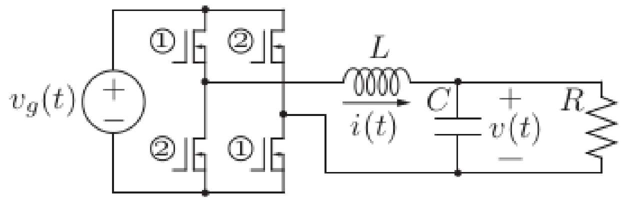
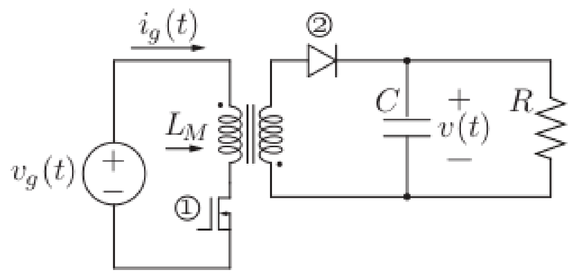
EE 458/533 – Power Electronics Controls, Winter 2022
Homework 1
Due Date: Thursday January 21 2022



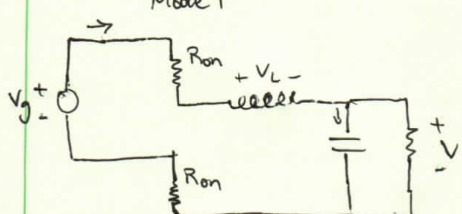
Circuit for Problem 1: The single-phase voltage source inverter.



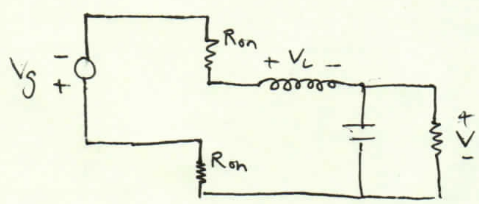
Circuit for Problem 2: The flyback converter.

1. a)

Mode 1



Mode 2



$$x(t) = \begin{bmatrix} f_1(x(t), u(t)) \\ f_2(x(t), u(t)) \end{bmatrix} = \begin{bmatrix} \frac{d}{dt} \langle i(t) \rangle_{T_s} \\ \frac{d}{dt} \langle v(t) \rangle_{T_s} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} \frac{1}{L}\left[\langle v_g(t)\rangle_{T_s} - \langle v(t)\rangle_{T_s} - 2R_{on}\langle i(t)\rangle_{T_s}\right] d(t) + \frac{1}{L}\left[-\langle v_g(t)\rangle_{T_s} - \langle v(t)\rangle_{T_s} - 2R_{on}\langle i(t)\rangle_{T_s}\right] d'(t) \\ \frac{1}{C}\left[\langle i(t)\rangle_{T_s} - \frac{\langle v(t)\rangle_{T_s}}{R}\right] d(t) + \frac{1}{C}\left[\langle i(t)\rangle_{T_s} - \frac{\langle v(t)\rangle_{T_s}}{R}\right] d'(t) \end{bmatrix}$$

b.) $L\frac{dI_L}{dt} = (V_g - V - 2R_{on}I_L)D + (-V_g - V - 2R_{on}I_L)D' = 0$

$$0 = DV_g - D'V_g - V - \frac{2VR_{on}D}{R} + \frac{2VR_{on}D'}{R}$$

$$0 = (2D-1)V_g - V\left(\frac{2R_{on}}{R} + 1\right)$$

$$0 = (2D-1)V_g - V\left(\frac{2R_{on}+R}{R}\right)$$

$$V = \frac{R(2D-1)V_g}{2R_{on}+R}$$

$$C\frac{dV_c}{dt} = \left(I - \frac{V}{R}\right)D + \left(I - \frac{V}{R}\right)D' = 0$$

$$0 = I - \frac{V}{R} \rightarrow I = \frac{V}{R} = \frac{(2D-1)V_g}{2R_{on}+R}$$

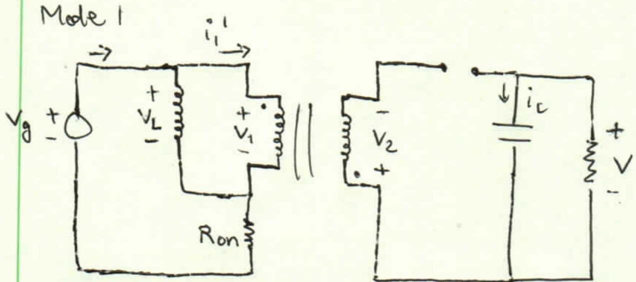
c.) $\dot{\tilde{x}} = A(x-\bar{x}) + B(u-\bar{u})$

$$A = \frac{\partial f}{\partial \tilde{x}}\Bigg|_{\substack{x=\bar{x},\\ u=\bar{u}}} = \begin{bmatrix} \frac{\partial f_1}{\partial \tilde{i}_L} & \frac{\partial f_1}{\partial \tilde{v}} \\ \frac{\partial f_2}{\partial \tilde{i}_L} & \frac{\partial f_2}{\partial \tilde{v}} \end{bmatrix}\Bigg|_{\substack{i_L=I_L,\ d=D\\ v_c=V_c,\ v_{in}=V_{in}}} = \begin{bmatrix} \frac{-2R_{on}}{L} & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix}$$

$$B = \frac{\partial f}{\partial \tilde{u}}\Bigg|_{\substack{x=\bar{x},\\ u=\bar{u}}} = \begin{bmatrix} \frac{\partial f_1}{\partial \tilde{d}} & \frac{\partial f_1}{\partial \tilde{v}_g} \\ \frac{\partial f_2}{\partial \tilde{d}} & \frac{\partial f_2}{\partial \tilde{v}_g} \end{bmatrix}\Bigg|_{\substack{i_L=I_L,\ d=D\\ v_c=V_c,\ v_{in}=V_{in}}} = \begin{bmatrix} \frac{2V_g}{L} & \frac{2D-1}{L} \\ 0 & 0 \end{bmatrix}$$

2. a)

Mode 1

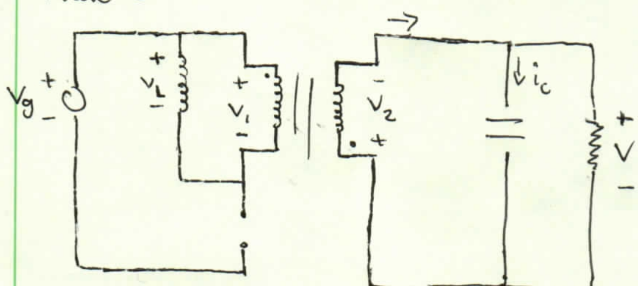


$$i_1' + \overset{0}{\cancel{n i_2}} = 0$$

$$i_1' = i_2 = 0$$

$$V_1 = \frac{V_2}{n} \rightarrow V_2 = nV_1 = nV_g$$

Mode 2



$$x(t) = \begin{bmatrix} f_1(x(t), u(t)) \\ f_2(x(t), u(t)) \end{bmatrix} = \begin{bmatrix} \frac{d}{dt} \langle i(t) \rangle_{T_s} \\ \frac{d}{dt} \langle v(t) \rangle_{T_s} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} \frac{1}{L}\left[\langle V_g(t) \rangle_{T_s} - R_{on} \langle i(t) \rangle_{T_s} \right] d(t) + \frac{1}{L}\left[-\frac{\langle v(t) \rangle_{T_s}}{n} \right] d'(t) \\ \frac{1}{C}\left[-\frac{\langle v(t) \rangle_{T_s}}{R} \right] d(t) + \frac{1}{C}\left[\frac{\langle i(t) \rangle_{T_s}}{n} - \frac{\langle v(t) \rangle_{T_s}}{R} \right] d'(t) \end{bmatrix}$$

b.) $$L\frac{dI_L}{dt} = (V_g - IR_{on})D + \left(\frac{-V}{n}\right)D' = 0$$

$$0 = V_g D - \frac{nD}{RD'}R_{on}V - \frac{D'}{n}V$$

$$0 = V_g D - V\left(\frac{nD}{RD'}R_{on} + \frac{D'}{n}\right)$$

$$V = \frac{V_g D}{\frac{nDR_{on}}{RD'} + \frac{D'}{n}}$$

$$C\frac{dV_c}{dt} = \left(\frac{-V}{R}\right)D + \left(\frac{I}{n} - \frac{V}{R}\right)D' = 0$$

$$0 = -\frac{V}{R} + \frac{I}{n}D'$$

$$I = \frac{nV}{RD'} = \frac{n}{RD'}\left(\frac{V_g D}{\frac{nDR_{on}}{RD'} + \frac{D'}{n}}\right)$$

c.) $$\dot{\tilde{x}} = A(x-\bar{x}) + B(u-\bar{u})$$

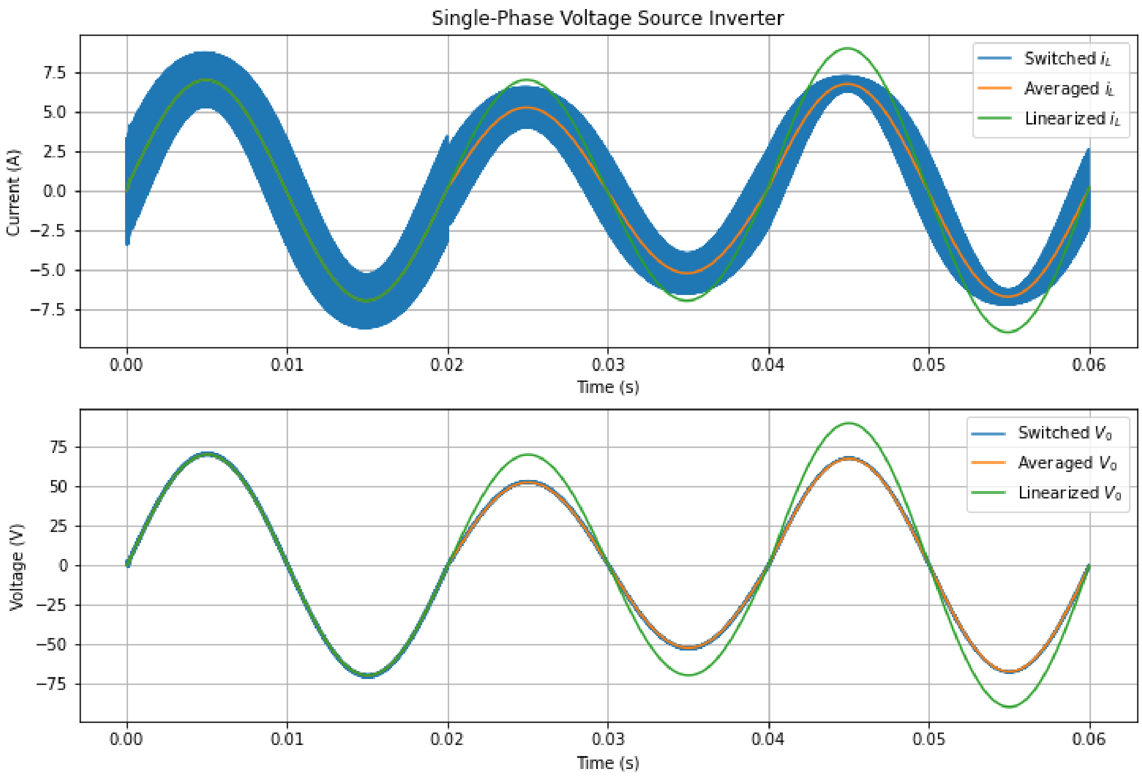
$$A = \frac{\partial f}{\partial \tilde{x}}\Bigg|_{\substack{x=\bar{x},\\ u=\bar{u}}} = \begin{bmatrix} \frac{\partial f_1}{\partial \tilde{i}_L} & \frac{\partial f_1}{\partial \tilde{v}} \\ \frac{\partial f_2}{\partial \tilde{i}_L} & \frac{\partial f_2}{\partial \tilde{v}} \end{bmatrix}\Bigg|_{\substack{i_L=I_L,\ d=D\\ v_c=V_c,\ v_{in}=V_{in}}} = \begin{bmatrix} -\frac{R_{on}D}{L} & \frac{-D'}{nL}\ \cancel{+\frac{D'}{2n}} \\ \frac{D'}{nC} & \frac{-1}{RC} \end{bmatrix}\Bigg|_{n=2}$$

$$A = \begin{bmatrix} -\frac{R_{on}D}{L} & \frac{-D'}{2L} \\ \frac{D'}{2C} & \frac{-1}{RC} \end{bmatrix}$$

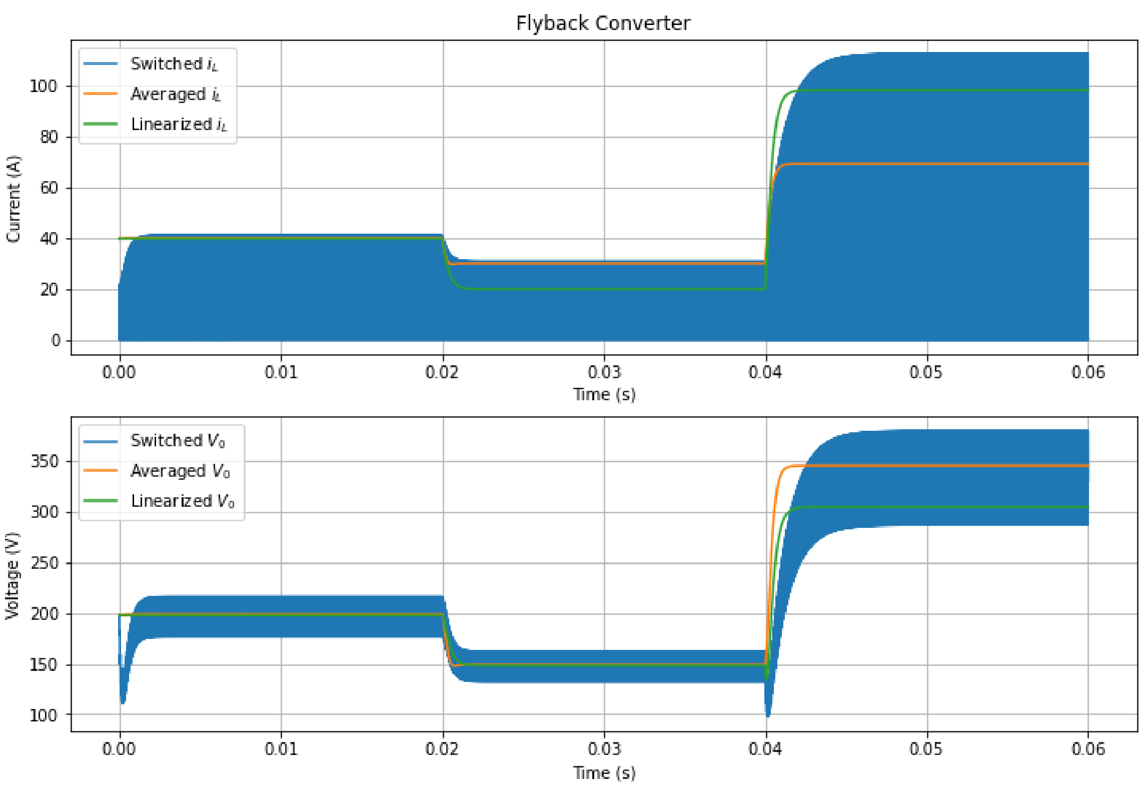
$$B = \left.\frac{\partial f}{\partial \tilde{u}}\right|_{\substack{x=\bar{x},\\ u=\bar{u}}} = \begin{bmatrix} \frac{\partial f_1}{\partial \tilde{i}} & \frac{\partial f_1}{\partial \tilde{v}_g} \\ \frac{\partial f_2}{\partial \tilde{i}} & \frac{\partial f_2}{\partial \tilde{v}_g} \end{bmatrix} =$$

$$B = \left.\begin{bmatrix} \frac{1}{L}\left(V_g - IR_{on} + \frac{V}{n}\right) & \frac{D}{L} \\ \cancel{\frac{-V}{RC}} - \frac{1}{nC} + \cancel{\frac{V}{RC}} & 0 \end{bmatrix}\right|_{n=2} = \begin{bmatrix} \frac{1}{L}\left(V_g - IR_{on} + \frac{V}{n}\right) & \frac{D}{L} \\ \frac{-1}{nC} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{L}\left(V_g - IR_{on} + \frac{V}{2}\right) & \frac{D}{L} \\ \frac{-1}{2C} & 0 \end{bmatrix}$$



Single-Phase Voltage Source Inverter
Switched i_L
Averaged i_L
Linearized i_L
Current (A)
Time (s)
Switched V_0
Averaged V_0
Linearized V_0
Voltage (V)
Time (s)



Flyback Converter
Switched i_L
Averaged i_L
Linearized i_L
Current (A)
Time (s)
Switched V_0
Averaged V_0
Linearized V_0
Voltage (V)
Time (s)

3 a.) $q = 0.02 = \frac{1}{N_r + 1} \rightarrow N_r = 49$

$T_{sw} = 2N_r \cdot T_{clk}$

$N_r = \frac{T_{sw}}{2T_{clk}}$

$q_r = \frac{1}{N_r + 1} = \frac{1}{\frac{T_{sw}}{2T_{clk}} + 1} = \frac{1}{\frac{f_{clk}}{2f_{sw}} + 1}$

$0.02 = \frac{1}{\frac{100\,MHz}{2f_{sw}} + 1}$ $\quad f_{sw} = 1.02\,MHz$

b.) $N_r = \frac{T_{sw}}{2T_{clk}} \rightarrow 2^{N_{pwm}} = \frac{f_{clk}}{2f_{sw}}$

$2^{16} = \frac{f_{clk}}{2f_{sw}} \rightarrow f_{sw} = \frac{f_{clk}}{2 \cdot 2^{16}} = \frac{100\,MHz}{2 \cdot 65{,}536} = 762\,Hz$

c.) $N_r \leq 2^{N_{pwm}} - 1$

$N_r \leq 2^{16} - 1$

$N_r \leq 65{,}535$

$q = \frac{1}{N_r + 1} = \frac{1}{65{,}536} = 1.5E-5$

$q = \Delta D = 0.0015\%$

d.) $V_0 = V_{in} \cdot \dfrac{D}{\dfrac{nDR_{on}}{RD'} + \dfrac{D'}{n}}$ and $n=2,\ R=10,\ R_{on} = 15\,m\Omega$

$$\Delta V_0 = V_{in} \dfrac{\Delta D}{\dfrac{n\Delta D R_{on}}{R(1-\Delta D)} + \dfrac{(1-\Delta D)}{n}} = V_{in} \dfrac{1.5E-5}{\dfrac{2(1.5E-5)(15E-3)}{10(1-1.5E-5)} + \dfrac{(1-1.5E-5)}{2}}$$

$\Delta V_0 = 3E-5\ V_{in}$