EE 458/533 – Power Electronics Controls, Winter 2022

Homework 4

Due Date: Wednesday February 17th 2021, 4:30 pm PT

Instructions. Please attach relevant simulation files.

Problem 1: Consider the buck-boost converter switching at 200 kHz using symmetrical carrier and peak-valley sampling (try to think how would you implement them in the PLECS simulation). We are interested in output dc voltage control. You are highly recommended to not spend time deriving the plant transfer function, directly use the transfer function derived in class in previous lectures .

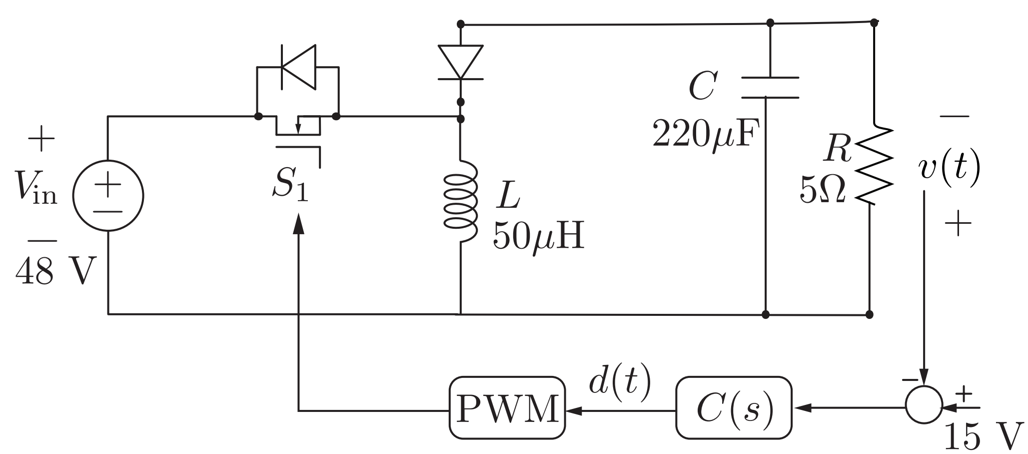


Figure 1: Closed loop control system for output voltage control of buck-boost converter

1. Draw a bode plot for the plant transfer function, clearly marking the gain and phase margin.
2. Design the best controller, $C(s)$, you can, that has the following specifications:
   - Has dc tracking capability with negligible steady state error (output voltage should be close to 15 V).
   - Has high bandwidth, but not more than 10% of the switching frequency.
   - Has a phase margin of at least 52°.
3. Verify your results in a simulation with a switched model of the buck-boost converter as shown in Fig. 1. In your simulation, the controller output should have a saturator based on the minimum and maximum permissible values of the signal $d(t)$.

   Now, add an anti-windup logic for your controller when the $d(t)$ goes beyond its permissible values. Overlap your plots with and without integral anti-windup. Your plot must have reference, output, error, duty (before and after saturation) block.

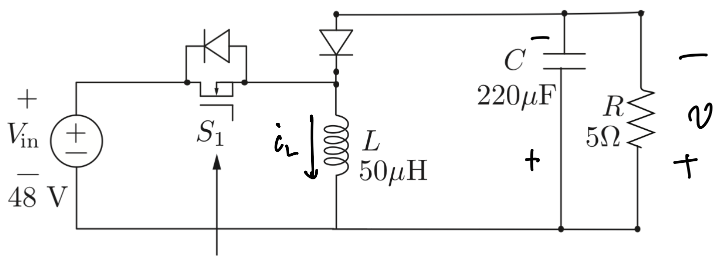
4. What happens to the system if you further increase your controller bandwidth? Report the largest controller bandwidth which meets your control requirement and also gives a stable simulation.

5. Discretize your controller by the Tustin or bilinear transformation (you can check my MATLAB but you have to show the complete working). Write the controller in the following form:

$$d[k] = \sum_{p=1}^{N} \alpha_p d[k-p] + \beta_0 e[k] + \sum_{p=1}^{N} \beta_p e[k-p] \tag{1}$$

where e is the error in the voltage and d is the duty ratio. You should outline the following values, $N, \alpha_p, \beta_p, \beta_0$.

We start by analyzing the circuit (we consider small esr r_L for the inductor)



TON

$$V_{in} = L\frac{di}{dt} + ir_L$$

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

TOFF

$$L\frac{di}{dt} + ir_L = -v$$

$$i = C\frac{dv}{dt} + \frac{v}{R}$$

$$\therefore \quad L\frac{d}{dt}\langle i\rangle = (V_{in} - ir_L)d + (-v - ir_L)d'$$

$$C\frac{d\langle v\rangle}{dt} = \left(-\frac{v}{R}\right)d + \left(i - \frac{v}{R}\right)d'$$

$\because$ $d + d' = 1$ we can write,

$$L\frac{d\langle i\rangle}{dt} = V_{in}d - ir_L - vd'$$

$$C\frac{d\langle v\rangle}{dt} = -\frac{v}{R} + id'$$

① Steady state solutions (for large signal terms)

$$V_{in}D - Ir_L - VD' = 0$$

$$-\frac{V}{R} + ID' = 0$$

or, $\boxed{I = \frac{v}{RD'}}$ & $V_{in} D - I r_L = V D'$

$$V_{in} D - \frac{V}{RD'} r_L = V D'$$

$$\therefore V = \frac{V_{in} D}{\left(D' + \frac{r_L}{RD'} \right)}$$

② Small signal

$$\frac{d\langle i \rangle}{dt} = \frac{1}{L}\left(V_{in} d - i r_L - v d' \right) = f_1$$

$$\frac{d\langle v \rangle}{dt} = \frac{1}{C}\left(-\frac{v}{R} + i d' \right) = f_2$$

$$\begin{bmatrix} \frac{d\langle i \rangle}{dt} \\ \frac{d\langle v \rangle}{dt} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

If $\langle i \rangle = I + \tilde{i}$; $\langle v \rangle = V + \tilde{v}$

So,

$$\begin{bmatrix} \frac{d\tilde{i}}{dt} \\ \frac{d\tilde{v}}{dt} \end{bmatrix} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial i} & \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial i} & \frac{\partial f_2}{\partial v} \end{bmatrix}\right|_{\substack{i = I, \\ v = V, \\ d = D,\, v_{in} = V_{in}}} \begin{bmatrix} i - I \\ v - V \end{bmatrix} + \left.\begin{bmatrix} \frac{\partial f_1}{\partial d} & \frac{\partial f_1}{\partial V_{in}} \\ \frac{\partial f_2}{\partial d} & \frac{\partial f_2}{\partial V_{in}} \end{bmatrix}\right|_{\substack{i = I, \\ v = V, \\ d = D, \\ v_{in} = V_{in}}} \begin{bmatrix} d - D \\ v_{in} - V_{in} \end{bmatrix}$$

$$\begin{bmatrix} \frac{d\tilde{i}}{dt} \\ \frac{d\tilde{v}}{dt} \end{bmatrix} = \left.\begin{bmatrix} -\frac{r_L}{L} & -\frac{d'}{L} \\ \frac{d'}{C} & -\frac{1}{RC} \end{bmatrix}\right|_{\substack{i=I,\\ v=V,\\ d=D,\, v_{in}=V_{in}}} \begin{bmatrix} \tilde{i} \\ \tilde{v} \end{bmatrix} + \left.\begin{bmatrix} \frac{V_{in}}{L} + \frac{v}{L} & \frac{d}{L} \\ -\frac{i}{C} & 0 \end{bmatrix}\right|_{\substack{i=I,\\ v=V,\\ d=D,\, v_{in}=V_{in}}} \begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}$$

[where, remember, D, V_{in} = input,

$I = \frac{V}{RD'}$ & $V = \frac{V_{in} D}{D' + \frac{r_L}{RD'}}$]

$$\therefore \underbrace{\begin{bmatrix} \frac{d\tilde{i}}{dt} \\ \frac{d\tilde{v}}{dt} \end{bmatrix}}_{\dot{\tilde{x}}} = \underbrace{\begin{bmatrix} -\frac{r_L}{L} & -\frac{D'}{L} \\ \frac{D'}{C} & -\frac{1}{RC} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \tilde{i} \\ \tilde{v} \end{bmatrix}}_{\tilde{x}} + \underbrace{\begin{bmatrix} \frac{V_{in}+V}{L} & \frac{D}{L} \\ -\frac{I}{C} & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}}_{\tilde{u}}$$

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$$

$$s\tilde{x}(s) = A\tilde{x}(s) + B\tilde{u}(s)$$

$$\tilde{x}(s) = (sI - A)^{-1} B\tilde{u}(s) = \begin{bmatrix} \tilde{i}_L(s) \\ \tilde{v}(s) \end{bmatrix}$$

O/P $\Rightarrow$ cap voltage = $\tilde{v}$

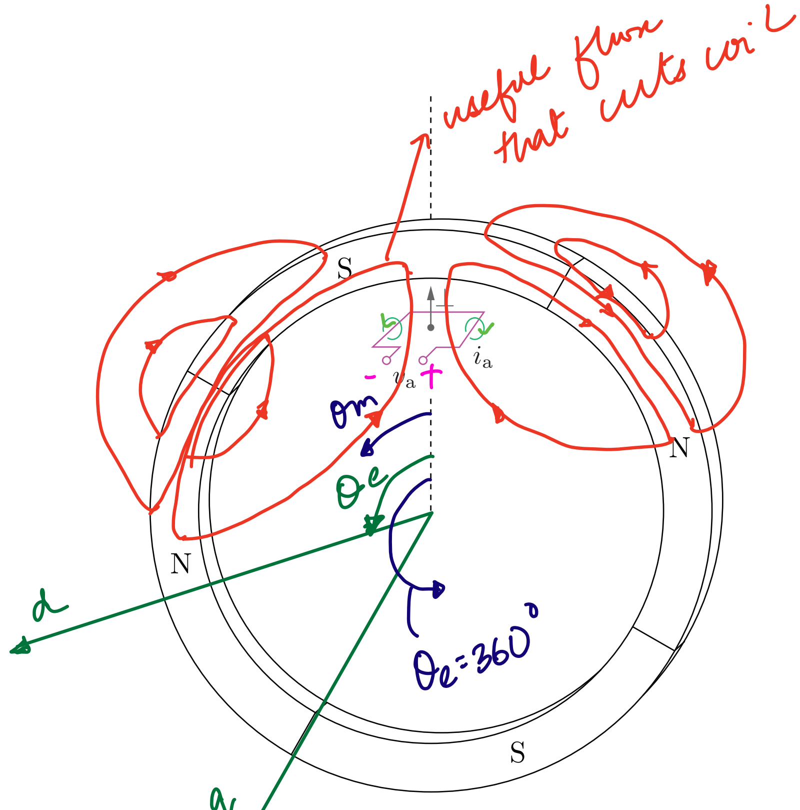
$$\tilde{v}(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} . \tilde{x}(s)$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} (sI - A)^{-1} B\tilde{u}(s)$$

Problem 2: Consider the three-phase permanent magnet motor where the outer ring of magnets can rotate relative to a set of stationary coils within its inner diameter. Note that this setup is very similar to the e-bike experiment. Only the a-phase winding has been drawn on Figure 2.

(a) Annotate Figure 2 with the items below:

- Draw the flux density B-field lines between all the magnetic poles. Also show how the field lines exist within the magnets.
- Define the d and q axes for the permanent magnets.
- Define the a-phase voltage polarity.
- Define the a-phase current positive direction such that its flux points toward the $\perp$ vector.
- Indicate the direction of the flux produced by the a-phase coil by adding arrows to the green flux circles.
- Label the mechanical angle, θ_{m}.
- Label one full electrical rotation of 360°. Write the equation of how θ_{e} and θ_{m} are related for this problem.
- Sketch the approximate placement of the b and c phase coils to form the full-three motor.



$\theta_m = \dfrac{\theta_e}{2}$

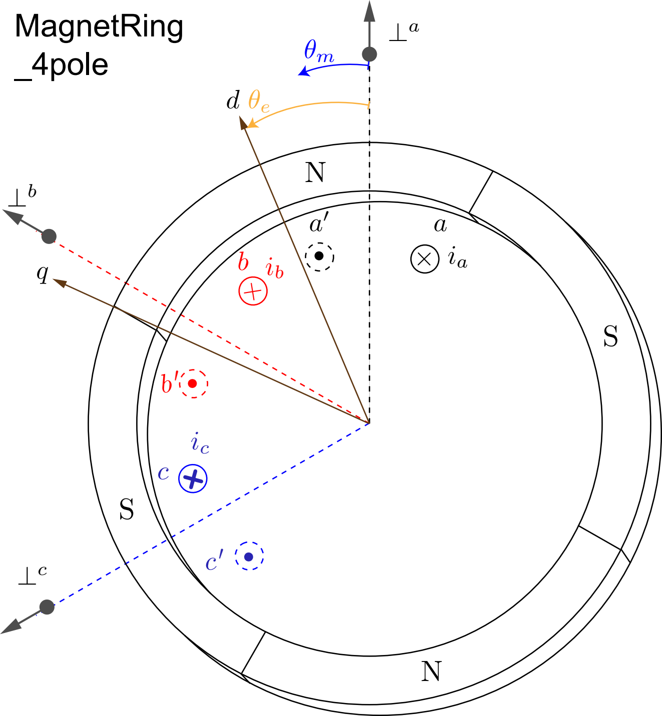
Figure 2: Permanent magnet motor setup.

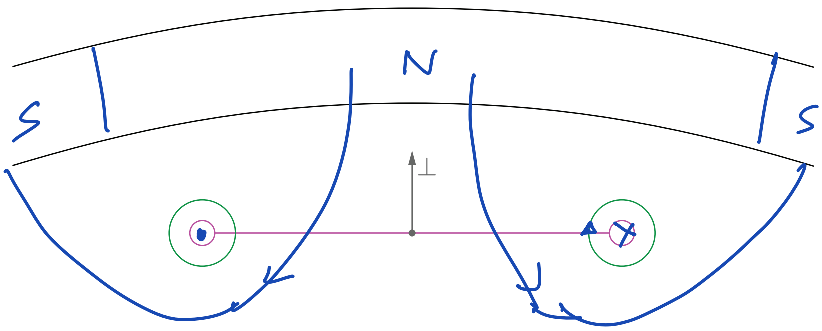
(b) Denote the flux produced by the magnetic ring which cuts through the phase-a loop area as $\Phi(\theta_e)$. The flux linkage through the coil, which has N turns, is denoted as $\lambda(\theta_e)$. Consider Figure 3 where we are looking at a side view of the phase-a coil in Figure 2. Annotate the diagram in Figure 3 by showing the following for each angle configuration:

- Label the position of the poles above the coil.
- Draw the flux cutting through the coil.
- Fill in the two pink circles with crosses or dots to indicate the positive flow of coil current i.
- Add arrows to the green circles to show the flux density direction produced by i.
- Fill in the flux and flux linkage equations in terms of N, i, and $\Phi_{\max}$, where $\Phi_{\max} > 0$ is the peak flux through the coil.

(c) Using the result from (b), give the form of the functions $\Phi(\theta_e)$ and $\lambda(\theta_e)$. Use Faraday's Law to compute the induced back electromotive force voltage $\mathcal{E}$.

MagnetRing
_4pole

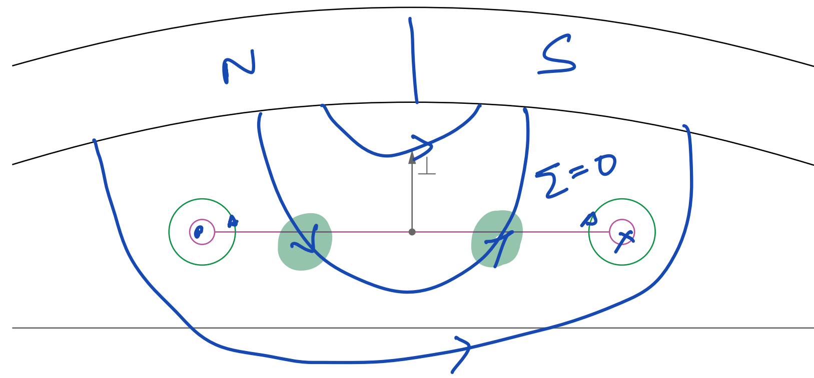




when $\theta_e = 0$, give the following:

$\Phi(\theta_e) = -\Phi_{max}$

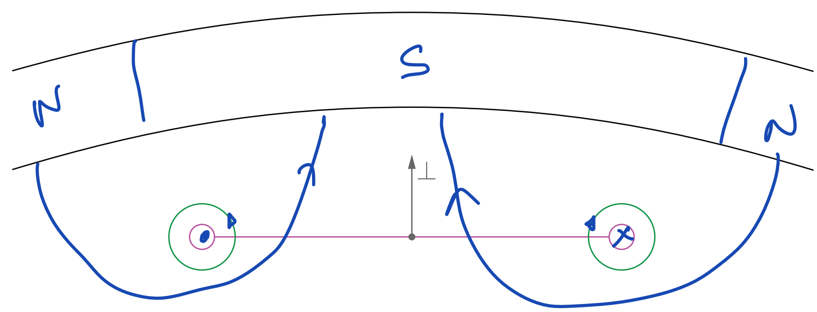
$\lambda(\theta_e) = -N\Phi_{max} + Li$



when $\theta_e = 90°$, give the following:

$\Phi(\theta_e) = 0$

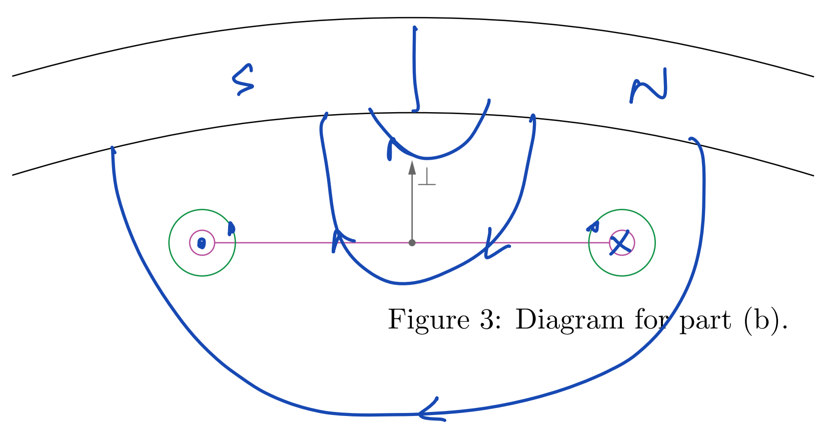
$\lambda(\theta_e) = Li$



when $\theta_e = 180°$, give the following:

$\Phi(\theta_e) = \Phi_{max}$

$\lambda(\theta_e) = N\Phi_{max} + Li$



when $\theta_e = 270°$, give the following:

$\Phi(\theta_e) = 0$

$\lambda(\theta_e) = Li$

Figure 3: Diagram for part (b).