Endogenous Joint Venture Formation in Procurement Auctions

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Abstract

We study welfare implications of prohibiting or promoting joint venture formation in procurement auctions. Given the number of participating firms, when firms form joint venture, it creates a trade-off between an anti-competitive effect caused by the reduced number of bidders and efficiency gain driven by the cost synergies. When considering some alternative policies of prohibiting or further encouraging joint venture, however, the number of participating firms would change according to the the entry incentive. We thus build and estimate a two-stage structural model with endogenous joint venture formation and bidding. When prohibiting joint venture, procurement efficiency would be worsen, because the decrease in anti-competitive effects would not be large enough to offset the foregone cost synergies. We also find that, mildly reducing entry costs would be a key to achieve better procurement efficiency, because a too much reduction in entry costs would invite competitive joint venture, which discourage other firms' entries.

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1 Introduction

In government procurement, many countries allow firms to participate in auctions as a joint venture, aiming to generate cost synergies among the forming firms and eventually decrease the procurement costs. Moreover, allowing joint venture may increase the

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number of total bidders and make the bids more competitive, because the small-sized firms might join the auction as a joint venture who might not be able to bid otherwise. However, allowing for joint venture formation may also have some anti-competitive effects. Conditional on the entry decision, allowing for a joint venture may reduce the number of bidders as multiple bidders jointly submit a single bid. Also, if joint ventures have cost synergies, single bidders may have less incentive to enter as they would face tougher competitors. Clarifying the trade-off between cost synergies and reduced competition from joint bidding is key to understand the welfare implication of joint ventures.

The literature has accumulated the empirical evidences for the pro-competitiveness of joint venture in several regions including Japan, EU, and Austria (Iimi, 2004; Estache and Iimi, 2009; Branzoli and Decarolis, 2015; Gugler, Weichselbaumer and Zulehner, 2021). While these researches the cost synergies of joint ventures leads to the more efficient procurement, few papers consider the effect on the incentive to entry of allowing joint venture at in an integrated model. Hence, our primary objective is to build an integrated model of entry, joint venture formation, and procurement auction bidding and estimate the model to disentangle where the source of pro-competitive and anti-competitive effects lie.

Another benefit of our structural model is that we are able to identify what hinders the formation of joint ventures and evaluate it from the social welfare view point. To begin with, though joint ventures are allowed, and even occasionally encouraged by the governments, joint bidding is not so frequently observed. (Gugler, Weichselbaumer and Zulehner, 2021) notes that joint bids account for only 5% of total bids in Austria. Since the literature finds that joint venture exhibits synergies and create capability to potential bidders to enter, the infrequent practice of joint bidding suggests the existence of countervailing power to form a joint venture. We quantify this obstacle for a joint venture as an additional entry cost which seems due to searching for a good partner or bargaining process among a joint venture.

Our model is composed of two stages: in the first stage, the potential entrants form joint venture or decide to enter as a single bidder, and, in the second stage, the participants play the first price auction with bid preference weighted by the attached scores. We call the first stage “entry stage” and the second stage “bidding stage”. The primal novelty of our model is the endogeneous joint venture formation: in an auction, the potential entrants decide either “try to form a joint venture” or “enter as a single bidder” or “not entry” considering the ex-ante expected payoffs in the bidding stage. Note that the success of joint venture formation makes a difference in the expected payoffs in the bidding stage. In this sense, the entry stage is a kind of group formation game with externality, for which, to the best of our knowledge, there is not an estimable model associated with an adequate equilibrium or solution concept to capture the behavior of the agents. We develop an equilibrium concept to describe the behavior of the potential entrants in the entry stage while the whole model is still estimable. We hope the method is applicable to the more broad research topics including merger and matching market.

Our data set covers all the procurement auction of construction site in Japan from 2018, 2019, and 2020. In each auction, we have at most third stage while we focus on the auctions which ends in the first stage. We observe bid and score for each firm and can identify whether or not the firm is a joint venture. For each auction, we know the location of the construction, the expected cost of the construction and the date of the first bid. In this auction, each firm is scored to evaluate the quality of the project and the submitted bid is weighted by this score. When we focus on the auction part, our paper estimate the auction with bid preference or scoring auction. Although we assume a specific distribution on the effective bid, which is the ratio of the score to the submitted bid, we retrieve the cost and the score for each firm. As we mentioned in the later section, this type of complicated bid preference is rarely studied in the existing literature.

The main findings in the estimation are as follows. First, we find that joint venture is more competitive than a single bidder in the bidding stage. In other words, when forming a joint venture, the participant is likely to get the higher score and the lower cost compared to a single bidder. Second, in the entry stage, forming a joint venture needs about 39 million yen as an additional entry cost. This amounts to 22.523% of the average expected cost of auctions in our sample. Furthermore, we estimate the matching function of joint ventures. This function maps the total number of potential participants who try to form a joint venture to the number of successfully formed joint ventures. By studying the estimated matching function, we find that even if there are 10 firms trying to form joint venture, just 2 joint ventures are formed eventually. This reflects the difficulty of the search for the good partner.

Based on the estimated parameters, we analyze two alternative policies: prohibiting joint venture formation and encouraging joint venture formation. For the prohibiting case, by setting the additional entry cost high enough to banish the incentive to form a joint venture, we simulate the procurement auctions when the auctioneer prohibit the joint venture formation. As a result, we find this policy increases the procurement efficiency in the smaller-sized auctions while this effect disappears in the larger-sized auctions. This implies that as the auction gets bigger, the cost synergies from joint venture plays a bigger role to achieve the lower procurement cost. When the auction is not so big, it is better to induce more entry by prohibiting joint venture formation.

To simulate the case of encouraging joint venture formation, we reduce the additional cost gradually. Our simulation clarifies that the mild encouragement is a key to achieve the more efficient procurement. When the additional cost is much reduced, then the more appearance of joint venture hinders the entry of the other potential entrants because joint venture is more likely to win the auction due to the cost synergy. And this anti-competitive effect dominates the pro-competitive effect from the cost synergy. However, by mildly decreasing the additional cost, we successfully benefit the cost synergy of joint venture. This observation is common for any size of auctions.

1.1 Related Literature

Traditionally, joint bidding is studied in the context of common value auction where the auctioneer sells a natural resource like petroleum. The empirical and theoretical results about the competitive effect of joint bidding are mixed. Mead (1967) and Levin (2004) argue that the joint bidding plays like a bidding rings or a collusion and hence impedes the competitive bids among the bidders. Millsaps and Ott (1985), Moody and Kruvant

(1988), and Hendricks and Porter (1992) indicate the pro-competitive effects of joint bidding caused by pooling information, relaxing the budget constraint and risk sharing.

The theoretical literature on joint bidding is extended to include another form of auctions like a patent auction (Asker, Baccara and Lee, 2021), the private value auctions (Cho, Jewell and Vohra, 2002; Cantillon, 2008; Chatterjee, Mitra and Mukherjee, 2017) and the auction with subcontracting (Bouckaert and Van Moer, 2021). But we do not apply these models to the current estimation problem, because these papers consider the more simple setting than the actual auctions. Empirical studies about the joint bidding are carried out almost independently from the theoretical literature (Iimi, 2004; Estache and Iimi, 2009; Branzoli and Decarolis, 2015). These papers show the reduced form evidence to support the pro-competitive effect of joint bidding.

The literature on merger or joint venture in other context considers the similar problem as the problem of joint venture in auction. Weese (2015); Akkus, Cookson and Horta¥ccsu (2016); Uetake and Watanabe (2020) consider the model of matching market in several different areas. These papers estimate the effect of matching using moment inequality approach developed by Fox (2018); Pakes (2010) due to the multiple equilibrium. In contrast to these literature, we develop a model which has unique equilibrium in the entry stage, i.e., joint venture formation stage. Miller and Weinberg (2017) is one example of papers which treats the competition after forming a group. This paper focus on the joint venture in beer industry which come to produce one item jointly after the formation. Their method is similar to the conduct parameter approach which is not applicable to the current context.

The closest study to ours is Gugler, Weichselbaumer and Zulehner (2021), which carries out a structural estimation of the auction with joint bidding. They focus on the types of the firms to form a joint venture and analyze the difference between the case of restricting all joint ventures and the case in which joint venture is allowed only to the small-sized firms to conclude that the joint venture by large-sized firms is the driving force of pro-competitive effect. In contrast to this study, we do not use the identity of the firm in our analysis. Instead we focus on the number of joint ventures and the single bidders in each auction. Though this is a data limitation, our model can assure the unique equilibrium in the entry stage which is not guaranteed in Gugler, Weichselbaumer and Zulehner (2021). Additionally, we put a focus on the adjustment cost to form a joint venture while Gugler, Weichselbaumer and Zulehner (2021)does not care about this point.

We consider the first-price auction with bid preference. Krasnokutskaya and Seim (2011) is the first structural study of this type of auction. They pint out that the bid preference in the bidding stage distorts the incentive to entry of the unfavored group and argue that the careful setting of the bid preference is necessary to achieve both the efficient procurement and improving the minority status. Other papers also insist on this point (Corns and Schotter, 1999; Marion, 2007; Athey, Coey and Levin, 2013; Nakabayashi, 2013; Rosa, 2019). All of these studies focus on the bid preference for the small-sized firms: in the bidding stage, the firms categorized as "small-sized" according to the number of employees or the size of the budget get favored by weighting their bid at a fixed rate. While our paper also considers the relationship between the bid preference in the bidding stage and the entry behavior in the entry stage, the situation is different from these existing ones: the first point is that the bid preference is based on the score which is determined by the characteristics of the entrant such as the size, whether or not the entrant is a local one, and whether or not the entrant is joint venture, i.e., our bid preference is much more complicated. The second point is that the potential bidders can decide to get favored in the bidding stage by forming a joint venture. This endogeneous nature is not considered in the previous literature.

The last contribution of this paper is the empirical evidence of the adjustment cost when two or more firms form a consortium. This is a well-studied topic in management science, surveyed in (Graebner et al., 2017). To generate synergy by forming a joint venture or after merger, it is essential to adjust the culture and the customs among the related firms (Chatterjee et al., 1992; Stahl and Voigt, 2008; Reus and Lamont, 2009). And the knowledge transfer, which is obviously necessary to make a synergy, requires a lot communication (Larsson and Finkelstein, 1999; Bresman, Birkinshaw and Nobel, 2010).

We think the adjustment cost of this paper is an expression of this type of behavior. While almost all the discussions in the existing literature is qualitative ones, we contribute a new quantitative evidence to the existence and the size of this cost to from a consortium.

2 Industry background and data

We focus on the public procurement auctions held by the Regional Development Bureaus in 8 different areas in Japan, which are the local offices of the Ministry of Land, Infrastructure, Transport, and Tourism. Their main task is to manage and develop the public infrastructures under the direct control of the Ministry of Land, Infrastructure, Transport, and Tourism, such as roads, rivers, and dams. Though the system to determine the firm to get the order is different based on the size and the goal of the project, most of them are determined by a variation of auctions which we describe below[1].

We collect the data of all the auctions held from 2018 to 2020 in all Regional Development Bureaus and focus on the orders whose expected cost is set above 60 million yen because the smaller orders are not necessarily the targets of the auction. This set of auctions is categorized into the Normal Competitive Bid System. The total number of such auctions in our data is 7280. For each auction, the Regional Development Bureaus set the expected cost based on the market prices of the necessary materials and the human resources. To achieve the procurement with the lowest price, this expected cost works as an upper bound for the bidding price in each auction. I.e., when all the bids are larger than this value, the same auction is held on another day.

Based on the expected cost, we have several categories of auctions. The first category used in this paper is *WTO type* and *non-WTO type*. Most of our samples, 7154 auctions, are called non-WTO type. This is the set of auctions whose expected cost is below 680 million yen. The auctions larger than this value, 126 auctions, are called WTO type. This distinction is based on Agreement on Government Procurement, which aims for the Most-Favored-Nation Treatment in the procurement held by the government. To remove the distinction based on the origin of the entrant, WTO-type auctions adopt different

[1] The rate of the projects which are allocated via non-auction way is ? in our sample period.

Table 1. Summary Statistics

	# Auctions	Rate of JV auction	Prob. JV winning	Log of Expected cost Mean	Min	Med	Max	# Bidders Mean	Min	Med	Max
Total											
Non-WTO	7154	0.007	0.896	18.845	17.910	18.831	20.346	3.928	1	3	28
JV allowed	237	0.105	0.560	20.712	20.034	20.385	22.935	7.570	1	7	20
JV allowed and Non-WTO	111	0.054	0.333	20.165	20.034	20.157	20.346	6.063	1	5	17
WTO	126	0.151	0.632	21.194	20.356	20.992	22.935	8.897	1	9	20
Construction types in Non-WTO											
asphalt	722	0.000	-	18.668	17.911	18.554	20.243	4.424	1	4	15
civilengineering	4001	0.009	0.946	18.949	17.912	18.978	20.339	4.779	1	4	28
building	121	0.000	-	18.803	17.912	18.642	20.341	1.702	1	1	7
dredging	45	0.000	-	19.208	18.025	19.353	20.152	3.111	1	2	8
airconditioning	24	0.000	-	18.754	17.942	18.881	19.401	1.958	1	2	6
slope	97	0.000	-	18.734	17.916	18.746	19.689	2.464	1	2	8
electricity	192	0.000	-	18.733	17.916	18.687	20.198	2.984	1	2	11
maintenance	1263	0.006	1.000	18.674	17.911	18.607	20.332	2.045	1	1	13
bridge	86	0.047	0.250	19.456	17.939	19.437	20.346	4.477	1	3	16
painting	37	0.000	-	18.418	17.910	18.326	19.719	4.108	1	3	13
prestressed	78	0.000	-	19.275	17.994	19.234	20.341	3.859	1	3	14
machinery	158	0.000	-	18.759	17.951	18.719	20.112	1.810	1	1	7
substation	52	0.000	-	18.504	17.916	18.367	20.071	2.673	1	2	10
cement	32	0.000	-	19.091	18.409	19.032	20.217	4.375	1	4	8
communication	173	0.000	-	18.523	17.916	18.433	20.265	2.069	1	2	7
landscaping	61	0.000	-	18.252	17.923	18.113	19.465	3.623	1	3	11
prefab	2	0.000	-	18.129	17.958	18.129	18.301	1.000	1	1	1
pipe	8	0.000	-	18.634	18.279	18.754	18.921	1.500	1	2	2
woodenbuilding	1	0.000	-	18.795	18.795	18.795	18.795	1.000	1	1	1
grout	1	0.000	-	19.087	19.087	19.087	19.087	1.000	1	1	1

Notes. Rate of JV auctions is the rate of the number of the auctions in which at least one joint venture participate to the total number of auctions in each construction type.

than the other auctions. Hence, our primary analysis focuses on the non-WTO type auctions.

In all the auctions of the Normal Competitive Bid System, the bid submitted by the entrant is weighted with the "score" of the entrant, which is determined by the characteristics of the firm according to the pre-determined scoring rule. In particular, the government compares the bids using "effective bid", which is the rate of the score to the submitted bid. Let b_i and s_i be the bid and the score of the firm i. Then the effective bid of this firm is $B_i \equiv \frac{s_i}{b_i}$. The firm with the highest effective bid wins the auction: i.e., the firm with the higher score and submitting the lower bid among the entrants is likely to win the auction. The score is ranged from the baseline $\underline{s} = 100$ to 200 in our data, while the bids are in order of 10^8, and the effective bids are tiny values. Hence, hereafter, we usually take the logarithm of both bids and effective bids in figures or analysis.

This system aims for the "best value" in the procurement auction. As the above rule of computing an effective bid suggests, the firm submitting the lowest bid does not necessarily win the auction when the other firm is evaluated with the larger score. The government can choose the best firm from a non-monetary point of view by setting an appropriate scoring rule. For example, when the government tries to minimize the gender gap in this area, the government makes the scoring rule giving the higher score to the firms whose director of the project is female. Other examples of the factors leading to a higher score are (1) more experience with similar projects, (2) an effective recovery plan under disasters, and (3) expertise in the area, which is evaluated based on the experience of the projects in the same area.

The second category is *JV allowed type*. The auction, whose expected cost is more extensive than 500 million yen, is categorized into this type. In this category, the potential entrants are allowed to form joint ventures to manage a large-scale project beyond the capability of just one firm due to technical limitations or budget constraints. In our data, 237 auctions are categorized into this group. The score in the bidding stage of a joint venture is computed based on the merged values of the forming firms; hence, in particular, a firm outside the area forms a joint venture with a local firm to get a higher score in the bidding stage. This distorts the incentive to form a joint venture in each local auction. When forming a joint venture, the firms can use the platform prepared by the government to find partners. This platform allows the candidates to compute the expected score when he forms a joint venture. Furthermore, after forming a joint venture, the government provides consulting services to the forming firms to advise how to proceed with the operation and the administrative tasks fluently. We find these services in all eight areas in Japan. This implies that the current amount of joint ventures is not satisfactory for the government.

Table 1 shows the auction level summary statistics. In the upper panel, we show the results for the four groups: non-WTO, JV allowed and non-WTO, and WTO. The second column shows the rate of the auctions in which at least one joint venture participates to all the auctions in each category. Only 10.5% of the auctions in which forming a joint venture is allowed has at least one joint venture. By comparing the third and the fourth

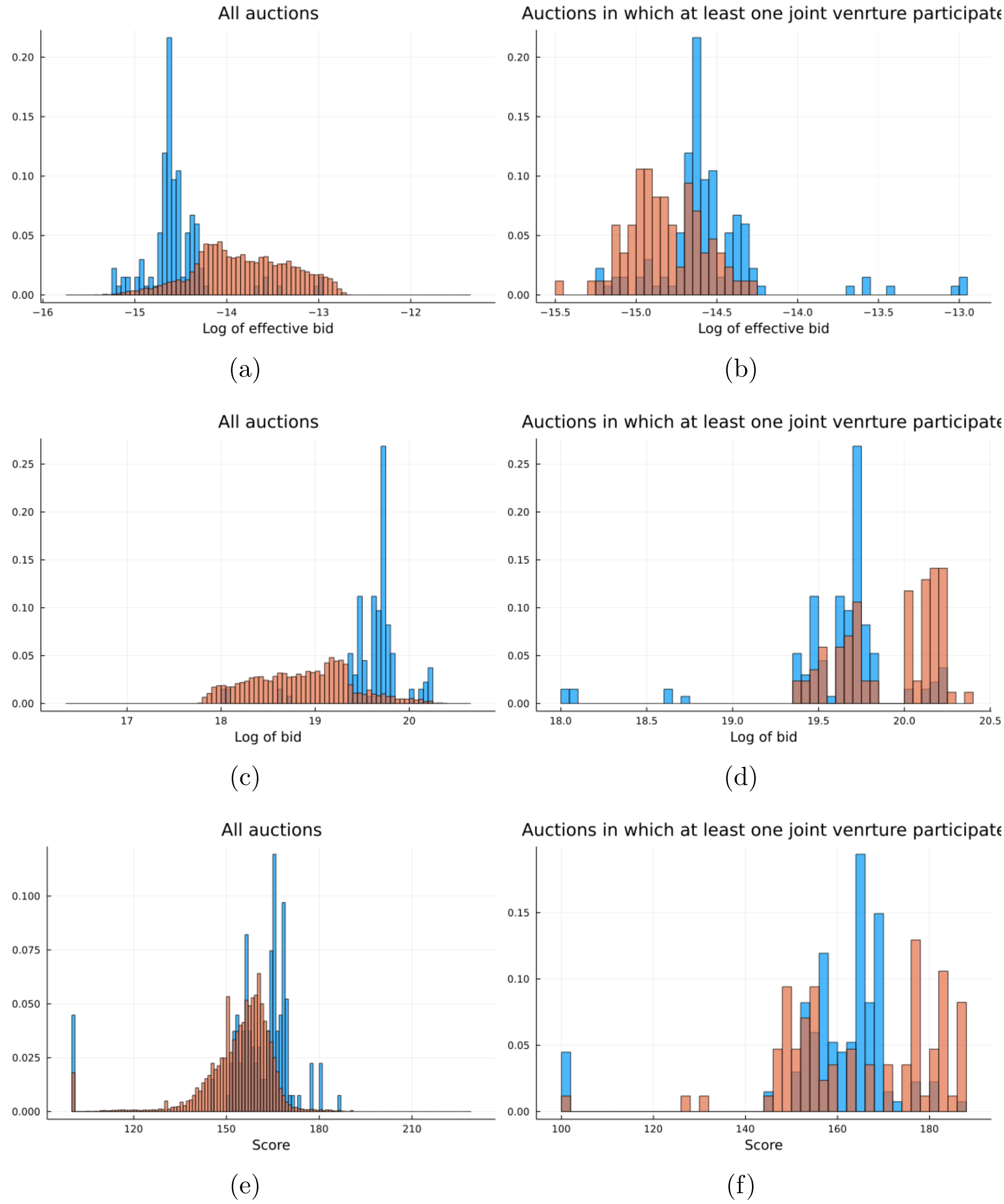


Figure 1. Histograms of effective bids, log of bids, and entrants' scores. Blue bins represent joint ventures, and red bins represent single bidders. In the left column, we use all the auctions. In the right column, we use the auctions in which at least one joint venture participates. The first row is about the log of effective bids, defined as the score's ratio to the bid. The second row is about the log of bids, and the third row is about the score.

row, joint ventures increase as the project size is bigger. Even in the largest class of the auctions, WTO type, 15.4% among them have at least one joint venture.

While the joint venture is rare, it is more likely to win the auction than a single bidder. This is shown in the third column, which computes the probability that a joint venture will win the auction in which at least one venture participates. Suppose the winning probability of a joint venture and one of a single bidder is not different. In that case, the probability of winning by a joint venture should be around the inverse of the mean of the number of the bidder in each category. The inverses are 0.255, 0.132, 0.165, and 0.112 for each four category. All of the actual probabilities are bigger than these inverses, which implies a joint venture is more likely to win the auction once it enters. This is motivational evidence for the severe cost of forming a joint venture in the entry stage because we can expect the high obstacle to forming a joint venture in the entry stage enough to cancel out the high expected payoff in the bidding stage.

The lower panel of Table 1 splits the non-WTO type auctions into subgroups based on the construction type. We find joint ventures in only three construction types: civil engineering, maintenance, and bridge. These three types consist 74.8% of all the auctions. As we see in the summary statistics about the expected cost, civil engineering and bridge are likely to belong to the group of expensive projects, and this is why joint ventures appear in them. In contrast, the projects grouped into maintenance are less expensive. Given that the maintenance projects are usually allocated to the local firms with expertise in the local area, the appearance of a joint venture in this group is due to the higher score given to the joint venture including local firms.

Figure 1 shows the distributions of firm-level bid information of non-WTO type: logarithm of effective bid ($\ln B_i$), logarithm of bid ($\ln b_i$), and score (s_i). Blue bins represent the bids of joint ventures, and red bins represent the bids of single bidders. The left column shows the histograms of the three variables of all the bids we observe in non-WTO type auctions, while the right column is about the bids observed in the auctions in which at least one joint venture exists. While we see the small effective bids by joint ventures in Panel (a), Panel (b) shows this is due to selective entry. As Panel

(c) shows that joint ventures enter the expensive project when we focus on the auctions in which at least one joint venture exists, the effective bids of joint ventures are more competitive than those of single bidders. As to the score, the distribution of the scores of joint ventures is bimodal, while one of the single bidders is unimodal. The right mode of the distribution of the scores of joint ventures is larger than the mode of the scores of single bidders. This gives a higher score for joint ventures on average.

3 Model

This paper describes procurement auctions in two stages: the entry stage and the bidding stage. These proceeds are as follows. In the entry stage, the potential entrants are the players. They decide whether or not to participate in the current auction and consider forming a joint venture with the other potential entrants. Given the choice of the potential entrants, the players in the bidding stage are determined. In the bidding stage, the entrants participate in an auction described above, given the score determined by the scoring rule. By comparing the effective bids, the auctioneer decides who wins the auction. In the following, we describe the details of the two stages.

3.1 Bidding stage model

We use the following notations for the auctions with bid-preference; given an auction, i denotes the index of the entrants in the auction, s_i denoted the score of the firm i, b_i denoted the bid of the firm i, and the effective bid is denoted by $B_i \equiv \frac{s_i}{b_i}$. The expected cost of the project is denoted by p. When the entrant i wins the auction, the payoff is as follows:

$$b_i - \tilde{c}_i p = \frac{s_i}{B_i} - \tilde{c}_i p,$$

where $\tilde{c}_i$ is the individual cost factor, i.e., the cost of the entrant i is equal to $\tilde{c}_i p$. The multiplication form in the cost follows the unobserved cost factor model of Krasnokutskaya and Seim (2011). We assume that the score and the individual cost factor are unobservable by the other entrants while we observe all the elements except the firm specific cost factor $\tilde{c}_{ij}$.

The number of the joint venture is denoted by m, and the number of single bidders is denoted by n. M, N represents the set of each type of bidders. We describe the optimization problem for a single bidder i in such an auction as follows: where we denote the effective bid of a single bidder and a joint venture by the upper script,

$$\max_B \; Pr\left(B_j^S \le B \; \forall j \in N, \; B_j^{JV} \le B \; \forall j \in M\right) \left(\frac{s_i}{B} - \tilde{c}_i p\right).$$

By taking the logarithm of the effective bids in the probability term, we have the following equivalent problem:

$$\max_B \; Pr\left(\ln B_j^S \le \ln B \; \forall j \in N, \; \ln B_j^{JV} \le \ln B \; \forall j \in M\right) \left(\frac{s_i}{B} - \tilde{c}_i p\right).$$

We assume the pair of score and individual cost factor is drawn independently from an identical distribution for both types of the entrant.

Assumption 1 *If the entrant i s a single bidder, the pair of the score and the individual cost factor, $(s_i, \tilde{c}_i)$, is drawn independently from G^S. If i is a joint venture, $(s_i, \tilde{c}_i)$, is drawn independently from G^{JV}. And we denote the distribution of $\frac{\tilde{c}_i}{s_i}$ of single bidders and the same of joint ventures by $\tilde{G}^S$ and $\tilde{G}^{JV}$.*

Under this assumption, and assuming that this auction has a unique equilibrium, the effective bids of single bidders and joint ventures follow two distinct distributions independently in the equilibrium. We denote these by F^S and F^{JV}. Then the payoff maximization problem of a single bidder i is written as follows:

$$\max_B \; F^S(\ln B)^{N-1} F^{JV}(\ln B)^M \left(\frac{s_i}{B} - \tilde{c}_i p\right). \tag{1}$$

We write the maximized expected payoff by $V^S(\frac{\tilde{c}_i}{s_i}; M, N, p)$ and the same one for joint venture by $V^{JV}(\frac{\tilde{c}_i}{s_i}; M, N, p)$.

3.1.1 Equilibrium

Here we discuss the existence and the uniqueness of the equilibrium in this auction according to the argument of Lebrun (1999), which argues about the existence and the uniqueness of the Bayesian Nash equilibrium in the first price asymmetric auction. Though the current situation is different, we adopt a similar argument to prove the equilibrium's existence and uniqueness.

First, we take the first order condition of the problem (1) given the distributions of effective bids to characterize the bidding strategy: for a single bidder and for joint venture,

$$\begin{cases} 1 - \frac{\tilde{c}_i}{s_i} pB = \frac{1}{(N-1)\frac{f^S(\ln B)}{F^S(\ln B)} + M\frac{f^{JV}(\ln B)}{F^{JV}(\ln B)}}, \\ 1 - \frac{\tilde{c}_i}{s_i} pB = \frac{1}{N\frac{f^S(\ln B)}{F^S(\ln B)} + (M-1)\frac{f^{JV}(\ln B)}{F^{JV}(\ln B)}}. \end{cases} \tag{2}$$

The optimal effective bid depends on the private information only through $\frac{\tilde{c}_i}{s_i}$. Hence, given the number of entrants of both types and the expected cost, bysolving the above two conditions, we get the optimal bidding strategies $B_S\left(\cdot\,; F^S, F^{JV}\right)$ and $B_{JV}\left(\cdot\,; F^S, F^{JV}\right)$ which map $\frac{\tilde{c}_i}{s_i}$ to the effective bid. We define an equilibrium in this game as follows.

Definition 1 *The pair of two invertible functions mapping $\frac{\tilde{c}}{s}$ to an effective bid, $B_S^\star(\cdot)$ and $B_{JV}^\star(\cdot)$, is an equilibrium, if it satisfies*

$$\begin{cases} B_S^\star(\cdot) = B_S\left(\cdot\,; F^S, F^{JV}\right) \\ B_{JV}^\star(\cdot) = B_{JV}\left(\cdot\,; F^S, F^{JV}\right) \end{cases} \tag{3}$$

where

$$\begin{cases} F^S(\ln B) \equiv Pr\left(B_S^\star\left(\frac{\tilde{c}_i}{s_i}\right) \le B\right) \\ F^{JV}(\ln B) \equiv Pr\left(B_{JV}^\star\left(\frac{\tilde{c}_i}{s_i}\right) \le B\right). \end{cases}$$

Assume there is an equilibrium in this game and the bidding strategy is decreasing in the fraction $\frac{\tilde{c}_i}{s_i}$. This means that when the firm faces a higher cost or the score is lower, i.e., the firm is more competitive, the firm submits lower effective bids. If the bidding strategy is increasing in $\frac{\tilde{c}_i}{s_i}$, the reverse is true; then each entrant has an incentive to deviate to submitting more competitive bids. Hence it is not restrictive that we assume the bidding strategy is decreasing in $\frac{\tilde{c}_i}{s_i}$. Under an equilibrium, the problem (1) is transformed into the following: note that here we use the fact that $B_S^\star(\cdot)$ and $B_{JV}^\star(\cdot)$ are decreasing,

$$\max_B \left(1 - \tilde{G}^S\left(B_S^{\star,-1}(B)\right)\right)^{N-1} \left(1 - \tilde{G}^{JV}\left(B_{JV}^{\star,-1}(B)\right)\right)^M \left(\frac{s_i}{B} - \tilde{c}_i p\right).$$

We have the analogous problem for a joint venture. By taking the first order conditions for both types and solving the system of differential equations, we have the following two differential equations:

$$\begin{cases} \frac{d}{dB} B_{JV}^{\star,-1}(B) = \frac{1}{N+M-1} \frac{1-\tilde{G}^{JV}\left(B_{JV}^{\star,-1}(B)\right)}{\tilde{g}^{JV}\left(B_{JV}^{\star,-1}(B)\right)} \frac{1}{B} \frac{1}{B_S^{\star,-1}(B)Bp-1}, \\ \frac{d}{dB} B_S^{\star,-1}(B) = \frac{1}{N+M-1} \frac{1-\tilde{G}^{S}\left(B_{S}^{\star,-1}(B)\right)}{\tilde{g}^{S}\left(B_{S}^{\star,-1}(B)\right)} \frac{1}{B} \frac{1}{B_S^{\star,-1}(B)Bp-1}. \end{cases}$$

Focus on the equation of joint venture. Note that the right hand side of the equation is continuous in B over $[0, \frac{s_i}{\tilde{c}_i p}]$ for each i. We further assume that the right hand side is Lipschitz continuous w.r.t. $B_S^{\star,-1}(B)$.

Assumption 2 $\frac{1-\tilde{G}^{JV}(\alpha)}{\tilde{g}^{JV}(\alpha)} \frac{1}{\alpha Bp-1}$ *is Lipschitz continuous w.r.t.* α.

We assume the same one for the single bidder. Under these assumptions, the solution of the differential equation of each type is unique given an initial condition. This implies that the equilibrium of this auction is unique up to the initial condition if there is at least one equilibrium. Note that no firm bids the bid bigger than the cost: for every entrant i, we have $b_i > \tilde{c}_i p \Leftrightarrow \frac{\tilde{c}_i}{s_i} pB_i - 1 < 0$. This implies that $\frac{d}{dB} B_{JV}^{\star,-1}(B)$ and $\frac{d}{dB} B_S^{\star,-1}(B)$ are negative. Hence, $B_{JV}^\star(\cdot)$ and $B_S^\star(\cdot)$ are decreasing, which does not contradict with our assumption.

3.1.2 Ex ante expected payoff

To consider the incentive in the entry stage, we compute the ex ante expected payoff in the bidding stage. The ex ante expected payoffs is defined for each pair of (M, N, p) and the type of the entrant and we denoted them by $u_j(M, N; p)$ and $u_s(M, N; p)$ where the upper script represents the type of the entrant.

$$\begin{cases} u_j(M, N; p) = E\left[V^{JV}(\frac{\tilde{c}_i}{s_i}; M, N, p)\right], \\ u_s(M, N; p) = E\left[V^{S}(\frac{\tilde{c}_i}{s_i}; M, N, p)\right]. \end{cases}$$

The expectations are taken with respect to the realization of the fraction $\frac{\tilde{c}_i}{s_i}$.

3.2 Entry stage

Let the upper bounds for M, N be $\bar{M}, \bar{N}$. Here, we fix an expected cost p and so u_{JV}, u_S are just matrices whose size is $\bar{M} \times \bar{N}$. When the firm enters an auction, he must pay the entry cost. Furthermore, if the firm forms a JV, he suffers more cost because JV formation needs negotiating cost. These two costs are the parameters we estimate in this entry stage. The baseline cost which emerges in the single bidder entry is denoted by c_S and the total cost merging in a JV entry is denoted by c_{JV}.

In the current model, we do not know the detail process of the joint venture formation hence we leave a part of the model unspecified and take a reduced form approach; we introduce a matching function ϕ. The second set of parameters we want to estimate is the parameters determining the form of the ϕ. This corresponds to the estimation of the pattern of JV formations.

Given c_j, c_s, α, the timing of the entry/joint venture formation game in one auction is as follow.

1. A set of potential bidder realizes exogenously. The number of them is L.

2. Each potential bidder takes an action from the following four candidate.

- Try to form a JV and enter as a Single if he does not form a JV, which is denoted by (JV, S),
- Try to form a JV and do not enter if he does not form a JV, which is denoted by (JV, N),
- Enter as a single bidder, which is denoted by (S), and
- No entry, which is denoted by (N).

Here the firm chooses the action giving the highest expected payoff given the prediction over the realizations of pair of (M, N) and the entry cost parameters c_{JV}, c_S and the action-firm specific individual cost term ϵ which is assumed to follow Type I error distribution. The prediction is determined in an equilibrium. The detail of this process and the equilibrium is in section **??**.

3. The number of potential bidders choosing each action is L_1, L_2, L_3 and $L - (L_1 + L_2 + L_3)$. Given the set of potential bidder trying to form a JV, $L_J \equiv L_1 + L_2$, the number of JV, M, realizes according to the matching function, ϕ, as

$$M = \phi(L_J; \alpha).$$

4. The number of single firm entrants is determined not deterministic. Given the number of JVs (M) and the set of actions chosen $((L_1, L_2, L_3))$, the distribution of the number of sindle bidders, which is denoted by N, set as in the section 3.2.3. In essence, we choose the firms who can form JVs randomly and the remaining firms who chooses (JV, S) enter as additional single bidders.

5. The total number of JV, M, and the total number of single bidder, N, realize.

Because we do not use the identity of each firm, what we must determine is the distribution over the pair of (M, N). And the above process is sufficient to specify the distribution.

3.2.1 Equilibrium in entry game

Consider the case of Seim (2006). The notations are as follows:

- $\mathcal{F}$ is the set of potential entrants.
- For market m, the possible location to enter is denoted by $l = 0, 1, \cdots, \mathcal{L}^m$ where 0 means "no entry".
- $f = 1, \cdots, \mathcal{F}$'s decision is $\mathbf{d}_f = (d_{f0}, d_{f1}, \cdots, d_{f\mathcal{L}^m})$ where $d_{fl} = 1$ when f enters the location l.

Hereafter, omit m representing the market. The profit for firm f when choosing l is

$$\Pi(l_f, l_{-f}, \epsilon_f) = \delta_{l_f} + \sum_b \gamma_b N_b(l_f, l_{-f}) + \epsilon_{fl}$$

where

- δ_l is the location specific profit term that is partly parametrized,
- $\sum_b \gamma_b N_b$ is the competition term, b denotes the location band from l and N_b is the number of firms entering the location placed within the location band b from l: i.e., when $d_{k,l}$ is the distance between location k and l,

$$N_b = \sum_k 1\{D_b < d_{k,l} < D_{b+1}\} n_k(l_f, l_{-f})$$

- ϵ_{fl} is the error term.

In each market, the firm does not know the value of the error terms for the other firms. So their decisions are uncertain for him. The pure strategy of firm f is denoted by $s_f(\epsilon_f) : \mathbb{R}^{\mathcal{L}} \to \mathcal{L}$ where $\epsilon_f = (\epsilon_{f1}, \cdots, \epsilon_{f\mathcal{L}})$. Now we define the expected payoff for the choice of l, given the strategies of the other firms,

$$\bar{\Pi}(l_f, s_{-f}, \epsilon_f) = \int \Pi(l_f, s_{-f}(\epsilon_{-f}), \epsilon_f) dP(\epsilon_{-f})$$

We consider Bayesian Nash equilibrium realizes in this market.

Definition 2 *The profile of strategies* $s^\star = (s_1^\star, \cdots s_{\mathcal{F}}^\star)$ *is BNE if and only if*

$$\bar{\Pi}(s^\star(\epsilon_f), s_{-f}^\star, \epsilon_f) \geq \bar{\Pi}(l_f, s_{-f}^\star, \epsilon_f),\ \forall f,\ \forall l_f,\ \forall \epsilon_f.$$

We consider another representation of BNE. We call the probability distribution over the action profile of the other firms "prediction" and denote it by $m_{-f}(l_{-f})$. Prediction-based strategy is denoted by $t_f(\epsilon_f, m_{-f})$ that specifies the optimal action for the firm f with ϵ_f thinks that the actions follow m_{-f}. Then we have the following equivalent expression of BNE.

Proposition 1 $s^\star = (s_1^\star, \cdots s_{\mathcal{F}}^\star)$ *is BNE, then there exists the set of* $t_f^\star$ *and* $m_f^\star$ *such that*

$$\sum_{l_{-f}} m_{-f}^\star(l_{-f}) \Pi(t_f^\star(\epsilon_f, m_{-f}^\star), l_{-f}, \epsilon_f) \geq \sum_{l_{-f}} m_{-f}^\star(l_{-f}) \Pi(l, l_{-f}, \epsilon_f),\ \forall f,\ \forall l,\ \forall \epsilon_f \tag{4}$$

and

$$m_f^\star(l) = Pr\left(t_f^\star(\epsilon_f, m_{-f}^\star) = l\right). \tag{5}$$

Furthermore, when we have $t_f^\star$ *and* $m_f^\star$ *satisfying the above two conditions, then we construct a BNE* $s^\star$*.*

Seim (2006) uses this another characterization to describe BNE. Note that the prediction in Seim (2006) is sufficient to describe the expected payoff., i.e.

$$\begin{aligned}\sum_{l_{-f}} m_{-f}^\star(l_{-f}) \Pi(l, l_{-f}, \epsilon_f) &= \sum_{l_{-f}} m_{-f}^\star(l_{-f}) \left(\delta_l + \sum_b \gamma_b N_b(l, l_{-f}) + \epsilon_{fl}\right) \\ &= \delta_l + \epsilon_{fl} + \sum_b \gamma_b \sum_{l_{-f}} m_{-f}^\star(l_{-f}) N_b(l, l_{-f}).\end{aligned}$$

However, this is not the case for the current model.

3.2.2 Equilibrium in the current model

In the current model, we do not know the detail process of the joint venture formation and it is stochastic whether or not the firm can form a JV when he chooses (JV, S) or (JV, N). This is the reason why we need a matching function to compute the expected utility for each action in contrast to the case of Seim (2006). First, we clarify this point.

When the firm chooses (JV, S), the prediction is the distribution over the pairs of number of JVs and singles and the success of forming JV, that is denoted by $H(\delta, M, N \mid JV, S)$ where δ is the dummy variable representing the success of forming a JV. We have the following decomposition:

$$
\begin{aligned}
& H(\delta, M, N \mid JV, S) \\
& = \sum_{L_1, L_2, L_3} Pr(\delta, M, N \mid L_1, L_2, L_3\ ;\ (JV, S)) \times\ Pr(L_1, L_2, L_3\ ;\ (JV, S)).
\end{aligned}
$$

Assuming that all the firms have the same prediction, $Pr(L_1, L_2, L_3\ ;\ (JV, S))$ can be computed based on the prediction. But the term $Pr(\delta, M, N \mid L_1, L_2, L_3\ ;\ (JV, S))$ is not determined then the prediction is not sufficient to specify the expected utility:

$$
\bar{E}(JV, S) = \int (\delta u_{JV}(M, N) + (1 - \delta) u_S(M, N))\, dF(\delta, M, N \mid JV, S).
$$

This is the difference from Seim (2006).

According to the above observation, we define an equilibrium. First we denote the prediction by p, which is $(H(\delta, M, N \mid (JV, S)), H(\delta, M, N \mid (JV, N)), H(M, N \mid (S)))_{\delta, M, N}$. We need expected utility maximization condition.

$$
\bar{E}(t_f(\epsilon_f, p)) + \epsilon_{f, t_f(\epsilon_f, p)} \geq \bar{E}(a) + \epsilon_{f,a},\ \forall f,\ \forall a,\ \forall \epsilon_f. \tag{6}
$$

Then, given p, we can compute the probability of each action. The probability of choosing action a is $m_f(a, p) \equiv Pr(t_f(\epsilon_f, p) = a)$ and then we can compute the following probability:

$$Q(L_1, L_2, L_3; p) = Pr\left(L_1 \text{ chooses (JV,S)}, L_2 \text{ chooses (JV,N)}, L_3 \text{ chooses S}\right).$$

Using this notation, the prediction is written as follows:

$$H(\delta, M, N \mid JV, S) = \sum_{L_1, L_2, L_3} r(\delta, M, N \mid L_1, L_2, L_3 \ ; \ (JV, S))Q(L_1, L_2, L_3 \ ; \ (JV, S), p),$$

where $r(\delta, M, N \mid L_1, L_2, L_3)$ is the conditional probability that is specified in section 3.2.3. Define a $5K \times L$ matrix R containing $r(\delta, M, N \mid L_1, L_2, L_3)$ and a $L \times 1$ vector $Q(p)$ containing $Q(L_1, L_2, L_3; p)$ where K is the number of all the cmobinations of (M, N) from $M \in [1, \cdots, \bar{M}]$ and $N \in [1, \cdots, \bar{N}]$ and L is the number of all the combinations of L_1, L_2 and L_3. Then, the condition like (5) is written as follows:

$$p = RQ(p). \tag{7}$$

We define an equilibrium:

Definition 3 *Given R, $(\{t_f\}_{f \in \mathcal{F}}, p)$ is an equilibrium if and only if the tuple satisfies (6) and (7).*

We can assure there exists a unique equilibrium when the number of potential entrants is not so large. This result is not specific to the specification of R we explain in section 3.2.3.

Proposition 2 *(1) $RQ(p)$ is a self-map on $\Delta_{2K} \times \Delta_{2K} \times \Delta_K$. (2) When $\ln L - \frac{1}{3} < \underline{u}(c_j, c_s)$, $RQ(p)$ is a contraction mapping, where $\underline{u}(c_j, c_s)$is the minimum value among the net expected payoffs, and then there exists a unique fixed point $p^\star$ in equation 7.*

3.2.3 How to construct R

Here we specify the way to define R. Let (L_1, L_2, L_3) be the number of firms which chooses (JV, S), (JV, N), and S.

Now we define the matching function as follows;

$$\phi(L_J) = \lfloor \alpha \ln \frac{L_J}{2} \rfloor.$$

While this specification is somewhat restrictive, as we mention later, this is sufficient to express all the reasonable patterns of JV formations.

Hereafter, we describe the way to compute $r(\delta, M, N \mid L_1, L_2, L_3; a)$ for each $a \in \{(JV, S), (JV, N), (S)\}$.

(1) When a firm chooses (JV, S)

Note that we consider the following decomposition:

$$\begin{aligned} r(\delta = 1, M, N \mid L_1, L_2, L_3; (JV, S)) &= r_1(M, N \mid L_1, L_2, L_3, \delta = 1; (JV, S)) \\ &\quad \times r_2(\delta = 1 \mid L_1, L_2, L_3; (JV, S)). \end{aligned}$$

Fix the parameters α. First, the number of formed JV is determined as $M_J = \lfloor \alpha \ln \frac{L_1 + L_2}{2} \rfloor$.

Second, we assume that all the combinations to form JVs are equally possible and we compute r_2 as follows:

$$\begin{aligned} r_2(\delta = 1 \mid L_1, L_2, L_3; (JV, S)) &= \frac{{}_{L_1+L_2-1}C_{2L_J-1} \times {}_{2L_J}C_2 \times \cdots \times {}_2C_2}{{}_{L_1+L_2}C_{2L_J} \times {}_{2L_J}C_2 \times \cdots \times {}_2C_2} \\ &= \frac{{}_{L_1+L_2-1}C_{2L_J-1}}{{}_{L_1+L_2}C_{2L_J}} = \frac{2L_J}{L_1 + L_2}. \end{aligned}$$

By the same computation, we have

$$r_2(\delta = 0 \mid L_1, L_2, L_3; (JV, S)) = \frac{L_1 + L_2 - 2L_J}{L_1 + L_2}$$

Third, we compute r_1 as follows:

$$r_1(M,N \mid L_1,L_2,L_3,\delta=1;(JV,S)) = \begin{cases} 0 \text{ if } M \neq L_J \text{ or } N-L_3<0 \\ \frac{{}_{L_1-1}C_{N-L_3} \times {}_{L_2}C_{2L_J-(L_1-(N-L_3))} \times {}_{2L_J}C_2 \times \cdots \times {}_2C_2}{{}_{L_1+L_2-1}C_{2L_J-1} \times {}_{2L_J}C_2 \times \cdots \times {}_2C_2} \text{ otherwise} \end{cases}$$

$$= \begin{cases} 0 \text{ if } M \neq L_J \text{ or } N-L_3<0 \\ \frac{{}_{L_1-1}C_{N-L_3} \times {}_{L_2}C_{2L_J-(L_1-(N-L_3))}}{{}_{L_1+L_2-1}C_{2L_J-1}} \text{ otherwise} \end{cases}$$

By the similar computation, we have

$$r_1(M,N \mid L_1,L_2,L_3,\delta=0;(JV,S)) = \begin{cases} 0 \text{ if } M \neq L_J \text{ or } N-L_3<0 \\ \frac{{}_{L_1-1}C_{N-L_3-1} \times {}_{L_2}C_{2L_J-(L_1-(N-L_3))}}{{}_{L_1+L_2-1}C_{2L_J}} \text{ otherwise} \end{cases}$$

Combine all the components described above to compute the elements of R.

(2) When a firm chooses (JV, N)

We also compute r_1 and r_2. r_2 is the same as in the case of (JV,S).

$$r_2\left(\delta=1 \mid L_1,L_2,L_3;(JV,N)\right) = \frac{2L_J}{L_1+L_2}$$

$$r_2\left(\delta=0 \mid L_1,L_2,L_3;(JV,N)\right) = \frac{L_1+L_2-2L_J}{L_1+L_2}$$

For r_1, we get the followings by the similar computation in the above case.

$$r_1(M,N \mid L_1,L_2,L_3,\delta=1;(JV,N)) = \begin{cases} 0 \text{ if } M \neq L_J \text{ or } N-L_3<0 \\ \frac{{}_{L_1}C_{N-L_3} \times {}_{L_2-1}C_{2L_J-(L_1-(N-L_3))-1}}{{}_{L_1+L_2}C_{2L_J}} \text{ otherwise} \end{cases}$$

$$r_1(M,N \mid L_1,L_2,L_3,\delta=0;(JV,N)) = \begin{cases} 0 \text{ if } M \neq L_J \text{ or } N-L_3<0 \\ \frac{{}_{L_1}C_{N-L_3} \times {}_{L_2-1}C_{2L_J-(L_1-(N-L_3))}}{{}_{L_1+L_2-1}C_{2L_J-1}} \text{ otherwise} \end{cases}$$

(3) When a firm chooses (S)

We need

$$F(M, N; (S)) = \sum_{L_1, L_2, L_3} r(M, N \mid L_1, L_2, L_3; (S)) \times Q\left(L_1, L_2, L_3; (S)\right).$$

Here r is specified as follows:

$$r(M, N \mid L_1, L_2, L_3; (S)) = \begin{cases} 0 \text{ if } M \neq L_J \text{ or } N - L_3 < 0 \\ \frac{{}_{L_1}C_{N-L_3} \times {}_{L_2}C_{2L_J-(L_1-(N-L_3))}}{{}_{L_1+L_2-1}C_{2L_J}} \text{ otherwise} \end{cases}.$$

The last probability is the probability that $N - L_3$ firms fail to form a JV among the firms which chooses (JV,S).

By stacking $r\left(\delta, M, N \mid L_1, L_2, L_3; a\right)$ for all $a \in \{(JV, S), (JV, N), (S)\}$ and all the combinations of (L_1, L_2, L_3), we can make a matrix R.

4 Estimation

Here, we describe the estimation strategies for the two stages separately. In the bidding stage, the estimation target is the distributions of the pair $(\tilde{c}_i, s_i)$ of a joint venture and a single bidder. This is achieved by retrieving the individual cost factor $\tilde{c}_i$ from the first order condition. Given the estimated distributions, we then estimate the ex-ante expected payoffs, u_{JV}, u_S, for the different levels of the expected cost. In the entry stage, we use the expected ex-ante payoffs to estimate the entry cost of a joint venture and a single bidder. Furthermore, we estimate the matching function, determining how many joint ventures are formed in an auction.

4.1 Estimation of bidding sage

Hereafter, we denote the index of auction by the subscript j. From equation (2), we have the following inversions for the cost of both types:

$$\tilde{c}_{ij} = \frac{s_{ij}}{p_j B_{ij}} \left(1 - \frac{1}{(N_j - 1)\frac{f^S(\ln B_{ij})}{F^S(\ln B_{ij})} + M_j \frac{f^{JV}(\ln B_{ij})}{F^{JV}(\ln B_{ij})}} \right) \tag{8}$$

for a single bidder, and

$$\tilde{c}_{ij} = \frac{s_{ij}}{p_j B_{ij}} \left(1 - \frac{1}{N_j \frac{f^S(\ln B_{ij})}{F^S(\ln B_{ij})} + (M_j - 1) \frac{f^{JV}(\ln B_{ij})}{F^{JV}(\ln B_{ij})}} \right) \tag{9}$$

for a joint venture. If we can estimate the two distributions of the effective bids for a single bidder and a joint venture, we can retrieve the individual cost factor for each firm using the above equation using these equations.

Now we describe the way to estimate the distributions of the effective bids. Note that the optimization problem implies that the optimal level of effective bid is dependent on the combination of (p, M, N) and whether or not the firm is a single bidder or a JV. First we assume that the logarithm of the effective bids follow a Normal distribution.

Assumption 3

$$\ln B_{ij}^{JV} \sim N\left(\theta_j^{JV}, \sigma_j^{2,JV}\right), \ \ln B_{ij}^{S} \sim N\left(\theta_j^{S}, \sigma_j^{2,S}\right)$$

We estimate these distributions by sieve methods, i.e., we estimate the mean and the variance using the following regerssion:

$$\ln B_{ij} = poly(d_{ij}^{JV}, p_j, M_j, N_j; \beta) + \epsilon_{ij}.$$

where d_{ij}^{JV} is the dummy variable for the joint venture and $poly(X, \beta)$ represents the polynomial function of the variables X with some degree when the coefficients are β. Using all the data, run the above regression and denote the estimated coefficients by $\hat{\beta}$.

Given this result, the estimated distributions in auction j are as follows:

$$\ln B_j^{JV} \sim N\left(poly(d_{ij}^{JV}=1, p_j, M_j, N_j; \hat{\beta}),\ \hat{\sigma}^{2,JV}\right), and$$
$$\ln B_j^{S} \sim N\left(poly(d_{ij}^{JV}=0, p_j, M_j, N_j; \hat{\beta}),\ \hat{\sigma}^{2,S}\right).$$

where the estimated variances are

$$\hat{\sigma}^{2,JV} = \frac{1}{\#JV} \sum_{ij:d_{ij}^{JV}=1} \left(\ln B_{ij} - poly(d_{ij}^{JV}=1, p_j, M_j, N_j; \hat{\beta})\right)^2, and$$
$$\hat{\sigma}^{2,S} = \frac{1}{\#Single} \sum_{ij:d_{ij}^{JV}=0} \left(\ln B_{ij} - poly(d_{ij}^{JV}=0, p_j, M_j, N_j; \hat{\beta})\right)^2.$$

Based on the estimated distributions, we retrieve the individual cost factors by the equation (8) and (9).

4.1.1 Estimate expected payoffs

Then we compute the ex-ante expected payoffs based on the estimated cost and the estimated distributions. In particular, we compute the followings: $u_{JV}(M, N, p)$ and $u_S(M, N, p)$. To do so, we split the space of p into 10 sub-groups and consider all the auctions whose p's are in the same sub-group and have the same value of p. In particular, we take the maximum and the minimum of the expected cost and denote them by $\bar{p}$ and $\underline{p}$. We take 10 representative points in the space of the logarithm of the expected cost: for $p_k \in \{0.05, 0.15, 0.25, \cdots, 0.85, 0.95\}$, we make a representative point $(\ln \bar{p} - \ln \underline{p}) \times p_k + \ln \underline{p}$. Let the upper bounds of M, N be $\bar{M}, \bar{N}$. Then the number of expected payoffs is $10 \times \bar{M} \times \bar{N} \times 2$.

When we focus on the single bidder, our computation procedure is as follows. For a given p_k,

1. Draw 1000 single bidder from the auctions whose logarithm of the expected costs are in $[(\ln \bar{p} - \ln \underline{p}) \times (p_k - 0.05) + \ln \underline{p},\ (\ln \bar{p} - \ln \underline{p}) \times (p_k + 0.05) + \ln \underline{p}]$.

2. For all the single bidder i, compute the maximized expected payoff for each combination

(M, N) given p_k and $(s_i, \tilde{c}_i)$. Note that the cost is estimated in the above process.

3. Take the average of the maximized expected payoffs among the drawn firms for each pair of (M, N). This corresponds to $u_S(M, N, p_k)$.

4.2 Estimation of entry model

We have 4 parameters: $\theta = (\alpha, c_{JV}, c_S, T)$. Given the estimation result (u_{JV}, u_S) in the bidding stage, our estimation strategy is as follows:

1. Solve the fixed point problem.

   (a) Set the initial value of prediction p^0, Denote the number of iteration by $s = 0, 1, \cdots$.

   (b) Compute the expected payoffs based on θ^t,

   $$\begin{cases} \bar{E}((JV, S); p^s) = \int \left(\delta(u_{JV}(M, N) - c_{JV}) + (1-\delta)(u_S(M, N) - c_s)\right) dF^s\left(\delta, M, N \mid (JV, S)\right), \\ \bar{E}((JV, N); p^s) = \int \delta(u_{JV}(M, N) - c_{JV}) dF^s\left(\delta, M, N \mid (JV, N)\right), \\ \bar{E}((S); p^s) = \int (u_S(M, N) - c_S) dF^s\left(M, N \mid (S)\right) \end{cases}$$

   (c) Compute the choice probability: for each a,

   $$m(a, p^s) \equiv Pr(a = \arg\max_{\hat{a}} \ \bar{E}(\hat{a}; p^s) + \epsilon_{\hat{a}}).$$

   where ϵ_a is a disturbance term and follows Extreme Value Type 1 distribution whose scale parameter is set to T.

   (d) Compute the distribution over the number of firms choosing each action (L_1, L_2, L_3), which is denoted by $Q(L_1, L_2, L_3; p^s)$, and stack them to make a vector $Q(p^s)$.

   (e) Update the prediction $p^{s+1} = R(\alpha)Q(p^s)$.

   (f) Repeat from step (ii) to step (v) until convergence. Denote the fixed point by p^{fp}, which is dependent on the parameter and so we write it by $p^{fp}(\theta)$.

2. Compute the log likelihood.

(a) Compute the distribution over the pairs of (M, N):

$$g(M, N; \theta) = \sum_{a \in \{(JV,S),(JV,N),(S).(N)\}} f^{fp}(M, N \mid a) m(a, p^{fp})$$

where $f^{fp}(M, N \mid a)$ is the conditional distribution over (M, N) when choosing a.

(b) Compute the log likelihood:

$$LL(\theta) = \sum_{(M,N)} T(M, N) \times \ln g(M, N; \theta)$$

3. Maximize $LL(\theta)$ w.r.t. θ while repeating the above two steps.

We have two remarks on the estimation process. The first one is that, in the step 2-(a), we have to specify the prediction of the firm who chooses (N), $f^{fp}(M, N \mid (N))$. We set $f^{fp}(M, N \mid (N)) = \frac{1}{K}$ for all the pair of (M, N). The second point is about the estimation of the matching function. By changing α we can express the possible non-decreasing matching functions which are below the upper bound, $\lfloor \frac{L_1 + L_2}{2} \rfloor$. In the current case, we have 12 potential entrants for each auction and then the number of possible matching functions is 32. Hence the estimation of α is reduced to the choice problem over these 32 discrete patterns. We assume that these patterns are sufficient to describe all the reasonable matching patterns.

4.2.1 Implementation

The above estimation process needs to compute the fixed point whenever updating the parameters. This takes much time; furthermore, this computation makes the log-likelihood maximization no differentiable task. To obtain the standard errors easily and allow many initial points search, we implement a differentiable version of NFXP by approximating the fixed point computation step by interpolation.

First, we restrict the parameter domains as $c_{JV} \in [0, 200]$ and $c_S \in [0, 200]$ and $\ln T \in [-3, 6]$. The units of the former two parameters are million yen. Before the estimation, we compute the fixed points for many grid points in $[0, 200] \times [0, 200] \times [-3, 6]$. Not that, when we fix a matching function and the class of auction, we can compute the foxed point p^{fp} for any pair of $(c_{JV}, c_S, \ln T)$. We adopt 20 grid points in the first two dimensions and 10 grid points in the last dimension. Then, based on the 4000 grid points, we interpolate the computed fixed points to obtain the differentiable function of $p^{fp}(c_{JV}, c_S, \ln T)$. By inserting this interpolated function into the above process, we get the differentiable version of NFXP.

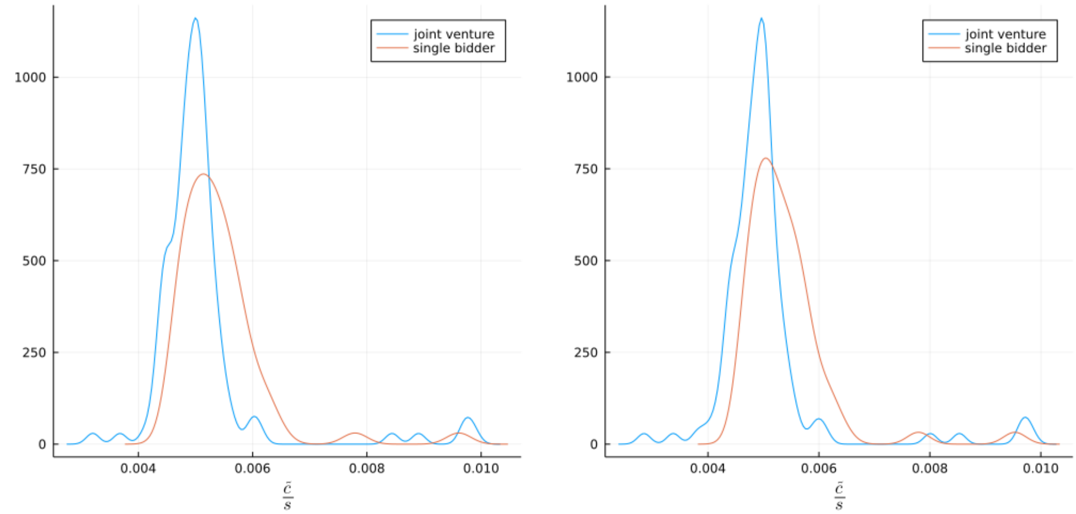
As we mentioned, this pre-computation of fixed points must be conducted for all pairs of matching function and the class of auctions. In our case, this requires $32 \times 10 = 320$ cases and for each case we need 4000 computations of fixed points.

5 Results

In the estimation, we restrict our sample to the auctions whose number of single bidders is below 11. This number is the 95 percentile of the number of single bidders in all the auctions. This restriction is aimed for removing the outliers like the auctions with too many participants.

5.1 Bidding stage

First, we show the estimation results for the bidding stage. We set the degree of the polynomials to 1 and 2 when we estimate the distributions of the logarithm of effective bids; then, we choose the results of the degree 1 considering the data fitting. Figure 2 is the estimated densities of $\frac{\tilde{c}_i}{s_i}$ for single bidder and joint venture. To make these, first, we retrieve $\tilde{c}_i$ and take the fraction $\frac{\tilde{c}_i}{s_i}$ for every i in the data and then plot the estimated densities based on these empirical data points. The blue line is the density for a joint venture, and the red line is the density for a single bidder. Panel (a) is the result for the homoskedasticity case, and panel (b) is the result for the type-dependent variance case. In both panels, we see the fraction $\frac{\tilde{c}_i}{s_i}$ of a joint venture is relatively smaller than the one of a single bidder. As the fraction $\frac{\tilde{c}_i}{s_i}$ is smaller when the entrant i is more competitive,



(a) Homoskedasticity case (b) Type dependent variance case

Figure 2. Density estimations of $\frac{\tilde{c}_i}{s_i}$. We focus on the entrants in the auctions in which at least one joint venture participates. The blue line represents the density of the joint venture, and the red line represents the density of a single bidder. Panel (a) is the result when we assume homoskedastic variances. Panel B is the result when we use type-dependent variance estimations.

both panels validate the fact that joint ventures are more likely to win the auctions in which we observe at least one joint venture. In the following estimations, we use the results obtained when we set the degree to 1 and allow the heteroskedastic variances in view of the model fitting.

We then estimate the ex-ante expected payoff in each auction based on the estimated distribution of $(\tilde{c}_i, s_i)$. Figure 8 shows the estimated ex-ante expected payoffs for a single bidder and a joint venture in each auction category. For each pair of the number of a joint venture, M, and the number of single bidders, N, we plot the estimated ex-ante expected payoff. In general, we find that the payoffs are bigger when the number of entrants is smaller. The value for the case when there is a single entrant is set to the corresponding expected cost in Figure 8.

To check the data fitting ability of this model, we compare the observed bids with the optimal bids obtained as solutions to the expected profit maximization problem, which is defined in (1). Figure 9 compares the simulated optimal effective bids with the observed effective bids in each category of auctions. The blue line is the estimated density of the simulated bids, and the red line is the estimated density of observed effective bids. We see the two densities are close to each other in every category of auctions. As we mentioned, this is the case when we use the polynomial of degree 1 to estimate the distributions of the logarithm of effective bids and allow the heteroskedastic variances. The other cases, like polynomial of degree 2 with homoskedastic assumption, do not perform as well as this case in comparing the simulated effective bids to the observed effective bids.

5.2 Entry stage

Here we show the estimation results for the entry stage. We have three estimation targets in our model. The first is the matching function, i.e., the relationship between the number of successfully formed joint ventures and the total number of potential entrants trying to form a joint venture with some other firm. The remaining two are the entry costs for a joint venture and a single bidder, which are denoted by (c_{JV}, c_S).

As we see in the model section, given the number of potential entrants, there is a finite discrete number of possible candidates for the function. In the current case, we have 12 potential entrants for each auction, which yields 32 potential functions. Within these patterns, we focus on the patterns which enable the firms to form at least two joint ventures for a sufficiently large number of potential entrants. This is because we often see 2 joint ventures in our data, and our model deterministically yields the number of joint ventures. Our total number of candidates for the matching function is 23. Hence, we estimate (c_{JV}, c_S) given one of these 23 patterns separately and pick the best pair of the estimates of (c_{JV}, c_S) and the matching function that yields the maximal loglikelihood.

Figure 3 depicts the estimated matching function in the blue line and the upper bound of the number of joint ventures in the orange line. This says no joint venture is formed until 4 potential entrants try to form a joint venture. When 4 or 5 firms try to form a joint venture, the successfully formed joint venture is just one. From 6 to 10, firms try to form a joint venture, 2 they form joint ventures. Lastly, when 12 potential entrants try to form a joint venture, there emerge 3 joint ventures in the auction. To form a joint venture, the process cost is necessary to search for the best partner. Hence, even if several firms hope to form a joint venture, it is not the case that all of them successfully form a

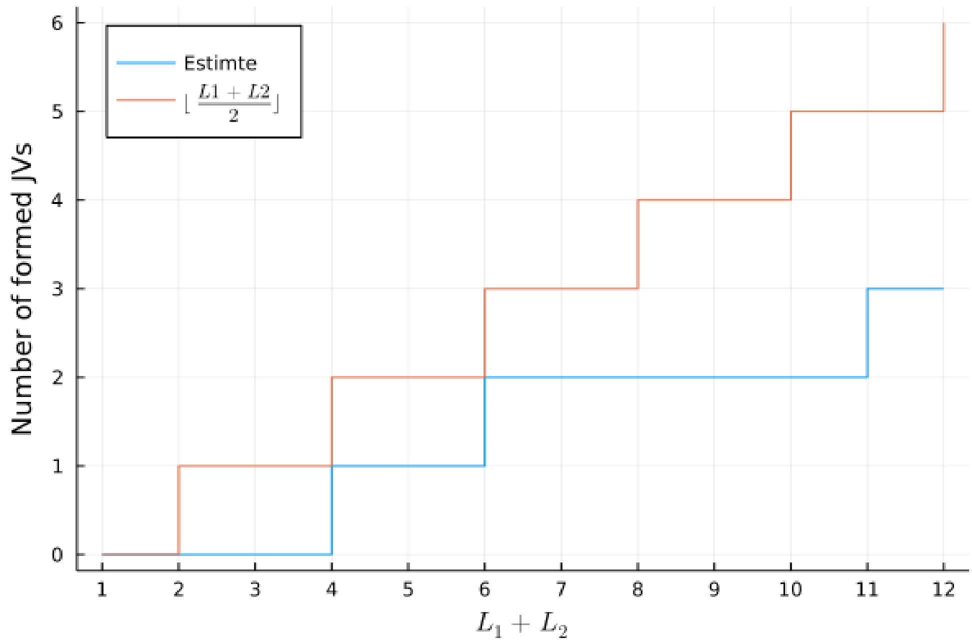


Figure 3. Estimated pattern of the formation of joint ventures. The horizontal axis is the total number of potential entrants who choose (JV, S) or (JV, N). The vertical line is the number of formed joint ventures. Orange line is the upper bound for the number of joint ventures, i.e., $\lfloor \frac{L_1+L_2}{2} \rfloor$.

joint venture.

Table 2 shows the estimation results of the other parameters. Remember that we have three remaining parameters: the cost to entry as a joint venture, c_{JV}, the cost to entry as a single bidder, c_S, and the scale parameter of the disturbance term in the decision T. The first three columns in Table 2 show the estimates for them. Note that we estimate the logarithm of the scale parameter. The estimates of c_{JV} and c_S are 107.164 and 68.064 in the unit of million yen. Because these are the estimates obtained when we set the payoff of the outside option to 0, we focus on the difference between these two values. The firm pays 39.100 million yen additionally when its entry as a joint venture compared to the case of a single bidder. For the standard error of $c_{JV} - c_S$, we compute

Table 2. Estimation Results in Entry Stage

Parameter	c_{JV}	c_S	$\ln T$	$c_{JV} - c_S$
Estimate	107.164	68.064	2.136	39.100
	(0.499)	(0.284)	(0.012)	(0.252)

The standard errors are shown in the parentheses.

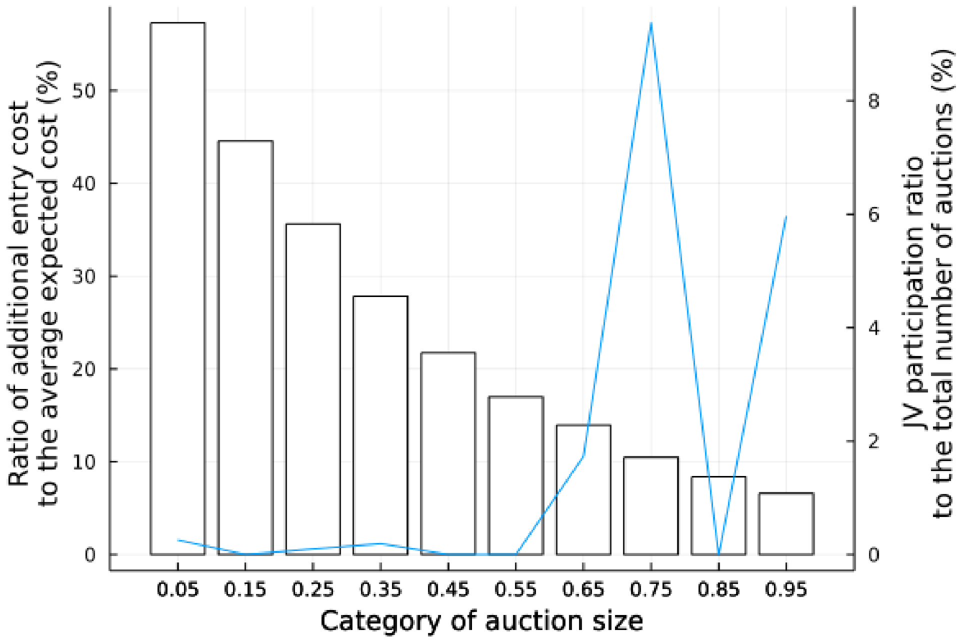


Figure 4. The bar plot is the ratio of the additional entry cost to the average level of the expected cost. The line plot is the ratio of the number of auctions in which at least joint ventures participate to the total number of auctions. We plot both of the values for each category of auction.

$[1\ -1][\hat{c}_{JV}\hat{c}_S]' \sim N\left(39.100, [1\ -1]\hat{\Omega}[1\ -1]'\right)$ where $\hat{\omega}$ is a variance-covariance matrix of $(\hat{c}_{JV}, \hat{c}_S)$. First, as we see in Table 2, the 95% confidence interval of this difference between $\hat{c}_{JV}$ and $\hat{c}_S$ does not include 0.

This additional entry cost amounts to 22.523% of the average expected cost level in our sample. By categorizing all the auctions with respect to the size of the expected cost, Figure 4 depicts the relative size of the additional cost to the average level of the expected cost in each category and the ratio of auctions in which at least one joint venture participates to the total number of auctions. In general, these two ratios negatively correlate, i.e., the entry of a joint venture is hindered by the relatively high additional cost to form a joint venture in small-sized auctions.

6 Counterfactual Simulation

First, we check the effect of prohibiting the formation of joint ventures. As a measure of the efficiency of the procurement, we consider *procurement efficiency* which is defined as

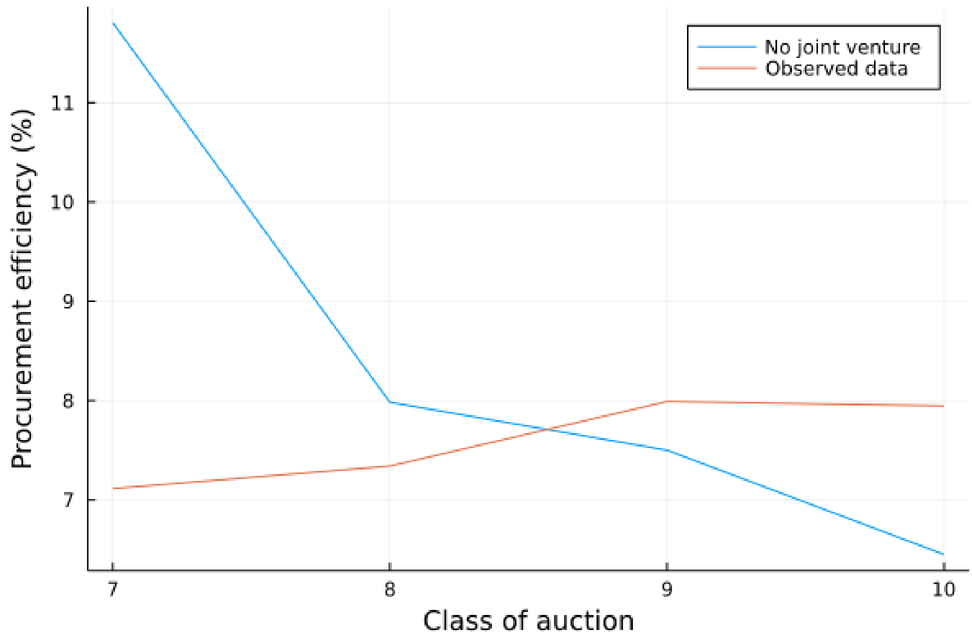


Figure 5. Procurement efficiency when prohibiting joint venture. The horizontal axis is the class of auction. We plot the procurement efficiencies for $p_k = 0.65, 0.75, 0.85, 0.95$, in which we find most of joint ventures in our data. The blue line corresponds to the no joint venture counterfactual simulation. The orange line corresponds to the observed data..

follows: for the set of auctions,

$$pe = \frac{\text{total amount of expected cost} - \text{total amount of winning bid}}{\text{total amount of expected cost}} \times 100.$$

For all the auctions in our data, this procurement efficiency is 7.048%. In the first set of counterfactual analysis, we compute this value for each class of auction when the government prohibits the entry of joint ventures.

We make the counterfactual set of auctions and their results as follows: for a fixed class of auction, p,

1. Count the number of auctions in the class.

2. Set the expected costs of all the auctions in the class to $p \times (p_{max} - p_{min})$, where p_{max}, p_{min} are the maximum and the minimum of the expected cost we observe in the data.

3. Based on the estimated values of (α, c_S, T) and a sufficiently high value of c_{JV},

compute the equilibrium prediction p^{fp} and the distribution over the patterns of entry.

4. Draw the entry pattern for each auction based on the above equilibrium distribution.

5. Draw the individual cost factor and the score for each firm from the estimated distribution of individual cost and score for single bidders.

6. Compute the optimal bid by solving the optimization problem for each bidder.

7. Determine the winner in each auction and decide the winning bid by comparing the effective bids.

We have two remarks on the above process. The first one concerns the value of c_{JV}. To remove the joint ventures in the above process, we set sufficiently high value to c_{JV}, like $c_{JV} = 5000$. By setting this value, the expected payoffs of choosing (JV, S) or (JV, N) get low so that no potential entrants try to form a joint venture. Figure 10 computed counterfactual probabilities on the entry patterns. We see no joint venture entry in all classes of auctions. Another remark is about the expected costs in the counterfactual auctions. Because we do not specify or estimate the distribution of the expected costs in each class, we fix one value.

Figure 5 shows the procurement efficiencies for each class of auctions from $p_k = 0.65, 0.75, 0.85, 0.95$, in which we find most of joint ventures in our data. The blue line corresponds to the counterfactual simulation in which we remove any entries of joint ventures, while the orange line represents the observed data.

The influence of prohibiting joint ventures is heterogeneous concerning the size of auctions. When we focus on smaller auctions, this restriction increases the procurement efficiency compared to the observed data. This is due to the increasing number of entrants by prohibiting the joint venture: i.e., the firms who would enter as a joint venture entry as single bidders separately. This leads to more competitive bids. On the other hand, for the larger auction classes, we find a decrease in procurement efficiency. This is because we cannot expect the cost reduction associated with the synergy of the joint venture. For

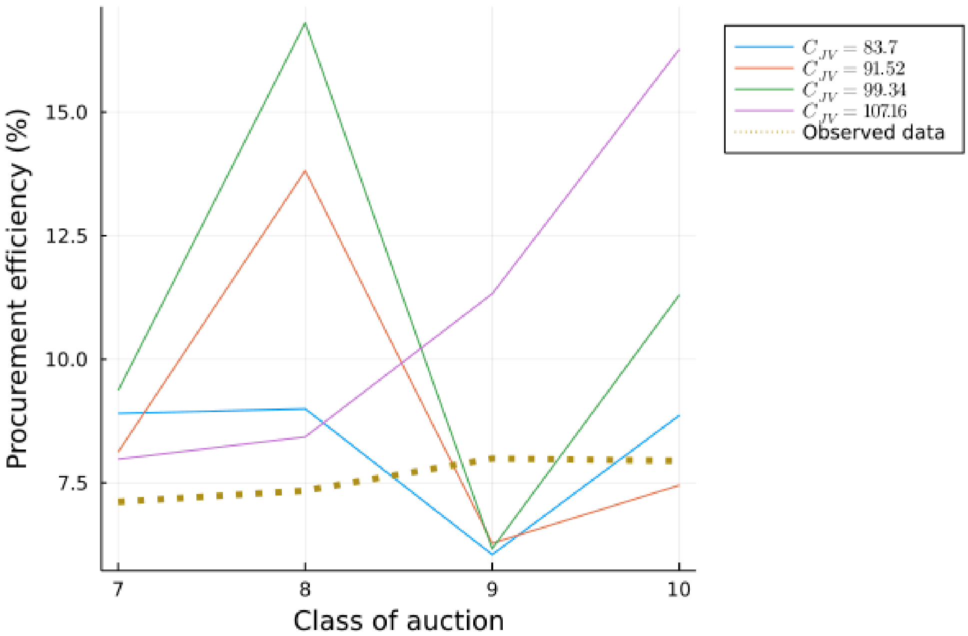


Figure 6. Counterfactual procurement efficiencies when the additional cost to entry as a joint venture decreased. The vertical line is the procurement efficiencies, and the horizontal line is the size of auctions. The dotted line expresses the procurement efficiencies which we observe in our data.

larger-sized auctions, the effect of decreasing the cost efficiency overwhelms the effect of increasing the number of entrants.

Next, we analyze the effect of encouraging joint venture formation by decreasing the additional entry cost. We follow the above simulation process for different levels of c_{JV} and compute the procurement efficiency for each auction size. Figure 6 shows the result. The purple line corresponds to the estimated value of c_{JV}. We have green, orange, and blue lines to decrease the entry cost. The green line performs more than the other reductions, and the observed efficiency level for all classes of auctions except the class 9. Too much reduction induces more entry of joint ventures, which threatens the other potential entrants. Then the pro-competitive effect of cost synergy is overwhelmed by the anti-competitive effect of the decreasing number of bidders. However, by decreasing the additional cost a little bit, for example, from the current level to the level of the green line, the auctioneer can benefit from the cost synergy of joint ventures, in particular, for the smaller-sized auctions.

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A Additional figures

A.1 Estimates of the density of $\frac{\tilde{c}_i}{s_i}$

Figure 7 shows the estimation results when we use the polynomials with degree 2. Basically, we see the same pattern as in the case of degree 1.

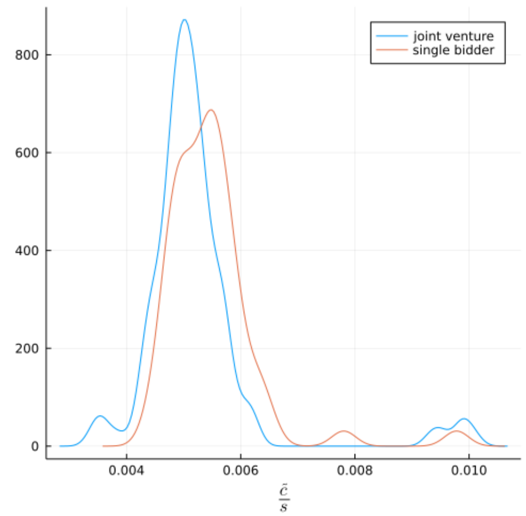
A.2 Exante expected payoff

Figure 8 shows the exante expected payoffs for all the classes of auctions. All the results are obtained based on the estimation which we obtain as we use the polynomial of degree 1.

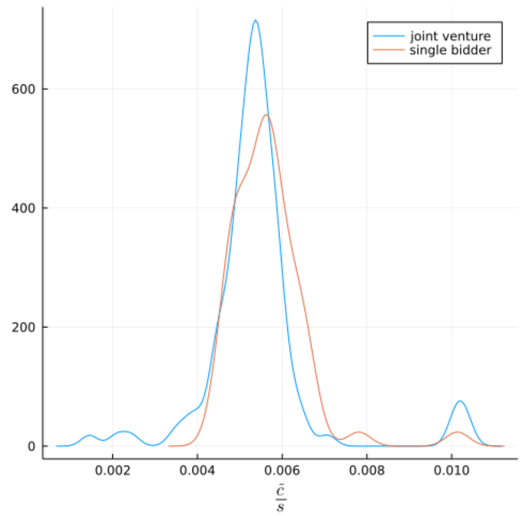
A.3 Model fit

Here we compare the simulated optimal effective bids with the observed effective bids. Figure 9 shows the comparison results for all the classes of auctions. From this result, we assume that the number of subgroup 10 is enough to make the model fit the data.

A.4 Counterfactual probabilities over the entry patterns

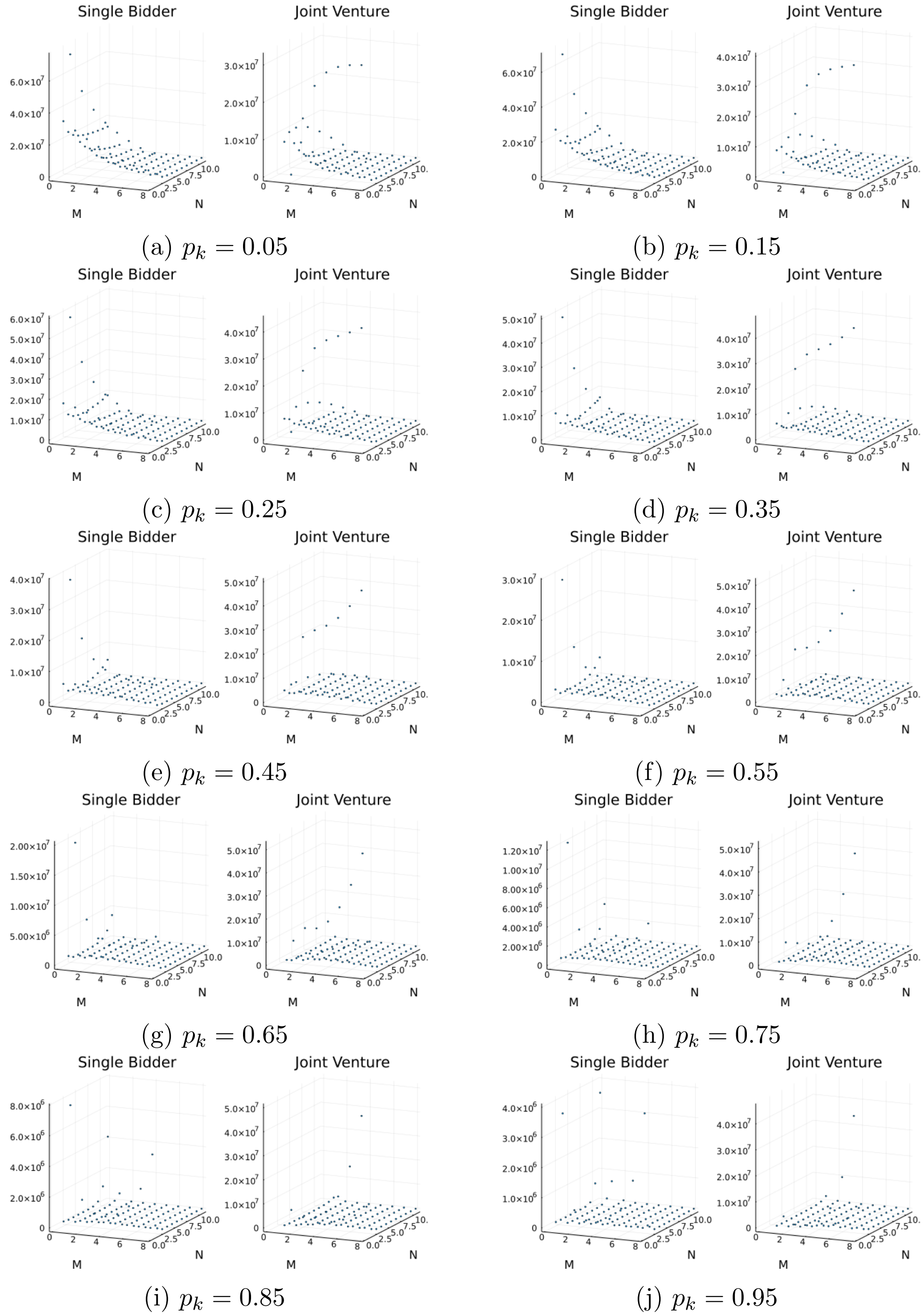


(a) Homoskedasticity case



(b) Type dependent variance case

Figure 7. Density estimations of $\frac{\tilde{c}_i}{s_i}$. We focus on the entrants in the auctions in which at least one joint venture participate. Blue line represents the density for joint venture and red line represents the density for single bidder. Panel (a) is the result when we assume homoskedastic variances. Panel B is the result when we use type-dependent variance estimations.



(a) $p_k = 0.05$ (b) $p_k = 0.15$

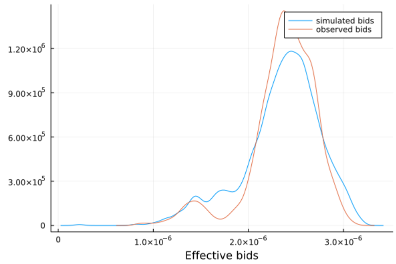
(c) $p_k = 0.25$ (d) $p_k = 0.35$

(e) $p_k = 0.45$ (f) $p_k = 0.55$

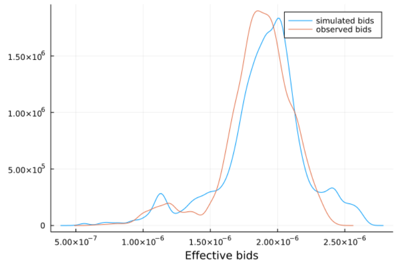
(g) $p_k = 0.65$ (h) $p_k = 0.75$

(i) $p_k = 0.85$ (j) $p_k = 0.95$

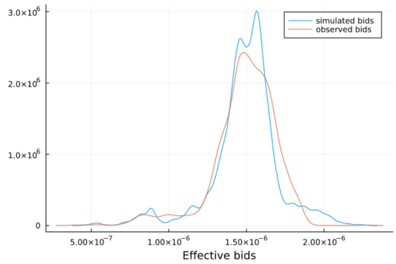
Figure 8. Estimates of the exante expected payoffs. For each category of auctions, $p_k = 0.05, 0.15, \cdots, 0.95$, we compute the expected payoffs by entering the auction for a single bidder and a joint venture given the number of entering single bidders and joint ventures.



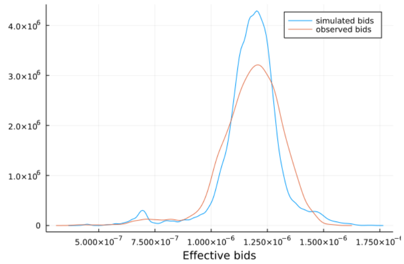
(a) $p_k = 0.05$



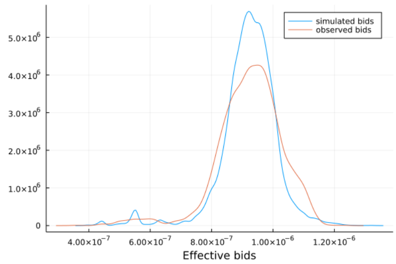
(b) $p_k = 0.15$



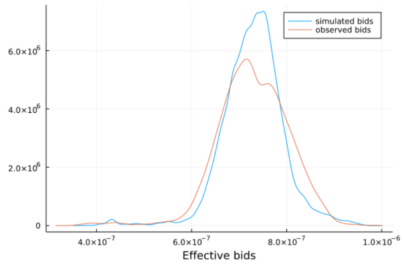
(c) $p_k = 0.25$



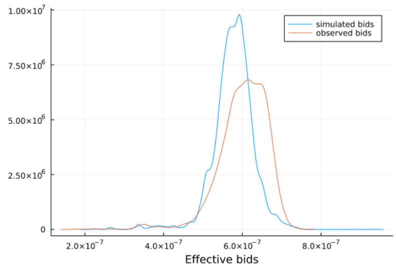
(d) $p_k = 0.35$



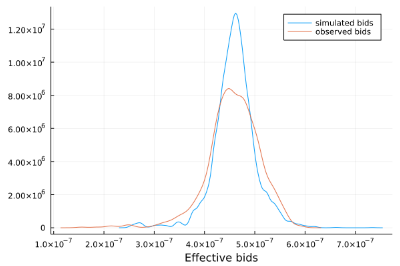
(e) $p_k = 0.45$



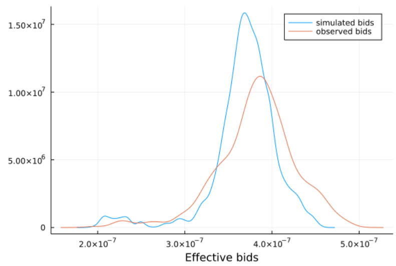
(f) $p_k = 0.55$



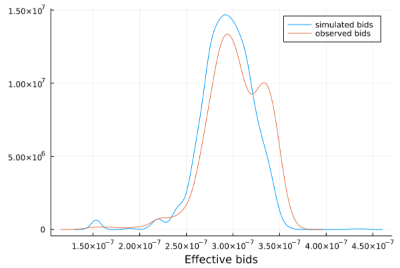
(g) $p_k = 0.65$



(h) $p_k = 0.75$

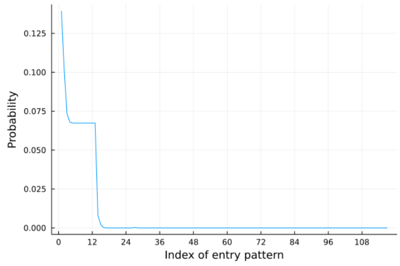


(i) $p_k = 0.85$

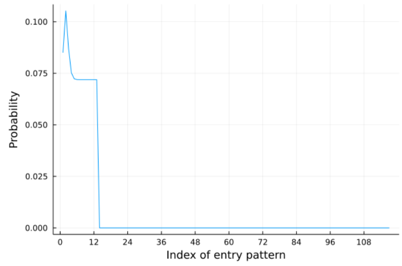


(j) $p_k = 0.95$

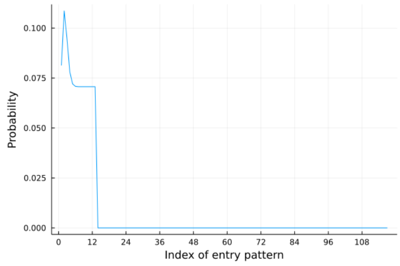
Figure 9. Comparison of the observed bids with the computed optimal bids. For each category of auctions, the blue line represents the estimated density function of computed optimal effective bids solving the profit maximization problem and the red line is the estimated density of the observed effective bids.



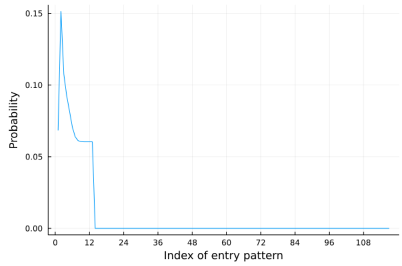
(a) $p_k = 0.05$



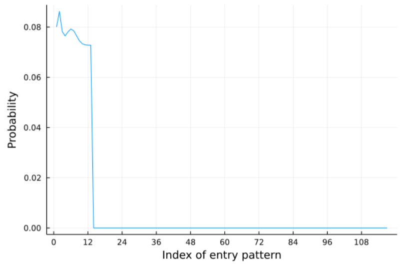
(b) $p_k = 0.15$



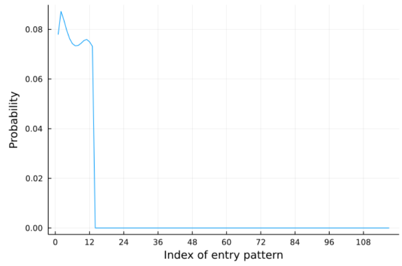
(c) $p_k = 0.25$



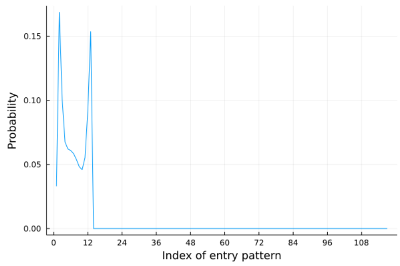
(d) $p_k = 0.35$



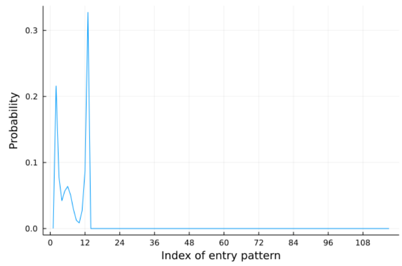
(e) $p_k = 0.45$



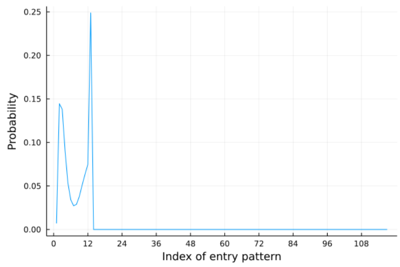
(f) $p_k = 0.55$



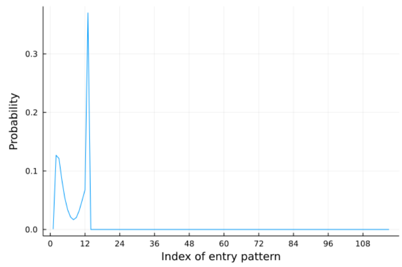
(g) $p_k = 0.65$



(h) $p_k = 0.75$



(i) $p_k = 0.85$



(j) $p_k = 0.95$

Figure 10. Counterfactual probability over the entry patterns. The horizontal axis is the index of the entry pattern. Since we have 12 potential entrants and set the maximal number of joint ventures to 8, all the possible entry patterns is $13 \times 9 = 117$. The first 13 entries correspond to the number of single bidders.