Macroeconomics 1 2018 S1S2
Homework 1

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1 Problem1

We need an additional assumption for stationarity.

$$\begin{aligned} & y_t = c + \phi y_{t-1} + \epsilon_t \\ \Leftrightarrow\ & (1-\phi L) y_t = c + \epsilon_t \\ \Leftrightarrow\ & y_t = (1-\phi L)^{-1}(c+\epsilon_t) = c + \sum_{u=0}^{\infty} \phi^u L^u \epsilon_t = c + \sum_{u=0}^{\infty} \phi^u \epsilon_{t-u} \end{aligned} \tag{1}$$

For stationarity, it is necessary the above limit exists. By Chaucy's convergence judgement, we know that it is necessary and sufficient for the convergence of series that the residual, i.e. $\sum_{u=n+1}^{\infty} \phi^u \epsilon_{t-u}$, converges to 0. Usually, in time series analysis, the Hilbert space, whose product is defined by covariance, is used when the convergence is discussed. Thus if we want to know whether $\sum_{u=n+1}^{\infty} \phi^u \epsilon_{t-u} \to 0$ or not, we must check the convergence w.r.t the norm induced by the covariance product. And for the below calculation we need as assumption that states $Var(y_t) < \infty$.

$$\| \sum_{u=n+1}^{\infty} \phi^u \epsilon_{t-u} \| = Var(\sum_{u=n+1}^{\infty} \phi^u \epsilon_{t-u}) = \sum_{n+1}^{\infty} |\phi|^{2u} \sigma^2 = 0$$

The second equality is followed by the additional assumption. The third equality is by the assumption $|\phi| < 1$. Now we have the result that $\sum_{u=0}^{\infty} \phi^u \epsilon_{t-u}$ exists, in other words, the time series is stationary.

Next, I calculate the mean, variance, j-th autocovariance.

Mean By (1), $E[y_t] = E\left[c + \sum_{u=0}^{\infty} \phi^u \epsilon_{t-u}\right] = c + 0 = c$

Variance By (1), $Var(y_t) = Var\left(\sum_{u=0}^{\infty} \phi^u \epsilon_{t-u}\right) = \sum_{u=0}^{\infty} |\phi|^{2u} \sigma^2 = \frac{\sigma^2}{1-\phi^2}$

Autocovariance $\gamma(t, t-j) = \phi Cov(Y_{t-1}, Y_{t-j}) = \cdots = \phi^j Var(Y_{t-j}) = \frac{\phi^u \sigma^2}{1-\phi^2}$

2 Problem2

3 Problem3

4 Problem4

5 Problem5