STAT6011/7611/6111/3317
COMPUTATIONAL STATISTICS (2016 Fall)

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1.1

Consider model 1 in section 4.1. I calculate the marginal likelihood by the algorithm in 3.2.2.

The prior distributions are as follows.

$$\begin{aligned}\alpha &\sim N(\mu_\alpha, \sigma_\alpha^2)\\ \beta &\sim N(\mu_\beta, \sigma_\beta^2)\\ \sigma^2 &\sim IG(a, b)\end{aligned}$$

First, I calculate α's full conditional distribution as follows.

$$\begin{aligned}p(\alpha|\mathbf{y}, \mathbf{x}, t, \beta, \sigma^2) &\propto p(\mathbf{y}|\alpha, \beta, \sigma^2)^t p(\alpha)\\
&= \left\{\Pi_{i=1}^N \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_i - \alpha - \beta(x_i - \bar{x}))^2}{2\sigma^2}\right)^t\right)\right\} \frac{1}{\sqrt{2\pi\sigma_\alpha}} \exp\left(-\frac{(\alpha - \mu_\alpha)^2}{2\sigma_\alpha^2}\right)\\
&\propto \exp\left(-\frac{1}{2\sigma^2}\left(t\sum_i (y_i - \beta(x_i - \bar{x}))^2 - 2t\alpha\sum_i (y_i - \beta(x_i - \bar{x})) + \alpha^2 Nt\right) + \frac{1}{2\sigma_\alpha^2}(\alpha^2 - 2\mu_{alpha}\alpha + \mu_\alpha^2)\right)\\
&= \exp\left(-\left(\frac{\sigma_\alpha^2 Nt + \sigma^2}{\sigma^2\sigma_\alpha^2}\alpha^2 - \left(\frac{\sigma_\alpha^2 t\sum_i (y_i - \alpha - \beta(x_i - \bar{x})) + \sigma^2\mu_\alpha}{\sigma^2\sigma_\alpha^2}\alpha\right)\right)\right)\\
&\propto \exp\left(-\frac{\sigma_\alpha^2 Nt + \sigma^2}{\sigma^2\sigma_\alpha^2}\left(\alpha - \frac{\sigma_\alpha^2 t\sum_i (y_i - \beta(x_i - \bar{x})) + \sigma^2\mu_\alpha}{\sigma_\alpha^2 Nt + \sigma^2}\right)^2\right)\end{aligned}$$

This means α's full conditional distribution is $N\left(\frac{\sigma_\alpha^2 t\sum_i (y_i - \beta(x_i - \bar{x})) + \sigma^2\mu_\alpha}{\sigma_\alpha^2 Nt + \sigma^2}, \frac{\sigma^2\sigma_\alpha^2}{\sigma_\alpha^2 Nt + \sigma^2}\right)$.

Secondly, I compute β's full conditional distribution as follows.

$$\begin{aligned}p(\beta|\mathbf{y}, \mathbf{x}, t, \alpha, \sigma^2) &\propto p(\mathbf{y}|\alpha, \beta, \sigma^2)^t p(\beta)\\
&\propto \exp\left(-\left(\frac{1}{2\sigma^2}t\sum_i (y_i - \alpha - \beta(x_i - \bar{x}))^2 + \frac{1}{2\sigma_\beta^2}(\beta - \mu_\beta)^2\right)\right)\\
&\propto \exp\left(-\left(\frac{\sigma_\beta^2 t\sum_i (x_i - \bar{x})^2 + \sigma^2}{2\sigma^2\sigma_\beta^2}\beta^2 - \frac{\sigma_\beta^2 t\sum_i (x_i - \bar{x})(y_i - \alpha) + \sigma^2\mu_\beta}{\sigma^2\sigma_\beta^2}\beta\right)\right)\\
&\propto \exp\left(-\frac{\sigma_\beta^2 t\sum_i (x_i - \bar{x})^2 + \sigma^2}{2\sigma^2\sigma_\beta^2}\left(\beta - \frac{\sigma_\beta^2 t\sum_i (x_i - \bar{x})(y_i - \alpha) + \sigma^2\mu_\beta}{\sigma_\beta^2 t\sum_i (x_i - \bar{x})^2 + \sigma^2}\right)^2\right)\end{aligned}$$

This means β's full conditional distribution is $N\left(\frac{\sigma_\beta^2 t\sum_i (x_i - \bar{x})(y_i - \alpha) + \sigma^2\mu_\beta}{\sigma_\beta^2 t\sum_i (x_i - \bar{x})^2 + \sigma^2}, \frac{\sigma^2\sigma_\beta^2}{\sigma_\beta^2 t\sum_i (x_i - \bar{x})^2 + \sigma^2}\right)$.

Third, I derive σ^2's full conditional distribution.

$$
\begin{aligned}
p(\sigma^2|\mathbf{y},\mathbf{x},t,\alpha,\beta) &\propto p(\mathbf{y}|\alpha,\beta,\sigma^2)^t p(\sigma^2) \\
&= \left\{\Pi_{i=1}^N \left(\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(y_i-\alpha-\beta(x_i-\bar{x}))^2}{2\sigma^2}\right)^t\right)\right\}\exp\left(-\frac{1}{b\sigma^2}\right)\frac{1}{\gamma(a)}b^{-a}(\sigma^2)^{-(a+1)} \\
&\propto (\sigma^2)^{-\frac{Nt}{2}+a+1}\exp\left(-\frac{bt\sum_i(y_i-\alpha-\beta(x_i-\bar{x}))^2+2}{2b\sigma^2}\right)
\end{aligned}
$$

This implies that the full conditional distribution of σ^2 is $IG\left(\frac{Nt}{2}+a, \frac{2b}{bt\sum_i(y_i-\alpha-\beta(x_i-\bar{x}))^2+2}\right)$.

I carry out Gibbs sampling by using these results.