# Econometrics 2 2017
# Problem set 1

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## 1 Problem 1

### 1.1 (a)

Let $\epsilon = y - \alpha - x^{'}\beta$, where $\alpha = E[y] - E[x^{'}]\beta$ and $\beta = \Sigma^{-1}\delta$, then I show $E[\epsilon] = 0$ and $E[\epsilon x] = 0$.

$$E[\epsilon] = E[y - E[y] + E[x^{'}]\beta - x^{'}\beta] = E[(y - E[y]) - (x^{'} - E[x^{'}])\beta] = (E[y] - E[y]) - (E[x^{'}] - E[x^{'}])\beta = 0$$

$$\begin{aligned} E[\epsilon x] &= E[x\epsilon] = E[x(y - E[y] + E[x^{'}]\beta - x^{'}\beta)] = E[x(y - E[y]) - x(x^{'} - E[x^{'}])\beta] \\ &= E[(x - E[x])(y - E[y]) + E[x](y - E[y]) - (x - E[x])(x^{'} - E[x^{'}])\beta - E[x](x^{'} - E[x^{'}])\beta] \\ &= \delta - E[(x - E[x])(x^{'} - E[x^{'}])]\Sigma^{-1}\delta = \delta - \delta = 0 \end{aligned}$$

So now I get the result.

### 1.2 (b)

This transformation is useful.

$$E[(y - a - x^{'}b)^2] = E[(y - E[y])^2] + 2E[(y - E[y])(E[y] - (a + x^{'}b))] + E[(E[y] - (a + x^{'}b))^2]$$

Then I get the FOC by differentiating by $a$ as follows.

$$E[-2(y - E[y])] + E[-2(E[y] - (a + x^{'}b))] = 0 \ \Rightarrow \ a = E[y] - E[x^{'}]b$$

Next, after inserting the above relationship to the original, I get the FOC by differentiating by $b$ as follows.

$$-2E[(y - E[y])(x^{'} - E[x^{'}])]^{'} + 2E[(x^{'} - E[x^{'}])^{'}(x^{'} - E[x^{'}])b] = 0 \ \Leftrightarrow \ b = \Sigma^{-1}\delta$$

And the second derivative by $b$ is $2\Sigma$, which is positive semi definite, then the second order condition for minimization is fulfilled. Thus I show $\alpha, \beta$ in (a) solves this minimization problem.

Then I show the second part. First I show the important property of the conditional expectation. If $y = E[y|x] + \epsilon$, then $E[\epsilon|x] = E[y - E[y|x]|x] = 0$ and for any function $h(x)$, $E[h(x)\epsilon] = E[E[h(x)\epsilon|x]] = E[h(x)E[\epsilon|x]] = 0$

### 1.3 (c)

### 1.4 (d)

### 1.5 (e)

## 2 Problem 2

### 2.1 (a)

### 2.2 (b)

### 2.3 (c)

### 2.4 (d)

## 3 Problem 3

### 3.1 (a)

### 3.2 (b)

### 3.3 (c)

### 3.4 (d)

### 3.5 (e)

## 4 Problem 4

### 4.1 (a)

### 4.2 (b)

### 4.3 (c)

### 4.4 (d)

## 5 Problem 5

### 5.1 (a)

### 5.2 (b)