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Incompressibility

we want $\nabla \cdot \vec{u} = 0$

in frequency domain $-ik_1\hat{u} - ik_2\hat{v} - ik_3\hat{w} = 0$

This can be enforced by removing the extra parts.

for each (k_1, k_2, k_3) $\qquad |\vec{k}|^2 = k_1^2 + k_2^2 + k_3^2$

let extra $= k_1\hat{u} + k_2\hat{v} + k_3\hat{w}$

$$\hat{u} \leftarrow \hat{u} - k_1(\text{extra})/|\vec{k}|^2$$

$$\hat{v} \leftarrow \hat{v} - k_2(\text{extra})/|\vec{k}|^2$$

$$\hat{w} \leftarrow \hat{w} - k_3(\text{extra})/|\vec{k}|^2$$

This should be satisfied for all time but we can do it when the compressibility has built up too much. At $t=0$ this should be enforced so we start from a divergence-free field.

Reality

For $\vec{u}$ a real vector field

$\hat{u}(0,0,0)$, $\hat{v}(0,0,0)$, $\hat{w}(0,0,0)$ should be real.

If not (when it cross some threshold), we can force reality by applying ifft to $\hat{u}, \hat{v}, \hat{w}$ and throw out the imagmary part, then fft back to the freguency field.

Actually, $\hat{u}(0,0,0) = \hat{v}(0,0,0) = \hat{w}(0,0,0) = 0 \quad \forall t$

since $\mathrm{Re}(\hat{u}(0,0,0)) = \sum u = \langle u \rangle \cdot (\#\text{ of nodes}) = 0$

turbulence fluctuating field should have zero average

Starting from a modal spectrum

$|\vec{k}|^2 = k_1^2 + k_2^2 + k_3^2$

If $|\vec{k}| \in [k\_start, k\_end]$

amp = amplitude formula

$\theta = 2\pi\,\text{rand}()$ ← rand is a uniform distribution over $[0,1]$

$\hat{u} = \text{amp}\, e^{i\theta} = \text{amp}\,[\cos\theta + i\sin\theta]$

$\theta = 2\pi\,\text{rand}()$

$\hat{v} = \text{amp}\, e^{i\theta}$

$\theta = 2\pi\,\text{rand}()$

$\hat{w} = \text{amp}\, e^{i\theta}$

After this we need to force incompressibility & force reality.