**Step 2: Optimize Beta Computation**

**Objective**

* Replace the computation of β=−(P′Q)−1⋅Q′Zi+1\beta = - (P'Q)^{-1} \cdot Q'Z\_{i+1}β=−(P′Q)−1⋅Q′Zi+1​ with solving the linear system (P′Q)⋅β=−Q′Zi+1(P'Q) \cdot \beta = - Q'Z\_{i+1}(P′Q)⋅β=−Q′Zi+1​.

**Implementation Steps**

1. **Compute Q′Zi+1Q'Z\_{i+1}Q′Zi+1​:**
   * Use multiply\_Den\_ClmM\_mtxT\_mtx to compute Q′Zi+1Q'Z\_{i+1}Q′Zi+1​.
2. **Modify Beta Computation:**
   * Use the same solve\_linear\_system function to solve (P′Q)⋅β=−Q′Zi+1(P'Q) \cdot \beta = - Q'Z\_{i+1}(P′Q)⋅β=−Q′Zi+1​.

**Code Implementation**

**A. Compute Q′Zi+1Q'Z\_{i+1}Q′Zi+1​ and Modify Beta Computation**

**Notes:**

* Ensure that mtxQTZ\_d is properly scaled by -1 before solving the linear system.
* For the rank-one case, perform updates using scalar operations.