

# Computational Applications in International Trade

## Lecture 3: Endogenous Production Networks

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# Motivation

- ▶ Few firms are directly participating in international trade
- ▶ Of course, firms can be indirectly exposed to international trade through the domestic network
- ▶ Want to take domestic firm-to-firm connections into account as well as how the domestic network responds to trade shocks

# Motivation: Evidence from the Firm-to-Firm network in Belgium

- ▶ Three main data sources:
  - ▶ NBB B2B transaction database: Panel of VAT-id to VAT-id transactions among the universe of Belgian firms, over years 2002-2014 (Dhyne et al., 2015).
  - ▶ Annual accounts: total sales, labor cost, ownership of other VAT-ids, zip code, 4-digit NACE code, etc.
  - ▶ Int'l trade dataset: VAT-year-country-product(CN8) level.
- ▶ We aggregate VAT-ids into firms. ▶ Aggregation details
- ▶ Selected sample includes firms in the non-financial private sector with at least one full-time employee. ▶ Sampling ▶ Sectoral composition

## Summary of descriptive results

1. Fact 1: Firms obtain a large share of their imports indirectly
2. Fact 2: By comparison, fewer firms are connected to the export market
3. Fact 3: Firms do not have to be large to obtain a large fractions of imports indirectly or to see most of their sales go to the foreign market
4. Fact 4 (older working paper version): Import supply and export demand shocks transmit through the production network

# Direct and indirect importing

- Firm  $j$ 's **direct** foreign input share:

network so 2 subscripts. Foreign import j

$$s_{Fj} = \frac{\text{Imports}_j}{\text{Labor costs}_j + \text{Dom. purchases}_j + \text{Imports}_j}$$

- Firm  $j$ 's **total** foreign input share:

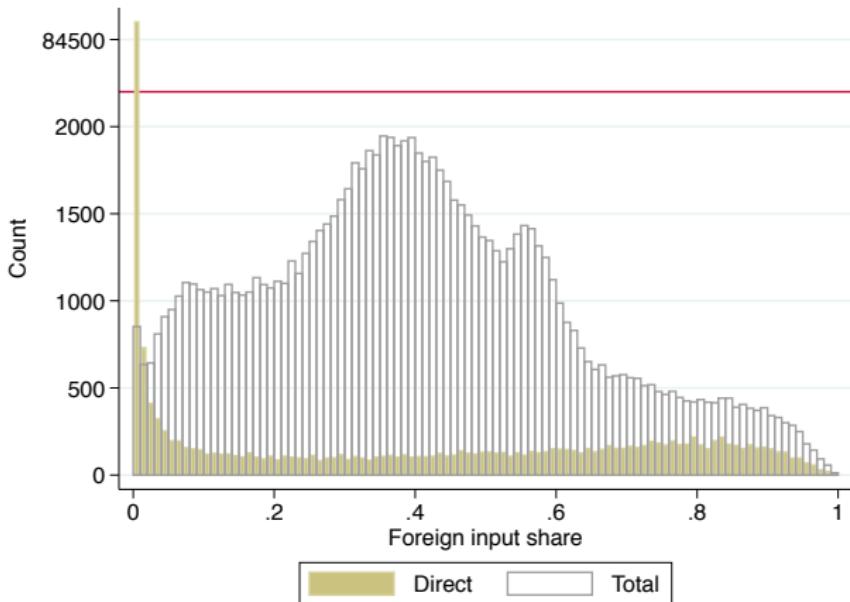
sum of all domestic suppliers.

$$s_{Fj}^{Total} = s_{Fj} + \sum_{i \in Z_j^D} s_{ij} \underbrace{\left[ s_{Fi} + \sum_{k \in Z_i^D} s_{ki} (s_{Fk} + \dots) \right]}_{\text{share of goods from firm i } s_{Fi}^{Total}}$$

**firm i also receives foreign import**

Leontif matrix inversion and measure asset j.

# Direct and indirect importing



Total foreign input share of about 40% average

99% of firms import directly or indirectly; Median: 39%; 80th percentile: 58%

▶ by sector

▶ excluding imports by wholesale and retail sectors

# Approaches to endogenous production networks

## 1. Stochastic processes approach:

- ▶ Jackson and Rogers (2007); Atalay, Hortacsu, Roberts, and Syverson (2011); Chaney (2014)
- ▶ No firm optimization in network formation

## 2. Extreme value approach:

- ▶ Oberfield (2018); Eaton, Kortum, and Kramarz (2018)
- ▶ Constant returns to scale, continuum of firms, select single supplier for each task. Difficulties in matching empirical relationship between firm size and outsourcing.

**Endogenous production network matters because if there is a shock maybe want to reconstruct network.**

**With fixed cost approach considering economies of scale.**

## 3. Fixed cost approach:

- ▶ Lim (2018); Tintelnot, Kikkawa, Mogstad, and Dhyne (2018); Taschereau-Dumouchel (2018)

# Challenges

firm's make guess about supplier's cost or customers demand from them. Based on this make decisions.  
In rational model need to be consistent with outcome. So could be messy with fixed point.

- ▶ Large fixed point problem: firms make choices based on cost / prices by other firms that themselves are equilibrium objects.
- ▶ Large discrete choice problem of which suppliers to add.
- ▶ Network effect of pair-wise choice: your choice to sell to firm  $a$  may affect you indirectly in your transaction with firm  $b$  if firm  $a$  is a supplier to firm  $b$ .

not easy to separate transaction relation among firms (could be strategic interaction). If firm a sells to firm b and firm c, where firm b and c are competitors.

- ▶ Strategic interaction: a typical firm has 33 suppliers and 9 customers.

## Overview: Lim (2018)

- ▶ Lim (2018) assumes that there are continuum of firms and that the seller makes the decision whether to offer the good to a supplier.
  - ▶ Seller can go independently through list of customer firms and decide whether it's worth incurring the fixed cost to sell
  - ▶ Everyone is measure zero, so stuff never comes back to you (almost surely)
  - ▶ No strategic interaction and no need to internalize the network effect  
**because continuum firms**

seller and not the user who can initiate transaction. This helps with:

1) removes large discrete choice problem because if seller initiate and seller believes (strong assumption) have constant marginal cost then can go customer by customer and decide if want to connect.

- ▶ TKMD allow the buyer firm to decide from whom to purchase inputs. Assume that there are is an ordering of firms, that determines who can purchase from whom.  
**don't let everyone connected to everyone but restriction ex ante.**
  - ▶ Solve large discrete choice problem via Jia (2008)'s algorithm
  - ▶ Assume no mark-ups in firm-to-firm trade (buyer has the full bargaining power), monopolistic competition in final goods trade
  - ▶ Acyclic network structure implies sold goods are not coming back to you or affect your profitability elsewhere

**Acyclic network: ordering of firms. Firm 1 can only hire labor. Firm 2 can hire labor and output of firm 1. Firm 3 can hire labor and import output from firm 1 and firm 2.**

**How to think about finite firm when assume continuum of firm. Aggregate network can still be analyzed.**

- ▶ Both papers create an artificial economy that looks like the actual network in some (but not all) observed dimensions
- ▶ Cannot perfectly replicate all the observed connections between firms in the model

- ▶ Exogenous unit continuum of firms producing differentiated goods
- ▶ Firms heterogeneous over states  $\chi \equiv (\phi, \delta)$ 
  - ▶  $\phi$ : **fundamental productivity** (labor input more productive)
  - ▶  $\delta$ : **fundamental quality** (household prefers product more)
  - ▶ exogenous distribution function  $G_\chi$  and support  $S_\chi \subset \mathbb{R}_+^2$

if have high delta, then households like your product better.

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- ▶ Representative household supplies  $L$  units of labor inelastically, with preferences:

$$U = \left[ \int_{S_\chi} [\delta x_H(\chi)]^{\frac{\sigma-1}{\sigma}} dG_\chi(\chi) \right]^{\frac{\sigma}{\sigma-1}}$$

- ▶ Conditional on prices, household demand  $x_H(\chi)$  is greater for firms with higher  $\delta$

# Lim (2018): Production Network

- ▶ Network specified by matching function  $m$ 
  - ▶  $m(\chi, \chi')$  = probability that  $\chi$ -firm buys from  $\chi'$ -firm
- ▶ Production CES in **labor** and **supplier inputs**, given matching function:

$$X(\chi) = \left[ [\phi l(\chi)]^{\frac{\sigma-1}{\sigma}} + \int_{S_\chi} m(\chi, \chi') \left[ \alpha x(\chi, \chi') \right]^{\frac{\sigma-1}{\sigma}} dG_\chi(\chi') \right]^{\frac{\sigma}{\sigma-1}}$$

**alpha: how important intermediates are**  
**because continuum, probability is fraction**

- ▶ Marginal cost:

$$\eta(\chi) = \left[ \phi^{\sigma-1} + \alpha^{\sigma-1} \int_{S_\chi} m(\chi, \chi') \left[ p(\chi, \chi') \right]^{1-\sigma} dG_\chi(\chi') \right]^{\frac{1}{1-\sigma}}$$

- ▶ Firms differ in labor productivity (not TFP); every firm has the same  $\alpha$ ; same elasticity of substitution as in utility function

- ▶ Market structure: monopolistic competition
- ▶ With continuum of sellers for each buyer, all firms charge constant CES markup,  $\mu = \frac{\sigma}{\sigma-1}$

# Lim (2018): Production Network

- ▶ Firms characterized by endogenous variables:

$$\Phi(\chi) \equiv \eta(\chi)^{1-\sigma} \quad (\text{inverse marginal cost})$$

$$\Delta(\chi) \equiv \frac{1}{Y_H/P_H^{1-\sigma}} X(\chi) \eta(\chi)^\sigma \quad (\text{demand shifter})$$

endogenous because depends on cost X and share nu

- ▶ Depend on **fundamental characteristics** and **characteristics of suppliers/customers** through matching function

$$\Phi(\chi) = \phi^{\sigma-1} + \mu^{1-\sigma} \alpha^{\sigma-1} \int_{S_\chi} m\left(\chi, \chi'\right) \Phi\left(\chi'\right) dF_\chi\left(\chi'\right)$$

$$\Delta(\chi) = \mu^{-\sigma} \delta^{\sigma-1} + \mu^{-\sigma} \alpha^{\sigma-1} \int_{S_\chi} m\left(\chi', \chi\right) \Delta\left(\chi'\right) dF_\chi\left(\chi'\right)$$

- ▶ Assume  $\alpha < 1$ . Decoupled contraction mappings in  $\Phi(\cdot)$  and  $\Delta(\cdot)$   
⇒ easily solved

- ▶ These endogenous variables determine all variables of interest:

$$\text{firm revenue: } R(\chi) \propto \Delta(\chi) \Phi(\chi)$$

$$\text{firm profit: } \Pi(\chi) \propto \Delta(\chi) \Phi(\chi)$$

$$\text{firm-to-firm sales: } r\left(\chi, \chi'\right) \propto \Delta(\chi) \Phi\left(\chi'\right)$$

$$\text{firm-to-firm profit: } \pi\left(\chi, \chi'\right) \propto \Delta(\chi) \Phi\left(\chi'\right)$$

- ▶ Unique market equilibrium for any matching function

$$m : S_\chi \times S_\chi \rightarrow [0, 1]$$

- ▶ Seller pays stochastic relationship fixed cost:  $\xi$  units of labor to activate relationship
- ▶ Static variable profit earned by a  $\chi'$ -firm from selling to  $\chi$ -firm:

$$\pi(\chi, \chi') \propto \Delta(\chi) \Phi(\chi')$$

- ▶ **Acceptance function** - probability that a relationship is selected:

$$a(\chi, \chi') = \Pr[\pi(\chi, \chi') \geq \xi] = F_{\xi,t}[\pi(\chi, \chi')]$$

where  $F_{\xi,t}$  is the unconditional distribution of  $\xi$

- ▶ Note: some relationships that buyers would like to form, do not get formed.

**Assumption that marginal cost of firms constant. Given current links cannot inference marginal cost.**

- ▶ Given guess of  $\{\Phi, \Delta\}$ , compute  $\pi$ , then compute  $a$  and iterate
- ▶ Earlier version of paper included dynamic frictions:
  - ▶  $1 - \nu$  chance for each relationship to be activated/terminated
  - ▶ introduces option values into relationship formation
  - ▶ requires with more complicated dynamic algorithm
- ▶ Efficiency
  - ▶ Static market equilibrium is inefficient: double marginalization
  - ▶ End. equilibrium: additional sources of inefficiency. Do not form all relationships that (buyer) firms would like to form + network externality

- ▶ As a proof of concept, paper study the role of endogenous network adjustments in business cycles
- ▶ Note: decision problem is static, but imposes AR(1) on the log cost to form links and log productivity
- ▶ Suppose the network had been hold fixed - what would real output be?

- ▶ Compustat data
  - ▶ publicly-listed firms in the US
  - ▶ standard financial/accounting firm-level information
  - ▶ records of firms' major customers ( $> 10\%$  of revenue)
  - ▶ panel data from 1979-2008, over 100,000 firm-year observations
  - ▶ firms account for just under 50% of US gross output in 2007
- ▶ Capital IQ data
  - ▶ both private and public firms
  - ▶ relationships recorded from multiple sources (publications, news reports)
  - ▶ select all firms in continental US with recorded relationship data and positive average revenue from 2003-2007
  - ▶  $\approx 9,000$  firms, accounting for around 65% of US gross output in 2007

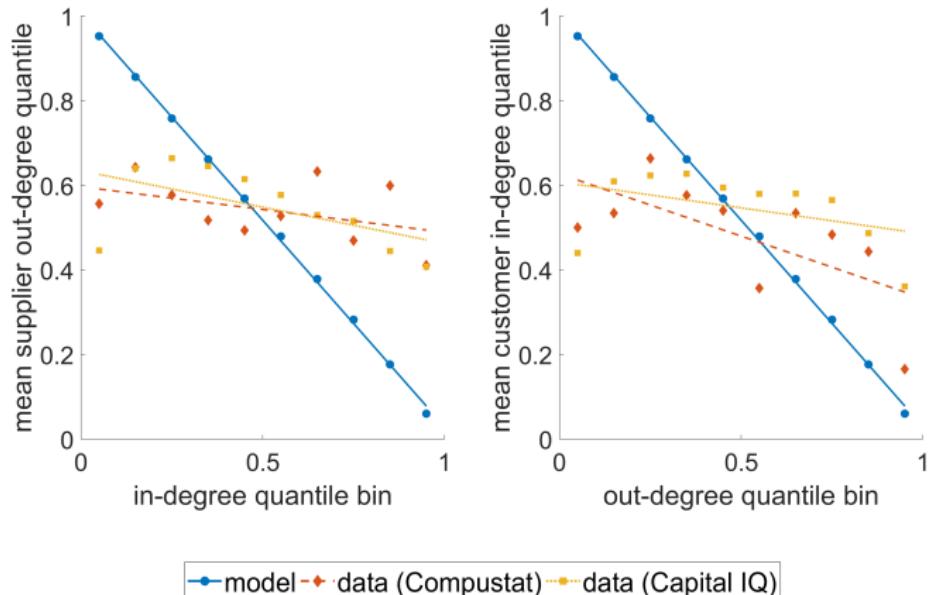
## Lim (2018): Mapping model to data

- ▶ Ignore demand differences,  $\delta = 1$
- ▶ AR(1) process for stochastic relationship cost to match persistence in links
- ▶ AR(1) process for productivity to match persistence of firm-level sales

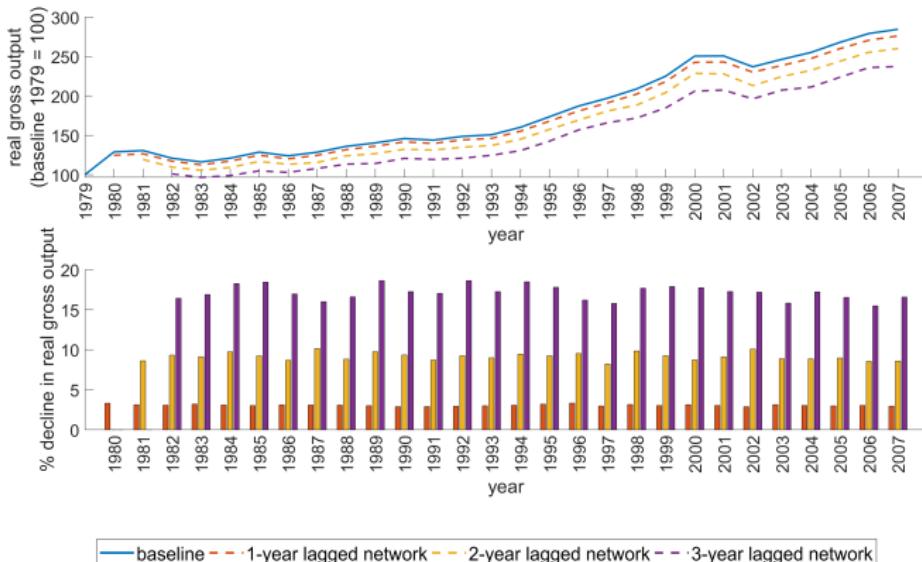
$$\vec{\phi}_t^{\sigma-1} = \frac{1}{\mu \Delta_{H,t}} \left[ I - (\alpha/\mu)^{\sigma-1} M \right]^{-1} \left( \overrightarrow{R/\Delta_t} \right)$$

- ▶ Value of general equilibrium term  $\Delta_{H,t}$  is equivalent to a normalization of level of  $\phi_t$
- ▶ choose arbitrary normalization in first period  $t = 1$
- ▶ calibrate level of  $\phi_t$  for  $t > 1$  to match US real GDP growth rates
- ▶ Not obvious how to match network characteristics (continuum of firms versus data)

# Lim (2018): Model fit



# Lim (2018): Counterfactual



- ▶ Restricting network adjustment always leads to lower output
- ▶ Endogenous network components of output are quantitatively large:
  - ▶ 4% at one-year horizons; 16% at three-year horizons
- ▶ No noticeable differences between booms or busts

- ▶ Applies the Lim (2018) model to study the role of endogenous networks in the adjustment of Chile to the great recession (foreign demand and supply shocks)
- ▶ Uses the version of Lim's model with the Calvo fairy (firms can adjust certain links only with probability  $\nu$ )
- ▶ Assumes firm productivity is time-invariant

- ▶ Use rich Belgian data to study the welfare effects of import price changes and their dependence on the domestic production network
- ▶ Comparison of:
  1. Fixed network: actual network versus roundabout production network
  2. Endogenous network versus Fixed Network
- ▶ Eaton and Kortum (2002), ACR (2012): Gains from trade are higher in models with intermediate goods than in models that abstract from intermediate inputs.
- ▶ Here: commonly observed firm-level and aggregate outcomes are all held fixed. Models may differ in the implied linkages between firms.

# Exogenous network

# Exogenous network Model

# Model environment

- ▶ Small open economy

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- ▶ Fixed number of domestic firms

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- ▶ Here: Take as given firms' suppliers, customers, import and export participation

# Consumers

- ▶ CES preferences (with  $\sigma > 1$ )

- ▶ Domestic demand:

$$q_{kH} = \beta_{kH}^{\sigma-1} \frac{p_{kH}^{-\sigma}}{P^{1-\sigma}} E$$

- ▶ Foreign demand:

$$q_{kF} = \beta_{kF}^{\sigma-1} \frac{p_{kF}^{-\sigma}}{P_F^{1-\sigma}} E_F,$$

# Production Technology

- ▶ Firms produce single products
- ▶ Marginal cost of final good producer  $j$ :

$$c_j(Z_j) = \frac{1}{\phi_j} \left( \sum_{k \in Z_j} \alpha_{kj}^{\rho-1} p_{kj}^{1-\rho} + \alpha_{Lj}^{\rho-1} w^{1-\rho} \right)^{1/(1-\rho)}$$

**firm specific product share**

- ▶ Assume  $\sigma > \rho > 1$

# Firm-level cost changes due to foreign price change

- Given fixed linkages between firms, the change in firm  $j$ 's unit cost,  $\hat{c}_j | \hat{p}_{F\cdot}$  due to an uniform change in foreign price,  $\hat{p}_{F\cdot}$ , is

$$\hat{c}_j | p_{F\cdot} = \left( (1 - s_{Fj}^{Total}) \hat{w}^{1-\rho} + s_{Fj}^{Total} \hat{p}_{F\cdot}^{1-\rho} \right)^{1/(1-\rho)}$$

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- $s_{Fj}^{Total}$  summarizes firm  $j$ 's exposure to foreign inputs.

# Change in real wage

- Given fixed linkages between firms, the change in the real wage,  $\frac{\hat{w}}{\hat{P}}$ , due to an uniform change in foreign price,  $\hat{p}_{F\cdot}$ , is:

$$\frac{\hat{w}}{\hat{P}} |^{p_{F\cdot}} = \hat{w} \left( \sum_i s_{iH} (\hat{c}_i |^{p_{F\cdot}})^{1-\sigma} \right)^{\frac{1}{\sigma-1}}$$

**weighted aggregate of firm level change and share of household consumption**

$$= \left( \sum_i s_{iH} \left( (1 - s_{Fi}^{Total}) + s_{Fi}^{Total} \frac{\hat{p}_{F\cdot}^{1-\rho}}{\hat{w}^{1-\rho}} \right)^{\frac{1-\sigma}{1-\rho}} \right)^{\frac{1}{\sigma-1}}$$

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# Implications for the gains from trade

- ▶ Effects of a change in the foreign price depends on:
  - ▶ Change in nominal wage
  - ▶ Firms' share in household demand,  $s_{jH}$
  - ▶ Firms' total share of foreign inputs,  $s_{Fj}^{Total}$

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- ▶ These objects depend on the network structure
- ▶ Two economies with the same firm-level domestic sales, value added, exports, and imports, may have different gains from trade ▶ Example

## Quantitative results

- ▶ Consider a 10 percent increase in the price of foreign goods
- ▶ Baseline:  $\sigma = 4$ ,  $\rho = 2$ .
- ▶ Median firm's cost increases by 2.7%. Nominal wage falls by 2%.
  - ▶ Cost changes histogram
- ▶ Real wage declines by 6%
- ▶ Calculation based on Hulten's theorem predicts a 8% decline in real wages (if no import content in exports) or 4.5% decline (if the same import content in exports and final demand).
  - ▶ Hulten
- ▶ Firm-level data with roundabout production assumption leads to similar results for 10 percent increase in foreign price (for autarky, real wage changes 5-10 percentage points different)
  - ▶ Table
  - ▶ Histogram

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- ▶ Firms also choose whether to import or export

# Sourcing strategy and export decision

- ▶ Total profits given  $Z_j$  and  $I_{jF}$

$$\pi_j(Z_j, I_{jF}) = \pi_j^{\text{var}}(Z_j, I_{jF}) - \sum_{k \in Z_j} f_{kj}w - I_{jF}f_{jF}w$$

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- ▶ Exogenously given set of eligible suppliers,  $\mathbf{Z}_j$
- ▶ Optimal sourcing strategy and on export participation:

$$\max_{Z_j, I_{jF}} \pi_j(Z_j, I_{jF}) \quad \text{s.t.} \quad Z_j \subseteq \mathbf{Z}_j, I_{jF} \in \{0, 1\}$$

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- ▶ Profit function is super-modular in elements of the sourcing strategy and in exporting

$$\begin{aligned} \pi_j^{\text{var}}(Z_j, I_{jF}) &= \frac{1}{\mu} \beta_{jD}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j(Z_j)^{(\sigma-1)/(\rho-1)} \frac{E}{P^{1-\sigma}} \\ &\quad + I_{jF} \frac{1}{\mu} \beta_{jF}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j(Z_j)^{(\sigma-1)/(\rho-1)} \tau^{1-\sigma} \frac{E_F}{P_F^{1-\sigma}} \end{aligned}$$

# Formation of domestic production network

- ▶ Set of eligible suppliers,  $Z_j$  ▶ Picture
- ▶ Order firms in an exogenous sequence
  - ▶ Firm 1 can only hire labor
  - ▶ Firm 2 can hire labor and purchase inputs from Firm 1
  - ▶ Firm 3 can hire labor and purchase inputs from Firm 1 and 2
  - ▶ ...

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- ▶  $Z_j$  has maximum 300 elements
  - ▶ Random draws of firms positioned earlier in the sequence
- ▶ Sequentially solve the problem of the firms
  - ▶  $Z_{k+1}$  does not affect firm  $k$  (other than through equilibrium objects  $w$  and  $A$ )

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# Equilibrium

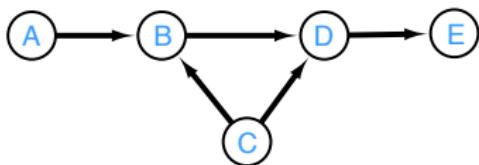
- ▶ Similar to previous section: small open economy
- ▶ Optimality of sourcing strategy and export decisions
- ▶ Fixed costs (paid in units of labor) enter the labor market clearing condition
- ▶ Trade balance condition is unchanged

# Endogenous network

## Shape of the network

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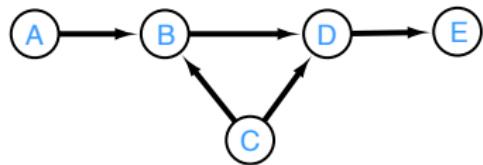
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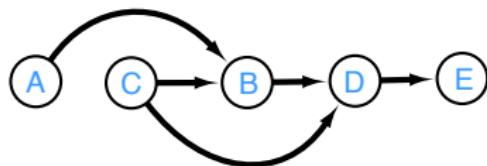
Directed acyclic network

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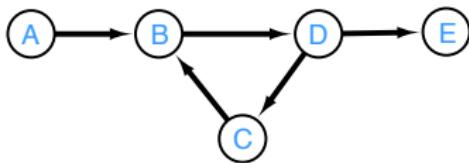
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Directed acyclic network, sorted

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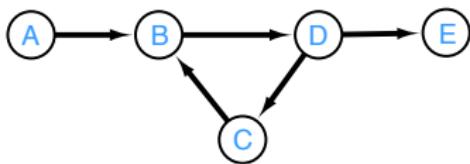
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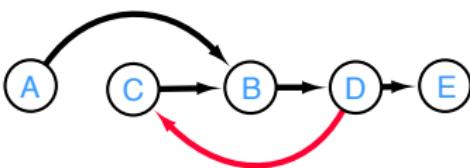
Directed cyclic network

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Directed cyclic network



Directed cyclic network, sorted

# Finding an ordering of firms

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- ▶ Let  $\nu(i)$  be an ordering of firms that maps firms  $\{i, j, k, \dots\} \in \Theta$  into numbers from  $\{1, \dots, N\}$ . We seek for the optimal  $\nu(\cdot)$  that minimizes the following objective function.

$$\min_{\{\nu(k)\}} \sum_{i,j} \mathbf{1}\{i \in Z_j\} \mathbf{1}\{\nu(i) > \nu(j)\}.$$

## Finding an ordering of firms

- ▶ Let  $\nu(i)$  be an ordering of firms that maps firms  $\{i, j, k, \dots\} \in \Theta$  into numbers from  $\{1, \dots, N\}$ . We seek for the optimal  $\nu(\cdot)$  that minimizes the following objective function.

$$\min_{\{\nu(k)\}} \sum_{i,j} \mathbf{1}\{i \in Z_j\} \mathbf{1}\{\nu(i) > \nu(j)\}.$$

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# Finding an ordering of firms

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- ▶ We obtain an ordering that produces around 18% of edges or 23% of the value of transactions in the whole firm-to-firm network in 2012 violating the ordering.
- ▶ Correlation between input-output table coefficients with 72 of full data and restricted data is 0.90 (unweighted ordering) / 0.97 (weighted ordering).

▶ Real wage changes with ordering data

# Endogenous network

## Empirical analysis

# Estimation with endogenous domestic production network

1. Recover the productivity distribution of firms (scaled by some general equilibrium objects) from the identity

$$\frac{x_{iH}}{s_{Li}^{(\sigma-1)/(1-\rho)}} = \phi_i^{\sigma-1} \frac{\mu^{1-\sigma} w^{1-\sigma}}{P^{1-\sigma}} E.$$

# Estimation with endogenous domestic production network

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$$\frac{x_{iH}}{s_{Li}^{(\sigma-1)/(1-\rho)}} = \phi_i^{\sigma-1} \frac{\mu^{1-\sigma} w^{1-\sigma}}{P^{1-\sigma}} E.$$

2. Estimate via simulated methods of moments distributional parameters of

- ▶ Firm-pair specific shifter in the production function,  $\alpha_{kj}$ ,  $\alpha_{Fj}$  ► details
- ▶ Firm-specific foreign demand shifter,  $\beta_{jF}$
- ▶ Fixed cost to add domestic supplier,  $wf_{kj}$
- ▶ Fixed cost to export,  $wf_{jF}$ , and import  $wf_{Fj}$
- ▶ Assume log-normal distributions

# Moments

- ▶ Domestic firm-to-firm sales,  $\hat{m}_1(\delta)$ :
  - ▶ Indegree
  - ▶ Outdegree
  - ▶ Labor share
  - ▶ Firm-to-Firm share
  - ▶ Calculate 25th, 50th, and 75th percentiles from data
- ▶ Imports and exports,  $\hat{m}_2(\delta)$ :
  - ▶ Share of importers and exporters
  - ▶ Direct and total foreign input share
  - ▶ Direct and total export share
  - ▶ Calculate 25th, 50th, and 75th percentiles from data
- ▶ Aggregate moments,  $\hat{m}_3(\delta)$ :
  - ▶ Ratio of agg. exports to agg. value added
  - ▶  $\sum_j s_{jH} s_{Fj}^{Total}$

## Estimated parameters

$\hat{\Phi}_{scale}^{\alpha_{dom}}$	$\hat{\Phi}_{scale}^{\alpha_F}$	$\hat{\Phi}_{scale}^{\beta_F}$	$\hat{\Phi}_{disp}^{\alpha,\beta}$	$\hat{\Phi}_{scale}^{f_{dom}}$	$\hat{\Phi}_{scale}^{f_{imp}}$	$\hat{\Phi}_{scale}^{f_{exp}}$	$\hat{\Phi}_{disp}^f$
-4.42	-2.22	-2.01	2.34	-3.21	2.64	6.75	6.98

## Model fit: targeted moments

	Data	Model
Number of dom. suppliers 25th percentile	19	17
Number of dom. suppliers 50th percentile	33	30
Number of dom. suppliers 75th percentile	55	49
Number of dom. buyers 25th percentile	2	2
Number of dom. buyers 50th percentile	9	13
Number of dom. buyers 75th percentile	34	46
Share of labor costs 25th percentile	0.17	0.12
Share of labor costs 50th percentile	0.34	0.25
Share of labor costs 75th percentile	0.54	0.49
Firm-to-Firm share 25th percentile	0.0002	0.0000
Firm-to-Firm share 50th percentile	0.0012	0.0003
Firm-to-Firm share 75th percentile	0.0053	0.0034

## Model fit: targeted moments

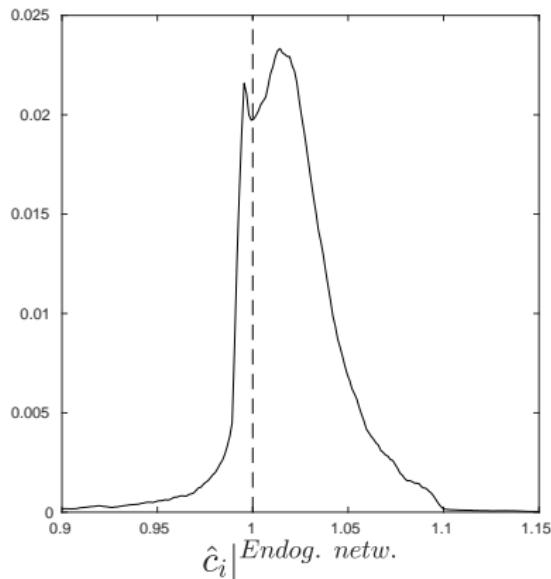
	Data	Model
Share of firms that export	0.12	0.10
Share of exports in total firm sales 25th percentile	0.01	0.02
Share of exports in total firm sales 50th percentile	0.10	0.14
Share of exports in total firm sales 75th percentile	0.62	0.50
Total export share 25th percentile	0.0008	0.0000
Total export share 50th percentile	0.0156	0.0328
Total export share 75th percentile	0.1005	0.2982
Share of firms that import	0.19	0.19
Share of imports in firm inputs 25th percentile	0.01	0.01
Share of imports in firm inputs 50th percentile	0.28	0.05
Share of imports in firm inputs 75th percentile	0.67	0.25
Total foreign input share 25th percentile	0.24	0.18
Total foreign input share 50th percentile	0.39	0.31
Total foreign input share 75th percentile	0.55	0.44
Ratio of aggregate exports to aggregate sales to domestic final demand	0.82	0.90
Import content of domestic final demand	0.58	0.65

## Model fit: non-targeted moments

	Data	Model
Corr (Indegree Buyer, Outdegree Seller)	-0.05	-0.13
Corr (Sales Buyer, Sales Seller)	-0.02	0.01

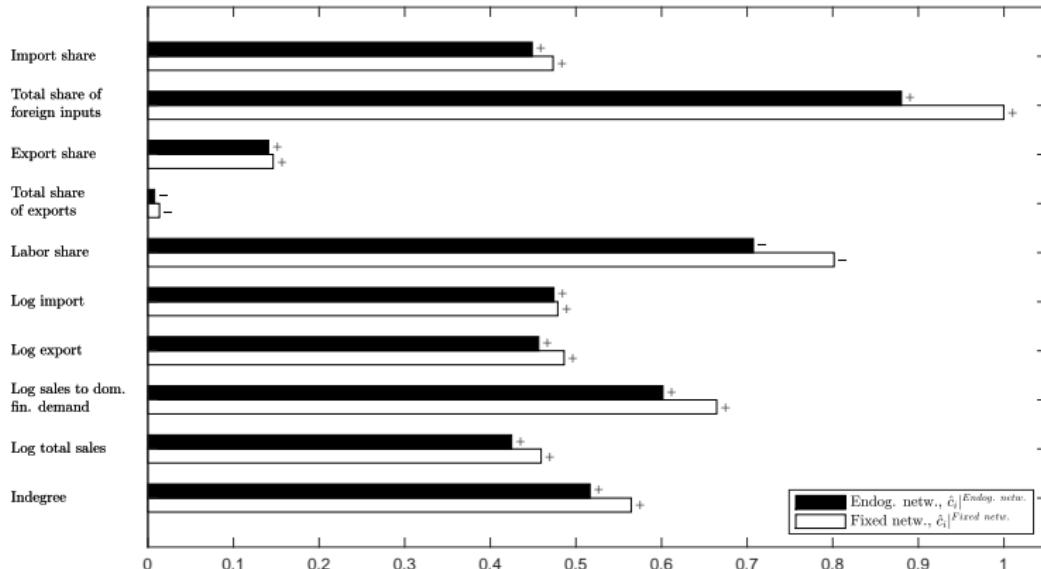
# Counterfactual (10% increase in foreign price)

Figure 1: Density of cost changes under endogenous networks



# Counterfactual (10% increase in foreign price)

Figure 2: Correlations with firm-level variables



# Counterfactual

Table 1: Changes in real wage upon change in foreign price

	Endogenous Network	Fixed Network
Autarky ( $\hat{p}_{F.} \rightarrow \infty$ )	0.4844	0.4157
50% decrease ( $\hat{p}_{F.} = 0.5$ )	2.2779	2.0170

# Counterfactual

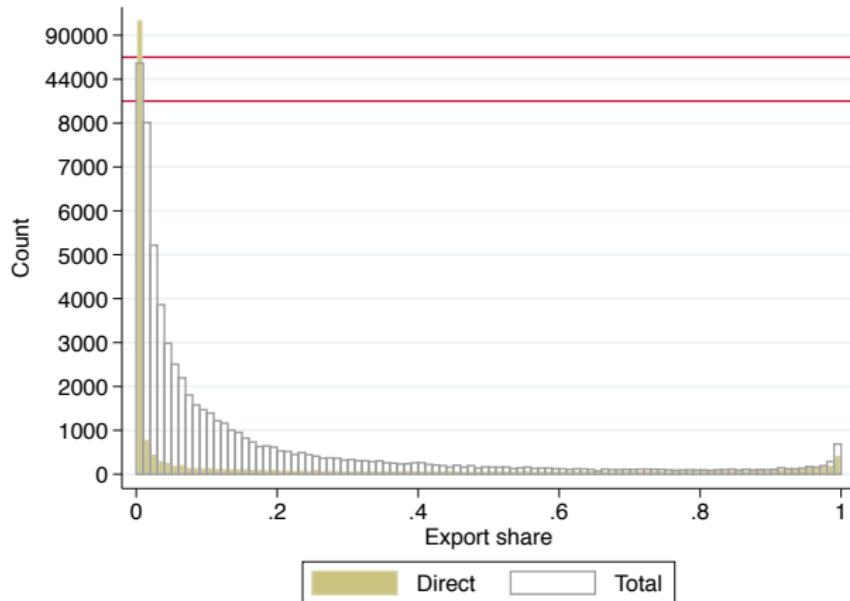
**Table 1:** Changes in real wage upon change in foreign price

	Endogenous Network	Fixed Network
Autarky ( $\hat{p}_F \rightarrow \infty$ )	0.4844	0.4157
50% decrease ( $\hat{p}_F = 0.5$ )	2.2779	2.0170
10% increase ( $\hat{p}_F = 1.1$ )	0.9388	0.9321
10% decrease ( $\hat{p}_F = 0.9$ )	1.0681	1.0881

# Back-up

# Direct and indirect exporting

$$r_{jF}^{Total} = r_{jF} + \sum_{k \in W_j} r_{jk} r_{kF}^{Total}$$



88% of firms export directly or indirectly; Median: 2%; 80th percentile: 14%

▶ Export share by sector

▶ Excluding exports wholesale and retail sectors

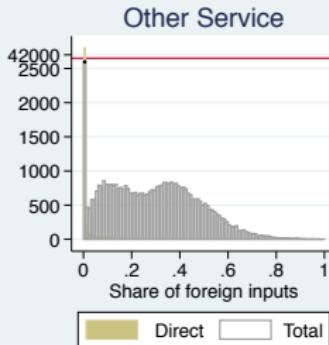
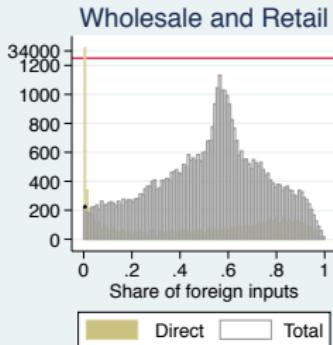
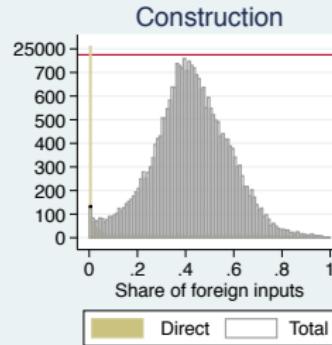
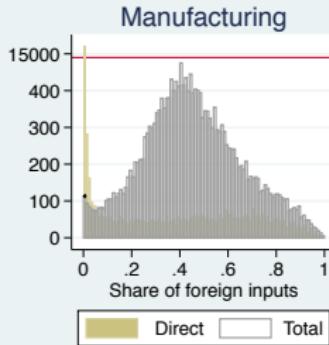
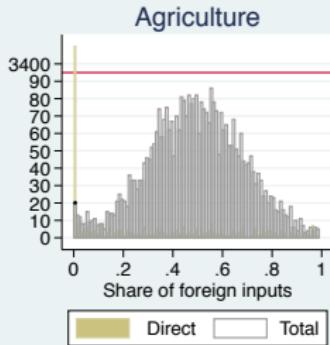
# Quantiles of indegree and outdegree distributions

	25th	Median	75th
Indegree	19	33	55
Outdegree	2	9	34

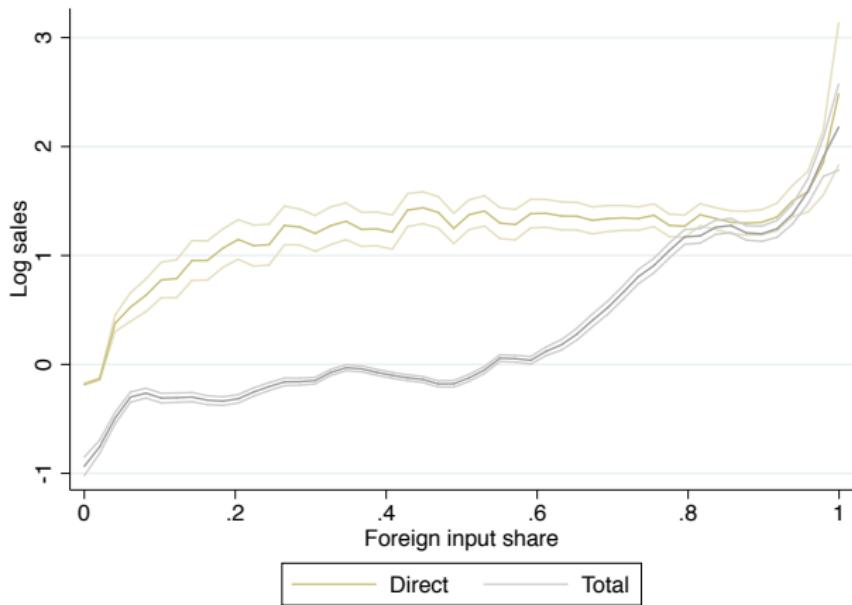
Size premium of direct and indirect linkages to Imports and Exports

► ICD and ICE by sector

# Total and Direct share of foreign inputs by private sector firms in Belgium

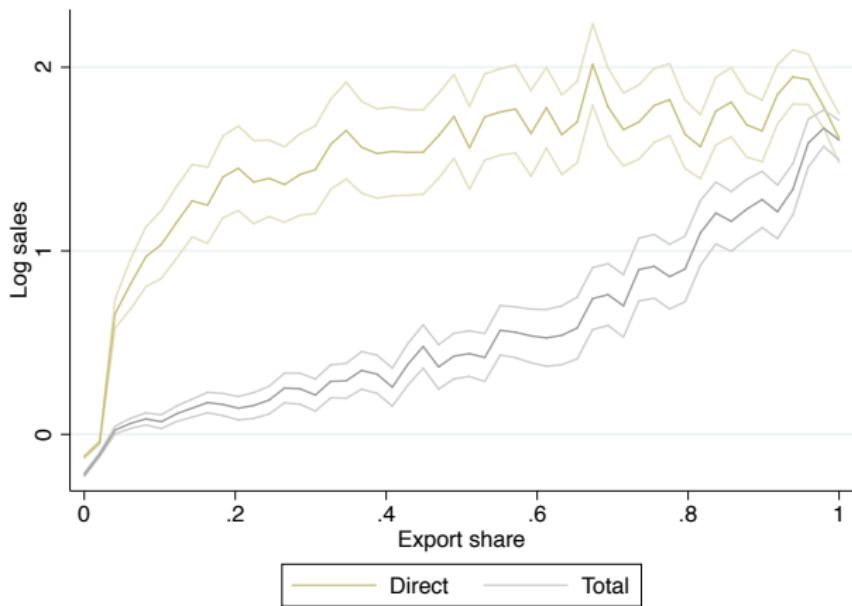


# Size premium of direct and indirect linkages to imports



▶ Back

# Size premium of direct and indirect linkages to exports



▶ Back

# Firm-level cost changes due to foreign price change

- ▶ Autarky ( $p_{F.} \rightarrow \infty$ ):

$$\hat{c}_j \mid p_{F.} \rightarrow \infty = (1 - s_{Fj}^{Total})^{1/(1-\rho)} \hat{w}^{aut}$$

- ▶ Small percentage point change in the foreign price,  $\frac{dp_{F.}}{p_{F.}}$ :

$$\frac{dc_j}{c_j} = (1 - s_{Fj}^{Total}) \frac{dw}{w} + s_{Fj}^{Total} \frac{dp_{F.}}{p_{F.}}$$

▶ Back

# Change in real wage

- ▶ Autarky ( $p_{F.} \rightarrow \infty$ ):

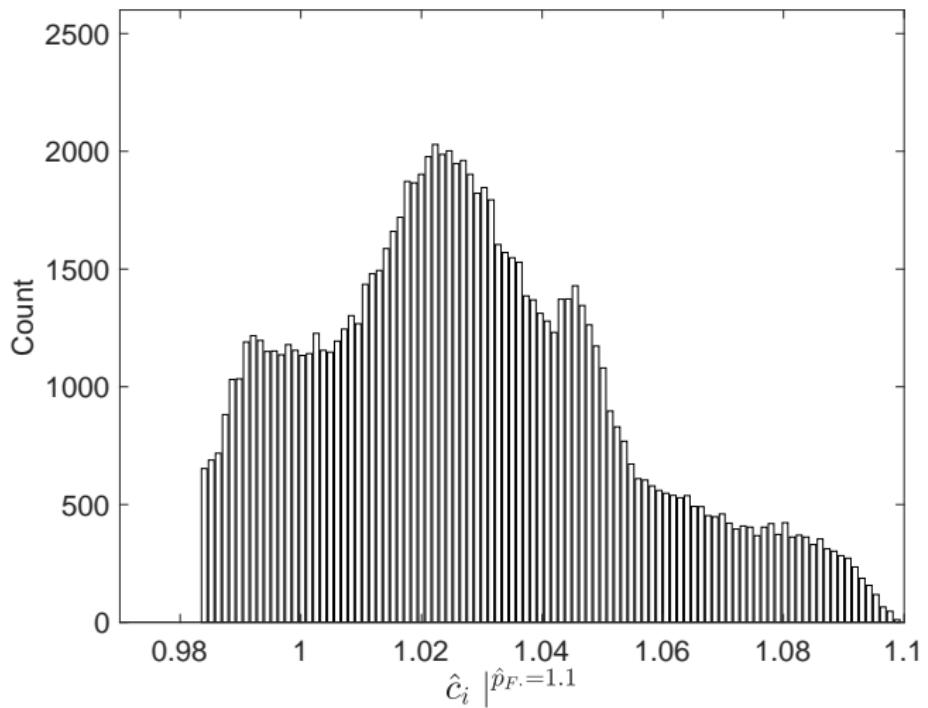
$$\frac{\hat{w}}{\hat{P}} \mid \hat{p}_{F.} \rightarrow \infty = \left( \sum_i s_{iH} (1 - s_{Fj}^{Total})^{\frac{1-\sigma}{1-\rho}} \right)^{\frac{1}{\sigma-1}}$$

- ▶ Small percentage point change in the foreign price,  $\frac{dp_{F.}}{p_{F.}}$ :

$$\begin{aligned} \frac{dw}{w} - \frac{dP}{P} &= \frac{dw}{w} - \sum_j s_{jH} \frac{dc_j}{c_j} \\ &= \left( \frac{dw}{w} - \frac{dp_{F.}}{p_{F.}} \right) \sum_j s_{jH} s_{Fj}^{Total} \end{aligned}$$

▶ Back

# Firm-level cost changes



▶ Back

# Alternative modeling approaches

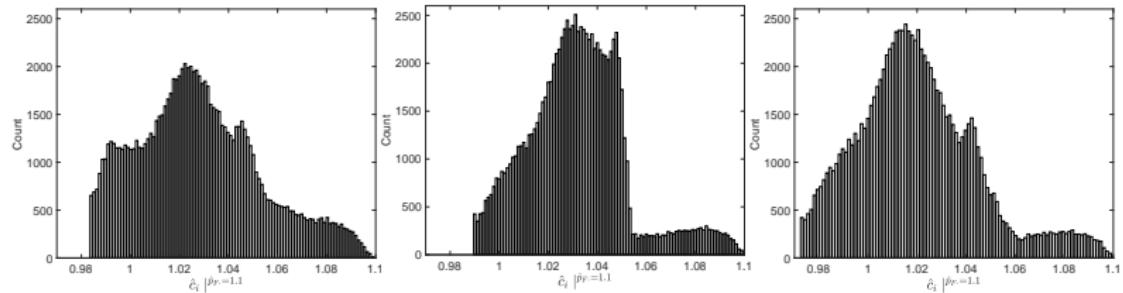
- ▶ Simple roundabout production economy:

$$c_j = \phi_i^{-1} \left( \alpha_{Dj}^{\rho-1} P^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{Lj}^{\rho-1} w^{1-\rho} \right)^{\frac{1}{1-\rho}}$$

- ▶ Sectoral roundabout production economy:

$$c_j = \phi_j^{-1} \left( \alpha_{Dj}^{\rho-1} \left( \prod_v P_v^{\gamma_{vu(j)}} \right)^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{Lj}^{\rho-1} w^{1-\rho} \right)^{\frac{1}{1-\rho}}$$

# Firm-level cost changes



a: Baseline model

b: Simple roundabout

c: Sectoral roundabout

► Sensitivity

► Back

# Comparison across models

a: Changes in real wage upon 10% increase in foreign price

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{w}}{\hat{P}} \mid \hat{p}_{F^*}$	0.940	0.941	0.931

b: Changes in real wage upon autarky

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{w}}{\hat{P}} \mid \hat{p}_{F^*} \rightarrow \infty$	0.558	0.596	0.441

► Real income

► Sensitivity

► Back

# Comparison with Hulten (1978)

- ▶ Real wage change under perfect competition

$$\underbrace{\frac{dw}{w} - \frac{dP}{P}}_{\text{Real wage change under perfect competition}} = \underbrace{\frac{\text{Imports}}{\text{VA}}}_{\text{Domar weight}} \left( \underbrace{\sum_j s_{jF} \left( (1 - s_{Fj}^{Total}) \frac{dw}{w} + s_{Fj}^{Total} \frac{dp_{F.}}{p_{F.}} \right)}_{\text{Change in export price}} - \underbrace{\frac{dp_{F.}}{p_{F.}}}_{\text{Change in import price}} \right) \\ = \frac{\text{Imports}}{\text{VA}} \left( \frac{dw}{w} - \frac{dp_{F.}}{p_{F.}} \right) \sum_j s_{jF} (1 - s_{Fj}^{Total})$$

- ▶ If no import content in exports:  $\sum_j s_{jF} (1 - s_{Fj}^{Total}) = 1$ .
- ▶ If same import content in exports and domestic consumption:

$$\sum_j s_{jF} (1 - s_{Fj}^{Total}) = \frac{\text{VA}}{\text{VA} + \text{Exports}}.$$

▶ Back

▶ Baseline model

# Sensitivity of firm-level cost change

a: Median  $\hat{c}_i$

$\rho$	$\sigma$	Baseline	Simple Roundabout	Sectoral Roundabout
1.5	2	1.035	1.038	1.030
1.5	4	1.017	1.025	1.008
1.5	6	1.009	1.019	0.998
2	2	1.046	1.047	1.041
2	4	1.027	1.032	1.018
2	6	1.017	1.024	1.006

b: 90th percentile  $\hat{c}_i$

$\rho$	$\sigma$	Baseline	Simple Roundabout	Sectoral Roundabout
1.5	2	1.069	1.058	1.059
1.5	4	1.061	1.047	1.045
1.5	6	1.057	1.042	1.038
2	2	1.075	1.064	1.065
2	4	1.065	1.052	1.051
2	6	1.060	1.046	1.043

▶ Back

# Changes in real income

a: Changes in real income upon 10% increase in foreign price

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{E}}{\hat{P}} \mid \hat{p}_F \cdot 1.1$	0.883	0.878	0.887

b: Changes in real income upon autarky

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{E}}{\hat{P}} \mid \hat{p}_F \rightarrow \infty$	0.374	0.390	0.291

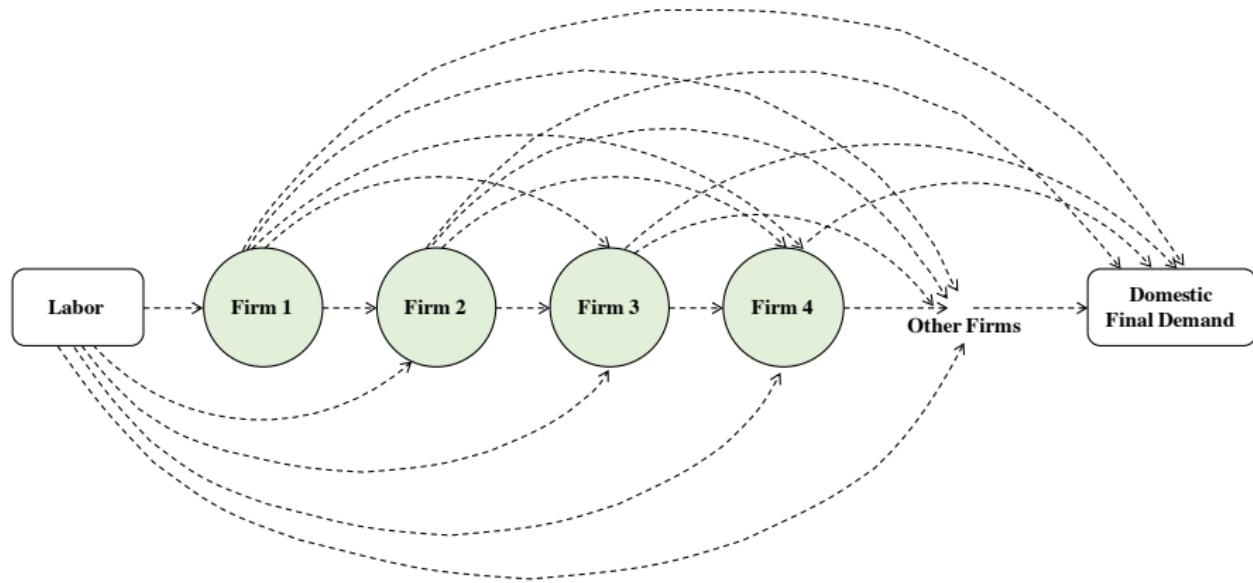
▶ Back

# Sensitivity of real wage change

$\rho$	$\sigma$	Baseline	Simple Roundabout	Sectoral Roundabout
1.5	2	0.945	0.944	0.939
1.5	4	0.931	0.934	0.922
1.5	6	0.927	0.932	0.915
2	2	0.955	0.953	0.949
2	4	0.940	0.941	0.931
2	6	0.934	0.937	0.923

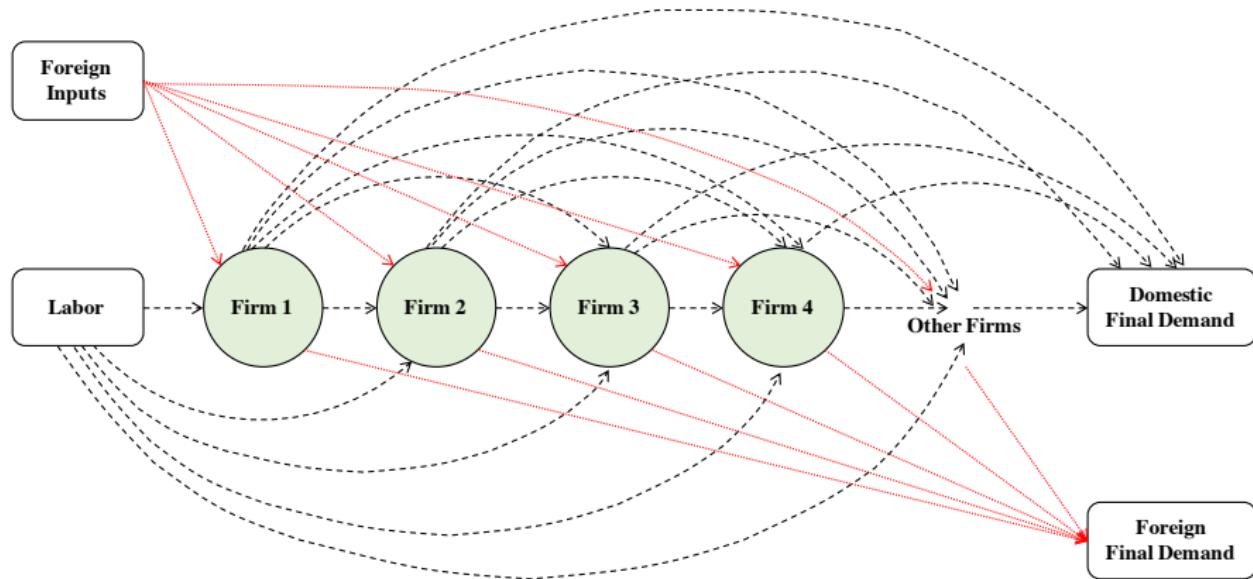
▶ Back

# Network formation



▶ back

# Network formation with trade



▶ back

# Aggregating vats to firms

- ▶ We group all VAT-id into firms that are either
  - ▶ linked with more than 50% of ownership (ownership filings).
  - ▶ have common foreign parent firm (FDI filings).
- ▶ In 2012, 896K VAT-ids collapsed to 860K firms. Of those firms, 842K firms consisted of single VAT-ids. The number of VAT-ids for multiple VAT-id firms are as below.

	Mean	10%	25%	50%	75%	90%	max
Num. VAT-id	3	2	2	2	3	5	372

- ▶ The 18K firms with multiple VAT-ids account for ~ 60% of the total output.

▶ Back

# Sample of analysis

- ▶ We restrict the sample of analysis according to the criteria below:
  - ▶ Belgian firms in industries other than government and finance.
  - ▶ File positive sales, positive labor cost, and at least one FTE employment
  - ▶ Following De Loecker et al. (2014) we also restrict the sample to firms with tangible assets of more than 100 euro, positive total assets for at least one year throughout the period.

Year	GDP (Excl. Gov. & Fin.)	Output (Excl. Gov. & Fin.)	Imports	Exports	Selected sample				
					Count	V.A.	Sales	Imports	Exports
2002	182	458	178	193	88,301	119	604	175	185
2007	230	593	254	267	95,941	152	782	277	265
2012	248	671	317	319	98,745	164	874	292	292

Notes: All numbers except for Count are denominated in billion Euro in current prices. Data for Belgian GDP, imports and exports are from Eurostat.

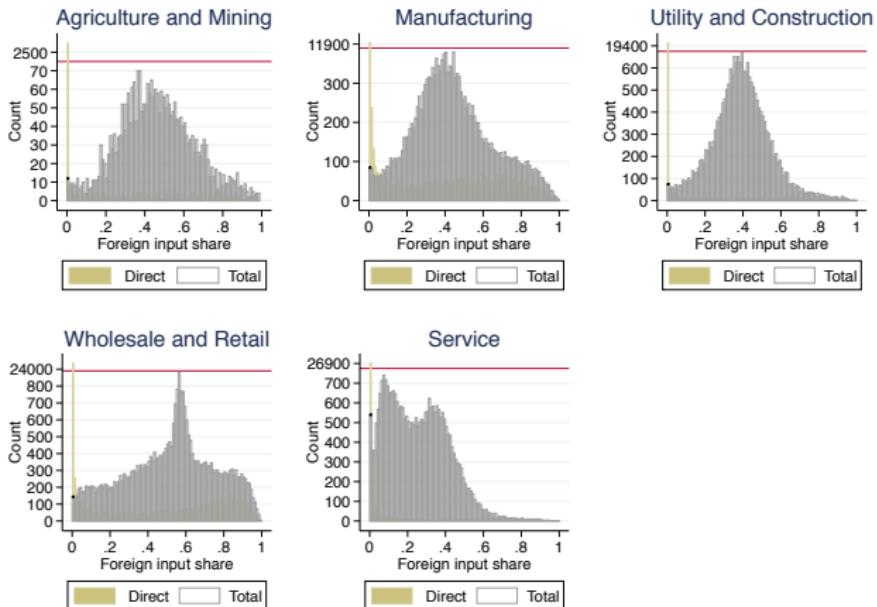
# Sectoral composition

Table 5: Sectoral composition in 2012

Sector	Count	V.A.	Output	Imports	Exports
Agriculture and Mining	2,805	28.5	49.4	16.9	10.9
Manufacturing	16,577	138	272	146	193
Utility and Construction	20,421	23.3	77.0	27.8	17.5
Wholesale and Retail	31,117	87.8	241	84.1	53.4
Service	27,825	79.1	127	17.6	16.9
Total	98,745	356	874	292	292

▶ Back

# Direct and total foreign input share by firms' sector



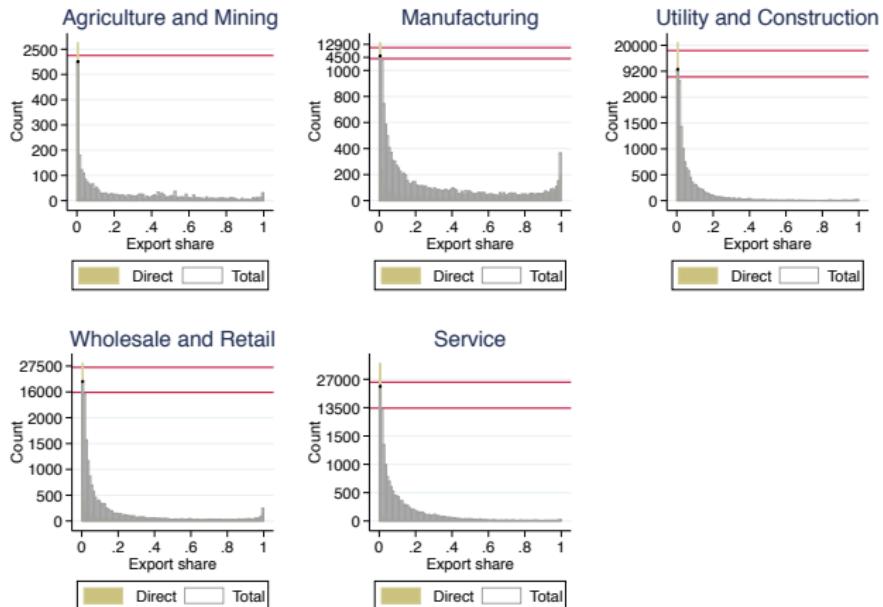
▶ Back

# Direct and total foreign input share by firms' sector

Sector	Direct			Total		
	Mean	Weighted Mean	Median	Mean	Weighted Mean	Median
Agriculture and Mining	0.05	0.70	0	0.46	0.85	0.45
Manufacturing	0.12	0.59	0	0.44	0.75	0.42
Utility and Construction	0.02	0.31	0	0.39	0.59	0.39
Wholesale and Retail	0.12	0.43	0	0.52	0.75	0.55
Service	0.01	0.19	0	0.25	0.41	0.24
Total	0.07	0.45	0	0.40	0.68	0.39

▶ Back

# Direct and total export share by firms' sector



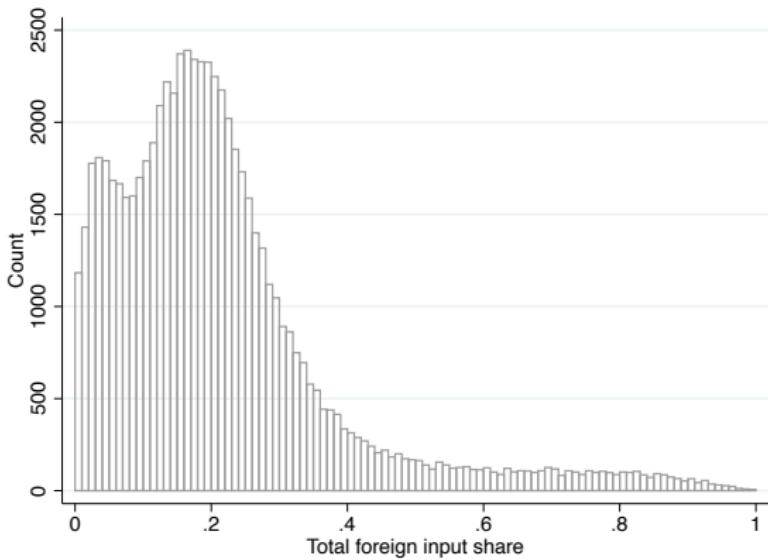
▶ Back

# Direct and total export share by firms' sector

Sector	Direct			Total		
	Mean	Weighted Mean	Median	Mean	Weighted Mean	Median
Agriculture and Mining	0.05	0.22	0	0.24	0.30	0.11
Manufacturing	0.11	0.56	0	0.23	0.62	0.07
Utility and Construction	0.01	0.19	0	0.06	0.25	0.01
Wholesale and Retail	0.04	0.21	0	0.09	0.26	0.01
Service	0.01	0.13	0	0.07	0.23	0.01
Total	0.04	0.33	0	0.11	0.40	0.02

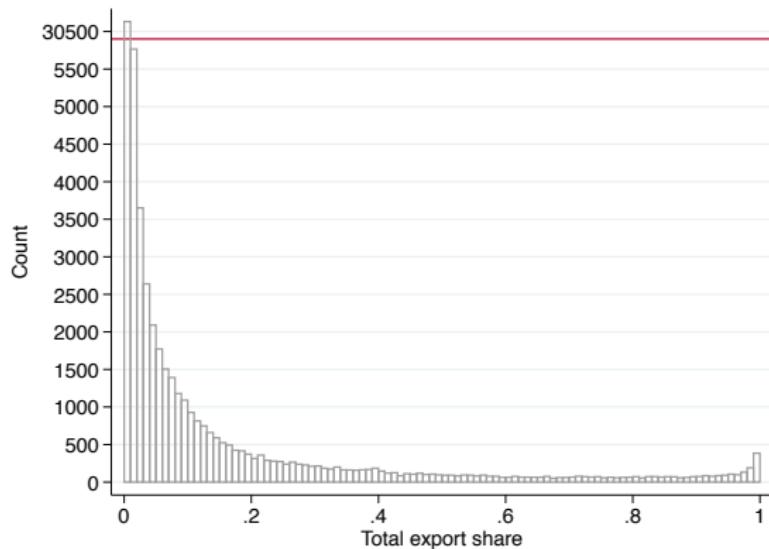
▶ Back

# Total foreign input share (excluding wholesale and retail sectors)



▶ Back

# Total export share (excluding wholesale and retail sectors)



▶ Back

# ICD and ICE across sectors

Sector	ICD	ICE
Agriculture and Mining	0.068	0.033
Manufacturing	0.158	0.522
Utility and Construction	0.059	0.052
Wholesale and Retail	0.237	0.155
Service	0.062	0.038
Total	0.584	0.800

▶ Back

# Gains from trade – example

- ▶ Consider two economies with the same level of GDP, gross production, and exports and imports
- ▶ Assume fixed network

	Economy 1			Economy 2		
	Firm 1	Firm 2	Firm 3	Firm 1	Firm 2	Firm 3
Imports	100	0	0			
Exports	0	0	100			
Gross production	200	200	200			
Domestic sales	200	200	100			
Labor costs	50	100	50			
Domestic purchases	0	50	100			
Profits	50	50	50			
Firm-to-firm sales	$x_{12} = 50$		$x_{23} = 50$			
	$x_{13} = 50$					
Sales to households	100	150	100			

# Gains from trade – example

- ▶ Consider two economies with the same level of GDP, gross production, and exports and imports
- ▶ Assume fixed network

	Economy 1			Economy 2		
	Firm 1	Firm 2	Firm 3	Firm 1	Firm 2	Firm 3
Imports	100	0	0	100	0	0
Exports	0	0	100	0	0	100
Gross production	200	200	200	200	200	200
Domestic sales	200	200	100	200	200	100
Labor costs	50	100	50	50	100	50
Domestic purchases	0	50	100	0	50	100
Profits	50	50	50	50	50	50
Firm-to-firm sales	$x_{12} = 50$ $x_{23} = 50$			$x_{23} = 100$ $x_{32} = 50$		
	$x_{13} = 50$					
Sales to households	100	150	100	200	100	50

## Gains from trade – example (continued)

- Direct and total shares of foreign inputs and shares in household consumption:

	Economy 1			Economy 2		
	Firm 1	Firm 2	Firm 3	Firm 1	Firm 2	Firm 3
$s_{Fi}$	2/3	0	0	2/3	0	0
$s_{Fi}^{Total}$	2/3	2/9	2/27	2/3	0	0
$s_{iH}$	2/7	2/7	3/7	4/7	1/7	2/7

## Gains from trade – example (continued)

- Direct and total shares of foreign inputs and shares in household consumption:

	Economy 1			Economy 2		
	Firm 1	Firm 2	Firm 3	Firm 1	Firm 2	Firm 3
$s_{Fi}$	2/3	0	0	2/3	0	0
$s_{Fi}^{Total}$	2/3	2/9	2/27	2/3	0	0
$s_{iH}$	2/7	2/7	3/7	4/7	1/7	2/7

- Plugging in  $\frac{\hat{w}}{\hat{P}} \mid \hat{p}_{F^*} \rightarrow \infty$  ( $\sigma = 4$ ,  $\rho = 2$ )

$$\frac{\hat{w}}{\hat{P}} \mid \hat{p}_{F^*} \rightarrow \infty = \left( \sum_i s_{iH} \left(1 - s_{Fi}^{Total}\right)^{\frac{1-\sigma}{1-\rho}} \right)^{\frac{1}{\sigma-1}} = \begin{cases} 0.79 & \text{if economy 1} \\ 0.77 & \text{if economy 2} \end{cases}$$

▶ Back

# Log-changes in real wage, FOA

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{dw}{w} - \frac{dp_{F.}}{p_{F.}}$	-0.118	-0.112	-0.130
$\sum_j s_j H^s F_j^{Total}$	0.584	0.609	0.607
$\frac{dw}{w} - \frac{dP}{P} = \left( \frac{dw}{w} - \frac{dp_{F.}}{p_{F.}} \right) \sum_j s_j H^s F_j^{Total}$	-0.069	-0.068	-0.079

# Log-changes in real wage, FOA

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{dw}{w} - \frac{dp_{F\cdot}}{p_{F\cdot}}$	-0.118	-0.112	-0.130
$\sum_j s_j H^s F_j^{Total}$	0.584	0.609	0.607
$\frac{dw}{w} - \frac{dP}{P} = \left( \frac{dw}{w} - \frac{dp_{F\cdot}}{p_{F\cdot}} \right) \sum_j s_j H^s F_j^{Total}$	-0.069	-0.068	-0.079

- ▶  $\frac{\text{Imports}}{\text{VA}} = 0.817$
- ▶  $\frac{\text{Imports}}{\text{VA} + \text{Exports}} = 0.451$

# $\frac{\hat{w}}{\hat{P}}$ under acyclic network

Table 6: Change in real wage  $\frac{\hat{w}}{\hat{P}}$  under acyclic network

$\rho$	$\sigma$	10% increase in $p_F$ .		Autarky	
		$\frac{\hat{w}}{\hat{P}}$ , Baseline	$\frac{\hat{w}}{\hat{P}}$ , acyclic	$\frac{\hat{w}}{\hat{P}}$ , Baseline	$\frac{\hat{w}}{\hat{P}}$ , acyclic
1.5	2	0.945	0.942	0.251	0.223
1.5	4	0.931	0.927	0.437	0.408
1.5	6	0.927	0.922	0.541	0.519
2	2	0.955	0.952	0.391	0.388
2	4	0.940	0.936	0.558	0.530
2	6	0.934	0.929	0.634	0.609

▶ Back

## Firm-pair specific shifters

- ▶ Firms' outdegree distribution is more skewed than firms' indegree distributions
- ▶ Assume that the  $\alpha_{kj}$  draw is a product of a supplier- $k$ -specific random variables and an i.i.d. firm-pair-specific random variable
- ▶ The supplier- $k$ -specific random variables are drawn from a Beta distribution with shape parameters 0.1 and 0.9

▶ Back