EE 458/ 533 – Power Electronics Controls
Midterm

Kevin Egedy

1 Conceptual Questions

1.1 False

$$N_r + 1 = \frac{T_s}{T_{clk}} = \frac{f_{clk}}{f_s}$$

$$N_r + 1 = \frac{1}{q}$$

$$q = \Delta d$$

The relationship of N_r to switching frequency is $N_r \propto \frac{1}{f_s}$, such that increasing N_r results in smaller switching frequency.
The relationship of N_r to quantization is $N_r \propto \frac{1}{q}$, such that increasing N_r results in smaller quantization of duty ratio.

1.2 False

Both MOSFETS experience conduction loss and switching loss.
Reference: Power MOSFET Basics by Alpha and Omega Semiconductor

$$P_{cond1} = \left(I_{mode1}^2\right) R_{on}$$

$$P_{sw1} = \frac{1}{2}\left(V_{L,mode1}\right)\left(I_{mode1}\right)\left(t_{d(on)} + t_r + t_{d(off)} + t_f\right)$$

$$P_{cond2} = \left(I_{mode2}^2\right) R_{on}$$

$$P_{sw2} = \frac{1}{2}\left(V_{L,mode2}\right)\left(I_{mode2}\right)\left(t_{d(on)} + t_r + t_{d(off)} + t_f\right)$$

$t_{d(on)}$: Turn on time delay. The time from when V_{gs} rises over 10% of the gate threshold to when the drain current rises past 10% of the specified current.

t_r: Rise Time. The time from when V_{gs} dops below 90% of the gate threshold to when the drain current drops below 90% of the load current.

$t_{d(off)}$: Turn off time delay. The time when V_{gs} drops below 90% of the gate threshold to when the drain current drops below 90% of the load current.

t_f: Fall time. The time between the drain current falling from 90% to 10% of load current.

1.3 True

Given sampling frequency f_{samp}, input frequency f_1, and sampled output frequency f_{out}

$$f_1 \leq \frac{f_{samp}}{2},\ f_{out} = f_1$$

$$f_1 > \frac{f_{samp}}{2},\ f_{out} = f_1 - k\frac{f_{samp}}{2} \text{ where } (k > 0)$$

$$0 \leq f_{out} \leq \frac{f_{samp}}{2}$$

Solving for $f_1 = 1001Hz$ and $f_1 = 2004Hz$:

$$\text{Given } f_1 = 1001, f_{out} = 1001 - 1\frac{2000}{2} = 1Hz$$

$$\text{Given } f_1 = 2004, f_{out} = 2004 - 2\frac{2000}{2} = 4Hz$$

Given $f_{samp} = 2000Hz$, the sampled output frequency of 900Hz, 1001Hz, and 2004Hz is 900Hz, 1Hz, and 4Hz respectively.

1.4 True

The system is sufficiently stable since the gain and phase margin are both greater than zero. In minimum phase systems, there can only exist a single gain crossover since there are no zeros to pull it up. In addition, the phase can't roll past -180 degrees until after the gain cross over frequency. Since the gain cross over is smaller than the phase cross over, the system will always yield a positive phase margin. Thus, the system magnitude will decrease at larger frequencies and provide negative feedback.

1.5 False

$$LC\frac{d^2v_g(t)}{dt^2} + \frac{L}{R}\frac{dv_g(t)}{dt} + v_g(t) = Dd(t)$$

$$\frac{L}{R}\frac{dv_g(t)}{dt} = Dd(t) - LC\frac{d^2v_g(t)}{dt^2} - v_g(t)$$

$$\frac{dv_g(t)}{dt} = \frac{R}{L}\left[Dd(t) - LC\frac{d^2v_g(t)}{dt^2} - v_g(t)\right]$$

2 Mathematical Methods for Dynamic Modeling

2.1 Differential equations

$$\dot{x} = f(x(t), u(t)) = \begin{bmatrix} f_1(x(t), u(t)) \\ f_2(x(t), u(t)) \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \langle i(t) \rangle \\ \langle v(t) \rangle \end{bmatrix} \text{ where } u = \begin{bmatrix} d'(t) \\ \langle v_g(t) \rangle \end{bmatrix}$$

Volt-Seconds

$$\langle v_L(t) \rangle = L\frac{d}{dt}\langle i(t) \rangle = \left[\Big(\langle v_g(t) \rangle - \langle i(t) \rangle R_{on}\Big)\Big(1 - d'(t)\Big) + \Big(\langle v_g(t) \rangle - \langle i(t) \rangle R_{on} - \langle v(t) \rangle\Big)d'(t)\right]$$

$$\langle v_L(t) \rangle = L\frac{d}{dt}\langle i(t) \rangle = \Big[\langle v_g(t) \rangle - \langle i(t) \rangle R_{on} - \langle v(t) \rangle d'(t)\Big]$$

$$\frac{d}{dt}\langle i(t) \rangle = \frac{1}{L}\Big[\langle v_g(t) \rangle - \langle i(t) \rangle R_{on} - \langle v(t) \rangle d'(t)\Big]$$

Charge Balance

$$\langle i_C(t) \rangle = C\frac{d}{dt}\langle v(t) \rangle = \left[\left(\frac{-\langle v(t) \rangle}{R}\right)\Big(1 - d'(t)\Big) + \left(\langle i(t) \rangle - \frac{\langle v(t) \rangle}{R}\right)d'(t)\right]$$

$$\langle i_C(t) \rangle = C\frac{d}{dt}\langle v(t) \rangle = \left[\frac{-\langle v(t) \rangle}{R} + \langle i(t) \rangle d'(t)\right]$$

$$\frac{d}{dt}\langle v(t) \rangle = \frac{1}{C}\left[\frac{-\langle v(t) \rangle}{R} + \langle i(t) \rangle d'(t)\right]$$

Differential Equations

$$\dot{x} = f(x(t), u(t)) = \frac{d}{dt}\begin{bmatrix} \langle i(t) \rangle \\ \\ \langle v(t) \rangle \end{bmatrix} = \begin{matrix} \frac{1}{L} \\ \frac{1}{C} \end{matrix}\begin{bmatrix} \langle v_g(t) \rangle - \langle i(t) \rangle R_{on} - \langle v(t) \rangle d'(t) \\ \\ \frac{-\langle v(t) \rangle}{R} + \langle i(t) \rangle d'(t)s \end{bmatrix} \text{ where } u = \begin{bmatrix} d'(t) \\ \\ \langle v_g(t) \rangle \end{bmatrix}$$

2.2 Linearized small-signal model

Volt-Seconds

$$\langle v_L(t)\rangle = L\frac{d}{dt}\langle i(t)\rangle = \Big[\langle v_g(t)\rangle - \langle i(t)\rangle R_{on} - \langle v(t)\rangle d'(t)\Big]$$

$$V_L(t) = L\frac{d}{dt}I = V_g - IR_{on} - VD' = 0$$

$$VD' = V_g - IR_{on}$$

$$V = \frac{V_g - IR_{on}}{D'}\bigg|_{I=\frac{V}{D'R}}$$

$$V = \frac{V_g - \frac{V}{D'R}R_{on}}{D'}$$

$$V = \frac{V_g}{D'} - \frac{VR_{on}}{D'^2R}$$

$$V + \frac{VR_{on}}{D'^2R} = \frac{V_g}{D'}$$

$$V = \frac{V_g}{D'} \cdot \frac{1}{1 + \frac{R_{on}}{D'^2R}}$$

Charge Balance

$$\langle i_C(t)\rangle = C\frac{d}{dt}\langle v(t)\rangle = \left[\frac{-\langle v(t)\rangle}{R} + \langle i(t)\rangle d'(t)\right]$$

$$I_C(t) = C\frac{d}{dt}V = \frac{-V}{R} + ID' = 0$$

$$ID' = \frac{V}{R}$$

$$I = \frac{V}{D'R}$$

$$I = \frac{1}{D'R} \cdot \frac{V_g}{D'} \cdot \frac{1}{1 + \frac{R_{on}}{D'^2R}}$$

$$I = \frac{V_g}{D'^2R} \cdot \frac{1}{1 + \frac{R_{on}}{D'^2R}}$$

Given

$$x(t) = \hat{x}(t) + X \qquad \hat{x}(t) = x(t) - X$$
$$u(t) = \hat{u}(t) + U \qquad \hat{u}(t) = u(t) - U$$

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t)$$
$$\dot{\hat{x}}(t) = A\left(x(t) - X\right) + B\left(u(t) - U\right)$$

$$A = \frac{d}{d\hat{x}(t)} f(x(t), u(t))\bigg|_{x=X,u=U} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}\bigg|_{x,u} = \begin{bmatrix} \frac{\partial f_1}{\partial \langle \hat{i}(t) \rangle} & \frac{\partial f_1}{\partial \langle \hat{v}(t) \rangle} \\ \frac{\partial f_2}{\partial \langle \hat{i}(t) \rangle} & \frac{\partial f_2}{\partial \langle \hat{v}(t) \rangle} \end{bmatrix}\bigg|_{x,u}$$

$$A = \begin{bmatrix} \frac{-R_{on}}{L} & \frac{-D'}{L} \\ \frac{D'}{C} & \frac{-1}{RC} \end{bmatrix}$$

$$B = \frac{d}{d\hat{u}(t)} f(x(t), u(t))\bigg|_{x=X,u=U} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \end{bmatrix}\bigg|_{x,u} = \begin{bmatrix} \frac{\partial f_1}{\partial \hat{d}'(t)} & \frac{\partial f_1}{\partial \langle \hat{v_g}(t) \rangle} \\ \frac{\partial f_2}{\partial \hat{d}'(t)} & \frac{\partial f_2}{\partial \langle \hat{v_g}(t) \rangle} \end{bmatrix}\bigg|_{x,u}$$

$$B = \begin{bmatrix} \frac{V}{L} & \frac{1}{L} \\ \frac{-I}{C} & 0 \end{bmatrix} = \begin{bmatrix} \frac{V_g}{D'L} \cdot \frac{1}{1 + \frac{R_{on}}{D'^2 R}} & \frac{1}{L} \\ \frac{-V_g}{D'^2 RC} \cdot \frac{1}{1 + \frac{R_{on}}{D'^2 R}} & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

2.3 Derive capacitor voltage

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t)$$

$$\dot{\hat{y}}(t) = C\hat{x}(t) + D\hat{u}(t)$$

Laplace Transform

$$s\hat{x}(s) = A\hat{x}(s) + B\hat{u}(s)$$

$$sI\hat{x}(s) - A\hat{x}(s) = B\hat{u}(s)$$

$$\hat{x}(s) = (sI - A)^{-1} B\hat{u}(s)$$

$$\hat{y}(s) = \Big(C(sI - A)^{-1}B + E\Big)\hat{u}(s) = \langle v(s)\rangle$$

$$\hat{y}(s) = \left(\begin{bmatrix} 0 & 1 \end{bmatrix} \left(s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{-R_{on}}{L} & \frac{-D'}{L} \\ \frac{D'}{C} & \frac{-1}{RC} \end{bmatrix} \right)^{-1} \begin{bmatrix} \frac{V_g}{D'L} \cdot \frac{1}{1 + \frac{R_{on}}{D'^2 R}} & \frac{1}{L} \\ \frac{-V_g}{D'^2 RC} \cdot \frac{1}{1 + \frac{R_{on}}{D'^2 R}} & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \right) \begin{bmatrix} D' \\ V_g \end{bmatrix}$$

$$(sI - A)^{-1} = \left(s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{-R_{on}}{L} & \frac{-D'}{L} \\ \frac{D'}{C} & \frac{-1}{RC} \end{bmatrix} \right)^{-1}$$

$$(sI - A)^{-1} = \begin{bmatrix} s + \frac{R_{on}}{L} & \frac{D'}{L} \\ \frac{-D'}{C} & s + \frac{1}{RC} \end{bmatrix}^{-1} = J^{-1}$$

$$(sI - A)^{-1} = \frac{1}{det(J)}(adj(J)) = \frac{1}{\Big((s + \frac{R_{on}}{L})(s + \frac{1}{RC})\Big) - \Big((\frac{D'}{L})(\frac{-D'}{C})\Big)} \begin{bmatrix} s + \frac{1}{RC} & \frac{-D'}{L} \\ \frac{D'}{C} & s + \frac{R_{on}}{L} \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{RLCs^2 + (L + RR_{on}C)s + (D'^2R + R_{on})}{RLC} \begin{bmatrix} s + \frac{1}{RC} & \frac{-D'}{L} \\ \frac{D'}{C} & s + \frac{R_{on}}{L} \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} \frac{RLCs + L}{RLCs^2 + (L + RR_{on}C)s + (D'^2R + R_{on})} & \frac{-RCD'}{RLCs^2 + (L + RR_{on}C)s + (D'^2R + R_{on})} \\ \frac{RLD'}{RLCs^2 + (L + RR_{on}C)s + (D'^2R + R_{on})} & \frac{RLCs + RR_{on}C}{RLCs^2 + (L + RR_{on}C)s + (D'^2R + R_{on})} \end{bmatrix}$$

$$C(sI-A)^{-1} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{RLCs+L}{RLCs^2+(L+RR_{on}C)s+(D'^2R+R_{on})} & \frac{-RCD'}{RLCs^2+(L+RR_{on}C)s+(D'^2R+R_{on})} \\ \frac{RLD'}{RLCs^2+(L+RR_{on}C)s+(D'^2R+R_{on})} & \frac{RLCs+RR_{on}C}{RLCs^2+(L+RR_{on}C)s+(D'^2R+R_{on})} \end{bmatrix}$$

$$C(sI-A)^{-1} = \begin{bmatrix} \frac{RLD'}{RLCs^2+(L+RR_{on}C)s+(D'^2R+R_{on})} & \frac{RLCs+RR_{on}C}{RLCs^2+(L+RR_{on}C)s+(D'^2R+R_{on})} \end{bmatrix}$$

$$C(sI-A)^{-1}B = \begin{bmatrix} \frac{RLD'}{RLCs^2+(L+RR_{on}C)s+(D'^2R+R_{on})} & \frac{RLCs+RR_{on}C}{RLCs^2+(L+RR_{on}C)s+(D'^2R+R_{on})} \end{bmatrix} \begin{bmatrix} \frac{V_g}{D'L} \cdot \frac{1}{1+\frac{R_{on}}{D'^2R}} & \frac{1}{L} \\ \frac{-V_g}{D'^2RC} \cdot \frac{1}{1+\frac{R_{on}}{D'^2R}} & 0 \end{bmatrix}$$

$$C(sI-A)^{-1}B = \Bigg[\left(RV_g - \frac{V_g(RLCs+RR_{on}C)}{D'^2RC} \right) \frac{1}{(1+\frac{R_{on}}{D'^2R})(RLCs^2+(L+RR_{on}C)s+D'^2R+R_{on})} \quad \frac{D'R}{RLCs^2+(RR_{on}C+L)s+(}$$

$$C(sI-A)^{-1}B = \begin{bmatrix} \left(\frac{RV_g}{D'^2R+R_{on}} \right) \frac{(D'^2R-R_{on})-Ls}{RLCs^2+(RR_{on}C+L)s+(D'^2R+R_{on})} & \frac{D'R}{RLCs^2+(RR_{on}C+L)s+(D'^2R+R_{on})} \end{bmatrix}$$

$$C(sI-A)^{-1}B+E = \begin{bmatrix} \left(\frac{RV_g}{D'^2R+R_{on}} \right) \frac{(D'^2R-R_{on})-Ls}{RLCs^2+(RR_{on}C+L)s+(D'^2R+R_{on})} & \frac{D'R}{RLCs^2+(RR_{on}C+L)s+(D'^2R+R_{on})} \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$C(sI-A)^{-1}B+E = G(s) = \begin{bmatrix} G_{vd}(s) & G_{vg}(s) \end{bmatrix}$$

$$\begin{bmatrix} G_{vd}(s) & G_{vg}(s) \end{bmatrix} = \begin{bmatrix} \left(\frac{RV_g}{D'^2R+R_{on}} \right) \frac{(D'^2R-R_{on})-Ls}{RLCs^2+(RR_{on}C+L)s+(D'^2R+R_{on})} & \frac{D'R}{RLCs^2+(RR_{on}C+L)s+(D'^2R+R_{on})} \end{bmatrix}$$

3 Controller Design Problem

3.1 Derive

Find $\frac{y(s)}{r(s)}$, given $d(s) = 0$

$$
\begin{aligned}
e &= r(s) - y(s) \\
y(s) &= C(s)P(s)e \\
y(s) &= -C(s)P(s)\left[r(s) - y(s)\right] \\
y(s) &= C(s)P(s)r(s) - C(s)P(s)y(s) \\
y(s)\left[1 + C(s)P(s)\right] &= C(s)P(s)r(s) \\
y(s) &= \frac{C(s)P(s)}{1 + C(s)P(s)}r(s) \\
\frac{y(s)}{r(s)} &= \frac{C(s)P(s)}{1 + C(s)P(s)}
\end{aligned}
$$

Find $\frac{y(s)}{d(s)}$, given $r(s) = 0$

$$
\begin{aligned}
e = r(s) - y(s)\Big|_{r(s)=0} &= y(s) \\
y(s) &= C(s)P(s)e + d(s) \\
y(s) &= -C(s)P(s)y(s) + d(s) \\
y(s)\left[1 + C(s)P(s)\right] &= d(s) \\
y(s) &= \frac{1}{1 + C(s)P(s)}d(s) \\
\frac{y(s)}{d(s)} &= \frac{1}{1 + C(s)P(s)}
\end{aligned}
$$

Given $T(s) = \dfrac{y(s)}{r(s)}$ and $S(s) = \dfrac{y(s)}{d(s)}$

3.1.1 False

$$
\begin{aligned}
T(s) &= -S(s) \\
\frac{C(s)P(s)}{1+C(s)P(s)} &= \frac{-1}{1+C(s)P(s)} \\
C(s)P(s) &\neq -1
\end{aligned}
$$

3.1.2 True

$$
\begin{aligned}
T(s) &= 1 - S(s) \\
\frac{C(s)P(s)}{1+C(s)P(s)} &= 1 - \frac{1}{1+C(s)P(s)} \\
\frac{C(s)P(s)}{1+C(s)P(s)} &= \frac{[1+C(s)P(s)] - 1}{1+C(s)P(s)} \\
\frac{C(s)P(s)}{1+C(s)P(s)} &= \frac{C(s)P(s)}{1+C(s)P(s)}
\end{aligned}
$$

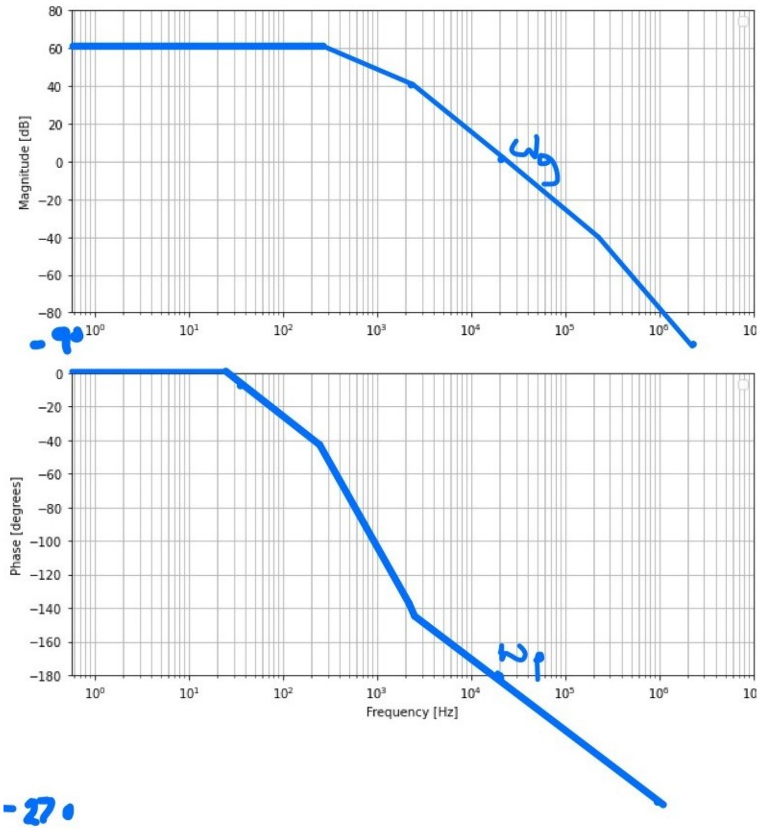
3.1.3 True

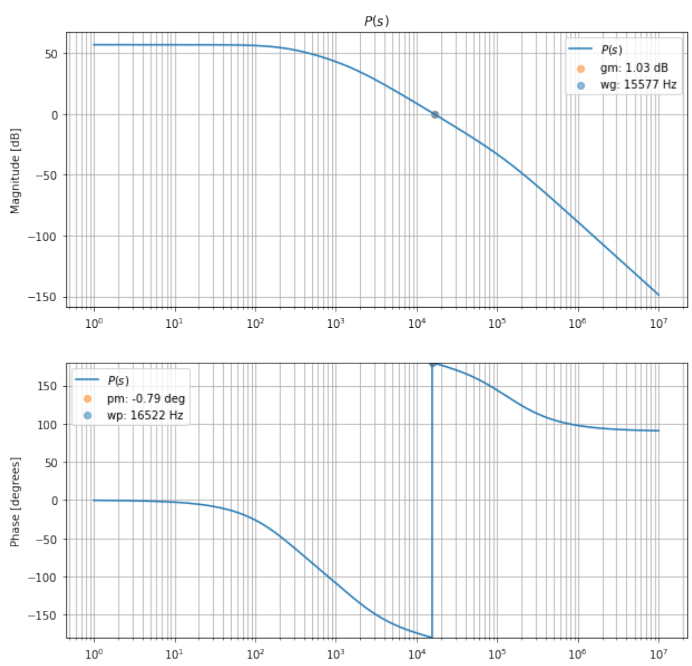
$$
\begin{aligned}
\frac{dT(s)}{ds} + \frac{dS(s)}{ds} &= 0 \\
\frac{d}{ds}\left(1 - S(s)\right) + \frac{d}{ds}S(s) &= 0 \\
-\frac{d}{ds}S(s) + \frac{d}{ds}S(s) &= 0 \\
0 &= 0
\end{aligned}
$$

3.2 Plant Bode Plot

Gain and phase margin are zero on my sketch for $p = 2026$.

System	GM	PM	ω_G	ω_P
$P(s)$	1.03dB	-0.79deg	15,577 Hz	16,522 Hz





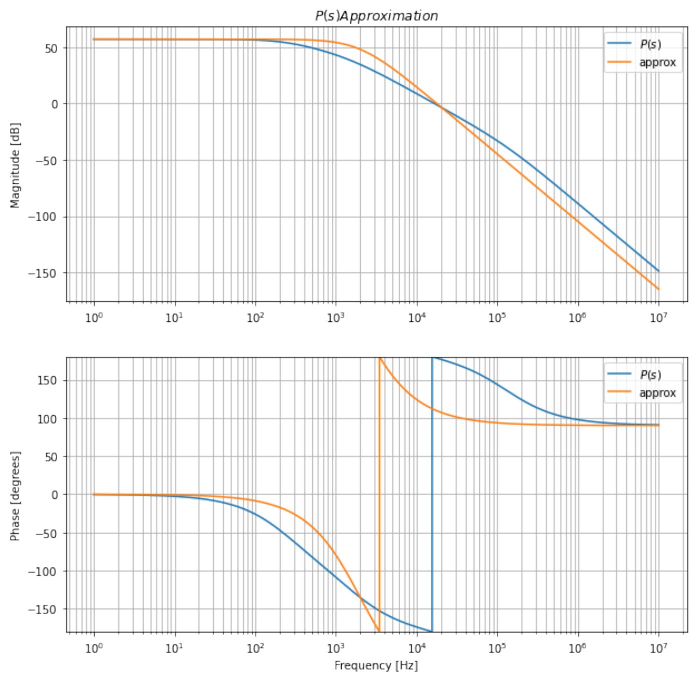
3.3 Design a controller (d=0)

Plant poles

Pole	f_c
$\omega_1 = \frac{3}{4}p$	242 Hz
$\omega_2 = 5p$	1612 Hz
$\omega_3 = \frac{p^2}{5}$	130,656 Hz

Plant is simplified to solve PI controller,

$$P'(s) = \frac{709}{\left(1 + \frac{s}{2\pi\ 2000}\right)^3}$$



Choose $f_c = 2000Hz$, where $\omega_c = 2\pi f_c$.

Solve $|C(s)P(s)|= 1$

$$\ell_i(s) = C(s)P(s) \approx \left(k_p + \frac{k_i}{s}\right)\frac{709}{\left(1 + \frac{s}{2\pi\ 2000}\right)^3}$$

$$|C(s)P(s)|\Big|_{\omega=\omega_c=2\pi 2000} = |\left(k_p + \frac{k}{j\omega_c}\right)\frac{709}{\left(1 + \frac{j\omega_c}{2\pi\ 2000}\right)^3}|= 1$$

$$1 = |\left(\frac{j\omega_c k_p + k_i}{j\omega_c}\right)\frac{709}{(1+j)^3}|$$

$$1 = \frac{\sqrt{(\omega_c k_p)^2 + (k_i)^2}}{\omega_c}\frac{709}{\sqrt{2}^3}$$

$$\frac{\sqrt{2}^3}{709}\omega_c = \sqrt{(\omega_c k_p)^2 + (k_i)^2}$$

$$\left(\frac{\sqrt{2}^3}{709}\right)^2 \omega_c^2 = (\omega_c k_p)^2 + (k_i)^2$$

Solve $\angle C(s)P(s) = -\pi + \Phi_m$

$$\angle C(s)P(s) = \angle C(s) + \angle P(s) = -\pi + \Phi_m$$

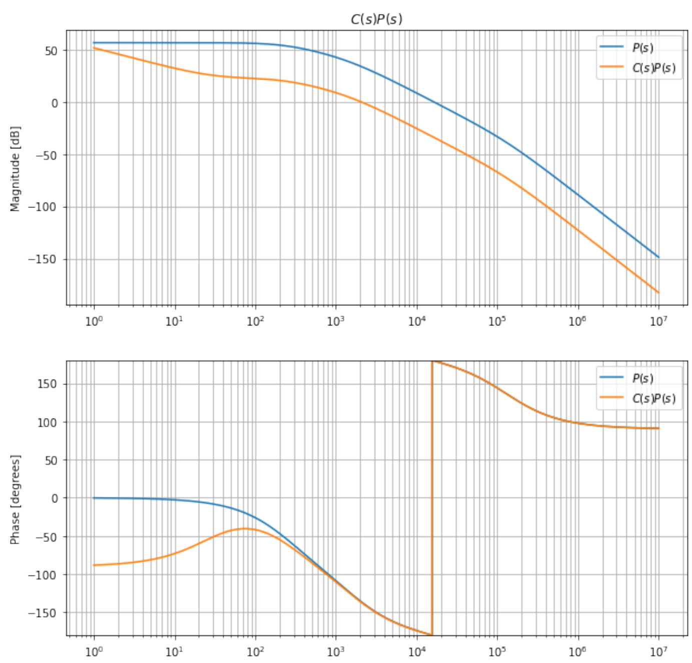
$$-\pi + \Phi_m = \angle C(s) - \frac{3}{2}\pi$$

$$-\pi + \Phi_m = \arctan\frac{\omega_c k_p}{k_i} - \frac{1}{2}\pi - \frac{3}{2}\pi$$

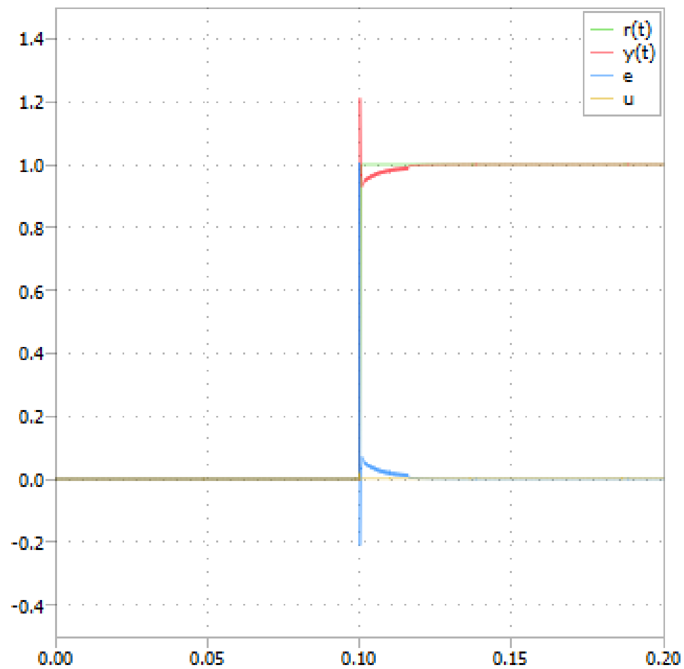
$$\pi + \Phi_m = \arctan\frac{\omega_c k_p}{k_i}$$

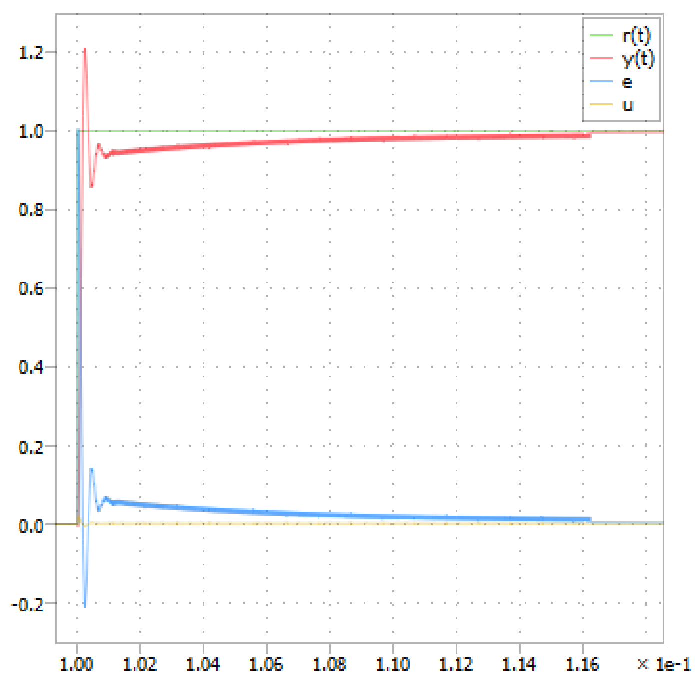
Loop gain $\ell(s) = C(s)P(s)$ where $C(s) = 0.02 + \dfrac{3.54}{s}$

System	GM	PM	ω_G	ω_P
$C(s)P(s)$	43.75dB	42.65deg	2084 Hz	15,457 Hz



3.4 Step Response





3.5 Time Domain Specifications

$$CLTF = K \frac{\omega_0^2}{s^2 + 2\zeta\omega_0^2 + \omega_0^2}$$

Overshoot

$$M_p = e^{-\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$0.3 = e^{-\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$\zeta = 1.13 \pm j0.065$$

Rise Time

$$t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}}{\omega_0\sqrt{1-\zeta^2}}$$

$$t_r \approx \frac{5}{3\omega_0\sqrt{1-\zeta^2}}$$

$$5ms \geq \frac{5}{3\omega_0\sqrt{1-\zeta^2}}$$

$$\omega_0 \geq \frac{5}{3 \cdot 5ms \cdot \sqrt{1-1.13^2}}$$

$$\omega_0 = 633 rad/s$$

Thus bandwidth is chosen to be approximately 100Hz and phase margin is 0.6 since $\zeta > 1$

4 Digital Systems

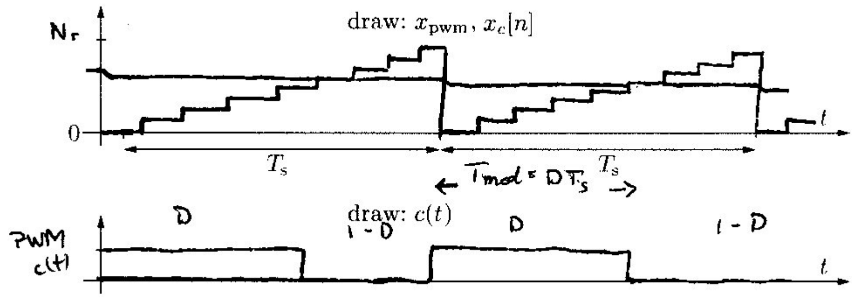


Figure 3: Template for PWM waveforms.

4.1

$$N_r + 1 = \frac{T_s}{T_{clk}} = \frac{f_{clk}}{f_s} = \frac{100MHz}{50kHz}$$

$$N_r = 2000 - 1 = 1999$$

4.2

Physical Voltage $v_{A/D}$

$$max(x_{A/D}[n]) = 2^{n_{A/D}} - 1 = 2^8 - 1 = 255$$

$$\text{Thus } 0 \leq x_{A/D}[n] \leq V_{FS}$$

$$x_{A/D}[n] = \frac{v_{A/D}[n] \cdot V_{FS}}{2^{n_{A/D}}}$$

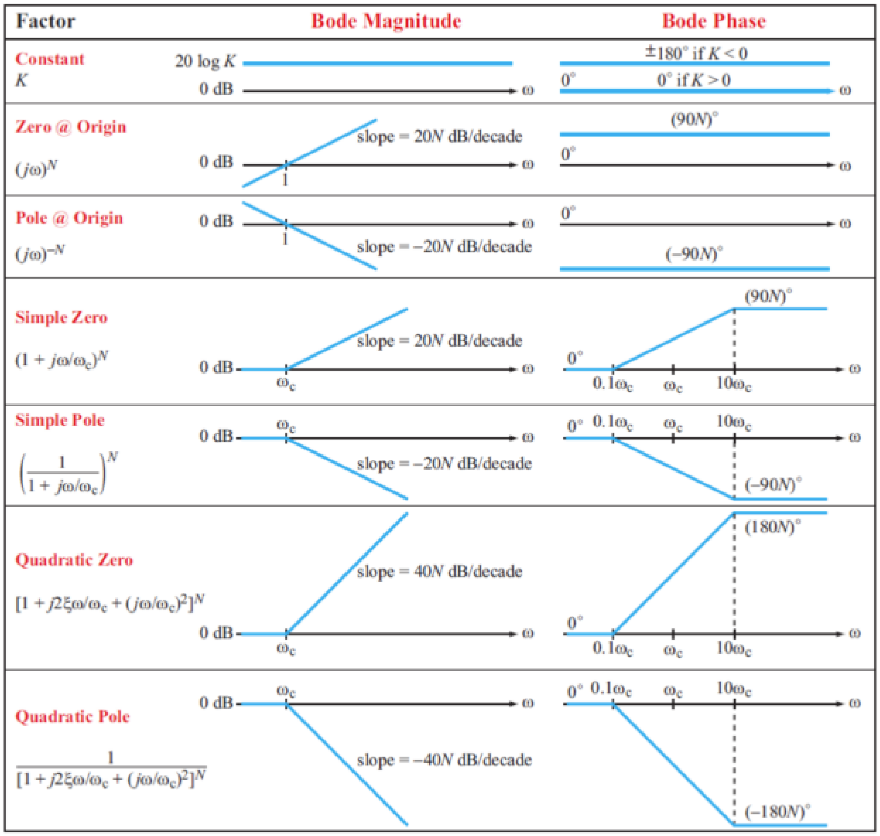
$$v_{A/D}[n] = \frac{x_{A/D}[n] \cdot V_{FS}}{2^{n_{A/D}}}$$

$$v_{A/D}[n] = \frac{107}{2^8} \cdot 1 = 0.418V$$

Voltage Resolution $q_{A/D}$

$$q_{A/D} = \frac{v_{FS}}{2^{n_{A/D}}} = \frac{1}{2^8} = 0.0039 = 3.9mV$$

5 Appendix



Factor
Bode Magnitude
Bode Phase
Constant
K
20 log K
0 dB
ω
±180° if K < 0
0°
0° if K > 0
Zero @ Origin
(jω)^N
slope = 20N dB/decade
(90N)°
Pole @ Origin
(jω)^−N
slope = −20N dB/decade
(−90N)°
Simple Zero
(1 + jω/ω_c)^N
ω_c
0.1ω_c
10ω_c
Simple Pole
(1/(1 + jω/ω_c))^N
Quadratic Zero
[1 + j2ξω/ω_c + (jω/ω_c)^2]^N
slope = 40N dB/decade
(180N)°
Quadratic Pole
1/[1 + j2ξω/ω_c + (jω/ω_c)^2]^N
slope = −40N dB/decade
(−180N)°