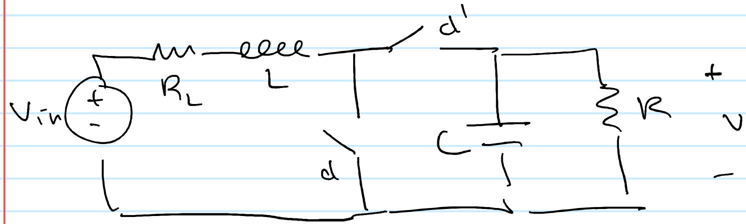
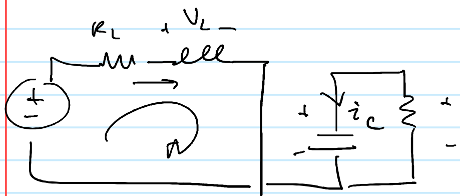
Prelab 2

Tuesday, January 18, 2022 7:45 AM

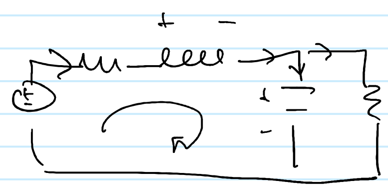


1: Steady State: Start w/ sw cycle avg:

M1:



M2:



M1:

$V_L - V_{in} + i_L R_L = 0$

$V_L = V_{in} - i_L R_L$

$L\frac{di_L}{dt} = v_{in} - i_L R_L$ * ✓

$i_C = \frac{-V_C}{R}$

$C\frac{dV_C}{dt} = \frac{-V_C}{R}$ * ✓

M2:

$-V_{in} + i_L R_L + V_L + V_C = 0$

$V_L = V_{in} - i_L R_L - V_C$

$L\frac{di_L}{dt} = V_{in} - i_L R_L - V_C$ *

$i_L = i_C + \frac{V_C}{R}$

$i_C = i_L - \frac{V_C}{R}$

$C\frac{dV_C}{dt} = i_L - \frac{V_C}{R}$ *

$$L\frac{d\langle i_L\rangle}{dt} = d(t)(V_{in} - i_L R_L) + (1-d)(V_{in} - i_L R_L - V_C)$$

$$= \cancel{dV_{in}} - \cancel{d i_L R_L} + V_{in} - i_L R_L - V_C - \cancel{dV_{in}} + \cancel{d i_L R_L} + dV_C$$

$$= v_{in} - i_L R_L - d' v_C$$

$$L\frac{d\langle i_L\rangle}{dt} = v_{in} - i_L R_L - d' v_C$$

$$\boxed{\frac{d\langle i_L\rangle}{dt} = \frac{1}{L}\left(v_{in} - i_L R_L - d' v_C\right)}$$

$$C\frac{d\langle v_C\rangle}{dt} = d\left(\frac{-V_C}{R}\right) + (1-d)\left(i_L - \frac{V_C}{R}\right)$$

$$= -\cancel{\frac{dV_C}{R}} + i_L - \frac{V_C}{R} - d i_L + \cancel{\frac{dV_C}{R}}$$

$$C\frac{d\langle v_C\rangle}{dt} = d' i_L - \frac{V_C}{R}$$

$$\boxed{\frac{d\langle v_C\rangle}{dt} = \frac{1}{C}\left(d' i_L - \frac{V_C}{R}\right)}$$

sw cycle averaged model

* NOTE: lower case above is $\langle\,\rangle_{T_S}$ and depends on t_1, upper case doesn't depend on t

Compute Steady State (just use VS / charge balance, setting dynamics to 0)

$L: \quad 0 = V_{in} - i_L R_L - d' v_C$

$d' v_C = V_{in} - i_L R_L$

$v_C = \frac{V_{in} - i_L R_L}{d'}$, since steady state: $V_C = \frac{V_{in} - \overset{\text{small term}}{I_L R_L}}{D'}$

$C: \quad 0 = d' i_L - \frac{v_C}{R} \qquad i_L = \frac{v_C}{R d'} \rightarrow \boxed{I_L = \frac{V_C}{RD'}}$

$0 = V_{in} - I_L R_L - D' V_C$

$= V_{in} - \frac{V_C}{RD'} R_L - D' V_C$

$= V_{in} - \left(\frac{R_L}{RD'} + D'\right) V_C$

$= V_{in} - \left(\frac{R_L + RD'D'}{RD'}\right) V_C$

$\rightarrow 0 = V_{in} - \left(\frac{R_L}{RD'} + D'\right) V_C$

$$\boxed{V_C = \frac{V_{in}}{\left(D' + \frac{R_L}{RD'}\right)}}$$

Steady State Model.

Linearize:

$$\bar{X} = \begin{bmatrix} I_L \\ V_C \end{bmatrix} = \begin{bmatrix} \frac{V_C}{RD'} \\ \frac{V_{in}}{D' + \frac{R_L}{RD'}} \end{bmatrix} \qquad \bar{U} = \begin{bmatrix} D' \\ V_{in} \end{bmatrix}$$

$$\dot{x} = f(x,u) = \begin{bmatrix} \langle \dot{i}_L \rangle \\ \langle \dot{V}_C \rangle \end{bmatrix} = \begin{bmatrix} \frac{1}{L}\left(\langle V_{in}\rangle - \langle i_L\rangle R_L - \overbrace{(1-d(t))}^{d'}\langle V_C\rangle\right) \\ \frac{1}{C}\left((1-d(t))\langle i_L\rangle - \frac{\langle V_C\rangle}{R}\right) \end{bmatrix}$$

$$\dot{x} = Ax + Bu \qquad A = \left.\frac{\partial f}{\partial x}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}} \qquad B = \left.\frac{\partial f}{\partial u}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}}$$

$$\begin{bmatrix} \frac{\partial f_1}{\partial \tilde{i}_L} & \frac{\partial f_1}{\partial \tilde{v}_C} \\ \frac{\partial f_2}{\partial \tilde{i}_L} & \frac{\partial f_2}{\partial \tilde{v}_C} \end{bmatrix} \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_C \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial \tilde{d}'} & \frac{\partial f_1}{\partial \tilde{v}_{in}} \\ \frac{\partial f_2}{\partial \hat{d}_1} & \frac{\partial f_2}{\partial \tilde{v}_{in}} \end{bmatrix} \begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}$$

$\downarrow$

$$\boxed{\begin{bmatrix} -\frac{R_L}{L} & -\frac{D'}{L} \\ \frac{D'}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_C \end{bmatrix} + \begin{bmatrix} \frac{V_C}{L} & \frac{1}{L} \\ -\frac{I_L}{C} & 0 \end{bmatrix} \begin{bmatrix} \tilde{d} \\ \tilde{V}_{in} \end{bmatrix}}$$

Linearized small signal model

Linearized small signal model.

Find Transfer Functions:

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$$

Laplace: $s\tilde{x}(s) = A\tilde{x}(s) + B\tilde{u}(s)$

$$sI\,\tilde{x}(s) = A\tilde{x}(s) + B\tilde{u}(s)$$

$$(sI - A)\,\tilde{x}(s) = B\tilde{u}(s)$$

$$\tilde{x}(s) = \underbrace{(sI-A)^{-1}}_{\text{tfr matrix}} B\tilde{u}(s)$$

$$\underbrace{\begin{bmatrix} \tilde{i}_L(s) \\ \tilde{v}_C(s) \end{bmatrix}}_{\tilde{x}} = \underbrace{\left(\underbrace{\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}}_{sI} - \underbrace{\begin{bmatrix} -\frac{R_L}{L} & -\frac{D'}{L} \\ \frac{D'}{C} & -\frac{1}{RC} \end{bmatrix}}_{A} \right)^{-1}} \underbrace{\begin{bmatrix} \frac{V_C}{L} & \frac{1}{L} \\ -\frac{I_L}{C} & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \tilde{d}(s) \\ \tilde{v}_{in}(s) \end{bmatrix}}_{\tilde{u}}$$

$$\tilde{x}(s) = \begin{bmatrix} s + \frac{R_L}{L} & \frac{D'}{L} \\ -\frac{D'}{C} & s + \frac{1}{RC} \end{bmatrix}^{-1} \begin{bmatrix} \frac{V_C}{L} & \frac{1}{L} \\ -\frac{I_L}{C} & 0 \end{bmatrix} \tilde{u}(s)$$

use matlab symbolic math package to solve inverse

$$\frac{1}{\frac{R_L + Ls + (D')^2 R + CLRs^2 + CRR_L s}{RLC}} \begin{bmatrix} s + \frac{1}{RC} & -\frac{D'}{L} \\ \frac{D'}{C} & \frac{s + R_L}{L} \end{bmatrix}$$

$$\underbrace{\frac{1}{s^2 + s\left(\frac{1}{RC} + \frac{R_L}{L}\right) + \frac{R_L}{RLC} + \frac{(D')^2}{LC}} \begin{bmatrix} s + \frac{1}{RC} & -\frac{D'}{L} \\ \frac{D'}{C} & \frac{s+R_L}{L} \end{bmatrix} \begin{bmatrix} \frac{V_C}{L} & \frac{1}{L} \\ -\frac{I_C}{C} & 0 \end{bmatrix}}$$

$$= \frac{1}{s^2 + s\left(\frac{1}{RC} + \frac{R_L}{L}\right) + \frac{R_L}{RLC} + \frac{(D')^2}{LC}} \begin{bmatrix} \frac{+V_C\left(s + \frac{1}{RC}\right)}{L} + \frac{D' I_L}{LC} & \frac{s + \frac{1}{RC}}{L} \\ \frac{+V_C D'}{LC} + \frac{-I_L\left(s + \frac{R_L}{L}\right)}{C} & \frac{D'}{LC} \end{bmatrix}$$

$$\underbrace{s^2 + s\left(\frac{1}{RC} + \frac{R_L}{L}\right) + \frac{R_L}{RLC} + \frac{(D')^2}{LC}}_{det(s)} \qquad \begin{bmatrix} +\frac{V_c D'}{2C} & + \frac{-I_L(s + \frac{R_L}{L})}{C} & \frac{D'}{2C} \end{bmatrix}$$

$v_g = V_{in}$

$$\tilde{x}(s) = \frac{1}{det(s)} \begin{bmatrix} G_{id}'(s) & G_{iv_g}'(s) \\ G_{vd}'(s) & G_{vv_g}'(s) \end{bmatrix} \begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}$$

$$G_{id} = \frac{G_{id}'(s)}{det(s)} \qquad G_{iv_g} = \frac{G_{iv_g}'(s)}{det(s)}$$

$$G_{vd} = \frac{G_{vd}'(s)}{det(s)} \qquad G_{vv_g}(s) = \frac{G_{vv_g}'(s)}{det(s)}$$

Normalized Transfer Fn:

$$G_{xd} = G_{do} \frac{\left(1 - \frac{s}{\omega_z}\right)}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$G_{id}(s) = \frac{\frac{V_c}{2}\left(s + \frac{1}{RC}\right) + \frac{D' I_L}{LC}}{s^2 + s\left(\frac{1}{RC} + \frac{R_L}{L}\right) + \frac{R_L}{RLC} + \frac{(D')^2}{LC}}$$

$$= \frac{\frac{V_c}{2}\left(s + \frac{1}{RC}\right) + \frac{D' I_L}{LC}}{s^2 + s\left(\frac{1}{RC} + \frac{R_L}{L}\right) + \frac{R_L + R(D')^2}{RLC}}$$

$$= \frac{RLC\left(\frac{V_C}{L}\left(s+\frac{1}{RC}\right)+\frac{D' I_L}{LC}\right)}{RLCs^2 + RLCs\left(\frac{1}{RC}+\frac{R_L}{L}\right)+R_L+R(D')^2}$$

$$= \frac{RLC\left(\frac{V_C}{L}\left(s+\frac{1}{RC}\right)+\frac{D' I_L}{LC}\right)}{(R_L+R(D')^2)\left[1+s\underbrace{\left(\frac{RLC}{R_L+R(D')^2}\right)\left(\frac{1}{RC}+\frac{R_L}{L}\right)}_{\frac{1}{Q\omega_0}}+s^2\underbrace{\left(\frac{RLC}{R_L+(RD')^2}\right)}_{\left(\frac{1}{\omega_0}\right)^2}\right]}$$

$$\boxed{\omega_0 = \sqrt{\frac{R_L+R(D')^2}{RLC}}}$$

$$Q\omega_0 = \frac{R_L+R(D')^2}{RLC\left(\frac{1}{RC}+\frac{R_L}{L}\right)}$$

$$Q\omega_0 = \frac{\omega_0^2}{\frac{1}{RC}+\frac{R_L}{L}} \qquad \boxed{Q = \left(\frac{\omega_0}{\frac{1}{RC}+\frac{R_L}{L}}\right)}$$

$$I_L = \frac{V_C}{RD'}$$

$$= \frac{RLC\left(\frac{V_C}{L}\left(s+\frac{1}{RC}\right)+\frac{D' I_L}{LC}\right)}{(R_L+R(D')^2)\left[1+\frac{s}{Q\omega_0}+\left(\frac{s}{\omega_0}\right)^2\right]}$$

numerator: $RLC\left(\frac{V_C s}{L}+\frac{V_C}{RLC}+\frac{V_C}{RLC}\right)$

$$\downarrow$$

$$+V_C sRC + 2V_C = \left[sRCV_C + 2V_C\right]$$

$$L \quad Q\omega_0 \quad (\frac{s}{\omega_0}) \quad)$$

numerator: $RLC\left(\frac{V_c s}{L} + \frac{V_c}{RLC} + \frac{V_c}{RLC}\right)$

$$+V_c s RC + 2V_c = \left[sRCV_c + 2V_c\right]$$

$$= 2V_c\left[s\frac{RC}{2} + 1\right]$$

$$= 2V_c\left[\underbrace{\frac{sRC}{2} + 1}\right] \rightarrow 1 - \frac{s}{\omega_z}$$

$$\boxed{\omega_{z,i} = \frac{-2}{RC}}$$

$$G_{id} = \underbrace{\frac{2V_c}{R_L + R(D')^2}}_{G_{d0}}\left[\frac{1 - \frac{s}{\omega_{z,i}}}{1 + \frac{s}{Q\omega_0} + (\frac{s}{\omega_0})^2}\right]$$

$$\boxed{G_{id} = G_{id_0}\left[\frac{1 - \frac{s}{\omega_{z,i}}}{1 + \frac{s}{Q\omega_0} + (\frac{s}{\omega_0})^2}\right]}$$

$$G_{vd} = \frac{G_{vd}'}{\det(s)} = \left(\frac{RLC}{R_L + R(D')^2}\right)\frac{+\frac{V_c D'}{LC} - \frac{I_L(s + \frac{R_L}{L})}{C}}{\left[1 + \frac{s}{Q\omega_0} + (\frac{s}{\omega_0})^2\right]}$$

numerator:

$$= RLC\left(\frac{+V_c D'}{LC} - \frac{I_L}{C}\left(s + \frac{R_L}{L}\right)\right)$$

$$= +RV_c D' - (RLC)\left(\frac{I_L s}{C} + \frac{I_L R_L}{LC}\right)$$

$$= +RV_c D' - RLI_L s - RI_L R_L$$

$$= +RV_c D' - RLI_L s - RI_L R_L$$

$$= +RV_c D' - \cancel{R}L\frac{V_c}{\cancel{R}D'} s - \cancel{R}\frac{V_c}{\cancel{R}D'} R_L$$

$$= +RV_c D' - s\frac{LV_c}{D'} - \frac{V_c R_L}{D'}$$

$$= V_c \left[+RD' - \frac{R_L}{D'} - s\frac{L}{D'} \right]$$

$$= V_c \left(\frac{-R_L}{D'} \; RD' \right) \left[1 - s\left(\frac{1}{-\left(\frac{R_L}{D'}\right) + RD'} \right) \frac{L}{D'} \right]$$

$$= V_c \left(\frac{-R_L}{D'} + RD' \right) \left[1 - s \left(\frac{L}{-R_L + R(D')^2} \right) \right]$$

$$\boxed{\omega_{Z,V} = \frac{R(D')^2 - R_L}{L}}$$

$$G_{vd} = \underbrace{\frac{V_c}{D'} \left[\frac{(R(D')^2 - R_L)}{(R_L + R(D')^2)} \right]}_{G_{vdo}} \left(\frac{1 - \frac{s}{\omega_Z}}{1 + \frac{s}{Q\omega_o} + \left(\frac{s}{\omega_o}\right)^2} \right)$$

$$\boxed{G_{vd} = G_{vdo} \left(\frac{1 - \frac{s}{\omega_{Z,V}}}{1 + \frac{s}{Q\omega_o} + \left(\frac{s}{\omega_o}\right)^2} \right)} \qquad \boxed{\omega_{Z,V} = \left(\frac{R(D')^2 - R_L}{L} \right)}$$

$$\boxed{G_{vi} = \frac{G_{vd}}{G_{id}} = \frac{G_{vdo}}{G_{ido}} \left(\frac{1 - \frac{s}{\omega_{Z,V}}}{1 - \frac{s}{\omega_{Z,i}}} \right)}$$

$$G_{id} = G_{id0}\left(\frac{}{1-\frac{s}{\omega_{zi}}}\right)$$

$$\boxed{G_{id} = G_{id_0}\left[\frac{1-\frac{s}{\omega_{z,i}}}{1+\frac{s}{Q\omega_0}+\left(\frac{s}{\omega_0}\right)^2}\right]}$$

$$\boxed{\omega_{z,i} = \frac{-2}{RC}}$$

$$\boxed{\omega_0 = \sqrt{\frac{R_L + R(D')^2}{RLC}}}$$

$$\boxed{Q = \left(\frac{\omega_0}{\frac{1}{RC}+\frac{R_L}{L}}\right)}$$

$$\boxed{G_{vd_0} = \frac{V_C}{D'}\left[\frac{(R(D')^2 - R_L)}{(R_L + R(D')^2)}\right]}$$

$$\boxed{G_{id_0} = \frac{2V_C}{R_L + R(D')^2}}$$

DC Gain: $s \to 0$, so G_{id_0} and G_{vd_0}!

zeroes: set $\left(1-\frac{s}{\omega_z}\right)=0$

ω_z is the zero!

poles: set $1+\frac{s}{Q\omega_0}+\left(\frac{s}{\omega_0}\right)^2 = 0$

$a = \left(\frac{1}{\omega_0}\right)^2 \quad b = \frac{1}{Q\omega_0} \quad c = 1$

poles: $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Use MATLAB to solve:

poles: $\omega_0 \sqrt{-(2Q-1)(2Q+1)} - 1$

$$\frac{}{2Q}$$

$$\frac{-\omega_0\sqrt{-(2Q-1)(2Q+1)+1}}{2Q}$$

Now we have symbolic, can just plug in #s

Bode plots: use MATLAB.