# Appendix

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7th October 2017

## 1 Eq.(24)

By (21),

$$
\begin{aligned}
g &= C_1 - w_1 \\
C_1 - w_1 &= \theta m - w_1 \ \because \text{Eq.(23)} \\
&\theta\left(w_1 + D + \frac{w_2}{R}\right) - w_1 \\
&\theta\left(\frac{w_2}{R} + D\right) - (1-\theta)w_1.
\end{aligned}
$$

## 2 Eq.(35)

By (34),

$$
\begin{aligned}
D = A\left(\frac{R^*}{q_2^*}\right) &\Rightarrow R^* = q_2^* \cdot A^{-1}(D) \\
&\Rightarrow \left.\frac{dR*}{dD}\right|_{LF} = q_2^* \cdot \frac{d}{dD}A^{-1}(D).
\end{aligned}
$$

Now,

$$
\begin{aligned}
\frac{dD}{dD} = 1 \Rightarrow \frac{dA(A^{-1}(D))}{dD} &= 1 \\
\Rightarrow \frac{dA^{-1}(D)}{dD} \cdot A'(A^{-1}(D)) &= 1 \\
\Rightarrow \frac{dA^{-1}(D)}{dD} &= \frac{1}{A'\left(A^{-1}\left(\frac{R*}{q_2^*}\right)\right)}.
\end{aligned}
$$

Then,

$$
\left.\frac{dR^*}{dD}\right|_{LF} = \frac{q_2^*}{A'}
$$

# 3 Eq.(38)

$$
\begin{aligned}
\frac{\mathrm{d}\theta^*}{\mathrm{d}D} &= \frac{\frac{d}{dD}\left(\frac{R^*}{q_2^*}\right) L^{'}\left(\frac{R^*}{q_2^*}\right) m^* - (L+w_1)\left(1 + w_2^* \frac{d}{dD}\frac{1}{R^*}\right)}{m^{*2}} \\
&= \frac{1}{m^{*2}}\left(\frac{1}{q_2^*}\left.\frac{dR^*}{dD}\right|_{LF} L^{'}m - (L+w_1)\left(1 - \frac{w_2^*}{R^{*2}}\left.\frac{dR^*}{dD}\right|_{LF}\right)\right)
\end{aligned}
$$

By (36),

$$
\begin{aligned}
&= \frac{1}{m^*}\left.\frac{dR^*}{dD}\right|_{LF}\frac{L^{'}}{q_2^*} - \frac{\theta^*}{m^{*2}}\left(\frac{w_2^*}{R^*} + D + w_1\right)\left(1 - \frac{w_2^*}{R^{*2}}\left.\frac{dR^*}{dD}\right|_{LF}\right) \\
&= \frac{1}{m^*}\left.\frac{dR^*}{dD}\right|_{LF}\frac{L^{'}}{q_2^*} - \frac{\theta^*}{m^*}\left(1 - \frac{w_2^*}{R^{*2}}\left.\frac{dR^*}{dD}\right|_{LF}\right) \\
&= \left(\frac{L^{'}}{q_2^*} + \frac{\theta^* w_2^*}{R^{*2}}\right)\frac{1}{m^{*2}}\left.\frac{dR^*}{dD}\right|_{LF} - \frac{\theta^*}{m^*}
\end{aligned}
$$

# 4 Eq.(31), Eq.(32) and Eq.(39)

When

$$
\begin{aligned}
U = &\int_0^{\theta^+}\{\theta\log(\tilde{C_1}) + (1-\theta)\log(\tilde{C_2})\}dF(\theta) \\
&+ \int_{\theta^*}^{1}\{\theta\log(w_1+X) + (1-\theta)\log(\underline{w})\}dF(\theta).
\end{aligned}
$$

F.O.C w.r.t D,

$$
\begin{aligned}
&\frac{d}{dD}\int_0^{\theta^*}\{\theta\log(\tilde{C_1}) + (1-\theta)\log(\tilde{C_2})\}f(\theta)d\theta \\
&+ \frac{d}{d\theta}\int_{\theta^*} 1\{\theta\log(w_1+X) + (1-\theta)\log(\underline{w})\}f(\theta)d\theta = 0
\end{aligned}
$$

Consider the first term. Let $G(\theta, D)$ be the indefinite intergral of the inside of the first term, then

$$
\begin{aligned}
\frac{d}{dD}[G(\theta, D)]_0^{\theta^*} &= \frac{d}{dD}G(\theta, D) \\
&= \frac{d\theta^*}{dD} \cdot \frac{\partial}{\partial \theta}G(\theta, D)|_{\theta=\theta^*} + \frac{\partial}{\partial D}G(\theta, D)|_{\theta=\theta^*} \\
&= \frac{d\theta^*}{dD} \cdot \{\theta^* \log(\theta^* m^*) + (1-\theta^*)\log((1-\theta^*)m^*)\} \cdot f(\theta^*) \\
&+ \int_0^{*} \left[\frac{\partial}{\partial \theta}\{\theta \log(\theta \cdot m) + (1-\theta)\log((1-\theta)m)\} \cdot f(\theta)\right].
\end{aligned}
$$

$\because$ we can change the order of the integer and the differentiation under some assumptions usually satisfied.

Because

$$
\frac{dm}{dD} = 1 + \frac{d}{dD}\frac{1}{R^*}w_2 = 1 - \frac{w_2}{(R^*)^2}\left.\frac{dR^*}{dD}\right|_{LF},
$$

we have

$$
\begin{aligned}
&\left.\frac{d\theta^*}{dD}\right|_{LF}\{\theta^* \log(\theta^* m^*) + (1-\theta^*)\log((1-\theta^*)m^*)\}f(\theta^*) \\
&+ \int_0^{\theta^*} \frac{1}{m}\left(1 - \frac{w_2}{R^2}\left.\frac{dR}{dD}\right|_{LF}\right) f(\theta)d\theta.
\end{aligned}
$$

Consider the second term.

$$
-\frac{d\theta^*}{dD}\{\theta^* \log(w_1 + X) + (1-\theta^*)\log(\underline{w})\}f(\theta^*)
$$

Then, the F.O.C. is

$$
\left\{\theta^* \log\left(\frac{w_1 + X}{\theta^* m^*}\right) + (1-\theta^*)\log\left(\frac{\underline{w}}{(1-\theta^*)m^*}\right)\right\} f(\theta^*) \left.\frac{d\theta^*}{dD}\right|_{LF} = \int_0^{\theta^*} \frac{1}{m}\left(1 - \frac{w_2}{R^2}\left.\frac{dR}{dD}\right|_{LF}\right) f(\theta)d\theta.
$$

This is different from (39). If, in (32),

$$
\tilde{C_2} = (1-\theta)\left(w_1 + D + \frac{w_2}{R}\right) \cdot R,
$$

then (39) is correct.