Lecture 2: Benchmark Heterogeneous Firm Model and Overview of Solution Methods

Thomas Winberry

July 24th, 2019

Motivating Facts: Doms and Dunne (1998)

Outcomes:
1) Economic model
2) structural model (handling hetrogeneity, considered as state variable)
3) Computation and how solve problem

Measurement

- Use Census data from LRD, 1972 - 1988
  - After 1988, stopped collecting data on sales and retirements

- Construct capital stock using perpetual inventory method
  - Focus on balanced panel

Get K_t from book value (sometimes adjust). Use law of motion of capital and data on investment to get K_(t+1).

- Analyze the growth rate of capital for plant *i* at time *t*

$$GK_{it} = \frac{i_{it} - \delta k_{it-1}}{0.5 \times (k_{it-1} + k_{it})}$$

i = plant, t = year.
Net change in capital stock. Growth rate of capital then divide by average.
Robustness measurement error. Entry and exit information incorporated.

Plant-Level Investment is Lumpy Across Plants

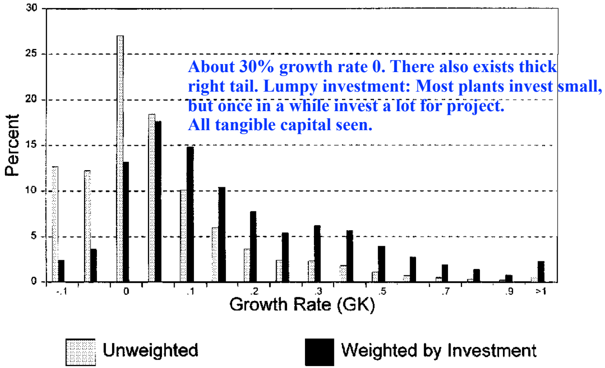
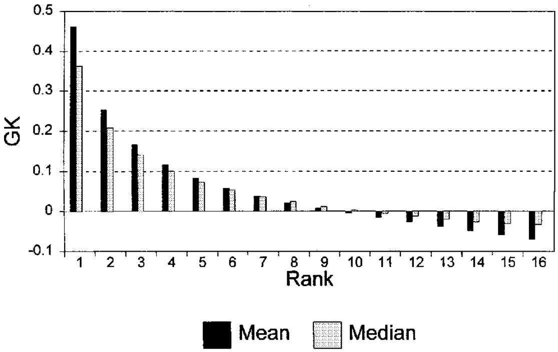


FIG. 1. Capital growth rate (GK) distributions: Unweighted and weighted by investment.

- 51.9% of plants increase capital $\leq 2.5\%$
- 11% of plants increase capital $\geq 20\%$

Plant-Level Investment is Lumpy Within Plants



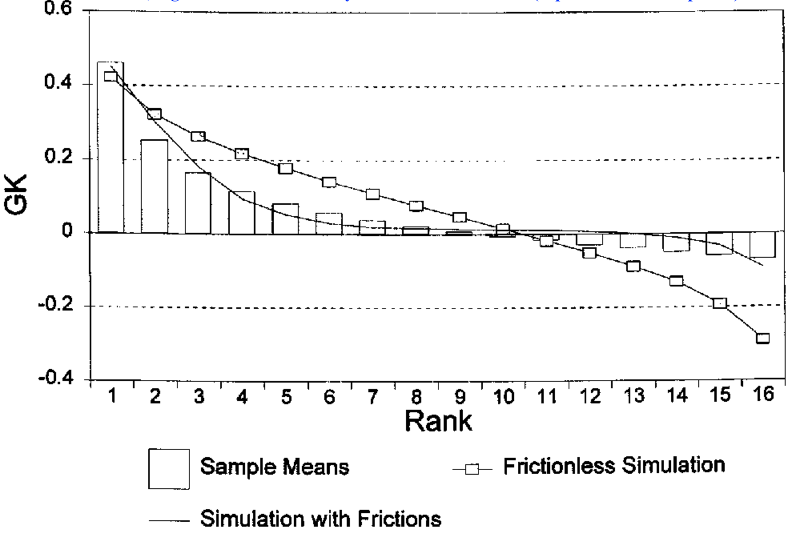
Shows most of time do nothing, but sometimes do high investment. Kurtosis investment.

- Capital growth in largest investment episode nearly 50%
- In median investment episode approximately 0%

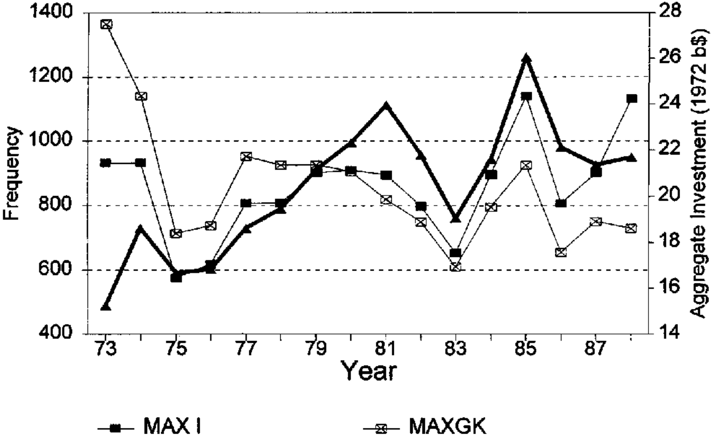
Investment at micro level:

Plant-Level Investment is Lumpy Within Plants

If shocks continuously distributed, capital stock proportional to shocks.
So will not get excess kurtosis. More continuous. Need non-convex capital stock.
Need pay adjustment cost so for low investment not worth the fixed cost.
Thus, big investment would only be worth for fixed cost (explanation for lumpiness).



Frequency of Spikes Correlated with Aggregate Investment



Benchmark Model: Khan and Thomas (2008)

Lumpiness of capital taken into account.

Model Overview

RBC model with heterogeneous firms.

Heterogeneous Firms

- Fixed mass
- Idiosyncratic + aggregate productivity shocks

Business cycle shocks based on aggregate productivity shocks only.

- Fixed capital adjustment costs

Representative Household

- Owns firms
- Supplies labor
- Complete markets

Heterogeneous Firms

Production technology $y_{jt} = e^{z_t} e^{\varepsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu}, \theta + \nu < 1$ **Decreasing returns. To have best firms not kick out small firms.**

- Idiosyncratic productivity shock $\varepsilon_{jt+1} = \rho_\epsilon \varepsilon_{jt} + \omega_{jt+1}^{\varepsilon}$ where $\omega_{jt+1}^{\varepsilon} \sim N(0, \sigma_\epsilon^2)$ **Independently distributed. Law of large numbers can be used!**
- Aggregate productivity shock $z_{t+1} = \rho_z z_t + \omega_{t+1}^{z}$ where $\omega_{t+1}^{z} \sim N(0, \sigma_z^2)$ **Common shock among firms. AR1 process .**

Capital is being inherited from previous period. Save some for next period.

Firms accumulate capital according to $k_{jt+1} = (1-\delta)k_{jt} + i_{jt}$

- If $\frac{i_{jt}}{k_{jt}} \notin [-a, a]$, pay fixed cost ξ_{jt} in units of labor
- Fixed cost $\xi_{jt} \sim U[0, \overline{\xi}]$

Investment rate (percent change in capital stock).
Small investment then don't have to pay fixed cost (from data seems small, maintenance cost).
Big investment implies pay high fixed cost.

Size of fixed cost is random. Randomness for fixed cost leads to more smooth capital. If fixed cost is fixed, they will always adjust at specific capital.

Firm Optimization Problem: Recursive Formulation

$$v(\varepsilon, k, \xi; \mathbf{s}) = \max_n e^z e^\varepsilon k^\theta n^\nu - w(\mathbf{s}) n$$
$$+ \max \left\{ v^A(\varepsilon, k; \mathbf{s}) - w(\mathbf{s}) \xi, v^N(\varepsilon, k; \mathbf{s}) \right\}$$

For heterogeneous models include: Individual state variable (specific to firm) and aggregate state variable (S, common to all firms).
Choose employment (n).

Firm Optimization Problem: Recursive Formulation

Once draw psi:

$$v(\varepsilon, k, \xi; \mathbf{s}) = \max_n e^z e^\varepsilon k^\theta n^\nu - w(\mathbf{s}) n$$
$$+ \max \left\{ v^A(\varepsilon, k; \mathbf{s}) - w(\mathbf{s}) \xi, v^N(\varepsilon, k; \mathbf{s}) \right\}$$

VA is always at least as big VN. If psi is small enough will choose VA.

Paying fixed cost value function:

Amount of investment i

Psi independent so no need to know transition!

$$v^A(\varepsilon, k; \mathbf{s}) = \max_{i \in \mathbb{R}} -i + \mathbb{E}\left[\Lambda(\mathbf{s}'; \mathbf{s}) v(\varepsilon', k', \xi'; \mathbf{s}') | \varepsilon, k; \mathbf{s}\right]$$

^: beta * marginal transition for firms!

Aggregate TFP (s') transition.

Not paying fixed cost value function: Invesstment (i) constrained to be small.

$$v^N(\varepsilon, k; \mathbf{s}) = \max_{i \in [-ak, ak]} -i + \mathbb{E}\left[\Lambda(\mathbf{s}'; \mathbf{s}) v(\varepsilon', k', \xi'; \mathbf{s}') | \varepsilon, k; \mathbf{s}\right]$$

Firm Optimization Problem: Recursive Formulation

$$v(\varepsilon, k, \xi; \mathbf{s}) = \max_n e^z e^\varepsilon k^\theta n^\nu - w(\mathbf{s})\, n + \max\left\{ v^A(\varepsilon, k; \mathbf{s}) - w(\mathbf{s})\,\xi,\ v^N(\varepsilon, k; \mathbf{s}) \right\}$$

Before drawing psi. By integrating out psi.

$$\widehat{v}(\varepsilon, k; \mathbf{s}) = \max_n e^z e^\varepsilon k^\theta n^\nu - w(\mathbf{s})\, n + \frac{\widehat{\xi}(\varepsilon, k; \mathbf{s})}{\overline{\xi}} \left(v^A(\varepsilon, k; \mathbf{s}) - w(\mathbf{s}) \frac{\widehat{\xi}(\varepsilon, k; \mathbf{s})}{2} \right)$$

psi distribution is uniform so probability of psi is psi/psi_upper bar.

$$+ \left(1 - \frac{\widehat{\xi}(\varepsilon, k; \mathbf{s})}{\overline{\xi}}\right) v^N(\varepsilon, k; \mathbf{s})$$

Household

Representative household who owns all firms in the economy

$$\max_{C(\mathbf{s}),N(\mathbf{s})} \frac{C(\mathbf{s})^{1-\sigma}-1}{1-\sigma} - \chi \frac{N(\mathbf{s})^{1+\alpha}}{1+\alpha} \text{ such that}$$

$$C(\mathbf{s}) = w(\mathbf{s})N(\mathbf{s}) + \Pi(\mathbf{s})$$

HH don't hold capital. So no intertemporal decisions made by HH.
Give this decision making to firms.
Decision for capital intertemporal decision taken away.
Decides how much to work and consumption only.

Household

Representative household who owns all firms in the economy

$$\max_{C(\mathbf{s}),N(\mathbf{s})} \frac{C(\mathbf{s})^{1-\sigma}-1}{1-\sigma} - \chi \frac{N(\mathbf{s})^{1+\alpha}}{1+\alpha} \text{ such that}$$

$$C(\mathbf{s}) = w(\mathbf{s})N(\mathbf{s}) + \Pi(\mathbf{s})$$

Intertemporal marginal substitution of consumption.
(Theory/intuition) HH own firms. HH value output different states of world based on marginal substitution of consumption.

Complete markets implies that $\Lambda(\mathbf{s}'; \mathbf{s}) = \beta \left(\frac{C(\mathbf{s}')}{C(\mathbf{s})}\right)^{-\sigma}$

- Firms maximize their market value
- Market value given by expected present value of dividends using stochastic discount factor
- With complete markets, SDF is household's intertemporal marginal rate of substitution

Defining Recursive Competitive Equilibrium

What is the aggregate state $\mathbf{s}$?

Defining Recursive Competitive Equilibrium

What is the aggregate state **s**?

- Aggregate shock z **TFP**

Defining Recursive Competitive Equilibrium

What is the aggregate state $\mathbf{s}$?

- Aggregate shock z
- Firm's individual states: productivity ε and capital k

Defining Recursive Competitive Equilibrium

What is the aggregate state **s**?

- Aggregate shock z
- Firm's individual states: productivity ε and capital k

$\implies$ need distribution of firms $g(\varepsilon, k)$

Market clearing condition. Aggregate labor demand = aggregate labor supply.
For aggregate need need to sum up in that case we need to have the distribution.

Defining Recursive Competitive Equilibrium

What is the aggregate state $\mathbf{s}$?

- Aggregate shock z
- Firm's individual states: productivity ε and capital k

  $\implies$ need distribution of firms $g(\varepsilon, k)$

What is the law of motion for the $\mathbf{s}$?

Defining Recursive Competitive Equilibrium

What is the aggregate state **s**?

- Aggregate shock z
- Firm's individual states: productivity ε and capital k

$\Longrightarrow$ need distribution of firms $g(\varepsilon, k)$

What is the law of motion for the **s**?

$$g'(\varepsilon', k') = \int \left[\begin{array}{c} \mathbb{1}\left\{\rho_\varepsilon \varepsilon + \sigma_\varepsilon \omega'_\varepsilon = \varepsilon'\right\} \\ \times \int \mathbb{1}\left\{k'(\varepsilon, k, \xi; \mathbf{s}) = k'\right\} dG(\xi) \end{array} \right]$$
$$\times\, p(\omega'_\varepsilon)\, g(\varepsilon, k)\, d\omega'_\varepsilon d\varepsilon dk$$

how many firms have some level of epsilon' and k'. Know initial distribution and policy rule so can guess where they end up. Then can figure out how many will end up at epsilon' and k'.
Look at all possible realization epsilon and k today. How many choose accumulation for k'.
How many start with epsilon, k that choose go to k'.

Recursive Competitive Equilibrium

A set of $\widehat{v}(\varepsilon, k; z, g)$, $C(z, g)$, $N(z, g)$, $w(z, g)$, $\Lambda(z'; z, g)$, and $g'(z, g)$ s.t.

stochastic discout factor

1. **Firm optimization**: Taking $\Lambda(z'; z, g)$ and $w(z, g)$ as given, $\widehat{v}(\varepsilon, k; z, g)$ solves Bellman equation
2. **Household optimization**: $w(z, g)C(z, g)^{-\sigma} = \chi N(z, g)^{\alpha}$
3. **Market clearing**:

labor supply from HH **aggregate labor demand from entire economy.**

$$N(z, g) = \int n(\varepsilon, k; z, g) g(\varepsilon, k) d\varepsilon dk$$

Need distribution of firms.

Aggregate consumption of HH

$$C(z, g) = \int (y(\varepsilon, k, \xi; z, g) - i(\varepsilon, k, \xi; z, g)) dG(\xi) g(\varepsilon, k) d\varepsilon dk$$

Asset market: $\Lambda(z'; z, g) = \beta \left(\dfrac{C(z', g'(z, g))}{C(z, g)} \right)^{-\sigma}$ **Arrow Debraw model**

intertemporal marginal rate of substitution

4. **Consistency**: $g'(\varepsilon, k)$ satisfies law of motion for distribution

hetrogeneous agent model so need to make sure law of motion for distribution is same

Model Parameterization

RBC targets | **Doms and Dons facts target (from hetrogeneity)**

	Desc.	Value		Desc.	Value
β	Discount factor	.961	ρ_z	Aggregate TFP AR(1)	.859
σ	Utility curvature	1	σ_z	Aggregate TFP AR(1)	.014
α	Inverse Frisch	$\lim \alpha \to 0$	$\bar{\xi}$	Fixed cost	.0083
χ	Labor disutility	$\implies N^* = \frac{1}{3}$	a	No fixed cost region	.011
ν	Labor share	.64	ρ_ε	Idiosyncratic TFP AR(1)	.859
θ	Capital share	.256	σ_ε	Idiosyncratic TFP AR(1)	.022
δ	Capital depreciation	.085			

consistent with macro (RBC model) | **consistent with micro**

Overview of Computational Methods

Computing Equilibrium

- Key challenge: aggregate state g is infinite-dimensional

Computing Equilibrium

- Key challenge: aggregate state g is infinite-dimensional

- Two steps:
  1. Compute steady state without aggregate shocks $\Longrightarrow$ distribution constant at g^*
  2. Compute full model with aggregate shocks $\Longrightarrow$ distribution varies over time

1) individual firms will still be growing and shrinking. But at aggregate will be constant. Distribution of firm is constant. Aggregate level things steady. But micro level churning happens. Simplifies the solution method.

SS in recursive form: aggregate state is constant over time.

Steady State Recursive Competitive Equilibrium

A set of $v^*(\varepsilon, k)$, C^*, N^*, w^*, and $g^*(\varepsilon, k)$ such that

1. **Firm optimization**: Taking w^* as given: $v^*(\varepsilon, k)$ solves Bellman equation
2. **Household optimization**: Taking w^* as given: $w^*(C^*)^{-\sigma} = \chi(N^*)^{\alpha}$
3. **Market clearing**: **stochastic factor drops out because C constant -> solve ^ easily.**

$$N^* = \int n(\varepsilon, k) g(\varepsilon, k) d\varepsilon dk$$

$$C^* = \int (y(\varepsilon, k, \xi) - i(\varepsilon, k, \xi)) dG(\xi) g^*(\varepsilon, k) d\varepsilon dk$$

4. **Consistency**:
   $g^*(\varepsilon, k)$ satisfies law of motion for distribution given g^*
   distribution of state constant over time (consistency). individual level state can be different.

Hopenhayn-Rogerson (1993) Algorithm

Start with guess of w^*

Hopenhayn-Rogerson (1993) Algorithm

Start with guess of w^*

- Solve firm optimization problem $\implies v^*(\varepsilon, k), n^*(\varepsilon, k), k'(\varepsilon, k, \xi)$

Hopenhayn-Rogerson (1993) Algorithm

Start with guess of w^*

- Solve firm optimization problem $\implies v^*(\varepsilon, k), n^*(\varepsilon, k), k'(\varepsilon, k, \xi)$
- Use $k'(\varepsilon, k, \xi)$ to compute stationary distribution $g^*(\varepsilon, k)$ by iterating on law of motion
- Compute implied labor demand $N^d = \int n^*(\varepsilon, k) g^*(\varepsilon, k) d\varepsilon dk$

Hopenhayn-Rogerson (1993) Algorithm

Start with guess of w^*

- Solve firm optimization problem $\implies v^*(\varepsilon, k), n^*(\varepsilon, k), k'(\varepsilon, k, \xi)$
- Use $k'(\varepsilon, k, \xi)$ to compute stationary distribution $g^*(\varepsilon, k)$ by iterating on law of motion
- Compute implied labor demand $N^d = \int n^*(\varepsilon, k) g^*(\varepsilon, k) d\varepsilon dk$
- Compute labor supply $N^s = \left(\frac{w^*(C^*)^{-\sigma}}{\chi}\right)^{\frac{1}{\alpha}}$

Hopenhayn-Rogerson (1993) Algorithm

Start with guess of w^*

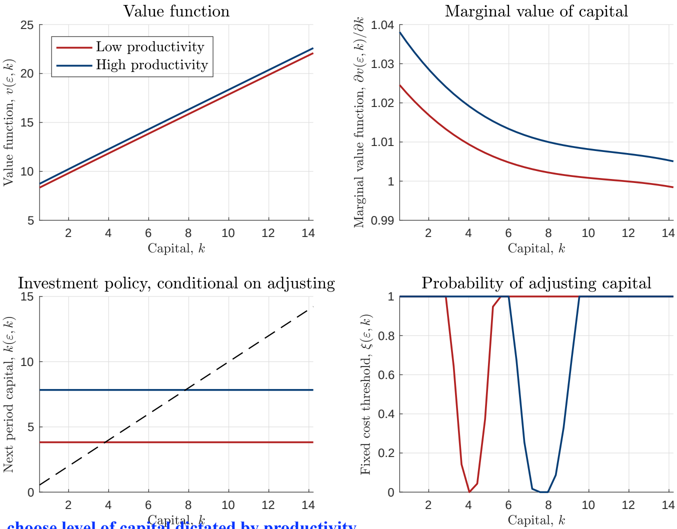
- Solve firm optimization problem $\implies v^*(\varepsilon, k), n^*(\varepsilon, k), k'(\varepsilon, k, \xi)$
- Use $k'(\varepsilon, k, \xi)$ to compute stationary distribution $g^*(\varepsilon, k)$ by iterating on law of motion
- Compute implied labor demand $N^d = \int n^*(\varepsilon, k) g^*(\varepsilon, k) d\varepsilon dk$
- Compute labor supply $N^s = \left(\frac{w^*(C^*)^{-\sigma}}{\chi} \right)^{\frac{1}{\alpha}}$

Update guess of w^* based on $N^d - N^s$

Walrass Law only need market clearing for n-1.
So only check labor market clearance.

Iterate to convergence

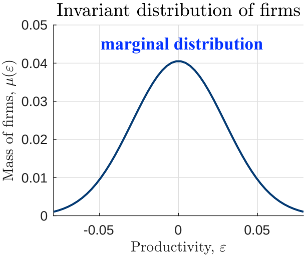
Steady State Decisions

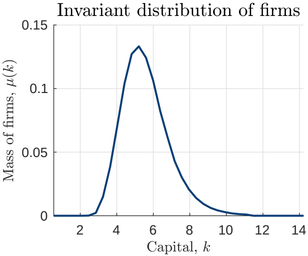


choose level of capital dictated by productivity and doesn't depend on current k.

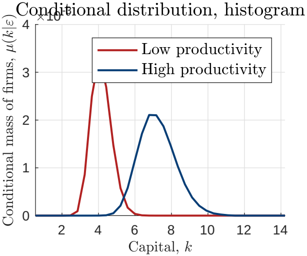
y-axis: psi_hat / psi (probability)

Steady State Distribution





capital proportional to productivity



random fixed cost shock create distribution

Full Model with Aggregate Shocks

- Outside of steady state,
  - Distribution g varies over time $\Longrightarrow$
    how to approximate distribution?
  - Law of motion for g is complicated $\Longrightarrow$
    how to approximate law of motion?
    infinite dimensional object from infinite dimensional input

Full Model with Aggregate Shocks

- Outside of steady state,
  - Distribution g varies over time $\Longrightarrow$ how to approximate distribution?
  - Law of motion for g is complicated $\Longrightarrow$ how to approximate law of motion?

- Will provide overview of three approaches in the literature:
  1. **Krusell-Smith**: approximate distribution with moments (used in Khan and Thomas 2008)
  2. **Reiter methods**: perturbation w.r.t. the distribution (used in Winberry 2018; will discuss details on Friday if time)
  3. **MIT shocks**: perfect foresight transition paths (used in Ottonello-Winberry 2018; will discuss on Friday)

Krusell and Smith (1998)

- Typical approach to dealing with challenge: Krusell-Smith (1998)
  - Approximate distribution with moments, e.g. $g(\varepsilon, k) \approx \overline{K}$
  - Approximate law of motion with parametric form
    $\log \overline{K}' = \alpha_0 + \alpha_1 z + \alpha_2 \log \overline{K}$
  - Approximate prices with parametric form
    $\log C = \gamma_0 + \gamma_1 z + \gamma_2 \log \overline{K}$ and $\log w = \eta_0 + \eta_1 z + \eta_2 \log \overline{K}$

All care about is prices they face (wage and discount factor). All want to do is forecast prices.
Simplify distribution is by agents don't care about whole distribution, but only know mean.
Law of motion of aggregate state by parametric form (law of linear forecast).
Solve for parameters during the solution method.
Stochastic discount factor so need logC.

Krusell and Smith (1998)

- Typical approach to dealing with challenge: Krusell-Smith (1998)
  - Approximate distribution with moments, e.g. $g(\varepsilon, k) \approx \overline{K}$
  - Approximate law of motion with parametric form $\log \overline{K}' = \alpha_0 + \alpha_1 z + \alpha_2 \log \overline{K}$
  - Approximate prices with parametric form $\log C = \gamma_0 + \gamma_1 z + \gamma_2 \log \overline{K}$ and $\log w = \eta_0 + \eta_1 z + \eta_2 \log \overline{K}$
- **Overview of the algorithm**:

  Simulate 100,000 firms as time series. And solve aggregate consumption, real wage. Don't have to match forecasting function. New time series so solve for alpha, gamma, and eita. Continue until coefficients converges.

  - Start with guess of α, γ, and η
    - Solve firm optimization $\Longrightarrow \widehat{v}(\varepsilon, k; z, \overline{K}), n(\varepsilon, k; z, \overline{K}), k'(\varepsilon, k, \xi; z, \overline{K})$
    - Simulate large panel of firms using $n(\varepsilon, k; z, \overline{K}), k'(\varepsilon, k, \xi; z, \overline{K})$
    - Compute aggregate time series $z_t, \overline{K}_t, C_t, w_t$
  - Update α, γ, and η using OLS on simulated series
  - Iterate to convergence on α, γ, and η
- Assess accuracy using, e.g., R^2 of forecasting rules

Winberry (2018)

- Approximate distribution with parametric family:

$$g(\varepsilon, k) \cong g_0 \exp\{g_1^1 \left(\varepsilon - m_1^1\right) + g_1^2 \left(\log k - m_1^2\right) + \sum_{i=2}^{n_g} \sum_{j=0}^{i} g_i^j \left[\left(\varepsilon - m_1^1\right)^{i-j} \left(\log k - m_1^2\right)^j - m_i^j\right]\}$$

$\rightarrow$ Aggregate state approximated by $(z, g(\varepsilon, k)) \approx (z, \mathbf{m})$

Winberry (2018)

- Approximate distribution with parametric family:

$$g(\varepsilon, k) \cong g_0 \exp\{g_1^1 \left(\varepsilon - m_1^1\right) + g_1^2 \left(\log k - m_1^2\right) + \sum_{i=2}^{n_g} \sum_{j=0}^{i} g_i^j \left[\left(\varepsilon - m_1^1\right)^{i-j} \left(\log k - m_1^2\right)^j - m_i^j\right]\}$$

$\rightarrow$ Aggregate state approximated by $(z, g(\varepsilon, k)) \approx (z, \mathbf{m})$

- Compute law of motion + prices directly by integration

Winberry (2018)

- Approximate distribution with parametric family:

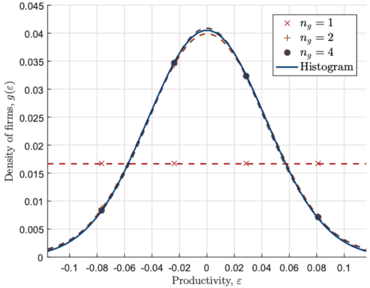
$$g\left(\varepsilon,k\right) \cong g_0 \exp\{g_1^1\left(\varepsilon - m_1^1\right) + g_1^2\left(\log k - m_1^2\right) + \sum_{i=2}^{n_g}\sum_{j=0}^{i} g_i^j\left[\left(\varepsilon - m_1^1\right)^{i-j}\left(\log k - m_1^2\right)^j - m_i^j\right]\}$$

$\rightarrow$ Aggregate state approximated by $(z, g(\varepsilon, k)) \approx (z, \mathbf{m})$

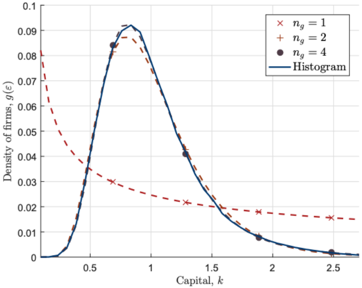
- Compute law of motion + prices directly by integration
- Compute aggregate dynamics using perturbation methods
  - Solve for steady state in `Matlab`
  - Solve for aggregate dynamics using `Dynare`

Winberry (2018)

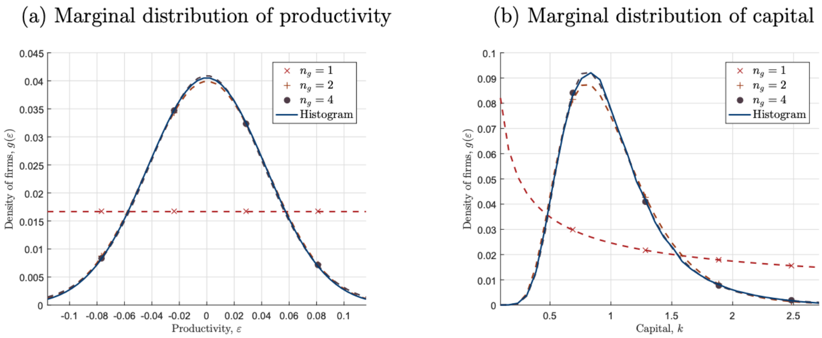
(a) Marginal distribution of productivity



(b) Marginal distribution of capital



Winberry (2018)



- Run time ≈ 20 - 40 seconds for accurate approximation
- Fast enough for likelihood-based estimation
- Codes at my `website`

MIT Shocks (aka Transition Paths)

Solve for SS. At t=0, innovation and positive TFP shock. Solve for transition path back to SS. Here, firms assume not other shock in future.

- **MIT shock** = unexpected innovation Δz_0 at $t = 0$
  + perfect foresign transition path back to steady state

Transition path of prices such that transition back to SS.

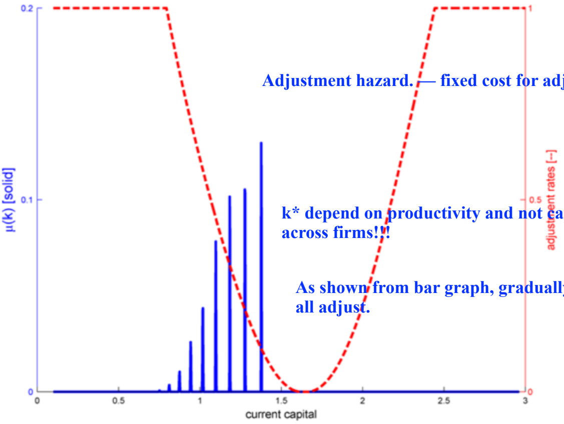
- In this model, characterized by $\{w_t\}_{t=0}^{\infty}$ and $\{C_t\}_{t=0}^{\infty}$
  1. Solve firm's problem by backward iteration
  2. Given $\{\widehat{v}_t(\varepsilon, k)\}_{t=0}^{\infty}$, simulate decisions to get $n_t^d(\varepsilon, k)$, $y_t(\varepsilon, k) - i_t(\varepsilon, k)$, and $g_t(\varepsilon, k) \implies$ get aggregates $N_t^d = \int n_t^d(\varepsilon, k) g_t(\varepsilon, k) d\varepsilon dk$ and $C_t^s = \int (y_t(\varepsilon, k) - i_t(\varepsilon, k)) g_t(\varepsilon, k) d\varepsilon dk$
  3. Equilibrium: $N_t^d = N_t^s$ and $C_t^s = C_t$

- Computational method: set $T =$ large enough and iterate over path of $\{w_t\}_{t=0}^{T}$ and $\{C_t\}_{t=0}^{T}$

- Converges to true IRF as $\Delta z_0 \to 0$

Results

Complicated Impulse Responses with Fixed Prices



Adjustment hazard. — fixed cost for adjustment cost

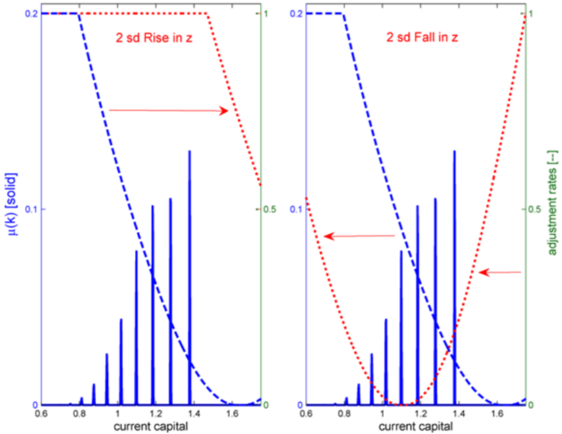
k* depend on productivity and not capital. Here no A so same across firms!!!

As shown from bar graph, gradually deccrease until all adjust.

Distribution in model with no idiosyncratic productivity shocks

Investment decision characterized by adjustment hazard

Complicated Impulse Responses with Fixed Prices

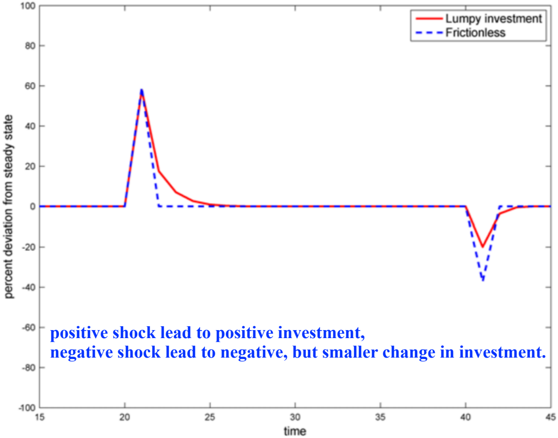


all firms adjust -> high investment

average firm closer to where it wants to be, so less adjustment -> less change in investment.

Response of aggregate investment to shock depends on interaction of initial distribution and adjustment hazards

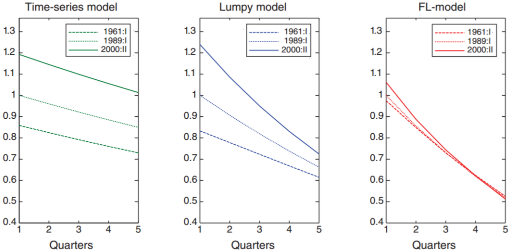
Implication: Sign Dependence



Aggregate investment more responsive to positive than negative shocks

Not true in frictionless model

Implication: State Dependence



recession when close to k*. So more response when far from recession.

From Bachmann, Caballero, and Engel (2013)

$$\frac{I_t}{K_t} = \sum_{j=1}^{p} \phi_j \frac{I_{t-j}}{K_{t-j}} + \sigma_t e_t$$

$$\sigma_t = \alpha_1 + \eta_1 \frac{1}{p} \sum_{j=1}^{p} \frac{I_{t-j}}{K_{t-j}}$$

Aggregate Nonlinearities with Fixed Prices

- Both of these are examples of nonlinear aggregate dynamics
    - Linear model has constant loading on aggregate shock

Nonlinearity because of non-linearity among adjustment, hazard and distributions. Smith says aggregate shock or mean enough information, but this suggests that more moments required to solve such non-linearity seen in the data.

Aggregate Nonlinearities with Fixed Prices

- Both of these are examples of nonlinear aggregate dynamics
    - Linear model has constant loading on aggregate shock
- Some evidence in aggregate data
    - Sign and state dependence $\rightarrow$ distribution of $\frac{I_t}{K_t}$ positively skewed
    - State dependence $\rightarrow$ dynamics of $\frac{I_t}{K_t}$ feature conditional heteroskedasticity

Aggregate Nonlinearities with Fixed Prices

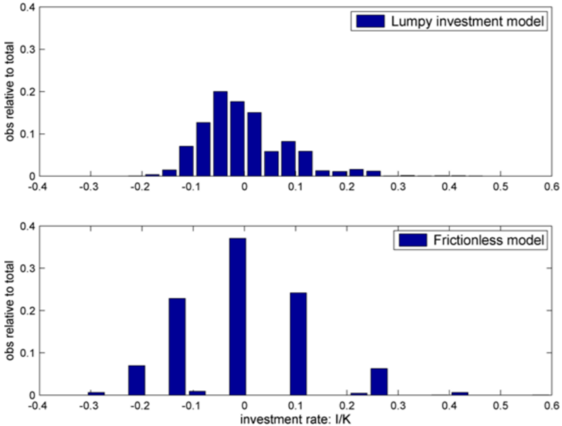
- Both of these are examples of nonlinear aggregate dynamics
  - Linear model has constant loading on aggregate shock

- Some evidence in aggregate data
  - Sign and state dependence $\rightarrow$ distribution of $\frac{I_t}{K_t}$ positively skewed
  - State dependence $\rightarrow$ dynamics of $\frac{I_t}{K_t}$ feature conditional heteroskedasticity

- My view: time series evidence is suggestive at best
  - Predictions are about extreme states, which are rare
  - But that is exactly when we care about these predictions!

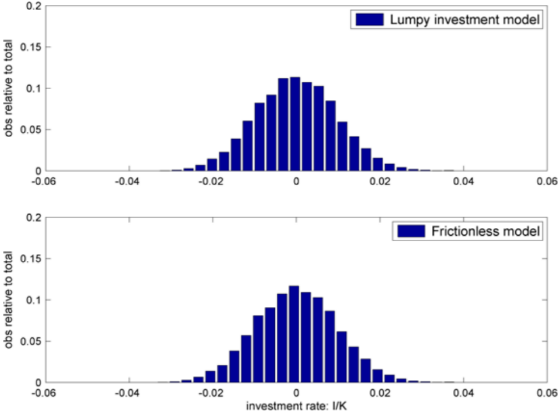
    $\Longrightarrow$ rely on cross-sectional data + carefully specified general equilibrium model

Distribution of Aggregate $\frac{I_t}{K_t}$ with Fixed Prices



once allow prices to change, non-linearity disappears. So similar to representative agent problem.
Aggregate investment rate compared.

Distribution of Aggregate $\frac{I_t}{K_t}$ in General Equilibrium



Return to same model as aggregate. In GE, hetrogeneity does not matter.

Distribution of Aggregate $\frac{I_t}{K_t}$ in General Equilibrium

TABLE III

ROLE OF NONCONVEXITIES IN AGGREGATE INVESTMENT RATE DYNAMICS

	Persistence	Standard Deviation	Skewness	Excess Kurtosis
Postwar U.S. data[a]	*0.695*	*0.008*	*0.008*	*−0.715*
A. Partial equilibrium models				
PE frictionless	−0.069	0.128	0.358	0.140
PE lumpy investment	0.210	0.085	1.121	2.313
B. General equilibrium models				
GE frictionless	0.659	0.010	0.048	0.048
GE lumpy investment	0.662	0.010	0.067	−0.074

[a] Data are annual private investment-to-capital ratio, 1954–2005, computed using Bureau of Economic Analysis tables.

Business Cycles Nearly Identical to Representative Firm

TABLE IV

AGGREGATE BUSINESS CYCLE MOMENTS

	Output	TFP[a]	Hours	Consump.	Invest.	Capital
A. Standard deviations relative to output[b]						
GE frictionless	(2.277)	0.602	0.645	0.429	3.562	0.494
GE lumpy	(2.264)	0.605	0.639	0.433	3.539	0.492
B. Contemporaneous correlations with output						
GE frictionless		1.000	0.955	0.895	0.976	0.034
GE lumpy		1.000	0.956	0.900	0.976	0.034

[a] Total factor productivity.

[b] The logarithm of each series is Hodrick–Prescott-filtered using a weight of 100. The output column of panel A reports percent standard deviations of output in parentheses.

Get same aggregate dynamics as RBC

Why Do the Nonlinearities Disappear?

General equilibrium price movements

- Time-varying elasticity comes from large movements in adjustment hazard
- Procyclical real interest rate and wage restrain those movements

$$1 + r_t = \frac{1}{\mathbb{E}_t[\Lambda_{t,t+1}]}$$

Why does heterogeneity not matter?
1) in shocks, which investment increases - > interest rate increases which reduce back to RBC when investment decrease vice versa.
2) adjustment cost is small, so investment increases. So adjustment cost should be larger so less sensitive to interest rate so will not play dampening effect.

Why Do the Nonlinearities Disappear?

General equilibrium price movements

- Time-varying elasticity comes from large movements in adjustment hazard
- Procyclical real interest rate and wage restrain those movements

$$1 + r_t = \frac{1}{\mathbb{E}_t[\Lambda_{t,t+1}]}$$

Specification of adjustment costs

- Calibrated adjustment costs small

Why Do the Nonlinearities Disappear?

General equilibrium price movements [Winberry (2018)]

- Time-varying elasticity comes from large movements in adjustment hazard
- Procyclical real interest rate and wage restrain those movements

$$1 + r_t = \frac{1}{\mathbb{E}[\Lambda_{t,t+1}]}$$

Specification of adjustment costs [Bachmann, Caballero, and Engel (2013)]

- Calibrated adjustment costs small

Bachmann, Caballero, and Engel (2013)

- Argue Khan and Thomas' calibration of adjustment costs responsible for irrelevance result
- Calibrate larger adjustment costs and recover aggregate nonlinearities

Bachmann, Caballero, and Engel (2013)

- Argue Khan and Thomas' calibration of adjustment costs responsible for irrelevance result
- Calibrate larger adjustment costs and recover aggregate nonlinearities
- Argument based on decomposition between AC smoothing and PR smoothing
  - Frictionless partial equilibrium model excessively volatile
  - AC smoothing: dampening due to adjustment costs
  - PR smoothing: dampening due to price movements
- Measure AC smoothing in data and target in calibration $\rightarrow$ higher adjustment costs

Model

Production technology $y_{jt} = e^{z_t} e^{\varepsilon_{st}} e^{\varepsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu}, \theta + \nu < 1$

- Idiosyncratic productivity shock $\varepsilon_{jt+1} = \rho_\epsilon \varepsilon_{jt} + \omega_{jt+1}^{\varepsilon}$ where $\omega_{jt+1}^{\varepsilon} \sim N(0, \sigma_\epsilon^2)$
- Aggregate productivity shock $z_{t+1} = \rho_z z_t + \omega_{t+1}^{z}$ where $\omega_{t+1}^{z} \sim N(0, \sigma_z^2)$
- Sectoral productivity shock $\varepsilon_{st+1} = \rho_\epsilon \varepsilon_{st} + \omega_{st+1}^{\varepsilon}$ where $\omega_{st+1}^{\varepsilon} \sim N(0, \sigma_{\epsilon_s}^2)$

Model

Production technology $y_{jt} = e^{z_t} e^{\varepsilon_{st}} e^{\varepsilon_{jt}} k_{jt}^{\theta} n_{jt}^{\nu}$, $\theta + \nu < 1$

- Idiosyncratic productivity shock $\varepsilon_{jt+1} = \rho_\epsilon \varepsilon_{jt} + \omega_{jt+1}^{\varepsilon}$ where $\omega_{jt+1}^{\varepsilon} \sim N(0, \sigma_\epsilon^2)$
- Aggregate productivity shock $z_{t+1} = \rho_z z_t + \omega_{t+1}^{z}$ where $\omega_{t+1}^{z} \sim N(0, \sigma_z^2)$
- Sectoral productivity shock $\varepsilon_{st+1} = \rho_\epsilon \varepsilon_{st} + \omega_{st+1}^{\varepsilon}$ where $\omega_{st+1}^{\varepsilon} \sim N(0, \sigma_{\epsilon_s}^2)$

Firms **accumulate capital** according to $k_{jt+1} = (1-\delta)k_{jt} + i_{jt}$

- If don't pay fixed cost, must undertake maintenance investment $\chi \times \delta k_{jt}$
- Otherwise, pay fixed cost ξ_{jt} in units of labor
- Fixed cost $\xi_{jt} \sim U[0, \overline{\xi}]$

Calibration

Set most parameters exogenously

Choose σ_z, $\bar{\xi}$, and χ to match degree of AC-smoothing

- Identify AC-smoothing using volatility of sectoral investment rates
    - Aggregated enough to capture interaction of distribution and hazards
    - Small enough to not generate price response

Calibration

Set most parameters exogenously

Choose σ_z, $\bar{\xi}$, and χ to match degree of AC-smoothing

- Identify AC-smoothing using volatility of sectoral investment rates
  - Aggregated enough to capture interaction of distribution and hazards
  - Small enough to not generate price response
- Targets:
  1. Volatility of aggregate investment rate
  2. Average volatility of sectoral investment rates
  3. Amount of conditional heteroskedasticity

AC vs. PR Smoothing Decomposition

TABLE 6—SMOOTHING DECOMPOSITION

Model	AC smoothing/total smoothing (in percent)		
	LB	UB	Average
Khan-Thomas-lumpy annual	0.0	16.1	8.0
Khan-Thomas-lumpy annual, our $\overline{\xi}$	8.1	59.2	33.7
Our model annual ($\chi = 0$), Khan and Thomas' $\overline{\xi}$	0.8	16.0	8.4
Our model annual ($\chi = 0$)	18.9	75.3	47.0
Our model annual ($\chi = 0.25$)	19.1	75.7	47.4
Our model annual ($\chi = 0.50$)	19.9	76.6	48.3
Our model quarterly ($\chi = 0$)	14.5	80.9	47.7
Our model quarterly ($\chi = 0.25$)	15.4	80.9	48.2
Our model quarterly ($\chi = 0.5$)	15.4	81.0	48.2

$$UB = \log[\sigma(\text{none})/\sigma(\text{AC})] \,/\, \log[\sigma(\text{none})/\sigma(\text{both})]$$

$$LB = 1 - \log[\sigma(\text{none})/\sigma(\text{PR})] \,/\, \log[\sigma(\text{none})/\sigma(\text{both})]$$

Calibrated Adjustment Costs

TABLE 4—THE ECONOMIC MAGNITUDE OF ADJUSTMENT COSTS—ANNUAL

Model	Adjustment costs/ unit's output (in percent) (1)	Adjustment costs/ unit's wage bill (in percent) (2)
This paper ($\chi = 0$)	38.9	60.9
This paper ($\chi = 0.25$)	12.7	19.8
This paper ($\chi = 0.50$)	3.6	5.6
Caballero-Engel (1999)	16.5	—
Cooper-Haltiwanger (2006)	22.9	—
Bloom (2009)	35.4	—
Khan-Thomas (2008)	0.5	0.8
Khan-Thomas (2008) "Huge Adj. Costs"	3.7	5.8

Notes: This table displays the average adjustment costs paid, conditional on adjustment, as a fraction of output (left column) and as a fraction of the wage bill (right column), for various models. Rows 4–6 are based on table IV in Bloom (2009). For Cooper and Haltiwanger (2006) and Bloom (2009) we report the sum of costs associated with two sources of lumpy adjustment: fixed adjustment costs and partial irreversibility. The remaining models only have fixed adjustment costs.

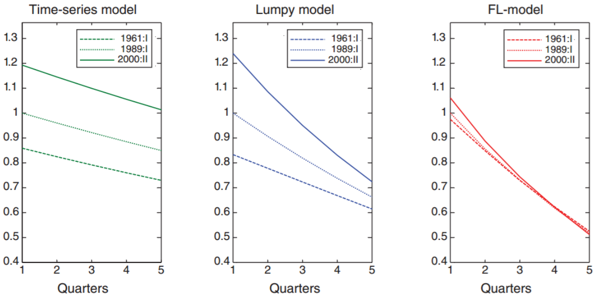
Aggregate Nonlinearities

TABLE 5—HETEROSCEDASTICITY RANGE

Model	$\log(\sigma_{95}/\sigma_5)$
Data	*0.3021*
This paper ($\chi = 0$)	0.1830
This paper ($\chi = 0.25$)	0.2173
This paper ($\chi = 0.50$)	0.2901
Quadratic adj. costs ($\chi = 0$)	0.0487
Quadratic adj. costs ($\chi = 0.25$)	0.0411
Quadratic adj. costs ($\chi = 0.50$)	0.0321
Frictionless	0.0539
Khan-Thomas (2008)	0.0468

Notes: This table displays heteroscedasticity range ($\log(\sigma_{95}/\sigma_5)$) for the data (row 1) and various model specifications that vary in terms of the maintenance parameter χ and the adjustment technology for capital: fixed adjustment costs (rows 2–4), quadratic adjustment costs (rows 5–7), a frictionless model, and the Khan-Thomas (2008) model. The adjustment costs for the models in rows 2–7 have been calibrated to match aggregate and sectoral investment rate volatilities.

Aggregate Nonlinearities



$$\frac{I_t}{K_t} = \sum_{j=1}^{p} \phi_j \frac{I_{t-j}}{K_{t-j}} + \sigma_t e_t$$

$$\sigma_t = \alpha_1 + \eta_1 \frac{1}{p} \sum_{j=1}^{p} \frac{I_{t-j}}{K_{t-j}}$$

Aggregate Nonlinearities

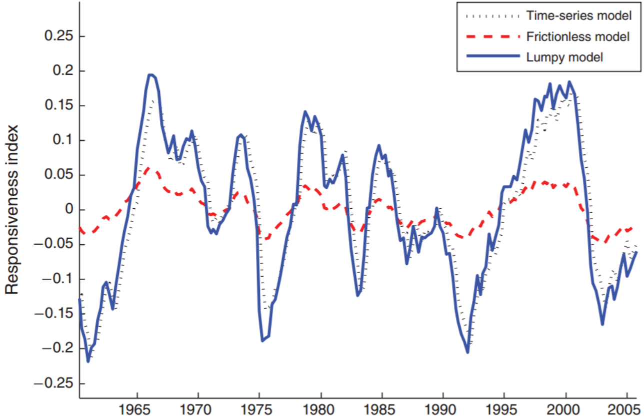


FIGURE 3. TIME PATHS OF THE RESPONSIVENESS INDEX

Aggregate Nonlinearities

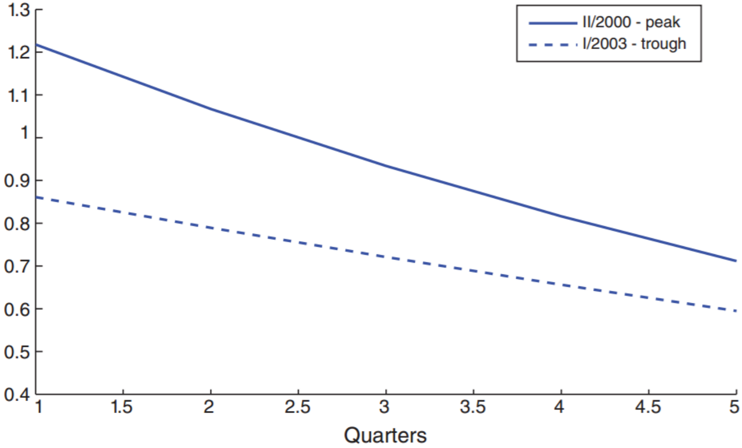


FIGURE 7. IMPULSE RESPONSES OF THE AGGREGATE INVESTMENT RATE IN THE 2000 BOOM-BUST CYCLE

Winberry (2018)

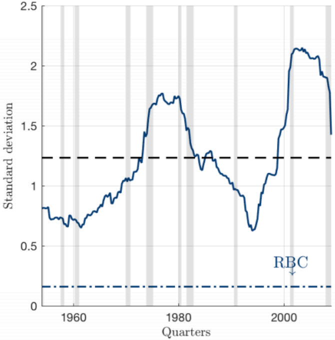
- Argues that procyclical interest rate inconsistent with data
- When consistent with data recover aggregate nonlinearities

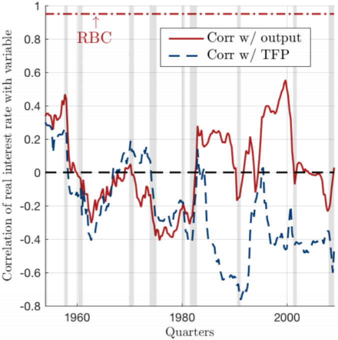
Winberry (2018)

- Argues that procyclical interest rate inconsistent with data
- When consistent with data recover aggregate nonlinearities

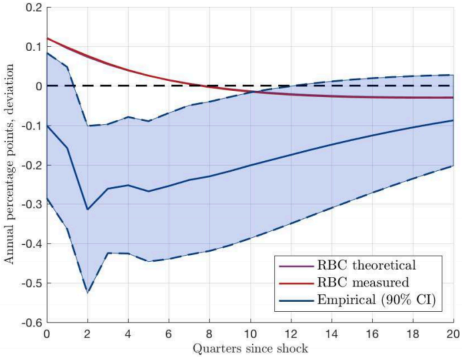
	$\sigma(r_t)$	$\rho(r_t, y_{t-1})$	$\rho(r_t, y_t)$	$\rho(r_t, y_{t+1})$
T-bill	2.18%	-0.08	-0.17	-0.251
AAA	2.34%	-0.29	-0.37	-0.40
BAA	2.43%	-0.32	-0.41	-0.45
Stock	24.7%	-0.24	-0.14	0.02
RBC	0.16%	0.61	0.97	0.74

Rolling Windows of r_t Dynamics





IRF of r_t to TFP Shock



Takeaways from this Lecture

- Specification of **benchmark heterogeneous firm model**
    - Individual vs. aggregate states
    - Role of the distribution in the aggregate state variable
    - Steady state: constant aggregates, but lots of churning at individual level

- Overview of how people **solve heterogeneous agent models**

- Response of aggregate investment to shocks **depends on distribution of firms**, which changes over the business cycle
    - Less responsive in recessions