# EE 458 – Power Electronics Controls
# Homework 3

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## 1 Boost Converter Computations

Find the small signal output voltage to duty ratio transfer function as defined,

$$G_{vd}(s) = G_0 \frac{1 - \dfrac{s}{w_z}}{1 + \dfrac{s}{Qw_0} + \left(\dfrac{s}{\omega_0}\right)^2}.$$

Volt-Seconds

$$\langle v_L(t)\rangle = L\frac{d}{dt}\langle i(t)\rangle = \left[\Big(\langle v_g(t)\rangle\Big)d(t) + \Big(\langle v_g(t)\rangle - \langle v(t)\rangle\Big)d'(t)\right]$$

$$\langle v_L(t)\rangle = L\frac{d}{dt}\langle i(t)\rangle = \Big[\langle v_g(t)\rangle - \langle v(t)\rangle d'(t)\Big]$$

$$V = \frac{V_g}{D'}$$

Charge Balance

$$\langle i_C(t)\rangle = C\frac{d}{dt}\langle v(t)\rangle = \left[\left(\frac{-\langle v(t)\rangle}{R}\right)d(t) + \left(\langle i(t)\rangle - \frac{\langle v(t)\rangle}{R}d'(t)\right)\right]$$

$$\langle i_C(t)\rangle = C\frac{d}{dt}\langle v(t)\rangle = \left[\frac{-\langle v(t)\rangle}{R} + \langle i(t)\rangle d'(t)\right]$$

$$I = \frac{V}{D'R} = \frac{V_g}{D'^2 R}$$

System Inputs

$$\dot{x} = \frac{d}{dt}\begin{bmatrix}\langle \hat{i}(t)\rangle \\ \langle \hat{v}(t)\rangle\end{bmatrix} = \begin{matrix}\dfrac{1}{L} \\ \dfrac{1}{C}\end{matrix}\begin{bmatrix}\langle v_g(t)\rangle - \langle v(t)\rangle d'(t) \\ \dfrac{-\langle v(t)\rangle}{R} + \langle i(t)\rangle d'(t)\end{bmatrix}$$

System Output $\hat{y}(s)$

$$\hat{y}(s) = \Big(C(sI-A)^{-1}B+E\Big)\hat{u}(s)$$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix} \frac{\partial f_1}{\partial\langle\hat{i}(t)\rangle} & \frac{\partial f_1}{\partial\langle\hat{v}(t)\rangle} \\ \frac{\partial f_2}{\partial\langle\hat{i}(t)\rangle} & \frac{\partial f_2}{\partial\langle\hat{v}(t)\rangle} \end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix} 0 & \frac{-D'}{L} \\ \frac{D'}{C} & \frac{-1}{RC} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix} \frac{\partial f_1}{\partial\hat{d}(t)} & \frac{\partial f_1}{\partial\langle\hat{v}_g(t)\rangle} \\ \frac{\partial f_2}{\partial\hat{d}(t)} & \frac{\partial f_2}{\partial\langle\hat{v}_g(t)\rangle} \end{bmatrix}\Bigg|_{x,u} = \begin{bmatrix} \frac{V}{L} & \frac{1}{L} \\ \frac{-I}{C} & 0 \end{bmatrix} = \begin{bmatrix} \frac{V_g}{D'L}. & \frac{1}{L} \\ \frac{-V_g}{D'^2RC} & 0 \end{bmatrix}$$

System Output expressed in equivalent $G(s)$

$$\hat{y}(s) = \Big(C(sI-A)^{-1}B+E\Big)\hat{u}(s)$$

$$\hat{y}(s) = G(s)u(s)$$

$$G(s) = \begin{bmatrix} G_{id}(s) & G_{ig}(s) \\ G_{vd}(s) & G_{vg}(s) \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{V_g}{D'}\right)\frac{RCs+2}{RLCs^2+Ls+D'^2R} & \frac{RCs+1}{RLCs^2+Ls+D'^2R} \\ \left(\frac{-V_g}{D'^2}\right)\frac{Ls-D'^2R}{RLCs^2+Ls+D'^2R} & \frac{D'R}{RLCs^2+Ls+D'^2R} \end{bmatrix}$$

Small signal output voltage to duty ratio transfer function

$$G_{vd}(s) = G_0\frac{1-\frac{s}{w_z}}{1+\frac{s}{Qw_0}+\left(\frac{s}{\omega_0}\right)^2}$$

$$G_{vd}(s) = \left(\frac{V_g}{D'^2}\right)\frac{D'^2R-Ls}{D'^2R+Ls+RLCs^2} = \left(\frac{V_g}{D'^2}\right)\frac{1-\frac{L}{D'^2R}s}{1+\frac{L}{D'^2R}s+\frac{RLC}{D'^2R}s^2} = \left(\frac{V_g}{D'^2}\right)\frac{1-\frac{s}{\frac{D'^2R}{L}}}{1+\frac{s}{\left(D'R\sqrt{\frac{C}{L}}\right)\left(\frac{D'}{\sqrt{LC}}\right)}+\frac{s^2}{\left(\frac{D'}{\sqrt{LC}}\right)^2}}$$

| Parameter | Value |
|---|---|
| $G_0$ | $\frac{V_g}{D'^2}$ |
| $\omega_z$ | $\frac{D'^2R}{L}$ |
| $Q$ | $D'R\sqrt{\frac{C}{L}}$ |
| $\omega_0$ | $\frac{D'}{\sqrt{LC}}$ |

## 2 Stability

Describe the stability of the system $G_{vd}(s)$, under the following conditions.

| Parameter | Value |
|---|---|
| $V_{in}$ | 24V |
| $V_{out}$ | 48V |
| $P_{out}$ | 250W |
| $f_{sw}$ | 100kHz |
| $L$ | 1.25mH |
| $C$ | 250 $\mu F$ |

With this system,

$$\begin{aligned} P_{out} = && I^2R = && \left(\frac{V}{D'R}\right)^2 R \\ R = && \frac{V^2}{P_{out}D'^2} = && 37\ \Omega \\ \omega_z = && \frac{D'^2R}{L} = && 7373\text{ Hz} \\ \omega_0 = && \frac{D'}{\sqrt{LC}} = && 894\text{ Hz} \end{aligned}$$

### 2.1 Reduce current ripple $i(t)$: stability decreases

Current ripple is defined by $\frac{i_L}{dt} \propto \frac{1}{L}$.

Increasing $L$ results in smaller $w_z$ since $w_z \propto \frac{1}{L}$.

Increasing $L$ results in smaller $w_0$ since $w_0 \propto \frac{1}{\sqrt{L}}$.

The phase margin decreases thus the system stability decreases.

### 2.2 Decrease output power to 100W: stability increases

Power is defined as $P_{out} = I^2R$ such that $I$ is 1.58x ($\sqrt{2.5}$) smaller than the original value.

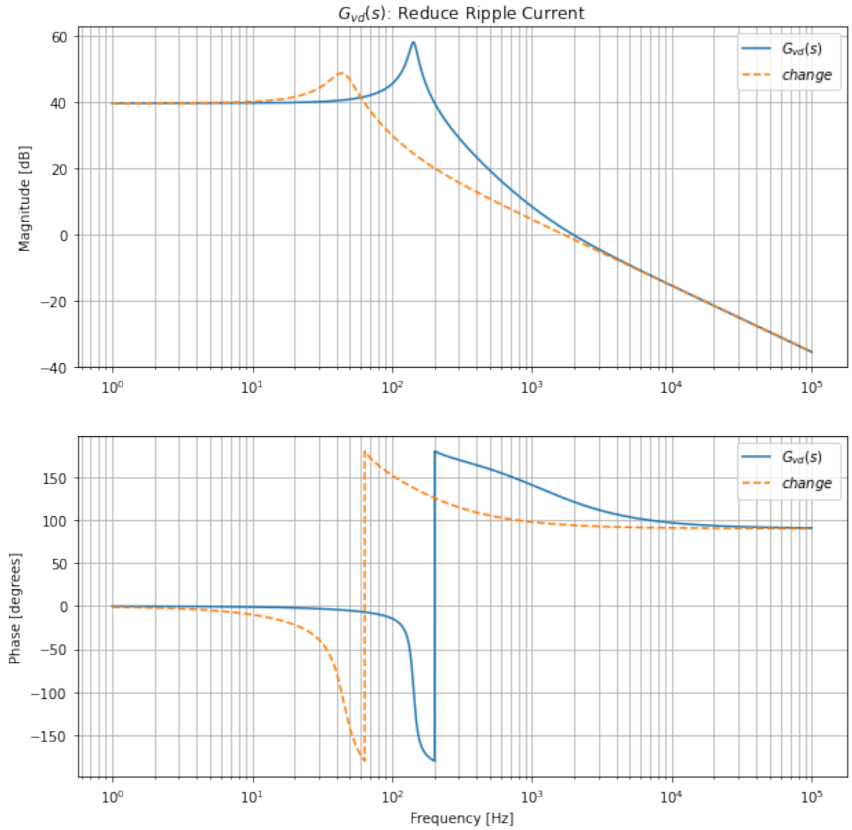
Output current $I$ is defined as $I = \frac{V}{D'R}$ where $R$ and $V$ is held constant (problem doesn't state output voltage has changed).

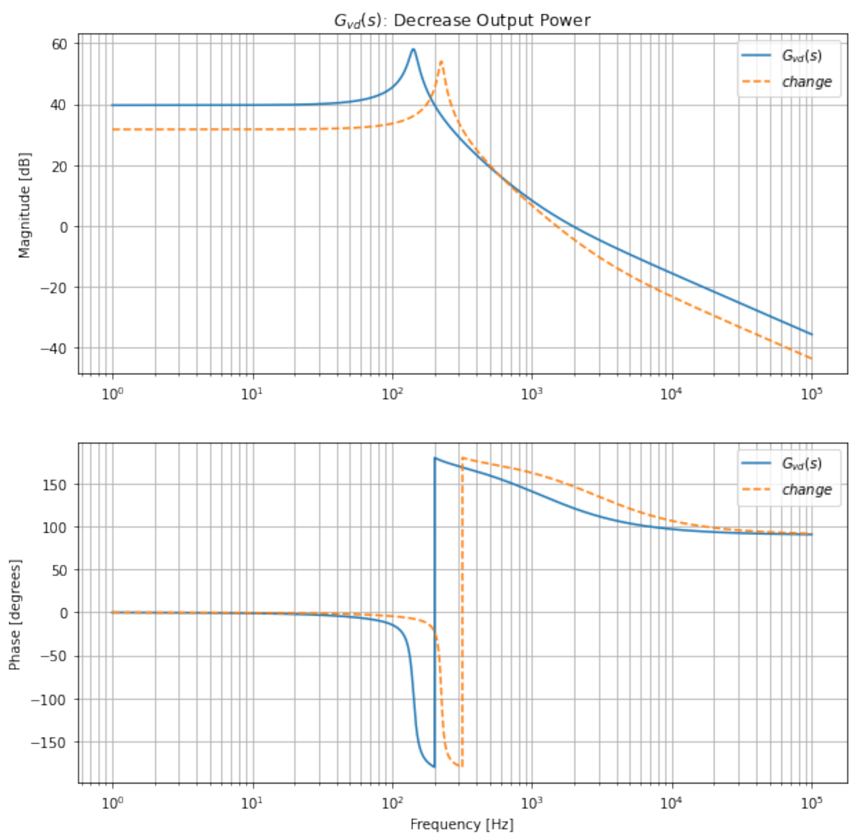
Then $D'$ is inversely proportional to $I$ such that it increases when power is decreased.

Increasing $D'$ results in larger $w_z$ since $w_z \propto D'^2$.

Increasing $D'$ results in larger $w_0$ since $w_0 \propto D'$.

The phase margin increases thus the system stability increases.


$G_{vd}(s)$: Reduce Ripple Current
Magnitude [dB]
Phase [degrees]
Frequency [Hz]
$G_{vd}(s)$
*change*


$G_{vd}(s)$: Decrease Output Power
Magnitude [dB]
Phase [degrees]
Frequency [Hz]
$G_{vd}(s)$
*change*

### 2.3 Decrease switching frequency to 10kHz: stability unchanged

The zeros or poles have no dependence on switching frequency and no ripple requirements are defined.

Thus stability remains unchanged.

### 2.4 Decrease output voltage to 30V: stability increases

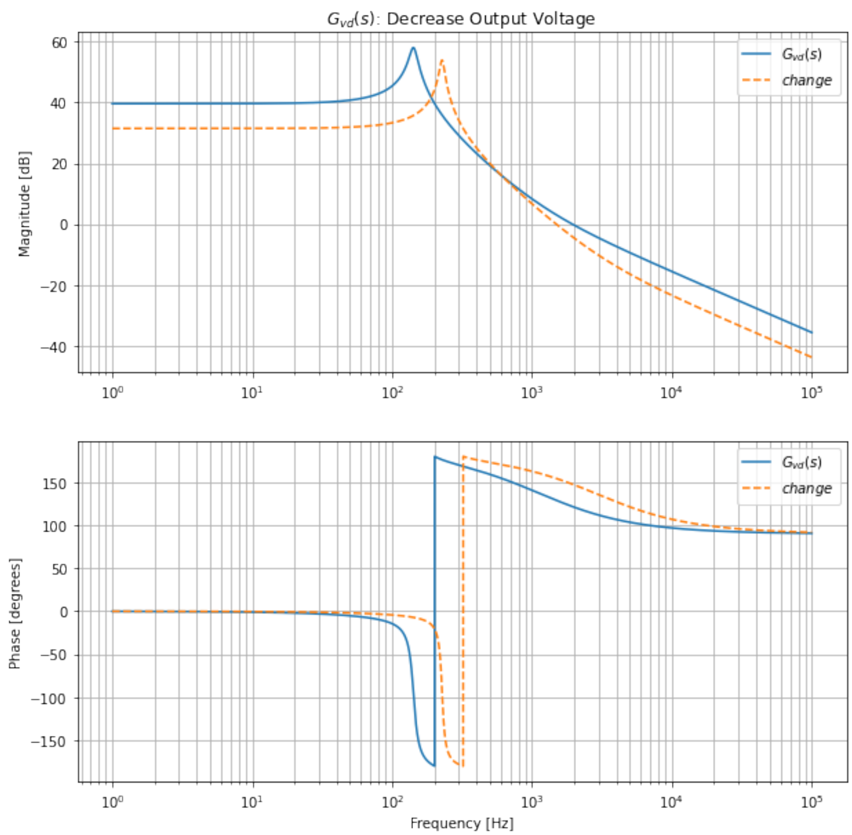
Output voltage is defined as $V = \dfrac{V_g}{D'}$ where $V_g$ is held constant (problem doesn't state it has changed).

Then $D'$ is inversely proportional to $V$ such that it increases when output voltage is decreased.

Increasing $D'$ results in larger $w_z$ since $w_z \propto D'^2$.

Increasing $D'$ results in larger $w_0$ since $w_0 \propto D'$.

The phase margin increases thus the system stability increases.

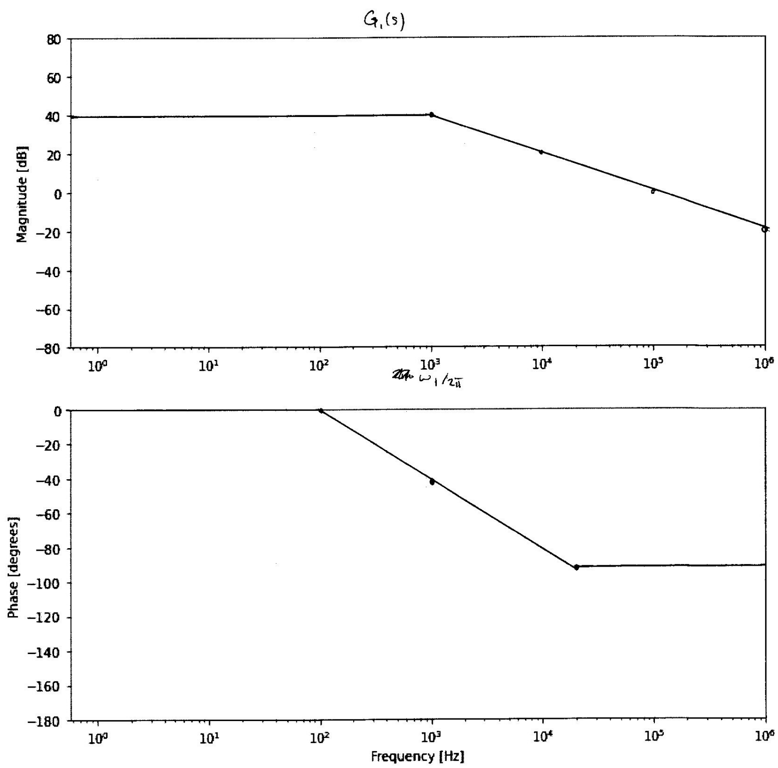


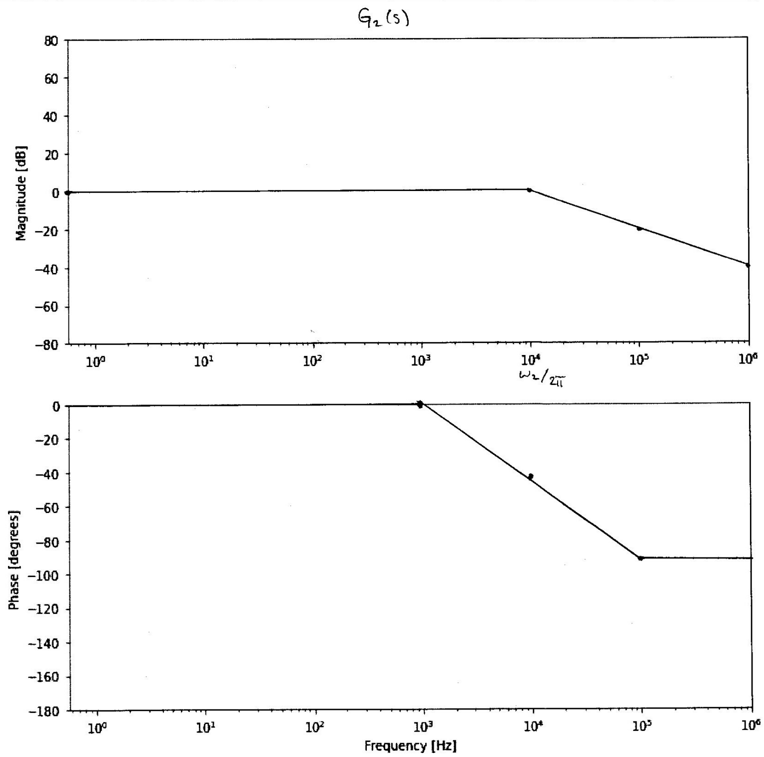
# 3 Transfer Functions

$$G_1(s) = 100 \frac{1}{1 + \dfrac{s}{2\pi \cdot 1kHz}}$$

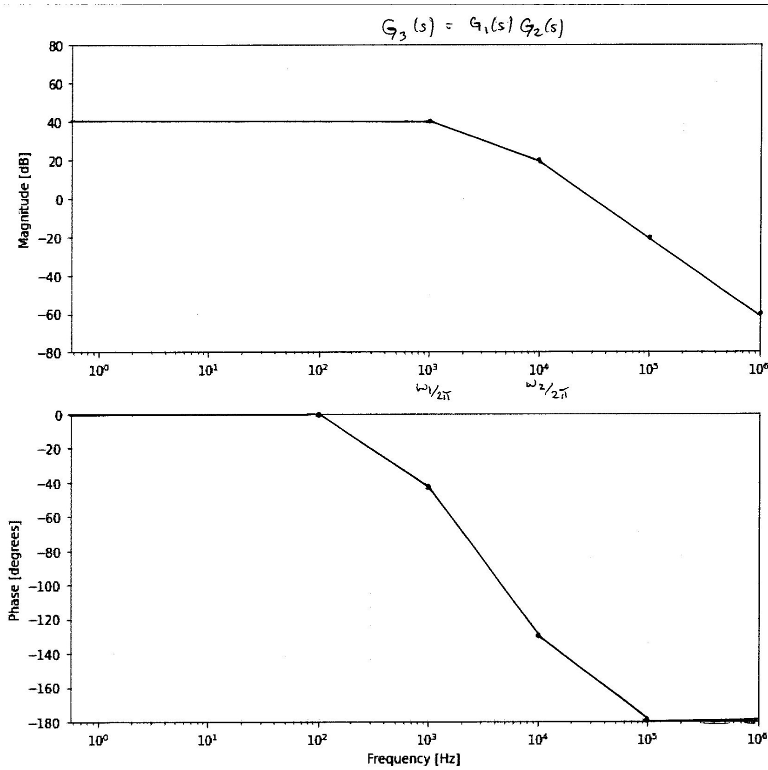
$$G_2(s) = \frac{1}{1 + \dfrac{s}{2\pi \cdot 10kHz}}$$

$$G_1(s)G_2(s) = G_0 \left( \frac{1}{1 + \dfrac{s}{\omega_1}} \right) \left( \frac{1}{1 + \dfrac{s}{\omega_2}} \right) = 100 \frac{1}{\left(1 + \dfrac{s}{2\pi \cdot 1kHz}\right)\left(1 + \dfrac{s}{2\pi \cdot 10kHz}\right)}$$





$G_2(s)$
Magnitude [dB]
$\omega_2/2\pi$
Phase [degrees]
Frequency [Hz]

$G_3(s) = G_1(s)\, G_2(s)$
Magnitude [dB]
$\omega_1/2\pi$
$\omega_2/2\pi$
Phase [degrees]
Frequency [Hz]

# 4 ADC and PWM

## 4.1 Digital Integer Value $x_{A/D}$

$$V_{FS} = 3V$$
$$n_{A/D} = 12bits$$
$$v_{A/D} = 0.75V$$
$$x_{A/D} = \left(\frac{v_{A/D}}{V_{FS}}\right) 2^{n_{A/D}} = \left(\frac{0.75}{3}\right) 2^{12} = 1024$$

## 4.2 Voltage Resolution $q_{A/D}$

$$q_{A/D} = \frac{v_{A/D}}{2^{n_{A/D}}} = 0.000732V = 0.732mV$$

## 4.3 $T_{hold}$

$$T_{sw} = 100kHz$$
$$T_{ctrl} = T_{A/D} + T_{comp} + T_{hold} = T_{sw}$$
$$\frac{1}{100kHz} = \frac{50}{100MHz} + \frac{500}{100MHz} + T_{hold}$$
$$T_{hold} = \frac{1}{100kHz} - \frac{50}{100MHz} - \frac{500}{100MHz}$$
$$T_{hold} = 10\mu s - 0.5\mu s - 5\mu s$$
$$T_{hold} = 4.5\mu s$$

$$\frac{T_{hold}}{T_{ctrl}} = \frac{4.5\mu s}{10\mu s} = 0.45$$

## 4.4 $T_{ctrl}, T_{mod}, T_d$

$$T_{clk} = 100Mhz$$
$$T_{ctrl} = 10\mu s = 1000T_{clk}$$
$$T_{mod} = D'T_{sw} = D' \cdot 1000T_{clk}$$
$$T_d = T_{A/D} + T_{comp} + T_{hold} + T_{mod}$$
$$Td = T_{ctrl} + T_{mod}$$
$$T_d = (1 + D')1000T_{clk} = (2 - D)1000T_{clk}$$

**4.5 see figure**

**4.6 see figure**

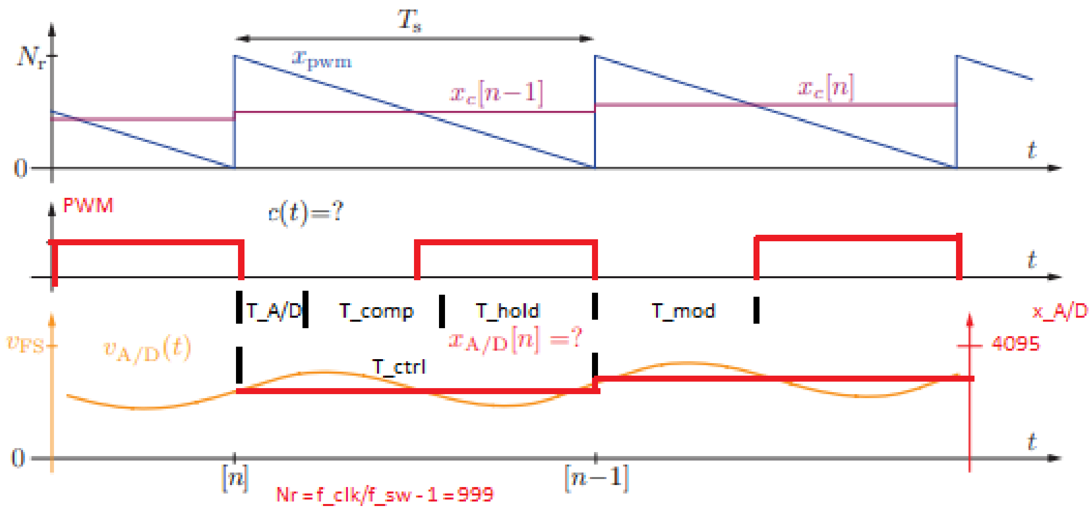



Figure 2: Digital system waveforms for Problem 3.