STAT3811/3955 Survival Analysis Assignment 1

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1 Q1

1.1 (a)

$$
\begin{aligned}
E[T|T>t] &= \int_t^\infty Tf(T|T>t)\mathrm{d}T = \int_t^\infty T\frac{f(T)}{1-F(t)}\mathrm{d}T \\
&= \frac{1}{1-F(t)}\left(\int_0^\infty Tf(T)\mathrm{d}T - \int_0^t Tf(T)\mathrm{d}T\right) = \frac{1}{1-F(t)}\left(\mu - tF(t) + \int_0^t F(T)\mathrm{d}T\right)
\end{aligned}
$$

So I get the below derivative of $m(t)$.

$$
m^{'}(t) = \frac{1}{(1-F(t))^2}\left\{\left(-F(t) - tf(t) + F(t)\right)(1-F(t)) + (\mu - tF(t) + \int_0^t F(T)\mathrm{d}T)f(t)\right\} - 1
$$

Then the following calculation leads to the result.

$$
\frac{1+m^{'}(t)}{m(t)} = \frac{f(t)\left(\mu - tF(t) + \int_0^t F(T)\mathrm{d}T - t(1-F(t))\right)}{(1-F(t))\left(\mu - tF(t) + \int_0^t F(T)\mathrm{d}T\right) - t(1-F(t))^2} = \frac{f(t)}{1-F(t)} = \lambda(t)
$$

1.2 (b)

Since $\int_0^t F(T)\mathrm{d}T = \int_0^t (-S(T)+1)\mathrm{d}T = -\int_0^t S(T)\mathrm{d}T + t$, then

$$
\begin{aligned}
m(t) &= \frac{1}{S(t)}\int_0^\infty (T-t)f(T)\mathrm{d}T = \frac{1}{S(t)}\left(\int_0^\infty (T-t)f(T)\mathrm{d}T - \int_0^t (T-t)f(T)\mathrm{d}T\right) \\
&= \frac{1}{S(t)}\left(\mu - t + \int_0^t F(T)\mathrm{d}T\right) = \frac{1}{S(t)}\left(\mu - \int_0^t S(u)\mathrm{d}u\right)
\end{aligned}
$$

Now, when T has an exponential distribution with $\mu = \frac{1}{\lambda}$,

$$
m(t) = \exp(\lambda t)\left(\mu + \frac{1}{\lambda}\exp(-\lambda t) - \frac{1}{\lambda}\right) = \frac{1}{\lambda} = \mu
$$

because $\int_0^\infty t\lambda\exp(-\lambda t)\mathrm{d}t = \frac{1}{\lambda}$.

1.3 (c)

First I consider the mean,

$$
\lim_{t\to 0} m(t) = \lim_{t\to 0} E[T|T>t] = E[T] = 1
$$

Now let $\delta = med(T)$, then $F(\delta) = \frac{1}{2}$ and $\lambda(\delta) = \frac{2}{\delta+1}$ due to (a). Then by using (b) I get the below calculation.

$$\frac{2}{\delta+1} = 2\left(1 - \int_0^{\delta}(1-F(u))\mathrm{d}u\right) \Leftrightarrow \int_0^{\delta}(1-F(u))\mathrm{d}u = \frac{\delta}{\delta+1}$$

By taking derivative of both sides about δ, I get the result as follows.

$$1 - F(\delta) = \frac{1}{(\delta+1)^2} \quad \Leftrightarrow \quad \frac{1}{2} = \frac{1}{(\delta+1)^2} \quad \Leftrightarrow \quad \delta = \sqrt{2} - 1$$

1.4 (d)

First I have the below representation of $m(t)$.

$$m(t)\frac{\mu - \int_0^t S(u)\mathrm{d}u}{S(t)} = \frac{\int_t^{\infty} S(u)\mathrm{d}u}{S(t)}$$

Since the limits of the both of enumerator and denominator are 0 as $t \to \infty$. By using L'Hopital's rule twice, I get the below result,

$$\lim_{t\to\infty} m(t) = \lim_{t\to\infty}\frac{-S(t)}{-f(t)} = \lim_{t\to\infty}\frac{f(t)}{-f'(t)} = \lim_{t\to\infty}\left(-\frac{\mathrm{d}}{\mathrm{d}t}\log f(t)\right)^{-1}$$

1.5 (e)

2 Q3

2.1 (a)

2.2 (b)

2.3 (c)

3 Q5

3.1 (a)

3.2 (b)

3.3 (c)

3.4 (d)

3.5 (e)

3.6 (f)

3.7 (g)

3.8 (h)

3.9 (i)

3.10 (j)