式いろいろ

ぴか

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1 Eq.(24)

By (21),

$$
\begin{aligned}
g &= C_1 - w_1 \\
C_1 - w_1 &= \theta m - w_1 \ \because \text{Eq.(23)} \\
&\theta\left(w_1 + D + \frac{w_2}{R}\right) - w_1 \\
&\theta\left(\frac{w_2}{R} + D\right) - (1-\theta)w_1.
\end{aligned}
$$

2 Eq.(35)

By (34),

$$
\begin{aligned}
D = A\left(\frac{R^*}{q_2^*}\right) &\Rightarrow R^* = q_2^* \cdot A^{-1}(D) \\
&\Rightarrow \left.\frac{dR*}{dD}\right|_{LF} = q_2^* \cdot \frac{d}{dD}A^{-1}(D).
\end{aligned}
$$

Now,

$$
\begin{aligned}
\frac{dD}{dD} = 1 &\Rightarrow \frac{dA(A^{-1}(D))}{dD} = 1 \\
&\Rightarrow \frac{dA^{-1}(D)}{dD} \cdot A'(A^{-1}(D)) = 1 \\
&\Rightarrow \frac{dA^{-1}(D)}{dD} = \frac{1}{A'\left(A^{-1}\left(\frac{R*}{q_2^*}\right)\right)}.
\end{aligned}
$$

Then,

$$
\left.\frac{dR^*}{dD}\right|_{LF} = \frac{q_2^*}{A'}
$$

3 Eq.(38)

$$
\begin{aligned}
\frac{\mathrm{d}\theta^*}{\mathrm{d}D} &= \frac{\frac{d}{dD}\left(\frac{R^*}{q_2^*}\right)L'\left(\frac{R^*}{q_2^*}\right)m^* - (L+w_1)\left(1+w_2^*\frac{d}{dD}\frac{1}{R^*}\right)}{m^{*2}} \\
&= \frac{1}{m^{*2}}\left(\frac{1}{q_2^*}\left.\frac{dR^*}{dD}\right|_{LF} L'm - (L+w_1)\left(1-\frac{w_2^*}{R^{*2}}\left.\frac{dR^*}{dD}\right|_{LF}\right)\right)
\end{aligned}
$$

4 Eq.(31), Eq.(32) and Eq.(39)

When

$$
\begin{aligned}
U = &\int_0^{\theta^+}\{\theta\log(\tilde{C_1}) + (1-\theta)\log(\tilde{C_2})\}dF(\theta) \\
&+\int_{\theta^*}^1\{\theta\log(w_1+X) + (1-\theta)\log(\underline{w})\}dF(\theta).
\end{aligned}
$$

F.O.C w.r.t D,

$$
\begin{aligned}
&\frac{d}{dD}\int_0^{\theta^*}\{\theta\log(\tilde{C_1}) + (1-\theta)\log(\tilde{C_2})\}f(\theta)d\theta \\
&+\frac{d}{d\theta}\int_{\theta^*}1\{\theta\log(w_1+X) + (1-\theta)\log(\underline{w})\}f(\theta)d\theta = 0
\end{aligned}
$$

Consider the first term. Let $G(\theta, D)$ be the indefinite intergral of the inside of the first term, then

$$
\begin{aligned}
\frac{d}{dD}[G(\theta,D)]_0^{\theta^*} &= \frac{d}{dD}G(\theta,D) \\
&= \frac{d\theta^*}{dD}\cdot\frac{\partial}{\partial\theta}G(\theta,D)|_{\theta=\theta^*} + \frac{\partial}{\partial D}G(\theta,D)|_{\theta=\theta^*} \\
&= \frac{d\theta^*}{dD}\cdot\{\theta^*\log(\theta^* m^*) + (1-\theta^*)\log((1-\theta^*)m^*)\}\cdot f(\theta^*) \\
&+\int_0^{*}\left[\frac{\partial}{\partial\theta}\{\theta\log(\theta\cdot m) + (1-\theta)\log((1-\theta)m)\}\cdot f(\theta)\right].
\end{aligned}
$$

$\because$ we can change the order of the integer and the differentiation under some assumptions usually satisfied.

Because

$$
\frac{dm}{dD} = 1 + \frac{d}{dD}\frac{1}{R^*}w_2 = 1 - \frac{w_2}{(R^*)^2}\left.\frac{dR^*}{dD}\right|_{LF},
$$

we have

$$\left.\frac{d\theta^*}{dD}\right|_{LF}\{\theta^*\log(\theta^* m^*) + (1-\theta^*)\log((1-\theta^*)m^*)\}f(\theta^*)$$
$$+\int_0^{\theta^*}\frac{1}{m}\left(1-\frac{w_2}{R^2}\left.\frac{dR}{dD}\right|_{LF}\right)f(\theta)d\theta.$$

Consider the second term.

$$-\frac{d\theta^*}{dD}\{\theta^*\log(w_1+X)+(1-\theta^*)\log(\underline{w})\}f(\theta^*)$$

Then, the F.O.C. is

$$\left\{\theta^*\log\left(\frac{w_1+X}{\theta^* m^*}\right)+(1-\theta^*)\log\left(\frac{\underline{w}}{(1-\theta^*)m^*}\right)\right\}f(\theta^*)\left.\frac{d\theta^*}{dD}\right|_{LF}=\int_0^{\theta^*}\frac{1}{m}\left(1-\frac{w_2}{R^2}\left.\frac{dR}{dD}\right|_{LF}\right)f(\theta)d\theta.$$

This is different from (39). If, in (32),

$$\tilde{C}_2=(1-\theta)\left(w_1+D+\frac{w_2}{R}\right)\cdot R,$$

then (39) is correct.