Problem 1.1 to 1.2 (Anisha Chutani)

Anisha Chutani
EE 452-HW 4
2/24/22

Problem 1:

Part 1

1\. 1) transfer function

From textbook:

$$G_{vd}(s) = G_{do} \cdot \frac{\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{Q\omega_o} + \left(\frac{s}{\omega_o}\right)^2\right)}$$

where

$$G_{do} = \frac{V}{DD'} \qquad W_z = \frac{D'^2 R}{DL}$$

$$Q = D'R\sqrt{\frac{C}{L}} \qquad \omega_o = \frac{D'}{\sqrt{LC}}$$

after deriving this transfer function & subbing in known values, I flipped the sign to match sign convention of lecture notes ~~to match~~ & HW,

$G_{vd} \to -G_{vd}(s)$

```
R = 5;
C = 220e-6;
L = 50e-6;
Vin = 48;
fs = 200e3;
V = -15; %this will be reversed in plecs simulation
D = V/(V-Vin) %equation would be different for reversed sign convention,
%but yield same duty of 0.2381
Dp = 1-D;

%Open loop plot - not stable
Gdo = V/(D*Dp);
wo = Dp/sqrt(L*C);
Q = Dp*R*sqrt(C/L);
wz = (Dp^2)*R/(D*L);
Gvd = tf([-Gdo/wz,Gdo],[1/(wo^2),1/(Q*wo),1]);
```

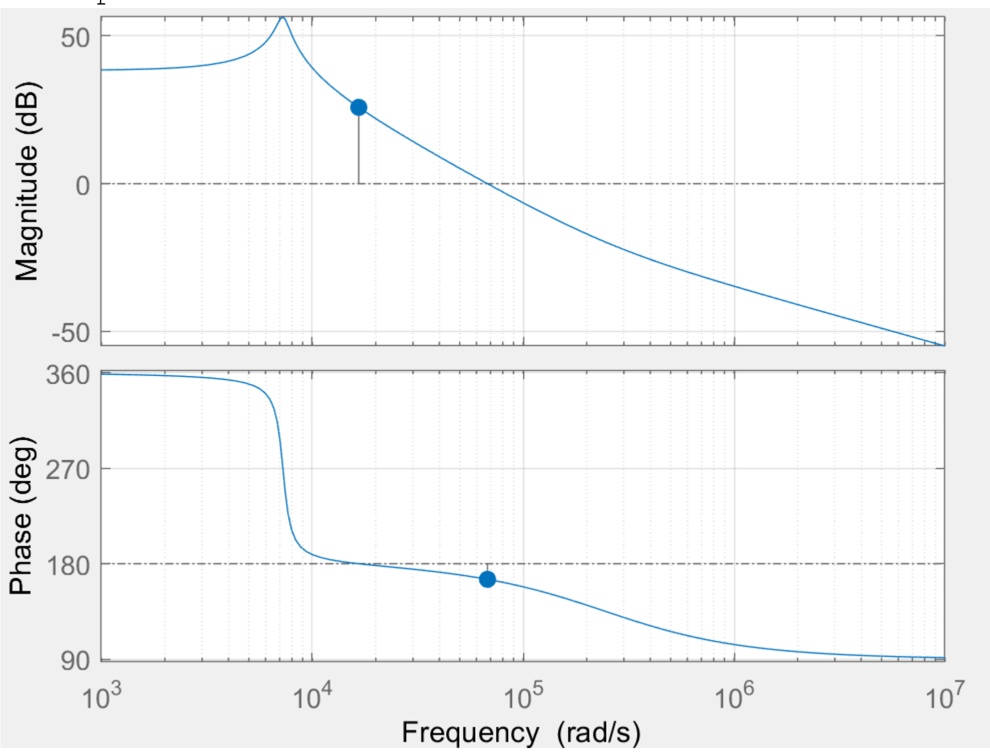
Transfer function after sign inversion and numerical values entered in:

```
Gvd =

      -0.0003391 s + 82.69
  ------------------------------
  1.895e-08 s^2 + 1.723e-05 s + 1

Continuous-time transfer function.
```

Bode plot:

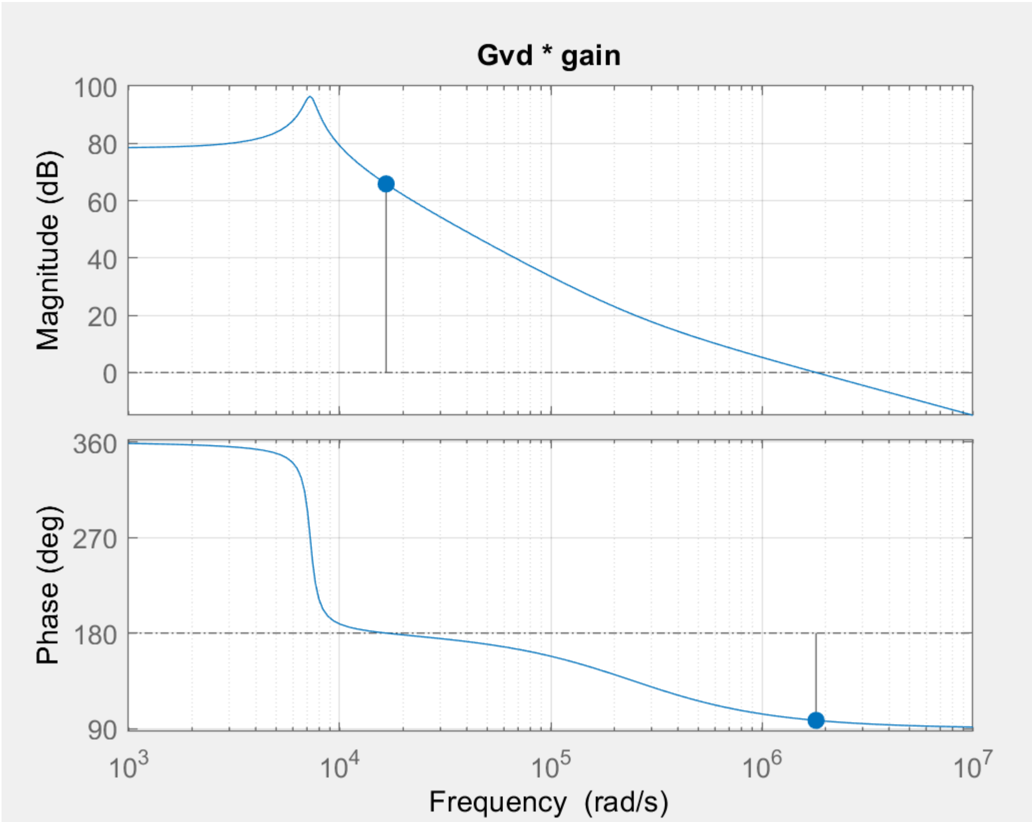


Phase margin: -14.7 degrees
Gain margin: -25.8dB

Part 2

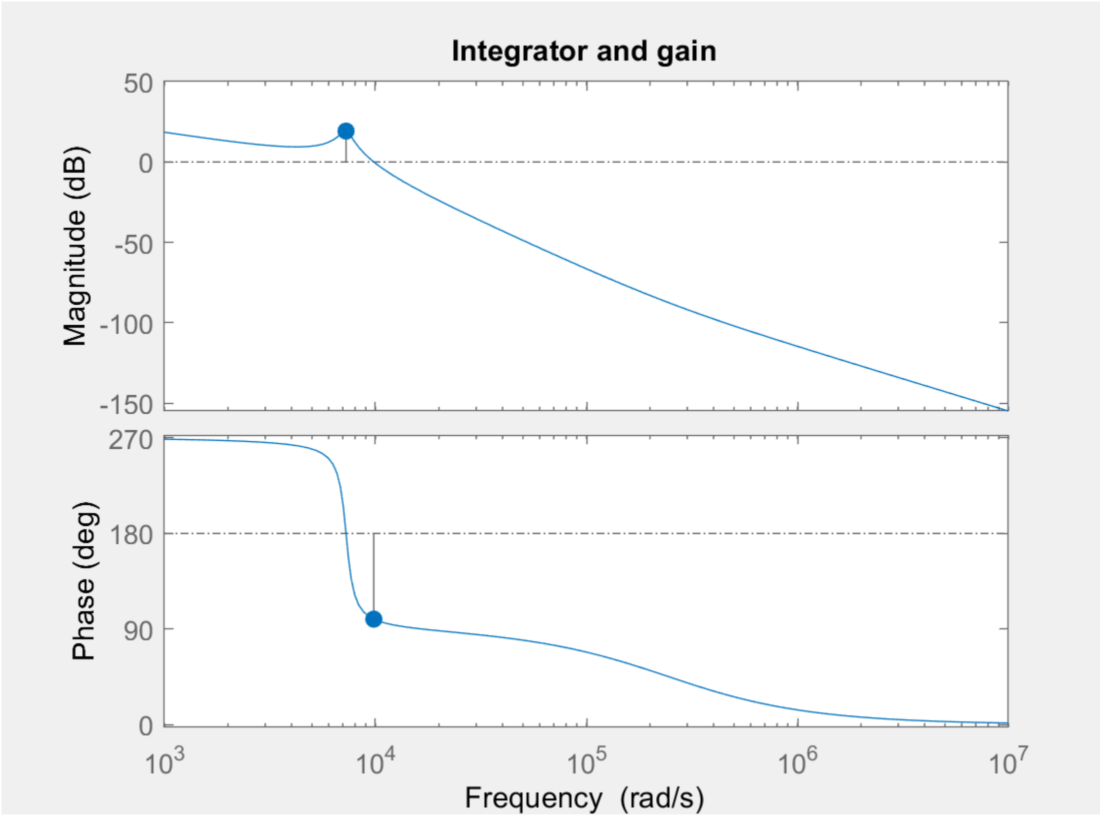
Controller design summary: PI controller + lead compensator
See Matlab script for detailed controller design.

Begin by just having a gain of 100.



This is still not stable. However, crossover frequency has increased (note: staying at this crossover frequency would probably be too high given the switching frequency)

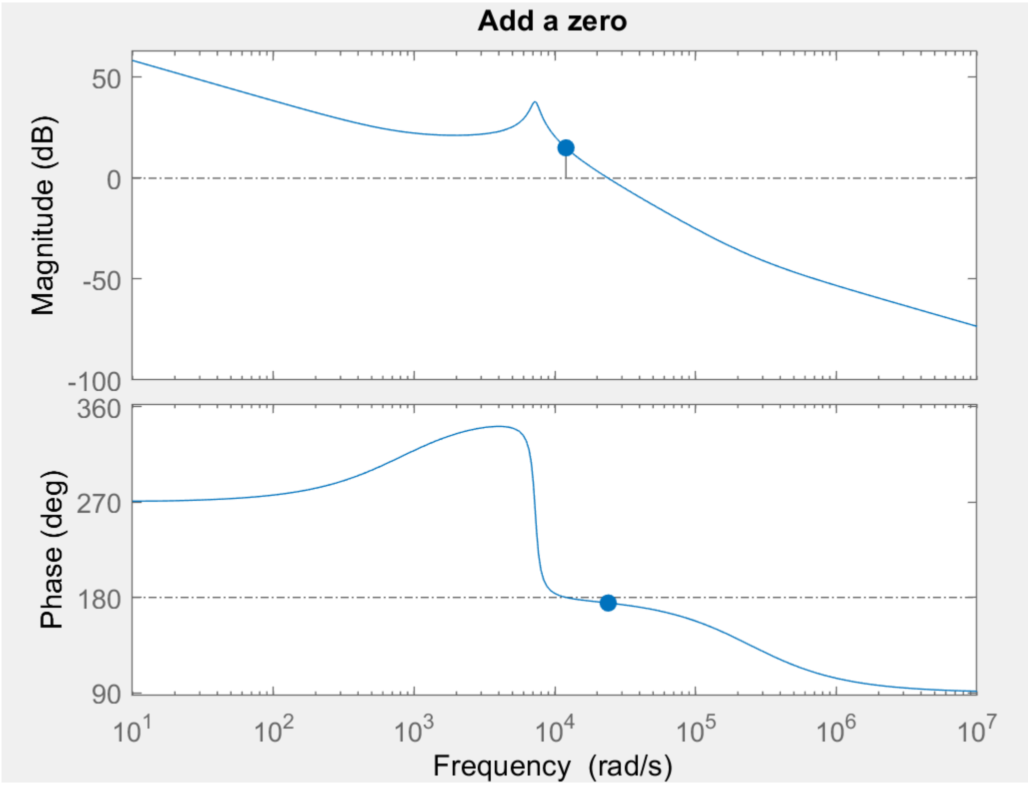
Next, I add an integrator. This means my controller is now 100/s



It is still not stable.

I add a zero near the crossover frequency; this makes my controller a PI controller.

```
 0.1192 s + 100
 --------------
       s
```



It is still not stable, but it is closer to being so. I do not want to increase gain further because I don't want to push bandwidth too much higher. Instead, I will add a lead compensator to bump up the phase.

I design my lead compensator to add 80 degrees of phase near the crossover.

lead compensator design

$$G_c(s) = G_{co} \frac{\left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)}$$

obtain ω_z, ω_p w/ following formulas:

$$f_z = f_c \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} \qquad f_p = f_c \sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}}$$

where θ is my desired phase increase of $80°$

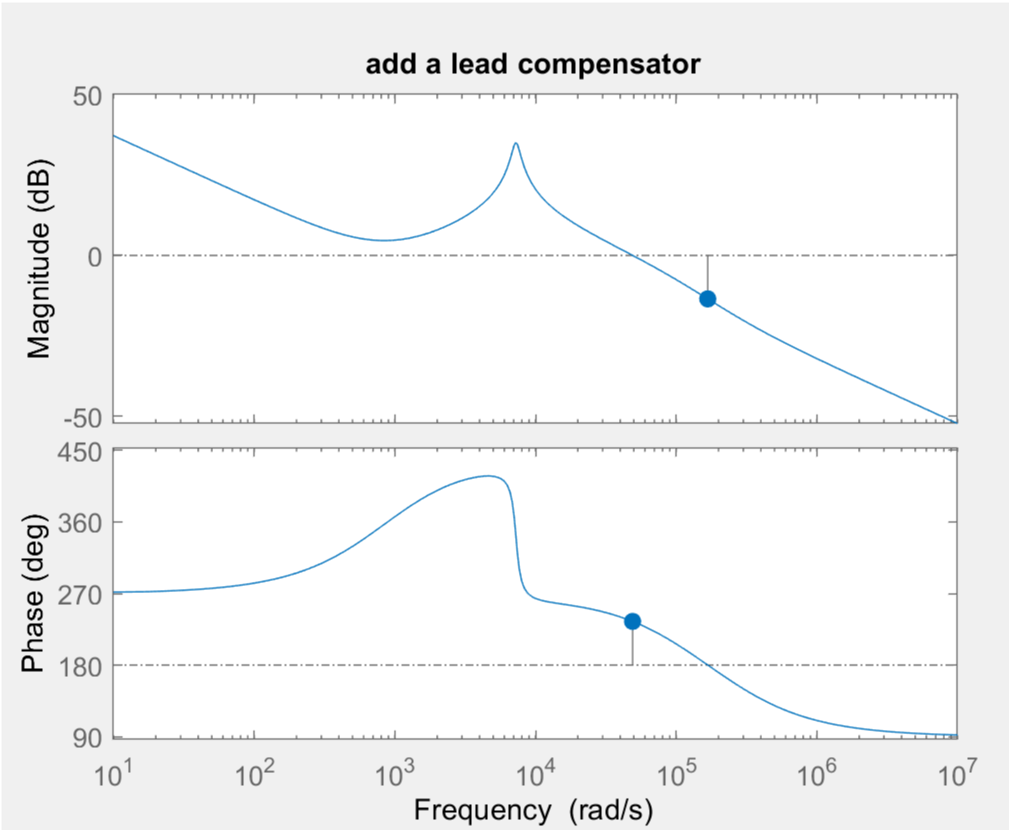
and

f_c is the frequency where I want to bump up phase (close to crossover)

$= 1.02e4$ rad/sec

once have f_z, f_p,

$$G_{co} = \sqrt{\frac{f_z}{f_p}}$$



It is now **stable**, hurray! Phase margin is 54.9 degrees, so **meets** the requirement. Gain margin is 13.5dB and crossover frequency is **49,000 rad/sec** which is decently high and less than 10% of switching frequency.

My final controller is:

```
 1.169e-05 s^2 + 0.02023 s + 8.749
 ---------------------------------
        8.577e-06 s^2 + s
```

Part 3

Plecs simulation without anti-windup. Duty is saturated from 0.05 to 0.95

Problem 1.1 to 1.2 (Tim Murphy)

<u>HW4 MATLAB Plots and Code</u> - Problem 1

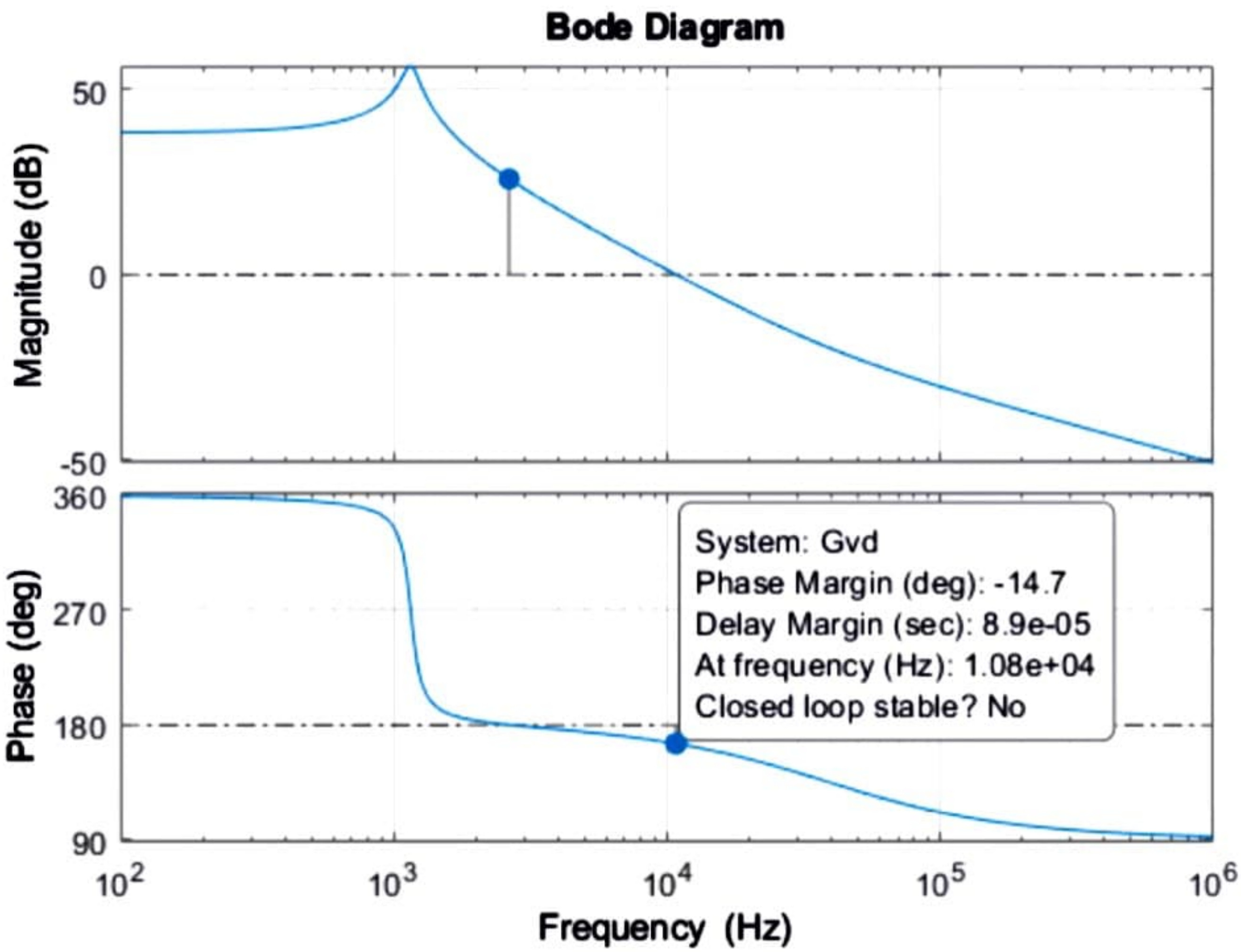


Figure 1: Uncompensated Plant Transfer Function

Starting out my control design I plotted the plant transfer function of the Buck-Boost converter. This way I can get an idea what the initial phase margin I am working with.

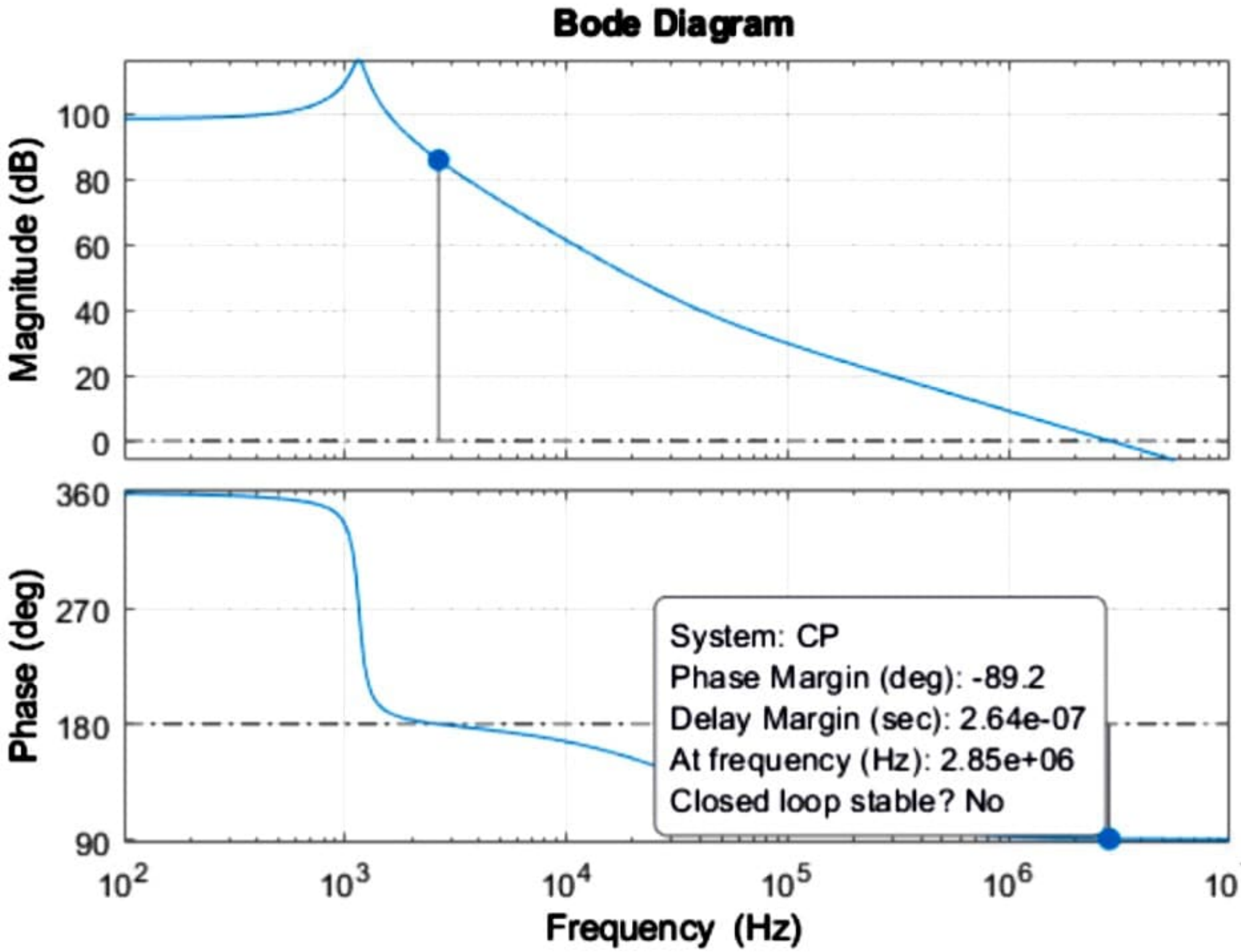


Figure 2: Plant Compensated with High Gain (x1k)

Next I incorporated gain compensation with my plant to increase bandwidth. As seen in figure 2, a gain of x1k pushes the crossover frequency into the megahertz range. So we now have larger bandwidth but our phase margin is significantly degraded.

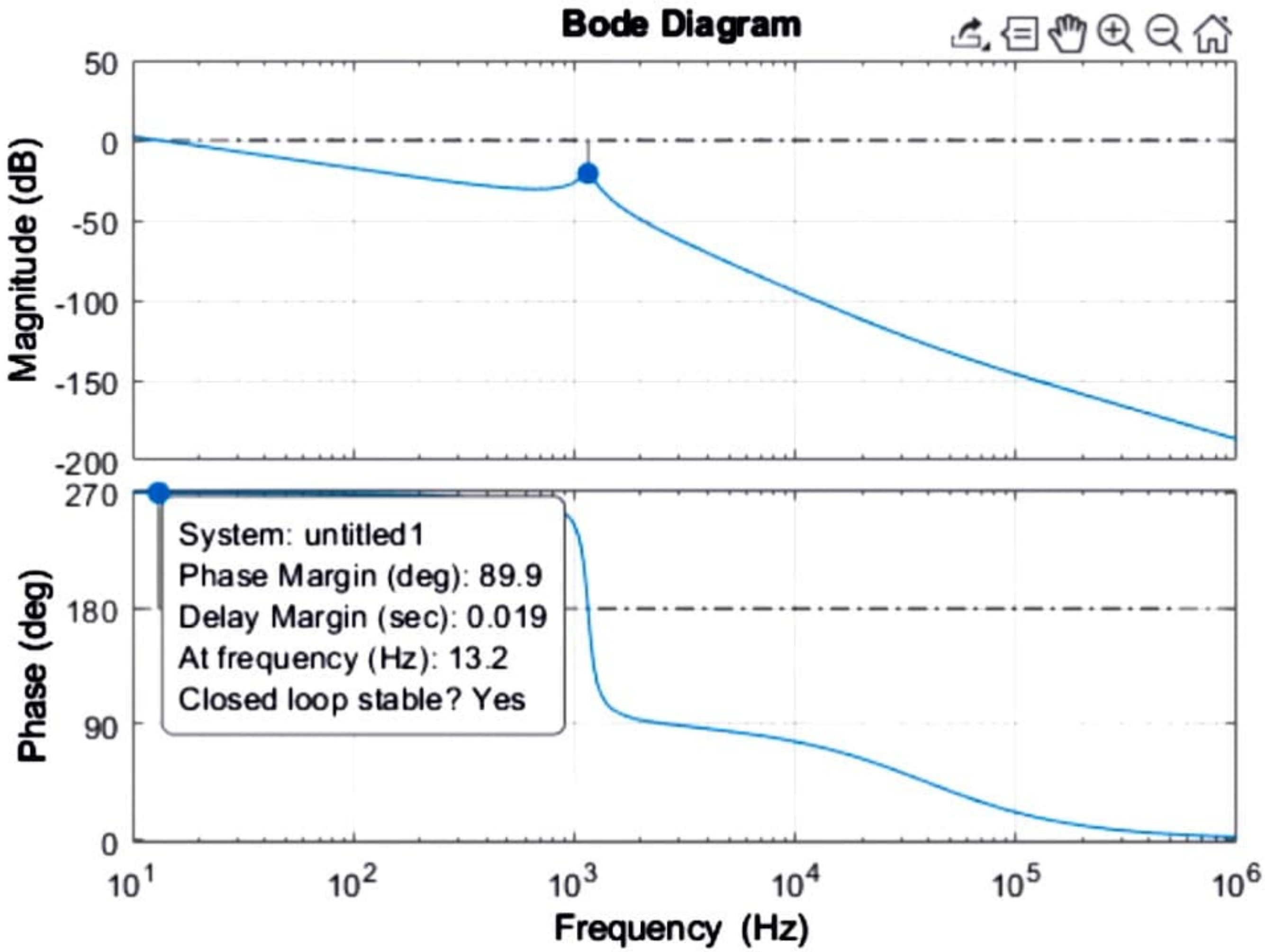


Figure 3: Compensated Plant with Integrator

Next I incorporated an integrator with no gain to the plant to see how this would affect phase margin and bandwidth. We now have better phase margin now, but our gain has been all but eliminated so now our controller will not track commands well. Now we need to incorporate the x1k gain to see if it improves.

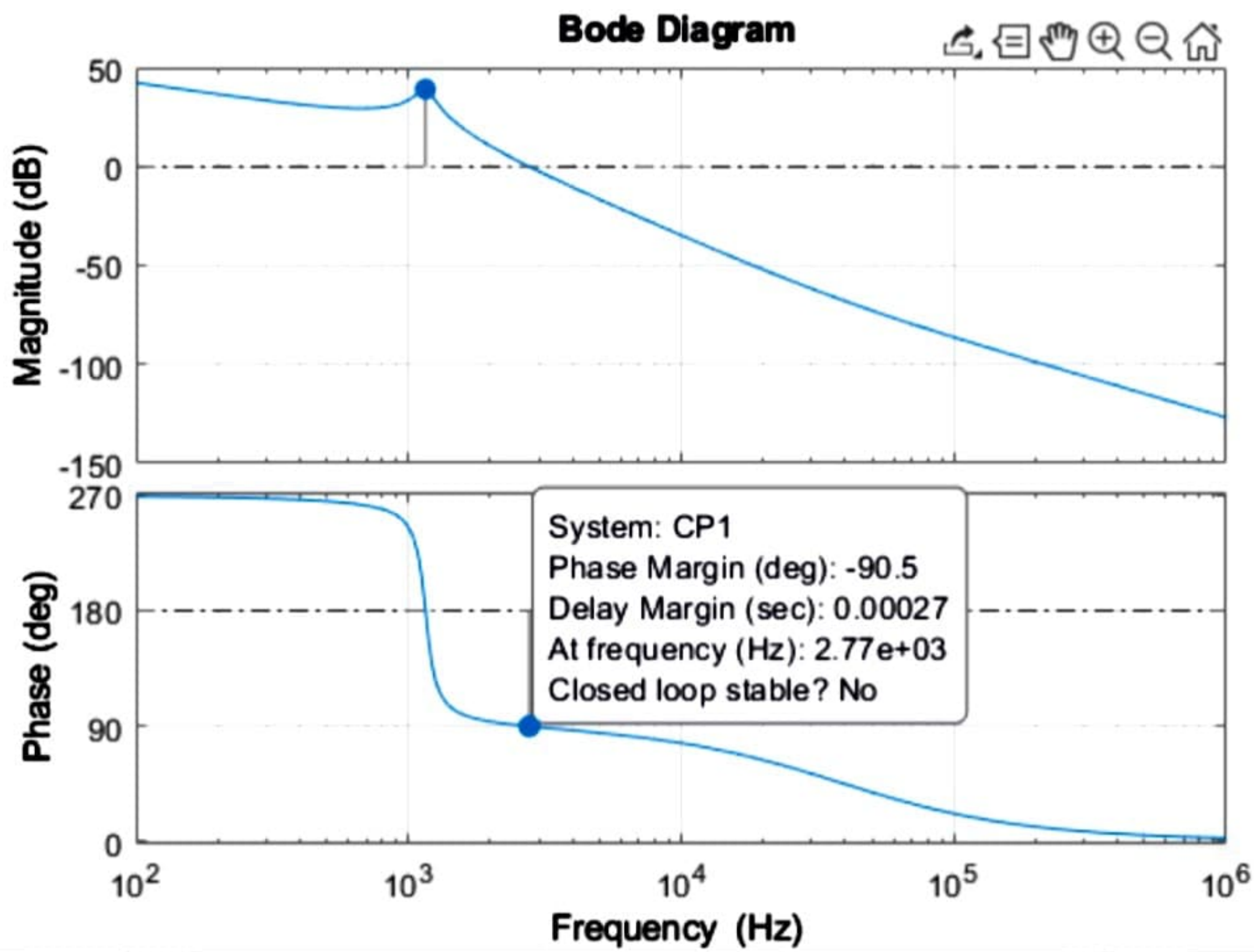


Figure 4: Compensated Plant with Integrator and Gain

Now my controller is starting to function more properly and I now have some decent gain to help with command tracking but once again my phase margin degraded. Ideally I needed something that would not affect my gain while improving the phase of my system. I tried adding a zero near the gain crossover frequency of 2.77 kHz to see if that would improve anything.

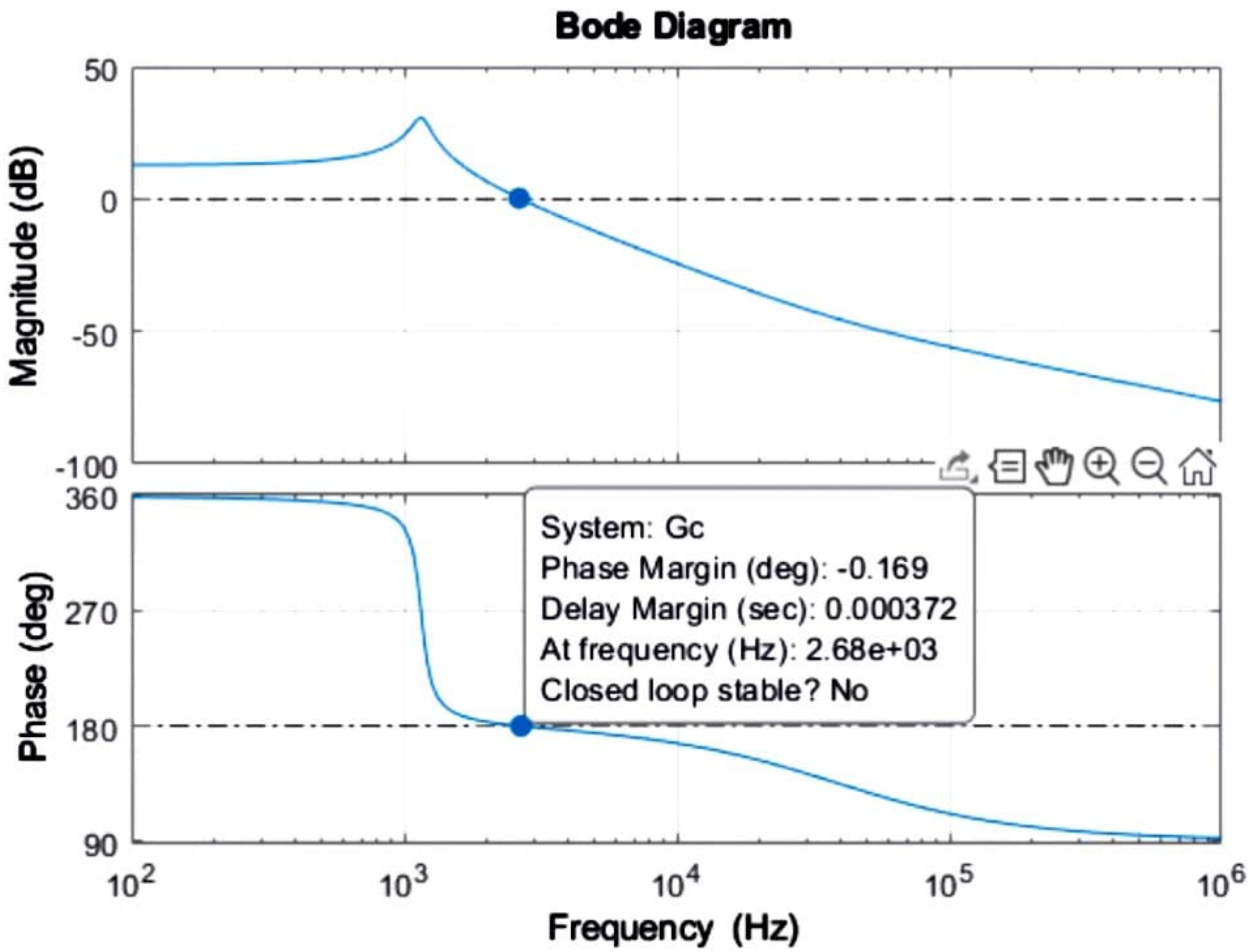


Figure 5: Compensated Plant with Integrator, Gain, and a Zero

Luckily the zero placement improved my phase margin considerably. My next strategy was to try to keep adding another nearby zero and a high frequency pole to see if that would improve my phase margin without impacting my gain plot. That ended up distorting my gain and accentuating the effects of the resonance seen in the gain curve at ω_0. Next I referred to the textbook and decided to implement a lead lag compensator. I figured to do so because now I have a target phase lead to add in order to improve phase margin, which is what a lead lag compensator would be ideal for as indicated in the book. So I went forth and designed a lag lead compensator around a cutoff frequency very close to 2.68 kHz (3 kHz) just utilizing the steps from the text. I wanted to try adding 55 degrees of phase to see if that would improve my phase margin to my desired target.

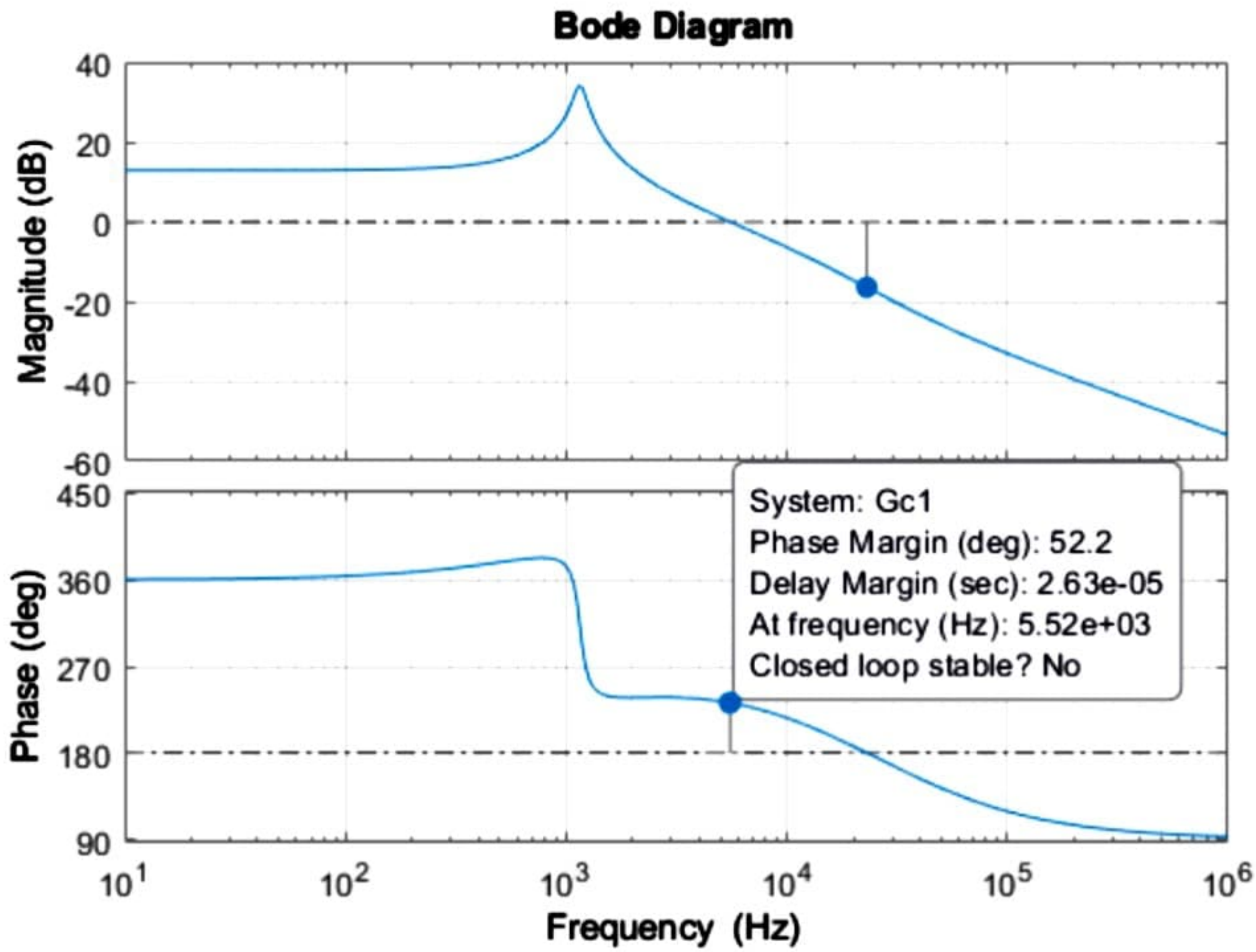


Figure 6: Compensated Plant with Gain, Integrator, Zero, and Lead/Lag Compensator

Sure enough adding the lead lag compensator improved the phase margin substantially. However it only pushed the phase margin to 39 degrees and not 52. I tried increasing the amount of phase added to 60 degrees but that did not afford much improvement, only 44 degrees of phase margin now. I tried one other tactic which was to shift the cutoff frequency of my lead lag compensator a little further to the right along the frequency spectrum of 3 kHz (now at 4 kHz). This adjustment was enough to push phase margin almost to exactly 52.2 degrees of phase margin while maintaining a bandwidth in the kHz range and have at least some “ok” reference tracking (referring to figure 6). The result seemed good, however I thought the closed loop stability warning saying “no” seemed suspect. So I put the new compensated transfer function into closed loop form and did a step response on it to see if was indeed closed loop stable.

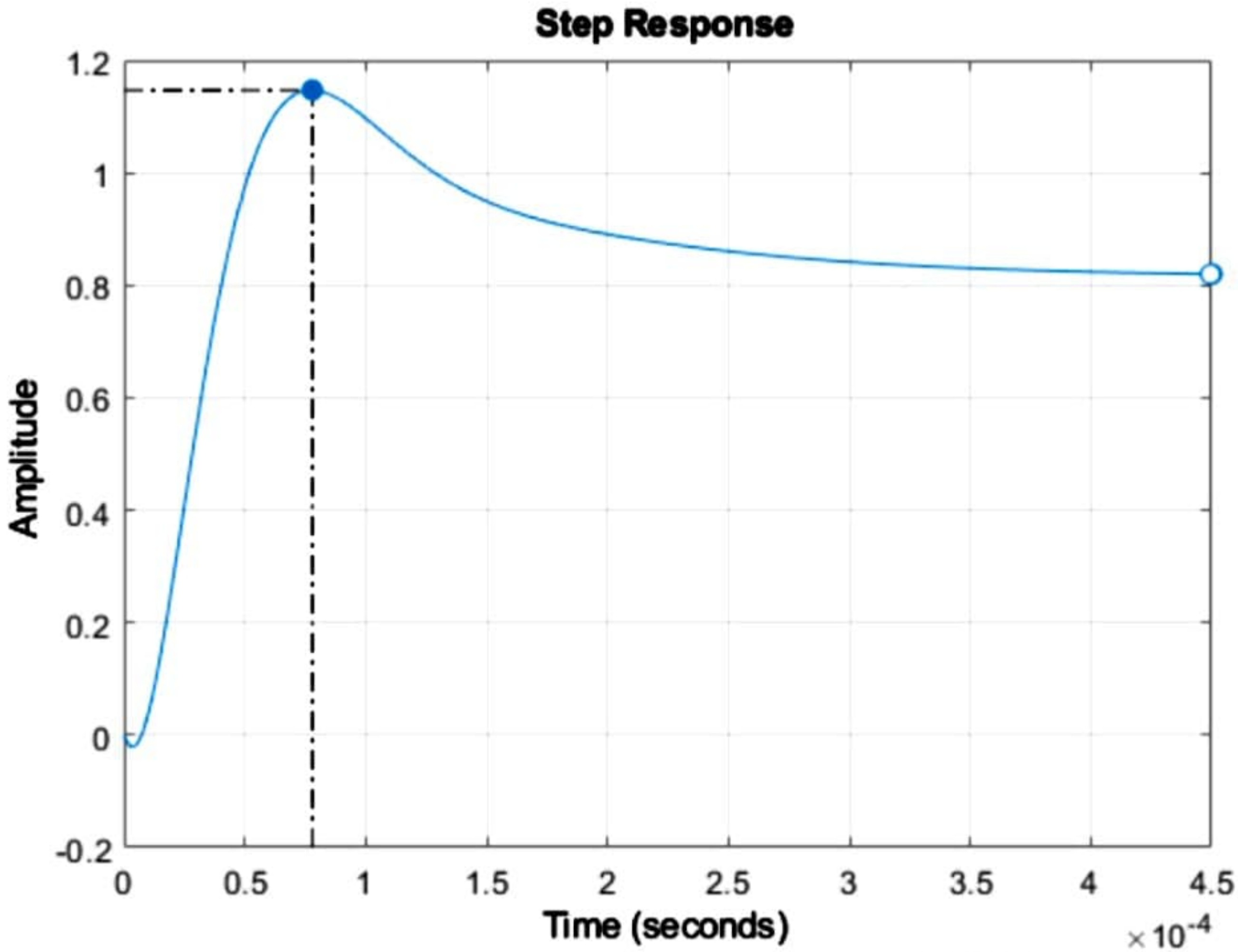


Figure 7: Compensated Plant with Gain, Integrator, Zero, and Lead/Lag Compensator Step Response

Using a step response as a check I concluded the controller seemed stable enough. The step response seemed quick, not much overshoot, and minimal oscillation to me which are signs of a stable system. From there I went and implemented this controller in PLECS.

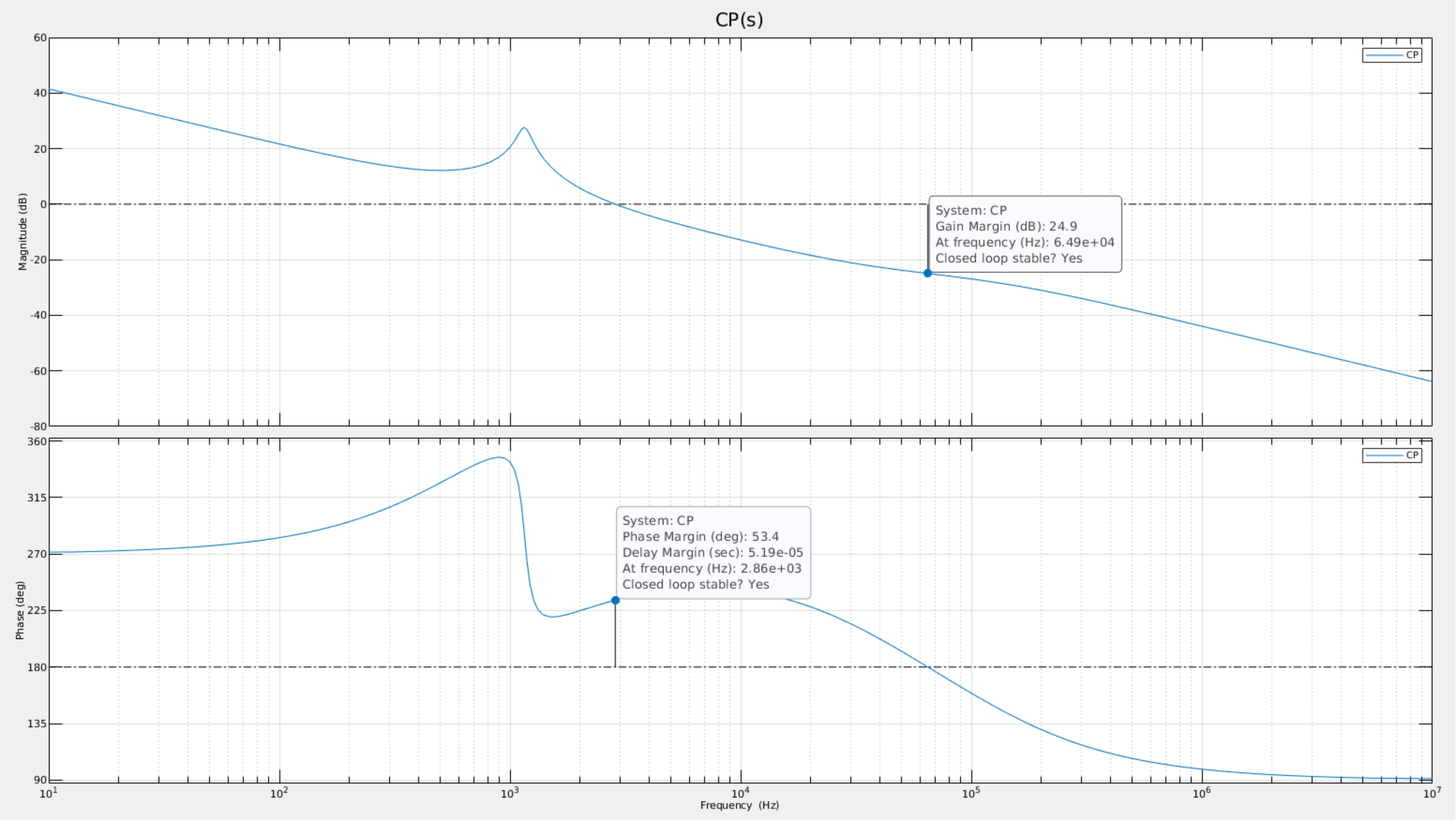
Problem 1.3 to 1.5 (Rohit Gujarathi)

2) Designed controller

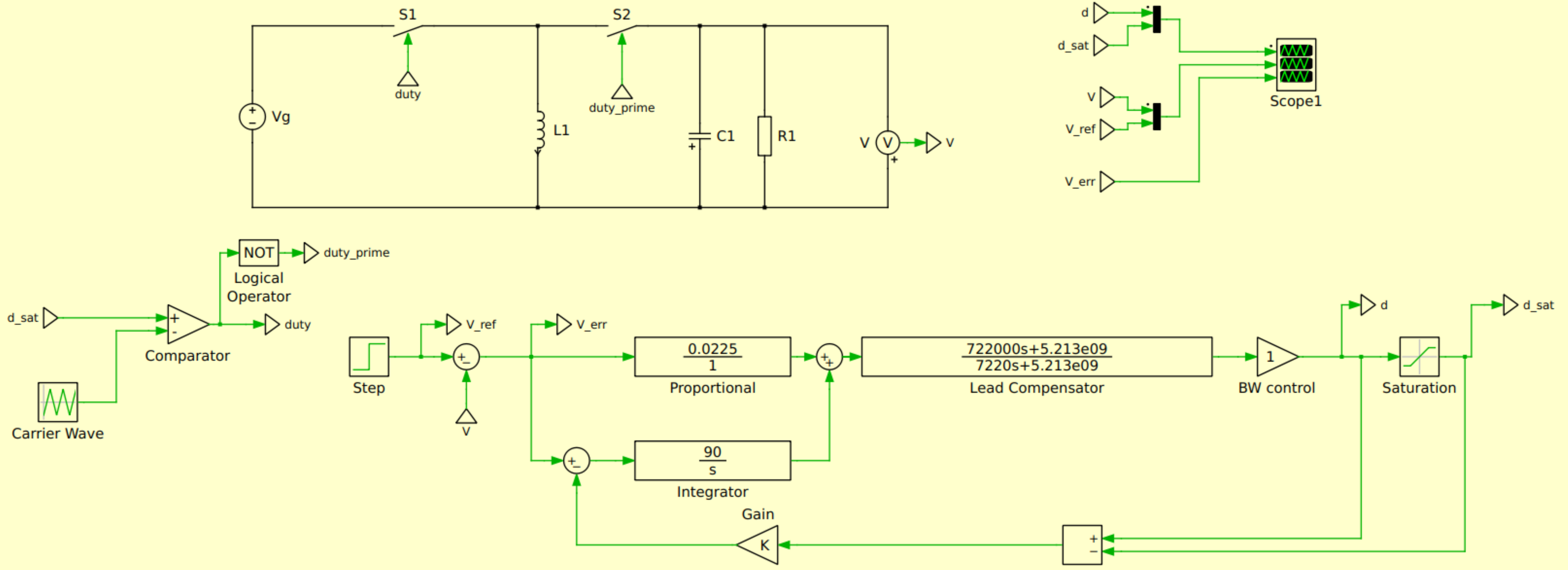
$$C(s) = \frac{6.498\times10^{7}\, s^{2} + 7.291\times10^{11}\, s + 1.877\times10^{15}}{2.888\times10^{7}\, s^{2} + 2.085\times10^{13}\, s}$$

Phase Margin = 53.4 degrees

Bandwidth = 2.8592 KHz

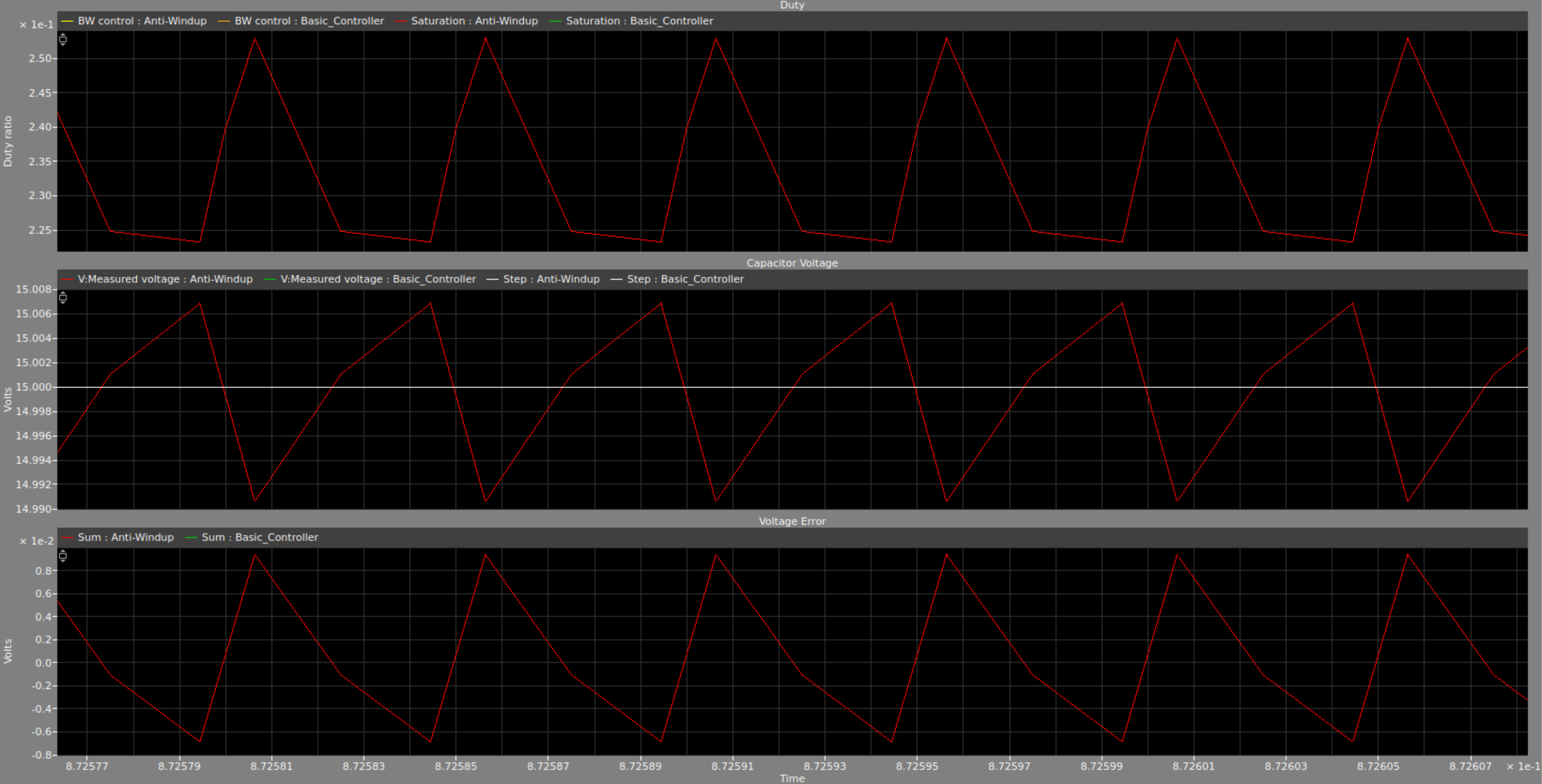


CP(s)
CP
Magnitude (dB)
60
40
20
0
-20
-40
-60
-80
System: CP
Gain Margin (dB): 24.9
At frequency (Hz): 6.49e+04
Closed loop stable? Yes
CP
Phase (deg)
360
315
270
225
180
135
90
System: CP
Phase Margin (deg): 53.4
Delay Margin (sec): 5.19e-05
At frequency (Hz): 2.86e+03
Closed loop stable? Yes
10^1
10^2
10^3
10^4
10^5
10^6
10^7
Frequency (Hz)



S1
S2
duty
duty_prime
Vg
L1
C1
R1
V
d
d_sat
V_ref
V_err
Scope1
NOT
Logical Operator
duty_prime
d_sat
duty
Comparator
Carrier Wave
V_ref
V_err
Step
0.0225 / 1
Proportional
722000s+5.213e09 / 7220s+5.213e09
Lead Compensator
1
BW control
Saturation
d
d_sat
90 / s
Integrator
Gain
K

Duty
BW control : Anti-Windup
BW control : Basic_Controller
Saturation : Anti-Windup
Saturation : Basic_Controller
Duty ratio
Capacitor Voltage
V:Measured voltage : Anti-Windup
V:Measured voltage : Basic_Controller
Step : Anti-Windup
Step : Basic_Controller
Volts
Voltage Error
Sum : Anti-Windup
Sum : Basic_Controller
Volts
Time

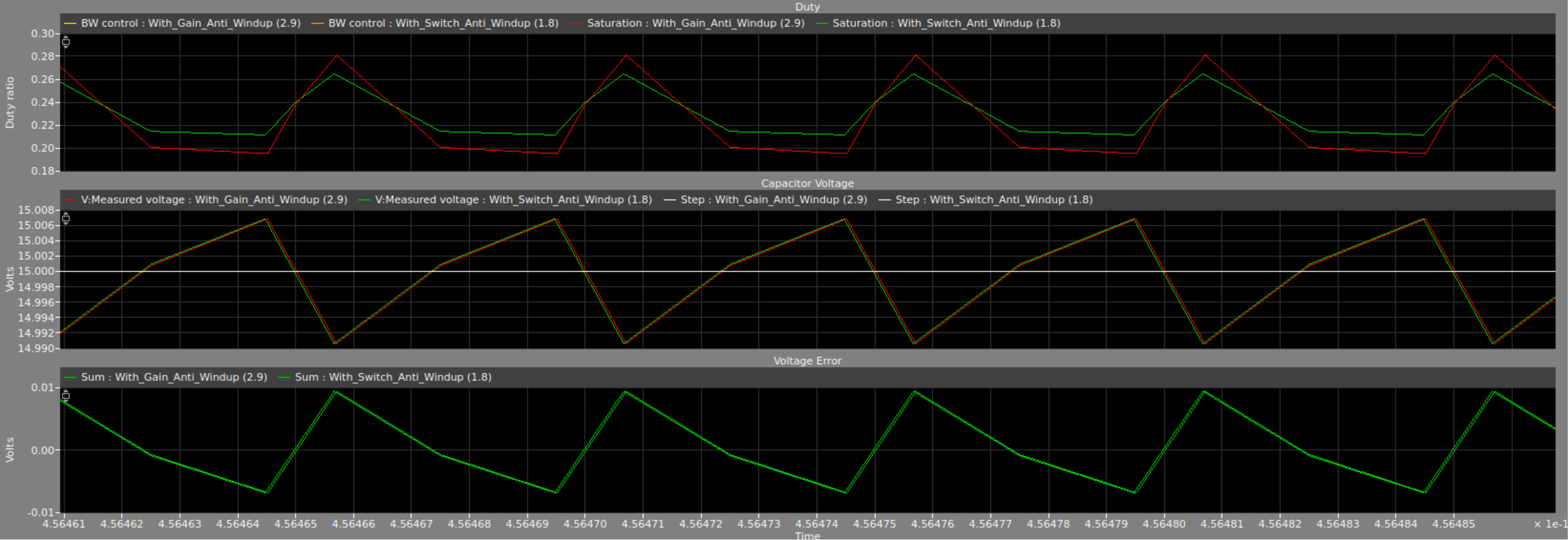

Duty
BW control : Anti-Windup
BW control : Basic_Controller
Saturation : Anti-Windup
Saturation : Basic_Controller
× 1e-1
Duty ratio
2.50
2.45
2.40
2.35
2.30
2.25
Capacitor Voltage
V:Measured voltage : Anti-Windup
V:Measured voltage : Basic_Controller
Step : Anti-Windup
Step : Basic_Controller
Volts
15.008
15.006
15.004
15.002
15.000
14.998
14.996
14.994
14.992
14.990
Voltage Error
Sum : Anti-Windup
Sum : Basic_Controller
× 1e-2
Volts
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
8.72577
8.72579
8.72581
8.72583
8.72585
8.72587
8.72589
8.72591
8.72593
8.72595
8.72597
8.72599
8.72601
8.72603
8.72605
8.72607
× 1e-1
Time

4) → If BW is increased, the rise time keeps decreasing. In this particular controller, as BW is increased PM also increases in turn increasing the peak overshoot.

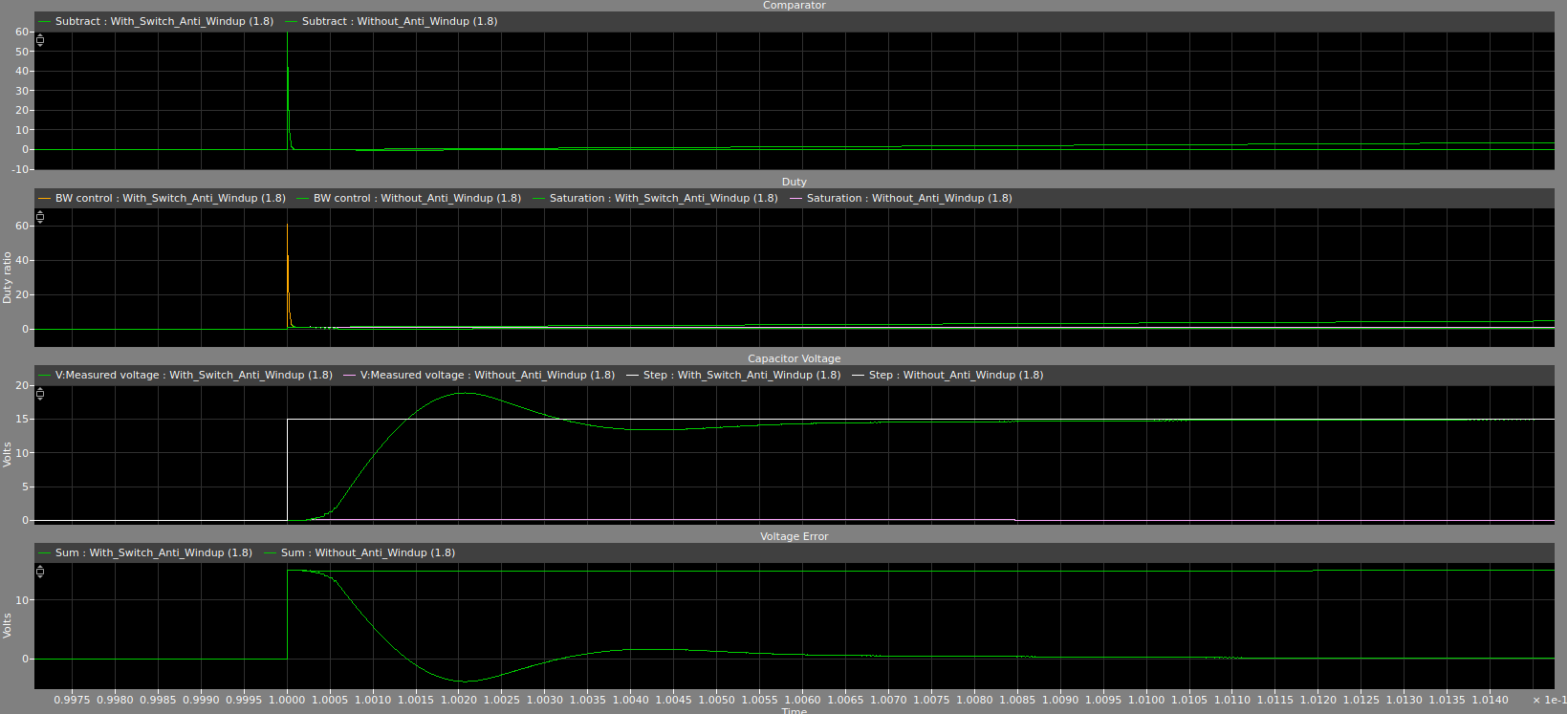
→ The largest BW is 6.6798 KHz. Increasing BW beyond this leads to unstable operation. The output duty cycle keeps increasing, ie it saturates to 1 and hence the inductor keeps charging and never dumps its energy to the capacitor.

→ Increasing BW to 6.6798 KHz is only possible due to anti-windup. without anti-windup the system does not operate correctly. ie duty is saturated to 1.

→ Moreover, increasing the BW, makes the system respond quickly. which makes it jerky. This can be seen as a greater peak to peak change in the duty cycle.



Duty
BW control : With_Gain_Anti_Windup (2.9)
BW control : With_Switch_Anti_Windup (1.8)
Saturation : With_Gain_Anti_Windup (2.9)
Saturation : With_Switch_Anti_Windup (1.8)
Duty ratio
0.30
0.28
0.26
0.24
0.22
0.20
0.18
Capacitor Voltage
V:Measured voltage : With_Gain_Anti_Windup (2.9)
V:Measured voltage : With_Switch_Anti_Windup (1.8)
Step : With_Gain_Anti_Windup (2.9)
Step : With_Switch_Anti_Windup (1.8)
Volts
15.008
15.006
15.004
15.002
15.000
14.998
14.996
14.994
14.992
14.990
Voltage Error
Sum : With_Gain_Anti_Windup (2.9)
Sum : With_Switch_Anti_Windup (1.8)
Volts
0.01
0.00
-0.01
4.56461
4.56462
4.56463
4.56464
4.56465
4.56466
4.56467
4.56468
4.56469
4.56470
4.56471
4.56472
4.56473
4.56474
4.56475
4.56476
4.56477
4.56478
4.56479
4.56480
4.56481
4.56482
4.56483
4.56484
4.56485
× 1e-1
Time


Comparator
Subtract : With_Switch_Anti_Windup (1.8)
Subtract : Without_Anti_Windup (1.8)
Duty
BW control : With_Switch_Anti_Windup (1.8)
BW control : Without_Anti_Windup (1.8)
Saturation : With_Switch_Anti_Windup (1.8)
Saturation : Without_Anti_Windup (1.8)
Duty ratio
Capacitor Voltage
V:Measured voltage : With_Switch_Anti_Windup (1.8)
V:Measured voltage : Without_Anti_Windup (1.8)
Step : With_Switch_Anti_Windup (1.8)
Step : Without_Anti_Windup (1.8)
Volts
Voltage Error
Sum : With_Switch_Anti_Windup (1.8)
Sum : Without_Anti_Windup (1.8)
Volts
Time

5) For bilinear transform, $s = \frac{2}{T_{samp}} \left[\frac{1 - z^{-1}}{1 + z^{-1}} \right]$

$$C(s) = \frac{6.498 \times 10^7 s^2 + 7.291 \times 10^{11} s + 1.877 \times 10^{15}}{2.888 \times 10^7 s^2 + 2.085 \times 10^{13} s}$$

$$\frac{d(s)}{e(s)} = \frac{6.498 \times 10^7 s^2 + 7.291 \times 10^{11} s + 1.877 \times 10^{15}}{2.888 \times 10^7 s^2 + 2.085 \times 10^{13} s}$$

let $p = 6.498 \times 10^7$ $\qquad s = 2.888 \times 10^7$

$q = 7.291 \times 10^{11}$ $\qquad t = 2.085 \times 10^{13}$

$r = 1.877 \times 10^{15}$

$$T_{samp} = \frac{1}{200 \times 10^3 \times 2} = 2.5 \times 10^{-6} \text{ s}$$

$$\rightarrow \frac{d(z)}{e(z)} = \frac{p\left(\frac{2}{T_{samp}}\right)^2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + q\left(\frac{2}{T_{samp}}\right)\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + \gamma}{s\left(\frac{2}{T_{samp}}\right)^2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + t\left(\frac{2}{T_{samp}}\right)\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$\rightarrow d(z)\,\frac{4s}{T_{samp}^2}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + d(z)\,\frac{2t}{T_{samp}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$= e(z)\,\frac{4p}{T_{samp}^2}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + e(z)\,\frac{2q}{T_{samp}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + \gamma e(z)$$

Multiplying by $(1+z^{-1})^2$

$$\rightarrow d(z)\,\frac{4s}{T_{samp}^2}(1-z^{-1})^2 + d(z)\,\frac{2t}{T_{samp}}(1-z^{-1})(1+z^{-1})$$

$$= e(z)\,\frac{4p}{T_{samp}^2}(1-z^{-1})^2 + e(z)\,\frac{2q}{T_{samp}}(1-z^{-1})(1+z^{-1}) + \gamma e(z)(1+z^{-1})$$

$$\rightarrow \frac{4s}{T_{samp}^2}\,d(z)(1-z^{-1})^2 + \frac{2t}{T_{samp}}\,d(z)(1-z^{-2})$$

$$= \frac{4p}{T_{samp}^2}\,e(z)(1-z^{-1})^2 + \frac{2q}{T_{samp}}\,e(z)(1-z^{-2}) + \gamma e(z)(1+z^{-1})$$

Taking inverse Z-transform

$$\to \frac{4s}{T_{samp}^2}\Big[d[n]-2d[n-1]+d[n-2]\Big]+\frac{2t}{T_{samp}}\Big[d[n]-d[n-2]\Big]$$

$$=\frac{4p}{T_{samp}^2}\Big[e[n]-2e[n-1]+e[n-2]\Big]$$

$$+\frac{2q}{T_{samp}}\Big[e[n]-e[n-2]\Big]+r\Big[e[n]+e[n-1]\Big]$$

$$\to \frac{4s}{T_{samp}^2}d[n]+\frac{4s}{T_{samp}^2}\Big(-2d[n-1]+d[n-2]\Big)+\frac{2t}{T_{samp}}d[n]-\frac{2t}{T_{samp}}d[n-2]$$

$$=e[n]\left(\frac{4p}{T_{samp}^2}+\frac{2q}{T_{samp}}+r\right)$$

$$+e[n-1]\left(\frac{-8p}{T_{samp}^2}+r\right)$$

$$+e[n-2]\left(\frac{4p}{T_{samp}^2}-\frac{2q}{T_{samp}}\right)$$

$$\rightarrow d[n]\left(\frac{4s}{T_{samp}^2} + \frac{2t}{T_{samp}}\right) =$$

$$\frac{4s}{T_{samp}^2}\left(2d[n-1] - d[n-2]\right) + \frac{2t}{T_{samp}} d[n-2]$$

$$+ e[n]\left(\frac{4p}{T_{samp}^2} + \frac{2q}{T_{samp}} + \gamma\right)$$

$$+ e[n-1]\left(\frac{-8p}{T_{samp}^2} + \gamma\right)$$

$$+ e[n-2]\left(\frac{4p}{T_{samp}^2} - \frac{2q}{T_{samp}}\right)$$

$$\rightarrow d[n]\left(\frac{4s}{T_{samp}^2} + \frac{2t}{T_{samp}}\right) = d[n-1]\frac{8s}{T_{samp}^2}$$

$$+ d[n-2]\left(\frac{-4s}{T_{samp}^2} + \frac{2t}{T_{samp}}\right) + e[n]\left(\frac{4p}{T_{samp}^2} + \frac{2q}{T_{samp}} + \gamma\right)$$

$$+ e[n-1]\left(\frac{-8p}{T_{samp}^2} + \gamma\right) + e[n-2]\left(\frac{4p}{T_{samp}^2} - \frac{2q}{T_{samp}}\right)$$

Substituting values:

$$\rightarrow \quad 3.5163 \times 10^{-6}\, d[n] = 3.6966 \times 10^{19}\, d[n-1]$$
$$- 1.8032 \times 10^{18}\, d[n-2]$$
$$+ 4.2172 \times 10^{19}\, e[n]$$
$$- 8.3173 \times 10^{19}\, e[n-1]$$
$$+ 4.1004 \times 10^{19}\, e[n-2]$$

dividing by 3.5163×10^{-6}

$$\rightarrow \quad d[n] = 1.0513\, d[n-1] - 0.0513\, d[n-2] + 1.1993\, e[n]$$
$$- 2.3653\, e[n-1] + 1.1661\, e[n-2]$$

Problem 1.3 to 1.5 (Prajakta Popat Wable)

Prajakta Wable

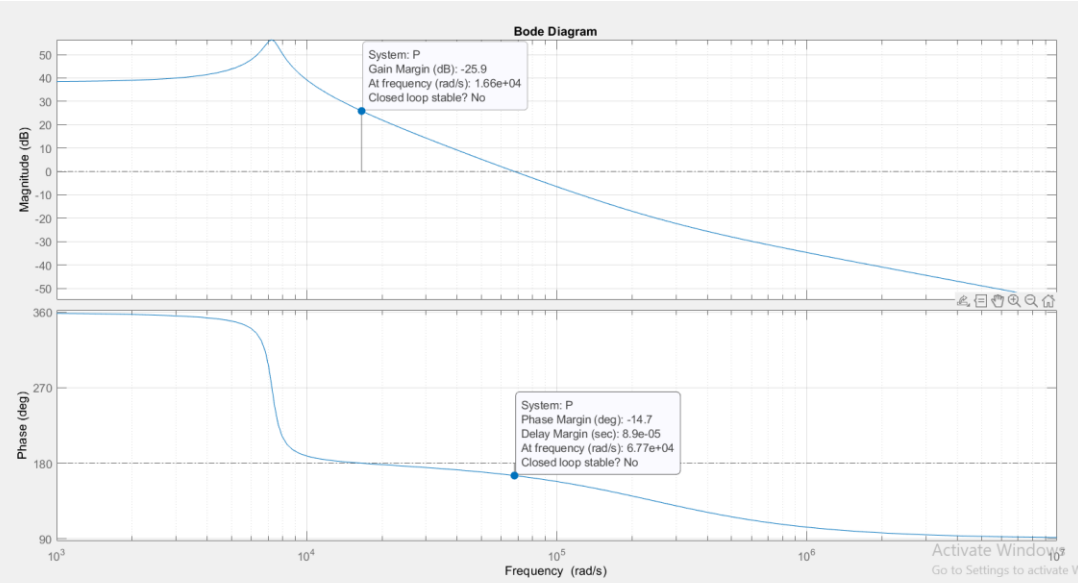
Homework 4

Question 1

Bode plot for the plant transfer function

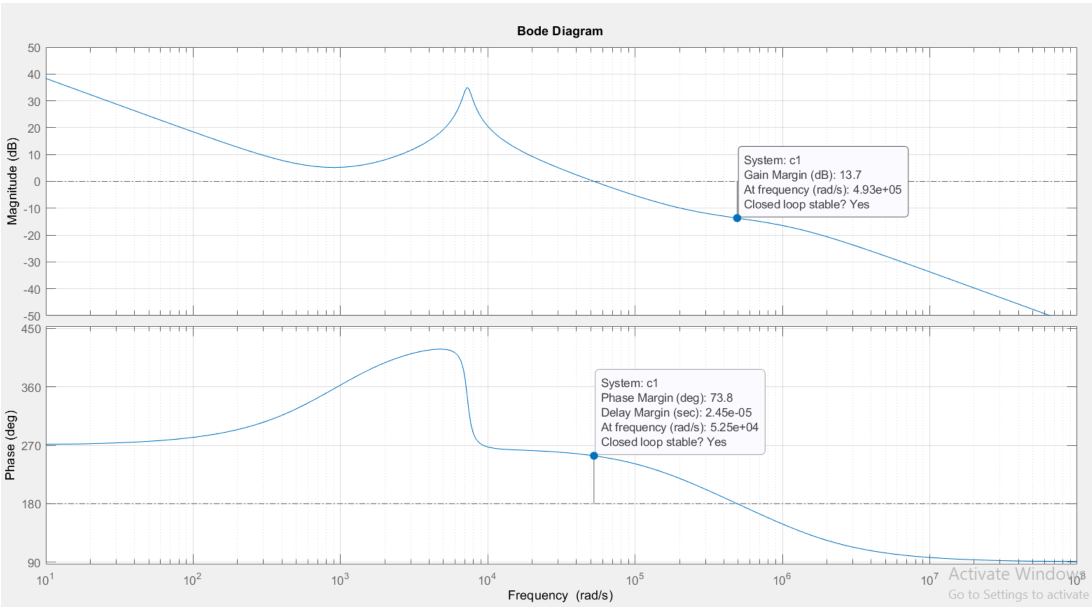
Gain margin= -25.9dB

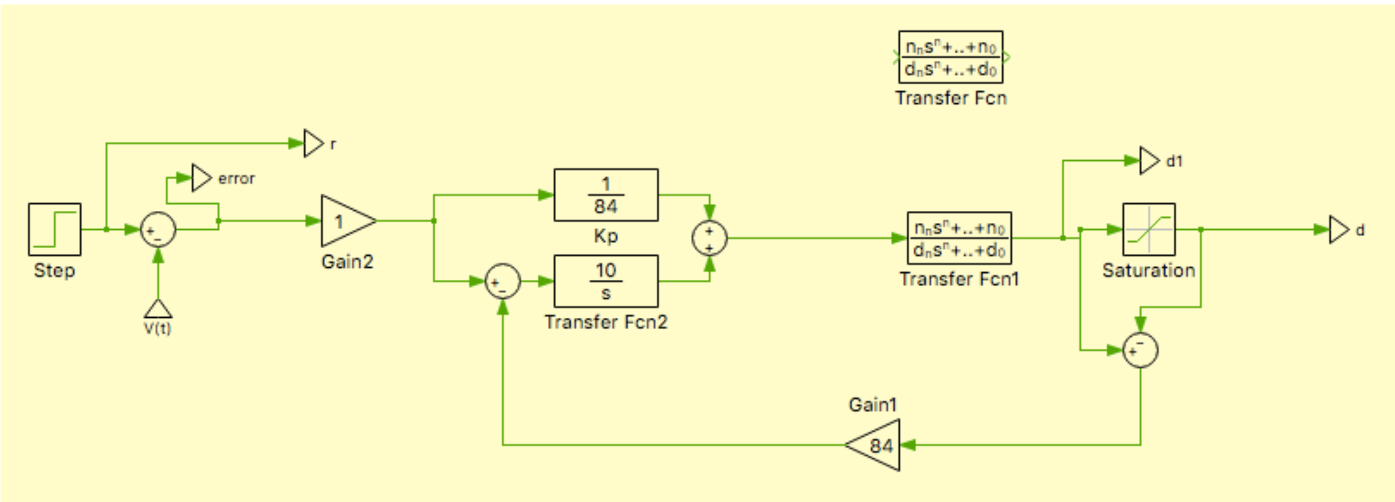
Phase margin= -14.7degree



Controller

$$C(s)=\frac{10*\left(\frac{s}{840}+1\right)*\left(\frac{s}{1030}+1\right)}{s*\left(\frac{s}{1000000}+1\right)}$$





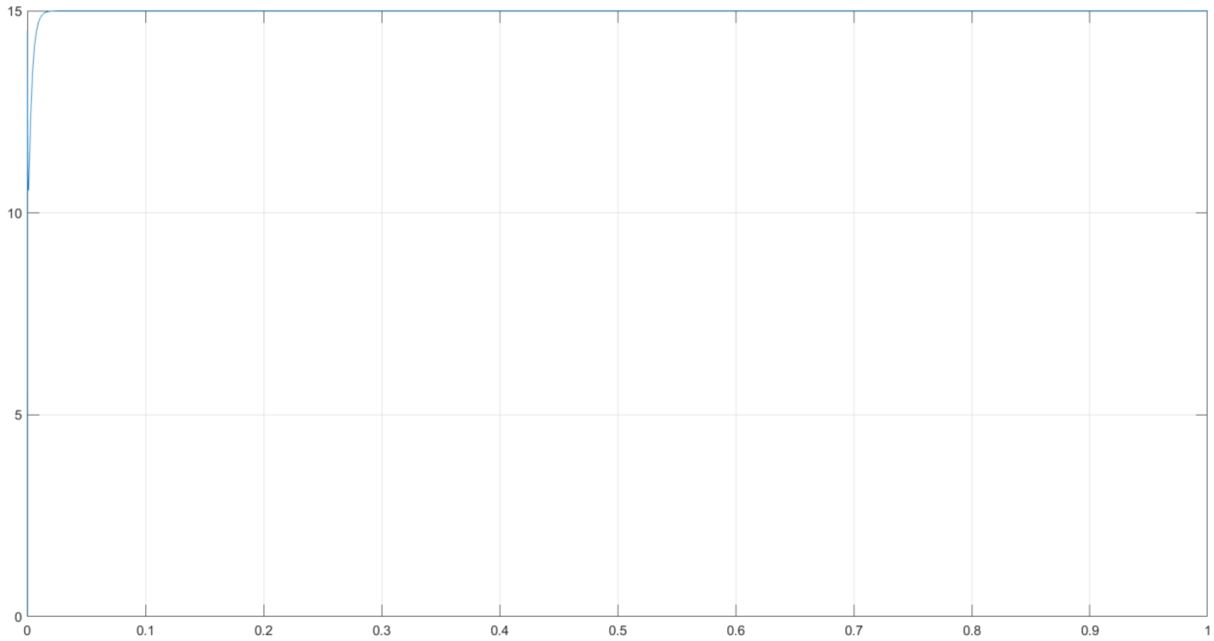
Transfer Fcn
r
error
Step
V(t)
1
Gain2
1
84
Kp
10
s
Transfer Fcn2
Transfer Fcn1
d1
Saturation
d
Gain1
84

Bandwidth= 5.25×10^4 rad/s which is less than 10% of switching frequency.

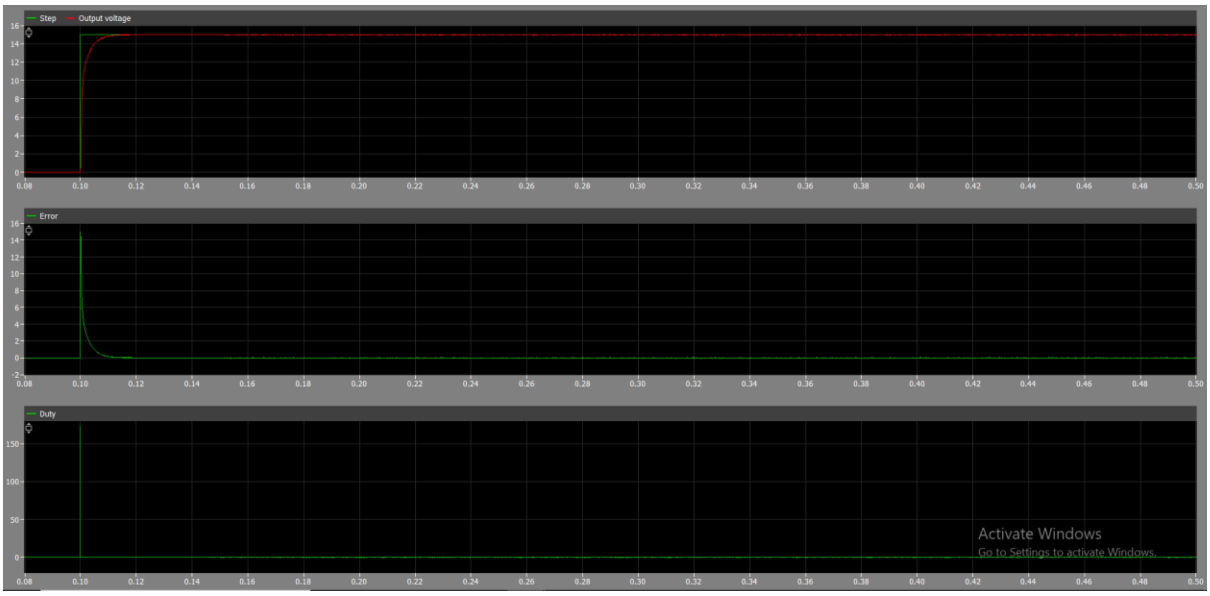
Phase margin= 73.8 degree

It has DC tracking capability.

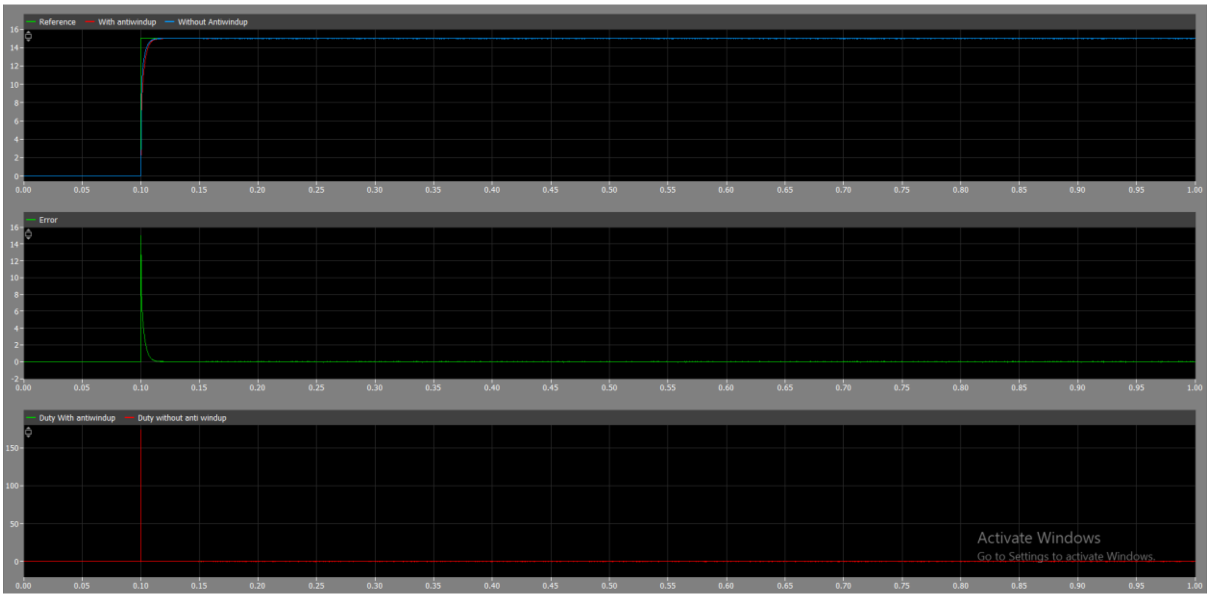
MATLAB step plot



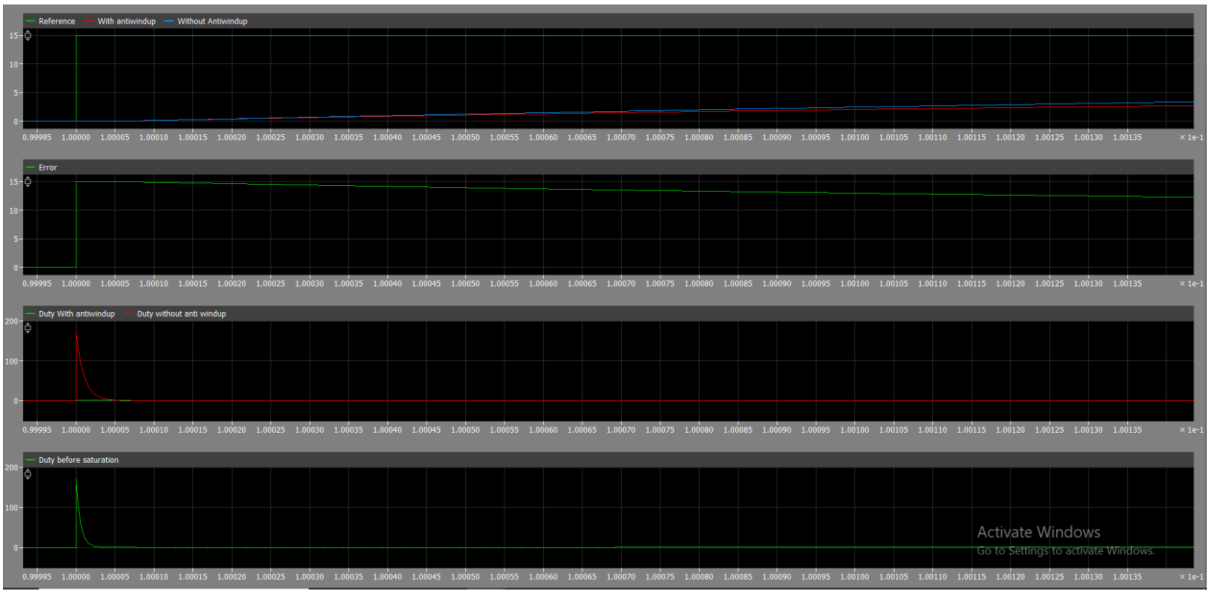
PLECS output. (Plecs file is attached with the homework)



With Anti Windup:



Zoomed transient



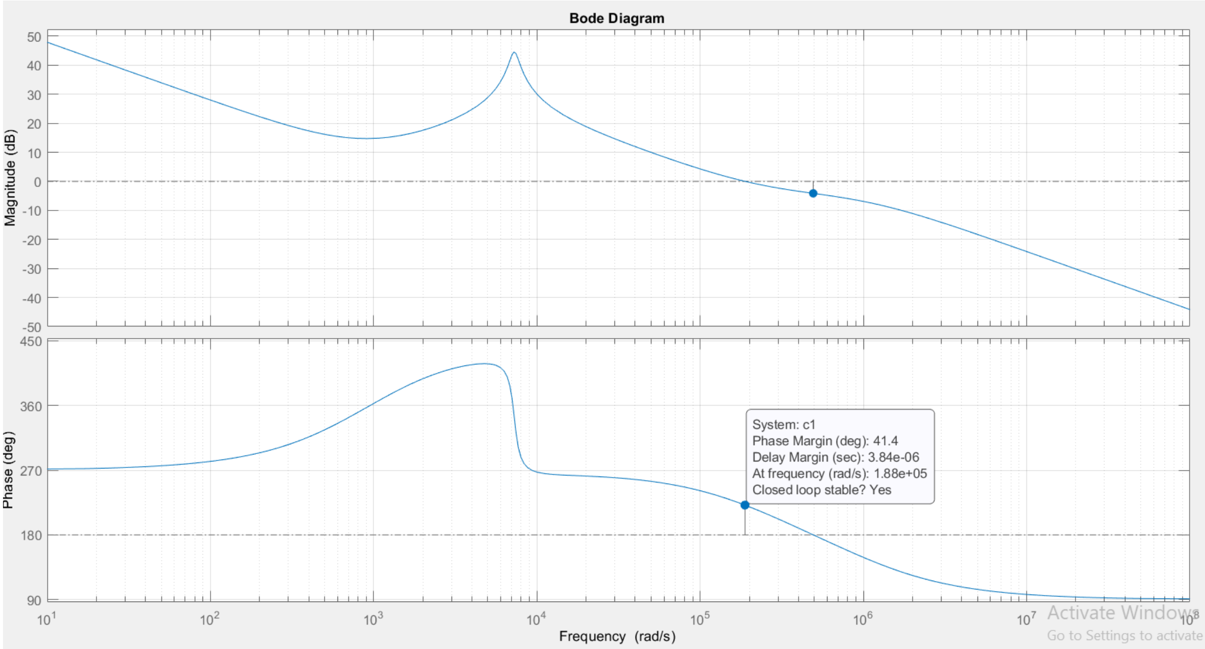
Effect of Increasing bandwidth:

When we increase bandwidth, output does not follow the input.

Maximum bandwidth= 1.88x10^5 rad/s

At $C(s)=3 * \frac{10*\left(\frac{s}{840}+1\right)*\left(\frac{s}{1030}+1\right)}{s*\left(\frac{s}{1000000}+1\right)}$

At this gain,



PLECS output at this frequency,

above this frequency, output does not follow the reference.

Prajakta

Homework 4.

Que. 1

(5) Tustin model,

$$C(s) = \frac{10 \times \left(\frac{s}{840}+1\right)\left(\frac{s}{1030}+1\right)}{s\left(\frac{s}{10^6}+1\right)} = \frac{d(s)}{e(s)}.$$

$$\frac{d(s)}{e(s)} = \frac{10^2 s^2 + 1.87\times10^5 s + 8.652\times10^7}{8.652 s^2 + 8.652\times10^6 s}$$

Denominator =

$$= 8.652 \times \frac{2}{T_{samp}}\left[\frac{2}{T_{samp}}\frac{(1-2z^{-1}+z^{-2})}{(1+z^{-1})^2} + 10^6 \frac{(1-z^{-1})(1+z^{-1})}{(1+z^{-1})^2}\right]$$

$(1-z^{-2})$

$$= \frac{8.652\times2}{T_{samp}(1+z^{-1})^2}\left[\left(\frac{2}{T_{samp}}+10^6\right) - \left(\frac{2}{T_{samp}}\right)2z^{-1} + \left(\frac{2}{T_{samp}}-10^6\right)z^{-2}\right].$$

Transfering $(1+z^{-1})^2$ to neumerator ——— (1).

$$= \frac{8.652\times2}{T_{samp}}\left[\left(\frac{2}{T_{samp}}+10^6\right) - \frac{4}{T_{samp}}z^{-1} + \left(\frac{2}{T_{samp}}-10^6\right)z^{-2}\right]$$

Numerator

$$= 10^2\left[\frac{4}{T_{samp}^2}(1-z^{-1})^2\right] + 1.87\times10^5\left[\frac{2}{T_{samp}}(1-z^{-2})\right]$$

$$+ 8.652\times10^7 \times (1+z^{-1})^2.$$

$$= \left(\frac{400}{Tsamp^2} \left(1 - 2z^{-1} + z^{-2}\right) + \frac{3.74 \times 10^5}{Tsamp} \left(1 - z^{-2}\right) + 8.652 \times 10^7 \left(1 + 2z^{-1} + z^{-2}\right) \right.$$

$$= \left[\frac{400}{Tsamp^2} + \frac{3.74 \times 10^5}{Tsamp} + 8.652 \times 10^7 \right] + \left(8.652 \times 10^7 - \frac{400}{Tsamp^2} \right) 2 z^{-1}$$

$$+ \left(\frac{400}{Tsamp^2} + 8.652 \times 10^7 - \frac{3.74 \times 10^5}{Tsamp} \right) z^{-2}.$$

$$Tsamp = \frac{1}{2 \times 200 \times 10^3} = 2.5 \times 10^{-6}.$$

$$\text{Denominator} = 1.24588 \times 10^{13} - 1.1075 \times 10^{13} z^{-1} - 1.384 \times 10^{12} z^{-2}$$

$$\text{Neumerator} = 6.4149 \times 10^{13} - 1.28 \times 10^{14} z^{-1} + 6.385 \times 10^{13} z^{-2}$$

$$\frac{d[z]}{e[z]} = \frac{6.4149 - 12.8 z^{-1} + 6.385 z^{-2}}{1.2458 - 1.1075 z^{-1} - 0.1384 z^{-2}} \quad \text{—②}$$

$$= \cancel{4.119 \times 10^6 z^2 - 8.219 \times 10^6 z + 4.1 \times 10^6} -$$

from eq^n ②. taking inverse z transform,

$$1.2458\, d[k] - 1.1075\, d[k-1] + \cancel{4.1} - 0.1384\, d[k-2]$$

$$= 6.4149\, e[k] - 12.8\, e[k-1] + 6.385\, e[k-2]$$

$$d[k] = \frac{1.1075\, d[k-1] + 0.1384\, d[k-2] + 6.4149\, e[k] - 12.8\, e[k-1] + 6.385\, e[k-2]}{1.2458}.$$

$$d[k] = 0.8889\, d[k-1] + 0.1110\, d[k-2] + 5.1492\, e[k] \cancel{10.27e} \cancel{[k-} - 10.27 e[k-1] + 5.125\, e[k-2].$$

$N = 2.$

$\alpha_1 = 0.8889$

$\alpha_2 = 0.111$

$\beta_1 = -10.27$

$\beta_2 = 5.125$

$\beta_0 = 5.1492.$

Also getting same using matlab.

$$\frac{d[z]}{e[z]} = \frac{5.149\, z^2 - 10.27\, z + 5.125}{z^2 - 0.889\, z - 0.1111}$$

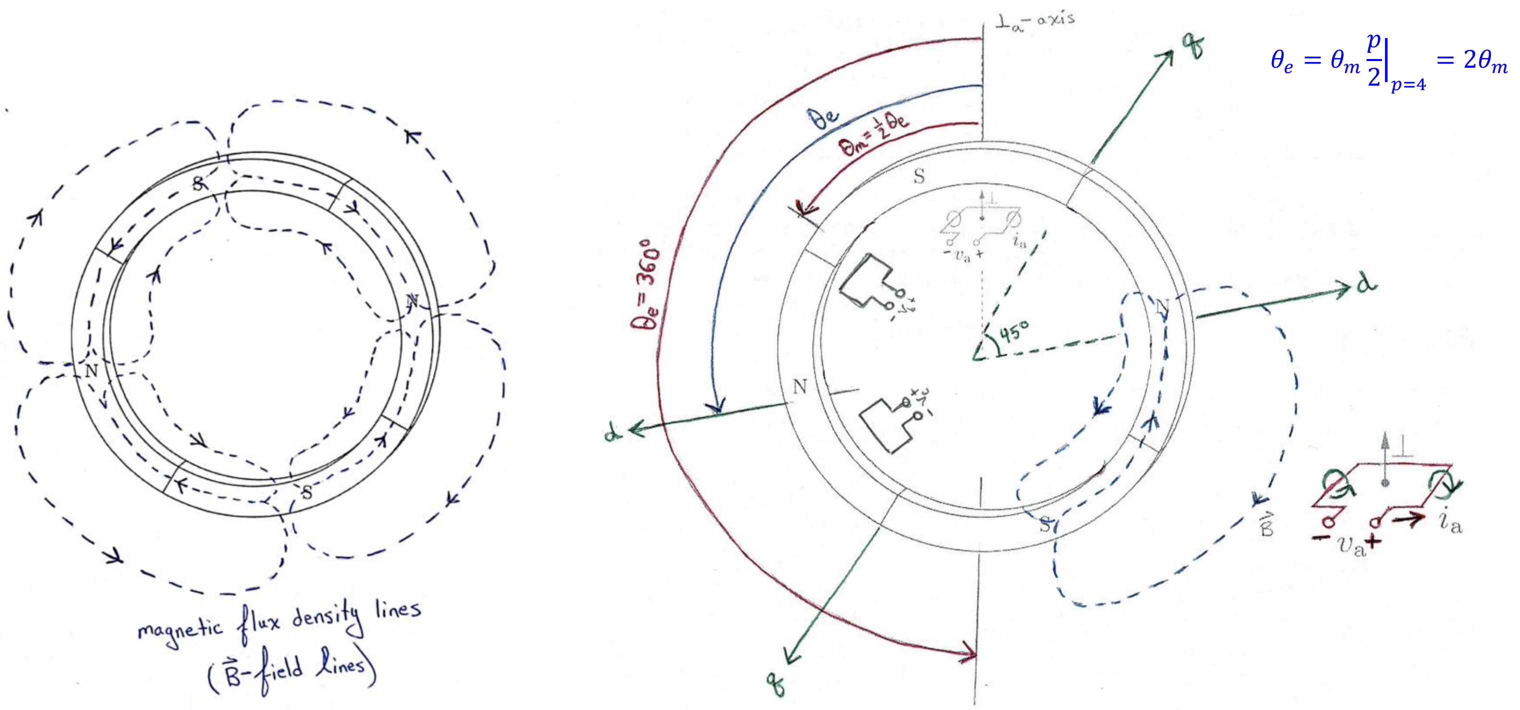
Problem 2 (David P Babin)

Problem 2: Consider the three-phase permanent magnet motor where the outer ring of magnets can rotate relative to a set of stationary coils within its inner diameter. Note that this setup is very similar to the e-bike experiment. Only the a-phase winding has been drawn on Figure 2.

(a) Annotate Figure 2 with the items below:

- Draw the flux density B-field lines between all the magnetic poles. Also show how the field lines exist within the magnets.
- Define the d and q axes for the permanent magnets.
- Define the a-phase voltage polarity.
- Define the a-phase current positive direction such that its flux points toward the $\perp$ vector.
- Indicate the direction of the flux produced by the a-phase coil by adding arrows to the green flux circles.
- Label the mechanical angle, θ_{m}.
- Label one full electrical rotation of 360°. Write the equation of how θ_{e} and θ_{m} are related for this problem.
- Sketch the approximate placement of the b and c phase coils to form the full-three motor.

$$\theta_e = \theta_m \frac{p}{2}\Big|_{p=4} = 2\theta_m$$



(b) Denote the flux produced by the magnetic ring which cuts through the phase-a loop area as $\Phi(\theta_e)$. The flux linkage through the coil, which has N turns, is denoted as $\lambda(\theta_e)$. Consider Figure 3 where we are looking at a side view of the phase-a coil in Figure 2. Annotate the diagram in Figure 3 by showing the following for each angle configuration:

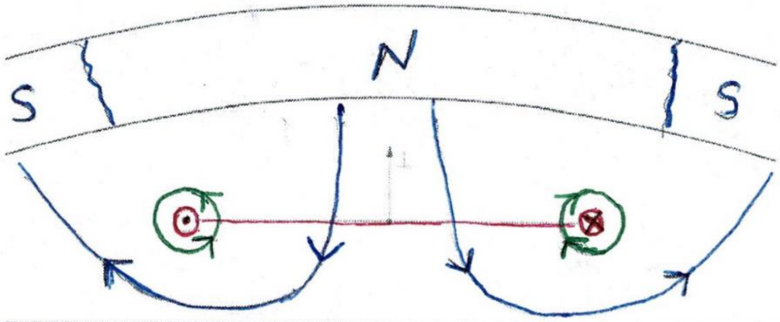
- Label the position of the poles above the coil.
- Draw the flux cutting through the coil.
- Fill in the two pink circles with crosses or dots to indicate the positive flow of coil current i.
- Add arrows to the green circles to show the flux density direction produced by i.
- Fill in the flux and flux linkage equations in terms of N, i, and $\Phi_{\max}$, where $\Phi_{\max} > 0$ is the peak flux through the coil.

Definitions:

- B → magnetic flux density
- H → magnetic field intensity
- ε → emf → electromotive force
- N → number turns of a loop of wire
- i → current through a loop of wire
- L → inductance of a loop of wire
- A → area of surface S created by a loop of a wire
- S → surface created by a loop of a wire
- μ → permeability

Notation:

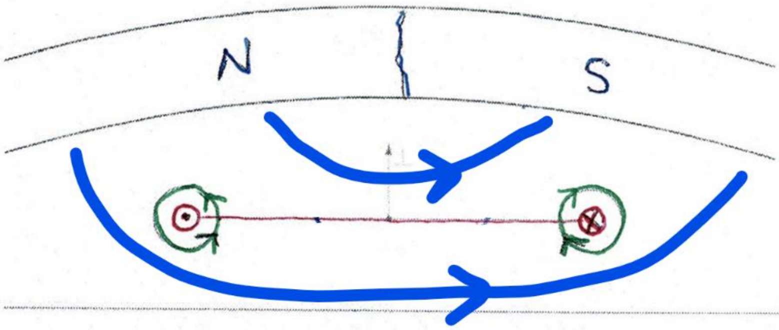
For this problem, ***we assume the positive direction of flux to be co-linear with (in the same direction as) the surface vector for the phase-a loop***.



when $\theta_e = 0$, give the following:

$\Phi(\theta_e) = -\Phi_{max}$

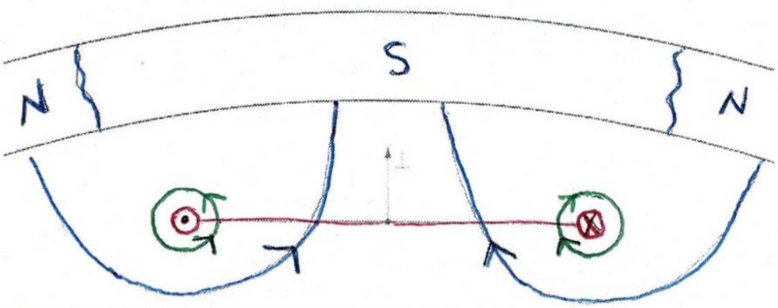
$\lambda(\theta_e) = -N\Phi_{max} + Li$



when $\theta_e = 90^\circ$, give the following:

$\Phi(\theta_e) = 0$

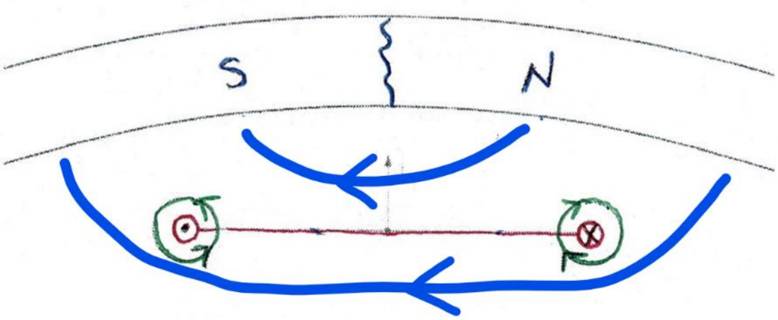
$\lambda(\theta_e) = Li$



when $\theta_e = 180^\circ$, give the following:

$\Phi(\theta_e) = \Phi_{max}$

$\lambda(\theta_e) = N\Phi_{max} + Li$



when $\theta_e = 270^\circ$, give the following:

$\Phi(\theta_e) = 0$

$\lambda(\theta_e) = Li$

(c) Using the result from (b), give the form of the functions $\Phi(\theta_e)$ and $\lambda(\theta_e)$. Use Faraday's Law to compute the induced back electromotive force voltage $\mathcal{E}$.

Flux linkage of magnetic ring with the phase-a coil – Φ:

Sequence of datapoints:

- $\Phi(\theta_e = 0°) = -\Phi_{max}$
- $\Phi(\theta_e = 90°) = 0$
- $\Phi(\theta_e = 180°) = \Phi_{max}$
- $\Phi(\theta_e = 270°) = 0$

$$\boxed{\boldsymbol{\Phi(\theta_e) = -\Phi_{max}\cos(\theta_e)}}$$

Flux Linkage of an *N* turns coil – *λ*:

Sequence of datapoints:

- $\lambda(\theta_e = 0°) = -N\Phi_{max} + Li$
- $\lambda(\theta_e = 90°) = Li$
- $\lambda(\theta_e = 180°) = N\Phi_{max} + Li$
- $\lambda(\theta_e = 270°) = Li$

$$\boxed{\boldsymbol{\lambda(\theta_e) = -N\phi_{max}\cos(\theta_e) + Li}}$$

Induced EMF – E:

$$\mathrm{E} = -\frac{d\lambda}{dt} = -\frac{d}{dt}\{-N\Phi_{max}\cos(\theta_e) + Li\} = N\Phi_{max}\frac{d}{dt}\{\cos(\theta_e)\} - L\frac{d}{dt}\{i\}$$

$$= -N\Phi_{max}\sin(\theta_e)\cdot\frac{d}{dt}\{\theta_e\} - L\frac{di}{dt} = -N\Phi_{max}\sin(\theta_e)\cdot\frac{d}{dt}\left\{\theta_m\frac{p}{2}\right\} - L\frac{di}{dt}$$

$$= -N\Phi_{max}\sin(\theta_e)\cdot\frac{p}{2}\cdot\frac{d}{dt}\{\theta_m\} - L\frac{di}{dt} = -N\Phi_{max}\sin(\theta_e)\frac{p}{2}\omega_m - L\frac{di}{dt}$$

$$\underline{\boldsymbol{\mathrm{E} = -\omega_e\cdot N\Phi_{max}\sin(\theta_e) - L\frac{di}{dt} = -\omega_m\frac{p}{2}\cdot N\Phi_{max}\sin(\theta_e) - L\frac{di}{dt}}}$$

- ***Back EMF – ε:***

$$\boxed{\boldsymbol{\varepsilon = \omega_e\cdot N\Phi_{max}\sin(\theta_e)}}$$

where: $\omega_e = \omega_m\frac{p}{2}$ *and* $\theta_e = \theta_m\frac{p}{2}$