Lecture 4 13th January 2022

HW1 has been uploaded (Files section)

Due on Thursday before class.

Revision: To obtain the small signal model.

① Draw the circuit as it appears in mode 1 & 2.

- Mode is based on switch / diode being turned ON/OFF.

② find the $\underbrace{\text{state equations}}$ of circuit in mode 1 & 2.

$$\text{mode I} \begin{cases} L\dfrac{di_L}{dt} = \ldots f_1 \ldots \\ C\dfrac{dv_c}{dt} = \ldots f_2 \ldots \end{cases}$$

$$\text{mode II} \begin{cases} L\dfrac{di_L}{dt} = \ldots f_3 \ldots \\ C\dfrac{dv_c}{dt} = \ldots f_4 \ldots \end{cases}$$

③ Obtain the $\underbrace{\text{switch cycle average}}$ model.

$$\langle x(t)\rangle_{T_s} = \int_t^{t+T_s} x(\tau)\,d\tau$$

$$L\frac{d}{dt}\langle i_L(t)\rangle_{T_s} = f_1 \cdot d(t) + f_3\, d'(t)$$

f_1, f_2 are functions of $i_L, v_c, V_{in}, D, t \ldots$

$$C\frac{d}{dt}\langle v_c(t)\rangle_{T_s} = f_2 d(t) + f_4 d'(t)$$

④ Write every input (duty ratio, input voltage),

state variable (capacitor voltage, inductor current)

as = sum of a large dc value + small ac peoturbation

eg. $d(t) = \underbrace{D}_{\text{not a fn of time}} + \tilde{d}(t) \rightarrow$ fun of time.

Substitute into eq. of step ③.

ⓐ Equate large signal on both sides &

→ notice $\frac{d}{dt}X = 0$ [EE 452 → Steady state solution]

ⓑ Equate small signal on both sides $\left(\frac{d\tilde{x}}{dt} = 0\right)$

[EE 458 - transient state solution]

$$L\frac{d\tilde{i}_L(t)}{dt} = f\left(t, \tilde{i}_L(t), \tilde{v}_c(t), \tilde{d}(t), \tilde{v}_{in}(t), I_L, V_c, D, V_{in}\right)$$

$$C\frac{d\tilde{v}_c(t)}{dt} = f\left(t, \tilde{i}_L(t), \tilde{v}_c(t), \tilde{d}(t), \tilde{v}_{in}(t), I_L, V_c, D, V_{in}\right)$$

⑤ We need to solve the eq. of Step ④.

One way to do is Laplace transform.

$$sL\tilde{i}_L(s) = F\left(s, \tilde{i}_L(s), \tilde{v}_c(s), \tilde{d}(s), \tilde{v}_{in}(s), I_L, V_c, D, V_{in}\right)$$

$$sC\tilde{v}_c(s) = F\left(s, \tilde{i}_L(s), \tilde{v}_c(s), \tilde{d}(s), \tilde{v}_{in}(s), I_L, V_c, D, V_{in}\right)$$

⑥ Obtain transfer functions.

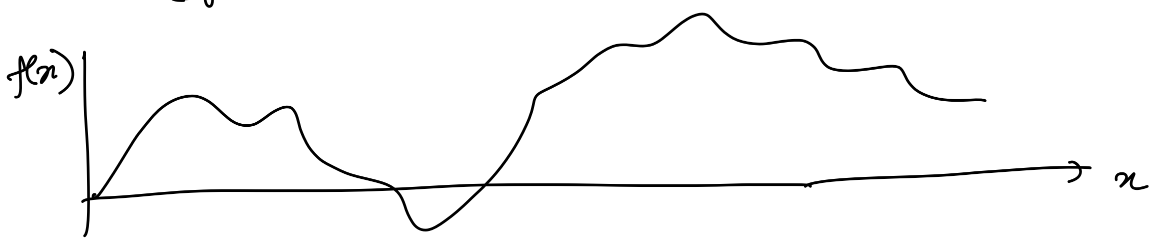
$\dfrac{\tilde{i}_L(s)}{\tilde{d}(s)} \longrightarrow$ set $\tilde{v}_{in}=0$, substitute away $\tilde{v}_c(s)$

$\left[\dfrac{\tilde{v}_c(s)}{\tilde{d}(s)}\right. \longrightarrow$ " , " $\tilde{i}_L(s)$

$\left[\dfrac{\tilde{i}_L(s)}{\tilde{v}_{in}(s)}\right. \longrightarrow$ set $\tilde{d}(s)=0$, ...

$\left[\dfrac{\tilde{v}_c(s)}{\tilde{v}_{in}(s)}\right. \longrightarrow$ " , . . .

Taylor Series



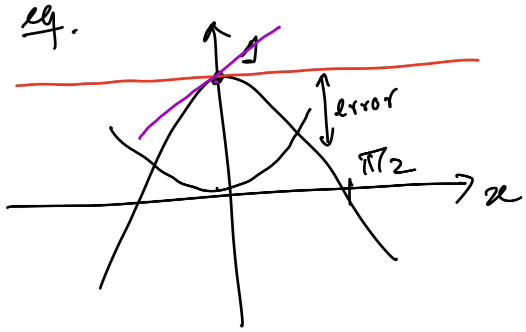
① Taylor series (Mc Laurin Series)

$$f(x) = \underline{c_0 + c_1 x} + c_2 x^2 + c_3 x^3 + \ldots$$

② Fourier series

$$f(x) = c_0 + \sum_{n=1}^{\infty} \left(a_n \sin(nx) + b_n \cos(nx) \right)$$

eg.



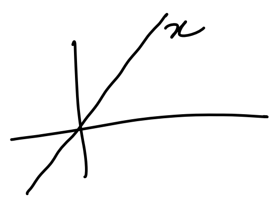
Q. How do you approximate $\cos(x)$ near $x=0$

$$f(x) = \boxed{}$$

$$\approx \underbrace{\cos x}$$

$-\sin x$

$$f(x) = 1 = \cos(0)$$



$$f(x) = \underline{c_0 + c_1 x + c_2 x^2}$$

$x = 0 \Rightarrow \quad f(0) = c_0 = 1.$

$$\left.\frac{\partial f}{\partial x}\right|_{x=0} = c_1 + 2c_2 x = 0 \qquad \therefore c_1 = 0$$

$$c_1 + 2c_2 . 0 = 0$$

$$f(x) = 1 + 0 . x$$

$$\frac{\partial^2 f}{\partial x^2} = 2c_2 = -\cos x \Big|_{x=0} \qquad \underline{-\cos x}$$

$$c_2 = -\frac{1}{2}$$

$$f(x) = 1 + 0 - \frac{1}{2}x^2 = 1 - \frac{x^2}{2}$$

$\underline{Q}$ What if I asked to find an approximation of $\cos x$ at $x = \pi/4$

$$f(x) = c_0 + c_1\left(x - \frac{\pi}{4}\right) + c_2\left(x - \frac{\pi}{4}\right)^2 \qquad (\ )^3$$

Equate the $\underline{fn}$ at $x = \pi/4$

$$3 \cdot 2 \ldots$$

$$\therefore f\left(\frac{\pi}{4}\right) = c_0 = \cos\left(\frac{\pi}{4}\right)$$

$$\frac{\partial f}{\partial x} = c_1 + 2c_2\left(x - \frac{\pi}{4}\right)$$

$$\frac{\partial f}{\partial x}\Big|_{x=\frac{\pi}{4}} = c_1$$

$$f(x) = f\left(\frac{\pi}{4}\right) + \frac{\partial f}{\partial x}\Big|_{x=\pi/4}\left(x - \frac{\pi}{4}\right) + \underbrace{\frac{1}{\lfloor 2}}_{=2\cdot 1}\frac{\partial^2 f}{\partial x^2}\left(x - \frac{\pi}{4}\right)^2$$

$$+ \frac{1}{\lfloor 3}\frac{\partial^3 f}{\partial x^3}\left(x - \frac{\pi}{4}\right)^3 \ldots\ldots$$

$$\langle v_L(t)\rangle_{T_s} = L\frac{d\langle i_L(t)\rangle_{T_s}}{dt} = \underline{\langle v_{in}(t)\rangle_{T_s} d(t) - \langle v_c(t)\rangle_{T_s} d'(t)}$$

$$\langle i_c(t)\rangle_{T_s} = C\frac{d}{dt}\langle v_c(t)\rangle_{T_s} =$$

$$\frac{-\langle v_c(t)\rangle_{T_s}}{R}\cdot d(t) + \left(\frac{-\langle v_c(t)\rangle_{T_s}}{R} + \langle i_L(t)\rangle_{T_s}\right)d'(t)$$

$$L\frac{d}{dt}\langle i_L(t)\rangle_{T_s} = f_1$$

$$C\frac{d}{dt}\langle v_c(t)\rangle_{T_s} = f_2$$

Let us call nominal operating point $= \overline{X}, \overline{u}$

$$\overline{X} = \text{states} = \begin{bmatrix} I_L \\ V_0 \end{bmatrix} = \begin{bmatrix} \frac{V_0}{D'R} \\ V_{in}\frac{D}{D'} \end{bmatrix} \qquad V_0 = V_C$$

$$\overline{u} = \text{input} = \begin{bmatrix} D \\ V_{in} \end{bmatrix}$$

$$L\frac{d\langle i_L(t)\rangle_{T_s}}{dt} = \underline{f_1(x,u)} = \langle V_{in}(t)\rangle_{T_s} d(t) - \langle v_c(t)\rangle_{T_s} d'(t) \checkmark$$

$$C\frac{d\langle v_c(t)\rangle_{T_s}}{dt} = \underline{f_2(x,u)} = \langle -\frac{v_c(t)\rangle_{T_s}}{R} + \langle i_L(t)\rangle_{T_s} d'(t)$$

$$\frac{\partial f}{\partial x} \qquad x = \begin{bmatrix} \langle i_L(t)\rangle_{T_s} \\ \langle v_c(t)\rangle_{T_s} \end{bmatrix}$$

$$f = \begin{bmatrix} f_1(x,u) \\ f_2(x,u) \end{bmatrix}$$

Derivative of a scalar w.r.t. vector

$$\underset{2\times 1 \text{ vector}}{\frac{\partial f}{\partial \underset{\text{vector}}{x}}} = \begin{bmatrix} \\ \\ \end{bmatrix} \begin{bmatrix} \langle i_L(t)\rangle_{T_S} - I_L \\ \langle v_C(t)\rangle_{T_S} - V_C \end{bmatrix} \quad V_C \equiv V_0$$

$$\frac{\partial \overset{[f_1\ f_2]}{f}}{\partial x} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial \tilde{i}_L} & \frac{\partial f_1}{\partial \tilde{v}_C} \\ \frac{\partial f_2}{\partial \tilde{i}_L} & \frac{\partial f_2}{\partial \tilde{v}_C} \end{bmatrix}\right|_{\substack{x=\bar{x},\\ u=\bar{u}}} \begin{bmatrix} \tilde{i}_L(t) \\ \tilde{v}_C(t) \end{bmatrix} +$$

$$\begin{bmatrix} \frac{\partial f_1}{\partial \tilde{d}} & \frac{\partial f_1}{\partial \tilde{v}_{in}} \\ \frac{\partial f_2}{\partial \tilde{d}} & \frac{\partial f_2}{\partial \tilde{v}_{in}} \end{bmatrix} \begin{bmatrix} \tilde{d}(t) \\ \tilde{v}_{in} \end{bmatrix}$$

$$f(x, u), \quad x \rightarrow \begin{bmatrix} i_L \\ v_C \end{bmatrix}, \quad u \rightarrow \begin{bmatrix} d \\ v_{in} \end{bmatrix}$$

$$\approx \left.\frac{\partial f}{\partial x}\right|_{\substack{x=\bar{x}\\ u=\bar{u}}} (x-\bar{x}) + \left.\frac{\partial f}{\partial u}\right|_{\substack{x=\bar{x},\\ u=\bar{u}}} (u-\bar{u}) +$$

$$\underbrace{\cancel{\left.\frac{\partial^2 f}{\partial x^2}\right|_{\substack{x=\bar{x}\\ u=\bar{u}}} (x-\bar{x})^2 + \left.\frac{\partial^2 f}{\partial u^2}\right|_{\substack{x=\bar{x}\\ u=\bar{u}}} (u-\bar{u})^2}}_{\text{ignore these.}} + \cdots$$

$$f(x) = \underbrace{c_0}_{?} + c_1 x + \underbrace{c_2 x^2}$$

$(x-\bar{x})$ ✓ $\quad (u-\bar{u})$

$$\begin{bmatrix} L\frac{d\langle i_L(t)\rangle_{T_s}}{dt} \\ C\frac{d\langle v_C(t)\rangle_{T_s}}{dt} \end{bmatrix} = \begin{bmatrix} f_1(x,u) \\ f_2(x,u) \end{bmatrix} = \cancel{\begin{bmatrix} f_1(\bar{x},\bar{u}) \\ f_2(\bar{x},\bar{u}) \end{bmatrix}} + \frac{\partial f}{\partial x}\cdot\tilde{x} + \frac{\partial f}{\partial u}\cdot\tilde{u}$$

0

why?

$v_L = L\frac{di_L}{dt} = 0 = f_1(\bar{x},\bar{u})$ (steady state soln)

$i_c = \frac{C dv_c}{dt} = 0 \; f_2(\bar{x},\bar{u})$ (")

$$\begin{bmatrix} \tilde{i}_L(t) \\ \tilde{v}_C(t) \end{bmatrix} = \begin{bmatrix} \langle i_L(t)\rangle_{T_s} - I_L \\ \langle v_c(t)\rangle_{T_s} - V_C \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \langle i_L(t)\rangle_{T_s} \\ \langle v_C(t)\rangle_{T_s} \end{bmatrix} = \begin{bmatrix} \tilde{i}_L(t) + I_L \\ \tilde{v}_C(t) + V_C \end{bmatrix}$$

$$\therefore\; L\frac{d}{dt}\langle i_L(t)\rangle_{T_s} = L\frac{d}{dt}\left(I_L + \tilde{i}_L(t)\right) = L\frac{d\tilde{i}_L(t)}{dt}$$

$$\because\; \frac{dI_L}{dt} = 0$$

$$C\frac{d}{dt}\langle v_C(t)\rangle_{T_s} = C\frac{d\tilde{v}_C(t)}{dt}.$$

$$\begin{bmatrix} L\dfrac{d\tilde{i}_L(t)}{dt} \\ C\dfrac{d\tilde{v}_c(t)}{dt} \end{bmatrix}_{2\times1} = \begin{bmatrix} f_1(\tilde{i}_L, \tilde{v}_c, \tilde{d}, \tilde{v}_{in}, I_L, V_c, D, V_{in}) \\ f_2(\;\cdots\cdots\;) \end{bmatrix}_{2\times1} = \underbrace{\begin{bmatrix} \dfrac{\partial f_1}{\partial \tilde{i}_L} & \dfrac{\partial f_1}{\partial \tilde{v}_c} \\ \dfrac{\partial f_2}{\partial \tilde{i}_L} & \dfrac{\partial f_2}{\partial \tilde{v}_c} \end{bmatrix}}_{\text{jacobians}\;2\times2} \underbrace{\begin{bmatrix} \tilde{i}_L \\ \tilde{v}_c \end{bmatrix}_{2\times1}}_{2\times1} + \underbrace{\begin{bmatrix} \dfrac{\partial f_1}{\partial \tilde{d}} & \dfrac{\partial f_1}{\partial \tilde{v}_{in}} \\ \dfrac{\partial f_2}{\partial \tilde{d}} & \dfrac{\partial f_2}{\partial \tilde{v}_{in}} \end{bmatrix}}_{2\times1} \begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}$$

$$\frac{d\langle i(t)\rangle_{T_s}}{dt} = \underline{f_1(x, u)} = \frac{1}{L}\left[\langle v_{in}(t)\rangle_{T_s}\, d(t) - \langle v_c(t)\rangle_{T_s}\, d'(t)\right] ✓$$

$$\frac{d\langle v_c(t)\rangle_{T_s}}{dt} = \underline{f_2(x, u)} = \frac{1}{C}\left[\langle -\frac{v_c(t)\rangle_{T_s}}{R} + \langle i_L(t)\rangle_{T_s}\, d'(t)\right]$$

$$\begin{bmatrix} \dfrac{d\tilde{i}_L(t)}{dt} \\ \dfrac{d\tilde{v}_c(t)}{dt} \end{bmatrix} = \begin{bmatrix} f_1(\;) \\ f_2(\;) \end{bmatrix} = \begin{bmatrix} 0 & -d'(t)/L \\ \dfrac{d'(t)}{C} & -\dfrac{1}{RC} \end{bmatrix}\Bigg|_{\substack{x=\bar{x} \\ u=\bar{u}}} \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_c \end{bmatrix} + \begin{bmatrix} \dfrac{\langle V_{in}(t)\rangle_{T_s} + \langle v_c(t)\rangle_{T_s}}{L} & \dfrac{d(t)}{L} \\ -\dfrac{\langle i_L(t)\rangle_{T_s}}{C} & 0 \end{bmatrix}\Bigg|_{\substack{x=\bar{x}, \\ u=\bar{u}}} \begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}$$

$$\frac{\partial \langle v_c(t)\rangle_{T_s}\, d'(t)}{\partial \tilde{v}_c} = \frac{\partial}{\partial v_c}\left(V_c + \tilde{v}_c(t)\right) d'(t)$$

$$\frac{1}{L}\left[\langle V_{in}(t)\rangle_{T_s}\, d(t) - \langle v_c(t)\rangle_{T_s}\,(1-d(t))\right]$$

$$d(t) = D + \tilde{d}$$

$$\frac{\partial}{\partial \tilde{d}}\,\frac{1}{L}\left[\langle V_{in}(t)\rangle_{T_s}(D+\tilde{d}) - \langle v_c(t)\rangle_{T_s}(1-D-\tilde{d})\right]$$

$$= \frac{\langle V_{in}(t)\rangle_{T_s}}{L} + \frac{\langle V_c(t)\rangle_{T_s}}{L}$$

$$\bar{x} = \begin{bmatrix} I_L \\ V_c \end{bmatrix} = \begin{bmatrix} \dfrac{V_o}{D'R} \\ V_{in}\dfrac{D}{D'} \end{bmatrix}$$

$$\bar{u} = \begin{bmatrix} D \\ V_{in} \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{d\tilde{i}_L(t)}{dt} \\ \dfrac{d\tilde{v}_c(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -D'/L \\ \dfrac{D'}{C} & -\dfrac{1}{RC} \end{bmatrix} \begin{bmatrix} \tilde{i}_L \\ \tilde{v}_c \end{bmatrix} + \begin{bmatrix} \dfrac{V_{in}+V_o}{L} & \dfrac{D}{L} \\ -\dfrac{I_L}{C} & 0 \end{bmatrix} \begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}$$

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$$

<u>Laplace</u>

$$s\tilde{x}(s) - \cancel{\tilde{x}(0)} = A\tilde{x}(s) + B\tilde{u}(s)$$

0

$$s\tilde{x}(s) = sI\tilde{x}(s)$$

$$s = sI\tilde{X}(s)$$

$$= sI\begin{bmatrix}\tilde{i}_L \\ \tilde{v}_c\end{bmatrix}$$

$$\begin{bmatrix}s\tilde{i}_L \\ s\tilde{v}_c\end{bmatrix} = s\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}\begin{bmatrix}\tilde{i}_L \\ \tilde{v}_c\end{bmatrix}$$

$$sI\tilde{x}(s) - A\tilde{x}(s) = B\tilde{u}(s)$$

$$(sI - A)\tilde{x}(s) = B\tilde{u}(s)$$

$$\underbrace{\tilde{x}(s)}_{\text{vector}} = \underbrace{(sI-A)^{-1}B}_{\text{transfer matrix}}\underbrace{\tilde{u}(s)}_{\text{vector}}$$

$$\begin{bmatrix}\tilde{i}_L(s) \\ \tilde{v}_c(s)\end{bmatrix} = \left\{\begin{bmatrix}s & 0 \\ 0 & s\end{bmatrix} - \begin{bmatrix}0 & -D'/L \\ D'/C & -\frac{1}{RC}\end{bmatrix}\right\}^{-1}\begin{bmatrix}\frac{V_{in}+V_o}{L} & \frac{D}{L} \\ -\frac{I_L}{C} & 0\end{bmatrix}\begin{bmatrix}\tilde{d}(s) \\ \tilde{v}_{in}(s)\end{bmatrix}$$

$$= \begin{bmatrix}s & +D'/L \\ -\frac{D'}{C} & s+\frac{1}{RC}\end{bmatrix}^{-1}\begin{bmatrix}\frac{V_{in}+V_o}{L} & \frac{D}{L} \\ -\frac{I_L}{C} & 0\end{bmatrix}\begin{bmatrix}\tilde{d}(s) \\ \tilde{v}_{in}(s)\end{bmatrix}$$

$$\begin{bmatrix}a & b \\ c & d\end{bmatrix}^{-1} = \frac{1}{(ad-bc)}\begin{bmatrix}d & -c \\ -b & a\end{bmatrix}^T$$

$$= \frac{1}{ab-bc}\begin{bmatrix}d & -b \\ -c & a\end{bmatrix}$$

$$= \frac{1}{\left\{s\left(s+\frac{1}{RC}\right)+\frac{D'^2}{LC}\right\}}\begin{bmatrix}s+\frac{1}{RC} & -D'/L \\ D'/C & s\end{bmatrix}\begin{bmatrix}\frac{V_{in}+V_o}{L} & \frac{D}{L} \\ -\frac{I_L}{C} & 0\end{bmatrix}\begin{bmatrix}\tilde{d}(s) \\ \tilde{v}_{in}(s)\end{bmatrix}$$

$$\begin{bmatrix} \tilde{i}_L(s) \\ \tilde{v}_c(s) \end{bmatrix} = \frac{1}{\left(s^2 + \frac{s}{RC} + \frac{D'^2}{LC}\right)} \begin{bmatrix} \left(s+\frac{1}{RC}\right)\left(\frac{V_{in}+V_o}{L}\right) + \frac{D' I_L}{LC} & \left(s+\frac{1}{RC}\right)\frac{D}{L} \\ \frac{D'}{C}\left(\frac{V_{in}+V_o}{L}\right) - \frac{s I_L}{C} & \frac{D'D}{LC} \end{bmatrix} \begin{bmatrix} \tilde{d}(s) \\ \tilde{v}_{in}(s) \end{bmatrix}$$

$$\tilde{i}_L(s) = \frac{\left(s+\frac{1}{RC}\right)\left(\frac{V_{in}+V_o}{L}\right) + \frac{D' I_L}{LC}}{s^2 + \frac{s}{RC} + \frac{D'^2}{LC}}$$

$$\frac{\tilde{v}_c(s)}{\tilde{v}_{in}(s)} = \frac{D'D/LC}{s^2 + \frac{s}{RC} + \frac{D'^2}{LC}}$$

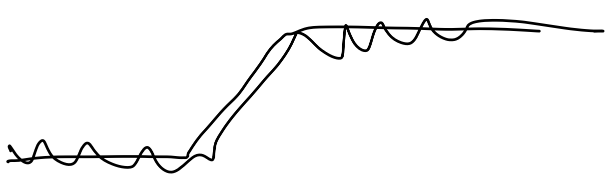
$$\frac{\tilde{i}_L(s)}{\tilde{d}(s)} = \frac{\left(sC+\frac{1}{R}\right)\left(V_{in}+V_o\right) + D' I_L}{LCs^2 + \frac{sL}{R} + D'^2}$$

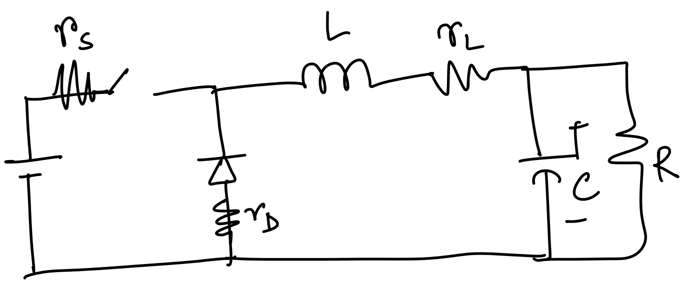
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}$$

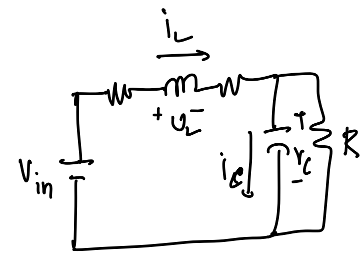
$\bar{x} = a\bar{p} + bq$ $\quad q = 0 \;\therefore\; \frac{x}{p} = a$

$y = cp + dq$

$$\frac{\tilde{i}_L(s)}{\tilde{d}(s)} = \frac{\left(V_{in}+V_c\right)\left(sC+\frac{1}{R}\right) + D' I_L}{s^2 LC + \frac{sL}{R} + D'^2}$$







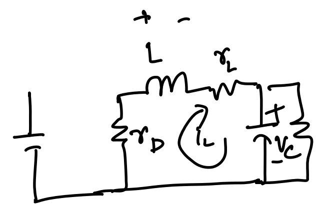
$v_C = v_o$

mode I

Mode I

$$L \frac{di_L}{dt} = V_{in} - V_C - r_s i_L - i_L \cdot r_L$$

$$C \frac{dv_C}{dt} = i_L - \frac{v_C}{R}$$



Mode II

Mode II

$$L \frac{di_L}{dt} = -i_L r_L - i_L r_D - V_C$$

$$C \frac{dv_C}{dt} = i_L - \frac{v_C}{R}$$

$$L \frac{d\langle i_L \rangle_{T_s}}{dt} = d(t) \left[V_{in} - V_C - r_s i_L - i_L \cdot r_L \right] + d'(t) \left[-i_L r_L - i_L r_D - V_C \right]$$

$$C \frac{d\langle v_C \rangle_{T_s}}{dt} = d(t) \left[i_L - \frac{v_C}{R} \right] + d'(t) \left[i_L - \frac{V_C}{R} \right]$$

$$= i_L - \frac{v_C}{R}.$$

$$\bar{u} = ? = \begin{bmatrix} D \\ V_{in} \end{bmatrix}$$

$$\bar{X} = ? = \begin{bmatrix} I_L \\ V_C \end{bmatrix}$$

$$D \left[V_{in} - V_C - r_s I_L - I_L r_L \right] + D' \left[-I_L r_L - I_L r_D - V_C \right] = 0 \quad \text{— (1)}$$

$$I_L = \frac{V_C}{R} \quad \text{— (2)}$$

$I_L =$ ______

$V_c =$ ______

$$\dot{\tilde{x}}(t) = A\Big|_{\substack{x=\bar{x}\\u=\bar{u}}} \tilde{x}(t) + B\Big|_{\substack{x=\bar{x}\\u=\bar{u}}} \tilde{u}(t)$$

$$\left.\begin{aligned}\frac{d\langle i_L\rangle_{T_s}}{dt} = \frac{1}{L}\Big\{ & d(t)\left[V_{in} - V_c - r_s i_L - i_L \cdot r_L \right] + \\ & d'(t)\left[-i_L r_L - i_L r_D - V_c \right] \Big\}\end{aligned}\right\} = f_1.$$

$$\frac{d\langle v_c\rangle_{T_s}}{dt} = \frac{1}{C}\left\{ i_L - \frac{v_c}{R} \right\} \Big\} f_2$$

$$\begin{bmatrix} \frac{d\tilde{i}_L}{dt} \\ \frac{d\tilde{v}_c}{dt} \end{bmatrix} = \left.\begin{bmatrix} \frac{\partial f_1}{\partial \tilde{i}_L}\ \frac{-d(t) r_s - r_L - d'(t) r_D}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}\begin{bmatrix} \tilde{i} \\ \tilde{v}_c \end{bmatrix}\right|_{x=\bar{x},\, u=\bar{u}} +$$

$$\left.\begin{bmatrix} \frac{1}{L}\left(V_{in} - V_c - r_s I_L - I_L r_L\right) \\ \quad \frac{+ i_L r_L + i_L r_D + V_c}{L} & \frac{d(t)}{L} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}\right|_{x=\bar{x},\, u=\bar{u}}$$

$$\dot{\tilde{x}} = \underbrace{\begin{bmatrix} \frac{-D(r_s + r_D) - r_L}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}}_{A} \tilde{x} + \underbrace{\begin{bmatrix} \frac{1}{L}\left(V_{in} + I_L(-r_s + r_D)\right) & \frac{D}{L} \\ 0 & 0 \end{bmatrix}}_{B} \tilde{u}$$

$$\tilde{x}(s) = (sI - A)^{-1} B\, u(s) \Rightarrow \begin{bmatrix} \tilde{i}_L(s) \\ \tilde{v}_c(s) \end{bmatrix} = (sI - A)^{-1} B \begin{bmatrix} \tilde{d}(s) \\ \hat{v}_{in}(s) \end{bmatrix}$$

$$\begin{bmatrix} \tilde{i}_L(s) \\ \tilde{v}_C(s) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} s + \frac{1}{RC} & \frac{1}{L} \\ -\frac{1}{C} & s + \frac{D(r_S + r_D) + r_L}{L} \end{bmatrix} \begin{bmatrix} \frac{1}{L}\left(V_{in} + I_L(-r_S + r_D)\right) & \frac{D}{L} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \end{bmatrix}$$

$$\Delta = \left(s + \frac{1}{RC}\right)\left(s + \frac{D(r_S + r_D) + r_L}{L}\right) + \frac{1}{LC}$$

$$\frac{\tilde{i}_L(s)}{\tilde{d}(s)} = \frac{\left(s + \frac{1}{RC}\right)\frac{1}{L}\left(V_{in} + I_L(-r_S + r_D)\right)}{\Delta}$$

$s^2 + \ldots$

$$\frac{\tilde{v}_c(s)}{\tilde{v}_{in}(s)} =$$

$$\frac{\tilde{i}_L(s)}{\tilde{v}_{in}(s)} =$$

$$\boxed{\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}}$$

Sample code for.

① symbolic inverse.

② solving ODE