EE 458-533
Experiment 2 :
Boost Converter Control

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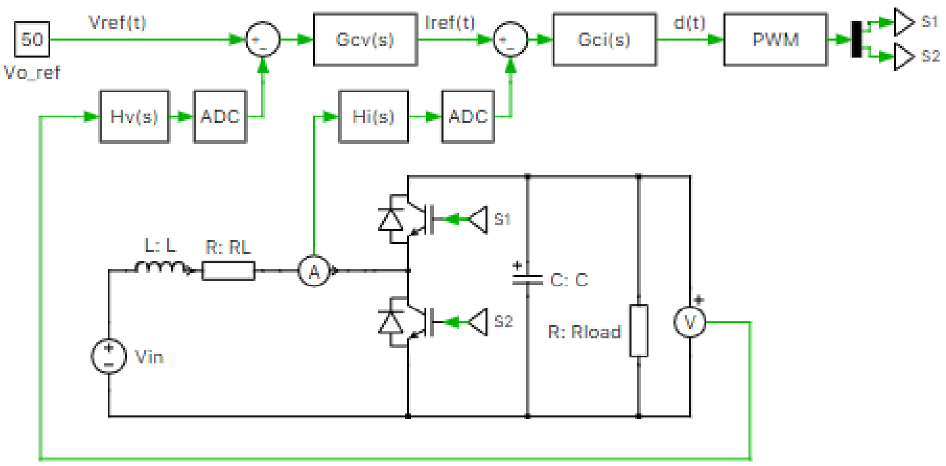
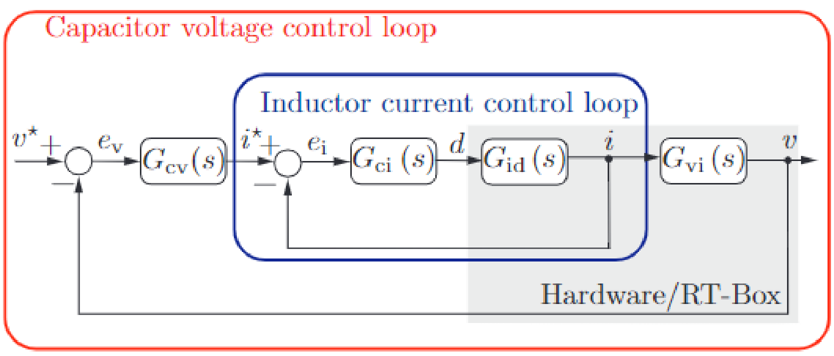




Tasks

2. Current Controller Design

1. Select a crossover frequency for the current controller, f_{ci}. This is where we obtain $|\ell_i(j2\pi f_{ci})| = 1$. Specify the phase margin, ϕ_m, $(\angle\ell_i(j2\pi f_{ci}) = -180 + \phi_m)$. Note that $f_{ci} \leq f_{sw}/10$ and $60° \leq \phi_m$ are good design practices. **[2 pts]**

2. Determine the gain and phase of the uncompensated open loop gain, $\ell_{u,i}(s)$, at the crossover frequency you have selected using $G_{id}(s)$. The uncompensated open loop gain in this case is just $G_{id}(s)$. Compare this value with the approximation, $G_{id,approx}(s)$. Note that we can use a PI Regulator when $\angle\ell_{u,i}(j2\pi f_{ci}) \geq -180° + \phi_m$. This is because a PI controller can only add additional phase lag. Determine the amount of phase lag you must introduce to achieve your target phase margin in $\ell_i(s)$. **[5 pts]**

3. Write an expression for the open loop gain magnitude, $|\ell_i(j2\pi f_{ci})|$. Write an expression for the open loop phase, $\angle\ell_i(j2\pi f_{ci})$. You may use $G_{id,approx}(s)$ in your design. Refer to 1.5.2 of [1] for additional guidance. Note that formula 1.83 has an error. **[5 pts]**

4. Use your expression for $\angle\ell_i(j2\pi f_{ci}) = -180 + \phi_m$ to derive ω_{PIi}.**[2 pts]**

5. Use your expression for $|\ell_i(j2\pi f_{ci})| = 1$ to derive $G_{PIi\infty}$.**[2 pts]**

6. Build a switched circuit model in PLECS to implement the inner current control of the boost converter. Using the Pulse Generator component, step the current controller set-point from rated input current to 50% of rated input current at a rate of 20 Hz. Based on the phase margin and bandwidth, verify the following time domain quantities : overshoot, rise time, settling time from the simulation. The theoretical values (use Table if needed) should match very closely to the predicted values from simulation. For the parameter verification, show plots of the duty ratio, inductor current, capacitor voltage.**[simulation : 20 pts, each parameter verification: 5 pts]**

7. In the simulation, show that the controller can reject a 10% change in the output resistance. **[10 pts]**

8. In your lab report, include plots of following transfer functions: $G_{id}(s)$, $G_{id,approx}(s)$, $G_{ci}(s)$, and $\ell_i(s)$. **[4X5 pts]**

9. **[Only for 559]**When modeling the load of the boost converter as a resistor, a change in current reference will lead to a change in output voltage. When the reference current is increased, what initially happens to the output voltage? What can be said about the initial trajectory of the output voltage in response to a change in inductor current? What transfer function characteristic does this type of relationship illustrate? **[8 pts]**

3. Voltage Controller Design

1. Design a PI regulator for the outer loop voltage controller. Document your design: **[8X2=16pts]**
   (a) Voltage controller crossover frequency, f_{cv}:____________________
   (b) Voltage controller phase margin design target, ϕ_m:____________________
   (c) Uncompensated open loop gain, $|\ell_{u,v}(j2\pi f_{cv})|$:____________________
   (d) Uncompensated open loop phase delay, $\angle\ell_{u,v}(j2\pi f_{cv})$:____________________
   (e) PI Controller zero frequency, ω_{PIv}:____________________
   (f) PI Controller high frequency asymptote, $G_{PIv\infty}$:____________________
   (g) Compensated open loop gain, $|\ell_v(j2\pi f_{cv})|$:____________________
   (h) Compensated open loop phase margin, $180° + \angle\ell_i(j2\pi f_{cv})$:____________________
2. Build a switched circuit model in PLECS to implement the outer voltage control of the boost converter. Using the Pulse Generator component, step the voltage controller set-point from rated voltage to 80% of rated voltage at a rate of 1 Hz. Based on the phase margin and bandwidth, verify the following time domain quantities : overshoot, rise time, settling time from the simulation. The theoretical values (use Table if needed) should match very closely to the predicted values from simulation. For the parameter verification, show plots of the duty ratio, inductor current reference, inductor current, capacitor voltage.**[simulation : 20 pts, each parameter verification: 5 pts]**
3. In your lab report, include bode plots of $G_{vi}(s), G_{vi,approx}(s), G_{cv}(s), \ell_v(s)$. **[4X5 pts]**
4. **[Only for 559]** Why do we have an inner current and an outer voltage control loop. Why not the reverse or, maybe the two loops in cascade or any other architecture you could think of?**[4 pts]**

4. Real-Time Implementation of Control of Boost Converter

- Configure one channel which will generate the PWM for S2 (Fig 1), and the complement of that PWM will be supplied to S1. Make sure that this channel number corresponds to the channel number in the PLECS module.
- Accept the inductor current and capacitor voltage in two float variables named, i^{adc} and v^{adc} respectively.
- Scale them appropriately and store the actual values of physical current and voltage values in new variables namely, i and v. The formulae relating the two is,

$$i = i^{\text{adc}}(3.3/4096)(1/\kappa_i)$$
$$v = v^{\text{adc}}(3.3/4096)(1/\kappa_v)$$

  where, κ_i, κ_v are factors you have already selected.
- To test the inner current loop first, set a global variable i_{ref}, which can be changed from watch window. Store the difference between i_{ref} and i in a new variable i_{err}. This will be the input to the PI controller.
- The output of the PI controller, yi is composed of two terms, yi_{pr} and yi_{int}. We obtain yi_{pr} by multiplying i_{err} with a static gain, called proportional gain. We obtain yi_{int} by integrating i_{err} and multiplying with an integrator gain.
- The output, yi is obtianed by adding yi_{pr} and yi_{int}. Note that "duty" in Lab 1 and yi are functionally identical. Follow similar steps to output the PWM from the channel that you have configured in both the c-code as well as the PLECS simulation.
- You should saturate this yi within an upper, $yi^{\max}$ and lower value, $yi^{\min}$. Select and report these values. You can select these based on the EE452 lab. **[2 pts]**
- Remember to saturate the output of the integrator to an upper, $yi_{\text{int}}^{\max}$ and lower value, $yi_{\text{int}}^{\min}$. This is required to obtain steady converter control. You can try without these too, and compare the results. These values are not random and depend strongly on your circuit operating conditions. Select these values according to nominal operating conditions and report them. **[4 pts]**
- Once the current controller seems to be working with the real time simulator, we next implement the voltage controller. We follow the same notation for variables, except replacing v for i to signify the voltage control. The major change that we now undertake is that, the output of the voltage controller, yv becomes the current reference, i_{ref}. Make sure to saturate the current reference based on the EE452 lab as well.
- Show waveforms for the inductor current, capcitor voltage and yi for a rated reference, i_{ref} to prove that the inner current control works. **[3X10 pts]**
- Show waveforms for the inductor current, inductor current reference, capcitor voltage and yi for a rated reference, v_{ref} to prove that the outer voltage control works. Note: you may use the CCS graphing feature to graph software variables, and PLECS to plot "analog" waveforms. Other methods are also acceptable. **[4X5 pts]**
- **[Only for 559]** You now need to implement a changing reference for i_{ref} and v_{ref} like you did in the simulations. Record the waveforms as in previous steps showing both the current and voltage control. **[10+10 pts]**
- **[Only for 559]** As a final step, implement an anti-windup to this controller. Considering the output yi can vary between $yi^{\min}$ and $yi^{\max}$, we need to take some corrective action every time it extends beyond that range. Whenever that happens, we saturate it at the limit it hits and then stop the integration part of the PI integrator, so that the huge error which led to saturation is no longer integrated and accumulated. Once the controller stabilizes the plant by only the proportional controller and the value of yi returns in the range $(yi^{\min}, yi^{\max})$, the integrator is reinstated. An easy way of removing the integrator is to multiply the input of the integrator with zero. **[5 pts]**

Assumptions:

- Because we are designing around the Phase Margin criterion, crossover frequency is implied to be gain crossover frequency:

$$\omega_c = \{\omega_{gc} = \omega_{gcf}\}$$

- Selecting crossover frequency:
  - current (gain) crossover frequency: $\omega_{ci} \leq \frac{\omega_{sw}}{10}$
  - voltage (gain) crossover frequency: $\omega_{cv} \leq \frac{\omega_{ci}}{10}$

- Bandwidth frequency ω_{BW} is approximately the (gain) crossover frequency:

$$\omega_{BW} \approx \omega_c$$

Parameters:

$$L = 1.3\ \mathrm{mH} \qquad R_L = 60\ \mathrm{m\Omega} \qquad C = 250\ \mu\mathrm{H} \qquad P_{load} = 250\ \mathrm{W}$$

$$V_{out} = 48\ \mathrm{V} \qquad V_{in} = 24\ \mathrm{V} \qquad f_{sw} = 10\ \mathrm{kHz} \quad D = 0.5134$$

$$R = R_{load} = \frac{V_{out}^2}{P_{load}} = 9.2160\ \Omega$$

DC (Steady State) Equations:

$$V = D'R \cdot \frac{V_{in}}{R_L + (D')^2 R} = 48\ \text{Volts}$$

$$I = \frac{V}{D'R} = \frac{V_{in}}{R_L + (D')^2 R} = 10.7031\ \text{Amps}$$

Plant Transfer Functions:

$$G(s) = \begin{bmatrix} G_{id}(s) & G_{ig}(s) \\ G_{vd}(s) & G_{vg}(s) \end{bmatrix}$$

- $\boldsymbol{G_{id}(s)}$

$$G_{id}(s) = \frac{\tilde{\imath}(s)}{\tilde{d}(s)} = \frac{V + D'IR + CRVs}{CLRs^2 + (L + CRR_L)s + R_L + (D')^2 R}$$

$$= \frac{V + D'IR}{R_L + (D')^2 R} \cdot \frac{1 + \frac{CRV}{V + D'IR}s}{\frac{CLR}{R_L + (D')^2 R}s^2 + \frac{L + CRR_L}{R_L + (D')^2 R}s + 1}$$

$$= G_{id,0} \cdot \frac{1 - \frac{s}{\omega_{z,i}}}{1 + \frac{2\zeta}{\omega_0}s + \left(\frac{s}{\omega_0}\right)^2}$$

- $\boldsymbol{G_{vd}(s)}$

$$G_{vd}(s)=\frac{\tilde{v}(s)}{\tilde{d}(s)}=\frac{-R\cdot(IR_L-D'V+ILs)}{CLRs^2+(L+CRR_L)s+R_L+(D')^2R}$$

$$=G_{vd,0}\cdot\frac{1-\frac{s}{\omega_{z,v}}}{1+\frac{2\zeta}{\omega_0}s+\left(\frac{s}{\omega_0}\right)^2}$$

- $\boldsymbol{G_{vi}(s)}$

$$G_{vi}(s)=\frac{\tilde{v}(s)}{\tilde{\imath}(s)}=\frac{G_{vd}(s)}{G_{id}(s)}=\frac{-R\cdot(IR_L-D'V+ILs)}{V+D'IR+CRVs}$$

$$=\frac{G_{vd,0}}{G_{id,0}}\cdot\frac{1-\frac{s}{\omega_{z,v}}}{1-\frac{s}{\omega_{z,i}}}$$

Gain, Poles, Zeros

$G_{id,0}$	$\frac{V+D'IR}{R_L+(D')^2R}=\frac{V+D'\left(\frac{V}{D'R}\right)R}{R_L+(D')^2R}=\frac{2V}{R_L+(D')^2R}$	$\mathbf{42.812}$
$G_{vd,0}$	$\frac{V}{D'}\cdot\frac{R(D')^2-R_L}{R(D')^2-R_L}$	$\mathbf{93.3606}$
$\omega_{z,i}$	$\frac{1}{-\frac{CRV}{V+D'IR}}=-\frac{V+D'IR}{CRV}=-\frac{V+D'\left(\frac{V}{D'R}\right)R}{CRV}=-\frac{2}{RC}$	$\mathbf{-868.056\frac{rad}{s}}$
$\omega_{z,v}$	$\frac{R(D')^2-R_L}{L}$	$\mathbf{1632.577\frac{rad}{s}}$
ω_0	$\frac{1}{\sqrt{\frac{CLR}{R_L+(D')^2R}}}=\sqrt{\frac{R_L+(D')^2R}{RLC}}$	$\mathbf{865.244\frac{rad}{s}}$
$\frac{2\zeta}{\omega_0}$	$\frac{L+CRR_L}{R_L+(D')^2R}\rightarrow\zeta=\frac{\omega_0}{2}\cdot\frac{L+CRR_L}{R_L+(D')^2R}=\frac{L+R_LRC}{2\sqrt{RLC(R_L+R(D')^2)}}$	$\mathbf{0.27748}$
Q	$\frac{1}{2\zeta}$	$\mathbf{1.8019}$

Section 2

From the set of transfer functions that you have derived in the Prelab, show how $G_{id}(s)$ can be approximated by V_{out}/sL. When would this approximation not hold true? **[5 pts]**

$$G_{id}(s) = \frac{\tilde{\imath}(s)}{\tilde{d}(s)} = \frac{V + D'IR + CRVs}{CLRs^2 + (L + CRR_L)s + R_L + (D')^2 R}$$

$$= \frac{V + D'\left(\frac{V}{D'R}\right)R + CRVs}{CLRs^2 + (L + CRR_L)s + R_L + (D')^2 R}$$

$$= \frac{CRVs + 2V}{CLRs^2 + (L + CRR_L)s + R_L + (D')^2 R}$$

The global behavior is dominated by the highest order polynomial terms of the numerator and denominator, as such:

$$G_{id}(s) \approx \lim_{s\to\infty} \frac{CRVs + 2V}{CLRs^2 + (L + CRR_L)s + R_L + (D')^2 R}$$

$$= \frac{CRVs}{CLRs^2} = \boxed{\frac{V}{sL} = G_{id,approx}(s)} \quad \textbf{for } s \gg \omega_0$$

This approximation would not hold true if s is **not** much larger than the ω_0

1. Select a crossover frequency for the current controller, f_{ci}. This is where we obtain $|\ell_i(j2\pi f_{ci})| = 1$. Specify the phase margin, ϕ_m, $(\angle\ell_i(j2\pi f_{ci}) = -180 + \phi_m)$. Note that $f_{ci} \le f_{sw}/10$ and $60° \le \phi_m$ are good design practices. **[2 pts]**

$$f_{ci} = \frac{f_{sw}}{10} = \frac{10\text{ kHz}}{10} = \boxed{1\text{ kHz} = f_{ci}} \quad \text{or} \quad \omega_{ci} = \frac{\omega_{sw}}{10} = 2\pi f_{ci} = \boxed{6.283 \times 10^3\,\frac{\text{rad}}{\text{s}} = \omega_{ci}}$$

$$\boxed{\phi_{mi} = 60° = \frac{\pi}{3}\text{ rad}}$$

2. Determine the gain and phase of the uncompensated open loop gain, $\ell_{u,i}(s)$, at the crossover frequency you have selected using $G_{id}(s)$. The uncompensated open loop gain in this case is just $G_{id}(s)$. Compare this value with the approximation, $G_{id,approx}(s)$. Note that we can use a PI Regulator when $\angle\ell_{u,i}(j2\pi f_{ci}) \ge -180° + \phi_m$. This is because a PI controller can only add additional phase lag. Determine the amount of phase lag you must introduce to achieve your target phase margin in $\ell_i(s)$. **[5 pts]**

$$\{\ell_{u,i}(s) = G_{id}(s)\}\big|_{s=j\omega_{ci}=j2\pi f_{ci}} = G_{id}(j6.283 \times 10^3)$$

```
G_id evaluated at 6.283185e+03 rad/s:
    Gain  = 6.03 = 15.60 dB
    Phase = -1.63 rad = -93.41°
```

$$\{\ell_{u,i,approx}(s) = G_{id,approx}(s)\}\big|_{s=j\omega_{ci}=j2\pi f_{ci}} = G_{id,approx}(j6.283 \times 10^3)$$

```
G_id_approx evaluated at 6.283185e+03 rad/s:
    Gain  = 5.88 = 15.38 dB
    Phase = -1.57 rad = -90.00°
```

It is evident that the uncompensated loop gain approximation

$$\ell_{u,i,approx}(s) = G_{id,approx}(s) = 15.38\ dB$$

is extremely close to the exact value of the uncompensated loop gain

$$\ell_{u,i}(s) = G_{id}(s) = 15.60\ dB.$$

Using the approximated (open) loop gain for all further analysis, the amount of **phase lag** we must introduce to achieve our target phase margin is $\boxed{\mathbf{30°}}$. I.e.:

$$\begin{aligned}
\angle\ell_i(s) &= \angle\{C(s)P(s)\} \\
&= \angle\{G_{ci}(s)G_{id}(s)\} \\
&= \angle G_{ci}(s) + \angle G_{id}(s) \\
&\approx \angle G_{ci}(s) + \angle G_{id,approx}(s)
\end{aligned}$$

$$\begin{aligned}
\angle\ell_i(s)|_{s=j\omega_{ci}} &= \angle\ell_i(\,j\omega_{ci}\,) \\
&= \angle\ell_i(\,j2\pi f_{ci}\,) \\
&= -180° + \phi_m \\
&= -180° + 60° \\
&= -120°
\end{aligned}$$

$$\begin{aligned}
\angle G_{id,approx}(s)\big|_{s=j\omega_{ci}} &= \angle G_{id,approx}(j\omega_{ci}) \\
&= -90°
\end{aligned}$$

So, $\angle G_{ci}(j\omega_{ci}) = \angle\ell_i(\,j\omega_{ci}\,) - \angle G_{id,approx}(j\omega_{ci})$

$$\begin{aligned}
&= (-120°) - (-90°) \\
&= -30°
\end{aligned}$$

3. Write an expression for the open loop gain magnitude, $|\ell_i(j2\pi f_{ci})|$. Write an expression for the open loop phase, $\angle\ell_i(j2\pi f_{ci})$. You may use $G_{id,approx}(s)$ in your design. Refer to 1.5.2 of [1] for additional guidance. Note that formula 1.83 has an error. **[5 pts]**

$$\begin{aligned}
\ell_i(s) &= G_{ci}(s)G_{id}(s) \\
&\approx G_{ci}(s)G_{id,approx}(s) \\
&= \left(k_{p,i} + \frac{k_{i,i}}{s}\right)\left(\frac{V}{sL}\right) \\
&= \frac{V}{sL}\left(k_{p,i} + \frac{k_{i,i}}{s}\right)
\end{aligned}$$

$$\|\ell_i(s)\|_{\omega=\omega_{BW}} = 1$$

$$\begin{aligned}
\left\|\frac{V}{sL}\left(k_{p,i} + \frac{k_{i,i}}{s}\right)\right\|_{\omega=\omega_{BW}\approx\omega_{ci}} &= 1 \\
\left\|\frac{V}{j\omega L}\left(k_{p,i} + \frac{k_{i,i}}{j\omega}\right)\right\|_{\omega=\omega_{BW}\approx\omega_{ci}} &= 1 \\
\left\|\frac{V}{j\omega_{ci}L}\left(k_{p,i} + \frac{k_{i,i}}{j\omega_{ci}}\right)\right\| &= 1 \\
\left\|\frac{V}{j\omega_{ci}L}\left(\frac{j\omega_{ci}k_{p,i} + k_{i,i}}{j\omega_{ci}}\right)\right\| &= 1 \\
\left\|\frac{V}{-\omega_{ci}^2 L}\left(j\omega_{ci}k_{p,i} + k_{i,i}\right)\right\| &= 1 \\
\frac{V}{L\omega_{ci}^2}\sqrt{\left(\omega_{ci}k_{p,i}\right)^2 + k_{i,i}^2} &= 1 \\
\sqrt{\left(\omega_{c,i}k_{p,i}\right)^2 + k_{i,i}^2} &= \frac{L\omega_{c,i}^2}{V}
\end{aligned}$$

$$\boxed{\boldsymbol{\frac{L^2\omega_{c,i}^4}{V^2} = \left(\omega_{ci}k_{p,i}\right)^2 + k_{i,i}^2}}$$

$$\angle \ell_i(s)|_{\omega=\omega_{BW}} = -180° + \phi_{mi}$$

$$\angle\left(\frac{V}{sL}\left(k_{p,i} + \frac{k_{i,i}}{s}\right)\right)\Big|_{\omega=\omega_{BW}=\omega_{ci}} = -180° + \phi_{mi}$$

$$\angle\left(\frac{V}{j\omega L}\left(k_{p,i} + \frac{k_{i,i}}{j\omega}\right)\right)\Bigg|_{\omega=\omega_{BW}=\omega_{ci}} = -180° + \phi_{mi}$$

$$\angle\left(\frac{V}{j\omega_{ci}L}\left(k_{p,i} + \frac{k_{i,i}}{j\omega_{ci}}\right)\right) = -180° + \phi_{mi}$$

$$\angle\frac{V}{j\omega_{ci}L} + \angle\left(k_{p,i} + \frac{k_{i,i}}{j\omega_{c,i}}\right) = -180° + \phi_{mi}$$

$$-\frac{\pi}{2} + \angle\left(k_{p,i} - j\frac{k_{i,i}}{\omega_{ci}}\right) = -180° + \phi_{mi}$$

$$-\frac{\pi}{2} + \tan^{-1}\left(\frac{-\frac{k_{i,i}}{\omega_{ci}}}{k_{p,i}}\right) = -180° + \phi_{mi}$$

$$-\frac{\pi}{2} - \tan^{-1}\left(\frac{k_{i,i}}{\omega_{ci}k_{p,i}}\right) = -180° + \phi_{mi}$$

$$-\frac{\pi}{2} - \left(\frac{\pi}{2} - \tan^{-1}\left(\frac{\omega_{ci}k_{p,i}}{k_{i,i}}\right)\right) = -180° + \phi_{mi}$$

Note: $\tan^{-1}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{x}\right)$

$$-\pi + \tan^{-1}\left(\frac{\omega_{ci}k_{p,i}}{k_{i,i}}\right) = -180° + \phi_{mi}$$

$$-180° + \tan^{-1}\left(\frac{\omega_{ci}k_{p,i}}{k_{i,i}}\right) = -180° + \phi_{mi}$$

$$\boxed{\boldsymbol{\phi_{mi} = \tan^{-1}\left(\frac{\omega_{ci}k_{p,i}}{k_{i,i}}\right)}}$$

4. Use your expression for $\angle\ell_i(j2\pi f_{ci}) = -180 + \phi_m$ to derive ω_{PIi}.**[2 pts]**
5. Use your expression for $|\ell_i(j2\pi f_{ci})| = 1$ to derive $G_{PIi\infty}$.**[2 pts]**

$$G_{ci}(s) = \left\{k_{p,i} + \frac{k_{i,i}}{s}\right\}$$

$$= G_{PIi\infty}\left(1 + \frac{\omega_{PIi}}{s}\right)$$

$$= G_{PIi\infty} + \frac{G_{PIi\infty}\,\omega_{PIi}}{s}$$

- $\boldsymbol{G_{PIi\infty} = k_{p,i}}$

- $G_{PIi\infty}\,\omega_{PIi} = k_{i,i}$

$$\omega_{PIi} = \frac{k_{i,i}}{G_{PIi\infty}} = \frac{\boldsymbol{k_{i,i}}}{\boldsymbol{k_{p,i}}} = \boldsymbol{\omega_{PIi}}$$

$$\angle \ell_i(s)|_{\omega=\omega_{BW}} = -180° + \phi_{mi}$$

$$\phi_{mi} = \tan^{-1}\left(\frac{\omega_{ci}k_{p,i}}{k_{i,i}}\right)$$

$$\tan(\phi_{mi}) = \frac{\omega_{ci}k_{p,i}}{k_{i,i}}$$

$$\underline{\frac{k_{i,i}}{k_{p,i}} = \frac{\omega_{ci}}{\tan(\phi_{mi})}}$$

Note:

$\tan(tan^{-1}(x)) = x$ & $\tan^{-1}(\tan(x)) = x - \pi \cdot \left(\frac{x}{\pi} + \frac{1}{2}\right)$

$$\|\ell_i(s)\|_{\omega=\omega_{BW}} = 1$$

$$\frac{L^2\omega_{ci}^4}{V^2} = (\omega_{ci}k_{p,i})^2 + k_{i,i}^2$$

$$\frac{L^2\omega_{ci}^4}{V^2} = (\omega_{ci}k_{p,i})^2 + \left(\frac{k_{p,i}\omega_{c,i}}{\tan(\phi_{mi})}\right)^2$$

$$k_{p,i}^2 = \frac{1}{\omega_{ci}^2 + \frac{\omega_{ci}^2}{\tan^2(\phi_{mi})}} \cdot \frac{L^2\omega_{ci}^4}{V^2}$$

$$\frac{1}{1 + \frac{1}{\tan^2(\phi_{mi})}} \cdot \frac{L^2\omega_{ci}^4}{\omega_{ci}^2 V^2} = \sin^2(\phi_{mi}) \cdot \frac{L^2\omega_{ci}^2}{V^2} = \left(\frac{L\omega_{ci}}{V}\sin(\phi_{mi})\right)^2$$

$$k_{p,i} = \pm\sqrt{\left(\frac{L\omega_{ci}}{V}\sin(\phi_{mi})\right)^2}$$

$$\underline{\boldsymbol{k_{p,i} = \frac{L\omega_{ci}}{V}\sin(\phi_{mi})}}$$

So we have:

$$\boxed{\boldsymbol{\omega_{PIi} = \frac{k_{i,i}}{k_{p,i}} = \frac{\omega_{ci}}{\tan(\phi_{mi})} = 3627.5987\,\frac{rad}{s}}}$$

$$\boxed{\boldsymbol{G_{PIi\infty} = k_{p,i} = \frac{L\omega_{ci}}{V}\sin(\phi_{mi}) = 0.14737}}$$

where: $k_{p,i} = 0.14737$ & $k_{i,i} = 534.60357$

6. Build a switched circuit model in PLECS to implement the inner current control of the boost converter. Using the Pulse Generator component, step the current controller set-point from rated input current to 50% of rated input current at a rate of 20 Hz. Based on the phase margin and bandwidth, verify the following time domain quantities : overshoot, rise time, settling time from the simulation. The theoretical values (use Table if needed) should match very closely to the predicted values from simulation. For the parameter verification, show plots of the duty ratio, inductor current, capacitor voltage.[**simulation : 20 pts, each parameter verification: 5 pts**]

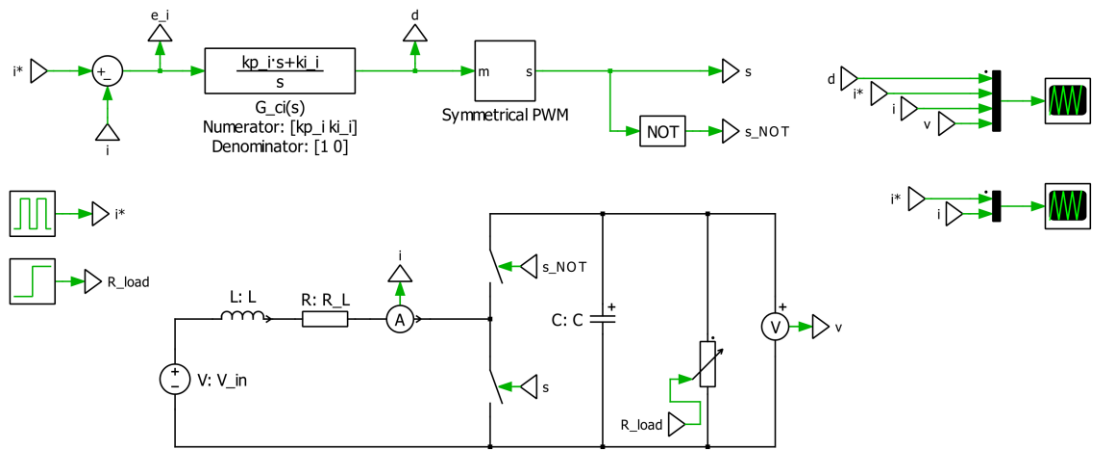


Figure: Current Control for Switched Circuit Boost Converter

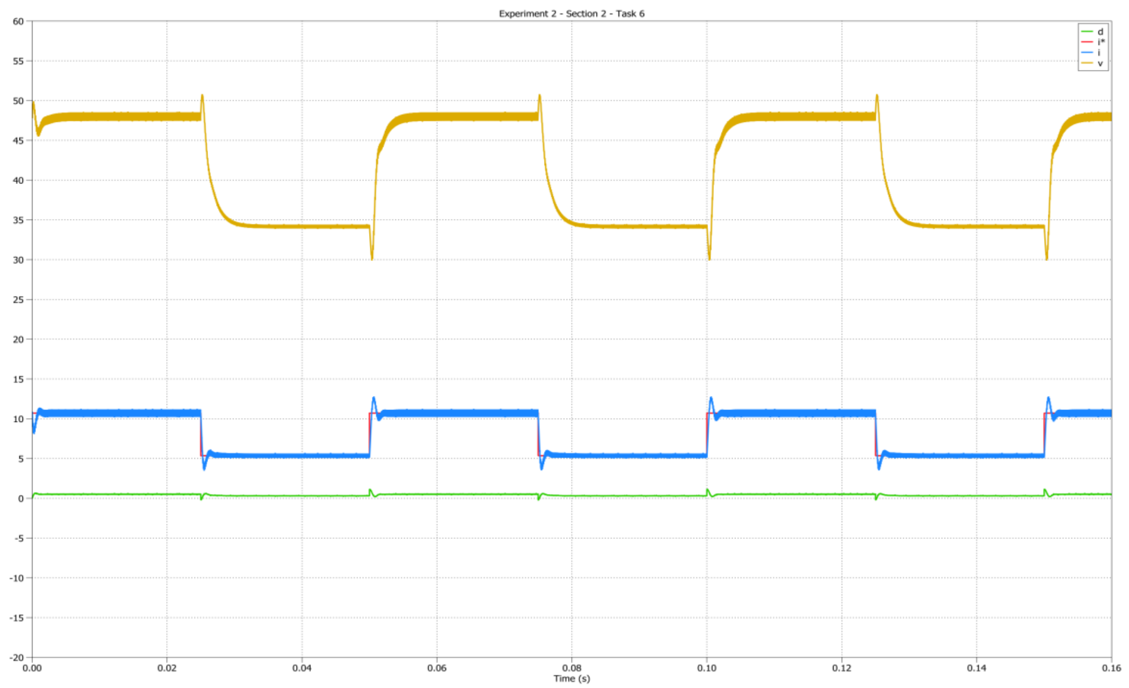


Figure: Current Control Waveforms: Tracking Pulse Generated I [20Hz]*

For $\ell_{i,approx}(s) = G_{ci}(s) \cdot G_{id,approx}(s)$, we have $PM_i = 60°$ & $\omega_{ci} = 6.283 \times 10^3 \frac{\text{rad}}{\text{s}}$, as such:

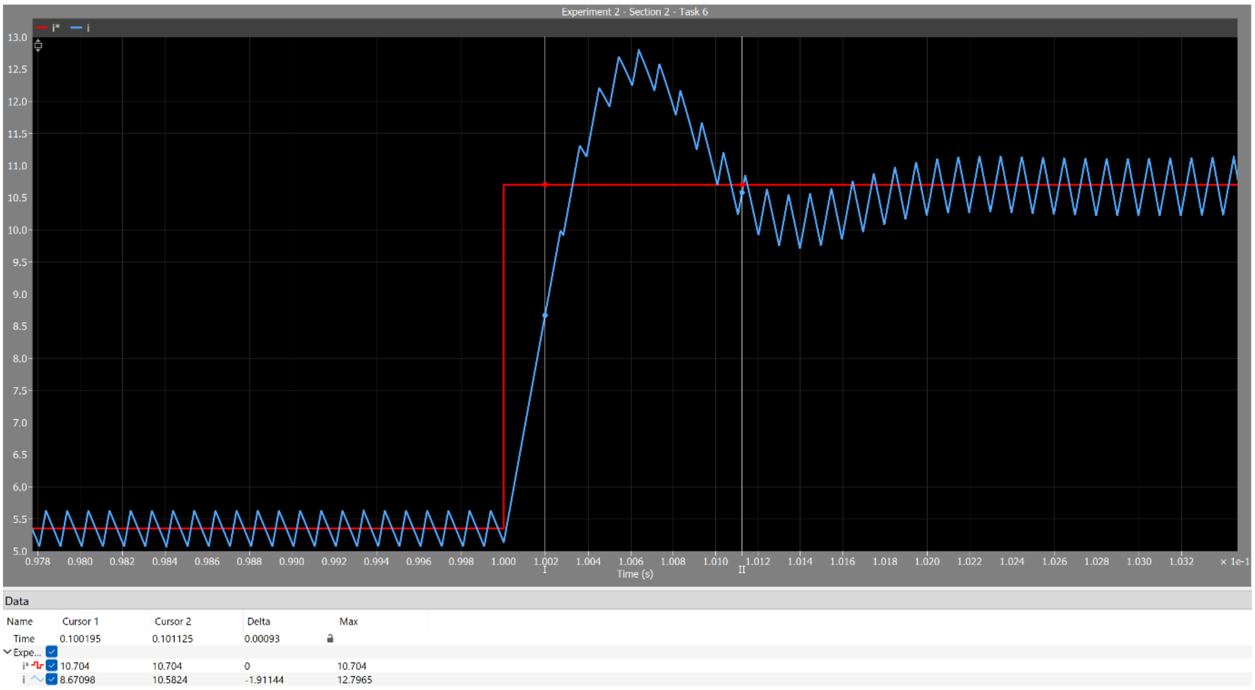
- $PM = \tan^{-1}\left(\frac{2\zeta}{\sqrt{1-2\zeta^2}}\right) \cdot \frac{180}{\pi}$

$$\underline{\boldsymbol{\zeta_i = \left|\frac{\tan\left(PM \cdot \frac{\pi}{180}\right)}{\sqrt{2 \cdot \tan^2\left(PM \cdot \frac{\pi}{180}\right) + 4}}\right|\Bigg|_{PM = PM_i = 60°} = 0.5477}}$$

- $\{\omega_c = \omega_{cg}\} = \omega_0\sqrt{1-2\zeta^2}$

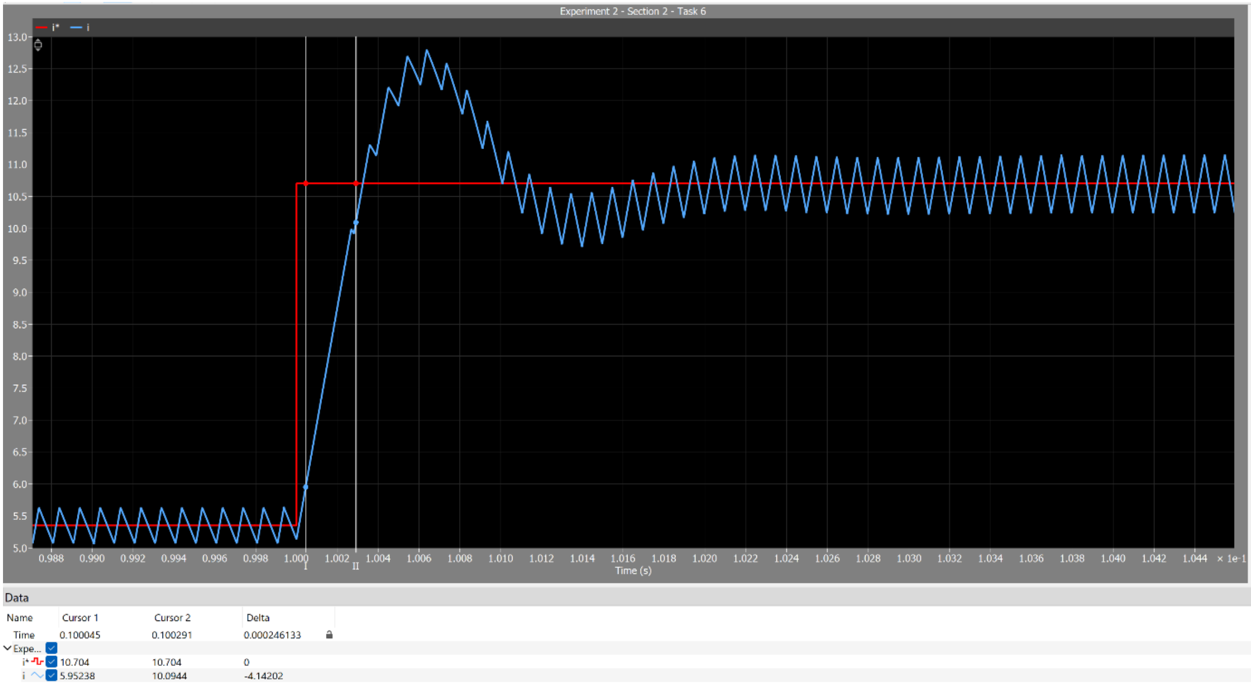
$$\underline{\boldsymbol{\omega_{0,i} = \frac{\omega_{cg} \approx \omega_{BW}}{\sqrt{1-2\zeta^2}}\Bigg|_{\substack{\omega_{cg} = \omega_{cg,i} = 6.283 \times 10^3 \frac{\text{rad}}{\text{s}} \\ \zeta_i = 0.5477}} = 9934.59 \frac{\text{rad}}{\text{s}}}}$$

Overshoot



$M_{p,PLECS} = 12.4965 - 10.704$	$\mathbf{1.7925\ Amps}$
$M_{p,\text{p.u.}} = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}\ \text{p.u.} = 0.1279\ \text{p.u.}$ $M_p = M_{p,\text{p.u.}} \cdot I_{steady-state} = (0.1279)(10.7031)$	$\mathbf{1.37\ Amps}$

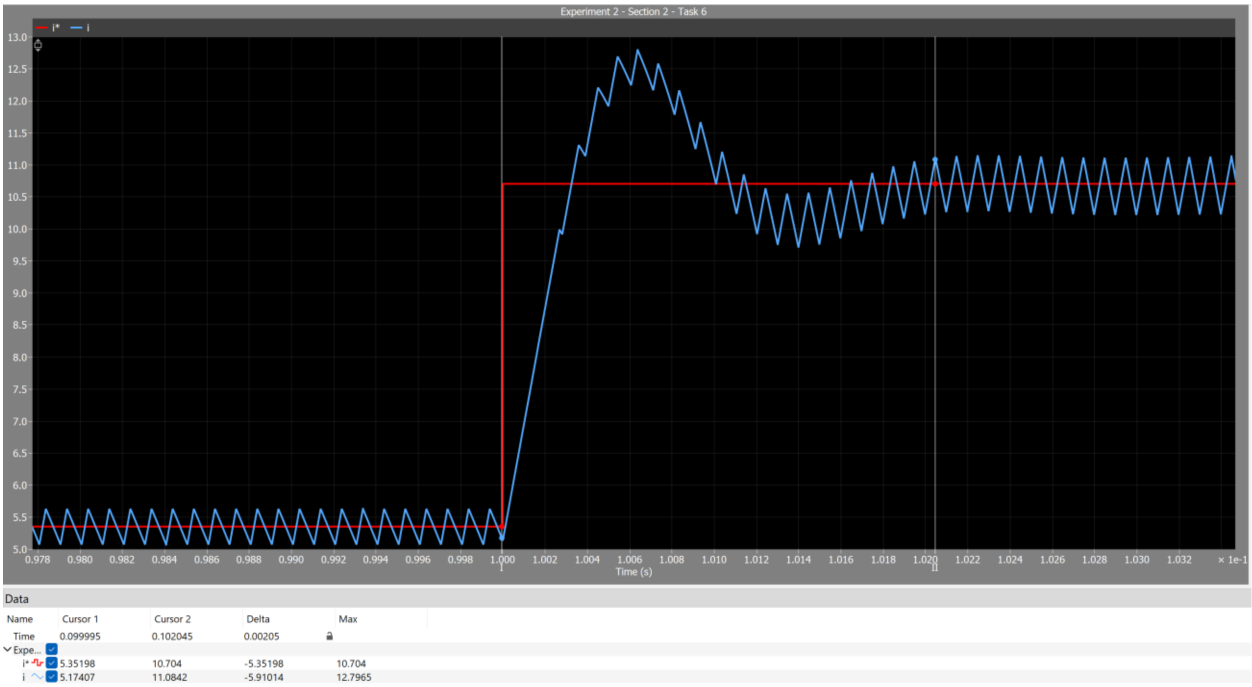
Rise Time



$t_{r,PLECS}$	$\mathbf{0.246\ ms}$
$t_r = \dfrac{\pi - \tan^{-1}\left(\dfrac{\sqrt{1-\zeta^2}}{\zeta}\right)}{\omega_0\sqrt{1-\zeta^2}} \approx \dfrac{5}{3\omega_0\sqrt{1-\zeta^2}}$	$\mathbf{0.2587\ ms}$

Settling Time

Settles to within 2% of the steady state value.



$t_{s,2\%,PLECS}$	$\mathbf{2.05\ ms}$
$t_{s,2\%} = \dfrac{4}{\zeta\omega_n}$	$\mathbf{0.735\ ms}$

7. In the simulation, show that the controller can reject a 10% change in the output resistance. **[10 pts]**

At time $t = 0.16$ seconds, the load resistance is changed to $R_{load,new} = 1.1R_{load}$. As is evident, our controller is able to reject the 10% change in output resistance and continue tracking our reference.

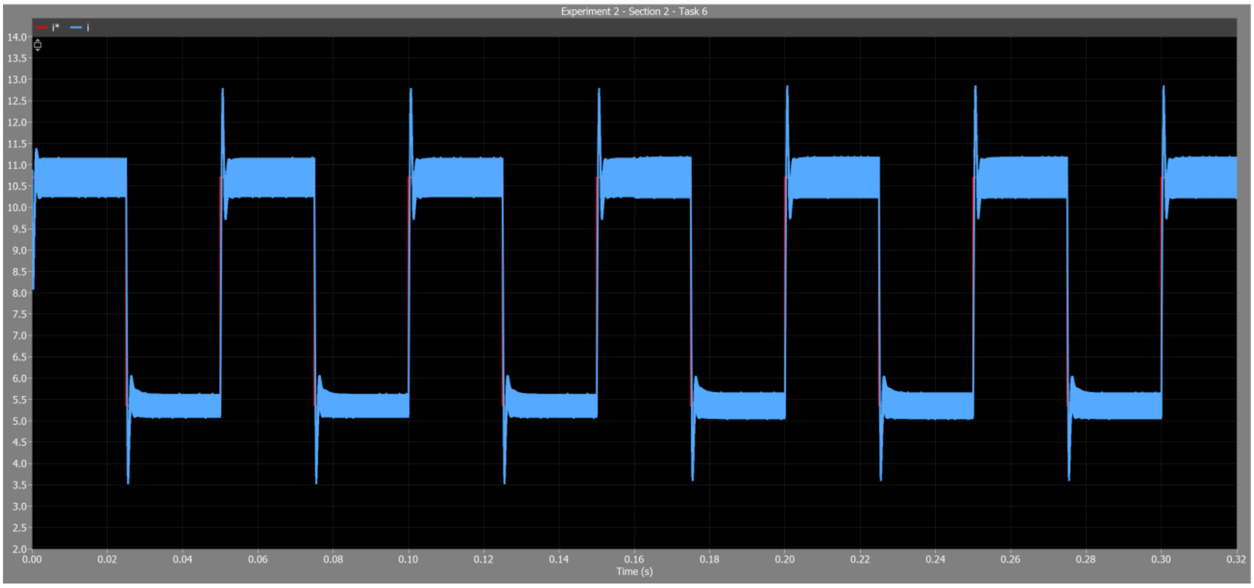
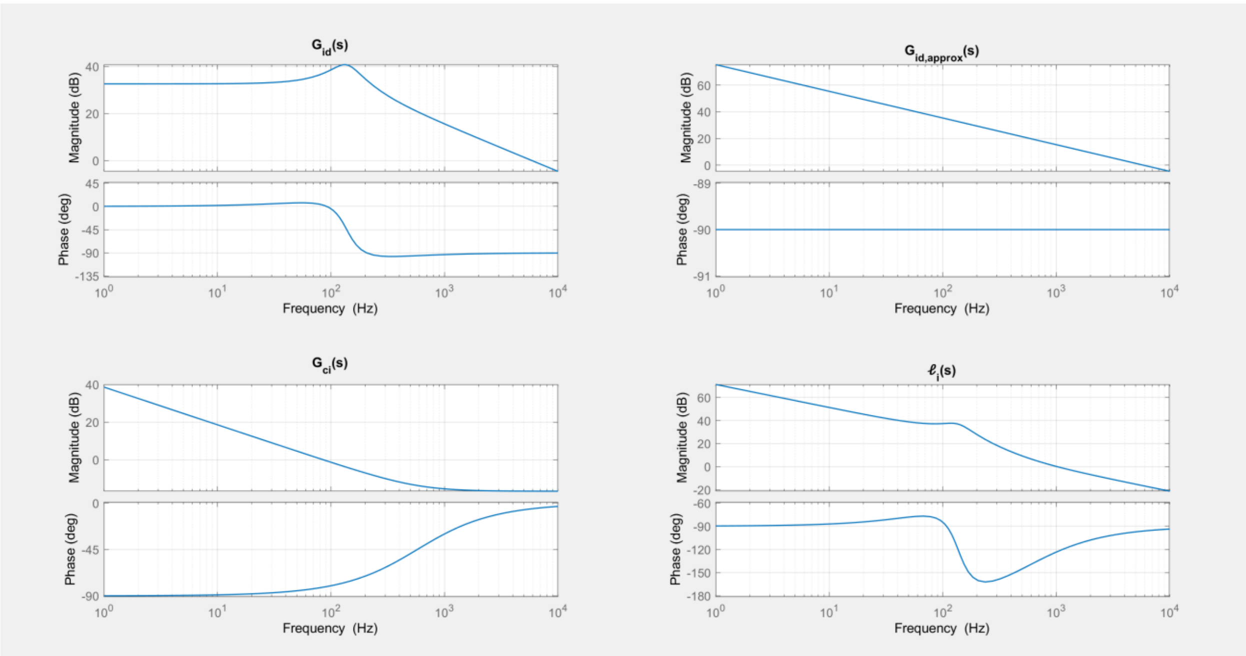
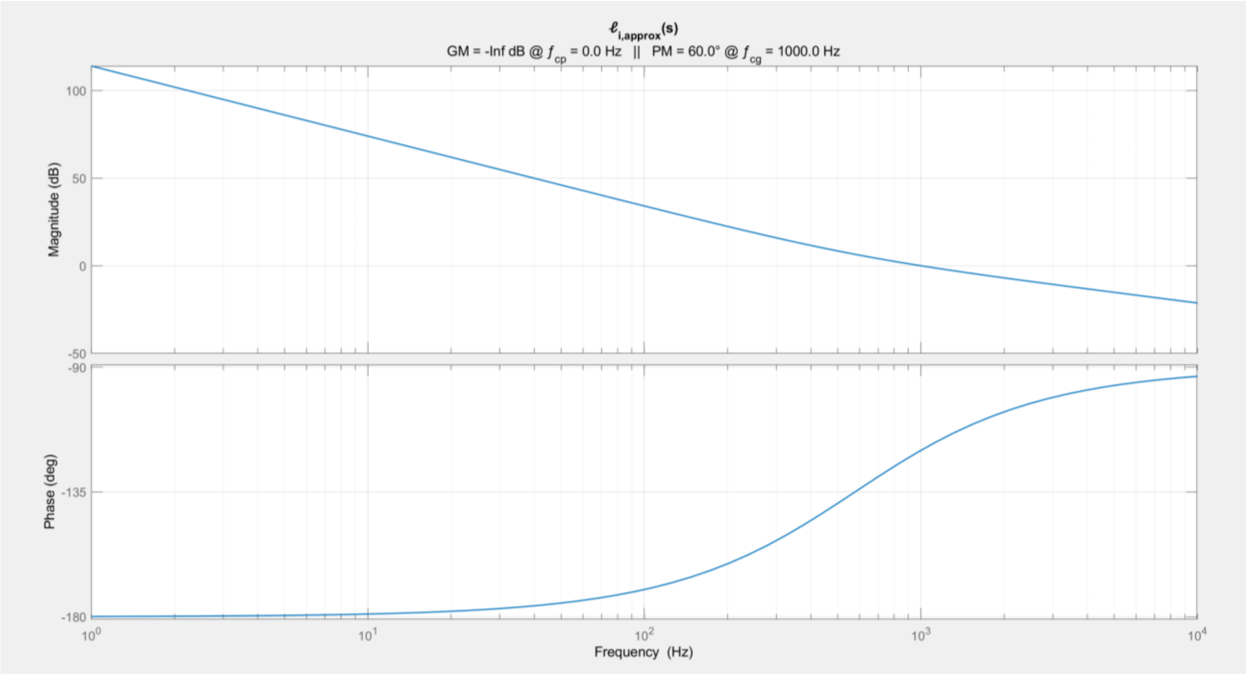


Figure: Current Controller Disturbance Rejection [10% change]

8. In your lab report, include plots of following transfer functions: $G_{id}(s)$, $G_{id,approx}(s)$, $G_{ci}(s)$, and $\ell_i(s)$. **[4X5 pts]**



The bode plot of the loop gain of the implemented current control system:



9. **[Only for 559]**When modeling the load of the boost converter as a resistor, a change in current reference will lead to a change in output voltage. When the reference current is increased, what initially happens to the output voltage? What can be said about the initial trajectory of the output voltage in response to a change in inductor current? What transfer function characteristic does this type of relationship illustrate? **[8 pts]**

- When we increase our reference current, the output voltage slightly dips down (decreases) before increasing.
- The initial trajectory of output voltage is opposite the change in inductor current.
- This relationship (of output voltage dipping slightly down when current reference increases and vice versa when current reference decreases) illustrates a **non**-minimum phase system. In other words, whenever our converter's transfer function has a right-half zero, then if we try to do any type of tracking, then we will observe that whatever it is we are trying to track will first go in the opposite direction of control before finally going in the direction of control.

Section 3

Referring to Chapter 4.2.3 of [1], obtain the following transfer function. **[5 pts]**

$$G_{vi}(s) = \frac{G_{vd}(s)}{G_{id}(s)} \approx \frac{D'R_{load}}{2} - \frac{R_L}{2D'}$$

$$G_{vi}(s) = \frac{\tilde{v}(s)}{\tilde{\imath}(s)} = \frac{-R\cdot(IR_L - D'V + ILs)}{V + D'IR + CRVs}$$

$$= \frac{R}{V(1+CRs) + ID'R}\cdot[D'V - I(R_L + Ls)]$$

$$= \frac{R}{V(1+CRs) + \left(\frac{V}{D'R}\right)D'R}\cdot\left[D'V - \left(\frac{V}{D'R}\right)(R_L + Ls)\right]$$

$$= \frac{R}{V(1+CRs)+V}\cdot\left[D'V - \left(\frac{V}{D'R}\right)(R_L + Ls)\right]$$

$$= \frac{1}{2+CRs}\cdot\frac{R}{V}\cdot\left[VD' - \left(\frac{V}{D'R}\right)(R_L + Ls)\right]$$

$$= \frac{1}{2+CRs}\cdot\frac{R}{V}\cdot\frac{1}{D'R}\cdot[V(D')^2R - V(R_L + Ls)]$$

$$= \frac{1}{VD'(2+CRs)}\cdot[V(D')^2R - V(R_L + Ls)]$$

$$= \frac{V(D')^2R}{VD'(2+CRs)} - \frac{V(R_L+Ls)}{VD'(2+CRs)}$$

$$= \frac{D'R}{2+CRs} - \frac{R_L + Ls}{D'(2+CRs)}$$

The approximation we can make is:

$$G_{vi}(s) \approx \lim_{s\to 0}\frac{D'R}{2+CRs} - \frac{R_L + Ls}{D'(2+CRs)}$$

$$= \boxed{\frac{\boldsymbol{D'R}}{\boldsymbol{2}} - \frac{\boldsymbol{R_L}}{\boldsymbol{2D'}} = \boldsymbol{G_{vi,approx}(s)}} \quad for\ \boldsymbol{s \ll \omega_0}$$

This approximation will not hold true if s is **not** much less than ω_0.

1. Design a PI regulator for the outer loop voltage controller. Document your design: **[8X2=16pts]**

   (a) Voltage controller crossover frequency, f_{cv}:______________________________

   $$f_{cv} = \frac{f_{ci}}{10} = \frac{1\text{ kHz}}{10} = \boxed{\mathbf{100\ Hz} = \boldsymbol{f_{cv}}} \quad \text{or} \quad \omega_{cv} = \frac{\omega_{ci}}{10} = 2\pi f_{cv} = \boxed{\mathbf{628.319}\,\frac{\mathbf{rad}}{\mathbf{s}} = \boldsymbol{\omega_{cv}}}$$

   (b) Voltage controller phase margin design target, ϕ_m:________________________

   $$\boxed{\boldsymbol{\phi_{mv}} = \mathbf{100° = 1.7453\ rad}}$$

(c) Uncompensated open loop gain, $|\ell_{u,v}(j2\pi f_{cv})|$:______________________________

(d) Uncompensated open loop phase delay, $\angle\ell_{u,v}(j2\pi f_{cv})$:______________________________

$$\{\ell_{u,v}(s) = G_{vi}(s)\}\big|_{s=j\omega_{cv}=j2\pi f_{cv}} = G_{vi}(j628.319)$$

```
G_vi evaluated at 6.283185e+02 rad/s:
   Gain  = 1.89 = 5.54 dB
   Phase = -0.99 rad = -56.95°
```

$$\{\ell_{u,v,approx}(s) = G_{vi,approx}(s)\}\big|_{s=j\omega_{cv}=j2\pi f_{cv}} = G_{vi,approx}(j628.319)$$

```
G_vi_approx evaluated at 6.283185e+02 rad/s:
   Gain  = 2.18 = 6.77 dB
   Phase = 0.00 rad = 0.00°
```

(e) PI Controller zero frequency, ω_{PIv}:______________________________

(f) PI Controller high frequency asymptote, $G_{PIv\infty}$:______________________________

$$G_{cv}(s) = \left\{k_{p,v} + \frac{k_{i,v}}{s}\right\} = \left\{G_{PIv\infty}\left(1 + \frac{\omega_{PIv}}{s}\right) = G_{PIv\infty} + \frac{G_{PIv\infty}\,\omega_{PIv}}{s}\right\}$$

- $\underline{\boldsymbol{G_{PIv\infty} = k_{p,v}}}$
- $G_{PIv\infty}\,\omega_{PIv} = k_{i,v} \Longrightarrow \omega_{PIv} = \frac{k_{i,v}}{G_{PIv\infty}} = \underline{\boldsymbol{\frac{k_{i,v}}{k_{p,v}} = \omega_{PIv}}}$

$$\ell_v(s) = G_{cv}(s)G_{vi}(s) \approx G_{cv}(s)G_{vi,approx}(s) = \left(k_{p,v} + \frac{k_{i,v}}{s}\right)\left(\frac{D'R}{2} - \frac{R_L}{2D'}\right)$$

$$\angle\ell_v(s)|_{\omega=\omega_{BW}} = -180° + \phi_{mv}$$

$$\Rightarrow \angle\left(\left(\frac{D'R}{2} - \frac{R_L}{2D'}\right)\left(k_{p,v} + \frac{k_{i,v}}{s}\right)\right)\Bigg|_{\omega=\omega_{BW}\approx\omega_{cv}} = -180° + \phi_{mv}$$

$$\Rightarrow \left\{\angle\left(\frac{D'R}{2} - \frac{R_L}{2D'}\right) + \angle\left(k_{p,v} + \frac{k_{i,v}}{j\omega_{cv}}\right) = 0 + \angle\left(k_{p,v} - j\frac{k_{i,v}}{\omega_{cv}}\right)\right\} = -180° + \phi_{mv}$$

$$\Rightarrow \left\{\tan^{-1}\left(\frac{-\frac{k_{i,v}}{\omega_{cv}}}{k_{p,v}}\right) = -\tan^{-1}\left(\frac{k_{i,v}}{\omega_{cv}\,k_{p,v}}\right)\right\} = -\pi + \phi_{mv}$$

$$\Rightarrow \phi_{mv} = \pi - \tan^{-1}\left(\frac{k_{i,v}}{\omega_{cv}\,k_{p,v}}\right) \Rightarrow \tan(\pi - \phi_{mv}) = \frac{k_{i,v}}{\omega_{cv}\,k_{p,v}} \Rightarrow \frac{k_{i,v}}{k_{p,v}} = \omega_{cv}\tan(\pi - \phi_{mv})$$

$$\Rightarrow \underline{\boldsymbol{\frac{k_{i,v}}{k_{p,v}} = -\omega_{cv}\tan(\phi_{mv})}}$$

$\|\ell_v(s)\|_{\omega=\omega_{BW}} = 1$

$$\Longrightarrow \left\{ \left\| \left(\frac{D'R}{2} - \frac{R_L}{2D'}\right)\left(k_{p,v} + \frac{k_{i,v}}{s}\right) \right\|_{\omega=\omega_{BW}\approx\omega_{cv}} = \left\| \left(\frac{D'R}{2} - \frac{R_L}{2D'}\right)\left(k_{p,v} + \frac{k_{i,v}}{j\omega}\right) \right\|_{\omega=\omega_{BW}\approx\omega_{cv}} \right\} = 1$$

$$\Longrightarrow \left\{ \left\| \left(\frac{D'R}{2} - \frac{R_L}{2D'}\right)\left(k_{p,v} + \frac{k_{i,v}}{j\omega_{cv}}\right) \right\| = \left\| \left(\frac{D'R}{2} - \frac{R_L}{2D'}\right)\left(\frac{j\omega_{cv}k_{p,v} + k_{i,v}}{j\omega_{cv}}\right) \right\| \right\} = 1$$

$$\Longrightarrow \left\{ \left(\frac{D'R}{2} - \frac{R_L}{2D'}\right) \left\| \left(\frac{j\omega_{cv}k_{p,v} + k_{i,v}}{j\omega_{cv}}\right) \right\| = \left(\frac{D'R}{2} - \frac{R_L}{2D'}\right) \frac{\sqrt{k_{i,v}^2 + (\omega_{cv}k_{p,v})^2}}{\omega_{c,v}} \right\} = 1$$

$$\Longrightarrow \left(\frac{D'R}{2} - \frac{R_L}{2D'}\right) \sqrt{\left(\frac{k_{i,v}}{\omega_{cv}}\right)^2 + k_{p,v}^2} = 1 \Longrightarrow \left(\frac{k_{i,v}}{\omega_{cv}}\right)^2 + k_{p,v}^2 = \frac{1}{\left(\frac{D'R}{2} - \frac{R_L}{2D'}\right)^2}$$

$$\Longrightarrow \left(\frac{-k_{p,v}\omega_{cv}\tan(\phi_{mv})}{\omega_{cv}}\right)^2 + k_{p,v}^2 = \frac{1}{\left(\frac{D'R}{2} - \frac{R_L}{2D'}\right)^2} \Longrightarrow k_{p,v}^2[\tan^2(\phi_{mv}) + 1] = \frac{1}{\left(\frac{D'R}{2} - \frac{R_L}{2D'}\right)^2}$$

$$\Longrightarrow k_{p,v}^2 \frac{1}{\cos^2(\phi_{mv})} = \frac{1}{\left(\frac{D'R}{2} - \frac{R_L}{2D'}\right)^2} \Longrightarrow k_{p,v} = \sqrt{\frac{\cos^2(\phi_{mv})}{\left(\frac{D'R}{2} - \frac{R_L}{2D'}\right)^2}} = \frac{\sqrt{\cos^2(\phi_{mv})}}{\frac{D'R}{2} - \frac{R_L}{2D'}}$$

$$\Longrightarrow \underline{\boldsymbol{k_{p,v} = |\cos(\phi_{mv})| \cdot \frac{2D'}{(D')^2R - R_L}}}$$

So we have:

$$\boxed{\boldsymbol{\omega_{PIv} = \frac{k_{i,v}}{k_{p,v}} = -\omega_{cv}\tan(\phi_{mv}) = 3563.37\,\frac{\mathrm{rad}}{\mathrm{s}}}} \qquad \boxed{\boldsymbol{G_{PIv\infty} = k_{p,v} = |\cos(\phi_{mv})| \cdot \frac{2D'}{(D')^2R - R_L} = 0.07963}}$$

where: $k_{p,v} = 0.0796$ & $k_{i,v} = 283.7495$

(g) Compensated open loop gain, $|\ell_v(j2\pi f_{cv})|$:______________________________

(h) Compensated open loop phase margin, $180° + \angle\ell_v(j2\pi f_{cv})$:________________________

```
l_v_approx evaluated at 6.283185e+02 rad/s:
   Gain  = 1.00 = -0.00 dB
   Phase = -1.40 rad = -80.00°
```

At $f_{cv} = 100$ Hz, the compensated open loop:

- Gain $= |\ell_v(j2\pi f_{cv})|\big|_{f_{cv}=100\text{ Hz}}$

  $= \boxed{\mathbf{0\ dB = Gain}}$

- Phase Margin $= 180° + \text{Phase}$

  $= 180° + \angle\ell_v(j2\pi f_{cv})|_{f_{cv}=100\text{ Hz}}$

  $= 180° + (-80°)$

  $= \boxed{\mathbf{100° = PM}}$

2. Build a switched circuit model in PLECS to implement the outer voltage control of the boost converter. Using the Pulse Generator component, step the voltage controller set-point from rated voltage to 80% of rated voltage at a rate of 1 Hz. Based on the phase margin and bandwidth, verify the following time domain quantities : overshoot, rise time, settling time from the simulation. The theoretical values (use Table if needed) should match very closely to the predicted values from simulation. For the parameter verification, show plots of the duty ratio, inductor current reference, inductor current, capacitor voltage.[**simulation : 20 pts, each parameter verification: 5 pts**]

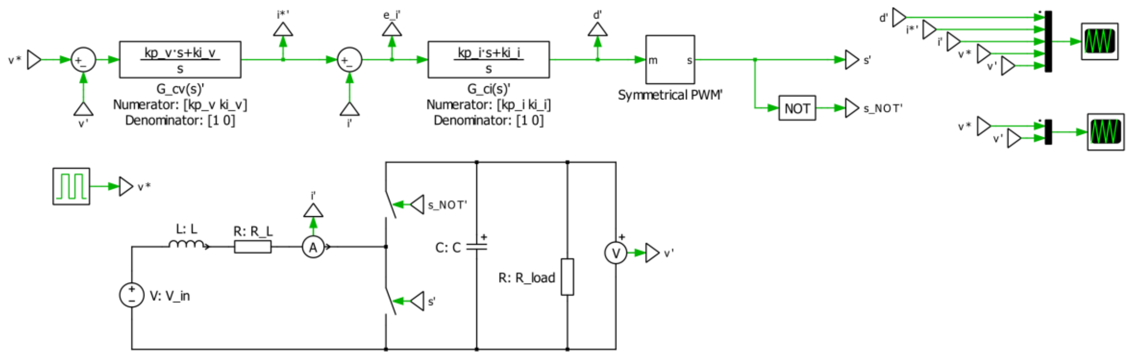


Figure: Voltage Control for Switched Circuit Boost Converter

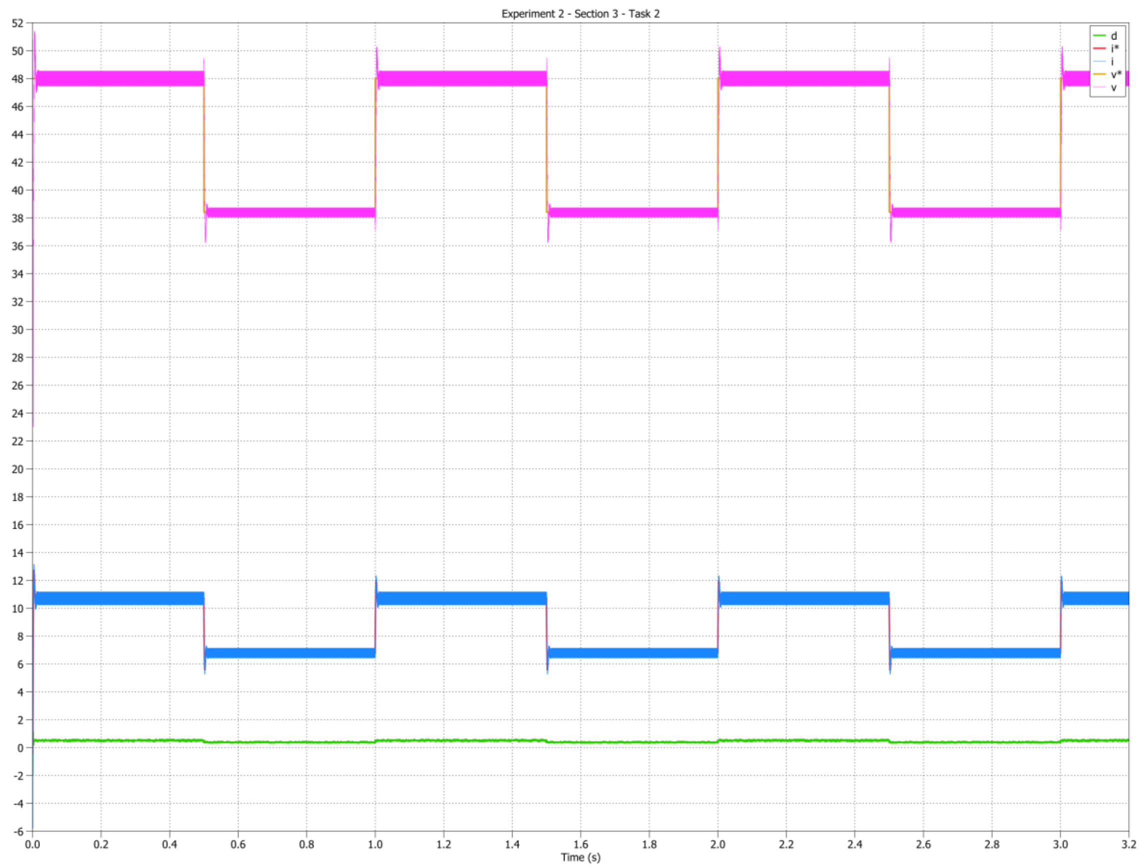


Figure: Voltage Control Waveforms: Tracking Pulse Generated V [1Hz]*

For $\ell_{v,approx}(s) = G_{cv}(s) \cdot G_{vi,approx}(s)$, we have $PM_v = 80°$ & $\omega_{cv} = 628.318 \frac{\text{rad}}{\text{s}}$, as such:

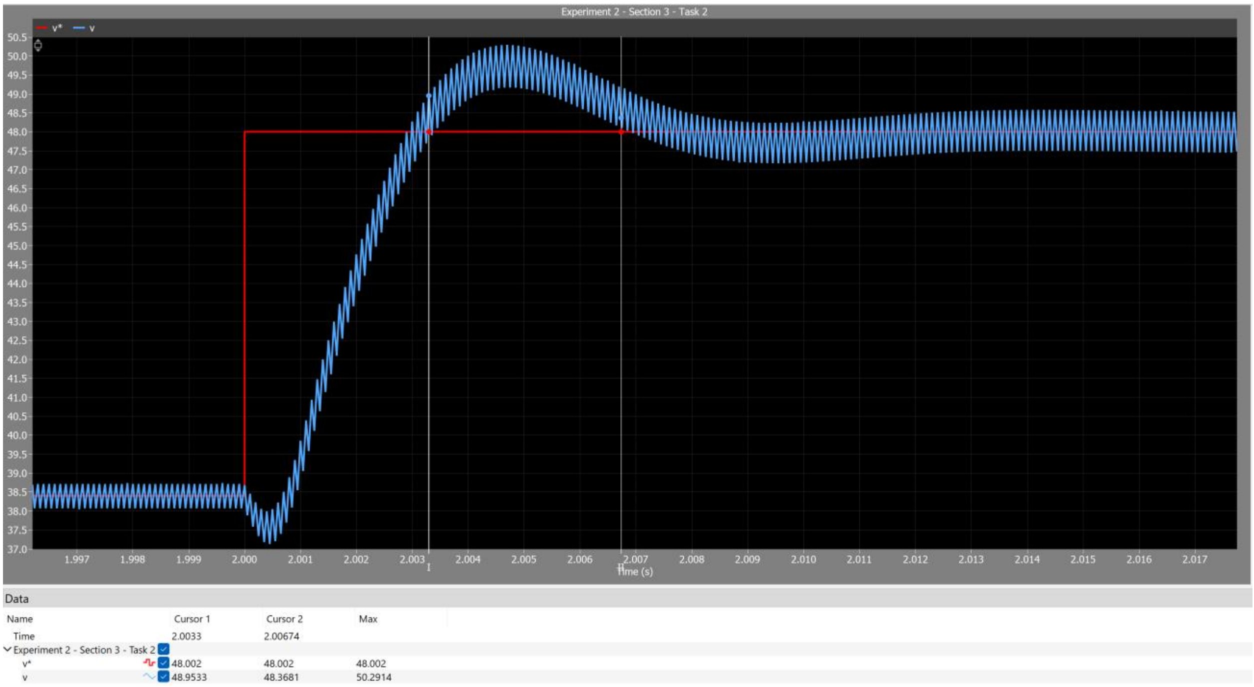
- $PM = \tan^{-1}\left(\frac{2\zeta}{\sqrt{1-2\zeta^2}}\right) \cdot \frac{180}{\pi}$

$$\underline{\boldsymbol{\zeta_v = \left|\left|\frac{\tan\left(PM \cdot \frac{\pi}{180}\right)}{\sqrt{2 \cdot \tan^2\left(PM \cdot \frac{\pi}{180}\right) + 4}}\right|\right|_{PM = PM_v = 80°} = 0.686}}$$

- $\{\omega_c = \omega_{cg}\} = \omega_0\sqrt{1-2\zeta^2}$

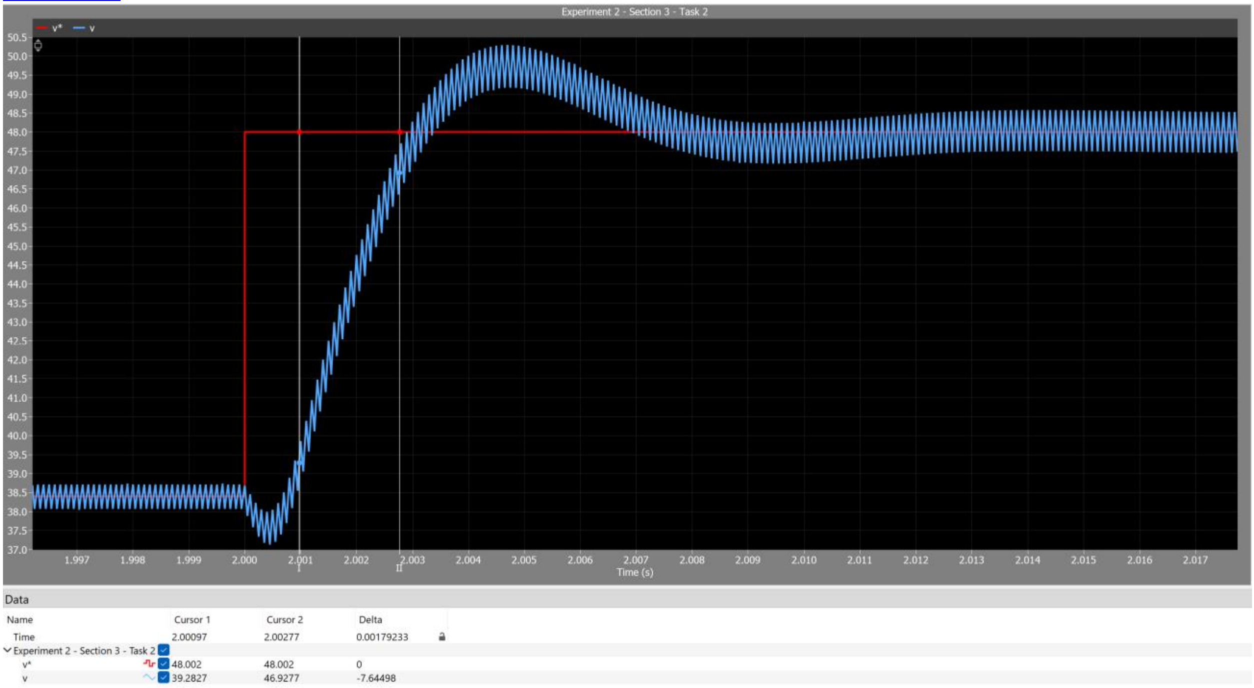
$$\underline{\boldsymbol{\omega_{0,v} = \frac{\omega_{cg} \approx \omega_{BW}}{\sqrt{1-2\zeta^2}}\Bigg|_{\substack{\omega_{cg} = \omega_{cg,v} = 628.318\frac{\text{rad}}{\text{s}} \\ \zeta_v = 0.686}} = 2596.843 \frac{\text{rad}}{\text{s}}}}$$

Overshoot



$M_{p,PLECS} = 50.2914 - 48.00$	$\mathbf{2.2914\ Volts}$
$M_{p,\text{p.u.}} = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}\ \text{p.u.} = 0.0517\ \text{p.u.}$ $M_p = M_{p,\text{p.u.}} \cdot V_{steady-state} = (0.0517)(48)$	$\mathbf{2.4816\ Volts}$

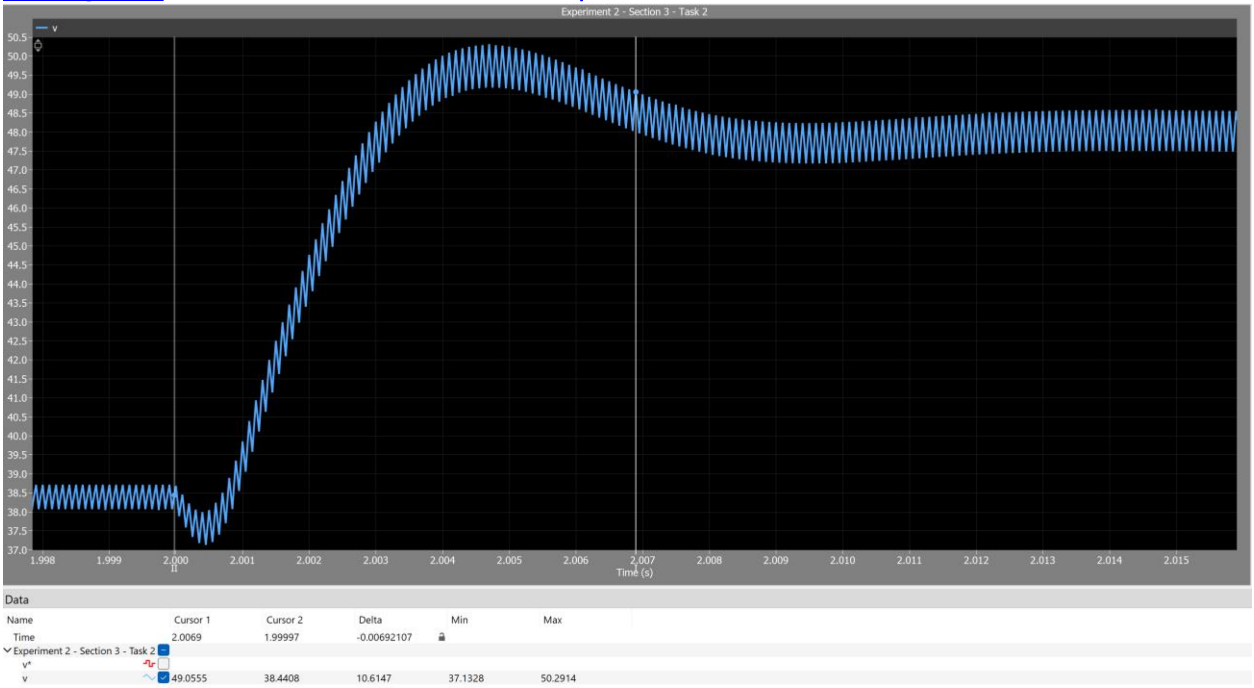
Rise Time



$t_{r,PLECS}$	$\mathbf{1.792\ ms}$
$t_r = \dfrac{\pi - \tan^{-1}\left(\dfrac{\sqrt{1-\zeta^2}}{\zeta}\right)}{\omega_0\sqrt{1-\zeta^2}} \approx \dfrac{5}{3\omega_0\sqrt{1-\zeta^2}}$	$\mathbf{1.23\ ms}$

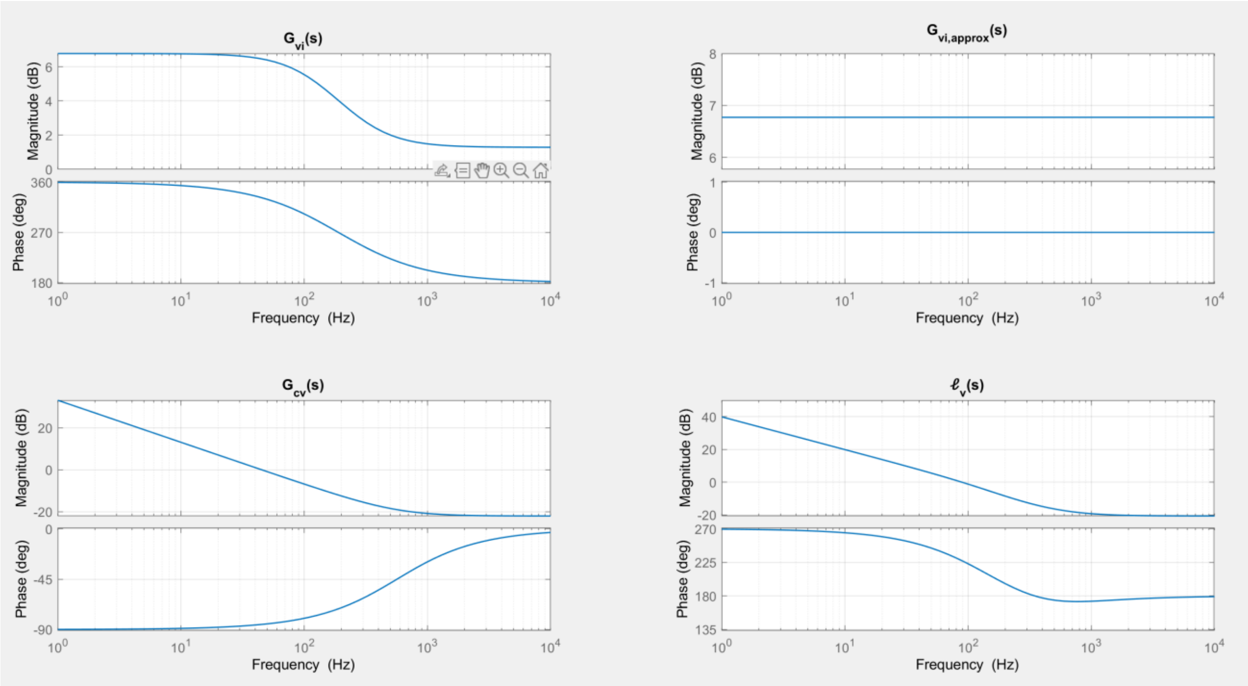
Settling Time

Settles to within 2% of the steady state value.

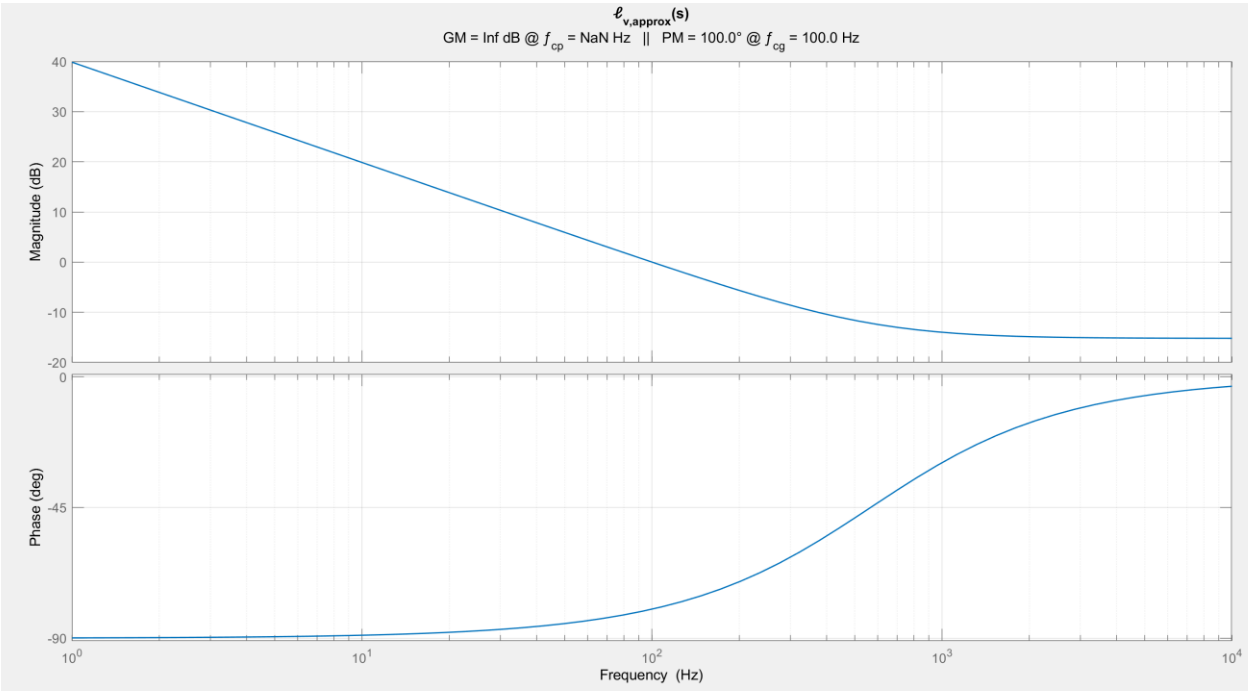


$t_{s,2\%,PLECS}$	$\mathbf{6.92\ ms}$
$t_{s,2\%} = \dfrac{4}{\zeta\omega_n}$	$\mathbf{2.25\ ms}$

3. In your lab report, include bode plots of $G_{vi}(s), G_{vi,approx}(s), G_{cv}(s), \ell_v(s)$. **[4X5 pts]**



The bode plot of the loop gain of the implemented voltage control system:



4. [**Only for 559**] Why do we have an inner current and an outer voltage control loop. Why not the reverse or, maybe the two loops in cascade or any other architecture you could think of?[**4 pts**]

The reason the voltage control loop is on the outside has to do with the fact that our objective is to control the capacitor (i.e., output) voltage and not the inductor current.

The reason we have the inner current loop is to better control the system (i.e., to improve performance of reference tracking) because it is actually the Inductor which stores energy in one cycle and then passes energy to the output in another cycle. So, if we are able to more effectively control the current through the inductor, then we in fact have better control of the output voltage.

We would **not** want two cascaded control loops because it sums the phases of both systems together, and the resulting phase margin would be low. Implementing a nested control system (i.e., an inner and outer control loop) and configuring the outside loop to have a bandwidth much smaller than the bandwidth of the inner loop (i.e., outer loop's bandwidth is approximately 1/10 of the inner loop's bandwidth), we are able to represent the inner control loop as a unity gain with respect to the outer control loop. In this way, we are able to avoid the reduction in phase margin as well as simplify system analysis.

Select appropriate scaling factors. Show calculations for them. **[3X3 = 9 pts]**

Formulas relating the integer ADC values of the inductor current and capacitor voltage to the actual (i.e., physically measured) values are:

$$i = i^{adc} \cdot \frac{V_{max-adc-input}}{2^{N_{adc}}} \cdot \frac{1}{\kappa_i} = i^{adc} \cdot \frac{3.3}{4096} \cdot \frac{1}{\kappa_i}$$

$$v = v^{adc} \cdot \frac{V_{max-adc-input}}{2^{N_{adc}}} \cdot \frac{1}{\kappa_i} = v^{adc} \cdot \frac{3.3}{4096} \cdot \frac{1}{\kappa_i}$$

The appropriate scaling factors are:

$$\kappa_{i_L} = i_{L,scale} = \frac{V_{max-ADC-input}}{i_{L,nominal} + i_{L,safety-margin}} = \frac{3.3\ \mathrm{V}}{10\ \mathrm{A} + 2\ \mathrm{A}} = 0.275$$

$$\kappa_{V_{in}} = V_{in,scale} = \frac{V_{max-ADC-input}}{V_{in,nominal} + V_{in,safety-margin}} = \frac{3.3\ \mathrm{V}}{24\ \mathrm{V} + 6\ \mathrm{V}} = 0.11$$

$$\kappa_{V_{out}} = V_{out,scale} = \frac{V_{max-ADC-input}}{V_{out,nominal} + V_{out,safety-margin}} = \frac{3.3\ \mathrm{V}}{48\ \mathrm{V} + 12\ \mathrm{V}} = 0.055$$

Note: $X_{safety-margin} \approx 0.25\, X_{nominal}$

You should saturate this yi within an upper, $yi^{\max}$ and lower value, $yi^{\min}$. Select and report these values. You can select these based on the EE452 lab. **[2 pts]**

yi^{max}	0.95
yi^{min}	0.05

Remember to saturate the output of the integrator to an upper, $yi^{\max}_{\text{int}}$ and lower value, $yi^{\min}_{\text{int}}$. This is required to obtain steady converter control. You can try without these too, and compare the results. These values are not random and depend strongly on your circuit operating conditions. Select these values according to nominal operating conditions and report them. **[4 pts]**

yi^{max}_{int}	0.95
yi^{min}_{int}	-0.95

Show waveforms for the inductor current, capcitor voltage and yi for a rated reference, i_{ref} to prove that the inner current control works. **[3X10 pts]**

Because the reference voltage $V_{ref} = 48$ V was defined in CCS using the *#define* keyword, it is not possible to graph it in CCS.

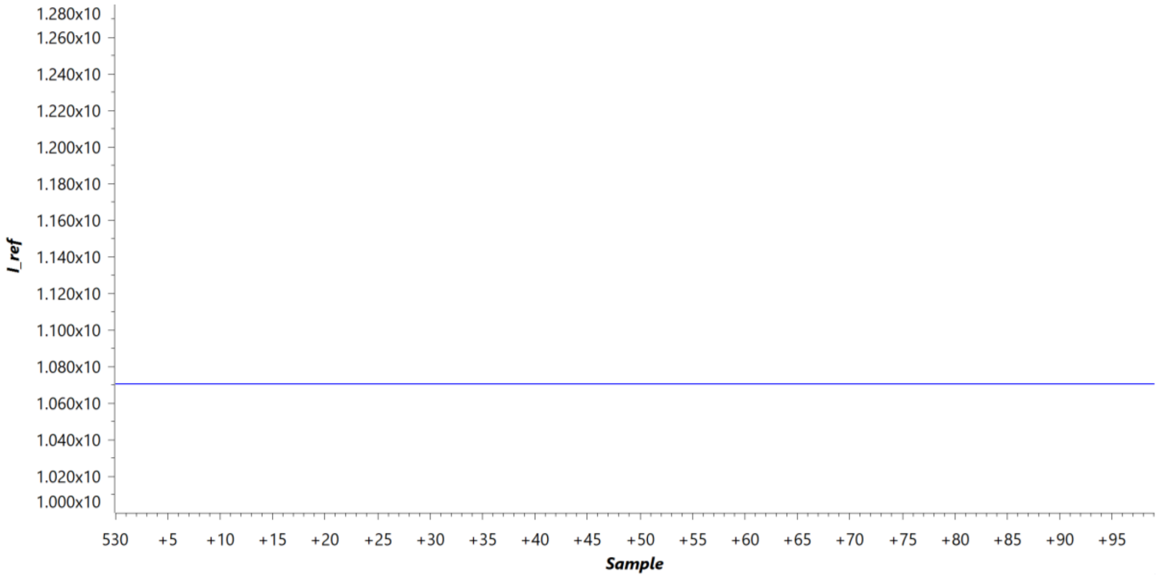


Figure: $I_{ref} = 10.7$ A

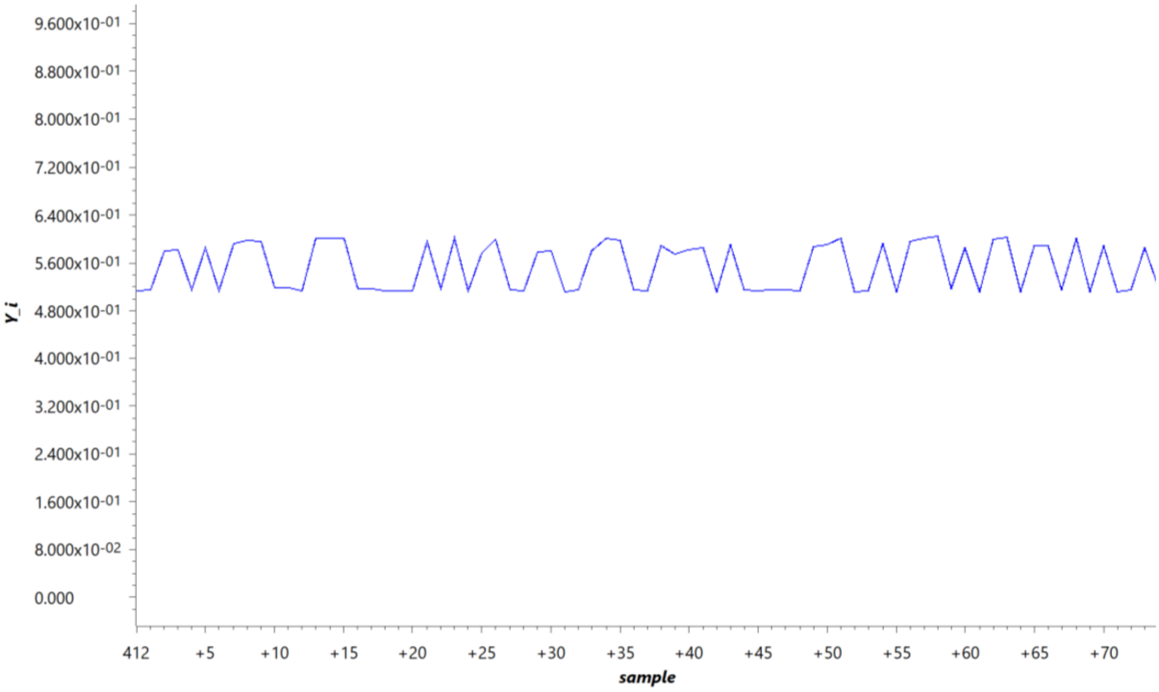


Figure: Current Controller Output (Y_i) for $I_{ref} = 10.7$ A

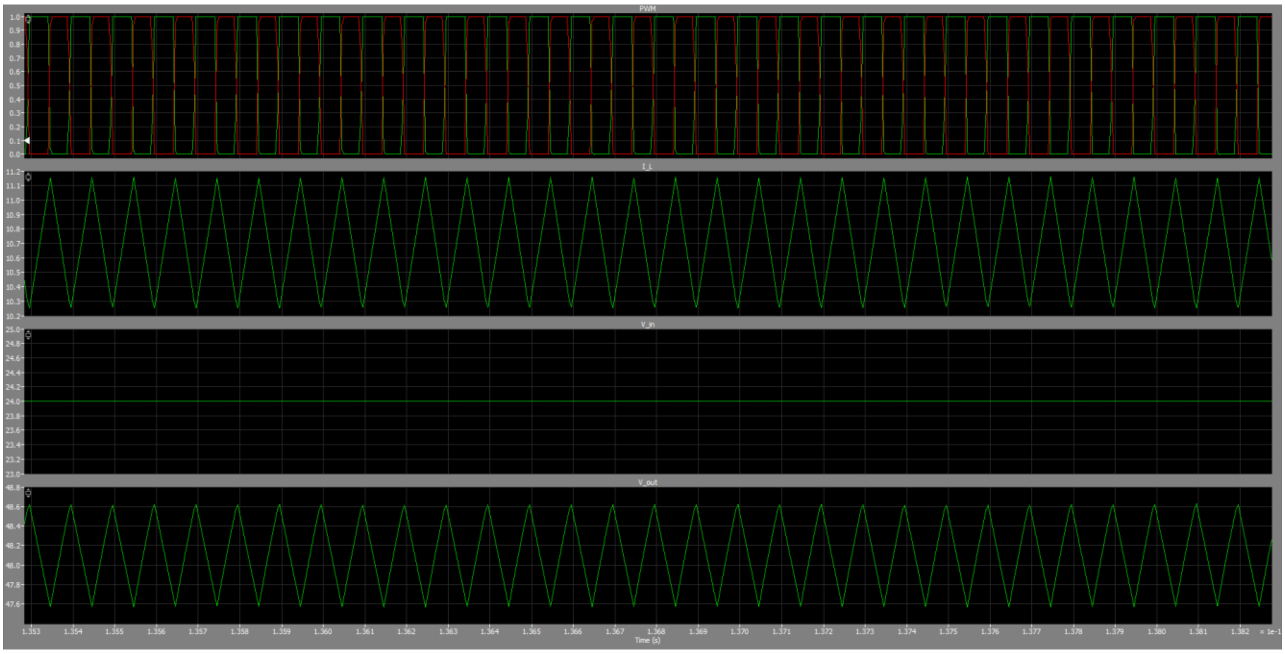


Figure: PLECS Waveforms for $I_{ref} = 10.7$ A

Show waveforms for the inductor current, inductor current reference, capcitor voltage and yi for a rated reference, v_{ref} to prove that the outer voltage control works. Note: you may use the CCS graphing feature to graph software variables, and PLECS to plot "analog" waveforms. Other methods are also acceptable. **[4X5 pts]**

Because the reference voltage $V_{ref} = 48$ V was defined in CCS using the *#define* keyword, it is not possible to graph it in CCS.

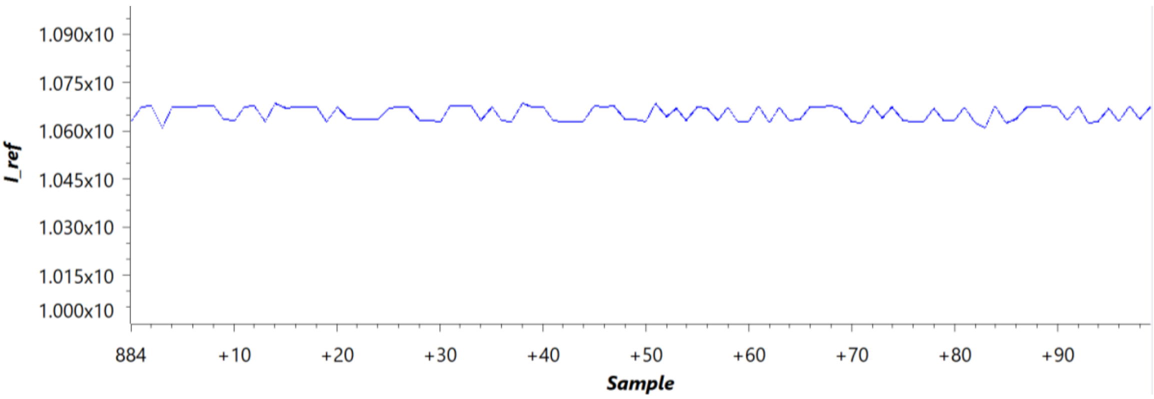


Figure: I_{ref} *Waveform for* $V_{ref} = 48$ V

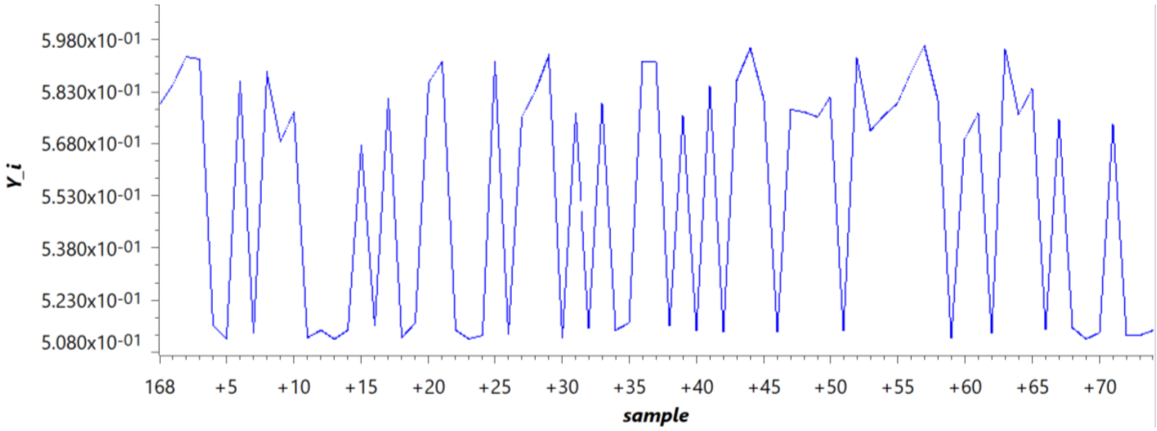


Figure: Current Controller Output (Y_i) for $V_{ref} = 48\text{ V}$

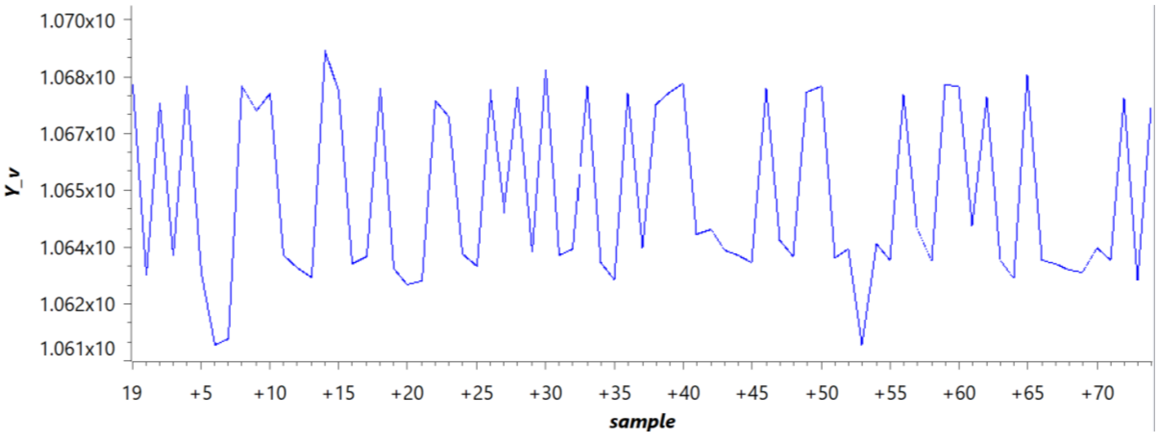


Figure: Voltage Controller Output (Y_v) for $V_{ref} = 48\text{ V}$

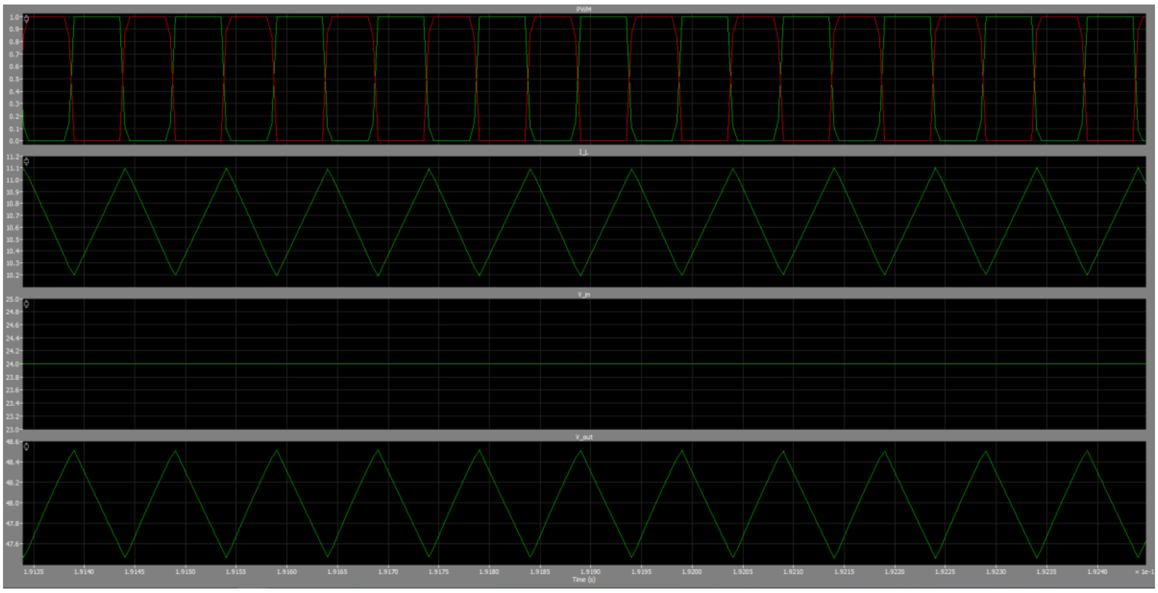


Figure: PLECS Waveforms for $V_{ref} = 48\text{ V}$

[**Only for 559**] You now need to implement a changing reference for i_{ref} and v_{ref} like you did in the simulations. Record the waveforms as in previous steps showing both the current and voltage control. [**10+10 pts**]

Changing Inductor Current Reference:

- $I_{ref,default} = 10.703$ A
- $I_{ref,new} = 9$ A

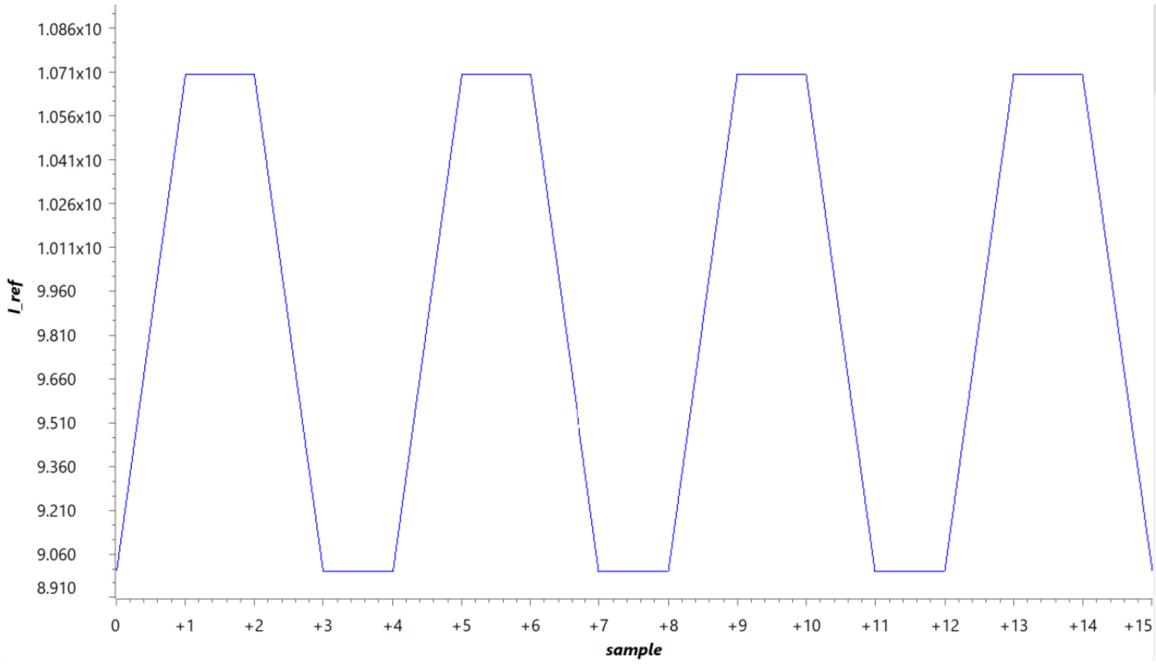


Figure: I_{ref} Pulse Waveform

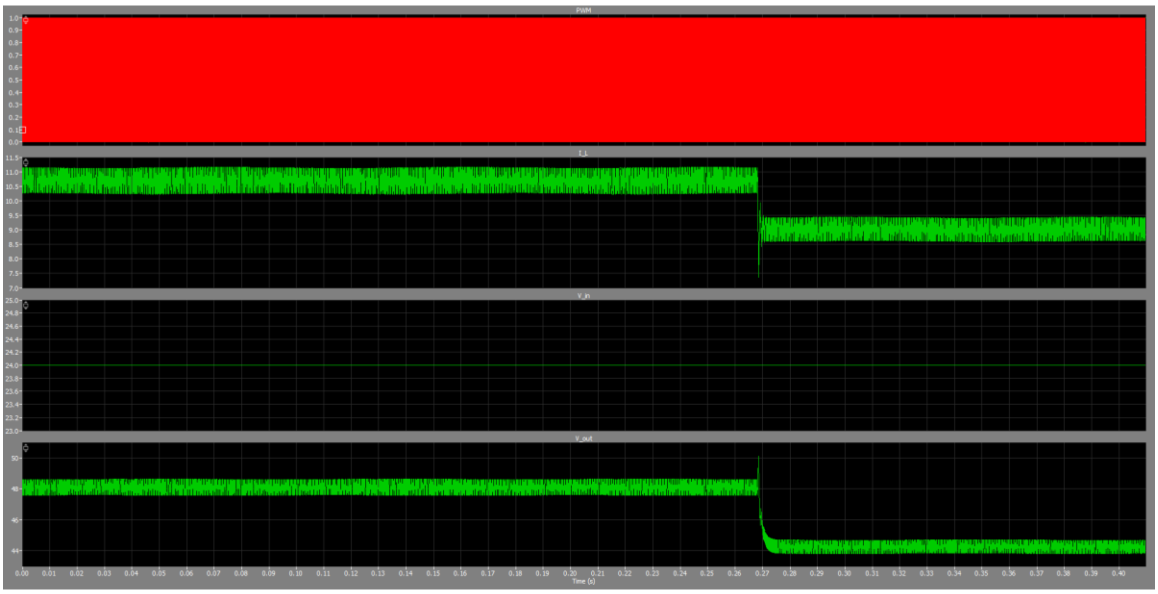


Figure: PLECS Waveforms with Pulse I_{ref} (Falling Edge)

Changing Output (Capacitor) Voltage Reference:

- $V_{ref,default} = 48\text{ V}$
- $V_{ref,new} = 50\text{ V}$

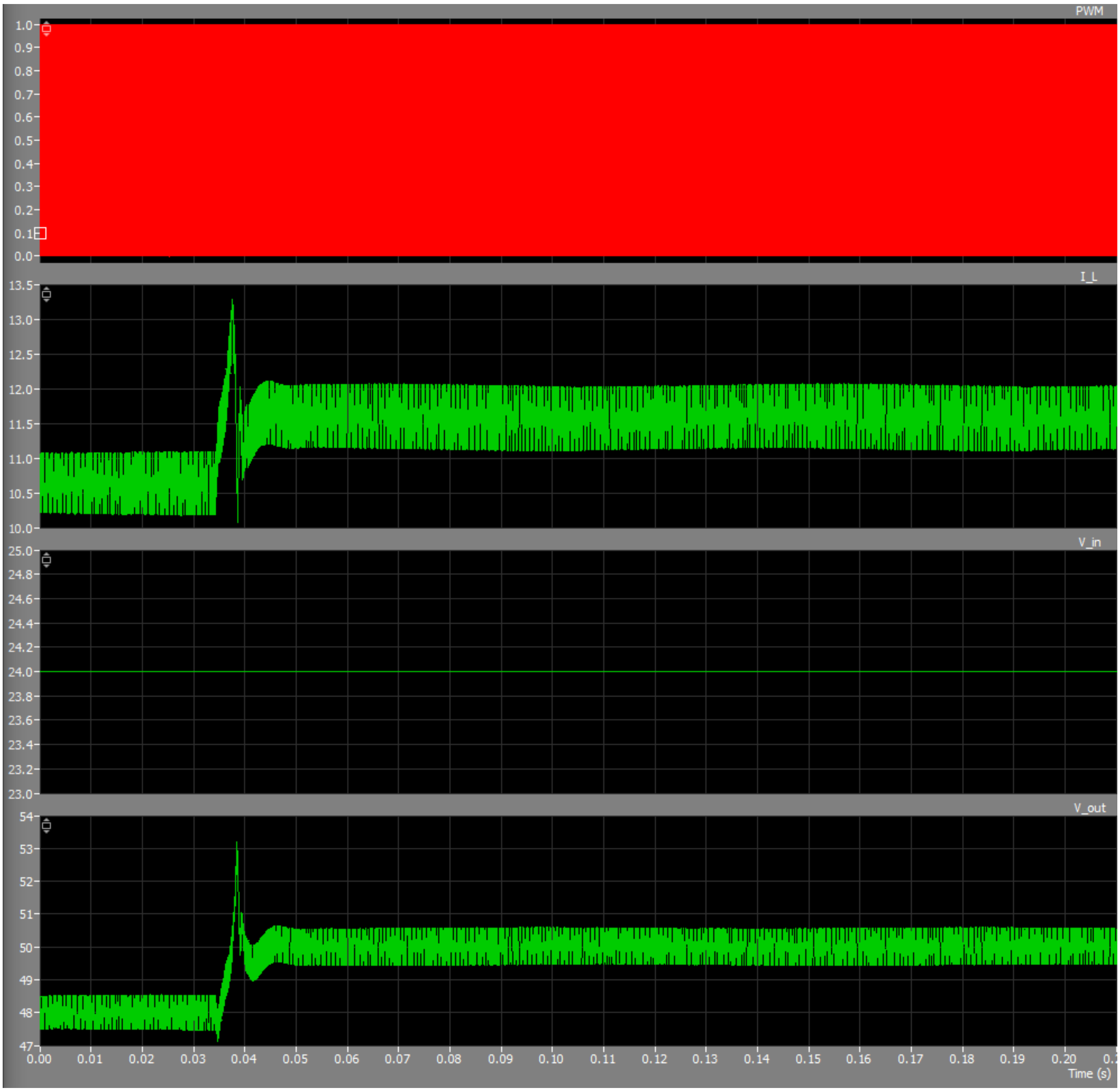


Figure: PLECS Waveforms with Pulse V_{ref} (Rising Edge)

[**Only for 559**] As a final step, implement an anti-windup to this controller. Considering the output yi can vary between $yi^{\min}$ and $yi^{\max}$, we need to take some corrective action every time it extends beyond that range. Whenever that happens, we saturate it at the limit it hits and then stop the integration part of the PI integrator, so that the huge error which led to saturation is no longer integrated and accumulated. Once the controller stabilizes the plant by only the proportional controller and the value of yi returns in the range $(yi^{\min}, yi^{\max})$, the integrator is reinstated. An easy way of removing the integrator is to multiply the input of the integrator with zero. [**5 pts**]

```c
__interrupt void adcA1ISR(void)
{

    // Add the latest result to the buffer
    // ADCRESULT0 is the result register of SOC0
    adcAResults[index++] = AdcaResultRegs.ADCRESULT0;

    // CUSTOM ASSIGNMENT
    temp_ADC = AdcaResultRegs.ADCRESULT0;
    DacaRegs.DACVALS.all = temp_ADC;

    // Accessing the measured digital quantities
    I_L_digital   = AdcaResultRegs.ADCRESULT0;
    V_in_digital  = AdcaResultRegs.ADCRESULT1;
    V_out_digital = AdcaResultRegs.ADCRESULT2;

    // Converting digital values to actual (real-world) quantities
    I_L   = I_L_digital   * ( 3.3 / (I_L_scale * 4096) ); // 2^N_adc = 2^12 = 4096
    V_in  = V_in_digital  * ( 3.3 / (V_in_scale * 4096) );
    V_out = V_out_digital * ( 3.3 / (V_out_scale * 4096) );


    ///////////////////////////////////////////////////////////////////////////////
    /// ALTERNATING REFRENCE
    ///////////////////////////////////////////////////////////////////////////////


    /// ALTERNATING VOLTAGE REFRENCE
    ///////////////////////////////////////////////////////////////////////////////

//    if (V_ref_index >= 1/T_samp) {
//        if (V_ref_toggle == 1){
//            V_ref = NEW_V_ref;
//
//        }
//        else {
//            V_ref = OLD_V_ref;
//
//        }
//        V_ref_index = 0;
//        V_ref_toggle = !V_ref_toggle;
//    }
//
//    V_ref_index++;


    /// ALTERNATING CURRENT REFRENCE
    ///////////////////////////////////////////////////////////////////////////////

//    if (I_ref_index >= 1/T_samp) {
//        if (I_ref_toggle == 1){
//            I_ref = NEW_I_ref;
//
```

```
//        }
//        else {
//            I_ref = OLD_I_ref;
//
//        }
//        I_ref_index = 0;
//        I_ref_toggle = !I_ref_toggle;
//    }
//
//    I_ref_index++;


    ///////////////////////////////////////////////////////////////////////////////
    /// VOLTAGE CONTROLLER
    ///////////////////////////////////////////////////////////////////////////////

    // Voltage Error
    V_err = V_ref - V_out;

    // PI Controller
        // Error propagated through the proportional controller
    Y_prop_v = V_err * k_p_v;
        // Error propagated through the integral controller
    Y_int_v = 0.5 * T_samp * (V_err + V_err_prev) * k_i_v + Y_int_v_prev;

    // Saturate Integral Controller Output
    if (Y_int_v < Y_int_v_min || Y_int_v > Y_int_v_max){
        if (Y_int_v < Y_int_v_min) {
            Y_int_v = Y_int_v_min;
        }
        else {
            Y_int_v = Y_int_v_max;
        }
    }

    // Update the voltage error
    Y_int_v_prev = Y_int_v;
    V_err_prev = V_err;

    // Voltage PI Controller Output
    if(disable_voltage_integrator) {
        Y_int_v = 0;
    }
    Y_v = Y_prop_v + Y_int_v;


    // Saturate PI Controller Output
    if ( (Y_v < Y_v_min || Y_v > Y_v_max) && !anti_windup) {
        if (Y_v < Y_v_min) {
            Y_v = Y_v_min;
        }
        else {
            Y_v = Y_v_max;
```

```
        }
    }

    // Anti-windup logic
    if ((Y_v < Y_v_min || Y_v > Y_v_max) && anti_windup) {
        disable_voltage_integrator = true;
    }
    else {
        disable_voltage_integrator = false;
    }

    // Reference of current controller
    I_ref = Y_v;

    ///////////////////////////////////////////////////////////////////////////////
    /// CURRENT CONTROLLER
    ///////////////////////////////////////////////////////////////////////////////

    ////// Current Error
    I_err = I_ref - I_L;

    // Current PI Controller
        // Error propagated through the proportional controller
    Y_prop_i = I_err * k_p_i;
        // Error propagated through the integral controller
    Y_int_i = 0.5 * T_samp * (I_err + I_err_prev) * k_i_i + Y_int_i_prev;

    // Saturate Integral Controller Output
    if (Y_int_i < Y_int_i_min || Y_int_i > Y_int_i_max){
        if (Y_int_i < Y_int_i_min) {
            Y_int_i = Y_int_i_min;
        }
        else {
            Y_int_i = Y_int_i_max;
        }
    }

    // Update the current error
    Y_int_i_prev = Y_int_i;
    I_err_prev = I_err;

    // Current PI Controller Output
    if(disable_current_integrator) {
        Y_int_i = 0;
    }
    Y_i = Y_prop_i + Y_int_i;

    // Saturate PI Controller Output
    if ((Y_i < Y_i_min || Y_i > Y_i_max) && !anti_windup) {
        if (Y_i < Y_i_min) {
            Y_i = Y_i_min;
        }
        else {
```

```
            Y_i = Y_i_max;
        }
    }

    // Anti-windup logic
    if ((Y_i < Y_i_min || Y_i > Y_i_max) && anti_windup) {
        disable_current_integrator = true;
    }
    else {
        disable_current_integrator = false;
    }

    // Output duty ratio
    duty = Y_i;

    EPwm1Regs.CMPA.bit.CMPA = duty * N_r;

    //
    // Set the bufferFull flag if the buffer is full
    //
    if(RESULTS_BUFFER_SIZE <= index)
    {
        index = 0;
        bufferFull = 1;
    }

    // CUSTOM CODE

    // TODO: UNCOMMENT

//    Ts = 1/fsamp;
//
//    dt = dt + Ts;
//    if (0.5 * Wn*dt >= 2*pi){
//        dt=0;
//    }
//    duty = 0.5*(1 + m*cos(0.5*Wn*dt));
//
//    EPwm1Regs.CMPA.bit.CMPA = duty * N_r;

    // CUSTOM CODE

    //
    // Clear the interrupt flag
    //
    AdcaRegs.ADCINTFLGCLR.bit.ADCINT1 = 1;

    //
    // Check if overflow has occurred
    //
    if(1 == AdcaRegs.ADCINTOVF.bit.ADCINT1)
    {
        AdcaRegs.ADCINTOVFCLR.bit.ADCINT1 = 1; //clear INT1 overflow flag
```

```
        AdcaRegs.ADCINTFLGCLR.bit.ADCINT1 = 1; //clear INT1 flag
    }

    //
    // Acknowledge the interrupt
    //
    PieCtrlRegs.PIEACK.all = PIEACK_GROUP1;
}
```