Rotation to a General Reference Frame

$$\boxed{\overline{f}^{\,s}_{qds} = e^{+j\theta}\,\overline{f}^{\,g}_{qds}} \quad \rightarrow \quad \boxed{\overline{f}^{\,g}_{qds} = e^{-j\theta}\,\overline{f}^{\,s}_{qds}}$$

General Reference Frame Referred, Complex Variable Equations:

$$\boxed{\begin{aligned} \overline{v}^{\,g}_{qds} &= r_s\,\overline{i}^{\,g}_{qds} + L_s(p + j\omega)\,\overline{i}^{\,g}_{qds} + L_m(p + j\omega)\,\overline{i}^{\,g}_{qdr} \\ \overline{v}^{\,g}_{qdr} &= r_r\,\overline{i}^{\,g}_{qdr} + L_r\left[p + j(\omega - \omega_r)\right]\overline{i}^{\,g}_{qdr} + L_m\left[p + j(\omega - \omega_r)\right]\overline{i}^{\,g}_{qds} \end{aligned}}$$

if $\omega = 0$ & $\theta = 0$ $\rightarrow$ Stator Reference Frame

if $\omega = \omega_r$ & $\theta = \theta_r$ $\rightarrow$ Rotor Reference Frame

if $\omega = \omega_e$ & $\theta = \theta_{e_0}$ $\rightarrow$ Synchronous (Excitation) Reference Frame

Clarke Transformation	Park Transformation
Forward Transformation:	*Stationary to Rotationg*
$\begin{bmatrix} v^s_{qs} \\ v^s_{ds} \\ v_{0s} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{-\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix}$	$\overline{f}^{\,g}_{qdx} = e^{-j\theta}\,\overline{f}^{\,s}_{qdx}$ $\begin{bmatrix} f^g_{qx} \\ f^g_{dx} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} f^s_{qx} \\ f^s_{dx} \end{bmatrix}$
Inverse Transformation:	*Rotating to Stationary*
$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ \frac{-1}{2} & \frac{-\sqrt{3}}{2} & 1 \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} & 1 \end{bmatrix}\begin{bmatrix} v^s_{qs} \\ v^s_{ds} \\ v_{0s} \end{bmatrix}$	$\overline{f}^{\,s}_{qdx} = e^{j\theta}\,\overline{f}^{\,g}_{qdx}$ $\begin{bmatrix} f^s_{qx} \\ f^s_{dx} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} f^g_{qx} \\ f^g_{dx} \end{bmatrix}$

<u>Power Flow in 3ϕ Machines:</u>

$$P_{s_{abc}} = v_{as}\,i_{as} + v_{bs}\,i_{bs} + v_{cs}\,i_{cs}$$

$$P_{r_{abc}} = v_{ar}\,i_{ar} + v_{br}\,i_{br} + v_{cr}\,i_{cr}$$

$$P_{s_{qd}} = v_{qs}^{s}\,i_{qs}^{s} + v_{ds}^{s}\,i_{ds}^{s} = \mathcal{Re}\left[\overline{v}_{qds}^{\,s}\,\overline{i}_{qds}^{\,s*}\right]$$

$$= \mathcal{Re}\left[\left(v_{qs}^{s} - j\,v_{ds}^{s}\right)\left(i_{qs}^{s} + j\,i_{ds}^{s}\right)\right] = \mathcal{Re}\left[\frac{2}{3}(v_{as} + a\,v_{bs} + a^2 v_{cs})\ \mathrm{x}\ \frac{2}{3}(i_{as} + a^2 i_{bs} + a\,i_{cs})\right]$$

$$= \frac{4}{9}(v_{as}\,i_{as} + v_{bs}\,i_{bs} + v_{cs}\,i_{cs}) + \frac{4}{9}\mathcal{Re}\left[v_{as}\left(a^2 i_{bs} + a\,i_{cs}\right) + a\,v_{bs}\left(i_{as} + a\,i_{cs}\right) + a^2 v_{cs}\left(i_{as} + a^2 i_{bs}\right)\right]$$

$$P_{s_{qd}} = \frac{6}{9}[v_{as}\,i_{as} + v_{bs}\,i_{bs} + v_{cs}\,i_{cs}] - \frac{2}{9}(3v_{0s})(3i_{0s})$$

$$\boxed{P_{s_{qd}} = \frac{2}{3}P_{s_{abc}} - 2\,v_{0s}\,i_{0s}}$$

$$\boxed{P_{s_{abc}} = \frac{3}{2}P_{s_{qd}} + 3\,v_{0s}\,i_{0s}}$$

Note: $P_{s_{qd}}$ is not equal to $P_{s_{abc}}$ because of choice of $\frac{2}{3}$ as a scaling factor during transformation. For a scaling of $\sqrt{\frac{2}{3}}$, this power in abc = qd (called the “power invariant transformation”).

$$P_{s_{abc}} = \frac{3}{2}\mathcal{Re}\left[\bar{v}^{s}_{qds}\,\bar{i}^{s*}_{qds}\right] + 3\,v_{0s}\,i_{0s} \qquad \text{(can ususually neglect zero sequence terms)}$$

$$= \frac{3}{2}\mathcal{Re}\left[\left(e^{j\theta}\bar{v}^{g}_{qds}\right)\left(e^{j\theta}\,\bar{i}^{g}_{qds}\right)^{*}\right] + 3\,v_{0s}\,i_{0s} = \frac{3}{2}\,\mathcal{Re}\left[\bar{v}^{g}_{qds}\,\bar{i}^{g*}_{qds}\right] + 3\,v_{0s}\,i_{0s}$$

$$P_{in_{total}} = \frac{3}{2}\mathcal{Re}\left[\bar{v}^{g}_{qds}\,\bar{i}^{g*}_{qds} + \bar{v}^{g}_{qdr}\,\bar{i}^{g*}_{qdr}\right] + 3\,v_{0s}\,i_{0s} + 3\,v_{0r}\,i_{0r}$$

Assuming no zero sequence components:

$$P_{in_{total}} = \frac{3}{2}\mathcal{Re}\Big\{\left[r_s\,\bar{i}^{g}_{qds} + L_s(p + j\,\omega)\,\bar{i}^{g}_{qds} + L_m(p + j\,\omega)\,\bar{i}^{g}_{qdr}\right]\bar{i}^{g*}_{qds} +$$

$$\left[r_r\,\bar{i}^{g}_{qdr} + L_r\left(p + j(\omega - \omega_r)\right)\bar{i}^{g}_{qdr} + L_m\left(p + j(\omega - \omega_r)\right)\bar{i}^{g}_{qds}\right]\bar{i}^{g*}_{qdr}\Big\}$$

$$= \frac{3}{2}\mathcal{Re}\Big\{r_s\left|\bar{i}^{g}_{qds}\right|^2 + L_s\,p\left|\bar{i}^{g}_{qds}\right|^2 + j\,\omega\,L_s\left|\bar{i}^{g}_{qds}\right|^2 + L_m\,p\;\bar{i}^{g}_{qdr}\,\bar{i}^{g*}_{qds} + j\,\omega\,L_m\,\bar{i}^{g}_{qdr}\,\bar{i}^{g*}_{qds} +$$

$$r_r\left|\bar{i}^{g}_{qdr}\right|^2 + L_r\,p\left|\bar{i}^{g}_{qdr}\right|^2 + j(\omega - \omega_r)L_r\left|\bar{i}^{g}_{qdr}\right|^2 + L_m\,p\;\bar{i}^{g}_{qds}\,\bar{i}^{g*}_{qdr} + j(\omega - \omega_r)L_m\,\bar{i}^{g}_{qds}\,\bar{i}^{g*}_{qdr}\Big\}$$

$$= \frac{3}{2}\mathcal{Re}\Big\{r_s\left|\bar{i}^{g}_{qds}\right|^2 + r_r\left|\bar{i}^{g}_{qdr}\right|^2 + L_{ls}\,p\left|\bar{i}^{g}_{qds}\right|^2 + L_{lr}\,p\left|\bar{i}^{g}_{qdr}\right|^2 + L_m\,p\left|\bar{i}^{g}_{qds} + \bar{i}^{g}_{qdr}\right|^2 +$$

$$j\,\omega\,L_s\left|\bar{i}^{g}_{qds}\right|^2 + j(\omega - \omega_r)L_r\left|\bar{i}^{g}_{qdr}\right|^2 + j\,\omega\,L_m\,\bar{i}^{g}_{qdr}\,\bar{i}^{g*}_{qds} + j(\omega - \omega_r)L_m\,\bar{i}^{g}_{qds}\,\bar{i}^{g*}_{qdr}\Big\}$$

$$= \frac{3}{2}\left(r_s\left|\bar{i}^{\,g}_{qds}\right|^2 + r_r\left|\bar{i}^{\,g}_{qdr}\right|^2\right) + \frac{3}{2}p\left(L_{ls}\left|\bar{i}^{\,g}_{qds}\right|^2 + L_{lr}\left|\bar{i}^{\,g}_{qdr}\right|^2 + L_m\left|\bar{i}^{\,g}_{qds} + \bar{i}^{\,g}_{qdr}\right|^2\right) +$$

$$\frac{3}{2}\mathcal{Re}\left\{j\,\omega\,L_s\left|\bar{i}^{\,g}_{qds}\right|^2 + j(\omega - \omega_r)L_r\left|\bar{i}^{\,g}_{qdr}\right|^2 + j\,\omega\,L_m\,\bar{i}^{\,g}_{qdr}\,\bar{i}^{\,g*}_{qds} + j(\omega - \omega_r)L_m\,\bar{i}^{\,g}_{qds}\,\bar{i}^{\,g*}_{qdr}\right\}$$

Therefore:

$$P_{in_{total}} = \frac{3}{2}\left(r_s\left|\bar{i}^{\,g}_{qds}\right|^2 + r_r\left|\bar{i}^{\,g}_{qdr}\right|^2\right) + \frac{3}{2}p\left(L_{ls}\left|\bar{i}^{\,g}_{qds}\right|^2 + L_{lr}\left|\bar{i}^{\,g}_{qdr}\right|^2 + L_m\left|\bar{i}^{\,g}_{qds} + \bar{i}^{\,g}_{qdr}\right|^2\right) +$$

$$\frac{-3}{2}\mathcal{Re}\left\{j\,\omega_r L_m\,\bar{i}^{\,g}_{qds}\,\bar{i}^{\,g*}_{qdr}\right\}$$

$$P_{em} = \frac{-3}{2}\mathcal{Re}\left\{j\,\omega_r L_m\,\bar{i}^{\,g}_{qds}\,\bar{i}^{\,g*}_{qdr}\right\}$$

or

$$\boxed{P_{em} = \frac{3}{2}\mathcal{Im}\left\{\omega_r L_m\,\bar{i}^{\,g}_{qds}\,\bar{i}^{\,g*}_{qdr}\right\}}$$

In scalar form:

$$\boxed{P_{em} = \frac{3}{2}\,\omega_r L_m\left(i^g_{qs}i^g_{dr} - i^g_{ds}i^g_{qr}\right)}$$

recall: $\omega_r = \frac{P}{2}\omega_{rm}$

$$\boxed{T_e = \frac{3}{2}\frac{P}{2}L_m\,Im\left\{\bar{i}_{qds}^{\,g}\,\bar{i}_{qdr}^{\,g*}\right\}} \leftarrow \text{Electromagnetic Torque}$$

Stator Flux Linkage is related to stator & rotor currents by

$$\bar{\lambda}_{qds}^{\,g} = L_s\,\bar{i}_{qds}^{\,g} + L_m\,\bar{i}_{qdr}^{\,g}$$

The torque in terms of stator currents and flux linkage can be expressed as:

$$T_e = \frac{3}{2}\frac{P}{2}\,Im\left\{\bar{i}_{qds}^{\,g}\,\bar{\lambda}_{qds}^{\,g*} - L_s\,\bar{i}_{qds}^{\,g}\,\bar{i}_{qds}^{\,g*}\right\}$$

$$\boxed{T_e = \frac{3}{2}\frac{P}{2}\,Im\left\{\bar{i}_{qds}^{\,g}\,\bar{\lambda}_{qds}^{\,g*}\right\}}$$

The torque can be viewed as the interaction of rotor flux and stator current. This is extremely useful for torque control, known as field oriented control.

$$\bar{\lambda}_{qdr}^{\,g} = L_r\,\bar{i}_{qdr}^{\,g} + L_m\,\bar{i}_{qds}^{\,g} \rightarrow \bar{i}_{qdr}^{\,g} = \frac{1}{L_r}\bar{\lambda}_{qdr}^{\,g} - \frac{L_m}{L_r}\bar{i}_{qds}^{\,g}$$

$$\boxed{T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}\,Im\left\{\bar{i}_{qds}^{\,g}\,\bar{\lambda}_{qdr}^{\,g*}\right\}}$$

Equation Summary for Simulation:

The following is a listing of state equations which can be used in simulating an AC machine:

$$p\lambda_{qs}^{s} = v_{qs}^{s} - r_s i_{qs}^{s}$$

$$p\lambda_{ds}^{s} = v_{ds}^{s} - r_s i_{ds}^{s}$$

$$p\lambda_{qr}^{s} = v_{qr}^{s} - r_r i_{qr}^{s} + \omega_r \lambda_{dr}^{s}$$

$$p\lambda_{dr}^{s} = v_{dr}^{s} - r_r i_{dr}^{s} - \omega_r \lambda_{qr}^{s}$$

$$T_e = \frac{3}{2}\frac{P}{2} L_m (i_{qs}^{s} i_{dr}^{s} - i_{qr}^{s} i_{ds}^{s})$$

where

$$\lambda_{qs}^{s} = L_s i_{qs}^{s} + L_m i_{qr}^{s}$$

$$\lambda_{ds}^{s} = L_s i_{ds}^{s} + L_m i_{dr}^{s}$$

$$\lambda_{qr}^{s} = L_r i_{qr}^{s} + L_m i_{qs}^{s}$$

$$\lambda_{dr}^{s} = L_r i_{dr}^{s} + L_m i_{ds}^{s}$$

so

$$i_{qs}^{s} = \frac{1}{\sigma L_s}\left(\lambda_{qs}^{s} - \frac{L_m}{L_r}\lambda_{qr}^{s}\right)$$

$$\sigma = 1 - \frac{L_m^2}{L_s L_r}$$ (coupling factor)

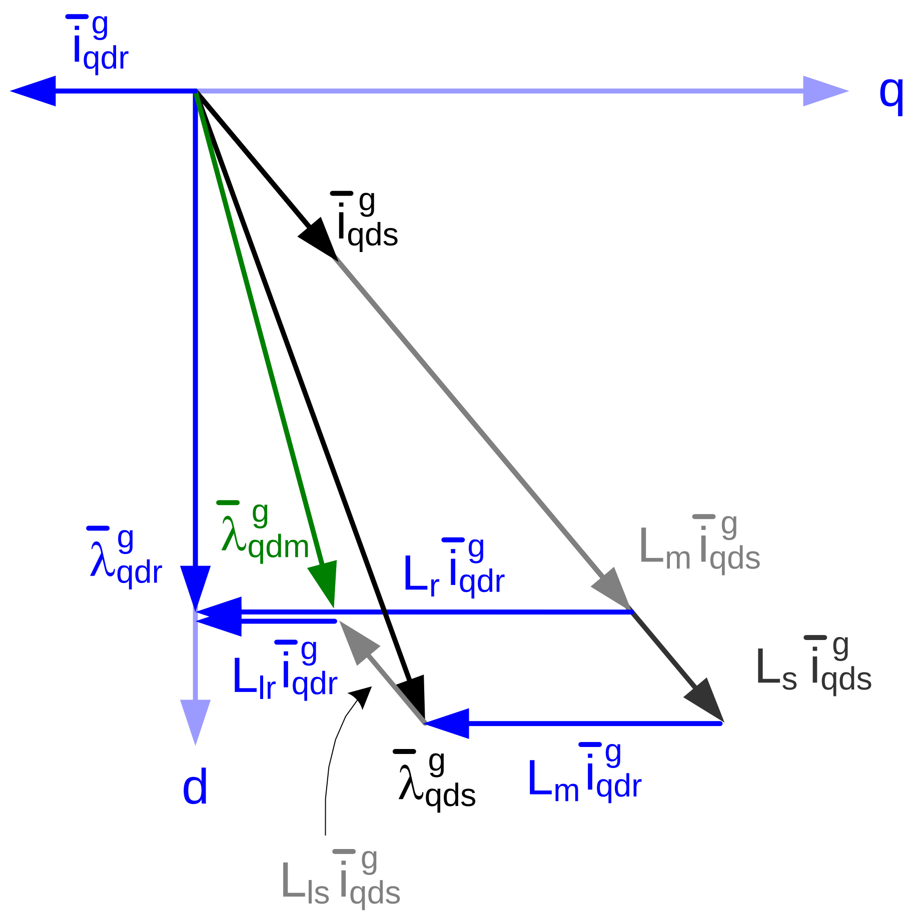
$$i_{ds}^{s} = \frac{1}{\sigma L_s}\left(\lambda_{ds}^{s} - \frac{L_m}{L_r}\lambda_{dr}^{s}\right)$$

$$i_{qr}^{s} = \frac{1}{L_r}\left(1 + \frac{L_m^2}{\sigma L_r L_s}\right)\lambda_{qr}^{s} - \frac{L_m}{\sigma L_r L_s}\lambda_{qs}^{s}$$

$$i_{dr}^{s} = \frac{1}{L_r}\left(1 + \frac{L_m^2}{\sigma L_r L_s}\right)\lambda_{dr}^{s} - \frac{L_m}{\sigma L_r L_s}\lambda_{ds}^{s}$$

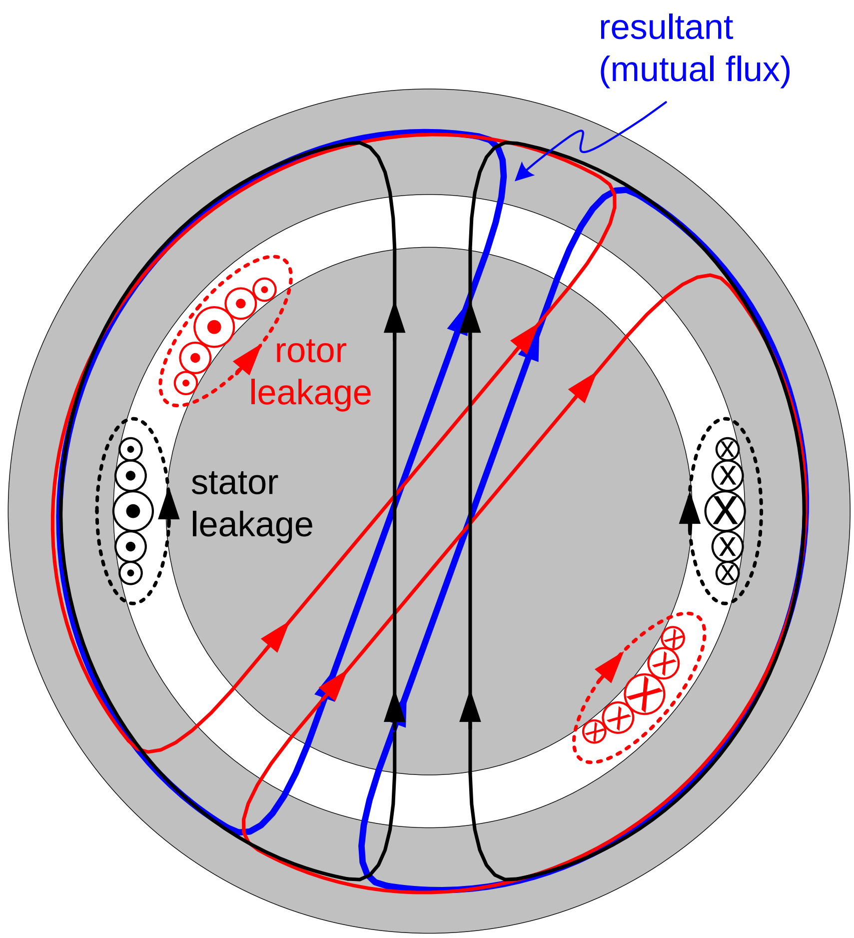
From above:

$$\boxed{T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r} Im\left\{\bar{i}^{\,g}_{qds}\,\bar{\lambda}^{g\,*}_{qdr}\right\}}$$



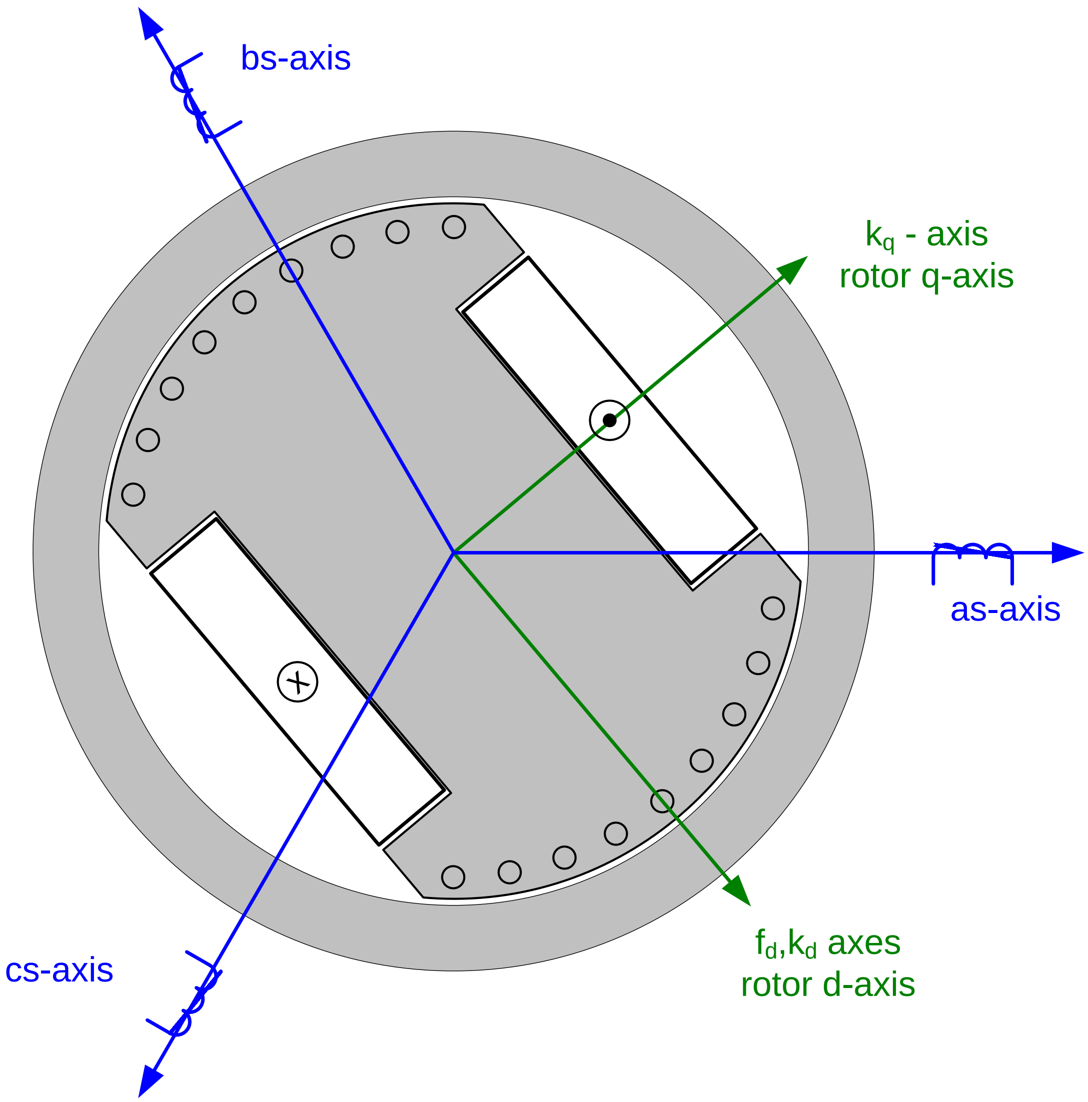
Vector Axis Representation of Complex Vector Equations (d-axis Aligned with Rotor Flux)

Physical interpretation of flux in an AC machine:



Physical Interpretation of Flux in an AC Machine

d,q,0 Theory Applied to Salient Pole Machines:



Salient Pole Wound Field Synchronous Machine

Self inductance is a function of rotor position. Neglecting higher order harmonics:

$$L_{as,as} = L_{ls} + L_{0s} - L_{2s}\cos(2\theta_r)$$

$$L_{0s} = \mu_o r \ell N_s^2 \frac{\pi}{8}\left(\frac{1}{g_{min}} + \frac{1}{g_{max}}\right)$$

$$L_{2s} = \mu_o r \ell N_s^2 \frac{\pi}{8}\left(\frac{1}{g_{min}} - \frac{1}{g_{max}}\right)$$

$$L_{bs,bs} = L_{ls} + L_{0s} - L_{2s}\cos(2\theta_r + 120^o)$$

$$L_{cs,cs} = L_{ls} + L_{0s} - L_{2s}\cos(2\theta_r - 120^o)$$

Mutual Inductances:

$$L_{as,bs} = L_{bs,as} = \frac{-1}{2}L_{0s} - L_{2s}\cos(2\theta_r - 120^o)$$

$$L_{as,cs} = L_{cs,as} = \frac{-1}{2}L_{0s} - L_{2s}\cos(2\theta_r + 120^o)$$

$$L_{bs,cs} = L_{cs,bs} = \frac{-1}{2}L_{0s} - L_{2s}\cos(2\theta_r)$$

The inductances cooresponding to the flux linking the field winding with the three stator phases are:

$$L_{as,fd} = L_{fd,as} = L_{sfd}\cos(\theta_r)$$

$$L_{bs,fd} = L_{fd,bs} = L_{sfd}\cos(\theta_r - 120^o)$$

$$L_{cs,fd} = L_{fd,cs} = L_{sfd}\cos(\theta_r + 120^o)$$

$$L_{sfd} = \mu_o r \ell N_s N_r \frac{\pi}{4}\frac{1}{g_{min}}$$

The inductances cooresponding to the flux linking the d-axis damper winding with the three stator phases are:

$$L_{as,kd} = L_{kd,as} = L_{skd}\cos(\theta_r)$$

$$L_{bs,kd} = L_{kd,bs} = L_{skd}\cos(\theta_r - 120^o)$$

$$L_{cs,kd} = L_{kd,cs} = L_{skd}\cos(\theta_r + 120^o)$$

$$L_{skd} = \mu_o r \ell N_s N_{kd} \frac{\pi}{4}\frac{1}{g_{min}}$$

The inductances cooresponding to the flux linking the q-axis damper winding with the three stator phases are:

$$L_{as,kq} = L_{kq,as} = -L_{skq}\sin(\theta_r)$$

$$L_{bs,kq} = L_{kq,bs} = -L_{skq}\sin(\theta_r - 120^o)$$

$$L_{cs,kq} = L_{kq,cs} = -L_{skq}\sin(\theta_r + 120^o)$$

$$L_{skq} = \mu_o\, r\, \ell\, N_s N_{kq} \frac{\pi}{4}\frac{1}{g_{max}}$$

Referring the stator variables to the rotor:

$$\overline{v}^r_{qds} = r_s\,\overline{i}^r_{qds} + (p + j\,\omega)\overline{\lambda}^r_{qds} \quad \text{or}$$

$$v^r_{qs} = r_s\, i^r_{qs} + p\,\lambda^r_{qs} + \omega_r\lambda^r_{ds}$$

$$v^r_{ds} = r_s\, i^r_{ds} + p\,\lambda^r_{ds} - \omega_r\lambda^r_{qs}$$

$$v^r_{fd} = r_s\, i^r_{fd} + p\,\lambda^r_{fd}$$

actual field voltage

actual currents, voltages, and flux linkages

$$v^r_{kd} = r_s\, i^r_{kd} + p\,\lambda^r_{kd}$$

$$v^r_{kq} = r_s\, i^r_{kq} + p\,\lambda^r_{kq}$$

use scalar equations instead of complex equations due to lack of symmetry

$$\lambda^r_{qs} = L_q\, i^r_{qs} + L_{mq}\left(i^r_{kq}\right)$$

$$\lambda^r_{ds} = L_d\, i^r_{ds} + L_{md}\left(i^r_{fd} + i^r_{kd}\right)$$

$$\lambda^r_{kq} = L_{kq}\, i^r_{kq} + L_{mq}\left(i^r_{qs}\right)$$

$$\lambda^r_{kd} = L_{kd}\, i^r_{kd} + L_{md}\left(i^r_{ds}\right)$$

Salient Pole Machine Torque

$$T_e = \frac{3}{2}\frac{P}{2} Im\left\{ \bar{i}^{\,r}_{qds}\, \bar{\lambda}^{\,r*}_{qds} \right\} = \frac{3}{2}\frac{P}{2}\left[\lambda^r_{ds} \cdot i^r_{qs} - \lambda^r_{qs} \cdot i^r_{ds} \right]$$

$$= \frac{3}{2}\frac{P}{2}\left[\underbrace{L_{md}\, i^r_{qs}\left(i^r_{fd} + i^r_{kd}\right)}_{\text{BLI torque (excitation or reaction torque)}} + \underbrace{-L_{mq}\, i^r_{ds}\left(i^r_{kq}\right)}_{\text{Damping (induction motor) torque}} + \underbrace{\left(L_{md} - L_{mq}\right) i^r_{qs}\, i^r_{qs}}_{\text{Reluctance torque}} \right]$$

Note: The rotor reference frame is the synchronous reference frame (rotor spins at synchronous speed).

<u>Extension of d,q,0 theory to PM Machines:</u>

This is a special case of salient pole machines

$$\Lambda_{mf} = L_{md}\, i^r_{fd}$$

Park's Equations for a PM Machine with a starting cage are:

$$v^r_{qs} = r_s i^r_{qs} + p\lambda^r_{qs} + \omega\lambda^r_{ds}$$

$$v^r_{ds} = r_s i^r_{ds} + p\lambda^r_{ds} - \omega\lambda^r_{qs}$$

$$v^r_{kd} = r_s i^r_{kd} + p\lambda^r_{kd}$$

$$v^r_{kq} = r_s i^r_{kq} + p\lambda^r_{kq}$$

where

$$\lambda^r_{qs} = L_{ls} i^r_{qs} + L_{mq}\left(i^r_{qs} + i^r_{kq}\right)$$

$$\lambda^r_{ds} = L_{ls} i^r_{ds} + L_{md}\left(i^r_{ds} + i^r_{kd}\right) + \Lambda_{mf}$$

$$\lambda^r_{kq} = L_{lkq} i^r_{kq} + L_{mq}\left(i^r_{kq} + i^r_{qs}\right)$$

$$\lambda^r_{kd} = L_{lkd} i^r_{kd} + L_{md}\left(i^r_{kd} + i^r_{ds}\right) + \Lambda_{mf}$$

Steady State Induction Machine Field Orientation Using dq Model:

Constraints:

1) Independent control of $\bar{i}^{e}_{qds}$

$$i^{e}_{qs} = I_{qs}, \; i^{e}_{ds} = I_{ds}$$

2) Orient Reference Frame to Rotor Flux such that i^{e}_{ds} is in direction of Rotor Flux

Rotor Flux Equation:

$$\bar{\lambda}^{e}_{qdr} =$$

$$\lambda^{e}_{qr} =$$

therefore $$\boxed{I_{qr} = \frac{-L_m}{L_r} I_{qs}}$$

$$\lambda^{e}_{dr} =$$

Rotor Voltage Equations:

$$\left[p + j(\omega_e - \omega_r)\right] \bar{\lambda}^{e}_{qdr} + r_r \bar{i}^{e}_{qdr} = 0$$

$$(\omega_e - \omega_r) \lambda^{e}_{dr} + r_r I_{qr} = 0$$

$$-(\omega_e - \omega_r) \lambda^{e}_{qr} + r_r I_{dr} = 0 \; \rightarrow$$

therefore $\boxed{\lambda_{dr}^{e} = L_m I_{ds}}$

$$\omega_e - \omega_r = \frac{-r_r}{\lambda_{dr}^{e}} I_{qr} = \frac{-r_r}{\lambda_{dr}^{e}}\left(\frac{-L_m}{L_r} I_{qs}\right) = \frac{r_r}{L_r}\frac{L_m I_{qs}}{\lambda_{dr}^{e}}$$

$$\omega_e - \omega_r = \boxed{s\omega_e = \frac{1}{\tau_r}\frac{I_{qs}}{I_{ds}}}$$

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r} Im\left\{\bar{i}_{qdr}^{e}\,\bar{\lambda}_{qds}^{e*}\right\} = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}\left[\lambda_{dr}^{e} i_{qs}^{e} - \lambda_{qr}^{e} i_{ds}^{e}\right] = \frac{3}{2}\frac{P}{2}\frac{L_m^2}{L_r} I_{ds} I_{qs}$$

Stator Voltage Equation:

$$\bar{v}_{qds}^{e} = r_s\,\bar{i}_{qds}^{e} + (p + j\,\omega_e)\bar{\lambda}_{qds}^{e}$$

$$\bar{V}_{qds} = r_s\,\bar{I}_{qds} + j\,\omega_e \bar{\lambda}_{qds}^{e}$$

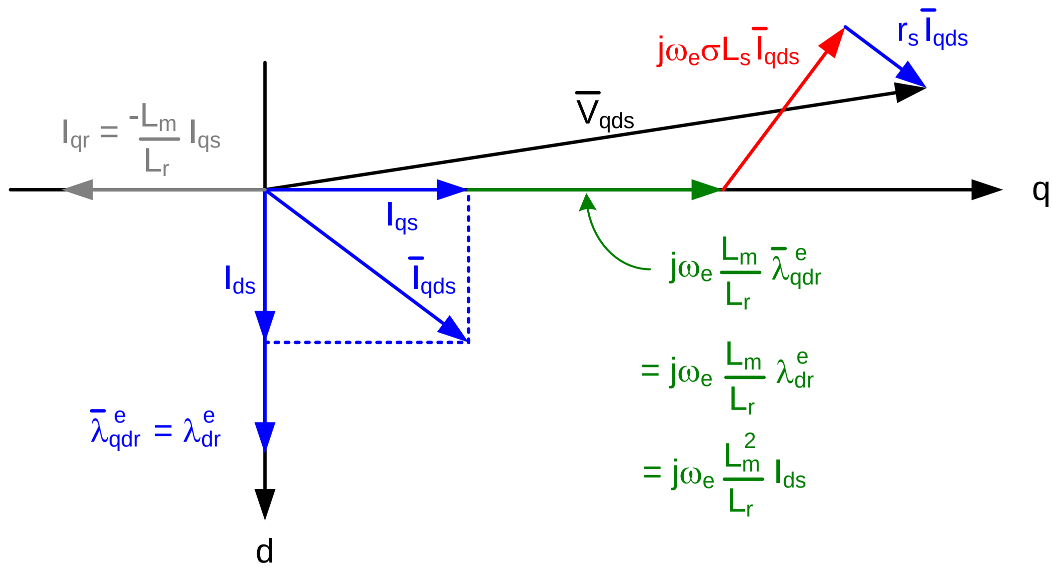
$$\bar{\lambda}_{qds}^{e} = L_s\,\bar{I}_{qds} + L_m\,\bar{I}_{qdr}$$

$$\bar{\lambda}_{qdr}^{e} = L_m\,\bar{I}_{qds} + L_r\,\bar{I}_{qdr}$$

$$\bar{\lambda}_{qds}^{e} = \sigma L_s\,\bar{I}_{qds} + \frac{L_m}{L_r}\bar{\lambda}_{qdr}^{e} \qquad \sigma = 1 - \frac{L_m^2}{L_s L_r}$$ (coupling factor)

$$\bar{V}_{qds} = (r_s + j\,\omega_e \sigma L_s)\,\bar{I}_{qds} + j\,\omega_e \frac{L_m}{L_r}\bar{\lambda}_{qdr}^{e}$$

I_{qr} is like the compensating winding in a DC machine



Vector Diagram of Steady State Operation of Induction Motor

<u>Steady State Induction Machine Field Orientation Summary:</u>

$$\lambda^e_{qr} = 0$$

1) $I^e_{qr} = \dfrac{-L_m}{L_r} I^e_{qs}$

2) $I^e_{dr} = 0$

3) $\lambda^e_{dr} = L_m I^e_{ds}$

4) $s\,\omega_e = \dfrac{r_r}{L_r}\dfrac{I^e_{qs}}{I^e_{ds}} = \dfrac{1}{\tau_r}\dfrac{I^e_{qs}}{I^e_{ds}}$

5) $T_e = \dfrac{3}{2}\dfrac{P}{2}\dfrac{L_m}{L_r}\lambda^e_{dr} I^e_{qs}$

$$= \frac{3}{2}\frac{P}{2}\frac{L_m^2}{L_r} I^e_{ds} I^e_{qs}$$