STAT3811/3955 Survival Analysis
Assignment 1

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1 Q1

1.1 (a)

$$
\begin{aligned}
E[T|T>t] &= \int_t^\infty T f(T|T>t)\mathrm{d}T = \int_t^\infty T \frac{f(T)}{1-F(t)}\mathrm{d}T \\
&= \frac{1}{1-F(t)}\left(\int_0^\infty T f(T)\mathrm{d}T - \int_0^t T f(T)\mathrm{d}T\right) = \frac{1}{1-F(t)}\left(\mu - tF(t) + \int_0^t F(T)\mathrm{d}T\right)
\end{aligned}
$$

So I get the below derivative of $m(t)$.

$$
m^{'}(t) = \frac{1}{(1-F(t))^2}\left\{(-F(t) - tf(t) + F(t))\,(1-F(t)) + (\mu - tF(t) + \int_0^t F(T)\mathrm{d}T)f(t)\right\} - 1
$$

Then the following calculation leads to the result.

$$
\frac{1+m^{'}(t)}{m(t)} = \frac{f(t)\left(\mu - tF(t) + \int_0^t F(T)\mathrm{d}T - t(1-F(t))\right)}{(1-F(t))\left(\mu - tF(t) + \int_0^t F(T)\mathrm{d}T\right) - t(1-F(t))^2} = \frac{f(t)}{1-F(t)} = \lambda(t)
$$

1.2 (b)

Since $\int_0^t F(T)\mathrm{d}T = \int_0^t (-S(T)+1)\mathrm{d}T = -\int_0^t S(T)\mathrm{d}T + t$, then

$$
\begin{aligned}
m(t) &= \frac{1}{S(t)}\int_0^\infty (T-t)f(T)\mathrm{d}T = \frac{1}{S(t)}\left(\int_0^\infty (T-t)f(T)\mathrm{d}T - \int_0^t (T-t)f(T)\mathrm{d}T\right) \\
&= \frac{1}{S(t)}\left(\mu - t + \int_0^t F(T)\mathrm{d}T\right) = \frac{1}{S(t)}\left(\mu - \int_0^t S(u)\mathrm{d}u\right)
\end{aligned}
$$

1.3 (c)

1.4 (d)

1.5 (e)

2 Q3

2.1 (a)

2.2 (b)

2.3 (c)

3 Q5

3.1 (a)

3.2 (b)

3.3 (c)

3.4 (d)

3.5 (e)

3.6 (f)

3.7 (g)

3.8 (h)

3.9 (i)

3.10 (j)