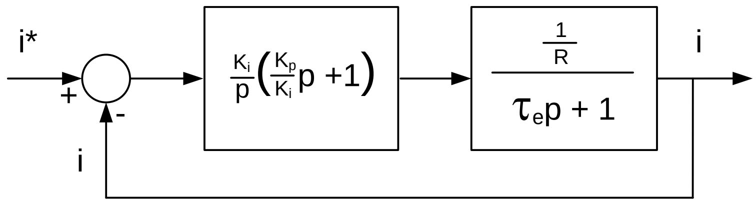
***Recap of Current Loop Controls:***

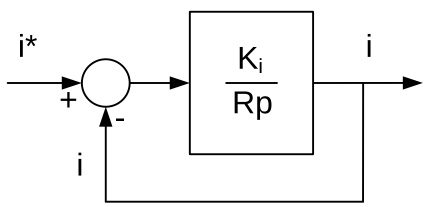



Current Regulation of DC Drive (with rotor locked)

Use Proportional plus Integral (PI) Control

$$G_c = K_p + \frac{K_i}{p} = \frac{K_i}{p}\left(\frac{K_p}{K_i}p + 1\right) = \frac{K_p\left(p+\frac{K_i}{K_p}\right)}{p}$$

With $\tau_c = \tau_e$:




Current Regulation of DC Drive (with Back EMF Decoupled) and Pole/Zero Cancellation

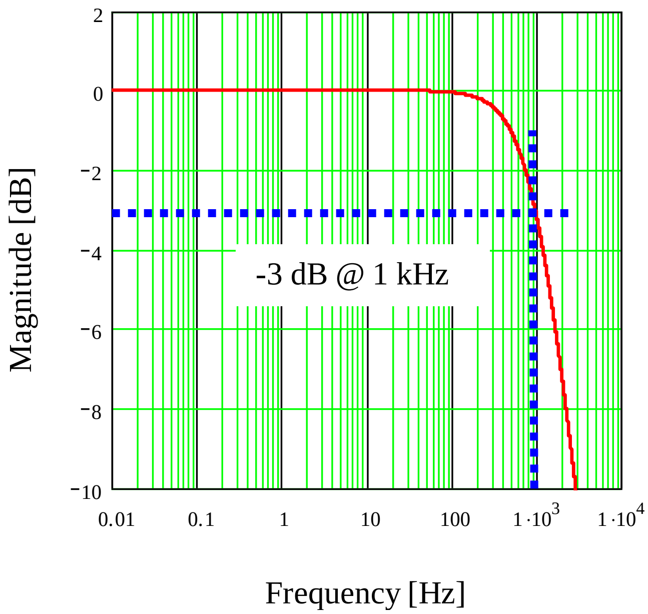
$\frac{I(p)}{I^*(p)} = \frac{K_i}{R_a p + K_i}$ why choose $K_i = 2\pi f_{desired} R_a$?

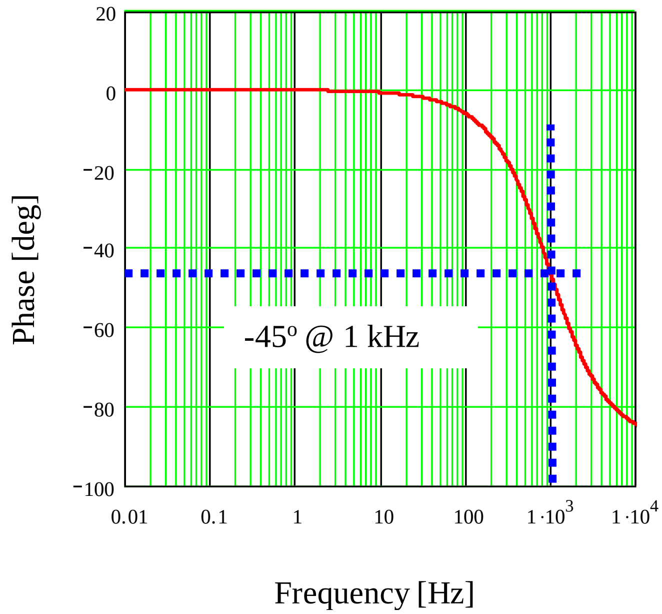
$\left|\frac{I(j\omega)}{I^*(j\omega)}\right| = \left|\frac{K_i}{R_a(j\omega)+K_i}\right| =$ $\angle\frac{I(j\omega)}{I^*(j\omega)} = 0^o - tan^{-1}\left(\frac{R_a\omega}{K_i}\right)$

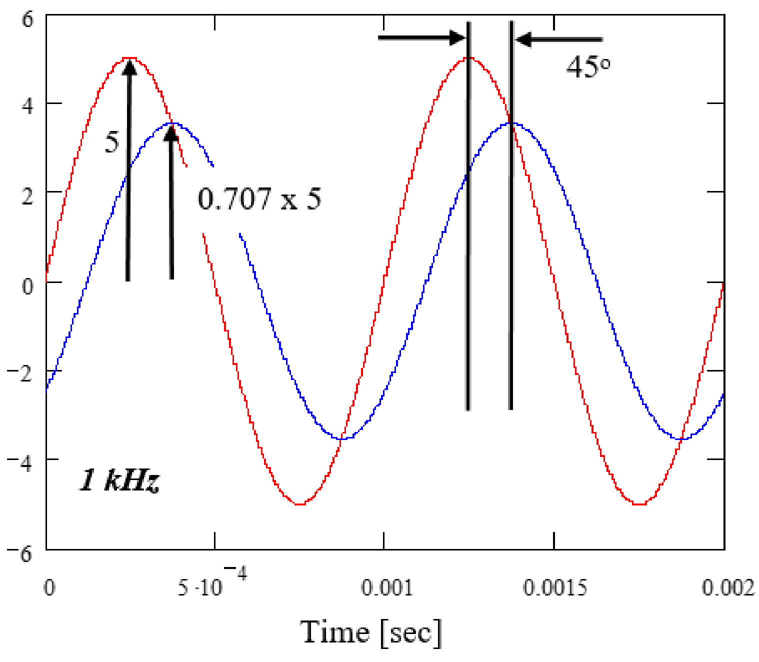
If $R_a\omega = K_i$

Then

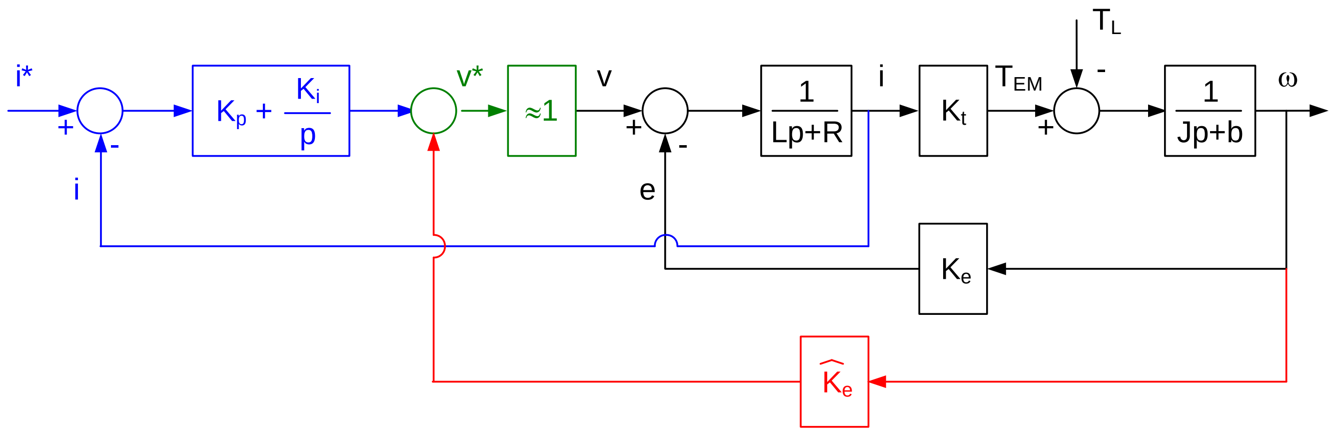
$\left|\frac{I(j\omega)}{I^*(j\omega)}\right| =$



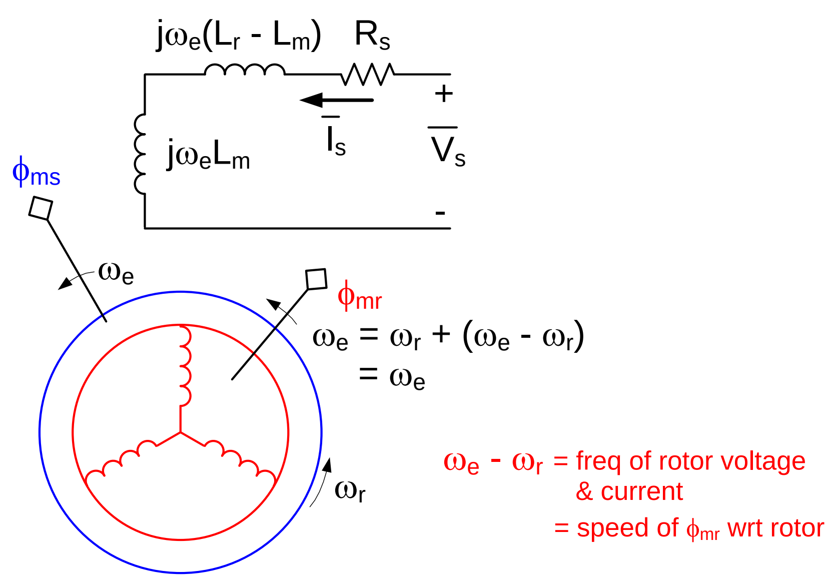

Note: By using Back EMF decoupling, the desired bandwidth can be achieved at non-zero speed conditions as well so long as the armature voltage is not saturated.




**DC Motor PI Current Regulator with Back EMF Decoupling**

**<u>Introduction to AC Induction Machines:</u>**

***Basic Theory of Squirrel Cage Induction Machines:***



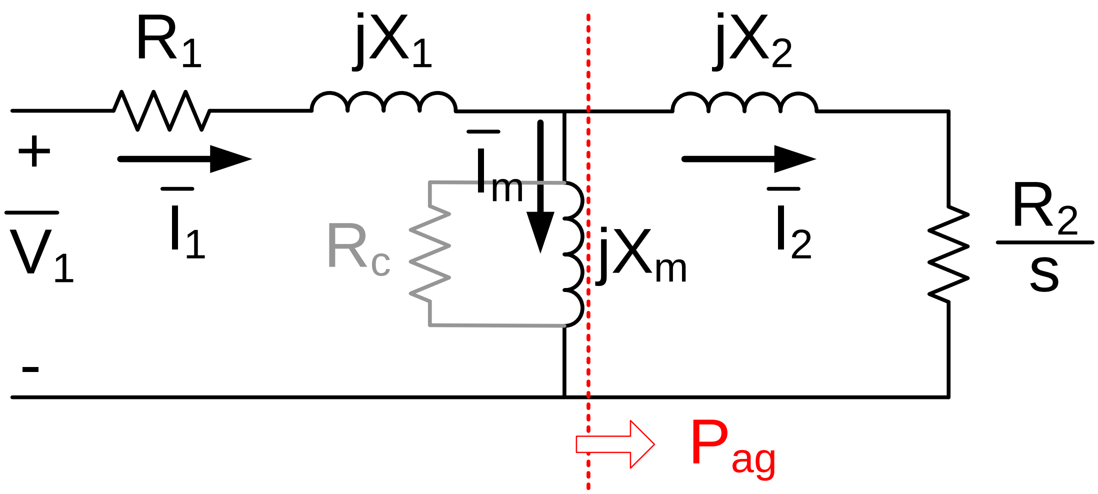

$\omega_e$ = excitation frequency

$\omega_r$ = rotor electrical speed

$$\omega_{em} = \frac{\omega_e}{P/2}$$

$$\omega_{rm} = \frac{\omega_r}{P/2}$$

***Conventional Per Phase Equivalent Circuit:***




Per Phase Equivalent Schematic of Induction Motor Reflected to Stator

Airgap Power:

$$P_{ag} = \frac{3I_2^2R_2}{s} \rightarrow$$

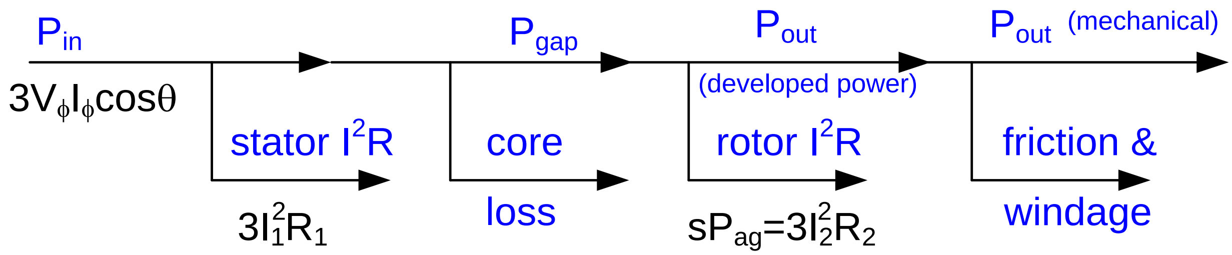
Rotor Loss =

$$P_{out} = \frac{3I_2^2R_2}{s}(1-s) =$$

A good (efficient) induction motor must run with low slip.
***Electromagnetic Torque:***

$$T_{em} = \frac{P_{out}}{\omega_{rm}} =$$

$$\boxed{T_{em} = \frac{3P}{2}\frac{I_2^2R_2}{s\omega_e}}$$




Power Flow in an Induction Motor

**<u>For a "good" motor</u>**

*for sure:*

$R_1 \rightarrow$

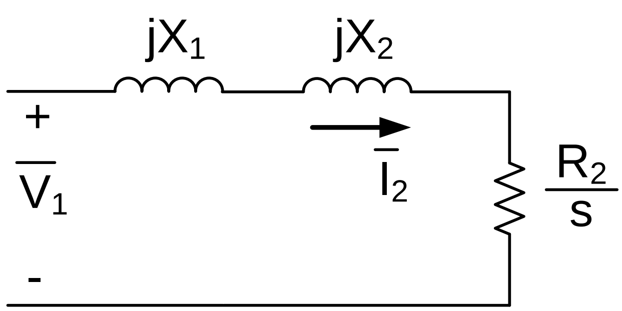
$X_m \rightarrow$

*probably:*

$R_2 \rightarrow$

$X_1, X_2 \rightarrow$

**<u>Max $P_{ag}$ (assuming small $R_1$ large $X_m$):</u>**



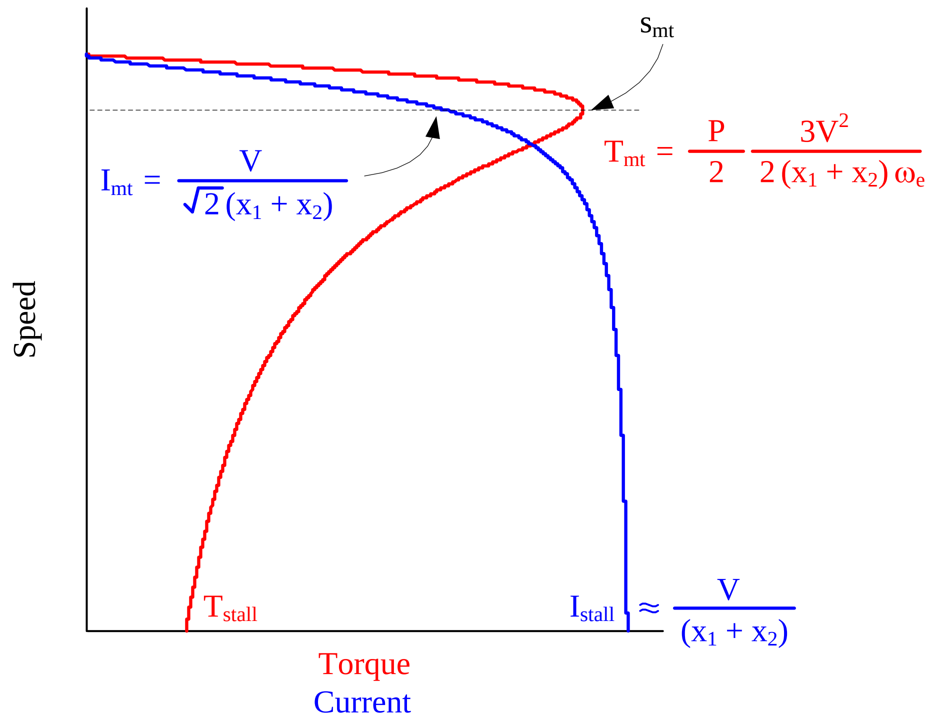

Per Phase Equivalent Schematic of Induction Assuming Small $R_1$ and Large $X_m$

Max $P_{ag}$ when:

$$\frac{R_2}{s_{mt}} = X_1 + X_2$$

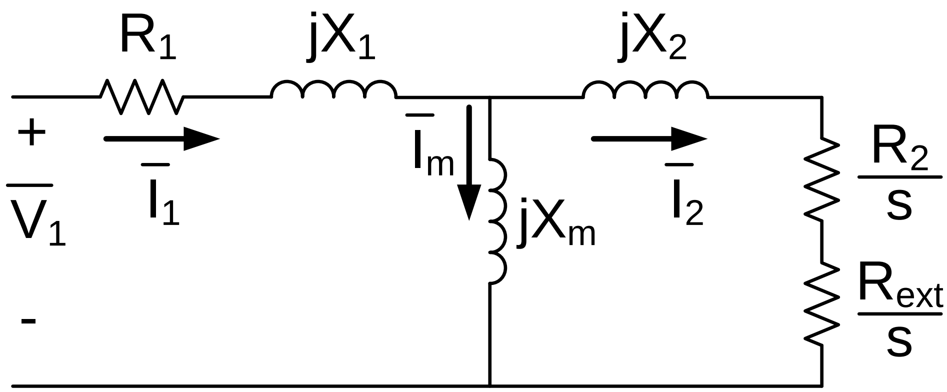
$$s_{mt} = \frac{R_2}{X_1 + X_2}$$

$$P_{ag\_mt} = 3I_2^2 \frac{R_2}{s_{mt}} =$$

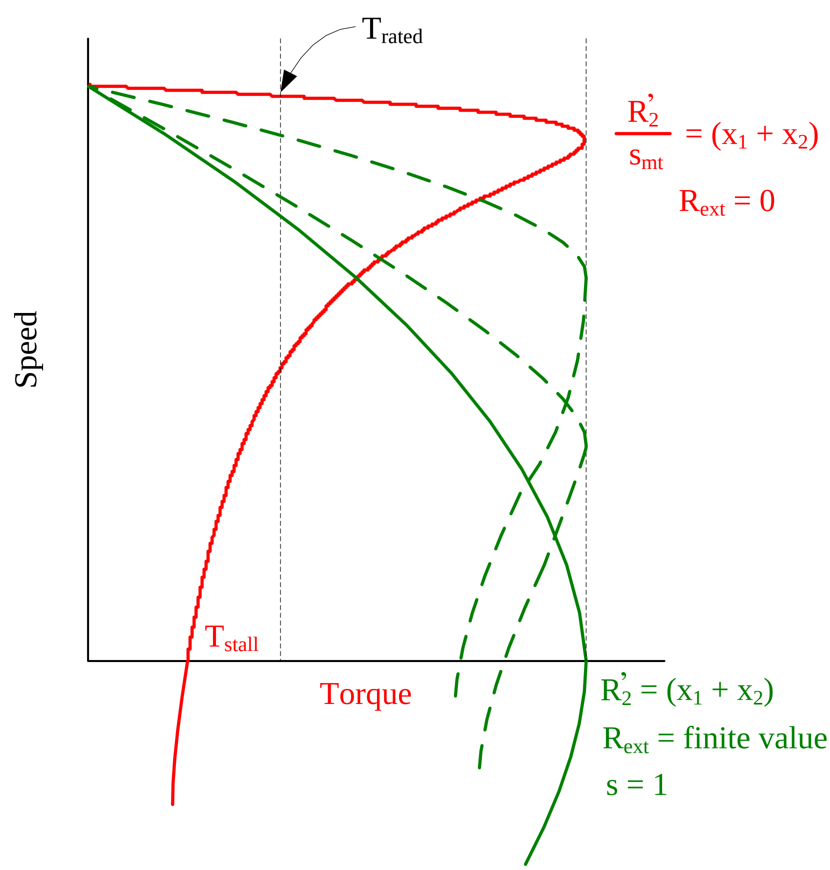



Speed vs. Torque and Current for Three Phase Induction Motor

***Wound Rotor Induction Motor:***




Per Phase Equivalent Schematic of Wound Rotor Induction Motor




Speed vs. Torque and Current for Wound Rotor Induction Motor

***Exact solution for slip for maximum torque:***

***Find slip for maximum torque analytically***

***total impedance***

$$z(s) = r_1 + j \cdot x_1 + \frac{\left(\frac{r_2}{s} + j \cdot x_2\right) \cdot j \cdot x_m}{\frac{r_2}{s} + j \cdot x_2 + j \cdot x_m}$$

***stator current***

$$I_1(s) = \frac{V_{s\_pu}}{r_1 + j \cdot x_1 + \frac{\left(\frac{r_2}{s} + j \cdot x_2\right) \cdot j \cdot x_m}{\frac{r_2}{s} + j \cdot x_2 + j \cdot x_m}}$$

***mutual voltage***

$$V_m(s) = V_{s\_pu} - \frac{V_{s\_pu}}{r_1 + j \cdot x_1 + \frac{\left(\frac{r_2}{s} + j \cdot x_2\right) \cdot j \cdot x_m}{\frac{r_2}{s} + j \cdot x_2 + j \cdot x_m}} \cdot \left(r_1 + j \cdot x_1\right)$$

***rotor current***

$$I_r(s) = \frac{\dfrac{V_{s\_pu} \cdot s \cdot x_2 \cdot x_m \cdot j^2 + V_{s\_pu} \cdot r_2 \cdot x_m \cdot j}{\left(s \cdot x_1 \cdot x_2 + s \cdot x_1 \cdot x_m + s \cdot x_2 \cdot x_m\right) \cdot j^2 + \left(r_2 \cdot x_1 + r_2 \cdot x_m + r_1 \cdot s \cdot x_2 + r_1 \cdot s \cdot x_m\right) \cdot j + r_1 \cdot r_2}}{\left(\frac{r_2}{s} + j \cdot x_2\right)} =$$

$$\frac{V_{s\_pu} \cdot s \cdot x_m \cdot j}{\left[s \cdot \left(x_2 + x_m\right) \cdot r_1 + \left(x_1 + x_m\right) \cdot r_2\right] \cdot j + \left[r_1 \cdot r_2 - \left[\left(x_1 \cdot x_2 + x_1 \cdot x_m + x_2 \cdot x_m\right) \cdot s\right]\right]}$$

***magnitude of rotor current***

$$\left|I_r\right| = \sqrt{\frac{\left(V_{s\_pu}\cdot s\cdot x_m\right)^2}{\left[s\cdot\left(x_2 + x_m\right)\cdot r_1 + \left(x_1 + x_m\right)\cdot r_2\right]^2 + \left[r_1\cdot r_2 - \left[\left(x_1\cdot x_2 + x_1\cdot x_m + x_2\cdot x_m\right)\cdot s\right]\right]^2}}$$

***Torque***

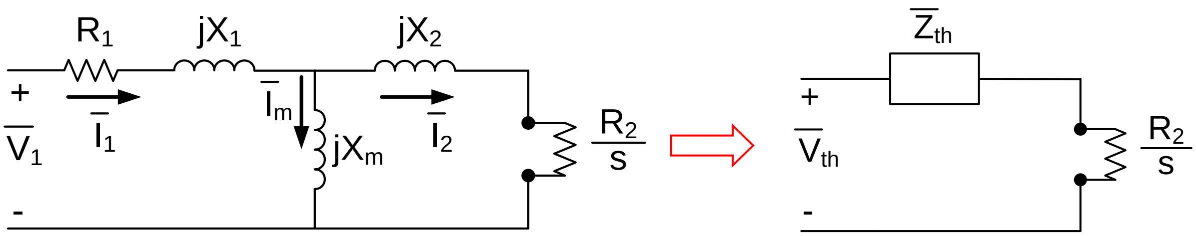
$$T_{pu}(s) = \frac{\left(\left|I_r\right|\right)^2\cdot r_2}{s\cdot\omega_b} = \frac{\dfrac{\left(V_{s\_pu}\cdot s\cdot x_m\right)^2}{\left[s\cdot\left(x_2 + x_m\right)\cdot r_1 + \left(x_1 + x_m\right)\cdot r_2\right]^2 + \left[r_1\cdot r_2 - \left[\left(x_1\cdot x_2 + x_1\cdot x_m + x_2\cdot x_m\right)\cdot s\right]\right]^2}\cdot r_2}{s\cdot\omega_b}$$

***Differentiate torque with respect to slip and set to zero to maximize***

$$s_{mt} = \sqrt{\frac{r_1^{\,2}\cdot r_2^{\,2} + r_2^{\,2}\cdot x_1^{\,2} + 2\cdot r_2^{\,2}\cdot x_1\cdot x_m + r_2^{\,2}\cdot x_m^{\,2}}{r_1^{\,2}\cdot x_2^{\,2} + 2\cdot r_1^{\,2}\cdot x_2\cdot x_m + r_1^{\,2}\cdot x_m^{\,2} + x_1^{\,2}\cdot x_2^{\,2} + 2\cdot x_1^{\,2}\cdot x_2\cdot x_m + x_1^{\,2}\cdot x_m^{\,2} + 2\cdot x_1\cdot x_2^{\,2}\cdot x_m + 2\cdot x_1\cdot x_2\cdot x_m^{\,2} + x_2^{\,2}\cdot x_m^{\,2}}}$$

Now, the easy way:

Thevenin Equivalent Circuit for a voltage source




Thevenin Equivalent Circuit

$\bar{Z}_{th} =$

$\frac{R_2}{s_{mt}} = |\bar{Z}_{th}| = |jX_2 + jX_m \parallel (R_1 + jX_1)|$

$T_{mt} =$

What's the Thevenin equivalent voltage?

$\bar{V}_{th} = \bar{V}_1 \frac{jX_m}{R_1 + jX_1 + jX_m}$

***Per Unit Representation of Machines***

$$P_b = P_{rated} \qquad V_b = V_{\varphi\_rated} \qquad \omega_b = \omega_{e\_rated}$$

$$P_b = 3V_b I_b \;\rightarrow\; I_b = \frac{P_b}{3V_b} = \frac{P_{rated}}{3V_{\varphi\_rated}} = \frac{P_{rated}}{\sqrt{3}V_{LL\_rated}}$$

$$Z_b = \frac{V_b}{I_b} = \frac{3V_b^2}{P_b} = \frac{V_{LL\_rated}^2}{P_b} \qquad \omega_{mb} = \frac{\omega_b}{P/2}$$

$$T_b = \frac{P_b}{\omega_{mb}} = \frac{P}{2}\frac{P_{rated}}{\omega_{e\_rated}}$$

$$I_{rated} = \frac{input\,VA}{V} = \frac{P_{rated}}{\eta cos\theta\sqrt{3}V_{LL\_rated}} =$$

$$T_{rated} = \frac{P}{2}\frac{P_{rated}}{\omega_{r\_rated}} = \frac{P}{2}\frac{P_{rated}}{(1-s_{rated})\omega_{e\_rated}} =$$

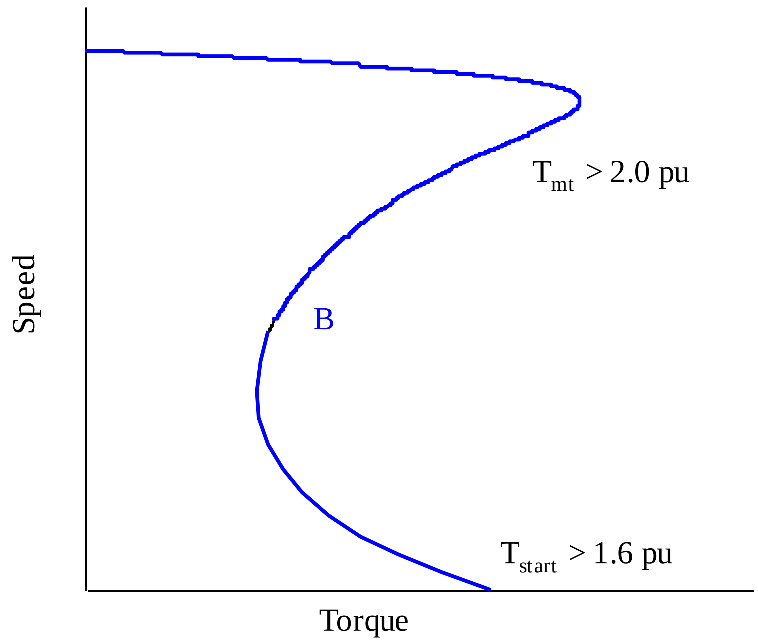
$$\frac{V}{V_b} = \frac{I}{I_b}\frac{Z}{Z_b} \qquad \frac{P}{P_b} = \frac{3V_\varphi V_\varphi cos\theta}{3V_b I_b}$$

$$V_{pu} = I_{pu}Z_{pu} \qquad P_{pu} = V_{pu}I_{pu}cos\theta$$

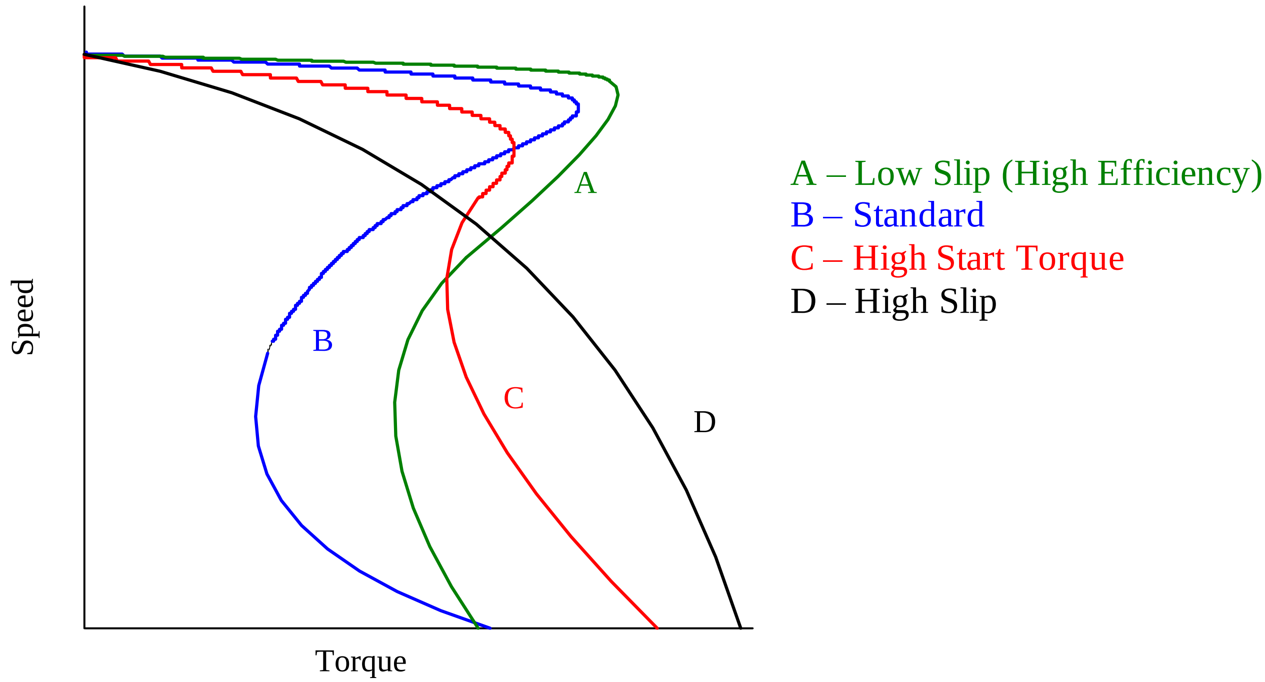
$$\frac{T}{T_b} = \frac{3\frac{P}{2}I_2^2\frac{R_2}{s\omega_e}}{3\frac{P}{2}\frac{V_b I_b}{\omega_{e\_rated}}} = \frac{3\frac{P}{2}I_2^2\frac{R_2}{s\omega_e}}{3\frac{P}{2}\frac{I_b^2 Z_b}{\omega_{e\_rated}}} = \frac{I_{pu}^2 R_{pu}}{s\omega_{e\_pu}}$$

for a good motor:

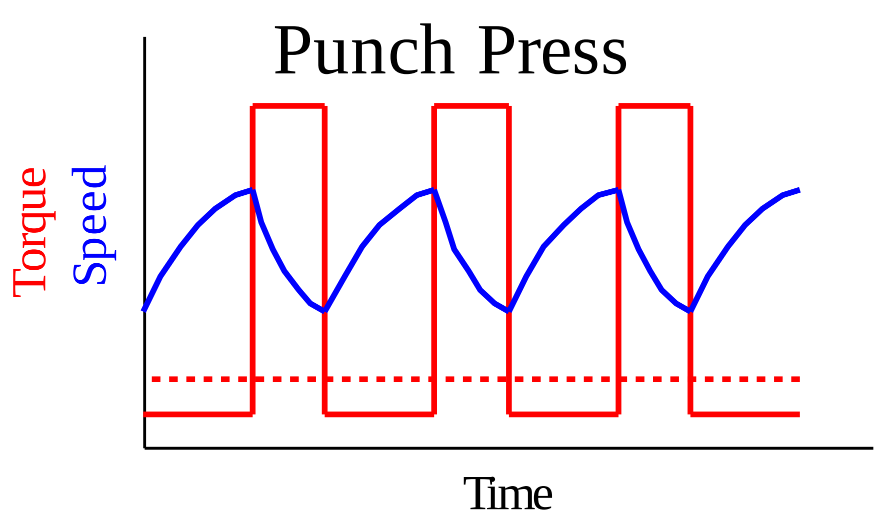
| | Large Machines | | Small Machines | |
|---|---|---|---|---|
| $R_1 \to 0$ | | $< R_1 <$ | | pu |
| $X_m \to \infty$ | | $> X_m >$ | | pu |
| $R_2$ | | $< R_2 <$ | | pu |
| $X_1, X_2$ | | $< X_1 + X_2 <$ | | pu |




Typical pu Speed Torque Curve for a NEMA Class B Induction Motor



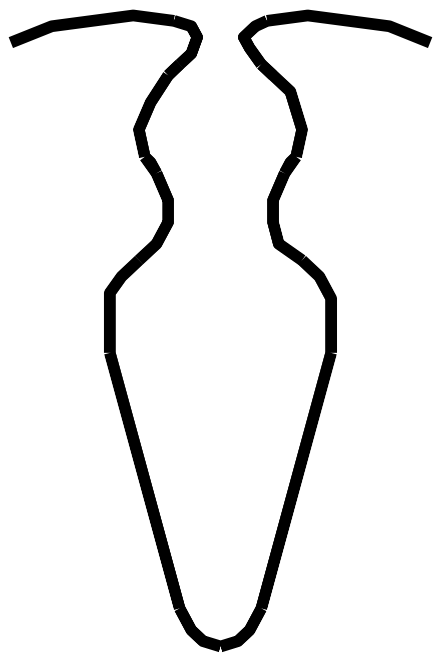

NEMA Standard Class Motors




NEMA Class D Typical Application

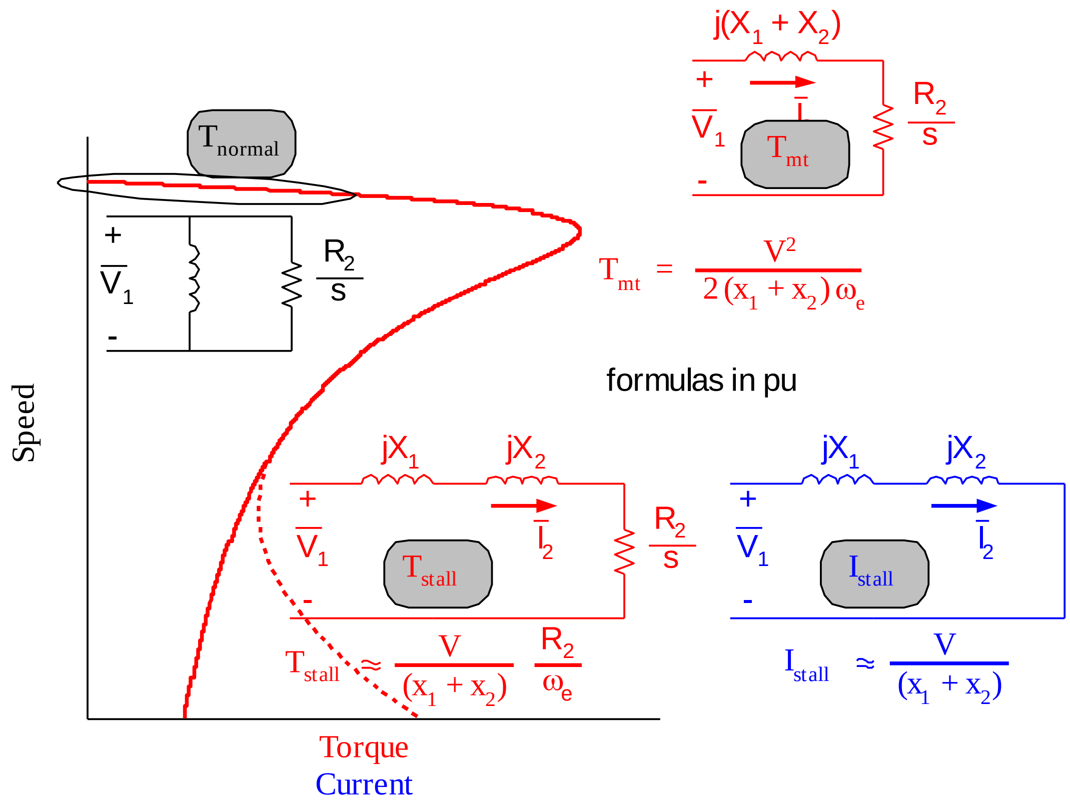
Induction Motor Parameter Variation:

1) Frequency Effects
    a. Rotor Slip Frequency Effects
        i. R increases with frequency (good)
        ii. L decreases with frequency (partly good)
    b. Stator Skin Effects (bad)
2) Flux Level (Saturation)

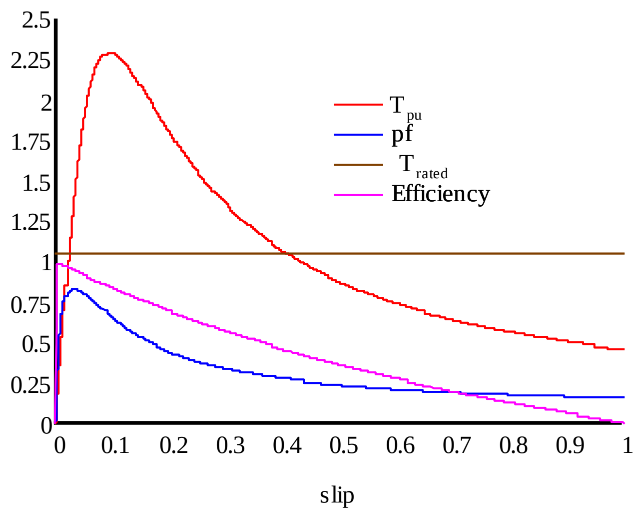


Double Cage Induction Motor Rotor Slot

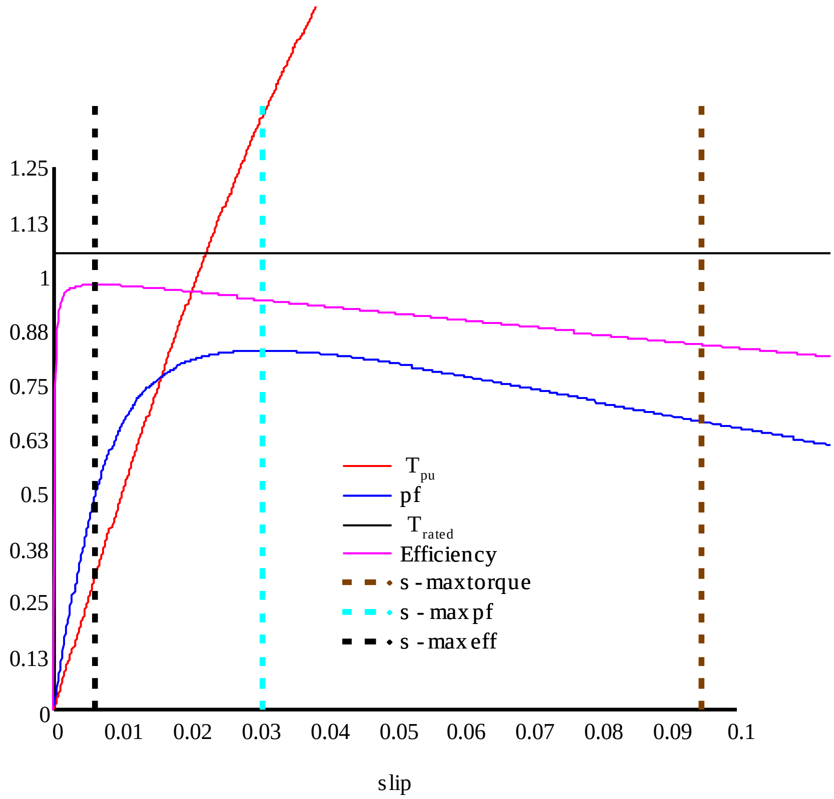
Induction Motor Performance Summary:




Induction Motor Performance Summary

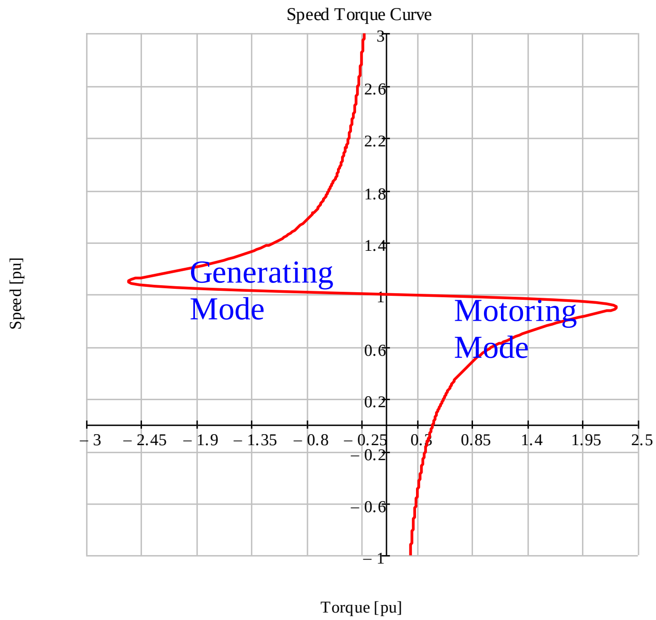


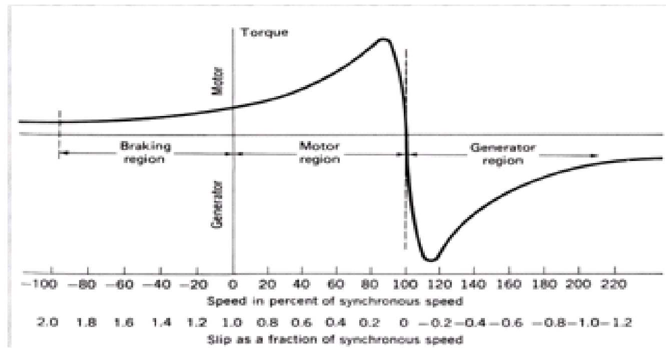

Induction Motor Performance vs. Slip




Zoomed-in Induction Motor Performance vs. Slip

**Operation of Induction Machine above $\omega_e$ (Induction Generator):**

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