Problem Set #1

OSE Computation

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Question 1: HPC Basics

$T(p, N) :=$ time to solve problem of total size N on p processors.

Parallel Speedup:
$$S(p, N) = \frac{T(1, N)}{T(p, N)}$$

Parallel Efficiency:
$$E(p, N) = \frac{S(p, N)}{p}$$

Amdahl's Law:
$$T(p, N) = fT(1, N) + (1 - f)\frac{T(1, N)}{p}, \text{ where f is sequential part of code}$$

Therefore, we can re-express $S(p, N)$ as the following:

$$S(p, N) = \frac{T(1, N)}{T(p, N)} = \frac{1}{(f + \frac{1-f}{p})}$$

If we increase the number of processors $p \to \infty$, we can see that speedup is limited by:

$$S(p, N) < \frac{1}{f}$$

Therefore, if we have 0.4% serial code, the theoretical maximum speedup we can obtain is

$$S(p, N) < \frac{1}{0.4} = \boxed{2.5}$$

If we have 100 CPU cores (processors), the maximum speedup we can obtain is:

$$S(p, N) = \frac{1}{(0.4 + \frac{1-0.4}{100})} \simeq \boxed{2.46}$$