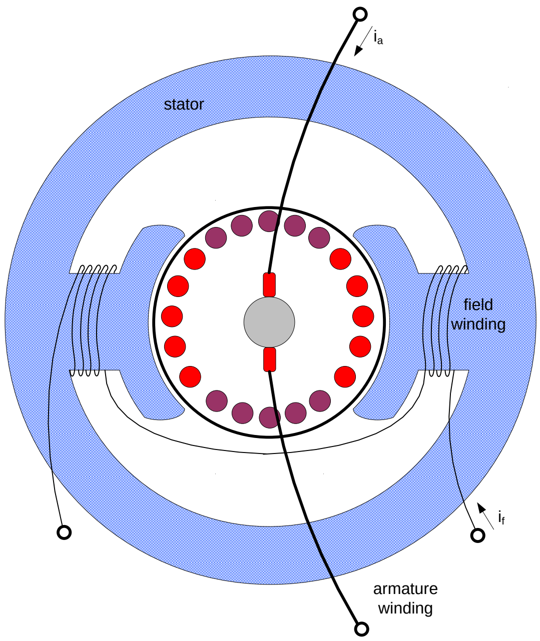
<u>DC Machines Overview</u>:

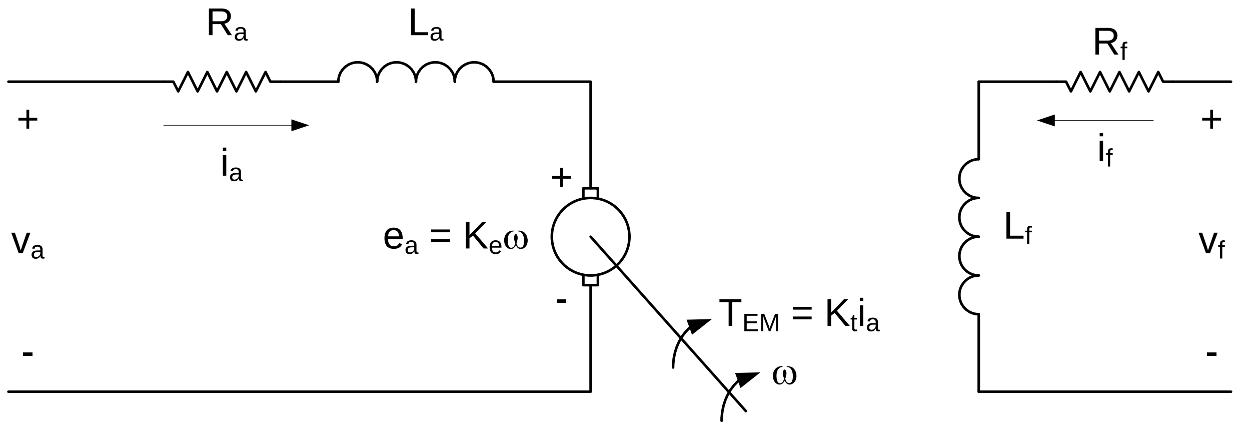
- Machines have simple (mechanical) commutation.
- Wound Field Machines have independent control of field and torque producing current.
- Can control speed by controlling armature voltage.
- Can control torque (and subsequently speed) by controlling armature current.

DC Machines Overview



Basic Structure of a Wound Field DC Machine

Wound Field DC Machine Equivalent Circuit



Armature Equations:

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a$$

Field Equations

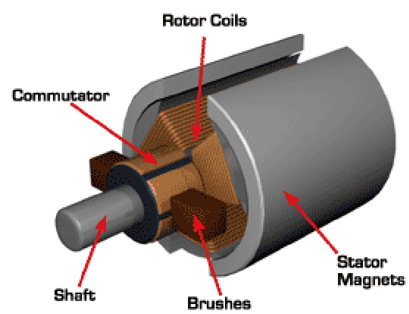
$$v_f = R_f i_f + L_f \frac{di_f}{dt} \quad \frac{d\varphi_f}{dt}$$

Constituent Equations:

$$e_a = K_f i_f \omega = K_e \omega$$

$$t_{em} = K_f i_f i_a = K_t i_a$$

Can replace field windings with PMs



Speed-Torque Characteristics:

$$v_a = V_a \qquad \varphi_f = constant \;\rightarrow\; K_e = K_t = constant$$

$$e_a = E_a \qquad I_a = \frac{T_{em}}{K_t}$$

$$\omega = \frac{E_a}{K_e} = \frac{V_a - R_a I_a}{K_e} = \frac{V_a}{K_e} - \frac{R_a}{K_e K_t} T_{em}$$

$$\boxed{\omega = \frac{V_a}{K_e} - \frac{R_a}{K_e K_t} t_{em}}$$

Speed as a function of Torque

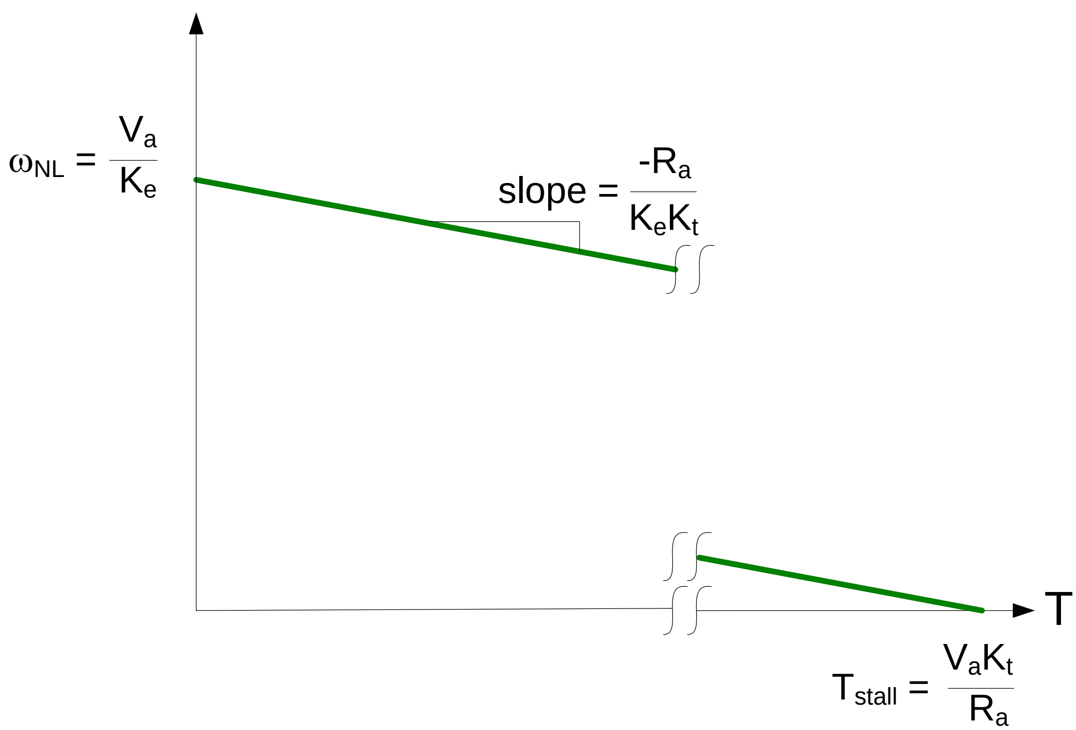
$$t_{em} = K_t I_a = K_t \frac{V_a - E_a}{R_a}$$

$$\boxed{t_{em} = \frac{K_t}{R_a} V_a - \frac{K_e K_t}{R_a} \omega}$$

Torque as a function of Speed

Motor Constant Squared (K_m^2)

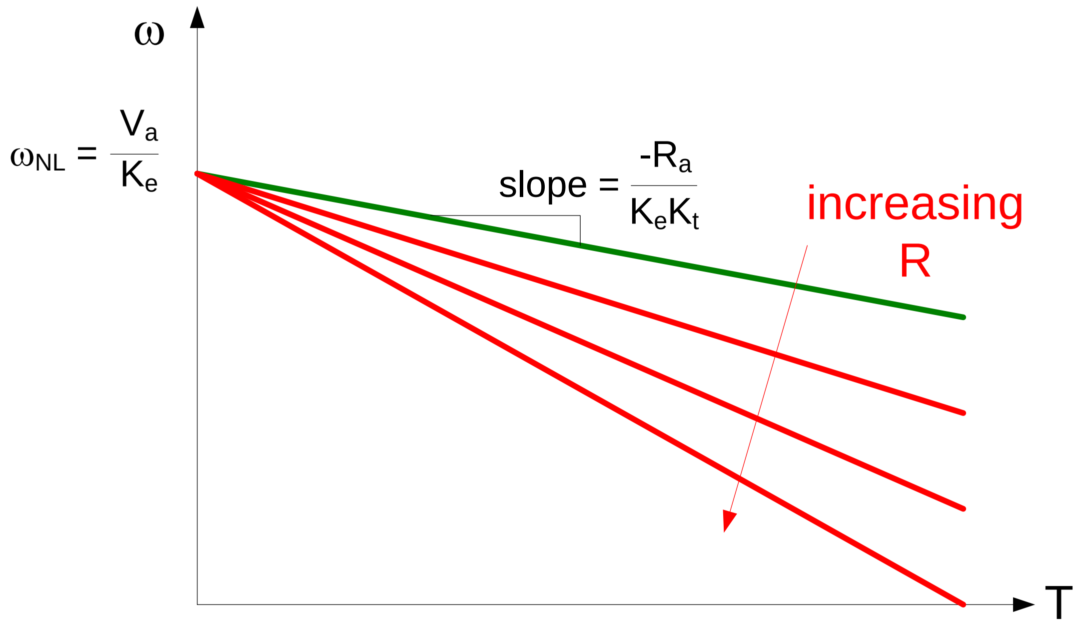
What does the speed-torque curve look like for the DC Machine?



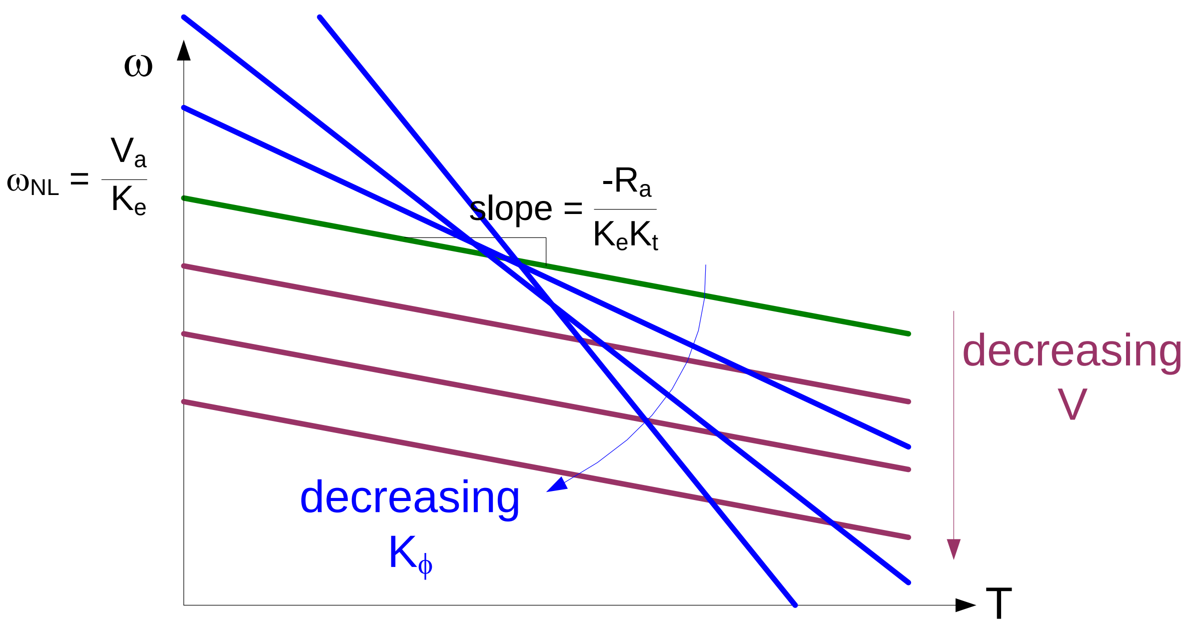
Speed Torque of a DC Machine with Constant Field Flux

Ways of varying speed:

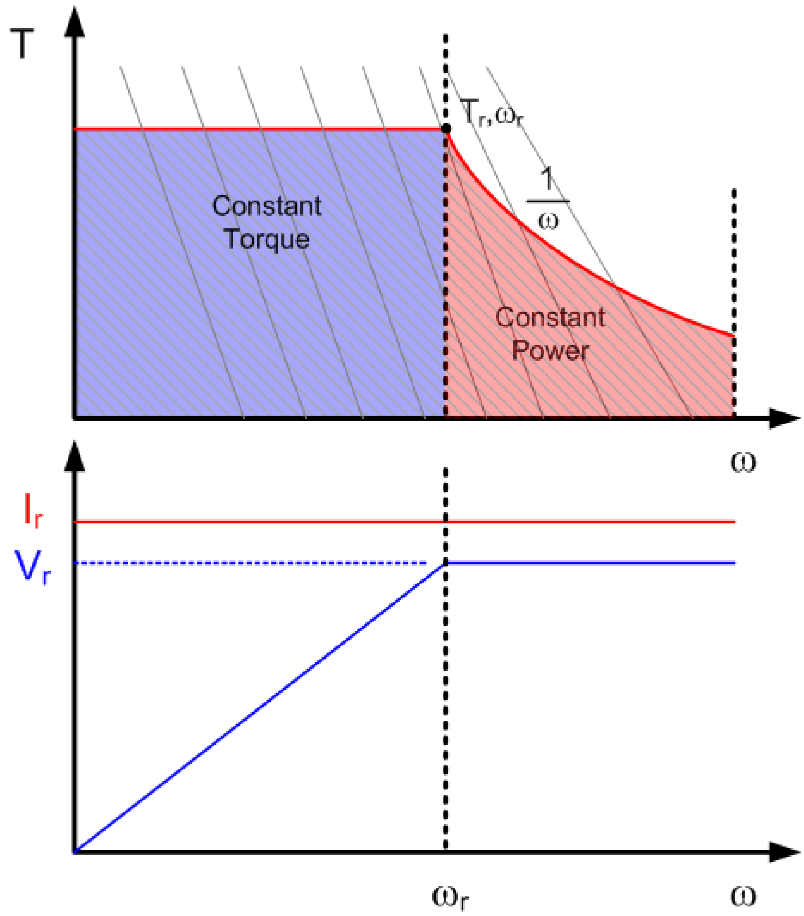
i) R – add external resistance (used in the "prehistoric days") ← Trolley Car
ii) V – standard for reducing speed (operating below rated speed)
iii) ϕ_f – standard for increasing speed (operating above rated speed)



Speed Torque of a DC Machine



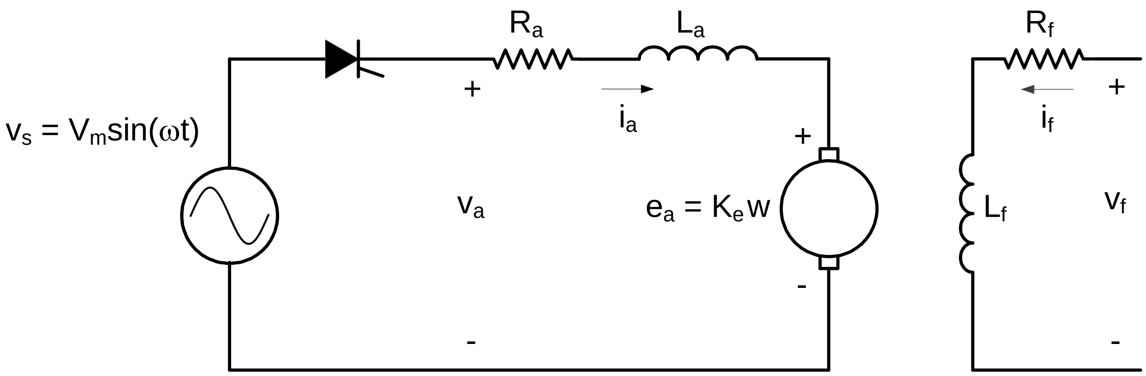
Speed Torque of a DC Machine



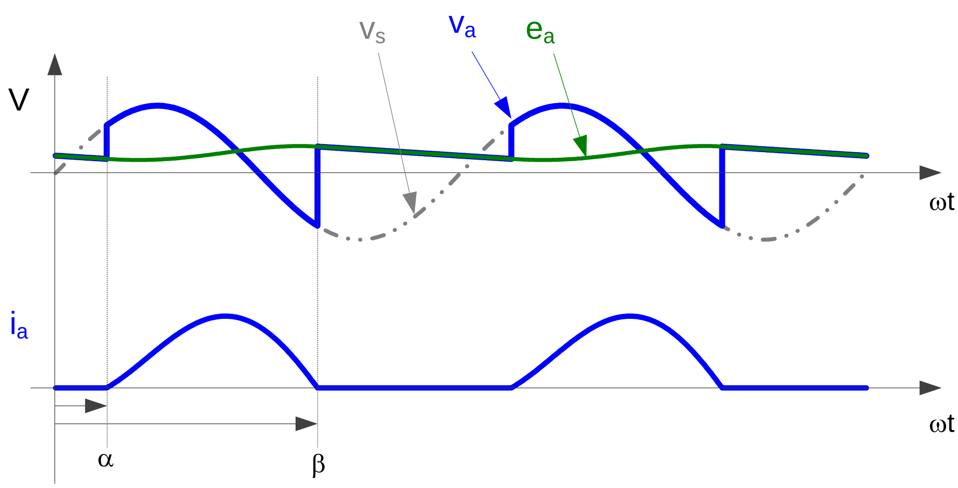
Capability curve for separately excited DC Machine

AC-DC Converters in Drives (late 1950's to 1980's)

Single Phase SCR AC/DC Converters – Half Wave Rectifier



Single Phase SCR Drive



Voltage and Current Waveforms of Single Phase SCR Drive

Critical parameters

- Switching angle – α
- Commutation angle – β (load commutated – reduces average voltage)
-

State Equations for a DC Motor

$$e_a(t) = K_e\omega(t)$$

$$v_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_a(t)$$

$$t_{em}(t) = K_t i_a(t) = t_L(t) + b\omega(t) + J\frac{d\omega(t)}{dt}$$

Armature Voltage

$$v_a(t) = \begin{cases} & 0 \le \omega t < \alpha \\ & \alpha \le \omega t < \beta \\ & \beta \le \omega t < 2\pi \end{cases}$$

Armature Current

$$i_a(t) = \begin{cases} & 0 \le \omega t < \alpha \\ & \alpha \le \omega t < \beta \\ & \beta \le \omega t < 2\pi \end{cases}$$

for $\alpha \le \omega t < \beta$

$v_a(t) = V_m sin(\omega t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_a(t)$ $\rightarrow$ solve differential equation for $i_a(t)$

Averaging Method:

$$\boldsymbol{E_a} = \overline{e_a(t)} = \frac{1}{T}\int_0^T e_a(t)dt = K_e\overline{\omega(t)}$$

$$\boldsymbol{T_{em}} = \overline{t_{em}(t)} = K_t\overline{\iota_a(t)}$$

$$\boldsymbol{I_a} = \overline{\iota_a(t)} = \frac{1}{T}\int_0^T i_a(t)dt = \frac{1}{2\pi}\int_\alpha^\beta i_a(\omega t)d(\omega t) \qquad \omega = \frac{2\pi}{T}$$

$$\boldsymbol{V_a} = \overline{v_a(t)} = \frac{1}{T}\int_0^T i_a(t)dt =$$

$$= \frac{1}{2\pi}\left[\alpha\boldsymbol{E_a} + V_m\big(-cos(\omega t)\big)\Big|_\alpha^\beta + (2\pi-\beta)\boldsymbol{E_a}\right]$$

$$= \frac{1}{2\pi}\left[\big(2\pi-(\beta-\alpha)\big)\boldsymbol{E_a} + V_m\big(cos(\alpha)-cos(\beta)\big)\right]$$

$$\boldsymbol{V_a} = \left[\left(1-\frac{\gamma}{2\pi}\right)\boldsymbol{E_a} + \frac{V_m}{2\pi}\big(cos(\alpha)-cos(\beta)\big)\right]$$

$$v_a(t) = R_a i_a(t) + L_a\frac{di_a(t)}{dt} + e_a(t)$$

$$\overline{\frac{di_a(t)}{dt}} = \frac{1}{T}\int_0^T \frac{di_a(t)}{dt}dt = \frac{1}{T}\int_0^T di_a(t) =$$

$$\boldsymbol{V_a} = R_a\boldsymbol{I_a} + \boldsymbol{E_a}$$

$$\left[\left(1-\frac{\gamma}{2\pi}\right)\boldsymbol{E_a} + \frac{V_m}{2\pi}\big(cos(\alpha)-cos(\beta)\big)\right] = R_a\boldsymbol{I_a} + \boldsymbol{E_a}$$

$$\frac{V_m}{2\pi}\big(cos(\alpha)-cos(\beta)\big) = R_a\boldsymbol{I_a} + \frac{\gamma}{2\pi}\boldsymbol{E_a}$$

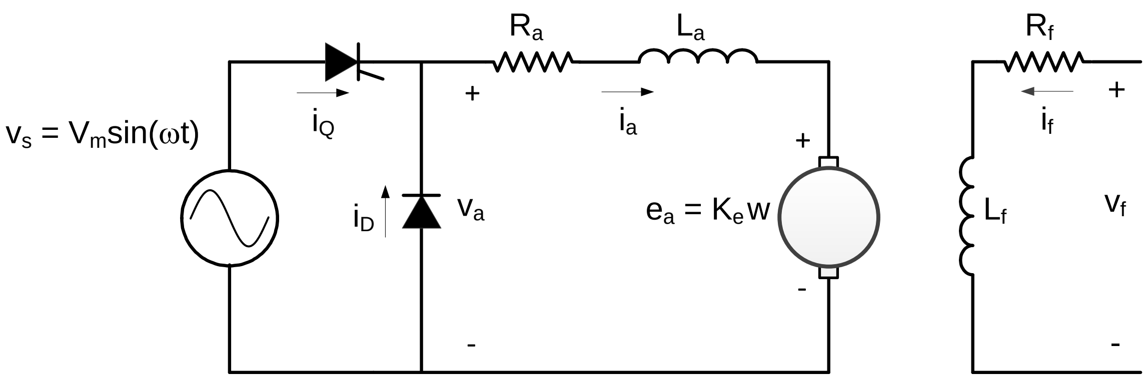
$$\boldsymbol{I_a} = \frac{\boldsymbol{1}}{R_a}\left[\frac{V_m}{2\pi}\big(cos(\alpha)-cos(\beta)\big) - \frac{\gamma}{2\pi}\boldsymbol{E_a}\right]$$

$$t_{em}(t) = K_t i_a(t) = J\frac{d\omega(t)}{dt} + b\omega(t) + t_L(t)$$

$$\overline{\frac{d\omega(t)}{dt}} = \frac{1}{T}\int_0^T \frac{d\omega(t)}{dt}dt = \frac{1}{T}\int_0^T d\omega(t) = \frac{1}{T}\big(\omega(T)-\omega(0)\big) = 0$$

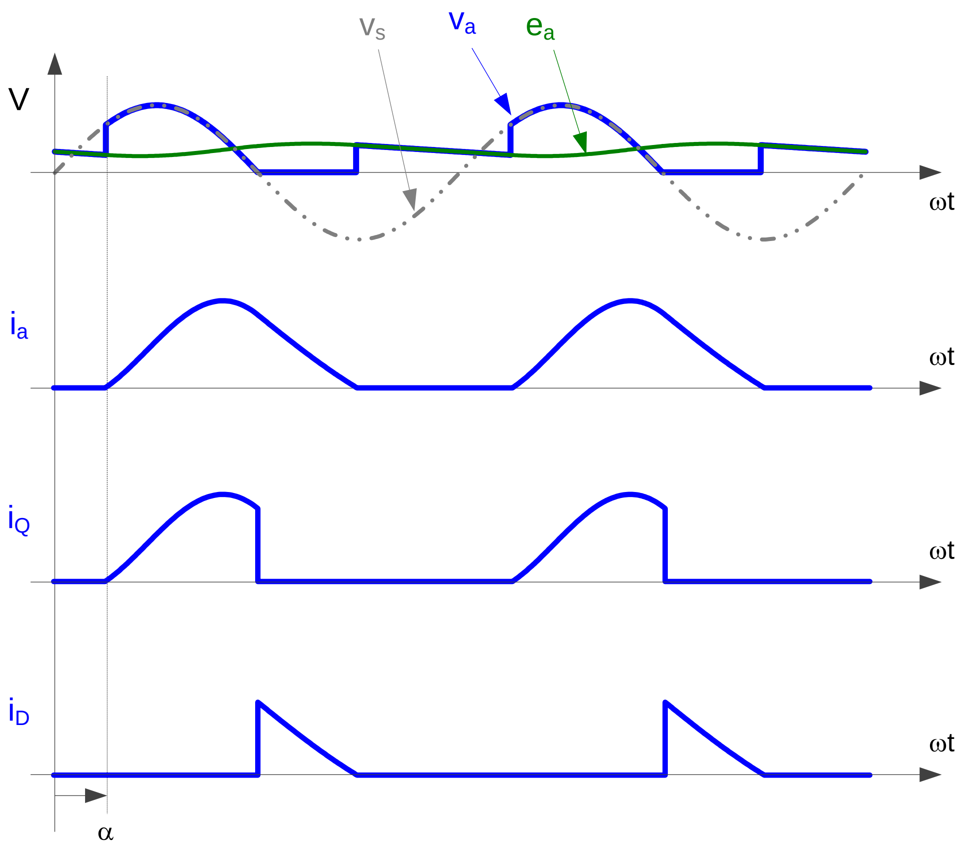
$$\boldsymbol{T_{em}} = K_t\boldsymbol{I_a} = b\overline{\omega(t)} + t_L(t)$$

Single Phase SCR Drive with Diode



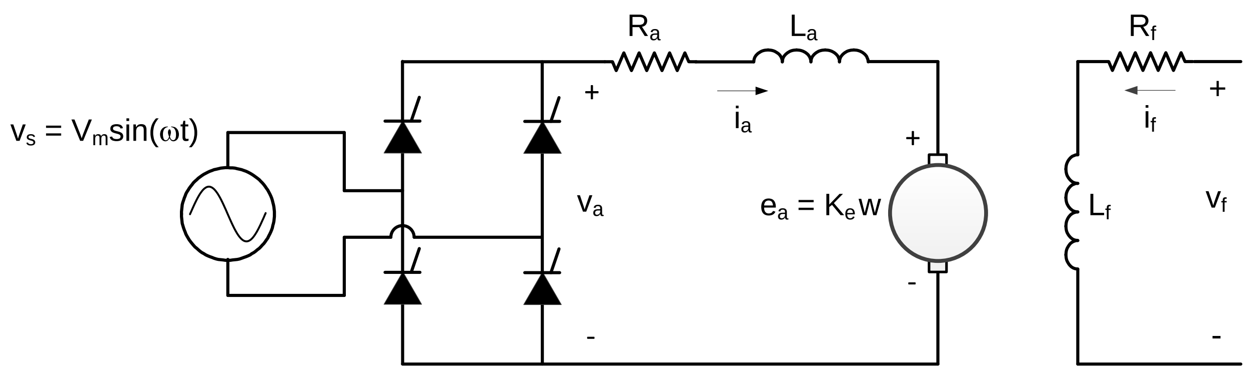
Single Phase SCR Drive with Free-Wheeling Diode

Eliminates negative voltage across load

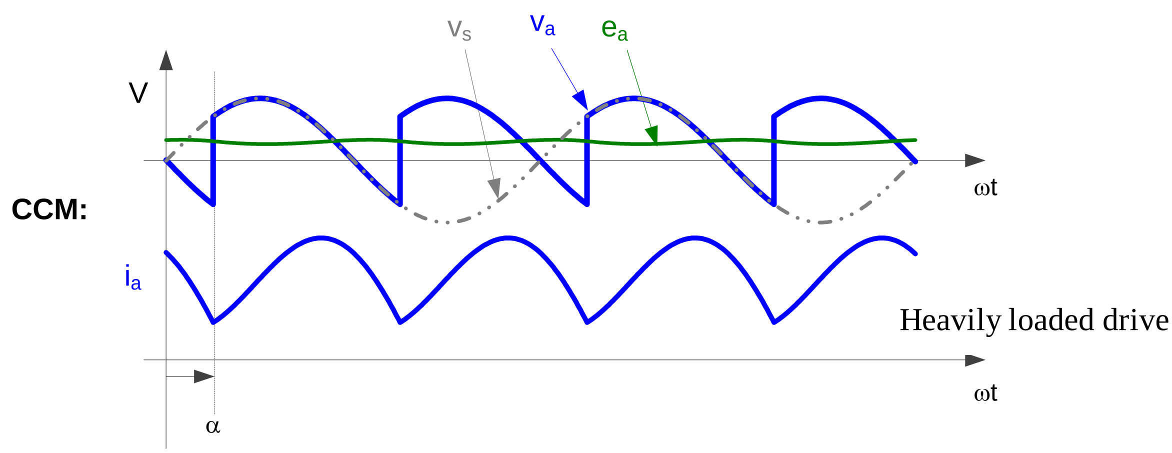


Voltage and Current Waveforms of Single Phase SCR Drive with Free-Wheeling Diode

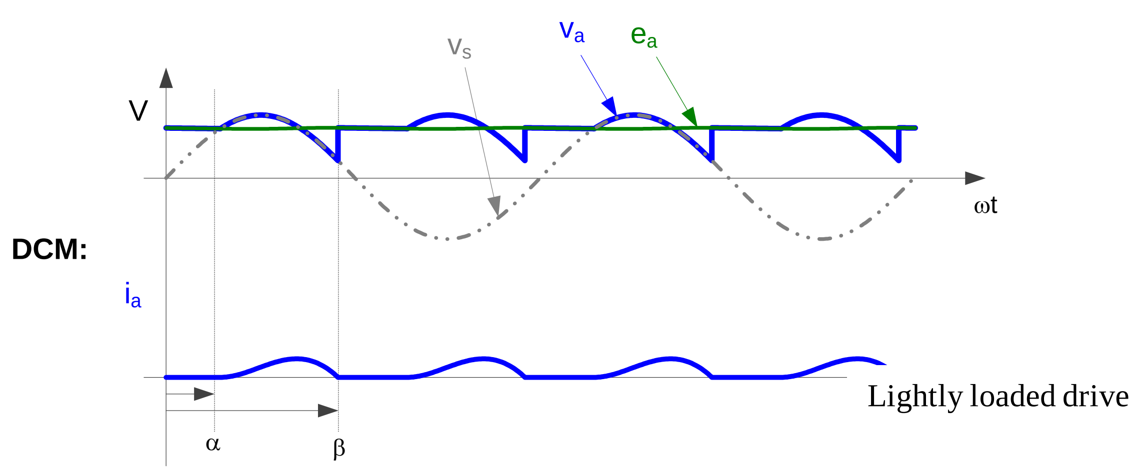
Single Phase SCR AC/DC Converters – Full Wave Rectifier



Single Phase Full Wave Rectified SCR Drive

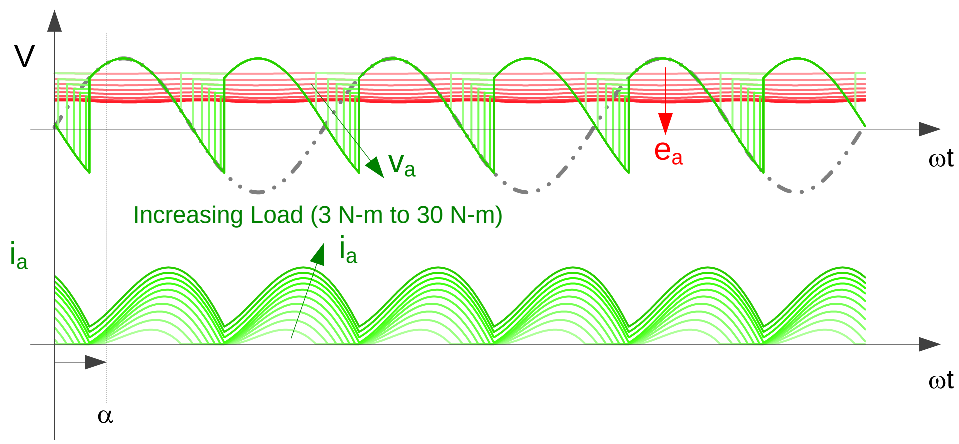


(a) CCM



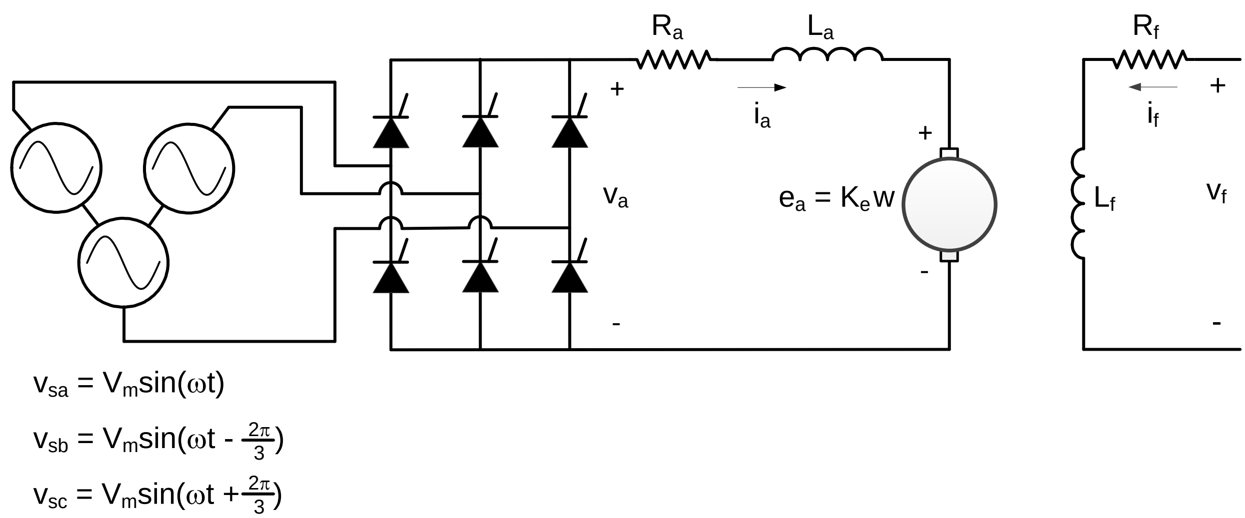
(b) DCM

Voltage and Current Waveforms of Full Wave Rectified Single Phase SCR Drive (a) Continuous Current Mode (CCM) and (b) Discontinuous Current Mode (DCM)

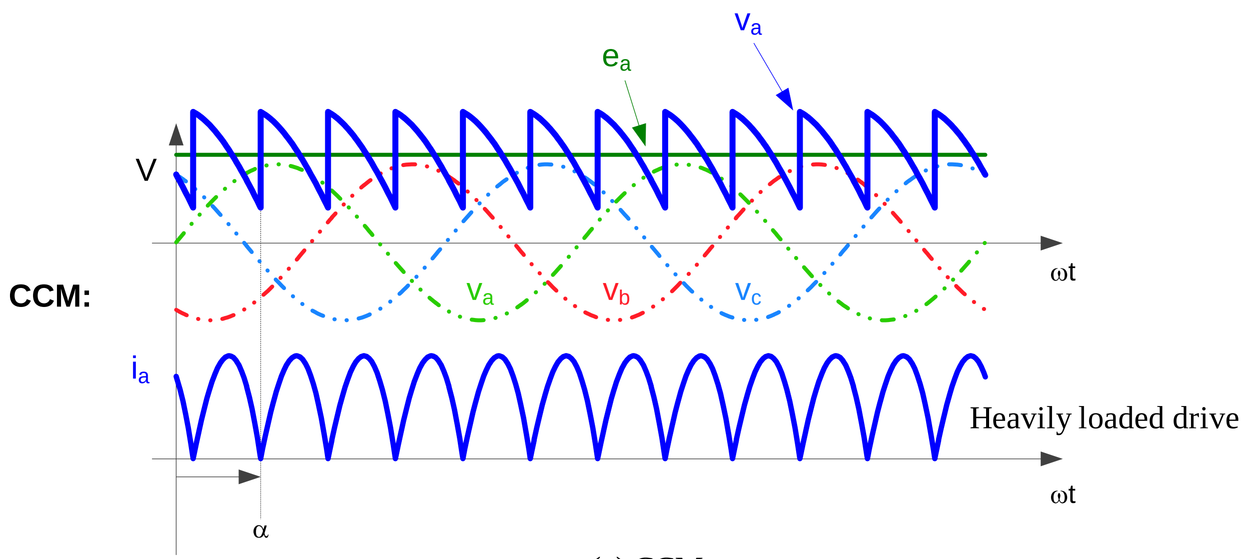


Voltage and Current Waveforms of Full Wave Rectified Single Phase SCR Drive with Increasing Load (Current Transitions from Discontinuous Mode when Lightly Loaded to Continuous Mode when Heavily Loaded)

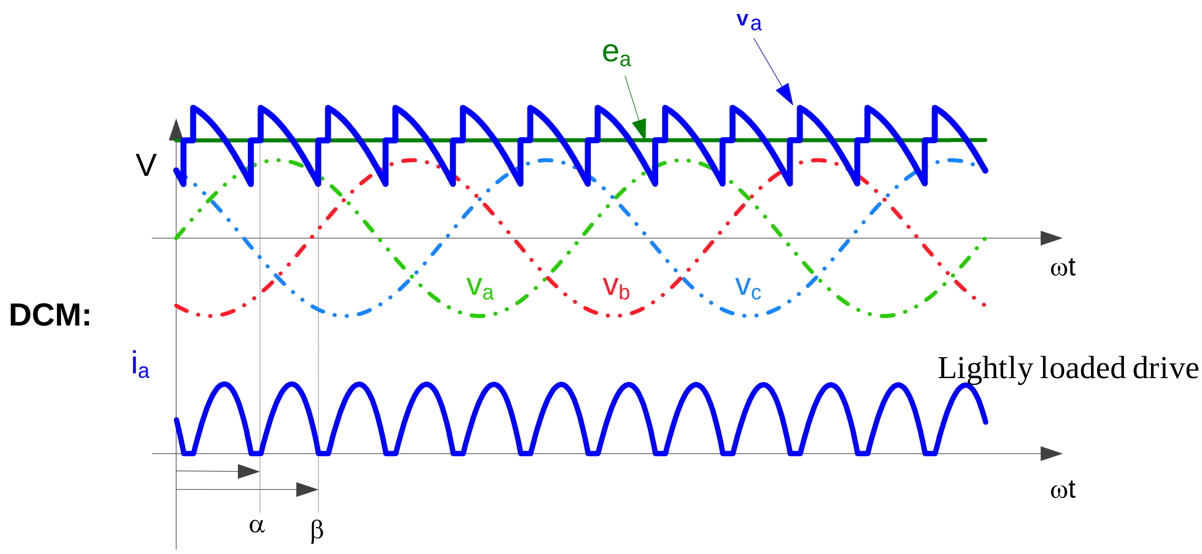
Three Phase SCR AC/DC Converters – Full Wave Rectifier



Three Phase Full Wave Rectified SCR Drive

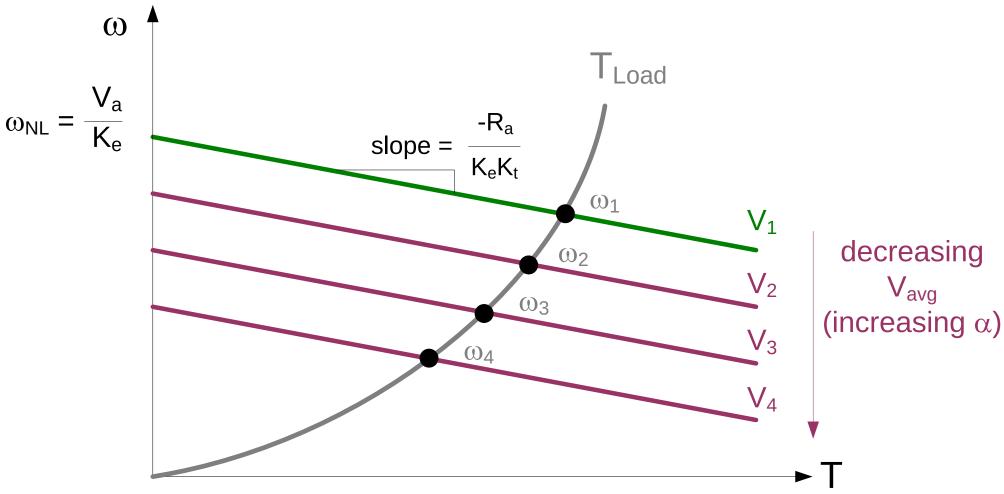


(a) CCM



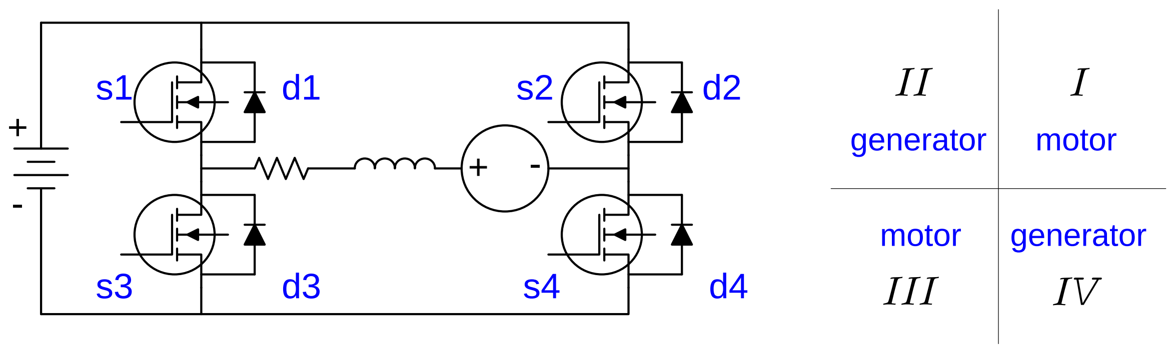
(b) DCM

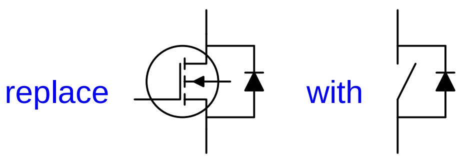
Voltage and Current Waveforms of Full Wave Rectified Three Phase SCR Drive (a) Continuous Current Mode (CCM) and (b) Discontinuous Current Mode (DCM)



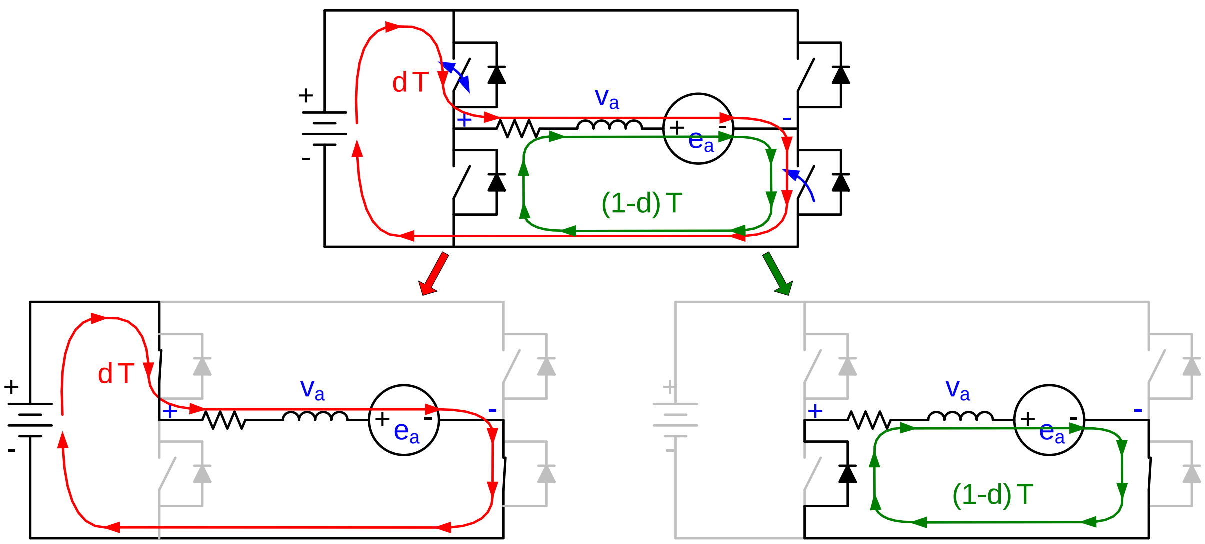
Speed vs. Torque Curve of a DC Machine

H-Bridge Inverter Configuration (1980's on)





Two quadrant operation in positive direction:



2Q Operation with S1 and S4 On

2Q Operation with S1 Off and S4 On

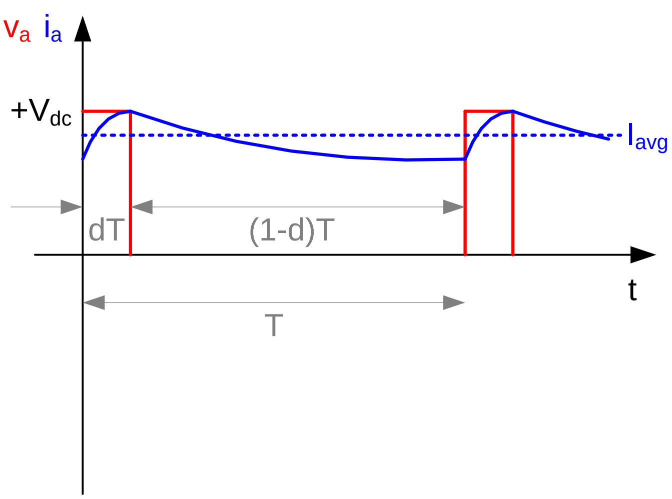
With S1 and S4 on (conducting):

$v_a =$

With S1 off and S4 on, D3 conducts (until current goes to zero):

$v_a =$

With S4 on, S1 on for $d \cdot T$, off for $(1-d) \cdot T$



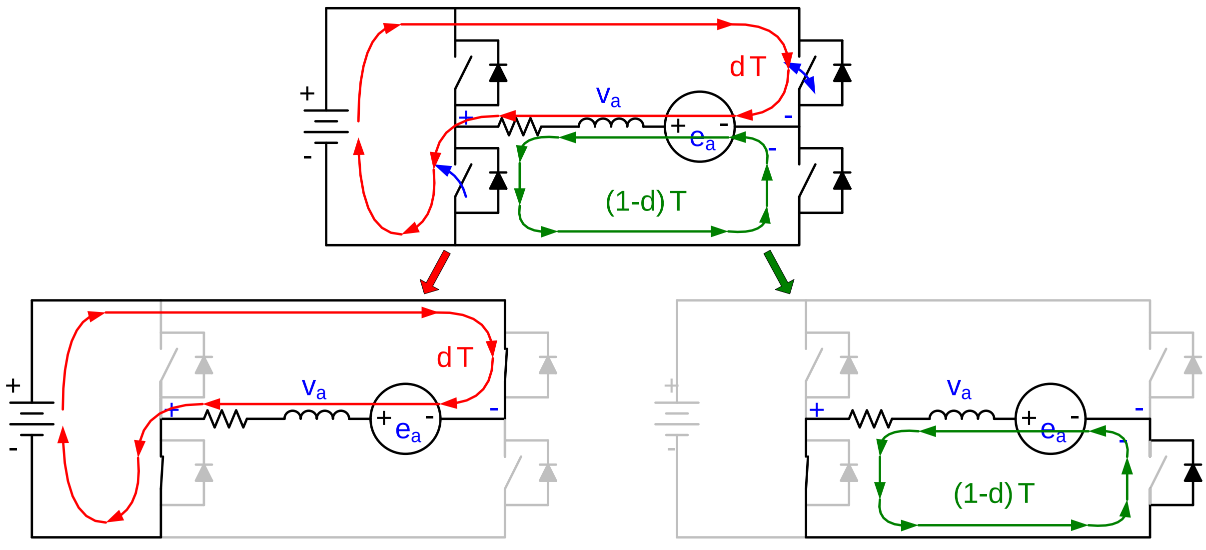
$$\overline{v_a} = \frac{1}{T}\int_0^T v_a(t)dt =$$

$$\overline{i_a} = \frac{\overline{v_a}-\overline{e_a}}{R_a}$$

2Q Operation

In order to apply voltage in opposite direction (negative voltage) to load, we have to stop switching S1, S4 and transition to switching S2, S3.

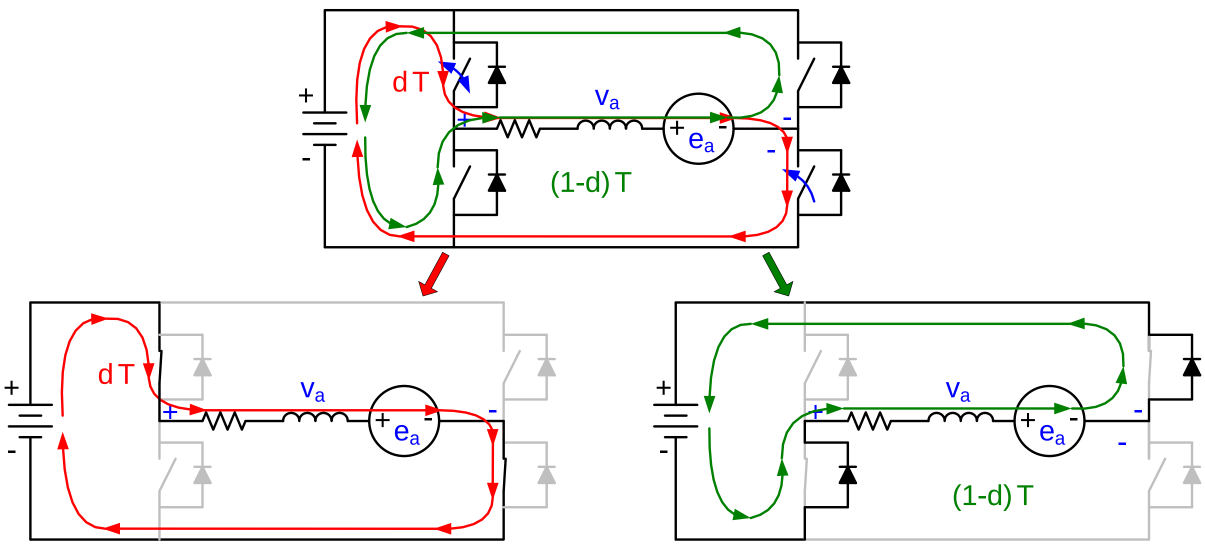
Two quadrant operation in opposite direction:

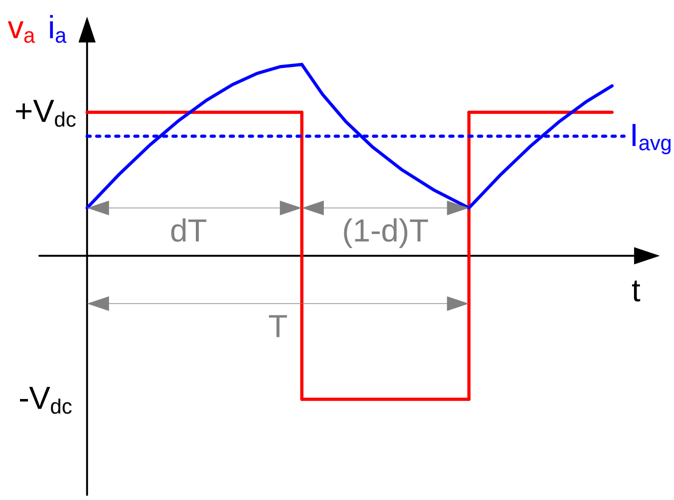


2Q Operation with S2 and S3 On

2Q Operation with S2 Off and S3 On

Four quadrant operation of a DC drive:



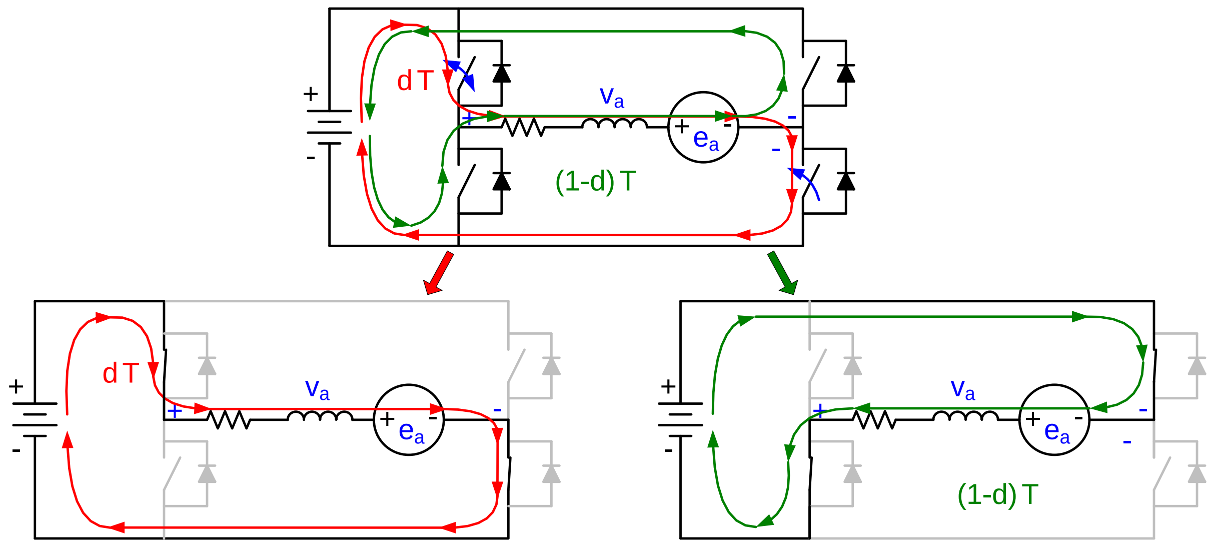


4Q Operation

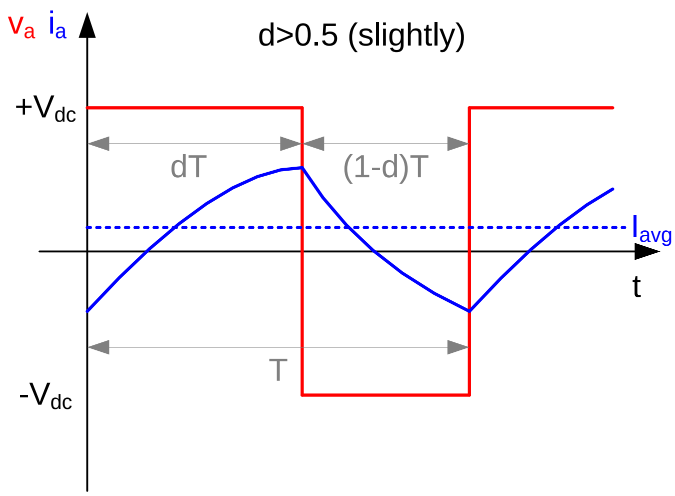
$$\overline{v_a} = \frac{1}{T}\int_0^T v_a(t)dt =$$

$$=$$

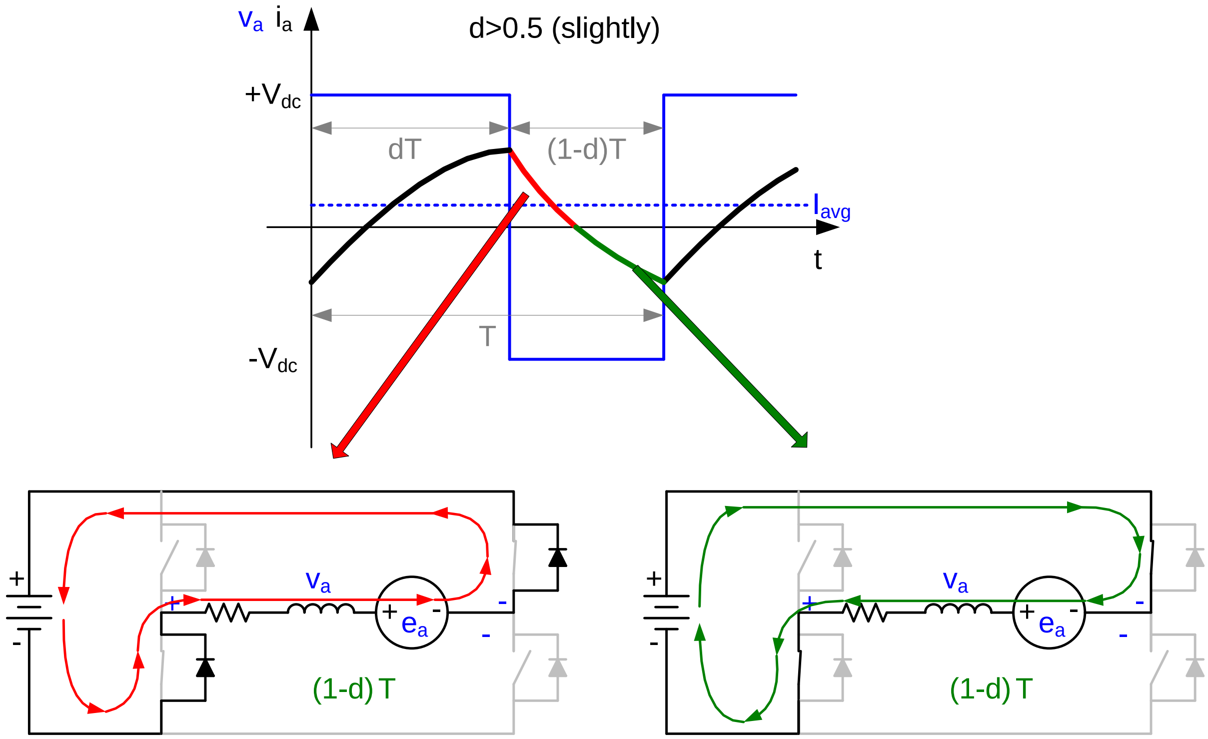
$$\overline{i_a} = \frac{\overline{v_a}-\overline{e_a}}{R_a}$$



Four quadrant operation of a DC drive when current goes negative:



What is current path when it's bipolar (positive and negative)?



Differences between 2Q and 4Q operation:

- 2Q has lower inverter losses due to use of fewer devices
- 2Q has lower motor losses due to lower switching ripple (which can be significant in low impedance machines)
- 4Q used for servo operation (can handle regenerative energy)

Example of switching ripple in 2Q and 4Q drives:

$R = 1\,\Omega$ $\quad L = 1\text{ mH}$ $\quad V_{dc} = 100\text{ V}$ $\quad I_{avg} = 10\text{ A}$

$f_s = 20\text{ kHz} \rightarrow T_s = 50\ \mu s$

2Q Operation at Stall:

$$I_{avg} = \frac{V_{avg}}{R} = \frac{d \cdot V_{dc}}{R} \rightarrow d = \frac{I_{avg} \cdot R}{V_{dc}} = 0.1$$

1st order approximation:

$$V = L\frac{\Delta i}{\Delta t} \rightarrow \Delta i = \frac{V \Delta t}{L} = \frac{(100 \text{ volts}) \cdot (0.1 \cdot 50 \times 10^{-6} \text{ sec})}{1 \times 10^{-3} \text{ H}} = 500 \times 10^{-3} \text{ A}$$

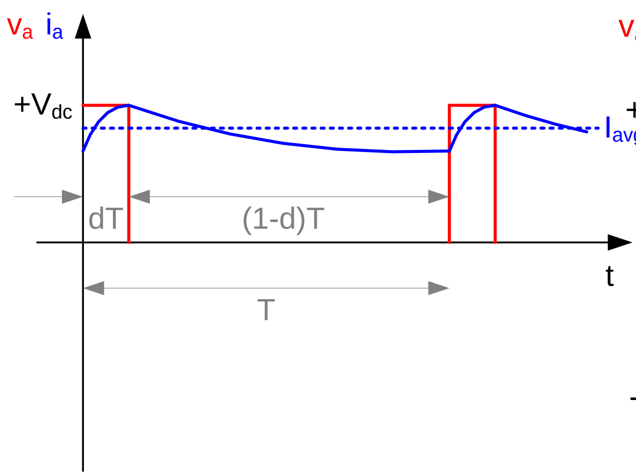
$$\Delta i_{2Q} = 500 \text{ mA}$$

4Q Operation at Stall:

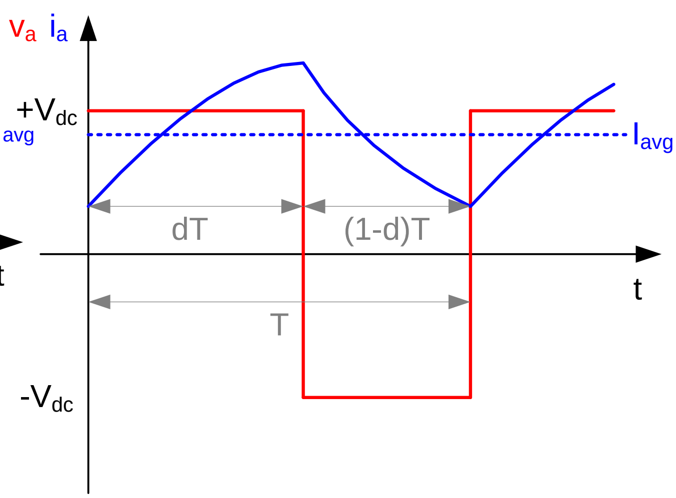
$$I_{avg} = \frac{V_{avg}}{R} = \frac{2(d - 0.5) \cdot V_{dc}}{R} \rightarrow d = \frac{I_{avg} \cdot R}{2 \cdot V_{dc}} + 0.5 = \frac{10}{200} + 0.5 = 0.55$$

$$\Delta i = \frac{V \Delta t}{L} = \frac{(100 \text{ volts}) \cdot (0.55 \cdot 50 \times 10^{-6} \text{ sec})}{1 \times 10^{-3} \text{ H}} = 2.75 \text{ A}$$

$$\Delta i_{4Q} = 2.525 \text{ A} > 5 \times \Delta i_{2Q}$$

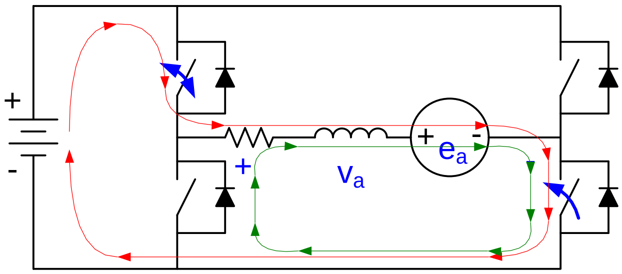


2Q Operation

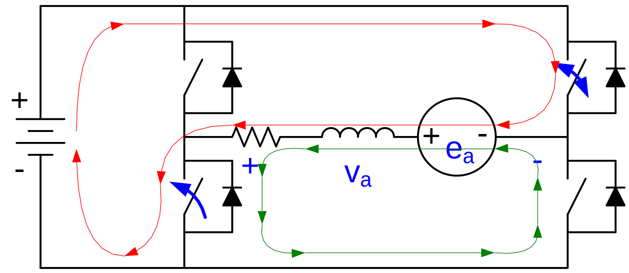


4Q Operation

Why can’t we dynamically brake in 2Q operation?



2Q Operation Motoring



2Q Operation Braking

Motoring

<u>*S1 closed (S4 closed)*</u>

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a$$

$$v_a - e_a = R_a i_a + L_a \frac{di_a}{dt}$$

<u>*S1 open (S4 closed)*</u>

$$-e_a = R_a i_a + L_a \frac{di_a}{dt}$$

Braking

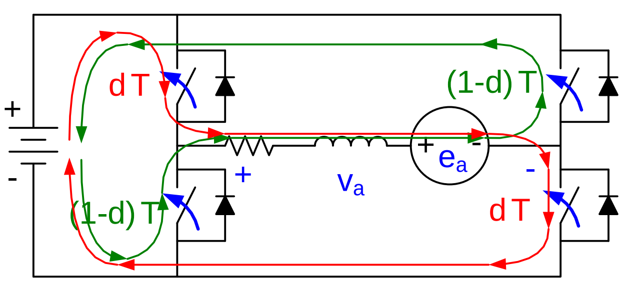
<u>*S2 closed (S3 closed)*</u>

$$v_a = R_a i_a + L_a \frac{di_a}{dt} - e_a$$

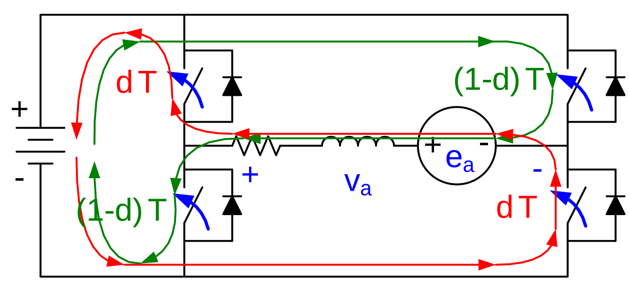
$$v_a + e_a = R_a i_a + L_a \frac{di_a}{dt}$$

<u>*S2 open (S3 closed)*</u>

$$+e_a = R_a i_a + L_a \frac{di_a}{dt}$$

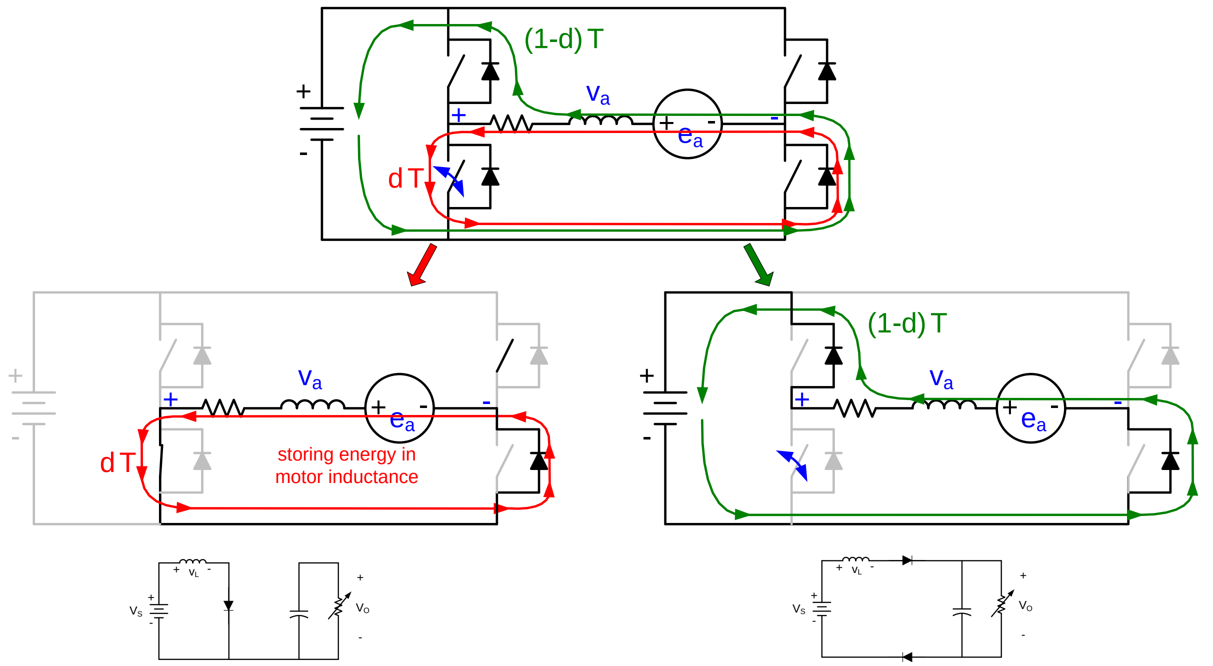


d > (1 – d)
4Q Operation Motoring

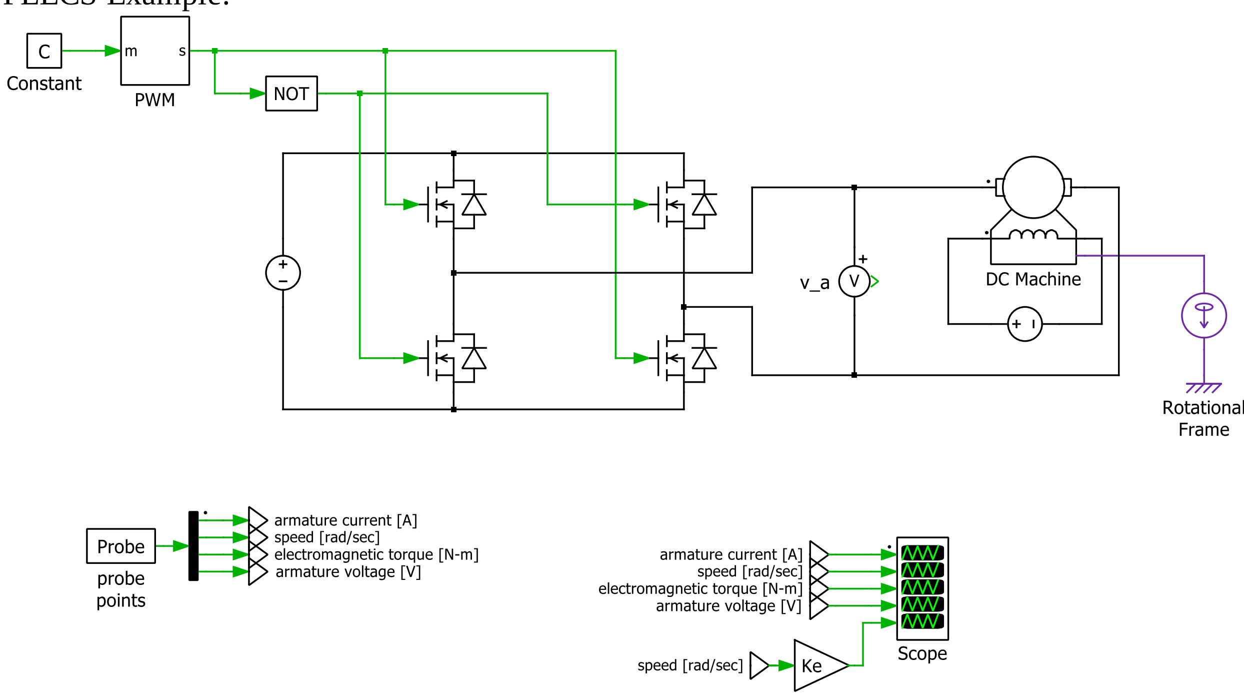


d < (1 – d)
4Q Operation Braking

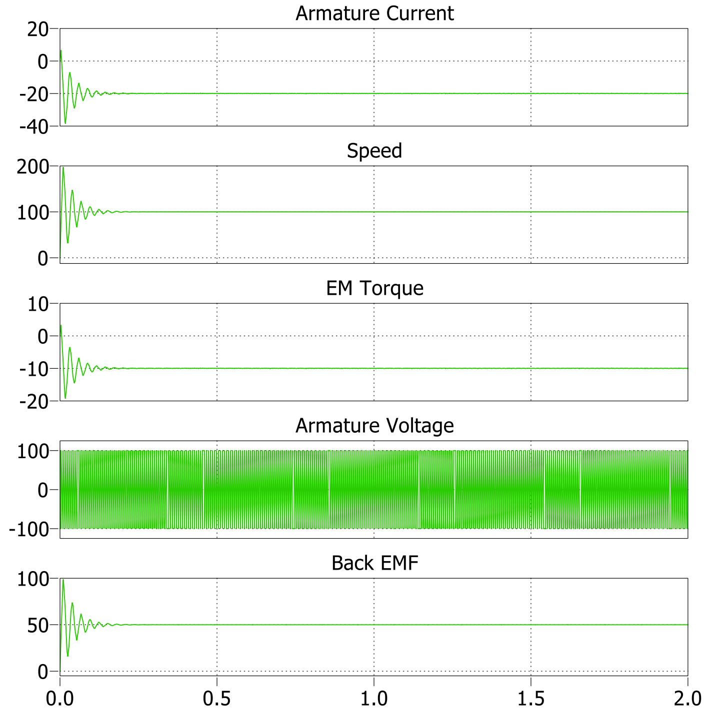
Boost Operation in a Motor Drive



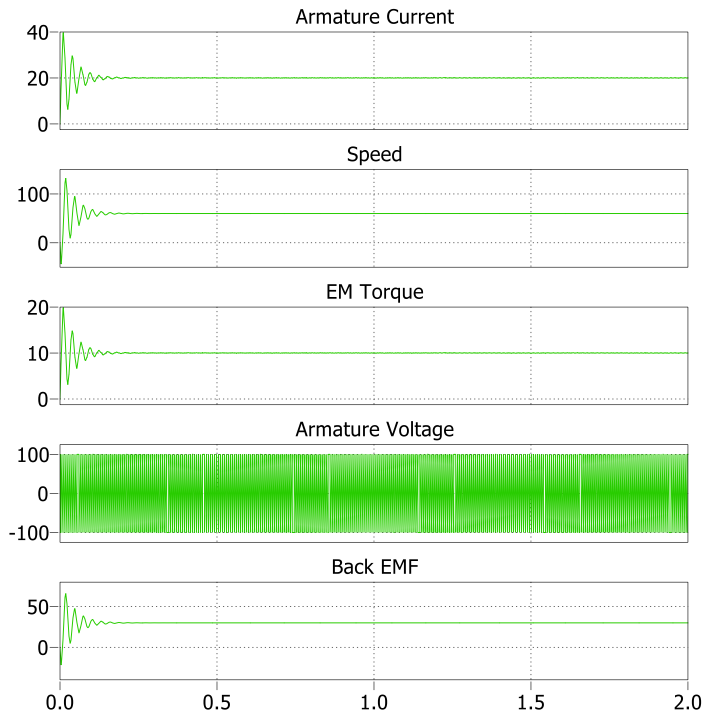
PLECS Example:



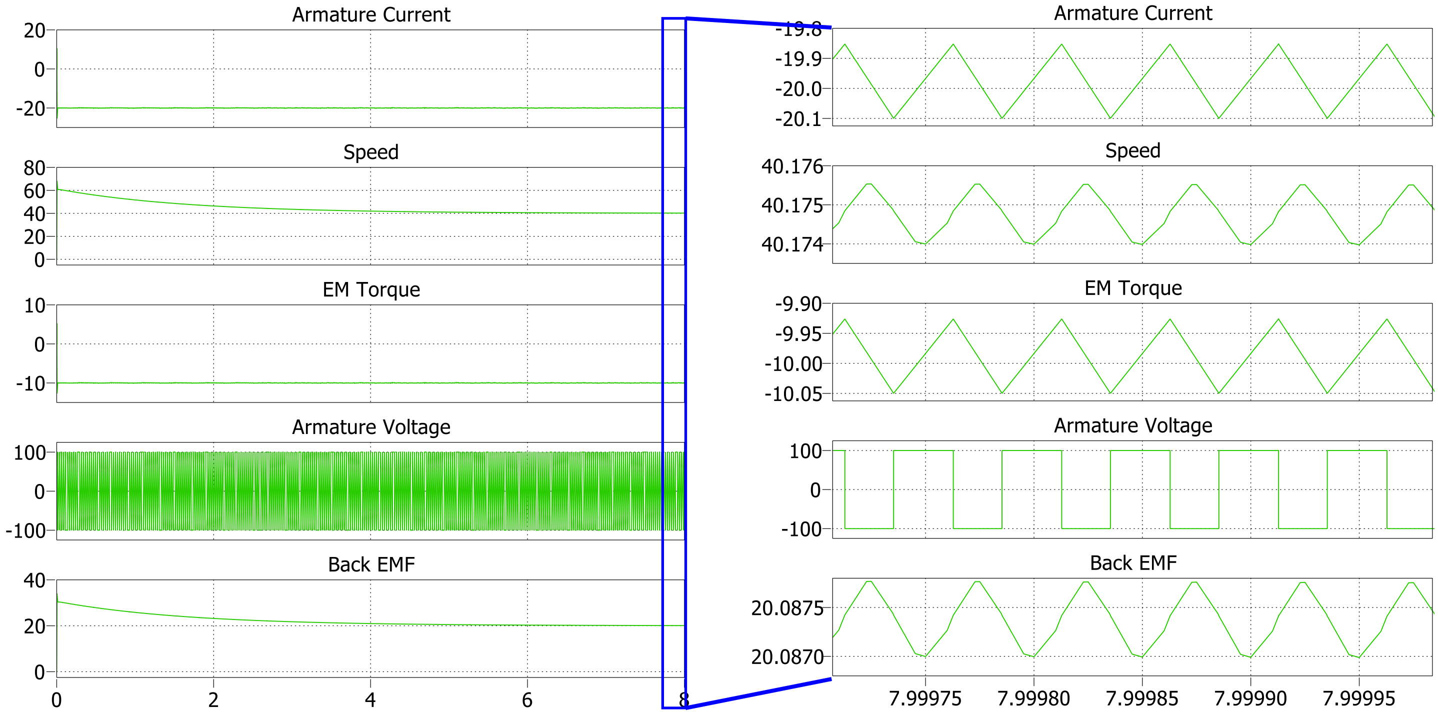
Aiding Load:



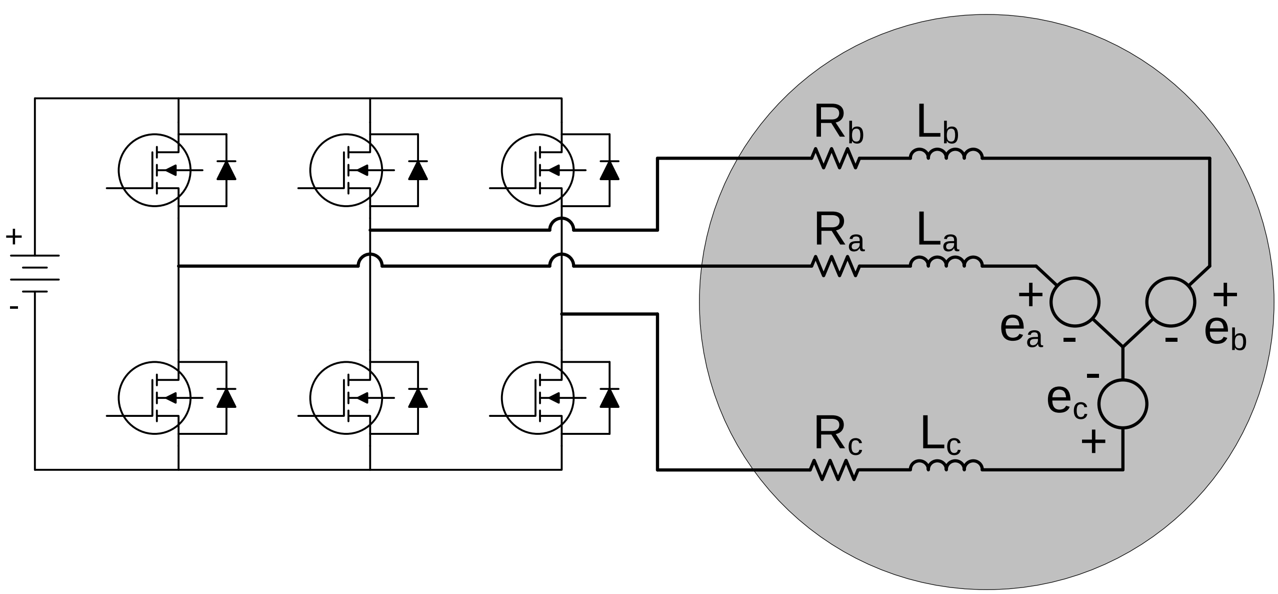
Opposing Load:



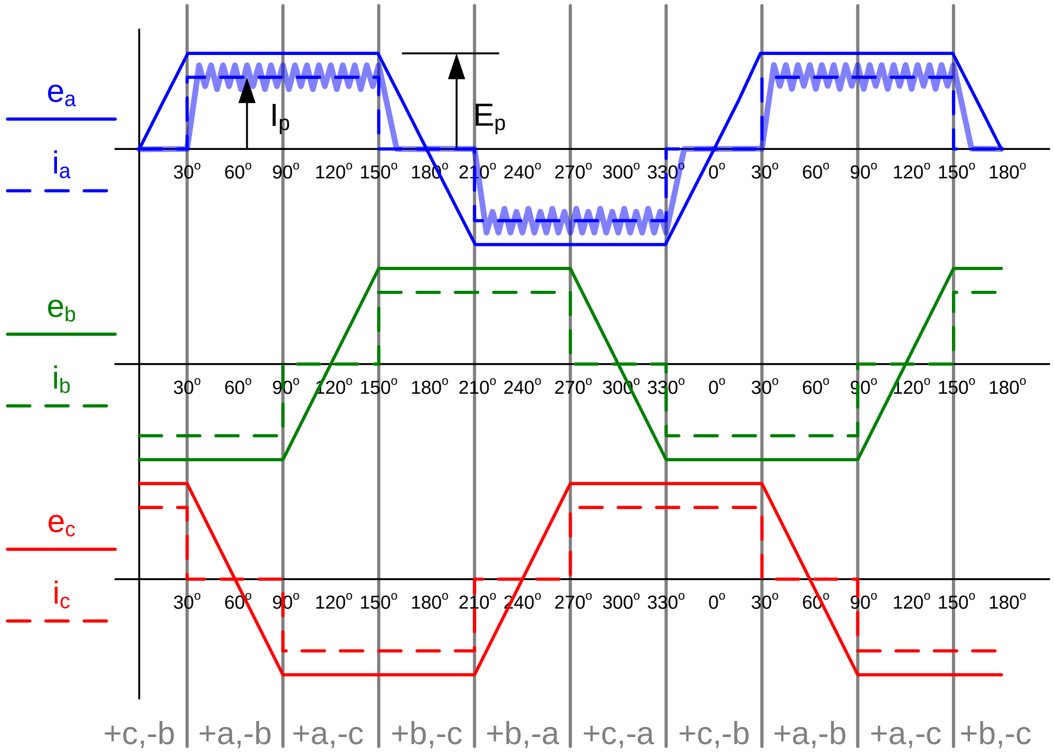
Example: Speed Controlled Motor with Aiding Load



Brushless DC (BLDC) Drives:

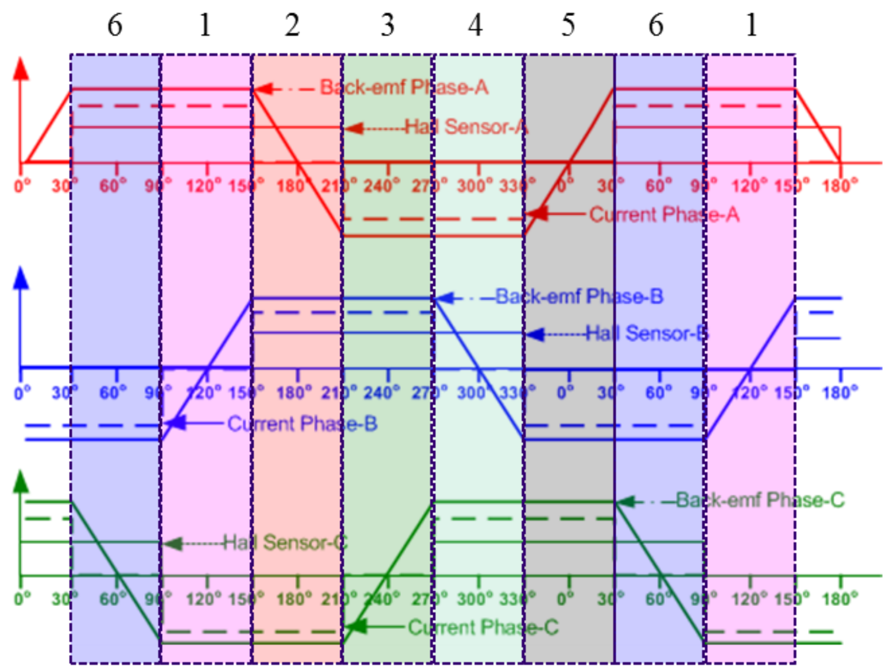


BLDC Drive (Motor and Inverter)



Trapezoidal Back EMF Waveforms

Hall Sequence for Positive Motoring (Positive Torque)



	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
Phase A	+	Z	-	-	Z	+
Phase B	Z	+	+	Z	-	-
Phase C	-	-	Z	+	+	Z
Hall A	1	1	0	0	0	1
Hall B	0	1	1	1	0	0
Hall C	0	0	0	1	1	1

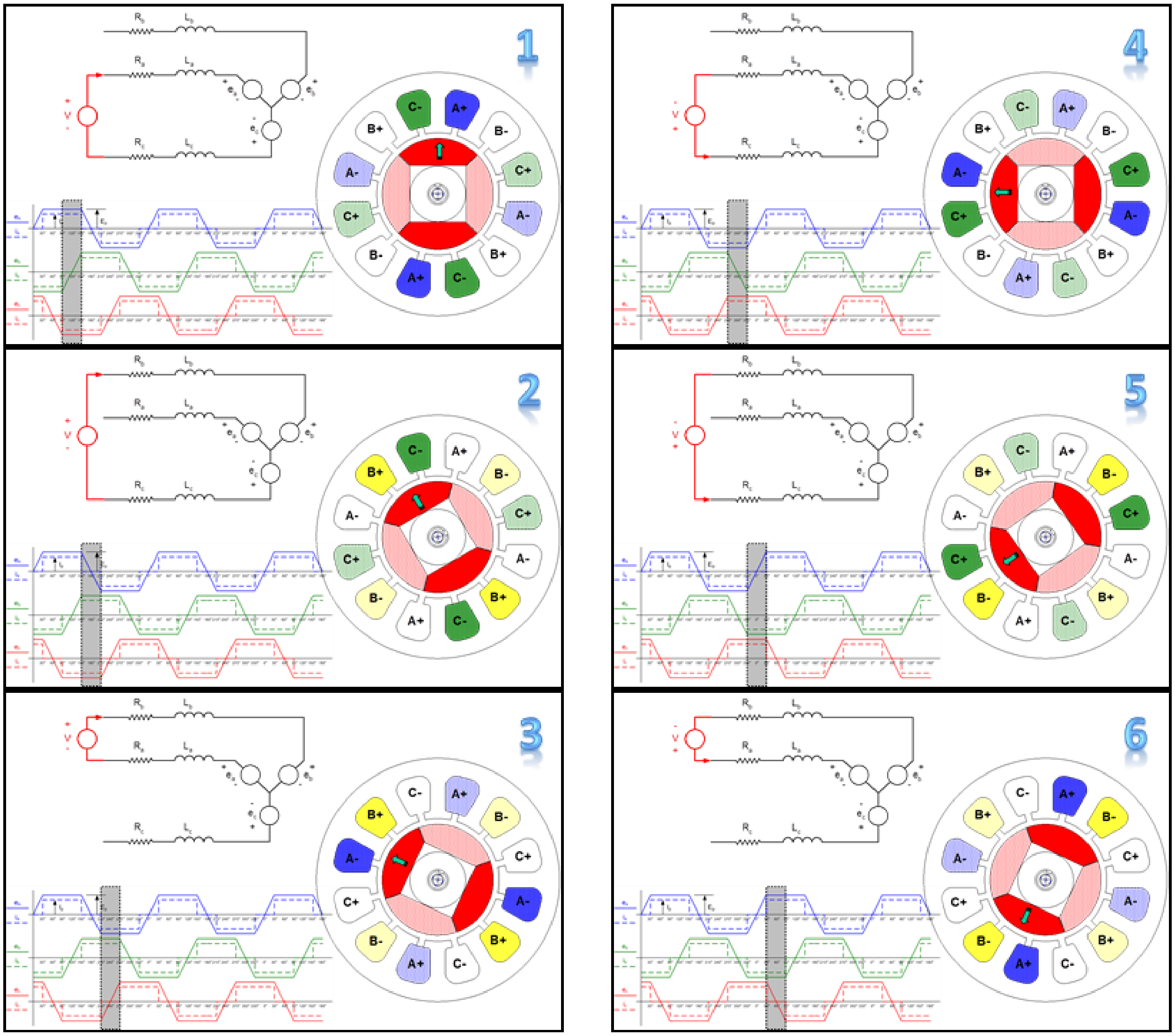
In order to produce torque in the opposite direction (run the motor in the opposite direction), reverse the "+" and "-" in the table above:

	Step					
	1	2	3	4	5	6
Phase A	+	Z	-	-	Z	+
Phase B	Z	+	+	Z	-	-
Phase C	-	-	Z	+	+	Z
Hall A	1	1	0	0	0	1
Hall B	0	1	1	1	0	0
Hall C	0	0	0	1	1	1

FWD

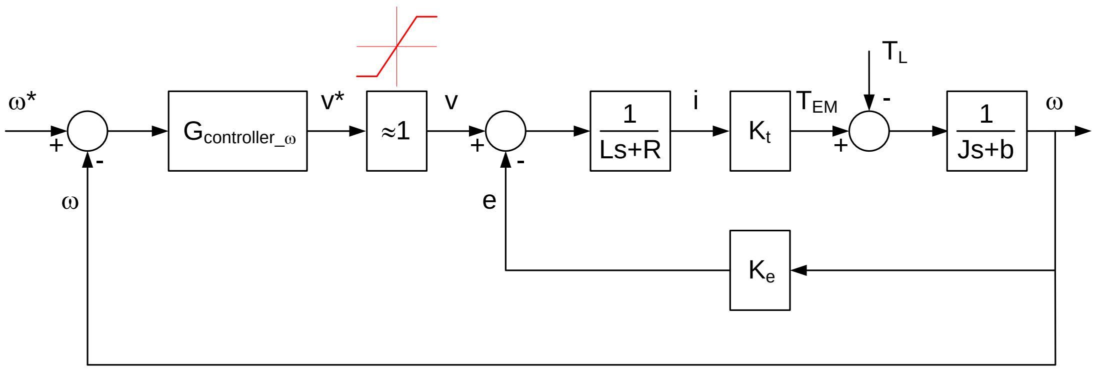
	Step					
	1	2	3	4	5	6
Phase A	-	Z	+	+	Z	-
Phase B	Z	-	-	Z	+	+
Phase C	+	+	Z	-	-	Z
Hall A	1	1	0	0	0	1
Hall B	0	1	1	1	0	0
Hall C	0	0	0	1	1	1

REV


1
4
2
5
3
6

Current Regulation in DC Machines:

For speed control in DC machines, we don't need to close a current loop. However, it is a preferred method for high performance motor controls. Closed loop speed can be controlled without a current loop. Assuming a speed control with an ideal voltage source using voltage as the manipulated variable, we have the following block diagram.



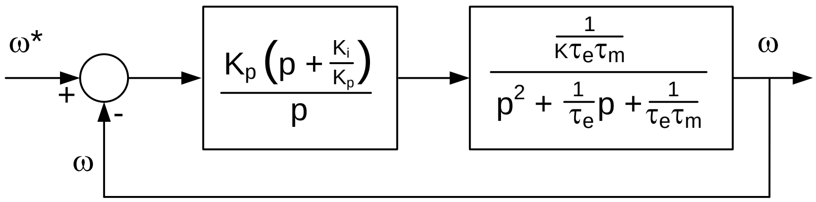
Speed Control of a DC Motor with Voltage as the Manipulated Variable

$$\frac{\Omega}{\Omega^*} = \frac{G_c K_t}{J L_a p^2 + (L_a b + J R_a) p + K_e K_t + b R_a + G_c K_t}$$

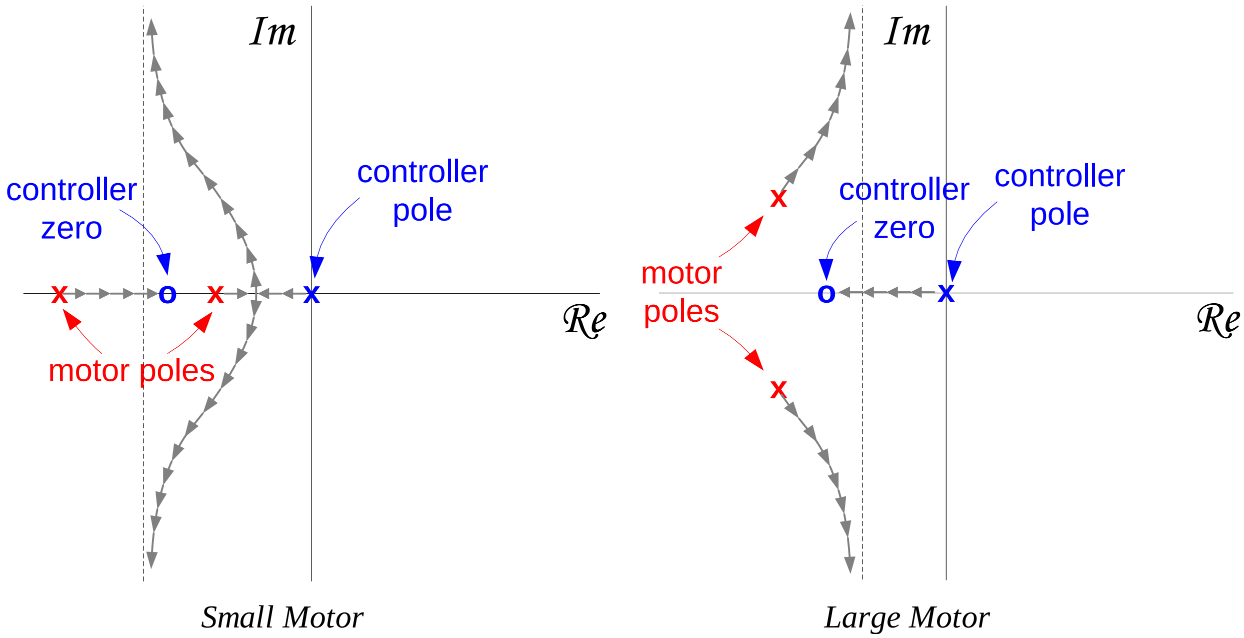
Difficult to design & tune G_c to get desired results. Alternative approach, control current with inner current loop.

Use Proportional plus Integral (PI) Control

$$G_c = K_p + \frac{K_i}{p} = \frac{K_i}{p}\left(\frac{K_p}{K_i} p + 1\right) = \frac{K_p\left(p + \frac{K_i}{K_p}\right)}{p}$$



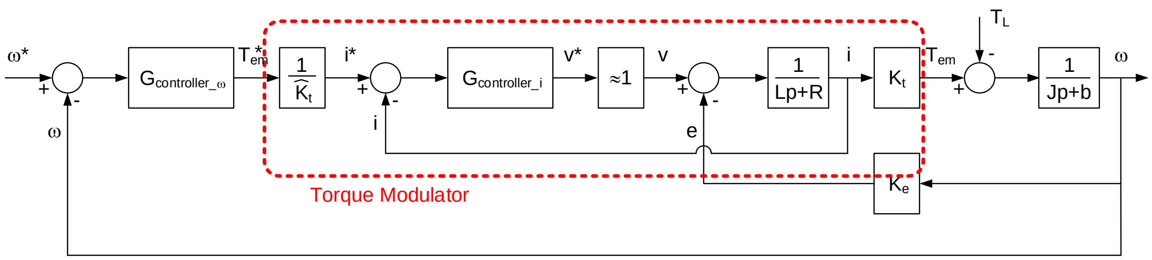
Speed Control of a DC Motor with Voltage as the Manipulated Variable with PI Controller



Root Locus of DC Motor with PI Speed Controller

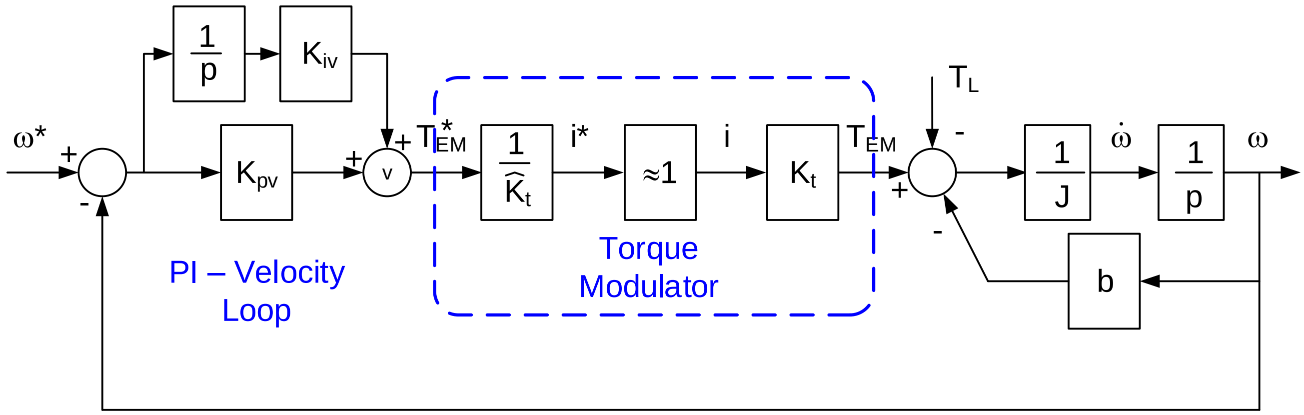
Closed Loop Speed Control Via Closed Loop Current Control

Closed loop speed control with ideal voltage source using voltage as manipulated variable with an inner current loop.

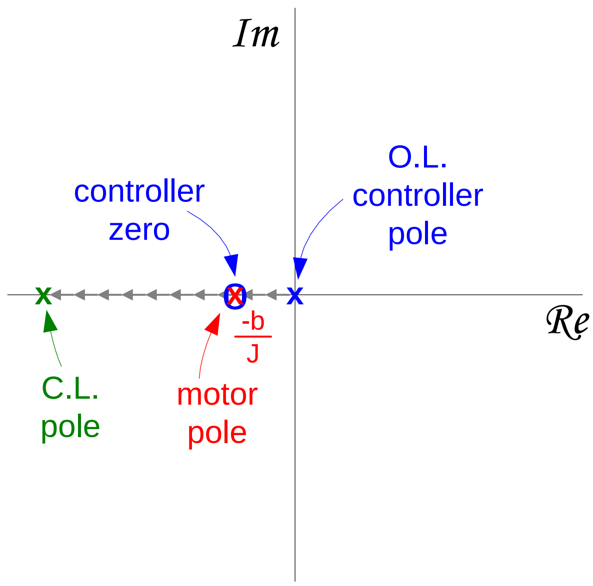


Speed Control of a DC Motor with Voltage as the Manipulated Variable

If current regulation is robust and significantly higher bandwidth than the desired rate loop dynamics, then we can assume that the current loop is ideal (i.e. actual current = commanded current).



Speed Control of a DC Motor with Voltage as the Manipulated Variable

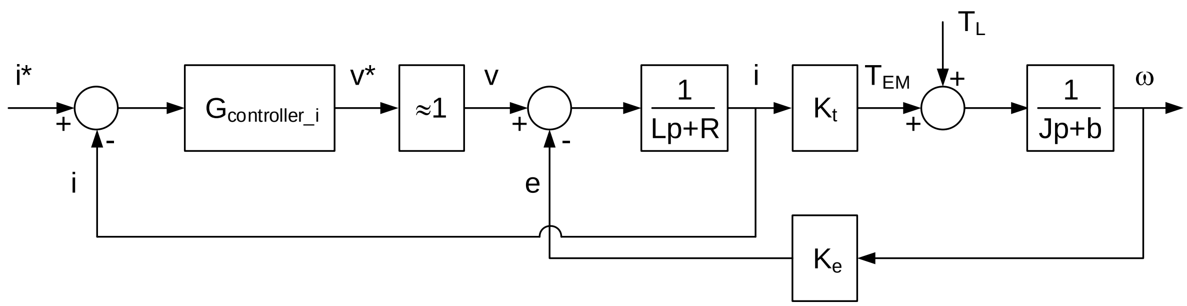


Root Locus of DC Motor with PI Speed Controller with Ideal Inner Current (Torque) Loop

Response not limited by τ_e and τ_m.

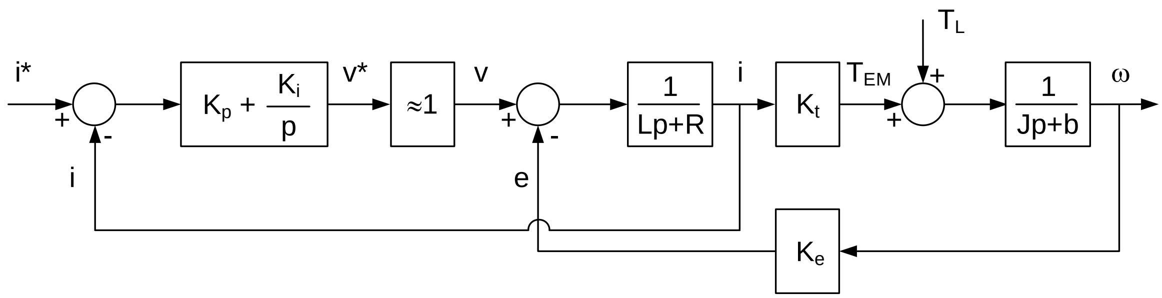
Only limited by feasible gains, system saturations.
(limited by noise) (voltage/current limits)

Current Loop Closure in DC Machines:



Current Regulator for DC Motor

Industry Standard is to use a Proportional Plus Integral (PI) Controller as shown below:



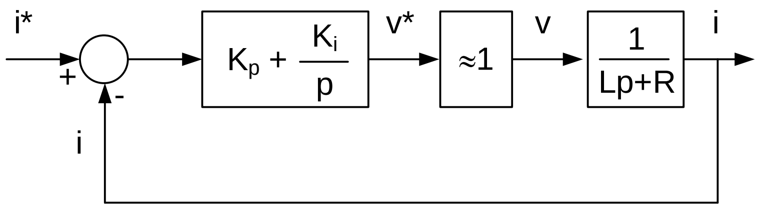
PI Current Regulator for DC Motor

Transfer function for current regulator:

$$\frac{I}{I^*} = \frac{(K_p p + K_i)(Jp+b)}{JL_a p^3 + [J(R_a+K_p)+bL_a]p^2 + [b(R_a+K_p)+JK_i+K_eK_t]p + bK_i}$$

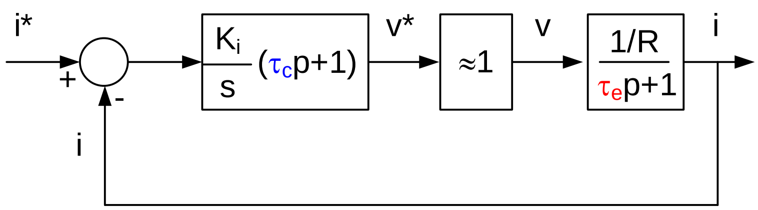
The challenge is how to tune current regulator gains, namely, how should we select K_p and K_i?
The answer is:

Tune current loop at locked rotor to decouple the effects of Back EMF:



PI Current Regulator for DC Motor

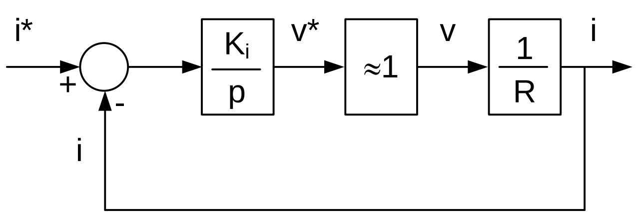
Which can be redrawn as:



$\tau_c = \frac{K_p}{K_i}$ $\tau_e = \frac{L}{R}$

PI Current Regulator for DC Motor Expressed as Time Constants

If $\tau_c = \tau_e$:



PI Current Regulator for DC Motor with Exact Pole-Zero Cancellation

The transfer function becomes:

$\frac{I}{I^*} =$ where $K_{tot} = \frac{K_i}{R_a}$

To determine bandwidth, set $K_{tot} = 2\pi f_{desired}$

where $f_{desired}$ = desired bandwidth in hertz.

Therefore, the gains to tune the current loop at locked rotor conditions are:

$\boxed{K_i = 2\pi f_{desired} R_a}$ and $\boxed{K_p = 2\pi f_{desired} L_a}$

Note: By using Back EMF decoupling, the desired bandwidth can be achieved at non-zero speed conditions as well.

PowerPoint Slideshow on Current Regulator Tuning.