Lecture 9 (1^{st} February 2022)

HW 3 due on Thursday (3^{rd} Feb)

Midterm will be given out on 4^{th} Feb.

Submission: Sunday midnight.

100 point

Q1 True/False - 10 point 5 × 2.

Q2 Linearizations, small signal 30/40

Q3 Controller design 30/40

Q4 ADC & PWM. 10/20

1 / 12

$$\langle v_L(t)\rangle_{T_s} = L\frac{d\langle i_L(t)\rangle_{T_s}}{dt} = \langle v_{in}(t)\rangle_{T_s} d(t) - \langle v_c(t)\rangle_{T_s} d'(t)$$

$$\langle i_c(t)\rangle_{T_s} = C\frac{d}{dt}\langle v_c(t)\rangle_{T_s} =$$

$$\underline{\frac{-\langle v_c(t)\rangle_{T_s}}{R}\cdot d(t)} + \left(\frac{-\langle v_c(t)\rangle_{T_s}}{R} + \langle i_L(t)\rangle_{T_s}\right)d'(t)$$

Large Signal

$$0 = \underline{V_{in}\cdot D - V_c\cdot D'}$$

$$0 = -\frac{V_c}{R}D - \frac{V_c}{R}\cdot D' + I_L D'$$

$$= \underline{-\frac{V_c}{R} + I_L D'}$$

$$V_{in}D = V_c D' \Rightarrow V_c = \frac{V_{in}D}{D'}$$

$$\& \quad I_L = \frac{V_c}{R\cdot D'} = \frac{V_{in}D}{RD'^2}$$

'D' , 'Vin'

$$f(x) = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0}\overbrace{(x-x_0)}^{\tilde{x}} + \frac{1}{L2}\left.\frac{\partial^2 f}{\partial x^2}\right|_{x=x_0}(x-x_0)^2$$

$$x_0 \to \bar{x}, \bar{u}$$

$$\bar{x} \to \begin{bmatrix} I_L \\ V_c \end{bmatrix} \quad \bar{u} \to \begin{bmatrix} D \\ V_{in} \end{bmatrix}$$

$$+ \cdots \frac{1}{Ln}\cdot\left.\frac{\partial^n f}{\partial x^n}\right|_{x=x_0}(x-x_0)^n$$

$$f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \frac{d\langle i_L \rangle}{dt} \\ \frac{d\langle v_c \rangle}{dt} \end{bmatrix}$$

$$\begin{array}{l} f_1 \to \\ f_2 \to \end{array} \begin{bmatrix} \frac{1}{L}\left\{ \langle v_{in}(t) \rangle_{T_s} d(t) - \langle v_c(t) \rangle_{T_s} d'(t) \right\} \\ \frac{1}{C}\left\{ -\frac{\langle v_c(t) \rangle}{R} + \frac{\langle i_L(t) \rangle d'(t)}{R} \right. \end{bmatrix}$$

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \frac{d\langle i_L \rangle}{dt} \\ \frac{d\langle v_c \rangle}{dt} \end{bmatrix} = \underline{\underline{f(\bar{x}, \bar{u})}} + \left.\frac{\partial f}{\partial x}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}} (x - \bar{x}) + \left.\frac{\partial f}{\partial u}\right|_{\substack{x=\bar{x} \\ u=\bar{u}}} (u - \bar{u}) + \dots$$

$$= \overbrace{\begin{bmatrix} \frac{1}{L}\left(V_{in} D - V_c D' \right) \\ \frac{1}{C}\left(-\frac{V_c}{R} + \frac{I_L D'}{R} \right) \end{bmatrix}}^{} + \underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial \langle i_L \rangle} & \frac{\partial f_1}{\partial \langle v_c \rangle} \\ \frac{\partial f_2}{\partial \langle i_L \rangle} & \frac{\partial f_2}{\partial \langle v_c \rangle} \end{bmatrix}}_{A_0^0 =} \begin{bmatrix} i_L - I_L \\ v_c - V_c \end{bmatrix}$$

0

$$x = \begin{bmatrix} \langle i_L \rangle \\ \langle v_c \rangle \end{bmatrix}$$

$$+ \underbrace{\begin{bmatrix} \frac{\partial f_1}{\partial d} & \frac{\partial f_1}{\partial v_{in}} \\ \frac{\partial f_2}{\partial d} & \frac{\partial f_2}{\partial v_{in}} \end{bmatrix}}_{B_0^0 =} \begin{bmatrix} d - D \\ v_{in} - V_{in} \end{bmatrix}$$

$$f_1 \to \frac{1}{L}\left\{ \langle v_{in}(t)\rangle_{T_s} d(t) - \langle v_c(t)\rangle_{T_s} d'(t) \right\}$$

$$f_2 \to \frac{1}{C}\left\{ -\frac{\langle v_c(t)\rangle}{R} + \frac{\langle i_L(t)\rangle d'(t)}{R} \right.$$

$$\frac{d}{d\langle v_c(t)\rangle}\left[-\frac{\langle v_c(t)\rangle d'(t)}{L} \right] = -\frac{d'(t)}{L}$$

$$A : \begin{bmatrix} \partial f_1/\partial\langle i_L\rangle & \partial f_1/\partial\langle v_c\rangle \\ \partial f_2/\partial\langle i_L\rangle & \end{bmatrix} = \begin{bmatrix} 0 & -\frac{d'(t)}{L} \\ \frac{d'(t)}{RC} & -\frac{1}{RC} \end{bmatrix}\Bigg|_{\substack{d = D \\ v = V_c \\ V_{i_L} = V_{in} \\ i_L = I_L}} = \underbrace{\begin{bmatrix} 0 & -D'/L \\ \frac{D'}{RC} & -\frac{1}{RC} \end{bmatrix}}_{A.}$$

$$\frac{\partial}{\partial\langle i_L\rangle}\left\{ \frac{\langle i_L\rangle d'}{CR} \right\} = \frac{d'(t)}{RC}$$

$$f_1 \to \frac{1}{L}\left\{ \langle v_{in}(t)\rangle_{T_s} d(t) - \langle v_c(t)\rangle_{T_s} d'(t) \right\}$$

$$f_2 \to \frac{1}{C}\left\{ -\frac{\langle v_c(t)\rangle}{R} + \frac{\langle i_L(t)\rangle d'(t)}{R} \right.$$

$$\dot{x} = f(x)$$

$$\dot{x} = f(\bar{x}) + \frac{\partial f}{\partial x}\Big|_{x=\bar{x}} (x - \bar{x})$$

$$\tilde{x} := x - \bar{x}$$

$$\therefore\ x = \bar{x} + \tilde{x}$$

$$\frac{d(\bar{x} + \tilde{x})}{dt} = f(\bar{x}) + \frac{\partial f}{\partial x}\Big|_{x=\bar{x}} \tilde{x}$$

$$\frac{d}{dt}(\bar{x}) = 0$$

$$\&$$

$$f(\bar{x}) = 0$$

$$\therefore\ \boxed{\frac{d}{dt}\tilde{x} = \frac{\partial f}{\partial x}\Big|_{x=\bar{x}} \cdot \tilde{x}}$$

$$\dot{x} = f(x)$$

introduce perturbation

$x = \bar{x} + \tilde{x}$ — small signal

large signal

$$\frac{d}{dt}(\bar{x}+\tilde{x}) = f(\bar{x}+\tilde{x})$$

$$\frac{d\tilde{x}}{dt} = f(\bar{x}+\tilde{x})$$

Eg: $f(x) = \underline{-k\cdot x}$; $\bar{x}$

$$\boxed{\frac{d\tilde{x}}{dt} = -k\tilde{x}}$$

$$\frac{d\tilde{x}}{dt} = -k(\bar{x}+\tilde{x})$$

$$= \underbrace{-k\bar{x}} - k\tilde{x} \quad \rightarrow \quad k\bar{x} = 0$$

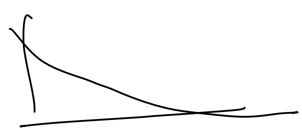
$$\bar{x} = 0$$

$$0 = -k\bar{x} \Rightarrow \bar{x} = 0$$

$$\frac{\tilde{i}_L}{\tilde{d}}$$

$$\boxed{\frac{d\tilde{x}}{dt} = -k\tilde{x}}$$

$$\dot{x} = -kx \Rightarrow x(t) = e^{-kt}x(0)$$

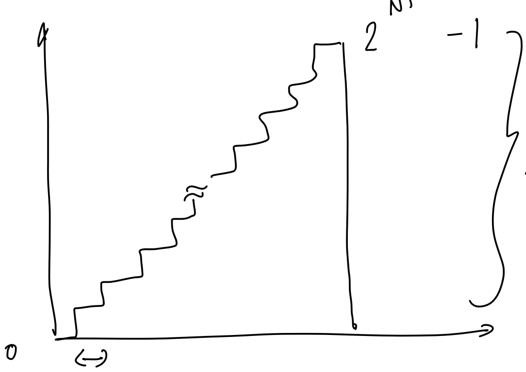


N^{pwm} = no of pwm bits

N^{adc} = no of adc bits

f_{clk} = system clock frequency.

PWM



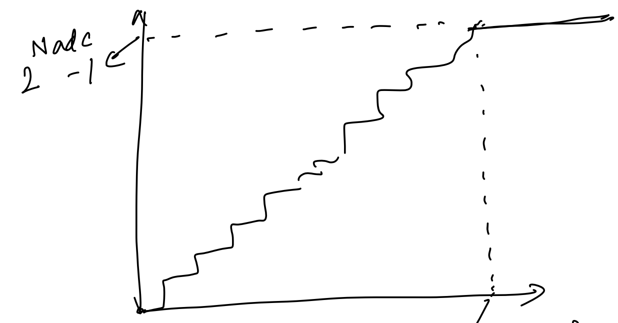
$t_{clk} = \frac{1}{f_{clk}}$

$T_{sw} = \frac{1}{f_{sw}}$

$$T_{sw} = 2^{N_{pwm}} t_{clk}$$

or $f_{sw} = \frac{1}{2^{N_{pwm}}} f_{clk}$

x[n]

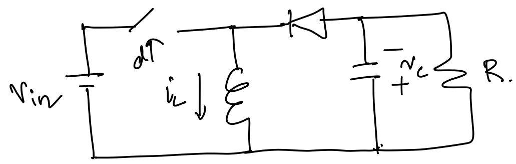


$v_{F.S} = 3V$

x(t)	x[n]
0	0
$0 < v < 3$	$\frac{v}{v_{F.S}} \cdot 2^{N_{adc}} = \frac{v}{3} \cdot 2^{N_{adc}}$
$v > 3$	$2^{N_{adc}} - 1$

$$\frac{v}{3} \cdot 2^{N_{adc}} = 2^{N_{adc}}$$

4 bit adc $\frac{3}{2^4}$



Given

V_{in} , f_{sw} , L, C

V_c / nominal V_{out}

P_{out}.

$R = ?$ $\quad P_{out} = \dfrac{V_c^2}{R}$

$D = ?$

① Large signal analysis $\quad I_L, V_c$

② Small s, "

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$$

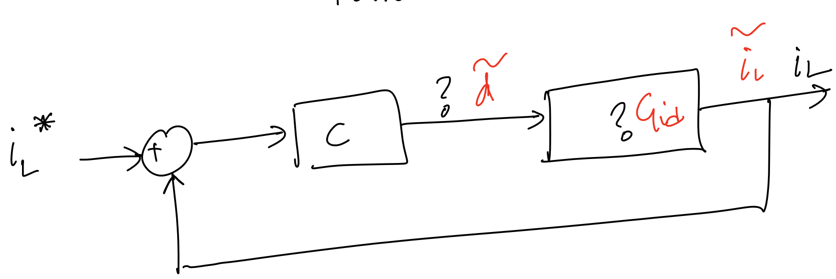
$$y = C\tilde{x}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad y = \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_c \end{bmatrix} = \underbrace{\begin{bmatrix} G_{id} & G_{ivg} \\ G_{vd} & G_{vvg} \end{bmatrix}}_{C(sI-A)^{-1}B} \begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}$$

$$[1 \quad 0] \qquad \tilde{i}_L = G_{id}\cdot\tilde{d} + G_{ivg}\cdot\tilde{v}_{in}$$

③ Current control of buck boost converter.

Plant TF $= G_{id}(s)$



4 Targets

Rise time, overshoot, settling time $\rightarrow$ Bandwidth $\approx$ gain cross over frequency of the compensated loop gain $= C \cdot P$

overshoot $\rightarrow$ Phase margin.

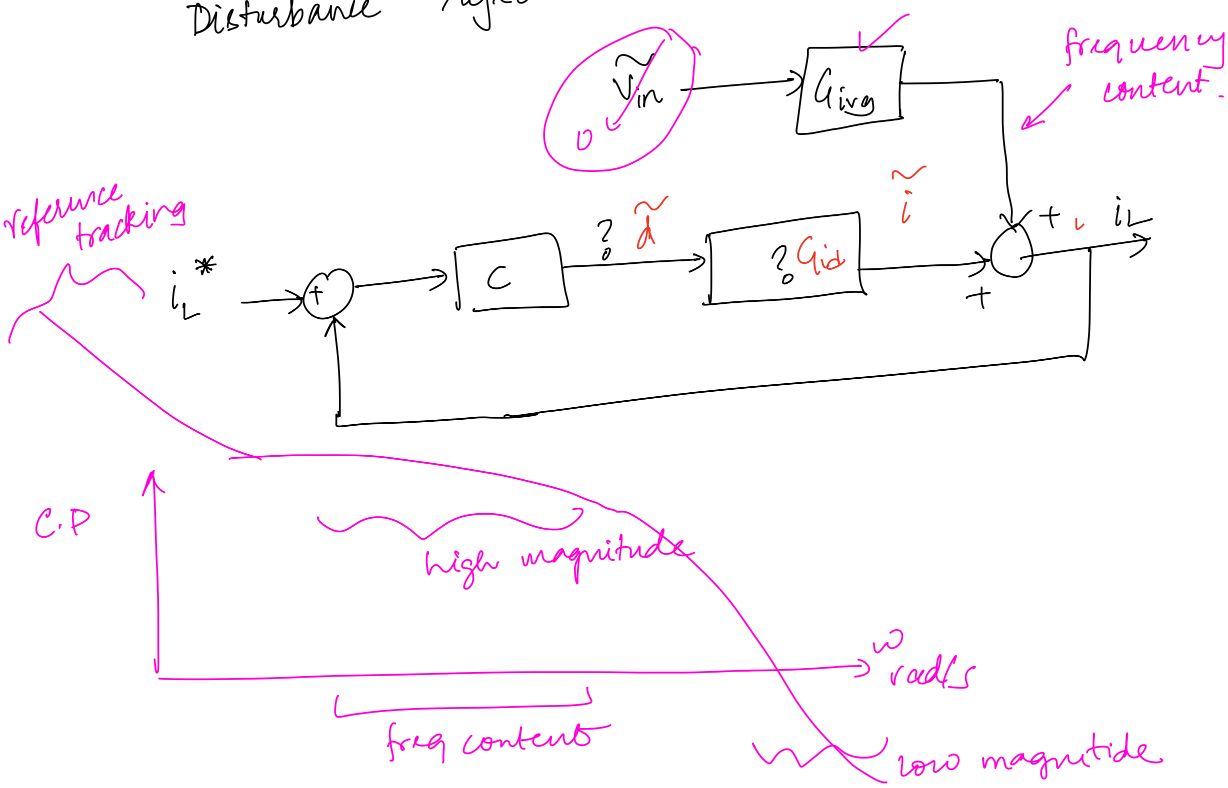
$$t_{rise},\ t_{settle} \propto \frac{1}{\text{gain cross over freq}}$$

$$\text{Overshoot} \propto \frac{1}{\text{Phase Margin}}$$

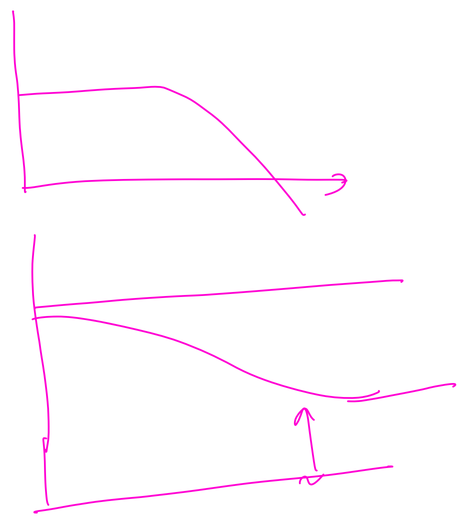
(a) Bandwidth & Phase margin requirement

(b) Reference tracking

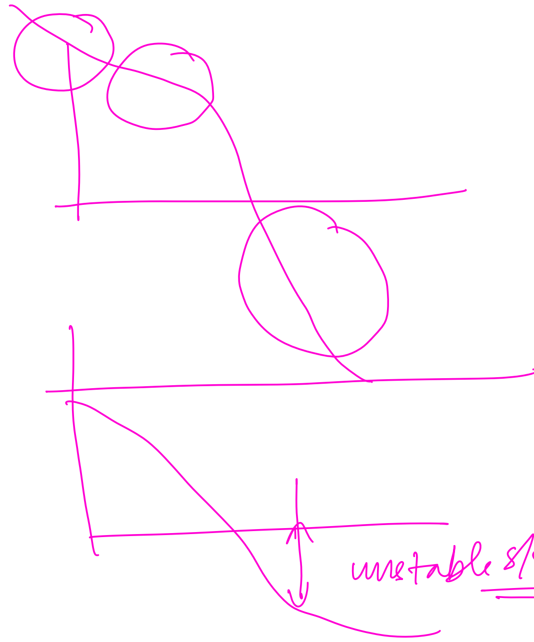
Disturbance rejection.



P



CP



5:35 pm

$$G_{id}(s) = \frac{\tilde{i}_L(s)}{\tilde{d}(s)} = \frac{(V_{in}+V_c)\left(sC+\frac{1}{R}\right) + D' I_L}{s^2 LC + \frac{sL}{R} + D'^2}$$

$$\frac{1}{D'^2}\,\frac{\overbrace{s.C\,(V_{in}+V_c)}^{b} + \overbrace{\frac{1}{R}.(V_{in}+V_c) + D' I_L}^{a}}{\frac{s^2 LC}{D'^2} + \frac{sL}{RD'^2} + 1}$$

$$\frac{1}{D'^2}\,\frac{sb + a}{\frac{s^2 LC}{D'^2} + \frac{sL}{RD'^2} + 1} = \boxed{\frac{a}{D'^2}} \cdot \frac{\left(s.\boxed{\frac{b}{a}}+1\right)}{\frac{s^2 LC}{D'^2} + \frac{sL}{RD'^2} + 1}$$

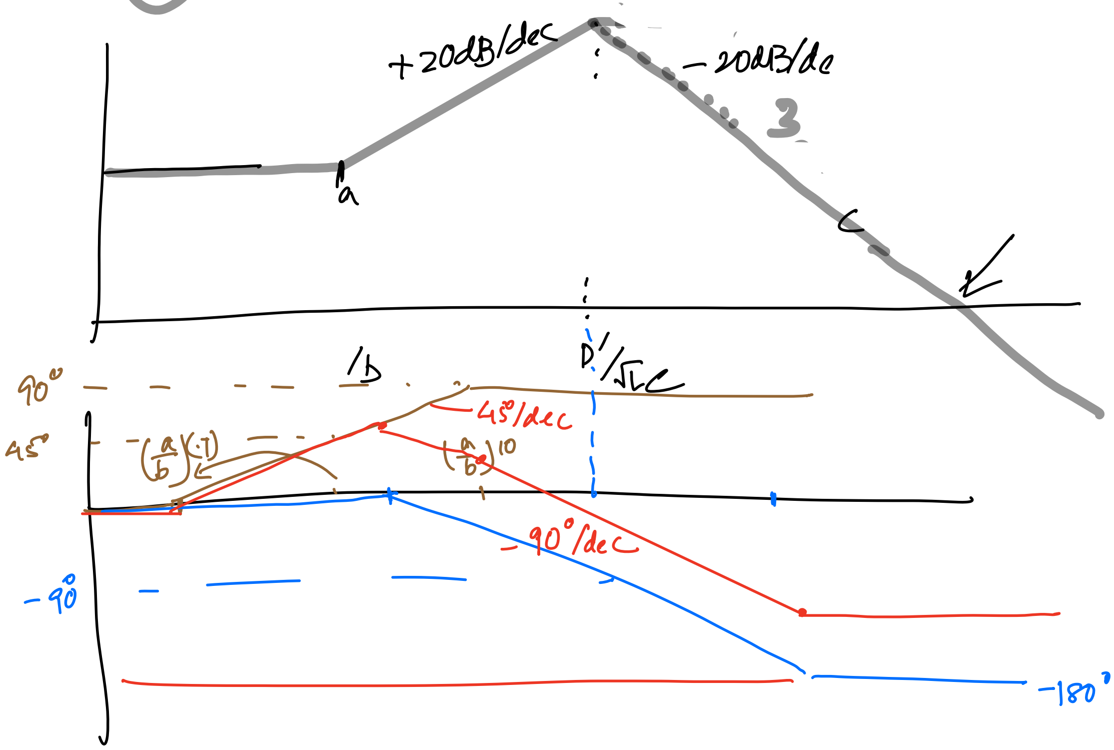
$$\left(\frac{s}{\omega_0}\right)^2 = \left(\frac{s}{D'/\sqrt{LC}}\right)^2 = \frac{s^2}{D'^2/LC} + \frac{sL}{RD'^2} + 1$$

DC gain = $\boxed{\frac{a}{D'^2}} + j\cdot 0$ $\qquad$ $\frac{a}{D'^2} > 1$

Zero = $\frac{a}{b}$ $\Big\}$ 1st order.

pole = $D'/\sqrt{LC}$ $\Big\}$ 2nd order

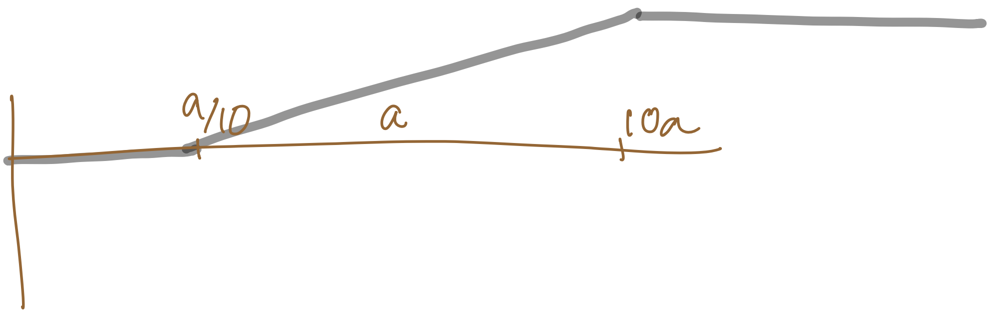
$\frac{a}{b} < \frac{D'}{\sqrt{LC}}$



$P_1 = \left(\frac{s}{a}+1\right)$ $\qquad$ $P_1 = 1 + \frac{j\omega}{a}$

$$|P_1(j\omega)| = \boxed{\sqrt{1+\frac{\omega^2}{a^2}}}$$

$$\angle P_1(j\omega) = \tan^{-1}\frac{\omega}{a}.$$



$P = \left(\frac{s}{2}+1\right)$ $\qquad$ $P = \underbrace{\left(\frac{s}{2j}+1\right)}\left(-\frac{s}{2j}+1\right)$