EE 560 – Electric Machines and Drives

Prof. N.J. Nagel, Autumn 2020 – Lecture 1

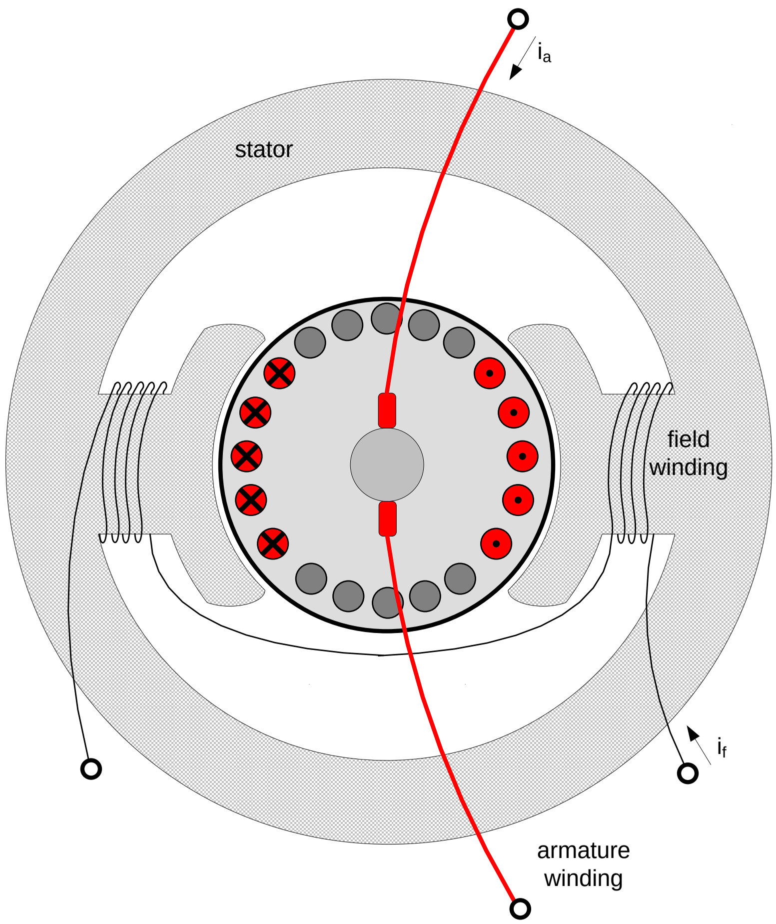
Big Picture:

- Course focuses on physics of machines.
- Will explore fundamentals of torque production in machines.
- Will explore high performance torque and speed control of adjustable speed drives.

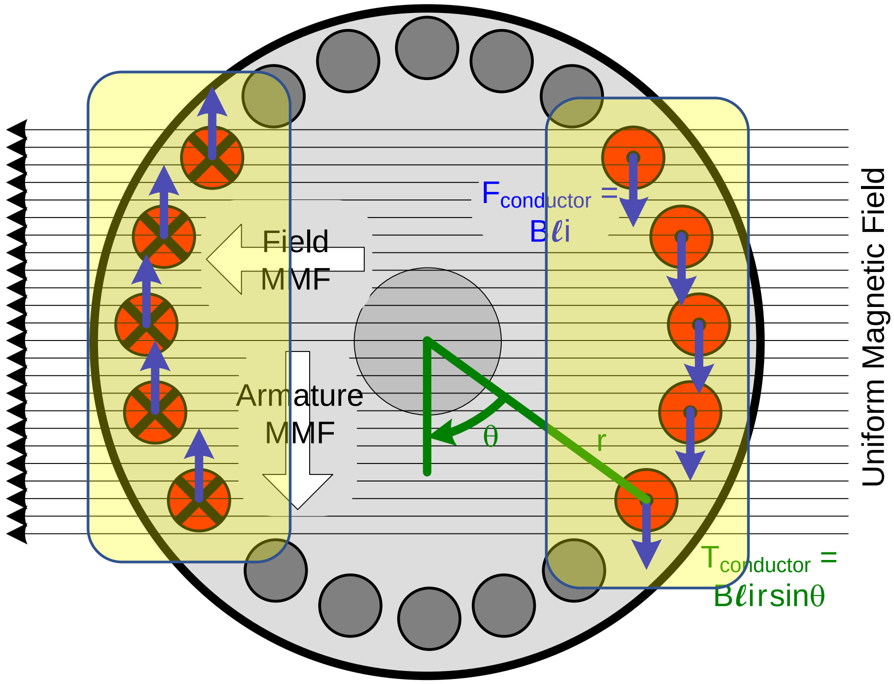
DC Machines Overview:

- Machines have simple (mechanical) commutation.
- Wound Field Machines have independent control of field and torque producing current.
- Can control speed by controlling armature voltage.
- Can control torque (and subsequently speed) by controlling armature current.

DC Machines



Basic Structure of a Wound Field DC Machine



Simplified Torque Production Mechanism in DC Machines

DC Machines Differential Equations:

Armature Circuit

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a$$

where

v_a – *armature voltage*
i_a – *armature current*
R_a – *armature winding resistance*
L_a – *armature winding inductance*
e_a – *back electro motive force (EMF)*

Back EMF

$$e_a = K_f i_f \omega = K_e \omega$$

where

e_a – *back electro motive force (EMF)*
K_f – *field constant (geometry dependent)*
i_f – *field current*
ω – *rotor speed*
K_e – *back EMF constant (at a fixed field current)*

Field Circuit

$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

where

v_f – *field voltage*
i_f – *field current*
R_f – *field winding resistance*
L_f – *field winding inductance*

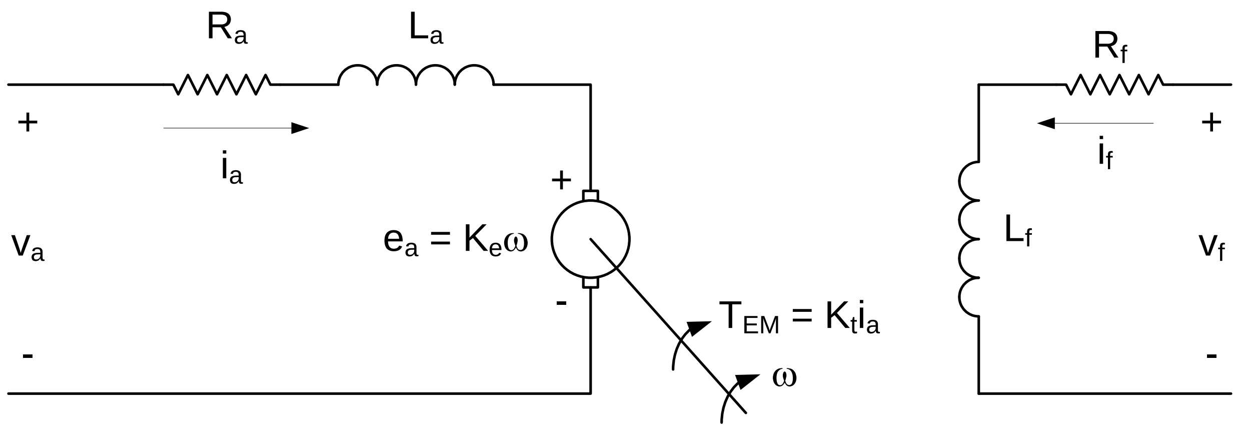
Torque Equation

$$t_{em} = K_f i_f i_a = K_t i_a$$

where

t_{em} –*electromagnetic torque produced by the motor*
K_f – *field constant (geometry dependent)*
i_f – *field current*
i_a – *armature current*
K_t – *torque constant (at a fixed field current)*

Wound Field DC Machine Equivalent Circuit



Armature Equations:

$v_a =$

Field Equations

$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

Constituent Equations:

$$e_a = K_f i_f \omega = K_e \omega$$

$$t_{em} = K_f i_f i_a = K_t i_a$$

Newton's 2nd Law:

$$\sum T_{mech} = T_{em}$$

Steady State Operation:

$$i_f = I_f\,,\ i_a = I_a\,,\ \omega = constant$$

$$V_f = R_f I_f$$

$$V_a = R_a I_a + E_a$$

$$T_{em} = K_t I_a$$

Example:

National Electrical Manufacturer's Association

100 Hp   1750 RPM   NEMA 503 frame (12.5in shaft centerline height)

$R_a = 0.0144\,\Omega$ @25 °C   ($0.0173\,\Omega$ @ operating temperature)

$L_a = 1.1$ mH

$$K_t = 0.885\,\frac{\text{ft-lb}_f}{\text{A}} \qquad K_e = 1.27\,\frac{\text{V}}{\left(\frac{\text{rad}}{\text{sec}}\right)}$$

$$\omega = 1750 \text{ RPM} = 183\,\frac{\text{rad}}{\text{sec}}$$

$$E_a = K_e\,\omega = 1.27\,\frac{\text{V}}{\left(\frac{\text{rad}}{\text{sec}}\right)} \text{ x } 183\,\frac{\text{rad}}{\text{sec}} = 233 \text{ volts}$$

rated power: $P_r = T_r\,\omega_r \rightarrow T_r = \dfrac{P_r}{\omega_r} = \dfrac{(100\text{ Hp})\left(746\,\frac{\text{W}}{\text{Hp}}\right)}{183\,\frac{\text{rad}}{\text{sec}}} = 407 \text{ N-m} = 300 \text{ ft-lb}_f$

$$I_{ar} = \frac{T_r}{K_t} = \frac{300 \text{ ft-lb}_f}{0.885\,\frac{\text{ft-lb}_f}{\text{A}}} = 339 \text{ A}$$

rated IR voltage drop: $R_a\,I_a = 339 \text{ A x } 0.0173\,\Omega = 5.9$ volts
(@ rated operating temperature)

rated machine voltage:

$V_{ar} = R_a\,I_{ar} + E_a = 238.9$ volts ($\cong$ 240 Vdc)

no load speed:

$$\omega_{nl} = 1750 \text{ RPM x } \frac{238.9 \text{ volts}}{233 \text{ volts}} = 1794 \text{ RPM}$$

What is the stall current?

$I_{a\_stall} = \dfrac{V_a}{R_a} = \dfrac{240 \text{ V}}{0.0144\,\Omega} = 16{,}590$ A!!!   *This is why we current regulate machines!*

Speed-Torque Characteristics:

$v_a = V_a \qquad \varphi_f = constant \;\rightarrow\; K_e = K_t = constant$

$e_a = E_a \qquad I_a = \frac{T_{em}}{K_t}$

$\omega = \frac{E_a}{K_e} =$

$$\boxed{\omega = \frac{V_a}{K_e} - \frac{R_a}{K_e K_t} t_{em}}$$

Speed as a function of Torque

$t_{em} = K_t I_a =$

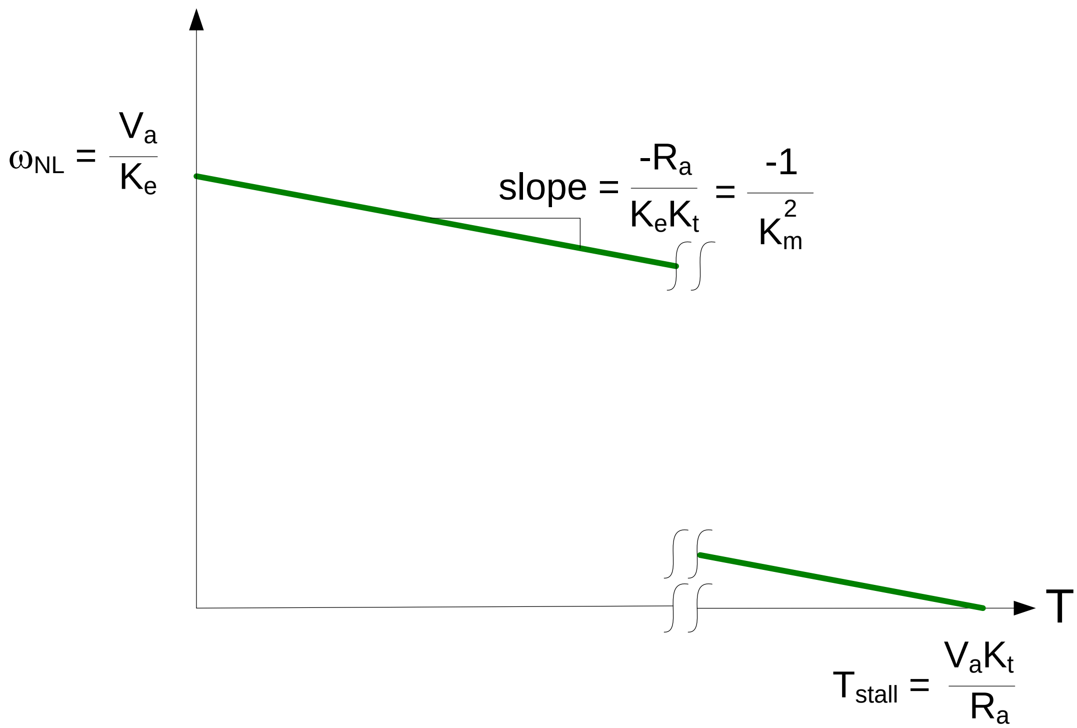
$$\boxed{t_{em} = \frac{K_t}{R_a} V_a - \frac{K_e K_t}{R_a} \omega}$$

Torque as a function of Speed

$K_m^2 =$

$K_m =$

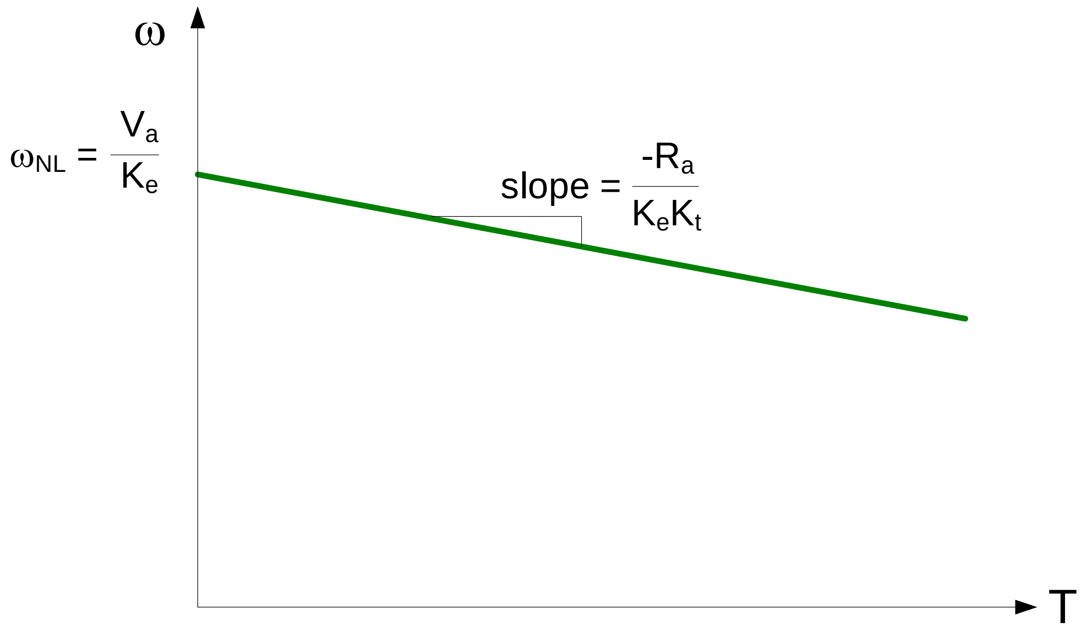
What does the speed-torque curve look like for the DC Machine?



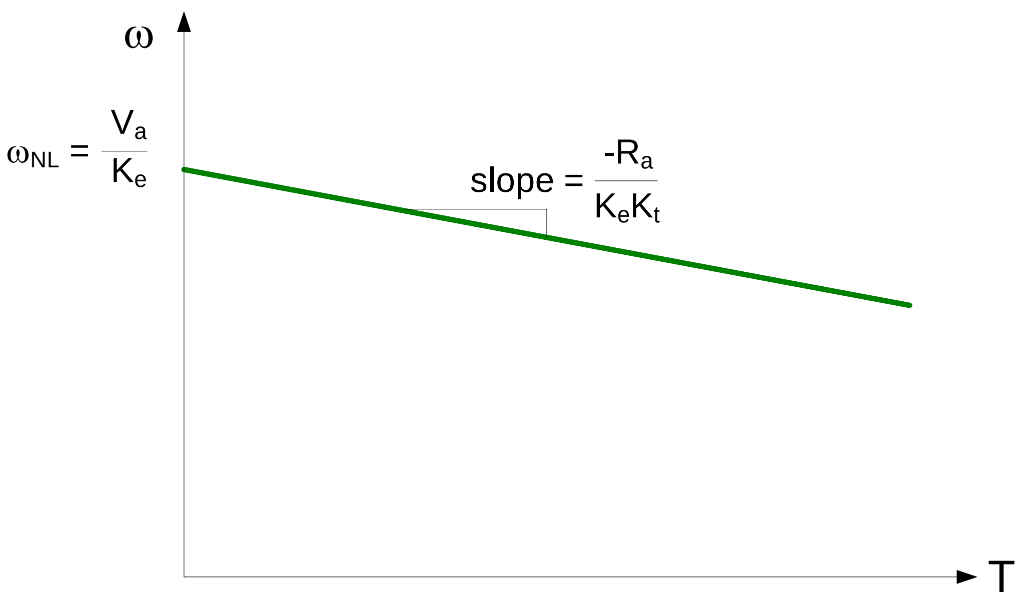
Speed Torque of a DC Machine with Constant Field Flux

Ways of varying speed:

i) R – add external resistance (used in the "prehistoric days") ← Trolley Car
ii) V – standard for reducing speed (operating below rated speed)
iii) ϕ_f – standard for increasing speed (operating above rated speed)

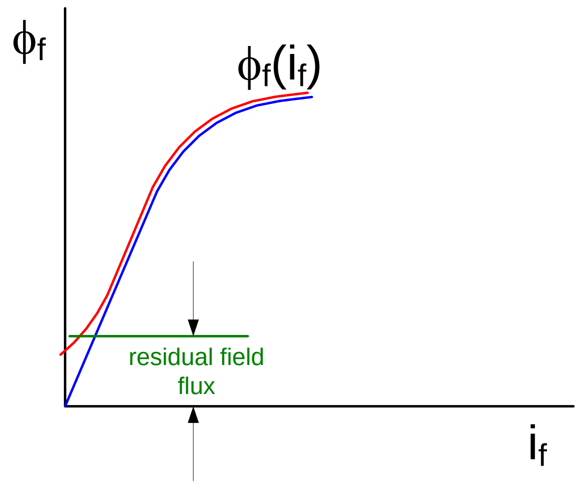


Speed Torque of a DC Machine

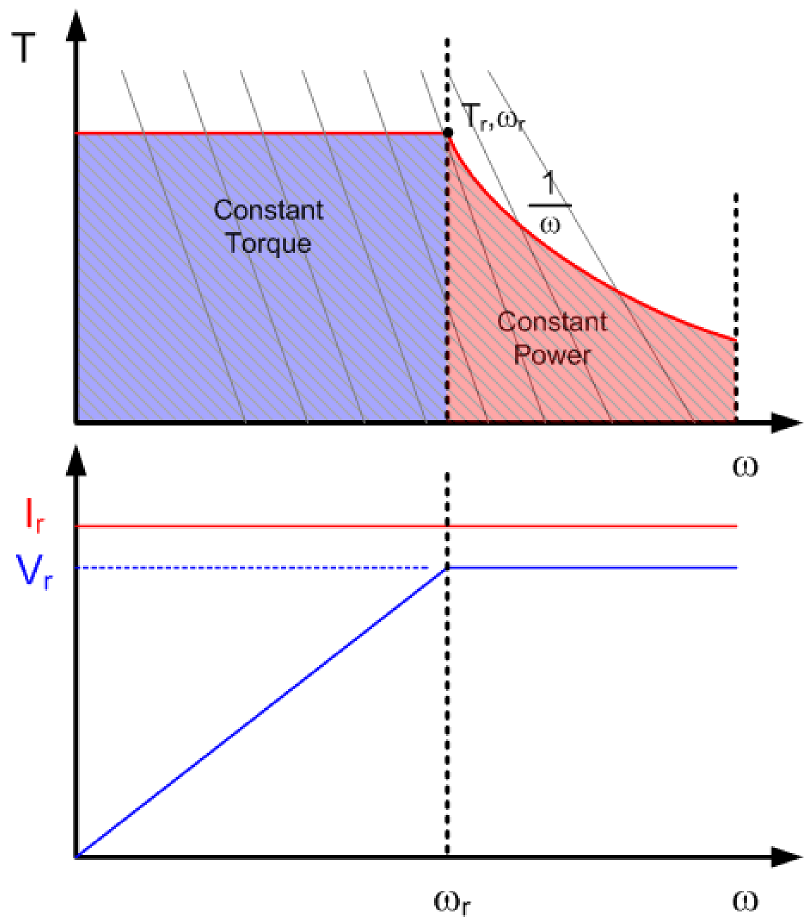


Speed Torque of a DC Machine

Note: catastrophic problem in lightly loaded or no load DC drives which loose field excitation

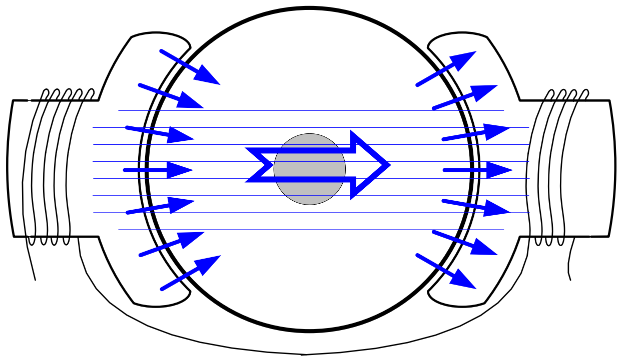


Field Flux vs. Field Current Showing Residual Field Flux

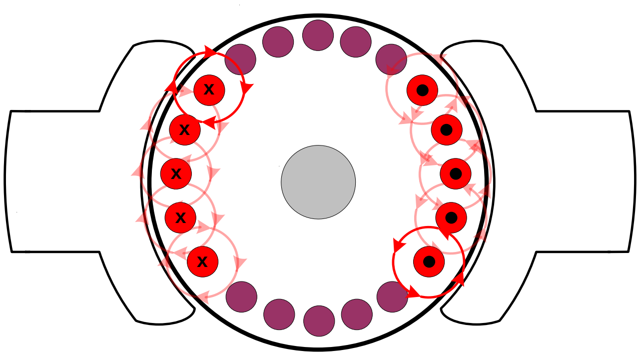


Capability curve for separately excited DC Machine

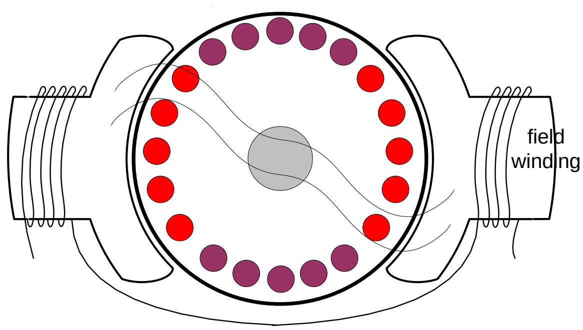
<u>Flux Distribution in DC Machines</u>



Flux with Field Excitation Only



Flux with Armature Excitation Only

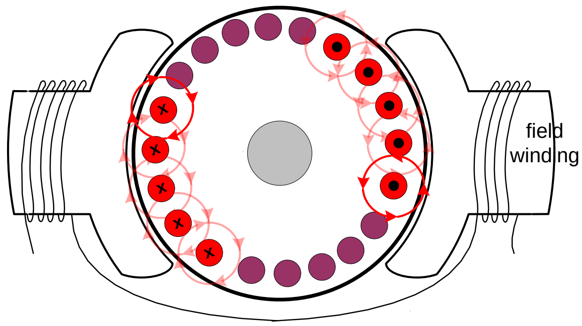


Flux with Field and Armature Excitation

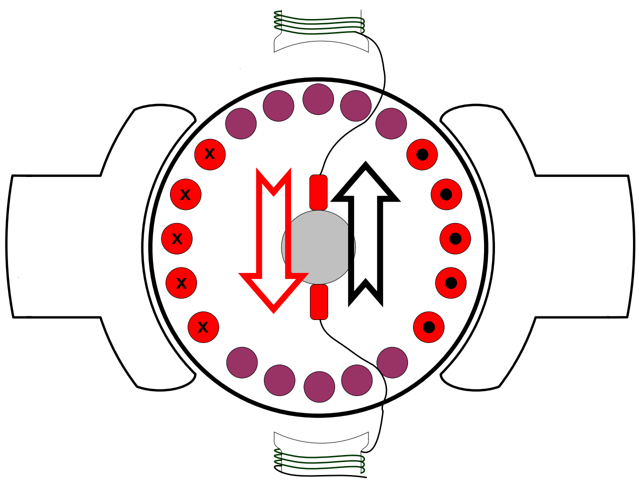
Problem: Skewing of the field flux is known as Armature Reaction.

Armature Reaction Mitigation Techniques:

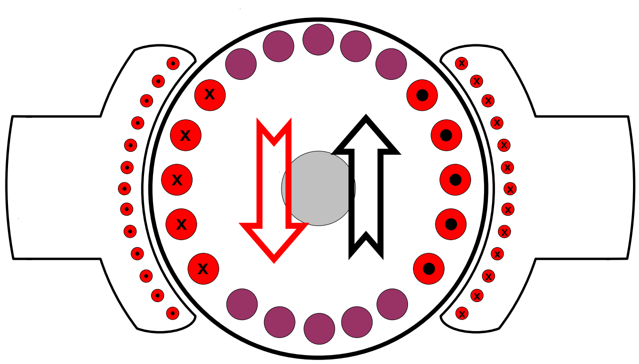
1) Rotate the brushes (helps for unidirectional applications only)
2) Add interpoles
3) Add compensating winding



Rotation of Brushes

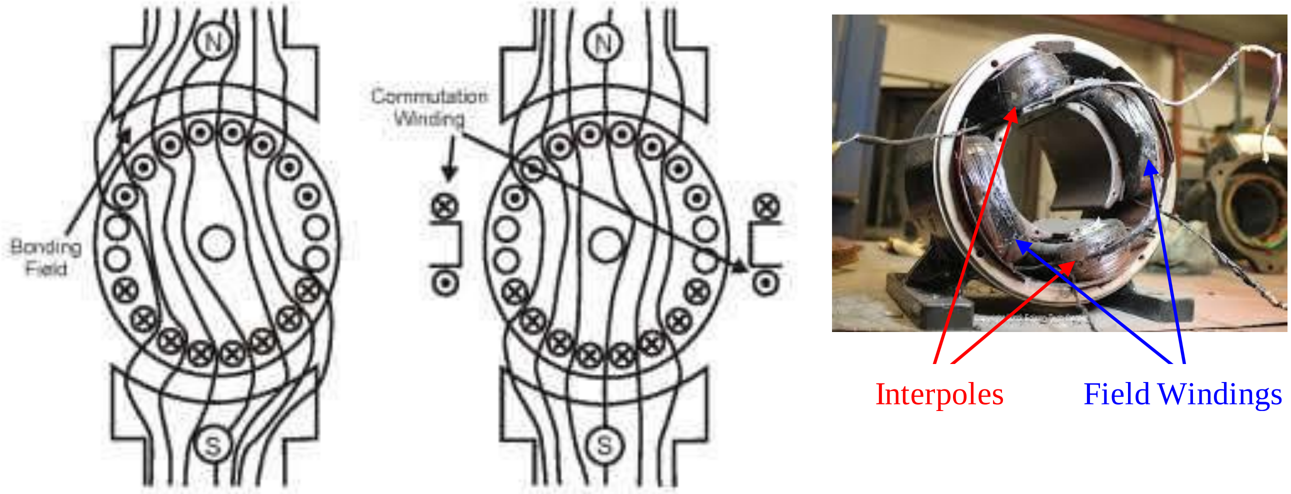


Interpoles

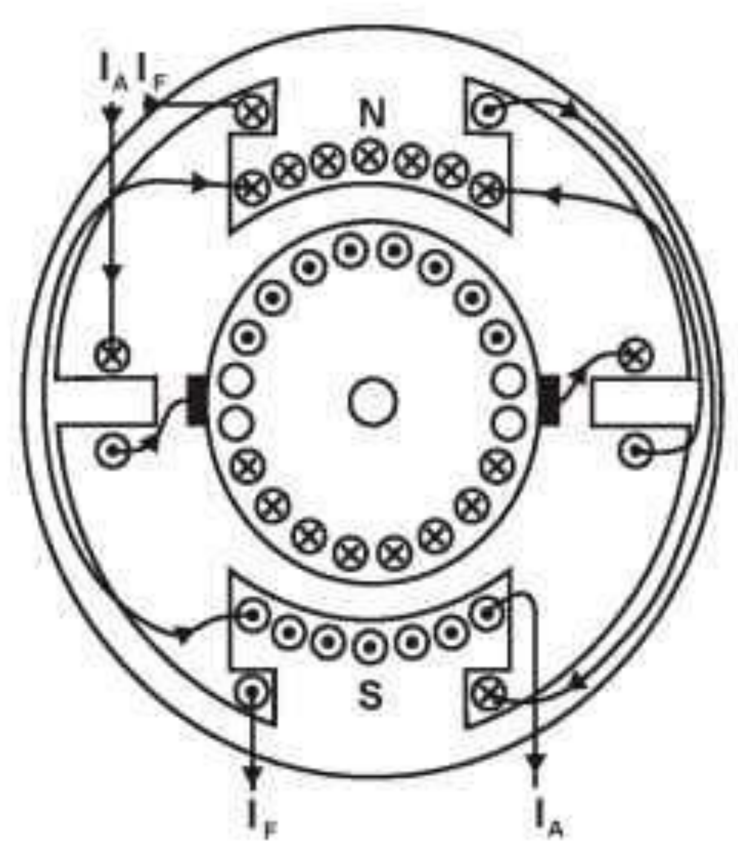


Armature Compensating Winding

Note: BLDC Motors also have armature reaction effects.



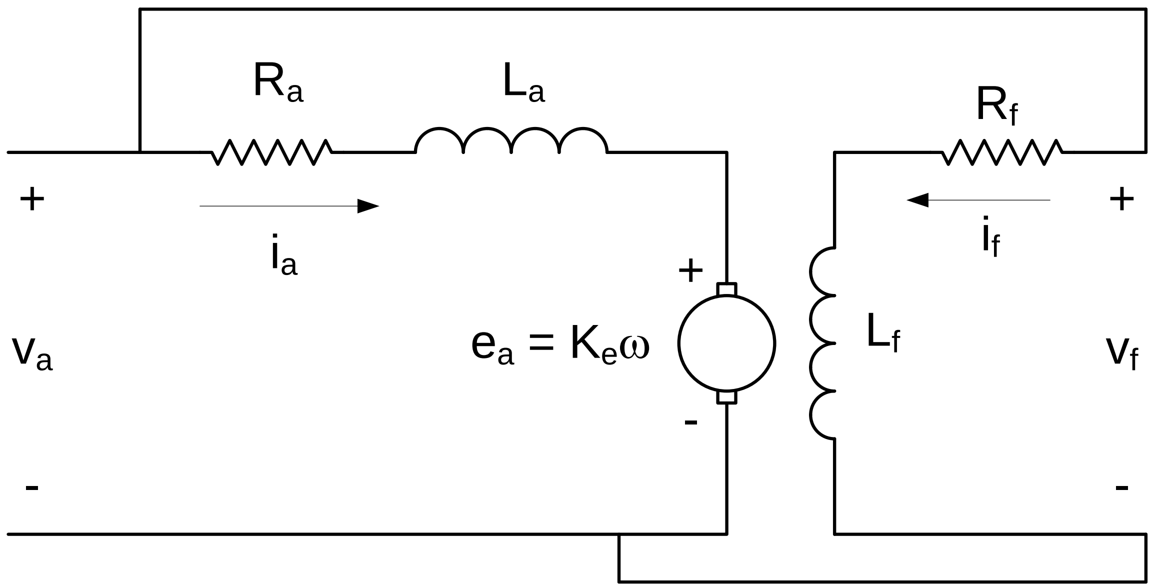
DC Machine with Interpoles (Commutation Windings)



DC Machine with Interpoles and Compensating Windings

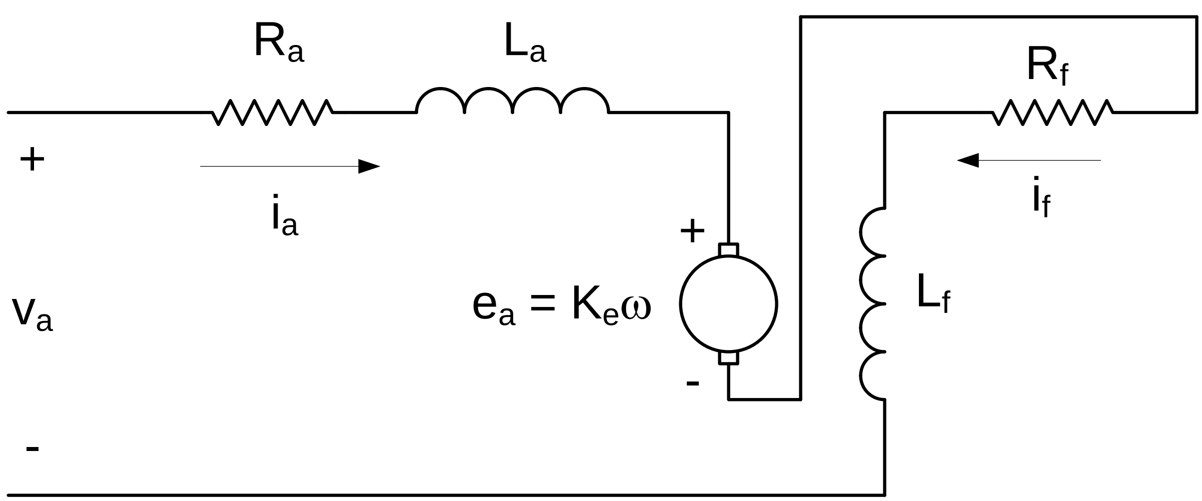
Used in large traction drives.

Shunt Wound DC Motor.



Shunt Wound DC Motor

Series Wound DC Motor.

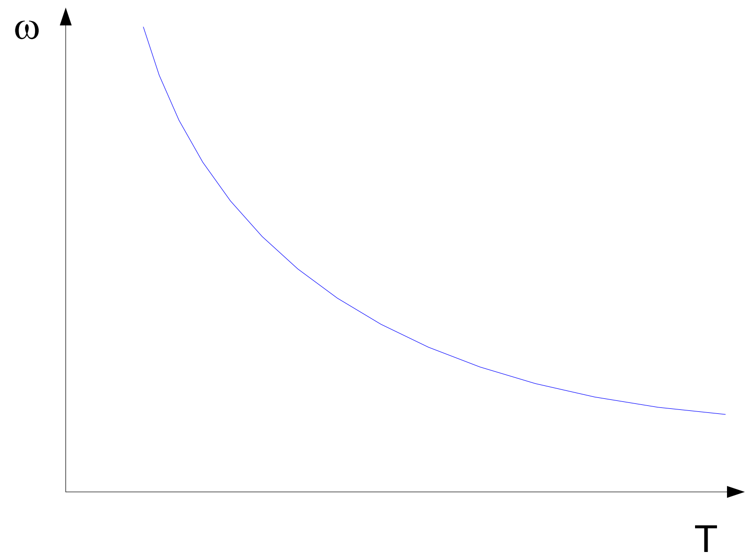


Series Wound DC Motor

What does speed-torque curve for series wound DC motor look like?

Series Wound DC Motor.

$T_{ss}(\omega) \propto \frac{V_a^2}{\omega^2}$ *torque inversely proportional to speed squared*

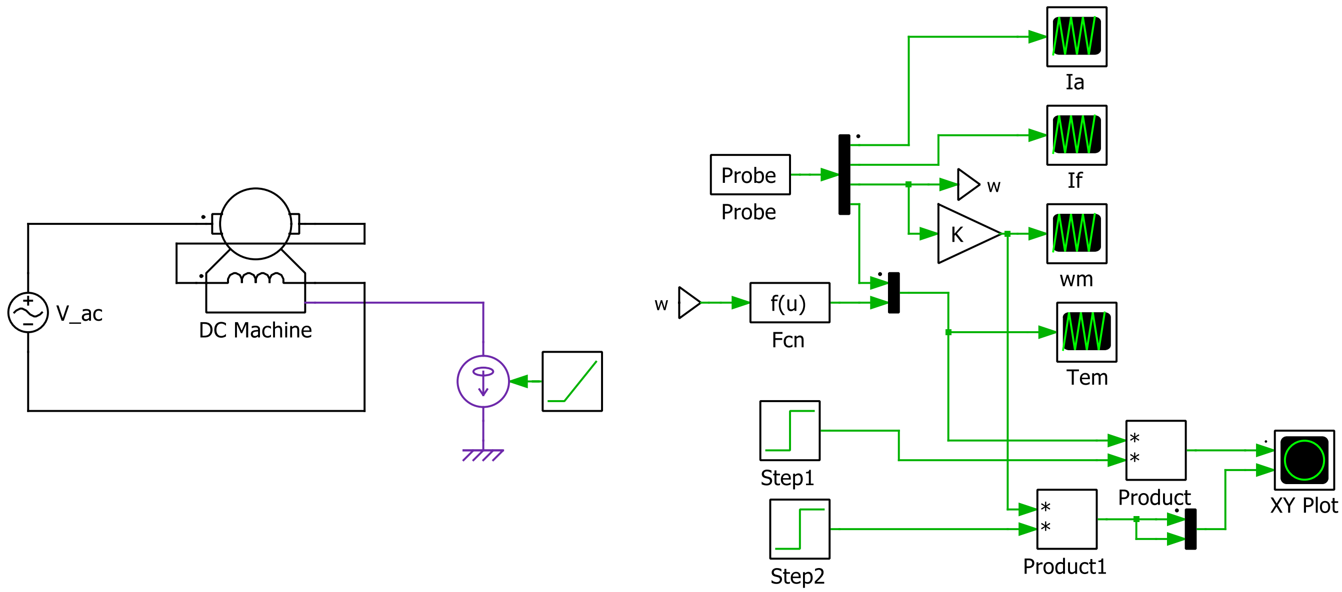


Series Wound DC Motor Speed-Torque Curve

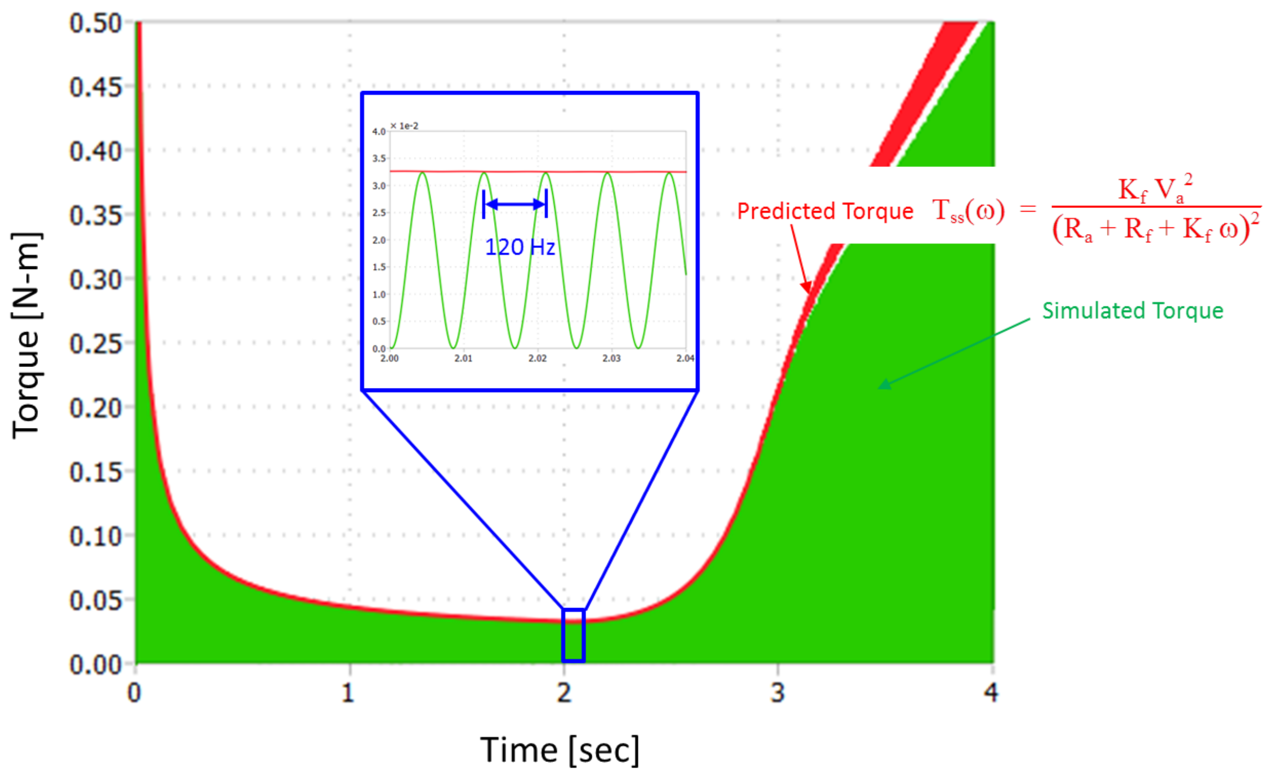
Also known as a Universal Motor (because it will run on AC or DC).

Good for drills, blenders, etc. *(high torque at low speed)*

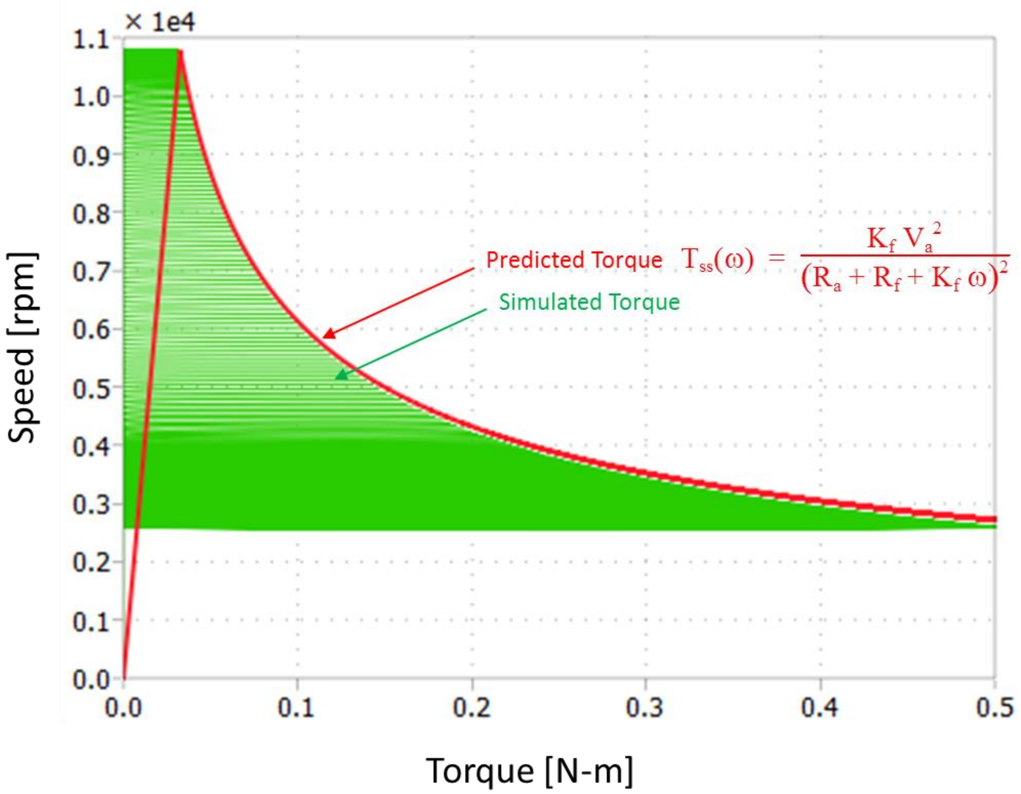
$T_{ss}(\omega) =$



PLECS Model of a Series Wound DC Motor



Series Wound DC Motor Simulated and Predicted Torque vs. Time Curves (60 Hz AC Excitation)



Series Wound DC Motor Simulated and Predicted Speed vs. Torque Curves

Limitations of DC Machines:

i) Continuous – Thermal Limits
ii) Occasional – Thermal Limits
– Load Cycle (Repeat Application of High Loads, Frequency, Duration)
iii) Instantaneous – Commutation

Torque Production in Wound Field DC Machines

$$t_{em} = K_t i_a = K_f i_f i_a$$

Why should we use armature current for speed/torque control instead of field current?

Dynamics of DC Machines

$v_a =$

$J\frac{d\omega}{dt} =$

introduce the LaPlace operator

$p \rightarrow \frac{d}{dt}$ (use 'p' instead of 's' in machines)

Equation	***Time Domain Representation***	***LaPlace Domain Representation***
Armature	$v_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_a(t)$	$V_a(p) = R_a I_a(p) + L_a \frac{dI_a(p)}{dt} + E_a(p)$
Field	$v_f(t) = R_f i_f(t) + L_f \frac{di_f(t)}{dt}$	$V_f(p) = R_f I_f(p) + L_f \frac{dI_f(p)}{dt}$
Mechanical	$J\frac{d\omega(t)}{dt} + b\omega(t) + t_L(t) = t_{em}(t)$	$J\frac{d\Omega(p)}{dt} + b\Omega(p) + T_L(p) = T_{em}(p)$

$V_a = R_a I_a + L_a p I_a + E_a$ (note : use capital letters and drop functional notation)

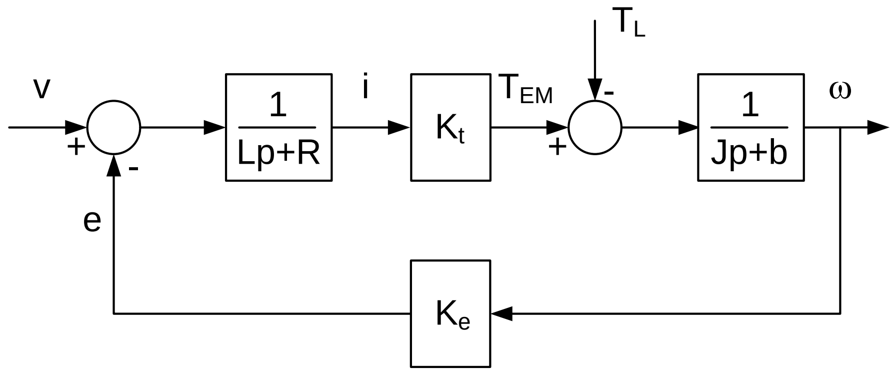
$E_a = K_e \Omega$

$T_{em} = K_t I_a$

$J\frac{d\Omega}{dt} = T_{em} - T_L - b\Omega$

$I_a =$ $\Omega =$

Block Diagram of DC Motor



Block Diagram of a DC Machine

$\Omega = \frac{1}{Jp+b}(T_{em} - T_L)$ $\qquad$ $T_{em} = K_t I_a$

$I_a = \frac{1}{L_a p+R_a}(V_a - E_a)$ $\qquad$ $E_a = K_e \Omega$

$\Omega =$

$$\Omega = \frac{K_t}{JL_a p^2+(L_a b+JR_a)p+K_e K_t+bR_a} V_a - \frac{L_a p+R_a}{JL_a p^2+(L_a b+JR_a)p+K_e K_t+bR_a} T_L$$

$\frac{\Omega}{V_a} = \frac{K_t}{JL_a p^2+(L_a b+JR_a)p+K_e K_t+bR_a}$ $\qquad$ speed/voltage TF

$\frac{\Omega}{T_L} = \frac{L_a p+R_a}{JL_a p^2+(L_a b+JR_a)p+K_e K_t+bR_a}$ $\qquad$ speed/load (disturbance) torque TF

Use $K = K_e = K_t$

for a voltage driven machine:

$$\frac{\Omega}{V_a} = \frac{\frac{K}{JL_a}}{p^2 + \frac{L_a b + JR_a}{JL_a}p + \frac{K^2 + bR_a}{JL_a}} =$$

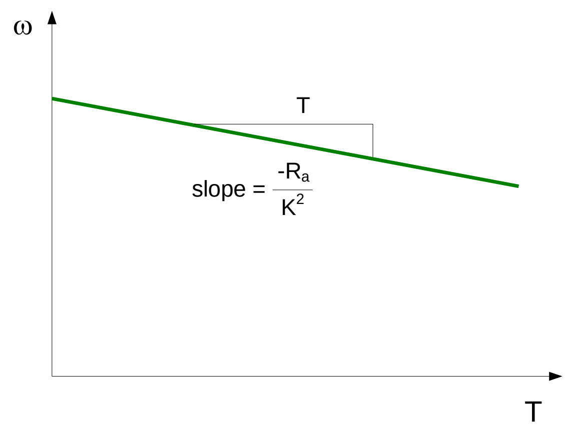
$\tau_e = \frac{L_a}{R_a}$ – electrical time constant

$\tau_m = \frac{JR_a}{K^2}$ – equivalent electromechanical time constant if electrical transients are very fast

$$J\frac{d\omega}{dt} = t_{em} - t_L - b\omega = K_t i_a - t_L - b\omega =$$

$$J\frac{d\omega}{dt} + \frac{K^2}{R_a}\omega = K_t\frac{v_a}{R_a} - t_L$$

$$\frac{d\omega}{dt} + \frac{K^2}{JR_a}\omega = K_t\frac{v_a}{JR_a} - \frac{t_L}{J}$$



Assuming electrical transients cannot be ignored:

with $b = 0$

$$\frac{\Omega}{V_a} = \frac{\frac{K}{JL_a}}{p^2 + \frac{R_a}{L_a}p + \frac{K^2}{JL_a}} = \frac{\frac{1}{K\tau_m\tau_e}}{p^2 + \frac{1}{\tau_e}p + \frac{1}{\tau_m\tau_e}}$$

Characteristic Equation: Denominator = 0

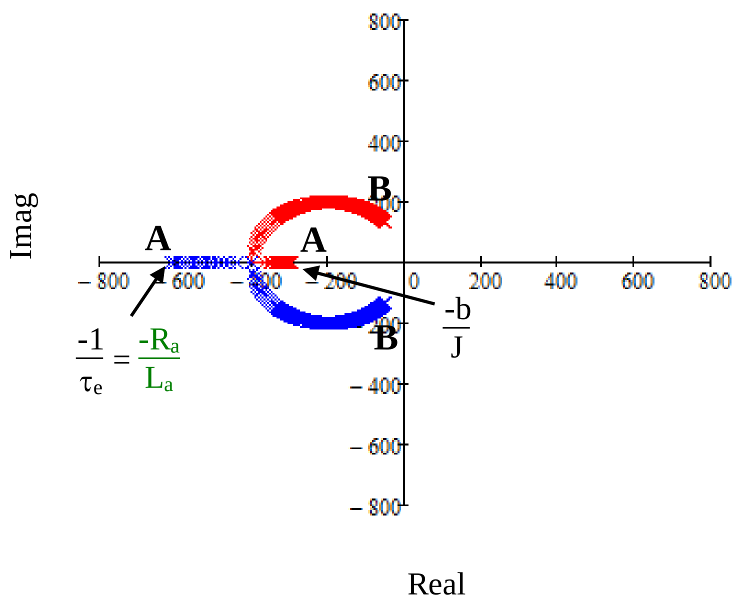
$$p^2 + \frac{1}{\tau_e}p + \frac{1}{\tau_m\tau_e} = 0$$

Eigenvalues (roots of C.E.)

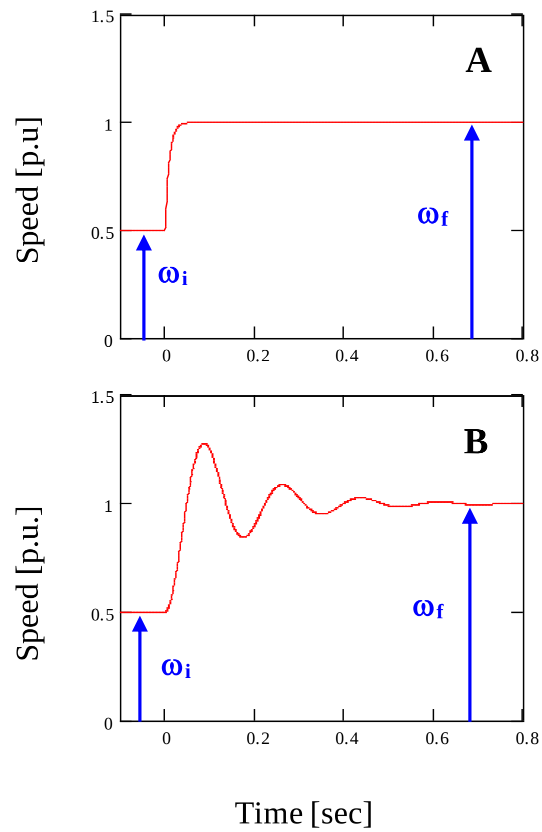
$$p = \frac{-1}{2\tau_e} \pm \frac{1}{\tau_e}\sqrt{\frac{1}{4} - \frac{\tau_e}{\tau_m}}$$

real roots if $\frac{1}{4} > \frac{\tau_e}{\tau_m} \rightarrow \tau_e < \frac{1}{4}\tau_m$ ***small machines***

complex roots if $\frac{1}{4} < \frac{\tau_e}{\tau_m} \rightarrow \tau_e > \frac{1}{4}\tau_m$ ***large machines***



Eigenvalue Plot of DC Machine



Time Response Plot of Speed with Step Change in Armature Voltage

Properties of 2nd order systems (a controls perspective)

$$\frac{C(p)}{R(p)} = \frac{\omega_n^2}{p^2+2\xi\omega_n p+\omega_n^2}$$

For step input:

$$C(p) = \frac{\omega_n^2}{p^2+2\xi\omega_n p+\omega_n^2} R(p) = \frac{\omega_n^2}{p^2+2\xi\omega_n p+\omega_n^2}\frac{1}{p}$$

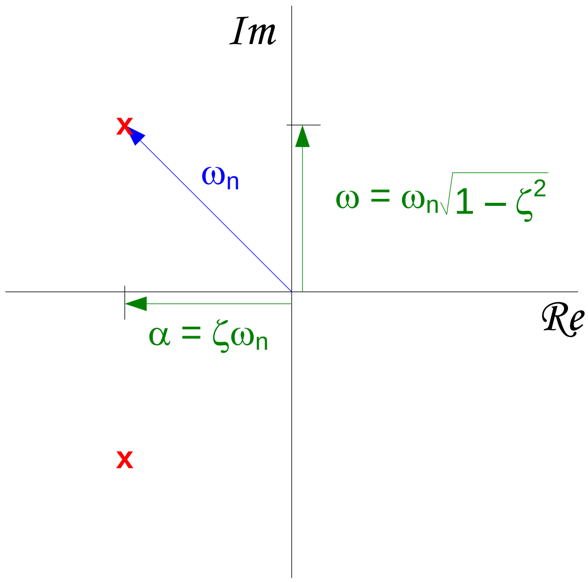
Unit Step Function

$$c(t) = \mathcal{L}^{-1}\left[\frac{\omega_n^2}{p^2+2\xi\omega_n p+\omega_n^2}\frac{1}{p}\right]$$

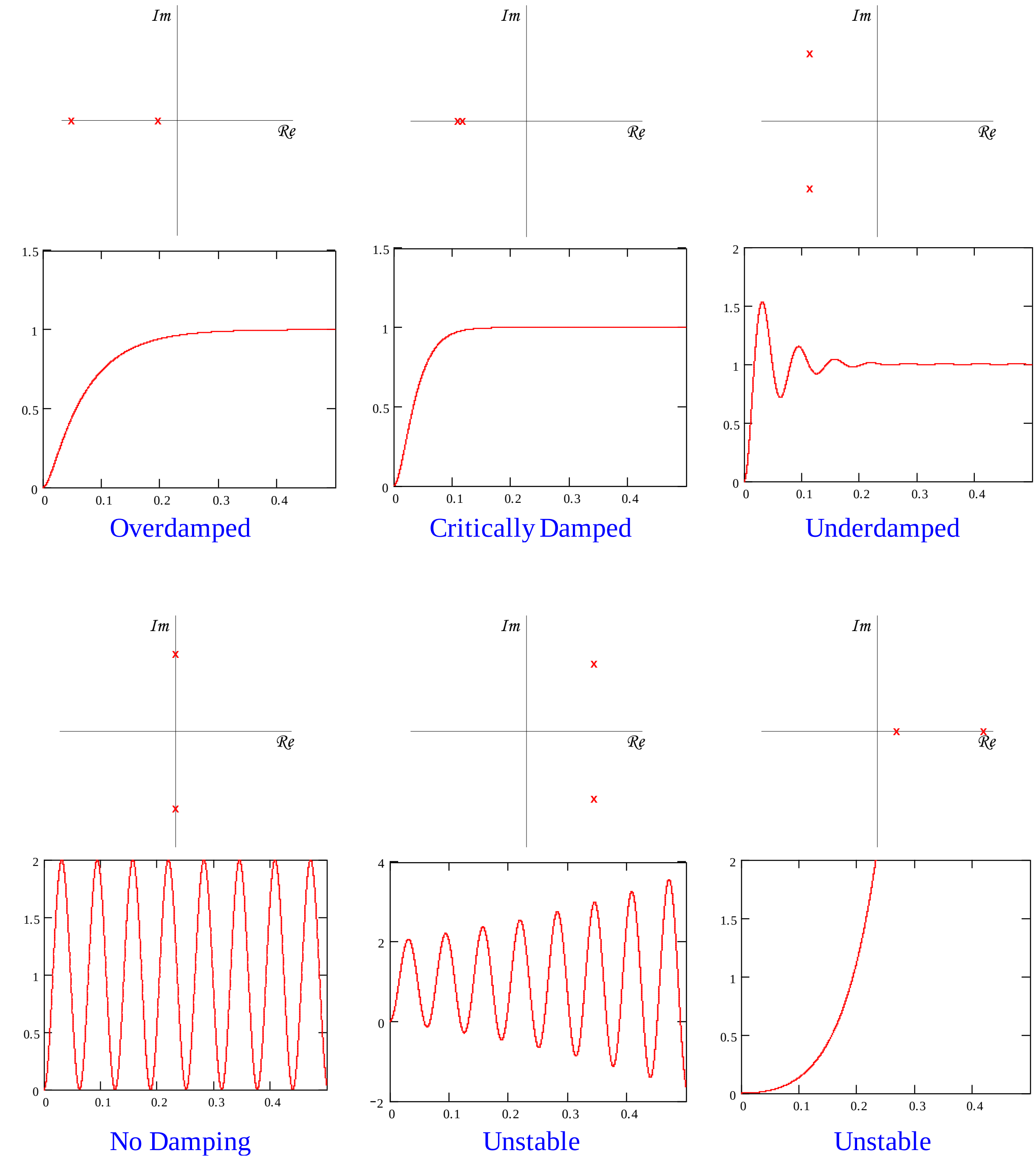
$$c(t) = 1 + \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} sin\left(\omega_n\sqrt{1-\xi^2}t - tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{-\xi}\right)\right) \quad t \geq 0$$

relationship between roots of CE and behavior of step response

$$p = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -\alpha \pm j\omega$$



Roots of a 2nd Order System



Roots of a 2nd Order System and the Corresponding Step Response

How does small perturbations in voltage effect speed?

$$\frac{\Omega(\mathrm{p})}{V_a(p)} = \frac{K_t}{JL_a p^2 + (L_a b + JR_a)p + K_e K_t + bR_a} \qquad \text{speed/voltage TF}$$

For sinusoidal inputs, set $p \rightarrow j\omega$

$$\frac{\Omega(j\omega)}{V_a(j\omega)} = \frac{K_t}{JL_a(j\omega)^2 + (L_a b + JR_a)(j\omega) + K_e K_t + bR_a} \qquad \begin{matrix}\Omega = \text{speed} \\ \omega = freq.\end{matrix}$$

$$|\Omega(j\omega)| = \left|\frac{K_t}{JL_a(j\omega)^2 + (L_a b + JR_a)(j\omega) + K_e K_t + bR_a}\right| |V_a(j\omega)|$$

$$|\Omega(j\omega)| = \frac{K_t}{\sqrt{[K_e K_t + bR_a - JL_a\omega^2]^2 + [(L_a b + JR_a)\omega]^2}} |V_a(j\omega)|$$