

## Steady State Induction Machine Field Orientation Summary:

$$\lambda_{qr}^e = 0$$

[Comparison to Transient Case](#)

$$1) \ I_{qr}^e = \frac{-L_m}{L_r} I_{qs}^e$$

$$2) \ I_{dr}^e = 0$$

$$3) \ \lambda_{dr}^e = L_m I_{ds}^e$$

$$4) \ s \omega_e = \frac{r_r}{L_r} \frac{I_{qs}^e}{I_{ds}^e} = \frac{1}{\tau_r} \frac{I_{qs}^e}{I_{ds}^e}$$

$$5) \ T_e = \frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} \lambda_{dr}^e I_{qs}^e$$

$$= \frac{3}{2} \frac{P}{2} \frac{L_m^2}{L_r} I_{ds}^e I_{qs}^e$$

## Dynamic State Induction Machine Field Orientation:

$$[p + j(\omega_e - \omega_r)] \bar{\lambda}_{qdr}^e + r_r \bar{i}_{qdr}^e = 0$$

$$r_r i_{qr}^e + p \lambda_{qr}^e + (\omega_e - \omega_r) \lambda_{dr}^e = 0$$

$$r_r i_{dr}^e + p \lambda_{dr}^e - (\omega_e - \omega_r) \lambda_{qr}^e = 0$$

$$\lambda_{qr}^e = 0 \rightarrow \boxed{i_{qs}^e = \frac{-L_r}{L_m} i_{qr}^e}$$

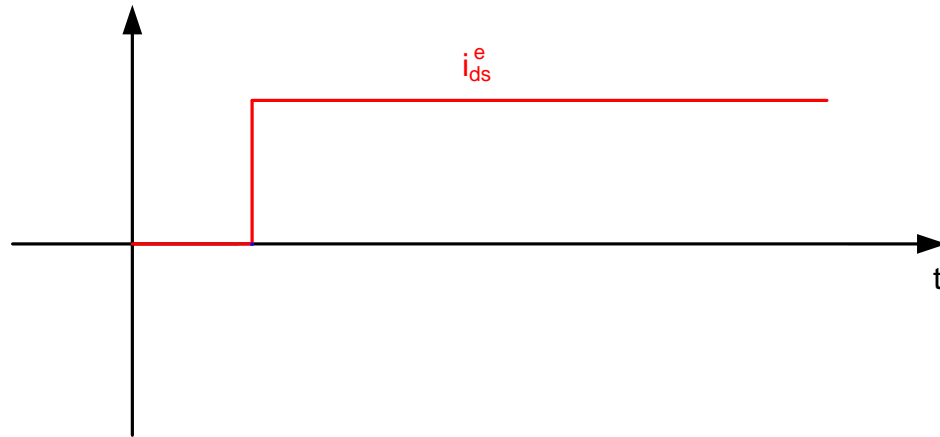
$$r_r i_{qr}^e + s \omega_e \lambda_{dr}^e = 0 \rightarrow \boxed{s \omega_e = \frac{-r_r i_{qr}^e}{\lambda_{dr}^e} = \frac{r_r}{L_r} \frac{L_m i_{qs}^e}{\lambda_{dr}^e}}$$

a) Eliminate  $\lambda_{dr}^e = L_m i_{ds}^e + L_r i_{dr}^e$

$$r_r i_{dr}^e + L_m p i_{ds}^e + L_r p i_{dr}^e = 0$$

$$(r_r + L_r p) i_{dr}^e = -p L_m i_{ds}^e$$

$$\frac{\dot{i}_{dr}^e}{i_{ds}^e} = \frac{-L_m p}{r_r + L_r p}$$

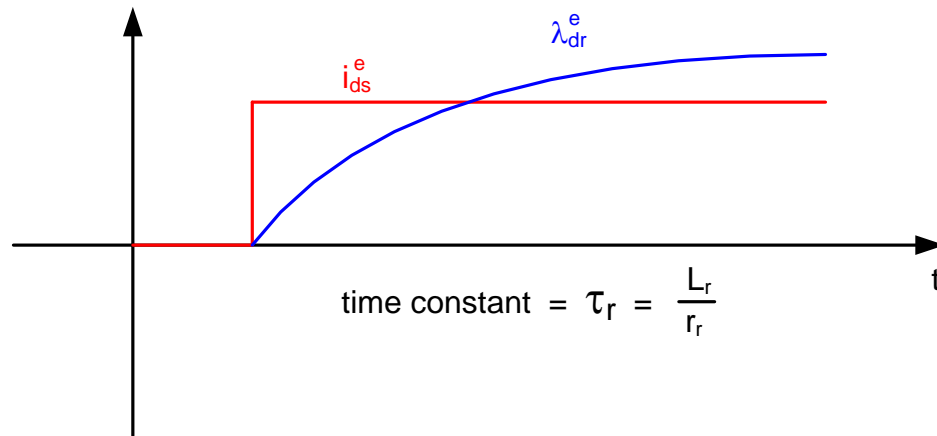


**Change in Rotor d-Axis Current with a Change in Stator d-Axis Current**

b) Eliminate  $i_{dr}^e = \frac{\lambda_{dr}^e}{L_r} - \frac{L_m}{L_r} i_{ds}^e$

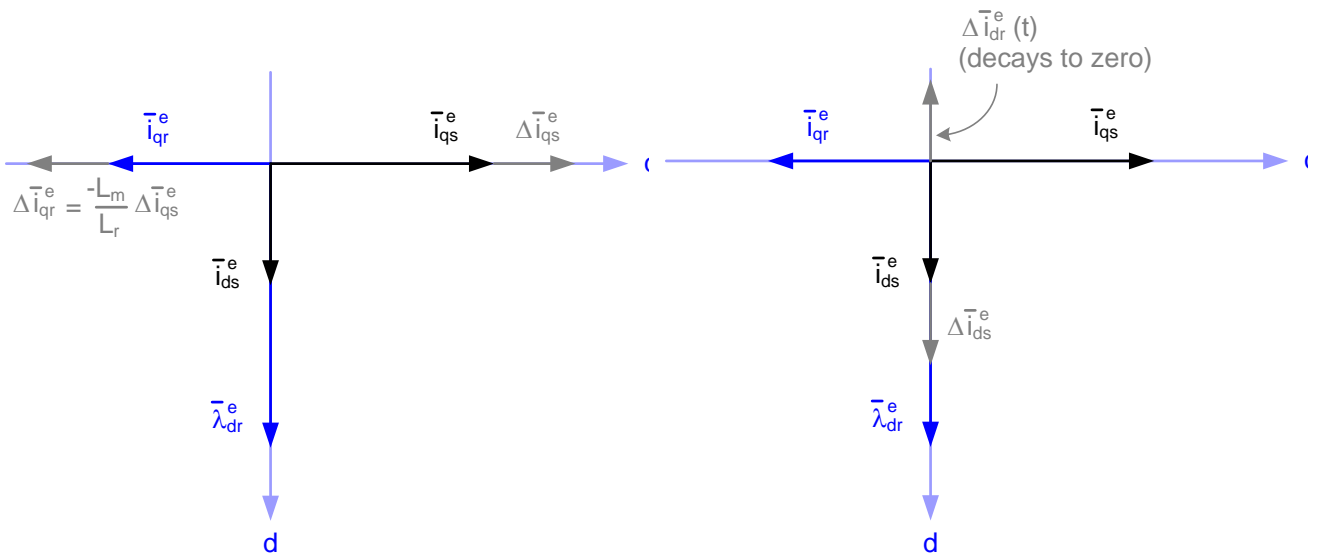
$$\frac{r_r}{L_r} \lambda_{dr}^e - \frac{r_r L_m}{L_r} i_{ds}^e + p \lambda_{dr}^e = 0$$

$$\left( p + \frac{r_r}{L_r} \right) \lambda_{dr}^e = \frac{r_r}{L_r} L_m i_{ds}^e$$



## Change in Rotor Flux with a Change in Stator d-Axis Current

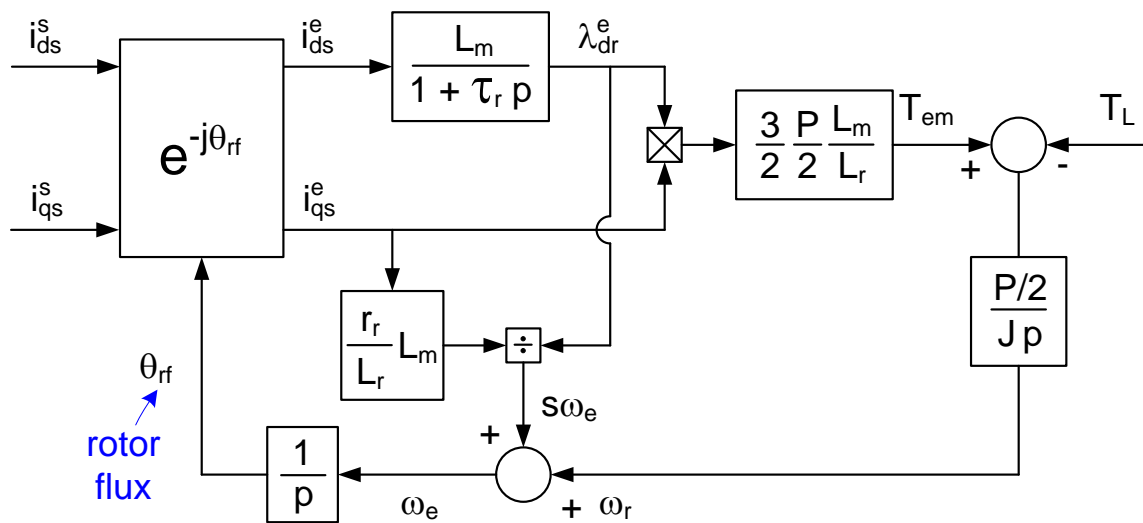
Major difference in transient case is that there can be a d-axis rotor current induced by any change in d-axis stator current that is not slip related.



## Fully Transient Indirect Field Orientation of Induction Machine with $i_{ds}^{e*}$ and $i_{qs}^{e*}$ :

$$\lambda_{dr}^e = \frac{L_m}{1 + p\tau_r} i_{ds}^e$$

$$s\omega_e^* = \frac{\left(\frac{1}{\hat{\tau}_r}\right) i_{qs}^{e*}}{\left(\frac{1}{1 + p\hat{\tau}_r}\right) i_{ds}^{e*}}$$

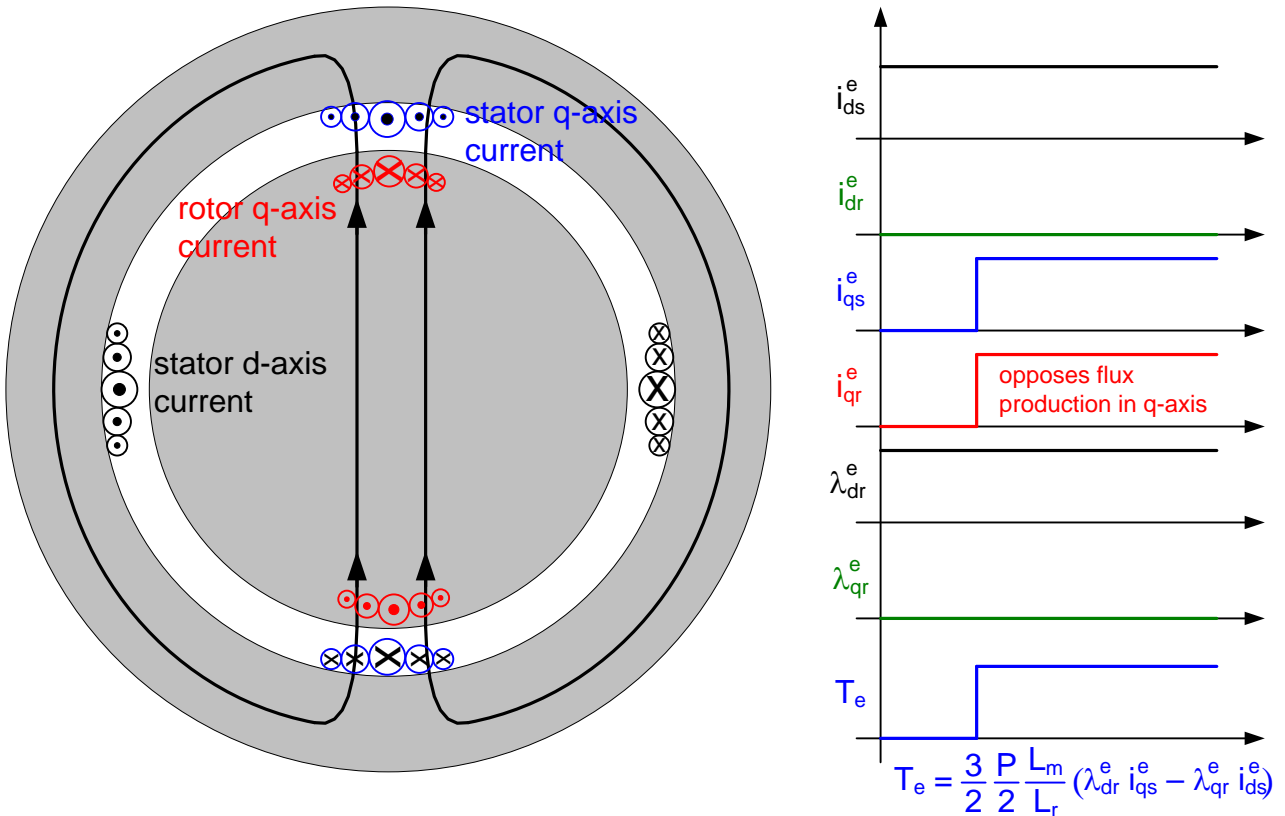


**Current Fed Induction Machine in Rotor Flux Oriented Reference Frame**

Indirect Field Orientation uses slip relationship to determine rotor flux

## Recap: Induction Machine Field Orientation Using dq Model

A static look at Field Orientation in an Induction Machine:



## **Torque Production in IM with Steady State d-Axis Current and Sudden Application of q-Axis Current**

So far we have discussed Indirect Field Orientation (IFO). IFO uses slip relationship to determine the spatial location of rotor flux.

## “Direct” Field Orientation:

Rotor flux angle is “directly” measured (or estimated) rather than calculating it using the slip relationship. The two methods are:

- a) Using air gap flux sensors (not common)

$$\bar{\lambda}_{qdm}^s = L_m (\bar{i}_{qds}^s + \bar{i}_{qdr}^s) \rightarrow$$

$$\bar{\lambda}_{qdr}^s = \bar{\lambda}_{qdm}^s + L_{lr} \bar{i}_{qdr}^s$$

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Problems

- 1) Sensors in airgap
- 2)  $L_{lr}$  variations (w/load)

- b) Use terminal voltage

$$\bar{\lambda}_{qds}^s = \frac{1}{p} (\bar{v}_{qds}^s - r_s \bar{i}_{qds}^s) \quad \text{integrate stator voltage to get stator flux}$$

$$\bar{\lambda}_{qdr}^s = L_m \bar{i}_{qds}^s + L_r \bar{i}_{qdr}^s$$

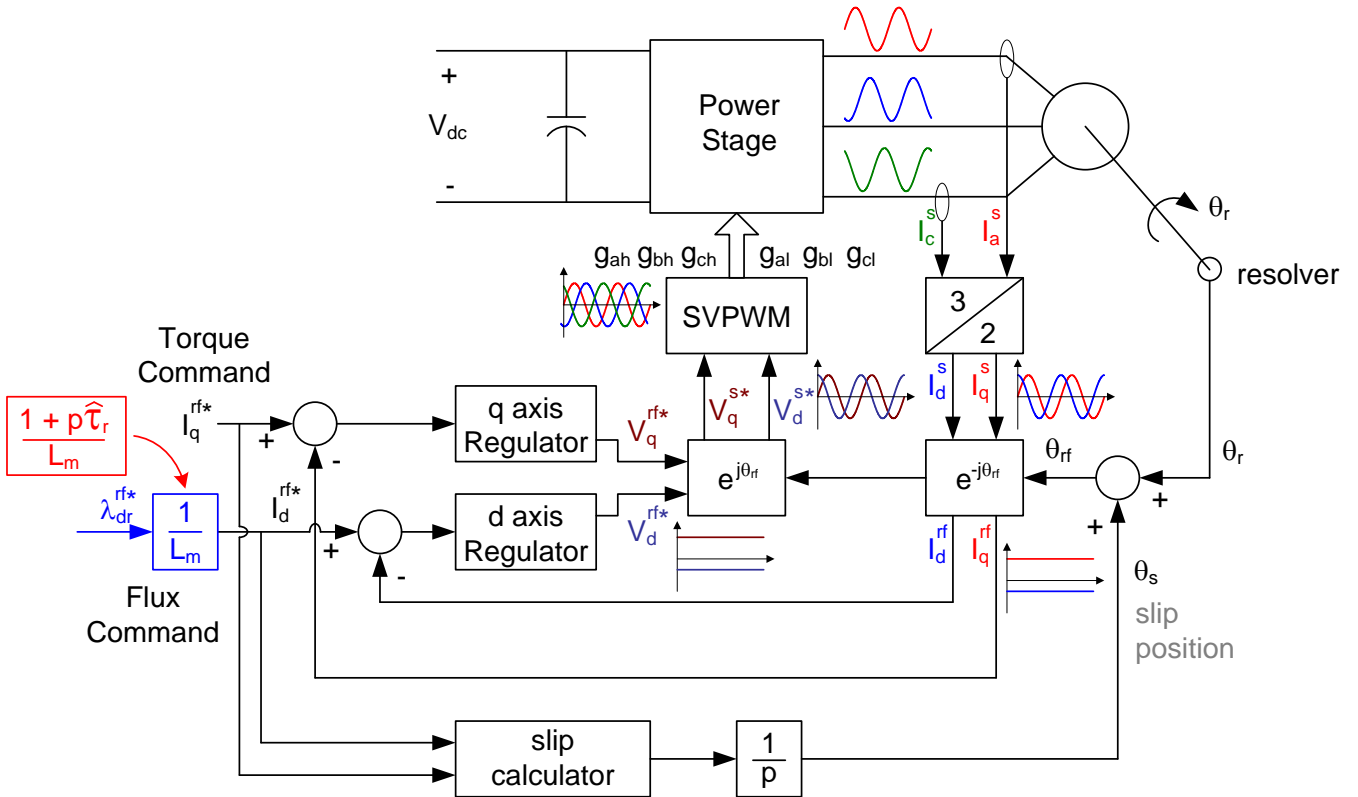
$$\bar{i}_{qdr}^s = \frac{\bar{\lambda}_{qds}^s}{L_m} - \frac{L_s}{L_m} \bar{i}_{qds}^s \quad \text{need rotor currents to get rotor flux}$$

$$\rightarrow \bar{\lambda}_{qdr}^s = \frac{L_r}{L_m} (\bar{\lambda}_{qds}^s - L_s' \bar{i}_{qds}^s) \quad \text{solve for rotor flux from stator flux and current}$$

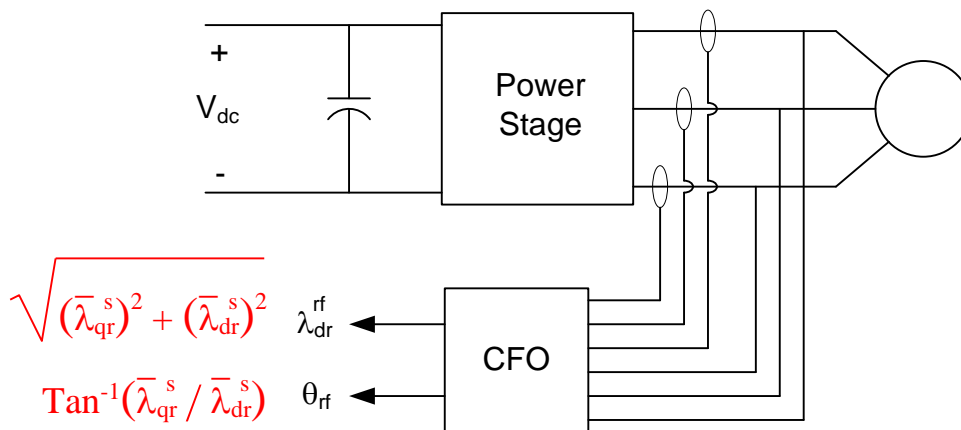
$L_s'$  = Stator Transient Inductance

$$L_s' = L_s - \frac{L_m^2}{L_r} \approx L_{ls} + L_{lr}$$

## Two Methods of Field Oriented Control (FOC) :

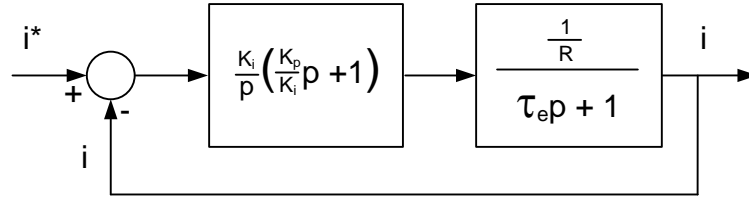


### Indirect Field Orientation – Rotor Flux Orientation Induction Machine Control



### Direct Field Orientation – Rotor Flux Orientation Induction Machine Control

## Recap of Current Loop Controls:



Current Regulation of DC Drive (with Back EMF Decoupled)

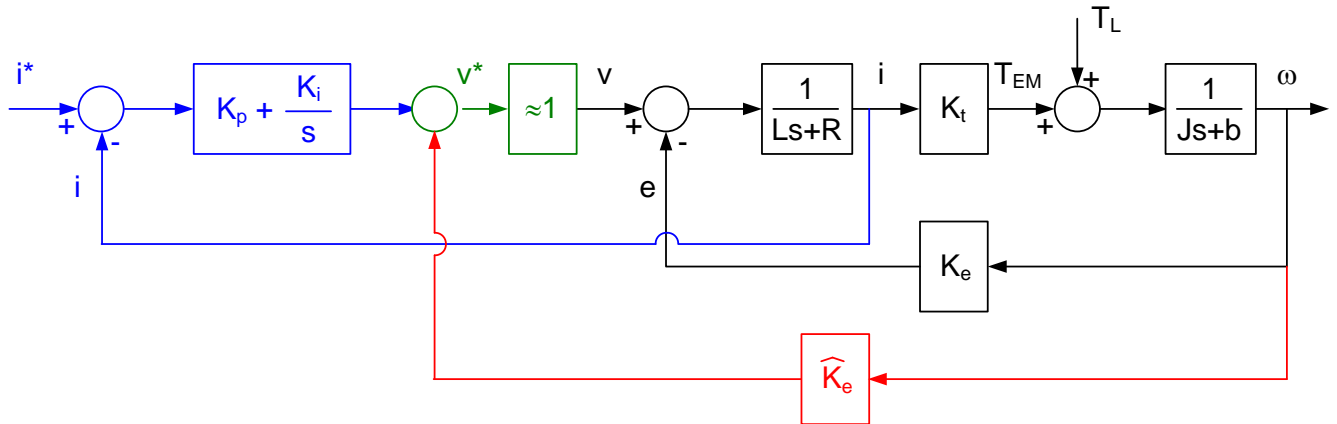
Use Proportional plus Integral (PI) Control

$$G_c = K_p + \frac{K_i}{p} = \frac{K_i}{p} \left( \frac{K_p}{K_i} p + 1 \right) = \frac{K_i}{p} (\tau_c p + 1)$$

The gains to tune the current loop at locked rotor conditions are thus:

$$K_i = 2\pi f_{\text{desired}} R \quad \text{and} \quad K_p = 2\pi f_{\text{desired}} L$$

Note: By using Back EMF decoupling, the desired bandwidth can be achieved at non zero speed conditions as well.



DC Motor PI Current Regulator with Back EMF Decoupling



## Induction Machine Current Regulation Tuning:

We can re-write stator voltage equations as:

$$\bar{v}_{qds}^e = \left[ r_s + (p + j\omega_e) \sigma L_s + \left( \frac{L_m}{L_r} \right)^2 r_r \right] \bar{i}_{qds}^e - \frac{L_m}{L_r} j(\omega_e - \omega_r) \bar{\lambda}_{qdr}^e + \frac{L_m}{L_r} j\omega_e \bar{\lambda}_{qdr}^e - \frac{L_m}{L_r} \frac{1}{\tau_r} \bar{\lambda}_{qdr}^e$$

$$\bar{v}_{qds}^e = \left[ r_s' + (p + j\omega_e) \sigma L_s \right] \bar{i}_{qds}^e + \frac{L_m}{L_r} \omega_{rb} \bar{\lambda}_{qdr}^e$$

where

$$r_s' = \left[ r_s + \left( \frac{L_m}{L_r} \right)^2 r_r \right] \quad \omega_{rb} = \left[ j\omega_r - \frac{1}{\tau_r} \right]$$

The scalar equations can be written as:

$$v_{qs}^e =$$

$$v_{ds}^e =$$

In the steady state,

$$\lambda_{dr}^e = L_m i_{ds}^e$$

So the equations can be rewritten as:

$$v_{qs}^e = (r_s' + \sigma L_s p) i_{qs}^e + \left( \sigma L_s \omega_e + \frac{L_m^2}{L_r} \omega_r \right) i_{ds}^e$$

$$v_{ds}^e = \left( \left[ r_s + \left( \frac{L_m}{L_r} \right)^2 r_r \right] + \sigma L_s p \right) i_{ds}^e - \sigma L_s \omega_e i_{qs}^e - \left( \frac{L_m}{L_r} \right)^2 r_r i_{ds}^e \quad \text{or}$$

$$v_{ds}^e = (r_s + \sigma L_s p) i_{ds}^e - \sigma L_s \omega_e i_{qs}^e$$

In summary:

$$v_{qs}^e = (\overset{\text{Transient Impedance}}{r_s' + \sigma L_s p}) i_{qs}^e + \left( \overset{\text{Synch Frame Coupling}}{\sigma L_s \omega_e} + \overset{\text{Back EMF}}{\frac{L_m^2}{L_r} \omega_r} \right) i_{ds}^e$$

$$v_{ds}^e = (\overset{\text{Transient Impedance}}{r_s + \sigma L_s p}) i_{ds}^e - \overset{\text{Synch Frame Coupling}}{\sigma L_s \omega_e} i_{qs}^e$$

## Indirect Field Oriented Control (IFOC):

$$T_e = \frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} [\lambda_{dr}^e i_{qs}^e - \lambda_{qr}^e i_{ds}^e] = \frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} \lambda_{dr}^e i_{qs}^e = \frac{3}{2} \frac{P}{2} \frac{L_m^2}{L_r} \overset{\text{field current}}{I_{ds}} \overset{\text{torque producing current}}{I_{qs}}$$

Transient case (when changing  $i_{ds}^e$ ):

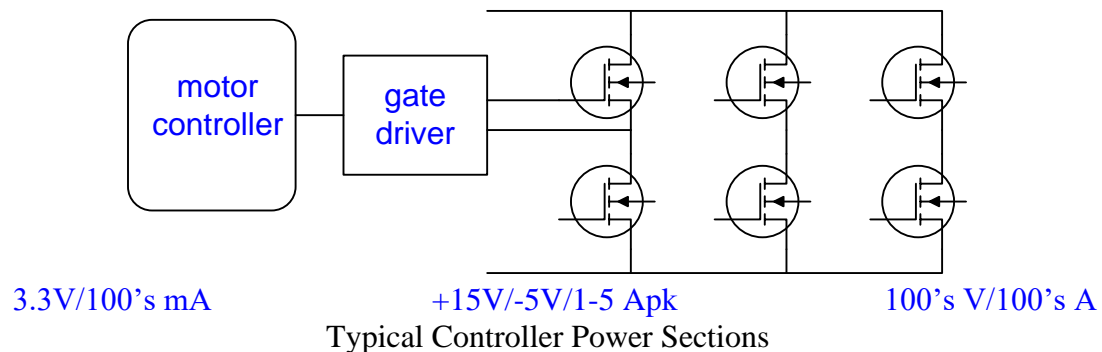
$$T_e = \frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} \lambda_{dr}^e i_{qs}^e = K_t i_{qs}^e \quad \rightarrow \quad K_t = \frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} \lambda_{dr}^e$$

Steady state (not changing  $i_{ds}^e$ ):

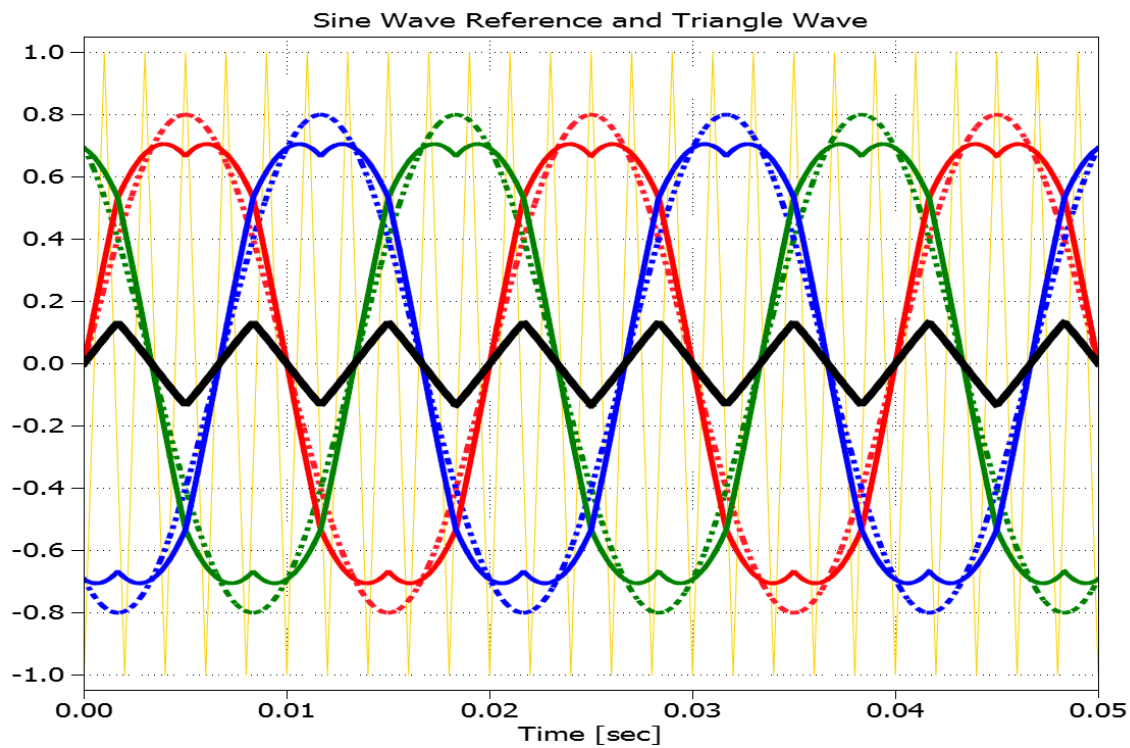
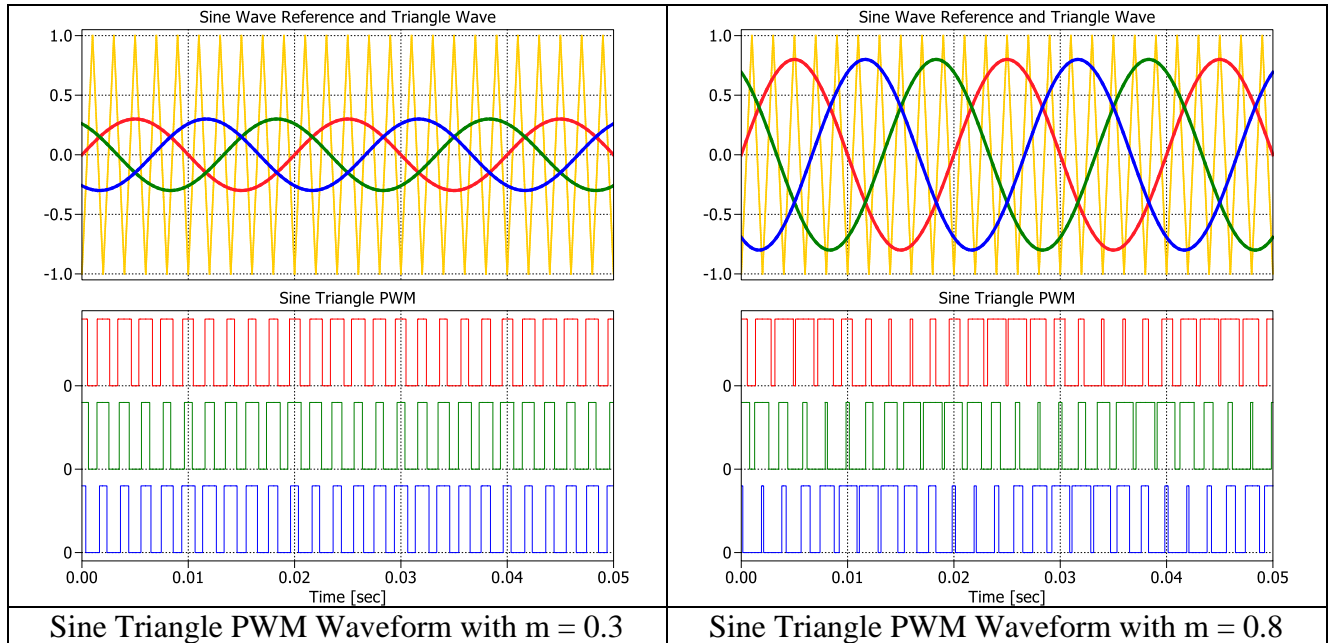
$$T_e = \frac{3}{2} \frac{P}{2} \frac{L_m^2}{L_r} I_{ds} I_{qs} = K_t i_{qs}^e \quad \rightarrow \quad K_t = \frac{3}{2} \frac{P}{2} \frac{L_m^2}{L_r} I_{ds}$$

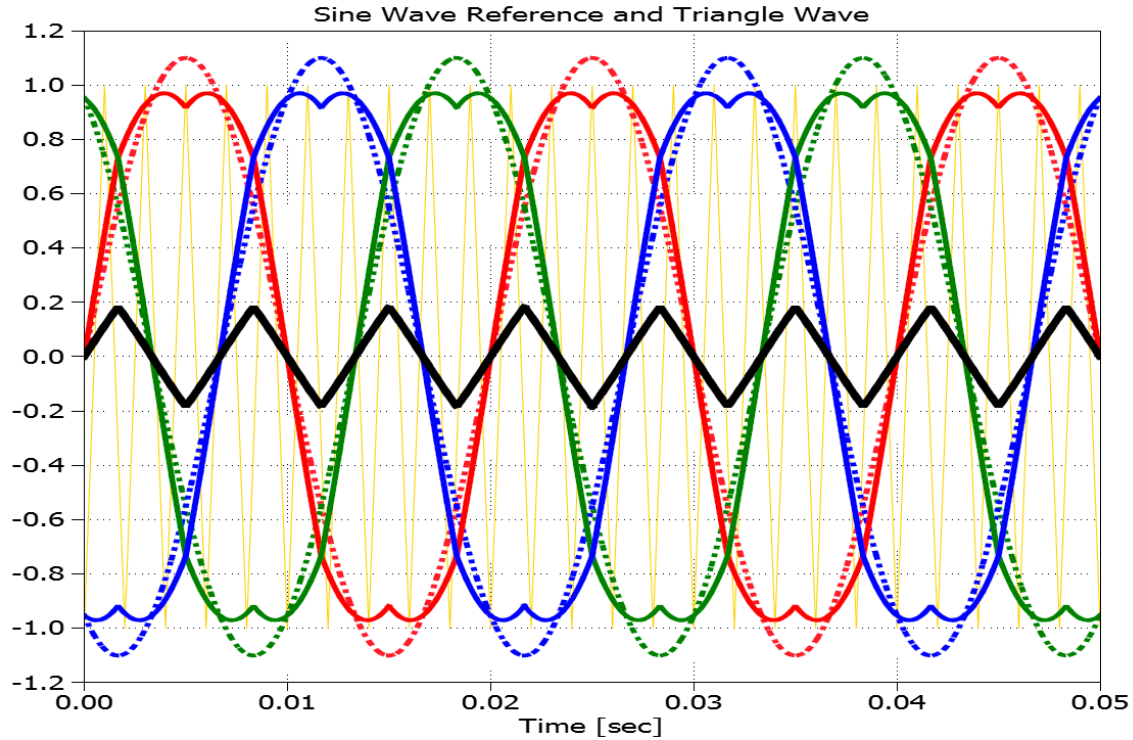
## Voltage control of AC machines

Typical inverter power levels:



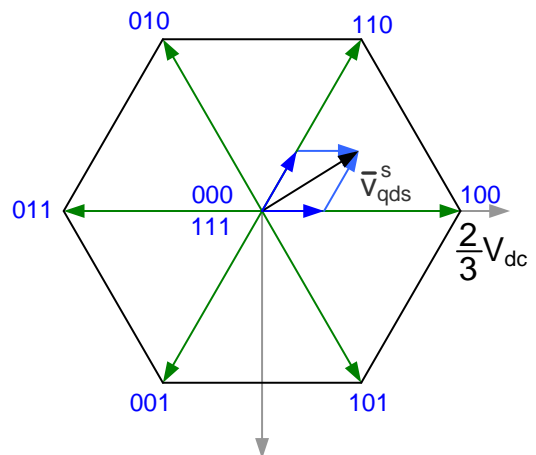
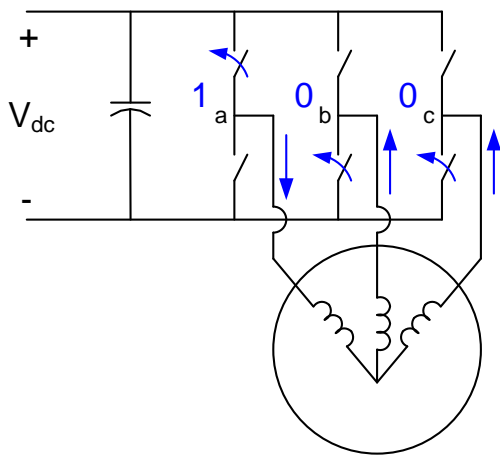
## Sine Triangle PWM:



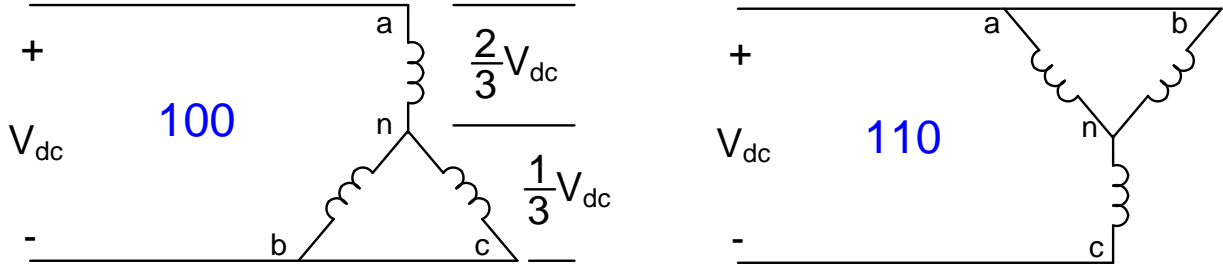


Sine Triangle and Modified Sine Triangle PWM Waveform with Zero Sequence Term with  $m = 1.1$

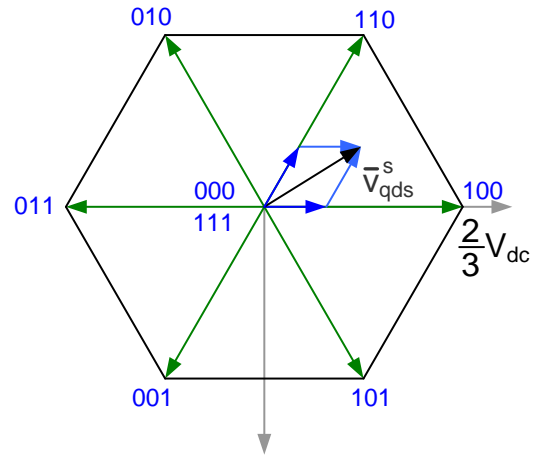
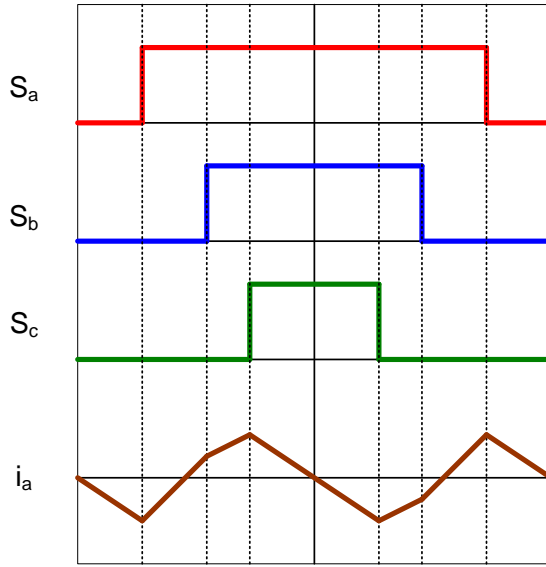
## Space Vector PWM:



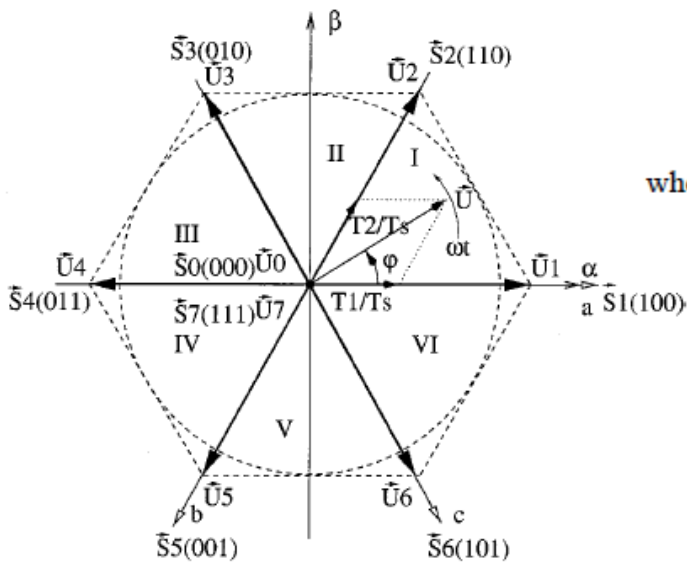
Switching States Used in Space Vector PWM



First Two States of Space Vector PWM



Switching States and Phase Current Ripple in Space Vector PWM Quadrant 1



$$\vec{U}(t) = \frac{t_0}{T_s} \vec{U}_0 + \frac{t_1}{T_s} \vec{U}_1 + \dots + \frac{t_7}{T_s} \vec{U}_7$$

$$\vec{U} = \frac{T_1}{T_s} \vec{U}_1 + \frac{T_2}{T_s} \vec{U}_2 + \frac{T_7}{T_s} \vec{U}_7 + \frac{T_0}{T_s} \vec{U}_0$$

where  $T_s - T_1 - T_2 = T_0 + T_7 \geq 0$ ,  $T_0 \geq 0$  and  $T_7 \geq 0$ .

Let the length of  $\vec{U}$  be  $m^*E$ , then we have

$$\frac{m^*}{\sin \frac{2\pi}{3}} = \frac{T_1}{T_s} \frac{1}{\sin(\frac{\pi}{3} - \varphi)} = \frac{T_2}{T_s} \frac{1}{\sin \varphi} \quad ($$

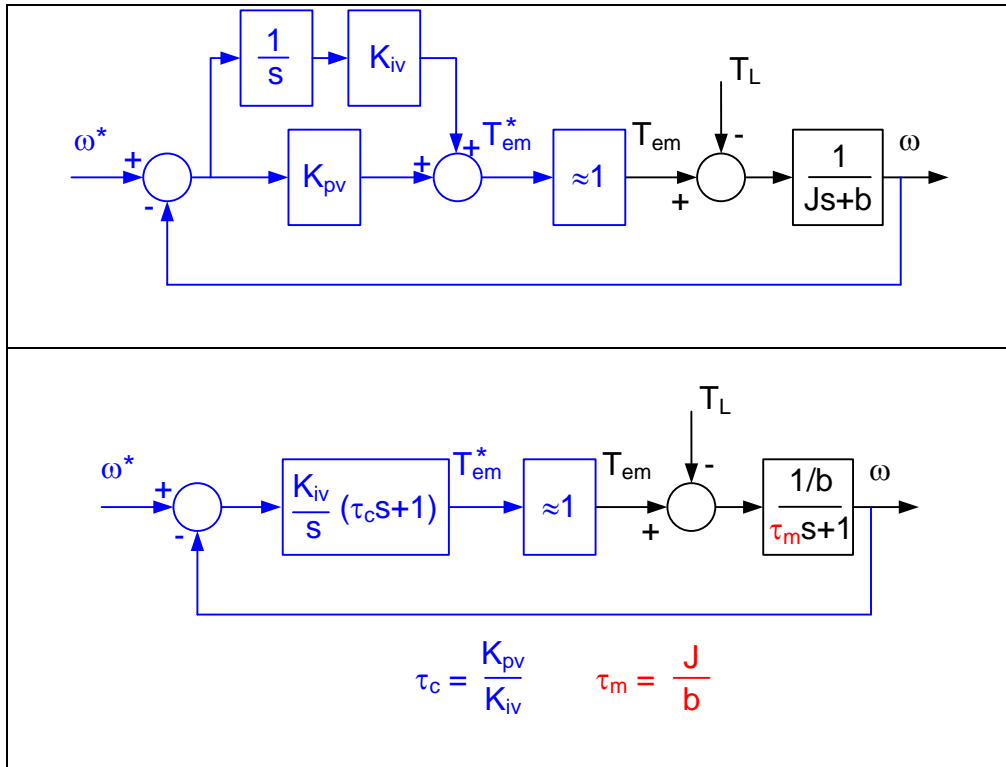
Thus,

$$\frac{T_1}{T_s} = \frac{2}{\sqrt{3}} m^* \sin(\frac{\pi}{3} - \omega t) = \frac{2}{\sqrt{3}} m^* \cos(\omega t + \frac{\pi}{6})$$

$$\frac{T_2}{T_s} = \frac{2}{\sqrt{3}} m^* \sin \omega t = \frac{2}{\sqrt{3}} m^* \cos(\omega t + \frac{3\pi}{2})$$

$$T_0 + T_7 = T_s - T_1 - T_2 \quad ($$

## Speed Control of Machines



Block Diagram of a DC Machine with Ideal Torque Regulator with a PI Speed Loop (Upper Figure)  
 Rewritten Using Mechanical and Controller Time Constants (Lower Figure)