Econometrics 2 2017
Problem set 1

Kei Ikegami (150012)

October 5, 2017

1 Problem 1

1.1 (a)

Let $\epsilon = y - \alpha - x^{'}\beta$, where $\alpha = E[y] - E[x^{'}]\beta$ and $\beta = \Sigma^{-1}\delta$, then I show $E[\epsilon] = 0$ and $E[\epsilon x] = 0$.

$$E[\epsilon] = E[y - E[y] + E[x^{'}]\beta - x^{'}\beta] = E[(y - E[y]) - (x^{'} - E[x^{'}])\beta] = (E[y] - E[y]) - (E[x^{'}] - E[x^{'}])\beta = 0$$

$$\begin{aligned} E[\epsilon x] &= E[x\epsilon] = E[x(y - E[y] + E[x^{'}]\beta - x^{'}\beta)] = E[x(y - E[y]) - x(x^{'} - E[x^{'}])\beta] \\ &= E[(x - E[x])(y - E[y]) + E[x](y - E[y]) - (x - E[x])(x^{'} - E[x^{'}])\beta - E[x](x^{'} - E[x^{'}])\beta] \\ &= \delta - E[(x - E[x])(x^{'} - E[x^{'}])]\Sigma^{-1}\delta = \delta - \delta = 0 \end{aligned}$$

So now I get the result.

1.2 (b)

This transformation is useful.

$$E[(y - a - x^{'}b)^2] = E[(y - E[y])^2] + 2E[(y - E[y])(E[y] - (a + x^{'}b))] + E[(E[y] - (a + x^{'}b))^2]$$

Then I get the FOC by differentiating by a as follows.

$$E[-2(y - E[y])] + E[-2(E[y] - (a + x^{'}b))] = 0 \ \Rightarrow \ a = E[y] - E[x^{'}]b$$

Next, after inserting the above relationship to the original, I get the FOC by differentiating by b as follows.

$$-2E[(y - E[y])(x^{'} - E[x^{'}])]^{'} + 2E[(x^{'} - E[x^{'}])^{'}(x^{'} - E[x^{'}])b] = 0 \ \Leftrightarrow \ b = \Sigma^{-1}\delta$$

And the second derivative by b is 2Σ, which is positive semi definite, then the second order condition for minimization is fulfilled. Thus I show α, β in (a) solves this minimization problem.

Then I show the second part. First I show the important property of the conditional expectation. If $y = E[y|x] + \epsilon$, then $E[\epsilon|x] = E[y - E[y|x]|x] = 0$ and for any function $h(x)$, $E[h(x)\epsilon] = E[E[h(x)\epsilon|x]] = E[h(x)E[\epsilon|x]] = 0$. Using this second property can easily prove the argument.

$$\begin{aligned} E[(y - a - x^{'}b)^2] &= E[(y - E[y|x] + E[y|x] - a - x^{'}b)^2] \\ &= E[(y - E[y|x])^2] + 2E[(y - E[y])(E[y|x] - a - x^{'}b)] + E[(E[y|x] - a - x^{'}b)^2] \\ &= E[(y - E[y|x])^2] + E[(E[y|x] - a - x^{'}b)^2] \end{aligned}$$

The second term in the first line vanishes since $E[(y - E[y])(E[y|x] - a - x^{'}b)] = E[\epsilon(E[y|x] - a - x^{'}b)] = 0$. This is because $E[y|x] - a - x^{'}b$ is just a function of x.

1.3 (c)

In the real econometric analysis, the relationship $E[\epsilon x] = 0$ is just an assumption. And that the correlation between the regressor and the error is zero needs the situation where all relevant variables are in the regression model, but this is unrealistic because certainly many unobservable variables exist. We can use IV for solving such a situation and get the consistent estimator of the coefficient of interesting variables.

1.4 (d)

1.5 (e)

2 Problem 2

2.1 (a)

2.2 (b)

2.3 (c)

2.4 (d)

3 Problem 3

3.1 (a)

3.2 (b)

3.3 (c)

3.4 (d)

3.5 (e)

4 Problem 4

4.1 (a)

4.2 (b)

4.3 (c)

4.4 (d)

5 Problem 5

5.1 (a)

5.2 (b)