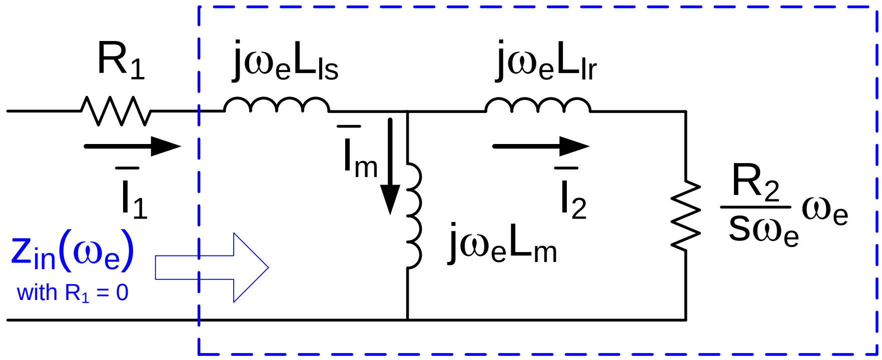
Constant Volts/Hz operation:



Induction Motor Per Phase Equivalent Circuit

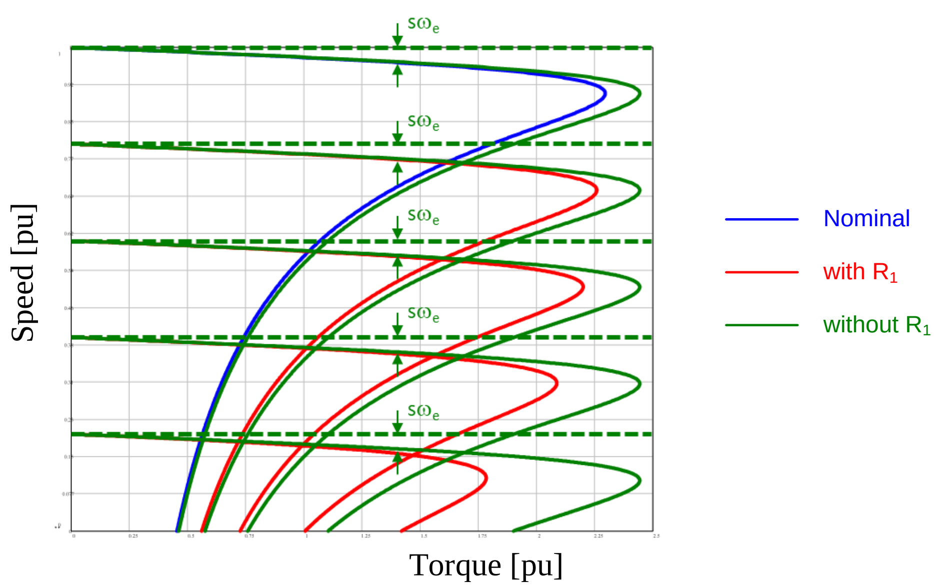
Neglecting R_1:

$$Z_{in}(\omega_e) = j\omega_e L_{ls} + j\omega_e L_m \Big/\!\!\Big/ \left(\frac{R_2}{s\omega_e}\omega_e + j\omega_e L_{lr}\right)$$

$$T_{em} = \frac{3P}{2}\frac{I_2^2 R_2}{s\omega_e}$$

Maintain constant V/f:

- Maintains circuit (machine) impedance
- Maintains stator current
- Maintains rotor current (and thus torque)
- Maintains magnetizing current (and thus flux)



Induction Motor Speed Torque Curves with Constant V/Hz

V/f control is often referred to as ‘scalar’ control.

Scalar Control:

- Low speed torque limitations
- Low dynamic response (low acceleration capability)
- Load torque determines speed transiently → does not directly control torque

Vector Control (Field Oriented Control):

- Full torque at zero speed
- High dynamic response (high acceleration capability)

Pioneers in FOC:

K. Hasse (1969) *Zur Dynamik drehzahlgeregelter Antriebe mit Stromrichtergespeisten Asynchronkurzschlufermotoren*, Ph.D. dissertation, T.U. Darmstadt

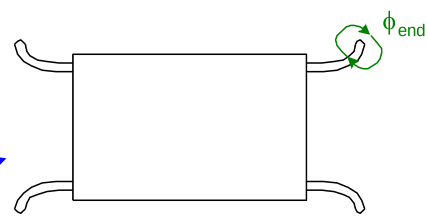
F. Blaschke (1972) *The principle of field orientation as applied to the new transvector closed loop control system for rotating field machines*, Siemens Rev

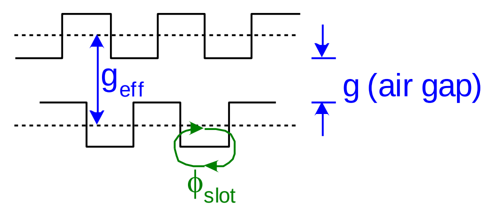
In order to understand this, we need a deeper understanding of the induction motor.

Coupled Circuit Modeling of AC Machines:

Begin with uniform airgap idealized model
Assumptions:

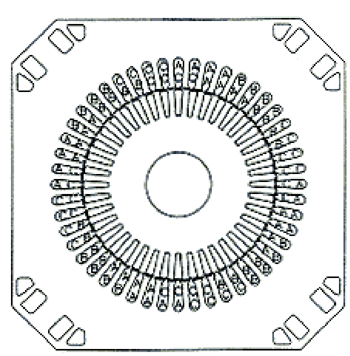
1) uniform airgap
2) ideal iron
    a. infinite permeability
    b. no core loss
3) no end effects
4) no slotting effects





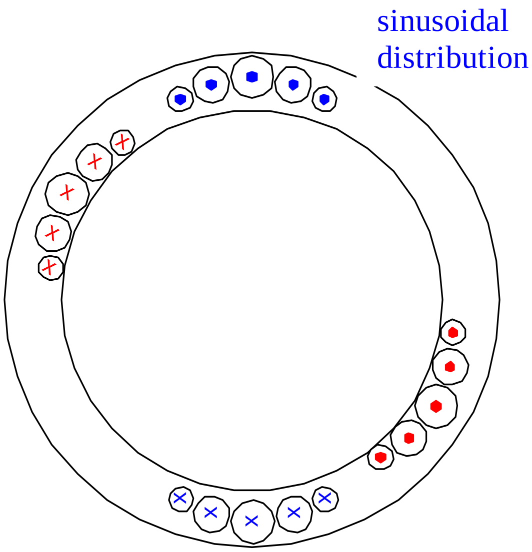
Note: assumptions 2,3, and 4 are handled by adding “correction terms”

stator and rotor structure of induction machine



http://www.reliance.com/prodserv/motgen/b7097_2.htm

Winding Inductances:



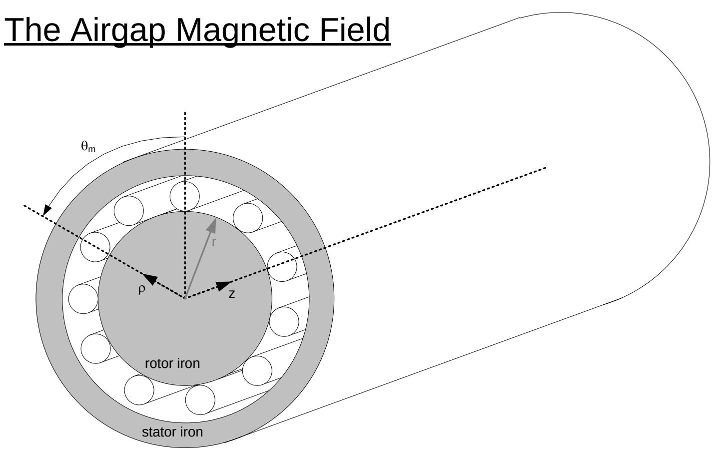
Sinusoidally Distributed Stator and Rotor Windings

Winding Inductances:

$$\lambda_a = L_{aa}\, i_a + L_{ab}\, i_b + L_{ac}\, i_c + \ldots$$

$$L_{aa} = \left.\frac{\lambda_a}{i_a}\right|_{i_b = i_c = \ldots = 0}$$

$$L_{ab} = \left.\frac{\lambda_a}{i_b}\right|_{i_a = i_c = \ldots = 0}$$



Cylindrical Coordinate System chosen for simplicity

Coordinates: ρ, θ_m, z

z-axis chosen along axis of machine

with ρ = r (rotor radius), θ_m defines a point in the air gap

In general, $H = H(\rho,\theta_m,z)$ - Magnetic Field Intensity in air gap will be a function of, or in other words, have components in each of the three directions. Since we're concerned with finding the flux linking a winding placed upon the surface of the rotor or stator, only the component normal to the surface (radial component) is significant. We need to find the radial component of $H(\rho,\theta_m,z)$.

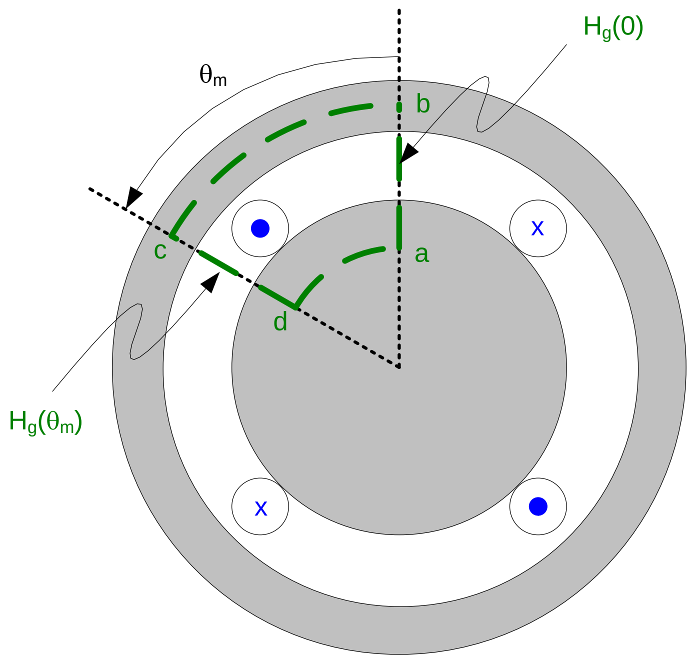
With assumption #3 (no end effects), magnetic field intensity is independent of the z coordinate.
$H = H(\rho,\theta_m)$.

If $r >> g$ (rotor radius much greater than air gap), variation with ρ will be negligibly small.
Therefore, $H = H(\theta_m)$.

Ampere's Law:

$$\oint \bar{H} \bullet d\bar{\ell} = \iint \bar{J} \bullet d\bar{A} = \text{current enclosed}$$

any closed path — current density — surface area



Integrate along abcda
Note: H_{bc} & H_{ad} must equal zero with assumption #2a (infinite permeability). Along air gap portions of path, MMF is simply air gap length (g) times air gap field intensity (H_g). Choose reference to be positive for H_g directed inward.

$$g[H_g(\theta) - H_g(0)] = \text{current enclosed} = \eta(\theta)\, i$$

net number of conductors carrying positive current

Radial component of air gap field intensity at θ_m

Note: $\eta(\theta)$ is dependent upon where you start

Guass' Law:

$$\oint_s \bar{B} \bullet d\bar{A} = \mu_o \oint_s \bar{H} \bullet d\bar{A} = 0$$

(total flux crossing air gap, or leaving rotor, is zero)

$dA =$

$$\oint_s \bar{B} \bullet d\bar{A} = \int_z \int_{\theta_m} B_g(\theta_m)\, r\, d\theta_m\, d_z = r\,\ell \int_{\theta_m} B_g(\theta_m)\, d\theta_m = \mu_o\, r\,\ell \int_0^{2\pi} H_g(\theta_m)\, d\theta_m$$

from above

$$H_g(\theta_m) = \frac{i}{g}\,\eta(\theta) + H_g(0)$$

$$\int_0^{2\pi}\left[\frac{i}{g}\eta(\theta_m) + H_g(0)\right]d\theta_m = 0 \;\rightarrow\; \frac{i}{g}\int_0^{2\pi}\eta(\theta_m)\,d\theta_m + 2\pi H_g(0) = 0$$

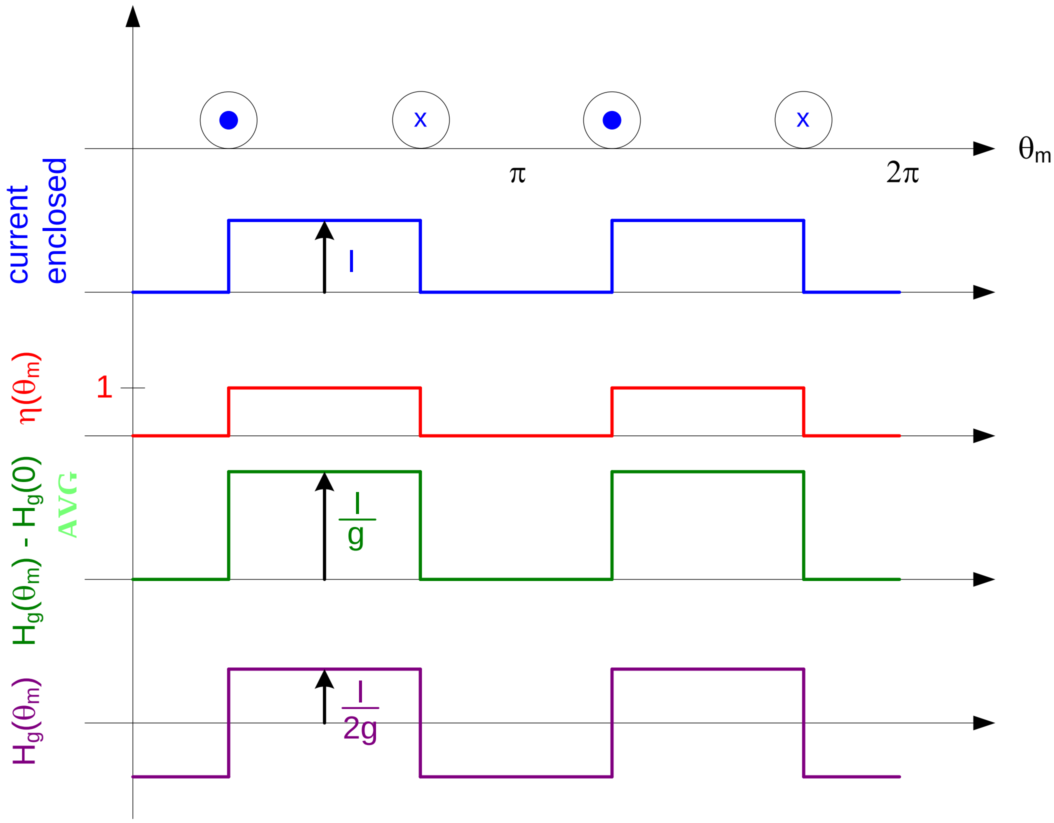
$$H_g(0) = -\frac{i}{g}\left\{\frac{1}{2\pi}\int_0^{2\pi}\eta(\theta_m)\,d\theta_m\right\} = -\frac{i}{g}\,\mathrm{AVG}\{\eta(\theta_m)\}$$

therefore $H_g(\theta_m) =$

define $N(\theta_m) = \left[\eta(\theta_m) - \mathrm{AVG}\{\eta(\theta_m)\}\right]$ (called the winding function)

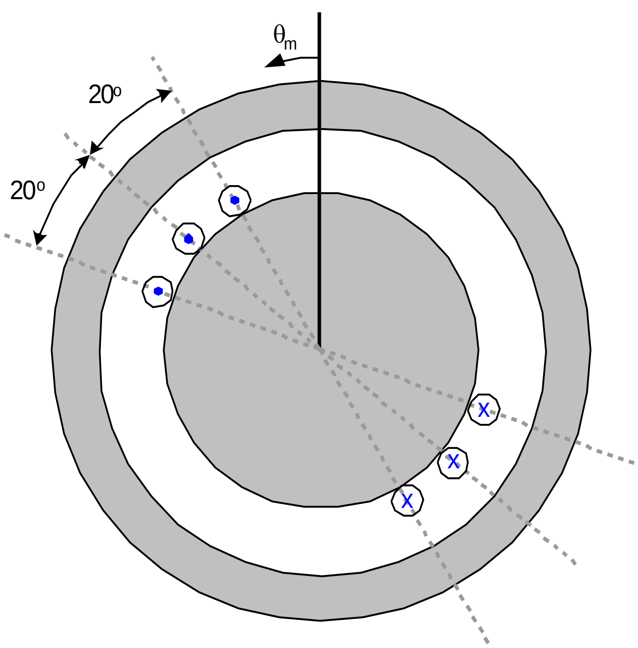
then

$$\boxed{H_g(\theta_m) \triangleq \frac{i}{g}N(\theta_m)}$$

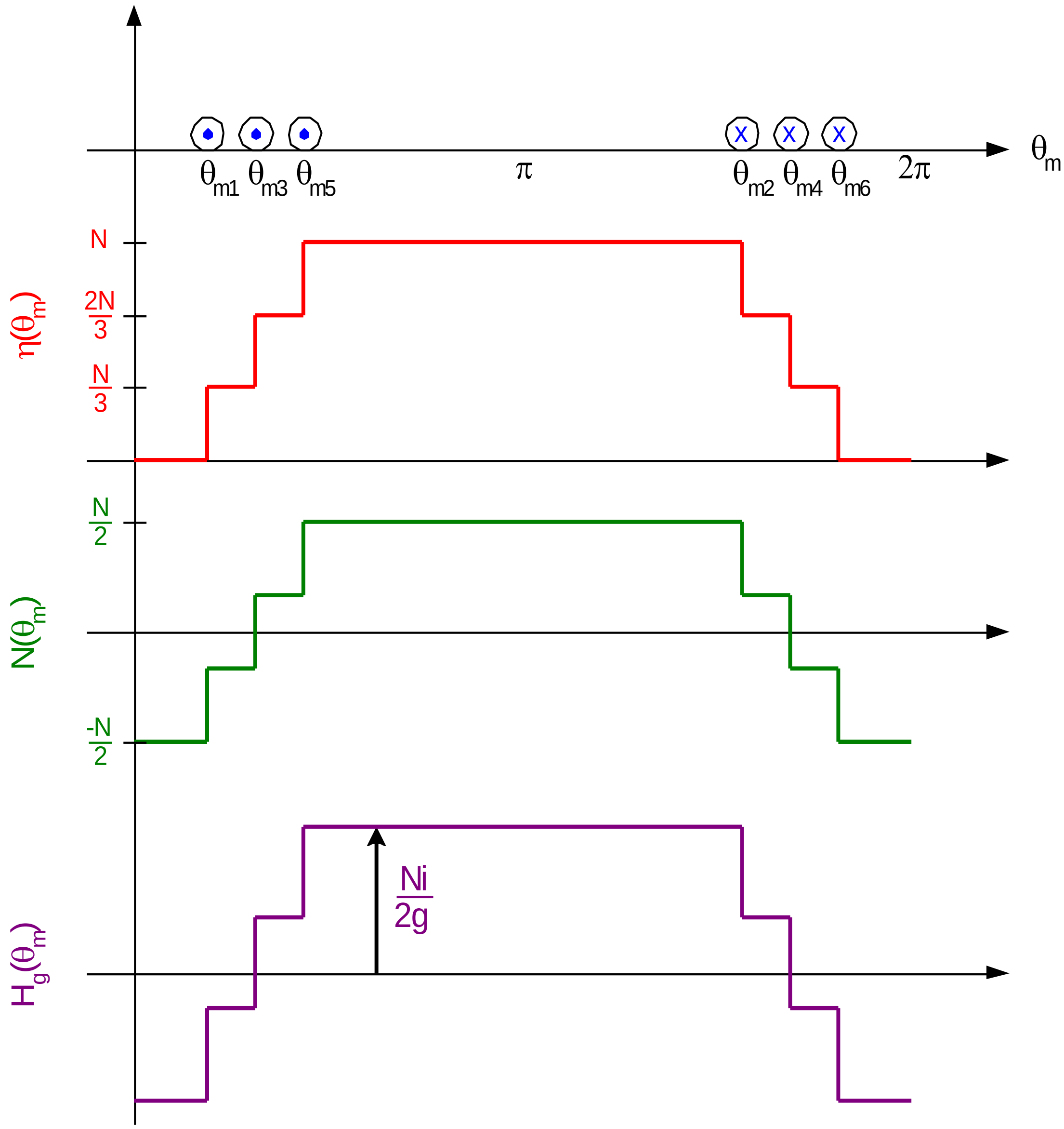


Winding Function from Winding Distribution Above

Example:



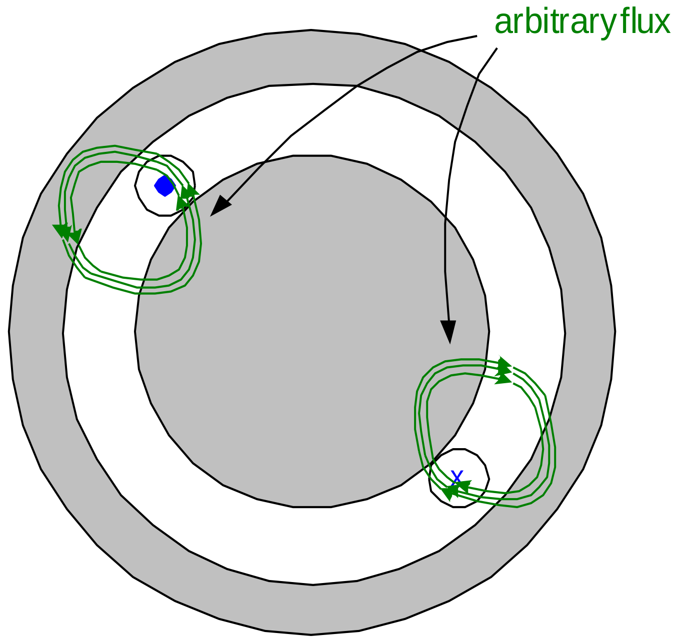
One phase of a three phase winding
3 slots/pole/phase (18 slot stator)

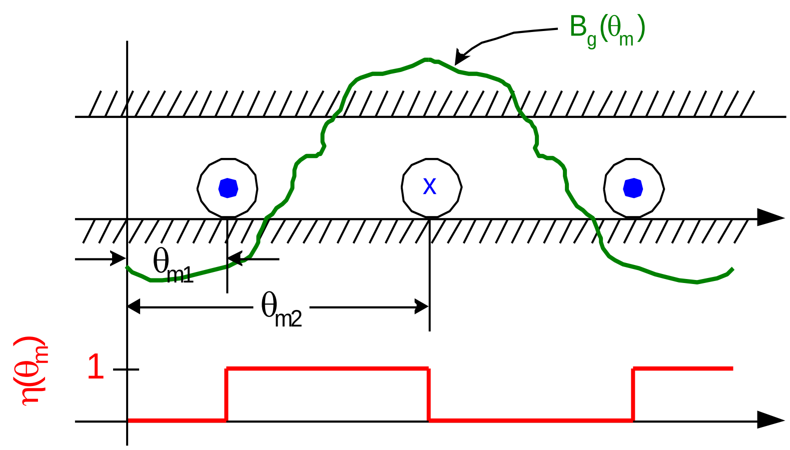


Peak value of $N(\theta_m)$:

$$N_{pk}(\theta_m) = \frac{\text{total \# of turns}}{\text{\# of poles}}$$

<u>Flux Linkage and Inductance:</u>



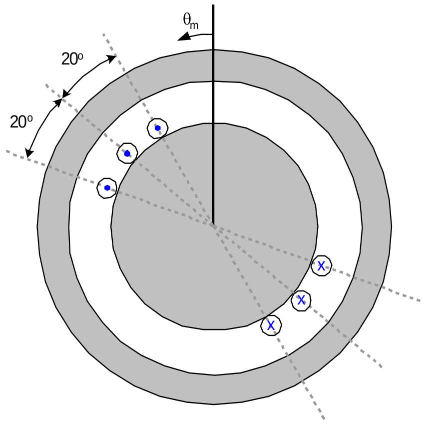


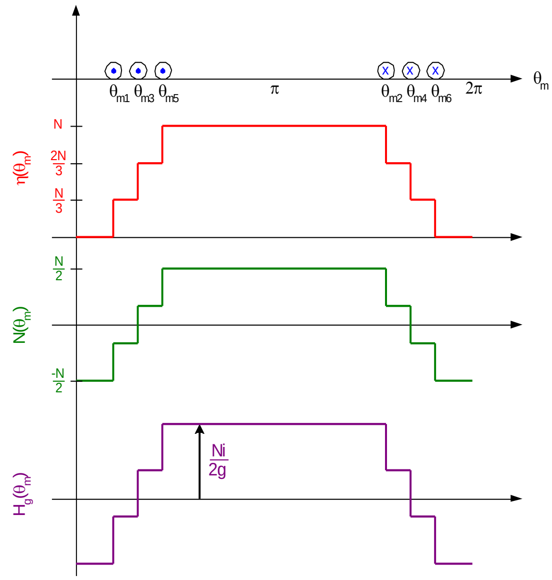
$$\lambda \;=\; r\,\ell \int_{\theta_1}^{\theta_2} B_g(\theta_m)\, d\theta_m \;=$$

$$\lambda \;=\; r\,\ell \int_{0}^{2\pi} \eta\big(\theta_m\big)\, B_g(\theta_m)\, d\theta_m$$

Counting Function

from previous example:





$$\lambda \;=$$

$$= r\,\ell\left[\int_{\theta_1}^{\theta_3} B_g(\theta_m)\,d\theta_m + 2\int_{\theta_3}^{\theta_5} B_g(\theta_m)\,d\theta_m + 3\int_{\theta_5}^{\theta_2} B_g(\theta_m)\,d\theta_m + 2\int_{\theta_2}^{\theta_4} B_g(\theta_m)\,d\theta_m + \int_{\theta_4}^{\theta_6} B_g(\theta_m)\,d\theta_m\right]$$

$$= r\,\ell\int_0^{2\pi} \eta(\theta_m)\,B_g(\theta_m)\,d\theta_m$$

but this is dependent upon where we choose our reference

$$N(\theta_m) = \left[\eta(\theta_m) - \mathrm{AVG}\{\eta(\theta_m)\}\right] \rightarrow \eta(\theta_m) = \left[N(\theta_m) + \mathrm{AVG}\{\eta(\theta_m)\}\right]$$

So

$$\lambda = r\,\ell\int_0^{2\pi} \eta(\theta_m)\,B_g(\theta_m)\,d\theta_m = r\,\ell\int_0^{2\pi}\left[N(\theta_m) + \mathrm{AVG}\{\eta(\theta_m)\}\right]B_g(\theta_m)\,d\theta_m$$

$$= r\,\ell\int_0^{2\pi} N(\theta_m)\,B_g(\theta_m)\,d\theta_m + r\,\ell\,\mathrm{AVG}\{\eta(\theta_m)\}\int_0^{2\pi} B_g(\theta_m)\,d\theta_m$$

$$\boxed{\lambda = r\,\ell\int_0^{2\pi} N(\theta_m)\,B_g(\theta_m)\,d\theta_m}$$

for the case of self inductance, $B_g(\theta_m)$ is the field produced by the winding itself:

$$B_{ga}(\theta_m) = \mu_o H_{ga}(\theta_m) = \frac{\mu_o i_a}{g} N_a(\theta_m)$$

The self-inductance is therefore:

$$\boxed{L_a = \frac{\lambda_{aa}}{i_a} = \frac{\mu_o r \ell}{g}\int_0^{2\pi} \left[N_a(\theta_m)\right]^2 d\theta_m}$$

for Mutual Inductance:

$$B_{ga}(\theta_m) = \frac{\mu_o i_b}{g} N_b(\theta_m)$$

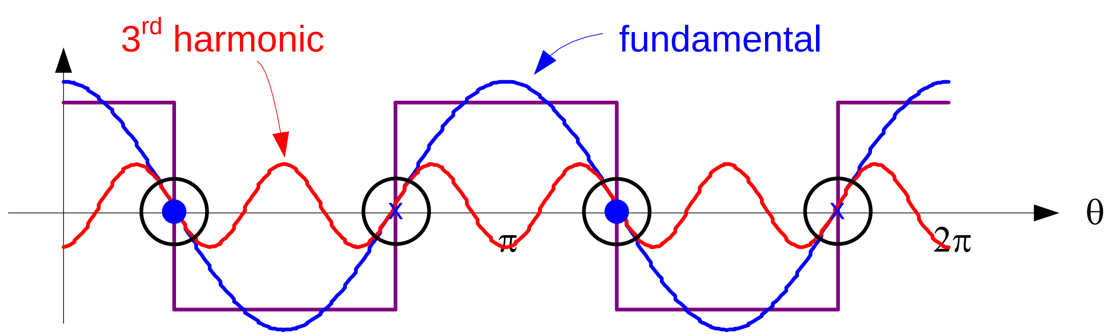
$$\boxed{L_{ab} = \frac{\lambda_{ab}}{i_b} = \frac{\mu_o r \ell}{g}\int_0^{2\pi} N_a(\theta_m) N_b(\theta_m)\, d\theta_m}$$

Recall for machines with more than 2 poles to express quantities in electrical (rather than mechanical) radians, we have:

$$\theta = \frac{P}{2}\theta_m$$

θ - electrical angle
θ_m – mechanical angle
P – number of motor poles

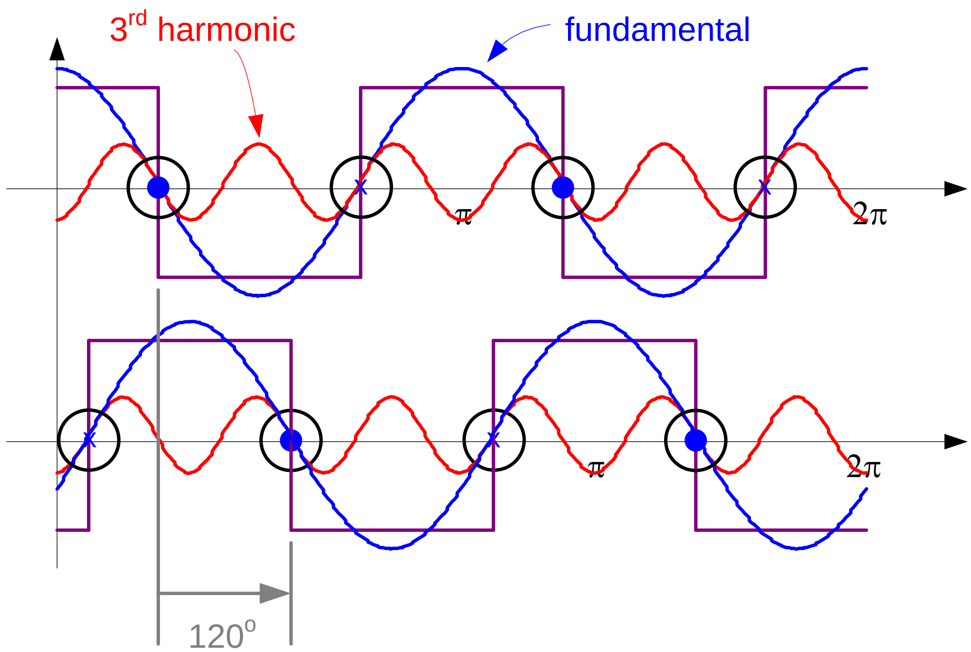
Concentrated Windings:



Winding Function for a Concentrated Winding

$$N(\theta) = \frac{4}{\pi} Np \left[\cos(\theta) - \frac{1}{3}\cos(3\theta) + \frac{1}{5}\cos(5\theta) \ldots\right]$$

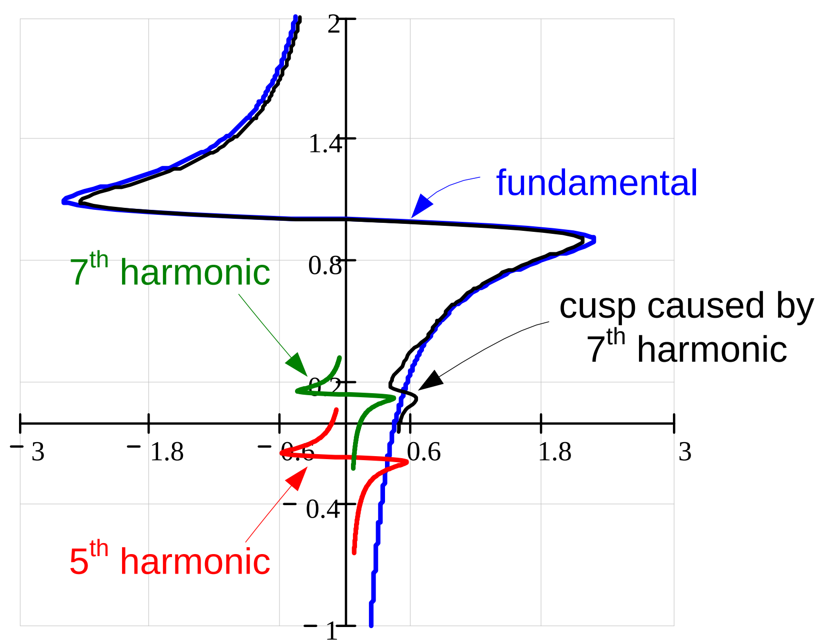
Elimination of 3rd Harmonic:



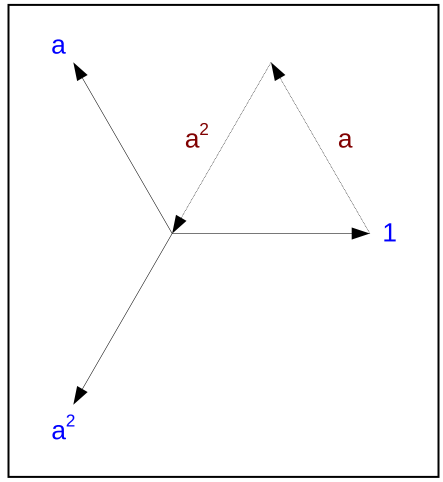
Winding Function for a Concentrated Winding of a 3 Phase Machine Showing 2 Phases

Displace turns by 120 (electrical) degrees to eliminate 3rd spatial harmonic

Side note on space harmonics with induction machines:



Speed Torque Curve of Induction Machine with Space Harmonics



Typically

a) No even harmonics (north pole is symmetric with south pole)
b) Only sine or only cosine waves (1/4 wave symmetry)

$$N(\theta) = \sum_{h=1}^{\infty} N_h \cos(h\,\theta + \phi_h)$$

For sinusoidal windings:

$$N(\theta) = N_h \cos(h\,\theta + \phi_h)$$

$$h\frac{P}{2}\theta_m$$

$$L_h = \frac{\mu_o r \ell}{g}\int_0^{2\pi} N_h^2 \cos^2(h\,\theta + \phi_h)\,d\theta_m$$

$$\boxed{L_h = \frac{\mu_o r \ell \pi}{g} N_h^2} \quad \textit{Self-Inductance}$$

$$L_{hk} = \frac{\mu_o r \ell}{g}\int_0^{2\pi} N_h \cos(h\,\theta + \phi_h)\, N_k \cos(k\,\theta + \phi_k)\,d\theta_m$$

$$= \frac{\mu_o r \ell N_h N_k}{g}\int_0^{2\pi} \cos\left(h\frac{P}{2}\theta + \phi_h\right)\cos\left(k\frac{P}{2}\theta + \phi_k\right)d\theta_m$$

$$= 0 \text{ for } h \neq k$$

There is no magnetic interaction between sinusoidal windings with different number of poles.

Mutual Inductances

$$N_a(\theta) = N_{ha}\cos(h\theta + \phi_{ha})$$

$$N_b(\theta) = N_{hb}\cos(h\theta + \phi_{hb})$$

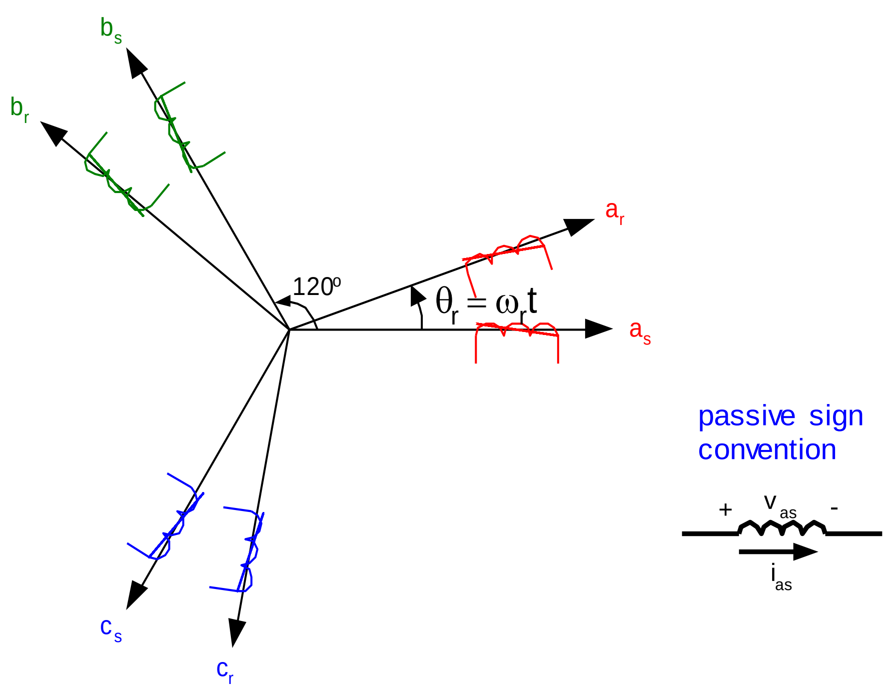
$$L_{hab} = \frac{\mu_o r \ell}{g}\int_0^{2\pi} N_{ha}\cos(h\theta + \phi_{ha})\, N_{hb}\cos(h\theta + \phi_{hb})\, d\theta_m$$

$$= \frac{\mu_o r \ell \pi N_{ha} N_{hb}}{g}\cos(\phi_{ha} - \phi_{hb})$$

$$\boxed{L_{hab} = \frac{\mu_o r \ell \pi N_{ha} N_{hb}}{g}\cos(h\delta_h)}$$ *Mutual Inductance*

angle between windings on fundamental scale

3φ Machine Model:



Space Vectors of a Three Phase Machine

- Assume sinusoidal winding distribution

$$N_a(\theta) = N_s \cos(\theta)$$
$$N_b(\theta) = N_s \cos(\theta - 120^o)$$
$$N_c(\theta) = N_s \cos(\theta + 120^o)$$

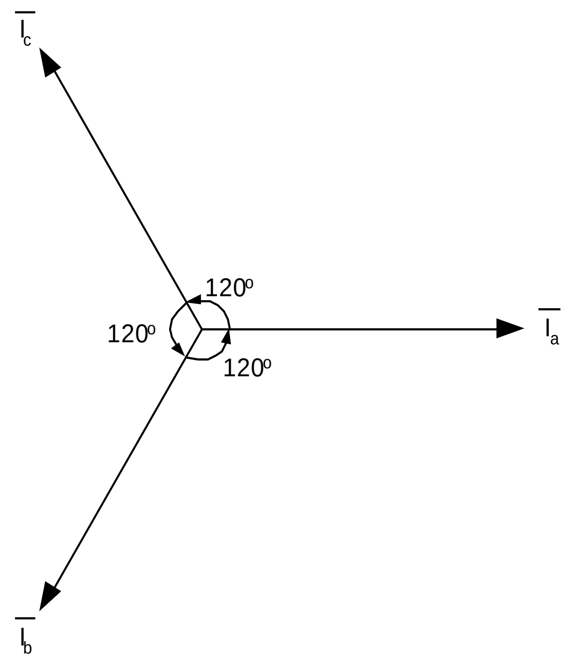
Note: Do not confuse phasors with space vectors (space vectors are spatial location of winding axis, phasors are temporal (time varying) vectors)

Example: for a time based system:

$$i_a(t) = \sqrt{2}I \cos(\omega t)$$
$$i_b(t) = \sqrt{2}I \cos(\omega t - 120^o)$$
$$i_c(t) = \sqrt{2}I \cos(\omega t + 120^o)$$

<u>Phasors:</u>

$$\bar{I}_a = I \angle 0^o$$
$$\bar{I}_b = I \angle -120^o$$
$$\bar{I}_c = I \angle 120^o$$



Phasor Representation of Temporal Signals

Total flux linkage for a 3ϕ machine in 1ϕ:

λ_{as} =

$v_{as} = r_s\, i_{as} + p\,\lambda_{as}$ → voltage equation

with sinusoidal windings,

Stator Self Inductance:

$$L_{as} = \frac{\mu_o r \ell \pi}{g} N_s^2 + L_{ls}$$

$$\frac{\mu_o r \ell \pi}{g} N_s^2 \triangleq \text{Magnetizing Inductance} = L_{ms}$$

$L_{ls} \triangleq$ Leakage Inductance (accounts for end turn, slot, etc.)

$$\boxed{L_{as} = L_{ms} + L_{ls} = L_{bs} = L_{cs}}$$

Stator Mutual Inductance (with other Stator Windings):

$$\boxed{L_{abs} = \frac{-1}{2} L_{ms} = L_{acs} = L_{bcs}}$$

Note: Negative sign denotes flux from other winding is opposite in direction of that produced by current in its own coil.

Stator Mutual Inductance (with Rotor Windings):

$$L_{asar} = \frac{\mu_o r \ell \pi}{g} N_s N_r \cos(\theta_r) \rightarrow \text{ from } L_{hab} = \frac{\mu_o r \ell \pi N_{ha} N_{hb}}{g} \cos(h\delta_h) \text{ with } h = 1,\ \delta = \theta_r$$

$$\boxed{L_{asar} = \frac{N_r}{N_s} L_{ms} \cos(\theta_r)}$$

$$\boxed{L_{asbr} = \frac{N_r}{N_s} L_{ms} \cos(\theta_r + 120^o)}$$

$$\boxed{L_{ascr} = \frac{N_r}{N_s} L_{ms} \cos(\theta_r - 120^o)}$$

Stator Voltage equation:

$$\lambda_{as} = L_{as} i_{as} + L_{abs} i_{bs} + L_{acs} i_{cs} + L_{asar} i_{ar} + L_{asbr} i_{br} + L_{ascr} i_{cr}$$

$$v_{as} = r_s i_{as} + p\,\lambda_{as}$$

$$= r_s i_{as} + L_{ls}\, p\, i_{as} + L_{ms}\, p\left(i_{as} - \frac{1}{2} i_{bs} - \frac{1}{2} i_{cs}\right) +$$

$$\frac{N_r}{N_s} L_{ms}\, p\left[i_{ar}\cos(\theta_r) + i_{br}\cos(\theta_r + 120^o) + i_{cr}\cos(\theta_r - 120^o)\right]$$

$$p\left(i_{ar}\cos(\theta_r)\right) =$$

Observations:

- These equations describe a set of nonlinear (because of product of current , position, and speed), coupled set of differential equations with time varying coefficients!
- Will show this is an 8^{th} order (with mechanical system) nonlinear, cross coupled set of differential equations with time varying coefficients!
- Special case: $\frac{d\theta_r}{dt} = \text{constant} = \omega_r$

  $$\theta_r = \omega_r t + \theta_{ro}$$

  equations are linear but still have time varying (periodic) coefficients