§7.

$q_i^a \to q_i^a + \varepsilon_i \quad (q^a \to q^a + \varepsilon)$ $\quad \delta L = \dfrac{df}{dt}$

i) $\delta L = 0$

$\sum_i \sum_a \dfrac{\partial L}{\partial q_i^a}\varepsilon_i$ Einsteinの縮約

$\dfrac{\partial L}{\partial q_i^a} = \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{q}_i^a}$

$\varepsilon_x = \varepsilon$

$\varepsilon_y = 0$

$\varepsilon_z = 0$ $\quad x_1, x_2$

$\delta L = \boxed{\sum_a \dfrac{\partial L}{\partial q_i^a}\varepsilon_i}$

$= \dfrac{d}{dt}\left(\boxed{\sum_a \dfrac{\partial L}{\partial \dot{q}_i^a}}\right)\varepsilon_i$

$p = \dfrac{\partial L}{\partial \dot{q}}$: $p_i^a = \dfrac{\partial L}{\partial \dot{q}_i^a}$

$f(x, y)$ $\quad f(x+\Delta x, y+\Delta y)$

$= f(x, y) + \dfrac{\partial f}{\partial x}\Delta x + \dfrac{\partial f}{\partial y}\Delta y + O((\Delta H)^2)$

$\delta L = 0$, ε_i : 任意

$\dfrac{d}{dt}\sum_a p_i^a = 0$

$(q_1, q_2, q_3) = q$

$\sum_a p_i^a$: 保存 (i : 任意)

$\sum_a \dfrac{dp_i^a}{dt} = \sum_a \dfrac{\partial L}{\partial q_i^a} = 0$ $\quad F_i^a = \dfrac{\partial L}{\partial q_i^a}$

$\delta L \overset{D}{=} \sum_a F_i^a = 0$ $\quad \sum_a p_i^a = \text{const.}$

$L(x, \dot{x}) = \sum_a \dfrac{1}{2} m_a \dot{x}_a^2 - U(x)$

$p_a = \dfrac{\partial L}{\partial \dot{x}_a} = m\dot{x}_a$: 運…

$F_a = \dfrac{\partial L}{\partial x_a} = -\dfrac{\partial U(x)}{\partial x_a}$

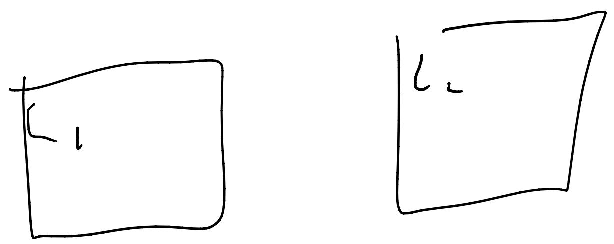
$\sum_a \mathbb{F}_a = 0$, $\sum_a \mathbb{P}_a$: 保存.

ii) $\delta L = \frac{df_i}{dt}\varepsilon_i$

$\delta L = \underline{\sum_a \frac{\partial L}{\partial q_i^a}}\varepsilon_i = \frac{d}{dt}\left(\sum_a p_i^a\right)\varepsilon_i = \underline{\frac{df_i}{dt}}\varepsilon_i$

$\frac{d}{dt}\left(\sum_a p_i^a - f_i\right) = 0$

$\boxed{\frac{df_i}{dt} = \sum_a F_i^a} \neq \delta L$



$L = L_1 + L_2 - U(q_1, q_2)$

$\sum \frac{\partial L}{\partial q} = 0$, $\frac{d}{dt}$

$L = \sum_a \frac{1}{2} m_a \dot{x}_a^2 - \sum_a m_a g z_a$

$\sum_a F_{x,y}^a = 0 \Rightarrow \frac{d}{dt}\sum_a P_{x,y}^a = 0$,

$\sum_a F_z^a = \sum_a \frac{\partial L}{\partial z^a} = -\sum_a m_a g = \frac{d}{dt}\left(-\sum_a m_a g t\right)$

$f_i = -\sum_a m_a g t$

$\sum_a P_z^a + \sum_a m_a g t = \sum_a m_a(\dot{z}_a + g t) = \text{const.}$

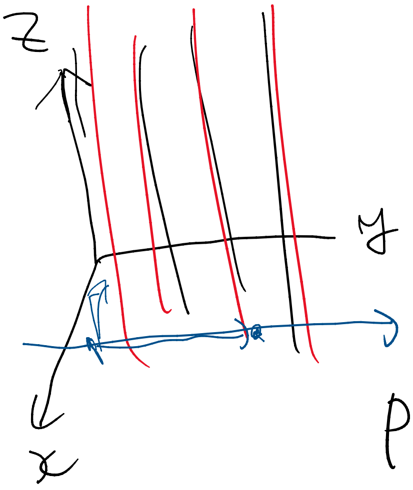
$\underset{0}{\overset{}{\dot{z}_a + gt}}$ $= 0$

$\dot{z}_a = -gt$

L : q_k^b に依らない

$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} \ldots\ p^b = \text{const.}$

$$\frac{\partial L}{\partial q_k^b} = 0 \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k^b} = 0 \qquad p_k^b = \text{const.}$$

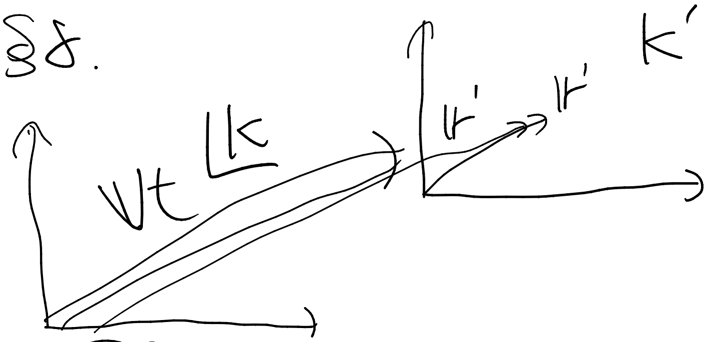


$$\mathbb{P} = (P_r, \underline{P_\varphi}, P_z) \qquad \dot{r}, \dot{\varphi}, \dot{z}$$

$$L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2)$$

$$P_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r} \qquad P_\varphi = \frac{\partial L}{\partial \dot{\varphi}} = \boxed{m r^2 \dot{\varphi}}$$

§ 8.



$$\mathbb{r}_a = \mathbb{r}_a' + \mathbb{V}t$$

$$\boxed{v_a = v_a' + \mathbb{V}}$$

$$\mathbb{P} = \sum_a m_a v_a$$

$$= \boxed{\sum_a m_a v_a'} + \sum_a m_a \mathbb{V}$$

$$= \mathbb{P}' + \mathbb{V}\sum_a m_a$$

$$\boxed{\mathbb{V} = \frac{\mathbb{P}}{\sum_a m_a}} \Rightarrow \mathbb{P}' = 0$$

$$\Rightarrow \mu = \sum_a m_a \text{ の粒子が } \mathbb{V}$$

$$\boxed{\mathbb{P} = \mu\mathbb{V}}$$

$$\boxed{\mathbb{R} = \frac{\sum_a m_a \mathbb{r}_a}{\mu} \quad \mathbb{V} = \frac{d\mathbb{R}}{dt}}$$

$$\mathbb{P} = \text{const}$$

$$\mathbb{V} = \text{const}$$

$\mathbb{R} = \text{const.}$ 全エネルギー：内部E

$\mathbb{R} = \mathbb{V}t$

$$E = \frac{1}{2}\sum_a m_a v_a^2 + U(x)$$

$\frac{1}{2}\mu \mathbb{V}^2$

$$= \frac{1}{2}\sum_a m_a (v_a' + \mathbb{V})^2 + U(x)$$

=慣性の運動量

$$= \frac{1}{2}\sum_a m_a v_a'^2 + \frac{1}{2}\sum_a m_a \mathbb{V}^2 + \cancel{\sum_a m_a v_a' \mathbb{V}} + U(x)$$

$\underset{\substack{\| \\ 0}}{P'} \cdot \mathbb{V}$

$\|$

E内部

$= E_{内部} + \frac{1}{2}\mu\mathbb{V}^2$