Psych 186B Quiz, 4/10/2020, Zili Liu

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I showed all of my work and included photos of them in the last two pages of the word document. Please let me know if you have any problems viewing them.

All questions have equal weight. A simple final answer is sufficient for full points, unless otherwise specified. Intermediate steps are not necessary, but may earn you partial credits if your final answer is wrong.

You don't need to find numerically approximate values for your answers. For example, you can leave √3 as is, not 1.732. But if it is √4, you are expected to know that √4 = 2. Therefore √28 = 2√7. As another example, if the answer is 10/5, you should write it as 2. But if it is 7/4, you can leave it as is.

Vector a = (3, 5, 4), vector b = (2, 6, 1).

1. Please compute 8a – 6b.

$$\mathbf{(12, 4, 26)}$$

2. Please compute a vector c that has length 1 and points in the direction opposite to that of b.

$$\boldsymbol{c = \left(\frac{-2}{\sqrt{41}}, \frac{-6}{\sqrt{41}}, \frac{-1}{\sqrt{41}}\right)}$$

3. What is the cosine of the angle between vector 98a and vector 76b?

$$\frac{\mathbf{8}}{\sqrt{\mathbf{82}}}$$

4. A vector e has length 32 in the x-y plane. Its dot product with the vector (-1, 0) is 16√2. What is the angle between vector e and the vector (-1, 0)?

$$\frac{\mathbf{1\sqrt{2}}}{\mathbf{2\sqrt{1}}}$$

5. Let vector d = a × b (cross product). Please compute the dot product between vector d and vector 54a + 37b.

$$\mathbf{0}$$

6. Is it possible for 6 non-zero vectors in a 5-dimensional space to be linearly independent? (Yes or No)

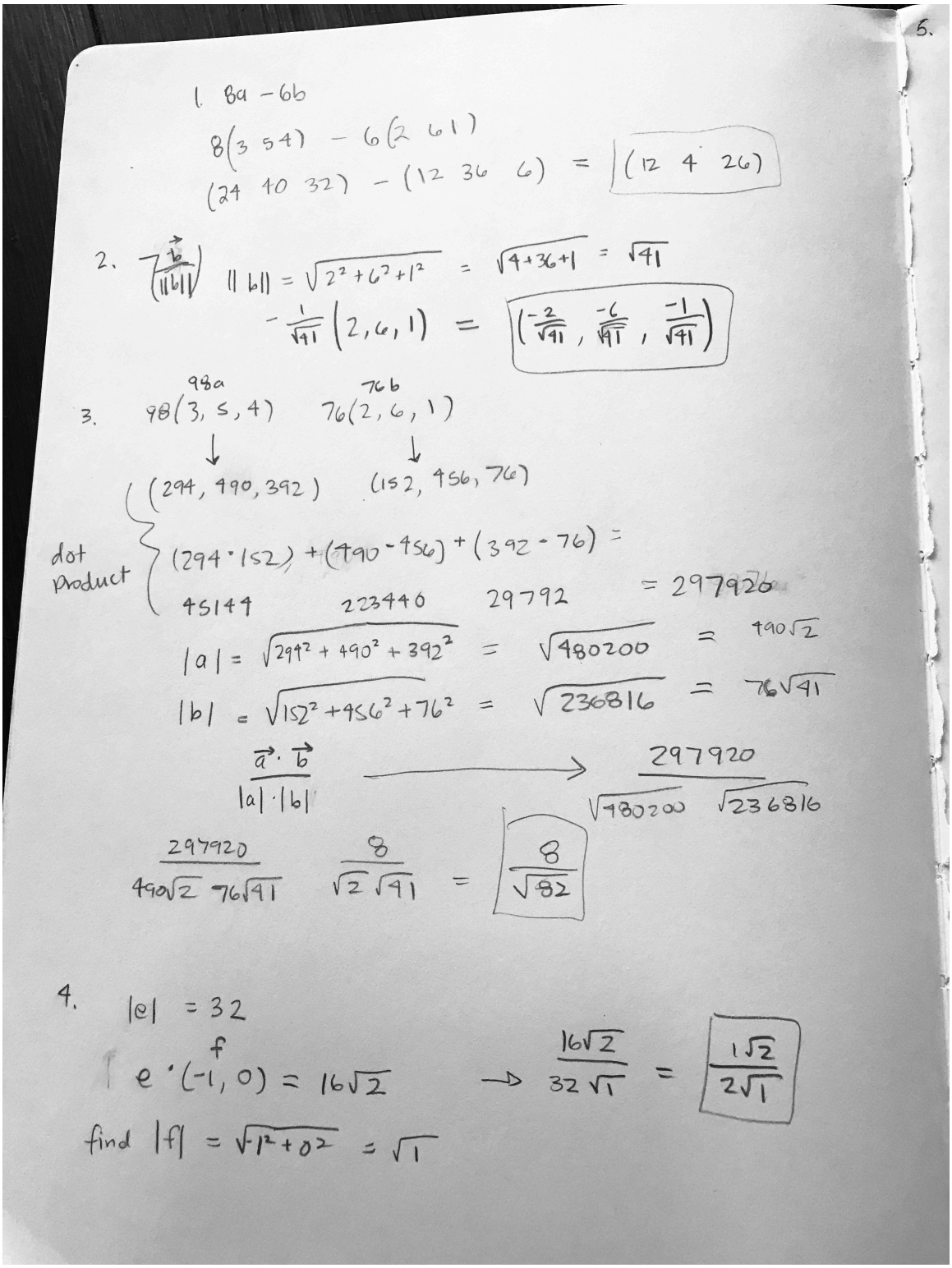
   No

7. Are 4 non-zero vectors in a 4-dimensional space always linearly independent?

   No

8. If the dot product of two vectors is zero, is the angle between them always 90 degrees? If yes, please prove it. If not, please provide a counter-example. (Hint: if a vector has zero length, its direction is undefined.

   Yes



5.

1. $8a - 6b$

$8(3, 5, 4) - 6(2, 6, 1)$

$(24, 40, 32) - (12, 36, 6) = \boxed{(12, 4, 26)}$

2. $\frac{\vec{b}}{\|b\|}$ $\quad \|b\| = \sqrt{2^2 + 6^2 + 1^2} = \sqrt{4+36+1} = \sqrt{41}$

$\frac{1}{\sqrt{41}}(2, 6, 1) = \boxed{\left(\frac{2}{\sqrt{41}}, \frac{6}{\sqrt{41}}, \frac{1}{\sqrt{41}}\right)}$

3. $98a \quad 98(3, 5, 4) \qquad 76b \quad 76(2, 6, 1)$

$\downarrow \qquad \downarrow$

$(294, 490, 392) \qquad (152, 456, 76)$

dot product: $(294 \cdot 152) + (490 \cdot 456) + (392 \cdot 76) =$

$45144 \qquad 223440 \qquad 29792 \qquad = 297926$

$|a| = \sqrt{294^2 + 490^2 + 392^2} = \sqrt{480200} = 490\sqrt{2}$

$|b| = \sqrt{152^2 + 456^2 + 76^2} = \sqrt{236816} = 76\sqrt{41}$

$\frac{\vec{a} \cdot \vec{b}}{|a| \cdot |b|} \longrightarrow \frac{297920}{\sqrt{480200}\ \sqrt{236816}}$

$\frac{297920}{490\sqrt{2}\ 76\sqrt{41}} \quad \frac{8}{\sqrt{2}\sqrt{41}} = \boxed{\frac{8}{\sqrt{82}}}$

4. $|e| = 32$

$e \cdot (-1, 0) = 16\sqrt{2} \longrightarrow \frac{16\sqrt{2}}{32\sqrt{1}} = \boxed{\frac{1\sqrt{2}}{2\sqrt{1}}}$

find $|f| = \sqrt{1^2 + 0^2} = \sqrt{1}$

5. $d = a \times b$

3 5 4 3 5 4
2 6 1 2 6 1

5 − 24 8 − 3 18 − 10

$d = (-19, 5, 8)$

$54a = (162, 270, 216)$

$37b = (74, 222, 37)$ $+= (236, 492, 253)$ (call it c)

$d \cdot c = (-19, 5, 8) \cdot (236, 492, 253)$

$(-19 \cdot 236) + (5 \cdot 492) + (8 \cdot 253) = \boxed{0}$

$-4484 + 2460 + 2024$