<p align="center">Econometrics 2 2017
Problem set 1</p>

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1 Problem 1

1.1 (a)

Let $\epsilon = y - \alpha - x'\beta$, where $\alpha = E[y] - E[x']\beta$ and $\beta = \Sigma^{-1}\delta$, then I show $E[\epsilon] = 0$ and $E[\epsilon x] = 0$.

$$E[\epsilon] = E[y - E[y] + E[x']\beta - x'\beta] = E[(y - E[y]) - (x' - E[x'])\beta] = (E[y] - E[y]) - (E[x'] - E[x'])\beta = 0$$

$$E[\epsilon x] = E[x\epsilon] = E[x(y - E[y] + E[x']\beta - x'\beta)] = E[x(y - E[y]) - x(x' - E[x'])\beta]$$

$$= E[(x - E[x])(y - E[y]) + E[x](y - E[y]) - (x - E[x])(x' - E[x'])\beta - E[x](x' - E[x'])\beta]$$

$$= \delta - E[(x - E[x])(x' - E[x'])]\Sigma^{-1}\delta = \delta - \delta = 0$$

So now I get the result.

1.2 (b)

This transformation is useful.

$$E[(y - a - x'b)^2] = E[(y - E[y])^2] + 2E[(y - E[y])(E[y] - (a + x'b))] + E[(E[y] - (a + x'b))^2]$$

Then I get the FOC by differentiating by a as follows.

$$E[-2(y - E[y])] + E[-2(E[y] - (a + x'b))] = 0 \Rightarrow a = E[y] - E[x']b$$

Next, after inserting the above relationship to the original, I get the FOC by differentiating by b as follows.

$$-2E[(y - E[y])(x' - E[x'])]' + 2E[(x' - E[x'])'(x' - E[x'])b] = 0 \Leftrightarrow b = \Sigma^{-1}\delta$$

And the second derivative by b is 2Σ, which is positive semi definite, then the second order condition for minimization is fulfilled. Thus I show α, β in (a) solves this minimization problem.

Then I show the second part. First I show the important property of the conditional expectation. If $y = E[y|x] + \epsilon$, then $E[\epsilon|x] = E[y - E[y|x]|x] = 0$ and for any function $h(x)$, $E[h(x)\epsilon] = E[E[h(x)\epsilon|x]] = E[h(x)E[\epsilon|x]] = 0$. Using this second property can easily prove the argument.

$$E[(y - a - x'b)^2] = E[(y - E[y|x] + E[y|x] - a - x'b)^2]$$

$$= E[(y - E[y|x])^2] + 2E[(y - E[y])(E[y|x] - a - x'b)] + E[(E[y|x] - a - x'b)^2]$$

$$= E[(y - E[y|x])^2] + E[(E[y|x] - a - x'b)^2]$$

The second term in the first line vanishes since $E[(y - E[y])(E[y|x] - a - x'b)] = E[\epsilon(E[y|x] - a - x'b)] = 0$. This is because $E[y|x] - a - x'b$ is just a function of x.