STAT6011/7611/6111/3317
COMPUTATIONAL STATISTICS (2016 Fall)
Assignment 3

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1

1.1

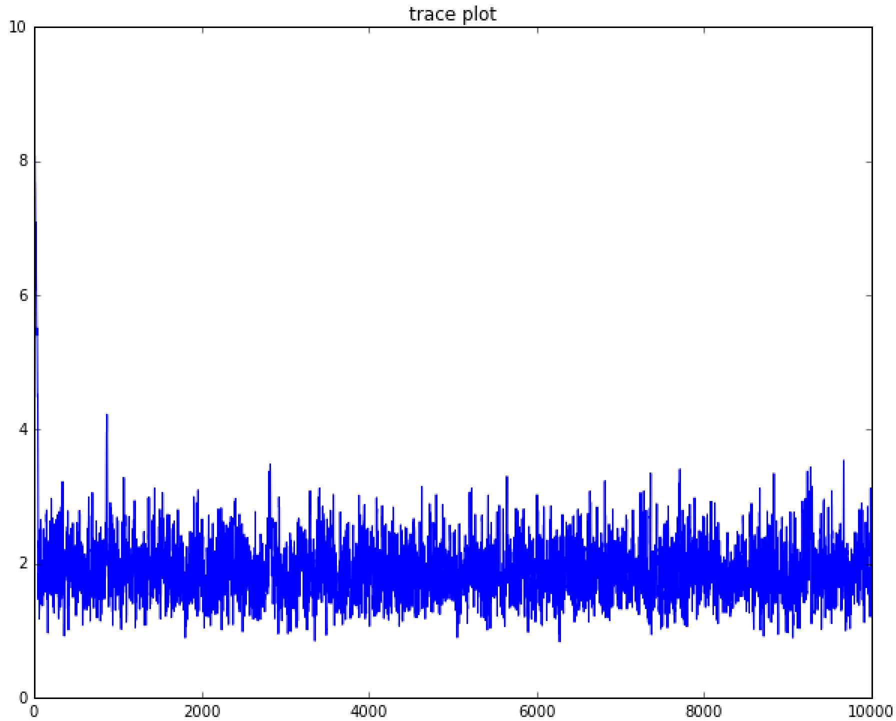
$$
\begin{aligned}
p(\mathbf{y}|\sigma^2) &= \int \cdots \int \Pi_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y_i-\theta_i)^2}{2}\right) \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\theta_i^2}{2\sigma^2}\right) \mathrm{d}\theta_1 \ldots \mathrm{d}\theta_n \\
&= \int \cdots \int \Pi_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sqrt{\sigma^2+1}} \exp\left(-\frac{y_i}{2} + \frac{\sigma^2 y_i^2}{2(\sigma^2+1)}\right) \frac{\sqrt{\sigma^2+1}}{\sqrt{2\pi}\sigma} \exp\left(\frac{\sigma^2+1}{\sigma^2}\left(\theta_i - \frac{\sigma^2}{\sigma^2+1} y_i\right)\right) \mathrm{d}\theta_1 \ldots \mathrm{d}\theta_n \\
&= \Pi_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sqrt{\sigma^2+1}} \exp\left(-\frac{y_i}{2} + \frac{\sigma^2 y_i^2}{2(\sigma^2+1)}\right) \\
&= \left(\frac{1}{\sqrt{2\pi}\sqrt{\sigma^2+1}}\right)^n \exp\left(-\frac{\sum_{i=1}^{n} y_i}{2} + \frac{\sigma^2 \sum_{i=1}^{n} y_i^2}{2(\sigma^2+1)}\right)
\end{aligned}
$$

1.2

$$
p(\sigma^2|\mathbf{y}) \propto \left(\frac{1}{\sqrt{2\pi}\sqrt{\sigma^2+1}}\right)^n \exp\left(-\frac{\sum_{i=1}^{n} y_i}{2} + \frac{\sigma^2 \sum_{i=1}^{n} y_i^2}{2(\sigma^2+1)}\right) \frac{1}{\sigma^2}
$$

Listing 1: 1-b

```
import numpy as np
import matplotlib.pyplot as plt
% matplotlib inline
import pandas as pd

y = q1.values[:, 1]
def pos(sigma):
    return np.exp(sigma*np.dot(y, y)/(2*(sigma + 1)) - sum(y)/2) / (sigma*(np.sqrt(2*np.pi * (sigma + 1)))**100)

np.random.seed(1234)
samples = [10]
for i in range(10000):
    u = np.random.uniform()
    propose = samples[-1] + np.random.normal(scale = 0.5)
    prob = min(1, pos(propose)/pos(samples[-1]))

    if prob < u:
        samples.append(samples[-1])
    else:
        samples.append(propose)

plt.figure(figsize = (10, 8))
plt.title("trace plot")
plt.plot(samples)
plt.show()
```



1.3

Let $\Theta = (\theta_1, \theta_2, \ldots, \theta_n)$, and let $\Theta_{-j} = (\theta_1, \ldots, \theta_{j-1}, \theta_{j+1}, \ldots, \theta_n)$.

$$p(\sigma^2|\mathbf{y}, \Theta) \propto \frac{1}{\sigma^2(\sqrt{2\pi\sigma^2})^n} \exp\left(-\frac{\sum_i^n \theta_i^2}{2\sigma^2}\right)$$

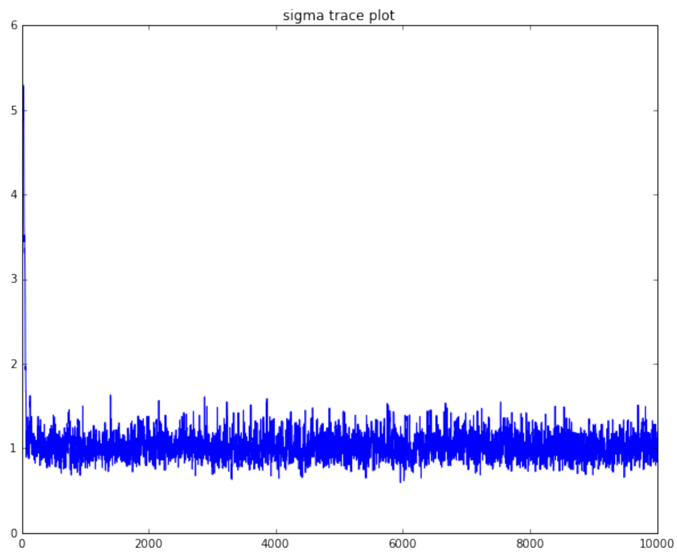
$$p(\theta_j|\mathbf{y}, \Theta_{-j}, \sigma^2) \propto N\left(\frac{\sigma^2 y_j}{\sigma^2+1}, \frac{\sigma^2}{\sigma^2+1}\right) \forall j \in \{1, 2, 3, \ldots, n\}$$

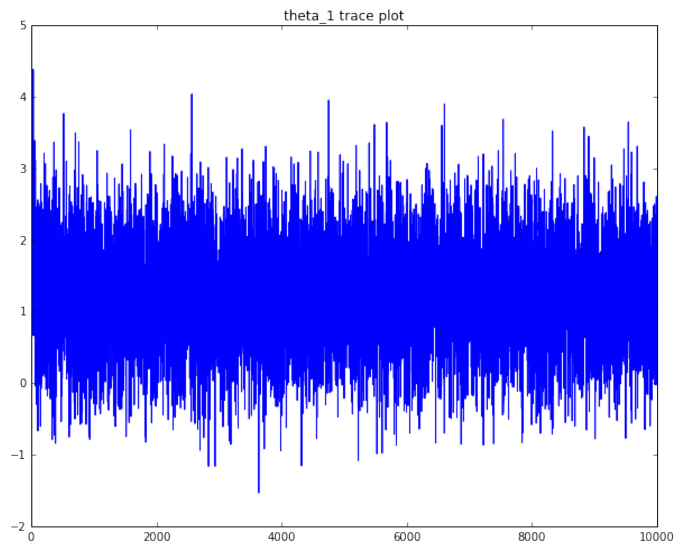
Listing 2: 1-c

```
def pos2(sigma, i):
    return np.exp(-np.dot(theta_sample[:, i], theta_sample[:, i])/(2*sigma)) / (sigma*(np.sqrt(2*np.pi * sigma))**100)

# Gibbs sampler
np.random.seed(1234)

sigma_sample = [5]
theta_sample = np.ones((len(y), 10000))

for i in range(10000):

    if i != 0:
        u = np.random.uniform()
        propose = sigma_sample[-1] + np.random.normal(scale = 0.2)
        prob = min(1, pos2(propose, i)/pos2(sigma_sample[-1], i))

        if prob < u:
            sigma_sample.append(sigma_sample[-1])
        else:
            sigma_sample.append(propose)

        for j in range(100):
            theta_sample[j, i] = np.random.normal(loc = sigma_sample[-1]*y[j]/(sigma_sample[-1] + 1), scale = np.
                sqrt(sigma_sample[-1]/(sigma_sample[-1] + 1)))

    else:
        for j in range(100):
```

```
            theta_sample[j, i] = np.random.normal(loc = sigma_sample[-1]*y[j]/(sigma_sample[-1] + 1), scale = np.
                sqrt(sigma_sample[-1]/(sigma_sample[-1] + 1)))


# plot sigma
plt.figure(figsize = (10, 8))
plt.title("sigma trace plot")
plt.plot(sigma_sample)
plt.xlim((0, 10000))
plt.show()

# plot first 3 theta
for i in range(3):
    plt.figure(figsize = (10, 8))
    plt.title("theta_1 trace plot")
    plt.plot(theta_sample[i, :])
    plt.xlim((0, 10000))
    plt.show()
```

Trace plot. σ^2 and θ_1.





2

2.1

2.2

2.3

2.4

3

3.1

3.2