Minimum Cost Diet Problem

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Introduction

The idea of this project is to solve the problem of finding the minimum cost diet satisfying a set of "Recommended Daily Allowances" (RDAs) for a particular set of nutrients and possible kind of food.

History

- Problem first "solved" by Stigler (1945) for a set of foods and prices and RDA requirements (see Dantzig (1990) for an entertaining discussion of what "solved" meant in that context).
- Times have changed: the variety of different kinds of food, food prices, and RDA requirements are all quite different from what they were for our grandparents.

Dietary Guidelines

A compilation of dietary guidelines are provided at in an appendix of https://www.dietaryguidelines.gov/sites/default/files/2021-03/Dietary_Guidelines_for_Americans-2020-2025.pdf these provide recommended levels of 31 different nutrients by age and sex. I've put some of these data into a google spreadsheet at https://docs.google.com/spreadsheets/d/1y95IsQ4HKspPW3HHDtH7QMtlDA66IUsCHJLutVL-MMc/.

Diet Cost

Stigler's insight was that the minimum cost diet (MCD) problem was most naturally posed as a linear program.

- Suppose n different kinds of food;
- Represent quantities consumed of these as a *vector* x with n elements. Many elements may be zero!
- Each kind of food has price; call this vector of prices p.

Total Cost

A consumer's diet costs $p'x$, where the prime indicates the inner or dot product of the two vectors.

Nutritional Recommendations

Each unit of a given kind of food is assumed to provide a set of nutrients.

- Suppose m nutrients, then let A be a matrix with m rows and n columns describing the nutritional content of a single unit of each of kind of food.
- Different sources of "recommendations" regarding nutrition.
  - Equalities: A female in her twenties 'should' consume:
    - 2000 kilo-calories
    - 46 grams of protein
    - 28 grams of fiber per day
- Inequalities:
  - Less than 23 grams of sodium

Matrix notation

We can write these constraints as something like

$$Ax \geq \underline{b}$$

where $\underline{b}$ is a vector of recommendations about *minimum* amounts of different nutrients. Similarly, if there are some things we want to make sure we eat *less* of (e.g., mercury, sodium, calories), that can be written as a set of linear inequalities

$$Ax \leq \bar{b},$$

where $\bar{b}$ is a vector of recommendations about *maximum* amounts of different nutrients. Note that this constraint can *also* be expressed as a greater than constraint by multiplying both sides by -1.

Linear Program

Putting this all together, the linear program to compute the minimum cost diet looks like

$$\min_x p'x$$

such that

$$\begin{bmatrix} A \\ -A \end{bmatrix} x \geq \begin{bmatrix} \underline{b} \\ -\bar{b} \end{bmatrix}.$$

We can express this more succinctly by denoting the stacked A matrices and b vectors by $\tilde{A}$ and $\tilde{b}$, respectively.

Solution

We'll introduce code which can quickly solve linear programs given inputs $(\tilde{A}, \tilde{b}, c)$. Your job in this project has more to do with finding interesting cases in which to use methods such as these.