

oo  
oo  
oo  
oooooooooooo

oooooooooooooooo  
oooooooooooo

# LEVEL-k AUCTIONS: CAN A NONEQUILIBRIUM MODEL OF STRATEGIC THINKING EXPLAIN THE WINNER'S CURSE AND OVERBIDDING IN PRIVATE-VALUE AUCTIONS?

Kei Ikegami

January 17, 2017

# Index

## Introduction

## Models

- Overview

- Equilibrium

- Cursed Equilibrium

- Nonequilibrium Level-k Models

## Comparing

- Optimal Bidding Strategy Comparing

- Econometrical Comparing

## Conclusion

oo  
oo  
oo  
ooooooo

oooooooooooo  
oooooooooooo

# Purpose

oo  
oo  
oo  
ooooooo

oooooooooooo  
oooooooooooo

# Precedents

oo  
oo  
oo  
ooooooo

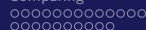
oooooooooooo  
oooooooooooo

# What's new

oo  
oo  
oo  
ooooooo

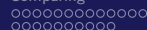
oooooooooooo  
oooooooooooo

# Result



## General Model

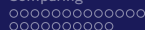
- ▶  $N$  bidders bid for a single object.
- ▶  $X_i$  is bidder  $i$ 's private signal.  $X = (X_1, \dots, X_N)$ .
- ▶  $S_j$  is additional random variable which is informative about the value of the object.  $S = (S_1, \dots, S_M)$ .
- ▶  $V_i = u_i(S, X)$  is bidder  $i$ 's value of the object, where  $u_i$  is symmetric across  $i$ .
- ▶  $V_i - p$  is the payoff for the bidder  $i$  winning the auction by paying  $p$ .
- ▶  $Y$  is the highest signal among bidders other than  $i$ .
- ▶  $v(x, y) = E[V_i | X_i = x, Y = y]$  is the expected value conditional on winning.
- ▶  $r(x) = E[V_i | X_i = x]$  is the unconditional expected value.



# Classification of Auctions

- ▶ First price auction vs Second price auction
- ▶ Independent private value auction(i.p.v) vs Common value auction(c.v)
- ▶ In i.p.v, the signals and values are independent among bidders.
- ▶ In c.v, the information of  $i$  and  $j$  is not independent and learning about the other bidders' information can cause the bidder to reassess his estimate of the value of the object. (e.g. Timber auction)





# First Price Auction

- ▶ In c.v, the optimal bidding strategy is calculated as (1).
- ▶ In i.p.v, the optimal bidding strategy is calculated as (2), because  $v(x, x) = x$  and  $Y$  is independent of  $X$ .

$$a_*(x) = v(x, x) - \int_{\underline{x}}^x \exp \left( - \int_y^x \frac{f_Y(t|t)}{F_Y(t|t)} dt \right) d(v(y, y)) \quad (1)$$

$$a_*(x) = x - \int_{\underline{x}}^x \frac{F_Y(y)}{F_Y(x)} dy = E[Y|Y < X] \quad (2)$$

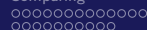


## Second Price Auction

- ▶ In c.v, the optimal bidding strategy is calculated as (3). This is not a weakly dominant strategy.
- ▶ In i.p.v, the optimal bidding strategy is calculated as (4), because  $v(x, x) = x$ . This is a weakly dominant strategy.

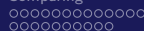
$$b_*(x) = v(x, x) \quad (3)$$

$$b_*(x) = x \quad (4)$$



# Points

- ▶  $\chi$  is the parameter denoting the probability that the bidder think the other bidders bid independently of signals.
- ▶ Cursed equilibrium for a given  $\chi$  value is called  $\chi$ -cursed equilibrium.
- ▶ In ER's(2002, 2006),  $\chi$ -cursed equilibrium is the same as one in a hypothetical " $\chi$ -virtual game", in which players believe that, with probability  $\chi$ , other's bid is independent of types.
- ▶ In i.p.v, the players bid independently of others' signals by definition, then the optimal bidding strategy in  $\chi$ -cursed equilibrium is the same as one in equilibrium ( $v(x, x) = r(x) = x$ ).
- ▶ In c.v, since  $v(x, x) \neq r(x)$ , cursed equilibrium differs from equilibrium.



# Common Value Auction

- ▶ In first price auction, the optimal bidding strategy is calculated as (5).
- ▶ In second price auction, the optimal bidding strategy is calculated as (6).
- ▶ These calculations are exactly the same as ones in equilibrium.

$$a_{\chi}(x) = \{(1 - \chi)v(x, x) + \chi r(x)\} \\ - \int_{\underline{x}}^x \exp\left(-\int_y^x \frac{f_Y(t|t)}{F_Y(t|t)} dt\right) d\{(1 - \chi)v(y, y) + \chi r(y)\} \quad (5)$$

$$b_{\chi}(x) = (1 - \chi)v(x, x) + \chi r(x) \quad (6)$$

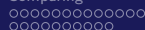


# Points

- ▶ Level-k model allows behavior to be heterogeneous, but assumes that each player's behavior is drawn from the common distribution over the  $k$  types.
- ▶ In this paper there are 3 types (denoted by  $L_k$ ) which best response to the lower type. i.e.  $L_1$  best responses to  $L_0$ , and  $L_2$  best responses to  $L_1$ .
- ▶ The key assumption is the behavior of  $L_0$ . One is Random  $L_0$ , in which the  $L_0$  bids uniformly randomly independent of its own signal. The other is Truthful  $L_0$ , in which  $L_0$  bids the value suggested by its own signal.
- ▶  $L_1$  and  $L_2$  are called Random  $L_1$  and  $L_2$  when  $L_0$  is set to Random  $L_0$ .
- ▶  $L_1$  and  $L_2$  are called Truthful  $L_1$  and  $L_2$  when  $L_0$  is set to Truthful  $L_0$ .
- ▶ In the latter slides I show the optimal bidding strategies of each player in each case one by one.

# Random L0

- ▶ This player bids i.i.d. uniformly over the range  $[\underline{z}, \bar{z}]$ , which is determined by its private signal and the value  $V_i = u_i(S, X)$ .



## Random L1 in First Price Auction

- ▶ Random L1 plays in the belief that all other players follow Random L0.
- ▶ Let  $Z$  be the highest bid among the others, the distribution function of  $Z$  be  $F_z(z)$ , and the pdf be  $f_z(z)$  (these two are from the ordered statistics).
- ▶ The optimal bidding strategy of L1 ( $a_1^r(x)$ ) solves (7) and is characterized by (8).

$$\max_a \int_z^a (r(x) - a) f_z(z) dz \quad (7)$$

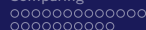
$$(r(x) - a) f_z(a) - F_z(a) = 0 \quad (8)$$



## Random L1 in First Price Auction

- ▶ This optimal bidding strategy is common in i.p.v and c.v.
- ▶ There are two differences from the optimal bidding strategy in equilibrium.
- ▶ One is that  $r(x)$  replaces  $v(x, x)$ , which reflects the fact that Random L1 think winning conveys no information about the value of the object even in c.v. (difference in value adjustment)
- ▶ Second is that the integral in (7) is over  $Z$  rather than  $Y$  (see (1) in paper). This is caused by L1 use nonequilibrium belief to evaluate the bidding trade-off. (difference in bidding trade-off)





## Random L1 in Second Price Auction

- ▶ The optimal bidding strategy ( $b_1^r(x)$ ) solves (9)'s maximization problem. And get (10).

$$\max_b \int_{\underline{z}}^b (r(x) - z) f_z(z) dz \quad (9)$$

$$b_1^r(x) = r(x) \quad (10)$$



## Random L1 in Second Price Auction

- ▶ One difference from equilibrium case is  $r(x)$  replaces  $v(x, x)$ . (difference in value adjustment)
- ▶ Second difference from equilibrium case is that the player use nonequilibrium belief. However note that this does not result in the deviating from truthful bidding as in first price auction.
- ▶ In other words, we have no bidding trade-off term in the optimal bidding strategy just as in the equilibrium case.
- ▶ (10) coincides with (6) when  $\chi = 1$ , i.e. fully cursed equilibrium.

# Random L2 in First Price Auction



○○  
○○  
○○  
○○  
○○○○○○○●○○○

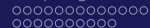
○○○○○○○○○○○○○○  
○○○○○○○○○○

# Random L2 in Second Price Auction



# Truthful L1 in First Price Auction





# Truthful L1 in Second Price Auction



# Truthful L2 in First Price Auction



○○  
○○  
○○  
○○  
○○○○○○○○○○

○○○○○○○○○○○○○○  
○○○○○○○○○○

# Truthful L2 in Second Price Auction





# Value Adjustment and Bidding Trade-Off

- ▶ 二つの言葉の定義を書く

# Summary Table

▶ Table 1 を挿入

# Equilibrium vs Cursed Equilibrium in First Price Auction

- ▶ i.p.v
- ▶ c.v.

# Equilibrium vs Cursed Equilibrium in Second Price Auction

- ▶ i.p.v
- ▶ c.v



# Equilibrium vs Random Level-k in First Price Auction

- ▶ i.p.v
- ▶ c.v

# Equilibrium vs Random Level-k in Second Price Auction

- ▶ i.p.v
- ▶ c.v



# Equilibrium vs Truthful Level-k in First Price Auction

- ▶ i.p.v
- ▶ c.v



# Equilibrium vs Truthful Level-k in Second Price Auction

- ▶ i.p.v
- ▶ c.v



# Cursed Equilibrium vs Random Level-k in First Price Auction

- ▶ i.p.v
- ▶ c.v



# Cursed Equilibrium vs Random Level-k in Second Price Auction

- ▶ i.p.v
- ▶ c.v



# Cursed Equilibrium vs Truthful Level-k in First Price Auction

- ▶ i.p.v
- ▶ c.v



# Cursed Equilibrium vs Truthful Level-k in Second Price Auction

- ▶ i.p.v
- ▶ c.v



# Summary: Where Level-k Model Can Improve?



○○  
○○  
○○  
○○  
○○○○○○○○○○

○○○○○○○○○○○○○○  
●○○○○○○○○○○

# Auction Examples: KL



○○  
○○  
○○  
○○○○○○○○○○

○○○○○○○○○○○○○○  
○●○○○○○○○○

# Auction Examples: AK





# Auction Examples: GHP







# Preparation for Comparing



○○  
○○  
○○  
○○○○○○○○○○

○○○○○○○○○○○○○○  
○○○○●○○○○○

# How to Compare



○○  
○○  
○○  
○○○○○○○○○○

○○○○○○○○○○○○○○  
○○○○○●○○○○

# Table3a



○○  
○○  
○○  
○○○○○○○○○○

○○○○○○○○○○○○○○  
○○○○○○●○○○

# Table3c



○○  
○○  
○○  
○○○○○○○○○○

○○○○○○○○○○○○○○  
○○○○○○○○●○○

# Table3d





# Table3b

- ▶ 他と比率が違う理由もかく

○○  
○○  
○○  
○○○○○○○○○○

○○○○○○○○○○○○○○  
○○○○○○○○○○●

# Summary: Could Level-k Model really Improve?



oo  
oo  
oo  
ooooooo

oooooooooooo  
oooooooooooo

# Summary





oo  
oo  
oo  
ooooooo

oooooooooooo  
oooooooooooo

# Implication

