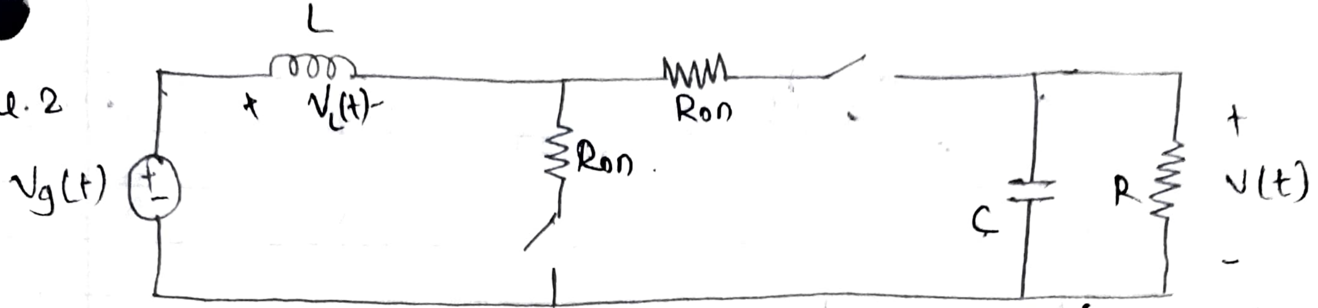
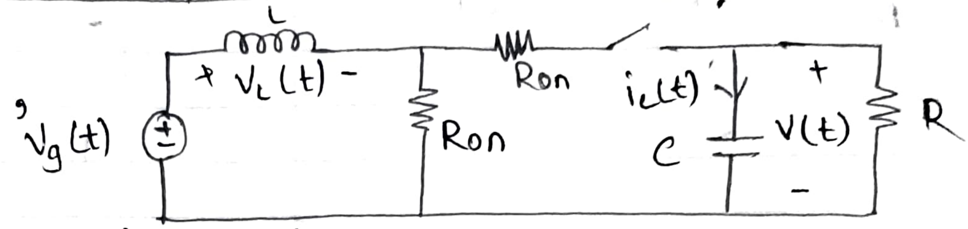
Problem 2 (Prajakta Popat Wable)

Que. 2

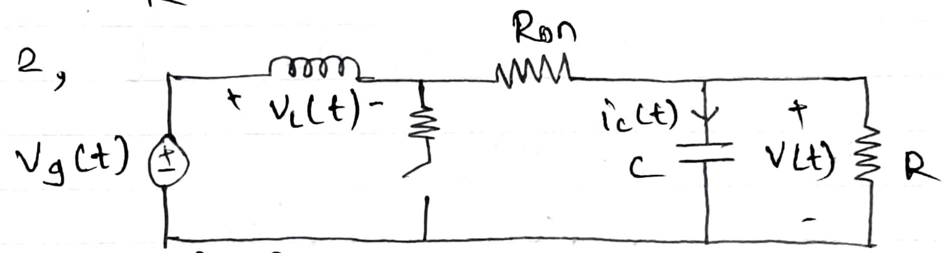


ⓐ for mode 1,



$$\left.\begin{aligned} V_L(t) &= V_g(t) - i(t) R_{on} \\ i_c(t) &= -\frac{V(t)}{R} \end{aligned}\right\} \text{ — (1)}$$

for mode 2,



$$\left.\begin{aligned} V_L(t) &= V_g(t) - i(t) R_{on} - V(t) \\ i_c(t) &= i(t) - \frac{V(t)}{R} \end{aligned}\right\} \text{ — (2)}$$

Eq^{n}s ① & ② can be written as,

$$\langle V_L(t)\rangle = L\frac{d\langle i_L(t)\rangle}{dt} = \langle V_g(t)\rangle - \langle i(t)\rangle R_{on} - \langle V(t)\rangle d'(t).$$

$$\langle i_c(t)\rangle = C\frac{d\langle V(t)\rangle}{dt} = \langle i(t)\rangle d'(t) - \frac{\langle V(t)\rangle}{R}.$$

We can write above eq^{n}s in the $\dot{x}(t) = f((x(t), u(t))$

$$\begin{bmatrix} \frac{d\langle i(t)\rangle}{dt} \\ \frac{d\langle V(t)\rangle}{dt} \end{bmatrix} = \begin{bmatrix} \frac{\langle V_g(t)\rangle}{L} - \frac{\langle i(t)\rangle}{L} R_{on} - \frac{V(t)}{L} d'(t) \\ \frac{\langle i(t)\rangle d'(t)}{C} - \frac{\langle V(t)\rangle}{RC} \end{bmatrix}$$

(b) $x(t) = \begin{bmatrix} I \\ V \end{bmatrix} + \begin{bmatrix} \tilde{i}(t) \\ \tilde{v}(t) \end{bmatrix}$ $\quad u(t) = \begin{bmatrix} D' \\ V_g \end{bmatrix} + \begin{bmatrix} \tilde{d}'(t) \\ \tilde{v}_g(t) \end{bmatrix}$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial \tilde{i}(t)} & \frac{\partial f_1}{\partial \tilde{v}(t)} \\ \frac{\partial f_2}{\partial \tilde{i}(t)} & \frac{\partial f_2}{\partial \tilde{v}(t)} \end{bmatrix} = \begin{bmatrix} \frac{-R_{on}}{L} & \frac{-D'}{L} \\ \frac{D'}{C} & \frac{-1}{RC} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial \tilde{d}'(t)} & \frac{\partial f_1}{\partial \tilde{v}_g(t)} \\ \frac{\partial f_2}{\partial \tilde{d}'(t)} & \frac{\partial f_2}{\partial \tilde{v}_g(t)} \end{bmatrix} = \begin{bmatrix} -V/L & 1/L \\ \frac{I}{C} & 0 \end{bmatrix}$$

$\tilde{y}(t) = \langle \tilde{v}(t) \rangle$.

$$\therefore \quad C = \begin{bmatrix} \frac{\partial \tilde{y}(t)}{\partial \tilde{i}(t)} & \frac{\partial \tilde{y}(t)}{\partial \tilde{v}(t)} \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} \frac{\partial \tilde{y}(t)}{\partial \tilde{d}'(t)} & \frac{\partial \tilde{y}(t)}{\partial \tilde{v}_g(t)} \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

We can write the $\dot{\tilde{x}}(t) = A\tilde{x}(t) + B\tilde{u}(t)$.

$$\begin{bmatrix} \frac{d\tilde{i}(t)}{dt} \\ \frac{d\tilde{v}(t)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R_{on}}{L} & \frac{-D'}{L} \\ \frac{D'}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} \tilde{i}(t) \\ \tilde{v}(t) \end{bmatrix} + \begin{bmatrix} -V/L & 1/L \\ I/C & 0 \end{bmatrix} \begin{bmatrix} \tilde{d}'(t) \\ \tilde{v}_g(t) \end{bmatrix}$$

& $\tilde{y}(t) = C\tilde{x}(t) + E\tilde{u}(t)$.

$$\begin{bmatrix} \tilde{v}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{i}(t) \\ \tilde{v}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{d}'(t) \\ \tilde{v}_g(t) \end{bmatrix}.$$

(c) $\hat{y}(s) = \langle v(s) \rangle = \left(C(sI-A)^{-1}B + E \right) \hat{u}(s)$.

By matlab,

$$G(s) = \left[\frac{R(I R_{on} - D'V + I L s)}{R_{on} + D'^2 R + (L + CR R_{on}) s + CLR s^2} \quad \frac{D'R}{R_{on} + D'^2 R + (L + CR R_{on}) s + CLR s^2} \right].$$

$$\tilde{y}(s) = \frac{1}{R_{on} + D'^2 R + (L + CR R_{on}) s + CLR s^2} \left[R(I R_{on} - D'V + I L s) \quad D'R \right] \begin{bmatrix} \tilde{d}'(s) \\ \tilde{v}_g(s) \end{bmatrix}$$

Problem 3 (a - e) (David P Babin)

Problem 3: Controller Design Problem [30 Points Total]- a,b,c,d are compulsory, choose one from e,f.

The theoretical model of a closed loop system is shown in Fig. 2 where $C(s)$ is the controller and $P(s)$ is the plant transfer function.

Also note, $r(s)$ is the reference, $y(s)$ is the output, $e(s)$ is the error and $u(s)$ is the control effort.

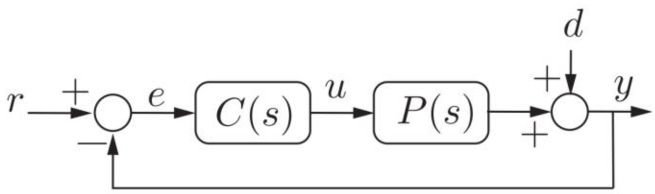


Figure 2: Closed loop control system

(a) **[10 Points]** Derive the following transfer functions:

1. $\frac{y(s)}{r(s)}$ (consider $d(s) = 0$ for this),**[4 Points]**
2. $\frac{y(s)}{d(s)}$ (consider $r(s) = 0$ for this),**[4 Points]**

in terms of $C(s)$ and $P(s)$.

Let $\frac{y(s)}{r(s)} := T(s)$ and $\frac{y(s)}{d(s)} := S(s)$, then which of the following is/are true? **[2 Points]**

1. $T(s) = -S(s)$,
2. $T(s) = 1 - S(s)$,
3. $\frac{dT(s)}{ds} + \frac{dS(s)}{ds} = 0$.

Derivation

1) $[r(s) - y(s)]C(s)P(s) = y(s)$

$$\Rightarrow y(s) + y(s)C(s)P(s) = r(s)C(s)P(s)$$

$$\Rightarrow y(s)\big(1 + C(s)P(s)\big) = r(s)C(s)P(s)$$

$$\Rightarrow y(s) = \frac{C(s)P(s)}{1 + C(s)P(s)} r(s)$$

$$\Rightarrow \boxed{\frac{\boldsymbol{y(s)}}{\boldsymbol{r(s)}} = \frac{\boldsymbol{C(s)P(s)}}{\boldsymbol{1 + C(s)P(s)}}}$$

2) $[0 - y(s)]C(s)P(s) + d(s) = y(s)$

$$\Rightarrow y(s) + y(s)C(s)P(s) = d(s)$$

$$\Rightarrow y(s)\big(1 + C(s)P(s)\big) = d(s)$$

$$\Rightarrow y(s) = \frac{1}{1 + C(s)P(s)} d(s)$$

$$\Longrightarrow \boxed{\frac{\boldsymbol{y(s)}}{\boldsymbol{d(s)}} = \frac{\mathbf{1}}{\mathbf{1} + \boldsymbol{C(s)P(s)}}}$$

True / False

1) $T(s) = -S(s) \rightarrow \boxed{\textbf{False}}$

2) $T(s) = 1 - S(s) \rightarrow \boxed{\textbf{True}}$

$$\boldsymbol{T(s)} = \frac{C(s)P(s)}{1 + C(s)P(s)} = \frac{-1 + 1 + C(s)P(s)}{1 + C(s)P(s)} = -\frac{1}{1 + C(s)P(s)} + \frac{1 + C(s)P(s)}{1 + C(s)P(s)}$$
$$= 1 - \frac{1}{1 + C(s)P(s)} = \mathbf{1} - \boldsymbol{S(s)}$$

3) $\frac{dT(s)}{ds} + \frac{dS(s)}{ds} = 0 \rightarrow \boxed{\textbf{True}}$

$$\frac{\boldsymbol{dT(s)}}{\boldsymbol{ds}} + \frac{\boldsymbol{dS(s)}}{\boldsymbol{ds}} = \frac{d}{ds}\{\,T(s) + S(s)\,\} = \frac{d}{ds}\{\,\big(1 - S(s)\big) + S(s)\,\} = \frac{d}{ds}\{1\} = \mathbf{0}$$

(b) [**5 Points**] Assume the transfer function $P(s)$ in Fig. 2 has the following form,

$$P(s) = \frac{0.35p}{\left(\frac{4s}{3p} + 1\right)\left(\frac{s}{5p} + 1\right)\left(\frac{5s}{p^2} + 1\right)}, \tag{2}$$

where p is the sum of day, month and year of your birth-date divided by ten: (dd+mm+yyyy)/10. (For example, for 10th December 1993, p = (10+12+1993)/10 = 201.5)

You should not reveal your date of birth, please proceed by mentioning the value of p and solving the subsequent questions.

Sketch the bode plot of the plant, $P(s)$ and find the gain and phase margin.

Show the following in your plot clearly: gain cross over frequency, phase cross over frequency, gain margin, phase margin.

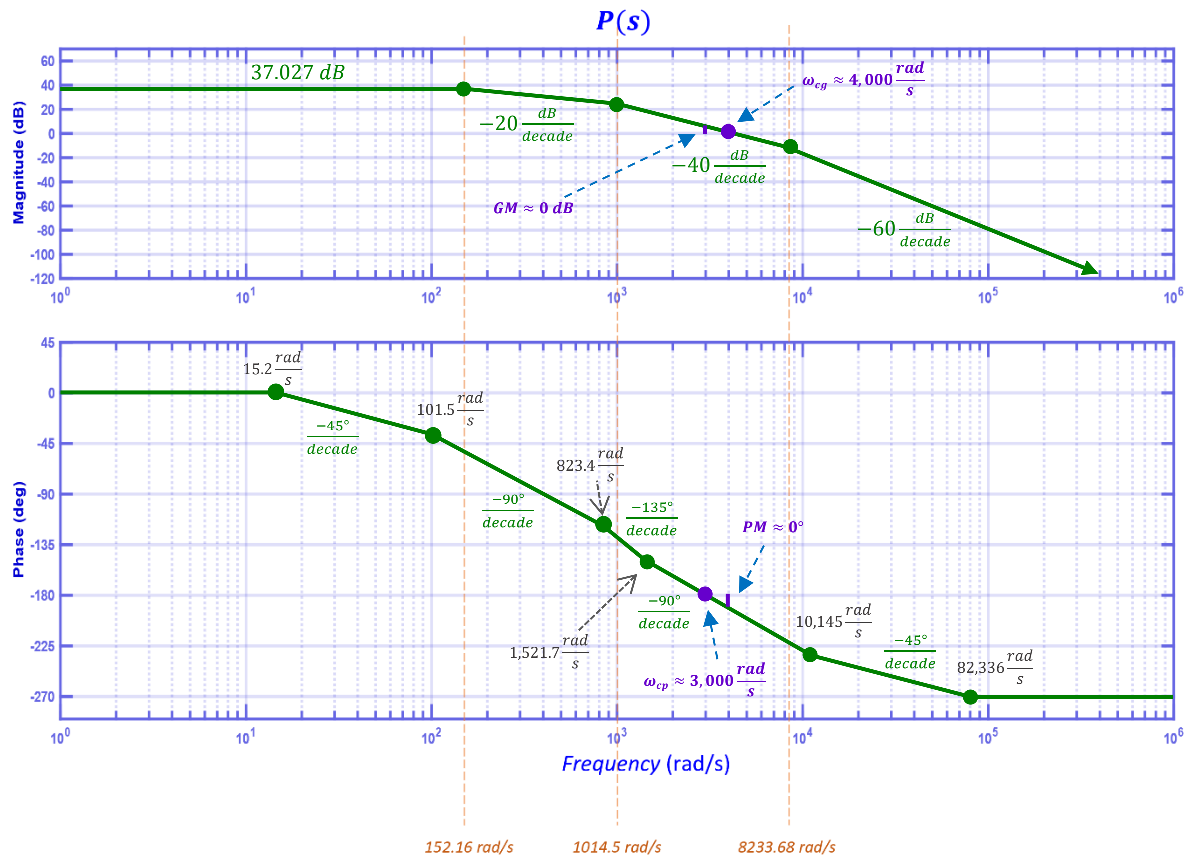
$$P(s) = \frac{0.35p}{\left(\frac{4s}{3p}+1\right)\left(\frac{s}{5p}+1\right)\left(\frac{5s}{p^2}+1\right)} = \frac{0.35p}{\left(\frac{s}{\frac{3p}{4}}+1\right)\left(\frac{s}{5p}+1\right)\left(\frac{s}{\frac{p^2}{5}}+1\right)} \text{ where } p = \frac{dd+mm+yyyy}{10} = 202.9$$

- System Poles:

$$\left\{\frac{3p}{4} = \mathbf{152.175},\ 5p = \mathbf{1014.5},\ \frac{p^2}{5} = \mathbf{8233.68}\right\}\ rad/s$$

- DC gain:

$$\text{DC gain} = 20\log_{10}(0.35p) = 20\log_{10}(71.015) = \mathbf{37.027\ dB}$$



Note: As shown in the linearized Bode plot, both the GM & PM are slightly negative. The notation used is:

$\omega_{cp} \rightarrow$ phase crossover frequency || $\omega_{cg} \rightarrow$ gain crossover frequency || $PM \rightarrow$ Phase Margin $GM \rightarrow$ Gain Margin

The exact values of the gain & phase margins are:

```
Gain Margin = 1.002 = 0.02 dB
Phase Margin = 0.001 rad = 0.04°
```

(c) **[5 Points]** Assume there is no disturbance in the system ($d = 0$). Design a controller $C(s)$ such that the closed loop system tracks a steady dc reference perfectly and the compensated loop gain, $\ell(s) = C(s)P(s)$, has a phase margin of *at least* 30°.

Report $C(s)$ and elucidate the steps of your controller design. Report the new gain and phase margins of the compensated system, $\ell(s)$. Overlay the bode plot of plant $P(s)$ and the loop gain $\ell(s) = C(s)P(s)$ on a single plot.

Without any control, the stability information of $P(s)$ is:

```
Stability Information for P:
   Gain Margin = 1.002 = 0.02 dB
   Phase Margin = 0.001 rad = 0.04°
   Gain Crossover Frequency = 3.120569e+03 rad/s = 4.966541e+02 Hz
   Phase Crossover Frequency = 3.124166e+03 rad/s = 4.972265e+02 Hz
```

To achieve perfect DC tracking, we can add an integral controller of the form $\frac{1}{s}$. As such, our controller is $C(s) = \frac{1}{s}$. The stability information of this compensated system $\ell(s) = C(s)P(s)$ is:

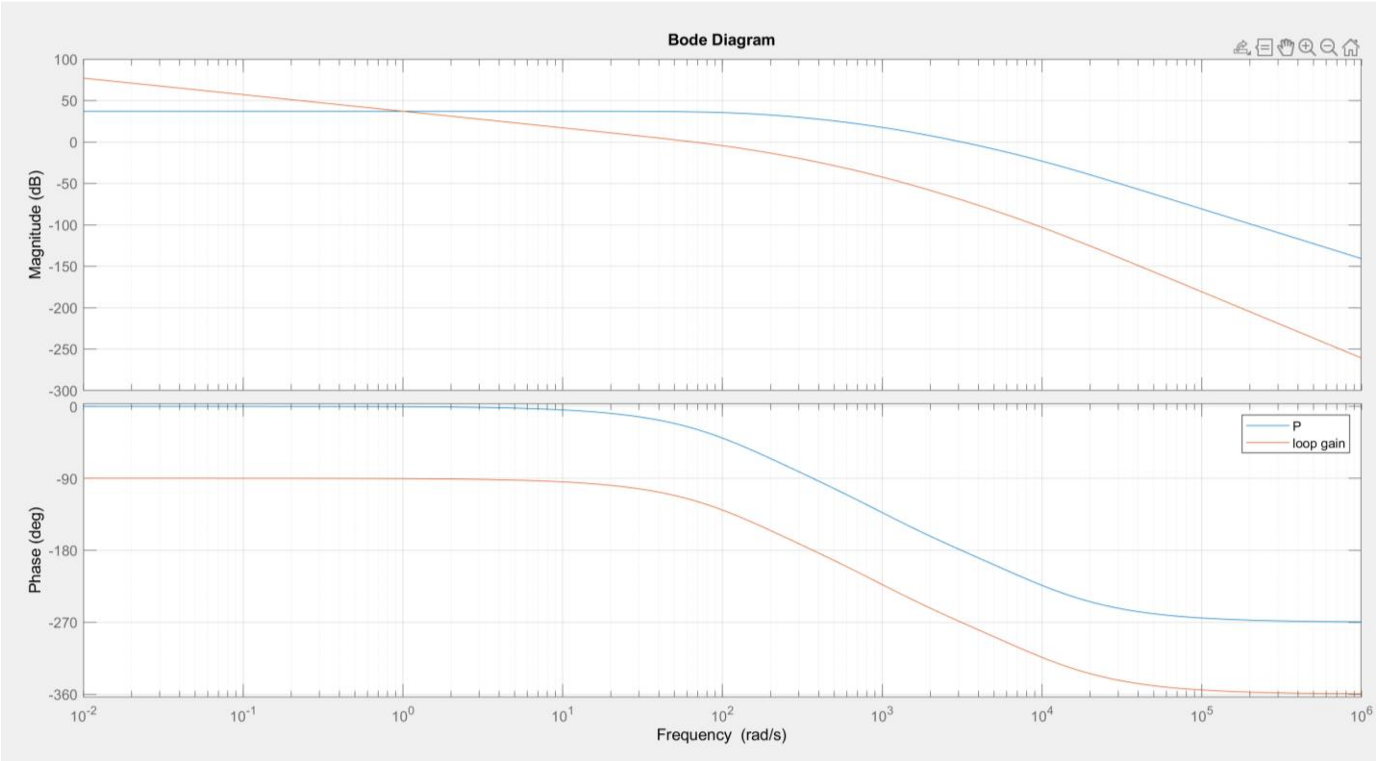
```
Stability Information for loop_gain:
   Gain Margin = 14.417 = 23.18 dB
   Phase Margin = 1.094 rad = 62.70°
   Gain Crossover Frequency = 6.514769e+01 rad/s = 1.036858e+01 Hz
   Phase Crossover Frequency = 3.677130e+02 rad/s = 5.852334e+01 Hz
```

To achieve a desired phase margin, we usually introduce a “zero” right after the gain crossover frequency $\omega_{cg} = 65.1476 \frac{rad}{s}$. However, in this case, adding a simple integral controller was sufficient to achieve both, the DC tracking and phase margin criteria. As such, our controller is:

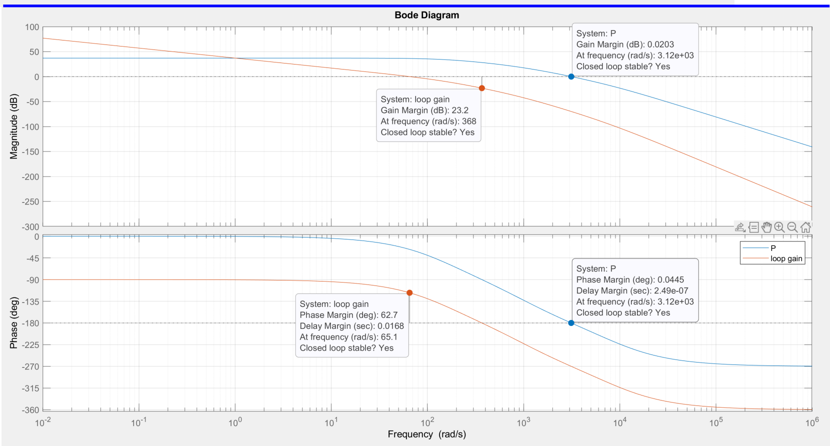
$$\boxed{C(s) = \frac{1}{s}}$$

The gain & phase margins of the compensated system $\ell(s) = C(s)P(s)$ are:

$$\boxed{PM = 62.7° \qquad GM = 23.18 \text{ dB}}$$



The following is the same Bode plot but with the stability margin data shown



(d) **[5 Points]** Plot the response of your closed loop system (with the controller derived in previous part) to a step function (r is a step function changing from 0 to 1 at 0.1 seconds) and show the tracking performance. Record reference (r), output (y), error (e) and control effort (u) on the same plot till the output, y converges to 1.

Zoom in considerably to convince that you have achieved zero steady-state error.

[Hint: You can use Transfer Function Blocks in Plecs to simulate the system.]

$$P(s) = \left.\frac{0.35p}{\left(\frac{4s}{3p}+1\right)\left(\frac{s}{5p}+1\right)\left(\frac{5s}{p^2}+1\right)}\right|_{p=202.9} = \frac{4.963e25}{5.498e14\, s^3+5.168e18\, s^2+5.366e21\, s+6.988e23}$$

(e) **[5 Points]** Again, assume $d = 0$. You are now asked to design a controller that has the following time domain specifications:

- Maximum permissible overshoot is 30% (so for a unit step reference, maximum output permissible is 1.3).
- Rise time to be less than 5 ms (the time taken for output to rise from 0.1 to 0.9 is at worst 5 ms).

[2 Points] From the time domain data, what target bandwidth and phase margin are you going to design your controllers for?

[3 points] Write down the steps of controller design and explicitly write the final form of controller $C(s)$. Plot the response of your closed loop system to a step function (r is a step function changing from 0 to 1 at 0.1 seconds) and show the tracking performance. Record reference (r), output (y), error (e) and control effort (u) on the same plot till the output, y converges to 1.

Time *to* Frequency Domain Requirements Conversion

- **Overshoot**

$$\left\{ M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \right\} \leq M_{p,max} \quad \textit{for } M_p \textit{ in p.u.}$$

$$\Rightarrow \zeta \geq \left\{ \sqrt{\frac{1}{1+\left(\frac{\pi}{\ln(M_{p,max})}\right)^2}} = \sqrt{\frac{1}{1+\left(\frac{\pi}{\ln(0.3)}\right)^2}} = 0.3579 \right\} \quad \textit{for } 0 \leq M_{p,max} \leq 1$$

$$\underline{\boldsymbol{\zeta = 0.3579}}$$

- **Rise Time**

$$\left\{ t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)}{\omega_0\sqrt{1-\zeta^2}} \approx \frac{2.5}{1.5\omega_0\sqrt{1-\zeta^2}} = \frac{5}{3\omega_0\sqrt{1-\zeta^2}} \right\} < t_{r,max}$$

$$\Rightarrow \omega_0 > \left\{ \frac{5}{3t_{r,max}\sqrt{1-\zeta^2}} = 356.98 \right\}$$

$$\underline{\boldsymbol{\omega_0 = 356.98 \frac{rad}{s}}}$$

- **Phase Margin**

$$PM = \tan^{-1}\left(\frac{2\zeta}{\sqrt{1-2\zeta^2}}\right)\cdot\frac{180}{\pi}\ \text{degrees}$$

As an approximation, for $0 < \zeta < 0.6$ (*or equivalently*, $0° < PM < 60°$):

$$PM \geq \{100\zeta = 100(0.3579) = 35.79\}$$

$$\boxed{\boldsymbol{PM = 35.79°}}$$

- **Bandwidth ≈ Gain Crossover Frequency**

$$\omega_c = \omega_{cg} > \left\{\omega_0\sqrt{1-2\zeta^2} = 356.98\sqrt{1-2(0.3579)^2} = 307.876\right\}$$

$$\boxed{\boldsymbol{\omega_{BW} \approx \omega_{cg} = 307.876\ \mathrm{rad/s}}}$$

Controller Design

Without any control, the stability information of $P(s)$ is:

```
Stability Information for P:
   Gain Margin = 1.002 = 0.02 dB
   Phase Margin = 0.001 rad = 0.04°
   Gain Crossover Frequency = 3.120569e+03 rad/s = 4.966541e+02 Hz
   Phase Crossover Frequency = 3.124166e+03 rad/s = 4.972265e+02 Hz
```

To achieve perfect DC tracking, we add an integral controller of the form $\frac{1}{s}$. As such, our controller is $C(s) = \frac{1}{s}$. The stability information of this compensated system $\ell(s) = C(s)P(s)$ is:

```
Stability Information for loop_gain:
   Gain Margin = 14.417 = 23.18 dB
   Phase Margin = 1.094 rad = 62.70°
   Gain Crossover Frequency = 6.514769e+01 rad/s = 1.036858e+01 Hz
   Phase Crossover Frequency = 3.677130e+02 rad/s = 5.852334e+01 Hz
```

To increase the gain crossover frequency ω_{cg}, and as such, increase the bandwidth ω_{BW}, we introduce an integral controller gain $K_I = 12.5$ to shift up the entire Bode magnitude plot. For $K_I = 12.5 \ \rightarrow\ \omega_{BW} \approx \omega_{cg} = 341.82\,\frac{\mathrm{rad}}{\mathrm{s}}$.

```
Stability Information for loop_gain:
   Gain Margin = 1.153 = 1.24 dB
   Phase Margin = 0.052 rad = 3.00°
   Gain Crossover Frequency = 3.418284e+02 rad/s = 5.440368e+01 Hz
   Phase Crossover Frequency = 3.677130e+02 rad/s = 5.852334e+01 Hz
```

To increase the Phase Margin, we introduce a "zero" $\omega_z = 342\,\frac{\text{rad}}{\text{s}}$ right after the new gain crossover frequency $\omega_{cg,new} = 341.82\,\frac{\text{rad}}{\text{s}}$ to shift up the Bode phase plot. For $\omega_z = 342\,\frac{\text{rad}}{\text{s}} \rightarrow PM = 44.87°$.

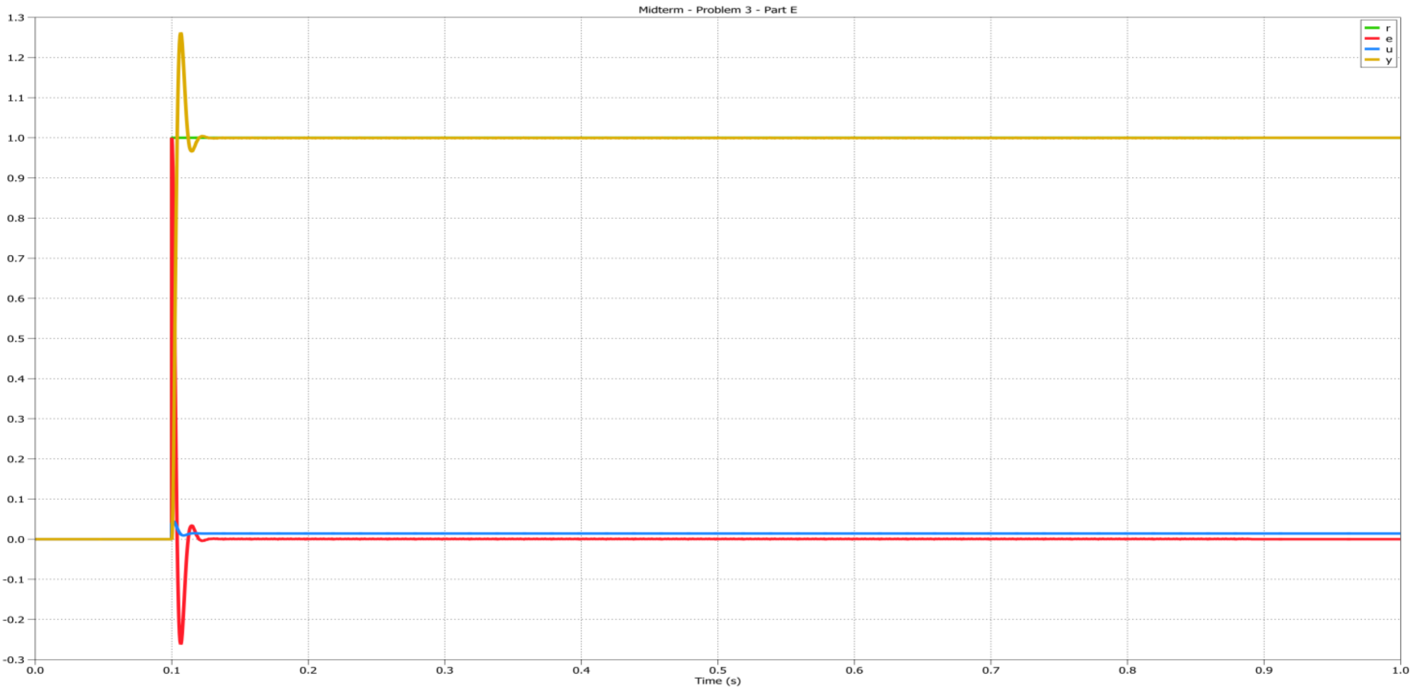
```
Stability Information for loop_gain:
   Gain Margin = 18.451 = 25.32 dB
   Phase Margin = 0.783 rad = 44.87°
   Gain Crossover Frequency = 4.351886e+02 rad/s = 6.926241e+01 Hz
   Phase Crossover Frequency = 2.571234e+03 rad/s = 4.092246e+02 Hz
```

As such, the controller necessary to achieve the imposed design criteria is:

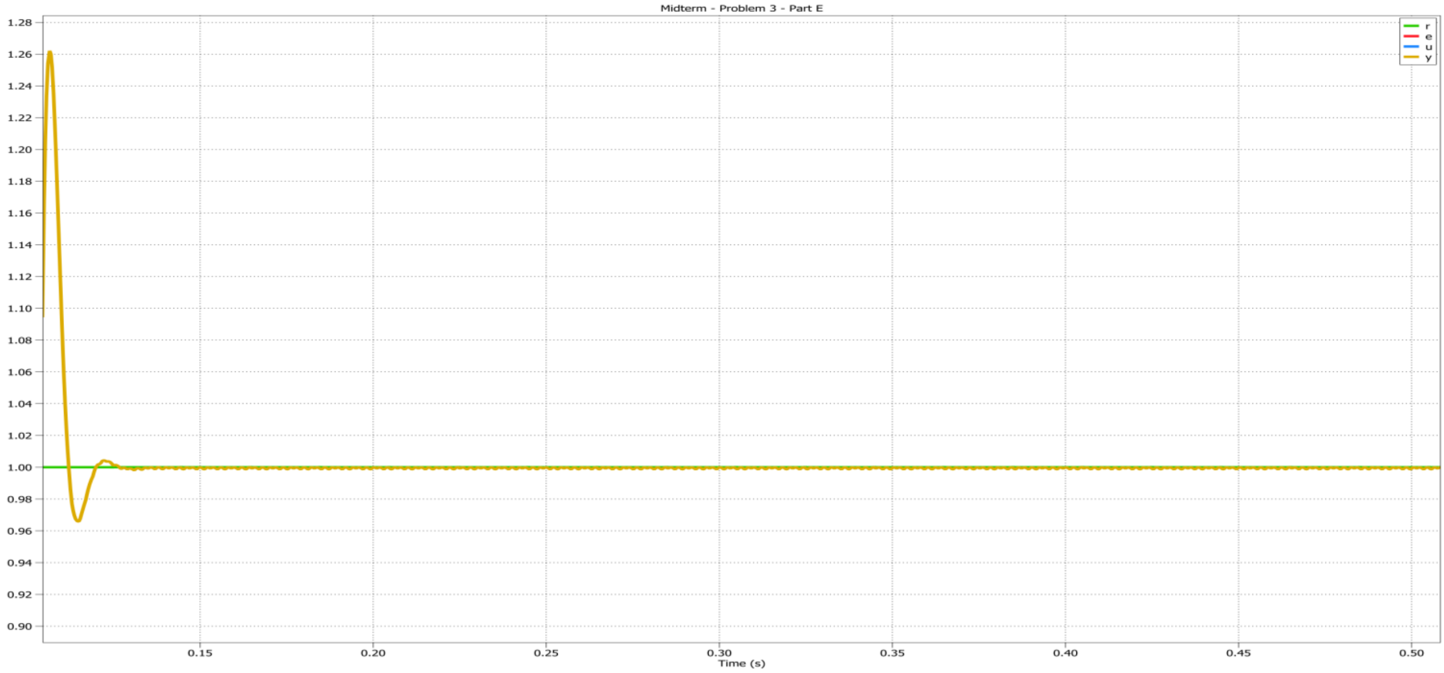
$$C(s) = \frac{12.5}{s} \cdot \frac{\frac{s}{342} + 1}{1} = \frac{0.03655\,s + 12.5}{s} = 0.03655 + \frac{12.5}{s} = K_P + \frac{K_I}{s}$$

$$\boxed{\boldsymbol{C(s) = \frac{0.03655\,s + 12.5}{s}}}$$

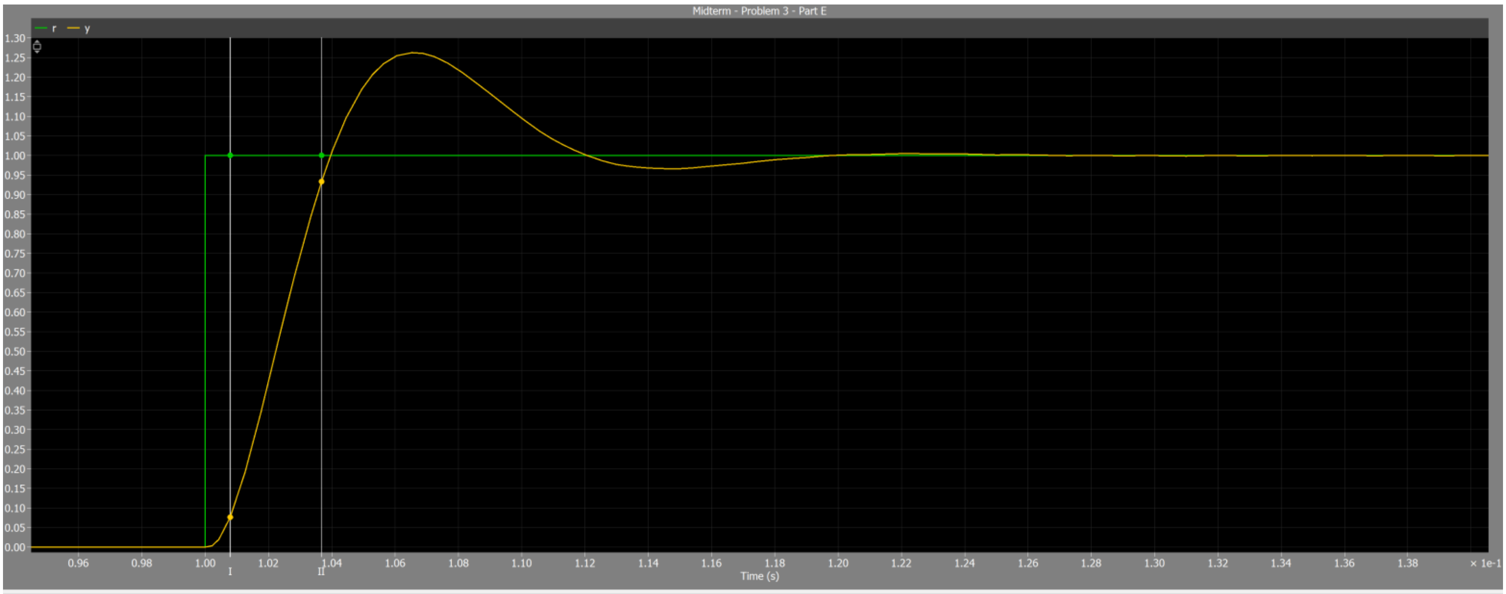
Step response of our closed loop system:



Condition of maximum permissible overshoot of 30% is satisfied:



Condition of a rise time less than 5 ms is satisfied:



Data

Name	Cursor 1	Cursor 2	Delta
Time	0.100789	0.103676	0.00288643

Problem 3 (f) (Rohit Gujarathi)

- **[3 Points]** In your original control system of Fig.2, now introduce a disturbance, $d = 0.5\ \sin(2\pi\, 10\, t)$ that is present at all times.
  Plot the response of your closed loop system to a step change in reference (r is a step function changing from 0 to 1 at 0.1 seconds), include disturbance as mentioned. Show the tracking performance by zooming appropriately. Record reference (r), output (y) and disturbance(d) on the same plot till the output, y converges around 1.
- **[2 Points]** Redesign your controller so that in addition to the design constraints of part (c), you now ensure that the oscillation in output voltage is reduced to less than 20% of the input disturbance magnitude.
  Report $C(s)$ and the new gain and phase margins of the compensated system, $\ell(s)$.
  Plot the response of your closed loop system to a step change in reference (r is a step function changing from 0 to 1 at 0.1 seconds), use disturbance as mentioned. Show the tracking performance and disturbance rejection by zooming appropriately. Record reference (r), output (y) and disturbance(d) on the same plot till the output, y converges around 1.

→ The original System without a $C(s)$ is unstable.

→ For DC tracking an integrator is used like what is done in problem 3C. Making $C(s) = \frac{1}{s}$ provides a PM = 62.7 but the disturbance is large.

→ Adding a PI controller $C(s) = 0.1s + \frac{1}{s} \Rightarrow \frac{0.15s+1}{s}$ fixes this issue. and reduces distrbance to ± 0.092 V which is less than 20% of its Magnitude. The PM is also sufficiently high at 43.1 deg.

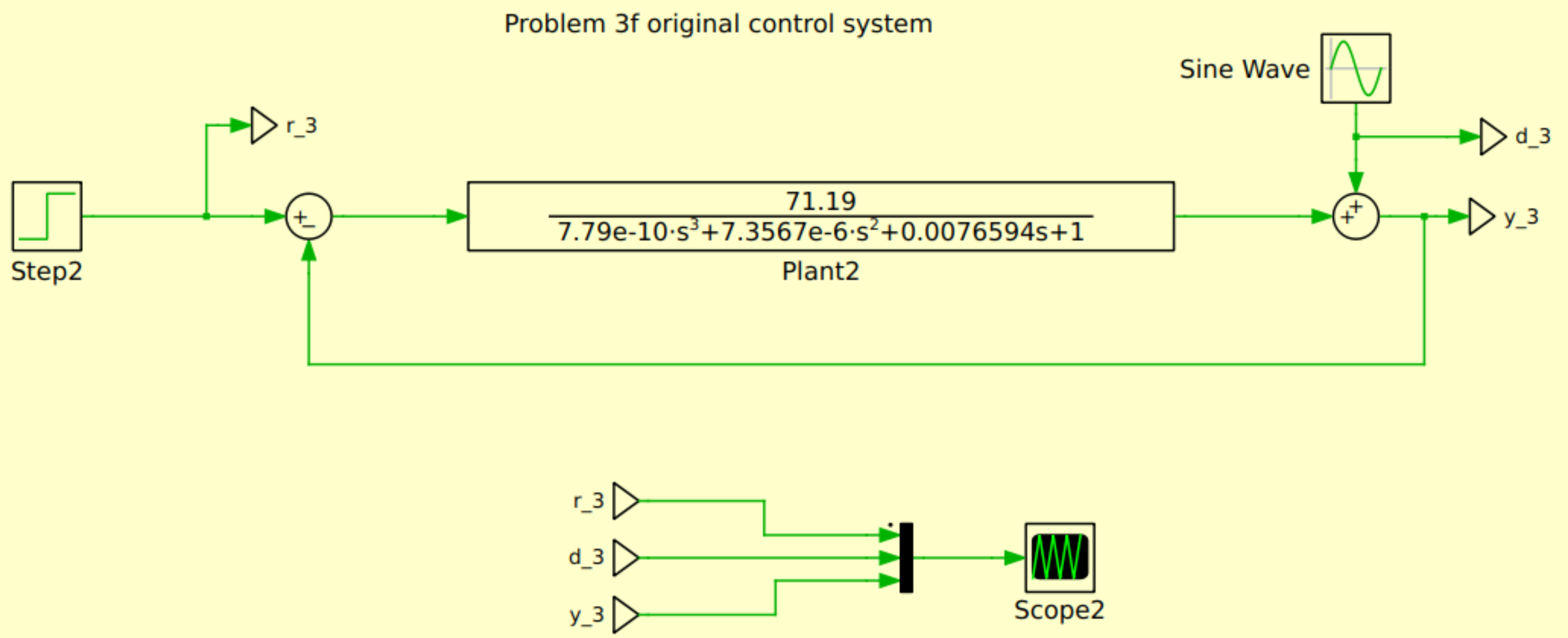
→ Specifications of new system:

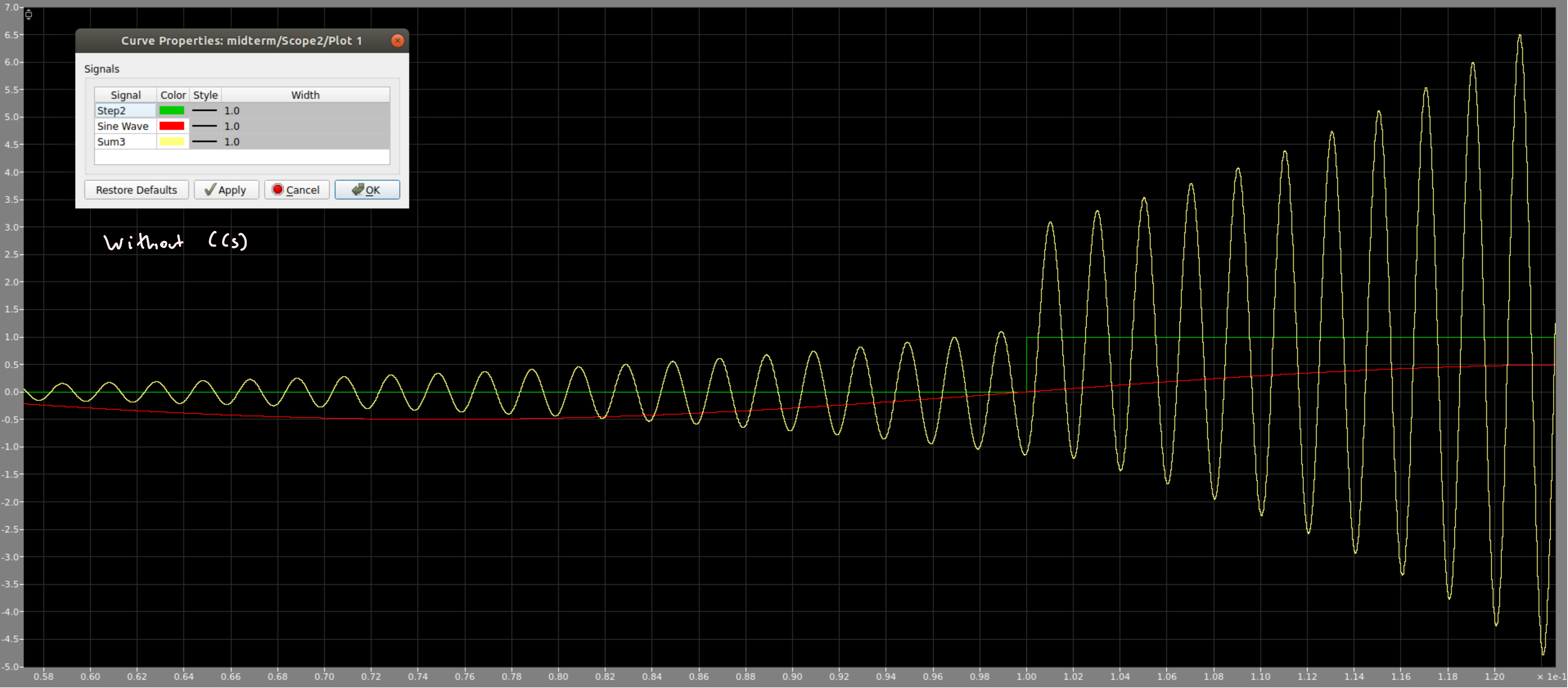
PM = 43.1 degrees

$\omega_{gc} = \omega_b = 174$ Hz

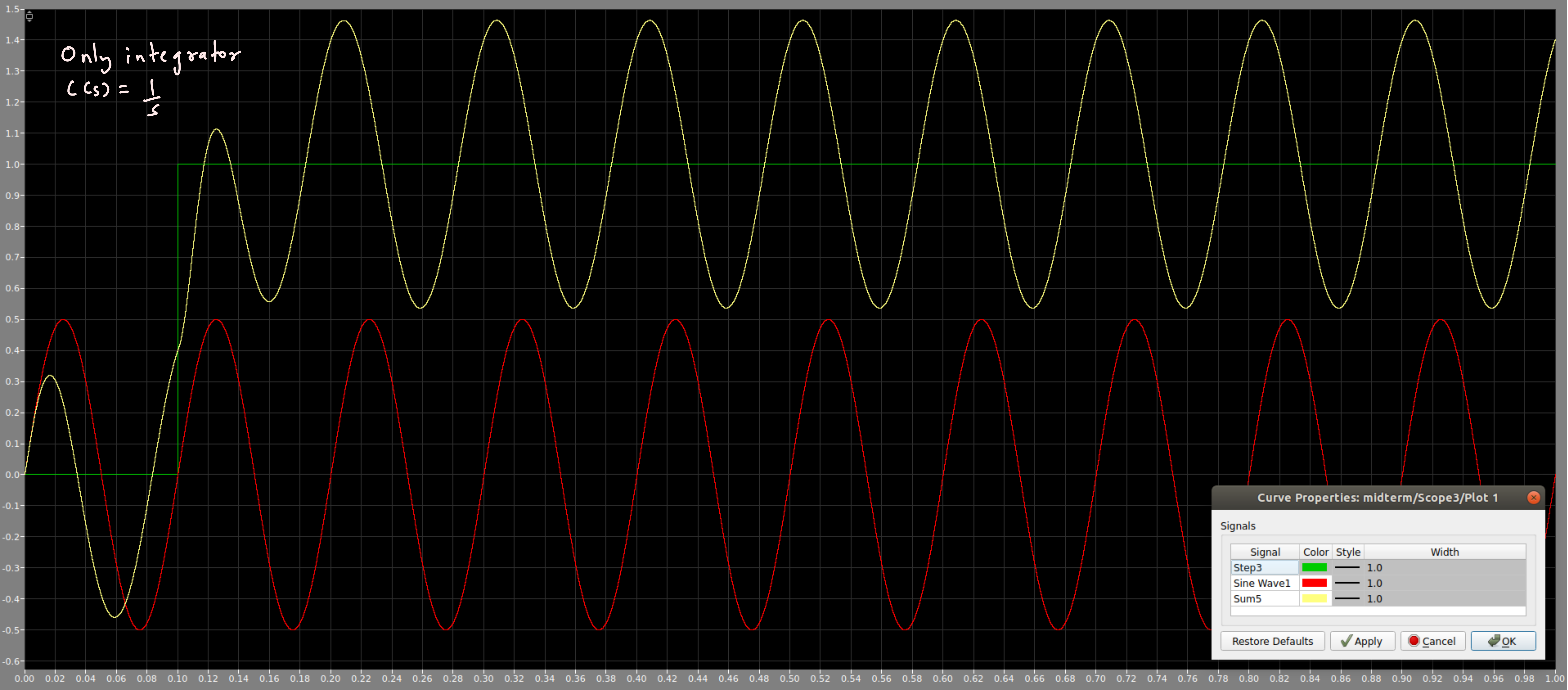
GM = 16.4 dB

$\omega_{pc} = 497$ Hz

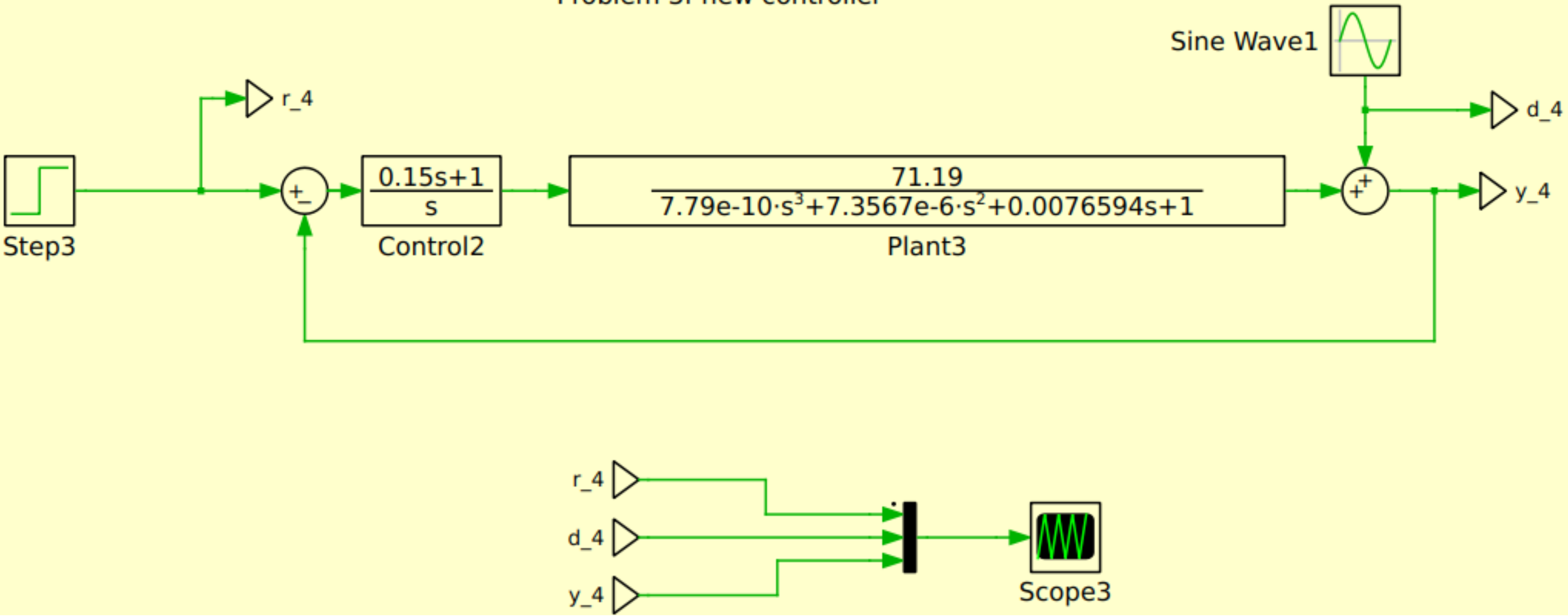

Problem 3f original control system
Sine Wave
r_3
d_3
y_3
71.19
7.79e-10·s3+7.3567e-6·s2+0.0076594s+1
Step2
Plant2
r_3
d_3
y_3
Scope2

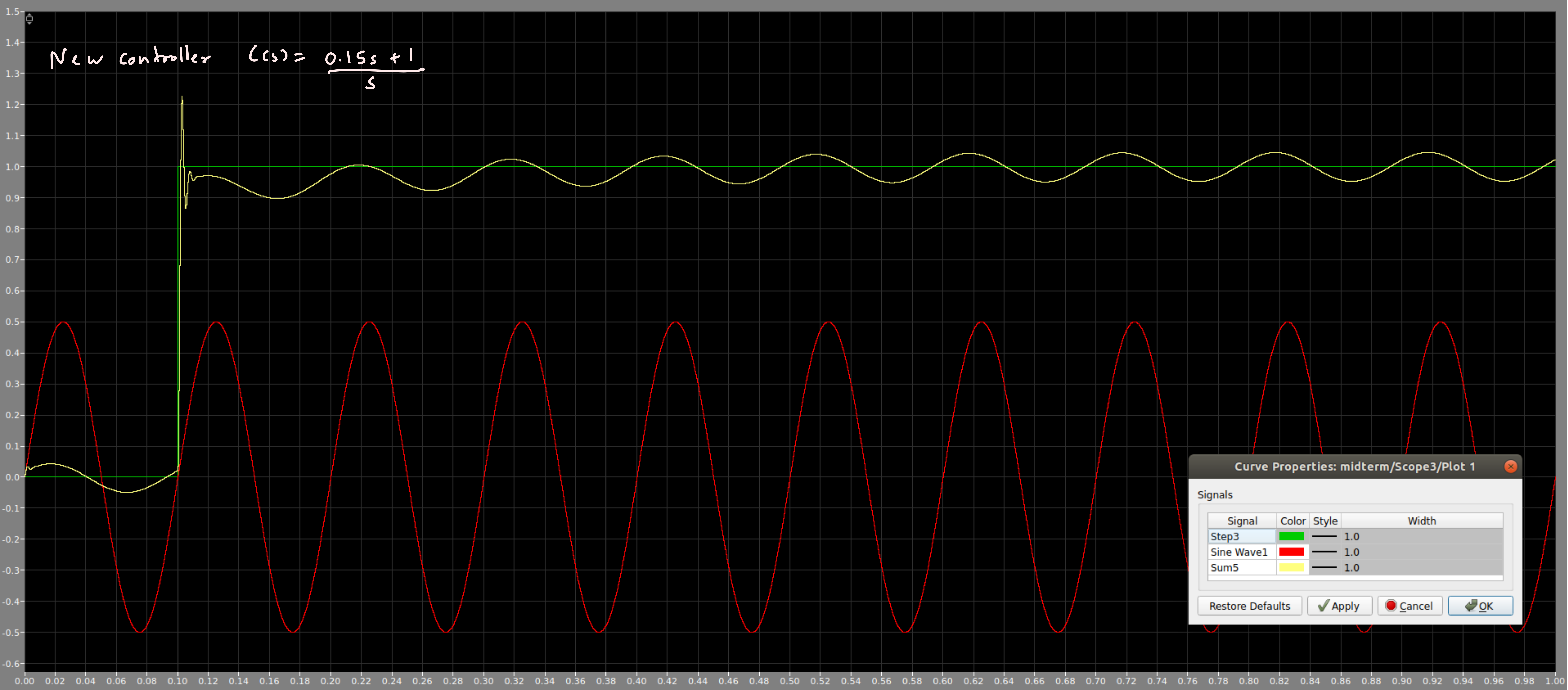


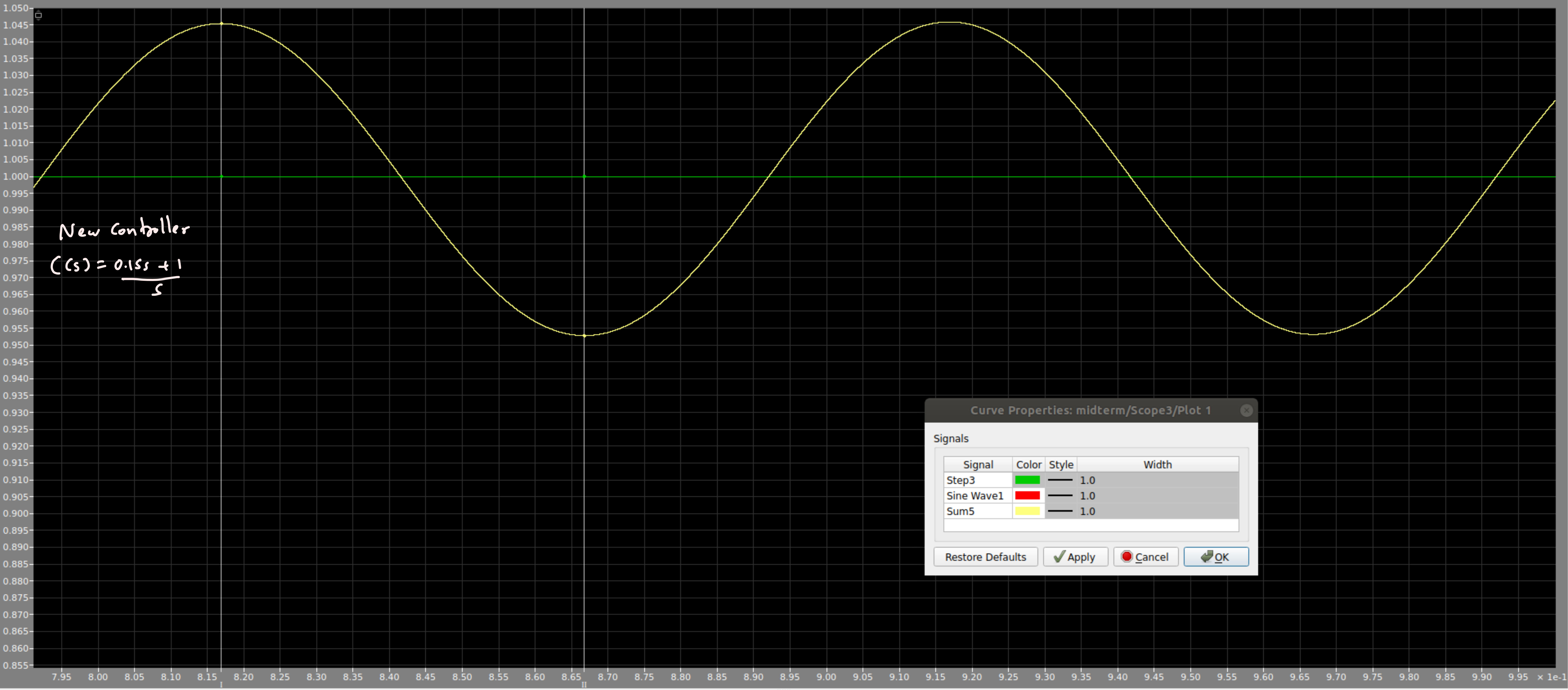
Curve Properties: midterm/Scope2/Plot 1
Signals
Signal
Color
Style
Width
Step2
1.0
Sine Wave
1.0
Sum3
1.0
Restore Defaults
Apply
Cancel
OK
Without C(s)
× 1e-1


Only integrator
C(s) = 1/s
Curve Properties: midterm/Scope3/Plot 1
Signals
Signal
Color
Style
Width
Step3
1.0
Sine Wave1
1.0
Sum5
1.0
Restore Defaults
Apply
Cancel
OK

Problem 3f new controller



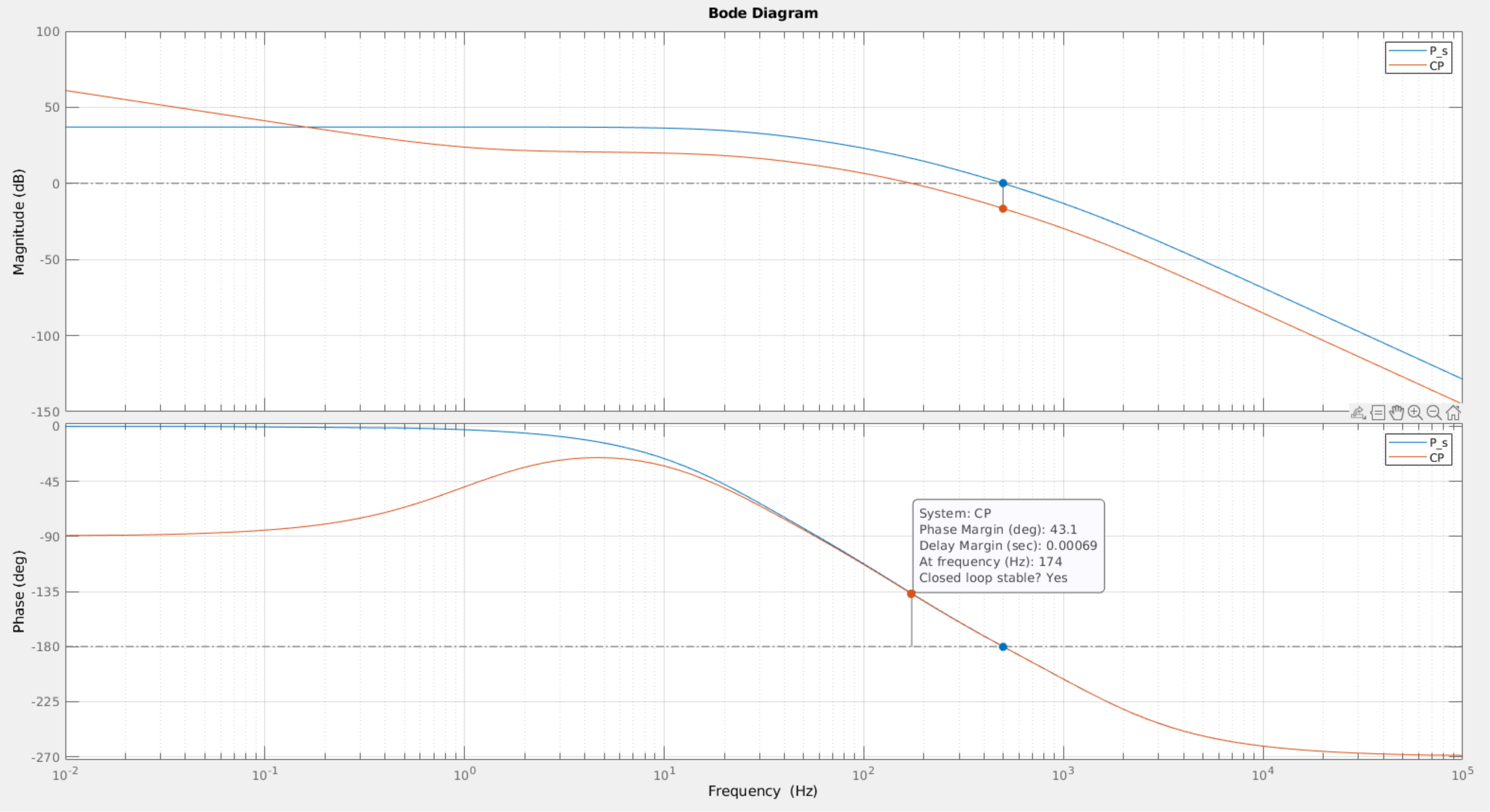

New controller $C(s) = \frac{0.15s + 1}{s}$
Curve Properties: midterm/Scope3/Plot 1
Signals
Signal
Color
Style
Width
Step3
1.0
Sine Wave1
1.0
Sum5
1.0
Restore Defaults
Apply
Cancel
OK

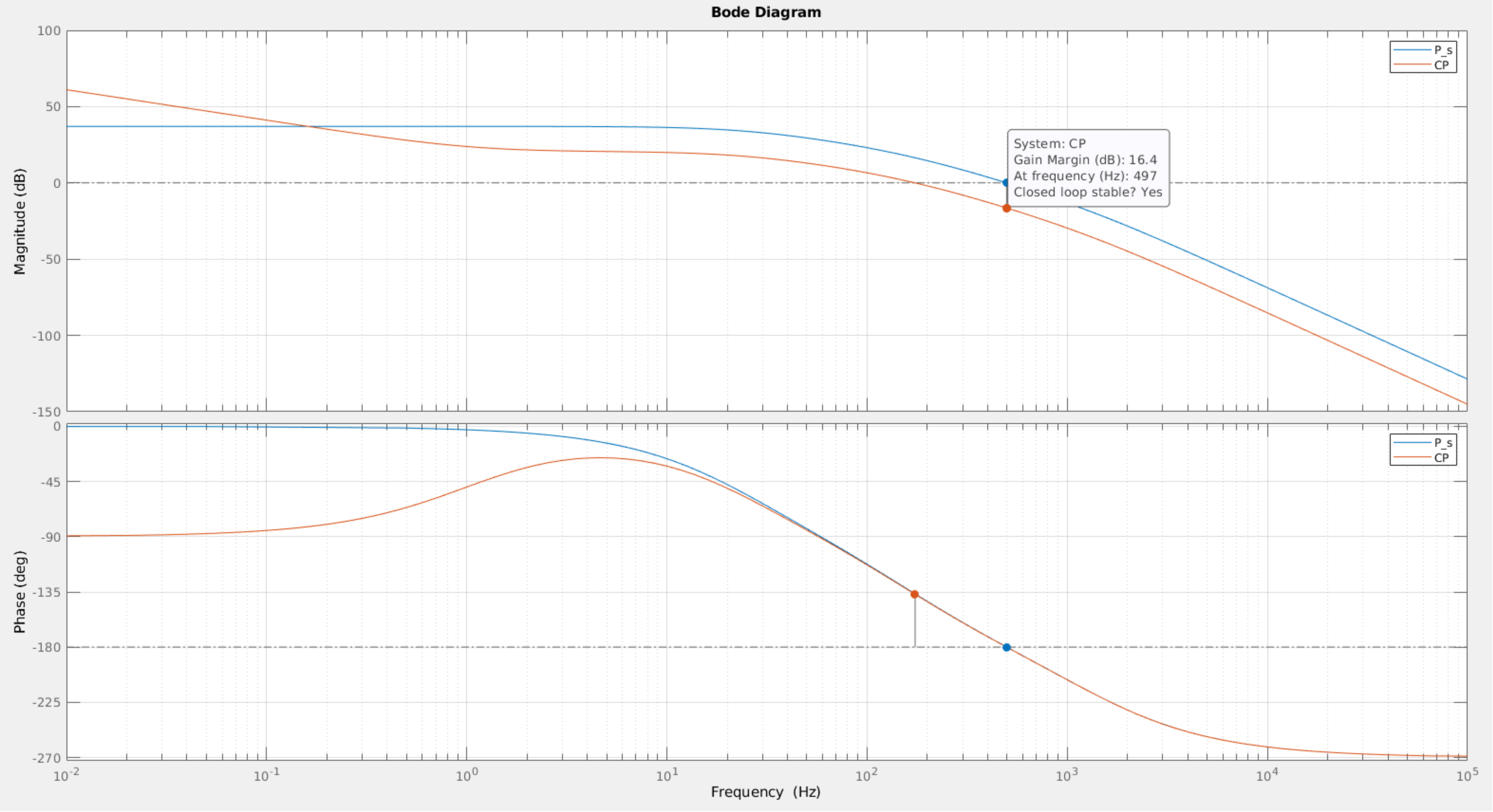


Data

Name	Cursor 1	Cursor 2	Delta
Time	0.81693	0.86674	0.04981
Plot 1			
Step3	1	1	0
Sine ...	0.43709	-0.43416	0.87125
Sum5	1.04538	0.952795	0.0925853

← within 20% of 0.5V


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (Hz)
P_s
CP
System: CP
Phase Margin (deg): 43.1
Delay Margin (sec): 0.00069
At frequency (Hz): 174
Closed loop stable? Yes



Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (Hz)
P_s
CP
System: CP
Gain Margin (dB): 16.4
At frequency (Hz): 497
Closed loop stable? Yes

Problem 4 (Kevin Douglas Egedy)

4 Digital Systems

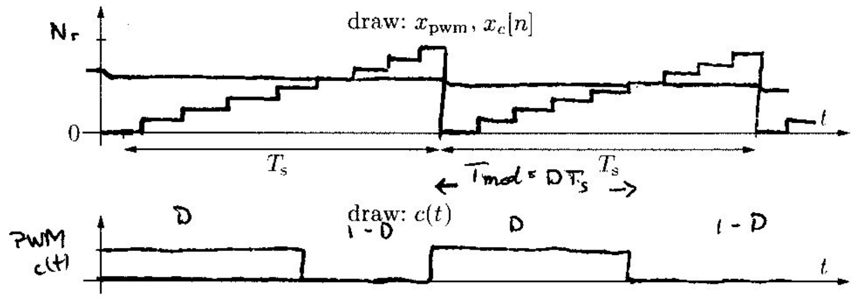


Figure 3: Template for PWM waveforms.

4.1

$$N_r + 1 = \frac{T_s}{T_{clk}} = \frac{f_{clk}}{f_s} = \frac{100MHz}{50kHz}$$

$$N_r = 2000 - 1 = 1999$$

4.2

Physical Voltage $v_{A/D}$

$$max(x_{A/D}[n]) = 2^{n_{A/D}} - 1 = 2^8 - 1 = 255$$

$$\text{Thus } 0 \leq x_{A/D}[n] \leq V_{FS}$$

$$x_{A/D}[n] = \frac{v_{A/D}[n] \cdot V_{FS}}{2^{n_{A/D}}}$$

$$v_{A/D}[n] = \frac{x_{A/D}[n] \cdot V_{FS}}{2^{n_{A/D}}}$$

$$v_{A/D}[n] = \frac{107}{2^8} \cdot 1 = 0.418V$$

Voltage Resolution $q_{A/D}$

$$q_{A/D} = \frac{v_{FS}}{2^{n_{A/D}}} = \frac{1}{2^8} = 0.0039 = 3.9mV$$