Bargaining over Leasing Contracts amid Shifting Power Balance*

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Abstract

This paper investigates the inter-firm contract terms by analyzing tenant leasing contracts in shopping malls, with a particular focus on controlling the time-varying power balance between the involved parties. Utilizing a novel dataset covering two shopping malls and their tenants over six years—including contract terms, daily sales, and negotiation records—we estimate a structural model to recover the evolution of power balances. The results reveal that the power balance varies over time due to the influence of tenants' past performance, such as sales per unit area. In our simulations, these temporal dynamics account for approximately 10% of the shopping mall's surplus. Based on these estimated power balance, we show that controlling for them allows valid empirical analysis of the contract terms. Our analysis demonstrates that the selection of contract terms aligns with the risk attitude of the shopping mall manager, and that up to twice the amount of rent can be collected by changing the way to pick a contract while keeping the power balance between the tenants fixed.

JEL Classification Codes: C71; C78; L81; R32; R33.

Keywords: Contracting; Nash Bargaining; Renegotiation; Shopping Center; Tenancy; Tenant.

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1 Introduction

Many transactions between firms are formalized through contracts, which outline various agreements between the involved parties. Although this requires a more complex process compared to fixed-price transactions, contracts are widely used, especially in long-term engagements. Economics has long studied their existence, the optimal structure of these contracts, and the determination of specific contractual terms. Various theories, such as contract theory, have been developed, and their predictions have been empirically tested using observed contracts. However, even now, the simple models adopted in the empirical studies and data limitations still do not allow researchers to fully investigate the mechanisms behind contracts in the filed[1][2].

Moreover, the unobserved and shifting power balance between the involved parties hinders the ideal situation to analyze the the contract terms by breaking the ceteris paribus condition, although many of the empirical analysis of contract assume exclusive power balance. For example, a CEO who significantly improves company performance during a particular period may gain greater influence compared to before. Similarly, the power dynamic between tenant farmers and landowners differs during times of famine compared to normal periods. If such factors are overlooked, empirical results may be biased. Capturing and accounting for shifts in these unobservable power relationships is crucial for understanding the mechanisms behind contract terms.

In this study, we construct a novel dataset of inter-firm contracts, specifically tenant leasing contracts in a shopping mall where monthly rents are determined using a type of share contract[3]. We estimate a tractable model to capture the evolution of power balance

[1]Many papers consider how firms choose a contract from a set of alternatives. When the focus is on the choice between two different contract forms, such as a share contract or a fixed-price contract, a binary choice model is used. For continuous variables in contract terms, regression models are employed. However, these simple models cannot fully handle endogeneity problems and cannot analyze multiple terms simultaneously. See Lafontaine and Slade (2013) for details.

[2]Data limitations are a severe problem preventing empirical work in the study of contracts between firms. This is because such contracts are usually confidential and rarely available to researchers. Linking contract information to other important variables, such as the performance of the contracting firms, is further complicated due to the difficulty in measuring performance.

[3]A share contract is a common form of agreement between firms. For instance, many business-format franchises adopt this type of contract. As the name suggests, the transfer amount between the parties is based on realized performance, such as sales or revenue. One key issue in the study of these contracts is the widespread use of simple affine forms, despite theoretical predictions allowing for more complex functional structures. There is substantial theoretical and empirical literature on the mechanisms behind contract forms Stiglitz (1974); Bhattacharyya and Lafontaine (1995); Lazear (2000); Carroll (2015). For surveys, see Lafontaine and Slade (2013); Lazear (2018).

between the involved parties. Using these estimates, we further analyze how contract terms are determined in this industry. Our results reveal expected patterns in contract terms that are not apparent without accounting for power dynamics. Moreover, we demonstrate that the selection of contract terms aligns with the preferences of the involved parties, particularly their risk attitudes. This study contributes to the existing literature by adding important elements from both data and modeling perspectives, providing new insights into the mechanisms of inter-firm contracts.

The leasing operations of the shopping mall primarily involve searching for new tenants and negotiating their contracts, as well as deciding whether to renew contracts with existing tenants and negotiating the terms during the renewal process. The main aspect of these contracts is the function used to determine the monthly rent based on sales, which often takes the form of an affine contract with a single kink. There is no fixed menu of leasing contracts, and different tenants often operate under varying contract terms. If the shopping mall's head office decides to retain a tenant, renewal negotiations are held before the end of the current contract period. In our analysis, we focus on these renewal negotiations where both parties share the same sales history in common and the concern about the asymmetric information is diminished.

As a peculiar feature compared to the usual bargaining over contract such as Kihlstrom and Roth (1982), both the shopping mall and the tenant must handle risk and uncertainty about the future sales yielded by the retail space through the renewal negotiation. In addition to the risk sharing given one distribution of the sales, they must first discuss the expected sales distribution[4]. This is not a unique problem to the current situation: it is well known that one of the fundamental motivations for engaging in the complex transactions, such as share contracts, is to deal with uncertainty (Goldberg, 1976; Lafontaine and Slade, 2013). Hence, we have to consider two different power balances between the involved parties when analyzing the mechanism of contract terms: one for risk sharing and the other for uncertainty resolution.

Our dataset covers two shopping malls and their tenants over approximately six years. For each tenant, in addition to the basic characteristics such as its area, we observe the realized sales and the number of customers on a daily basis, which are used as performance

[4]Another clear deviation is the complexity of the contract. Kihlstrom and Roth (1982) consider the simplest insurance contract which specifies the amount of transfer in the case of accident. Our case handles a type of share contract which determines the amount of transfer as a function of some performance measure.

measures. Additionally, for each tenant and shopping mall pair, we observe the ongoing contract. We also have records of meeting minutes between the shopping mall and both potential and incumbent tenants, which are used to construct measures of the intensity of search behavior and identify the contents of the initial offer. This dataset is particularly rare in empirical analysis because contracts between firms are usually kept confidential from other potential contractors, and sales information within a shopping mall is typically highly private, as it significantly impacts the shopping mall's market strength.

From the contract data and the sales data, we can calculate the rent paid to the shopping mall by every tenants each month. These data reveal several key points. First, comparing rent and sales across tenants shows a disparity in the relative size of tenants within the mall. This difference is primarily driven by variations in sales per unit of area, which is likely to influence the balance of power between the tenant and the shopping mall. In fact, tenants with relatively lower sales per unit area are often forced to exit the mall. Furthermore, even among tenants who successfully renew their contracts, those with higher relative sales per unit area tend to have lower expected rent in the new contract. This demonstrates that the power balance between the shopping mall and its tenants affects both the external margin and the internal margin.

However, there are several limitations to the observational analysis. First, we must care about the measurement error caused by using only the relative ranking of sales per unit area as a measure of the power balance between the parties. As widely recognized in the literature, there are other elements, such as preferences, that are assumed to influence the power balance. Second, there is the issue of how to account for the risk attitudes of both the shopping mall and the tenants. Even if a tenant accepts a renewal with higher average rent, if the new contract increases the variance in rents, it could be less favorable depending on their degree of risk aversion. To address these issues, this paper introduces a structural model in which the equilibrium set of contracts is determined through bargaining between risk-averse agents. Using this model, we recover the power balances and risk attitudes of both parties based on the contract terms observed agreed upon in renewal negotiations.

We briefly highlight the key deviations of our model from existing empirical bargaining models. First, our model simultaneously handles the complexity of contract terms. The bargaining solution does not specify the exact contract terms. In this sense, our model is incomplete. Our estimation relies on characterization of the set of equilibrium contract terms, and the selection mechanism from this equilibrium set is estimated separately. Second, as mentioned earlier, the parties face two distinct bargaining issues: risk sharing and uncertainty resolution. We model this with two bargaining stages: in the first stage, the tenant negotiates with the shopping mall's manager over the uncertainty surrounding the sales distribution. In the subsequent stage, given the agreement on the sales distribution, the tenant negotiates the contract terms to share the risk with the shopping mall's negotiation representative. In our model, the existence of these two bargaining processes is endogenously justified as the optimal institutional design from the view point of the shopping mall's manager, who strategically divides the authority within the firm. We adopt the Nash-in-Nash solution to describe the sequential resolution of uncertainty over the sales distribution and risk-sharing through the contract.

We estimate the model via maximum likelihood estimation under the constraint that the observed contract terms are included in the equilibrium set of contracts. The identification relies on the fact that the condition of the Nash bargaining solution over the terms of contract allows us to recover the corresponding bargaining powers for all the continuing tenants. From this recovered bargaining powers, we estimate the marginal effects of several variables. For the value of the outside option, we estimate the marginal effect of the search behavior. For this point, we use the control function approach to handle the endogeneity of the intensity of searching.

Our estimation results show that the balance of bargaining power in renewal negotiations varies over time. This variation is due to its dependence on past performance, such as the relative ranking of sales per unit area within a shopping mall. In our counterfactual analysis, where we remove all time-dependent components from the balance of bargaining power, the shopping malls would increase their share of the surplus. The impact, measured as the change in certainty equivalence from the actual surplus, is around 10%, although it varies among tenants. This result indicates that it is not enough to rely on fixed effect analysis even when the researcher has access to a panel data.

Lastly, based on these estimates, we separately estimate the selection mechanisms of contract terms for the two shopping malls. Our estimation results, obtained when controlling for the balance of bargaining power between the involved parties, align with the expected relationships between contract terms suggested by anecdotal evidence. Furthermore, from the results, we can identify differences in how the malls formulate contracts. This difference is consistent with the difference in the degree of risk aversion between the two shopping malls. From the perspective of the shopping mall management company, the risk attitude of the manager might be an obstacle to collect higher rents. In our counterfactual analysis, one of the malls can increase its rent revenue by adopting the contract-making strategy of the other mall.

1.1 Related literature

Our research primarily contributes to the studies on tenant leasing in shopping malls[5]. Notably, Gould, Pashigian and Prendergast (2005) uses rent data to identify the existence of preferential treatment for *anchor tenants*, who attract neighbors to the mall and induce higher spending at other stores. The focus is on the team problem in shopping malls: every tenants, including the mall itself, exerts effort to attract consumers, but this effort is difficult to observe. From this perspective, the study finds that anchor tenants, such as well-known brands, often pay little or no rent. This point is also discussed in Pashigian and Gould (1998)[6].

In a broader scope, the effects and incidences of share contract have been studied both theoretically and empirically. The main body of literature addresses moral hazard as the rationale behind share contracts, analyzing their incidence and effects. Key studies in this area include Lafontaine and Shaw (1999); Ackerberg and Botticini (2002); Mortimer (2008); D'Haultfœuille and Février (2020); Conlon and Mortimer (2021), which range from franchise contracts to leasing contracts in the video rental industry. A recent survey on this topic is provided by Lafontaine and Slade (2013)[7].

In most of the aforementioned studies, bargaining power is typically excluded, with

[5]There are both empirical and theoretical studies. Theory papers on rent structures in shopping malls include the following: Benjamin, Boyle and Sirmans (1992) and Brueckner (1993), who model rent determination based on the demand for retail space. Lee (1995) proposes a principal-agent model to determine rent, focusing on hidden actions. The optimal rent structure in this context is either (1) a linear revenue share, purely for risk-sharing when there is no moral hazard, or (2) a fixed rent plus a linear revenue share when there is moral hazard. In my data, we observe an additional rent structure: the multi-kink rent. Wheaton (2000) summarizes several models of rent determination in shopping malls. Recently, Monden, Takashima and Zennyo (2021) built a parametric model to study rent determination, arguing that in equilibrium, larger tenants receive contracts with lower revenue shares, i.e., they benefit from preferential treatment.

[6]While we focus on a specific group of tenants in shopping malls that are relatively small and do not benefit from preferential treatment, the rent structure and dynamics of anchor tenants represent a separate topic that warrants investigation.

[7]As mentioned in Lafontaine and Slade (2013), there are many papers that examine other types of contracts, such as cost-plus and fixed-price contracts. While many of these studies are based on reduced-form analysis, Gagnepain, Ivaldi and Martimort (2013) analyzes cost-plus contract using a structural model.

contracts modeled as take-it-or-leave-it offers. In contrast, our study incorporates bargaining power into the model and investigates the dynamic changes in bargaining power that drive contract modifications.

Our research also stands in the literature of empirical bargaining. One type of the empirical bargaining literature adopts the Nash bargaining solution or the Nash-in-Nash solution to describe the equilibrium. Nash-in-Nash solution has been widely adopted to describe the vertical contracts in recent industrial organization studies, including Crawford et al. (2018); Gowrisankaran, Nevo and Town (2015); Crawford and Yurukoglu (2012). For a comprehensive overview, see the recent survey by Lee, Whinston and Yurukoglu (2021)[8].

Unlike previous studies, this research considers a situation where bargainers face uncertainty over a part of the bargaining problem. For example, many of the studies modeling vertical contracts assume that downstream demand is mutually agreed upon by both parties, and a common demand function is used to define and solve the bargaining problem. However, in many bargaining scenarios, including our case, there is no agreement on the environment of the problem itself[9]. In our case, the bargainers do not totally agree on the future distribution of sales, and the contract negotiation must begin from the stage of reaching such an agreement. We conceptualize this situation as model uncertainty and propose a tractable model in which bargaining takes place both in the model space and the contract space.

Finally, our study aligns with classical bargaining research, seeking to elucidate the sources of bargaining power. This study adds another evidence to this literature that the past performance is an important factor determining the balance of bargaining powers. Building on the theoretical work of Rubinstein (1982, 1985), which highlights the discount factor as a key determinant of bargaining power, several empirical studies have explored the construction of bargaining power. Recently, Backus et al. (2020) conducted an empirical analysis using eBay bargaining data, identifying customer experience and

[8]Another type of this research involves building specific structural models of bargaining problems to include particular information structures or unique dynamics. Sieg (2000); Watanabe (2005); Silveira (2017) analyze dispute resolution, Larsen (2020) studies bilateral bargaining over the price of used cars, Ambrus, Chaney and Salitskiy (2018) investigates delays in bargaining over captives ransomed from Barbary pirates, and Merlo and Wilson (1995); Merlo and Tang (2012) propose a framework for stochastic sequential bargaining applied to various fields.

[9]As stated early in Goldberg (1976) and thoroughly discussed in subsequent literature, one of the rationales behind the use of share-type contracts is the uncertainty over future demand. The situation is similar to a common value auction: avoiding the commitment to ex-post overly optimistic or pessimistic transfer, the bargainers want to make the transfer depend on the realized outcome, i.e., sales.

patience as factors influencing bargaining outcomes. Research on incomplete contracts also investigates how past actions such as investment or changes in the environment affecting the value of outside options relate to the negotiation result. This literature begins with classical arguments like those of Williamson (1983) and extends through empirical studies such as Joskow (1987, 1990). More recent empirical research has identified several decisive factors in renegotiation outcomes, as seen in Benmelech and Bergman (2008); Gagnepain, Ivaldi and Martimort (2013); Ater et al. (2022).

2 Tenant leasing in a shopping mall

Here, we describe the details of leasing operations in the shopping mall industry. The specific practices, industry norms, and typical processes mentioned here are based on information obtained through interviews with leasing officers at the shopping mall management company, which provided the data used in this paper, as well as real estate professionals involved in the data provision.

A shopping mall, managed by a real estate company, functions as a commercial complex where the company generates revenue through the rents paid by its tenants. Since the number of tenants directly impacts foot traffic, shopping malls typically aim to house a large number of tenants, striving to keep commercial spaces fully occupied. While the number and types of tenants vary depending on each mall's operational strategy, they usually feature a mix of small to medium-sized tenants, such as restaurants, apparel brands, and specialty stores, alongside larger tenants like multiplex cinemas, bookstores, and supermarkets.

We review the overall operations of the shopping mall. The management can be understood in three distinct phases: the search for new tenants, the negotiation and signing of lease agreements, and the ongoing relationship maintenance after tenants have moved in. The company continuously scouts for potential tenants, targeting existing stores in the vicinity or promising retail businesses, thereby maintaining a pool of prospective tenants. Whenever a new shopping mall opens or a vacancy arises due to the existing tenant, the manager taps into this pool to secure and sign contracts with new tenants. The lease agreements are not off-the-shelf contracts but rather are the result of negotiations between the shopping mall side and the prospective tenant, often involving lengthy discussions to reach a mutual agreement. Even after a tenant has moved in, the mall management conducts regular meetings with tenants to share business updates and discuss local market conditions. Additionally, it takes the actions such as conducting mystery shopping evaluations to assess customer service and share the results with the tenants. These steps are part of the continuous effort to foster a positive and ongoing relationship with the tenants and to monitor the tenants' effort on sales promotion.

The real estate company delegates the management of the shopping mall to a manager. However, due to the large number of tenants, the manager must rely on subordinates to handle these tasks. The mall manager rarely participates directly in contract negotiations or business meetings with potential new tenants. Instead, the negotiation is typically led by a younger field staff member, who sits at the table to discuss the detailed contract terms. Hereafter, we call these staff by negotiation representative.

In contracts between shopping malls and their tenants, the negotiation is typically over the lease period, restoration obligations upon exiting, and rent. As with many business negotiations, much of the contract details are determined by industry norms, resulting in a somewhat automatic decision-making process. In the current context, for most tenants, excluding those with a special status in the mall, such as cinemas, the lease period and restoration obligations are generally set according to these established norms because these two points do not cause severe conflict or disagreement between the two parties. From the tenant's perspective, once the costs of renovations have been amortized, the lease period becomes less of a concern. Regarding restoration obligations upon exiting, the standard principle is that tenants bear the cost of returning the space to its original condition. While there may be discussions over specifics, such as the selection of contractors, most small to medium-sized tenants have little choice but to use the contractors specified by the mall, making this a relatively minor issue.

In contrast to the above two points, rent negotiations often involve significant disagreement and intense conflict between the parties. The primary challenge lies in the difficulty in agreeing on future sales expectation. Interviews with field representatives revealed that once a mutual sales forecast is agreed, determining the rent becomes relatively straightforward, guided by market norms and each party's market strength. This point is reflected in the common commercial practice of adopting a variant of share contracts for the rent. In order to deal with the uncertainty over the future sales, the rent is not fixed but varies according to sales performance.

This paper focuses on the renegotiation process with existing tenants, rather than negotiations with new tenants, due to the assumed greater information asymmetry and the presence of non-quantifiable factors in the latter case. The process of renegotiation typically begins six months to a year before the existing lease term ends. In most cases, the shopping mall management initiates the process by informing the tenant whether they wish to terminate the lease or propose a renewal. If the mall management requests termination, the tenant generally has little room to oppose it. If a renewal is proposed, negotiations over the terms of the new lease commence.

These negotiations usually involve representatives from both sides, although senior executives from the head office may also participate depending on the importance of the issues being discussed. This involvement reflects the limited authority of the initial representatives, as it is uncommon for decisions to be made on the spot during these meetings. Instead, negotiations typically proceed with weekly meetings, where each party takes the proposed terms back to their head offices for consideration before reconvening the following week with responses. While the current lease end date serves as a nominal deadline, it is rare for the final agreement on the renegotiated terms to be delayed until the last moment. Additionally, it is important to note that once negotiations for renewal begin, the possibility of the tenant vacating due to a breakdown in negotiations is generally not even considered a likely outcome.

It is noteworthy that the shopping mall's efforts to search for new tenants are closely related to the renegotiation of existing leases. Fundamentally, the intensity of these search activities is determined endogenously by the mall's current business conditions and the surrounding environment. During renegotiations, it is common practice for the mall to specifically target the commercial space in question and actively market it to potential tenants within its pool. This strategy is expected not only to provide leverage in negotiations with the existing tenant but also as a contingency plan in case the negotiations fall through. Thus, the extent of new tenant search efforts is strongly influenced by the particular commercial space undergoing renegotiation.

2.1 Rent structure

In shopping malls, the rent is determined by a type of share contract. This form of contract largely represents a compromise between the conflicting interests of the shopping mall and the tenant regarding fixed costs. Given a projected sales, tenants generally prefer a purely commission-based arrangement, while the mall seeks to maximize fixed rent. The

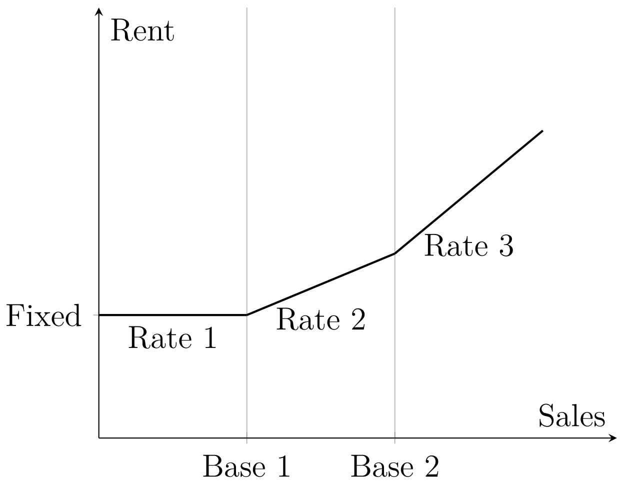


Figure 1. Rent Structure with Two Kinks

adopted rent structure is a commercially customary solution to this fundamental conflict.

The rent structure is defined by a set of parameters: *Fixed*, which represents the fixed amount of rent; *Base*, which is the set of thresholds at which the commission rate changes; and *Rate*, which is the commission rate applicable between these thresholds. An example of a rent structure with two kinks is illustrated in Figure 1. Until sales exceed Base_1, the rent is set to *Fixed*. If sales are greater than Base_1 but less than Base_2, the rent increases at a rate of Rate_2. Beyond that point, the rent increases at a rate of Rate_3[10].

The space of this function includes extreme contracts as special cases, such as the perfect commission contract, the fixed rent contract, and the affine contract. However, these extreme cases are relatively less observed in actual lease agreements. In particular, the perfect commission contract and fixed rent contract are considered special arrangements typically reserved for anchor tenants, such as large apparel stores that significantly influence foot traffic in the shopping mall. Generally, the shopping mall management does not permit such contracts.

[10] A rent structure with K kinks is characterized by $K+2$ bases and $K+1$ rates. We denote the bases by Base_i for $i = 0, 1, \cdots, K+1$, and the rates by Rate_i for $i = 1, 2, \cdots$, where $\text{Base}_0 = 0$ and $\text{Base}_{K+1} = \infty$. Based on them, the rent is computed as follows

$$\begin{aligned}\text{Rent} =& \text{Fixed} \\ &+ \sum_{i=1} 1\left\{Sales \in [\text{Base}_{i-1}, \text{Base}_i]\right\} \\ &\times \left\{\text{Rate}_i \times (\text{Sales} - \text{Base}_{i-1}) + \sum_{j=1}^{i-1} \text{Rate}_j \times (\text{Base}_j - \text{Base}_{i-1})\right\}.\end{aligned}$$

In our main analysis, we focus on a rent structure characterized by a single kink with $Rate_1 = 0$, which we term the *mixed-type contract*[11]. As discussed in Section 3, this type of rent structure is the most prevalent. Hereafter, for clarity, we will refer to the $Fixed$, $Base_1$, and $Rate_2$ of the mixed-type contract as *Fixed*, *Base*, and *Rate*, respectively.

The presence of Base is a significant difference in the form of contract compared to the well-known affine contracts. In practice, the existence of Base plays a crucial role in lease negotiations because many tenants impose certain constraints on Fixed: such as a cap on the percentage of fixed amount of rent to their sales. Negotiating with tenants over these company policies can be time-consuming. Therefore, while Fixed is also a negotiable object, the primary focus in negotiation over rent structure is on its Base and Rate. Naturally, shopping malls prefer to lower the base to ensure that the monthly sales exceed it and yield the part of commission, whereas tenants aim for the opposite. Within this conflict of interests, a key aspect of negotiation between malls and tenants involves finding an optimal combination of Base and Rate, balancing the risks associated with sales fluctuations. For instance, a mall may agree to lower Base in exchange for a higher Rate.

3 Data

Our data covers two shopping malls, both managed by the same real estate company, for six years from 2017 to 2023. These malls are located in the western region in Japan but are geographically somewhat distant from each other, and each managed by different managers[12]. One is situated in the downtown area of a city, while the other is located in the suburbs. In this paper, we refer to the former as *Mall 0* and the latter as *Mall 1*. Mall 0 was built at 2011 and Mall 1 was built at 2009. Both are mixed-use shopping malls, featuring a balanced mix of tenants, ranging from large-scale tenants such as cinemas and major apparel brands to small and medium-sized tenants, including apparel, dining, and miscellaneous goods stores.

[11]The term "mixed-type contract" refers to the fact that its functional form combines aspects of an affine contract and a fixed rent contract.

[12]The two shopping malls are located 12.32 km apart in a straight line. They are situated in different prefectures, and more importantly, each mall is adjacent to the central station of its respective region. In Japan, commercial areas are typically defined with the train station as the focal point. From the company's perspective, these malls are considered to belong to distinct commercial zones, and so each is managed independently by separate administrators.

For the tenants in these two shopping malls, we observe basic characteristics such as area in addition to the daily performance measure such as daily realized sales and the number of customers who make purchases. In our main analysis, we aggregate these performance measures on a monthly basis. Since this sales data is used to calculate the monthly rent, the data is highly reliable. A limitation of our data is the absence of information regarding the floor location and the distance from the main entrance—factors that is expected to influence the value of retail space.

We can also track the contract information determining the rent agreed by a shopping mall and its tenant. This data covers the parameters of the tenant: Fixed, Bases, and Rates defined in Section 2.1. These are used to compute the monthly rent and so there is little doubt on its quality and attrition problem.

We have access to the records of meeting minutes documented by the representatives of the shopping mall for meetings with both potential and incumbent tenants. Every meeting minutes is categorized by the primary objective of the meeting, with the main categories being 'New Tenant Search' and 'Renewal Negotiation.' All records are dated and include detailed notes for each meeting. However, not all discussions or offers are documented, as some of these conversations, particularly those conducted over the phone, are not directly recorded in the minutes. We use this meeting data to analyze the following two aspects: the initial offer made by the shopping mall and the intensity of search behavior. The initial offer is almost always recorded in the minutes, which typically includes the form of a list detailing Fixed, Bases, and Rates. Finally, we quantify the intensity of the shopping mall's search efforts by counting the number of 'New Tenant Search' meetings each month.

This dataset can link the performance with the contract terms and track the change in the contract terms as a panel data. This kind of dataset is particularly rare in empirical analysis because contract terms between firms are usually kept confidential from other potential contractors, and sales information within a shopping mall is typically highly private, as it significantly impacts the shopping mall's market strength.

3.1 Performance data

We present several sets of figures and tables to capture the actual leasing operations. First, we show a summary about the total sales and rents by malls. The first panel in Figure 2 shows the transitions of the number of tenants by malls: Mall 0 is always

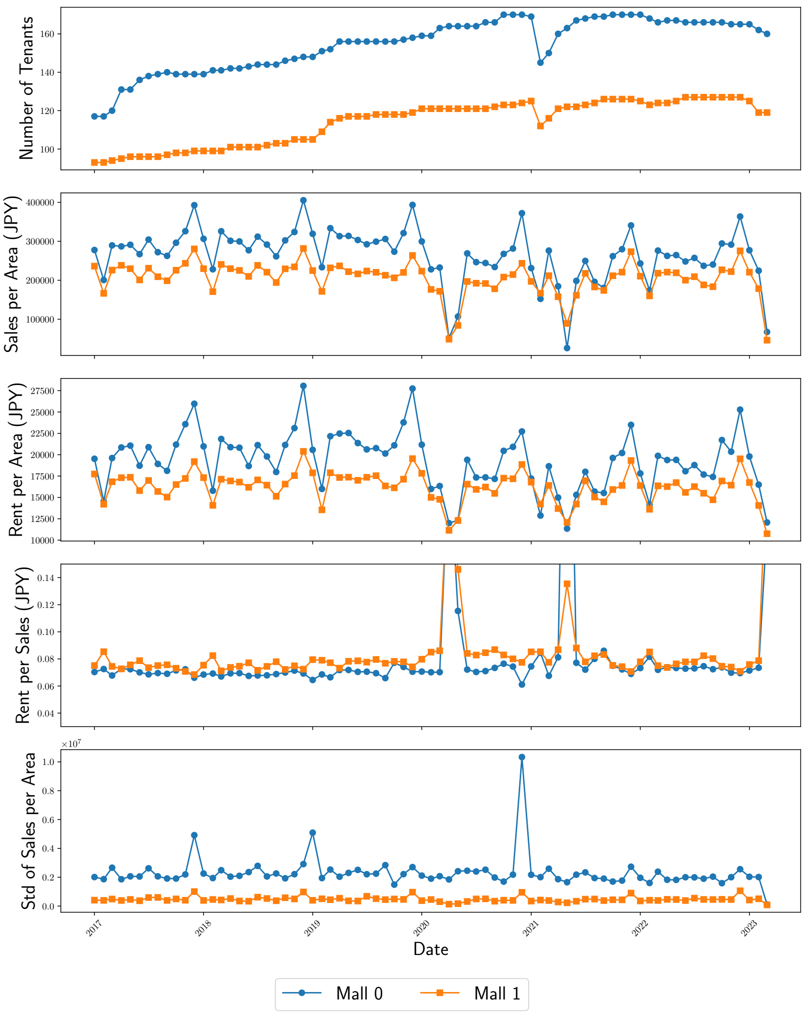


Figure 2. Metrics by Malls

Note: Trends in key performance indicators of Mall 0 and Mall 1 (2017–2023). The figures represent (a) the number of tenants, (b) sales per area (JPY), (c) rent per area (JPY), and (d) the rent-to-sales ratio across both shopping malls. Each metric highlights the changes in mall performance over time, with Mall 0 and Mall 1 being compared in each sub-figure.

larger than Mall 1. The second two panels in Figure 2 show that the sales per area and rent per area are both higher in Mall 0 compared to Mall 1: Mall 0 located in downtown consistently achieves higher sales throughout nearly all the months in our data period, resulting in higher total rents. However, the rent structures seem to differ between these two shopping malls. As illustrated in the fourth panel of Figure 2, the rent collected per unit of sales is higher in Mall 1 than in Mall 0. The last panel shows that the variance in sales per area among tenants remains relatively constant throughout the period, indicating that the uncertainty in sales arises primarily from its expected sales rather than from its variance.

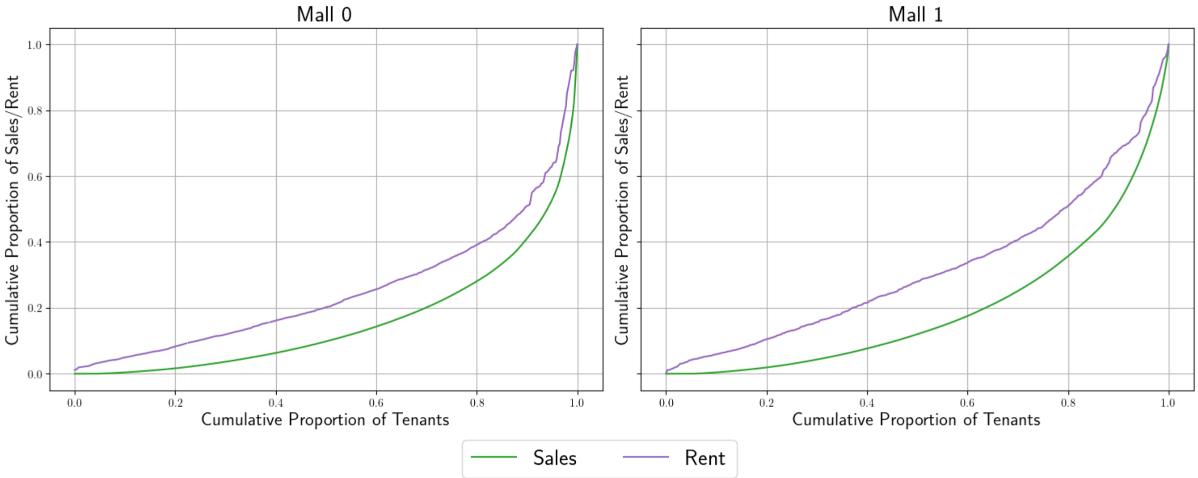


Figure 3. Lorentz Curves of Sales and Rents by Mall in 2023

Note: In both plots, the horizontal axis represents the cumulative proportion of tenant-month pairs, starting from those that yield the lowest sales (for the green line) or rent (for the purple line). The vertical axis indicates the cumulative proportion of total sales (green line) or total rent (purple line) within the shopping mall.

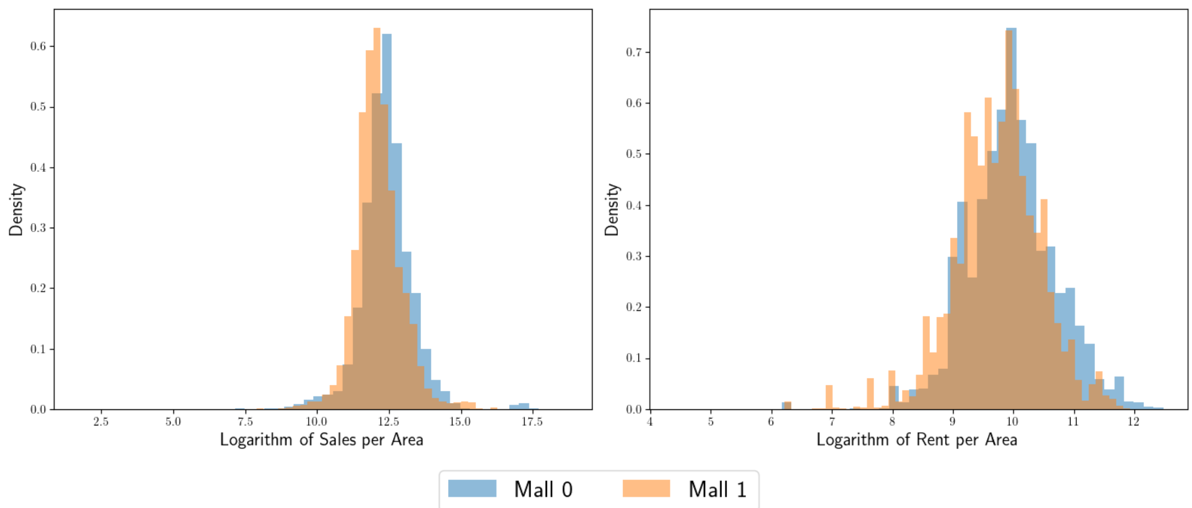
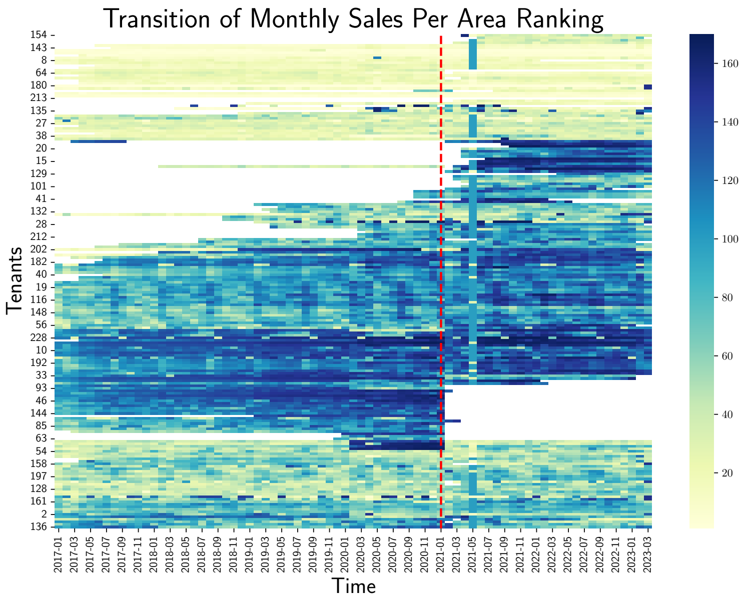
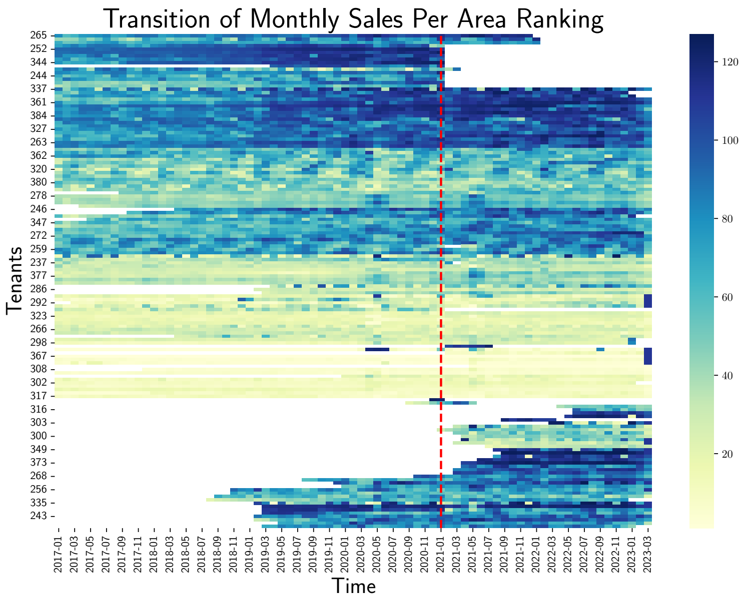


Figure 4. Histograms of Logarithm of Monthly Sales and Rent per Area by Mall

Note: In the left histograms, we omit samples that yield zero monthly sales, which constitute about 8% of all samples. For the histograms, we omit samples whose ratio of rent to sales exceeds 200%, accounting for about 6% of all samples.



(a) Mall 0



(b) Mall 1

Figure 5. Transition of Monthly Sales per Area Ranking by Malls

Note: Panel (a) shows the transitions of the sales per area ranks for each tenant in Mall 0, while Panel (b) displays the transitions for tenants in Mall 1. In these heat maps, each row represents a tenant, and each column represents a month from January 2017 to March 2023. Each cell indicates the rank of the tenant in the corresponding month. Lighter colors denote higher ranks. The tenants are ordered based on hierarchical clustering to group tenants with similar ranking patterns in adjacent rows.

Next, we examine the performance data at the tenant level. As previously mentioned, the tenants within the shopping mall vary widely in size. Figure 3 presents the Lorenz curves for sales and rent by tenant for the fiscal year 2023. It is evident that revenue from a few large tenants is significant for the shopping mall in terms of both sales and rent. This pattern holds consistent across different fiscal years.

Furthermore, the disparities in sales and rent are not merely due to differences in store area. The left and central panels of Figure 4 display histograms of the logarithm of monthly sales and monthly rent per unit area for each store. Clearly, both sales and rent per unit area vary significantly among stores. Hence, the substantial differences in store sales within a shopping mall emerge through two channels: differences in each store's potential and differences in their size.

All the above patterns are common in both shopping malls 0 and 1. And, as shown in the left panel of Figure 4, there is no significant difference in the distribution of tenants' sales potential between the two malls.

Anecdotally, when shopping malls assess the value of tenants, they tend to rely on sales per unit area rather than absolute sales or rent levels[13]. Naturally, tenants that can generate sales more efficiently are more valuable to the shopping mall, which is likely to give them a stronger position in negotiations.

This point can be observed in terms of the external margin: tenants with lower sales per unit area are more likely to be forced out of the shopping mall. Figure 5 illustrates the transition of the ranks of sales per area for each tenant within their respective shopping malls, where Panel (a) shows Mall 0 and Panel (b) shows Mall 1. In these heat maps, each row corresponds to a tenant and each column corresponds to a month. Each month, we rank all the tenants in a shopping mall by their sales per area. Darker colors represent lower-ranked tenants. An empty cell indicates that the tenant was not present during that month. It is clear that the higher-ranked tenants are more likely to continue their leases with the shopping mall. At the start of 2021 (red dotted line in the figures), both shopping malls decided to remove a set of tenants for an overall renewal, and at that time, the exiting tenants seem to have been chosen from those with relatively darker colors.

[13]Since the required floor space for tenants is somewhat constrained by their type of business, it is common during negotiations, including renewal negotiations, to refer to sales per unit area when discussing appropriate rent agreements.

	# Samples	Mean	Std	Min	25%	50%	75%	Max
Fixed per Area	442	17.40	13.59	0.50	9,06	14.92	20.00	91.53
Base per Area	442	205.04	143.02	10.65	140.00	180.00	200.00	1,440.00
Rate (%)	442	0.91	0.30	0.20	0.80	1.00	1.00	3.50

Table 1. Descriptive Stats of the Parameters of Mixed-type Contract

Note: "Fixed per Area" and "Base per Area" is scaled by 1,000 JPY.

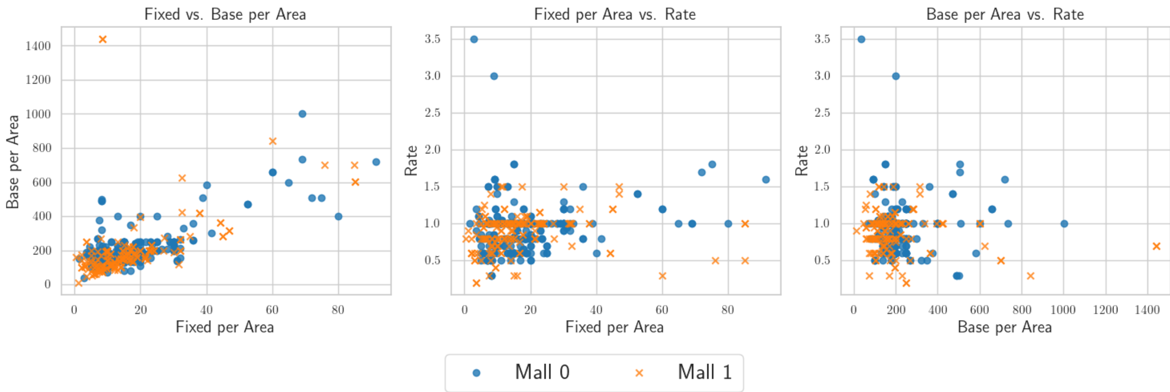


Figure 6. Scatter Plots of Parameters of Mixed-type Contract

Note: "Fixed per Area" and "Base per Area" is scaled by 1,000 JPY. "Rate" is percentile scale. Blue circle represents the sample in Mall 0 and orange cross represents the sample in Mall 1.

3.2 Contract data and meeting minutes data

Here, we examine the rent structures in static and dynamic perspectives. Our contract data allows us to observe how the terms of contracts have changed with each renewal for each tenant. To capture these dynamic changes, we introduce the concept of *contract number*. For each tenant, the initial contract period in our dataset is assigned a contract number of 0, which increases by 1 with each subsequent renewal[14]. Thus, our contract data is structured as a panel data, with tenants as the identifiers and the contract number serving as the time variable. The data includes parameters used to determine rent as well as the contract duration.

Static aspects We have a total of 619 contracts. As shown in Figure A.1, these contracts are qualitatively grouped based on the values of the parameters—Fixed, Bases, and Rates—into eight distinct groups, with the number of contracts in each group indicated at each leaf of the figure. We name the four major types of contracts: perfect-commission

[14]It is important to note that our data does not cover the opening periods of the shopping malls. Therefore, even if the contract number is 0, it does not necessarily imply that the contract is the initial agreement made when the tenant first entered the shopping mall.

Initial Offer \ Next Contract	**exit**	**same**	**up**	**down**	**updown**	**TOTAL**
exit	52	3	1	2	2	60
same	15	137	11	13	13	189
up	6	60	49	5	6	126
down	5	6	5	11	5	32
updown	1	1	3	5	9	19
TOTAL	79	207	69	36	35	426

Table 2. Initial Offer and Resulting Contract

Note: The table presents the relationship between the first initial offer (rows) and the resulting contract type (columns). Both the initial offer and the resulting contracts are categorized into five groups: exit, same, up, down, and updown. The numbers in the cells indicate the count of contracts corresponding to each combination of initial offer and resulting contract.

contract, fixed-rent contract, mixed-type contract, and dual-kinked contract. We focus on the most observed type of contract, the *mixed-type contract*: which has one kink and a positive fixed rent with $Rate_1 = 0$, composing 72% of all the contracts.[15] For this mixed-type contract, the average contract duration is 1,830 days, with a standard deviation of 1,189.13 days. The histogram of contract durations is shown in Figure A.2.

The rent structure varies within the mixed-type contract. Table 1 presents the descriptive statistics for the three parameters of the mixed-type contract: Fixed, Base, and Rate. For the first two, we consider their values per unit area. The scatter plots in Figure 6 illustrate the interrelationships among the three parameters. Fixed and Base show a positive correlation, which is expected since shopping malls aim to increase the likelihood of earning commission from tenants with a lower Fixed value[16]. In contrast, no clear relationship is observed between Rate and either Fixed or Base, even though they should be positively correlated based on the roles of Base and Rate, as discussed in Section 2. In Appendix B.1, we further verify that these variations cannot be fully attributed to fixed differences between the shopping malls or their tenants by focusing on brands that are common to both shopping malls.

Dynamic aspects We then describe the dynamic changes in the terms of mixed-type contract. The frequency of contract renewal negotiations varies among tenants. Ex-

[15] As explained in Section 2, perfect-commission contract and fixed-rent contract are considered as special treatment for special tenants. Dual-kinked contract is also adapted to the larger-sized tenants relative to the tenants with mixed-type contracts.

[16] This is also partly due to a common commercial practice: the Base is typically set to 10 times the Fixed amount at the initial offer.

cluding the negotiations in which the tenant is determined to exit, Figure A.3 presents histograms of the number of contract renewals for each mall. The average number of renewals is 1.00. Naturally, tenants with shorter contract durations tend to have a higher number of renewals.

To analyze each renewal negotiation, we introduce a method to represent qualitative changes from the ongoing contract terms. When a set of parameters of a mixed-type contract is given, by checking the change in the amount of rent, the relationship from the ongoing ones is classified into the following four groups: *same* refers to the case where all three parameters are the same; *up* indicates the case where the new mixed-type contract increases the expected rent—such as when the fixed increases, the base decreases, the rate increases, or combinations of these changes; *down* represents the opposite of the case of up; and *updown* refers to cases combining both upward and downward adjustments, such as when the rate increases while the fixed component decreases. Using this classification, Table 2 shows the results of categorizing the initial offers observed in the meeting minutes and the negotiation results, termed the *Next contracts*. Note that *exit* denotes an offer for the tenant to exit.

First, we find that, on average, shopping malls prefer to make some changes to the contract terms. In the initial offers, the proportion of "same" contracts, excluding exits, is 51.64%. We also see that exit offers are not very negotiable; 86.67% of exit offers actually result in an exit. Furthermore, among the negotiations that resulted in an exit, 65.82% were due to exit offers, and only 7.38% resulted in an exit among the negotiations starting with some category except exit. Offers other than exit are negotiable, with the proportion where the initial offer and the next contract belong to the same category being 56.28%. Particularly, in cases where the initial offer is "up," excluding cases that resulted in an exit, the next contract is "same" in 50% of the samples. Looking only at the next contract, among samples where the next contract is not exit, the proportion that settled on "same" is 59.65%, indicating that the majority remain persistent over time. Our data indicates that the involved parties intend some form of change in the contract terms whereas the terms actually appear persistent when comparing the current contract with the next one[17].

Based on the above categorization, we again find an evidence that the change in

[17] This persistence in contract terms was also observed in existing literature, such as in Lafontaine and Shaw (1999)

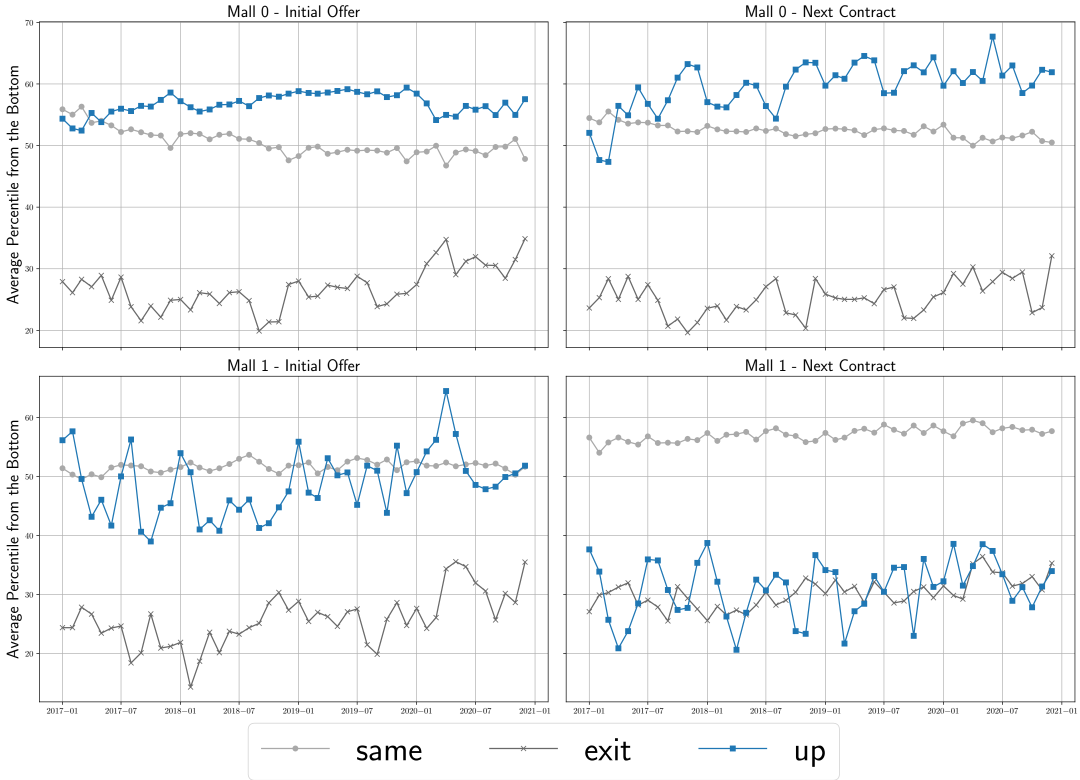


Figure 7. Average Monthly Rank of Sales per Area by Initial Offer and Next Contract

Note: The top plot shows the average sales per area ranking in Mall 0, while the bottom plot represents Mall 1. Each plot tracks the changes in average rankings over time, distinguishing between tenants' initial offer and their next contract (same, exit, or up). The vertical axis represents the average rank of sales per area, with lower values indicating higher ranks, and the horizontal axis covers the years 2017 to 2021, which is before the throughout renewal of both shopping malls.

contract terms reflect the past performance: which corresponds to the internal margin version of Figure 5. Figure 7 presents four panels depicting the monthly trajectories of tenants' average ranks of sales per area within the shopping mall, categorized by the type of initial offer and the next contract. The upper two panels correspond to Mall 0, while the lower two panels represent Mall 1. In the left panels, tenants are categorized by their initial offers in the renewal negotiation, and in the right panels, they are grouped by the type of the next contract. For Mall 1, tenants who received an up offer were ranked similarly to those whose initial offer was "same". However, when examining the next contract, the up offer was agreed only with tenants ranked similarly to those exiting the shopping mall who is obviously worse performers. At the same time, for Mall 0, we do not find the similar patterns.

Figure 7 suggests that changes in contracts reflect performance in some way, but from this alone, we cannot understand the paths and the reasons. It is natural to think that it affects bargaining strength as suggested in the professional interview, but that is not necessarily the case. This is because we are not capturing the benefits of contract changes due to changes in risk entailed with it. Even if the amount of rent increases, if the contract also significantly increases the variance of the rent, such changes may not be desirable for a risk-averse shopping mall. Furthermore, it is concerning that the patterns between Mall 0 and Mall 1 are clearly different. We cannot distinguish whether this is because each mall reacts differently in how they change contracts based on attributes like past sales, or because each mall has its own tendencies in the contracts they use—for example, changes that increase the amount may not be a significant issue in Mall 0. Reflecting these concerns, in our main analysis, we build a structural model and estimate it to clarify the determinants of the terms of contract and their changes.

3.3 Motivating observations

Before going onto the model discussion, we further review the several observations mainly about the dynamic changes of terms of contract.

First, we investigate the quantitative change made to the distribution of monthly rents. For this purpose, we calculate the changes in the average rent amount and its standard deviation between the ongoing contract and the subsequent contract, using the realized sales during the period of the ongoing contract. Figure 8 shows the distribution of these two changes by malls. Notably, there are no changes grouped in the upper left region of the figure, indicating that changes involving a lower rent amount and higher variance are never agreed upon. Both shopping malls generally face trade-offs between the rent amount and its variance: most changes in contract terms either involve higher rent with higher variance (the upper right region) or lower rent with lower variance (the lower left region). This figure also works to check if the risk, or the variance of the rent, matters to the involved parties. The observation directly suggests that the shopping malls and tenants are not risk-neutral agents and so we must rely on a type of utility function handling both the risk and the amount of rent to evaluate a contract.

Second, we consider what drives changes in contract terms. One potential factor is a change in the tenant's potential as a retailer. For example, a longer tenure in the same shopping mall may help a tenant build a loyal customer base, potentially increasing expected sales with each successive contract number. Consequently, the "appropriate" contract terms might also change. This aspect is examined in Figure A.4, which presents

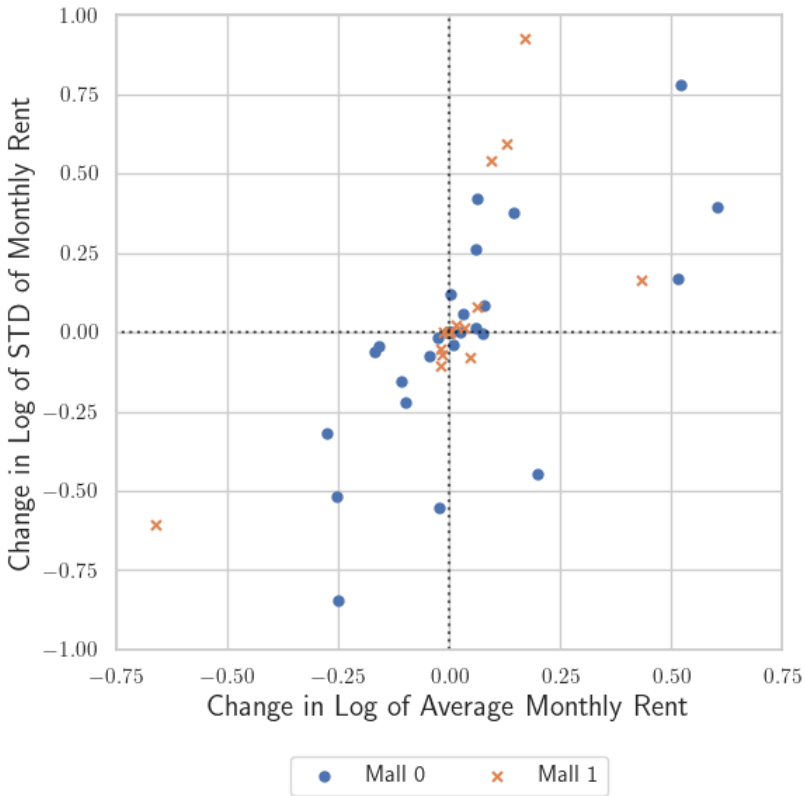


Figure 8. Change in Average and Std of Log of Monthly Rent

Note: The average and the standard deviation of the monthly rents is computed using the realized sales during the ongoing contract period for the ongoing contract and the next contract. The scatter plots show the difference of them by malls.

box plots of sales, rent, and their values per unit area for each contract number. The distribution of sales and rent does not show a clear trend as the contract number increases. In this sense, the fundamental aspects of a tenant, such as their sales potential, appear to remain consistent over time.

If a tenant's sales potential does not change significantly over time, what determines contract terms? To begin with, it is challenging to identify any clear descriptive patterns in the change in the terms of contract. Regarding changes over time, Figure A.5 presents box plots of the mixed-type contract parameters against the contract number. Here no obvious patterns emerge in the changes of the parameters themselves[18][19]. These

[18]Remember that, as shown in Figure 6, even among the parameters of a single mixed-type contract, the expected relationships are not visibly apparent.

[19]As shown in the lower middle panel of Figure A.5, the ratio of months in which sales exceed the base remains almost unchanged over time. This suggests the difficulty of accurately forecasting future sales. Unforeseen factors, such as shifts in commercial trends, play a dominant role, making it challenging for the shopping mall to learn about a tenant's sales potential and achieve uncertainty resolution during

	(1) Level	(2) Diff	(3) Diff / Change
Rate	−0.063	−0.049	−0.180
	(0.219)	(0.222)	(0.374)
Base	0.069	0.000	0.000
	(0.206)	(0.000)	(0.000)
Fixed	2.839**	0.000	0.000
	(1.319)	(0.000)	(0.000)
Observations	197	197	156
Adjusted R^2	0.992	0.964	0.967

Table 3. Effects on Performance

Note: This table presents the estimated effects of Rate, Base, and Fixed components on performance across three models: (1) Level, (2) Diff, and (3) Diff/Change. The coefficients show the impact of each variable, with standard errors in parentheses. The asterisks indicate levels of statistical significance: *p<0.1; **p<0.05; ***p<0.01.

observations suggest that we need to control the relevant situations in order to identify the determinants of the contract terms and their changes. At this point, we need to care about the presence of confounding factors, particularly the bargaining strength during negotiations which likely plays a significant role in these decisions. This is why we build a structural model to recover such elements from data.

Third, we check the existence of *moral hazard*. We check if the contract terms influence on the performance of the tenants. Our analysis focuses on tenants who have experienced at least one renewal in our dataset and have had mixed-type contracts in both successive leases. For these samples, we regress the average monthly sales during a lease period on the parameters of the mixed-type while including covariates such as past average sales, the total sales of the shopping mall, and the parameters of the previous contract.

Table 3 presents the regression results: Column (1) shows the result when the dependent variable is the level of average sales, Column (2) displays the result of the first difference version of the Column (1), and Column (3) shows the result when we focus on samples where the contracts differed between the two successive leases. The significant influence of Fixed observed in the first column disappears when differences are considered. This suggests the possibility of selection: higher Fixed value is set for tenants expected

renewal negotiations.

to achieve higher sales. In contrast, we do not observe any indication of moral hazard: such as higher rates leading to reduced effort from tenants. Furthermore, as discussed in Section 2, the shopping mall can directly monitor tenant efforts through daily consumer interactions and conducts mystery surveys to assess service quality, sharing the results with tenants. This oversight reduces the likelihood of moral hazard in this context[20].

Finally, Figure 9 illustrates the time trajectories of the counts of renewal negotiations and new tenant searches. The renewal contracts are clustered around specific periods in both shopping malls. Because both malls experienced overall renewals at the beginning of 2021, a peak in renewal negotiations occurrs in mid-2020. After 2021, since many of the new tenants have the similar lease terms, we observe the synchronization of the start of the renewal negotiations. In contrast, the counts of new tenant searches remain relatively steady over time. However, as mentioned in Section 2, the counts of these two different types of meetings are positively correlated, suggesting that new tenant searches are an endogenous process.

4 Model and solution

Here, we describe a two-stage model: in the first stage, the shopping mall decides which tenants to retain, and in the second stage, renewal negotiations are held with the continuing tenants regarding the new contract terms. The first stage involves a discrete choice by the mall, comparing the outside option with the expected utility of entering renewal negotiations. The main focus of this section is the second stage, where we propose a model for renewal negotiations along with the solution concept. Note that the model described here only considers decisions up to the mall manager level. Decisions made at the management company level are not included. As discussed in Section 2, the management company delegates authority to the mall manager.

We start by defining the monthly rent structure under mixed-type contract. We denote a monthly sales of a tenant by S and we write a rent structure as follows, $R(S;\theta)$ where $\theta \in \Theta$ represents the parameters and Θ is the parameter space of mixed-type

[20]While we do not find the evidence of moral hazard problem in our situation, our framework can be applied to the situation where the principal faces an agency problem when contracting. Several set of researches analyze the bargaining over contracts when there is an agency problem: for example, Pitchford (1998); Balkenborg (2001); Demougin and Helm (2006); Yao (2012); Li, Xiao and Yao (2013). If we describe the bargaining problem according to these models, it is possible to include moral hazard in our framework.

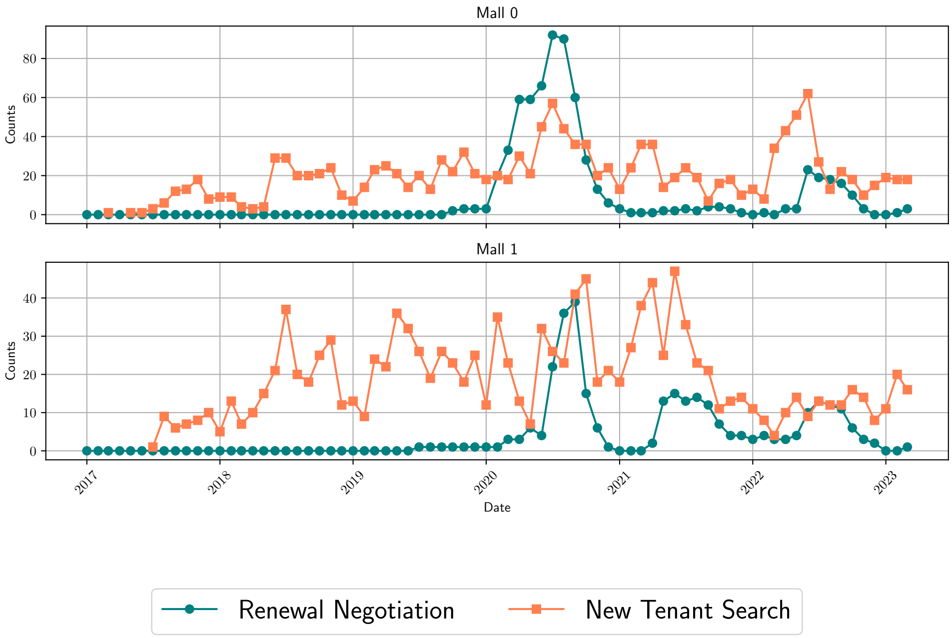


Figure 9. Counts of Renewal Negotiations and New tenant Searches over Time by Malls

Note: This figure shows the time series of the counts of renewal negotiations (green line) and new tenant searches (orange line) by Mall 0 and Mall 1 from 2017 to 2023. The top plot represents the counts for Mall 0, while the bottom plot shows the counts for Mall 1.

contract: $\theta = (f, b, r) \in \mathbb{R}_+^3$, f is Fixed, b is Base$_1$ and r is Rate$_2$ of a mixed-type contract. Given θ, the rent is computed as follows:

$$R(S; \theta) = f + r \max\{0, S - b\}.$$

Determining the rent for a retail space involves deciding how to divide the surplus generated by sales in that space between the shopping mall and the tenant. Since this surplus is not realized at the time of contract negotiation, both parties face risk and uncertainty, which must be addressed through an appropriate contract term. This is why they often resort to a share contract, rather than a simple fixed-price agreement[21]. Risk refers to the fluctuation in rent due to probabilistic variance when a distribution of sales is assumed. Uncertainty, on the other hand, arises in situations where the distribution of sales is unknown. Even in the case of renewal negotiation, where both parties have observed the sales history under the current agreement, future sales distributions may deviate from empirical ones due to unexpected factors such as macroeconomic shocks,

[21] See Lafontaine and Slade (2013) for a discussion on how risk and uncertainty motivate the use of share contracts.

changes in consumer trends, or tenant-specific circumstances.

Formally, let $\mathcal{G}$ represent the space of possible sales distributions, with g denoting a specific distribution within this space. Given a model g, we can define a bargaining problem aimed at risk-sharing and compute a bargaining solution as usual[22]. The challenge lies in that neither party agrees on which g is true. In general, under such uncertainty, one would consider bargaining under incomplete information and predict the bargaining outcome using a reasonable solution concept of a Bayesian manner[23]. However, these solutions require strong restrictions on the information structure and the way offers are made, making them difficult to apply to the current situation[24].

In this study, we consider another way of resolving uncertainty rather than the Bayesian manner. As a starting point, imagine a situation where the head office of the shopping mall, denoted by H, has exclusive power to set the contract terms with the tenant, denoted by T. We model the head office's decision-making under uncertainty as maximizing the expected utility of the worst case following Gilboa and Schmeidler (1989). Specifically, H solves the following maxmin problem:

$$\max_{\theta \in \Theta} \min_{g \in \mathcal{G}} \mathbb{E}_g \left[U_H(S; \theta) \right] - f\left(g, \mathcal{D}\right)$$

where $U_H(S;\theta)$ is the utility of H when the monthly sales are S under contract term θ. Let $\mathcal{D}$ represent the observed monthly sales data during the ongoing contract. The function $f(g, \mathcal{D})$ denotes the transaction cost associated with selecting g as the model for future sales, given that the head office has observed $\mathcal{D}$. For example, if model g deviates significantly from a "reference point," such as the average sales during the ongoing contract, substantial evidence would be required to justify adopting g. This process incurs considerable time and cost to gather the necessary information. Note that g is not considered a true model, in contrast to the Bayesian approach. Instead, g is selected as one

[22]The standard empirical modeling is demonstrated in Lee, Whinston and Yurukoglu (2021).

[23]In a non-cooperative game analysis, parties having a prior belief over the model space are expected to form a posterior belief based on signals or the behavior of the other party, such as the contract offer. The bargaining outcome is expected to constitute a kind of Bayesian equilibrium(Rubinstein, 1985; Deneckere and Liang, 2006; Loertscher and Marx, 2022). Ausubel, Cramton and Deneckere (2002) provide a comprehensive summary. In cooperative game analysis, the existing literature propse a set of axioms and the corresponding axiomatic solution as an expected outcome in the bargaining (Harsanyi and Selten, 1972; Myerson, 1984; de Clippel, Fanning and Rozen, 2022).

[24]Furthermore, from an empirical point of view, it is problematic that multiple equilibria usually remain when the span of offers is not diminished. Since bargaining between firms is typically time-consuming due to internal discussions within each firm, we have to introduce some arbitrary selection mechanism to pick an equilibrium to implement.

way to address uncertainty from $\mathcal{G}$.

We introduce two deviations from this maxmin problem. First, due to the large number of tenants, H must rely on negotiation representatives, employees working as field staff, to make contracts with tenants. For each tenant T, we denote the representative by M. We use $U_M(S;\theta)$ and $U_T(S;\theta)$ to represent their utility functions, similar to H's utility function. However, H does not fully delegate contract approval authority to M, because their preferences are only partially aligned. Specifically, H is concerned with the transaction cost of selecting a model g, denoted by $f(g,\mathcal{G})$, while M is not. Instead, H allows M to negotiate with T based on a predetermined model g. As we will see later, this division of authority is supported as the optimal institutional design in our model[25]. We denote the contract terms agreed upon between M and T, given a model g, as $\theta(g)$. This agreement requires bargaining between M and T, which we refer to as *bargaining for risk sharing*, and will be discussed in detail later. Given a model space $\mathcal{G}$ and $\theta(g)$, H still selects the model that represents the worst-case scenario when evaluating the situation. Therefore, H's value function is expressed as follows:

$$v_H\left(\mathcal{G}\right) \equiv \min_{g\in\mathcal{G}} \ \mathbb{E}_g\left[U_H(S;\theta(g))\right] - f\left(g,\mathcal{G}\right),$$

and the selected contract term in the bargaining for risk sharing is denoted by $\theta\left(\mathcal{G}\right)$.

As the second deviation, H does not have exclusive power over T in defining the problem. While H's behavioral model remains non-negotiable, the model space itself can be influenced by T. In other words, through negotiation, T attempts to convince H that the worst-case scenario is not as severe as initially thought, in order to achieve a better outcome. This process is referred to as *bargaining for uncertainty resolution.* Specifically, the bargaining problem between H and T concerns the subset $\mathcal{G}_0 \subset \mathcal{G}$, and the possible set of the pair of expected utilities is defined as follows:

$$\left\{\left(v_H(\mathcal{G}_0), \mathbb{E}\left[U_T\left(S;\theta\left(\mathcal{G}_0\right)\right)\right]\right) \mid \mathcal{G}_0 \subset \mathcal{G}\right\}. \tag{1}$$

In summary, a tenant faces two inter-related sequential bargaining problems during a renewal negotiation: risk sharing and uncertainty resolution. The goal of the bargaining for uncertainty resolution is to select a model space through negotiations with the shop-

[25]This also mirrors the observed pattern of negotiations: M must submit the current proposal to the head office for approval before returning to the negotiation table with the decision.

ping mall's head office. The goal of the bargaining for risk sharing is to determine the detailed contract terms through negotiations with the mall's representative based on the model picked by the head office as the worst case scenario.

In Subsection 4.1, given the above preferences, we describe the detailed of the bargaining problems in the renewal negotiation, including their bargaining frontiers and the solution concept to determine the new contract terms for the continuing tenant. In Section 4.2, given the problem for the above continuing tenant, we model how the head office of the shopping mall decides whether a tenant should continue to exist.

4.1 Renewal negotiation

In Section 4.1.1, we introduce a solution concept for the dual bargaining tables: Nash-in-Nash. In Section 4.1.2, we specify the bargaining problem in the second table between the tenant and the negotiation representative of the shopping mall. In Section 4.1.3, we specify the bargaining problem in the first table between the tenant and the head office of the shopping mall.

4.1.1 Nash-in-Nash solution

The renewal negotiation involves two bargaining problems. Both of these are considered complete-information bargainings, in the sense that the structure—including the bargaining set, the disagreement point, and the balance of bargaining powers—is common knowledge between the parties.

Given a model g, in the bargaining for risk sharing, T and M can compute the expected surplus split for every $\theta \in \Theta$. This constitutes the bargaining set for this negotiation. For any given disagreement point, we have a formal complete-information bargaining problem, denoted by $\mathcal{B}(g)$. $\theta(g)$, which is the agreed contract term given a model g, is chosen as a bargaining solution to $\mathcal{B}(g)$. Expecting this following contract $\theta(g)$, in the bargaining for the uncertainty resolution, H and T bargains over the subset of the model space.

Therefore, the two bargainings are not independent issues but are interrelated. To solve the interrelated bargaining problems, we use the *Nash-in-Nash solution*. This solution concept assumes that each bargaining achieves the Nash bargaining solution, given the outcomes of the other bargaining. In our model, we employ a type of Nash-in-Nash solution where the two bargaining processes are solved sequentially[26].

This model of dual bargaining tables is grounded in the optimal institutional design of the shopping mall. As we see later, at the bargaining for risk sharing, M and T tend to overestimate future expected sales which requires a lot of transaction costs. To prevent this higher transaction cost, the shopping mall strategically divides authority as follows: the risk-sharing decision is delegated to the negotiation representative, while decision regarding the model space is handled by the head office[27].

4.1.2 Bargaining for risk sharing

At this bargaining table, the two parties are the negotiation representatives of the shopping mall and the tenant. They share a common distribution of sales, g, which is picked by the head office. Given this distribution, their task is to negotiate over the space of contract terms, denoted by Θ, to allocate the risk between themselves.

Preference The tenant and the negotiation representative of the shopping mall are both risk-averse agents. Their utility functions are CRRA functions with different levels of risk aversion:

$$U_M(S;\theta) = -e^{-\rho_M(I+R(S;\theta))}$$

$$U_T(S;\theta) = -e^{-\rho_T(J+S-R(S;\theta))},$$

where ρ_M and ρ_T are the parameters of risk aversions, and I is the initial wealth level of M and J is the same one for the tenant. To evaluate a contract term, both of M and T takes the expectation of their utilities with respect to a distribution of the sales.

Bargaining frontier To characterize the bargaining solution, we first consider the bargaining set at this table. We define the bargaining set as a subset of contract terms,

[26] Although many empirical papers consider multiple bargaining processes to be solved simultaneously, Crawford and Yurukoglu (2012) adopts a sequential version of the Nash-in-Nash solution.

[27] This division of authority is related to the classical assumption of delegated bargaining agents which works as a microfoundation for Nash-in-Nash solution. Although Collard-Wexler, Gowrisankaran and Lee (2019) provide another microfoundation of the Nash-in-Nash solution, it requires shrinking the time between bargaining periods. This is not the case in bargaining between firms.

denoted by $\tilde{\Theta} \subset \Theta$, in the following way: given a set of disagreement points $(d_1, d_2) \in \mathbb{R}^2$,

$$BS\left(\tilde{\Theta}\right) \equiv \{(x, y) \in \mathbb{R}^2 \,| x \geq d_x, y \geq d_y, x = \mathbb{E}\left[U_M(S;\theta)\right] - q_x,$$
$$y = \mathbb{E}\left[U_T(S;\theta)\right] - q_y, (q_x, q_y) \in \mathbb{R}^2_+, \theta \in \Theta\}.$$

The corresponding bargaining frontier is denoted by $BF(\tilde{\Theta})$ and the function which expresses this frontier is denoted by $F(x; \tilde{\Theta})$.

For a tractable expression of the expected utilities for both the shopping mall and the tenants, we restrict the space of the distributions of sales: $\mathcal{G}$ is the set of Gaussian distributions[28]. We assume that sales follow a Gaussian distribution with parameters (μ, σ). At this bargaining table, both parties are aware of this distribution.

Assumption 1. $S \sim N(\mu, \sigma^2)$.

Even under Assumption 1, because the mixed-type contract involves terms that require maximization, directly characterizing the bargaining set is challenging. To derive the bargaining set induced by the mixed-type contract, we consider an auxiliary problem where the agents negotiate over affine contracts. Let $\Theta_r = \{(f, b, r) \in \Theta_o \mid b = 0\}$. Given a particular $\theta_r \in \Theta_r$, an affine rent structure is specified as $R(S; \theta_r) = f + rS$. We can then demonstrate that the bargaining set in this auxiliary problem is equivalent to the bargaining set in the original problem. At the same time, this result implies that the mixed-type contract is a redundant contract form; in other words, we can achieve the same pair of expected utilities using a corresponding contract with $b = 0$.

Lemma 1. *Under Assumption 1, $BS(\Theta) = BS(\Theta_r)$.*

Proof. See Appendix C. □

Hence, the bargaining frontier of this auxiliary problem is identical to that of the bargaining problem involving mixed-type contracts. The bargaining frontier is obtained by maximizing the tenant's expected utility, subject to the constraint that the shopping mall's expected utility remains above a certain threshold level. We can derive a tractable expression of the bargaining frontier, as stated in Lemma 2.

[28]This assumption is for tractability. It is possible to use another set of distributions by replacing the explicit form of the bargaining frontier obtained below by the numerical ones.

Lemma 2. *Let*

$$c_1 \equiv U_M\left(I + \frac{\mu^2}{2\sigma^2\rho_M} - \frac{1}{2\sigma^2\rho_M}\left(\frac{\rho_T+\rho_M}{\rho_T}\right)^2\left(\mu - \sigma^2\frac{\rho_T\rho_M}{\rho_T+\rho_M}\right)^2\right)$$

$$c_2 \equiv U_M\left(I + \frac{\mu^2}{2\sigma^2\rho_M} - \frac{1}{2\sigma^2\rho_M}\left(\mu - \sigma^2\frac{\rho_T\rho_M}{\rho_T+\rho_M}\right)^2\right)$$

Then, under Assumption 1, the Pareto frontier for the auxiary problem is obtained as follows:

$$F(u_M;\Theta_r) = \begin{cases} U_T\left(J + \frac{\mu^2}{2\sigma^2\rho_T}\right) & \text{if } u_M < c_1 \\ U_T\left(J + (1-r^\star(u_M))\mu - \frac{\sigma^2}{2}\rho_T(1-r^\star(u_M))^2\right) & \text{if } c_1 \le u_M < c_2 \\ U_T\left(J + I + \mu - \frac{\sigma^2}{2}\frac{\rho_T\rho_M}{\rho_T+\rho_M} + \frac{1}{\rho_M}\ln(-u_M)\right) & \text{otherwise} \end{cases}$$

where

$$r^\star(u_M) = \begin{cases} \frac{\mu - \sqrt{\mu^2 + 2\sigma^2\rho_M\left(I + \frac{1}{\rho_M}\ln(-u_M)\right)}}{\sigma^2\rho_M} & \text{if } \frac{\mu}{\sigma^2} > \frac{\rho_T\rho_M}{\rho_T+\rho_M} \\ \frac{\mu + \sqrt{\mu^2 + 2\sigma^2\rho_M\left(I + \frac{1}{\rho_M}\ln(-u_M)\right)}}{\sigma^2\rho_M} & \text{otherwise} \end{cases}$$

Proof. See Appendix C. □

We can compute the corresponding contract terms at every point on the frontier. The case where $c_1 \le u_M \le c_2$ corresponds to the scenario where $f = 0$; in other words, within this range, the contract term on the frontier is a perfect commission contract. This is illustrated in Figure 10, which shows two Pareto frontiers—one for perfect commission contracts and another for affine contracts under a specific set of parameter values. For the region where u_M is small, the affine contract is dominated by the perfect commission contract.

As noted in Section 2, the shopping mall prohibits the usage of the extreme type of contract as the commercial customery. Hence, in the following analysis, we extends the bargaining frontier of affine contracts to the region where affine contracts are dominated by a perfect commission contract. This extended function is the bargaining frontier of this table. Specifically, the function representing the bargaining frontier is written as

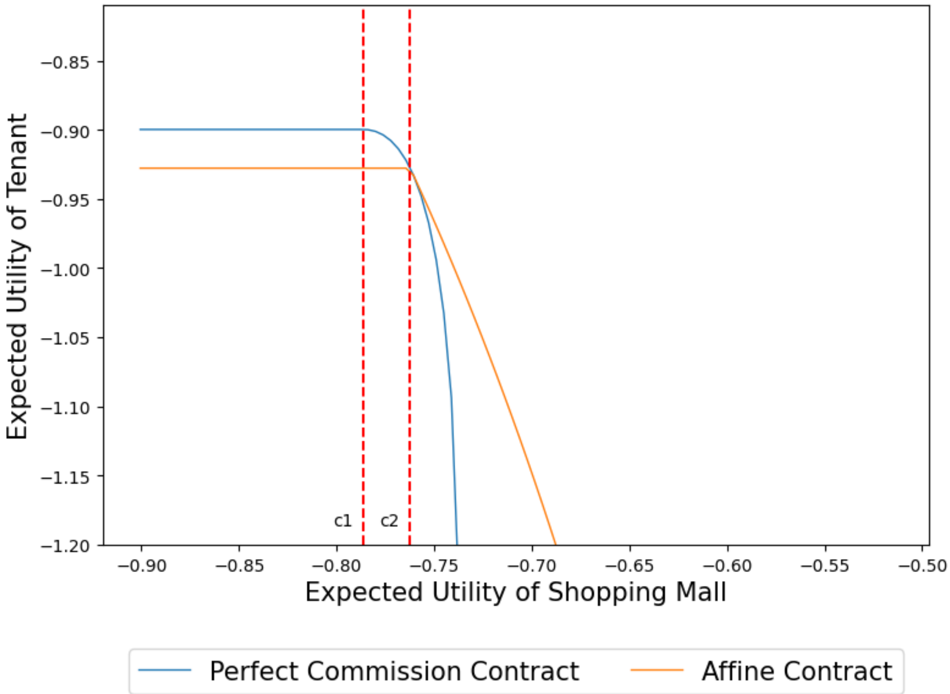


Figure 10. Pareto Frontiers for Perfect Commission Contracts and Affine Contracts

Note: Setting is as follows: $I = 1.0, \sigma^2 = 2.0, \mu = 1.5$. The risk aversions are set to $\rho_M = 0.2$ and $\rho_T = 0.5$.

follows:

$$F(u_M) = -e^{-\rho_T\left(I+J+\mu-\frac{\rho_T\rho_M}{\rho_T+\rho_M}\frac{\sigma^2}{2}\right)}(-u_M)^{-\frac{\rho_T}{\rho_M}}. \tag{2}$$

Nash bargaining solution We consider a scenario where the bargaining power is uneven and the breakup point is not set at $(0, 0)$. First, we introduce the concept of *bargaining power* for the two parties: $\beta_M > 0$ for the shopping mall and $\beta_T > 0$ for the tenant, which determines the form of the generalized Nash product. We denote the breakup points as c_M for the shopping mall and c_T for the tenants, and assume their values as specified in Assumption 2. For the tenants, since most of them have only one location in the area, it is natural to assume that the breakup point results in no gain for them. For the shopping mall, there may be externalities between the sales and the rents of tenants: if a tenant with strong consumer drawing power exits, the expected rents after their departure could decrease from the current total rent levels. To capture this effect, we multiply a constant $c > 0$ by the initial wealth I to determine the break up utility.

Assumption 2. $c_T = U_T(0)$ *and for the shopping mall, for some constant* c, $c_M =$

$U_M(cI)$.

It is well known that the generalized Nash bargaining solution is characterized as the point where the slope of the bargaining frontier equals the slope of the generalized Nash product. In our case, using (2), this condition is described in Proposition 1.

Proposition 1. *For any contract term $\theta \in \Theta$ obtained in the Nash bargaining solution when the bargaining powers are (β_M, β_T) and the utilities at the break-up point are (c_M, c_T), the following condition is satisfied*

$$\frac{\beta_M}{\beta_T} \frac{\mathbb{E}\left[U_T(S;\theta)\right] - c_T}{\mathbb{E}\left[U_T(S;\theta)\right]} = \frac{\rho_T}{\rho_M} \frac{\mathbb{E}\left[U_M(S;\theta)\right] - c_M}{\mathbb{E}\left[U_M(S;\theta)\right]}. \tag{3}$$

It is noteworthy that this model is incomplete in the sense that there are a set of equilibrium mixed-type contracts and we do not assume any selection mechanism among them. This is because the mixed-type contract is redundant, meaning there is a set of mixed-type contracts that can achieve this surplus split. As shown in Lemma 1, for any mixed-type contract, there always is a counter part in affine contract which is also included in the space of mixed-type contracts. Nonetheless, note that all contracts agreed upon at this table must satisfy the condition mentioned above and we observe at least one selected mixed-type contract in our data.

Lastly, for the ease of computation in the other bargaining, we set the ratio of the bargaining powers in this table to the inverse ratio of the risk aversions. This assumption coincides with the classical analysis on the relationship between the risk attitude and the bargaining power: the more risk averse agent has less bargaining strength in the bargaining (Sobel, 1981; Roth and Rothblum, 1982). From empirical point of view, as mentioned in Section 2, the bargaining over the contract is for risk sharing given one distribution of the sales and so it is natural that the risk aversions mainly drives the bargaining powers.

Assumption 3. $\frac{\beta_M}{\beta_T} = \frac{\rho_T}{\rho_M}$

Hereafter, we use $u_M(\mu)$ and $u_T(\mu)$ to denote the achieved equilibrium utility when μ is the model in the bargaining for risk sharing. By combining (2) and (3) under

Assumption 3, we have the explicit form of them: where $\varrho = \frac{\rho_T \rho_M}{\rho_T + \rho_M}$,

$$
\begin{aligned}
u_M(\mu) &= -e^{-\varrho\left((c+1)I - \varrho\frac{\sigma^2}{2}\right)} e^{-\varrho\mu} \\
u_T(\mu) &= -e^{-\rho_T\left(I - \varrho\frac{\sigma^2}{2}\right) + \frac{\rho_T^2}{\rho_T+\rho_M}\left((c+1)I - \varrho\frac{\sigma^2}{2}\right)} e^{-\varrho\mu}
\end{aligned} \tag{4}
$$

From (4), it is easy to check that both of expected utilities of M and T at the equilibrium increases with μ. This implies that the preferences of M and T are aligned in the sense that both of them prefer the higher μ when making a contract.

Corollary 1. $\frac{\partial}{\partial\mu} u_M(\mu) > 0$ *and* $\frac{\partial}{\partial\mu} u_T(\mu) > 0$.

4.1.3 Bargaining for uncertainty resolution

At this bargaining table, the two parties negotiate the space of the distribution of sales. Under Assumption 1, the distribution is characterized by the mean, μ, and the variance, σ^2. As shown in Figure 2, the variance remains stable across both firms, allowing us to treat σ^2 as a known quantity, estimated from sales data during the previous lease period. Therefore, the bargaining process focuses on the space of the expected sales, μ. In other words, the total model space is $\mathcal{M} \equiv \mathbb{R}_{++}$ and the subset $\mathcal{M}_0 \subset \mathcal{M}$ is the model space held by H, which is bargained between T.

Preference Since H and M belong to the same company and share the profits, H essentially derives the same utility from the rent as M: $U_H(S;\theta) = U_M(S;\theta)$. We model the transaction cost as the personnel cost required to gather the necessary information to accept μ, given that $\mathcal{D}$ has been observed. Let $N_e(\mu, \mathcal{D})$ denote the amount of necessary evidence: the number of evidence collected following a Poisson distribution with parameter ϑ. Using w to represent the personnel cost per unit time, the expected transaction cost is given by:

$$
f(\mu, \mathcal{D}) = w \times \frac{N_e(\mu, \mathcal{D})}{\vartheta}.
$$

We assume that the required amount of information increases quadratically with the deviation of the equilibrium utility from the utility at a reference point, defined as the utility level under the observed average sales, denoted by $\hat{\mu}$, and the ongoing contract term, denoted by $\hat{\theta}$. For some $h > 0$, this can be expressed as:

$$N_e(\mu, \mathcal{D}) \equiv h \times \Big(u_M(\mu) - U_M(\hat{\mu}; \hat{\theta})\Big)^2 .$$

For the later use, we define a composite of the parameters by $\lambda \equiv \frac{wh}{\vartheta}$. Furthermore, we denote the expected utility of H given one μ by $u_H(\mu) \equiv u_M(\mu) - f(\mu, \mathcal{D})$. We assume that H faces the sufficiently high cost of gathering the evidence, which makes H prefer smaller μ.

Assumption 4. *$\lambda > 1$*

Lemma 3. *Under Assumption 4, $\frac{\partial}{\partial \mu} u_H(\mu) < 0$.*

Bargaining frontier To specify the bargaining frontier of this problem, we first consider the value function of H. Let $\mathcal{M}_0$ be the model space and $\bar{\mu} = \sup \mathcal{M}_0$. Then, by Lemma 3, the value function of H is attained when $\mu = \bar{\mu}$:

$$v_H(\mathcal{M}_0) = \min_{\mu \in \mathcal{M}_0} \; u_H(\mu) = u_H(\bar{\mu}).$$

When we use the same break-up point in the bargaining for risk sharing, based on the general expression (1), the bargaining set of this bargaining problem is specified as follows[29]:

$$\left\{ (u_H(\bar{\mu}), u_T(\bar{\mu})) \mid \bar{\mu} \in \mathcal{M}, \; u_H(\bar{\mu}) \geq c_M, \; u_T(\bar{\mu}) \geq c_T \right\}.$$

This set also constitutes the bargaining frontier, which is denoted by $F^U(u_H)$. It is easy to show that the bargaining frontier is concave.

Lemma 4. *$F^U(u_H)$ is a concave function.*

According to Corollary 1 and Lemma 3, H and T face a conflict of interest on this bargaining frontier: H prefers the lower $\bar{\mu}$ whereas T prefers the higher $\bar{\mu}$. In words, the situation is that T is presenting its sales potential to convince H, who believes that there

[29]For the utility level at the break-up point, we assume that H does not feel the transaction cost for the case of disagreement. This is because we do not specify the reason of the break up and so we do not specify the transaction cost accompanied with the result. Instead, we assume that there is no transaction cost for this case. This assumption is the most preferred case to the shopping mall side. It is easy to change this assumption in the latter formulation.

is no benefit in raising the projected sales if it requires the effort of gathering additional information, to adopt a higher sales projection.

Nash bargaining solution As shown in Lemma 4, the bargaining set is concave. Then, as same in the case of the bargaining for risk sharing, we can characterize the Nash bargaining solution. Let B^U be the balance of the bargaining powers: $B^U \equiv \frac{\beta_H^U}{\beta_T^U}$.

Proposition 2. *At the Nash bargaining solution,*

$$B^U = -\frac{1 - \lambda\left(u_M + c_M - 2\hat{u}_M\right)}{1 - 2\lambda\left(u_M - \hat{u}_M\right)} \frac{u_T}{c_T} \frac{c_M}{u_M},$$

where $\hat{u}_M = U_M\left(\hat{\mu}; \hat{\theta}\right)$.

Proof. See Appendix C. □

Furthermore, under Assumption 3, we can compute the achieved expected utility of the shopping mall. The right hand side of Corollary 2 is defined as a function $u_M(B^U)$.

Corollary 2. *Under Assumption 3,*

$$u_M = \frac{\left(1 + 2\lambda\tilde{u}_M\right)\left(1 + B^U\right) - \lambda c_M}{\lambda\left(1 + 2B^U\right)}.$$

Proof. See Appendix C. □

4.1.4 Optimal institutional design

From Corollary 1, it is obvious that both of the shopping mall and the tenants can increase the expected utilities as the accepted $\bar{\mu}$ increases. Hence, if M has the authority to define the model space $\mathcal{M}_0$, there is no reason to cap the supremum of the expected sales in the future. This would decrease the expected utility of H. To avoid this over optimistic view on the future sales and the following over optimistic contracts, the shopping mall management company in advance divides the authority to the head office and the representative: the head office controls the model space to regulate the optimism in the bargaining for risk sharing.

4.2 Continuing decision

As observed in the data, the exit decision is typically not a subject of negotiation. The shopping mall's head office decides whether a tenant should continue by comparing the expected utility with the value of the outside option available at that time.

Let ψ denote the monetary value of the outside option for the targeted retail space, which follows a distribution. The game proceeds as follows: First, ψ are realized, and then, before B^U realizes, the shopping mall determines the expected utility that would be obtained in the renewal negotiation, as described in Lemma **??**. If the expected utility is smaller than $U_M(cI + \psi)$, the head office offers the tenant an exit offer. Otherwise, both parties proceed to the renewal negotiation stage, and B^U realizes. After this realization of the bargaining powers, the negotiation is resolved as outlined in Section 4.1.

In the above game, the shopping mall must decide the exit before knowing B^U. This corresponds to the case where ε^u works as the unknown information held by the tenant that will change the bargaining strength of the two parties. This interpretation allows us to understand why the negotiation is time-consuming: through negotiation ε^u gradually realizes and after its realization, the bargaining is immediately resolved. This is why we adopt this information structure on the continuation decision of the shopping mall[30].

In our empirical analysis, we account for the shopping mall's search behavior, which may be correlated with unobserved factors influencing the value of the outside option. However, we do not explicitly model the shopping mall's search behavior. Instead, we address the endogeneity issue using a control function approach, as explained in Section 5.1.

5 Empirical strategy

The target of our estimation are categorized into four sets: (1) the parameters built into the model, e.g. risk aversions of the shopping mall and the tenants, (2) the parameters which describes the bargaining powers, (3) the parameters which determines the value

[30]The case is similar to studies of auctions with entry, where we have multiple options for the information structure. In one model, the potential entrant decides whether to enter or not with information about their private type in the following auction stage (Levin and Smith, 1994; Krasnokutskaya and Seim, 2011). In another model, they have to enter without knowing the specific value of their types (Samuelson, 1985). In yet another, they only observe signals of their types (Marmer, Shneyerov and Xu, 2013). The information structure itself is generally an empirical question, and even in our case of bargaining, further study is required.

of outside options, and (4) the distribution of unobserved disturbances. In Section 5.1, we introduce the definition of variables and the parameterized model. In Section 5.2, we describe how we estimate the parameters.

5.1 Parametrization

We use i to denote a tenant and k to denote a shopping mall. An ongoing contract between tenant i and mall k is indexed by contract number τ, starting from 1. Each contract has a duration in months, denoted by $T_{ik\tau}$. The monthly performance of tenant i in mall k during month t is represented by $\tilde{X}_{ikt}$ for all $t \in 1, \ldots, T_{ik\tau}$. For an ongoing contract, we measure performance as the average monthly performance, denoted by $X_{ik\tau} \equiv \frac{1}{T_{ik\tau}} \sum_{t=1}^{T_{ik\tau}} \tilde{X}_{ikt}$. Additionally, we calculate the monthly average of basic characteristics of the shopping mall and its tenants, such as the tenant's area and the total number of tenants in the mall, for use in our estimation.

The variables, apart from the covariates, are defined as follows: for the initial wealth of shopping mall k at time τ, we use the average of the total monthly rents collected in the mall. $\mu_{ik\tau}$ denotes the expected sales agreed upon by tenant i and mall k at time τ. $\sigma^2_{ik\tau}$ represents the variance of sales for tenant i in mall k at time τ. As mentioned, the variance of sales is stable over time, so $\sigma^2_{ik\tau}$ is set to the estimated variance from past sales data, $\hat{\sigma}^2_{ik\tau}$. The observed contract term is also indexed by (i, k, τ).

Built-in parameters We use ρ^k to denote the risk preference parameter of shopping mall k. For tenants, $\rho_{ik\tau}$ represents the risk preference parameter of tenant i in shopping mall k at time τ. Additionally, c_k denotes the coefficient associated with the initial wealth when calculating the wealth at the disagreement point.

Parameters for bargaining powers Using the performance measure, we parametrize the logarithm of the ratio of the bargaining powers in the bargaining for uncertainty resolution:

$$\ln B_{ik\tau} = \mathbf{X}'_{ik\tau}\gamma + \varepsilon^u_{ik\tau},$$

where $\mathbf{X}_{ik\tau}$ is the vector of the covariates and $\varepsilon^u_{ik\tau}$ is the term of unobserved balance of bargaining powers. In our analysis, there are three covariates specific to the tenant: *tenant* *area*, *previous sales per unit area*, and *previous sales ranking per unit area.* Additionally, there are four variables related to the shopping mall: *previous total mall sales*, *total number of tenants*, *total number of customers*, and *monthly new tenant searches.*

Parameters for value of outside options For each contract, we denote the value of the outside option for the shopping mall by $\psi_{ik\tau}$. This value should captures the market demand for the space: In the local tenant leasing market, any vacant or occupied retail space is sought after by potential service providers. In our case, each retail space in a shopping mall is also in demand by potential tenants, and this demand essentially constitutes the value of the outside option. Hence, we consider there are three components to determine the value of $\psi_{ik\tau}$: public information regarding the retail space, search behavior conducted by the shopping mall, and a random disturbance that captures the market's unobserved demand.

The public information includes the total sales of the shopping mall, the total number of customers, the total number of tenants, and the size of the space—all of which are monthly average values observed during the contract period. These are represented by $\mathbf{Z}_{ik\tau}$. As a direct measure of search behavior, we use the number of meetings held to search for new tenants. Since we cannot observe which meeting corresponds to which retail space, we also consider the average number of monthly meetings for new tenant searches throughout the contract period. The logarithm of this average is denoted by $d_{ik\tau}$. We consider the following linear model for the monetary value of the outside option:

$$\psi_{ik\tau} = \mathbf{Z}'_{ik\tau}\gamma_o + \beta_o d_{ik\tau} + \tilde{\varepsilon}^o_{ik\tau}.$$

We have an endogeneity concern with $d_{ik\tau}$, in the sense that knowledge of a better outside option might reduce the incentive to engage in search behavior. To address this issue, we adopt the control function approach (Petrin and Train, 2010; Wooldridge, 2015). We model search behavior as follows:

$$d_{ik\tau} = \mathbf{Z}'_{ik\tau}\tilde{\gamma}o + \mathbf{W}'_{ik\tau}\delta + \nu^o ik\tau, \tag{5}$$

where $\mathbf{W}_{ik\tau}$ is a vector of instrumental variables that influence the intensity of search behavior but do not directly affect the market demand for retail space. We propose using realized sales and the number of customers in the retail space as such instruments. The rationale for this choice is that this information is confidential between the shopping mall and the tenant, and is not accessible to potential tenants. We model the unobserved market demand, $\tilde{\varepsilon}^o$, as follows:

$$\tilde{\varepsilon}^o = \kappa \nu^o + \varepsilon^o,$$

where κ captures the correlation between search behavior and unobserved market demand, and ε^o is a mean-zero random disturbance.

We first regress (5) to obtain estimates of ν^o. Then, in the subsequent structural estimation, we include the estimated $\hat{\nu}^o$ as a covariate in the model for the monetary value of the outside option, i.e.,

$$\psi_{ik\tau} = \mathbf{Z}'_{ik\tau}\gamma_o + \beta_o d_{ik\tau} + \kappa \hat{\nu}^o_{ik\tau} + \varepsilon^o_{ik\tau}.$$

This ensures that the variation in d, given the control variables and $\hat{\nu}^o$, arises from the exogenous variables $\mathbf{W}_{ik\tau}$, allowing us to identify βo.

Parameters for disturbances In order to account for the selection of continuing contracts, we introduce a distribution for the unobserved disturbances in the balance of bargaining powers and the value of the outside option. For each contract, there are two random terms: $(\varepsilon^u, \varepsilon^o)$, where ε^u represents the random element in bargaining power, and ε^o represents the disturbance in the value of the outside option. We assume that these terms follow a bivariate Gaussian distribution: $(\varepsilon^u, \varepsilon^o) \sim N(0, \Sigma)$.

5.2 Estimation and identification

For concise notation, we drop the notation of i, k, τ. We characterize each contract by a tuple $\mathcal{D} \equiv (\chi, (f, b, r), \mathbf{X}, \mathbf{Z}, I)$, where $\chi \in \{0, 1\}$ is a dummy variable indicating continuation. When $\chi = 0$, the components (f, b, r) do not exist, and we denote this null contract by ϕ. For a given shopping mall k, the full set of parameters is represented by $\xi = (\gamma, \gamma_o, \beta_o, \kappa, \Sigma, c_k, \rho_k, \{\rho_{ik\tau}\}_{i\tau})$.

First of all, we define a function computing the surplus split of the shopping mall given one balance of bargaining powers:

$$u_M\left(\mathbf{X},\varepsilon^u\right)=u_M(e^{\mathbf{X}'\gamma+\varepsilon^u}),$$

where $u_M(B^U)$ is defined as the right hand side of Lemma **??**,

The likelihood for a tenant which exits after at least one period of contract is computed as follows:

$$L\left(\left(\chi=0,\phi\right),\mathbf{X},\mathbf{Z},I;\xi\right)=1-Pr\left(\mathbb{E}\left[u_M(\mathbf{X},\varepsilon^u)\mid\varepsilon^o\right]>-e^{-\rho_k(cI+\mathbf{Z}'\gamma_o+\beta_o d+\kappa\hat{\nu}^o+\varepsilon^o)}\right),$$

where the probability is taken with ε^o and the expectation is taken with respect to ε^u. The expected payoff of the renewal contract is computed using the conditional distribution: $\varepsilon^u\mid\varepsilon^o\sim N\left(\rho\sigma_u\frac{\varepsilon^o}{\sigma_o},\ \sigma_u^2(1-\rho^2)\right)$,

$$\mathbb{E}\left[u_M(\mathbf{X},\varepsilon^u)\mid\varepsilon^o\right]=\int u_M(\mathbf{X},\varepsilon^u)\frac{1}{\sqrt{2\pi}\sqrt{\sigma_u^2(1-\rho^2)}}e^{-\frac{\left(\varepsilon^u-\rho\sigma_u\frac{\varepsilon^o}{\sigma_o}\right)}{2\sigma_u^2(1-\rho^2)}}\mathrm{d}\varepsilon^u.$$

This value is approximated by simulated draw of conditional ε^u. ε_l^u is the simulated sample of ε^u, which is drawn from $N\left(\rho\sigma_u\frac{\varepsilon^o}{\sigma_o},\ \sigma_u^2(1-\rho^2)\right)$. Let L be the total number of simulated draws.

$$\frac{1}{L}\sum_l u_M(\mathbf{X},\varepsilon_l^u).$$

Let $p(\mathbf{X},\mathbf{Z},I;\xi)$ be the probability of continuation seen above. Then the frequency estimator for this probability is as follows

$$\hat{p}(\mathbf{X},\mathbf{Z},I;\xi)=\frac{1}{L}\sum_l\mathbf{1}\left\{\mathbb{E}\left[u_M(\mathbf{X},\varepsilon^u)\mid\varepsilon_l^o\right]>-e^{-\rho_k\left(cI+\mathbf{Z}'\gamma_o+\beta_o d+\kappa\hat{\nu}^o+\varepsilon_l^o\right)}\right\}.$$

Then, the likelihood of the exiting tenant is

$$L\left(\left(\chi=0,\phi\right),\mathbf{X},\mathbf{Z},I;\xi\right)=1-\hat{p}(\mathbf{X},\mathbf{Z},I;\xi).$$

For the likelihood of a continuing tenant, we use the observed contract terms. From Proposition 2, we can calculate the value of the balance of bargaining power for each continuing tenant. This allows us to infer the unobserved component of the balance of bargaining power, ensuring that the observed contract terms represent an equilibrium.

$$\hat{\varepsilon}^u \equiv \ln\left(-\frac{1-\lambda\left(u_M+c_M-2\tilde{u}_M\right)}{1-2\lambda\left(u_M-\tilde{u}_M\right)}\frac{u_T}{c_T}\frac{c_M}{u_M}\right)-\mathbf{X}'\gamma,$$

where u_M is the directly computed expected utility of the shopping mall:

$$u_M=\frac{1}{L}\sum_l -e^{-\rho_k(I+R(S_l;\theta)))}.$$

by the observed terms of contract, θ, where S_l is the simulated draw of sales from $N\left(\mu_{ik\tau},\hat{\sigma}^2_{ik\tau}\right)$. And u_T is computed by the function of the bargaining frontier (2).

The likelihood for a continuing tenant is

$$L\left((\chi=1,(f,b,r)),\mathbf{X},\mathbf{Z},I;\xi\right)=\int_{\mathbb{E}[u_M(\mathbf{X},\varepsilon^u)|\varepsilon^o]>-e^{-\rho_k\left(cI+\mathbf{Z}'\gamma_o+\beta_o d+\kappa\hat{\nu}^o+\varepsilon^o\right)}} f\left(\hat{\varepsilon}^u\mid\varepsilon^o\right)\times f(\varepsilon^o)\mathrm{d}\varepsilon^o.$$

The simulated value of this is

$$\frac{1}{L}\sum_l \mathbf{1}\left\{\mathbb{E}\left[u_M(\mathbf{X},\varepsilon^u)\mid\varepsilon^o_l\right]>-e^{-\rho_k\left(cI+\mathbf{Z}'\gamma_o+\beta_o d+\kappa\hat{\nu}^o+\varepsilon^o_l\right)}\right\}\frac{1}{\sqrt{2\pi}\sqrt{\sigma_u^2(1-\rho^2)}}e^{-\frac{\left(\hat{\varepsilon}^u-\rho\sigma_u\frac{\varepsilon^o_l}{\sigma_o}\right)^2}{2\sigma_u^2(1-\rho^2)}}.$$

The log likelihood function of all the observations is constructed as follows:

$$LL(\xi)=\sum_{(i,k,\tau)}\ln L\left(\mathcal{D}_{ik\tau};\xi\right).$$

We maximize this function under the constraint that all the equilibrium surplus split is the Nash bargaining solution in the bargaining for risk sharing. Our estimator is one example of MPEC introduced by Su and Judd (2012). We try 500 estimations with different initial points which are sampled from the feasible regions and pick the estimate which attains the highest likelihood.

Identification The identification argument can be decomposed into two parts: one concerning continuing tenants and the other concerning exiting tenants.

For the continuing tenants, as shown in Proposition 2, we can compute the balance of bargaining powers using the model's built-in parameters and the observed contract terms. Once the necessary parameters for this computation are identified, the coefficients associated with the covariates explaining the balance of bargaining powers can be identified by regressing the recovered balances on those covariates. Therefore, the key task is to identify the parameters required for computing the balance of bargaining powers. Specifically, given one shopping mall k, for each pair (i, τ) which specifies a continuing tenant, we need to know $\rho_{ik\tau}, \rho_k, c_k, \lambda_k$ as parameters and one endogenous object $\mu_{ik\tau}$ to compute the balance of bargaining powers.

Now we have two restrictions per such continuing tenant: (i) the realized surplus split (u_M, u_T) must be on the frontier specified by (2), and (ii) the realized surplus split (u_M, u_T) must be the Nash bargaining solution in the table for risk sharing, i.e., it must satisfy (3) under Assumption 3. When we denote the total number of continuing tenants by N_C, the total number of parameters, which includes the endogenous object $\mu_{ik\tau}$, is $2N_C + 3$, and the total number of restrictions is $2N_C$. Hence, without any assumption, we cannot identify all the parameters.

We assume that tenants' risk aversion is constant over time; that is, $\rho_{ik\tau} = \rho_{ik}$. If we have at least three continuing tenants each observed in two or more periods, the number of parameters does not exceed the number of restrictions. Therefore, using the Nash bargaining solution conditions in the risk-sharing framework, we can recover the corresponding balances of bargaining power in the uncertainty resolution context.

We cannot use the contract terms for the exiting tenants. Therefore, we need to impose some structure on their parameters. In our current estimation, we assume that exiting tenants who have never continued in the shopping mall share the same risk aversion. Under this assumption, we can recover the risk aversion for these exiting tenants. Assumption 5 summarizes the necessary assumptions for identification results in our model.

Assumption 5. *For continuing tenants, $\rho_{ik\tau} = \rho_{ik}$, $\forall\ i, k$ and, for exiting tenants which have never experienced continuation, $\rho_{ik\tau} = \rho_k^{exit}$ for some value of ρ_k^{exit}.*

Given all the identification results, we can determine the expected surplus obtained when starting a renewal negotiation for every tenant. Using this and the observed continuation decisions, we can infer the value of the outside option and the coefficients associated with the explanatory covariates.

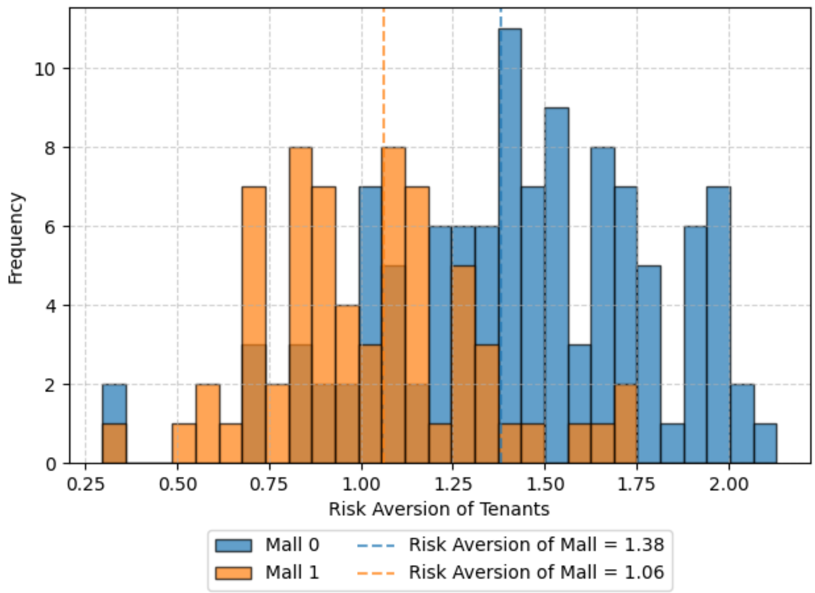


Figure 11. Histogram of Risk Aversions of Tenants

Note: Along with the histograms of the estimated risk aversions of tenants, the dotted lines represent the estimated risk aversions of the two malls. The current value of risk aversion is obtained when we scale the rent by 1 billion JPY. This scale is chosen to avoid the overflow when we compute the exponential terms in the model.

6 Results

Here, we review the estimation results. In Section 6.1, we present the estimated parameter values along with the recovered endogeneous objects. This serves as a kind of model validation check. We examine how the results align with the fact that Mall 0 is located in a more urban area than Mall 1 and is more attractive to both tenants and customers. Additionally, based on the estimation results, we clarify how the balance of bargaining power changes over time. Our findings suggest that assuming bargaining power is constant over time—like a fixed effect—is insufficient for controlling bargaining power when analyzing the detail terms of contract. In Section **??**, based on the estimation results, we conduct simulation analysis to examine how the terms of contract are chosen and changed over time.

6.1 Estimation results

We begin by examining the estimates of risk aversion. Figure 11 presents histograms of the estimated risk aversion for each tenant. The dotted line represents the risk aversion of the shopping malls. The ranges of the estimated risk aversions are almost overlapping, which is expected since the tenants are not significantly different across malls. The risk aversion levels of the shopping malls fall within these ranges, being nearly equal to the medians of the risk aversions of the tenants within each shopping mall. The two shopping malls are estimated to have different levels of risk aversion, which aligns with the fact that they are managed by different managers.

Our model allows us to assess the balance of bargaining power in negotiations for uncertainty resolution. This balance determines how the surplus is split between the shopping mall and the tenants. By comparing the distribution of these estimates for each shopping mall, we can evaluate their relative strengths in renewal negotiations. Figure A.7 presents histograms of the logarithm of the balance of bargaining power for each tenant and the shopping mall to which they belong. Mall 0 exhibits a higher proportion of larger values compared to Mall 1, indicating that Mall 0 tends to hold a relatively stronger position in relation to its tenants. This tendency aligns with the location differences between the two malls: Mall 0 is situated in the downtown area, which has greater customer-drawing power, while Mall 1 is located in the suburbs, which appears to have less market influence than Mall 0. This disparity in potential may be reflected in the balance of bargaining power.

Figure 12 shows the scatter plot between the observed average monthly sales of each contract and the estimated upper bound of the sales mean, which is considered for the next contract[31]. The latter value is an endogenous variable that is subject to negotiation during the bargaining process for uncertainty resolution. In most cases, the upper bound is set higher than the observed averages. Notably, the estimated upper bound does not increase as the past average of sales rises. When regressing the estimated upper bound on the observed average, the coefficients are estimated to be -0.20 for Mall 0 and -3.02 for Mall 1, neither of which are statistically significant. Instead, the upper bound tends to be set around a mall-specific constant: among the samples depicted in Figure 12, the mean of the upper bounds is 0.039 for Mall 0 and 0.019 for Mall 1. This difference in the means of the upper bounds aligns with the location differences between the two shopping malls.

The estimates of the interesting parameters are summarized in Table A.1. First,

[31] Outliers located outside the ranges shown in Figure 12 are excluded. There are 6 outliers for Mall 0 and 3 outliers for Mall 1.

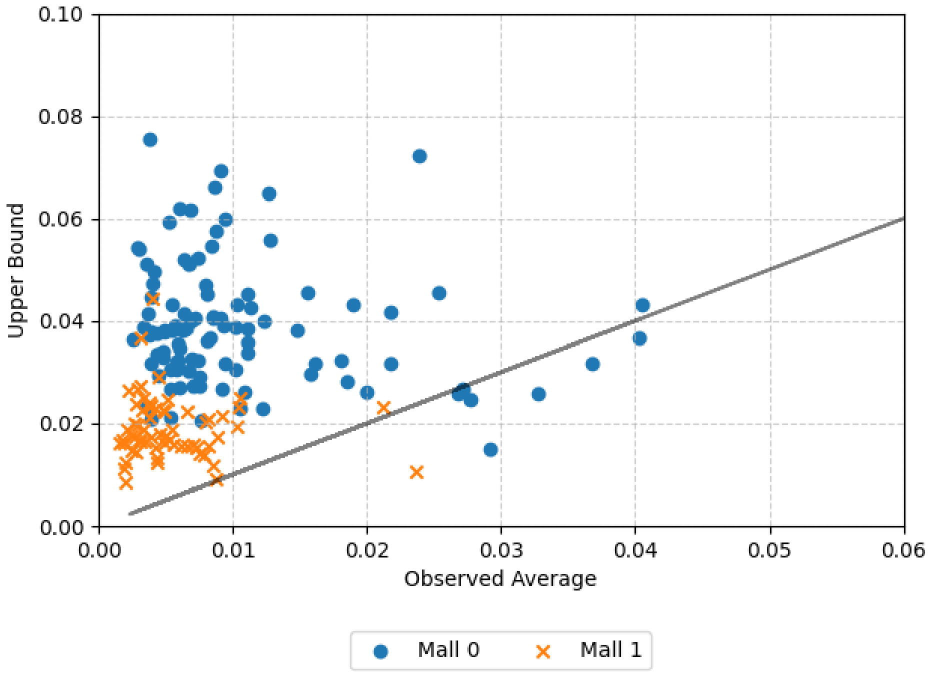


Figure 12. Observed Average Sales vs Estimated Cap on the Mean of the Sales by Mall

Note: Each dot represents a contract. Black line is 45 degree line. The scale of both horizontal and vertical axis is set to 1 billion JPY. This scale is chosen to avoid the overflow when we compute the exponential terms in the model. This figure removes the outliers which are located outside of the the range: 6 samples for Mall 0, and 3 samples for Mall 1.

we examine the parameters related to the balance of bargaining power. As previously indicated, the higher a tenant's relative rank in terms of sales per unit area within the shopping mall, the more the balance of bargaining power shifts unfavorably for the mall. As for the mall specific variables, the bargaining power of the mall strengthens when the number of tenants and the number of customers making purchases in the mall increase. About the tenant specific variation, larger store areas is correlated with a stronger position against the mall. All of these trends align straightforwardly with expectations[32].

Regarding the parameter associated with the value of outside options, we find that increased tenant search intensity reduces the value of outside options. At first glance, this result may appear counterintuitive. However, it could be driven by the longer negotiation processes with potential entrants. When a shopping mall engages in more negotiations

[32]A natural question is whether the directly observed rank of tenants is a valid variable affecting the balance of bargaining power. It is well known that, in a shopping mall, certain types of tenants generate externalities that influence the number of customers and the sales of nearby tenants. A well-known example is anchor tenants, as studied in Gould, Pashigian and Prendergast (2005). In Appendix B, we explore such latent potential among tenants have influenced the balance of bargaining powers.

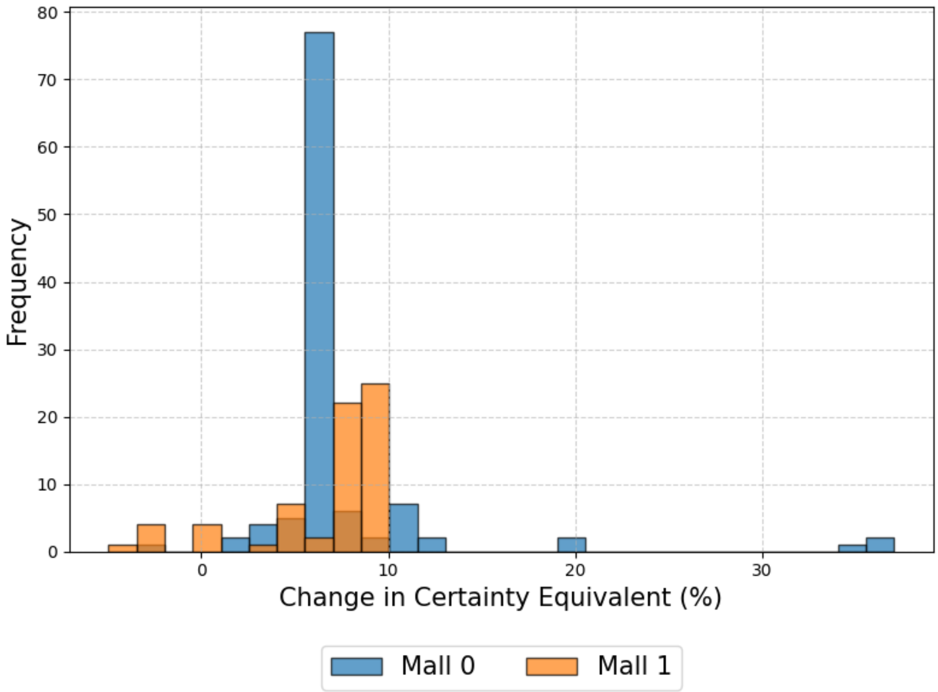


Figure 13. Change in Certainty Equivalence (%)

with prospective tenants, this often implies prolonged discussions and multiple meetings. These extended negotiations provide tenants with ample opportunity to gather sufficient evidence to demonstrate their productivity[33].

As for the disturbance structure, our estimates indicate a negative correlation between the error term in the value of outside options and the error term in the balance of bargaining power. This suggests that retail spaces with higher unobserved demand tend to be leased to tenants in stronger bargaining positions relative to the mall. This finding can be interpreted as evidence of selection: more desirable retail spaces are assigned to more stronger tenants.

The above estimation results confirm that the balance of bargaining power changes over time. Next, we discuss the magnitude of this change. To measure the impact, we consider a counterfactual scenario where the balance of bargaining power remains constant over time. Using the estimated parameters, we calculate the counterfactual surplus split between the shopping mall and its tenants. By comparing this with the realized surplus split, we demonstrate how significantly the change in bargaining power

[33]Figure A.6 supports the hypothesis that longer negotiations benefit the tenant. Specifically, in renewal negotiations, focusing on cases that begin with an up offer, negotiations that do not result in an up offer for the next contract tend to take longer. This suggests that extended negotiations work in favor of the tenant, as they are more likely to secure a better contract through discussions.

Mall	(1) Both	(2) Mall 0	(3) Mall 1	(4) Mall 0	(5) Mall 1	(6) Mall 0	(7) Mall 1
Base	-0.000000661	0.000131	-0.000575	-0.000691	0.00307	-0.000517	0.00227***
	(0.000605)	(0.000670)	(0.00166)	(0.00200)	(0.00266)	(0.000785)	(0.000737)
Contract Number				0.00777*	-0.00545	0.00804	-0.00120
				(0.00453)	(0.00494)	(0.00488)	(0.00238)
Area				0.000183	0.000210	0.000149	0.000109
				(0.000319)	(0.000243)	(0.000114)	(0.000227)
Avg. Sales				-0.00106	-0.00296	0.00197***	0.00428***
				(0.000942)	(0.00292)	(0.000626)	(0.00116)
Var. Sales				0.00191	-0.0105	0.00123	-0.00277
				(0.00241)	(0.00640)	(0.000933)	(0.00297)
Log BP (Uncertainty)						-0.0495***	-0.0330**
						(0.0164)	(0.0160)
Log BP (Risk)						0.124***	0.102***
						(0.0170)	(0.0140)
N	152	90	62	90	62	90	62
adj. R^2	-0.007	-0.011	-0.014	-0.015	0.072	0.702	0.852

Standard errors in parentheses
* p<0.1, ** p<0.05, *** p<0.01

Table 4. Tradeoff between Rate and Base

over time affects this situation.

Specifically, when computing the expected logarithm of balance of bargaining power in the counterfactual scenario, we only include the constant term, the dependence on risk aversion, and fixed variables such as area. We then compute the surplus split and its certainty equivalent for the shopping mall under this counterfactual steady balance of bargaining power. Figure 13 shows the resulting change in the certainty equivalent of the surplus splits compared to the actual situation. In this measure, the influence of past performance ranges from 0% to around 30%, with most changes concentrated around $5 \sim 10\%$[34]. This simulation also indicates that the change is almost always positive for both Mall 0 and Mall 1. In other words, as the balance of bargaining power adjusts over time, the shopping mall's strength at the bargaining table decreases.

[34]Figure A.8 presents the change in surplus in percentage terms for both Mall 0 and Mall 1, based on the realized surplus. The change is relatively small, from 0% to 8%.

6.2 Selection mechanism

Here we exploit the estimation results, in particular the recovered value of the balance of bargaining powers, to examine the determinants of the terms of contract. From the model perspective, the question about the mechanism behind the terms of contract corresponds to the empirical analysis of the equilibrium selection mechanism which determines the terms of contract from the set of equilibrium contracts.

Before analyzing the selection mechanism, I first revisit the interrelationship between contract terms to highlight the importance of controlling the balance of bargaining powers. As discussed in Section 2, Base and Rate should be positively correlated. However, as shown in Figure 6, no clear pattern emerges. Table 4 presents the regression results of Rate on Base. The first five columns show no significant relationship between them. When we account for the estimated balance of bargaining powers for uncertainty resolution and risk sharing, as shown in the last two columns, we observe that Mall 1 compensates a higher Rate with a higher Base. Additionally, for both shopping malls, the positive coefficients associated with average sales align with intuition: higher tenant sales lead to a higher Rate, given the same Base value. Moreover, the fit of the model, measured by the adjusted R squared, improves in the last two columns, which also indicates the importance of controlling the balance of bargaining powers when analyzing the determination of terms of contract.

The terms of the contract, including Fixed, Base, and Rate, are determined simultaneously. We use a simple Seemingly Unrelated Regression (SUR) model to estimate the selection mechanism. This model treats the contract terms as outcome variables, with regressors including the balance of bargaining powers and other covariates, such as the contract number. It is important to note that we do not impose any structural model on this selection process. The SUR model should be viewed as an approximation of the actual selection mechanism.

The estimation results are summarized in Table A.2 for Mall 0 and Table A.3 for Mall 1. First of all, except for the Fixed term of Mall 0, the balance of bargaining powers influences the terms of the contract. A common pattern observed is that the Base decreases when the shopping mall holds a stronger position, which is typically accompanied by a reduction in the Rate. For Mall 1, this reduction in Base appears to be preferred even at the expense of a decrease in the Fixed term.

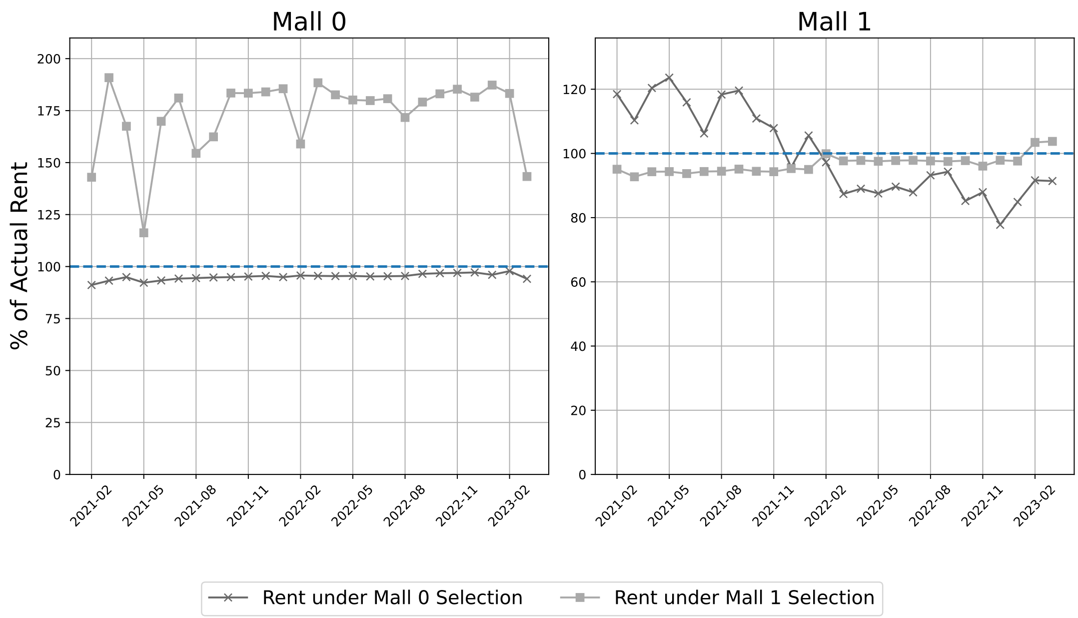


Figure 14. Simulated Total Monthly Rent Compared to the Actual Values by Malls

Note: The thick gray line with cross markers represents the rent collected under the counterfactual contract terms derived from the selection mechanism for Mall 0. The light gray line with circle markers represents the rent under the same conditions for Mall 1. The vertical axis shows the percentage of the total computed rent relative to the actual collected rent, with the blue horizontal dotted line representing 100%. Since all the contracts used in this analysis are from the post-2021 overall renewal, the horizontal axis spans from the beginning of 2021 to the end of our data period, March 2023.

The two malls appear to adopt different selection mechanisms regarding the use of realized sales history. Given a balance of bargaining powers, Mall 0 tends to set a higher Fixed when sales are more volatile, implementing this higher Fixed in exchange for a higher Base. In contrast, Mall 1's contracts do not account for sales variance. Instead, the level of sales is reflected in the contract terms: higher average sales lead to a higher Base and a higher Rate.

Furthermore, the coefficients associated with the contract number indicate the basic strategy for contract adjustments over time. Mall 0 starts with a low Rate and gradually increases it over time while keeping the Fixed steady, whereas Mall 1 gradually increases the Fixed while maintaining a steady Rate.

As discussed above, the two shopping malls employ different methods for using sales history and adjusting contracts. A natural question arises: is this difference "rational"? To explore this, I simulate counterfactual contract terms using the estimated selection mechanisms to determine what would happen if each mall adopted the other's selection mechanism[35]. Details of the counterfactual simulation procedure are provided in Appendix B. I begin by examining changes in the total rent, the primary concern for the shopping mall management company. Then, I analyze the distributional changes in monthly rent to see if the current selection mechanism aligns with the managers' risk preferences, which may not always be in the best interest of the management company.

Figure 14 compares the simulated rent to the actual rent by mall. In the left panel, the simulated rent under Mall 0's selection mechanism (thick gray line with cross markers) closely matches the actual rent. Similarly, in the right panel, the simulated rent for Mall 1 (light gray line with circle markers) also aligns with the actual rent. This demonstrates a good fit of the estimated selection mechanisms in both malls.

In terms of total rent collected, Mall 0 underperformed throughout the entire period, and Mall 1 did so in the earlier period compared to the counterfactual rent. By adjusting the selection mechanism when choosing contract terms, they could increase rent even with the same power balance between the mall and the tenants. As shown in Figure A.9, the gradual underperformance of Mall 0's selection mechanism in the latter period is due to its inability to keep up with the rapid increase in actual rent. Table A.3 shows that Mall 1 gradually increases its Fixed rent as contract numbers rise. Some of Mall 1's one-year contracts are affected by this pattern in the later period, whereas Mall 0's selection mechanism lacks this feature, preventing it from adapting to the rapid rent increase.

Next, I examine the distributional changes in monthly rents when applying the other mall's strategy. Figure 15 shows the relationship between the increase in average monthly rent and the increase in its standard deviation for each tenant, comparing the counterfactual contract terms to the actual ones. Mall 0's counterfactual terms are generated using Mall 1's selection mechanism, and vice versa for Mall 1. The change in average rent is mixed for both Mall 0 and Mall 1. However, when it comes to rent volatility, Mall 0 sees more variation under Mall 1's selection mechanism, while Mall 1 experiences less volatility under Mall 0's mechanism. This suggests that Mall 0's selection mechanism tends to choose contract terms that lead to less variation. As illustrated in Figure 11, Mall 0's manager appears to be more risk-averse than Mall 1's, aligning the selection mechanism with the manager's risk preferences. But, as you have seen in Figure 14, the selection mechanism set by the manager does not always maximize total rent, which is

[35] This assumes that the selection mechanism is non-negotiable from the tenants' perspective, with the mall determining how to select a contract from the set of equilibrium contracts.

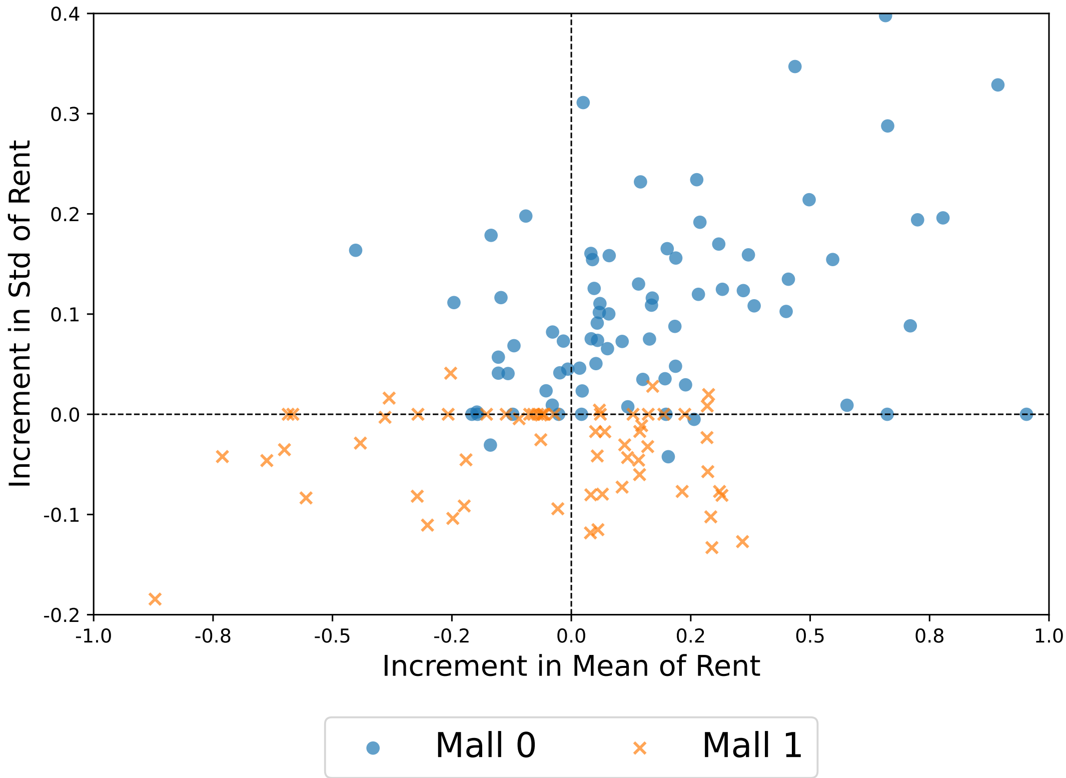


Figure 15. Increment in the Average and Std of Monthly Rent when Using the Other Mall's Selection Mechanism

the primary concern for the management company.

7 Concluding remarks

This study examines the impact of the balance of power between parties on the leasing contract terms for retail spaces in shopping malls. We model the negotiation over contract terms using a two-stage bargaining framework and estimate it with newly collected data. First, we find that the balance of power in negotiations changes over time, suggesting that a fixed-effects analysis alone is insufficient, even with access to panel data. Second, we demonstrate that controlling for the balance of bargaining power is crucial for identifying patterns in contract terms. Finally, we simulate counterfactual rents under different contract selection mechanisms. The results indicate that the mall manager's risk preferences influence the chosen contract terms, and that the management company could increase rent revenues by adopting an alternative contract selection mechanism, even with a fixed balance of power with tenants.

The retail space leasing agreements analyzed in this paper are just one example, and the central point of this study—namely, that a more detailed empirical investigation of contract terms can be achieved by considering the power balance between the parties—applies to a wide range of cases. In the literature, insurance contracts are frequently studied, but there are numerous other cases, such as leasing agreements or contracts between sports players and teams, where the contents of the contracts are diverse and difficult to handle in empirical research. Exploring these types of contracts is one potential future direction.

Moreover, when considering such extensions, it becomes essential to incorporate moral hazard into the model. While the importance of power dynamics between parties under moral hazard is already well recognized in the literature, the findings of this study suggest that this aspect must also be seriously considered in empirical research.

Another issue that this paper does not address is the team problem. In shopping malls, sports teams, and even regular companies, it is common to make contracts with multiple agents simultaneously. When these contracts are interrelated through externalities, rather than being independent, adjusting contract terms to manage incentives becomes more complex. In fact, some papers, such as Gould, Pashigian and Prendergast (2005), have pointed out the existence of the team problem in shopping malls leasing operation. It is natural that the Nash-in-Nash framework could be a valuable tool for analyzing such situations.

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A Additional figures and tables

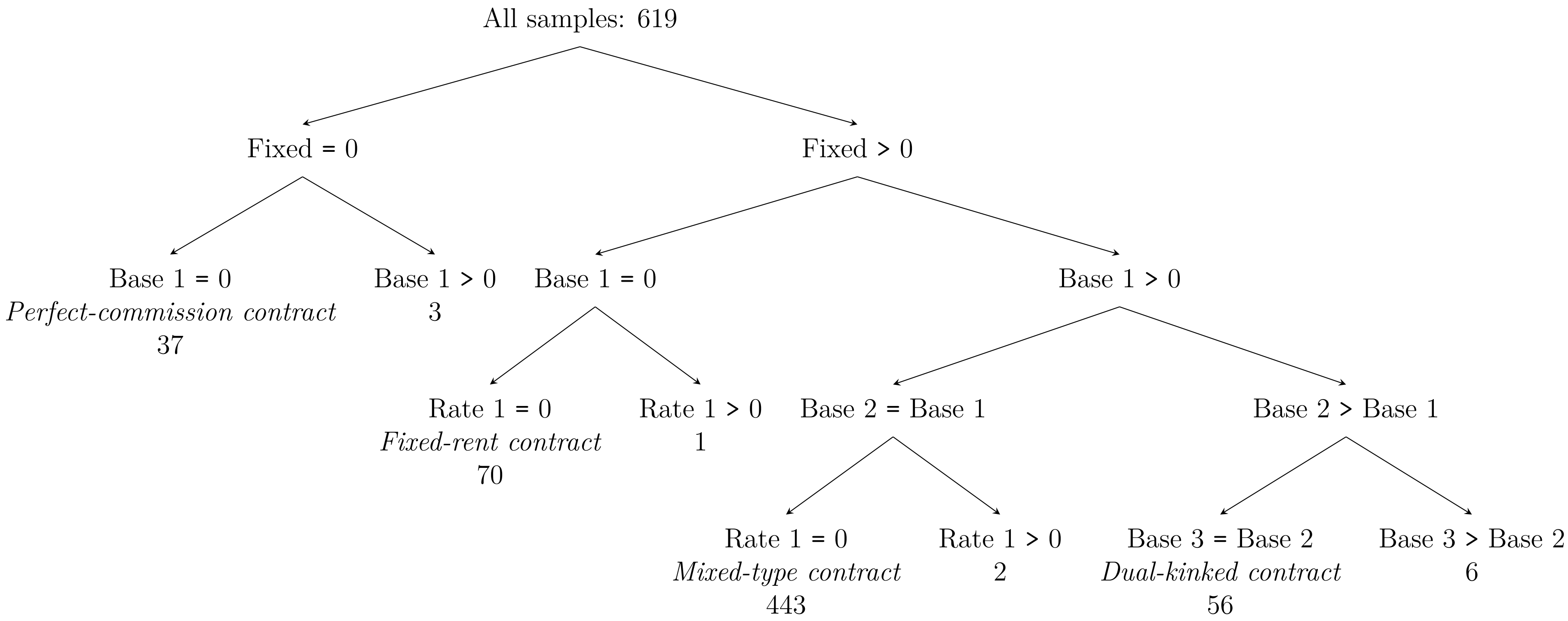

Figure A.1. Group of Contracts

Note: This figure illustrates the hierarchical structure of contracts across 618 samples. The tree splits samples based on Fixed, Bases and Rates, with nested conditions on additional variables (Base 1, Base 2, Base 3, and Rate 1). Each branch represents distinct contract groups, showing how various base and rate parameters define the segmentation. The numbers on each node represent the number of contracts in each subgroup.

	Mall 0	Mall 1
Panel 1: Built-in parameters		
ρ_M	1.381	1.062
	(0.000)	(0.000)
ρ_i^{exit}	1.816	2.285
	(0.000)	(0.000)
Panel 2: Balance of bargaining powers		
constant	−0.196	0.327
	(0.000)	(0.000)
Log # of Tenants	0.209	0.270
	(0.000)	(0.000)
Log Total Customer	0.205	0.103
	(0.000)	(0.000)
Observed rank (% from the Above)	0.758	0.067
	(0.000)	(0.000)
Area	−0.434	−0.503
	(0.000)	(0.000)
Panel 3: Value of outside options		
constant	−0.423	−0.298
	(0.000)	(0.000)
Log new tenant search	−0.049	−0.102
	(0.000)	(0.000)
Residual in First Stage	0.100	0.409
	(0.000)	(0.000)
Panel 4: Disturbance		
σ_o	0.000	0.016
	(0.000)	(0.000)
σ_u	0.002	0.007
	(0.000)	(0.000)
Correlation	−0.317	−0.828
	(0.000)	(0.000)

Table A.1. Estimates of the Structural Model by Malls

		(1)			(2)			(3)	
	Rate	Base	Fixed	Rate	Base	Fixed	Rate	Base	Fixed
Contract Number	0.00840*	-0.354	-0.0250	0.00782*	-0.0735	0.00493	0.00807***	-0.0602	0.00526
	(0.00435)	(0.339)	(0.0486)	(0.00455)	(0.292)	(0.0463)	(0.00244)	(0.286)	(0.0454)
Area	0.0000762	0.134***	0.00923***	0.000120	0.0921***	0.00459***	0.000105	0.0857***	0.00468***
	(0.000111)	(0.00863)	(0.00124)	(0.000160)	(0.0103)	(0.00163)	(0.0000901)	(0.0106)	(0.00168)
Avg. Sales				-0.00102	-0.0573	0.000418	0.00204***	-0.135	0.00922
				(0.00114)	(0.0729)	(0.0116)	(0.000720)	(0.0843)	(0.0134)
Var. Sales				0.00147	0.631***	0.0574***	0.000881	0.671***	0.0552**
				(0.00215)	(0.137)	(0.0218)	(0.00116)	(0.136)	(0.0216)
Log BP (Uncertainty)							-0.0480***	-2.987*	-0.0569
							(0.0136)	(1.590)	(0.253)
Log BP (Risk)							0.125***	-2.154	0.341
							(0.0121)	(1.416)	(0.225)
N	90	90	90	90	90	90	90	90	90

Table A.2. SUR estimation Results (Mall 0)

	(1)			(2)			(3)		
	Rate	Base	Fixed	Rate	Base	Fixed	Rate	Base	Fixed
Contract Number	0.000532	-0.477	0.0494	-0.00494	0.164	0.0910	-0.000441	0.335	0.144***
	(0.00615)	(0.433)	(0.0611)	(0.00612)	(0.326)	(0.0595)	(0.00254)	(0.320)	(0.0535)
Area	-0.0000297	0.0773***	0.00558***	0.000353*	0.0466***	0.00501***	0.000121	0.00507	-0.00616*
	(0.000157)	(0.0110)	(0.00156)	(0.000189)	(0.0101)	(0.00184)	(0.000157)	(0.0198)	(0.00330)
Avg. Sales				-0.00152	0.469***	0.0161	0.00533***	0.461***	0.0274
				(0.00217)	(0.115)	(0.0211)	(0.000962)	(0.121)	(0.0202)
Var. Sales				-0.0115	-0.346	-0.0422	-0.00307	-0.130	0.0299
				(0.00769)	(0.410)	(0.0749)	(0.00321)	(0.404)	(0.0675)
Log BP (Uncertainty)							-0.0400***	-3.086**	-0.874***
							(0.0101)	(1.275)	(0.213)
Log BP (Risk)							0.0985***	-1.350	-0.157
							(0.00789)	(0.994)	(0.166)
N	62	62	62	62	62	62	62	62	62

Table A.3. SUR estimation Results (Mall 1)

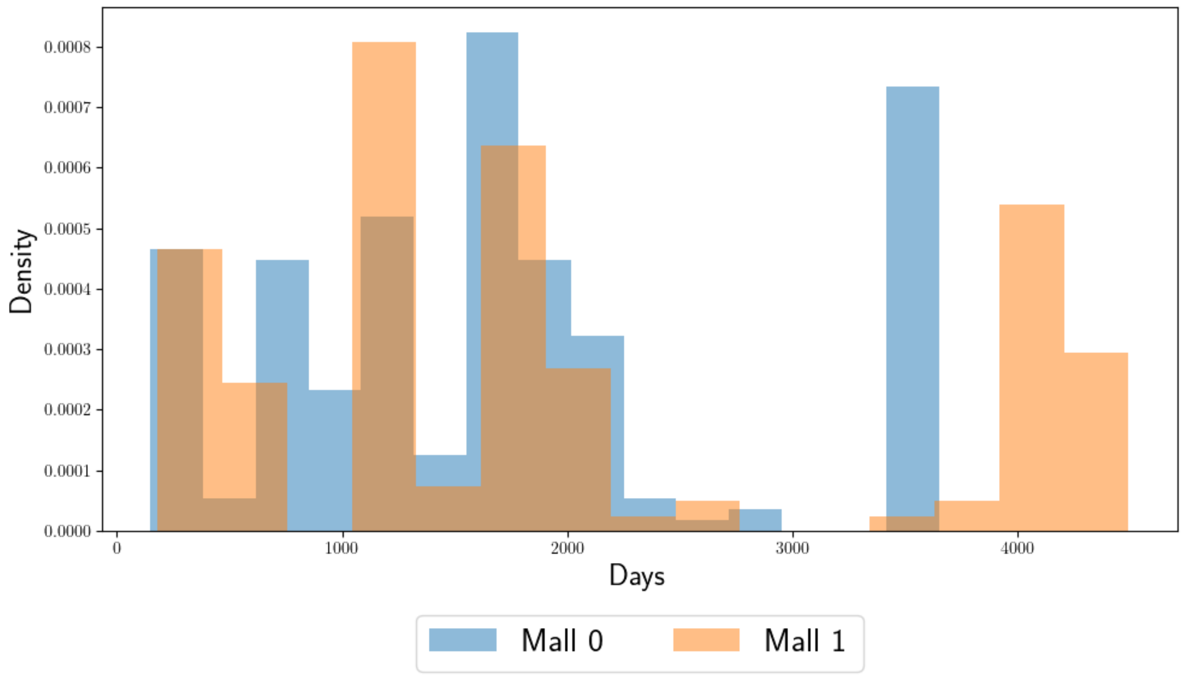

Figure A.2. Histogram of Contract Duration by Mall

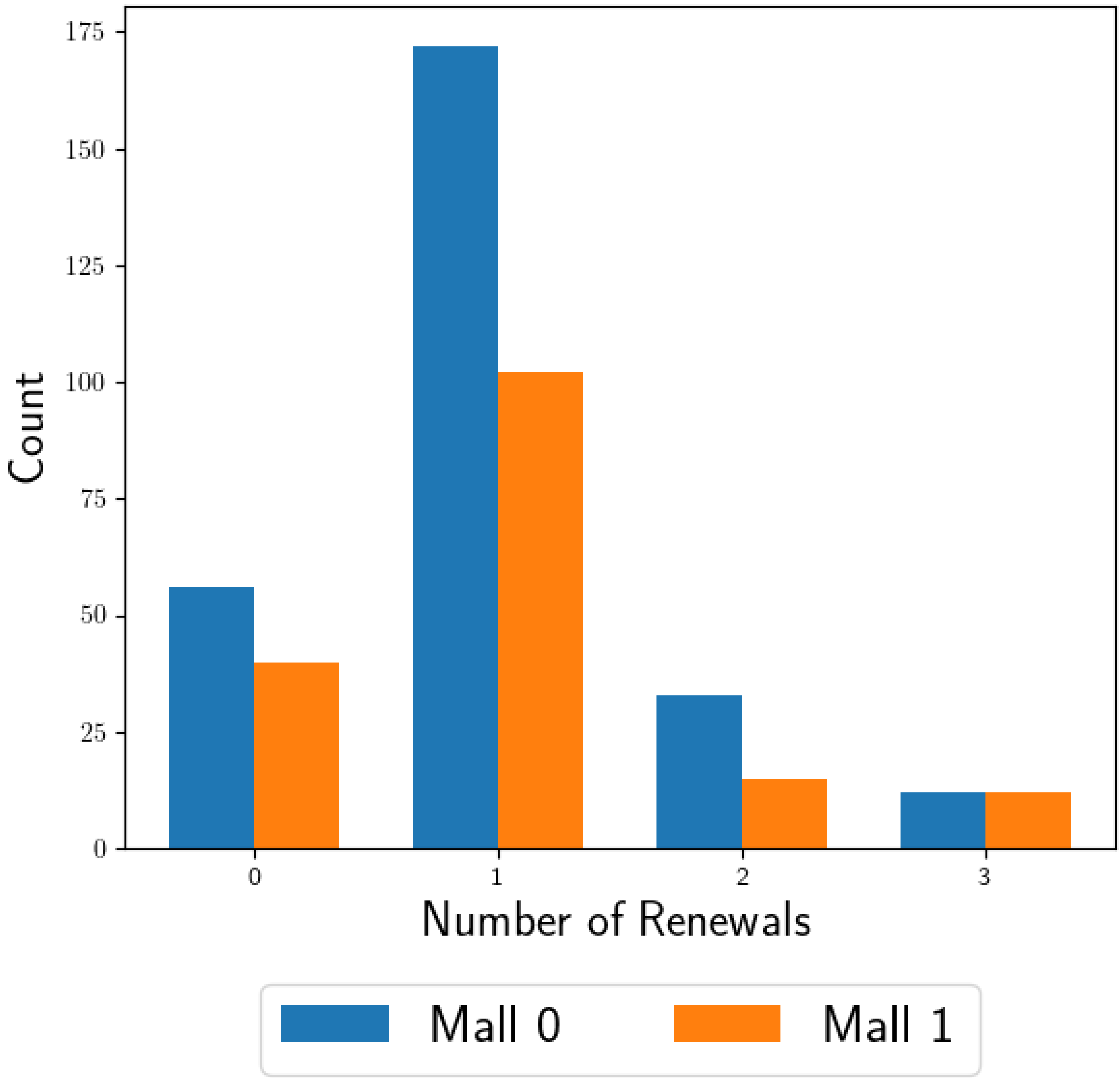

Figure A.3. Histogram of Number of Renewals by Mall

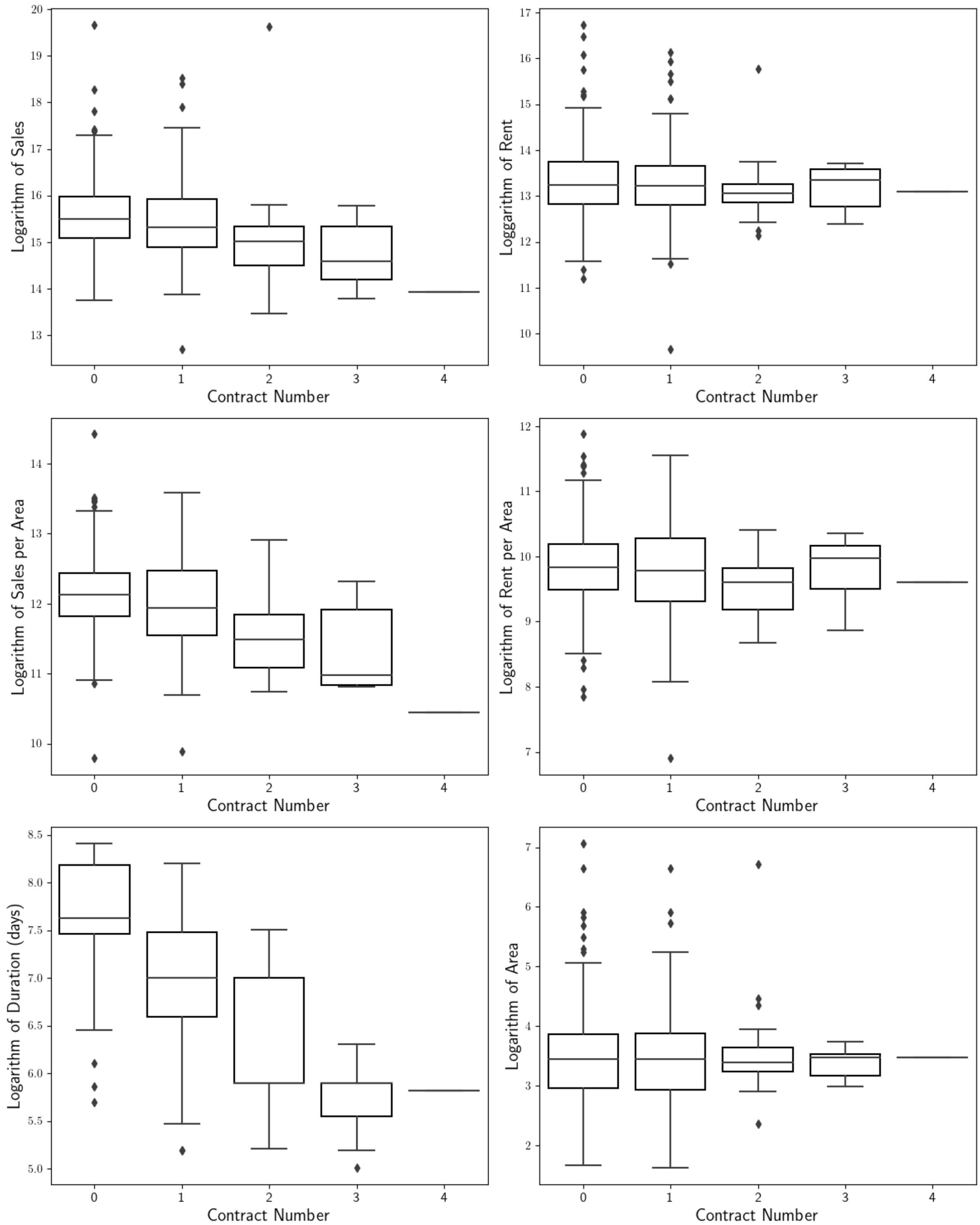

Figure A.4. Box Plots of Performance Measures and Characteristics

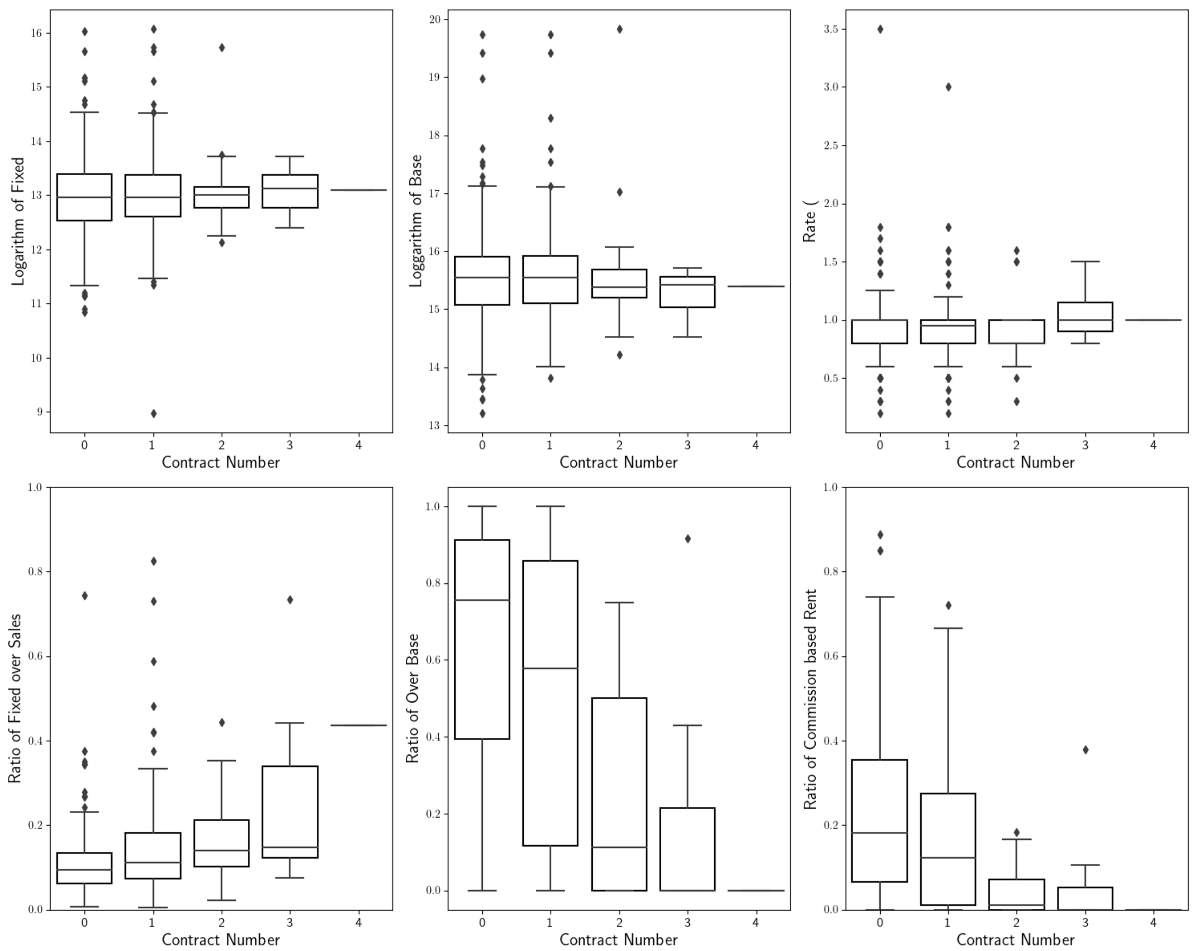

Figure A.5. Dynamic Change in Terms of Contract

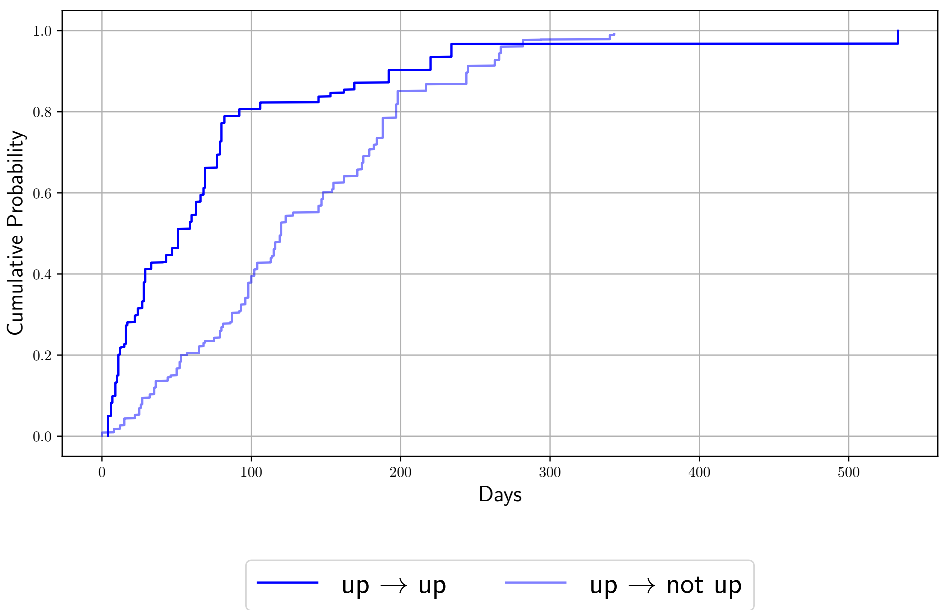

Figure A.6. Empirical Cumulative Distribution Functions of Length of Negotiations

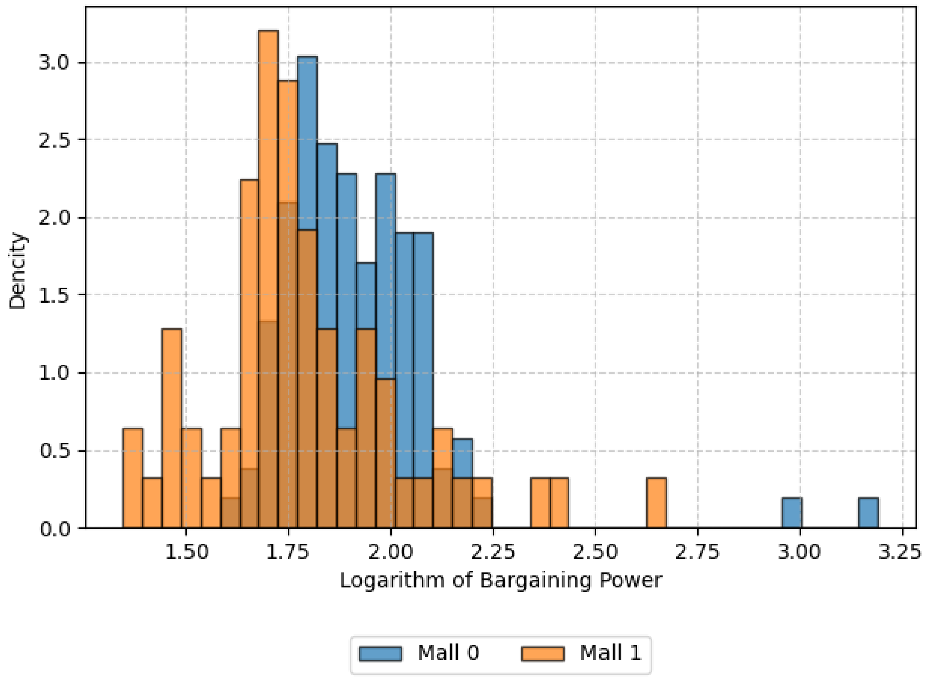

Figure A.7. Histograms of Logarithm of Balance of Bargaining Powers

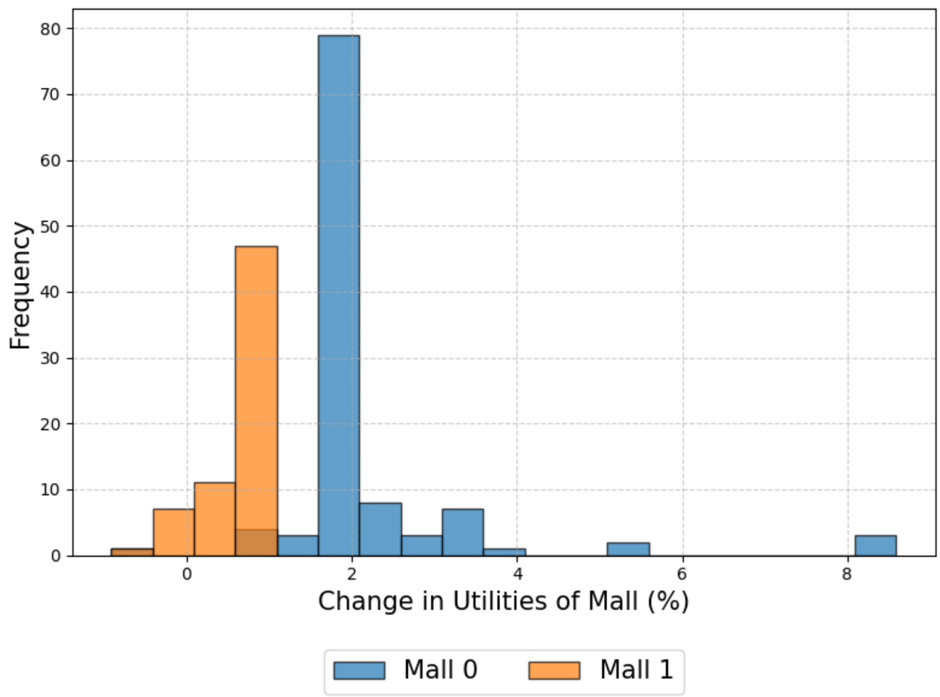

Figure A.8. Change in Surplus (%)

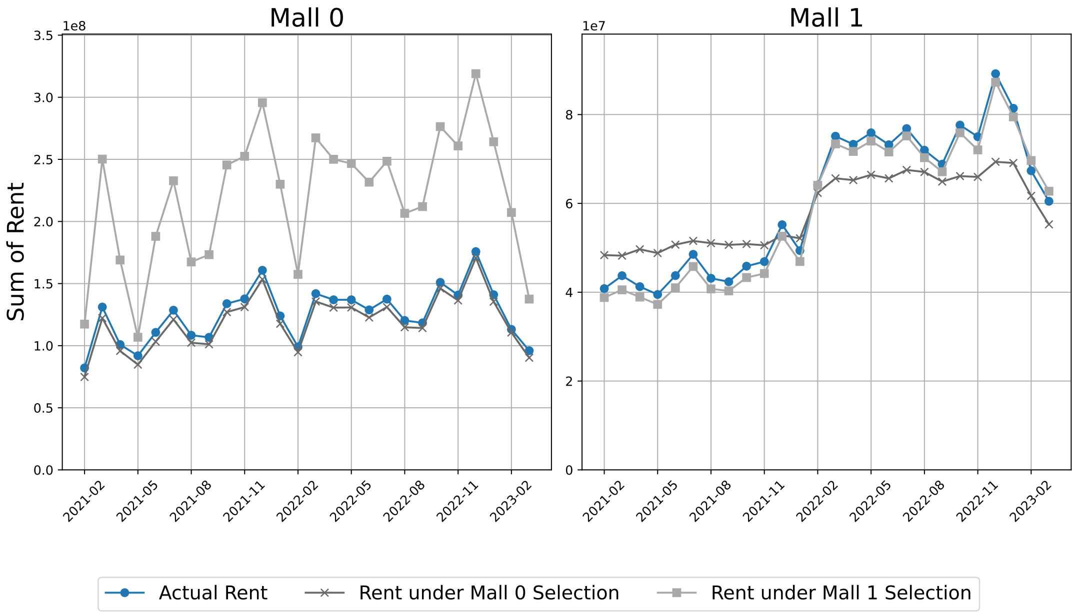

Figure A.9. Simulated Total Monthly Rents and Actual Rents

B Additional analysis

B.1 Variation within common brands

The observed variation in the rent structures cannot be fully attributed to fixed differences in preferences between the shopping malls or their tenants. When analyzing common brands in both malls, as summarized in Table A.4, the rent structure distributions appear comparable: where the contract type is numerated according to the order of leaves in Figure A.1 from the left. This suggests that the mall-specific factors do not significantly influence the rent structures for these brands. Moreover, for common tenants, the parameters of the rent structures vary between the two shopping malls. Figure A.10 illustrates the parameters of the mixed-type contracts across both malls, highlighting these differences.

Contract Type	**1**	**2**	**3**	**5**	**6**	**7**	**8**
Mall 0	2	0	10	58	2	1	2
Mall 1	7	1	11	44	0	7	0

Table A.4. Distribution of Rent Structures of Common Brands by Malls

Note: This table shows the distribution of rent structures for common brands across Mall 0 and Mall 1. The contract types are listed from 1 to 8, which are indexed according to the leaves in Figure A.1.

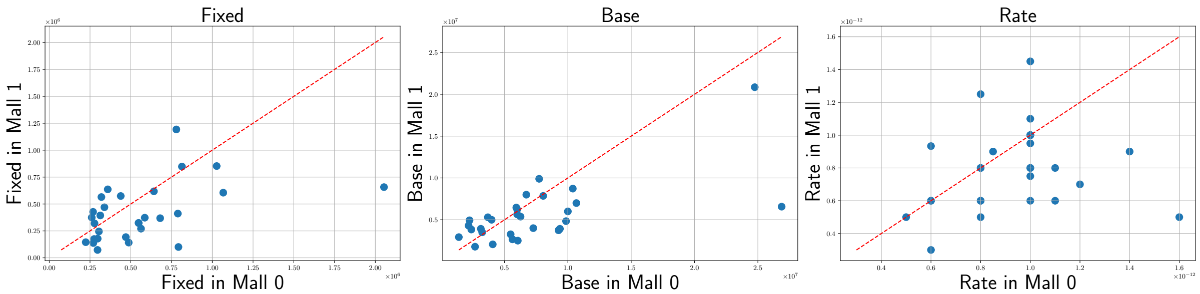

Figure A.10. Difference in Contracts of Common Tenants by Malls

Note: Comparison of the parameters of a contract between Mall 0 and Mall 1. The figure consists of three scatter plots, each comparing the Fixed, Bases, and Rates of common tenants across the two malls. The horizontal axis represents the values for Mall 0, while the vertical axis represents the corresponding values for Mall 1. The plots demonstrate how similar or different the contract parameters are for the same brands in the two malls.

B.2 Misperception about balance of bargaining powers

Due to externalities between tenants, sales or sales per unit area may not accurately reflect a tenant's true sales potential. Specifically, one professional noted: "Certain types of tenants seem to have a unique ability to draw in customers, but not all of those customers make purchases at that tenant. Some may only window shop and then be reminded to buy something from a different tenant. While we typically rely on sales per unit area to assess tenant performance, we are interested in capturing this kind of underlying potential if possible." In this section, we use a simple decomposition method to recover such a true potential of the tenants and then check if the estimated balance of bargaining powers reflect them.

To avoid the complexity introduced by differences in the average spend per customer, we use the number of customers as a performance measure. The number of customers (x_{ikt}) is modeled as follows:

$$x_{ikt} = \delta_i + \sum_{j \neq i, j \in \mathcal{T}_{kt}} \alpha_{ij} \delta_j + \varepsilon_{ikt},$$

where:

- δ_i: baseline customer drawing power of tenant i
- α_{ij}: the proportion of customers who visit tenant j and also stop by tenant i for some reason
- ε_{ikt}: disturbance term

When we assume that α is constant, the first equation is

$$x_{ikt} = \delta_i + \alpha \sum_{j \neq i, j \in \mathcal{T}_{kt}} \delta_j + \varepsilon_{ikt}.$$

For any pair (i, j) in Mall k, we have the following equation for the difference of the two number of customers:

$$x_{ikt} - x_{jkt} = (\delta_i - \delta_j)(1 - \alpha) + \varepsilon_{ikt} - \varepsilon_{jkt}.$$

When there are more than four tenants, we can identify and estimate $(1 - \alpha)\delta_i$ for all the

	(1)	(2)	(3)	(4)
Potential (% from the Bottom)	-0.110*	-0.0791	0.0109	0.0218
	(0.0660)	(0.0696)	(0.0726)	(0.0745)
ln B	-0.0473	-0.0601	-0.455***	-0.479***
	(0.114)	(0.110)	(0.119)	(0.116)
Contract Number	0.0499*	0.0942*	0.0165	0.0418
	(0.0261)	(0.0477)	(0.0224)	(0.0362)
N	152	152	152	152
adj. R^2	0.184	0.216	0.456	0.468
Mall controls		√		√
Tenant controls			√	√

Standard errors in parentheses
* p<0.1, ** p<0.05, *** p<0.01

Table A.5. Bargaining Powers and True Potential

tenants. This is enough to determine the relative rank of the tenants based on this "true customer drawing power."

To estimate the decomposition above, we rely on the number of customers aggregated weekly for each tenant. Using weekly-aggregated data increases the sample size, which is a challenge in any fixed-effect model. We rank the tenants within each shopping mall based on the recovered value of $(1-\alpha)\delta_i$.

In our main analysis, we regress the estimated balance of bargaining power on this rank. Specifically, we calculate the percentile rank of each tenant within a shopping mall, starting from the bottom, and use it as an independent variable in the regression. The regression results are summarized in Table A.5. When controlling the covariates relevant with the tenant and the shopping mall itself, which are described in our main analysis section, the true potential is not correlated with the estimated balance of bargaining powers as shown in the first row.

This indicates that the shopping mall's concern is valid. They cannot rely on the "true potential" of tenants when defining the balance of powers with them. One possible future direction is to analyze the counterfactual scenario when they can use this true potential when determining the power balance.

B.3 Detail procedure of counterfactual analysis

Here we describe the detail procedure of the estimation of the selection mechanism and the subsequent counterfactual analysis.

When estimating the selection mechanism, we basically follow the same SUR model which generates Table A.2 and Table A.3. We have the two deviations. First, we use the logarithm values of Fixed, Base, and Rate as the dependent variables in the SUR model. This assures the non-negativity of the simulated values of them. Second, we add several set of covariates in addition to the covariates used in Table A.2 and Table A.3. Specifically, we add the polynomial terms of the balance of bargaining powers and the year dummy variable[36]. This inclusion is to make the estimated selection mechanism fit well to the actual amount of rent.

When simulating counterfactual rent structures, we first use the estimated parameters from the SUR model to recover the expected values of the logarithms of Fixed, Base, and Rate. Next, using the estimated variance-covariance matrix, we simulate 100 error terms for each of the three values to generate the simulated rent structures. For each simulated rent structure, we compute the counterfactual rent based on the actual sales history, and then average these values to determine the counterfactual rent collected for each month.

[36]This variables takes binary values: before 2022 or after 2022. This is because our data period is 2021, 2022, and the beginning two months of 2023.

C Proofs

Lemma 5. *For any affine contract (f, r), the expected utility of T and M are written as follows:*

$$\mathbb{E}\left[U_T\left(-f+(1-r)S\right)\right] = -e^{-\rho_T\left(-f+(1-r)\mu-\frac{\sigma^2}{2}(1-r)\rho_T\right)}$$

and

$$\mathbb{E}\left[U_M\left(I+f+rS\right)\right] = -e^{-\rho_M\left(I+f+r\mu-\frac{\sigma^2}{2}\rho_M\right)}.$$

Proof. For M,

$$\begin{aligned}\mathbb{E}\left[U_M(I+f+rS)\right] &= e^{-\rho_M(I+f)}\mathbb{E}\left[-e^{-\rho_M rS}\right] \\ &= -e^{-\rho_M(I+f)}e^{-\rho_M(r\mu-\frac{\sigma^2r^2}{2}\rho_M)} = U_M\left((I+f+r\mu-\frac{\sigma^2r^2}{2}\rho_M\right).\end{aligned}$$

For tenant side, from the same computation, we get the result. □

Proof of Lemma 1 Pick a mixed type contract (f, b, r). By Lemma 5, what we want to show is that the following equalities hold for some pair $(\tilde{f}, \tilde{r})$:

$$\begin{cases} EU_T(f,b,r) \equiv \mathbb{E}\left[U_T\left(S-f-r\max\{0,S-b\}\right)\right] = -e^{-\rho_T\left(-\tilde{f}+(1-\tilde{r})\mu-\frac{\sigma^2}{2}(1-\tilde{r})\rho_T\right)} \\ EU_M(f,b,r) \equiv \mathbb{E}\left[U_M\left(I+f+r\max\{0,S-b\}\right)\right] = -e^{-\rho_M\left(I+\tilde{f}+\tilde{r}\mu-\frac{\sigma^2}{2}\rho_M\right)} \end{cases}$$

From the above equations, if there is such an affine contract, we have

$$-\frac{\sigma^2}{2}(\rho_T+\rho_M)\left(\tilde{r}-\frac{\rho_T}{\rho_T+\rho_M}\right)^2 + I + \mu - \frac{\sigma^2}{2}\frac{\rho_T\rho_M}{\rho_T+\rho_M} + \frac{1}{\rho_T}\ln(-EU_T) + \frac{1}{\rho_M}\ln(-EU_M) = 0.$$

Hence, if the above quadratic equation with respect to $\tilde{r}$ has at least one solution, we can determine a $\tilde{r}$ and the corresponding $\tilde{f}$ is also determined by the initial equations.

As the worst case, we think about the case of $I = 0$. We argue that the following term must be positive for all the triplet of (f, b, r):

$$\mu - \frac{\sigma^2}{2}\frac{\rho_T \rho_M}{\rho_T + \rho_M} + \frac{1}{\rho_T}\ln(-EU_T(f,b,r)) + \frac{1}{\rho_M}\ln(-EU_M(f,b,r)).$$

We can remove the fixed component from the above:

$$\mu - \frac{\sigma^2}{2}\frac{\rho_T \rho_M}{\rho_T + \rho_M} + \frac{1}{\rho_T}\ln \mathbb{E}\left[e^{-\rho_T(S - r\max\{0, S-b\})}\right] + \frac{1}{\rho_M}\ln \mathbb{E}\left[e^{-\rho_M(r\max\{0,S-b\})}\right]. \quad (6)$$

We firs show that (6) is positive for all r when $b \to \infty$:

$$\mu - \frac{\sigma^2}{2}\frac{\rho_T \rho_M}{\rho_T + \rho_M} + \frac{1}{\rho_T}\ln \mathbb{E}\left[e^{-\rho_T S}\right] = \frac{\sigma^2}{2}\frac{\rho_T^2}{\rho_T + \rho_M} > 0.$$

In the following, we show that (6) is monotonically decreasing with respect to b for any value of r. If so, (6) is always positive for all the mixed-type contract.

This is directly proven by computing the derivative of (6) with respect to b. By simple computation, the sign of the derivative is equal to the sign of the following term:

$$\mathbb{E}\left[\mathbf{1}\{S < b\}e^{-\rho_T S}\right]e^{\rho_M r b}\mathbb{E}\left[\mathbf{1}\{S > b\}e^{-\rho_M r S}\right] - \mathbb{E}\left[\mathbf{1}\{S < b\}\right]e^{-\rho_T r b}\mathbb{E}\left[\mathbf{1}\{S > b\}e^{-\rho_T(1-r)S}\right]$$

The equivalent condition that the above term is negative is

$$\mathbb{E}\left[e^{-\rho_T S} \mid S > b\right] < \mathbb{E}\left[e^{-\rho_T S}\frac{e^{\rho_T(S-b)}}{\mathbb{E}\left[e^{-\rho_M r(S'-b)} \mid S' > b\right]} \mid S > b\right].$$

This condition is true because $e^{\rho_T(S-b)} > 1 \geq e^{-\rho_M(S-b)}$ for all the case where $S > b$. So we are done.

Proof of Lemma 2 The bargaining frontier is characterized as the solution of the following maximization problem: for some value of u_M,

$$\max_{(f,r)\in\mathbb{R}_+^2} \ \mathbb{E}\left[e^{-\rho_T(J-f+(1-r)S)}\right]$$
$$\text{s.t. } \mathbb{E}\left[e^{-\rho_M(I+f+rS)}\right] \geq u_M.$$

Here, we directly solve this maximization problem to characterize the bargaining frontier of this problem.

From Lemma 5, the problem is written as follows:

$$
\begin{aligned}
&\max_{(f,r)\in\mathbb{R}^2_+} \; -\frac{\sigma^2\rho_T}{2}\left(r-\left(1-\frac{\mu}{\sigma^2\rho_T}\right)\right)^2 - f + \frac{\mu^2}{2\sigma^2\rho_T} \\
&\text{s.t. } \; -\frac{\sigma^2\rho_M}{2}\left(r-\frac{\mu}{\sigma^2\rho_M}\right)^2 + I + f + \frac{\mu^2}{2\sigma^2\rho_M} + \frac{1}{\rho_M}\ln(-u_M) \geq 0.
\end{aligned}
\tag{7}
$$

We solve this problem in the following sequence: first we fix f and compute the optimal r and the corresponding frontier. then we compute the envelope of the frontier by searching over f. This sequential analysis provides us with the intuition about the connection to the perfect commission contract.

We fix a f. Let $r^\star$ be the optimal value of r and obj be the value of the objective function in (7). Then, by direct calculation, the solution of the above maximization problem is described as follows: there are three cases about the relative size of $\frac{\mu}{\sigma^2}$ with respect to the risk aversions.

1. $\frac{\mu}{\sigma^2} > \rho_T$

   (a) $u_M \leq U_M(I+f)$

   $$r^\star = 0, \; obj = \mu - \frac{\sigma^2}{2}\rho_T - f$$

   (b) $U_M(I+f) < u_M \leq U_M\left(I+f+\frac{\mu^2}{2\sigma^2\rho_M}\right)$

   $$r^\star = \hat{r}^-, \; obj = -\frac{\sigma^2\rho_T}{2}\left(\hat{r}^- - \left(1-\frac{\mu}{\sigma^2\rho_T}\right)\right)^2 - f + \frac{\mu^2}{2\sigma^2\rho_T}$$

2. $\rho_T \geq \frac{\mu}{\sigma^2} > \frac{\rho_T\rho_M}{\rho_T+\rho_M}$

   (a) $u_M \leq U_M\left(I+f+\frac{\mu^2}{2\sigma^2\rho_M} - \frac{1}{2\sigma^2\rho_M}\left(\frac{\rho_T+\rho_M}{\rho_T}\right)^2\left(\mu-\sigma^2\frac{\rho_T\rho_M}{\rho_T+\rho_M}\right)\right)$

   $$r^\star = 1-\frac{\mu}{\sigma^2\rho_T}, \; obj = -f + \frac{\mu^2}{2\sigma^2\rho_T}$$

   (b) $U_M\left(I+f+\frac{\mu^2}{2\sigma^2\rho_M} - \frac{1}{2\sigma^2\rho_M}\left(\frac{\rho_T+\rho_M}{\rho_T}\right)^2\left(\mu-\sigma^2\frac{\rho_T\rho_M}{\rho_T+\rho_M}\right)\right) < u_M \leq U_M\left(I+f+\frac{\mu^2}{2\sigma^2\rho_M}\right)$

   $$r^\star = \hat{r}^-, \; obj = -\frac{\sigma^2\rho_T}{2}\left(\hat{r}^- - \left(1-\frac{\mu}{\sigma^2\rho_T}\right)\right)^2 - f + \frac{\mu^2}{2\sigma^2\rho_T}$$

3. $\frac{\rho_T \rho_M}{\rho_T+\rho_M} \geq \frac{\mu}{\sigma^2}$

(a) $u_M \leq U_M \left(I + f + \frac{\mu^2}{2\sigma^2\rho_M} - \frac{1}{2\sigma^2\rho_M} \left(\frac{\rho_T+\rho_M}{\rho_T}\right)^2 \left(\mu - \sigma^2 \frac{\rho_T\rho_M}{\rho_T+\rho_M}\right)\right)$

$$r^\star = 1 - \frac{\mu}{\sigma^2\rho_T}, \ obj = -f + \frac{\mu^2}{2\sigma^2\rho_T}$$

(b) $U_M \left(I + f + \frac{\mu^2}{2\sigma^2\rho_M} - \frac{1}{2\sigma^2\rho_M} \left(\frac{\rho_T+\rho_M}{\rho_T}\right)^2 \left(\mu - \sigma^2 \frac{\rho_T\rho_M}{\rho_T+\rho_M}\right)\right) < u_M \leq U_M \left(I + f + \frac{\mu^2}{2\sigma^2\rho_M}\right)$

$$r^\star = \hat{r}^+, \ obj = -\frac{\sigma^2\rho_T}{2}\left(\hat{r}^+ - \left(1 - \frac{\mu}{\sigma^2\rho_T}\right)\right)^2 - f + \frac{\mu^2}{2\sigma^2\rho_T}$$

where

$$\hat{r}^- = \frac{\mu - \sqrt{\mu^2 + 2\sigma^2\rho_M\left(I + f + \frac{1}{\rho_M}\ln(-u_M)\right)}}{\sigma^2\rho_M}, \ \hat{r}^+ = \frac{\mu + \sqrt{\mu^2 + 2\sigma^2\rho_M\left(I + f + \frac{1}{\rho_M}\ln(-u_M)\right)}}{\sigma^2\rho_M}$$

Based on the above results, for every cases, by envelope theorem, we can characterize the optimal $r^\star$ and $f^\star$ as follows

$$r^\star = \frac{\rho_T}{\rho_T+\rho_M}, \ f^\star = -I - \frac{1}{2\sigma^2\rho_M} + \frac{1}{2\sigma^2\rho_M}\left(\mu - \frac{\rho_T\rho_M}{\rho_T+\rho_M}\sigma^2\right)^2 - \frac{1}{\rho_M}\ln(-u_M).$$

This gives the condition where the optimal contract term includes positive fixed component:

$$u_M \geq c_2,$$

where

$$c_2 \equiv U_M\left(I + \frac{\mu^2}{2\sigma^2\rho_M} - \frac{1}{2\sigma^2\rho_M}\left(\mu - \frac{\rho_T\rho_M}{\rho_T+\rho_M}\sigma^2\right)^2\right).$$

And the envelop is directly computed as follows:

$$U_T\left(I + \mu - \frac{\sigma^2}{2}\frac{\rho_T\rho_M}{\rho_T+\rho_M} + \frac{1}{\rho_M}\ln(-u_M)\right).$$

Then, we have to consider the case where $u_M < c_2$: as we have seen above, this corresponds to the case where the optimal contract is a perfect commission contract. The Pareto frontier is characterized by the following maximization problem:

$$\begin{aligned}&\max_{r\in\mathbb{R}_+}\ \mathbb{E}\left[U_T(J+(1-r)S)\right]\\ &\text{s.t. } \mathbb{E}\left[U_M(I+rS)\right]\geq u_M.\end{aligned}$$

From Lemma 5, we transform the above into the following form:

$$\begin{aligned}&\max_{r\in\mathbb{R}_+}\ U_T(J+(1-r)\mu-\frac{\sigma^2(1-r)^2}{2}\rho_T)\\ &\text{s.t. } U_M(I+r\mu-\frac{\sigma^2r^2}{2}\rho_M)\geq u_M.\end{aligned}\tag{8}$$

The objective function is increasing in it input, and so we have the following problem

$$\begin{aligned}&\max_{r\in\mathbb{R}_+}\ -\frac{\sigma^2\rho_T}{2}r^2+(\sigma^2\rho_T-\mu)r-\frac{\sigma^2\rho_T}{2}+\mu+J\\ &\text{s.t. } -\frac{\sigma^2\rho_M}{2}r^2+\mu r+I+\frac{1}{\rho_M}\ln(-u_M)\geq 0.\end{aligned}$$

We can solve (8) directly. In high sales case, i.e. $\frac{\mu}{\sigma^2}>\rho_T$, depending on the value of u_M, the solution $r^\star(u_M)$ to the problem and the value of the problem, which is written as $u_T^\star(u_M)$, is determined as follows:

$$\begin{aligned}r^\star(u_M)&=\begin{cases}0 & \text{if } u_M\leq U_M(I)\\ r^-(u_M) & \text{otherwise}\end{cases},\\ u_T^\star(u_M)&=U_T(J+(1-r^\star(u_M))\mu-\frac{\sigma^2}{2}(1-r^\star(u_M))^2\rho_T),\end{aligned}$$

where

$$r_2^-(u_M)\equiv\frac{\mu-\sqrt{\mu^2+2\sigma^2\rho_M\left(I+\frac{1}{\rho_M}\ln(-u_M)\right)}}{\sigma^2\rho_M}.$$

In the cases of upper and lower middle sales, i.e. $\frac{\rho_M}{\rho_M+\rho_T}\leq\frac{\mu}{\sigma^2}\leq\rho_T$, the solutions are obtained as follows:

$$r^\star(u_M) = \begin{cases} 1 - \frac{\mu}{\sigma^2 \rho_T} & \text{if } u_M \le U_M\left(I + \frac{\mu^2}{2\sigma^2\rho_M} - \left(\frac{\rho_T+\rho_M}{\rho_T}\right)^2 \frac{1}{2\sigma^2\rho_M}\left(\mu - \frac{\rho_T\rho_M}{\rho_T+\rho_M}\sigma^2\right)^2\right) \\ r^-(u_M) & \text{otherwise} \end{cases},$$

$$u_T^\star(u_M) = U_T(J + (1 - r^\star(u_M))\mu - \frac{\sigma^2}{2}(1 - r^\star(u_M))^2 \rho_T).$$

In the low sales case, i.e. $\frac{\mu}{\sigma^2} \le \frac{\rho_M}{\rho_M+\rho_T}$, the solutions are obtained as follows:

$$r^\star(u_M) = \begin{cases} 1 - \frac{\mu}{\sigma^2 \rho_T} & \text{if } u_M \le U_M\left(I + \frac{\mu^2}{2\sigma^2\rho_M} - \left(\frac{\rho_T+\rho_M}{\rho_T}\right)^2 \frac{1}{2\sigma^2\rho_M}\left(\mu - \frac{\rho_T\rho_M}{\rho_T+\rho_M}\sigma^2\right)^2\right) \\ r^+(u_M) & \text{otherwise} \end{cases},$$

$$u_T^\star(u_M) = U_T(J + (1 - r^\star(u_M))\mu - \frac{\sigma^2}{2}(1 - r^\star(u_M))^2 \rho_T),$$

where

$$r^+(u_M) \equiv \frac{\mu + \sqrt{\mu^2 + 2\sigma^2\rho_M\left(I + \frac{1}{\rho_M}\ln(-u_M)\right)}}{\sigma^2\rho_M}.$$

By summarizing this result, we get the claim.

Proof of Proposition 2 First of all, from Proposition 1, we have the explicit locus of the equilibrium surplus splits as follows: when we denote the expected utilities of M and T by u_M and u_T,

$$u_T = \frac{-Bu_M}{\frac{\rho_T}{\rho_M}c_M + \left(B - \frac{\rho_T}{\rho_M}\right)u_M}. \tag{9}$$

Using (9),

$$
\begin{aligned}
\frac{\partial u_T}{\partial u_M} &= \frac{-B\frac{\rho_T}{\rho_M}c_M}{\left(\frac{\rho_T}{\rho_M}c_M + \left(B - \frac{\rho_T}{\rho_M}\right)u_M\right)^2} \\
&= \frac{-Bu_M}{\frac{\rho_T}{\rho_M}c_M + \left(B - \frac{\rho_T}{\rho_M}\right)u_M} \frac{\frac{\rho_T}{\rho_M}\frac{c_M}{u_M}}{\frac{\rho_T}{\rho_M}c_M + \left(B - \frac{\rho_T}{\rho_M}\right)u_M} \\
&= \frac{1}{1 + \left(\frac{\rho_M}{\rho_T}B - 1\right)\frac{u_M}{c_M}} \frac{u_T}{u_M} \\
&= \frac{u_T - c_T}{u_M - c_M} \frac{u_T}{c_T} \frac{c_M}{u_M}.
\end{aligned}
$$

The model indicates $u_H = u_M - \lambda\left(u_M - \hat{u}_M\right)^2$. Let $h(u_H)$ be the value of u_M which satisfies the above equation for a u_H. By implicit function theorem, we have the derivative of h:

$$
h'(u_H) = \frac{u_M}{u_H} = \frac{1}{1 - 2\lambda(u_M - \hat{u}_M)}.
$$

Now the condition of Nash bargaining solution is written as follows:

$$
\begin{aligned}
\frac{\partial u_T}{\partial u_H} &= -B^U \frac{u_T - c_T}{u_H - c_H} = -B^U \frac{u_T - c_T}{u_M - \lambda\left(u_M - \hat{u}_M\right)^2 - \left(c_M - \lambda\left(c_M - \hat{u}_M\right)^2\right)} \\
&= -B^U \frac{1}{1 - \lambda\left(u_M + c_M - 2\hat{u}_M\right)} \frac{u_T - c_T}{u_M - c_M}.
\end{aligned}
$$

By chain rule, we have the following

$$
\frac{\partial u_T}{\partial u_H} = \frac{\partial u_T}{\partial u_M} \frac{\partial u_M}{\partial u_H}.
$$

This implies that

$$
B^U = -\frac{1 - \lambda(u_M + c_M - 2\hat{u}_M)}{1 - 2\lambda(u_M - \hat{u}_M)} \frac{u_T}{c_T} \frac{c_M}{u_M}.
$$

So we are done.

Proof of Corollary 2 By Proposition 1 and Proposition 2, we have

$$B = \frac{\rho_T}{\rho_M} \frac{(u_M - c_M)\,(1 - 2\lambda(u_M - \hat{u}_M))\,B^U}{(1 - 2\lambda(u_M - \hat{u}_M))\,B^U u_M + (1 - \lambda(u_M + c_M - \hat{u}_M))c_M}.$$

Under Assumption 3, we have

$$\frac{(1 + 2\lambda\tilde{u}_M)\left(1 + B^U\right) - \lambda c_M}{\lambda\,(1 + 2B^U)}.$$

So we are done.