From last lecture:

Stator Voltage equation:

$$\lambda_{as} = L_{as}\, i_{as} + L_{abs}\, i_{bs} + L_{acs}\, i_{cs} + L_{asar}\, i_{ar} + L_{asbr}\, i_{br} + L_{ascr}\, i_{cr}$$

$$v_{as} = r_s\, i_{as} + p\,\lambda_{as}$$

$$= r_s\, i_{as} + L_{ls}\, p\, i_{as} + L_{ms}\, p\left(i_{as} - \frac{1}{2} i_{bs} - \frac{1}{2} i_{cs}\right) +$$
$$\frac{N_r}{N_s} L_{ms}\, p\left[i_{ar}\cos(\theta_r) + i_{br}\cos\left(\theta_r + 120^o\right) + i_{cr}\cos\left(\theta_r - 120^o\right)\right]$$

$$p\left(i_{ar}\cos(\theta_r)\right) = \underbrace{\cos(\theta_r)\, p\, i_{ar}}_{\textit{transformer voltage}} - \underbrace{i_{ar}\sin(\theta_r)\frac{d\theta_r}{dt}}_{\textit{speed dependent voltage}}$$

Observations:

- These equations describe a set of nonlinear (because of product of current , position, and speed), coupled set of differential equations with time varying coefficients!
- Will show this is an 8^{th} order (with mechanical system) nonlinear, cross coupled set of differential equations with time varying coefficients!
- Special case: $\frac{d\theta_r}{dt} = \text{constant} = \omega_r$

  $$\theta_r = \omega_r t + \theta_{ro}$$

  equations are linear but still have time varying (periodic) coefficients

It gets more complicated from here, we need to introduce notation which will simplify analysis and understanding, but it is something new and will take effort to understand.

System Equations for a 3ϕ Machine in the Stationary a,b,c Reference Frame:

$$\underline{V}_{abcs} = r_s \underline{I}_{abcs} + p \underline{\Lambda}_{abcs}$$ stator equation

$$\underline{V}_{abcr} = r_r \underline{I}_{abcr} + p \underline{\Lambda}_{abcr}$$ rotor equation

$$\underline{V}_{abcs} = \begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} \qquad \underline{I}_{abcs} = \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \qquad \underline{\Lambda}_{abcs} = \begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix}$$

but there is coupling between the stator and rotor so we can separate the flux linkage into 2 matrices:

$$\underline{\Lambda}_{abcs} = \underline{\Lambda}_{abcs(s)} + \underline{\Lambda}_{abcs(r)}$$

$$\underline{\Lambda}_{abcr} = \underline{\Lambda}_{abcr(s)} + \underline{\Lambda}_{abcr(r)}$$

$$\underline{\Lambda}_{abcs(s)} = \begin{bmatrix} L_{as} & L_{abs} & L_{acs} \\ L_{abs} & L_{bs} & L_{bcs} \\ L_{acs} & L_{bcs} & L_{cs} \end{bmatrix} \underline{I}_{abcs} \qquad \underline{\Lambda}_{abcs(r)} = \begin{bmatrix} L_{asar} & L_{asbr} & L_{ascr} \\ L_{bsar} & L_{bsbr} & L_{bscr} \\ L_{csar} & L_{csbr} & L_{cscr} \end{bmatrix} \underline{I}_{abcr}$$

$$\underline{\Lambda}_{abcr(s)} = \begin{bmatrix} L_{aras} & L_{arbs} & L_{arcs} \\ L_{bras} & L_{brbs} & L_{brcs} \\ L_{cras} & L_{crbs} & L_{crcs} \end{bmatrix} \underline{I}_{abcs} \qquad \underline{\Lambda}_{abcr(r)} = \begin{bmatrix} L_{ar} & L_{abr} & L_{acr} \\ L_{abr} & L_{br} & L_{bcr} \\ L_{acr} & L_{bcr} & L_{cr} \end{bmatrix} \underline{I}_{abcr}$$

Flux Linkages of *stator phase windings* resulting from currents flowing in the *stator windings*:

$$L_{as} = L_{ms} + L_{ls} = L_{bs} = L_{cs} \qquad L_{abs} = L_{acs} = L_{bcs} = \frac{-1}{2} L_{ms}$$

$$\underline{\Lambda}_{abcs(s)} = \begin{bmatrix} L_{ms} + L_{ls} & \frac{-L_{ms}}{2} & \frac{-L_{ms}}{2} \\ \frac{-L_{ms}}{2} & L_{ms} + L_{ls} & \frac{-L_{ms}}{2} \\ \frac{-L_{ms}}{2} & \frac{-L_{ms}}{2} & L_{ms} + L_{ls} \end{bmatrix} \underline{I}_{abcs}$$

Flux Linkages of *stator phase windings* resulting from currents flowing in the *rotor windings*:

$$L_{asar} = L_{bsbr} = L_{cscr} = \frac{N_r}{N_s} L_{ms} \cos(\theta_r)$$

$$L_{asbr} = L_{bscr} = L_{csar} = \frac{N_r}{N_s} L_{ms} \cos(\theta_r + 120^o)$$

$$L_{ascr} = L_{bsar} = L_{csbr} = \frac{N_r}{N_s} L_{ms} \cos(\theta_r - 120^o)$$

$$\underline{\Lambda}_{abcs(r)} = \frac{N_r}{N_s} L_{ms} \begin{bmatrix} \cos(\theta_r) & \cos(\theta_r + 120^o) & \cos(\theta_r - 120^o) \\ \cos(\theta_r - 120^o) & \cos(\theta_r) & \cos(\theta_r + 120^o) \\ \cos(\theta_r + 120^o) & \cos(\theta_r - 120^o) & \cos(\theta_r) \end{bmatrix} \underline{I}_{abcr}$$

For rotor flux linkages due to stator and rotor currents:

$$\underline{\Lambda}_{abcr(r)} = \begin{bmatrix} \left(\frac{N_r}{N_s}\right)^2 L_{ms} + L_{lr} & \frac{-1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms} & \frac{-1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms} \\ \frac{-1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms} & \left(\frac{N_r}{N_s}\right)^2 L_{ms} + L_{lr} & \frac{-1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms} \\ \frac{-1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms} & \frac{-1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms} & \left(\frac{N_r}{N_s}\right)^2 L_{ms} + L_{lr} \end{bmatrix} \underline{I}_{abcr}$$

$$\underline{\Lambda}_{abcr(s)} = \frac{N_r}{N_s} L_{ms} \begin{bmatrix} \cos(\theta_r) & \cos(\theta_r - 120^o) & \cos(\theta_r + 120^o) \\ \cos(\theta_r + 120^o) & \cos(\theta_r) & \cos(\theta_r - 120^o) \\ \cos(\theta_r - 120^o) & \cos(\theta_r + 120^o) & \cos(\theta_r) \end{bmatrix} \underline{I}_{abcs}$$

Complex Vector Notation

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$ *Euler Identity*

$e^{-j\theta} = \cos(\theta) - j\sin(\theta)$

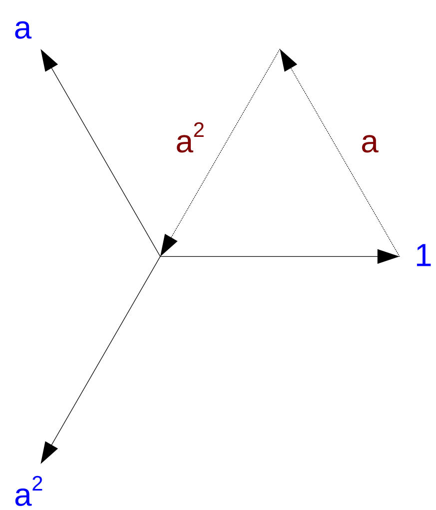
$\cos(\theta) =$

common notation: Let $a = e^{+j120^o} =$

$a^2 = e^{+j240^o} =$

$a^3 = e^{+j360^o} =$

$1 + a + a^2 = 0$



Define Complex Vector Voltage:

$$\bar{v}_{abcs} = v_{as} + a\,v_{bs} + a^2 v_{cs}$$

Define Complex Vector Current:

$$\bar{i}_{abcs} = i_{as} + a\,i_{bs} + a^2 i_{cs}$$

3ϕ Machine:

$$\bar{v}_{abcs} = v_{as} + a\,v_{bs} + a^2 v_{cs} \qquad \bar{i}_{abcs} = i_{as} + a\,i_{bs} + a^2 i_{cs}$$

1) Stator Resistive Terms

$$i_{as} r_s + a\,i_{bs} r_s + a^2 i_{cs} r_s =$$

2) Stator Self Inductance Terms

$$(L_{ms} + L_{ls})\,p\,(i_{as} + a\,i_{bs} + a^2 i_{cs}) =$$

3) Mutual Terms (Stator)

$$\frac{-1}{2} L_{ms}\,p\left[(i_{bs} + i_{cs}) + a(i_{as} + i_{cs}) + a^2(i_{as} + i_{bs})\right] =$$

$$\frac{-1}{2} L_{ms}\,p\left[i_{as}(a + a^2) + i_{bs}(1 + a^2) + i_{cs}(1 + a)\right] =$$

$$\frac{-1}{2} L_{ms}\,p\left[-i_{as} + -a\,i_{bs} + -a^2 i_{cs}\right] =$$

4) Mutual Terms (Rotor)

$$\frac{N_r}{N_s} L_{ms}\, p \left[\cos(\theta_r)(i_{ar} + a\, i_{br} + a^2\, i_{cr}) + \cos\left(\theta_r + 120^o\right)(i_{br} + a\, i_{cr} + a^2\, i_{ar}) + \cos\left(\theta_r - 120^o\right)(i_{cr} + a\, i_{ar} + a^2\, i_{br})\right]$$

$$= \frac{N_r}{N_s} L_{ms}\, p \left[\bar{i}_{abcr} \cos(\theta_r) + a^2\, \bar{i}_{abcr} \cos\left(\theta_r + 120^o\right) + a\, \bar{i}_{abcr} \cos\left(\theta_r - 120^o\right)\right]$$

$$= \frac{N_r}{N_s} \frac{L_{ms}}{2}\, p \left\{\bar{i}_{abcr} \left[e^{j\theta r} + e^{-j\theta r} + a^2\left(e^{j(\theta r + 120^o)} + e^{-j(\theta r + 120^o)}\right) + a\left(e^{j(\theta r - 120^o)} + e^{-j(\theta r - 120^o)}\right)\right]\right\}$$

$$= \frac{N_r}{N_s} \frac{L_{ms}}{2}\, p \left\{\bar{i}_{abcr} \left[3 e^{j\theta r} + e^{-j\theta r}(1 + a + a^2)\right]\right\}$$

$$=$$

Stator Voltage Equation:

$$\bar{v}_{abcs} = r_s\, \bar{i}_{abcs} + \left(\frac{3}{2} L_{ms} + L_{ls}\right) p\, \bar{i}_{abcs} + \frac{3}{2} \frac{N_r}{N_s} L_{ms}\, p \left[\bar{i}_{abcr}\, e^{j\theta r}\right]$$

Apply Turns Ratio

$$\bar{i}'_{abcr} = \frac{N_r}{N_s}\, \bar{i}_{abcr} \qquad r'_r = \left(\frac{N_r}{N_s}\right)^2 r_r \qquad L_m = \frac{3}{2} L_{ms}$$

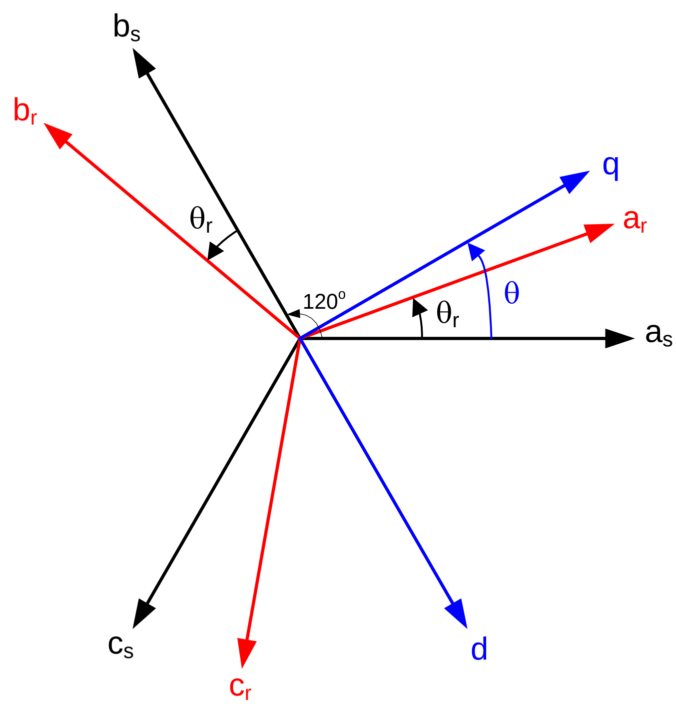
$$\bar{v}'_{abcr} = \frac{N_s}{N_r}\, \bar{v}_{abcr} \qquad L'_r = \left(\frac{N_r}{N_s}\right)^2 L_{mr} + L'_{lr}$$

The final complex vector equations with turns ratio transformations:

$$\bar{v}_{abcs} = r_s \bar{i}_{abcs} + L_s p\, \bar{i}_{abcs} + L_m p\left[\bar{i}\,'_{abcr}\, e^{j\theta r}\right]$$

$$\bar{v}\,'_{abcr} = r_r'\, \bar{i}\,'_{abcr} + L_r' p\, \bar{i}\,'_{abcr} + L_m p\left[\bar{i}_{abcs}\, e^{-j\theta r}\right]$$

Transformations to a Rotating Reference Frame:



General Reference Frame of a Three Phase Machine

Define:

$$f_{qs} = \frac{2}{3}\left[f_{as}\cos(\theta) + f_{bs}\cos\left(\theta - 120^{o}\right) + f_{cs}\cos\left(\theta + 120^{o}\right)\right]$$

$$f_{ds} = \frac{2}{3}\left[f_{as}\sin(\theta) + f_{bs}\sin\left(\theta - 120^{o}\right) + f_{cs}\sin\left(\theta + 120^{o}\right)\right]$$

f denotes **v**, **i**, or $\boldsymbol{\lambda}$

Note: Scale factor is used to preserve magnitude relationship for sinusoidal steady state quantities. Another common choice is $\sqrt{\frac{2}{3}}$ to preserve power calculations between abc & dq coordinates. Third variable is necessary to define a unique transformation (since there are three variables):

$f_{0s} =$ zero sequence component (normal to dq plane)

Note: Under most conditions, $f_{0s} = 0$ (but not always!).

Define New Complex Vector:

$$\overline{f}_{qds} = f_{qs} - j\,f_{ds} = \frac{2}{3}\left[f_{as}e^{-j\theta} + f_{bs}e^{-j(\theta - 120^{O})} + f_{cs}e^{-j(\theta + 120^{O})}\right]$$

$$= \frac{2}{3}e^{-j\theta}\left[f_{as} + a\,f_{bs} + a^2 f_{cs}\right]$$

The **a** operator handles transformation from 3ϕ to 2ϕ and the $e^{-j\theta}$ handles the rotation.

$$\overline{f}_{qdr} = \frac{2}{3}e^{-j(\theta - \theta r)}\left[f_{ar} + a\,f_{br} + a^2 f_{cr}\right]$$

Recall Complex Variable form for stator given by:

$$\bar{v}_{abcs} = r_s \bar{i}_{abcs} + L_s p\, \bar{i}_{abcs} + L_m p\left[\bar{i}'_{abcr} e^{j\theta_r}\right]$$

multiply by $e^{-j\theta} \rightarrow$

$$e^{-j\theta}\bar{v}_{abcs} = r_s e^{-j\theta}\, \bar{i}_{abcs} + L_s e^{-j\theta} p\, \bar{i}_{abcs} + L_m e^{-j\theta} p\left[\bar{i}'_{abcr} e^{j\theta_r}\right]$$

From the chain rule of differentiation:

$$x\frac{dy}{dt} = \frac{d}{dt}(xy) - y\frac{dx}{dt}$$

Therefore:

$$e^{-j\theta}\bar{v}_{abcs} = r_s e^{-j\theta}\, \bar{i}_{abcs} + L_s p\left(e^{-j\theta}\, \bar{i}_{abcs}\right) + L_m p\left[\bar{i}'_{abcr} e^{-j(\theta - \theta_r)}\right] + j\omega\left[L_s e^{-j\theta} \bar{i}_{abcs} + L_m \bar{i}'_{abcr} e^{-j(\theta - \theta_r)}\right]$$

$$\bar{v}_{qds} = r_s \bar{i}_{qds} + L_s p\, \bar{i}_{qds} + L_m p\, \bar{i}'_{qdr} + j\omega\left[L_s \bar{i}_{qds} + L_m \bar{i}'_{qdr}\right]$$

for Rotor:

$$\bar{v}'_{qdr} = r'_r\, \bar{i}'_{qdr} + L'_r p\, \bar{i}'_{qdr} + L_m p\, \bar{i}_{qds} + j(\omega - \omega_r)\left[L'_r \bar{i}'_{qdr} + L_m \bar{i}_{qds}\right]$$

For zero sequence:

$$v_{as} + v_{bs} + v_{cs} = (r_s + pL_{ls})(i_{as} + i_{bs} + i_{cs}) \xrightarrow{x\frac{1}{3}}$$

$v_{0s} =$

$v'_{0r} =$

Summary of Equations (Can Implement in Simulations):

For Stator:

$$v_{qs} = r_s i_{qs} + \frac{d\lambda_{qs}}{dt} + \omega \lambda_{ds}$$

$$v_{ds} = r_s i_{ds} + \frac{d\lambda_{ds}}{dt} - \omega \lambda_{qs}$$

$$v_{0s} = r_s i_{0s} + \frac{d\lambda_{0s}}{dt}$$

where

$$\lambda_{qs} = L_{ls} i_{qs} + L_m (i_{qs} + i_{qr}')$$

$$\lambda_{ds} = L_{ls} i_{ds} + L_m (i_{ds} + i_{dr}')$$

$$\lambda_{0s} = L_{ls} i_{0s}$$

For Rotor:

$$v_{qr}' = r_r' i_{qr}' + \frac{d\lambda_{qr}'}{dt} + (\omega - \omega_r) \lambda_{dr}'$$

$$v_{dr}' = r_r' i_{dr}' + \frac{d\lambda_{dr}'}{dt} - (\omega - \omega_r) \lambda_{qr}'$$

$$v_{0r}' = r_r' i_{0r}' + \frac{d\lambda_{0r}'}{dt}$$

where

$$\lambda_{qr}' = L_{lr}' i_{qr}' + L_m (i_{qs} + i_{qr}')$$

$$\lambda_{dr}' = L_{lr}' i_{dr}' + L_m (i_{ds} + i_{dr}')$$

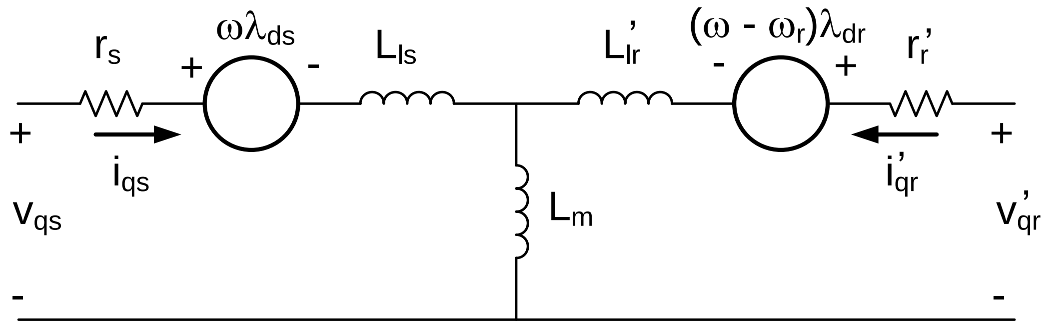
$$\lambda_{0r}' = L_{lr}' i_{0r}'$$

As stated, in most cases machine is connected in Δ or Y such that no neutral current flows. In this case, v_{0s} & v_{0r}' equal zero. The four remaining equations are often assembled in matrix form as:

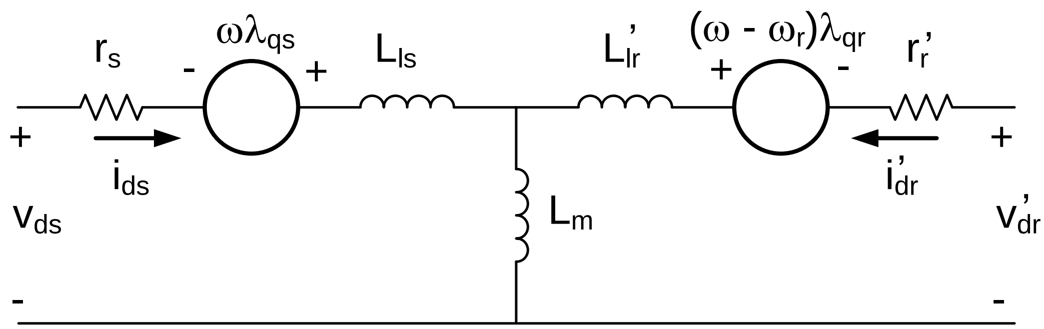
$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{qr}' \\ v_{dr}' \end{bmatrix} = \begin{bmatrix} r_s + L_s p & \omega L_s & L_m p & \omega L_m \\ -\omega L_s & r_s + L_s p & -\omega L_m & L_m p \\ L_m p & (\omega - \omega_r) L_m & r_r' + L_r' p & (\omega - \omega_r) L_r' \\ -(\omega - \omega_r) L_m & L_m p & -(\omega - \omega_r) L_r' & r_r' + L_r' p \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr}' \\ i_{dr}' \end{bmatrix}$$

where $\quad L_s = L_{ls} + L_m \qquad L_r' = L_{lr}' + L_m \qquad L_m = \frac{3}{2} L_{ms}$

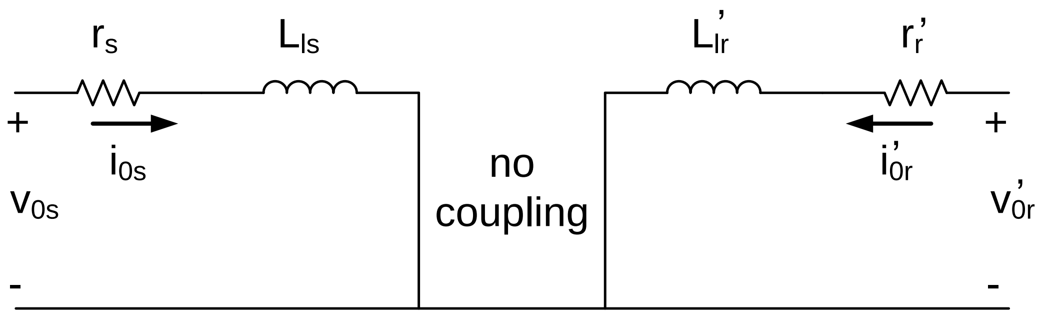
Three Phase Machine dq0 equivalent circuit:



q – Axis Equivalent Circuit



d – Axis Equivalent Circuit



0 – Axis Equivalent Circuit

Note: zero sequence components undesirable in machines because they can only produce heat, not torque.

Formalize and simplify notation:

$\bar{f}_{qdx}^{y}$ — f is a variable (v, i, λ)
x is where the variable comes from (r – rotor, s – stator)
(actual location of variables)
y is where variables are referred to (r – rotor, s – stator, g - general)

Stator Refered Complex Variable Equations ($\theta, \omega = 0$):
(Note: Now dropping ' notation, it is assumed)

$$\bar{v}_{qds}^{s} = r_s \bar{i}_{qds}^{s} + L_s p \bar{i}_{qds}^{s} + L_m p \bar{i}_{qdr}^{s}$$

$$\bar{v}_{qdr}^{s} = r_r \bar{i}_{qdr}^{s} + L_r p \bar{i}_{qdr}^{s} + L_m p \bar{i}_{qds}^{s} - j\omega_r \left(L_r \bar{i}_{qdr}^{s} + L_m p \bar{i}_{qds}^{s} \right)$$

$$= r_r \bar{i}_{qdr}^{s} + L_r (p - j\omega_r) \bar{i}_{qdr}^{s} + L_m (p - j\omega_r) \bar{i}_{qds}^{s}$$

which can be re-written as:

$$\bar{v}_{qds}^{s} = r_s \bar{i}_{qds}^{s} + p \bar{\lambda}_{qds}^{s}$$

$$\bar{v}_{qdr}^{s} = r_r \bar{i}_{qdr}^{s} + (p - j\omega_r) \bar{\lambda}_{qdr}^{s}$$

where

$$\bar{\lambda}_{qds}^{s} =$$

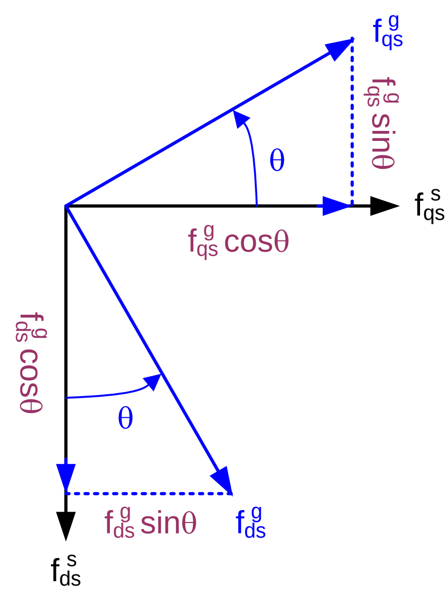
$$\bar{\lambda}_{qdr}^{s} =$$

$$\begin{bmatrix} v_{qs}^s \\ v_{ds}^s \\ v_{qr}^s \\ v_{dr}^s \end{bmatrix} = \begin{bmatrix} r_s + L_s p & 0 & L_m p & 0 \\ 0 & r_s + L_s p & 0 & L_m p \\ L_m p & -\omega_r L_m & r_r + L_r p & -\omega_r L_r \\ \omega_r L_m & L_m p & \omega_r L_r & r_r + L_r p \end{bmatrix} \begin{bmatrix} i_{qs}^s \\ i_{ds}^s \\ i_{qr}^s \\ i_{dr}^s \end{bmatrix}$$

Rotation from a General Reference Frame

$$f_{qs}^s = f_{qs}^g \cos\theta + f_{ds}^g \sin\theta$$

$$f_{ds}^s = f_{ds}^g \cos\theta - f_{qs}^g \sin\theta$$



$$\overline{f}_{qds}^s = f_{qs}^s - j f_{ds}^s$$

$$= f_{qs}^g \cos\theta + f_{ds}^g \sin\theta - j f_{ds}^g \cos\theta + j f_{qs}^g \sin\theta$$

$$= f_{qs}^g (\cos\theta + j\sin\theta) + f_{ds}^g (\sin\theta - j\cos\theta)$$

$$= f_{qs}^g (\cos\theta + j\sin\theta) - j f_{ds}^g (\cos\theta + j\sin\theta)$$

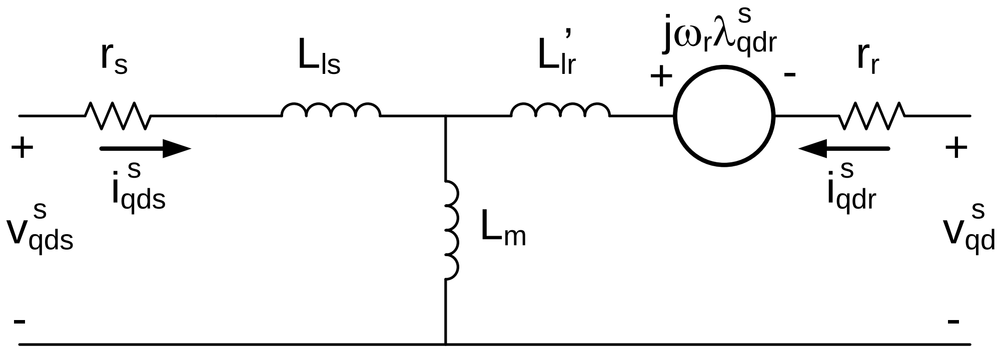
$$= \left(f_{qs}^g - j f_{ds}^g\right)(\cos\theta + j\sin\theta)$$

$$\boxed{\overline{f}_{qds}^s = e^{+j\theta}\,\overline{f}_{qds}^g} \quad \rightarrow$$

Return to Stator Refered Complex Variable Equations ($\theta, \omega = 0$):

$$\bar{v}^s_{qds} = r_s \bar{i}^s_{qds} + p\bar{\lambda}^s_{qds}$$

$$\bar{v}^s_{qdr} = r_r \bar{i}^s_{qdr} + (p - j\omega_r)\bar{\lambda}^s_{qdr}$$



Complex, Stator Referred Model

Rotation to a General Reference Frame

$$\boxed{\bar{f}^s_{qds} = e^{+j\theta}\bar{f}^g_{qds}} \quad \rightarrow \quad \boxed{\bar{f}^g_{qds} = e^{-j\theta}\bar{f}^s_{qds}}$$

General Reference Frame Referred, Complex Variable Equations:

$$e^{-j\theta}\bar{v}^s_{qds} = r_s e^{-j\theta}\bar{i}^s_{qds} + L_s e^{-j\theta} p\,\bar{i}^s_{qds} + L_m e^{-j\theta} p\,\bar{i}^s_{qdr}$$

$$\boxed{\begin{aligned} \bar{v}^g_{qds} &= r_s \bar{i}^g_{qds} + L_s (p + j\omega)\bar{i}^g_{qds} + L_m (p + j\omega)\bar{i}^g_{qdr} \\ \bar{v}^g_{qdr} &= r_r \bar{i}^g_{qdr} + L_r\left[p + j(\omega - \omega_r)\right]\bar{i}^g_{qdr} + L_m\left[p + j(\omega - \omega_r)\right]\bar{i}^g_{qds} \end{aligned}}$$

if $\omega = 0$ & $\theta = 0$ → Stator Reference Frame

if $\omega = \omega_r$ & $\theta = \theta_r$ → Rotor Reference Frame

if $\omega = \omega_e$ & $\theta = \theta_{e_0}$ → Synchronous (Excitation) Reference Frame

<u>Inverse abc – dq Transformation:</u>

<u>Transformation:</u>

$$\bar{v}_{qds}^{s} = \frac{2}{3}\left[v_{as} + a v_{bs} + a^2 v_{cs}\right]$$

$$v_{0s} = \frac{1}{3}\left[v_{as} + v_{bs} + v_{cs}\right]$$

if $v_{as} + v_{bs} + v_{cs} = 0 = v_{0s}$

recall:

$$a = e^{+j120^o} = \frac{-1}{2} + j\frac{\sqrt{3}}{2}$$

$$a^2 = e^{+j240^o} = \frac{-1}{2} - j\frac{\sqrt{3}}{2}$$

$$\bar{v}_{qds}^{s} = \frac{2}{3}\left[v_{as} + \left(\frac{-1}{2} + j\frac{\sqrt{3}}{2}\right)v_{bs} + \left(\frac{-1}{2} - j\frac{\sqrt{3}}{2}\right)v_{cs}\right]$$

$$= \frac{2}{3}\left[v_{as} + \frac{-1}{2}(v_{bs} + v_{cs}) + j\frac{\sqrt{3}}{2}(v_{bs} - v_{cs})\right]$$

$$= \frac{2}{3}\left[\frac{3}{2}v_{as} - j\frac{\sqrt{3}}{2}\left(v_{cs} - v_{bs}\right)\right] = v_{as} - j\frac{v_{cb}}{\sqrt{3}}$$

therefore

$$v_{qs}^{s} = v_{as}$$

$$v_{ds}^{s} = \frac{v_{cb}}{\sqrt{3}} = -\frac{v_{bc}}{\sqrt{3}}$$

Inverse Transformation:

$$\bar{v}_{qds}^{s} = \frac{2}{3}\left[v_{as} + \frac{-1}{2}(v_{bs} + v_{cs}) + j\frac{\sqrt{3}}{2}(v_{bs} - v_{cs})\right]$$

$$= \frac{2}{3}\left[v_{as} + \frac{-1}{2}(3v_{0s} - v_{cs}) + j\frac{\sqrt{3}}{2}(v_{bs} - v_{cs})\right]$$

$$= v_{as} - v_{0s} + j\frac{1}{\sqrt{3}}(v_{bs} - v_{cs})$$

$$v_{as} = \mathcal{Re}\left[\bar{v}_{qds}^{s}\right] + v_{0s}$$

$$v_{bs} = \mathcal{Re}\left[a^{2}\bar{v}_{qds}^{s}\right] + v_{0s}$$

$$v_{cs} = \mathcal{Re}\left[a\bar{v}_{qds}^{s}\right] + v_{0s}$$

Summary

Clarke Transformation

<u>Forward Transformation:</u>

$$\begin{bmatrix} v_{qs}^{s} \\ v_{ds}^{s} \\ v_{0s} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{-\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix}$$

From Wikipedia

Edith Clarke (February 10, 1883 – October 29,1959) was the first female electrical engineer and the first female professor of electrical engineering at the University of Texas at Austin. She specialized in electrical power system analysis and wrote *Circuit Analysis of A-C Power Systems*.

<u>Inverse Transformation:</u>

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ \frac{-1}{2} & \frac{-\sqrt{3}}{2} & 1 \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} & 1 \end{bmatrix}\begin{bmatrix} v_{qs}^{s} \\ v_{ds}^{s} \\ v_{0s} \end{bmatrix}$$

From Wikipedia

Robert H. Park (March 15, 1902 – February 18,1994) was an American electrical engineer and inventor, best known for the Park's transformation, used to simplify the analysis of three-phase electric circuits. His related 1929 concept paper ranked second, when looking at the impact of all twentieth century power enegineering papers. Park was an IEEE Fellow and a member of the National Academy of Engineering

Park Transformation

<u>Stationary to Rotationg</u>

$$\overline{f}_{qdx}^{\,g} = e^{-j\theta}\,\overline{f}_{qdx}^{\,s}$$

$$\begin{bmatrix} f_{qx}^{g} \\ f_{dx}^{g} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} f_{qx}^{s} \\ f_{dx}^{s} \end{bmatrix}$$

<u>Rotating to Stationary</u>

$$\overline{f}_{qdx}^{\,s} = e^{j\theta}\,\overline{f}_{qdx}^{\,g}$$

$$\begin{bmatrix} f_{qx}^{s} \\ f_{dx}^{s} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} f_{qx}^{g} \\ f_{dx}^{g} \end{bmatrix}$$