

# Appendix

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7th October 2017

## 1 Eq.(24)

By (21),

$$\begin{aligned} g &= C_1 - w_1 \\ C_1 - w_1 &= \theta m - w_1 \quad \because \text{Eq.(23)} \\ &\theta \left( w_1 + D + \frac{w_2}{R} \right) - w_1 \\ &\theta \left( \frac{w_2}{R} + D \right) - (1 - \theta)w_1. \end{aligned}$$

## 2 Eq.(35)

By (34),

$$\begin{aligned} D = A \left( \frac{R^*}{q_2^*} \right) &\Rightarrow R^* = q_2^* \cdot A^{-1}(D) \\ &\Rightarrow \left. \frac{dR^*}{dD} \right|_{LF} = q_2^* \cdot \frac{d}{dD} A^{-1}(D). \end{aligned}$$

Now,

$$\begin{aligned} \frac{dD}{dD} = 1 &\Rightarrow \frac{dA(A^{-1}(D))}{dD} = 1 \\ &\Rightarrow \frac{dA^{-1}(D)}{dD} \cdot A'(A^{-1}(D)) = 1 \\ &\Rightarrow \frac{dA^{-1}(D)}{dD} = \frac{1}{A' \left( A^{-1} \left( \frac{R^*}{q_2^*} \right) \right)}. \end{aligned}$$

Then,

$$\left. \frac{dR^*}{dD} \right|_{LF} = \frac{q_2^*}{A'}$$

### 3 Eq.(38)

$$\begin{aligned}\frac{d\theta^*}{dD} &= \frac{\frac{d}{dD} \left( \frac{R^*}{q_2^*} \right) L' \left( \frac{R^*}{q_2^*} \right) m^* - (L + w_1) \left( 1 + w_2^* \frac{d}{dD} \frac{1}{R^*} \right)}{m^{*2}} \\ &= \frac{1}{m^{*2}} \left( \frac{1}{q_2^*} \frac{dR^*}{dD} \Big|_{LF} L' m - (L + w_1) \left( 1 - \frac{w_2^*}{R^{*2}} \frac{dR^*}{dD} \Big|_{LF} \right) \right)\end{aligned}$$

By (36),

$$\begin{aligned}&= \frac{1}{m^*} \frac{dR^*}{dD} \Big|_{LF} \frac{L'}{q_2^*} - \frac{\theta^*}{m^{*2}} \left( \frac{w_2^*}{R^*} + D + w_1 \right) \left( 1 - \frac{w_2^*}{R^{*2}} \frac{dR^*}{dD} \Big|_{LF} \right) \\ &= \frac{1}{m^*} \frac{dR^*}{dD} \Big|_{LF} \frac{L'}{q_2^*} - \frac{\theta^*}{m^*} \left( 1 - \frac{w_2^*}{R^{*2}} \frac{dR^*}{dD} \Big|_{LF} \right) \\ &= \left( \frac{L'}{q_2^*} + \frac{\theta^* w_2^*}{R^{*2}} \right) \frac{1}{m^{*2}} \frac{dR^*}{dD} \Big|_{LF} - \frac{\theta^*}{m^*}\end{aligned}$$

### 4 Eq.(31), Eq.(32) and Eq.(39)

When

$$\begin{aligned}U &= \int_0^{\theta^+} \{ \theta \log(\tilde{C}_1) + (1 - \theta) \log(\tilde{C}_2) \} dF(\theta) \\ &\quad + \int_{\theta^*}^1 \{ \theta \log(w_1 + X) + (1 - \theta) \log(\underline{w}) \} dF(\theta).\end{aligned}$$

F.O.C w.r.t D,

$$\begin{aligned}&\frac{d}{dD} \int_0^{\theta^*} \{ \theta \log(\tilde{C}_1) + (1 - \theta) \log(\tilde{C}_2) \} f(\theta) d\theta \\ &+ \frac{d}{d\theta} \int_{\theta^*}^1 \{ \theta \log(w_1 + X) + (1 - \theta) \log(\underline{w}) \} f(\theta) d\theta = 0\end{aligned}$$

Consider the first term. Let  $G(\theta, D)$  be the indefinite integral of the inside of the first term, then

$$\begin{aligned}
\frac{d}{dD}[G(\theta, D)]_0^{\theta^*} &= \frac{d}{dD}G(\theta, D) \\
&= \frac{d\theta^*}{dD} \cdot \frac{\partial}{\partial \theta}G(\theta, D)|_{\theta=\theta^*} + \frac{\partial}{\partial D}G(\theta, D)|_{\theta=\theta^*} \\
&= \frac{d\theta^*}{dD} \cdot \{\theta^* \log(\theta^* m^*) + (1 - \theta^*) \log((1 - \theta^*) m^*)\} \cdot f(\theta^*) \\
&\quad + \int_0^{\theta^*} \left[ \frac{\partial}{\partial \theta} \{\theta \log(\theta \cdot m) + (1 - \theta) \log((1 - \theta)m)\} \cdot f(\theta) \right] d\theta.
\end{aligned}$$

$\therefore$  we can change the order of the integer and the differentiation under some assumptions usually satisfied.

Because

$$\frac{dm}{dD} = 1 + \frac{d}{dD} \frac{1}{R^*} w_2 = 1 - \frac{w_2}{(R^*)^2} \frac{dR^*}{dD} \Big|_{LF},$$

we have

$$\begin{aligned}
&\frac{d\theta^*}{dD} \Big|_{LF} \{\theta^* \log(\theta^* m^*) + (1 - \theta^*) \log((1 - \theta^*) m^*)\} f(\theta^*) \\
&+ \int_0^{\theta^*} \frac{1}{m} \left( 1 - \frac{w_2}{R^2} \frac{dR}{dD} \Big|_{LF} \right) f(\theta) d\theta.
\end{aligned}$$

Consider the second term.

$$-\frac{d\theta^*}{dD} \{\theta^* \log(w_1 + X) + (1 - \theta^*) \log(\underline{w})\} f(\theta^*)$$

Then, the F.O.C. is

$$\left\{ \theta^* \log \left( \frac{w_1 + X}{\theta^* m^*} \right) + (1 - \theta^*) \log \left( \frac{\underline{w}}{(1 - \theta^*) m^*} \right) \right\} f(\theta^*) \frac{d\theta^*}{dD} \Big|_{LF} = \int_0^{\theta^*} \frac{1}{m} \left( 1 - \frac{w_2}{R^2} \frac{dR}{dD} \Big|_{LF} \right) f(\theta) d\theta.$$

This is different from (39). If, in (32),

$$\tilde{C}_2 = (1 - \theta) \left( w_1 + D + \frac{w_2}{R} \right) \cdot R,$$

then (39) is correct.