Mitigating Congestion through Information Revelation

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1 Introduction

2 Model

I consider a dynamic game, whose first stage is voting for trains and second one is boarding. In the first stage players are asked to vote for the most preferable train, and they can tell a lie in this phase. The result of voting is revealed after all players have voted, and then they choose which train they really board according to the population density estimated by the result, considering the tradeoff between disutility from congestion and the utility from getting on the appropriate train. This situation is modeled as a nonatomic infinite player game and solved by Perfect Bayesian Equilibrium concept.

At the station in suburb area, there is K (even number) trains going to the center of the city in rush hour. All residents near by the station have to get on one of the K trains in the same period like 7.a.m. 9.a.m. Because there is a lot of commuters, they always suffer severe congestion everyday. They are the players in this model.

All the players have their first best train and second best train. The first best train means the player choose the train if there is no restriction or disutility effects like congestion. The second best train is the train they choose if they refrain from catching the first best train. I assume that all the player must board either the first best or the second best. In this case, we have ${}_KC_2$ types of players, but I additionally put assumptions for the sake of reality and computation.

To begin with I split the trains into two subgroups by their arriving time. Let $k = \{1, 2, \cdots, K\}$ represent earlier trains by earlier numbers, i.e. the earliest train is indexed by 1, and the last train is by K. Then the former half train group is composed of $k = \left\{1, 2, \cdots, \frac{K}{2}\right\}$ and the later half train group is $k = \left\{\frac{K}{2} + 1, \frac{K}{2} + 2, \cdots, K\right\}$. Furthermore I divide the players into two subgroups according to their type. Those who have the first best train in the former half train group is named as the former group, and the later half is defined analogously.

The first assumption is that the players in the former group have the one train before as their second best train and the player whose best train is $k = 1$ always get on the train. And the same restriction is put on the later half group. By this assumption we denote the type of the players by their first best train. For example, if $K = 6$, there are 6 types players. Type 1 is the player who always board the first train, in other words the players both of whose first best train and the second best train is train 1. Type 2 player is the player whose first best is train 2 and second best is 1. Type 5 is the player whose first best is train 5 and second best is train 6.

The main reason why there exists super crowded trains in rush hour is that such trains arrive at the time when both groups of players are in the station. I model this situation by overlapping the two groups of trains. That is the last train in the former half train group is the first train in the later half train group. For example, when $K = 6$, there are 5 trains. And the player in the first half player group prefer $1, 2$, or 3, while the player in the second half player group prefer $3, 4$, or 5. This is the second assumption.

The third assumption is the symmetric distribution of types in both of the groups. This absolutely simplify the computation. And I think this is not so unrealistic that it diminishes the importance of the model, because the commuters deriving the biased distribution should move their residence to the next station since they have the subsidy from their company. Furthermore the players are assumed to be divided into the first half and the second half with the same fraction, i.e. $\frac{1}{2}$. Then It is sufficient to put focus on the first half group. I denote the type distribution by $\{q_t\}_{t=1}^{\frac{K}{2}}$ where $\sum_{i=1}^{\frac{K}{2}} q_t = \frac{1}{2}$.

Next we model the preference of players.