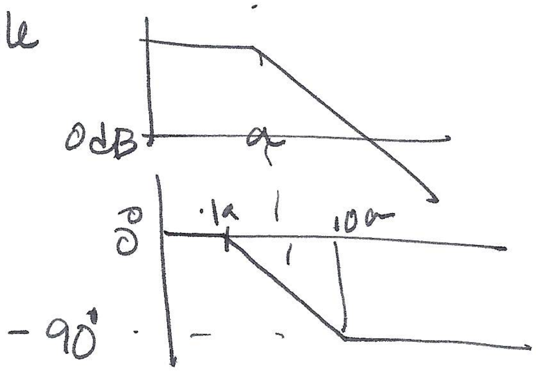
Problem 1

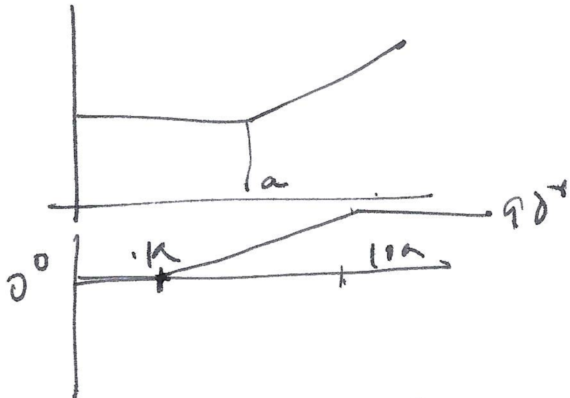
(1) LHP Pole



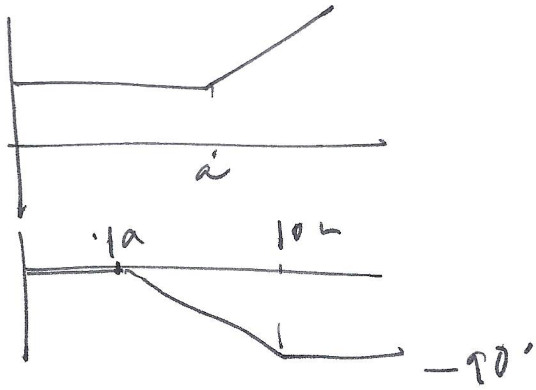
Minimum phase s/s

you can know phase plot from gain

LHP zero



RHP zero



$(s-a)$

$-\tan^{-1}\frac{j\omega}{a}$

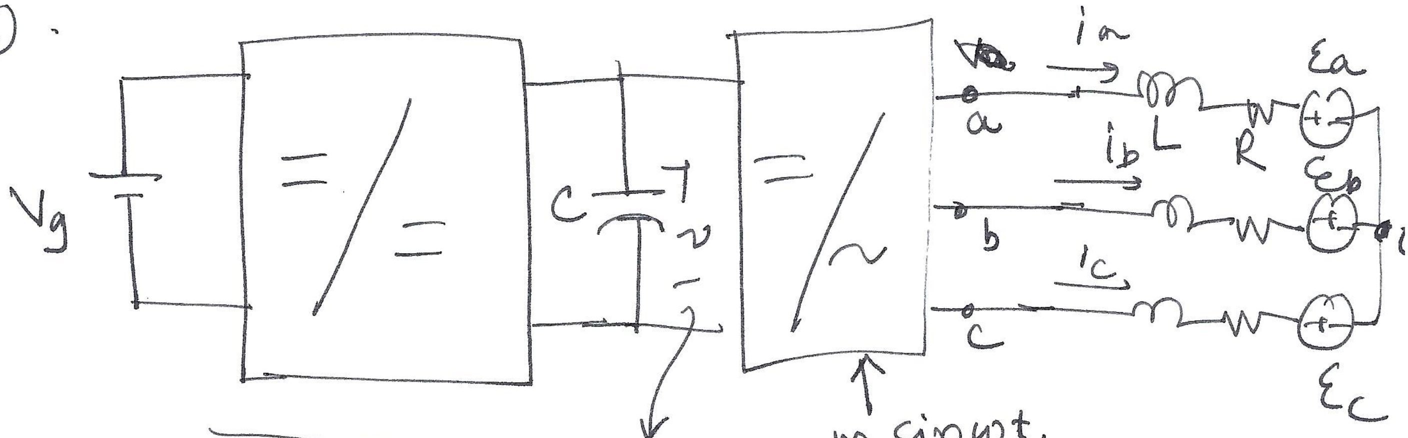
Same gain plot, diff phase plot.

for any <u>minimumphase</u> TF $G(s)$

$$\underbrace{\overbrace{\angle G(j\omega)}^{\text{phase}} = -\frac{\pi}{2}\cdot\frac{d\,\log_{10}\overbrace{|G(j\omega)|}^{\text{gain}}}{d\,\log_{10}\omega}}_{\text{one-to-one mapping for minimum phase s/s}} \quad \text{radian}$$

Problem 1

(2).



m sin ωt,
m sin(ωt − 2π/3)
m sin(ωt + 2π/3)

We know,

$$\tilde{v}(t) = G_{vd}\,\tilde{d}(t) + G_{vg}\,\tilde{v}_g(t)$$

$$\& \quad v_{ao}(t) = v(t)\, m \sin\omega t.$$

Let, $$\tilde{v}_g(t) = V_g + \underbrace{\mathring{v}_g \sin(\omega_d t)}_{\text{disturbance oscillation}}$$

$$\therefore v_{ao}(t) = \left[G_{vd}\,d(t) + G_{vg}\,v_g(t)\right] m \sin\omega t$$

we dont neeed to study this now ~ 0

$$v_{ao}(t) = G_{vg} \cdot (V_g + \mathring{v}_g \sin(\omega_d t))\, m \sin\omega t$$

$$= G_{vg} \cdot V_g \cdot m \sin\omega t + G_{vg} \cdot v_g\, m \cdot \sin\omega t \sin(\omega_d t)$$

$$= G_{vg} V_g\, m \sin\omega t + \frac{G_{vg}\,\mathring{v}_g\, m}{2}\left[\cos(\omega - \omega_d)t - \cos(\omega + \omega_d)t\right]$$

$G_{vg} \approx 1$.

$\therefore$ $v_{ao}(t)$

has a component we all know,

$v_g m \sin\omega t$

&

now has another disturbance component,

$$\frac{G_{vg} v_g m}{2}\left[\cos(\omega-\omega_d)t - \cos(\omega+\omega_d)t\right]$$

we know,

$$i = \frac{v_{ao}(t) - E_a(t)}{sL+R}$$

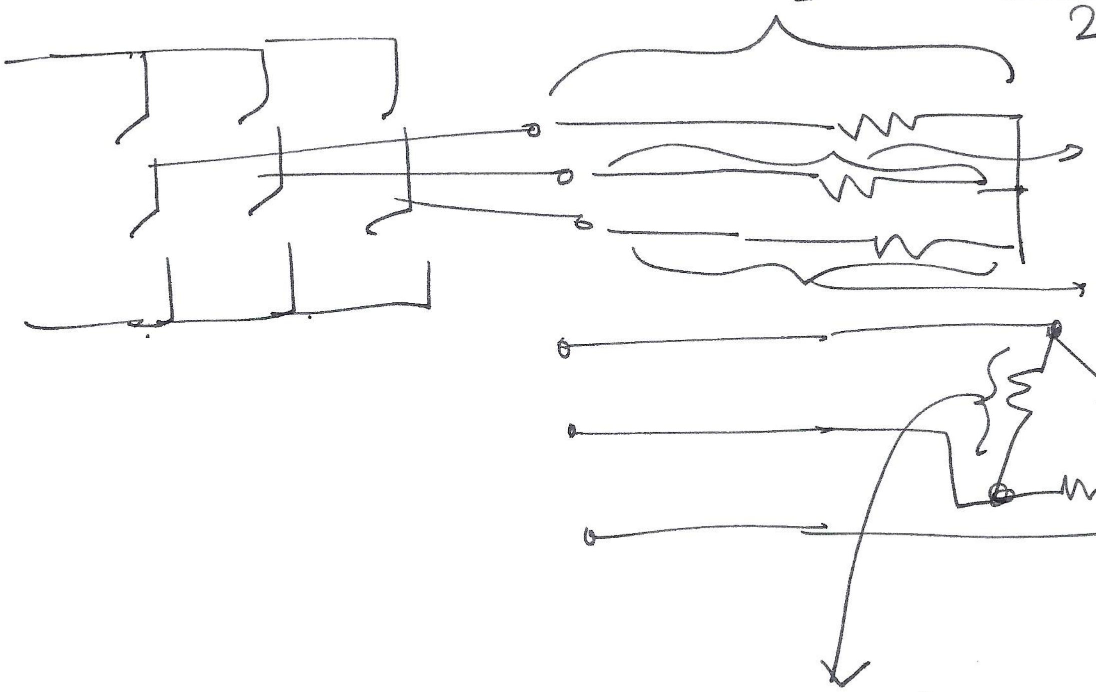
If the current controller loop gain,

$$l_i = \left(k_{p,m} + \frac{k_{i,m}}{s}\right)\frac{1}{sL+R}$$

has high gain at $(\omega-\omega_d)$ & $(\omega+\omega_d)$ frequency we can compensate disturbances at $v_g(t)$.

Problem (3).

$\frac{mV_{dc}}{2}\angle 0^\circ \rightarrow \frac{48 \times 0.9}{2}$



$\frac{mV_{dc}}{2}\angle -120^\circ$

$\frac{mV_{dc}}{2}\angle -240^\circ$

$$\frac{mV_{dc}}{2}\angle 0^\circ - \frac{mV_{dc}}{2}\angle -120^\circ$$

$$= \sqrt{3}\,\frac{mV_{dc}}{2}\angle 30^\circ$$

Ans

$$\text{Star} = \frac{48 \times 0.9}{2}$$

$$\text{Delta} = \frac{48 \times 0.9\sqrt{3}}{2}$$

$$C\frac{dv}{dt}\left(1+\frac{r_{esr}}{R}\right) = i - \frac{V}{R}$$

$$C\frac{dv}{dt} = i \cdot \frac{1}{1+\frac{r_{esr}}{R}} - \frac{V}{R} \cdot \frac{1}{1+\frac{r_{esR}}{R}}$$

$$L\frac{di}{dt} = V_{in} - i r_L - V - r_{esr} \cdot C\frac{dv}{dt}$$

$$= V_{in} - \hat{i} r_L - V - r_{esr} \cdot \left[i \cdot \frac{1}{1+\frac{r_{esr}}{R}} - \frac{V}{R}\frac{1}{1+}\right.$$

$$\dot{x} = Ax + Bu$$

$$\begin{bmatrix} \frac{di}{dt} \\ \frac{dv}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{r_L}{L} - \frac{r_{esr}}{L\left(1+\frac{r_{esr}}{R}\right)} & -\frac{1}{L} - \frac{1}{RL\left(1+\frac{r_{esr}}{R}\right)} \\ \frac{1}{C} \cdot \frac{1}{1+\frac{r_{esr}}{R}} & -\frac{1}{RC} \cdot \frac{1}{1+\frac{r_{esr}}{R}} \end{bmatrix} \begin{bmatrix} i \\ v \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & \frac{1}{L} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d \\ V_{in} \end{bmatrix}$$

$$\left(\frac{sK_{pv} + k_{iv}}{s}\right) \cdot \left(\frac{1}{1+s\tau_i}\right) \frac{1}{sC}.$$

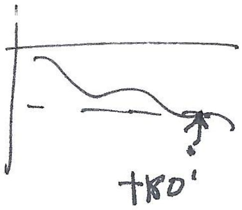
$s = j\omega$

$$l_v(j\omega) = \left(\frac{K_{iv} - j\omega K_{pv}}{j\omega}\right)\left(\frac{1}{1+j\omega\tau_i}\right)\frac{1}{j\omega C}$$

① at $\omega = \omega_g$ (gain cross over freq).

$$\left| l_v(j\omega) \right|_{\omega=\omega_g} = 1$$

$$\left| \left(\frac{k_{iv} - j\omega_g k_{pv}}{j\omega_g}\right)\left(\frac{1}{1+j\omega_g\tau_i}\right)\frac{1}{j\omega_g C} \right| = 1$$



$$\boxed{\frac{\sqrt{k_{iv}^2 + (\omega_g k_{pv})^2}}{\omega_g} \frac{1}{\sqrt{1+(\omega_g\tau_i)^2}} \frac{1}{\omega_g C} = 1} \quad ①$$

② at $\omega = \omega_g$ $\quad \measuredangle\, l_v(j\omega)\big|_{\omega=\omega_g} + 180° = PM°$

$+\tan^{-1}\frac{\omega k_{pv}}{k_{iv}} \leftarrow \frac{sk_{pv}+k_{iv}}{s}$ $\quad \frac{1}{1+s\tau_i} \quad \frac{1}{sC}$

$\frac{sk_{pv}+k_{iv}}{s} \to -90°$

$\frac{1}{1+s\tau_i} \to -\tan^{-1}\omega.\tau_i$

$\frac{1}{sC} \to -90°$

$$\boxed{\tan^{-1}\frac{\omega_g k_{pv}}{k_{iv}} - 90^\circ - \tan^{-1}\omega\tau_i - 90^\circ + 180^\circ = PM^\circ}$$

2 eq.

2 unknown - k_{pv}, k_{iv}

Use fsolve/ in MATLAB / python
solve.

A

Controller design. Alternate solution.

$$\frac{L_v}{1+L_v} = \frac{k_{pv} s + k_{iv}}{C\tau_i s^3 + C s^2 + k_{pv} s + k_{iv}}$$

$$= \frac{1 + \frac{k_{pv}}{k_{iv}} s}{1 + \frac{k_{pv}}{k_{iv}} s + \frac{C}{k_{iv}} s^2 + \frac{C\tau_i}{k_{iv}} s^3}$$

$$:= \frac{1 + a_1 s}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$

$$a_1 = \frac{k_{pv}}{k_{iv}} \qquad a_2 = \frac{C}{k_{iv}} \qquad a_3 = \frac{C\tau_i}{k_{iv}}$$

Look up Maksimovic, Erickson's book

$$1 + \tau_1 s + \tau_1\tau_2 s^2 + \tau_1\tau_2\tau_3 s^2$$

$$= 1 + (\tau_1 + \tau_2 + \tau_3)s + (\tau_1\tau_2 + \tau_2\tau_3 + \tau_1\tau_3)s^2 + \tau_1\tau_2\tau_3 s^3$$

if and only if $|\tau_1| \gg |\tau_2| \gg |\tau_3|$

For us, B

$$a_1 \approx \tau_1$$

$$a_2 \approx \tau_1 \tau_2$$

$$a_3 \approx \tau_1 \tau_2 \tau_3$$

$$\tau_1 = \frac{k_{pv}}{k_{iv}}$$

$$\tau_2 = \frac{a_2}{\tau_1} = \frac{c/k_{iv}}{k_{pv}/k_{iv}} = \frac{c}{k_{pv}}$$

$$\tau_3 = \frac{a_3}{\tau_1 \tau_2} = \tau_i$$

$\boxed{\text{If}}$ we assume, $\tau_1 \gg \tau_2 \gg \tau_3$

$$\frac{k_{pv}}{k_{iv}} \gg \frac{c}{k_{pv}} \gg \tau_i$$

So, $\frac{c}{k_{pv}} \gg \tau_i$

$$\therefore \boxed{k_{pv} \ll \frac{c}{\tau_i}} \rightarrow \boxed{M_1 \gg 1 \quad k_{pv} = \frac{c}{M_1 \tau_i}}$$

$$\frac{k_{pv}}{k_{iv}} \gg \frac{c}{k_{pv}} \quad \therefore \quad k_{iv} \ll \frac{k_{pv}^2}{c} \quad \boxed{k_{iv} = \frac{k_{pv}^2}{M_2 c}} \quad M_2$$

$M_1 = M_2 = 5$ chooses $k_{pv} = \frac{c}{5\tau_i}$; $k_{iv} = \frac{k_{pv}^3}{5c} = \frac{c}{5^3 \tau_i^2}$

So then,

$$\frac{l_v}{1+\lambda_v} = \frac{1+\cancel{a_1} s}{(1+\cancel{a_1 s})\left(1+\frac{a_2}{a_1}s\right)\left(1+\frac{a_3}{a_2}s\right)}$$

$$= \frac{1}{1+\left(\frac{a_2}{a_1}+\frac{a_3}{a_2}\right)s+\frac{a_3}{a_1}s^2}$$

should remind you of

$$= \frac{1}{1+\frac{2\zeta}{\omega_0}s+\frac{1}{\omega_0^2}s^2}.$$

$$\omega_0 = \sqrt{\frac{a_1}{a_3}} = \sqrt{\frac{K_{pv}}{c\,\tau_i}}$$

& since, we know, $K_{pv} = \frac{c}{M_1 \tau_i}$

$$\therefore \omega_0 = \frac{1}{\tau_i}\sqrt{\frac{1}{M_1}}.$$

& Damping $= \zeta = \frac{\omega_1}{2}\left(\frac{a_2}{a_1}+\frac{a_3}{a_2}\right)$

$$\zeta = \frac{1}{2}\sqrt{\frac{K_{pv}}{c\,\tau_i}}\left(\frac{c}{K_{pv}}+\tau_i\right) \Rightarrow$$

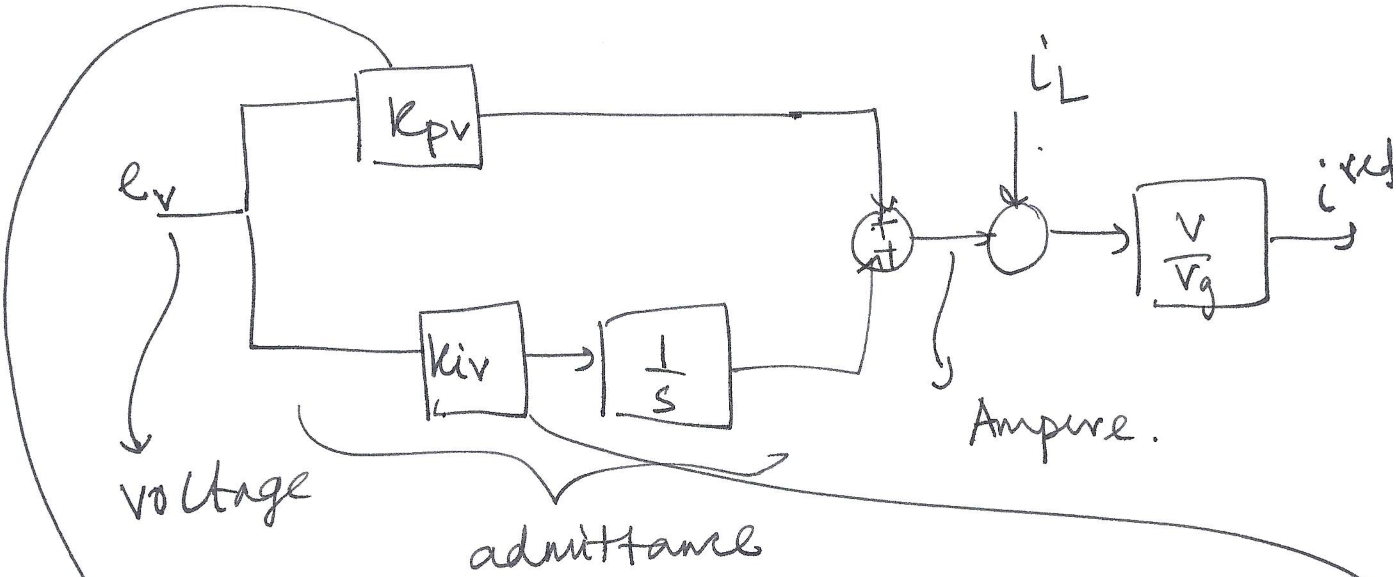
D

$$\rho^2 = \frac{k_p v}{4 c \tau_i}\left(\frac{c^2}{k_p v^2} + \tau_i^2 + \frac{2 c \tau_i}{k_p v}\right)$$

$$= \frac{c}{4 \tau_i k_p v} + \frac{\tau_i k_p v}{4c} + \frac{1}{2}$$

$$= \frac{M_1}{4} + \frac{1}{4 M_1} + \frac{1}{2}$$

or, $$\rho = \sqrt{\frac{1}{4}\left(M_1 + \frac{1}{M_1}\right) + \frac{1}{2}}$$



k_{pv} := ohm^{-1} or mho or siemens (Ω^{-1} / ℧ / S)

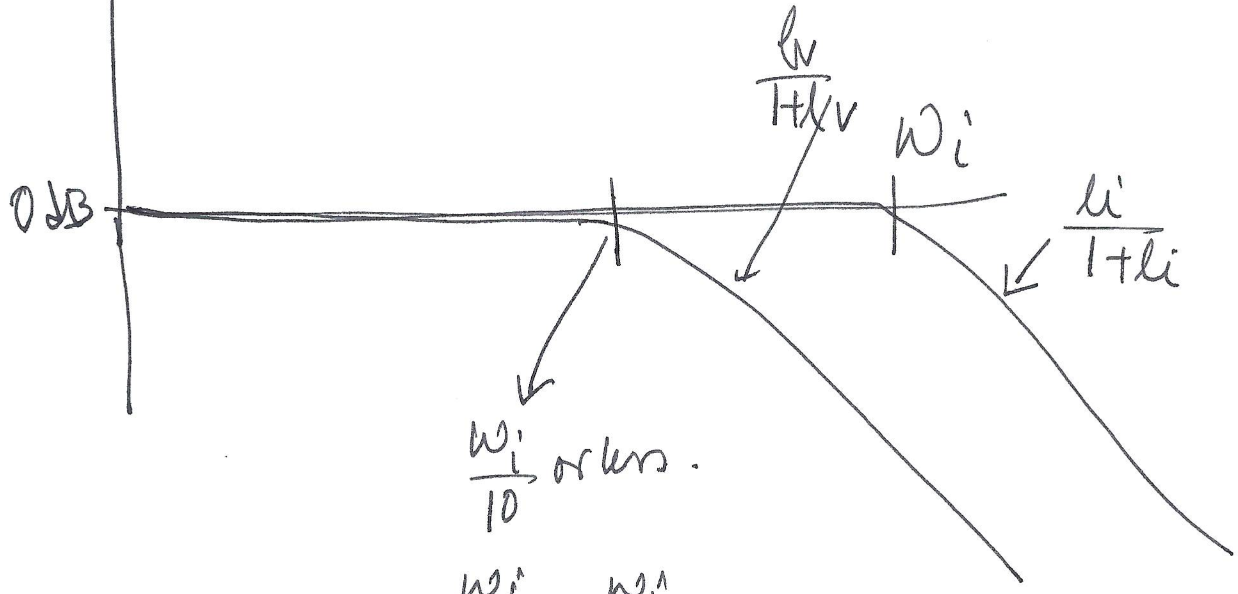
K_{iv} := Admittance = $\frac{k_{iv}}{s}$

Impedance = $\frac{s}{k_{iv}} \equiv s.(L)$

$\therefore \frac{1}{k_{iv}} \equiv L$

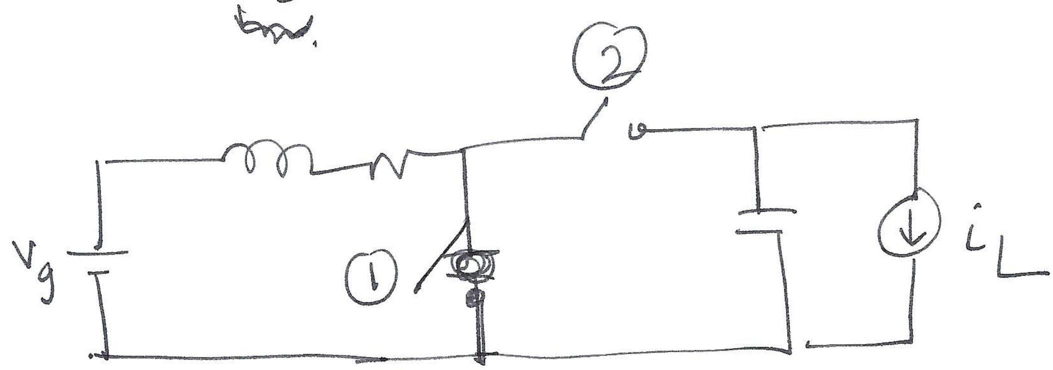
$k_{iv} = L^{-1}$ (Henry)$^{-1}$.

CLTF

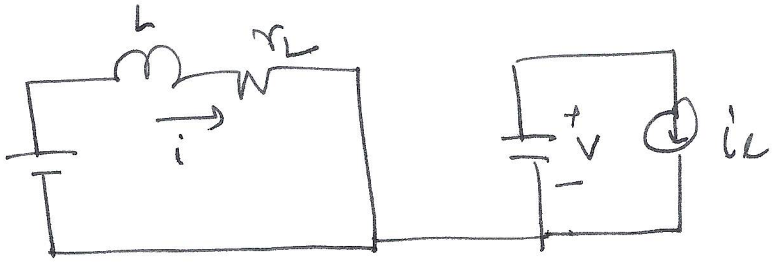


$\frac{\omega_i}{3}$, $\frac{\omega_i}{5}$ too.

2 (b)

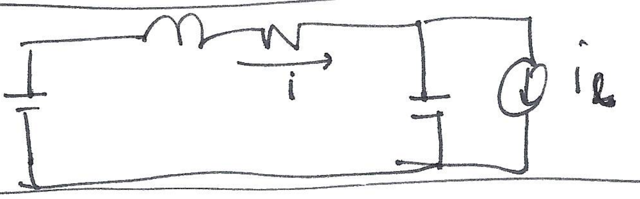


ON



$$V_{in} = L\frac{di}{dt} + i r_L \qquad C\frac{dv}{dt} = -i_R \qquad (1)$$

OFF



$$V_{in} = L\frac{di}{dt} + i r_L + V \qquad i = C\frac{dv}{dt} + i_R \qquad (2)$$

At steady state,

$$\left(L\frac{di}{dt}\right)_{1} \cdot D + \left(L\frac{di}{dt}\right)_{2} D' = 0$$

$$(V_{in} - i r_L) D + (V_{in} - i r_L - V) D' = 0$$

$$\Rightarrow \boxed{V_{in} - i r_L = V \cdot D'} \quad (3)$$

$$\left(C\frac{dv}{dt}\right)_{1} D + \left(C\frac{dv}{dt}\right)_{2} \cdot D' = 0$$

$$(-i_L) D + (i - i_L) D' = 0$$

$$i D' = i_L \qquad \boxed{i = \frac{i_L}{D'}} \quad (4)$$

~~Subst (4) in (3).~~

$$\cancel{V_{in} - \frac{i_L}{D'} r_L = V \cdot D'}$$

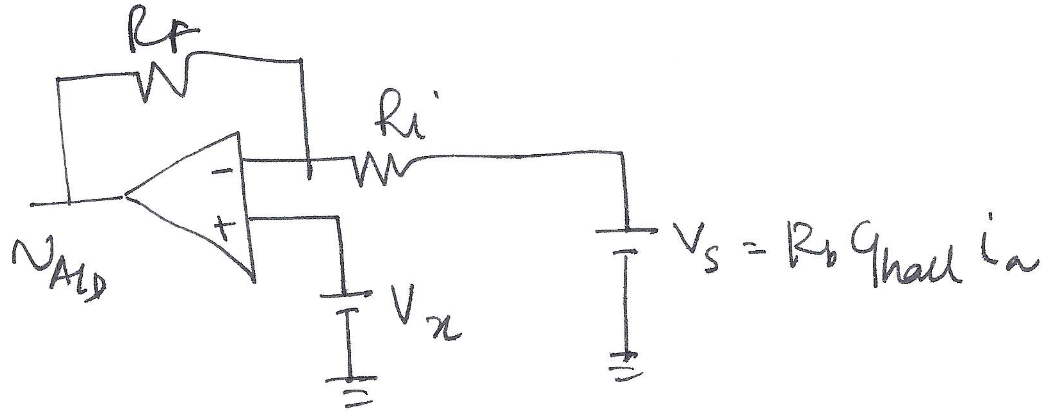
Subst (3) in (4).

$$i = \frac{i_L}{\frac{V_{in} - i r_L}{V}} = \underbrace{i \cdot (V_{in} - i r_L) = V \cdot i_L}_{\text{quadratic}}$$

$$\boxed{i^2 r_L - i \cdot V_{in} + V \cdot i_L = 0.}$$

$V_{in} \to$ 24 V, $V \to$ Assume 48 V, $i_L \to$ 2A

solve $\to$ obtain i, Use (4) to obtain D'



$$v_{A/D} = v_x\left(1+\frac{R_f}{R_i}\right) - \frac{R_f}{R_i}\cdot R_b\, q_{hall}\, i_a$$

@ $i_a = 0$, $\quad v_{A/D} = V_{FS}/2$

$$\boxed{v_x\cdot\left(1+\frac{R_f}{R_i}\right) = \frac{\overset{3}{V_{FS}}}{2}} \quad — ①$$

$v_{A/D} = \varepsilon \quad$ if $\quad i_a = i_{pk}$

$$\varepsilon = \underbrace{v_x\left(1+\frac{R_f}{R_i}\right)}_{\text{from ①} \; = V_{FS}/2} - \frac{R_f}{R_i}\cdot R_b\cdot q_{hall}\cdot i_{pk}$$

$$\underset{\downarrow \atop .1}{\varepsilon} = \underset{\downarrow \atop 3/2}{\frac{V_{FS}}{2}} - \underset{\downarrow \atop ?}{\frac{R_f}{R_i}}\cdot \underset{\downarrow \atop 75}{R_b}\cdot \underset{\downarrow \atop \frac{1}{1000}}{q_{hall}}\cdot \underset{\downarrow \atop 40}{i_{pk}}$$

$R_f = \frac{R_f}{R_i}\cdot 100\,k\Omega$

$? \longrightarrow v_x = \dfrac{V_{FS}/2}{1+R_f/R_i}$

or,

$\cancel{i_a}$

$$i_a \simeq -\underbrace{m \frac{R_f}{R_i} R_b G_{hall} \frac{2^{12}}{3}}_{=1} i_a + \underbrace{m v_x \left(1+\frac{R_f}{R_i}\right) \frac{2^{12}}{3} + b}_{=0}$$

$$\therefore b = -m v_x \left(1+\frac{R_f}{R_i}\right) \frac{2^{12}}{3}$$

$$\&, \quad m = -\frac{1}{\frac{R_f}{R_i} R_b G_{hall} \frac{2^{12}}{3}} = -\frac{V_{FS}}{\frac{R_f}{R_i} R_b G_{hall} 2^{n}}$$

$$b = +\left(+\frac{1}{\frac{R_f}{R_i} R_b G_{hall} \cancel{\frac{2^{12}}{3}}}\right) \cdot v_x \left(1+\frac{R_f}{R_i}\right) \cancel{\frac{2^{12}}{3}}$$

$$= \frac{v_x \left(1+R_f/R_i\right)}{\frac{R_f}{R_i} R_b G_{hall}}$$

$$P_e = -\frac{3}{2}\lambda_{pk}\,\omega_e\, i_q$$

$$\boxed{\tau_e = -\frac{3}{2}\underbrace{\lambda_{pk}}_{\text{const}}\ \underbrace{i_q}_{\text{const}}\ \frac{P}{2}}$$

$$J\dot{\omega}_m + B\omega_m = \tau_e - \cancelto{0}{\tau_\ell}$$

$$\dot{\omega}_m = \frac{\overbrace{\tau_e}^{\text{constant}} - B\omega_m}{J}.$$

$$\begin{cases} J_1\dot{\omega} + B\omega = \tau \\ J_2\dot{\omega} + B\omega = \tau \end{cases}$$

$\cancel{J\omega = \text{constant}}$

$$(sJ + B)\,\omega(s) = \tau(s)$$

$$\omega(s) = \frac{\tau(s)}{sJ+B}.$$

for step change in i_q, we get step change in torque $\therefore\ \tau(s) = \frac{T_e}{s}$ $\quad\therefore\ \omega(s) = \frac{T_e}{s(sJ+B)}$

Final value theorem

given, $y(s)$

$$\underset{t\to\infty}{Lt}\ y(\infty) = \underset{s\to 0}{Lt}\ s.y(s)$$

$$\omega(\infty) = \underset{t\to\infty}{Lt}\ \omega(t) = \underset{s\to 0}{Lt}\ s.\omega(s)$$

step input for torque

$$= .s.\ \frac{T_e}{s}.\frac{1}{sJ+B} = \frac{T_e}{B}.$$

proves that irrespective of J, the final speed is same.

Initial rate of change.

$$\frac{d\omega}{dt} \rightarrow s\,\omega(s).$$

Initial value theorem

given $y(s)$

$$\boxed{\underset{t\to 0}{Lt}\ y(0) = \underset{s\to\infty}{Lt}\ s\,y(s)}$$

$$\left.\frac{d\omega}{dt}\right|_{t=0} = \underset{t\to 0}{Lt}\ \frac{d}{dt}\omega(t) = \underset{s\to\infty}{Lt}\ s.\frac{T_e}{sJ+B}$$

$$= \underset{s\to\infty}{Lt}\ \frac{T_e}{J+B/s} = \frac{T_e}{J}$$

$\therefore$ $\boxed{\text{initial slope} = \frac{T_e}{J}}$ higher J, lower slope

$$\omega(s) = \frac{\tau(s)}{sJ+B}$$

step change in torque

$$\omega(s) = \frac{T_e}{s(sJ+B)}$$

$$\therefore \frac{d\omega}{dt} \rightarrow s\,\omega(s) = \frac{T_e}{sJ+B}$$

(13)

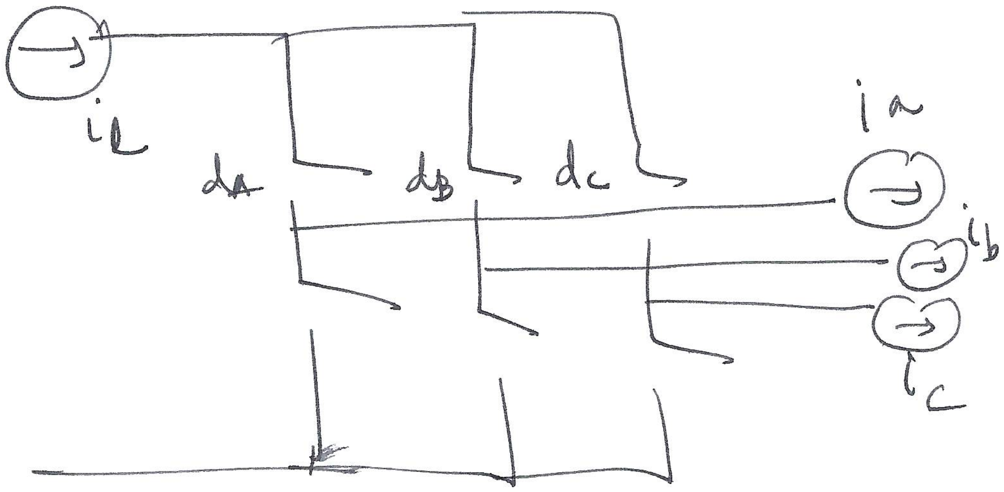
Prob 4

(a)

Initial value :-

$\boxed{i_L = 0}$;

(b)



when $d_A = 1$, $i_L = i_a$ (top switch ON)

$d_A = 0$ $i_L = 0$ (bot switch ON)

$d_B = 1$, $i_L = i_b$

$d_B = 0$, $i_L = 0$

$d_C = 1$, $i_L = i_C$

$d_C = 0$ $i_L = 0$

Superposition

$$\underbrace{i_L = d_A i_a + d_B i_b + d_C i_C}_{\text{saves a sensor !!}}$$

K.

$\boxed{4c}$

$$\longrightarrow \underbrace{\boxed{\dfrac{v}{v_g}}}_{\text{sensed value}} \longrightarrow$$

vs

$$\longrightarrow \boxed{2} \longrightarrow$$

does not have dynamics of v, v_g

so, "quieter" — less noisy.