EE 458/ 533 – Power Electronics Controls, Winter 2022
Midterm – Cover Page
Due: Sunday February 6th 2022, 11:59 pm Pacific Time

Name: ____________________________

Instructions.

- Show all your work and clearly indicate your final answer for each problem.
- Open-book, use any resource you want.
- Submit your solution and all the PLECS/ Simulink models and Matlab/ Python scripts you might have used. Comment the script as well as simulation models to help us evaluate what you have tried to do.
- While overlapping waveforms and recording bode plots, ensure the axis, legends and other relevant details are clear. Provide zoomed in waveforms in addition to the normal waveform to show the crossover frequencies and tracking / disturbance rejection etc.
- Please do not discuss this exam with anyone else until submission.
- There is a choice between Prob 3 (e) and (f). Do only one. You *might* get extra credit if you solve both.

Problem 1: Conceptual Questions [10 Points Total]
For each of the statements below, state if they are true or false.
If true, provide justification, if false provide a counter-example or a logical explanation. No marks will be rewarded for only true or false identification without proper reasoning.

1. For the same PWM clock frequency as I keep increasing the number of bits available for PWM (assume $N_r = 2^{\text{no of PWM bits}}$), both switching frequency and quantization of duty ratio increases.

2. For a synchronous boost converter in Fig. 1, for unidirectional power transfer (power flows from v_g to R), the MOSFET used for storing energy in L suffers both switching and conduction loss, whereas the MOSFET used for transfering energy from L to C suffers only conduction loss.

3. For an input signal $x(t)$,

   $$x(t) = a_1 \sin(2\pi\, 900t) + a_2 \sin(2\pi\, 1001t) + a_3 \sin(2\pi\, 2004t),$$

   sampled at a frequency of 2000 Hz, the reconstructed signal will have frequencies of the following three components: $1, 4, 900$ Hz.

4. For a minimum phase system (system with no right half zero, right half pole or delay) a *sufficient* condition of stability is that the system's gain cross over frequency has a value smaller than the system's phase cross over frequency.

5. For a buck converter, the input voltage-to-duty ratio transfer function can be described by the following system of differential equation,

   $$LC\frac{d^2 v_g(t)}{dt^2} + \frac{L}{R}\frac{dv_g(t)}{dt} + v_g(t) = Dd(t), \tag{1}$$

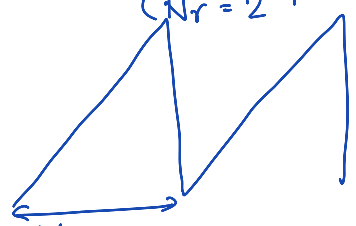
   where $d(t)$ is the duty ratio input and $v_g(t)$ is the input voltage.

   We can claim that if the duty ratio input, $d(t)$ is a function with magnitude of 1 at t = 0 and zero elsewhere (in literature we call this the delta dirac function), the same system can be described as, $\frac{dv_g(t)}{dt}|_{t=0} = D$.

Problem 1

① FALSE
Let us choose asymmetrical trailing edge carrier.

$(N_r = 2^{N_{pwm}} - 1)$



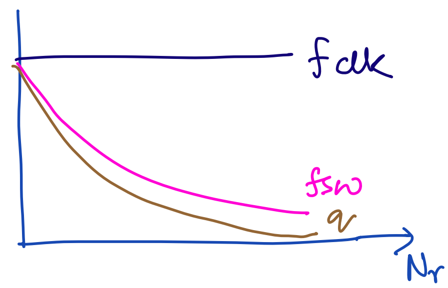
$1/f_{sw}$

$T_{sw} = N_r \cdot T_{clk}$

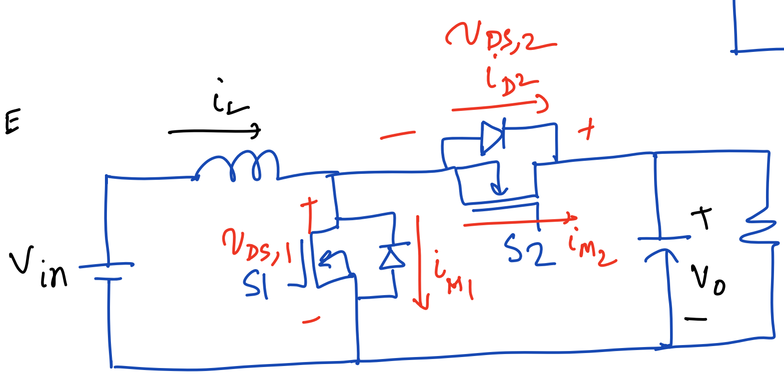
$\therefore \; f_{sw} = \dfrac{f_{clk}}{N_r}$ as $N_r \uparrow$, f_{clk} fixed

$\therefore \; f_{sw} \downarrow$

$q = \dfrac{1}{N_r}$. As $N_r \uparrow$, $q \downarrow$



② TRUE

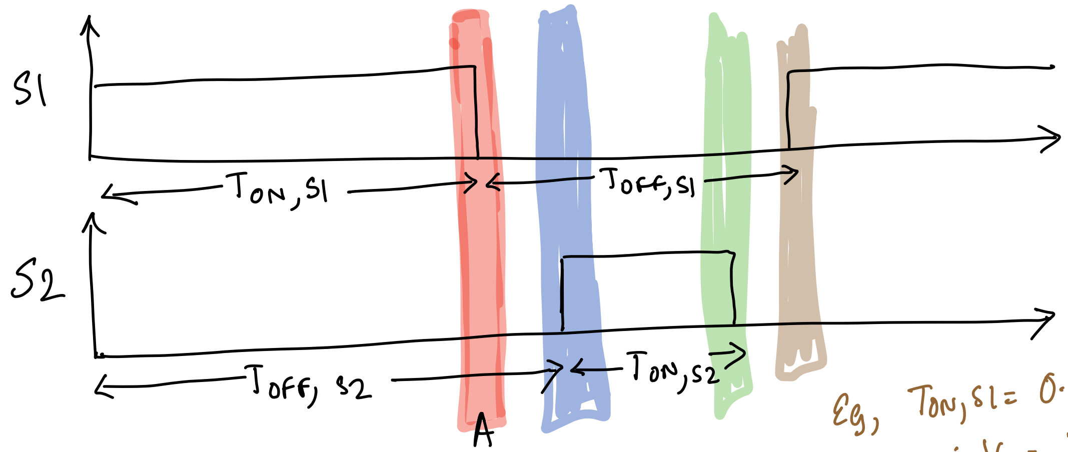


$(V_o > V_{in})$

S1 has turn ON & turn OFF losses & conduction losses
S2 only has conduction loss

i_M = mosfet current
i_D = diode current

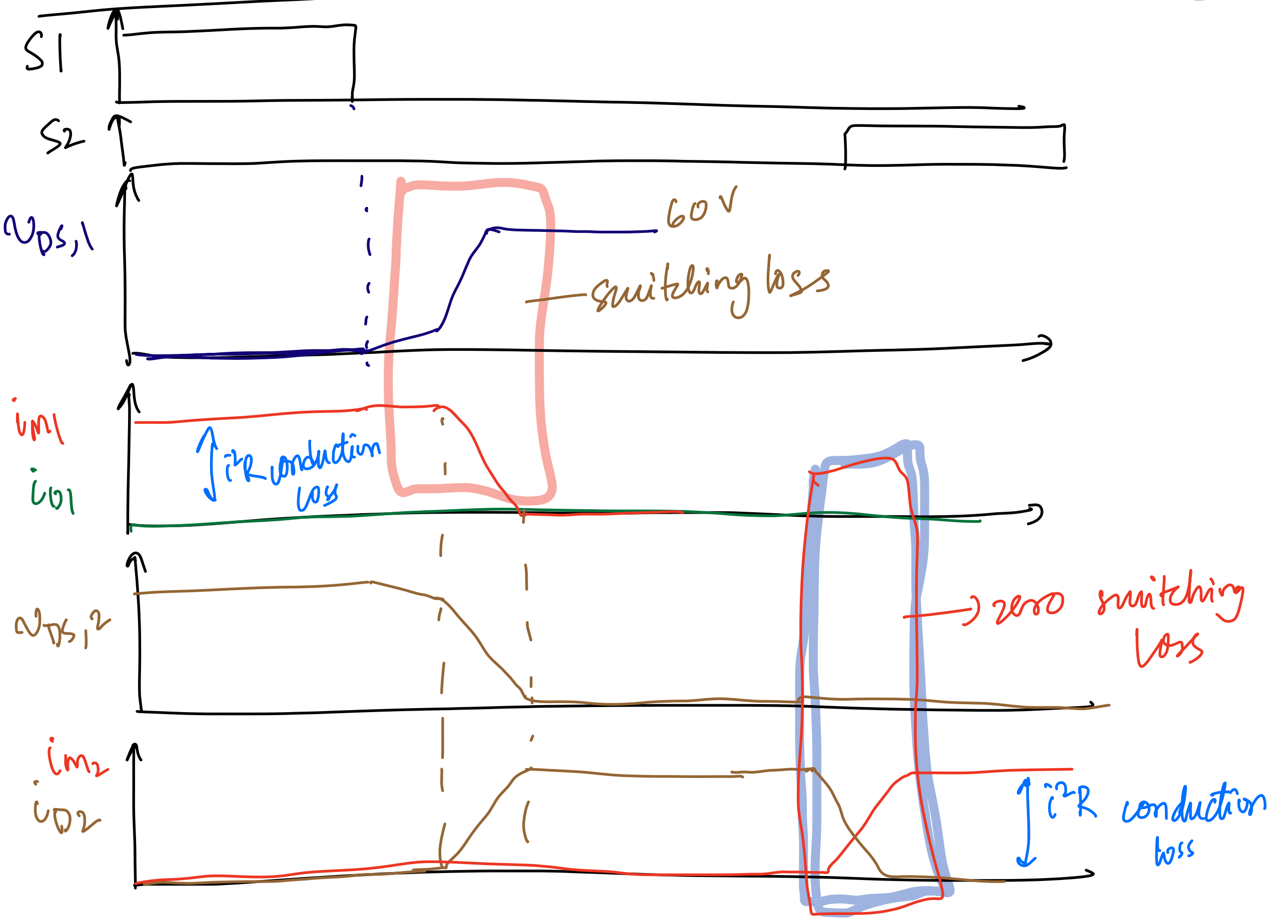
v_{DS} = voltage across mosfet/diode.

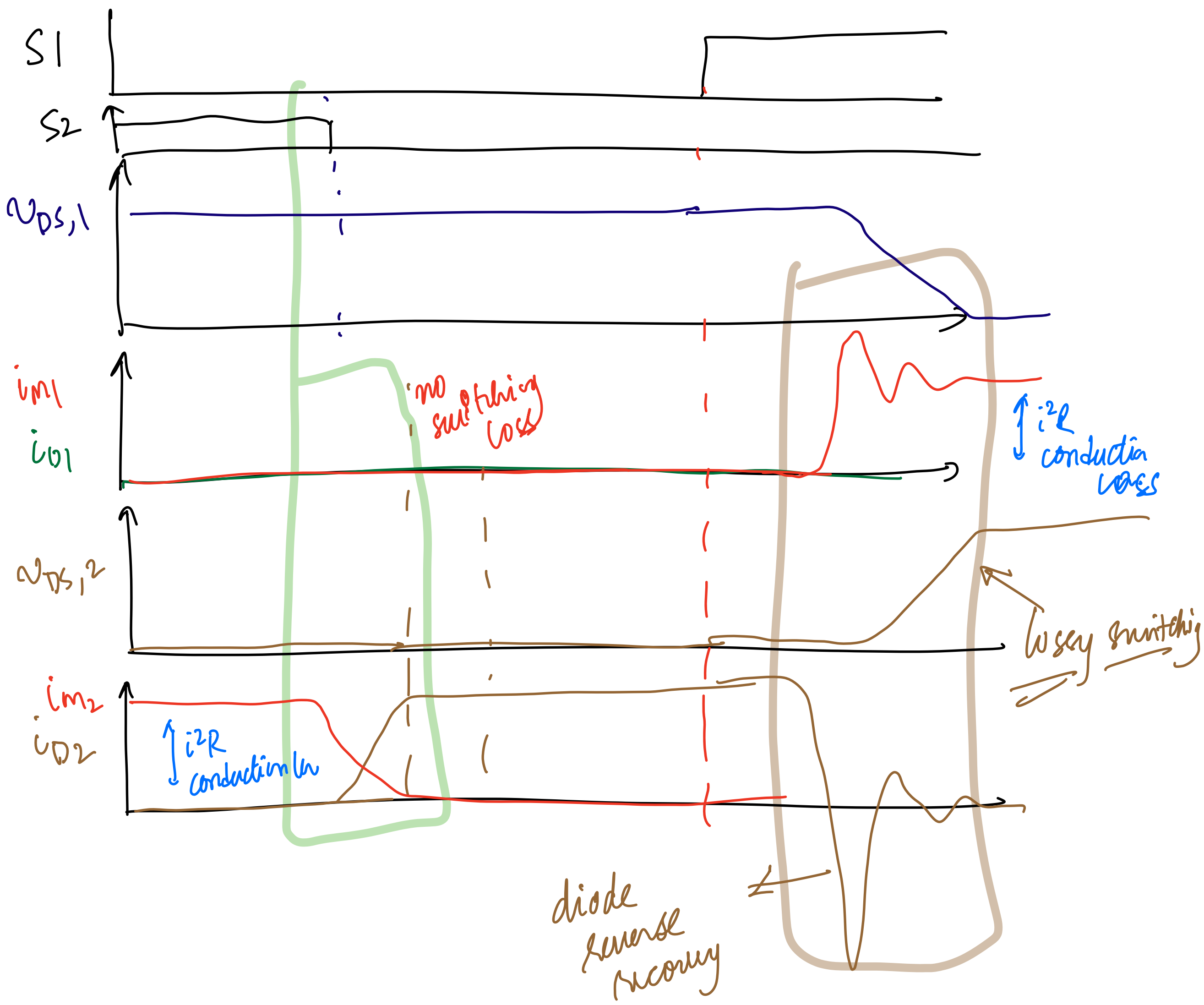


Eg, $T_{ON,S1} = 0.6$

$\therefore V_o = \frac{V_{in}}{1-.6} = \frac{240}{4} = 60V$

Focus on transition A

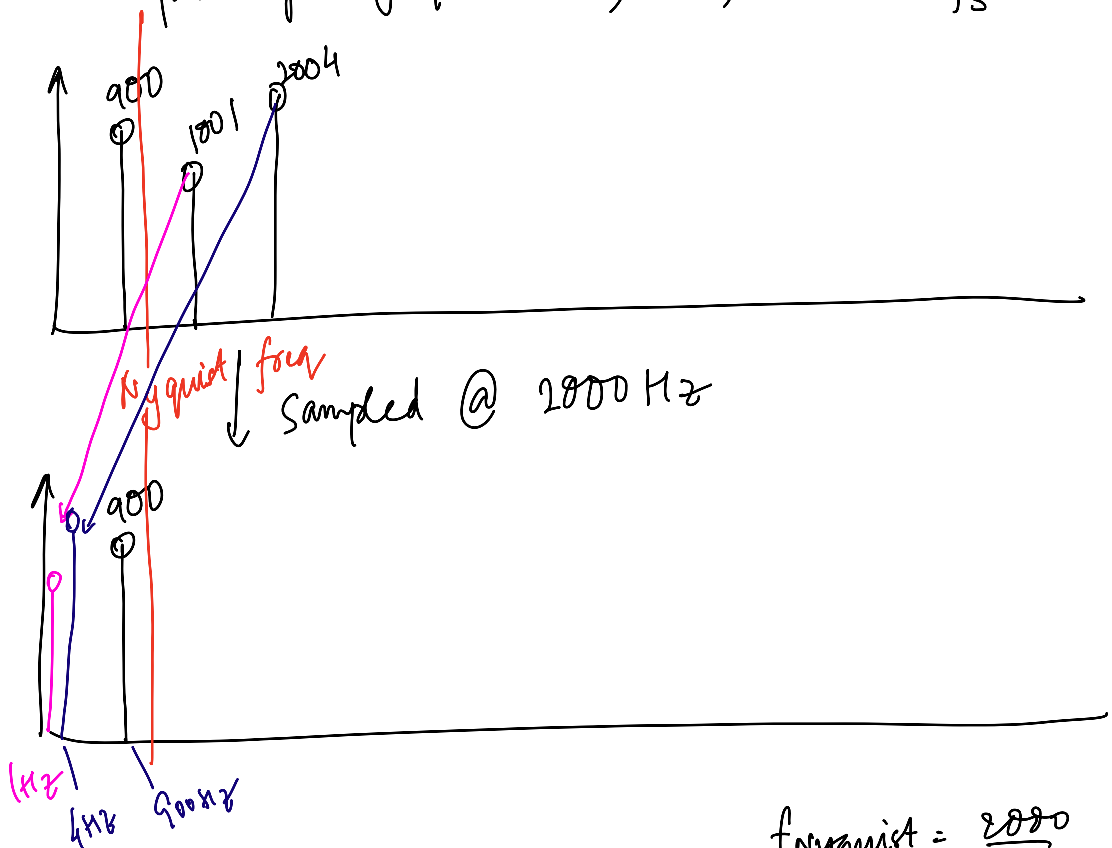




S1
S2
v_DS,1
i_M1
i_D1
no switching loss
i²R conduction losses
v_DS,2
lossy switching
i_M2
i_D2
i²R conduction loss
diode reverse recovery

(3) TRUE

Input signal freq = 900, 1001, 2004 .; $f_s = 2000\,Hz$



$f_{Nyquist} = \frac{2000}{2} = 1000\,Hz$

$900\,Hz < f_{Nyquist}$

$f_1 = 1001\,Hz - k \cdot f_{Nyquist}$ $\quad k = 0, 1, \ldots$

(a) $k = 0$ $\quad f_1 = 1001\,Hz$; is $0 < f_1 < f_{Nyquist}$? NO

(b) $k = 1$ $\quad f_1 = 1001 - 1 \times f_{Nyquist} = 1\,Hz$

$0 < f_1 < f_{Nyquist}$? YES

$\therefore$ 1 Hz

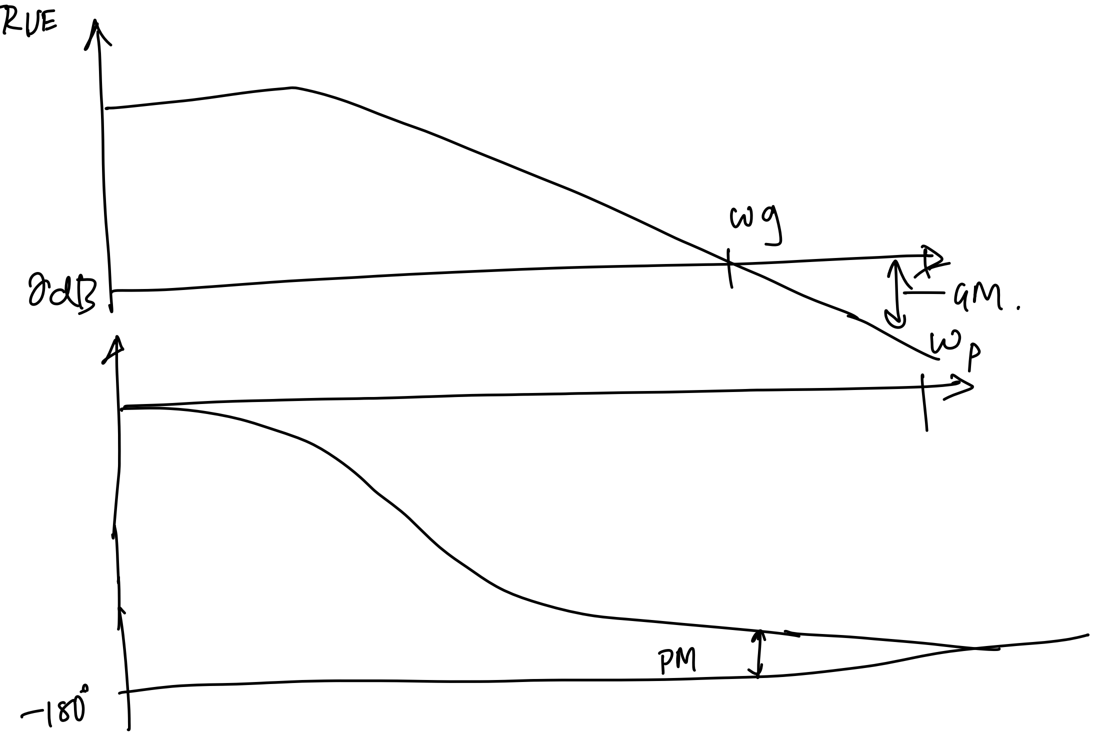
$f_2 = 2004\,Hz - k \cdot f_{Nyquist}$

(a) $k=0$ $\quad f_2 = 2004\,Hz \rightarrow$ No

$k=1$ $\quad f_2 = 2004 - 1000 = 1004 \rightarrow$ No.

$k=2$ $\quad f_2 = 2004 - 2000 = 4\,Hz \rightarrow$ Yes.

④ TRUE



"Hand waving proof"

Absence of RHP / RHZ means phase & gain plot have an unique relationship.

$$\angle L(j\omega) = -\frac{\pi}{2} K \frac{d}{d\omega}\left(\log_{10}|L(j\omega)|\right) \text{ rad.} \qquad K = \text{constant}$$

So, a falling gain plot ⇒ falling phase plot.

So there is a "monotonic" relationship between them.

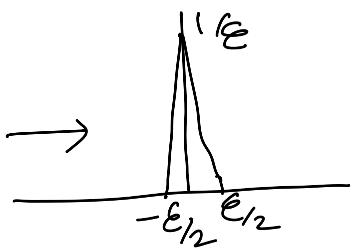
This means. if $\omega_g < \omega_p$, PM & GM are positive.

Hence, stable system.

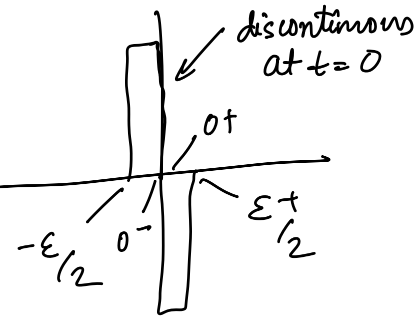
(e) FALSE

$$LC\frac{d^2v_g(t)}{dt^2} + \frac{L}{R}\frac{dv_g(t)}{dt} + v_g(t) = Dd(t),$$

say, $\frac{dv_g(t)}{dt} = d(t) \longrightarrow$ impulse. $\longrightarrow$



so, $\therefore \frac{d^2 v_g(t)}{dt} = \frac{d}{dt}[d(t)] \longrightarrow$



this is not possible.

so the only solution for this is,

$$LC\frac{d^2v_g(t)}{dt^2} = D\delta(t)$$

$$\Rightarrow \int_0^t LC\frac{d^2v_g(\tau)}{d\tau^2}d\tau = D\underbrace{\int_{-\varepsilon/2}^{\varepsilon/2}\delta(\tau)\,d\tau}_{=D.}$$

$$\therefore LC\left.\frac{dv_g(t)}{dt}\right|_{t=0} = D$$

$$\left.\frac{dv_g(t)}{dt}\right|_{t=0} = \frac{D}{LC}$$

Problem 2: Mathematical Methods for Dynamic Modeling [40 Points Total]
Consider the boost converter in Fig. 1. The input voltage is $v_g(t)$, and output voltage is $v(t)$. Configuration ① has a duty ratio of $d(t)$, and ② has a duty ratio of $d'(t) = 1 - d(t)$. Each MOSFET has an on-state resistance of R_{on} (all other elements are lossless). The states are the capacitor voltage $v(t)$, and current $i(t)$ through the inductor.

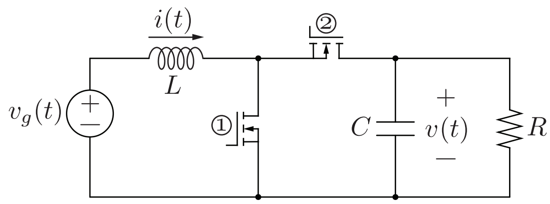


Figure 1: Boost converter for Problems 2 and 3.

(a) **[10 Points]** Derive the differential equations for the averaged inductor current, $\langle i(t)\rangle$, and averaged capacitor voltage, $\langle v(t)\rangle$. Put your result into the form

$$\dot{x}(t) = f(x(t), u(t)),$$

where

$$x(t) = \begin{bmatrix}\langle i(t)\rangle \\ \langle v(t)\rangle\end{bmatrix}, \quad u(t) = \begin{bmatrix} d'(t) \\ \langle v_g(t)\rangle\end{bmatrix}, \quad f(x(t),u(t)) = \begin{bmatrix} f_1(x(t),u(t)) \\ f_2(x(t),u(t))\end{bmatrix}.$$

$$L \frac{d}{dt}\langle i\rangle(t) = d(t)\left(\langle v_g\rangle(t) - R_{on}\langle i\rangle(t)\right) + d'(t)\left(\langle v_g\rangle(t) - R_{on}\langle i\rangle(t) - \langle v\rangle(t)\right)$$

$$= \langle v_g\rangle(t) - R_{on}\langle i\rangle(t) - d'(t)\langle v\rangle(t)$$

$$C \frac{d}{dt}\langle v\rangle(t) = d(t)\left(-\frac{\langle v\rangle(t)}{R}\right) + d'(t)\left(\langle i\rangle(t) - \frac{\langle v\rangle(t)}{R}\right)$$

$$= d'(t)\langle i\rangle(t) - \frac{\langle v\rangle(t)}{R}$$

$$\Rightarrow \boxed{\dot{x}(t) = \frac{d}{dt}\begin{bmatrix}\langle i\rangle(t) \\ \langle v\rangle(t)\end{bmatrix} = \begin{bmatrix}\frac{1}{L}\left(\langle v_g\rangle(t) - R_{on}\langle i\rangle(t) - d'(t)\langle v\rangle(t)\right) \\ \frac{1}{C}\left(d'(t)\langle i\rangle(t) - \frac{\langle v\rangle(t)}{R}\right)\end{bmatrix}}$$

(b) **[15 Points]** Derive a linearized small-signal model of the form

$$\dot{\widehat{x}}(t) \approx \mathcal{A}\widehat{x}(t) + \mathcal{B}\widehat{u}(t),$$
$$\widehat{y}(t) = \mathcal{C}\widehat{x}(t) + \mathcal{E}\widehat{u}(t),$$

where $\mathcal{A}, \mathcal{B} \in \mathbb{R}^{2\times 2}$ are real-valued square 2×2 matrices, and

$$\widehat{x}(t) = x(t) + X, \quad \widehat{u}(t) = u(t) + U, \quad X = \begin{bmatrix} I \\ V \end{bmatrix}, \quad U = \begin{bmatrix} D' \\ V_g \end{bmatrix}.$$

$\widehat{y}(t)$ is chosen to correspond to the capacitor state such that

$$\widehat{y}(t) = \langle v(t) \rangle.$$

Express all entries in $\mathcal{A}$ and $\mathcal{B}$ in terms of $I, V, D', V_g, R, L, C, R_{\text{on}}$ and give $\mathcal{C}$ and $\mathcal{E}$.

$$\mathcal{A} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial \langle i \rangle} & \frac{\partial f_1}{\partial \langle v \rangle} \\ \frac{\partial f_2}{\partial \langle i \rangle} & \frac{\partial f_2}{\partial \langle v \rangle} \end{bmatrix}\right|_{\substack{x=X \\ u=U}} = \left.\begin{bmatrix} -\frac{R_{on}}{L} & -\frac{d'(t)}{L} \\ \frac{d'(t)}{C} & -\frac{1}{RC} \end{bmatrix}\right|_{\substack{x=X \\ u=U}} = \boxed{\begin{bmatrix} -\frac{R_{on}}{L} & -\frac{D'}{L} \\ \frac{D'}{C} & -\frac{1}{RC} \end{bmatrix} = \mathcal{A}}$$

$$\mathcal{B} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial d'} & \frac{\partial f_1}{\partial \langle v_g \rangle} \\ \frac{\partial f_2}{\partial d'} & \frac{\partial f_2}{\partial \langle v_g \rangle} \end{bmatrix}\right|_{\substack{x=X \\ u=U}} = \left.\begin{bmatrix} -\frac{\langle v \rangle}{L} & \frac{1}{L} \\ \frac{\langle i \rangle}{C} & 0 \end{bmatrix}\right|_{\substack{x=X \\ u=U}} = \boxed{\begin{bmatrix} -\frac{V}{L} & \frac{1}{L} \\ \frac{I}{C} & 0 \end{bmatrix} = \mathcal{B}}$$

- For $\hat{y}(t) = \langle \hat{v} \rangle$, pick $\mathcal{C}$ & $\mathcal{E}$ below

$$\boxed{\mathcal{C} = \begin{bmatrix} 0 & 1 \end{bmatrix} \in \mathbb{R}^{1\times 2}, \quad \mathcal{E} = \mathcal{O}_{1\times 2} = \begin{bmatrix} 0 & 0 \end{bmatrix} \in \mathbb{R}^{1\times 2}}$$

$$\Rightarrow \quad \begin{aligned} \hat{x}(t) &= \mathcal{A}\hat{x} + \mathcal{B}\hat{u} \\ \hat{y}(t) &= \mathcal{C}\hat{x} + \mathcal{D}\hat{u} \end{aligned}$$

(c) **[15 Points]** Given the matrices $\mathcal{A}, \mathcal{B}, \mathcal{C}$, and $\mathcal{E}$, derive the capacitor voltage in the frequency domain using the following formula:

$$\widehat{y}(s) = \langle v(s) \rangle = \underbrace{\left(\mathcal{C}(s\mathcal{I} - \mathcal{A})^{-1}\mathcal{B} + \mathcal{E}\right)}_{G(s)} \widehat{u}(s),$$

where $\mathcal{I}$ is the 2×2 identity matrix, $\widehat{u}(s) \in \mathbb{C}^{2\times1}$ is a complex column vector, and $G(s) \in \mathbb{C}^{1\times2}$ is a complex 1×2 row vector given by

$$G(s) = \begin{bmatrix} G_{vd}(s) & G_{vg}(s) \end{bmatrix}.$$

$$G(s) = \left(\mathcal{C}(s\mathcal{I} - \mathcal{A})^{-1} \mathcal{B} + \overset{0_{2\times2}}{\cancel{\mathcal{E}}} \right) \hat{u}(s)$$

$$= \mathcal{C}\left(s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -\frac{R_{on}}{L} & -\frac{D'}{L} \\ \frac{D'}{C} & -\frac{1}{RC} \end{bmatrix} \right)^{-1} \mathcal{B}\,\hat{u}(s)$$

$$= \mathcal{C} \begin{bmatrix} s + \frac{R_{on}}{L} & \frac{D'}{L} \\ -\frac{D'}{C} & s + \frac{1}{RC} \end{bmatrix}^{-1} \mathcal{B}\,\hat{u}(s)$$

$\ast$ recall

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

introduce $\mathcal{C}$

$$= \frac{1}{\underbrace{\left(s + \frac{R_{on}}{L}\right)\left(s + \frac{1}{RC}\right) - \left(\frac{D'}{L}\right)\left(-\frac{D'}{C}\right)}_{\text{side calc.}}} \underbrace{\overset{1\times2}{\begin{bmatrix} 0 & 1 \end{bmatrix}} \overset{2\times2}{\begin{bmatrix} s + \frac{1}{RC} & -\frac{D'}{L} \\ \frac{D'}{C} & s + \frac{R_{on}}{L} \end{bmatrix}}}_{\ast\ 1\times2} \mathcal{B}\,\hat{u}(s)$$

side calc. $\rightarrow$ $\text{denom} = s^2 + s\left(\frac{1}{RC} + \frac{R_{on}}{L}\right) + \frac{R_{on}}{RLC} + \frac{D'^2}{LC}$ } save for later

$$= \frac{1}{\text{denom}} \overbrace{\begin{bmatrix} \frac{D'}{C} & s + \frac{R_{on}}{L} \end{bmatrix}}^{\ast} \begin{bmatrix} -\frac{V}{L} & \frac{1}{L} \\ \frac{I}{C} & 0 \end{bmatrix} \hat{u}(s) = \frac{1}{\text{denom}} \begin{bmatrix} \frac{D'}{C}\left(-\frac{V}{L}\right) + \left(s + \frac{R_{on}}{L}\right)\left(\frac{I}{C}\right) & \frac{D'}{C}\frac{1}{L} \end{bmatrix} \begin{bmatrix} \hat{d}(s) \\ \langle \hat{v}_g \rangle(s) \end{bmatrix}$$

$$\boxed{= \underbrace{\frac{1}{s^2 + s\left(\frac{1}{RC} + \frac{R_{on}}{L}\right) + \frac{R_{on}}{RLC} + \frac{D'^2}{LC}} \begin{bmatrix} \frac{D'}{C}\left(-\frac{V}{L}\right) + \left(s + \frac{R_{on}}{L}\right)\left(\frac{I}{C}\right) & \frac{D'}{C}\frac{1}{L} \end{bmatrix}}_{G(s)} \begin{bmatrix} \hat{d}(s) \\ \langle \hat{v}_g \rangle(s) \end{bmatrix} = \langle \hat{v} \rangle(s)}$$

Problem 4: Digital Systems [20 Points Total]

(a) **[10 Points]** On the top, draw an asymmetric trailing-edge carrier with period T_s and whose counter varies between $0 \leq x_{pwm} \leq N_r$. Sketch a compare value, x_c, that updates each time the counter equals zero. On the bottom, draw the PWM output. Clearly mark the modulation delay, t_{mod}, and express it in terms of D and T_s.

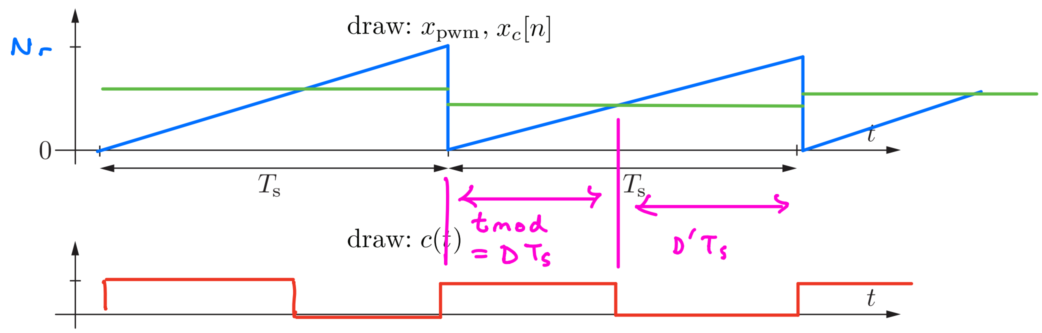


Figure 2: Template for PWM waveforms.

(b) **[5 Points]** Given the carrier in (a) and clock period $T_{clk} = (100\,\text{MHz})^{-1}$, compute N_r such that the $f_s = 50\,\text{kHz}$.

$$T_s = (N_r + 1)\, T_{clk}$$

$$\Rightarrow N_r = \frac{f_{clk}}{f_s} - 1 = \frac{10\cancel{0} \times 10^{\cancel{6}\,3}}{5\cancel{0} \times 10^{\cancel{3}}} - 1 = \boxed{1999 = N_r}$$

(c) **[5 Points]** Consider an 8-bit ADC with full-scale range $V_{FS} = 1\,\text{V}$. If the digital integer produced by the ADC is $x_{A/D} = 107$, then what is the physical voltage, $v_{A/D}$, on the ADC pin? Hint: You can neglect the rounding behavior of the ADC for this calculation.

$$x_{A/D} = 0 \quad \text{if} \quad v_{A/D} \leq 0$$

$$= \text{round}\left(v_{A/D} \cdot \frac{2^{N_{A/D}}}{v_{FS}} \right) \quad \text{if} \quad 0 < v_{A/D} < v_{FS}$$

$$= 2^{N_{A/D}} \quad \text{if} \quad v_{A/D} \geq V_{FS}$$

$$\therefore \quad v_{A/D} = \frac{x_{A/D} \cdot v_{FS}}{2^{n_{A/D}}} = \frac{107}{\boxed{2^8}} = 0.418\,V$$

many people did it $2^8 - 1$
(that is wrong)