EE 458/533 – Power Electronics Controls, Winter 2022

Homework 2

Due Date: Thursday January 20th 2021

Instructions. You must scan your completed homework assignment into a pdf file, and upload your file to the Canvas Assignment page by 11:59 pm Pacific Time on the due date above. All pages must be gathered into a single file of moderate size, with the pages in the correct order. Please note that the grader will not be obligated to grade your assignment if the file is unreadable or very large.

Problem 1: The objective of this problem is to build on the results from Homework 1 to obtain an analytical model in the frequency domain. We only consider the voltage source inverter in Figure 1.

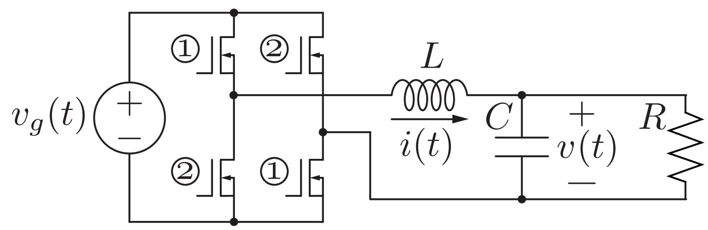

Figure 1: The single-phase voltage source inverter.

As in Homework 1, the input voltage is $v_g(t)$, and output voltage is $v(t)$. Configuration ① has a duty ratio of $d(t)$, and ② has a duty ratio of $d'(t) = 1 - d(t)$. Each MOSFET has an on-state resistance of R_{on} (all other elements are lossless). The states are the capacitor voltage $v(t)$, and current $i(t)$ through the inductor. For the circuit in Figure 1, do the following:

(a) Derive the small-signal model of the circuit. To obtain your result, express the states as

$$\langle i(t)\rangle = I + \langle \widehat{i}(t)\rangle$$
$$\langle v(t)\rangle = V + \langle \widehat{v}(t)\rangle,$$

and the inputs as

$$d(t) = D + \widehat{d}(t)$$
$$\langle v_g(t)\rangle = V_g + \langle \widehat{v}_g(t)\rangle.$$

After linearizing the state equations, discard the dc terms I, V, D, and V_g to obtain the finalized small-signal circuit.

Obtain an equivalent circuit representation of the small signal circuit obtained (you may use transformers to model the equivalent circuit when necessary).

(b) Given your small-signal model, derive the time-domain state equations for the inductor current and capacitor voltage. In other words, compute $\frac{d}{dt}\langle\widehat{i}(t)\rangle$ and $\frac{d}{dt}\langle\widehat{v}(t)\rangle$. Confirm that it matches $\dot{\widehat{x}}(t) \approx A\widehat{x}(t) + B\widehat{u}(t)$ from Homework 1 where

$$\widehat{x}(t) = x(t) - X = \begin{bmatrix} \langle\widehat{i}(t)\rangle \\ \langle\widehat{v}(t)\rangle \end{bmatrix}, \quad \dot{x}(t) = \dot{\widehat{x}}(t), \quad \widehat{u}(t) = u(t) - U = \begin{bmatrix} \widehat{d}(t) \\ \langle\widehat{v}_g(t)\rangle \end{bmatrix}.$$

(c) To define a complete state-space model, we pick the output as the states. Hence,

$$\widehat{y}(t) = \widehat{x}(t) = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} \langle\widehat{i}(t)\rangle \\ \langle\widehat{v}(t)\rangle \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{E} \begin{bmatrix} \widehat{d}(t) \\ \langle\widehat{v}_g(t)\rangle \end{bmatrix}.$$

Given the matrices A, B, C, and E, derive the frequency domain model using the following formula:

$$\widehat{y}(s) = \underbrace{\left(C(sI - A)^{-1}B + E\right)}_{G(s)} \widehat{u}(s),$$

where $\widehat{y}(s), \widehat{u}(s) \in \mathbb{C}^{2\times1}$ are complex column vectors and $G(s) \in \mathbb{C}^{2\times2}$ is a complex 2×2 matrix given by

$$G(s) = \begin{bmatrix} G_{id}(s) & G_{ig}(s) \\ G_{vd}(s) & G_{vg}(s) \end{bmatrix}.$$

(d) Using your favorite software and the parameters specified in Homework 1, create Bode plots for the transfer functions $G_{id}(s), G_{ig}(s), G_{vd}(s)$, and $G_{vg}(s)$. Assume $V_g = 100\,\mathrm{V}$ and $D = 0.5$. Clearly place a title above each plot, the x-axis should be in units of Hz, and the x-axis should span from 1 Hz to half the switching frequency.

From the curves, obtain the following information and try to rationalize with only that information if each of the system is stable in closed loop with an unity gain controller.

- Gain cross over frequency
- Phase cross over frequency
- Gain margin
- Phase margin