform

$$\underbrace{\begin{bmatrix} x_d \\ x_q \end{bmatrix}}_{\text{dc quantities}} = \frac{2}{3} \underbrace{\begin{bmatrix} \cos\theta_d & \cos\left(\theta_d - \frac{2\pi}{3}\right) & \cos\left(\theta_d + \frac{2\pi}{3}\right) \\ -\sin\theta_d & -\sin\left(\theta_d - \frac{2\pi}{3}\right) & -\sin\left(\theta_d + \frac{2\pi}{3}\right) \end{bmatrix}}_{:= \Gamma_{dq}(\theta_d)} \underbrace{\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}}_{\text{ac quantities}}$$

$=: x_{dq}$

if $\theta_d = \theta$

for eg $x_a = X \sin\theta$

$x_b = X \sin\left(\theta - \frac{2\pi}{3}\right)$

$x_c = X \sin\left(\theta + \frac{2\pi}{3}\right)$

$$x_{dq} := \begin{bmatrix} x_d \\ x_q \end{bmatrix}$$

$$x_{abc} := \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}$$

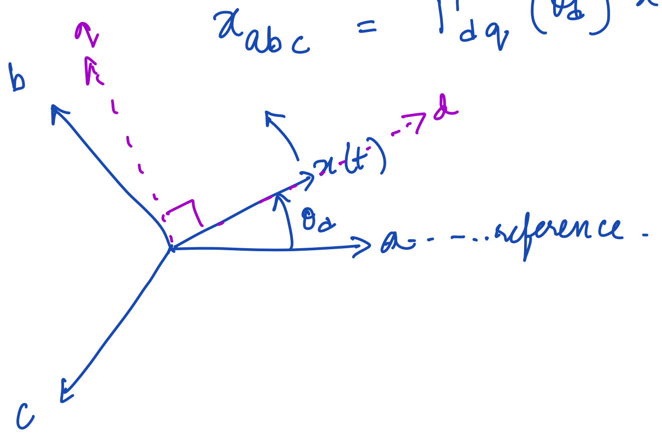
angle of rotation

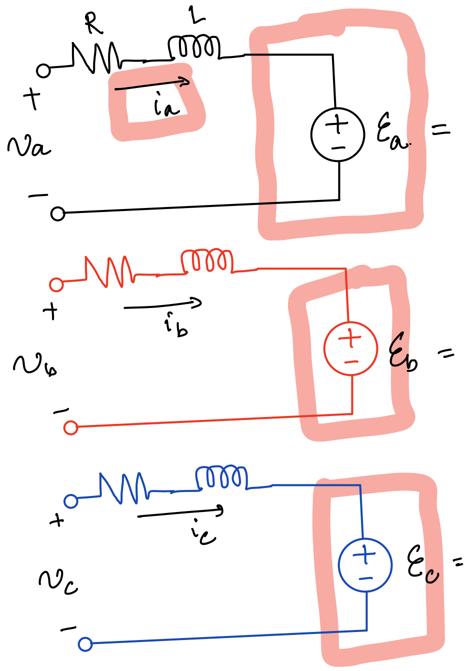
vector $\mathbb{R}^{3\times 1}$

real number

$$\underbrace{x_{dq}}_{\text{vector } \in \mathbb{R}^{2\times 1}} = \frac{2}{3} \underbrace{\Gamma_{dq}(\theta_d)}_{\text{matrix } \in \mathbb{R}^{2\times 3}} \cdot x_{abc}$$

$$x_{abc} = \Gamma_{dq}^{T}(\theta_d)\, x_{dq}$$





$\mathcal{E}_a = \lambda_{pk}\, w_e \sin(\theta_e)$

$\mathcal{E}_b = \lambda_{pk}\, w_e \sin\left(\theta_e - \frac{2\pi}{3}\right)$

$\mathcal{E}_c = \lambda_{pk}\, w_e \sin\left(\theta_e - \frac{4\pi}{3}\right)$

$$\begin{cases} v_a = L\dfrac{di_a}{dt} + Ri_a + \mathcal{E}_a & \text{—— 1} \\ v_b = L\dfrac{di_b}{dt} + Ri_b + \mathcal{E}_b & \text{—— 2} \\ v_c = L\dfrac{di_c}{dt} + Ri_c + \mathcal{E}_c & \text{—— 3} \end{cases}$$

Combine eq 1, 2, 3 into a single eq by using linear algebra

$$x_{abc} = \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}$$

$$v_{abc} = L\frac{d}{dt}\left(i_{abc}\right) + R\, i_{abc} + \mathcal{E}_{abc}$$

$$\boxed{x_{dq} = \frac{2}{3}\underbrace{\Gamma_{dq}(\theta_d)}_{\text{matrix } \in \mathbb{R}^{2\times 3}} \cdot x_{abc}}$$

vector $\in \mathbb{R}^{2\times 1}$

$$x_{abc} = \Gamma_{dq}^{T}(\theta_d)\, x_{dq}$$

← Identities derived in Lec 13.

$$\boxed{x_{dq} = \left\{\Gamma_{dq}^{T}(\theta_d)\right\}^{-1} x_{abc}}$$

$$\Rightarrow \left[\Gamma_{dq}^{T}(\theta_d)\right]^{-1} = \frac{2}{3}\Gamma_{dq}(\theta_d)$$

$$v_{abc} = \Gamma_{dq}^{T}(\theta_d)\, v_{dq} \rightarrow \begin{bmatrix} v_d \\ v_q \end{bmatrix}$$

$$\Gamma_{dq}^{T}(\theta_d)\, v_{dq} = L\frac{d}{dt}\left(\Gamma_{dq}^{T}(\theta_d)\, i_{dq}\right) + R\,\Gamma_{dq}^{T}(\theta_d)\, i_{dq} + \Gamma_{dq}^{T}(\theta_d)\,\varepsilon_{dq}$$

premultiply both sides of eq with $\left[\Gamma_{dq}^{T}(\theta_d)\right]^{-1}$

$$v_{dq} = L\left[\Gamma_{dq}^{T}(\theta_d)\right]^{-1}\frac{d}{dt}\left(\Gamma_{dq}^{T}(\theta_d)\, i_{dq}\right) + R\,\underbrace{\left[\Gamma_{dq}^{T}(\theta_d)\right]^{-1}\cdot\Gamma_{dq}^{T}(\theta_d)}\, i_{dq}$$

$$+ \underbrace{\left[\Gamma_{dq}^{T}(\theta_d)\right]^{-1}\Gamma_{dq}^{T}(\theta_d)}_{= I}\,\varepsilon_{dq}$$

$$v_{dq} = \underbrace{L\left[\Gamma_{dq}^{T}(\theta_d)\right]^{-1}\frac{d}{dt}\left(\Gamma_{dq}^{T}(\theta_d)\, i_{dq}\right)}_{①} + R\, i_{dq} + \varepsilon_{dq}.$$

where, $\varepsilon_{dq} = \frac{2}{3}\Gamma_{dq}(\theta_d)\,\varepsilon_{abc}.$

focus on ①

$$L\left[\Gamma_{dq}^{T}(\theta_d)\right]^{-1}\frac{d}{dt}\left(\overbrace{\Gamma_{dq}^{T}(\theta_d)\, i_{dq}}\right)$$

$$L\left[\Gamma_{dq}^{T}(\theta_d)\right]^{-1}\cdot\left[\left\{\frac{d}{dt}\left(\Gamma_{dq}^{T}(\theta_d)\right)\right\} i_{dq} + \Gamma_{dq}^{T}(\theta_d)\frac{d}{dt}(i_{dq})\right]$$

$$= \underbrace{L\left[\Gamma_{dq}^{T}(\theta_d)\right]^{-1}\left\{\frac{d}{dt}\left[\Gamma_{dq}^{T}(\theta_d)\right]\right\}}_{②} i_{dq} + L\cdot\frac{d}{dt}(i_{dq})$$

②:– $L\left[\Gamma_{dq}^{T}(\theta_d)\right]^{-1}\left\{\underbrace{\frac{d}{dt}\left[\Gamma_{dq}^{T}(\theta_d)\right]}\right\} i_{dq}$

inductance

$$L \cdot \frac{2}{3}\Gamma_{dq}(\theta_d)$$

$$\underbrace{\begin{bmatrix} \cos\theta_d & \cos\left(\theta_d-\frac{2\pi}{3}\right) & \cos\left(\theta_d+\frac{2\pi}{3}\right) \\ -\sin\theta_d & -\sin\left(\theta_d-\frac{2\pi}{3}\right) & -\sin\left(\theta_d+\frac{2\pi}{3}\right) \end{bmatrix}}_{:=\ \Gamma_{dq}(\theta_d)}$$

$$\underbrace{\frac{d}{dt}\left(\Gamma_{dq}(\theta_d)\right)} = \begin{bmatrix} \frac{d}{dt}\cos(\theta_d) & \frac{d}{dt}\left(\cos\left(\theta_d-\frac{2\pi}{3}\right)\right) & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} -\sin(\theta_d)\,\frac{d\theta_d}{dt}\ (\to \omega_d) & -\sin\left(\theta_d-\frac{2\pi}{3}\right)\cdot\omega_d & -\sin\left(\theta_d-\frac{4\pi}{3}\right)\omega_d \\ -\cos(\theta_d)\,\omega_d & -\cos\left(\theta_d-\frac{2\pi}{3}\right)\omega_d & -\cos\left(\theta_d-\frac{4\pi}{3}\right)\omega_d \end{bmatrix}}$$

$$L\left\{\Gamma_{dq}^{T}(\theta_d)\right\}^{-1}\left\{\frac{d}{dt}\left(\Gamma_{dq}^{T}(\theta_d)\right)\right\} i_{dq}$$

$$= L\cdot\frac{2}{3}\Gamma_{dq}(\theta_d)\left[\quad\quad\right]\cdot i_{dq}$$

$$= L\ \frac{2}{3}\underbrace{\begin{bmatrix} \cos\theta_d & \cos\left(\theta_d-\frac{2\pi}{3}\right) & \cos\left(\theta_d+\frac{2\pi}{3}\right) \\ -\sin\theta_d & -\sin\left(\theta_d-\frac{2\pi}{3}\right) & -\sin\left(\theta_d+\frac{2\pi}{3}\right) \end{bmatrix}}\begin{bmatrix} -\sin(\theta_d) & -\cos(\theta_d) \\ -\sin\left(\theta_d-\frac{2\pi}{3}\right) & -\cos\left(\theta_d-\frac{2\pi}{3}\right) \\ -\sin\left(\theta_d-\frac{4\pi}{3}\right) & -\cos\left(\theta_d-\frac{4\pi}{3}\right) \end{bmatrix}\omega_d\, i_{dq}$$

$$\left[\frac{d}{dt}M^T = \left(\frac{dM}{dt}\right)^T\right]$$

$$= L\cdot\frac{2}{3}\begin{bmatrix} 0 & -3/2 \\ 3/2 & 0 \end{bmatrix}\omega_d\, i_{dq} = L\cdot\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\omega_d\, i_{dq}$$

$$v_{dq} = \underbrace{L\cdot\frac{d}{dt}i_{dq} + \overbrace{\boxed{L\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\omega_d\, i_{dq}}}^{②}}_{①} + \mathcal{E}_{dq} + R i_{dq}.$$

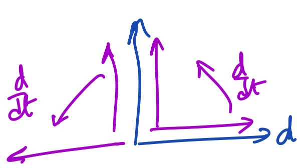
$$L\omega_d\cdot\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

$$\underbrace{\mathcal{E}_{dq} = \frac{2}{3}\Gamma_{dq}(\theta_d)\,\mathcal{E}_{abc}.}$$

$$\mathcal{E}_{dq} = \frac{2}{3}\Gamma_{dq}(\theta_d)\begin{bmatrix} \lambda_{pk}\omega_e\sin(\theta_e) \\ \lambda_{pk}\omega_e\sin\left(\theta_e-\frac{2\pi}{3}\right) \\ \lambda_{pk}\omega_e\sin\left(\theta_e-\frac{4\pi}{3}\right) \end{bmatrix}$$

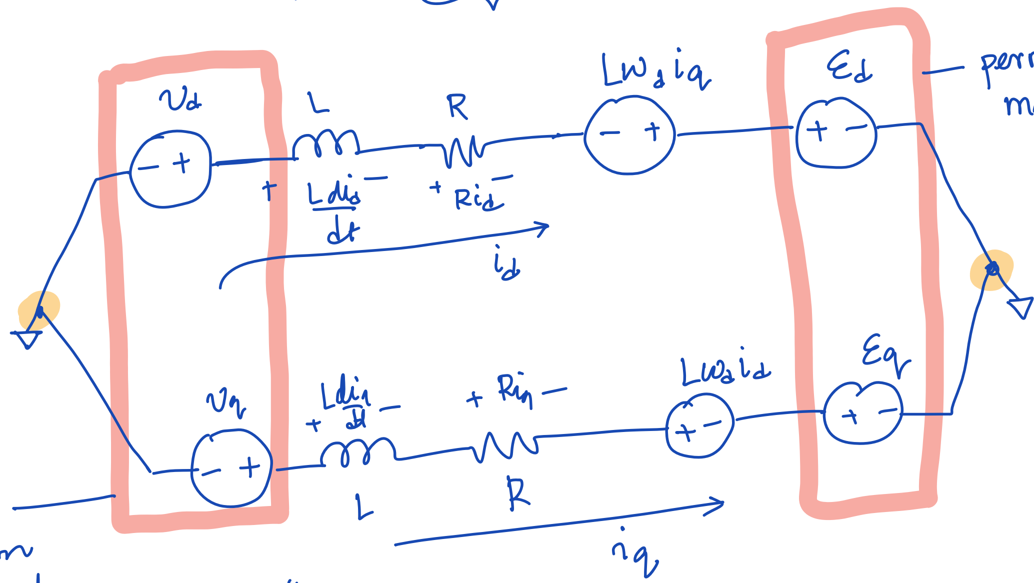
$$v_d = L\frac{di_d}{dt} - L\omega_d i_q + R i_d + \mathcal{E}_d$$

$$v_q = L\frac{di_q}{dt} + \underbrace{L\omega_d i_d}_{\text{cross-coupling term.}} + R i_q + \mathcal{E}_q$$



$$v_d = L\frac{di_d}{dt} - L\omega_d i_q + R i_d + \mathcal{E}_d$$

$$v_q = L\frac{di_q}{dt} + \underbrace{L\omega_d i_d} + R i_q + \mathcal{E}_q$$



permanent magnet causes this

You apply an external voltage "controllable"

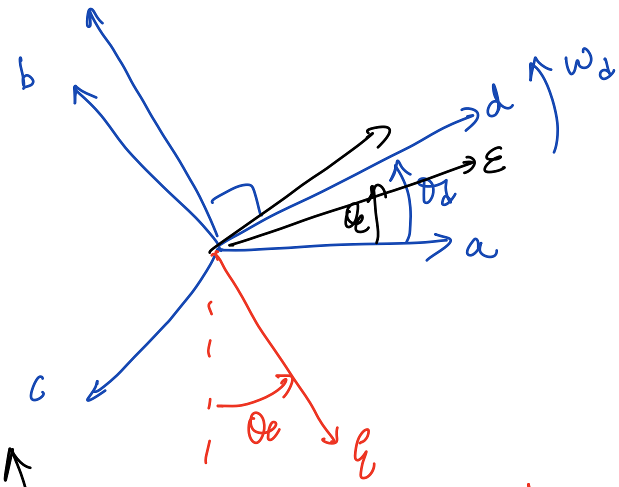
$$\boxed{i_d + i_q = 0}$$

$$i_a + i_b + i_c = 0$$

$$\boxed{\underbrace{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = 0} \rightarrow \Gamma(\theta_d)\, i_j$$

$$\mathcal{E}_{dq} = \frac{2}{3}\Gamma_{dq}(\theta_d)\underbrace{\begin{bmatrix} \lambda_{pk}\omega_e \sin(\theta_e) \\ \lambda_{pk}\omega_e \sin(\theta_e - \frac{2\pi}{3}) \\ \lambda_{pk}\omega_e \sin(\theta_e - \frac{4\pi}{3}) \end{bmatrix}}$$

$$= \frac{2}{3}\lambda_{pk}\omega_e \begin{bmatrix} \cos(\theta_d) & \cos(\theta_d - \frac{2\pi}{3}) & \cos(\theta_d - \frac{4\pi}{3}) \\ -\sin(\theta_d) & -\sin(\theta_d - \frac{2\pi}{3}) & -\sin(\theta_d - \frac{4\pi}{3}) \end{bmatrix} \cdot \underbrace{\begin{bmatrix} \sin(\theta_e) \\ \sin(\theta_e - \frac{2\pi}{3}) \\ \sin(\theta_e - \frac{4\pi}{3}) \end{bmatrix}}$$

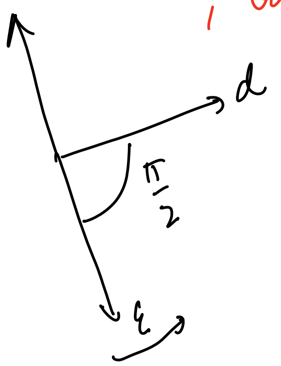


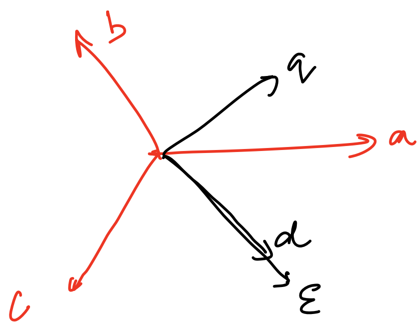
$$\boxed{\theta_d = \theta_e = \frac{\pi}{2}}$$

$$\boxed{\theta_d = \theta_e}$$

$$\dot{\theta}_d = \dot{\theta}_e$$

$$\omega_d = \omega_e$$

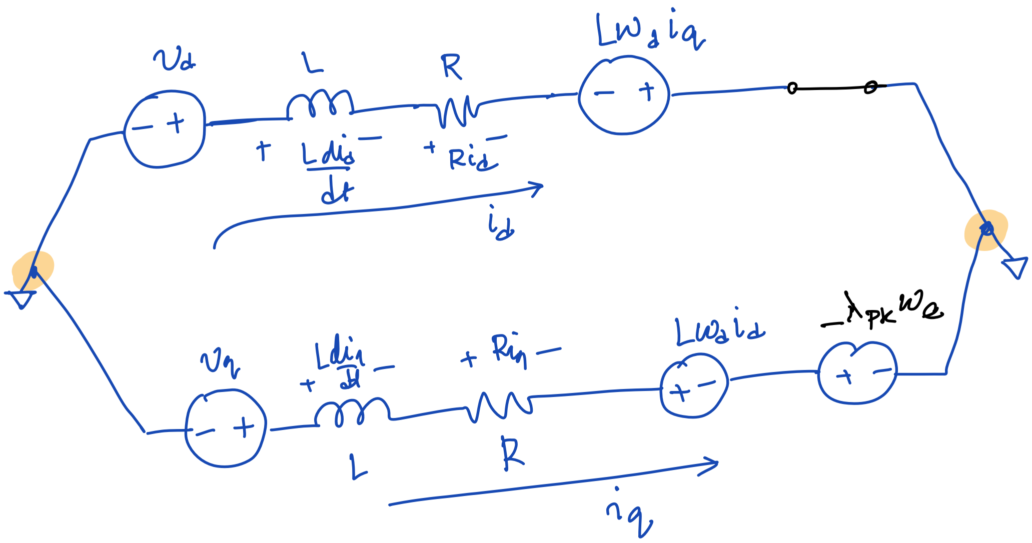




$$\begin{bmatrix}\varepsilon_d \\ \varepsilon_q\end{bmatrix} = \frac{2}{3}\lambda_{pk}\omega_e \begin{bmatrix} \cos(\theta_d) & \cos(\theta_d - \frac{2\pi}{3}) & \cos(\theta_d - \frac{4\pi}{3}) \\ -\sin(\theta_d) & -\sin(\theta_d - \frac{2\pi}{3}) & -\sin(\theta_d - \frac{4\pi}{3}) \end{bmatrix} \cdot \begin{bmatrix} \sin(\theta_e) \\ \sin(\theta_e - \frac{2\pi}{3}) \\ \sin(\theta_e - \frac{4\pi}{3}) \end{bmatrix}$$

$$\varepsilon_d = 0$$

$$\varepsilon_q = \frac{2}{3}\lambda_{pk}\cdot\omega_e\left(-\frac{3}{2}\right) = -\lambda_{pk}\omega_e$$



Power delivered to the motor

$$P = \underline{\varepsilon_a \cdot i_a + \varepsilon_b i_b + \varepsilon_c i_c}$$

$$\varepsilon_{abc} = \begin{bmatrix} \varepsilon_a \\ \varepsilon_b \\ \varepsilon_c \end{bmatrix}$$

$$= \varepsilon_{abc}^T \, i_{abc}$$

$$i_{abc} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$= \left(\Gamma_{dq}^T(\theta_d)\varepsilon_{dq}\right)^T \cdot \left(\Gamma_{dq}^T(\theta_d)\, i_{dq}\right)$$

$$\varepsilon^T_{abc} = [\varepsilon_a \ \ \varepsilon_b \ \ \varepsilon_c]$$

$$= \varepsilon_{dq}^T \left(\Gamma_{dq}^T(\theta_d)\right)^T \left(\Gamma_{dq}^T(\theta_d)\, i_{dq}\right)$$

$$v_{abc} = \Gamma_{dq}^T(\theta_d)\, v_{dq}$$

$$= \varepsilon_{dq}^T \cancel{\underbrace{\Gamma_{dq}(\theta_d)\,\Gamma_{dq}^T(\theta_d)}_{I}} \, i_{dq}$$

x

$$x_{dq} = \frac{2}{3}\Gamma_{dq}(\theta_d)\, x_{abc}$$

$$x_{abc} = \underbrace{\Gamma_{dq}^T}\, x_{dq}$$

$$= \left(\frac{2}{3}\Gamma_{dq}\right)^{-1}$$

$$\frac{3}{2}\Gamma_{dq} = \Gamma_{dq}^T$$

$$= \varepsilon_{dq}^T \cdot i_{dq} = [\varepsilon_d \ \ \varepsilon_q]\begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

$$= \underbrace{\varepsilon_d i_d + \varepsilon_q i_q}.$$

Correct derivation

$$x_{dq} = \frac{2}{3}\Gamma_{dq}(\theta)\, x_{abc}$$

$$x_{abc} = \left(\frac{2}{3}\Gamma_{dq}(\theta)\right)^{-1} x_{dq}$$

$$= \frac{3}{2}\Gamma_{dq}^{-1}(\theta)\, x_{dq}$$

$\vdots$ some math

$$= \Gamma_{dq}^{T}(\theta)\, x_{dq}.$$

$$\therefore \quad \frac{3}{2}\Gamma_{dq}^{-1}(\theta) = \Gamma_{dq}^{T}(\theta) \quad \Rightarrow \quad \boxed{\frac{3}{2} = \Gamma_{dq}(\theta)\Gamma_{dq}^{T}(\theta)}$$

power at machine output :-

$$P = \mathcal{E}_a i_a + \mathcal{E}_b i_b + \mathcal{E}_c i_c$$

$$= \mathcal{E}_{abc}^{T}\, i_{abc}$$

$$= \left(\Gamma_{dq}^{T}(\theta)\, \mathcal{E}_{dq}\right)^{T} \left(\Gamma_{dq}^{T}(\theta)\, i_{dq}\right)$$

$$= \mathcal{E}_{dq}^{T}\, \Gamma_{dq}(\theta) . \Gamma_{dq}^{T}(\theta)\, i_{dq}$$

$$= \frac{3}{2}\, \mathcal{E}_{dq}^{T}\, i_{dq}$$

$$= \frac{3}{2}\begin{bmatrix} \mathcal{E}_d & \mathcal{E}_q \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

$$= \frac{3}{2}\left(\mathcal{E}_d i_d + \mathcal{E}_q i_q\right)$$

$$P = \frac{3}{2}\left(\mathcal{E}_d i_d + \mathcal{E}_q i_q\right)$$

$\mathcal{E}_d = 0$

$\mathcal{E}_q = \frac{2}{3}\lambda_{pk} \cdot w_e \left(-\frac{3}{2}\right) = -\lambda_{pk} w_e$

$$P = \frac{3}{2}\left(0 \cdot i_d + (-\lambda_{pk} w_e)\, i_q\right)$$

$w_e = \dot{\theta}_e$

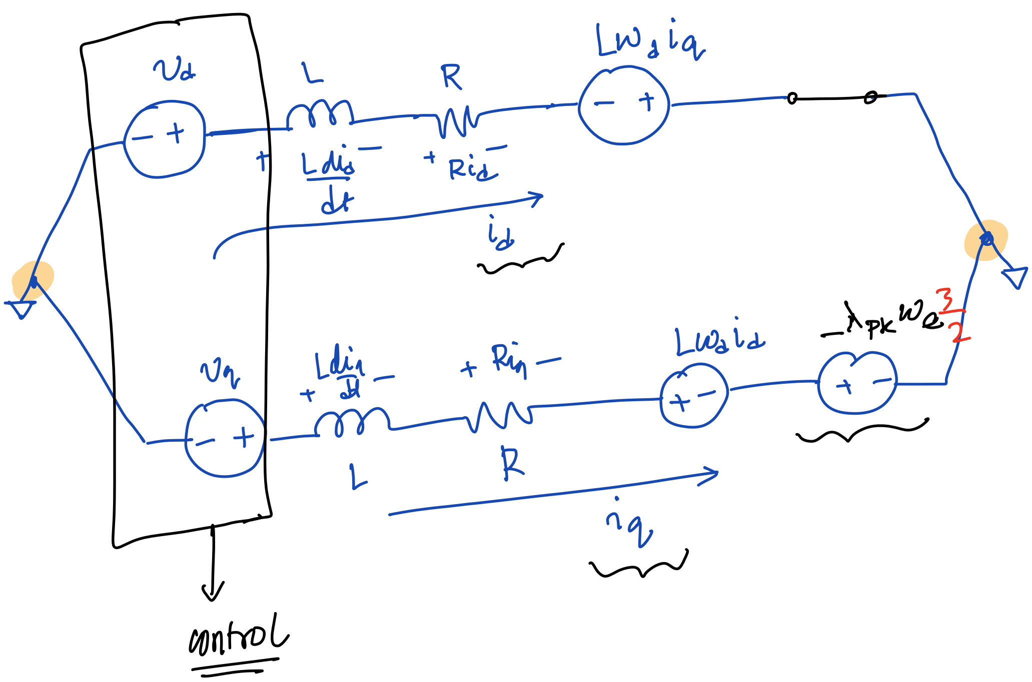
$$P = -\frac{3}{2}\lambda_{pk} \frac{P}{2} \cdot w_m i_q$$

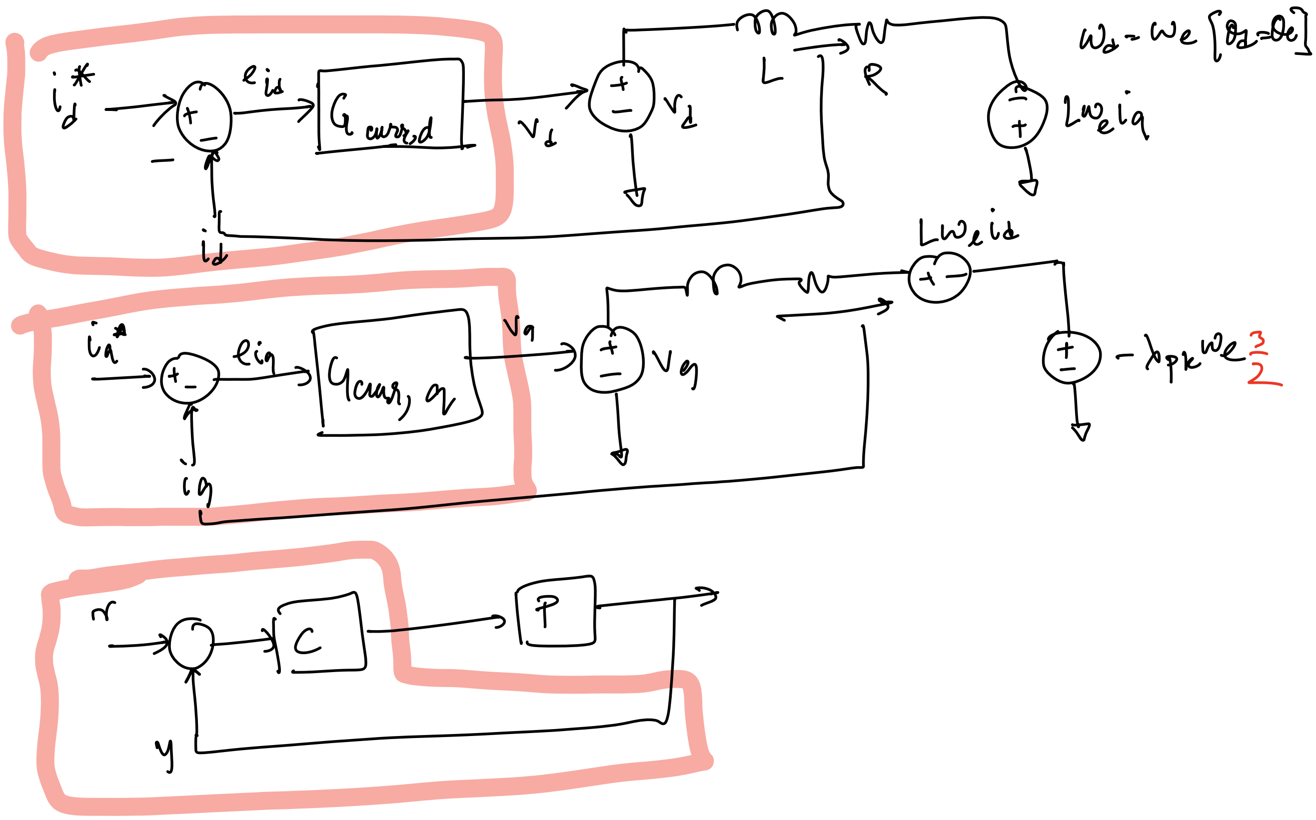
$\theta_e = \frac{P}{2}\theta_m$

$$\text{Torque} = \frac{P}{w_m} = -\lambda_{pk}\frac{3}{2}\cdot\frac{P}{2}\cdot i_q$$

$w_e = \frac{P}{2} w_m$

$$\boxed{\tau = -\lambda_{pk}\frac{3}{2}\cdot\frac{P}{2}\, i_q}$$ ✓

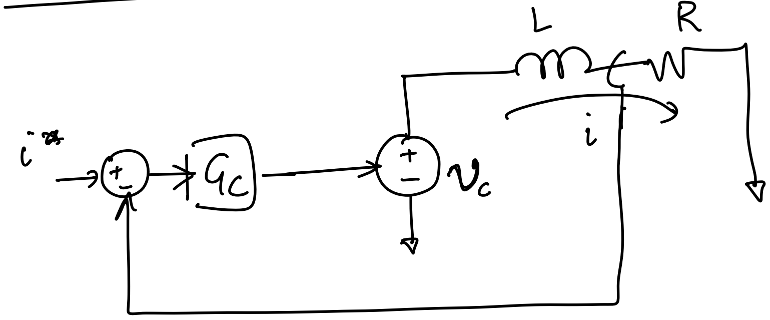




i_d*
e_id
G curr,d
v_d
L
R
Lω_e i_q
ω_d = ω_e [θ_d = θ_e]
i_d
Lω_e i_d
i_q*
e_iq
G curr, q
v_q
−λ_pk ω_e 3/2
r
C
P
y

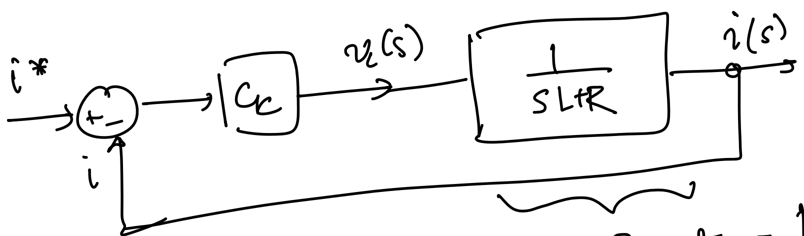
5:35

Simpler "toy" circuit



$$v_c = L\frac{di}{dt} + iR \Rightarrow v_c(s) = i(s)[sL+R]$$

$$i(s) = \frac{v_c(s)}{sL+R}$$



$$\text{Plant} = \boxed{P(s) = \frac{1}{sL+R}}.$$

Design

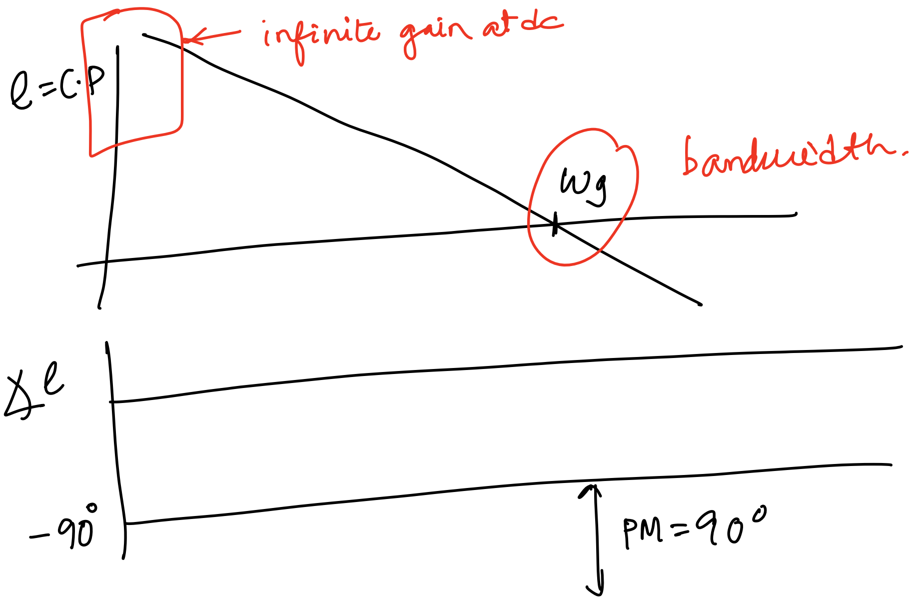
$G_c(s)$ such that

1. perfect DC tracking
2. 'w_n' some bandwidth.
3. PM > 60°

Internal model principle / plant inversion method

Goal! Make the loop gain $(\ell = C\cdot P) = \frac{\omega_g}{s}$.

Pick C such that $C\cdot P = \frac{\omega_g}{s}$.



$$C\cdot P = \frac{\omega_g}{s}.$$

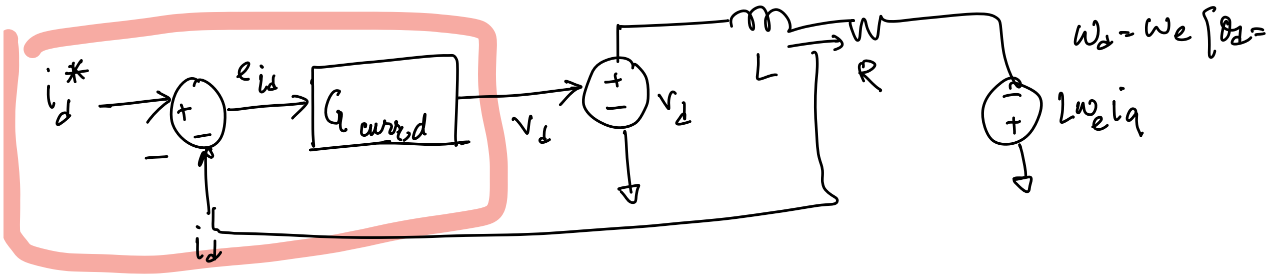
$$C = \frac{\omega_g}{s}\cdot\frac{1}{P}$$

$$= \frac{\omega_g}{s}\cdot\frac{1}{1/(sL+R)}$$

$$C(s) = \frac{\omega_g}{s}(sL+R)$$

$$C(s) = \omega_g L + \frac{\omega_g R}{s} \quad := k_p + \frac{k_i}{s}$$

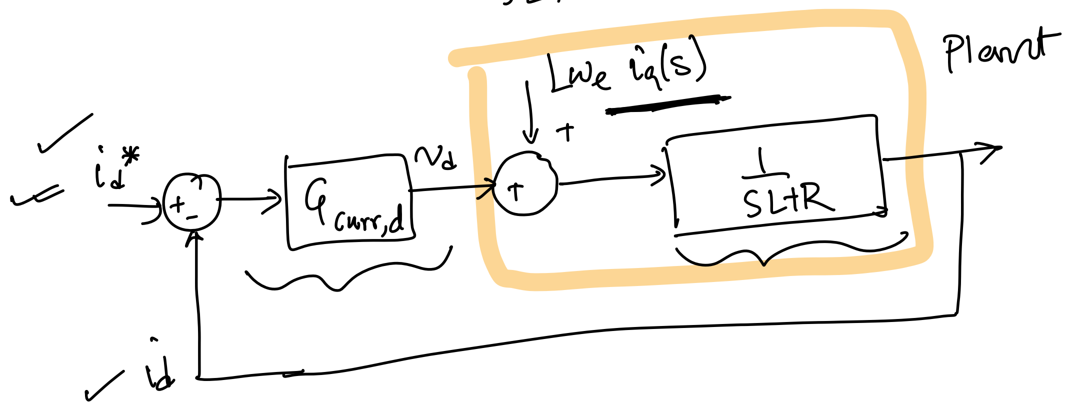
$$\boxed{\begin{aligned} k_p &= \omega_g L \\ k_i &= \omega_g R \end{aligned}}$$

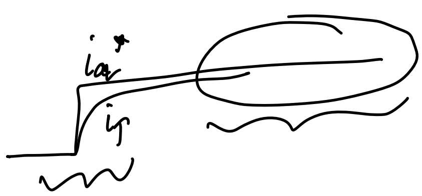


D axis controller

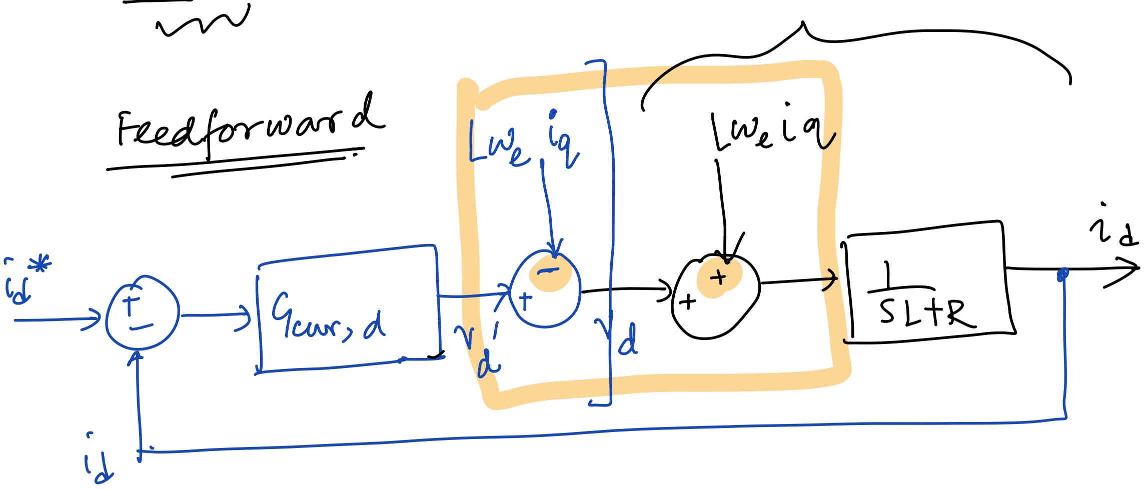
$$v_d(s) = (sL + R)\, i_d(s) - L\omega_e i_q(s)$$

$$i_d(s) = \frac{v_d(s) + L\omega_e i_q(s)}{sL + R}$$





Feedforward

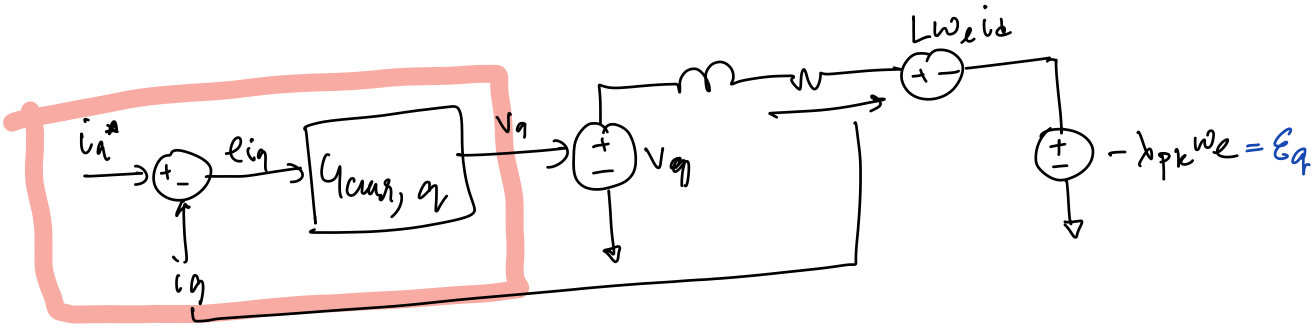


$$\underbrace{V_d' - L\omega_e i_q}_{} = V_d$$

$$i_d(s) = \frac{v_d(s) + L\omega_e i_q(s)}{sL+R}$$

$$i_d(s) = \frac{V_d'(s) - \cancel{L\omega_e i_q(s)} + \cancel{L\omega_e i_q(s)}}{sL+R}$$

$$i_d(s) = \frac{V_d'(s)}{sL+R}.$$

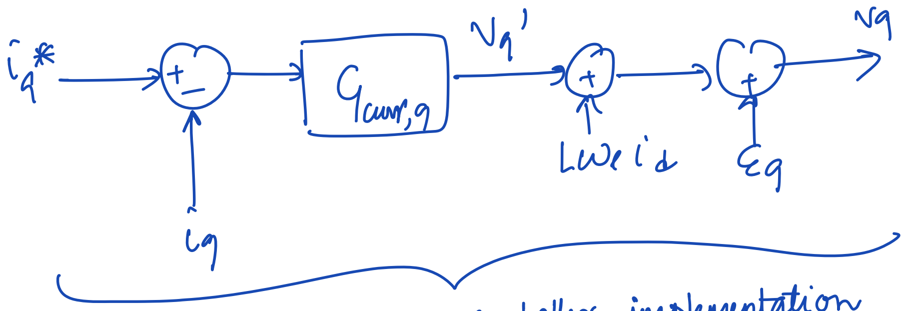


$$v_q = L\frac{di_q}{dt} + Ri_q + L\omega_e i_d + \mathcal{E}_q$$

$$i_q(s) = \frac{\overbrace{v_q - L\omega_e i_d - \mathcal{E}_q}}{sL+R}.$$

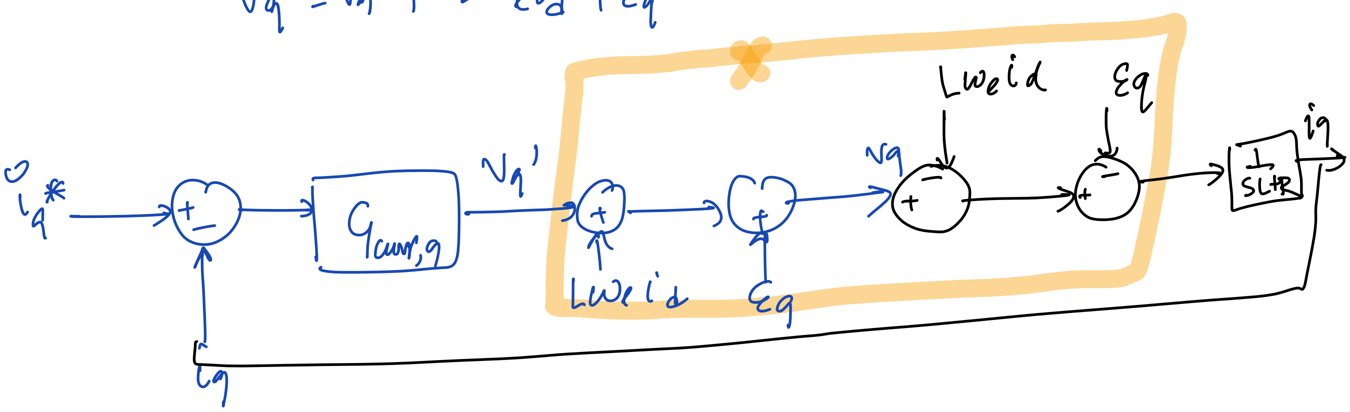
controller output $= \dfrac{(v_q' + L\omega_e i_d + \mathcal{E}_q) - L\omega_e i_d - \mathcal{E}_q}{sL+R}$

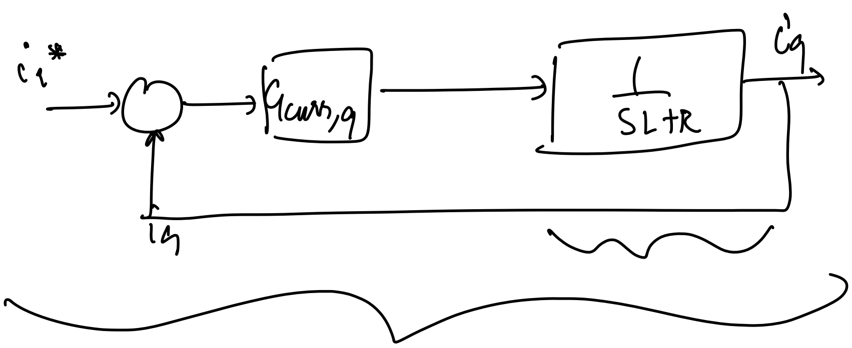
$$= \frac{v_q'}{sL+R}$$



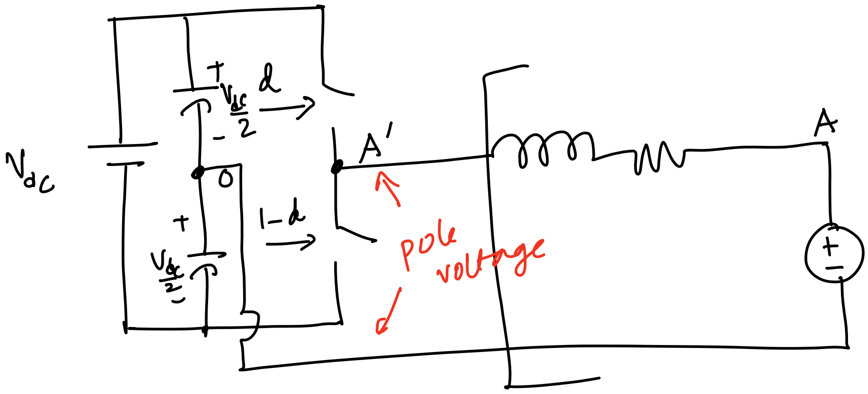
Controller implementation

$$V_q = V_q' + L\omega_e i_d + \varepsilon_q$$



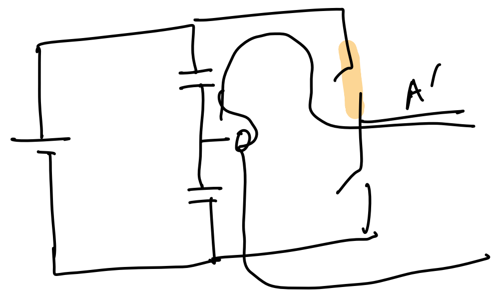


Inverter (half bridge / single leg)



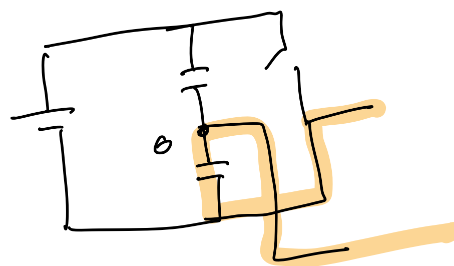
$$\langle v_{A'O}\rangle_{T_s} =$$

when $0 < t < T_{on}$



$$v_{A'O} = \frac{v_{DC}}{2}$$

$0 < t < T_{off}$



$$v_{A'O} = -\frac{v_{DC}}{2}$$

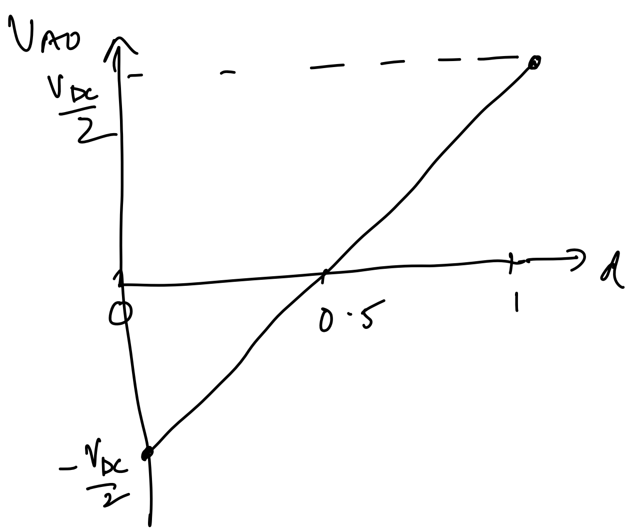
$$\langle v_{A'O}\rangle_{T_s} = \frac{v_{DC}}{2}\cdot d + \left(-\frac{v_{DC}}{2}\right)(1-d)$$

$$= \frac{v_{DC}}{2}(2d-1)$$

$d=0 \qquad \langle V_{A'O} \rangle_{T_S} = -\frac{V_{DC}}{2}$

$d=1 \qquad \langle V_{A'O} \rangle_{T_S} = \frac{V_{DC}}{2}$

$d=0.5 \qquad \langle V_{A'O} \rangle_{T_S} = 0$



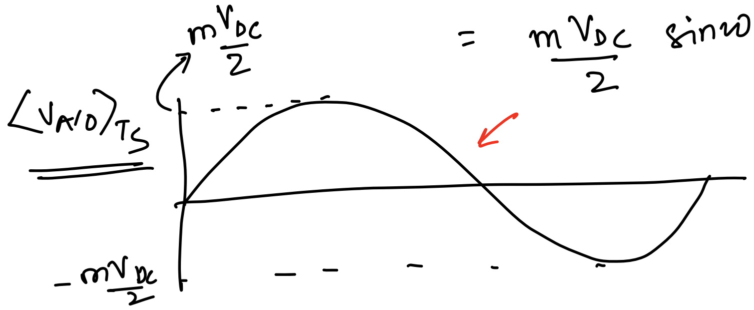
$d = 0.5 + 0.5\, m \sin\omega t$

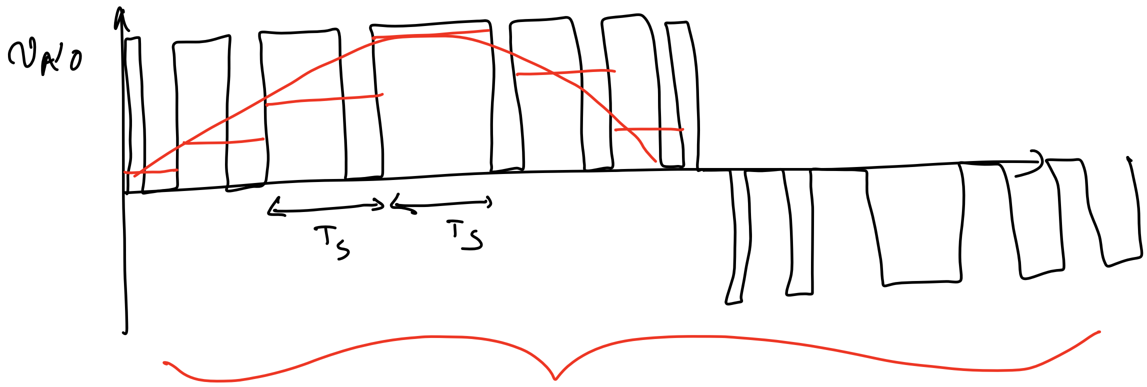
$$\langle v_{A'O} \rangle_{T_S} = \frac{v_{DC}}{2} \cdot (2d-1)$$

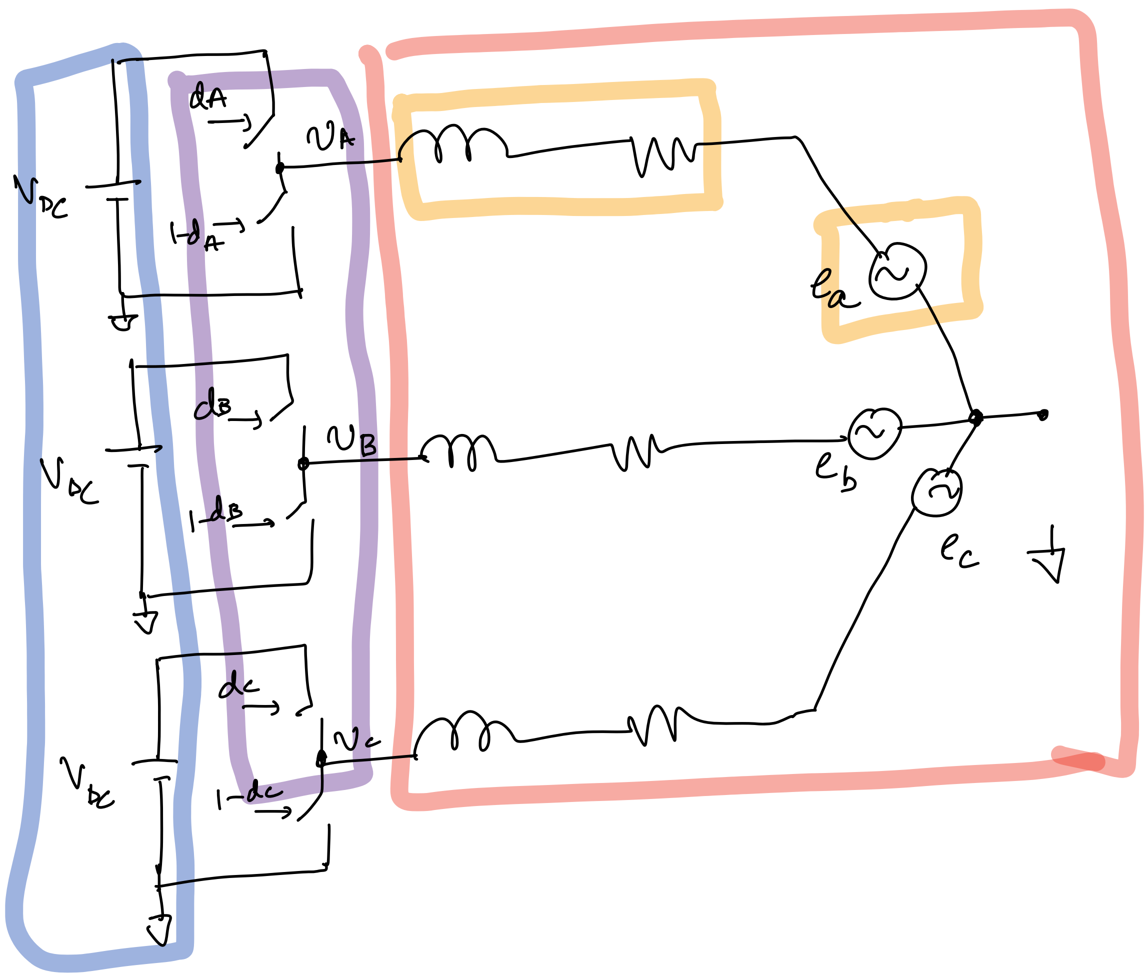
$$= \frac{v_{DC}}{2} \left(2\,(0.5 + 0.5 m \sin\omega t) - 1 \right)$$

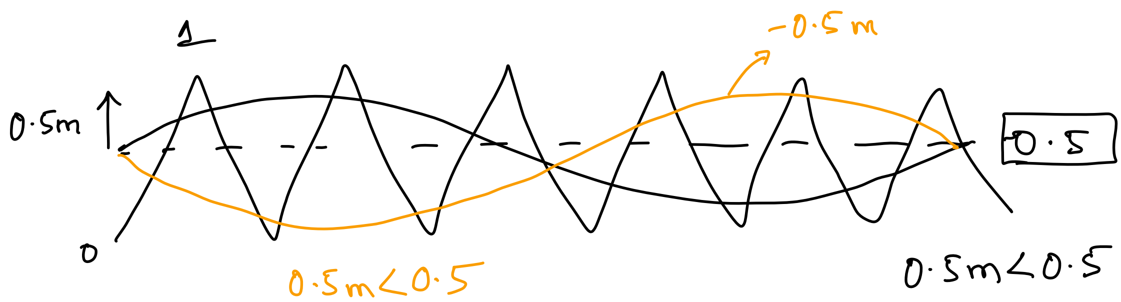
$$= \frac{v_{DC}}{2} \left(m \sin\omega t \right)$$

$$= \frac{m V_{DC}}{2} \sin\omega t$$









$0.5m < 0.5 \therefore \boxed{m < 1}$

$d_A \in [0, 1]$

$$\boxed{d_A = 0.5 + 0.5\, m \sin\omega t}$$

$\boxed{-1 < m < 1}$

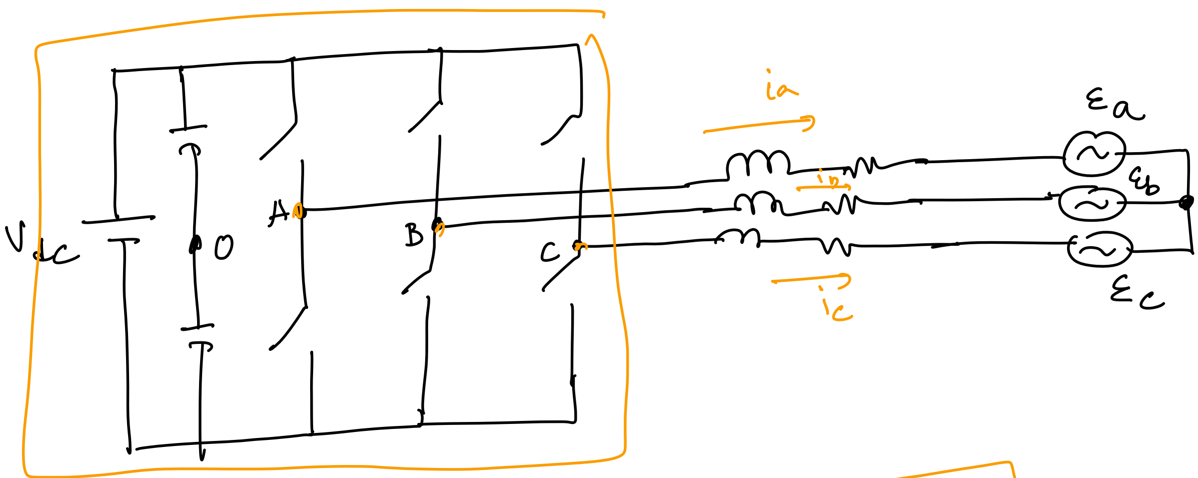
$$d_B = 0.5 + 0.5\, m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$0.5m < |.5|$

$$d_C = 0.5 + 0.5 m \sin\left(\omega t - \frac{4\pi}{3}\right)$$

$-.5 < 0.5m < .5$

$-1 < m < 1$



$$\langle v_{AO} \rangle_{T_s} = \frac{m V_{dc}}{2} \sin\omega t$$

$\boxed{-1 < m < 1}$

$$\langle v_{BO} \rangle_{T_s} = \frac{m V_{dc}}{2} \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$\langle v_{CO} \rangle_{T_s} = \frac{m V_{dc}}{2} \sin\left(\omega t - \frac{4\pi}{3}\right)$$

$$-\frac{V_{DC}}{2} < v_{AO} < \frac{V_{DC}}{2}$$


V_{dc}
dV_{dc}