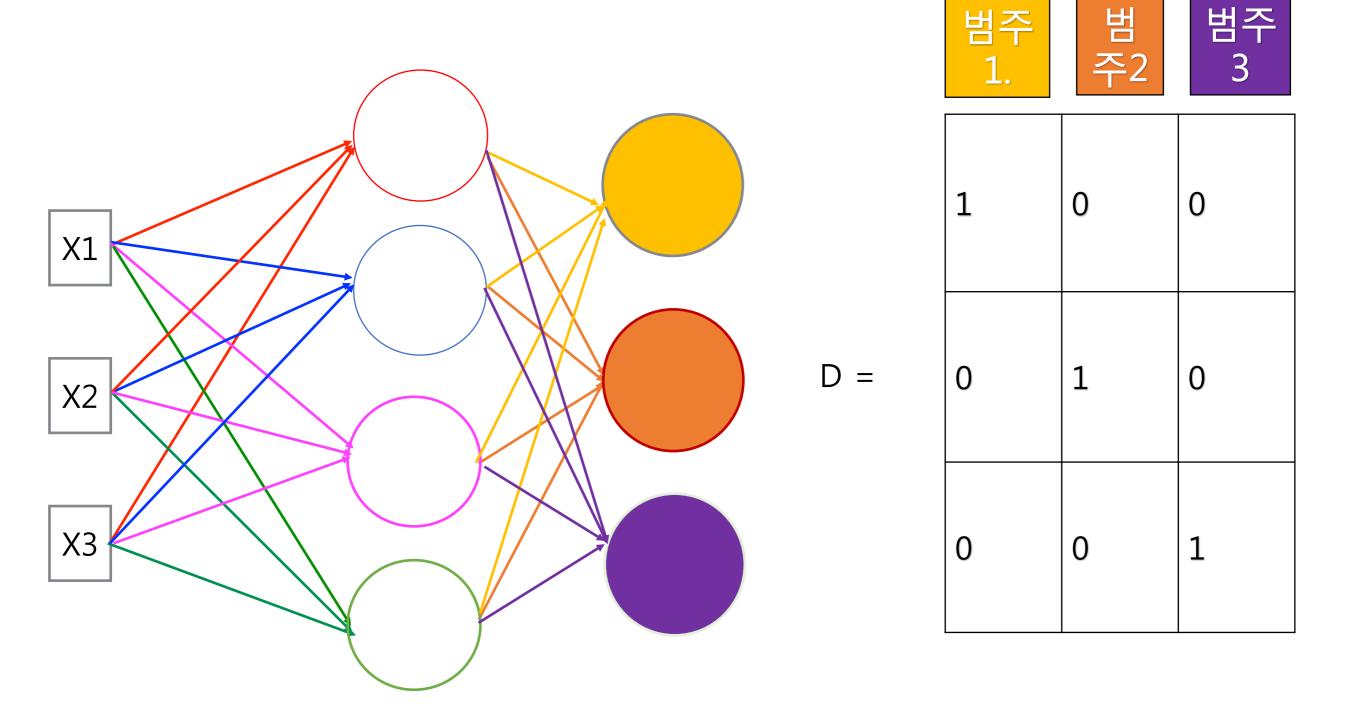
딥러닝 첫걸음

4. 신경망과 분류 (MultiClass)

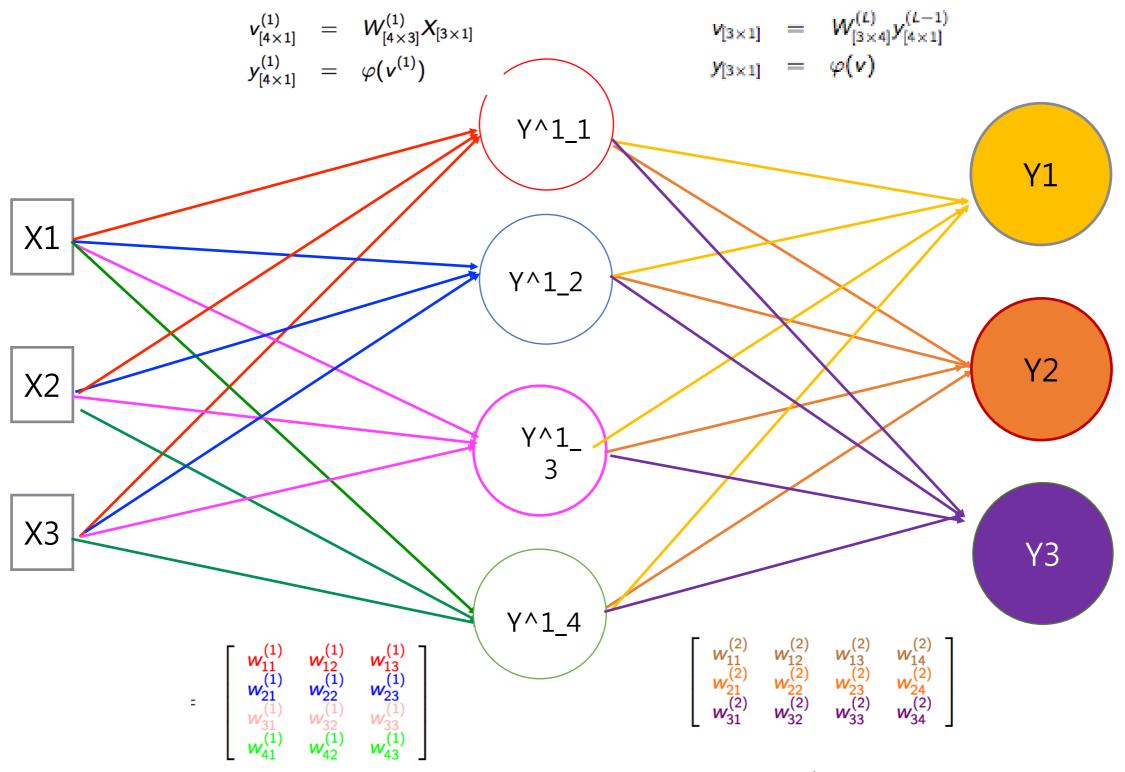
다범주 분류 신경망

- Categorization(분류): 예측 대상 = 범주
- 이진분류: 예측 대상 범주가 2가지인 경우
 - 출력층 node 1개 다층신경망 분석 (3장의 내용)
- 다범주 분류: 예측 대상 범주가 3가지 이상인 경우
 - 출력층 node 2개 이상 다층신경망 분석
 - 비용함수 : Softmax 함수 사용

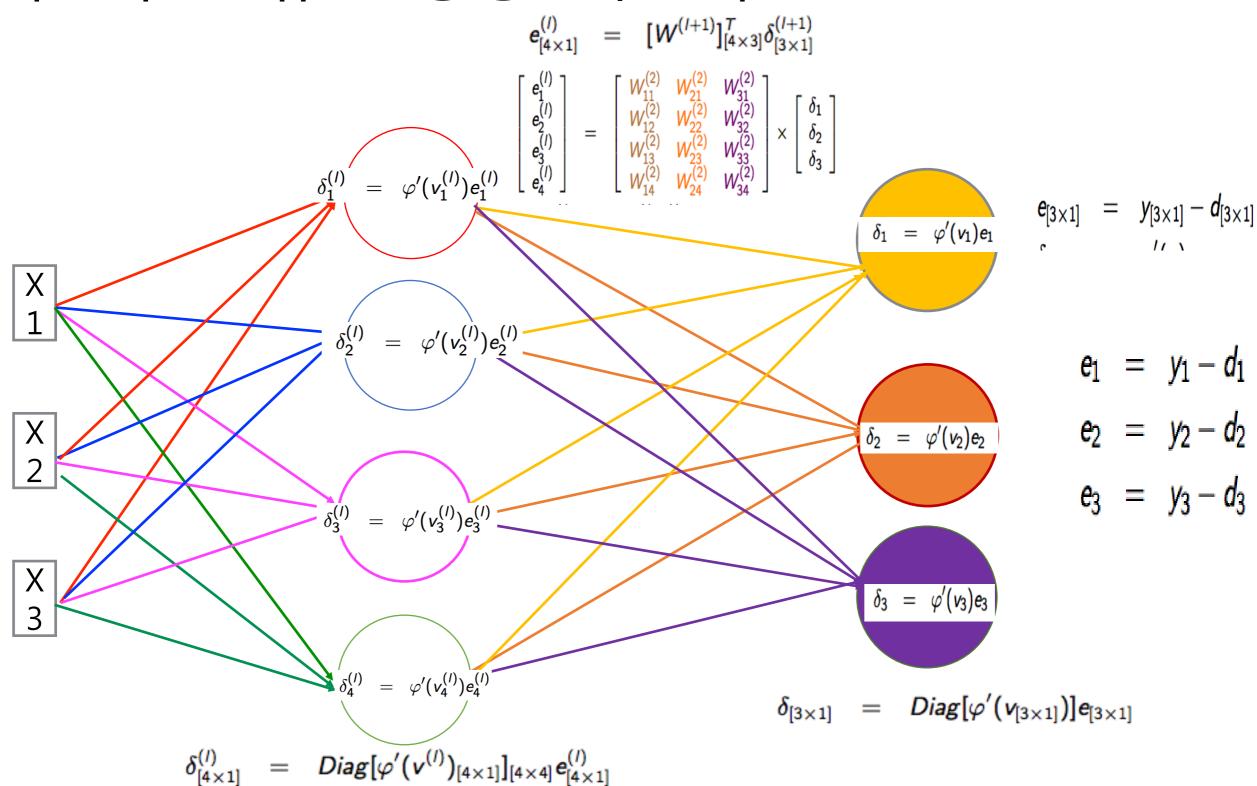
다범주 분류신경망



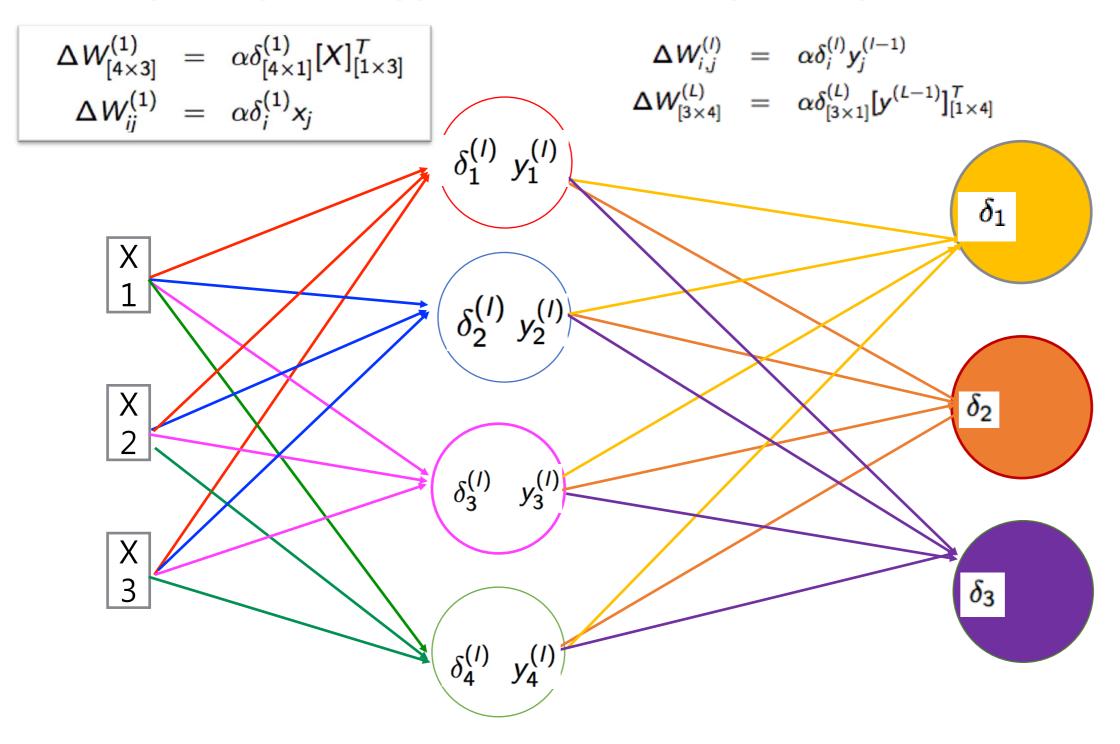
다범주 분류 신경망: 입출력



다범주 분류 신경망: 역전파



다범주 분류 신경망: 가중치 조정



다범주신경망 학습

STEP 1 가중치 초기화

$$W_{[4\times3]}^{(1)} = \begin{bmatrix} w_{11}^{(1)} & w_{12}^{(1)} & w_{13}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} & w_{23}^{(1)} \\ w_{31}^{(1)} & w_{32}^{(1)} & w_{33}^{(1)} \\ w_{41}^{(1)} & w_{42}^{(1)} & w_{43}^{(1)} \end{bmatrix}$$

$$W_{[3\times4]}^{(2)} = \begin{bmatrix} w_{11}^{(2)} & w_{12}^{(2)} & w_{13}^{(2)} & w_{14}^{(2)} \\ w_{21}^{(2)} & w_{22}^{(2)} & w_{23}^{(2)} & w_{24}^{(2)} \\ w_{31}^{(2)} & w_{32}^{(2)} & w_{33}^{(2)} & w_{34}^{(2)} \end{bmatrix}$$

다범주신경망 학습(2)

STEP 2 입력데이터 → 결과

1. 입력층

$$X_{[3\times 1]} = [x_1, x_2, x_3]^T$$

2. 은닉층

$$v_{[4\times1]}^{(1)} = W_{[4\times3]}^{(1)} X_{[3\times1]}$$
 $y_{[4\times1]}^{(1)} = \varphi(v^{(1)})$
 $v_{[3\times1]}^{(l+1)} = W_{[3\times4]}^{(l+1)} y_{[4\times1]}^{(l)}$
 $y_{[3\times1]}^{(l+1)} = \varphi(v^{(l+1)})$

3. 출력층

$$v_{[3\times1]} = W_{[3\times4]}^{(L)} y_{[4\times1]}^{(L-1)}$$

 $y_{[3\times1]} = \varphi(v_{[3\times1]})$

다범주신경망 학습(3)

STEP3 오차, δ

1. 출력층

$$e_{[3\times 1]} = y_{[3\times 1]} - d_{[3\times 1]}$$

 $\delta_{[3\times 1]} = Diag[\varphi'(v_{[3\times 1]})]e_{[3\times 1]}$

2. 은닉층

$$e_{[4\times1]}^{(I)} = [W^{(I+1)}]_{[4\times3]}^{T} \delta_{[3\times1]}^{(I+1)}$$

$$\delta_{[4\times1]}^{(I)} = Diag[\varphi'(v^{(I)})_{[4\times1]}]_{[4\times4]} e_{[4\times1]}^{(I)}$$

STEP4 가중치조정

$$\Delta W_{i,j}^{(l)} = \alpha \delta_i^{(l)} y_j^{(l-1)}
\Delta W_{[3\times4]}^{(L)} = \alpha \delta_{[3\times1]}^{(L)} [y^{(L-1)}]_{[1\times4]}^T
\Delta W_{[4\times3]}^{(l)} = \alpha \delta_{[4\times1]}^{(l)} [X]_{[1\times3]}^T
W'^{(l)} = W^{(l)} + \Delta W^{(l)}$$

Softmax function

$$\varphi : R^n \to R^n$$

$$\varphi(v_i) = \frac{\exp(v_i)}{\sum_{k=1}^n \exp(v_k)}$$

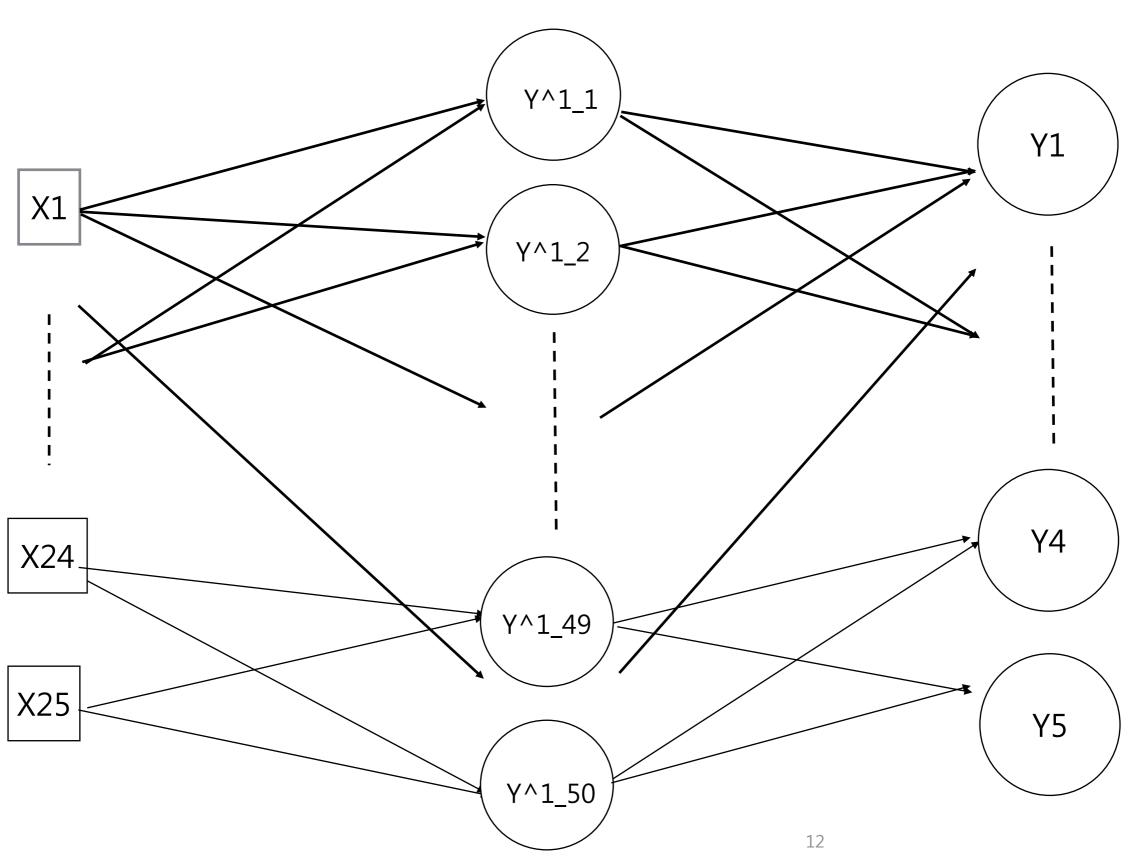
$$\frac{\partial \varphi(v_i)}{\partial v_i} = \frac{\exp(v_i) \sum_{k=1}^n \exp(v_k) - \exp(v_i) \exp(v_i)}{(\sum_{k=1}^n \exp(v_k))^2} \\
= \frac{\exp(v_i)}{\sum_{k=1}^n \exp(v_k)} \cdot \frac{\sum_{k=1}^n \exp(v_k) - \exp(v_i)}{\sum_{k=1}^n \exp(v_k)} \\
= \frac{\exp(v_i)}{\sum_{k=1}^n \exp(v_k)} \cdot \left[1 - \frac{\exp(v_i)}{\sum_{k=1}^n \exp(v_k)}\right] \\
= \varphi(v_i)(1 - \varphi(v_i))$$

1계 도함수는 Sigmoid 함수와 동일

Softmax.m

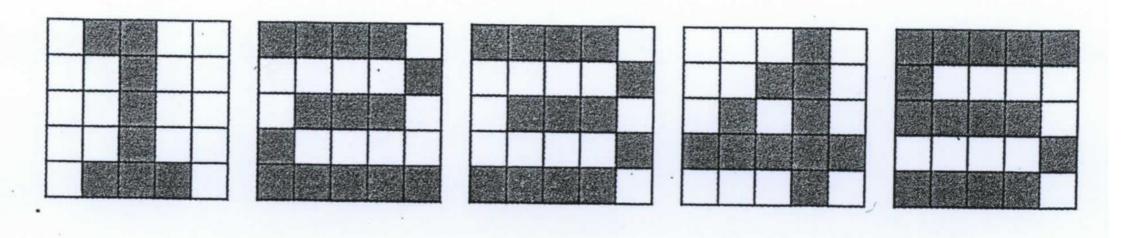
```
function y = Softmax(x)
ex=exp(x);
y=(ex)/sum(ex);
end
```

예제: 글자맟추기 입력 25 ->은닉 50 ->출력5

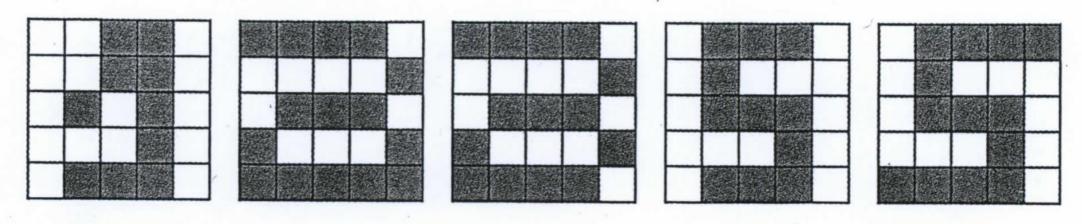


학습데이터, 검증데이터

학습데이터



검증데이터



training function: Multiclass.m

function [W1, W2] = Multiclass(W1,W2,X,D)

1. Learning Rate

alpha=0.9;

2. Loop Setup.: SGD

N=5;

for k=1:N

3. Input-Output

x=reshape(X(:,:,k),25,1); %25*1 d=D(k,:)'; %5*1

v1=W1*x; %(50*1=50*25 x 25*1

y1=Sigmoid(v1);

 $v=W2*y1; %(5*1=5*50 \times 50*1)$

y=Softmax(v);

4. Back Propagation

e=d-y; delta=e; %(5*1) e1=W2'*delta; %(50*5)*(5*1) delta1=y1.*(1-y1).*e1; %(50*1)

5. Update

dW1=alpha*delta1*x'; %(50*1)*(1*25) W1=W1+dW1; dW2=alpha*delta*y1'; %(5*1)*(1*50) W2=W2+dW2;

End % end of SGD loop end % end of function

TestMulticlass .m

```
clear all
rng(3);
                                                          2. 가중치 초기화
1. Data loading
                                                          W1=2*rand(50,25)-1;
                           X(:,:,4)=[0\ 0\ 0\ 1\ 0;
X = zeros(5,5,5);
                                   0 0 1 1 0;
                                                          W2=2*rand(5,50)-1;
                                   0 1 0 1 0;
                                                         W10=W1;
X(:,:,1)=[0\ 1\ 1\ 0\ 0;
       0 0 1 0 0;
                                                          W20=W2;
                                   1 1 1 1 1;
       0 0 1 0 0;
                                   00010
       0 0 1 0 0;
                                                          3. 기계 학습
       0 1 1 1 0
                                                          for epoch = 1:1000
                           X(:,:,5)=[1\ 1\ 1\ 1\ 1;
                                   10000;
                                                             [W1,
                                   1 1 1 1 0;
                                                          W2]=Multiclass(W1,W2,X,D);
X(:,:,2)=[1\ 1\ 1\ 1\ 0;
                                   00001;
                                                          end
       00001;
                                   11110
       0 1 1 1 0;
       1 0 0 0 0;
                                                         4. 추정
       11111
                           D=[1 \ 0 \ 0 \ 0];
                                                          N=5;
                             0 1 0 0 0;
                                                         for k=1:N
X(:,:,3)=[1\ 1\ 1\ 1\ 0;
                             0 0 1 0 0;
                                                            x = reshape(X(:,:,k),25,1);
       00001;
                             0 0 0 1 0;
                                                            v1=W1*x;
       0 1 1 1 0;
                              00001
                                                            y1=Sigmoid(v1);
       00001;
                                                            v = W2*y1;
                             ];
       11110
                                                            y=Softmax(v)
       ];
                                                          end
```

RealMulticlass .m : 입력데이터

```
clear all
rng(3);
                            X(:,:,3)=[1\ 1\ 1\ 1\ 0;
                                    00001;
X = zeros(5,5,5);
                                    0 1 1 1 0;
                                    1 0 0 0 1;
1. 기계학습
TestMultiClass;
                                    11110
2. Data loading
X = zeros(5,5,5);
                            X(:,:,4)=[0\ 1\ 1\ 1\ 0;
                                    0 1 0 0 0;
X(:,:,1)=[0\ 0\ 1\ 1\ 0;
                                    0 1 1 1 0;
       0 0 1 1 0;
                                    0 0 0 1 0;
        0 1 0 1 0;
                                    01110
        0 0 0 1 0;
        01110
                            X(:,:,5)=[0\ 1\ 1\ 1\ 1;
X(:,:,2)=[1\ 1\ 1\ 1\ 0;
                                    0 1 0 0 0;
        00001;
                                    0 1 1 1 0;
        0 1 1 1 0;
                                    00010;
        10001;
                                    11110
        11111
        ];
```

```
3. 추정
N=5;
for k=1:N
x=reshape(X(:,:,k),25,1);
v1=W1*x;
y1=Sigmoid(v1);
v=W2*y1;
y=Softmax(v)
end
```

실습 . Matlab => R

- Softmax.m
- Multiclass.m
- TestMulticlass.m
- RealMulticlass.m

• .m 에서 과업 특성 파악 => 과업 list 작성 => R code

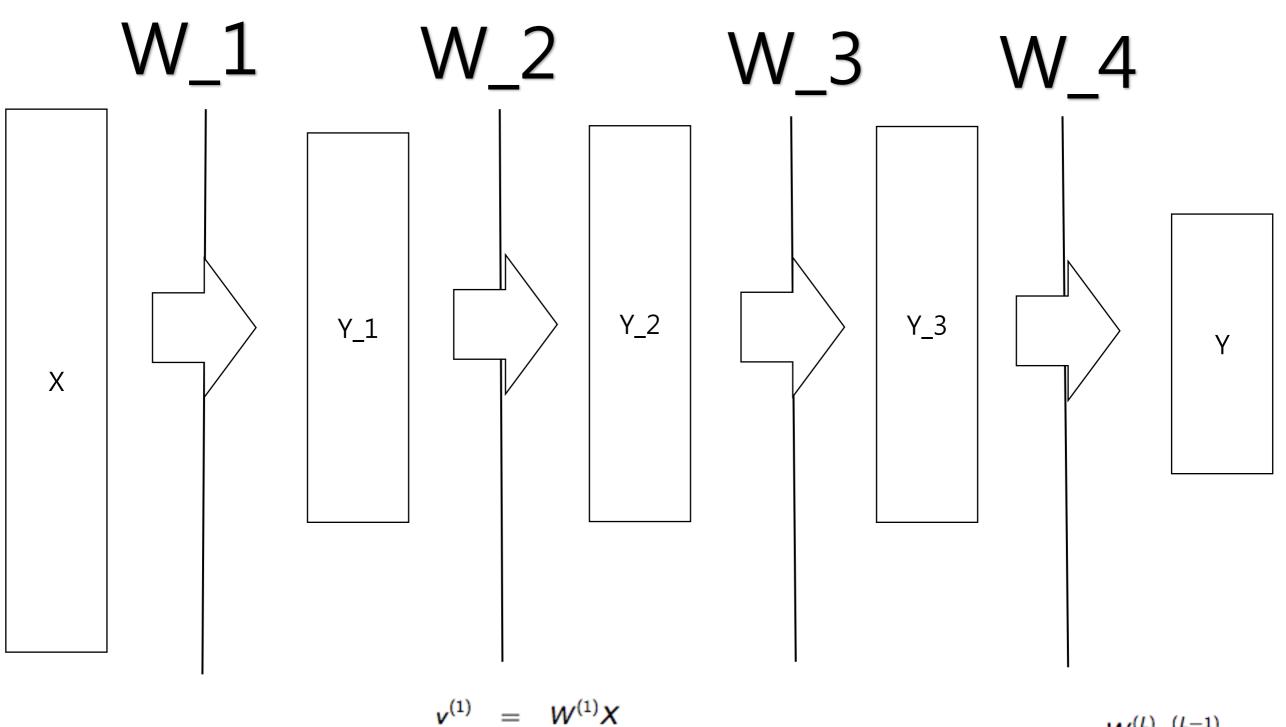
딥러닝 첫걸음

5. 딥러닝(심층신경망)

심층신경망: 은닉층 2개 이상 다층신경망

- 심층신경망의 문제점: 그래디언트 소실, 과적합, 계산부담
 - 그래디언트 소실 : 입력층에 가까운 은닉층에 출력층의 오차 정보 소실
 - ReLU 함수를 활성화 함수로 활용
 - 과적합
 - Dropout으로 해소: 무작위로 일부 node를 제외
 - 계산부담: 하드웨어의 발달 + 병렬처리로 해소
- 예제: 글자맟추기
 - 입력층 (25) → 제1은닉층(20) → 제2은닉층(20) → 제3은닉층(20) → 출력층(5)

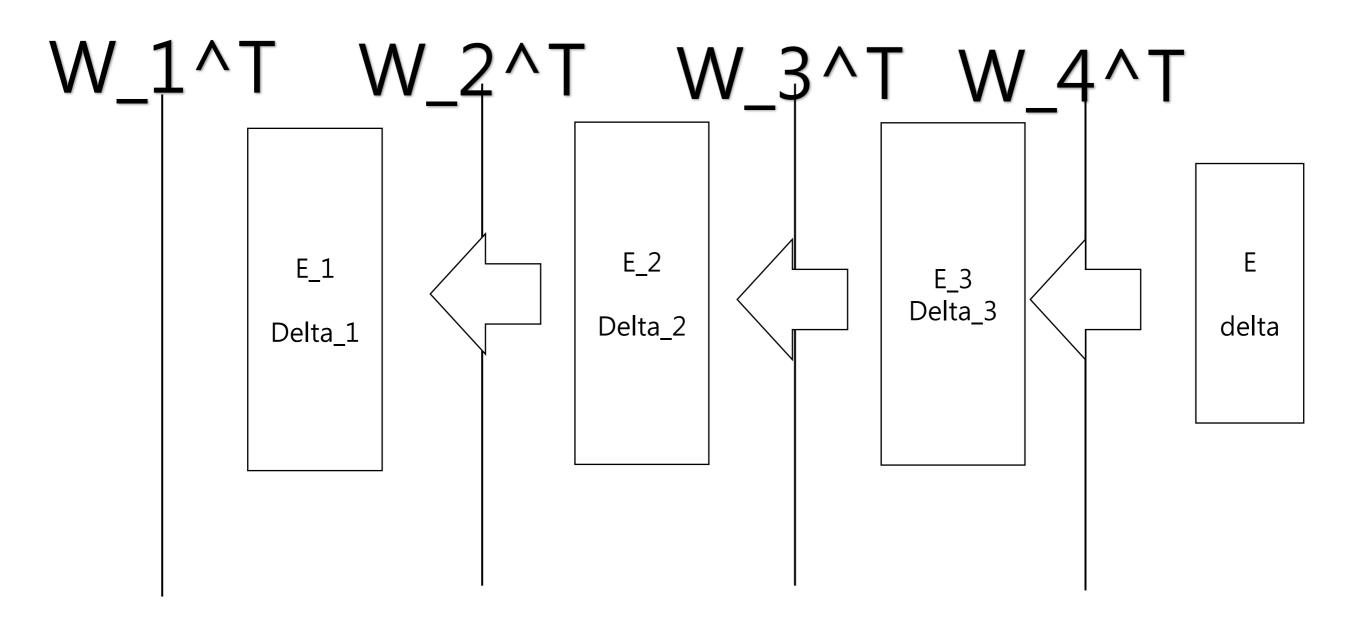
예제: 글자맞추기 입출력



$$v^{(1)} = W^{(1)}X$$
 $y^{(1)} = \varphi(v^{(1)})$
 $v^{(l+1)} = W^{(l+1)}y^{(l)}$
 $y^{(l+1)} = \varphi(v^{(l+1)})$

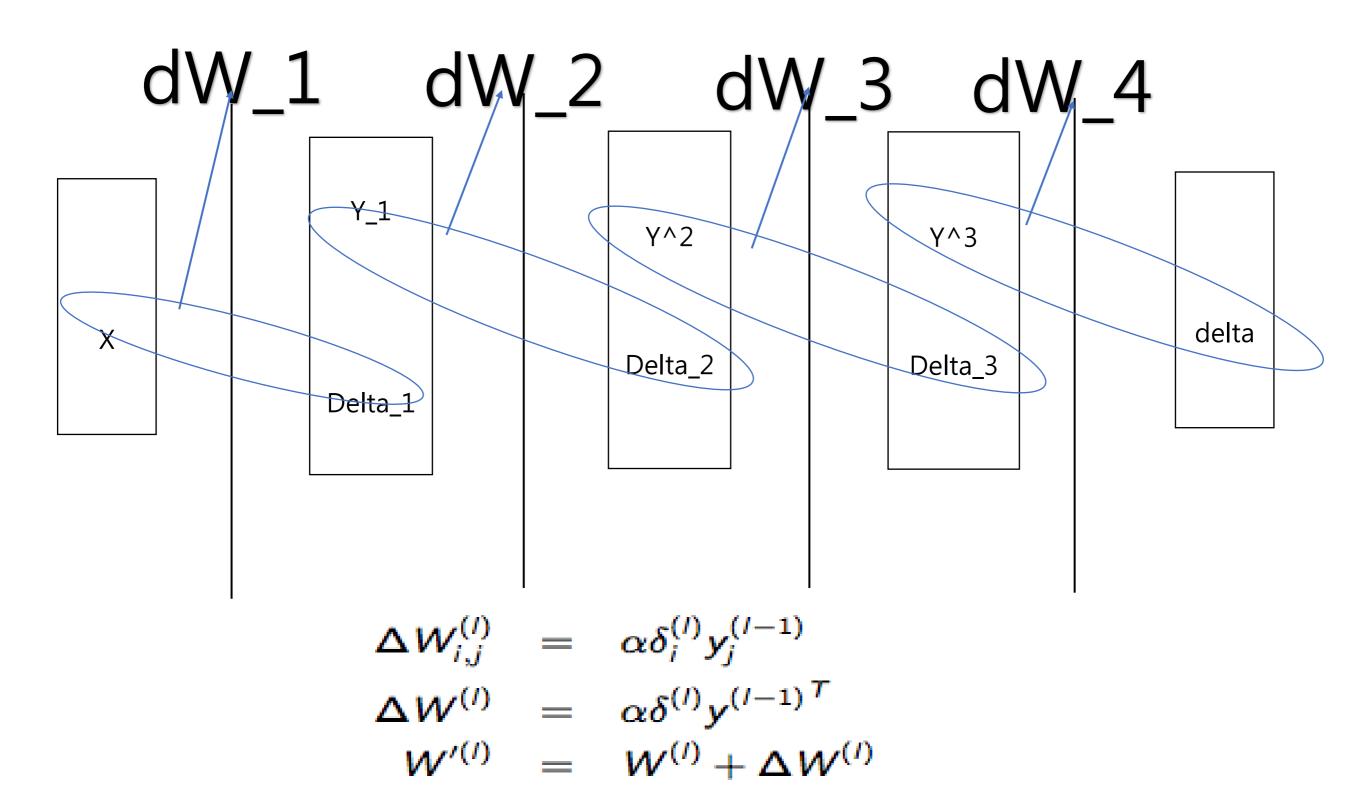
$$v = W^{(L)}y^{(L-1)}$$
$$y = \varphi(v)$$

예제: 글자맞추기-BackProp



$$e^{(l)} = [W^{(l+1)}]^T \delta^{(l+1)}$$
 $e = y - d$
 $\delta^{(l)} = Diag[\varphi'(v^{(l)})]e^{(l)}$ $\delta = Diag[\varphi'(v)]e$

예제: 글자맞추기-가중치조정



심층신경망 학습

STEP 1 가중치 초기화

$$W_{[20\times25]}^{(1)}$$
 $W_{[20\times20]}^{(2)}$
 $W_{[20\times20]}^{(3)}$
 $W_{[20\times20]}^{(4)}$

심층신경망 학습 (2)

STEP 2 입력데이터 → 결과

1. 입력층

$$X_{[25\times1]}$$

2. 은닉층

$$v_{[20\times1]}^{(1)} = W_{[20\times25]}^{(1)} X_{[25\times1]}$$
 $y_{[20\times1]}^{(1)} = \varphi(v^{(1)})$
 $v_{[20\times1]}^{(2)} = W_{[20\times20]}^{(2)} y_{[20\times1]}^{(1)}$
 $y_{[20\times1]}^{(2)} = \varphi(v^{(2)})$
 $v_{[20\times1]}^{(3)} = W_{[20\times20]}^{(3)} y_{[20\times1]}^{(2)}$
 $y_{[20\times1]}^{(3)} = \varphi(v^{(3)})$

3. 출력층

$$v_{[5\times1]} = W_{[5\times20]}^{(4)} y_{[20\times1]}^{(3)}$$

 $y_{[5\times1]} = \varphi(v_{[5\times1]})$

심층신경망 학습 (3)

STEP3 오차, δ

1. 출력층

$$e_{[5\times1]} = y_{[5\times1]} - d_{[5\times1]}$$

 $\delta_{[5\times1]} = Diag[\varphi'(v_{[5\times1]})]e_{[5\times1]}$

2. 은닉층

$$\begin{array}{lll} e_{[20\times1]}^{(3)} & = & [W^{(4)}]_{[20\times5]}^T \delta_{[5\times1]} \\ \delta_{[20\times1]}^{(3)} & = & Diag[\varphi'(v^{(3)})_{[20\times1]}]e_{[20\times1]}^{(3)} \\ e_{[20\times1]}^{(2)} & = & [W^{(3)}]_{[20\times20]}^T \delta_{[20\times1]}^{(3)} \\ \delta_{[20\times1]}^{(2)} & = & Diag[\varphi'(v^{(2)})_{[20\times1]}]e_{[20\times1]}^{(2)} \\ e_{[25\times1]}^{(1)} & = & [W^{(2)}]_{[25\times20]}^T \delta_{[20\times1]}^{(2)} \\ \delta_{[25\times1]}^{(1)} & = & Diag[\varphi'(v^{(1)})_{[25\times1]}]e_{[25\times1]}^{(1)} \end{array}$$

심층신경망 학습 (4)

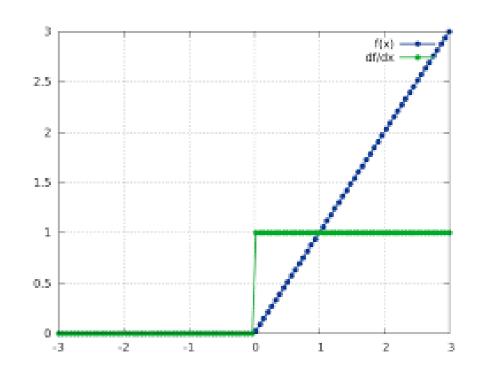
STEP4 가중치조정

$$\Delta W_{i,j}^{(I)} = \alpha \delta_i^{(I)} y_j^{(I-1)}
\Delta W_{[5 \times 20]}^{(4)} = \alpha \delta_{[5 \times 1]} [y^{(3)}]_{[1 \times 20]}^T
\Delta W_{[20 \times 20]}^{(3)} = \alpha \delta_{[20 \times 1]}^{(3)} [y^{(2)}]_{[1 \times 20]}^T
\Delta W_{[20 \times 20]}^{(2)} = \alpha \delta_{[20 \times 1]}^{(2)} [y^{(1)}]_{[1 \times 20]}^T
\Delta W_{[20 \times 25]}^{(1)} = \alpha \delta_{[20 \times 1]}^{(1)} [X]_{[1 \times 25]}^T
W'^{(I)} = W^{(I)} + \Delta W^{(I)}$$

ReLU function

$$\varphi(v) = \max\{0, v\} = \begin{cases} v & v > 0 \\ 0 & v \le 0 \end{cases}$$

$$\varphi'(v) = \begin{cases} 1 & v > 0 \\ 0 & v \le 0 \end{cases}$$

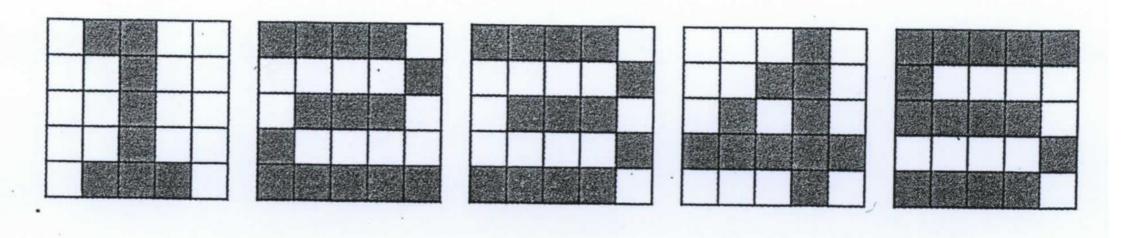


ReLU.m

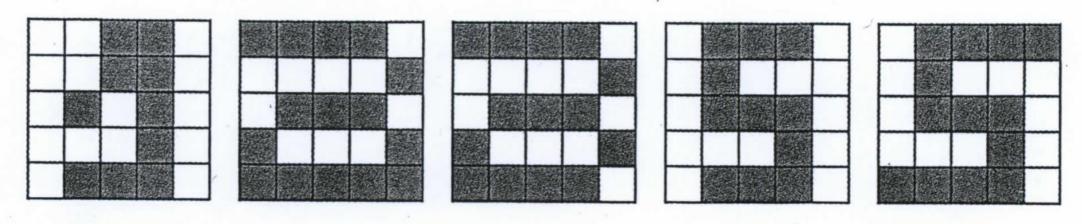
```
function y=ReLU(x)
y=max(0,x);
end
```

학습데이터, 검증데이터

학습데이터



검증데이터



training function: DeepReLu.m

```
function[W1,W2,W3,W4]=DeepReLU(W1,W2,W
3,W4,X,D)
```

1. Learning Rate

```
alpha=0.01;
```

2. SGD

```
N=5;
for k=1:N
    x=reshape(X(:,:,k),25,1);
    v1=W1*x;
    y1=ReLU(v1);
    v2=W2*y1;
    y2=ReLU(v2);
    v3=W3*y2;
    y3=ReLU(v3);
    v=W4*y3;
    y=softmax(v);
```

4. Back Propagation

```
d=D(k,:)';
    e=d-y;
    delta=e;
    e3=W4'*delta;
    delta3=(v3>0).*e3;
    e2=W3'*delta3;
    delta2=(v2>0).*e2;
    e1=W2'*delta2;
    delta1=(v1>0).*e1;
```

5. Update

```
dW4=alpha*delta*y3';
W4=W4+dW4;
dW3=alpha*delta3*y2';
W3=W3+dW3;
dW2=alpha*delta2*y1';
W2=W2+dW2;
dW1=alpha*delta1*x';
W1=W1+dW1;
End % end of SGD loop
end % end of function
```

TestDeepRelu .m

clear all

```
1. Data loading
X = zeros(5,5,5);
X(:,:,1)=[0\ 1\ 1\ 0\ 0;
       0 0 1 0 0;
       0 0 1 0 0;
       0 0 1 0 0;
       01110
X(:,:,2)=[1\ 1\ 1\ 1\ 0;
       00001;
       0 1 1 1 0;
       10000;
       11111
X(:,:,3)=[1\ 1\ 1\ 1\ 0;
       00001;
       0 1 1 1 0;
       00001;
       11110
       ];
```

```
X(:,:,4)=[0\ 0\ 0\ 1\ 0;
       0 0 1 1 0;
       0 1 0 1 0;
        1 1 1 1 1;
       00010
X(:,:,5)=[1\ 1\ 1\ 1\ 1;
        10000;
        1 1 1 1 0;
       00001;
        11110
D=[1\ 0\ 0\ 0\ 0;
  0 1 0 0 0;
  0 0 1 0 0;
  0 0 0 1 0;
  00001
  ];
```

```
2. 가중치 초기화
W1=2*rand(20,25)-1;
W2=2*rand(20,20)-1;
W3=2*rand(20,20)-1;
W4=2*rand(5,20)-1;
3. 기계 학습
for epoch = 1:1000
  [W1, W2, W3, W4]
=DeepReLU(W1,W2,W3,W4,X,D);
end
4. 추정
N = 5;
for k=1:N
  x = reshape(X(:,:,k),25,1);
v1=W1*x;
y1=ReLU(v1);
v2=W2*y1;
y2=ReLU(v2);
v3=W3*y2;
y3=ReLU(v3);
v = W4*y3;
y=softmax(v)
   end
```

RealRelu.m : 입력데이터 실험

```
clear all
rng(3);
                                                        3. 추정
                           X(:,:,3)=[1\ 1\ 1\ 1\ 0;
                                                        N = 5;
                                   00001;
X = zeros(5,5,5);
                                                        for k=1:N
                                   0 1 1 1 0;
                                                           x = reshape(X(:,:,k),25,1);
                                   10001;
1. 기계학습
                                                           v1=W1*x;
TestDeepReLU;
                                   11110
                                                           y1=ReLU(v1);
2. Data loading
X = zeros(5,5,5);
                                                           v2=W2*y1;
                           X(:,:,4)=[0\ 1\ 1\ 1\ 0;
                                                           y2=ReLU(v2);
                                   0 1 0 0 0;
X(:,:,1)=[0\ 0\ 1\ 1\ 0;
                                   0 1 1 1 0;
       0 0 1 1 0;
                                                           v3=W3*y2;
                                   0 0 0 1 0;
       0 1 0 1 0;
                                                           y3 = ReLU(v3);
                                   01110
       0 0 0 1 0;
       01110
                                                           v = W4*y3;
                                                           y=softmax(v)
                           X(:,:,5)=[0\ 1\ 1\ 1\ 1;
X(:,:,2)=[1\ 1\ 1\ 1\ 0;
                                                        end
                                   0 1 0 0 0;
       00001;
                                   0 1 1 1 0;
       0 1 1 1 0;
                                   00010;
       10001;
                                   11110
       11111
```

실습 . Matlab => R

- Relu.m
- DeepReLu.m
- TestdeepReLu.m

• .m 에서 과업 특성 파악 => 과업 list 작성 => R code

Dropout: 일부 node를 임의로 제외

- 과적합 문제: 학습데이터에서 지나치게 많은 정보가 들어간다
- Dropout: 학습데이터에서 무작위로 일부 정보를 제외한다.
- Dropout 과정
 - 1. Random sampling으로 제외할 node를 고른다 (Dropout.m)
 - 2. 1.에서 제외한 node를 빼고 입력-> 출력 과정을 거친다. (y)
 - 3. 2.에서 얻은 y를 이용하여 역전파 과정을 거친다. (delta, e)
 - 4. 2.3 에서 얻은 y와 delta를 이용하여 가중치를 조정한다.
 - 5. 1.-4.를 반복한다.
 - (2-5: DeepDropout.m)
- (실습하지 않습니다.)

Dropout.m

```
function ym=Dropout(y1,ratio)
[m,n]=size(y1);
ym=zeros(m,n);

num=round(m*n*(1-ratio));
idx=randperm(m*n,num);
ym(idx)=1/(1-ratio);
```

end

training function: DeepDropout.m

```
function[W1,W2,W3,W4]=DeepDropout(W1,W2
,W3,W4,X,D)
1. Learning Rate
alpha=0.01;
2. SGD
N = 5;
for k=1:N
v1=W1*x;
  y1=Sigmoid(v1);
  y1=y1.*Dropout(y1,0.2);
  v2=W2*y1;
  y2=Sigmoid(v2);
  y2=y2.*Dropout(y2,0.2);
  v3=W3*y2;
  y3=Sigmoid(v3);
  y3=y3.*Dropout(y3,0.2);
  v = W4*y3;
  y=softmax(v);
```

4. Back Propagation

```
d=D(k,:)';
e=d-y;
delta=e;
e3=W4'*delta;
delta3=y3.*(1-y3).*e3;
e2=W3'*delta3;
delta2=y2.*(1-y2).*e2;
e1=W2'*delta2;
delta1=y1.*(1-y1).*e1;
```

5. Update

```
dW4=alpha*delta*y3';
W4=W4+dW4;
dW3=alpha*delta3*y2';
W3=W3+dW3;
dW2=alpha*delta2*y1';
W2=W2+dW2;
dW1=alpha*delta1*x';
W1=W1+dW1;
end % end of SGD loop
end % end of function
```

TestDeepDropout .m

clear all

1. Data loading

```
X = zeros(5,5,5);
X(:,:,1)=[0\ 1\ 1\ 0\ 0;
       0 0 1 0 0;
       0 0 1 0 0;
       0 0 1 0 0;
       01110
X(:,:,2)=[1\ 1\ 1\ 1\ 0;
       00001;
       0 1 1 1 0;
        10000;
       11111
X(:,:,3)=[1\ 1\ 1\ 1\ 0;
       00001;
       0 1 1 1 0;
       00001;
       11110
        ];
```

```
X(:,:,4)=[0\ 0\ 0\ 1\ 0;
       0 0 1 1 0;
       0 1 0 1 0;
        1 1 1 1 1;
       00010
X(:,:,5)=[1\ 1\ 1\ 1\ 1;
        10000;
        1 1 1 1 0;
       00001;
        11110
D=[1\ 0\ 0\ 0\ 0;
  0 1 0 0 0;
  0 0 1 0 0;
  0 0 0 1 0;
  00001
  ];
```

```
2. 가중치 초기화
W1=2*rand(20,25)-1;
W2=2*rand(20,20)-1;
W3=2*rand(20,20)-1;
W4=2*rand(5,20)-1;
3. 기계 학습
for epoch = 1:1000
  [W1, W2, W3, W4]
=DeepDropout(W1,W2,W3,W4,X,D);
end
4. 추정
N = 5;
for k=1:N
  x = reshape(X(:,:,k),25,1);
v1=W1*x;
y1=Sigmoid(v1);
v2=W2*y1;
y2=Sigmoid(v2);
v3=W3*y2;
y3=Sigmoid(v3);
v = W4*y3;
y=softmax(v)
   end
```

Realdropout.m :입력데이터 실험

```
clear all
                                                        3. 추정
rng(3);
                           X(:,:,3)=[1\ 1\ 1\ 1\ 0;
                                                        N = 5;
                                   00001;
X = zeros(5,5,5);
                                                        for k=1:N
                                   0 1 1 1 0;
                                                           x = reshape(X(:,:,k),25,1);
                                   10001;
1. 기계학습
TestDeepDropout;
                                   11110
                                                           v1=W1*x;
                                                           y1=Sigmoid(v1);
2. Data loading
X = zeros(5,5,5);
                           X(:,:,4)=[0\ 1\ 1\ 1\ 0;
                                                           v2=W2*y1;
                                   0 1 0 0 0;
                                                           y2=Sigmoid(v2);
X(:,:,1)=[0\ 0\ 1\ 1\ 0;
                                   0 1 1 1 0;
       0 0 1 1 0;
                                   00010;
       0 1 0 1 0;
                                                           v3=W3*y2;
                                   01110
       0 0 0 1 0;
                                                           y3=Sigmoid(v3);
       01110
                                                           v = W4*y3;
                           X(:,:,5)=[0\ 1\ 1\ 1\ 1;
                                                           y=softmax(v)
X(:,:,2)=[1\ 1\ 1\ 1\ 0;
                                   0 1 0 0 0;
                                                        end
       00001;
                                   0 1 1 1 0;
       0 1 1 1 0;
                                   00010;
       10001;
                                   11110
       11111
```